Brane-worlds and low energy heterotic M-theory

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ABSTRACT

The current understanding of theoretical physics tells us that there exists a unique, nonperturbative quantum theory living in 11D spacetime (M-theory), from which five 10D superstring theories arise as perturbative limits. Finding the explicit form of this M-theory is one of the greatest theoretical challenges of the twenty first century. In this thesis, we shed the light on some important aspects, vacuum energy, moduli stabilization and gaugino condensates in the framework of 5D heterotic M-theory. The central question we are trying to answer in this thesis is: *what is the mechanism for radion stabilization?*. To answer this question we calculate the total bulk vacuum energy, which is the difference between the twisted and untwisted fermion vacuum energies, in both flat and curved spaces. It is found that this bulk vacuum energy alone doesn't stabilize the radion field. We then try to add and investigate some non-perturbative effects such as the gaugino condensates and use the technique of dimensional reduction to reach an effective superpotential. Dimensional reduction is a necessary step required to know how our real 4D world is described by a higher dimensional theory. After performing the dimensional reduction, we have a look at the resulting effective superpotential for a 4D gravitino with ghost fields. The importance of the ghost vacuum energy is in its positive sign which is helpful in the stabilization proplem when added to the total fermionic bulk vacuum energy with its ordinary negative sign.

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1. INTRODUCTION

1.1 Extra dimensions and Brane-worlds

Most theoretical physicists believe that at high enough energies, classical General Relativity fails to describe gravity and must be unified with quantum field theory. The supposed quantum gravity theory should contain significant corrections as the fundamental energy scale (the Planck scale) is approached. Superstrings are a good candidate, where all particles in nature are just different vibrations of strings of the string scale ($\sim 10^{-33} cm$).

A class of models has been inspired in the context of branes in string theory, called 'brane-world models' (see [9] for a review). In such models, the observable universe is regarded as a $3+1-$dimensional surface (the brane) embedded in a $3+1+d-$dimensional spacetime (the bulk). Standard model particles and fields are trapped on the brane and only gravity is free to access the bulk. At low energies, gravity is localized at the $3+1-$dimensional brane allowing General Relativity to be recovered. At high energies, gravity leaks into the higher dimensional bulk, behaving in a truly higher dimensional theory. These models may differ from traditional Kaluza-Klein models in that the extra dimensions are not necessarily small compared to the length scales accessible to modern accelerators.

Although the idea that lower dimensional hypersurfaces constitute the visible world had been suggested before [4, 25], the idea only became popular in 1998 when the model of Arkani-Hamed, Dimopoulos and Dvali (ADD)[11] was proposed. This model is an attempt to attack the long standing hierarchy problem (that is why gravity is much weaker than all other forces) through the idea of *large* extra-dimensions.

An important common feature of all extra-dimensional models is that they have additional scalar fields. These scalar fields couple to the 4D energy-momentum tensor modifying the 4D gravity (and so sometimes called gravi-scalars). However, there are strong experimental constraints on such 'scalar-tensor theories' of gravity. For example, in the case of only one compact extra-dimension (5D bulk), by calculating the slowing down of binary pulsars due to the radiation of these gravi-scalars, it could be shown that [87, 88] the presence of the gravi-scalars leads to a modification of Einsteins quadrupole formula by 20%, but observations agree with the quadrupole formula by better than 0.5%. For more extra-dimensions there will be more gravi-scalars and the problem gets worse.

1.2 Kaluza-Klein basics

In 1919 (published only in 1921), Kaluza proposed that gravity and electromagnetism could be unified by adding one extra dimension [90]. His main aim was to unify the Hilbert-Einstein action with the action of electromagnetism. He started from a pure $5D$ gravitational action. Then, after integrating out, he could get the equations of General Relativity, Maxwell's equations and a scalar field coupled to the electromagnetic field tensor. This means that the additional part in the 5D metric g_{AB} gives the Maxwell field and a scalar field (the dilaton field).

In 1926, Klein [91] suggested that the extra dimension has a circular topology so that the extra coordinate y is periodic. The compactification of the direction y with radius L means y and $y + 2\pi L$ are identified. The space then has a topology $R^4 \times S^1$, which means that there is a little circle at each point in four-dimensional spacetime [see Fig.1.1].

The gravity action in 5D could be written as

$$S^{(5)} = \frac{M_5^3}{2} \int d^4x \int_0^{2\pi L} dy \sqrt{g^{(5)}} R^{(5)} \tag{1.1}$$

Where

$$\frac{1}{M_5^3} \equiv 8\pi G_5 \equiv \kappa_5^2 \tag{1.2}$$

The 5D metric could be expressed in $4+1$ form as

$$g_{AB} = e^{\phi/\sqrt{3}} \begin{pmatrix} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_\mu A_\nu & e^{-\sqrt{3}\phi} A_\mu \\ \\ e^{-\sqrt{3}\phi} A_\nu & e^{-\sqrt{3}\phi} \end{pmatrix} \tag{1.3}$$

Where $g_{\mu\nu}$, A_μ and ϕ are tensor, vector and scalar fields respectively. The periodicity in y means that the components of the five dimensional metric can be expanded in terms of Fourier series [92]

$$g_{\mu\nu}(x,y) = \sum_{n=-\infty}^{n=\infty} g_{\mu\nu}^{(n)}(x) e^{(2n\pi y/L)} \tag{1.4}$$

$$A_\mu(x,y) = \sum_{n=-\infty}^{n=\infty} A_\mu^{(n)}(x) e^{(2n\pi y/L)} \tag{1.5}$$

$$\phi(x,y) = \sum_{n=-\infty}^{n=\infty} \phi^{(n)}(x) e^{(2n\pi y/L)} \tag{1.6}$$

So, the theory describes an infinite number of four-dimensional fields. The mass of the mode n becomes $m_n^2 = \frac{n^2}{L}$ which means that the smaller the size L the higher the energy required to probe it. Only the zero (massless) mode (1.3) is effective at low energies and massive modes will be important at higher energies.

After integrating out the extra dimension, the low-energy 5D action (1.1) becomes

$$S = \frac{(2\pi L) M_5^3}{2} \int d^4x \sqrt{-g}[R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}e^{-\sqrt{3}\phi} F_{\mu\nu}F^{\mu\nu}] \tag{1.7}$$

By comparing the above action with the 4D action we can get a relation between the 4D Planck scale and the 5D one as

$$M_4^2 = (2\pi L) M_5^3 \tag{1.8}$$

The additional scalar field worried Kaluza and Klien, but now physicists expect to see new scalar fields in their theories. Modern higher dimensional theories don't imply the compactification manifold to be a circle.

In spite of the beautiful unification of gravity and electromagnetism, Kaluza-Klein theory failed to include other forces. Also, it doesn't explain the weakness of gravity in comparison to electromagnetism. The Kaluza-Klein theory was essentially abandoned until the advent of supergravity and string theory, where the idea of higher-dimensional theories was reintroduced in physics.

1.3 ADD model - large extra dimensions.

The ADD model was proposed in 1998 [11, 23] to solve the hierarchy problem between the Planck scale and the weak scale. The basic idea is that large volume compact extra dimensions would lower the fundamental Planck scale to the weak scale, leaving a single scale M_{ew}. We summarize this in the following equation

$$M_{ew} \sim 1TeV \approx M_{Pl(4+d)}. \tag{1.9}$$

As in Kaluza Klein theories, the geometry is factorized (meaning that the 4-dimensional part of the metric does not depend on extra-dimensional coordinates), and the metric reads:

$$ds^2 = g_{\mu\nu}(x^\alpha)dx^\mu dx^\nu + g_{ij}(x^5)dx^i dx^j. \tag{1.10}$$

The space-time is $R^4 \times M_n$, where M_n is an n dimensional compact manifold of radius R and volume R^n. The Plank scale $M_{Pl(4+n)}$ of this $(4+n)$ dimensional theory is taken to be $\sim M_{ew}$.

By Gauss law in $4+n$ dimensions, for small separation $r \ll R$, the Newtonian potential between two particles of masses m_1 and m_2 will be given by

$$V_r(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{r^{n+1}}, \qquad r \ll R. \tag{1.11}$$

The usual $1/r$ could be obtained when the masses are placed at distances $r \gg R$, that is

$$V_r(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2} R^n} \frac{1}{r}, \qquad (r \gg R). \tag{1.12}$$

We can write now the effective 4-dimensional Planck scale M_P as

$$M_P^2 \sim M_{P(4+n)}^{n+2} R^n \tag{1.13}$$

So, if we put $M_{P(4+n)} \sim M_{ew}$ and demand that R be chosen to give the observed M_P we get

$$R \sim 10^{\frac{30}{n}-17} cm \times \left(\frac{1TeV}{M_{ew}}\right)^{1+\frac{2}{n}}. \tag{1.14}$$

The case for $n = 1$ is empirically excluded as $R \sim 10^{13} cm$ which implies modifications for Newton's law over solar system distances. For $n = 2$, $R \sim 10^{-2} cm$ which suggests modifications on the submillimeter scale. Since the experimental capabilities are limited, the knowledge of the validity of these laws of nature is limited. For example, very little is known about the behaviour of gravity at distances $< 10^{-4} cm$ or $> 10^{28} cm$ [27].

Unfortunately, while the ADD model solves the hierarchy between the Planck and weak scale, it replaces this with a hierarchy between the fundamental Planck scale M_{4+n} and the compactification scale $\mu_c = R^{-1}$ ($\mu_c = 1/r_c$ for RS model) [35]. As we will see, in the Randall-Sundrum model the hierarchy between the Planck and weak scales could be resolved without the need to introduce a large hierarchy between M_{4+n} and μ_c.

Reducing the fundamental scale to the weak scale gives some hope for the experimental tests of quantum gravity. Theories of quantum gravity, string theory for example, might be accessible at modern colliders such as the LHC.

1.4 The hierarchy problem

Despite being in a very good agreement with experiments, the standard model of elementary particles (based on the $SU(3) \times SU(2) \times U(1)$ gauge group) suffers several unattractive features. One of these unattractive features is the gauge hierarchy problem, the standard model cannot consistently accommodate the weak energy scale $\mathcal{O}(1TeV)$ and a much higher scale such as the Planck mass scale $\mathcal{O}(10^{19}) GeV$. This is why it has been suggested that the standard model is only an effective low energy theory embedded in some more fundamental high scale theory that could contain gravity.

There are in fact two long standing fine tuning problems, the hierarchy problem

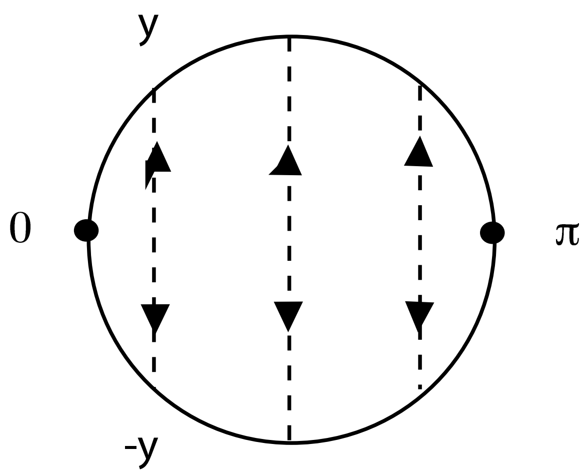


Fig. 1.1: The orbifold S_1/Z_2 on which the extra dimension y is compactified. It is just a circle with two fixed points 0 and π identified and z_2 symmetry imposed.

and the cosmological constant problem. In both of them there are two fundamental scales; an experimentally observed scale and a theoretically expected scale, which are many orders of magnitude apart.

In the hierarchy problem, the observed scale is the energy scale at which the electromagnetic interaction unifies with the weak interaction around $1TeV$. The theoretical scale is set by quantum correction to the Higgs mass.

The Planck energy scale (at which a theory of quantum gravity should be revealed) is theoretically calculated to lie at $10^{19}GeV$ or $10^{-35}m$. The hierarchy of sixteen orders of magnitude between these two scales is called the hierarchy problem. The model that solves the problem most 'economically' is the RS model with a single extra dimension [2, 3].

1.5 The predecessors of brane-worlds

The idea of the universe as a domain wall was first proposed by Rubakov and Shaposhnikov in 1983 [25], who imagined particles confined by a 3D potential well at low energy. A system of two branes of equal and opposite tension bounding a fifth dimension which contains bulk scalar fields first received serious attention after the compactification of Horava-Witten theory to 5 dimensions [12].

It is widely accepted that the weakly coupled $E_8 \times E_8$ heterotic string is one of the most phenomenologically viable of five superstring theories. Unfortunately, the predicted value for Newton's constant in this theory is too large. Witten [17] has shown that this situation can be resolved in the strong coupling limit of the heterotic string, which is believed to be equivalent at low energy to eleven-dimensional supergravity, with E_8 Super-Yang-Mills gauge theories on two branes [4]. This theory can be compactified to get a 5D theory. It is known that in order for the theory to predict the correct values of Newton's constant and grand unification gauge couplings, the orbifold radius must be an order of magnitude or so larger than the compactification scale. Hence, at some intermediate energy scale, the theory has a consistent five-dimensional description.

Lukas et al. [6, 12] have derived the five-dimensional effective action from Horava-Witten theory. They have shown that the resulting theory is a gauged version of N = 1 supergravity in five dimensions, with a non-abelian set of E_8 gauge fields on one brane, and spontaneously broken to E_6 on the other. The vacuum solution for this theory has a curved bulk metric. This was the true predecessor of most brane-world scenarios.

The 5D solution gives rise to an effective four dimensional theory in which the separation of the domain walls becomes one of the moduli fields. It is important to identify effects which can provide a potential for the brane separation and fix this particular modulus. This is discussed further in chapter (2). One possible mechanism is that quantum fluctuations of the bulk fields stabilise the branes at phenomenologically acceptable positions. This has been discussed extensively in the context of the Randall-Sundrum brane world scenario (see for example [18]). Previous work of this kind in five-dimensional heterotic M-theory has been done for scalar fields by Garriga et al. [19].

1.6 Einstein Equations on The Brane

How to reach a 4D effective theory on the brane is an important question that should be answered in any braneworld model. For a single brane system, Shiromizu, Maeda and Sasaki reached a useful set of equations by projecting the higher dimensional Einstein equations onto a Z_2 symmetric brane. That means, as in the original Horava-Witten theory [4], there exists a Z_2 reflection symmetry along the extra dimension $z \to -z$.

According to [1], we live on 4D brane $(M, q_{\mu\nu})$ in a 5D spacetime $(V, g_{\mu\nu})$ with the induced metric

$$q_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \tag{1.15}$$

Where n^α is a unit vector on M.
To relate the 5D and 4D quantities we make use of Gauss' equation

$$^{(4)}R^\alpha_{\beta\gamma\delta} = ^{(5)} R^\mu_{\nu\rho\sigma} q^\alpha_\mu q^\nu_\beta q^\rho_\gamma q^\sigma_\delta + K^\alpha_\gamma K_{\beta\delta} - K^\alpha_\delta K_{\beta\gamma}, \tag{1.16}$$

and the Codacci equation

$$D_\nu K^\nu_\mu - D_\mu K = ^{(5)} R_{\rho\sigma} n^\sigma q^\rho_\mu. \tag{1.17}$$

In these equations,
$K_{\mu\nu} = q^\alpha_\mu q^\beta_\nu \nabla_\alpha n_\beta \equiv$ extrinsic curvature on $\mathcal{M}$.
$K = K^\mu_\mu$ is the trace.
D_μ=covariant derivative with respect to $q_{\mu\nu}$.

Contracting (1.16) and using the 5D Einstein equation (The idea here is that they are trying to eliminate the 5D quantities to an equation restricted to the brane, but this will not be entirely successful),

$$^{(5)}R_{\rho\sigma} - \frac{1}{2} g^{(5)}_{\rho\sigma} R = \kappa_5^2 T_{\rho\sigma} \tag{1.18}$$

We get

$$\begin{aligned} ^{(4)}G_{\mu\nu} = \frac{2\kappa_5^2}{3}[T_{\rho\sigma} q^\rho_\mu q^\sigma_\nu + (T_{\rho\sigma} n^\rho n^\sigma - \frac{1}{4} T^\rho_\rho) q_{\mu\nu}] + K K_{\mu\nu} \\ -K^\sigma_\mu K_{\nu\sigma} - \frac{1}{2} q_{\mu\nu}(K^2 - K^{\alpha\beta} K_{\alpha\beta}) - E_{\mu\nu} \end{aligned} \tag{1.19}$$

where $E_{\mu\nu}$ is a traceless tensor given in terms of the 5D Weyl tensor $C^{\alpha}_{\beta\rho\sigma}$ as $E_{\mu\nu} \equiv^{(5)} C^{\alpha}_{\beta\rho\sigma} n_\alpha n^\rho q^\beta_\mu q^\sigma_\nu$ and carries information about the gravitational field in the 5D bulk. The 5D metric can be put into the form

$$ds^2 = d\chi^2 + q_{\mu\nu} dx^\mu dx^\nu, \tag{1.20}$$

with the brane located at $\chi = 0$. The $5D$ energy momentum tensor is

$$T_{\mu\nu} = -\Lambda g_{\mu\nu} + S_{\mu\nu}\delta(\chi) \tag{1.21}$$

Where

$$S_{\mu\nu} = -\lambda q_{\mu\nu} + \tau_{\mu\nu} \tag{1.22}$$

The reason for including Λ, a bulk cosmological constant, will be explained in the next section. Clearly the $\delta(\chi)$ function is introduced to restrict matter to the brane. λ is the brane vacuum energy (brane tension) and $\tau_{\mu\nu}$ is the brane energy-momentum tensor. This singular behaviour in the energy momentum tensor leads to Israel's junction conditions [108] i.e. a discontinuity (a jump) in the extrinsic curvature $K_{\mu\nu}$ across a hypersurface (embedded in a higher dimensional space) is related to the energy momentum tensor on that hypersurface. This reminds us with what happens in electromagnetism when the jump of the normal component of D across two different media is related to the charge density on the separation surface of the two media. These conditions could then be written as

$$[K_{\mu\nu}] = K^+_{\mu\nu} - K^-_{\mu\nu} = -\kappa_5^2 (S_{\mu\nu} - \frac{1}{3} q_{\mu\nu} S), \tag{1.23}$$

where $K^\pm_{\mu\nu} = \lim_{y\to\pm 0} K_{\mu\nu}$. Applying Z_2 symmetry allows us to write

$$K^+_{\mu\nu} = -K^-_{\mu\nu} = -\frac{1}{2}\kappa_5^2 (S_{\mu\nu} - \frac{1}{3} q_{\mu\nu} S) \tag{1.24}$$

Plugging this into the equation for the 4D Einstein tensor we get

$${}^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} + 8\pi G_N \tau_{\mu\nu} + \kappa_5^4 \pi_{\mu\nu} - E_{\mu\nu}, \tag{1.25}$$

where

$$\Lambda_4 = \frac{1}{2}\kappa_5^2(\Lambda + \frac{1}{6}\kappa_5^2\lambda^2) \quad (1.26)$$

$$G_N = \frac{\kappa_5^4\lambda}{48\pi} \quad (1.27)$$

$$\pi_{\mu\nu} = -\frac{1}{4}\tau_{\mu\alpha}\tau_\nu^\alpha + \frac{1}{12}\tau\tau_{\mu\nu} + \frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}\tau^2 \quad (1.28)$$

The brane cosmological constant Λ_4 depends on the brane tension and the bulk cosmological constant. That means a fine tuning is required to get viable solutions. The 4D Newton's constant is directly proportional to the brane tension. There is also unusual term $\pi_{\mu\nu}$ which is quadratic in the energy momentum tensor and can produce a significant change in the cosmological evolution.

1.7 Randall-Sundrum models and the geometrical origin of the hierarchy

Randall and Sundrum suggested a set up to solve the hierarchy problem in which the extra dimensions are small, but the background metric is not flat along the extra coordinate; it is a slice of Anti de Sitter (AdS_5) space. This curved space causes the energy scales on the two branes to be different, one scale is exponentially suppressed on the negative tension brane. This exponential suppression can then naturally explain why the physical scales observed are so much smaller than the Planck scale [36].

According to articles [2, 3], the elementary particles except for the graviton are localized on a 3+1 dimensional brane or branes. There are two popular models. The first one (RS1) [3] has a finite size for the extra dimension with two branes with positive and negative tensions respectively [see fig.1.2]. It attempts to address the hierarchy problem geometrically, where the warping of the extra dimension generates a large ratio of energy scales so that the natural energy scale at one end of the extra dimension is much larger than at the other end.

In the second model (RS2) [2], the negative tension brane has been placed infinitely far away (the extra dimension is infinite in size) so that there is only one

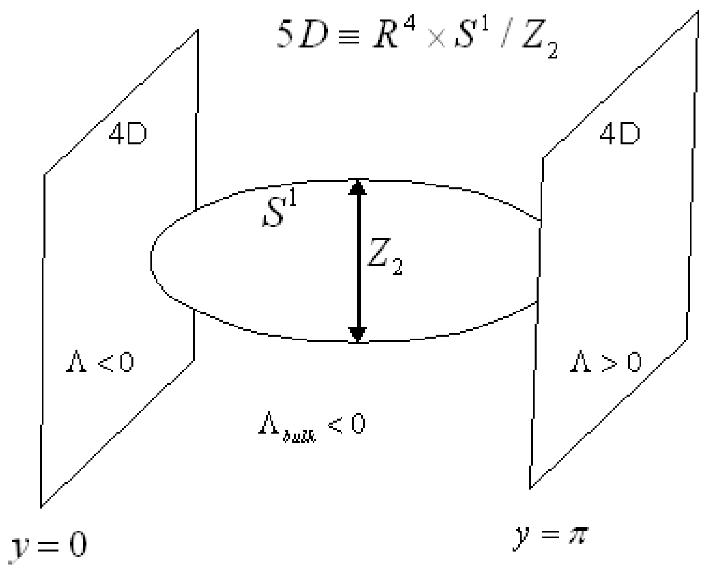


Fig. 1.2: The topology in RS model is R^4 multiplied by a line element which is taken to be a circle with Z_2 symmetry in RS1 and an infinite real line R^+ in RS2. The topology in the 5D reduced heterotic M-theory is the same as that of RS1.

brane left in the model. The generalized RS1 scenario with radion stabilization seems more realistic than the RS2 model. An important feature that has been pointed out by the RS2 model is that there is an alternative to compactification, meaning that we don't necessarily have to compactify the extra dimension. The action of the RS1 model is given by

$$\begin{aligned} S &= S_{gravity} + S_{vis} + S_{hid} \\ S_{gravity} &= \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G}\{-\Lambda + 2M^3\mathcal{R}\} \\ S_{vis} &= \int d^4x \sqrt{-g_{vis}}\{\mathcal{L}_{vis} - \lambda_{vis}\} \\ S_{hid} &= \int d^4x \sqrt{-g_{hid}}\{\mathcal{L}_{hid} - \lambda_{hid}\}. \end{aligned} \tag{1.29}$$

Where Λ and M are the $5D$ cosmological constant and Planck scale respectively. A constant vacuum energy for both branes has been separated out which can act as a gravitational source. In order to obtain a Minkowiski brane, we have to set

$\Lambda_4 = 0$ and then (1.26) implies

$$\Lambda_5 = -\frac{\lambda^2 \kappa_5^2}{6}, \quad \kappa_5^2 = M^{-3}. \tag{1.30}$$

meaning that the bulk space is AdS. Since AdS is conformally flat, $E_{\mu\nu} = 0$ in (1.25). Also, a Minkowiskian brane implies that $\tau_{\mu\nu} = 0$ and that gives ${}^{(4)}G_{\mu\nu} = 0$.

The above relation is the RS fine tuning condition which ensures the zero value of the effective cosmological constant on the brane so that the brane has the induced geometry of Minkowski spacetime. This condition is the main unattractive feature of the RS model [22] and it seems unlikely as a relation between two independent quantities, without a physical basis. The RS1 model is unstable under small deviations from this fine tuning between the brane tension and the bulk cosmological constant. The bulk metric is given by

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2 \tag{1.31}$$

Where k is the curvature of the AdS. Noting that $\sqrt{-G} = r_c\sqrt{-g^{(4)}}$ and $R = e^{2kr_c\phi}R^{(4)}$, the gravitational part $S_{gravity}$ in (1.29) gives the $4D$ Planck scale as

$$M_{Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}]. \tag{1.32}$$

In order to investigate the physically observed masses of matter fields we assume a Higgs field with mass m_H on the hidden brane. The metric on the visible brane is $g_{\mu\nu}^{vis} = e^{-2kr_c\pi}\overline{g}_{\mu\nu}$ with $\overline{g} = g_{\mu\nu}^{hid}$. To get the mass we normalize the Higgs field as follows. The action for the Higgs field on the visible brane is

$$S_{vis} = \int d^4x\sqrt{-g_{vis}}\{g_{vis}^{\mu\nu}D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - m_o^2)^2\}, \tag{1.33}$$

Where λ is an arbitrary coupling constant. We redefine the Higgs field to absorb the warp factor i.e. $H \to e^{kr_c\pi}H$, the action becomes

$$S_{vis} = \int d^4x\sqrt{-\overline{g}}\{\overline{g}^{\mu\nu}D_\mu \widetilde{H}^\dagger D_\nu \widetilde{H} - \lambda(|\widetilde{H}|^2 - e^{-2kr_c\pi}m_o^2)^2\}, \tag{1.34}$$

Where $\widetilde{H} = e^{-kr_c\pi}H$. So, the observer located on the visible brane will measure the Higgs mass as $m = e^{-kr_c\pi}m_o$. This is a general result; i.e. any field on the visible brane with a fundamental mass parameter m_o will appear to have the physical mass $m = e^{-kr_c\pi}m_o$. For example if $m_0 \sim M_{Pl}$ then $kr_c \simeq 12$ leads to $m \sim m_{ew}$.

In order to get an appropriate hierarchy between the Planck scale and the electroweak scale in RS1 model, the distance between the two branes must be set to about 50 times the bulk curvature scale. Of course, this would be more satisfactory if this value could be explained by a dynamical mechanism [37].

The massless degree of freedom in RS model called the radion. Since the geometrical interpretation of the radion is the distance between the two branes, this means that the radius of the extra dimension is not fixed.

There have been several attempts in the literature to generate the radion mass, as we will see later on. The simplest radion stabilization mechanism by Goldberger and Wise [35, 93] stabilized the radion without any severe fine-tuning of the parameters in the full theory. It has been applied to the two brane RS model [53, 63] to recover gravity consistent with observation. The collider signatures for the RS1 model have been studied in detail in [65].

An interesting result was found in [119], where the higher KK modes of the graviton in the RS1 model couple to the standard model fields on the brane with a much larger strength ($e^{kr_c\pi}M_{Pl}^{-1}$) than the zero mode graviton (M_{Pl}^{-1}). It is much easier then to observe the KK excitations in modern colliders than to observe the graviton!. The supersymmetric extension of the Randall Sundrum scenario has been considered in [49–52].

1.8 DGP model (braneworlds with infinite volume extra dimensions)

RS2 [2] is an example of an infinite size extra-dimension brane-world ($V_N \equiv \int d^N y\sqrt{G} \to \infty$). Another infinite size extra-dimension model has been suggested in [120] (GRS model) in which gravity is five dimensional both at short and large distance scales, but it is a conventional 4D-gravity at intermediate length scales. However, this last model is considered to be inconsistent due to the existence of ghost fields (see [86] and the references therein).

In the DGP model, a 3-brane is embedded in 5D Minkowiski bulk where gravity in the bulk is taken to be very strong. The Lagrangian for the model is

$$\begin{aligned} S &= \frac{M^3}{2}\int d^5x\sqrt{-g^{(5)}}R^{(5)} - \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g^{(4)}}R^{(4)} \\ &+ \int d^4x\sqrt{-g^{(4)}}L_m + M^3\int_{\partial\mathcal{M}} d^4x\sqrt{-g^{(4)}}K \end{aligned} \tag{1.35}$$

In the above action, because of the different mass scales M (the 5D Planck scale) and M_{pl} (4D Planck scale), gravity propagates differently on the brane and on the bulk. When $M \to 0$ and M_{Pl} is finite, the above action describes $4D$ gravity on the brane. When $M_{Pl} \to 0$ and M is finite, it describes $5D$ gravity in the bulk. The two different pre-factors in front of the bulk and the brane actions give rise to a characteristic length scale $r_c = M_{Pl}^2/M^3$, called crossover scale. At distance scales much smaller than this characteristic distance, we have the usual $4D$ gravitational physics. On scales larger than r_c the 5D physics is recovered. The brane Ricci scalar is possibly generated by one loop corrections of massive scalars and fermions localized on the brane [see fig.1.3].

The higher dimensional Planck scale M in this model is much smaller than in other extra dimensional models. For example, we have seen before that (see equation (1.13)) $M_P^2 \sim M_{P(4+n)}^{n+2}V_n$, with V_n the volume of the extra dimension and n the number of the extra dimensions. But for the case of $V_n \to \infty$ this relation doesn't hold, and M can be much smaller than the TeV scale, making gravity in the bulk much stronger.

The higher dimensional theory is assumed to be supersymmetric, whilst SUSY is spontaneously broken on the brane. These breaking effects can be localized on the brane without affecting the bulk, Only when the infinite volume gives a large enough suppression factor. The infiniteness of the extra dimension means there's no need to stabilize the size of the extra dimension as it is neither compactified nor warped.

The existence of a critical length scale r_c below which $4D$ Newtonian gravity is recovered on the brane and above which modified gravity dominates looks very

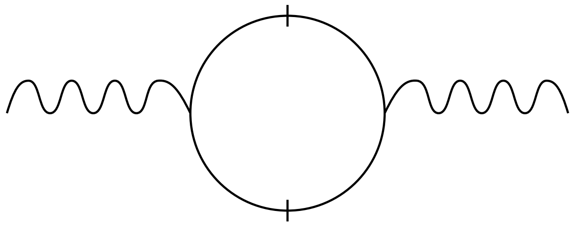


Fig. 1.3: The one-loop diagram with massive scalars and fermions (brane matter fields) in the loop which generates the brane Ricci scalar (Ricci scalar for 4D graviton). Matter fields indicated by solid line and gravitons by wave lines. vertical short lines on matter fields propagator indicate that they are massive.

interesting for cosmologists. Several attempts have been made to get a consistent extension of general relativity that modifies gravity at cosmological distances while remains in an agreement with observations at shorter distances (example [120]). One of the motivations of these models is to explain something that happens at very large scales, i.e. the expansion of the universe is accelerating! This is usually explained by introducing a cosmological constant, or a form of mysterious dark energy with negative pressure called dark energy. The DGP model allows a cosmological solution in which accelerated expansion of the universe is realized without introducing a cosmological constant [121]. Based on this model, a mechanism that dilutes the cosmological constant was also proposed [122].

Cosmology in the DGP model is governed by the modified Friedmann equation [121]

$$H^2 = \frac{\kappa_4^2 \rho}{3} \mp \frac{H}{r_c}, \tag{1.36}$$

Where H is the Hubble parameter and ρ is the matter density on the brane. The two possible choices of sign lead to two branches of cosmological evolution. The negative sign corresponds to a decelerating expanding universe (of course in the absence of cosmological constant on the brane). This branch of solutions is called the FRW branch. The positive sign corresponds to an accelerating expanding universe, this branch of solutions is called self-accelerating branch.

Because the DGP model is very complicated, it is often not easy to solve the

Einstein equations in the higher dimensional spacetime. The model is controversial and its viability is in question [138, 139] .

1.9 Extra time-like dimensions

The extra dimensions in almost all extra dimensional models are assumed to be space-like. This is because several difficulties appear in the presence of more than one time-like direction. The main problem with time-like compactified dimensions is the existence of tachyonic modes, which implies violations of causality. If we consider a five dimensional space-time with a signature $(1,1,-1,-1,-1)$ and attempted to compactify τ (the extra time coordinate) on a circle of radius L, the standard KK excitations become tachyonic states with imaginary masses, quantized in units of i/L. Various issues arising in brane-world scenarios with time-like extra dimensions were discussed in [89].

1.10 M-theory story in a nutshell

Around 1995, it was found that the five distinct supersymmetric 10-dimensional string theories: type I, type IIA, type IIB, $SO(32)$ heterotic, and $E_8 \times E_8$ heterotic are related to each other via S, T and U duality transformations. These dualities express an exact quantum equivalence, which means that the two dual theories are just two different descriptions of a single theory.

The S duality relates the weak coupling limit of one string theory with the strong coupling limit of another string theory. Type I and $SO(32)$ heterotic are related by S duality, where one of them evaluated at strong coupling is equivalent to the other one evaluated at weak coupling. The S duality is a symmetry of type IIB string theory, and we say that it is self-dual. Because of the existence of such duality, the strong coupling behaviour of type I, type IIB and $SO(32)$ can be determined by a weak coupling analysis. The behaviour of type IIA and $E_8 \times E_8$ heterotic at strong coupling is very different. It is believed that they grow an eleventh dimension [115].

On the other hand, the T duality relates different compactifications of different theories. If the compact dimension is a circle, and there are two theories A and B with compact dimension radius R_A and R_B, then they are T dual to each other if they are equivalent and $R_A R_B = (l_s)^2$ where l_s is a fundamental length scale. This relation means that shrinking the compactified dimension to zero in one theory corresponds to decompactification of the dual theory. The two theories IIA and IIB are T dual and so are the two heterotic theories. Finally, there's a U duality between two theories A and B if theory A compactified on a space of large (or small) volume is equivalent to theory B at strong (or weak coupling) [116].

The 10D string theories are connected to the 11D supergravity as well. Carrying out a dimensional reduction of 11D supergravity to 10D gives type I, IIA or IIB supergravity, which are the low energy limits of I, IIA and IIB superstrings respectively. In chapter 3 we will describe the original 11D supergravity and the Horava-Witten theory in detail. Although Witten gave the name M-theory to the unknown 11D quantum theory whose low energy effective description is 11D supergravity, this term is used by many authors to refer to the single 11D theory that gives the 5 superstring theories as special limits.

1.11 Organization of the Thesis

The dissertation is organized as follows:

Chapter One We give a review of different extra-dimensional theories and illustrate the basic idea, advantages and disadvantages of all of them. Unification of fundamental interactions and solution of the hierarchy problem are the main motivations. In this context, we explained the meaning of the hierarchy problem and the moduli stabilization problem.

Chapter Two We present a detailed review of the moduli stabilization problem and classify the attempts to attack it into four main mechanisms: bulk massive scalars, vacuum energy, nonperturbative contributions and non-zero flux contributions. We start this chapter with a section about SUSY breaking which is a necessary step in any supersymmetric theory to reach a description of our SUSY-broken 4D world.

Chapter Three We discuss the original 11D Horava-Witten theory and its reduction to 5D. The study of the deeply rich structure of the lower dimensional theory is an active area. The useful technique of moduli space approximation is also illustrated and a BPS solution of a dilatonic brane-world is presented. We end the chapter by giving a summary for some possible moduli systems we are going to use through the thesis.

Chapter Four We calculate the total bulk Casimir energy by calculating the difference between twisted and the untwisted fermion fields. We do the case of flat space first and then the curved space case. We also prove the attractivity of the bulk Casimir energy.

Chapter Five We start by deriving the gaugino condensate potential in the framework of the improved heterotic M-theory suggested by Ian Moss in 2005. In the second part of this chapter, we reach the gaugino condensate superpotential by reducing the 11D Rarita-Schwinger field to 4D. The form obtained agrees with the standard known form of this superpotential in most theories.

Chapter Six We add two terms to our gaugino condensate superpotential derived in chapter five, the flux term and another non-perturbative term that depends on the Calabi-Yau volumes V_1 and V_2. The two toy models have an AdS KKLT minimum. We then try to use the bulk vacuum energy to turn this into a dS minimum.

Chapter Seven We perform a 5D reduction for the gravitino field. We review the BRST formalism and make use of it to remove the $\Gamma^I \psi_I$ term using a gauge fixing function. This will result in two new ghost fields, which are important for dealing with the stabilization topic. The vacuum energy of the ghost fields has a (+ve) sign (that leads to a repulsive force) while for the real fermions (as we have got in chapter 4) it has a (-ve) sign. We end this chapter by expressing the SUSY breaking parameter θ in terms of the condensate using the twisted boundary conditions of the improved heterotic M-theory.

Chapter Eight We calculate the vacuum energy of the ghost fields obtained in chapter 7 for the case of flat space first and then the curved space case.

Conclusion and Further Work We summarize our results and point out various ways with which one can proceed in future research.

2. MODULI STABILIZATION

One of the main theoretical issues in theories with extra dimensions is that of determining their size. As we mentioned in the discussion of the RS1 model, a solution to the hierarchy problem has been proposed in which the observable universe is a 3-brane at an orbifold fixed point of the non-factorizable geometry given by (1.31). The orbifold has fixed points at $y = 0$ and $y = \pi r_c$. However, the dynamics does not determine the value of r_c, leaving it a free parameter. This means there is no mechanism to ensure the stability of the system.

If we are interested only in one extra dimension, then the scalar degree of freedom governing the separation is called radion. A solution to the so called radion stabilization problem in the RS1 model has been found by adding a bulk scalar field, which has five-dimensional dynamics, to the model [93]. The mechanism does not involve any fine-tuning and it gives the radion a mass somewhat below the TeV scale. A complete calculation of the radion mass has been given by Tanaka and Montes [53], where they obtained the TeV-scale. However, since there is no knowledge about the origin and actual form of the stabilization potential, very little can be said about radion masses without further assumptions. A phenomenological guess for the radion potential has been discussed in [54]. In the literature, phenomenological aspects of the radion have been studied such as its decay modes (massive radions may decay into visible particles [54]) [56, 57], its signatures at present and future colliders [58, 60] and its effects on electroweak precision measurements [61, 62]. The phenomenology of the radion depends on the strength of its coupling to the brane fields.

Radion stabilization raises an important question in cosmology, i.e. how do we stabilize the large extra dimensions while keeping all the virtues and predictions of the big bang and inflationary cosmology? This has been discussed in [64].

2.1 SUSY and SUSY breaking

As it is well known, the Higgs scalar in the standard model acquires a non-vanishing vacuum expectation value and therefore breaks the electroweak symmetry. However, the loop corrections to the masses of scalar particles are quadratically divergent and this makes the electroweak symmetry breaking scale unstable against radiative corrections. Supersymmetric theories are free from quadratic divergences due to cancellations between boson and fermion loop corrections and this can stabilize the hierarchy between the Planck scale and the electroweak scale.

The unification of gauge couplings is considered to be one of the most attractive features of the supersymmetric extension of the standard model. If we plot the effective coupling constants as a function of the energy scale, we find the three couplings in the standard model don't unify very precisely. However, after the addition of SUSY i.e. within the supersymmetric extension of the standard model, they do approach a common value (see [123]).

Unfortunately, on the other hand, SUSY doesn't explain the origin of the electroweak scale and the mechanism of electroweak symmetry breaking is still mysterious. The standard model explains the electroweak symmetry breaking by assuming the existence of a scalar field (Higgs filed) that gives masses to the vector bosons and fermions, but there is no answer as to why the Higgs field should have a non-zero vacuum expectation value. It is 'too strong' to say that the standard model explains the electroweak scale.

Another point is that SUSY introduces new particles which are the supersymmetric partners of the standard model particles. As a requirement of particle phenomenology SUSY must be broken. In other words, if SUSY plays a role in low energy physics, it must be broken. The resulting theory is a supersymmetric extension of the standard model with SUSY broken a little above the electroweak scale.

SUSY breaking then is a necessary step in any supersymmetric theory to reconcile SUSY with actual experiments. This could be achieved by adding to the Lagrangian, defined by the $SU(3) \times SU(2) \times U(1)$ gauge symmetry and superpotential W, some extra terms which respect the gauge symmetry but break supersymmetry in a specific manner such that no quadratic terms appear. These extra terms are called soft SUSY breaking terms. They may arise if SUSY is broken in a hidden high energy sector, but this affects the visible sector indirectly. By the hidden sector we mean all fields and particles which don't directly interact with the standard model fields and particles (gluons, photons, W^+, W^- and Z bosons).

2.2 Mechanisms for radion stabilization.

There have been numerous studies of moduli stabilization in general and various stabilization mechanisms were suggested. We summarize some of these as follows:

2.2.1 Introducing a massive scalar field to the bulk.

This mechanism has been proposed by Goldberger and Wise [93]. In their article they introduced a 5D scalar field. The 5D bulk field appears to a 4D observer as an infinite tower of scalar fields with masses m_n, as in usual Kaluza Klein compactification. They started with the 5D action

$$S = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G}(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2), \tag{2.1}$$

where G_{AB} is given by the RS metric (1.31) and m is of order of M_{pl}. After integration by parts and performing Kaluza Klien decomposition, this leads to the 4D action

$$S = \frac{1}{2}\sum_n \int d^4x(\partial^\mu\phi^n\partial_\mu\phi^n - m_n^2\phi_n^2) \tag{2.2}$$

For a Randall Sundrum model, the masses m_n are given by the solutions of the transcendental equation

$$y_\nu(ax_n)j_\nu(x_n) - j_\nu(ax_n)y_\nu(x_n) = 0 \tag{2.3}$$

where $a = e^{-\pi k r_c}$, $m_n = kax_n$ and x_n is the n'th positive solution to (2.3). The functions j_ν and y_ν are given by the following combinations of Bessel functions

$$j_\nu(z) = 2J_\nu(z) + zJ_\nu^{'}(z) \tag{2.4}$$

$$y_\nu(z) = 2Y_\nu(z) + zY_\nu^{'}(z) \tag{2.5}$$

where the order ν of the Bessel functions is given by

$$\nu = \sqrt{4 + \frac{m^2}{k^2}} \tag{2.6}$$

m is the mass of the 5D scalar field.

The introduction of a scalar field creates an attractive force between the two branes which would ensure equilibrium when the distance between them is precisely the radius r_c required to generate the required hierarchy. The potential has a minimum at r_c without fine tuning of parameters. Examples of this trend are [38–48].

The addition of scalar fields in the bulk is favorable from a string theory viewpoint because in general a compactification from 10 or 11 dimensions to 5 dimensions introduces many 5 dimensional scalar fields [37].

2.2.2 Casimir energy approach.

Instead of introducing an ad-hoc classical interaction between the branes (through the bulk scalar field), one may ask whether the Casimir energy of bulk fields may be sufficient to stabilize the radion. In fact, before branes, Candelas and Weinberg in 1984 [76] found that the quantum effects from matter fields, or gravity, can be used to fix the size of compact extra dimensions. Other examples of this mechanism are [18, 19, 30–32, 66, 68–75].

In [30] it was shown that the contributions of the Casimir energy of bulk gauge fields depend logarithmically on the radion. These contributions stabilize the radion and generate a large hierarchy of scales without fine tuning. The Casimir effect on the background of conformally flat braneworld geometries has been investigated in [74].

The Casimir effect is a macroscopic quantum effect, i.e. it is a quantum effect which can be measured in the laboratory. It is an amazing success of quantum field theory and comes from the half quanta of the harmonic oscillator $\hbar\omega_k/2$. The fields in QFT are an infinite set of oscillators labelled by the wave number k. The n'th excitation of a single oscillator k corresponds to a state with n field quanta and energy

$$E_n^k = \hbar\omega_k(n + 1/2). \tag{2.7}$$

This means the state with no real quanta has a nonzero energy

$$E_o^k = \frac{\hbar\omega_k}{2}, \tag{2.8}$$

which leads to an infinite total energy of the vacuum,

$$E_{Casimir} = \frac{\hbar}{2}\sum_k \omega_k. \tag{2.9}$$

This divergent sum must be regularized to get a finite expression. This results in the Casimir effect [29], namely the dependence of the vacuum energy on the boundary conditions for the field. The famous attractive force between two electrical conductors in three dimensions is

$$F(d) = \frac{\pi^2}{240}\frac{\hbar c}{d^4}A, \tag{2.10}$$

where A is the area of the plates separated by a distance d. The electric charge e does not appear in this expression, which means that this is not an effect of coupling the electromagnetic field to the material plates. Instead of that the attractive force is due to the change in zero point or vacuum field energy (2.8). Vacuum energy is related to the concept of virtual particles coming from the uncertainty principle.

This result was confirmed and extended by many researchers who used different approaches to learn more about this force and related quantum phenomena [98]. Casimir [29] and other authors [99] proposed that this force could be regarded as a radiation pressure from the vacuum field. In general, this Casimir force arising from vacuum radiation pressure can be either attractive or repulsive [100]. As in [101], the subject of whether it is attractive or repulsive may depend on many factors including the space-time dimensionality, the boundary conditions, the space-time metric and so on.

In most practical examples the Casimir effect is considered for the electromagnetic field just because it is strong enough to produce measurable effect. But, in general, this effect is not restricted to the electromagnetic field and can occur for any quantum field.

In braneworld scenarios the fields obey boundary conditions on the boundary branes and hence one expects a Casimir-type effect if we treat the fields as quantum fields. The force between the branes will vary according to the separation of the branes and the Casimir effect will induce a potential for the radion in the dimensionally reduced theory. The Casimir effect has been used for radion stabilization in a number of models [19, 22, 30–32].

2.2.3 Gaugino condensation approach - nonperturbative effects.

Gaugino condensation is a non-perturbative effect that may break supersymmetry. The lack of understanding of the mechanism by which SUSY breaking happens is the most important missing part of any supersymmetric unification theory, and constructing a realistic scheme of SUSY breaking is one of the big challenges to SUSY phenomenology. Consequently, we need a dynamical mechanism that explains naturally (without any ad-hoc assumptions) the transition to the non-supersymmetric case. The dynamical formation of Gaugino condensates is a natural source of SUSY breaking, The original idea was suggested in Ref. [77].

The gaugino condensation mechanism has been discussed in many papers and it is believed to play a crucial role for moduli stabilization and SUSY breaking in string theory [77–82]. The SUSY breaking scale could then be set by the condensate scale. In the context of low energy heterotic M-theory, the most likely candidate for forming a fermion condensate is the gaugino on the hidden brane, since the effective gauge coupling on the hidden brane is larger and runs much more rapidly into a strong coupling regime than the gauge coupling on the visible brane. Gaugino condensation gives a potential depending on the Calabi-Yau volume [128–131].

The condensate potential is generally a function of several moduli fields [82]. The size of the moduli fields should be determined upon the minimization of the potential over the moduli space. A typical gaugino condensate potential is [142]

$$V(S,T) \sim \frac{1}{ST^3} e^{-3S/4\pi b}, \tag{2.11}$$

with b is the coefficient of the one-loop beta function of the hidden sector group. This potential has a runaway behaviour for both S and T where S and T are moduli (taken here to be real). Some attempts have been made to avoid the runaway behaviour, such as multiple gaugino condensate (or racetrack) models or adding a non-perturbative correction to Kähler potential. In the multiple gaugino condensate case, the superpotential is given as a sum of exponential terms which generate a potential with a local minimum.

In 2003, Kachru, Kallosh, Linde and Trivedi (KKLT, [134]) introduced the first explicit model in which all moduli are fixed within type IIB string theory. This was done by turning on fluxes as a first step (see below), which fix the complex moduli and the dilaton S, and introducing non-perturbative superpotentials in a second step to stabilise the Kähler modulus T. For a detailed study of the phenomenology of these models, see [141]. Unfortunately, the resulting potential for T has an AdS vacuum which needs to be uplifted and that means a third step is needed. We give some details in the next section.

2.2.4 Flux compactification approach.

A partial solution to the moduli problem lies in turning on background fluxes in the vacuum [143–145]. Turning on a non-vanishing flux warps the compactification space away from a pure Calabi-Yau threefold [94] and generates a superpotential of the form [134]

$$W_f = \int_{\mathcal{M}} G \wedge \Omega \tag{2.12}$$

where G is a three-form flux and Ω is the holomorphic three-form Ω of the Calabi-Yau threefold. In general, this flux superpotential is difficult to calculate except for special cases. The idea here is that when the relevant moduli are stabilized, Ω

is constant and then W_f can take any integer +ve or -ve (the different choices of Calabi-Yau manifolds and the different values of fluxes leads to the string theory landscape, which refers to the large number of false vacua in string theory). As has been pointed out in [146], the presence of background fluxes in the compactified space (i.e. non-zero vacuum expectation values of certain field strengths) leads to fixing all complex structure moduli as well as the dilaton. Unfortunately, it was found that this mechanism doesn't apply to the modulus parametrizing the size of the compact manifold. The KKLT model used nonperturbative effects such as gaugino condensation on $D7$ branes to stabilize the remaining modulus. The KKLT setup requires the presence of a number of $D7/D3$ branes and an anti $D3$ brane. The major achievements are that all moduli are fixed and the cosmological constant is small and positive.

The model starts with a 4D supergravity scalar potential which is given by

$$V_s = M_{Pl}^{-2} e^K \left(K^{I\overline{J}} D_I W D_{\overline{J}} \overline{W} - 3|W|^2 \right). \tag{2.13}$$

Where $D_I W = \partial_I W + W \partial_I K$ is the Kähler covariant derivative of the superpotential and $K^{I\overline{J}} = (\partial_I \partial_{\overline{J}} K)^{-1}$. The first term represents SUSY breaking and the second term represents the gravitino mass $m_{3/2}$. After the minimization of this potential, we can have SUSY broken in the vacuum ($D_i W \neq 0$) or not.

The total KKLT scalar potential is

$$V_{KKLT} = V_s + V_u, \tag{2.14}$$

where V_u is the SUSY breaking contribution required to uplift an AdS minimum to a de Sitter one. The correct Kähler potential

$$K = -\ln(S + \overline{S}) - 3\ln(T + \overline{T}), \tag{2.15}$$

leaves the volume modulus T un-stabilized. To stabilize it, the following T-dependent superpotential is added

$$W = w_0 - Ce^{-aT}. \tag{2.16}$$

w_0 is a constant induced by the fluxes and C is a model dependent coefficient and a is related to the beta function of gaugino condensation on the $D7$ branes. T is stabilized with $D_T W = 0$. The third step is the uplifting of the minimum. The uplifting potential due to the presence of the anti $D3$ brane is

$$V_{uplift} = \frac{D}{(T+\overline{T})^2}, \tag{2.17}$$

where D is a tuning constant allowing to obtain de Sitter vacuum. The effect of the uplifting term is to change the vacuum energy to a small positive or zero value. This is achieved with $D \sim m_{3/2}^2 M_{Pl}^2 \sim 10^{-26} M_{Pl}^4$. Since the background geometry of the KKLT model is warped, the desired value of D can be obtained by placing the anti $D3$ brane at the appropriate point in the compact space.

3. THE 5D REDUCTION OF HORAVA-WITTEN THEORY: 5D HETEROTIC M-THEORY

After the discovery of the duality transformations which relate the five distinct 10-dimensional superstring theories with each other and with 11-dimensional supergravity theory, people started to think that all of these theories arise as different limits of a mother 11-dimensional theory known as M-theory. The size of the 11th dimension in M-theory is related to the string coupling strength and grows as the coupling becomes strong [9]. Details of M-theory are unknown, but its low energy limit is thought to be 11-dimensional supergravity.

3.1 Horava-Witten theory: the strong coupling behaviour

In the HoravaWitten formulation of M-theory [4, 5], the gauge fields of the standard model are confined on two 9-branes located at the end points of an S^1/Z_2 orbifold. The 6 extra dimensions on the branes are compactified on a very small scale, close to the fundamental scale, and their effect on the dynamics is felt through moduli fields, i.e. 5D scalar fields. A 5D reduction of the HoravaWitten theory and the corresponding brane-world cosmology is given in [6–8].

We can only speak about the low energy limit of M-theory, which is supergravity plus two boundaries. Horava and Witten showed that M-theory on the orbifold $R^{10} \times S^1/Z_2$ is dual to the strong coupling limit of the 10D $E_8 \times E_8$ heterotic string. This duality says that M-theory on $R^{10} \times S^1/Z_2$ of radius R_{11} is equivalent to the $E_8 \times E_8$ heterotic string with coupling constant g_s, where [4, 113]

$$R_{11} = g_s^{2/3} l_P \tag{3.1}$$

This allows us to say that the low energy effective theory must approach 11D

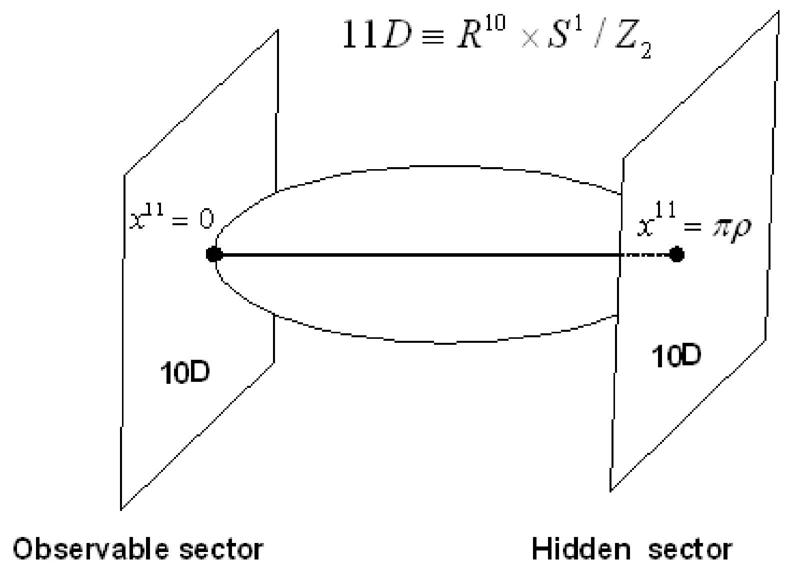


Fig. 3.1: Horava-Witten set up. The orbifold radius ρ is connected via the string coupling g_s by $\rho = g_s^{2/3} l_P$. The eleventh dimension is only accessible in the strong coupling limit.

supergravity in the strong coupling limit. Relation (3.1) means that when R_{11} is small, the string picture is a good description, and when R_{11} is large, supergravity is a good description. This is also the same relation that one finds between the M-theory on $R^{10} \times S^1$ and Type IIA superstring theory, in the low energy limit.

Just like in the case of the Randall-Sundrum models, the orbifold S_1/Z_2 is equivalent to an interval, and so in Horava-Witten theory the space is 11D bounded by two 10D orbifold planes with a Z_2 reflection symmetry in the eleventh dimension. The eleven dimensional supergravity lives in the bulk. Horava-Witten theory is usually reduced to a 5D world $R^4 \times S^1/Z_2$ via compactification on a Calabi-Yau space with the residual effects of the CY manifold being described by their moduli.

In order to cancel the gauge and gravitational anomalies that arise and keep the gauge and local SUSY invariance, an E_8 gauge group is required to act on each of the two 10-dimensional planes at the orbifold fixed points $x^{11} = 0, \pi\rho$, where ρ is the length scale of the bulk.

The 11D Yang-Mills gauge coupling constant g is fixed in terms of the 11D gravitational constant κ_{11} via

$$g^2 = 2\pi(4\pi\kappa_{11}^2)^{2/3} \tag{3.2}$$

This leads to [17]

$$G_N = \frac{\kappa_{11}^2}{16\pi^2 V\rho}; \quad \alpha_G = \frac{(2\pi\kappa_{11}^2)^{2/3}}{2V} \tag{3.3}$$

Where V is the CY volume and α_G is the GUT scale coupling constant. Note that here $\kappa_{11}^{2/9}$ is the 11D Planck scale [124, 125]. For $V = 1/M_G^6$ with $M_G = 3 \times 10^{16}$ GeV s the GUT mass and $\alpha_G = 1/24$, one finds $\kappa_{11}^{-2/9} = M_G$ and $1/\pi\rho \cong 4.7 \times 10^{15}$. This explains the Planck scale-Gut scale hierarchy. In other words, this gives us a natural explanation for grand unification occurring below the $4D$ Planck scale, since it is the 11D Planck scale that is fundamental and its mass scale is $\simeq M_G$.

So, as one probes to higher energy, our 4-dimensional world first goes through an intermediate regime where the orbifold dimension becomes visible, the universe thus appearing five dimensional with two boundary branes. Only at energies of the order of string scale would the universe look 11-dimensional.

3.2 The 11D low energy action

As we have described in the previous section, the low energy limit of M-theory is 11D supergravity with two boundaries, 11D supergravity, was constructed 30 years ago [117] and it contains three kinds of fields (that form the supergravity multiplet): the graviton field or the metric g, the gravitino field ψ_I and a three index antisymmetric gauge field C_{IJK} with a field strength G.

We have to mention that this theory is non-renormalizable. (This can be shown easily by calculating the mass dimension of its action. It is not equal to 4). This destroyed the hopes to be a fundamental theory!

The usual supergravity action is:

$$S_{SG} = \frac{2}{\kappa^2}\int_{\mathcal{M}_{11}} d^{11}x\sqrt{g}\left[-\frac{1}{2}\mathcal{R} - \frac{1}{2}\overline{\psi}_I\Gamma^{IJK}D_J\psi_K - \frac{1}{48}G_{IJKL}G^{IJKL}\right. \\ -\frac{\sqrt{2}}{192}\left(\overline{\psi}_I\Gamma^{IJKLMN}\psi_N + 12\overline{\psi}^J\Gamma^{KL}\psi^M\right)G_{JKLM} \\ \left. -\frac{\sqrt{2}}{3456}\epsilon^{I_1I_2...I_{11}}C_{I_1I_2I_3}G_{I_4...I_7}G_{I_8...I_{11}}\right], \tag{3.4}$$

where the capital indices $I, J, ... = 0, ..., 9, 11$ are used for the 11D space $\mathcal{M}_{11}$. The orbifold S^1/Z_2 has radius ρ and the coordinate x^{11} is restricted to $x^{11} \in [0, \pi\rho]$. The gamma matrices satisfy $\{\Gamma_I, \Gamma_J\} = 2g_{IJ}$ and $\Gamma^{I...K} = \Gamma^{[I}...\Gamma^{K]}$. The spinors are Majorana, and $\overline{\psi} = \psi^T\Gamma^0$.

The total 11D Horava-Witten action then is the supergravity one plus a Yang-Mills action describing the two E_8 Yang-Mills theories on the two boundaries. The bosonic part of the boundary action is

$$S_{YM} = \frac{-1}{8\pi\kappa^2}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[\int_{\mathcal{M}_{10}^{(1)}}\sqrt{-g}\left(tr(F^{(1)})^2 - \frac{1}{2}tr\mathcal{R}^2\right)\right. \\ \left. -\int_{\mathcal{M}_{10}^{(2)}}\sqrt{-g}\left(tr(F^{(2)})^2 - \frac{1}{2}tr\mathcal{R}^2\right)\right] \tag{3.5}$$

where the Yang-Mills coupling constant is expressed in terms of κ according to [5] and the boundary trR^2 terms are required by supersymmetry [12]. The action of the low energy limit of M-theory also includes extrinsic curvature terms [33, 136].

The bulk fields in the total action are the 11D metric g_{IJ}, the three-form C_{IJK} with bulk field strength $G_{IJKL} = 24\partial_{[I}C_{JKL]}$ and the gravitino ψ_I. The two E_8 gauge fields A_I^i, $i = 1, 2$ with field strengths F_{IJ}^i and their gaugino superpartners χ^i live on the 10D hypersurfaces $\mathcal{M}_{10}^i$.

3.3 The 5D reduced Horava-Witten theory

The question now is: how do we reduce the 11 dimensional theory? The existence of 10 dimensions in string theory is incompatible with the observed dimensionality of space time, which is 4. Therefore we have to hide the extra 6 dimensions.

When we do this in the case of 11D M-theory, we end up with the interesting 5D system of two branes (that became so popular after the RS model) but with many interesting new particles arising from the reduction. We can get the 4D effective theory easily by integrating out the 5'th dimension. The resulting 4D effective theory is interesting from the point of view of particle physics phenomenology [6, 12, 17, 97, 127]. In chapter (5) we shall describe the full reduction to 4 dimensions.

The reduction of the 11D action to 5 dimensions has been done in Ref. [6]. In the 11D theory, the supergravity multiplet consists of the graviton, gravitino and the field C. The total bulk field content of this 5 dimensional theory is given by the gravity multiplet $(g_{\alpha\beta}, A_\alpha, \psi^i_\alpha)$ together with the universal hypermultiplet $(V, \sigma, \zeta, \bar{\zeta})$. V is the Calabi-Yau volume. After the dualization, the three-form $C_{\alpha\beta\gamma}$ produces a scalar field σ. The 5 dimensional effective action can be written as [7]

$$S_5 = S_{bulk} + S_{bound} \tag{3.6}$$

Where

$$\begin{aligned} S_{bulk} \;=\; & \frac{-1}{2\kappa_5^2}\int_{\mathcal{M}_5}\sqrt{-g}\left[\mathcal{R} + \frac{3}{2}\bar{F}_{\alpha\beta}\bar{F}^{\alpha\beta} + \frac{1}{\sqrt{2}}\epsilon^{\alpha\beta\gamma\delta\epsilon}A_\alpha\bar{F}_{\beta\gamma}\bar{F}_{\delta\epsilon} + \right. \\ & \frac{1}{2V^2}\partial_\alpha V\partial^\alpha V + \frac{1}{2V^2}\left[(\partial_\alpha\sigma - i(\zeta\partial_\alpha\bar{\zeta} - \bar{\zeta}\partial_\alpha\zeta) - 2\alpha\epsilon(x^{11})A_\alpha)\right]^2 \\ & \left. + \frac{2}{V}\partial_\alpha\zeta\partial^\alpha\bar{\zeta} + \frac{\alpha^2}{3V^2}\right] \end{aligned} \tag{3.7}$$

And

$$\begin{aligned} S_{bound} \;=\; & \frac{\sqrt{2}}{\kappa_5^2}\left[\int_{M_4^{(1)}}\sqrt{-g}V^{-1}\alpha - \int_{M_4^{(2)}}\sqrt{-g}V^{-1}\alpha\right] \\ & - \frac{1}{16\pi\alpha_{GUT}}\sum_{i=1}^{2}\int_{M_4^{(i)}}\sqrt{-g}\left(V\mathrm{tr}F^{(i)}_{\mu\nu}F^{(i)\mu\nu} - \sigma\mathrm{tr}F^{(i)}_{\mu\nu}\widetilde{F}^{(i)\mu\nu}\right). \end{aligned} \tag{3.8}$$

where $\widetilde{F}^{(i)\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^{(i)}_{\rho\sigma}$ and the expansion coeffecients α_i are

$$\alpha_i = \frac{\pi}{\sqrt{2}}\left(\frac{\kappa}{4\pi}\right)^{2/3}\frac{1}{v^{2/3}}\beta_i, \quad \beta_i = -\frac{1}{8\pi^2}\int_{C_i} tr(\mathcal{R}\wedge\mathcal{R}). \tag{3.9}$$

with the Calabi-Yau volume V defined as

$$V = \frac{1}{v}\int_X \sqrt{g^{(6)}} \tag{3.10}$$

where $g^{(6)}$ is the determinant of the Calabi-Yau metric.

3.3.1 BPS solution for a simple system of two branes

The spectra of string theories often contain a special class of states called BPS states (Bogomol'nyi-Prasad-Sommerfield). BPS states are stable in the sense that they cannot decay into other states [126]. The corresponding solutions are BPS solutions, described by a set of moduli.

In the previous section we have seen that there are a large number of fields in the 5D heterotic M-theory action. It is almost not possible to find a general solution to all the resulting equations of motions. The simplest case one can try is the vacuum solution obtained by setting as many fields as we can to zero. The system then contains only gravity and a scalar field. The relevant part of the action then is [8, 15, 16]

$$S = \int d^5x\sqrt{g}\left(-\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \mathcal{V}(\phi)\right). \tag{3.11}$$

where the potential $\mathcal{V}(\phi)$ is an exponential potential of the form

$$\mathcal{V}(\phi) = \frac{\alpha^2}{6\kappa^2}e^{-2\sqrt{2}\kappa\phi} \tag{3.12}$$

The dilaton ϕ is related to the Calabi-Yau volume by $V = e^{\Phi}$, $\Phi = \sqrt{2}\kappa\phi$. This simple model is called the dilatonic braneworld [15] with the scalar field called dilaton. The constant α has units of energy. We are not considering moving branes; our branes are stationary and we will be looking for static BPS solutions.

Potentials of this form arise in many theories of the fundamental interactions including superstring and higher dimensional theories [37]. The action (3.11) leads to the following field equations

$${}^{(5)}R_{\mu\nu} - \frac{1}{2}g^{(5)}_{\mu\nu}R = \kappa^2[\phi_{,\mu}\phi_{,\nu} - g^{(5)}_{\mu\nu}(\frac{1}{2}\phi_{,\rho}\phi^{,\rho} + \mathcal{V}(\phi)] \tag{3.13}$$

where the energy-momentum tensor is given by

$${}^{(5)}T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - g^{(5)}_{\mu\nu}(\frac{1}{2}\phi^{,\rho}\phi_{,\rho} + \mathcal{V}(\phi)) \tag{3.14}$$

To find a solution for these equations, we make an ansatz

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + d\tilde{z}^2 \tag{3.15}$$

$\sigma = \sigma(\widetilde{z})$ and $\phi = \phi(\widetilde{z})$ are ansatz. With this ansatz, the Einstein equations give [see Appendix (C.1.1)]

$$\begin{aligned} 6\sigma'^2 - \frac{\kappa^2}{2}\phi'^2 + \kappa^2\mathcal{V}(\phi) &= 0, \\ 3\sigma'' + 6\sigma'^2 + \kappa^2(\frac{\phi'^2}{2} + \mathcal{V}(\phi)) &= 0, \end{aligned} \tag{3.16}$$

where the prime denotes differentiation with respect to $\widetilde{z}$. We now need boundary conditions for the scalars ϕ and σ. The boundary condition on the dilaton field can be found from the variation of the action (3.8) with respect to ϕ and requiring that the surface variation vanishes. This gives

$$\phi' = \frac{\alpha}{\kappa}e^{-\sqrt{2}\kappa\phi} \tag{3.17}$$

The boundary conditions for the radion can be found from the junction conditions. For flat branes, the trace of the extrinsic curvature

$$K = 4\sigma' \quad \text{at } z = z_1 \quad , \quad K = -4\sigma' \quad \text{at } z = z_2 \tag{3.18}$$

Tracing junction condition $[K_{\mu\nu} - g_{\mu\nu}K] = -\frac{\kappa^2}{6}T_{\mu\nu}$ gives

$$K = \frac{\kappa^2}{6}T \tag{3.19}$$

The 4-dimensional energy-momentum tensor can be calculated from the boundary action (3.8), and after substituting in (3.19) we get the boundary condition as

$$\sigma' = \frac{\alpha}{3\sqrt{2}}e^{-\sqrt{2}k\phi} \tag{3.20}$$

The solution to (3.16), (3.17) and (3.20) is

$$\sigma = -\frac{1}{6}\ln(1 - \sqrt{2}\alpha\widetilde{z}) \tag{3.21}$$

It is useful to have expressions for the metric in different coordinate systems. For a conformally flat metric, Substitute back in (3.15)

$$\begin{aligned} ds^2 &= (1 - \sqrt{2}\alpha\widetilde{z})^{\frac{1}{3}}\eta_{\mu\nu}dx^\mu dx^\nu + d\widetilde{z}^2 \\ &= (1 - \sqrt{2}\alpha\widetilde{z})^{\frac{1}{3}}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), \end{aligned} \tag{3.22}$$

where

$$d\tilde{z} = dz \left(1 - \sqrt{2}\alpha\tilde{z}\right)^{\frac{1}{6}}. \tag{3.23}$$

This gives

$$\begin{aligned} \left(1 - \sqrt{2}\alpha\tilde{z}\right) &= \left(\frac{5\alpha}{3\sqrt{2}}z\right)^{\frac{6}{5}} \\ &\equiv \left(\frac{z}{z_1}\right)^{\frac{6}{5}}, \quad z_1 = \frac{3\sqrt{2}}{5\alpha}. \end{aligned} \tag{3.24}$$

The metric (3.15) could then be written in a conformally flat form as

$$\begin{aligned} ds^2 &= e^{-2\sigma}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \\ &= \left(\frac{z}{z_1}\right)^{\frac{2}{5}} (\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \end{aligned} \tag{3.25}$$

The dilaton for the conformally flat metric (3.25) (also found in [15]) is

$$\phi(z) = \frac{3\sqrt{2}}{5\kappa} \ln \frac{z}{z_1} + \phi_o \tag{3.26}$$

The values of z on the two branes, z_1 and z_2 can be used as the moduli parameters of the background solution as we will see in the next section.

The linear dependence of Calabi-Yau volume on the extra dimension $\tilde{z}$ makes it interesting to compare the metric (3.40) with the one used by Curio and Krause [132]

$$ds^2 = \left(\frac{V}{V_1}\right)^{-\frac{1}{3}} \eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{V}{V_1}\right)^{\frac{1}{3}} \left(g_{lm}(x^n)dx^l dx^m + (dx^{11})^2\right) \tag{3.27}$$

where

$$V = (1 - \mathcal{S}_1 x^{11})^2 V_1 \tag{3.28}$$

The quadratic dependence of V on x^{11} is because of the definition of x^{11} is different from the definition for $\tilde{z}$ due to the different metric background. $\mathcal{S}_1$ can be expressed as a power series in $\kappa^{2/3}$, i.e. $\mathcal{S}_1 = \mathcal{S}_1^{(1)}\kappa^{2/3} + \mathcal{S}_1^{(2)}\kappa^{5/3} + \ldots$ and only for the first term we get a linear volume dependence

$$V(x^{11}) = (1 - 2\mathcal{S}_1^{(1)}\kappa^{2/3}x^{11})V_1 + \mathcal{O}(\kappa^{4/3}) \tag{3.29}$$

This was also found before in [140].

3.4 The moduli space approximation

The moduli space approximation is an another approach (different from the one used in section (1.6)) used to get a 4D effective theory from the higher dimensional one. The moduli space here could be defined as the collection of the vacuum expectation values of massless scalar fields [114]. In [14], a 4D low energy theory was derived from a supergravity-inspired 5D theory using this approach. The moduli space approximation is a good approximation only when the time-variation of the moduli fields is small (the low-velocity assumption). In the context of braneworlds, this approximation was also used in [106, 107].

In the framework of 5-dimensional compactification of M-theory [8, 12], the moduli space approximation describes, through a 4-dimensional effective action, a system of two branes of opposite tension embedded in a 5-dimensional warped space-time. Besides the fields living on the positive tension brane (assumed to be our universe), the moduli associated with the position of the branes in the fifth dimension act as two scalar fields thereby leading to an effective biscalar-tensor theory of gravity [13]. This means that for an observer in 4D, the branes are realized as moduli *massless* fields.

In RS1 there's a single modulus, called the radion, related to the thickness of the AdS slice. In dilatonic brane-worlds (5D heterotic M-theory), there are two moduli, one related to the distance between the branes and the another related to volume of the Calabi-Yau space.

To reach a 4D effective theory using this approach, the following assumptions are made:

1. The brane positions z_1 and z_2 become dependent on the 4D coordinates, $z_1(x^\mu)$ and $z_2(x^\mu)$. They are then non-constant brane-world moduli.
2. The 4D Minkowiskian flat metric $\eta_{\mu\nu}$ is promoted to 4D curved metric $g^{(4)}_{\mu\nu}$.
3. Terms involving more than two derivatives of the brane positions are ignored (a good approximation if the branes are slowly moving). This means we

neglect terms like $(\partial z_1)^3$ in constructing the effective four-dimensional theory.

4. Finally, the massive Kaluza Klein states are not included.

In Ref. [14], the ansatz was inserted into the 5D action

$$S_{bulk} = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}} d^5x \sqrt{-g^{(5)}} (\mathcal{R} - \frac{3}{4}[(\partial\psi)^2 + U]). \tag{3.30}$$

The background metric is

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + d\widetilde{z}^2 \tag{3.31}$$

The bulk potential energy of the scalar field ψ is related to the boundary superpotential U_B by

$$U = \left(\frac{\partial U_B}{\partial \psi}\right)^2 - U_B^2 \tag{3.32}$$

The boundary potential is an exponential function of the field ψ

$$U_B = 16(\widehat{\alpha}^2 - 1) k^2 e^{2\widehat{\alpha}\psi} \tag{3.33}$$

Comparing this potential with the potential for the heterotic M-theory in (3.12), we get $\widehat{\alpha} = \sqrt{\frac{3}{2}}$ and $k = \frac{\alpha}{3\sqrt{2}}$ which we will be using. The positions of the first and the second brane $\widetilde{z}_1$ and $\widetilde{z}_2$ are denoted by $\phi(x^\mu)$ and $\sigma(x^\mu)$. After redefining these two moduli by

$$\widetilde{\phi}^2 = (1 - 6k\phi)^{\frac{4}{3}} \tag{3.34}$$

$$\widetilde{\sigma}^2 = (1 - 6k\sigma)^{\frac{4}{3}}, \tag{3.35}$$

In Ref. [14] the 4D action was given in terms of $\widetilde{\sigma}$ and $\widetilde{\phi}$ in the Jordan frame.

$$\widetilde{\phi} = Q \ \cosh \ R \tag{3.36}$$

$$\widetilde{\sigma} = Q \ \sinh \ R \tag{3.37}$$

The final effective action given in [14] has the form of multiscalar tensor theory

$$S_{bulk} = \frac{1}{16\pi G} \int d^4x \sqrt{-g^{(4)}} \left(\mathcal{R} - \frac{9}{2}\frac{(\partial Q)^2}{Q^2} - \frac{3}{2}(\partial R)^2\right). \tag{3.38}$$

where $16\pi G = 8\alpha\sqrt{\frac{2}{3}}\kappa_5^2$.

The moduli are massless at the classical level, but quantum corrections will add a potential term of the form [19]

$$S = -\int d^4x\sqrt{-g^{(4)}}V(Q,R) \tag{3.39}$$

generated at one loop.

3.5 Possible moduli systems

In this section we list some useful moduli systems which we are going to use to describe the brane positions for different situations.

1. The moduli $(\tilde{z}_1, \tilde{z}_2)$ for the Einstein frame metric

$$ds^2 = V^{\frac{1}{3}}\eta_{\mu\nu}dx^\mu dx^\nu + d\tilde{z}^2 \tag{3.40}$$

   where $V = (1-\sqrt{2}\alpha\tilde{z})$ is the volume of Calabi-Yau space.

2. The moduli (V_1, V_2) with V_1 and V_2 are the Calabi-Yau volumes at z_1 and z_2 respectively.

3. The moduli (z_1, z_2) for the conformally flat metric

$$ds^2 = \left(\frac{z}{z_1}\right)^{\frac{2}{5}}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu) \tag{3.41}$$

   This system will be used in chapters 4.

4. The moduli (Q, R), related to the conformally flat coordinates z_1 and z_2 by

$$z_1 = Q\,\sinh(R) \tag{3.42}$$

$$z_2 = Q\,\cosh(R) \tag{3.43}$$

   They are connected with V_1 and V_2 by

$$Q = \sqrt{V_1^{4/3} - V_2^{4/3}} \tag{3.44}$$

$$R = \tanh^{-1}\left(\frac{V_2}{V_1}\right)^{2/3} \tag{3.45}$$

5. The Kähler moduli (S, T) which are related to V_1 and V_2 by

$$S = \frac{\sqrt{2}}{4}\left(V_1^{2/3} + V_2^{2/3}\right)^{3/2} \tag{3.46}$$

$$T = \frac{3}{4\alpha}\left(V_1^{2/3} + V_2^{2/3}\right)^{1/2}\left(V_1^{2/3} - V_2^{2/3}\right) \tag{3.47}$$

Note that when $V_1 \approx V_2 \approx V$, $S \approx V$ and $T \approx \alpha^{-1}(V_1 - V_2)$, i.e. S becomes the volume modulus and T becomes the radion.

4. CASIMIR ENERGY FOR TWISTED FERMION FIELDS

In this chapter we calculate the difference in the Casimir potential for the case of twisted and untwisted fermions in heterotic $M-$theory. Twisted fermions were introduced by Antoniadis and Quiros as an explicit means of SUSY breaking [153], and they calculated the vacuum energy in the flat space limit. The Casimir potential for untwisted fermions in the warped heterotic $M-$theory background was calculated in [15]. The work presented in this chapter is original research done in collaboration with Prof. Ian G. Moss.

4.1 Introduction

The identity

$$\prod_p (1-p^{-s})^{-1} = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s > 1, \tag{4.1}$$

which holds for every prime number $p \neq 1$ (s is a real variable) was found by Euler while investigating prime numbers [102]. Later, Riemann realized that s should be extended into a complex variable and denoted the resulting function by $\zeta(s)$. Since that time it is called Riemann zeta function,

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s \in C, \quad \Re(s) > 1. \tag{4.2}$$

The series is convergent only when the real part of s, $\Re(s)$, is greater than one.

Studies of complex analytic manifolds led to the definition of a zeta function associated with a type of Laplacian operator [102, 103]. The zeta function for an elliptic operator Δ is defined by the functional trace,

$$\zeta_\Delta(s) = tr(\Delta^{-s}). \tag{4.3}$$

When the operator has a discrete set of eigenvalues λ_n, we could write

$$\zeta_\Delta(s) = \sum_{n=1}^{\infty} \lambda_n^{-s} \tag{4.4}$$

For 4D space time, this sum only converges for $\Re(s) > 2$. This restriction could be removed by analytic continuation to values of s in the complex plane.

The vacuum energy in a static background has been calculated in many applications [104], where the eigenvalues of Δ in these applications are of the form $k^2 + \omega_n^2$. If we used a compactification length L and take the limit $L \to \infty$ at the end we get for n-dimensional case

$$\zeta_\Delta(s) = L^n \int \frac{d^n k}{(2\pi)^n} \sum_{m=1}^{\infty} (k^2 + \omega_m^2)^{-s}. \tag{4.5}$$

This gives

$$\zeta_\Delta(s) = \frac{L^n}{2^n \pi^{\frac{n}{2}}} \frac{\Gamma(s - \frac{n}{2})}{\Gamma(s)} \sum_{m=0}^{\infty} \omega_m^{n-2s} \tag{4.6}$$

The vacuum energy then will be

$$V_C = \mp L^{-n} \zeta'(0) \tag{4.7}$$

The minus sign is for bosons and the plus sign is for fermions. Note that for $\omega_n \propto l^{-1}$, where l is the finite length scale in the problem, then $V_C \propto l^{-n}$. Casimir effect calculations are probably the most notable example for the use of Zeta function regularization to remove divergencies in quantum field theory.

4.2 Twisted and untwisted fermions in five dimensions

In this chapter, we will concentrate on the boundary conditions common in supersymmetric theories where the 5D fermions are usually represented as two four component spinors, ψ_a, $a = 1, 2$, related by a symplectic transformation. The symplectic Majorana condition is

$$\psi^{aT} C = \overline{\psi}_a. \tag{4.8}$$

where C is the charge conjugation matrix. The index a is raised with the antisymmetric metric ϵ_{ab}, so that

$$\psi_1 = -\psi^2, \quad \psi_2 = \psi^1. \tag{4.9}$$

These two four spinors can be grouped into a single eight-component Majorana spinor

$$\Psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix} \tag{4.10}$$

and eight-component γ matrices can be formed

$$\Gamma = \begin{pmatrix} \gamma_A & 0 \\ 0 & -\gamma_A \end{pmatrix}. \tag{4.11}$$

The Majorana condition on the eight-component fermion is

$$\Psi^T \mathcal{C} = \overline{\Psi}, \tag{4.12}$$

where

$$\mathcal{C} = \begin{pmatrix} 0 & C \\ C & 0 \end{pmatrix}. \tag{4.13}$$

and $\overline{\Psi} = \Psi^\dagger \Gamma^0$ is the usual Dirac adjoint. Assuming that SUSY is broken only on the hidden brane at z_2, introduce projection operators on both branes $P_+ = \frac{1}{2}(1+\Gamma_5)$ and $P_\chi = \frac{1}{2}(1+\chi\Gamma_5)$ respectively, where the matrix χ depends on a real parameter θ so that

$$\chi = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{4.14}$$

The twisted (antiperiodic) boundary conditions for twisted bulk fermions are then

$$P_+\Psi = 0 \quad \text{on} \quad \mathcal{M}^{(1)} \tag{4.15}$$

$$P_\chi\Psi = 0 \quad \text{on} \quad \mathcal{M}^{(2)} \tag{4.16}$$

where the angle θ determines how much the fermions are twisted. Later, we will relate χ with the gaugino condensate.

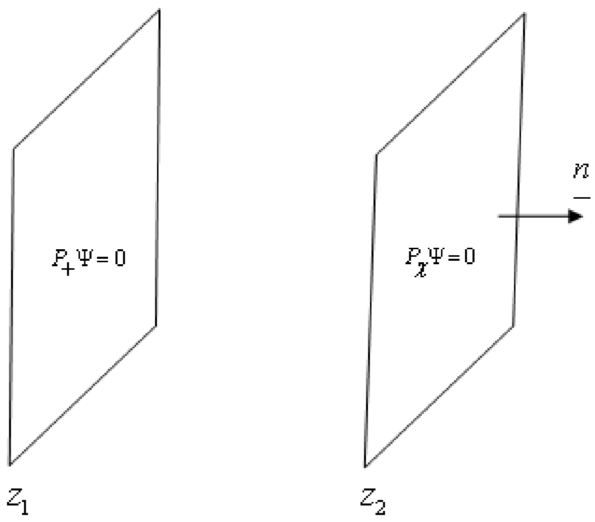


Fig. 4.1: On the visible brane at z_1 we have $P_+\psi = 0$ and $(\partial_z z + \frac{1}{2}K + m)P_-\psi = 0$. On the hidden brane at z_2 we have $P_\chi\psi = 0$ and $(\partial_z z + \frac{1}{2}K + m)P_{-\chi}\psi = 0$.

The aim of this chapter is to calculate the total Casimir energy, which is equivalent to the difference between the twisted and untwisted fermion cases. This can be illustrated as follows. SUSY implies that the total vacuum energy of the untwisted fermions and untwisted bosons is zero. This means that

$$V_C(\text{ untwisted bosons}) = -V_C(\text{untwisted fermions}).$$

Now, the total vacuum energy ΔV_C of the twisted fields is equal to

$$\Delta V_C = V_C(\text{twisted fermions}) + V_C(\text{twisted bosons}).$$

But, since there are no known bosons with twisted boundary conditions, the vacuum energy of the twisted bosons is just the vacuum energy of the untwisted bosons. It then follows directly from this discussion that the total vacuum energy is equal to

$$\Delta V_C = V_C(\text{twisted fermions}) - V_C(\text{untwisted fermions}).$$

4.2.1 Fermion modes

The Dirac eigenfunctions are solutions to

$$D^2\Psi = \lambda\Psi, \tag{4.17}$$

where D is the Dirac operator for mass m and D^2 is a second order Laplacian,

$$D^2 = -\partial^2 + \frac{1}{4}R + m^2 + \partial\!\!\!/ m. \tag{4.18}$$

According to Lukas et al. [12], the fermion masses in reduced heterotic $M-$theory are typically of the form

$$m = -\frac{\alpha}{\sqrt{2}}\gamma V^{-1}, \tag{4.19}$$

where V is the Calabi-Yau volume and the value of γ depends on which fermion is being discussed. We use the conformally flat metric

$$ds^2 = e^{-2\sigma}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \tag{4.20}$$

then

$$\Delta_0 = -\partial_z^2 + k^2 + m_0^2 + \Gamma^5 m_0'. \tag{4.21}$$

The 0 index is just a reminder that the operator has been rescaled from curved to flat space. m_0 is the rescaled fermion mass m and given by

$$m_0 = e^{-\sigma}m = e^{-\sigma}\frac{\gamma\alpha}{\sqrt{2}}V^{-1} = \frac{3\gamma}{5}z^{-1}, \tag{4.22}$$

using (3.26) in chapter 3, where the Calabi-Yau volume V was expressed in terms of z. The value of γ depends on the choice of the fermion field. Later in chapter 7 we will give a detailed example for the gravitino and other fermion fields with different values of γ.

The eigenvalue equation is then

$$\left(-\partial_z^2 + k^2 + \frac{9\gamma^2}{25}z^{-2} \pm \frac{3}{5}\gamma z^{-2}\right)\Psi = \lambda\Psi \tag{4.23}$$

for $\Gamma^5\Psi = \pm\Psi$ and $m_n^2 = \lambda - k^2$. Hence,

$$\Psi'' + m_n^2\Psi - \left(\frac{9\gamma^2}{25} \pm \frac{3}{5}\gamma\right) z^{-2}\Psi = 0. \tag{4.24}$$

Comparing with Bessel's equation [20]

$$W'' + \left(\lambda^2 - \frac{\nu^2 - \frac{1}{4}}{z^2}\right) W = 0, \tag{4.25}$$

then Ψ is a Bessel function with index is given by

$$\nu = \left(\frac{1}{2} \pm \frac{3}{5}\gamma\right). \tag{4.26}$$

The hypermultiplet fermion, for example, has $\gamma = \frac{1}{6}$. This means $\nu = \frac{2}{5}$ or $\frac{3}{5}$. The solution gives the wave function for fermions in z direction as [15]

$$\psi_n(z) = \sqrt{z}\left(A^- J_{\frac{2}{5}}(m_n z) + B^- Y_{\frac{2}{5}}(m_n z) + A^+ J_{\frac{3}{5}}(m_n z) + B^+ Y_{\frac{3}{5}}(m_n z)\right) \tag{4.27}$$

where $A^{\mp}$ and $B^{\mp}$ are constant spinors (integration constants). We need the eigenvalue equation which defines implicitly the discrete spectrum m_n. We apply the twisted boundary conditions to the wave function above.

Recalling the twisted boundary conditions (4.15) and (4.16), we can write (4.16) as

$$CP_-\Psi - iJSP_+\Psi = 0 \tag{4.28}$$

where

$$C = \cos\frac{\theta}{2}, \quad S = \sin\frac{\theta}{2}, \quad J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4.29}$$

The normal or z derivative (denoted by a prime) flips P_+ and P_-, as described in Ref.[15]

$$C(P_+\Psi)' - iJS(P_-\Psi)' = 0 \tag{4.30}$$

Applying these four boundary conditions on the wave function

$$\Psi(z) = \sqrt{z}\left(A^- J_\nu(m_n z) + B^- Y_\nu(m_n z) + A^+ J_{\overline{\nu}}(m_n z) + B^+ Y_{\overline{\nu}}(m_n z)\right) \tag{4.31}$$

where $\overline{\nu} = 1 - \nu$ and $P_\pm A^\pm = 0$, we get a system of four equations,

$$A^+ J_{\overline{\nu}}(m_n z_1) + B^+ Y_{\overline{\nu}}(m_n z_1) = 0 \tag{4.32}$$

$$A^- Y_{\overline{\nu}}(m_n z_1) + B^- J_{\overline{\nu}}(m_n z_1) = 0 \tag{4.33}$$

$$A^- SJ_\nu(m_n z_2) + B^- SY_\nu(m_n z_2) + iJA^+ CJ_{\overline{\nu}}(m_n z_2) + iJB^+ CY_{\overline{\nu}}(m_n z_2)) = 0 \tag{4.34}$$

$$A^- CY_{\overline{\nu}}(m_n z_2) - B^- CJ_{\overline{\nu}}(m_n z_2) - iJA^+ SY_{\overline{\nu}}(m_n z_2) + iJB^+ SJ_{\nu}(m_n z_2)) = 0 \quad (4.35)$$

Non-trivial solutions occur only when

$$\begin{vmatrix} J_{\overline{\nu}}(m_n z_1) & Y_{\overline{\nu}}(m_n z_1) & 0 & 0 \\ 0 & 0 & Y_{\overline{\nu}}(m_n z_1) & -J_{\overline{\nu}}(m_n z_1) \\ CJ_{\overline{\nu}}(m_n z_2) & CY_{\overline{\nu}}(m_n z_2) & SJ_{\nu}(m_n z_2) & SY_{\nu}(m_n z_2) \\ -SY_{\nu}(m_n z_2) & SJ_{\nu}(m_n z_2) & CY_{\overline{\nu}}(m_n z_2) & -CJ_{\overline{\nu}}(m_n z_2) \end{vmatrix} = 0.$$

We then get the eigenvalue equation for the twisted fermions as

$$J_{\overline{\nu}}(m_n z_1)(CY_{\overline{\nu}}(m_n z_2) \pm SJ_{\nu}(m_n z_2)) - Y_{\overline{\nu}}(m_n z_1)(CJ_{\overline{\nu}}(m_n z_2) \mp SY_{\nu}(m_n z_2)) = 0 \quad (4.36)$$

Making use of the linear relation

$$Y_{\nu}(x) = \frac{J_{\nu}(x)\cos(\nu\pi) - J_{-\nu}(x)}{\sin(\nu\pi)} \quad (4.37)$$

(4.36) becomes

$$J_{\overline{\nu}}(m_n z_1)(CJ_{-\overline{\nu}}(m_n z_2) \pm SJ_{\nu}(m_n z_2)) - J_{-\overline{\nu}}(m_n z_1)(CJ_{\overline{\nu}}(m_n z_2) \mp SJ_{-\nu}(m_n z_2)) = 0 \quad (4.38)$$

Later we will consider $\nu = 2/5$ and $\overline{\nu} = 3/5$.

4.3 Casimir potential in flat space

For flat space, the warping factor $e^{-2\sigma} = 1$, and the metric is

$$ds_5^2 = dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu. \quad (4.39)$$

The operator (4.18) in flat space with zero mass is

$$\Delta = -\nabla^2. \quad (4.40)$$

The Dirac equation becomes

$$\frac{\partial^2}{\partial z^2} u_n = (k^2 - m_n^2) u_n, \quad (4.41)$$

which has the solution

$$u_n = A\sin\left(\frac{\pi n z}{l_5}\right) + B\cos\left(\frac{\pi n z}{l_5}\right). \quad (4.42)$$

For flat space and when the branes are very close to each other we could choose relevant masses m_n such that $m_n z$ is very large. For untwisted fermions in flat space,

$$\sin(m_n z_1)\ \cos(m_n z_2) - \sin(m_n z_2)\ \cos(m_n z_1) = 0. \quad (4.43)$$

This gives the fermion masses as

$$m_n = \frac{n\pi}{z_1 - z_2}, \qquad n = 0, 1, 2, \ldots \quad (4.44)$$

We now turn to the twisted bulk fermions where the eigenvalue equation is (4.38). We remember the following relation [20] when $|z| \to \infty$

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}}\left[\cos(z - \frac{\nu\pi}{2} - \frac{\pi}{4}) + e^{\varphi z} O(|z^{-1}|)\right], \quad (|arg\ z| < \pi). \quad (4.45)$$

For flat space, the eigenvalue equation could now be simplified to

$$\sin(m_n z_1)\ \cos\left(\frac{\theta}{2} \mp m_n z_2\right) - \cos(m_n z_1)\ \sin\left(m_n z_2 \mp \frac{\theta}{2}\right) = 0. \quad (4.46)$$

This leads to two equations for fermion masses

$$m_n^{(-)} = \frac{n\pi - \frac{\theta}{2}}{z_1 - z_2}, \qquad n = 0, 1, 2, ... \quad (4.47)$$

$$m_n^{(+)} = \frac{n\pi + \frac{\theta}{2}}{z_1 - z_2}, \qquad n = 0, 1, 2, ... \quad (4.48)$$

The ζ function in flat space with a volume Ω could now be written as

$$\zeta(s) = \Omega \int \frac{d^4k}{(2\pi)^4}\left[\sum_n (m^{(+)2} + k^2)^{-s} + \sum_n (m^{(-)2} + k^2)^{-s}\right]. \quad (4.49)$$

This k integral diverges for $s < 2$ and was evaluated already in [105]. Introducing $x = |k^2|/m^2$ for both integrals we get

$$\begin{aligned} \zeta(s) &= \quad (4.50) \\ & \sum_n \frac{\Omega}{16\pi^2}\left[m^{(+)4-2s}\int_0^\infty dx\ x(x+1)^{-s} + m^{(-)4-2s}\int_0^\infty dx\ x(x+1)^{-s}\right] \\ &= \sum_n \frac{\Omega}{16\pi^2}\frac{m^{(+)4-2s}}{(s-2)(s-1)} + \sum_n \frac{\Omega}{16\pi^2}\frac{m^{(-)4-2s}}{(s-2)(s-1)} \end{aligned}$$

The last expression can be analytically continued to a function with poles at $s = 1$ and $s = 2$. We still need to evaluate the sum, for this we use

$$\zeta(s,q) = \frac{2\Gamma(1-s)}{(2\pi)^{(1-s)}}\left[\sin\frac{\pi s}{2}\sum_{n=1}^{\infty}\frac{\cos 2\pi nq}{n^{1-s}} + \cos\frac{\pi s}{2}\sum_{n=1}^{\infty}\frac{\sin 2\pi nq}{n^{1-s}}\right] \quad s<0. \quad (4.51)$$

Then

$$\zeta(s) = \frac{\Omega}{4\pi^2}\frac{1}{(s-1)(s-2)}\left(\frac{\pi}{z_1-z_2}\right)^{4-2s}\frac{\Gamma(5-2s)}{(2\pi)^{5-2s}}\sin\pi(s-2)\sum_{n=1}^{\infty}\frac{\cos n\theta}{n^{5-2s}}. \quad (4.52)$$

The Casimir energy for twisted fermions, untwisted fermions and the difference are respectively

$$\begin{aligned} V_C(\theta) &= \zeta'(0) = \frac{3}{32\pi^2}\frac{1}{l_5^4}\sum_{n=1}^{\infty}\frac{\cos n\theta}{n^5}, & (4.53)\\ V_C(0) &\equiv \zeta'(0)|_{\theta=0} = \frac{3}{32\pi^2}\frac{1}{l_5^4}\sum_{n=1}^{\infty}\frac{1}{n^5}, & (4.54)\\ \Delta V_C &= \frac{3}{32\pi^2}\frac{1}{l_5^4}\left(\sum_{n=1}^{\infty}\frac{\cos n\theta}{n^5} - \sum_{n=1}^{\infty}\frac{1}{n^5}\right). & (4.55)\end{aligned}$$

This means that in flat space the Casimir energy is definitely attractive, since $\partial V/\partial l_5 < 0$ implies $F_{Casimir} > 0$ (attractive). We have to investigate this point as well in curved space.

4.4 Small θ limit (small twist)

Eq. (4.55) could be written as

$$\begin{aligned} \Delta V_C &= \frac{3}{32\pi^2}\frac{1}{l_5^4}\left(\sum_{n=1}^{\infty}\frac{1}{n^5}(\cos n\theta - 1)\right) & (4.56)\\ &= -\frac{3\zeta(3)}{16\pi^2 l_5^4}\sin^2(n\theta/2). & (4.57)\end{aligned}$$

For $\theta << 1$, $\cos n\theta - 1 \simeq -n^2\theta^2/2$. We then have

$$\Delta V_C = -\frac{3}{64\pi^2}\frac{\theta^2}{l_5^4}\zeta(3). \quad (4.58)$$

The small θ limit here means small twist. Later we will relate θ with the gaugino condensate on the hidden brane and the small θ limit will be interpreted as a small value of gaugino condensate.

4.5 Casimir potential in curved space

4.5.1 A Review for the untwisted case

Before we discuss the twisted fermions case, we describe the case of untwisted fermions calculated in [15]. This work was based on the method invented by Garriga et al [19, 66, 67] and by Flachi et al [32, 68].

The zeta functions we are interested in have the form,

$$\zeta(s) = \int d^4x \int \frac{d^4k}{(2\pi)^4} \sum_n \left(\frac{k^2 + m_n^2}{\mu_R^2} \right)^{-s} \tag{4.59}$$

Introducing $\tau = z_1/z_2$ and defining $\mu_n = z_2 m_n$, then we have the implicit equation for μ_n from (4.36),

$$F_{untwisted}(\mu_n, \tau) = J_{2/5}(\mu_n \tau) Y_{2/5}(\mu_n) - J_{2/5}(\mu_n) Y_{2/5}(\mu_n \tau) = 0 \tag{4.60}$$

Performing the momentum k integrals by changing to polar coordinates gives

$$\zeta(s) = \mu_R^{2s} \int d^4x \frac{\Gamma(s-2)}{(4\pi)^2 \Gamma(s)} \widehat{\zeta}(2s-4) z_2^{2s-4} \tag{4.61}$$

For the masses μ_n we have only an implicit equation which makes it complicated to evaluate the sum over them. Fortunately, the residue theorem allows us to write the sum over the positive zeros of $F(z)$ as a contour integral,

$$\widehat{\zeta}(2s-4) = \int_C dz z^{4-2s} \frac{d}{dz} \ln |F(z)| \tag{4.62}$$

Where the contour C is any contour encloses the positive zeros of $F(z)$ [see figure (4.2)].

For the implicit eigenvalue equation (4.60) we must restrict s to lie in the range $5/2 < \Re(s) < 3$. The contribution to the integral (4.62) from the large semi circle vanishes (just because the function inside the contour vanishes for large z), and we are left with the contribution along the imaginary axis and the small semi circle. This results in

$$\widehat{\zeta}(2s-4) = \frac{\sin(\pi s)}{\pi} \int_\epsilon^\infty dx x^{4-2s} \frac{d}{dx} \ln \left| P^0(x) \right| + \int_{C_\epsilon} \frac{dz}{2\pi i} z^{4-2s} \frac{d}{dz} \ln |F(z)| \tag{4.63}$$

Where $P^0(x) = F(ix)$ and C_ϵ is a small semi circle around the origin. Using formulae for the analytic continuation of Bessel functions,

$$P^0(x) = I_\nu(\tau x)K_\nu(x) - I_\nu(x)K_\nu(\tau x) \tag{4.64}$$

The leading order term for large x is denoted by P_a^0,

$$P_a^0(x) = I_\nu(x)K_\nu(\tau x). \tag{4.65}$$

The asymptotic expansion of the Bessel functions for large x gives

$$I_\nu(x) \sim \frac{e^x}{\sqrt{2\pi x}}, \qquad K_\nu(x) \sim \sqrt{\frac{\pi}{2x}}e^{-x}, \tag{4.66}$$

so that

$$P^0(x) \sim -P_a^0(x) \sim \frac{e^{x(1-\tau)}}{2x\sqrt{\tau}}. \tag{4.67}$$

We can now write the following equation

$$\begin{aligned}&\int_\epsilon^\infty dx x^{4-2s}\frac{d}{dx}\ln\left|P^0(x)\right| = \\ &\int_\epsilon^\infty dx x^{4-2s}\frac{d}{dx}\ln\left|\frac{P^0(x)}{P_a^0(x)}\right| + \int_\epsilon^\infty dx x^{4-2s}\frac{d}{dx}\ln\left|P_a^0(x)\right|\end{aligned} \tag{4.68}$$

We need these two integrals at $s = 0$. Analytic continuation can provide finite expressions for divergent integrals. The main idea here is that the integral on the LHS cannot be evaluated analytically or numerically at $s = 0$ as it diverges. So we divide it into two integrals the first one could be evaluated numerically and the second one could be evaluated analytically at $s = 0$. Actually, for large x, the first term on the RHS vanishes and we will have only the second one, i.e.

$$x^{4-2s}\frac{d}{dx}\ln\left|P^0(x)\right| \approx x^{4-2s}\frac{d}{dx}\ln\left|P_a^0(x)\right|. \tag{4.69}$$

Unfortunately, one integral on the RHS still diverges and we still need to do more to regularize it. If we can express $I_\nu(x)$ and $K_\nu(x)$ in terms of power series, by redefining them, then after substitution back in the integral we will be able to subtract off the undesirable terms that leads to divergence.

We define new functions $\Sigma_\nu^I(x)$ and $\Sigma_\nu^K(x)$ through

$$I_\nu(x) = \frac{e^x}{\sqrt{2\pi x}}\Sigma_\nu^I(x), \qquad K_\nu(x) = \sqrt{\frac{\pi}{2x}}e^{-x}\Sigma_\nu^K(x), \tag{4.70}$$

and define constants β_n by

$$\ln\left|\Sigma_\nu^I(x)\right| = \sum_{n=1}^{\infty} \beta_n x^{-n}. \tag{4.71}$$

Note also that

$$\Sigma_\nu^I(x) \simeq \Sigma_\nu^K(-x). \tag{4.72}$$

Explicit expressions for the β_n can be found in [68]. Now regularized functions can be defined by subtracting off the terms which cause the integrand to diverge at large x,

$$U_I(x) = \frac{d}{dx}\ln\left|\Sigma_\nu^I(x)\right| + \sum_{n=0}^{3} n\beta_n x^{-n-1} + 4\beta_4 x^{-5} e^{-k/x} \tag{4.73}$$

$$U_K(x) = \frac{d}{dx}\ln\left|\Sigma_\nu^K(x)\right| + \sum_{n=0}^{3} (-1)^n n\beta_n x^{-n-1} + 4\beta_4 x^{-5} e^{-k/x} \tag{4.74}$$

Now we can write the RHS of (4.69) using (4.73), (4.74),(4.71) and (4.72). After taking the limit $\epsilon = 0$ we get finally:

$$\widehat{\zeta}(2s-4) = -\frac{4\sin\pi s}{\pi}\left\{g_\nu(s) + b_\nu(s) + a_\nu(s)\tau^{2s-4} + \beta_4 k^{-2s}\Gamma(2s)(1+\tau^{2s-4})\right\} \tag{4.75}$$

where the functions $g_\nu(s)$, $b_\nu(s)$ and $a_\nu(s)$ are defined as

$$g_\nu(s) = -\frac{1}{4}\int_0^\infty dx x^{4-2s}\frac{d}{dx}\ln\left|\frac{P^o(x)}{P_a^o(x)}\right| \tag{4.76}$$

$$b_\nu(s) = -\frac{1}{4}\int_0^\infty dx x^{4-2s} U_I(x) \tag{4.77}$$

$$a_\nu(s) = -\frac{1}{4}\int_0^\infty dx x^{4-2s} U_K(x). \tag{4.78}$$

At $s = 0$, the vacuum energy is given by

$$\zeta'(0) = \frac{-1}{8\pi^2}\left(\frac{G_\nu(\tau)}{z_2^4} + \frac{B_\nu}{z_2^4} + \frac{A_\nu}{z_1^4}\right) - \frac{\beta_4}{8\pi^2}\left(\frac{\ln(z_1\mu_R)}{z_1^4} + \frac{\ln(z_2\mu_R)}{z_2^4}\right), \tag{4.79}$$

where $B_\nu = b_\nu(0)$, $A_\nu = a_\nu(0)$, and $G_\nu(\tau) = g_\nu(0)$. After integration by parts,

$$G_\nu(\tau) = \int_0^\infty dx x^3 \ln\left(1 - \frac{I_\nu(\tau x)K_\nu(x)}{K_\nu(\tau x)I_\nu(x)}\right), \tag{4.80}$$

which has a negative numerical value. The τ dependence in the vacuum energy (4.79) depends on the term

$$\zeta^{'}(0) \sim -\frac{1}{8\pi^2}\frac{G_\nu(\tau)}{z_2^4} \tag{4.81}$$

which has positive numerical value. The positive sign here could be interpreted as a repulsive force which is not useful for the stabilization problem. This just expresses the fact that the untwisted bulk fermions don't produce the *ordinary* attractive Casimir energy. However, the twisted bulk fermions do produce an attractive Casimir energy as we are going to find in the next section.

4.5.2 The case of twisted fermions

In this section we would like to calculate the difference between the twisted and untwisted bulk fermions cases. In other words, we require the difference between the two ζ functions

$$\widehat{\zeta}_{twisted}(0) - \widehat{\zeta}_{untwisted}(0) \tag{4.82}$$

When calculating the difference (4.82), the contribution to the integral (4.63) from the small semi circle in figure (4.2) vanishes, as well as that from the large semi circle, and then we are left with the contribution along the imaginary axis only.

Eq. (4.38) leads to two twisted fermion masses m_n^+ and m_n^- which, unfortunately, are given implicitly. We denote the twisted version of Eq. (4.64) with positive sign by P_+^θ and the one with negative sign by P_-^θ. Since P_-^θ is just the complex conjugate $\overline{P_+^\theta}$, we will get for the integral (4.62)

$$\begin{aligned} \int_\epsilon^\infty dx x^{4-2s}\frac{d}{dx}(\ln\left|P_+^\theta(x)\right| + \ln\left|P_-^\theta(x)\right|) &= \\ \int_\epsilon^\infty dx x^{4-2s}\frac{d}{dx}\ln\left|P_+^\theta(x)\overline{P_+^\theta}(x)\right| &= 2\int_\epsilon^\infty dx x^{4-2s}\frac{d}{dx}\ln\left|P_+^\theta(x)\right| \end{aligned} \tag{4.83}$$

From now on, we drop the (+) and the (−) and continue with P^θ. The zeta function for the twisted case has the form,

$$\widehat{\zeta}(2s-4) = \frac{\sin(\pi s)}{\pi}\left(\int_0^\infty dx x^{4-2s}\frac{d}{dx}\ln\left(\frac{P^\theta\overline{P^\theta}}{P_a^\theta\overline{P_a^\theta}}\right) + \int_0^\infty dx x^{4-2s}\frac{d}{dx}\ln\left(P_a^\theta\overline{P_a^\theta}\right)\right) \tag{4.84}$$

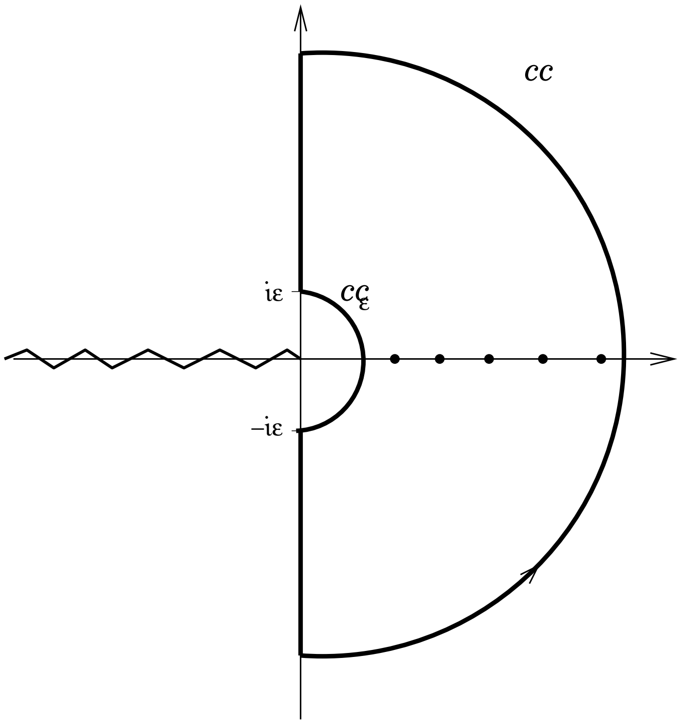


Fig. 4.2: Contour used for the contour integral in (4.62)

$P^\theta(x)$ denotes $P(x)$ at $\theta \neq 0$ and is defined from (4.38) as

$$\begin{aligned} P^\theta(x) &= C\left(I_{\overline{\nu}}(\tau x)K_{\overline{\nu}}(x) - I_{\overline{\nu}}(x)K_{\overline{\nu}}(\tau x)\right) \\ &- iS(I_{\overline{\nu}}(\tau x)K_\nu(x) + I_\nu(x)K_{\overline{\nu}}(\tau x) + (2/\pi)\sin(\nu\pi)K_{\overline{\nu}}(\tau x)K_\nu(x)) \end{aligned} \quad (4.85)$$

$P^\theta_a(x)$ is the most divergent part, defined as

$$P^\theta_a(x) = I_{\overline{\nu}}(\tau x)\left(CK_{\overline{\nu}}(x) - iSK_\nu(x)\right). \quad (4.86)$$

The first integral in (4.84) is convergent, but the second one is divergent. To regularize it we follow the same procedure used for the untwisted case calculations with a small difference due to the non-zero value of θ. We set

$$I_\nu(x) = \frac{e^x}{\sqrt{2\pi x}}\Sigma^I_\nu(x), \quad K_\nu(x) = \sqrt{\frac{\pi}{2x}}e^{-x}\Sigma^K_\nu(x), \quad (4.87)$$

where

$$\ln \Sigma^K_\nu(x) = \Sigma\beta^K_\nu x^{-n}, \quad \beta^\theta_n = C\beta^K_n(\overline{\nu}) - iS\beta^K_n(\nu). \quad (4.88)$$

Then, defining the two regularized functions

$$U_K^\theta(x) = \frac{d}{dx}\ln\left(C\Sigma_{\overline{\nu}}^K - iS\Sigma_\nu^K\right) + \sum_{n=1}^{3} n\beta_n^\theta x^{-n-1} + 4\beta_4^\theta x^{-5}e^{-k/x} \tag{4.89}$$

and

$$U_I(x) = \frac{d}{dx}\ln\Sigma_{\overline{\nu}}^I + \sum_{n=1}^{3} n\beta_n^I + 4\beta_4^I x^{-5}e^{-k/x} \tag{4.90}$$

we get finally,

$$\widehat{\zeta}(2s-4) = \frac{-4\sin(\pi s)}{\pi}\left[g_\nu(S) + b_\nu\tau^{2s-4} + a_\nu^\theta(s) + k^{-2S}\Gamma(2s)(\beta(\theta) + 2\beta_4\tau^{2s-4}))\right] \tag{4.91}$$

where

$$\begin{aligned} g_\nu(s) &= -\frac{1}{8}\int_0^\infty dx\ x^{4-2s}\frac{d}{dx}\ln\frac{P^\theta\overline{P^\theta}}{P_a^\theta\overline{P_a^\theta}} \qquad (4.92)\\ b_\nu(s) &= -\frac{1}{8}\int_0^\infty dx\ x^{4-2s}2U_I(x)\\ a_\nu^\theta(s) &= -\frac{1}{8}\int_0^\infty dx\ x^{4-2s}(U_K^\theta + \overline{U_K^\theta})\\ \beta(\theta) &= \beta_4^\theta + \overline{\beta_4^\theta} = 2\beta_4 C, \quad C = \cos\theta \end{aligned}$$

The vacuum energy of the twisted fermions is then

$$\zeta'(0,x) = \frac{-1}{8\pi^2}\left(\frac{g_\nu(0)}{z_2^4} + \frac{b_\nu(0)}{z_1^4} + \frac{a_\nu^\theta(0)}{z_2^4}\right) - \frac{\beta_4}{4\pi^2}\frac{\ln(\mu_R z_2)}{z_2^4} - \frac{\beta_4}{4\pi^2}\frac{\ln(\mu_R z_1)}{z_1^4}. \tag{4.93}$$

4.6 The 5D effective potential

We now subtract the $\theta = 0$ case to calculate the difference and get the 5D effective potential. Assuming that the SUSY breaking happens on the hidden brane located at z_2 we can ignore the z_1 terms and get the 5D effective potential as

$$\Delta V_C = -\frac{1}{8\pi^2}\left(\frac{\Delta G^\theta(\tau)}{z_2^4} + \frac{B(\theta)}{z_2^4}\right) - \frac{\beta_4(C-1)}{4\pi^2}\frac{\ln(\mu_R z_2)}{z_2^4} \tag{4.94}$$

Where

$$\begin{aligned} \Delta G^\theta(\tau) &= \frac{-1}{8}\int_0^\infty x^4\frac{d}{dx}\left(\ln\frac{P^\theta\overline{P^\theta}}{P_a^\theta\overline{P_a^\theta}} - \ln\left(\frac{P^0}{P_a^0}\right)^2\right) \qquad (4.95)\\ B(\theta) &= \frac{-1}{8}\int_0^\infty x^4\frac{d}{dx}\left(\left(U_K^\theta + \overline{U_K^\theta}\right) - 2U_I(x)\right) \qquad (4.96) \end{aligned}$$

The difference function $\Delta G^{\theta}(\tau)$ could now be written as (after integration by parts),

$$\Delta G_{\nu}^{\theta}(\tau) = \int_{0}^{\infty} dx x^{3} \ln \left| \frac{P^{\theta}(x) P_{a}^{0}(x)}{P_{a}^{\theta} P^{0}(x)} \right| \tag{4.97}$$

which has positive numerical value. The dependence on τ is contained in the term

$$\zeta^{'}(0) \sim -\frac{1}{8\pi^{2}} \frac{\Delta G^{\theta}(\tau)}{z_{2}^{4}}. \tag{4.98}$$

4.6.1 Numerical evaluation of $\Delta G^{\theta}(\tau)$

At this point we would like to investigate the value of the integral (4.97) at different values of τ and θ. The Bessel function orders ν and $\overline{\nu}$ are $\frac{3}{5}$ and $\frac{2}{5}$ respectively. Clearly for the supersymmetric case ($\theta = 0$), the Casimir energy is zero and the integral vanishes.

For the non-supersymmetric case as τ approaches 1, the two branes are getting more closer and the Casimir energy becomes stronger. For example, for $\tau = 1.2$ and $\theta = \pi$ the value of the integral (evaluated numerically using Maple) is 466.0. For the same θ and $\tau = 1.8$ the value is 1.79. For $\theta = \pi$ and $\tau = 1.01$, $\Delta G^{\pi}(1.01) = 7.52 \times 10^{7}$. As one approaches supersymmetry, i.e. as θ approaches zero, the Casimir energy is getting lower and lower. In general, for $\theta = n\pi$ and n is an integer, the Casimir energy is zero for even n and has the same value for odd n. The integral is large (and positive) for small brane separation means that the vacuum energy is large (and negative)and vise versa.

The function $\Delta G_{\nu}^{\theta}(\tau)$ is a part of the Casimir potential (4.94) that becomes dominant at small brane separation ($\tau \to 1$), but it doesn't represent all of the physics. To do that we have to take into consideration the functions A_{ν} and B_{ν}.

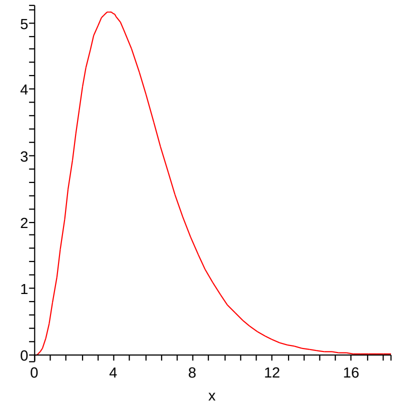
The integrand (4.97) is plotted as a function of x for several values of τ in figures 4.3(a) - 4.3(f) to show that the integral we have got is well-behaved. Values of $G^{\theta}(\tau)$ and $\Delta G^{\theta}(\tau)$ have been tabulated in table 4.1 for $\theta = \pi, \pi/2$ to show the effect of the change of the value of the SUSY parameter θ. For the twisted case, Fig. 4.4(a) shows that the function $G^{\theta}(\tau)$ decreases as θ decreases and as the separation between the two branes increases. For the difference case, Fig. 4.4(b) also shows that the function $\Delta G^{\theta}(\tau)$ decreases as θ decreases and as the separation between the two branes increases. Figures 4.6 and 4.5 shows a 3D plot of the total Casimir energy, approximated by (4.98), in both (z_2, τ) and (z_1, z_2) directions. The potential goes to $-\infty$, it has no minimum. 2D plots of (4.98) in z_2 direction for different values of τ has been shown in Fig.4.7.

4.7 summary

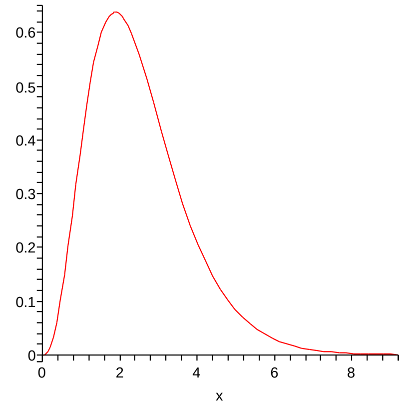
We have calculated the total bulk vacuum energy due to twisted fermion fields, which is the difference between the twisted and the untwisted case, for flat and curved space. The total 5D effective potential doesn't have a minimum and other bulk effects need to be added to stabilize the radion.

$\tau > 1$	$G^{\theta}(\tau)$		$\Delta G^{\theta}(\tau)$	
	$G^{\pi}(\tau)$	$G^{\pi/2}(\tau)$	$\Delta G^{\pi}(\tau)$	$\Delta G^{\pi/2}(\tau)$
1.1	3600.725813	110.3501650	7489.713486	3999.375267
1.15	707.1716823	21.41545529	1475.631231	789.8898863
1.2	222.6566867	6.6841251	465.9992946	250.0297253
1.25	90.7699894	2.719147221	190.4921799	102.3868650
1.3	43.61064998	1.315347407	91.70708754	49.42757998
1.35	23.44124127	0.7204989719	49.40907749	26.68828901
1.4	13.69962736	0.4342920396	28.91960127	15.65389601
1.45	8.530711681	0.2822211976	18.02691424	9.774664917
1.5	5.585122742	0.1949612382	11.80896505	6.419418004
1.55	3.809722071	0.1415799066	8.051331216	4.384794838
1.6	2.687671769	0.1071208312	5.680682348	3.099581383
1.65	1.950597487	0.08381729896	4.116361023	2.250408830
1.7	1.450282869	0.06693527726	3.058630712	1.675175090
1.75	1.101264244	0.05496962259	2.31789662	1.271815291
1.8	0.8516970012	0.04591905370	1.788883642	0.9835005356
1.85	0.6692604758	0.03938748791	1.402156975	0.7720705772
1.9	0.5334835670	0.03356146523	1.114560720	0.6149169512
1.95	0.4306798501	0.02947137658	0.8967776417	0.4954818484

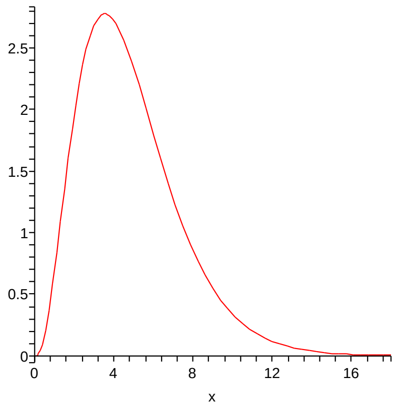
Tab. 4.1: The numerical values of $G^{\theta}(\tau)$ and $\Delta G^{\theta}(\tau)$ for different values of τ. all values have been evaluated with $\nu = \frac{3}{5}$ and $\overline{\nu} = \frac{2}{5}$.



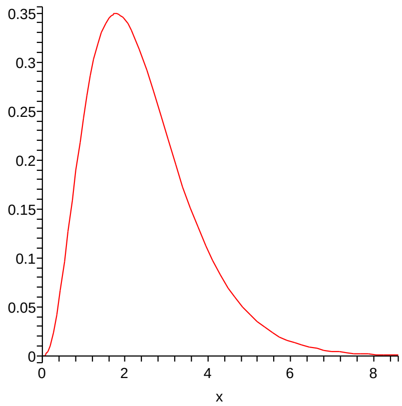
(a) $\Delta G^{\pi}(1.4) = 28.91960127$



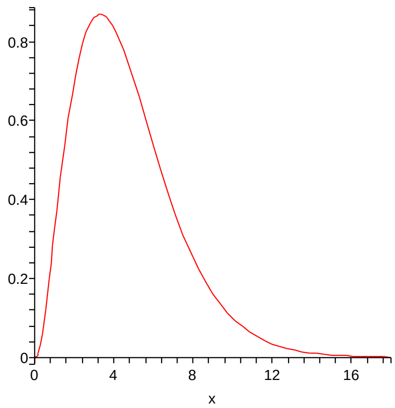
(b) $\Delta G^{\pi}(1.8) = 1.788883642$



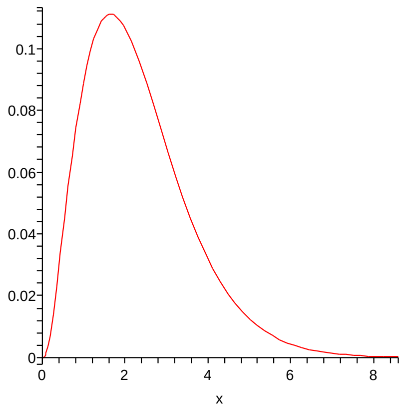
(c) $\Delta G^{\pi/2}(1.4) = 15.65389601$



(d) $\Delta G^{\pi/2}(1.8) = 0.9835005356$

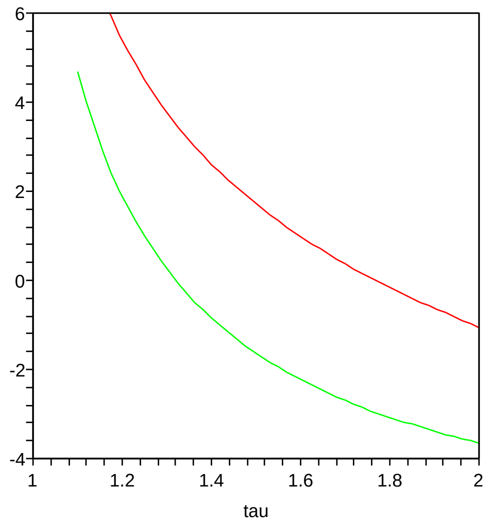


(e) $\Delta G^{\pi/4}(1.4) = 4.9757831$

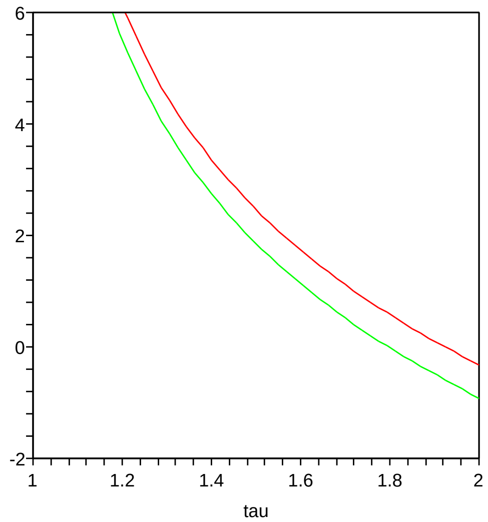


(f) $\Delta G^{\pi/4}(1.8) = 0.3170365018$

Fig. 4.3: Different plots of the integrand used to evaluate $\Delta G^{\theta}(\tau)$ for different θ and τ. These show that the integral is well-behaved.

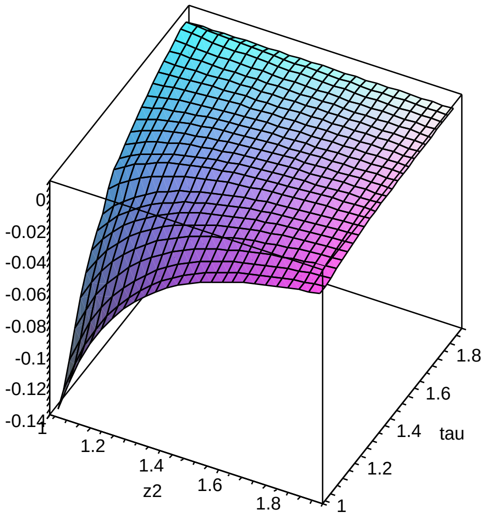


(a) A plot of $\ln G^{\pi}(\tau)$ (red) and $\ln G^{\pi/2}(\tau)$ (green). This shows that $G^{\theta}(\tau)$ decreases as θ decreases, i.e. as we approaches SUSY, and as the separation between the two branes increases.



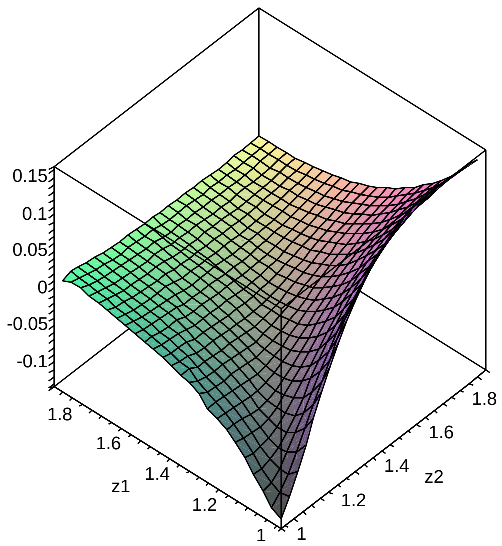
(b) A plot of $\ln \Delta G^{\pi}(\tau)$ (red) and $\ln \Delta G^{\pi/2}(\tau)$ (green) shows that $\Delta G^{\theta}(\tau)$ decreases as SUSY is approached (θ decreases) and as the separation between the two branes increases.

Fig. 4.4:



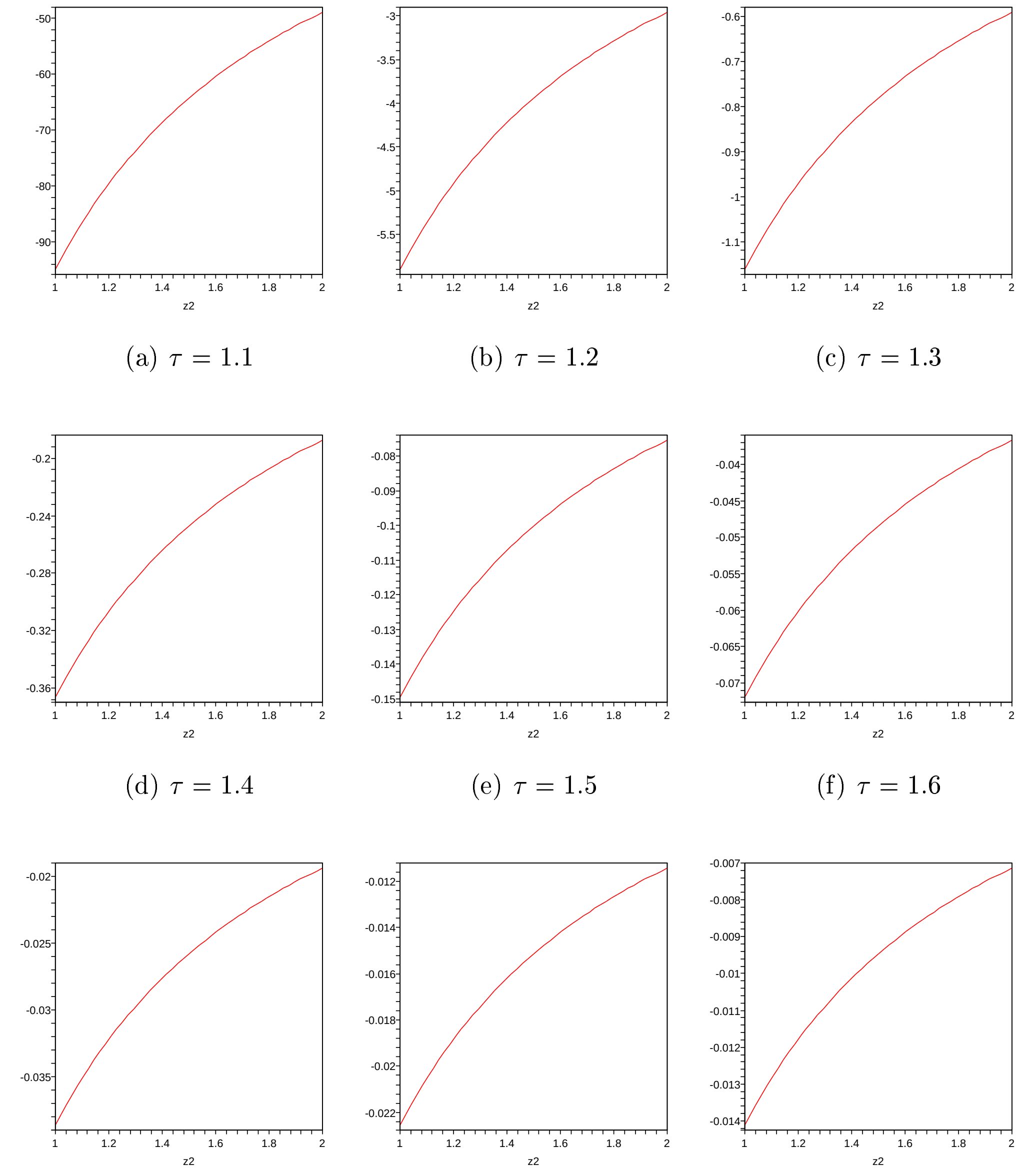
(a) A 3D plot of ΔV_C (eq. 4.98) showing no minimum in z_2 or τ directions.

Fig. 4.5:



(a) A 3D plot of ΔV_C (eq. 4.98) showing no minimum in z_1 or z_2 directions.

Fig. 4.6:



(a) $\tau = 1.1$ (b) $\tau = 1.2$ (c) $\tau = 1.3$

(d) $\tau = 1.4$ (e) $\tau = 1.5$ (f) $\tau = 1.6$

(g) $\tau = 1.7$ (h) $\tau = 1.8$ (i) $\tau = 1.9$

Fig. 4.7: The potential (4.98) for different τ and $\theta = \pi$.

5. GAUGINO CONDENSATION IN AN IMPROVED HETEROTIC M-THEORY

This chapter on gaugino condensation contains some new results for the effective potential and the superpotential which arise from gaugino condensation. The work in this chapter was done in collaboration with Prof. i.G. Moss.

5.1 *Improved heterotic M-theory and its new boundary conditions*

We start with a quick review of the improved heterotic M-theory [33, 136, 137] which we shall use as our framework. As we have seen in section (3.2), the original formulation of Horava and Witten of Heterotic $M-$theory has the following structure (see eq. 3.4 and 3.5)

$$S = S_{SG} + S_{YM} \tag{5.1}$$

S_{SG} contains a factor κ_{11}^{-2}, Whilst the matter action S_{YM} has a factor $\epsilon\kappa_{11}^{-2}$, where ϵ is a scaling parameter. Anomaly cancellation requires that $\epsilon = \mathcal{O}\left(\kappa^{2/3}\right)$ which means that S_{YM} is of order $\kappa_{11}^{2/3}$ compared to S_{SG}. At order κ_{11}^{2} singular terms depending on the square of the delta-function start to arise. This problem has been overcome [33, 136, 137] by modifying the boundary conditions on the gravitino and the supergravity 3-form, so that now an action can be constructed which is non-singular and supersymmetric to higher orders.

The theory is formulated on a manifold $\mathcal{M}$ with a boundary consisting of two disconnected components $\mathcal{M}_1$ and $\mathcal{M}_2$ with identical topology. The eleven-dimensional part of the action is the conventional action for supergravity, with metric g_{IJ} , gravitino ψ_I and antisymmetric tensor C_{IJK} [117]. The original formulation of Horava and Witten contained an extra '$\chi\chi\chi\Psi$' term, but it is not present in the new version.

The specification of the theory is completed by boundary conditions. For the tangential anti-symmetric tensor components,

$$C_{ABC} = \mp\frac{\sqrt{2}}{12}\epsilon\left(\omega^{Y}_{ABC} - \frac{1}{2}\omega^{L}_{ABC}\right) \mp \frac{\sqrt{2}}{48}\epsilon\,\mathrm{tr}\bar{\chi}\Gamma_{ABC}\chi. \tag{5.2}$$

where ω^Y and ω^L are the Yang-Mills and Lorentz Chern-Simons forms. These boundary conditions replace the modified Bianchi identity in the old formulation (see for example [12]). For the gravitino,

$$\Gamma^{AB}\left(P_{\pm} + \epsilon\Gamma P_{\mp}\right)\Psi_A = \epsilon\left(J_Y{}^A - \frac{1}{2}J_L{}^A\right), \tag{5.3}$$

where $P_{\pm}$ are chiral projectors using the outward-going normals and

$$\Gamma = \frac{1}{96}\mathrm{tr}(\bar{\chi}\Gamma_{ABC}\chi)\Gamma^{ABC}. \tag{5.4}$$

J_Y is the Yang-Mills supercurrent and J_L is a gravitino analogue of the Yang-Mills supercurrent. The resulting theory is supersymmetric to all orders in the parameter ϵ. The gauge, gravity and supergravity anomalies vanish if

$$\epsilon = \frac{1}{4\pi}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}. \tag{5.5}$$

A useful relation for the condensate on the boundary can be obtained by recalling that in heterotic M-theory, we can relate the spin connection to the Yang-Mills field so that $\omega^Y = \omega^L = \omega$ on the hidden brane, and $\omega^Y = 0$ on the visible brane. Then,

$$C_{ABC} = \frac{\sqrt{2}}{24}\epsilon\omega_{ABC} - \frac{\sqrt{2}}{48}\epsilon\bar{\chi}\Gamma_{ABC}\chi \quad \text{on} \quad \partial\mathcal{M}_1 \tag{5.6}$$

$$C_{ABC} = \frac{\sqrt{2}}{24}\epsilon\omega_{ABC} + \frac{\sqrt{2}}{48}\epsilon\bar{\chi}\Gamma_{ABC}\chi \quad \text{on} \quad \partial\mathcal{M}_2 \tag{5.7}$$

Where the term $\epsilon\bar{\chi}\Gamma\chi$ is non-vanishing for the gaugino condensate.

5.2 Background metric and flux

The $11D$ background metric ansatz is based on the product $M \times S_1/Z_2 \times X$ where X is a Calabi-Yau space. In this metric there are two copies of the 4-dimensional manifold M, M_1 and M_2, separated by a distance l_{11}. The value for the inverse radius of the Calabi-Yau space is supposed to be of order the Grand Unification scale 10^{16}GeV and the inverse separation would be of order 10^{14} GeV. The explicit form of the metric is

$$ds^2 = V^{-2/3}\left(d\tilde{z}^2 + V^{1/3}\eta_{\mu\nu}dx^\mu dx^\nu\right) + V^{1/3}\tilde{g}_{AB}dx^A dx^B \tag{5.8}$$

Where $\eta_{\mu\nu}$ is the Minkowiski metric on M, g_{AB} is the Calabi-Yau metric on X which is independent of $\tilde{z}$ and $V \equiv V(z)$, $z_1 \leq z \leq z_2$. The tilde denotes quantities in Einstein frame.

This background metric ansatz is similar to one used by Curio and Krause [132], except that we use a different coordinate z in the S^1/Z_2 direction. For simplicity, we will restrict the class of Calabi-Yau spaces to those with only one harmonic $(1,1)$ form (see appendix C.2.1). To allow for gravity in 4D, the metric is replaced by

$$ds^2 = V^{-2/3}\left(d\tilde{z}^2 + V^{1/3}Q^{-2}\tilde{g}_{\mu\nu}dx^\mu dx^\nu\right) + V^{1/3}\tilde{g}_{AB}dx^A dx^B \tag{5.9}$$

Where the factor Q^{-2} is required to put the metric $\tilde{g}_{\mu\nu}$ into the Einstein frame and is given by (3.44).

The volume function $V = (1 - 6k\tilde{z})$ (see 3.40) is the exact solution of the zz component of the Einstein equations [1]. For the field strength G we use the ansatz

$$G_{ab\bar{c}\bar{d}} = \frac{1}{3}\alpha\left(\tilde{g}_{c\bar{c}}\tilde{g}_{b\bar{d}} - \tilde{g}_{a\bar{d}}\tilde{g}_{b\bar{c}}\right) \tag{5.10}$$

This ansatz solves the field equation $\nabla.G = 0$.

5.2.1 Condensates

The ansatz for a gaugino condensate on the boundary M_i is [129]

$$\overline{\chi}_i\Gamma_{abc}\chi_i = \Lambda_i\tilde{\varepsilon}_{abc} \tag{5.11}$$

[1] Our solution for V is equivalent to the one used by Lukas et al. in Ref. [12] when adapted to our coordinate system. They express the solution as $V = b_0H^3$. It is also equivalent to the background used by Curio and Krause in Ref. [132], $V = (1 - \mathcal{S}_1x^{11})^2$, when their $\mathcal{S}_1 = \alpha V_1^{-2/3}/\sqrt{2}$. See chapter 3

This is the standard expression for covariantly constant condensates [79, 112]. χ_i is the spinor represents the gaugino field, Λ_i is the condensation scale and depends only on the modulus V_i, $\widetilde{\varepsilon}_{abc}$ is a covariantly constant three-form on the Calabi-Yau space (on any given Calabi-Yau three-fold X, we have a covariantly constant holomorphic three-form ε_{abc} and its anti-holomorphic complex conjugate $\overline{\varepsilon}_{\overline{a}\overline{b}\overline{c}}$). The gaugino condensate appears in the boundary conditions for the anti-symmetric tensor field and induces non-vanishing components C_{abc}.

Let

$$C_{abc} = \frac{1}{6}\xi\widetilde{\varepsilon}_{abc}. \tag{5.12}$$

where ξ is a complex scalar field. The field strength associated with these tensor components is

$$G_{abcz} = -(\partial_z\xi)\widetilde{\varepsilon}_{abc}. \tag{5.13}$$

The boundary conditions for the C_{abc} field from Eqs. (5.6) and (5.7) is

$$C_{abc} = \begin{cases} \frac{\sqrt{2}}{48}\epsilon\Lambda_i\widetilde{\varepsilon}_{abc} & \text{on} \quad z_2 \\ \\ 0 & \text{on} \quad z_1 \end{cases} \tag{5.14}$$

and the field equation is

$$\nabla.G = 0 \tag{5.15}$$

Equation (5.15) could be written explicitly as

$$\partial_z\left(g^{a\overline{a}}g^{b\overline{b}}g^{c\overline{c}}g^{55}G_{abc5}\right) = 0 \tag{5.16}$$

where the fifth dimension z is real and fixed. This implies that $G_{abcz} \propto V^{1/3}$ which means $C_{abc} \propto V^{4/3}$, we then have

$$C_{abc} = A\widetilde{\varepsilon}_{abc}V^{4/3} + B\widetilde{\varepsilon}_{abc} \tag{5.17}$$

where A and B are constants that could be determined easily using the two boundary conditions (5.14). We get

$$A = \frac{\sqrt{2}}{48}\epsilon\Lambda\frac{1}{V_1^{4/3} - V_2^{4/3}} \tag{5.18}$$

$$B = \frac{-\sqrt{2}}{48}\epsilon\Lambda\frac{V_1}{V_1^{4/3} - V_2^{4/3}} \tag{5.19}$$

So

$$C_{abc} = \frac{\sqrt{2}}{48}\epsilon\Lambda\widetilde{\varepsilon}_{abc}\frac{V^{4/3} - V_1^{4/3}}{Q^2} \tag{5.20}$$

where $Q^2 = V_1^{4/3} - V_2^{4/3}$. For the field strength, we get

$$G_{abcz} = \frac{\sqrt{2}\Lambda\epsilon k}{12Q^2}\widetilde{\varepsilon}_{abc}V^{1/3}. \tag{5.21}$$

The non-zero flux depends on V_1 and V_2 through the Q term and through Λ, which depends on the volume factors V_1 and V_2 in the gaugino couplings.

We now consider the 11D action

$$S = -\frac{1}{2\kappa_{11}^2}\int_{M_{11}} G^2\sqrt{|g^{(11)}|}d^{11}x \tag{5.22}$$

The relation between the 11D and the 4D metric follows directly from (5.9) as

$$\sqrt{|g^{(11)}|} = \sqrt{|\widetilde{g}^{(4)}|}Q^{-4} \tag{5.23}$$

The action (5.22) then could be written in 5D as

$$S = -\frac{1}{2\kappa^2}\int_{M_5}\frac{\epsilon^2k^2\Lambda^2}{36Q^8}V^{1/3}(z)\sqrt{|\widetilde{g}^{(4)}|}dx^4dz \tag{5.24}$$

After integrating out the extra dimension we get the 4D action as

$$S = -\frac{1}{2\kappa^2}\int_{M_4}\frac{\epsilon^2k^2\Lambda^2}{48Q^6}\sqrt{|\widetilde{g}^{(4)}|}dx^4 \tag{5.25}$$

So, the G^2 term in the action (5.22) reduces to a potential V_G in the Einstein frame, where

$$V_G = -\frac{\epsilon^2k^2\Lambda^2}{48Q^6} \tag{5.26}$$

In section 5.2.4 we shall attempt to find the potential by a better method, using a reduction of the fermion sector.

5.2.2 Condensate scale

We now try to evaluate the condensate scale Λ. After the reduction to 5D, the Yang-Mills action becomes [12]

$$S_{YM} = \sum_i \frac{1}{4g^2}\int_{M_4} V_iF^2dV \tag{5.27}$$

where F is the Yang-Mills field strength and g is the Yang-Mills coupling if $V = 1$. If $V \neq 1$, we can absorb it into g such that $V_2/g^2 = 1/g^2_{eff}$. The coupling g_{eff} changes with energy. Assume the condensates happen at a scale $\Lambda = M^3$, where M is the mass scale at which $g_{eff}(M) \approx 1$. From the renormalization group equation, we have

$$\frac{dg_{eff}}{dt} = \beta(g_{eff}) \approx \beta_1 g^3_{eff}. \tag{5.28}$$

This gives

$$g^{-2}_{eff} = \beta_1 t + const., \tag{5.29}$$

where $t = \ln(E/\mu)$ and μ is the renormalization scale. We have then

$$g^{-2}_{eff} - g^{-2}_{(0)eff} = \beta_1 \ln(E/\mu). \tag{5.30}$$

If $\mu = E$, then $g_{eff} = g_{(0)eff}$. If $E = M$ then (5.30) leads directly to

$$M = \mu e^{-(\beta_1 g^2)^{-1} V_2}. \tag{5.31}$$

The condensate scale is just M^3, hence

$$\Lambda = \mu^3 e^{-3(\beta_1 g^2)^{-1} V_2}. \tag{5.32}$$

5.2.3 Superpotential

Since any supergravity Lagrangian is expected to contain the Einstein-Hilbert Lagrangian and the Rarita-Schwinger Lagrangian for the gravitino field, we try to reduce the 11D Rarita-Schwinger Lagrangian

$$L_{RS} = \frac{1}{2\kappa^2_{11}} \left(\overline{\psi}_I \Gamma^{IJK} D_J \psi_K\right). \tag{5.33}$$

In chapter 7, we shall see that this can be re-written in a form that is more suitable for the reduction as

$$L_{RS} = \frac{1}{2\kappa^2_{11}} \overline{\lambda}^I \left(\not{\nabla} - \frac{\sqrt{2}}{96} G_{PQRS} \Gamma^{PQRS} \right) \lambda_I. \tag{5.34}$$

We use the notation $I, P, \ldots$ for eleven–dimensional indices, $\mu, \nu, \ldots$ for four-dimensional ones, and $a, b, \ldots$ for Calabi-Yau space. $\overline{\lambda}_I = \lambda^+_I(\gamma_0 \otimes \mathbb{1})$. The 11-dimensional gamma matrices satisfy $\{\Gamma_M, \Gamma_N\} = 2g_{MN}$.

5.2.4 Effective superpotential from the 11D theory.

When reducing on a metric with no warp factor, the higher dimensional gamma matrices are decomposed as

$$\Gamma_\mu = \gamma_\mu \otimes \mathbb{1}, \quad \Gamma_a = \gamma_5 \otimes \gamma_a \quad \text{and} \quad \Gamma_{11} = \gamma_5 \otimes \gamma_7 \tag{5.35}$$

Where $\gamma^5 = i\gamma^1\gamma^2\gamma^3\gamma^4$ is the 4D chirality operator.

It is useful to consider a particular representation for the Dirac matrices γ^μ. A useful choice of these matrices is given by Majorana representation in which the γ matrices are either purely real γ^α or purely imaginary (γ^5 and γ^a). The Majorana condition on the spinor then is just a reality constraint.

When reducing on a warped metric of the general form

$$ds^2 = e^{2b}ds^2_{(4)} + e^{2f}ds^2_{(6)} + e^{2k}(dx^{11})^2, \tag{5.36}$$

in order to retain $\{\gamma_\mu, \gamma_\nu\} = 2g^{(4)}_{\mu\nu}$ etc., (5.35) becomes

$$\Gamma_\mu = e^b\gamma_\mu \otimes \mathbb{1}, \quad \Gamma_a = \gamma_5 \otimes e^f\gamma_a \quad \text{and} \quad \Gamma_{11} = \gamma_5 \otimes e^k\gamma_7 \tag{5.37}$$

So, for the metric (5.9), we have

$$\Gamma_\mu = V^{-1/6}Q^{-1}\gamma_\mu \otimes \mathbb{1}, \quad \Gamma_a = \gamma_5 \otimes V^{1/6}\gamma_a \quad \text{and} \quad \Gamma_{11} = \gamma_5 \otimes V^{-1/3}\gamma_7. \tag{5.38}$$

For the raised indices,

$$\Gamma^\mu = V^{1/6}Q\gamma^\mu \otimes \mathbb{1}, \quad \Gamma^a = \gamma_5 \otimes V^{-1/6}\gamma^a \quad \text{and} \quad \Gamma^{11} = \gamma_5 \otimes V^{1/3}\gamma_7. \tag{5.39}$$

See also appendix (C.3).

To perform the dimensional reduction we need the metric ansatz given in (5.9) and a spinor ansatz for the embedding of the 4D gravitino ψ_μ in the 11D one λ_μ with the help of the internal spinors (the 6D Calabi-Yau spinors in appendix B.4). This could be written in general as

$$\lambda_\mu = \theta^+_\mu \otimes u_+ + \gamma_5\theta^-_\mu \otimes u_- \tag{5.40}$$

Where $\theta_\mu \equiv \theta_\mu(x^\nu, z)$ and $u_\pm$ are covariantly constant 6D spinors. A specific choice of the function $\theta^\pm_\mu$ gives

$$\lambda_\mu = a\psi_\mu(x^\nu)V^{-1/6} \otimes u_+ + a^*\psi^*_\mu(x^\nu)V^{-1/6} \otimes u_- \tag{5.41}$$

Where a is a complex number used for normalization. Then,

$$\overline{\lambda}^\mu = (a\psi_\nu \cdots + a^*\psi^*_\nu \dots)^\dagger Q^2 V^{1/3} (\gamma_0 \otimes 1\!\!1) \widetilde{g}^{\mu\nu} \tag{5.42}$$

Now, the covariant derivative transforms to the Einstein frame as

$$\nabla\!\!\!\!/ \to QV^{1/6} (\gamma^\nu \otimes 1\!\!1) \widetilde{\nabla}, \tag{5.43}$$

and we have

$$\begin{aligned} \Gamma^{zabc} G_{zabc} &= 4V^{1/6}(1\!\!1 \otimes \gamma_7)\gamma^{abc} G_{zabc} \\ &= 4V^{1/6}(1\!\!1 \otimes \gamma_7)\gamma^{abc} \frac{\sqrt{2}\Lambda\epsilon}{36Q^2} \widetilde{\varepsilon}_{abc} V^{1/3}, \end{aligned} \tag{5.44}$$

where

$$\begin{aligned} \Gamma^{zabc} &= (\gamma_5 \otimes \gamma_7)(\gamma_5 \otimes \gamma_{\overline{a}})(\gamma_5 \otimes \gamma_{\overline{b}})(\gamma_5 \otimes \gamma_{\overline{c}}) \\ &= (1\!\!1 \otimes \gamma_7 \gamma_{\overline{a}\overline{b}\overline{c}}). \end{aligned} \tag{5.45}$$

Inserting all that into (5.33) with the help of the relations (B.15) appendix (B) We get the 5D Lagrangian

$$\begin{aligned} L^{(5)}_{RS} = a^2 \overline{\psi}^\mu \frac{V^{1/6}}{\left(V_1^{4/3} - V_2^{4/3}\right)^{1/2}} \nabla\!\!\!\!/_\nu \psi_\mu + \frac{i\epsilon\Lambda a^2 V^{1/6}}{3\left(V_1^{4/3} - V_2^{4/3}\right)^2} \times \\ \left(\overline{\psi}^\mu \psi^*_\mu + \overline{\psi}^{*\mu}\psi_\mu\right) + \frac{a^2 V^{1/6}}{\left(V_1^{4/3} - V_2^{4/3}\right)^{1/2}} \overline{\psi}^{*\mu} \nabla\!\!\!\!/_\nu \psi^*_\mu \end{aligned} \tag{5.46}$$

Integrating out the extra dimension, we get the 4D Lagrangian

$$\begin{aligned} L_{RS} &= \frac{6a^2}{7} \frac{\left(V_1^{7/6} - V_2^{7/6}\right)}{\left(V_1^{4/3} - V_2^{4/3}\right)^{1/2}} \left(\overline{\psi}^\mu \nabla\!\!\!\!/_\nu \psi_\mu + \overline{\psi}^{*\mu} \nabla\!\!\!\!/_\nu \psi^*_\mu\right) \\ &+ \frac{i\epsilon\Lambda a^2}{3} \frac{\left(V_1^{7/6} - V_2^{7/6}\right)}{\left(V_1^{4/3} - V_2^{4/3}\right)^2} \left(\overline{\psi}^\mu \psi^*_\mu + \overline{\psi}^{*\mu}\psi_\mu\right) \end{aligned} \tag{5.47}$$

To get the superpotential we just compare (5.47) with the general form of the RS Lagrangian in 4D

$$L_{RS} = \frac{1}{2\kappa_P^2} e^{K/2} \left(\overline{\psi}^\mu \, \nabla\!\!\!/ \psi_\mu + W \overline{\psi}^\mu \psi_\mu \right) \tag{5.48}$$

For the correct normalization of the kinetic term we have to pick a^2 such that

$$a^2 = \frac{7}{6} \frac{\left(V_1^{4/3} - V_2^{4/3} \right)^{1/2}}{\left(V_1^{7/6} - V_2^{7/6} \right)} \tag{5.49}$$

From the mass term, using (5.32) for Λ,

$$W = \frac{i\epsilon}{3} \mu^3 e^{-3(\beta_1 g^2)^{-1} V_2} \tag{5.50}$$

This final superpotential contains no surprises as it takes the standard form expected for a gaugino condensate in any supersymmetric theory [135]. It becomes clear also that the condensate superpotential contains no corrections due to the warping of the metric in higher dimensions.

Most discussions of the condensate induced superpotential do not take the warping of the metric into account. We have found that the warping of the metric background has had no effect on the superpotential as none of the three warping factors of the metric appears in (5.50). Krause [133] also finds that the warping does not affect the condensate contribution to the superpotential, but he claims a warping dependence in the flux term. In [148], Anguelova and Zoubos extracted the flux-induced superpotential from the gravitino mass term of the 4D effective theory after the dimensional reduction of the fermionic terms in the 11D action.

5.3 summary

We have calculated the Gaugino condensate potential in the framework of the improved heterotic $M-$theory after introducing a metric ansatz and a flux ansatz. The condensate scale has been evaluated using the renormalization group equation. We then derived the gaugino condensate superpotential from the reduction of the $11D$ Rarita-Schwinger Lagrangian. We then start in the next chapter to make use of this superpotential to calculate the potential in two models.

6. KKLT ADS VACUUM AND CASIMIR ENERGY.

Moduli stabilisation can be achieved by following a similar pattern to moduli stabilisation in type IIB string theory [134]. The first stage involves finding a suitable superpotential which fixes the moduli but leads to an Anti-de Sitter vacuum. The negative energy of the vacuum state is then raised by adding a non-supersymmetric contribution to the energy. The potential is given in terms of the Kähler potential K and the superpotential W,

$$V = \kappa_4^{-2} e^K \left(K^{i\bar{j}} D_i W \overline{D_j W} - 3 |W|^2 \right), \tag{6.1}$$

With

$$D_i W = e^{-K} \partial_{V_i} \left(e^K W \right). \tag{6.2}$$

Minima of the potential occur when $D_i W = 0$. If these minima exist, their location is fixed under supersymmetry transformations. However, the boundary conditions at the potential minima are not generally preserved by supersymmetry and the theory at a supersymmetric minimum is not necessarily supersymmetric. We shall examine the supersymmetric minima of the potential for two toy models concentrating on general features rather than obtaining a precise fit with particle phenomenology.

6.1 Model A: Double-condensate

Following the type IIB route, we assume the existence of a flux term W_f in the superpotential which stabilises the $(2,1)$ moduli, and then remains largely inert whilst the other moduli are stabilised.

The gauge coupling on the hidden brane runs to large values at moderate energies and this is usually taken to be indicative of the formation of a gaugino condensate. Local supersymmetry restricts the form of this condensate to [135]

$$\Lambda_2 = B_2 V_{CY}^{-1/2} e^{-\mu V_2} \tag{6.3}$$

where B_2 is a constant and μ is related to the renormalization group β-function by

$$\mu = \frac{6\pi}{b_0 \alpha_{GUT}}, \quad \beta(g) = -\frac{b_0}{16\pi^2} g^3 + \dots . \tag{6.4}$$

The gauge coupling on the visible brane is supposed to run to large values only at low energies to solve the hierarchy problem, and a low energy condensate would have a negligible effect on moduli stabilisation. There might, however, be a separate gauge coupling from part of the E_6 symmetry on the visible brane which becomes large at moderate energies with a significant condensate term. The requirement for this to happen is a large β-function, possibly arising from charged scalar field contributions. The total superpotential for such a model is

$$W = be^{-\mu V_2} + ce^{-\tau V_1} - d, \tag{6.5}$$

where $d = -W_f$ and b, c are constants, which we assume to be real but not necessarily positive.

The fields at the minimum of the potential could be complex, and we therefore separate real and imaginary parts,

$$V_i = u_i + iv_i. \tag{6.6}$$

With the Kähler potential

$$K = -3 \ln \left((V_1 + \overline{V_1})^{4/3} - (V_2 + \overline{V_2})^{4/3} \right) \tag{6.7}$$

The super derivatives of the potential are

$$D_{V_1} W = -c\tau e^{-\tau V_1} - 2 \left(u_1^{4/3} - u_2^{4/3} \right)^{-1} u_1^{1/3} W, \tag{6.8}$$

$$D_{V_2} W = -b\mu e^{-\mu V_2} - 2 \left(u_1^{4/3} + u_2^{4/3} \right)^{-1} u_2^{1/3} W. \tag{6.9}$$

Solving for the values of V_1 and V_2 at the minimum of the potential is not very informative. Instead, we express the parameters b, c and d in terms of the values

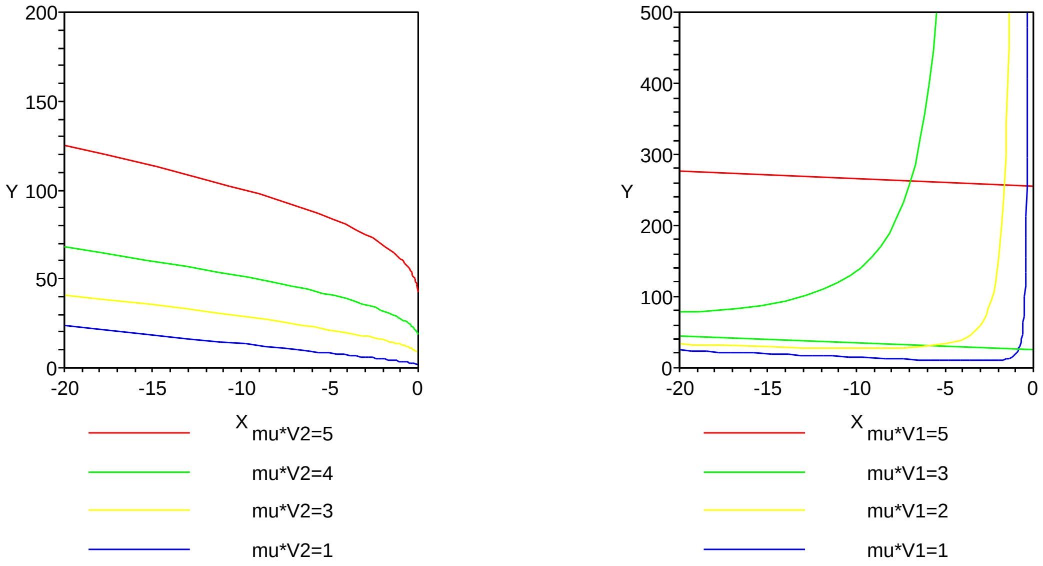


Fig. 6.1: The values of the volume moduli V_1 and V_2 at the minimum of the potential with two condensates and $\tau/\mu = 1.2$. Here $X = b/d$ and $Y = c/d$. The left panel shows values of V_1 and the right panel shows values of V_2.

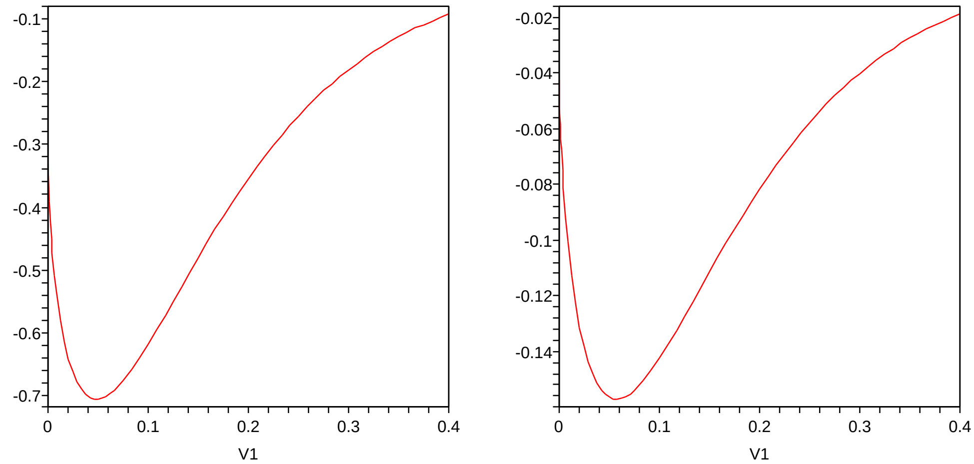
of V_1 and V_2 at the supersymmetric minimum by solving (6.5), (6.8) and (6.9),

$$\frac{b}{d} = \frac{-2u_2^{1/3}e^{\mu V_2}\mu^{-1}}{u_1^{4/3} - u_2^{4/3} - 2\mu^{-1}u_2^{1/3} + 2\tau^{-1}u_1^{1/3}}, \tag{6.10}$$

$$\frac{c}{d} = \frac{2u_1^{1/3}e^{\tau V_1}\tau^{-1}}{u_1^{4/3} - u_2^{4/3} - 2\mu^{-1}u_2^{1/3} + 2\tau^{-1}u_1^{1/3}}. \tag{6.11}$$

We conclude from these expressions that, if b/d and c/d are real, then V_1 and V_2 are both real. (If b and c are not real, then it becomes difficult to satisfy the background field equations on the antisymmetric tensor field with the resulting complex boundary conditions).

Supersymmetric minima exist for $b < 0$ and $c > 0$. The values of V_1 and V_2 at the minima are shown in Fig. 6.1. At the minima of the potential, the flux term $|W_f|$ is larger than the gauge condensate terms. This is consistent with the idea that we consider the stabilisation of the $(2,1)$ moduli independently of the other moduli.



(a) 2D plot of (6.12) for $V_2 = 3$ with minimum at around -0.7

(b) 2D plot of (6.12) for $V_2 = 6$ with minimum at around -1.16

Fig. 6.2: Plots for model A showing the minimum of the potential (6.12).

The supergravity KKLT potential with AdS minimum is

$$\begin{aligned} V_{KKLT1} &= \left((2V_1)^{4/3} - (2V_2)^{4/3}\right)^{-3} \left[\frac{c^2}{4}\tau^2 (2V_1)^{2/3}\left((2V_1)^{4/3} + 3(2V_2)^{4/3}\right) \times \right. \\ & e^{-2\tau V_1} + \frac{1}{4}(2V_2)^{2/3}\left(3(2V_1)^{4/3} + (2V_2)^{4/3}\right)\left(-\mu b e^{-\mu V_2}\right)^2 \\ & \left. -8c\tau\mu V_1 V_2 e^{-(\tau V_1 + \mu V_2)} + 4W\left(c\tau e^{-\tau V_1} + b e^{-\mu V_2}\right) + W^2\right]. \end{aligned} \tag{6.12}$$

This potential has AdS minimum and is plotted in figure (6.2) for the following values of the parametrs: c=μ=τ=5 and b=d=1.

6.2 *Model B: Other non-perturbative terms*

If there are no high energy condensates on the visible brane, then we can replace the condensate with another non-perturbative effect. The usual candidate for this is a membrane which stretches between the two boundaries. The area of the membrane $\propto V_1 - V_2$ and the type of contribution this gives to the superpotential is

$$W_{np} = c e^{-\tau(V_1 - V_2)}. \tag{6.13}$$

The total superpotential for this model is given by

$$W = be^{-\mu V_2} + ce^{-\tau(V_1-V_2)} - d \tag{6.14}$$

$$= W_g + W_{np} + W_f. \tag{6.15}$$

Where W_g is the condensate potential on the 'hidden' brane.

The supergravity KKLT potential with AdS minimum is

$$\begin{aligned} V_{KKLT2} = & \left((2V_1)^{4/3} - (2V_2)^{4/3}\right)^{-3} \left[\frac{1}{4}c^2\tau^2(2V_1)^{2/3}\left((2V_1)^{4/3} + 3(2V_2)^{4/3}\right) \times \right. \\ & e^{-2\tau(V_1-V_2)} + \frac{1}{4}(2V_2)^{2/3}\left(3(2V_1)^{4/3} + (2V_2)^{4/3}\right)\left(-\mu b e^{-\mu V_2} - c\tau e^{-\tau(V_1-V_2)}\right)^2 \\ & -8c\tau V_1 V_2 e^{-\tau(V_1-V_2)}\left(-\mu b e^{-\mu V_2} - c\tau e^{-\tau(V_1-V_2)}\right) \\ & \left. +4W\left(c\tau V_1 e^{-\tau(V_1-V_2)} + V_2(\mu b e^{-\mu V_2} + c\tau e^{-\tau(V_1-V_2)})\right) + W^2\right]. \end{aligned} \tag{6.16}$$

This potential has AdS minimum and is plotted in figure 6.3(c), 6.3(d) for c=μ=5, $\tau = 0.5$ and b=d=1.

Recalling that for a supersymmetric minimum, $\partial_K V = 0$ if $D_i W = 0$. where D_i is the Kähler covariant derivative, the value of the minimum of the potential energy

$$V = e^K(K^{i\bar{j}} D_i W \overline{D_j W} - 3\,|W|^2) \tag{6.17}$$

is

$$V_{min.} = -3e^K\,|W|^2 \tag{6.18}$$

The system of equations

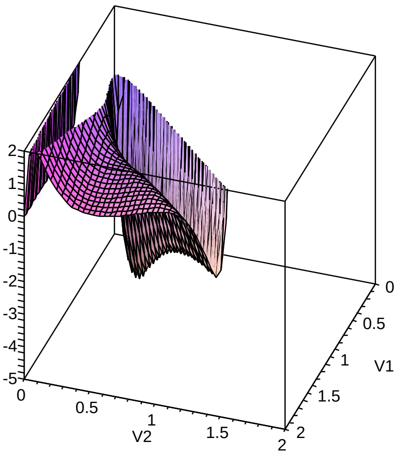
$$D_i W = 0 \tag{6.19}$$

should have a solution in the correct phenomenological range for V_1 and V_2. The superderivatives are

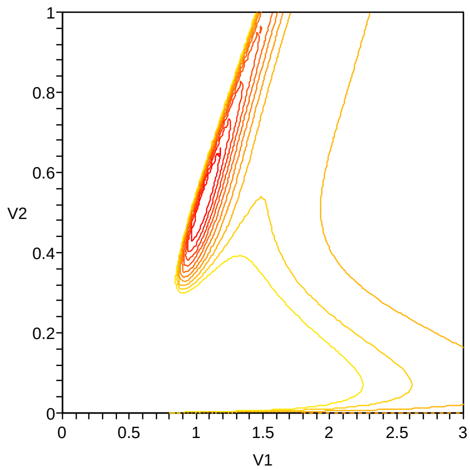
$$D_{V_1} W = -c\tau e^{-\tau(V_1-V_2)} - 2u_1^{1/3}\left(u_1^{4/3} - u_2^{4/3}\right)^{-1} W, \tag{6.20}$$

$$D_{V_2} W = -b\mu e^{-\mu V_2} + c\tau e^{-\tau(V_1-V_2)} + 2u_2^{1/3}\left(u_1^{4/3} - u_2^{4/3}\right)^{-1} W. \tag{6.21}$$

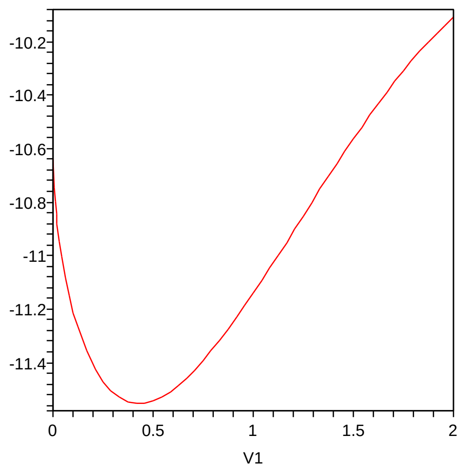
This time the parameters b, c and d given in terms of the values of V_1 and V_2 at



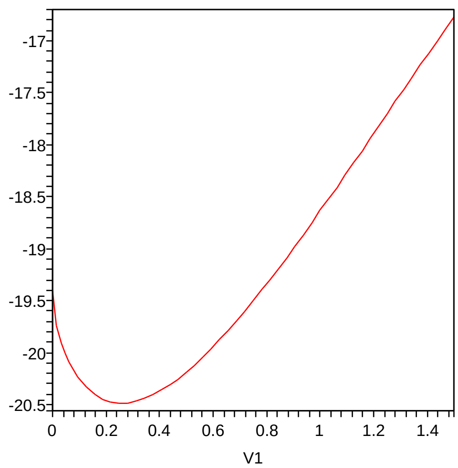
(a) 3D plot of (6.16) for $c = \mu = \tau = 5$ and $b = d = 1$.



(b) Contour plot of (6.16) for $c = \mu = \tau = 5$ and $b = d = 1$.



(c) 2D plot of (6.16) for $V_2 = 0.6$ with minimum at around -11.6.



(d) 2D plot of (6.16) for $V_2 = 0.7$ with minimum at around -20.5.

Fig. 6.3: Plots of (6.16).

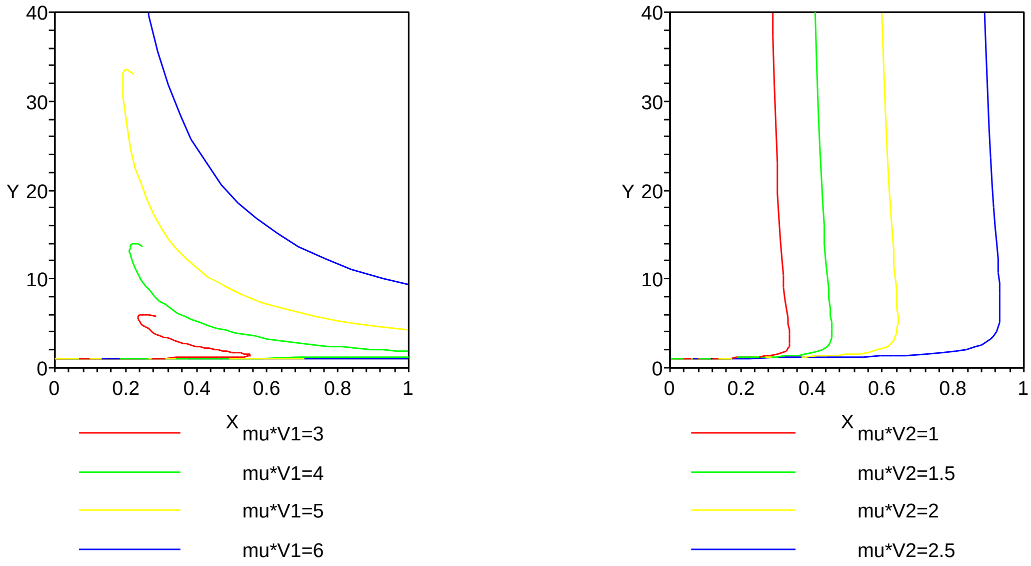


Fig. 6.4: The values of the volume moduli V_1 and V_2 at the minima of the potential with W_{np} and $\tau = \mu$. $X = b/d$ and $Y = c/d$. The left panel shows values of V_1 and the right panel shows values of V_2.

the supersymmetric minimum are

$$\frac{b}{d} = \frac{-2u_2^{1/3}e^{\mu V_2}\mu^{-1}}{u_1^{4/3} - u_2^{4/3} + 2\mu^{-1}(u_1^{1/3} - u_2^{1/3}) + 2\tau^{-1}u_1^{1/3}}, \tag{6.22}$$

$$\frac{c}{d} = \frac{2u_1^{1/3}e^{\tau V_1}\tau^{-1}}{u_1^{4/3} - u_2^{4/3} + 2\mu^{-1}(u_1^{1/3} - u_2^{1/3}) + 2\tau^{-1}u_1^{1/3}}. \tag{6.23}$$

Again we conclude from these expressions that V_1 and V_2 are both real.

The values of the moduli at the supersymmetric minima of the potential are shown in Fig.(6.4), where we have taken $\tau = \mu$. Other values of τ give a qualitatively similar figure.

Adding an extra term to the KKLT potential (6.16) can uplift the AdS minimum to a stable dS one. In the next chapter, we try the ghost vacuum energy as this extra potential.

6.3 Uplifting the KKLT dS vacuum

In this section we try to see how much the dS minimum, we have got in the previous section, can be raised via Casimir energy. Recall the 5D metric (5.8),

$$ds^2 = V^{1/3}Q^{-2}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + d\tilde{z}^2, \quad V = 1 - \alpha\sqrt{2}\tilde{z} \tag{6.24}$$

The warping factor is equal to one for flat space, but when the space is approximately flat we have the 5D distance between the branes

$$l_5 = \frac{1}{\alpha\sqrt{2}}(V_1 - V_2) \tag{6.25}$$

So, when $V_1 \approx V_2$, the bulk Casimir energy is (recall Eq. (4.58))

$$\Delta V_C(V_1, V_2) = C\alpha^4 \left(V_1 - V_2\right)^{-4} \tag{6.26}$$

The constant C is going to be determined in the following chapters. This expression needs to be expressed in the 4D Einstein frame. We do that by comparing the volumes, we have

$$\Delta\widetilde{V}_C = \Delta V_C \frac{\sqrt{|g^{(4)}|}}{\sqrt{|\tilde{g}^{(4)}|}} \tag{6.27}$$

That means that the Casimir energy in the 4D Einstein frame is

$$(\Delta\widetilde{V}_C) = (\Delta V_C)V_1^{2/3}Q^{-4} \tag{6.28}$$

In the limit of small warping,

$$Q^2 \approx \frac{4}{3}V_1^{\frac{1}{3}}\left(V_1 - V_2\right) \tag{6.29}$$

Adding that to (6.16) we have

$$V_{total} = V_{KKLT} + \frac{9}{16}C\alpha^4 \left(V_1 - V_2\right)^{-6} \tag{6.30}$$

Fig. 6.5 shows the total potential (6.30) for the gaugino condensate model. Unfortunately, it is clear from the plots (6.5(a)-6.5(d)) that the contribution of the ghost vacuum energies is only enough to rise the AdS minimum to dS one when C is large. We get the same result for the non-perturbative model (see Fig. 6.6). In the next section we investigate this analytically by comparing the AdS minimum of the potential with the vacuum energy. However, when the branes are very close to each other the Casimir energy is overwhelming. In the following chapter we will evaluate C and find it is connected with the condensates. This is not surprising because the Casimir energy depends on broken SUSY and it vanishes if there are no condensates.

6.4 Comparing V_{min} and V_C.

We now would like to compare the minimum of the potential with the Casimir energy 5D expression to see the possible values that the constant C must have to be able to uplift the AdS minimum to a dS one. The minimum of the potential in terms of the 4D Planck scale is (see (6.18))

$$V_{min} = -3e^K |W|^2 \tag{6.31}$$

In model B, from (6.20) and (6.21),

$$|W| = \frac{1}{2} Q^2 \left(V_1^{1/3} - V_2^{1/3} \right)^{-1} \mu W_g \tag{6.32}$$

Hence

$$V_{min} = -\frac{32}{3} \mu^2 Q^{-2} \left(V_1^{1/3} - V_2^{1/3} \right)^{-2} \kappa_P^2 \alpha^2 \epsilon^2 \Lambda^2 \tag{6.33}$$

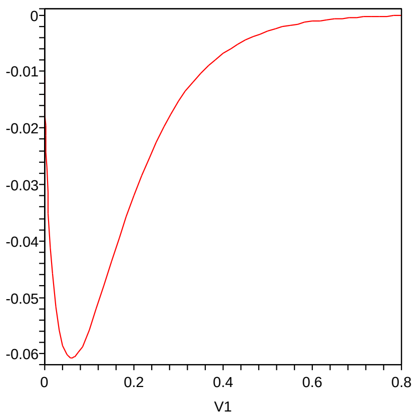
So

$$\frac{\Delta \widetilde{V}_C}{|V_{min}|} \sim V_1^{-1} (V_1 - V_2)^{-3} \frac{\alpha^2 \kappa_P^2}{\epsilon^2 \Lambda^2} C \tag{6.34}$$

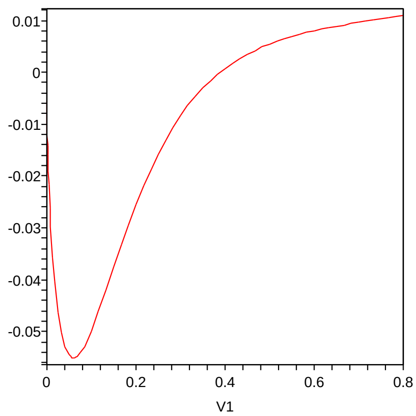
But $\alpha^2 \kappa_P^2$ is related to α_{GUT}^2 by (see [12])

$$\alpha^2 \kappa_P^2 = \frac{4}{3\pi} \beta^2 \alpha_{GUT}^2 \tag{6.35}$$

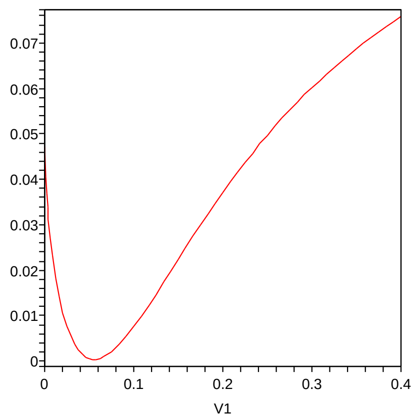
$\alpha_{GUT} = 1/40$. For V_C to be comparable to V_{min} we need C to be of order α_{GUT}^{-2} or the two branes are very close to each other. Alternatively, we have to consider the case of large warping.



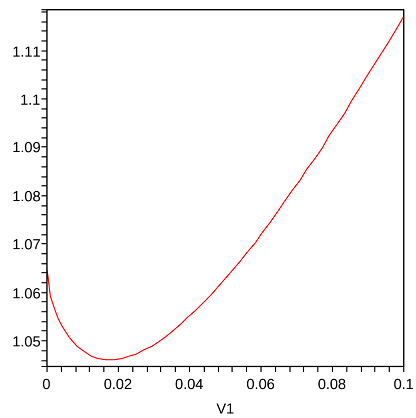
(a) 2D plot of (6.30) at $V_2 = 7$ for $C = 5$ with minimum at around -0.06.



(b) 2D plot of (6.30) at $V_2 = 7$ for $C = 500$ with minimum at around -0.055.

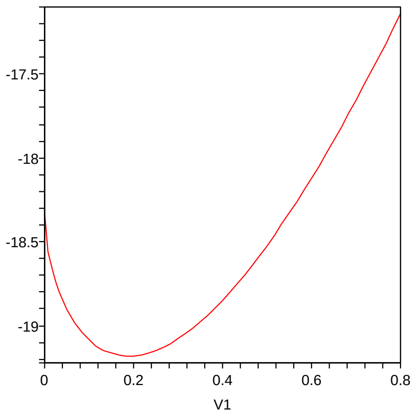


(c) 2D plot of (6.30) at $V_2 = 7$ for $C = 5400$ with minimum at 0.

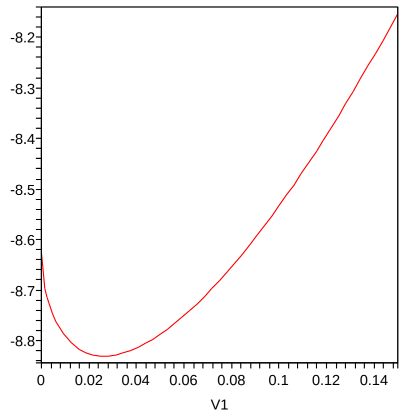


(d) 2D plot of (6.30) at $V_2 = 7$ for $C = 10^5$ with minimum at around $+1.1046$

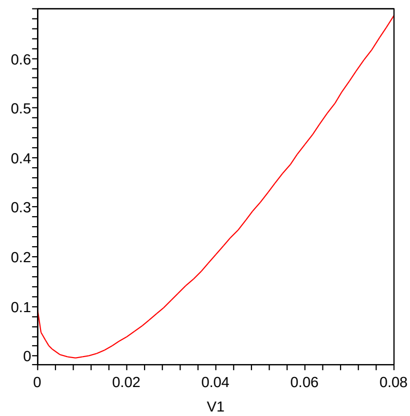
Fig. 6.5: Plots of (6.30) for model A: Positive Casimir energy of the ghost fields can uplift the AdS vacuum to dS one only at an undesirably large values of C.



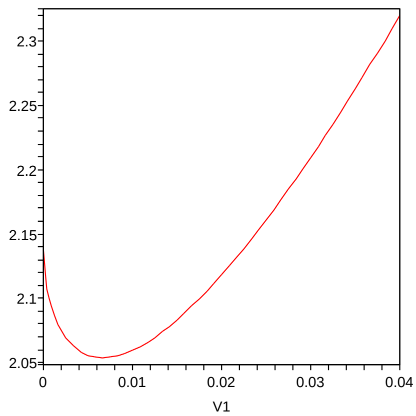
(a) 2D plot of (6.30) at $V_2 = 7$ for $C = 10^5$ with minimum at around -19.3.



(b) 2D plot of (6.30) at $V_2 = 7$ for $C = 10^6$ with minimum at around -8.83.



(c) 2D plot of (6.30) at $V_2 = 7$ for $C = 181 \times 10^4$ with minimum at 0.



(d) 2D plot of (6.30) at $V_2 = 7$ for $C = 2 \times 10^6$ with minimum at around $+2.055$

Fig. 6.6: Plots of (6.30) for model B.

6.5 summary

We made use of the gaugino condensate superpotential calculated in the previous chapter and constructed two models. In each of them we add another non-perturbative term with a flux term. Both models lead to AdS minima which need to be raised to dS minima by some extra effect. The ghosts associated to the gravitino field have a positive vacuum energy which may be helpful in obtaining dS minima. We start calculating these positive vacuum energy of the ghost fields in the next chapters.

7. 5D REDUCTION OF THE GRAVITINO

7.1 Introduction

This chapter and the next one discuss the Casimir energy contribution for the gravitino field. In this chapter we make a 5D reduction to the gravitino field starting by performing the gauge fixing and applying the BRST transformation. This gives two new ghost fields [150, 151]. We then perform the dimensional reduction for these three fields and express the boundary conditions in terms of the gaugino condensates. In the next chapter we calculate the Casimir energy contribution from the gravitino and ghost fields in flat and curved spaces.

The subjects of Gauge fixing and dimensional reduction for the gravitino field are interesting on their own. Dimensional reduction is a necessary step that must be performed to reach the effective theory, while gauge fixing is required when quantizing a field theory with gauge symmetry. Previous work on gauge fixing for the 11D gravitino has been done by Lukic and Moore [152], but most of the work presented in this chapter is original research done in collaboration with Prof. Ian G. Moss.

We now start from the 11D gravitino action (5.33) and try at first to simplify it by making the following redefinition

$$\psi_I = \lambda_I - \frac{1}{9}\Gamma_I\left(\Gamma^J\lambda_J\right) \tag{7.1}$$

Then

$$\overline{\psi}_I = \overline{\lambda}_I + \frac{1}{9}\left(\overline{\lambda}_J\Gamma^J\right)\Gamma_I \tag{7.2}$$

This means

$$\Gamma^I\psi_I = -\frac{2}{9}\Gamma^I\lambda_I, \quad \overline{\psi}_I\Gamma^I = \frac{2}{9}\overline{\lambda}_I\Gamma^I \tag{7.3}$$

With the help of the identities derived in appendix (B) for the products of gamma matrices, we finally get

$$L_{RS} = \frac{1}{2\kappa_{11}^2}\left[\overline{\lambda}_I\Gamma^J D_J\lambda^I + \frac{9}{4}(\overline{\psi}_I\Gamma^I)(\Gamma^J D_J)(\Gamma^K\psi_K)\right]. \tag{7.4}$$

This agrees with Lukic and Moore [152]. For the terms containing the field strength G_{PQRS}, using the same redefinition for ψ_I, we get the total Lagrangian as

$$\begin{aligned} L_\psi = \overline{\lambda}_I\left(\Gamma^J D_J - \frac{\sqrt{2}}{96}G_{PQRS}\Gamma^{PQRS}\right)\lambda^I + \frac{\sqrt{2}}{4}G_{PQRS}\overline{\lambda}^P\Gamma^{QR}\lambda^S \\ -\frac{9}{4}(\overline{\psi}_I\Gamma^I)\left(\Gamma^J D_J + \frac{\sqrt{2}}{32}G_{PQRS}\Gamma^{PQRS}\right)(\Gamma^K\psi_K). \end{aligned} \tag{7.5}$$

The full result does not agree with Lukic and Moore [152]. We would like to remove the $\Gamma^I\psi_I$ term using a gauge fixing function. In order to achieve this task, we are going to use the BRST mechanism which will result in two new ghost fields.

7.2 A review to the BRST formalism for the case of electromagnetism

When using the path integral formalism to generate propagators, one faces a difficulty due to gauge freedom. For example, for the generating functional

$$Z = \int \mathcal{D}A_\mu e^{i\int Ldx}, \tag{7.6}$$

with L invariant under gauge transformations $A_\mu \to A_\mu + \nabla_\mu\Lambda$, the integration is taken over all A_μ including those that are related only by a gauge transformation. This gives an infinite factor in Z and problems for the Green's functions obtained by the functional differentiation of Z. The simplest solution is to *fix a particular gauge* such that the integral over A_μ doesn't include values related by the gauge transformation. This can be done simply by imposing a Lorentz gauge condition $\nabla_\mu A^\mu = 0$, and including the gauge fixing term (for general α)

$$L_{gf} = -\frac{\alpha}{2}(\nabla_\mu A^\mu)^2 \tag{7.7}$$

The total Lagrangian now becomes

$$L = L_g + L_{gf}, \tag{7.8}$$

where $L_g = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$. The case for $\alpha = 1$ is called Feynman gauge.

Ensuring that the physics of any gauge theory doesn't depend on the choice of the gauge fixing terms is a basic requirement that must be fulfilled. To confirm that the addition of the gauge fixing terms doesn't change the theory we can follow the BRST approach and ensure that the BRST symmetry is not broken. The BRST approach is based on the addition of extra fields, called ghosts, to the theory which cancel any extra degrees of freedom introduced by the gauge fixing. We obtain the BRST transformation by replacing the gauge parameter with a new field and adding extra terms to the action. So, under BRST symmetry, the variation of the wave function ψ and A_μ is

$$s\psi = igc\psi \qquad sA_\mu = \nabla_\mu c, \tag{7.9}$$

where c an anticommuting scalar. However, the variation of the gauge fixing term will not vanish

$$sL_{gf} = -\alpha(\nabla_\mu A^\mu)\nabla^2 c. \tag{7.10}$$

We can cancel it by adding another term for the ghost field,

$$L_{gh} = \overline{c}\nabla^2 c. \tag{7.11}$$

where

$$s\ c = 0, \quad s\overline{c} = \alpha(\nabla_\mu A^\mu). \tag{7.12}$$

The total action then will be

$$I = -\int d\mu(x)(L_g + L_{gh} + L_{gf}). \tag{7.13}$$

The BRST transformations must satisfy the nilpotency restriction $s^2 = 0$. This only happens when the ghost fields satisfy $\nabla^2 c = 0$. We can remove this restriction ($\nabla^2 c = 0$) by introducing a new ghost field (called an antighost) b. The complete set of transformations then is

$$sA_\mu = \nabla_\mu c \quad s\ c = 0 \quad s\ \overline{c} = ib \quad s\ b = 0. \tag{7.14}$$

The gauge fixing Lagrangian which is invariant under this symmetry is

$$L_{gf} = -ib(\nabla_\mu A^\mu) - \frac{1}{2\alpha}b^2. \tag{7.15}$$

In the Landau gauge ($\alpha \to \infty$), b resembles a Lagrange multiplier. We can integrate b out of the theory to recover (7.13).

7.3 BRST symmetry for 11D Rarita-Schwinger Field

Now we are going to carry out the same procedure for the 11D Rarita-Schwinger Lagrangian (7.5). While, for the case of electromagnetism, the gauge fixing term depended on $\nabla_\mu A^\mu$ and the BRST transformation of the vector field was $sA_\mu = \nabla_\mu c$, the gauge fixing condition here depends on $\Gamma^I \psi_I$ and the BRST transformation of the fermion field is $s\psi_I = D_I \eta$, with η a ghost. An extra complication in this case is that the gauge fixing Lagrangian is not simply the square of the gauge fixing term, since now we place an operator in between to match (7.5), i.e.

$$L_{GF} \sim \frac{9}{4}(\overline{\psi}_I \Gamma^I)\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)(\Gamma^K \psi_K). \tag{7.16}$$

As we will see, this will lead to two ghost fields, instead of one. To illustrate this we start by recalling the usual supersymmetry transformation for the 11D supergravity (BRST transformations are the same as supersymmetry transformations but with the parameter η refers to a ghost field)

$$\delta e^{\hat{I}}{}_J = \frac{1}{2}\bar{\eta}\Gamma^{\hat{I}}\psi_J \tag{7.17}$$

$$\delta\psi_I = D_I(\hat{\Omega})\eta + \frac{\sqrt{2}}{288}\left(\Gamma_I{}^{JKLM} - 8\delta_I{}^J\Gamma^{KLM}\right)\eta\hat{G}_{JKLM} \tag{7.18}$$

$$\delta C_{IJK} = -\frac{\sqrt{2}}{8}\bar{\eta}\Gamma_{[IJ}\psi_{K]}. \tag{7.19}$$

where C_{IJK} is a three-form which can be dualized to a scalar. The total Lagrangian is $L_{total} = L + L_{GF} + L_\eta$ and we require $sL_{total} = 0$. Similar to (7.15), the gauge fixing Lagrangian which is invariant under BRST symmetry is

$$\begin{aligned} L_{GF} &= \frac{9}{2}\overline{b}\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)(\Gamma^K \psi_K) \\ &- \frac{9}{4}\overline{b}\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right) b. \end{aligned} \tag{7.20}$$

For the ghost field,

$$L_\eta = \overline{\eta}\left(\Gamma^J D_J - \frac{\sqrt{2}}{288} G_{PQRS}\Gamma^{PQRS}\right)\eta. \tag{7.21}$$

The variation gives

$$\begin{aligned} sL_{GF} &= \frac{9}{2}\overline{b}\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)(\Gamma^K s\psi_K) \\ &= \frac{9}{2}\overline{b}\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)\left(\Gamma^J D_J - \frac{\sqrt{2}}{288} G_{PQRS}\Gamma^{PQRS}\right)\eta \\ sL_\eta &= -\frac{9}{2}\overline{b}\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)\left(\Gamma^J D_J - \frac{\sqrt{2}}{288} G_{PQRS}\Gamma^{PQRS}\right)\eta. \end{aligned} \tag{7.22}$$

Where we used

$$\begin{aligned} s(\Gamma^K\psi_K) &= \left(\Gamma^J D_J - \frac{\sqrt{2}}{288} G_{PQRS}\Gamma^{PQRS}\right) \\ s\overline{\eta} &= -\frac{9}{2}\overline{b}\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right) \\ sb &= s\overline{b} = s\eta = 0. \end{aligned} \tag{7.23}$$

Note $s^2 = 0$, and

$$sL_{GF} + sL_\eta = 0 \tag{7.24}$$

Equation (7.20) could be rewritten as

$$\begin{aligned} L_{GF} &= -\frac{9}{4}\left(\overline{b} - \overline{\Gamma\psi}\right)\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)(b - \Gamma\psi) \\ &+ \frac{9}{4}\overline{\psi}\Gamma\left(\Gamma^J D_J + \frac{\sqrt{2}}{32} G_{PQRS}\Gamma^{PQRS}\right)\Gamma\psi \end{aligned} \tag{7.25}$$

The new theory now has extra fields (antighosts or antifields) $\overline{b}$ and b which are commuting variables, so we shall integrate these fields out in the path integral,

$$\int db\, d\overline{b}\; e^{i\int(L+L_{GF}+L_\eta)} \;=\; \det{}^{1/2}\left(\Gamma^J D_J+\frac{\sqrt{2}}{32}G_{PQRS}\Gamma^{PQRS}\right)\times \quad e^{i\int L+L_\eta+\frac{9}{4}\overline{\psi}\Gamma\left(\Gamma^J D_J+\frac{\sqrt{2}}{32}G_{PQRS}\Gamma^{PQRS}\right)\Gamma\psi} \tag{7.26}$$

Replace the determinant by a new field c,

$$\det{}^{1/2}\left(\Gamma^J D_J+\frac{\sqrt{2}}{32}G_{PQRS}\Gamma^{PQRS}\right)=\int dc\, d\overline{c}\; e^{i\int\overline{c}\left(\Gamma^J D_J+\frac{\sqrt{2}}{32}G_{PQRS}\Gamma^{PQRS}\right)c} \tag{7.27}$$

The $\Gamma\psi$ terms in (7.25) cancel the $\Gamma\psi$ terms in L_ψ. Therefore

$$L_{total}=L_\lambda+L_\eta+L_c, \tag{7.28}$$

where

$$L_\lambda \;=\; \overline{\lambda}_I\left(\Gamma^J D_J-\frac{\sqrt{2}}{96}G_{PQRS}\Gamma^{PQRS}\right)\lambda^I+\frac{\sqrt{2}}{4}G_{PQRS}\overline{\lambda}^P\Gamma^{QR}\lambda^S \tag{7.29}$$

$$L_c \;=\; \overline{c}\left(\Gamma^J D_J-\frac{\sqrt{2}}{32}G_{PQRS}\Gamma^{PQRS}\right)c \tag{7.30}$$

$$L_\eta \;=\; \overline{\eta}\left(\Gamma^I D_I-\frac{\sqrt{2}}{288}G_{PQRS}\Gamma^{PQRS}\right)\eta \tag{7.31}$$

The additional ghost terms (7.30) and (7.31) here make very important contributions to Casimir energy stabilization. The importance of the 'ghost' part is that they give a positive sign for the vacuum energy (which leads to a repulsive force) while the real fermions (as we have seen before) give a negative sign for the vacuum energy.

7.4 Reduction to 5 dimensions

Reduction to 5D means that we are going to use the 5D Einstein frame. The metric (5.9) will then be written as

$$ds^2=V^{-2/3}\left(\widetilde{g}_{\alpha\beta}dx^\alpha dx^\beta\right)+V^{1/3}\left(\widetilde{g}_{a\overline{b}}dx^a dx^{\overline{b}}+\widetilde{g}_{\overline{a}b}dx^{\overline{a}}dx^b\right). \tag{7.32}$$

where $\widetilde{g}_{a\bar{b}}$ is the Calabi-Yau metric. The gamma matrices are given by

$$\Gamma_\mu = V^{-1/3}\widetilde{\gamma}_\mu \otimes \mathbb{1}, \quad \Gamma_a = V^{1/6}\widetilde{\gamma}_5 \otimes \widetilde{\gamma}_a \quad \text{and} \quad \Gamma_5 = V^{-1/3}\widetilde{\gamma}_5 \otimes \widetilde{\gamma}_7 \tag{7.33}$$

For raised indices,

$$\Gamma^\mu = V^{1/3}\widetilde{\gamma}_\mu \otimes \mathbb{1}, \quad \Gamma^a = V^{-1/6}\widetilde{\gamma}_5 \otimes \widetilde{\gamma}_a \quad \text{and} \quad \Gamma^5 = V^{1/3}\widetilde{\gamma}_5 \otimes \widetilde{\gamma}_7 \tag{7.34}$$

The metric (7.32) also implies

$$\sqrt{|g^{(11)}_{AB}|} = V^{-2/3}\sqrt{|\widetilde{g}^{(5)}_{\alpha\beta}|}. \tag{7.35}$$

Here, we will include some background values of the field strength G, so that the term $\Gamma^{PQRS}G_{PQRS}$ is

$$\Gamma^{PQRS}G_{PQRS} = 4V^{-1/6}\mathbb{1} \otimes \widetilde{\gamma}^7\widetilde{\gamma}^{\overline{a}\overline{b}\overline{c}}G_{abcz} + 6V^{-2/3}\mathbb{1} \otimes \widetilde{\gamma}^{ab\overline{c}\overline{d}}G_{ab\overline{c}\overline{d}}, \tag{7.36}$$

where

$$G_{ab\overline{c}\overline{d}} = \frac{\alpha}{3}\left(\widetilde{g}_{a\overline{c}}\widetilde{g}_{b\overline{d}} - \widetilde{g}_{a\overline{d}}\widetilde{g}_{b\overline{c}}\right). \tag{7.37}$$

We use the ansatz for the gravitino

$$\lambda_\alpha = V^{-1/6}\theta^+_\alpha \otimes u_+ + V^{-1/6}\gamma_5\theta^-_\alpha \otimes u_- \tag{7.38}$$

where $u_\pm$ are covariantly constant spinors on the Calabi-Yau space. The conjugate spinor is

$$\overline{\lambda}^\alpha = (V^{-1/6}\overline{\theta^{+\alpha}} \otimes u^\dagger_+ - V^{-1/6}\gamma_5\overline{\theta^{-\alpha}} \otimes u^\dagger_-)V^{2/3} \tag{7.39}$$

The covariant derivative acting on spinors of the form (7.38) reduces to

$$\Gamma^J D_J \to V^{1/3}\left(\gamma^\nu \otimes \mathbb{1}\right)\Gamma^J\widetilde{D}_J. \tag{7.40}$$

The factor $V^{-1/6}$ in (7.38) has been chosen to cancel the V's in the kinetic terms in the field equations. All other terms will have V's raised to some power, as we will see.

After making use of the identities in appendix B.4, the Lagrangian (7.28) reduces to the 5D Lagrangian

$$L_\theta = \frac{1}{2\kappa_{11}}\Big[\left(\overline{\theta^{+\alpha}}\Gamma^J D_J\theta^+_\alpha - \overline{\theta^{-\alpha}}\Gamma^J D_J\theta^-_\alpha\right) + V^{-1/3}\left(\overline{\theta^{+\alpha}}\theta^{+\alpha} - \overline{\theta^{-\alpha}}\theta^{-\alpha}\right)$$
$$+\frac{\sqrt{2}}{24}V^{-1/6}\left(\overline{\theta^{+\alpha}}\gamma_5\theta^-_\alpha + \overline{\theta^{-\alpha}}\gamma_5\theta^+_\alpha\right)\times i\widetilde{\varepsilon}^{abc}G_{abcz}$$
$$-\frac{\sqrt{2}}{48}V^{-1}\left(\overline{\theta^{+\alpha}}\theta^+_\alpha - \overline{\theta^{-\alpha}}\theta^-_\alpha\right)\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}}\Big] \quad (7.41)$$

$$L_c = \frac{1}{2\kappa_{11}}\Big[\left(\overline{c^{+\alpha}}\Gamma^J D_J c^+_\alpha - \overline{c^{-\alpha}}\Gamma^J D_J c^-_\alpha\right) + V^{-1/3}\left(\overline{c^{+\alpha}}c^{+\alpha} - \overline{c^{-\alpha}}c^{-\alpha}\right)$$
$$+\frac{\sqrt{2}}{8}V^{-1/6}\left(\overline{c^{+\alpha}}\gamma_5 c^-_\alpha + \overline{c^{-\alpha}}\gamma_5 c^+_\alpha\right)\times i\widetilde{\varepsilon}^{abc}G_{abcz}$$
$$-\frac{\sqrt{2}}{16}V^{-1}\left(\overline{c^{+\alpha}}c^+_\alpha - \overline{c^{-\alpha}}c^-_\alpha\right)\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}}\Big] \quad (7.42)$$

$$L_\eta = \frac{1}{2\kappa_{11}}\Big[\left(\overline{\eta^{+\alpha}}\Gamma^J D_J\eta^+_\alpha - \overline{\eta^{-\alpha}}\Gamma^J D_J\eta^-_\alpha\right) + V^{-1/3}\left(\overline{\eta^{+\alpha}}\eta^{+\alpha} - \overline{\eta^{-\alpha}}\eta^{-\alpha}\right)$$
$$+\frac{\sqrt{2}}{72}V^{-1/6}\left(\overline{\eta^{+\alpha}}\gamma_5\eta^-_\alpha + \overline{\eta^{-\alpha}}\gamma_5\eta^+_\alpha\right)\times i\widetilde{\varepsilon}^{abc}G_{abcz}$$
$$-\frac{\sqrt{2}}{144}V^{-1}\left(\overline{\eta^{+\alpha}}\eta^+_\alpha - \overline{\eta^{-\alpha}}\eta^-_\alpha\right)\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}}\Big] \quad (7.43)$$

where $\overline{\psi} = \psi^\dagger\gamma_0$. From the Lagrangian, we can derive the field equations for the fields $\theta^{\pm\alpha}$,$c^{\pm\alpha}$,$\eta^{\pm\alpha}$ as

$$\Gamma^J D_J\theta^{+\alpha} + V^{-1/3}\theta^{+\alpha} + \frac{\sqrt{2}}{24}V^{-1/6}\gamma_5\theta^{-\alpha}i\widetilde{\varepsilon}^{abc}G_{abcz} - \frac{\sqrt{2}}{48}V^{-1}\theta^{+\alpha}\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}} = 0 \quad (7.44)$$

$$\Gamma^J D_J\theta^{-\alpha} + V^{-1/3}\theta^{-\alpha} + \frac{\sqrt{2}}{24}V^{-1/6}\gamma_5\theta^{+\alpha}i\widetilde{\varepsilon}^{abc}G_{abcz} - \frac{\sqrt{2}}{48}V^{-1}\theta^{-\alpha}\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}} = 0 \quad (7.45)$$

$$\Gamma^J D_J c^{+\alpha} + V^{-1/3}c^{+\alpha} + \frac{\sqrt{2}}{8}V^{-1/6}\gamma_5 c^{-\alpha}i\widetilde{\varepsilon}^{abc}G_{abcz} - \frac{\sqrt{2}}{16}V^{-1}c^{+\alpha}\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}} = 0 \quad (7.46)$$

$$\Gamma^J D_J c^{-\alpha} + V^{-1/3}c^{-\alpha} + \frac{\sqrt{2}}{8}V^{-1/6}\gamma_5 c^{+\alpha}i\widetilde{\varepsilon}^{abc}G_{abcz} - \frac{\sqrt{2}}{16}V^{-1}c^{-\alpha}\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}} = 0 \quad (7.47)$$

$$\Gamma^J D_J\eta^{+\alpha} + V^{-1/3}\eta^{+\alpha} + \frac{\sqrt{2}}{72}V^{-1/6}\gamma_5\eta^{-\alpha}i\widetilde{\varepsilon}^{abc}G_{abcz} - \frac{\sqrt{2}}{144}V^{-1}\eta^{+\alpha}\widetilde{g}^{a\overline{c}}\widetilde{g}^{b\overline{d}}G_{ab\overline{c}\overline{d}} = 0 \quad (7.48)$$

$$\Gamma^J D_J \eta^{-\alpha} + V^{-1/3}\eta^{-\alpha} + \frac{\sqrt{2}}{72}V^{-1/6}\gamma_5\eta^{+\alpha} i\widetilde{\varepsilon}^{abc}G_{abcz} - \frac{\sqrt{2}}{144}V^{-1}\eta^{-\alpha}\widetilde{g}^{a\bar{c}}\widetilde{g}^{b\bar{d}}G_{ab\bar{c}\bar{d}} = 0 \tag{7.49}$$

These equations could be greatly simplified by removing the 'mass' term containing $\widetilde{\varepsilon}^{abc}G_{abcz}$ by a certain rescaling, as we will see later.

7.4.1 Boundary conditions

We now need boundary conditions for the modes. We take the following boundary conditions on the hidden brane (see section 5.1)

$$\left(P_{-} - \epsilon\Gamma P_{+}\right)\lambda_\mu = 0, \tag{7.50}$$

where

$$P_{\mp} = \frac{1}{2}\left(1\!\!1 \otimes 1\!\!1 \mp \widetilde{\gamma}_5 \otimes \widetilde{\gamma}_7\right). \tag{7.51}$$

We assume $\overline{\chi}\Gamma_{ABC}\chi \propto \widetilde{\varepsilon}_{abc}$, then

$$\Gamma = \frac{1}{96}\overline{\chi}\Gamma_{ABC}\chi\Gamma^{ABC} = \frac{1}{2}CI, \tag{7.52}$$

where

$$I = -\frac{i}{48}\gamma_5 \otimes \left(\widetilde{\varepsilon}_{abc}\widetilde{\gamma}^{abc} + \widetilde{\varepsilon}_{\bar{a}\bar{b}\bar{c}}\widetilde{\gamma}^{\bar{a}\bar{b}\bar{c}}\right) \tag{7.53}$$

The constant C is related to the gaugino condensate $\Lambda = CV_2^{-1/2}$ (see chapter 6). Substituting from (7.38), (7.51) and (7.52) into (7.50), taking $\varepsilon_{abc}\varepsilon^{abc} = 48$, we get the boundary conditions

$$P_-^{(4)}\theta_\alpha^+ - \frac{iC\epsilon}{2}P_+^{(4)}\theta_\alpha^- = 0 \tag{7.54}$$

$$P_+^{(4)}\theta_\alpha^- - \frac{iC\epsilon}{2}P_-^{(4)}\theta_\alpha^+ = 0 \tag{7.55}$$

where $P_\pm^{(4)} = (1 \pm \gamma_5)$. We assume that the boundary conditions on the visible brane are untwisted ($C = 0$), while they are twisted on the hidden brane where there is a gaugino condensate. We would like to compare these boundary conditions with the one we used for the twisted fermions calculations in chapter 4,

$$P_\chi \Psi = 0, \tag{7.56}$$

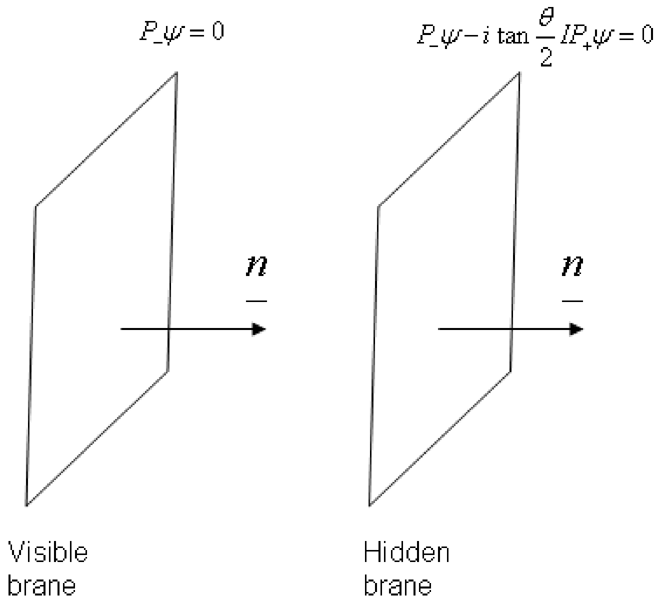


Fig. 7.1: The twisted boundary conditions on the visible and hidden brane with the direction of n taken outward.

where $P_\chi = \frac{1}{2}(1 + \chi\Gamma_5)$. Let

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{7.57}$$

then χ could be expressed as

$$\chi = \cos\theta\ \mathbb{1} + iJ\sin\theta \tag{7.58}$$

Define $P_\pm^{(5)} = \frac{1}{2}(1 \pm \gamma_5)$, then (7.56) becomes

$$\frac{1}{2}\left(1 + \cos\theta\ \mathbb{1} + iJ\sin\theta\ \right) P_-^{(5)}\Psi + \frac{1}{2}\left(1 - \cos\theta\ \mathbb{1} - iJ\sin\theta\ \right) P_+^{(5)}\Psi = 0 \tag{7.59}$$

Then

$$\left(\cos^2\frac{\theta}{2}\ \mathbb{1} + iJ\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) P_-^{(5)}\Psi + \left(\sin^2\frac{\theta}{2}\ \mathbb{1} - iJ\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) P_+^{(5)}\Psi = 0 \tag{7.60}$$

Multiplying both sides by $\cos^2\frac{\theta}{2} - i\sin\frac{\theta}{2}\cos\frac{\theta}{2}$, noting that $J^2 = 1$ we finally reach

$$P_-^{(5)}\psi - i\left(\tan\frac{\theta}{2}\right) JP_+^{(5)}\psi = 0 \tag{7.61}$$

Comparing with (7.54), we get the relationship between the angle θ in the two boundary conditions as

$$\tan\frac{\theta}{2} = \frac{C\epsilon}{2} \tag{7.62}$$

This allows us to express θ in terms of the condensates,

$$\theta = 2\tan^{-1}\left(\frac{C\epsilon}{2}\right) \tag{7.63}$$

This equation tells us that the Gaugino condensates on the hidden brane leads to a non-vanishing θ which breaks supersymmetry. When C vanishes, θ vanishes and supersymmetry is retained.

7.5 Summary

We have reviewed the BRST formalism and made use of it to remove the $\Gamma^I\psi_I$ term using a gauge fixing function. This process gave two ghost fields which are important for dealing with the stabilization topic. We then performed a dimensional reduction for the total 11D Lagrangian to 5D and got the 5D field equations which can be simplified by eliminating the mass term which we do in the next chapter. We ended by expressing the SUSY breaking parameter θ in terms of the condensate using the twisted boundary conditions of the improved heterotic M-theory.

8. GRAVITINO AND GHOST FIELD VACUUM ENERGIES

8.1 Eliminating the 'mass' term

Going back to section (7.4), we would like to find eigenmodes for the fields $\theta^{\pm\alpha}$, $c^{\pm\alpha}$ and $\pm\eta^{+\alpha}$. This would be easier if we could omit the 'mass' term containing $\widetilde{\varepsilon}^{abc}G_{abcz}$. We can do this by rescaling λ_α in the gravitino Lagrangian, but there will be a price because we will have a modified boundary condition, as we will see. Before we do this, we recall the two ansatzë for the flux

$$G_{ab\overline{c}\overline{d}} = \frac{\alpha}{3}\left(\widetilde{g}_{a\overline{c}}\widetilde{g}_{b\overline{d}} - \widetilde{g}_{a\overline{d}}\widetilde{g}_{b\overline{c}}\right), \tag{8.1}$$

$$G_{zabc} = -(\partial_z\xi)\widetilde{\varepsilon}_{abc}. \tag{8.2}$$

We now start from

$$L_\lambda = \overline{\lambda}^I\left(\Gamma^J D_J - \frac{\sqrt{2}}{96}G_{PQRS}\Gamma^{PQRS}\right)\lambda_I \tag{8.3}$$

The mass term could be written as

$$G_{PQRS}\Gamma^{PQRS} = 4\ G_{\alpha ABC}\Gamma^\alpha\Gamma^{ABC} + 6\ G_{ab\overline{c}\overline{d}}\Gamma^{ab\overline{c}\overline{d}} \tag{8.4}$$

where $G_{\alpha ABC}$ and $G_{ab\overline{c}\overline{d}}$ are 5D and 6D objects respectively. The factor 4 comes because we have four equal terms with four different arrangements for the index α, and the factor 6 for the six equal terms with six different arrangements for the holomorphic indices $\overline{a}$ and $\overline{b}$. The following rescaling can cancel the $G_{\alpha ABC}$ term in the field equations

$$\left(\Gamma^J D_J - \frac{4\sqrt{2}}{96}G_{\alpha ABC}\Gamma^\alpha\Gamma^{ABC} - \frac{6\sqrt{2}}{96}G_{ab\overline{c}\overline{d}}\Gamma^{ab\overline{c}\overline{d}}\right)\lambda_\mu \tag{8.5}$$

$$= S^{-1}\left(\Gamma^J D_J - \frac{6\sqrt{2}}{96}G_{ab\overline{c}\overline{d}}\Gamma^{ab\overline{c}\overline{d}}\right)S\lambda_\mu.$$

This means we rescale λ_μ into $\lambda_\mu^{'} = S\lambda_\mu$, with

$$S = e^{i\Phi} \tag{8.6}$$

The derivative of the rescaled $\lambda_\mu^{'}$ gives

$$S^{-1}\Gamma^J D_J(S\lambda_\mu) = i(\Gamma^J D_J \Phi)\lambda_\mu + \Gamma^J D_J \lambda_\mu \tag{8.7}$$

Which means that we require for (8.5) that

$$-\frac{4\sqrt{2}}{96} G_{\alpha ABC}\Gamma^\alpha \Gamma^{ABC} = i(\Gamma^J D_J \Phi) \tag{8.8}$$

To satisfy this with (8.2) and (7.53), we choose

$$\Phi = 2\sqrt{2}\xi I. \tag{8.9}$$

Using the expression for C_{abc} in (5.20), the value of Φ on the hidden brane is

$$\Phi = -\frac{\epsilon C}{2} I. \tag{8.10}$$

Note that $I^2 = 1$. This achieves the required simplification of (8.5). However, the boundary condition (7.50) becomes

$$\left(P_- - \epsilon \Gamma P_+\right) S^{-1}\lambda_\mu^{'} = 0. \tag{8.11}$$

To obtain the new boundary conditions we substitute from (8.6) and (8.9) into (8.11) with $\Gamma = \frac{C}{2} I$. This finally gives

$$P_- \left(1 - i\frac{tan\left(tan^{-1}(\frac{\epsilon C}{2}) - \frac{\epsilon C}{2}\right)}{1 + \frac{\epsilon C}{2} tan(\frac{\epsilon C}{2})} I\right)\lambda_\mu^{'} = 0. \tag{8.12}$$

For small $C\epsilon$, the *twist* part in (8.12) will be cancelled up to order $(C\epsilon)^3$ and we get the untwisted boundary conditions $P_-\lambda_\mu^{'} = 0$. This means, that after rescaling the gravitino mass, the Casimir energy for the graviton multiplet is given by the untwisted value $V_C = 0$. The situation, however, is expected to be different for the ghost fields because of the different coefficients in the mass term. We now use the same rescaling in (7.30) and (7.31), but for the c and η fields we have to take $\Phi = 3\epsilon CI/2$ and $\Phi = \epsilon CI/6$ respectively. This gives the twisted boundary conditions

$$P_{-}\left(1 + i\frac{4\epsilon C}{4 + 3\epsilon^2 C^2} I\right) c' = 0 \tag{8.13}$$

$$P_{-}\left(1 + i\frac{4\epsilon C}{12 + \epsilon^2 C^2} I\right) \eta' = 0 \tag{8.14}$$

to leading order in ϵC. Comparing with (7.61) we get the vacuum energy for the $c-$field and $\eta-$field, calculated as the difference between the twisted and untwisted cases, as (recall eq. (4.58))

$$\Delta V_c = \frac{3C^2\epsilon^2}{16\pi^2 l_5^4}\zeta(3) \tag{8.15}$$

$$\Delta V_\eta = \frac{C^2\epsilon^2}{192\pi^2 l_5^4}\zeta(3) \tag{8.16}$$

with $\zeta(3) = 1.2020569032$.

We now have a formula for the constant C which appeared in the discussion of radion stabilization in section 6.3. So far, we have calculated the Casimir energy for the twisted fermions between the two branes. In chapter 5 we wrote down the formula for the gaugino condensate potential energy. The aim now is to see if the addition of the vacuum energy of the ghosts c and η can help in stabilization. C is related to the condensates, though $C \approx e^{-\mu V_2}$. This means we have the ghost vacuum energies in terms of l_5 and V_2 as

$$\Delta V_c(l_5, V_2) = \frac{3e^{-2\mu V_2}\epsilon^2}{16\pi^2 l_5^4}\zeta(3) \tag{8.17}$$

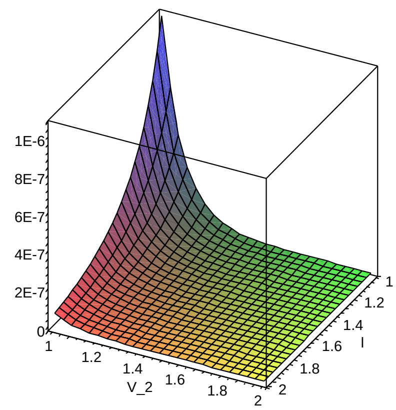
$$\Delta V_\eta(l_5, V_2) = \frac{e^{-2\mu V_2}\epsilon^2}{192\pi^2 l_5^4}\zeta(3) \tag{8.18}$$

So, in flat space, the twisted ghost fields lead to a *positive* vacuum energy which leads to a repulsive force. In the following section we turn to the case of warped bulk and calculate the ghost vacuum energies in curved space.

8.2 Warped bulk case

For warped bulk, the distance l_5 is given by

$$l_5 = \int_{z_1}^{z_2} \left(\frac{z}{z_1}\right)^{(1/5)} = \frac{5}{6} z_2 \tau^{-\frac{1}{5}}(1 - \tau^{6/5}). \tag{8.19}$$



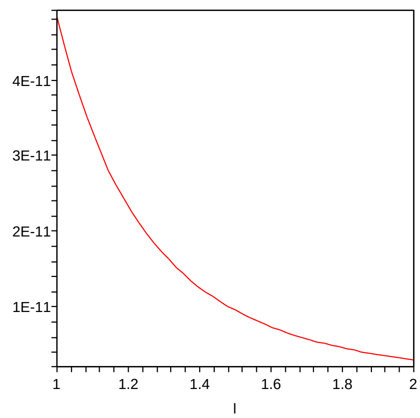


Fig. 8.1: (a) 3D plot of the sum of the ghosts potential shows l_5 and V_2 directions. (b) The sum of the ghosts potential at constant V_2 ($V_2 = 2$).

We use the conformally flat metric

$$ds^2 = \left(\frac{z}{z_1}\right)^{2/5} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \tag{8.20}$$

For the twisted bulk fermions we had

$$J_{\overline{\nu}}(m_n z_1)(CJ_{-\overline{\nu}}(m_n z_2) \pm SJ_\nu(m_n z_2)) - J_{-\overline{\nu}}(m_n z_1)(CJ_{\overline{\nu}}(m_n z_2) \mp SJ_{-\nu}(m_n z_2)) = 0, \tag{8.21}$$

where $\nu + \bar{\nu} = 1$, $C = \cos\theta/2$, $S = \sin\theta/2$ and θ is now related to the condensate by (8.12). We recall the expression (4.26) for the Bessel function index

$$\nu = \pm\left(\frac{1}{2} + \frac{3}{5}\gamma\chi\right) \tag{8.22}$$

We determine the value of γ for the θ^α, c^α and η^α fields from their mass terms. For the gravitino we have

$$\frac{\sqrt{2}}{48} V^{-1} \theta^{+\alpha} \widetilde{g}^{a\bar{c}} \widetilde{g}^{b\bar{d}} G_{ab\bar{c}\bar{d}} = \frac{\alpha\sqrt{2}}{24} V^{-1} \theta^{+\alpha}, \tag{8.23}$$

where we made use of (8.1). For the ghost fields we have respectively

$$\frac{\sqrt{2}}{16} V^{-1} c^{+\alpha} \widetilde{g}^{a\bar{c}} \widetilde{g}^{b\bar{d}} G_{ab\bar{c}\bar{d}} = \frac{\alpha\sqrt{2}}{8} V^{-1} c^{+\alpha} \tag{8.24}$$

$$\frac{\sqrt{2}}{144} V^{-1} \eta^{+\alpha} \widetilde{g}^{a\bar{c}} \widetilde{g}^{b\bar{d}} G_{ab\bar{c}\bar{d}} = \frac{\alpha\sqrt{2}}{72} V^{-1} \eta^{+\alpha} \tag{8.25}$$

Comparing the RHS of these equations with the fermion mass $M = \gamma\alpha V^{-1}/\sqrt{2}$ in (4.22), we get $\gamma = 1/6, -13/2, -1/2$ for the θ^α, c^α and η^α fields respectively. This leads to the following values for the Bessel function index

$$\nu_\theta = \pm\left(\frac{2}{5}, \frac{3}{5}\right), \quad \nu_c = \pm\left(-\frac{4}{5}, \frac{9}{5}\right), \quad \nu_\eta = \pm\left(\frac{1}{5}, \frac{4}{5}\right). \tag{8.26}$$

The vacuum energy can then be calculated for those fields using

$$\zeta_\lambda^{'}(0) \sim -\frac{1}{8\pi^2}\frac{\Delta G_\lambda(\tau)}{z_2^4}, \quad \zeta_C^{'}(0) \sim +\frac{1}{8\pi^2}\frac{\Delta G_c(\tau)}{z_2^4}, \quad \zeta_\eta^{'}(0) \sim +\frac{1}{8\pi^2}\frac{\Delta G_\eta(\tau)}{z_2^4}, \tag{8.27}$$

where

$$\begin{aligned} \Delta G_\lambda(\tau) &= \int_0^\infty dx x^3 \ln\left|\frac{P_\lambda^\theta(x)P_a^0(x)}{P_{\lambda a}^\theta P^0(x)}\right| \\ \Delta G_c(\tau) &= \int_0^\infty dx x^3 \ln\left|\frac{P_c^\theta(x)P_a^0(x)}{P_{ca}^\theta P^0(x)}\right| \\ \Delta G_\eta(\tau) &= \int_0^\infty dx x^3 \ln\left|\frac{P_\eta^\theta(x)P_a^0(x)}{P_{\eta a}^\theta P^0(x)}\right| \end{aligned} \tag{8.28}$$

The functions P_a^0 and P^0 are defined in chapter 4, and

$$\begin{aligned} \left|\frac{P_\lambda^\theta(x)}{P_{\lambda a}^\theta(x)}\right| &= \Big[C\left(I_{\frac{3}{5}}(\tau x)K_{\frac{3}{5}}(x) - I_{\frac{3}{5}}(x)K_{\frac{3}{5}}(\tau x)\right) \mp iS\left(I_{\frac{3}{5}}(\tau x)K_{\frac{2}{5}}(x)\right. \\ &\left.+I_{\frac{2}{5}}(x)K_{\frac{3}{5}}(\tau x) + \frac{2}{\pi}\sin\left(\frac{2}{5}\pi\right)K_{\frac{3}{5}}(\tau x)K_{\frac{3}{5}}(x)\right)\Big] \Big/ \Big[\Big(CI_{\frac{3}{5}}(x)K_{\frac{3}{5}}(\tau x) \\ &iSI_{\frac{2}{5}}(x)K_{\frac{3}{5}}(\tau x)\Big)\Big] \end{aligned} \tag{8.29}$$

$$\begin{aligned} \left|\frac{P_c^\theta(x)}{P_{ca}^\theta(x)}\right| &= \Big[C\left(I_{\frac{9}{5}}(\tau x)K_{\frac{9}{5}}(x) - I_{\frac{9}{5}}(x)K_{\frac{9}{5}}(\tau x)\right) \mp iS\left(I_{\frac{9}{5}}(\tau x)K_{-\frac{4}{5}}(x)\right. \\ &\left.+I_{-\frac{4}{5}}(x)K_{\frac{9}{5}}(\tau x) + \frac{2}{\pi}\sin\left(-\frac{4}{5}\pi\right)K_{\frac{9}{5}}(\tau x)K_{\frac{9}{5}}(x)\right)\Big] \Big/ \Big[\Big(CI_{\frac{9}{5}}(x)K_{\frac{9}{5}}(\tau x) \\ &iSI_{-\frac{4}{5}}(x)K_{\frac{9}{5}}(\tau x)\Big)\Big] \end{aligned} \tag{8.30}$$

$$\begin{aligned} \left|\frac{P_\eta^\theta(x)}{P_{\eta a}^\theta(x)}\right| &= \Big[C\left(I_{\frac{4}{5}}(\tau x)K_{\frac{4}{5}}(x) - I_{\frac{4}{5}}(x)K_{\frac{4}{5}}(\tau x)\right) \mp iS\left(I_{\frac{4}{5}}(\tau x)K_{\frac{1}{5}}(x)\right. \\ &\left.+I_{\frac{1}{5}}(x)K_{\frac{4}{5}}(\tau x) + \frac{2}{\pi}\sin\left(\frac{\pi}{5}\right)K_{\frac{4}{5}}(\tau x)K_{\frac{4}{5}}(x)\right)\Big] \Big/ \Big[\Big(CI_{\frac{4}{5}}(x)K_{\frac{4}{5}}(\tau x) \\ &iSI_{\frac{1}{5}}(x)K_{\frac{4}{5}}(\tau x)\Big)\Big] \end{aligned} \tag{8.31}$$

where the regularization process goes as has been done in chapter 4.

The integrals (8.28) for the gravitino and its ghosts are exactly the same integral (4.97) we have got in chapter 4 for the spin 1/2 twisted fermion case. According to what we found in this chapter, the gravitino integral vanishes. For the ghost integrals, the values of these two integrals are tabulated in table (8.1) and plotted in Fig.(8.3). The analysis following from Fig.(8.3) is similar to the spin 1/2 twisted case where the ghost Casimir energy becomes stronger as the two branes are getting closer. For small brane separation, the integral is large and positive and the effective vacuum energy is large and positive as well. The total ghost vacuum energy then is

$$\zeta'_{gh}(0) \sim +\frac{1}{8\pi^2}\frac{\Delta G_c(\tau)+\Delta G_\eta(\tau)}{z_2^4}, \tag{8.32}$$

Fig. (8.2(c)) shows no minimum for (8.32). The 5D effective potentials for the ghosts c and η are

$$\Delta V_c = \frac{1}{8\pi^2}\left(\frac{\Delta G_c^\theta(\tau)}{z_2^4}+\frac{B_c(\theta)}{z_2^4}\right)-\frac{\beta_4(C-1)}{4\pi^2}\frac{\ln(\mu_R z_2)}{z_2^4} \tag{8.33}$$

$$\Delta V_\eta = \frac{1}{8\pi^2}\left(\frac{\Delta G_\eta^\theta(\tau)}{z_2^4}+\frac{B_\eta(\theta)}{z_2^4}\right)-\frac{\beta_4(C-1)}{4\pi^2}\frac{\ln(\mu_R z_2)}{z_2^4} \tag{8.34}$$

where $B(\theta)$ is defined as in chapter 4.

In terms of z_2 and τ, Eqs. (8.17) and (8.18) can be expressed as

$$\Delta V_c(z_2,\tau) = \frac{3\epsilon^2\zeta(3)}{16\pi^2}\frac{e^{\frac{-\mu 5\alpha}{3\sqrt{2}}z_2\tau^{-1/5}}}{z_2^4(1-\tau)^4} \tag{8.35}$$

$$\Delta V_\eta(z_2,\tau) = \frac{\epsilon^2\zeta(3)}{192\pi^2}\frac{e^{\frac{-\mu 5\alpha}{3\sqrt{2}}z_2\tau^{-1/5}}}{z_2^4(1-\tau)^4} \tag{8.36}$$

The sum of (8.35) and (8.36) is shown in Fig. 8.2(a). The warped case tends to the flat case as τ tends to 1.

8.3 Summary

The mass term we have got in the field equations in the previous chapter can be eliminated by rescaling the gravitino field. However, this rescaling modifies the

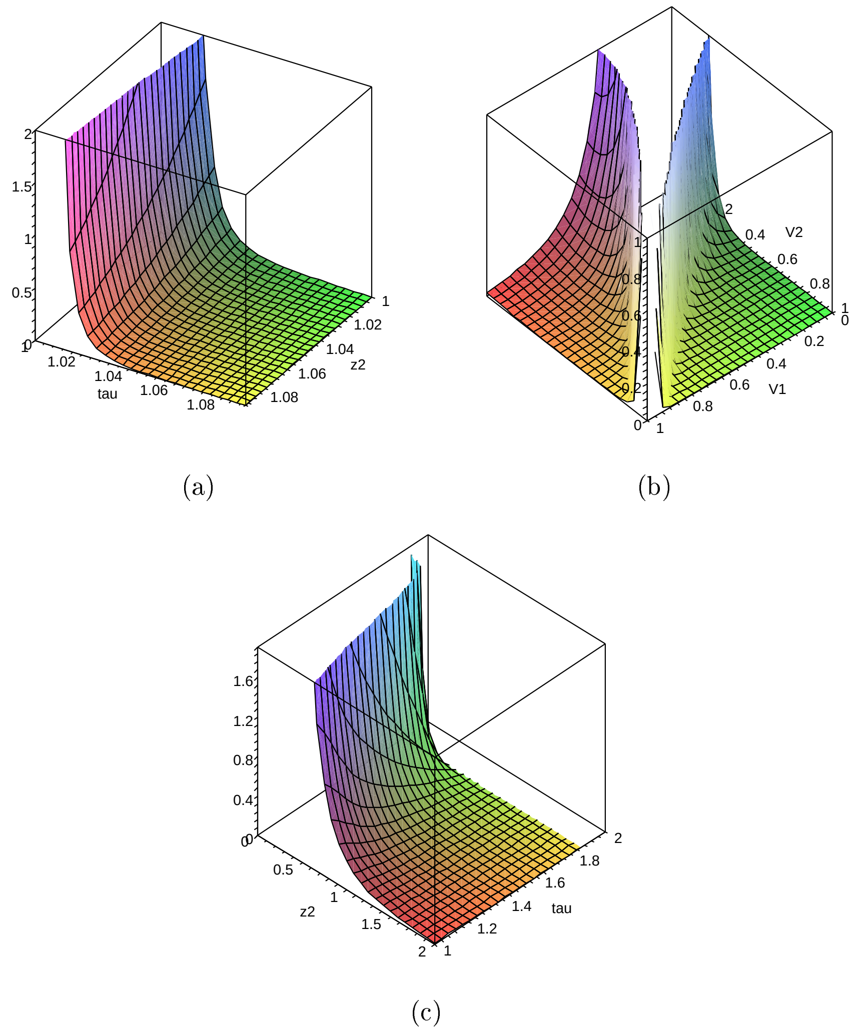
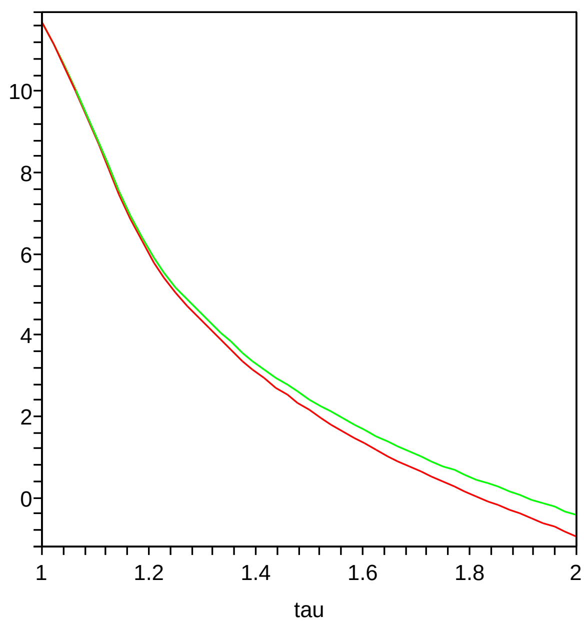


Fig. 8.2: (a) A 3D plot of the sum of the flat ghost potentials expressed in terms of z_2 and τ. (b) A 3D plot of the sum of the flat ghost potentials expressed in terms of V_1 and V_2. (c) A 3D plot of the warped total ghost Casimir energy (8.32) showing z_2 and τ directions.

τ	$\Delta G_c^\pi(\tau)$	$\Delta G_\eta^\pi(\tau)$
1.1	6980.571826	7407.466538
1.2	405.9747437	456.4954922
1.3	74.904937707	89.07993796
1.4	22.19804089	27.87835147
1.5	8.543430981	11.30532187
1.6	3.880314033	5.40405996
1.7	1.977367300	2.892661516
1.8	1.097112055	1.683017233
1.9	0.6495109547	1.043306447

Tab. 8.1: The total vacuum energy $\Delta G^\pi(\tau)$ for the two ghosts c and η evaluated numerically at different τ.



(a) The plot of $\ln \Delta G_c^\pi(\tau)$ (red) and $\ln \Delta G_\eta^\pi(\tau)$ (green).

Fig. 8.3:

boundary conditions and leads to zero vacuum energy of the gravitino but not for the ghost fields. We then calculated the ghost vacuum energy for the flat and curved space making use of our general calculations in chapter 4.

9. CONCLUSION AND FURTHER WORK

9.1 Conclusion

The final conclusion of the work done in this thesis can be summarized in the following main points:

- The total bulk Casimir potential, calculated in the framework of the improved 5D heterotic M-theory, does not have a minimum and it is unable alone to stabilize the radion field. Other bulk contributions must be considered to get a stabilization.

- Considering some non-perturbative effects, like gaugino condensates and others, an AdS supersymmetric minimum can be obtained which has to be raised to a dS stable minimum by adding a non-supersymmetric contribution. The non-supersymmetric contribution we considered in this thesis was the ghost field vacuum energy.

- The dimensional reduction of the gravitino field to 5D gives rise to two new ghost fields. The boundary conditions of the 5D reduced gravitino field can be expressed in terms of the gaugino condensate on the hidden brane.

- The gravitino field λ gives a zero contribution to the Casimir energy when the warping is small, and only its ghosts contribute to the Casimir energy. The twisted ghost fields lead to a positive vacuum energy.

- The contribution of the ghosts vacuum energy was too small to uplift the AdS minimum to a dS one in the case we examined and when the warping is small.

9.2 Further Work

The following work is recommended as a follow-up to this study:

- We can make more use of the deeply rich structure of the 5D reduced theory by studying the contributions from other hypermultiplets and the graviphoton to the bulk Casimir energy.

- For the gravitino and moduli masses, more can be done regarding the phenomenology. The MSSM soft masses are controlled by the F-terms, and then one expects the soft masses to be of the order of the gravitino mass. The moduli masses are found from derivatives of V_{SUGRA} at the minimum and are also within one-two orders of magnitude from $m_{3/2}$ [147]...

- In chapter 5, it remains to be seen how the other ingredients of low energy heterotic M-theory, which we have neglected, enter into the mix, for example five-branes and anti five-branes may play a role in a realistic model. Some features of the present calculation may be helpful in these generalisations. Expressing the superpotential in terms of other moduli systems like the five dimensional S and T superfields or the Calabi-Yau volumes V_1 and V_2 may be helpful. The inclusion of five-branes in the improved formalism for heterotic M-theory still remains to be developed.

- It really looks interesting to investigate the possible relation that might exists between the gaugino condensates we have studied in this thesis and the Bose Einstein condensates, this can shed more light on the connection between superfluidity and high energy physics; for example super Yang-Mills theory [149]...

APPENDIX

A. ENERGY, SCALES AND DIMENSIONS

eV (electron-Volt): The amount of energy gained by an electron dropping through a potential difference of one volt, which is 1.6×10^{-19} joules.

MeV (megaelectron-Volt): $10^6 eV$.

GeV (gigaelectron-Volt) scale: $10^9 eV$.

TeV (teraelectron-Volt) scale : $10^{12} eV$.

Plank scale: $1.22 \times 10^{19} GeV$.

Electroweak scale: $10^2 GeV$.

GUT scale: $10^{16} GeV$.

Dimensions:

Brane charge α: L^{-1}.

$\kappa_4^2 = 8\pi G$: L^2.

$\kappa_5^2 = 8\pi G_5$: L^3.

$\kappa_n^2 = 8\pi G_n$: L^{n-2}.

Bulk length scale ρ: L.

Energy: L^{-1}.

ds^2: L^2.

$d^n x\sqrt{|g|}$:L^n, n is the number of dimensions.

Ricci scalar R: L^{-2}.

cosmological constant Λ: L^{-2}.

Potential (energy/unit volume): $1/L^4$.

Moduli fields: All moduli fields are dimensionless and measure the form of the internal manifold relative to the dimensionful quantities α and ρ.

B. SPINOR IDENTITIES

B.1 Gamma matrices

Flat space Gamma matrices satisfy

$$\left\{\Gamma^A, \Gamma^B\right\} = 2\eta^{AB}. \tag{B.1}$$

B.1.1 Identities for the products of gamma matrices

$$\Gamma^{I_1 \ldots I_n}\Gamma_{J_1 \ldots J_m} = \sum_{r=0}^{min(n,m)} \binom{n}{r}\binom{m}{r}(-1)^{nr}\sigma_r r! \delta^{[I_1}_{[J_1} \ldots \delta^{I_r}_{J_r}\Gamma^{I_{r+1}\ldots I_n]}{}_{J_{r+1}\ldots J_m]}, \tag{B.2}$$

$$\Gamma^{I_1 \ldots I_n}\Gamma_{J_1 \ldots J_m}\Gamma_{I_1 \ldots I_n} = \sum_{r=0}^{min(n,m)} \binom{d-m}{n-r}\binom{m}{r}(-1)^{r+(m-1)n}\sigma_n \Gamma_{J_1 \ldots J_m}. \tag{B.3}$$

where $\sigma_r = (-1)^{r(r+1)/2} = +, -, -, +$ for $r = 0, 1, 2, 3$. In our 11D case, this gives for example

$$\begin{aligned}
\Gamma^I \Gamma_K &= \Gamma^I{}_K + \delta^I{}_K, \\
\Gamma^{IJ}\Gamma_K &= \Gamma^{IJ}{}_K - 2\delta_K{}^{[I}\Gamma^{J]}, \\
\Gamma_K \Gamma^{IJ} &= \Gamma^{IJ}{}_K + 2\delta_K{}^{[I}\Gamma^{J]}, \\
\Gamma^{IJK}\Gamma_L &= \Gamma^{IJK}{}_L + 3\delta^I{}_D \Gamma^{JK]}, \\
\Gamma_L \Gamma^{IJK} &= \Gamma_L{}^{IJK} + 3\delta^{[I}{}_L \Gamma^{JK]},
\end{aligned} \tag{B.4}$$

Results for contractions depend on the number of dimensions. In 11 dimensions,

$$\begin{aligned}
\Gamma^{IJK}\Gamma_K &= 9\Gamma^{IJ}, \\
\Gamma_L \Gamma^{LJK} &= 9\Gamma^{JK}, \\
\Gamma^{IJ}\Gamma_J &= 10\Gamma^I, \\
\Gamma^J \Gamma_J &= 11.
\end{aligned} \tag{B.5}$$

B.2 Other 11D identities

$$\begin{aligned}
\Gamma_I \Gamma^{IJKLMP} &= 6\Gamma^{JKLMP}, \\
\Gamma^{IJKLMP}\Gamma_P &= 6\Gamma^{IJKLM}, \\
\Gamma_I \Gamma^{IJKLMP}\Gamma_P &= 7\Gamma^{JKLM}, \\
\overline{\lambda}_I \Gamma^{IJKLMP} &= -\overline{\lambda}\Gamma^{JKLMP} - \overline{\lambda}^P \Gamma^{JKLM} - 4\overline{\lambda}^J \Gamma^{KLMP}, \\
\overline{\lambda}_I \Gamma^{IJKLM} &= -\overline{\lambda}\Gamma^{JKLM} - 4\overline{\lambda}^J \Gamma^{KLM}, \\
\Gamma^{JKLMP}\lambda_P &= \Gamma^{JKLM}\lambda - 4\Gamma^{JKL}\lambda^P, \\
\Gamma^{KLMP}\lambda_P &= \Gamma^{KLM}\lambda - 3\Gamma^{KL}\lambda^M.
\end{aligned} \tag{B.6}$$

with $\lambda \equiv \Gamma^I \lambda_I$.

B.3 Rarita-Schwinger equation

B.3.1 The pure fermionic term

In this appendix we derive Eq. (7.4) from (5.33). With the help of the 11D gamma matrices identities (B.4), we can write

$$\begin{aligned}
\overline{\psi}_I \Gamma^{IJK} D_J \psi_K &= \overline{\lambda}_I \Gamma^{IJK} D_J \lambda_K - \frac{1}{9}\overline{\lambda}_I \Gamma^{IJK} D_J \Gamma_K \left(\Gamma^L \lambda_L\right) \\
&+ \frac{1}{9}\left(\overline{\lambda}_M \Gamma^M\right)\Gamma_I \Gamma^{IJK} D_J \lambda_K - \frac{1}{81}\left(\overline{\lambda}_M \Gamma^M\right)\Gamma_I \Gamma^{IJK} D_J \Gamma_K \left(\Gamma^L \lambda_L\right)
\end{aligned} \tag{B.7}$$

where we used (7.3). Using (B.4) and (B.5), we get

$$\frac{1}{9}\overline{\lambda}_I \Gamma^{IJK} D_J \Gamma_K \left(\Gamma^L \lambda_L\right) = \left(\overline{\lambda}_L \Gamma^L\right)\left(\Gamma^J D_J\right)\left(\Gamma^K \lambda_K\right) - \overline{\lambda}^J D_J \left(\Gamma^K \lambda_K\right), \tag{B.8}$$

$$\frac{1}{9}\left(\overline{\lambda}_M \Gamma^M\right)\Gamma_I \Gamma^{IJK} D_J \lambda_K = \left(\overline{\lambda}_I \Gamma^I\right)\left(\Gamma^J D_J\right)\left(\Gamma^K \lambda_K\right) - \left(\overline{\lambda}_I \Gamma^I\right) D_J \lambda^J, \tag{B.9}$$

$$\frac{1}{81}\left(\overline{\lambda}_M \Gamma^M\right)\Gamma_I \Gamma^{IJK} D_J \Gamma_K \left(\Gamma^L \lambda_L\right) = \frac{10}{9}\left(\overline{\lambda}_I \Gamma^I\right)\left(\Gamma^J D_J\right)\left(\Gamma^L \lambda_L\right),$$

Inserting that in (B.7) yields

$$\overline{\psi}_I \Gamma^{IJK} D_J \psi_K = \overline{\lambda}_I \Gamma^J D_J \lambda^I + \frac{9}{4}(\overline{\psi}_I \Gamma^I)(\Gamma^J D_J)(\Gamma^K \psi_K). \tag{B.10}$$

B.3.2 The term containing ψ with G

The Lagrangian is

$$L_G = \frac{\sqrt{2}}{192} G_{JKLM} \left(\overline{\psi}_I \Gamma^{IJKLMP} \psi_P + 12 \overline{\psi}^J \Gamma^{KL} \psi_M \right) \tag{B.11}$$

Making use of the identities (B.6), the first term is

$$\begin{aligned} \overline{\psi}_I \Gamma^{IJKLMP} \psi_P &= \left(\overline{\lambda}_I + \frac{1}{9} \overline{\lambda} \Gamma_I \right) \Gamma^{IJKLMP} \left(\lambda_P - \frac{1}{9} \Gamma_P \lambda \right) \\ &= \left(-\overline{\lambda} \Gamma^{JKLMP} - \overline{\lambda}^P \Gamma^{JKLM} - 4 \overline{\lambda}^J \Gamma^{KLMP} \right) \lambda_P \\ &+ \frac{2}{3} \overline{\lambda} \Gamma^{JKLMP} \lambda_P - \frac{2}{3} \overline{\lambda}_I \Gamma^{IJKLM} \lambda - \frac{14}{27} \overline{\lambda} \Gamma^{JKLM} \lambda \\ &= -\frac{1}{3} \overline{\lambda} \left(\Gamma^{JKLM} \lambda - 4 \Gamma^{JKL} \lambda^M \right) - \overline{\lambda}^P \Gamma^{JKLM} \lambda_P \\ &- 4 \overline{\lambda}^J \left(\Gamma^{KLM} \lambda - 3 \Gamma^{KL} \lambda^M \right) + \frac{2}{3} \overline{\lambda} \Gamma^{JKLM} \lambda \\ &+ \frac{8}{3} \overline{\lambda}^J \Gamma^{KLM} \lambda - \frac{14}{27} \overline{\lambda} \Gamma^{JKLM} \lambda \\ &= -\overline{\lambda}_P \Gamma^{JKLM} \lambda_P - \frac{5}{27} \overline{\lambda} \Gamma^{JKLM} \lambda + 12 \overline{\lambda}^J \Gamma^{KL} \lambda^M . \end{aligned} \tag{B.12}$$

and the second term is

$$\begin{aligned} 12 \overline{\psi}^J \Gamma^{KL} \psi_M &= 12 \left(\overline{\lambda}^J + \frac{1}{9} \overline{\lambda} \Gamma^J \right) \Gamma^{KL} \left(\lambda^M - \frac{1}{9} \Gamma^M \lambda \right) \\ &= 12 \overline{\lambda}^J \Gamma^{KL} \lambda^M + \frac{4}{3} \overline{\lambda} \Gamma^{JKL} \lambda^M - \frac{4}{3} \overline{\lambda}^J \Gamma^{KLM} \lambda - \frac{4}{27} \overline{\lambda} \Gamma^{JKLM} \lambda \\ &= 12 \overline{\lambda}^J \Gamma^{KL} \lambda^M - \frac{4}{27} \overline{\lambda} \Gamma^{JKLM} \lambda . \end{aligned} \tag{B.13}$$

The Lagrangian then becomes

$$L_G = \frac{\sqrt{2}}{192} G_{JKLM} \left(-\overline{\lambda}^P \Gamma^{JKLM} \lambda_P - \frac{1}{3} \overline{\lambda} \Gamma^{JKLM} \lambda + 24 \overline{\lambda}^J \Gamma^{KL} \lambda^M \right) . \tag{B.14}$$

B.4 Six dimensional identities

The covariantly constant spinors are denoted by $u_\pm$

$$\begin{aligned} u_\pm^\dagger u_\mp &= 0, \quad u_\pm^\dagger u_\pm = 1, \quad \nabla u_\pm = 0, \quad u_\pm^\dagger \gamma_{abc} u_\pm = 0, \\ u_\pm^\dagger \gamma_{abc} u_\mp &= \pm i \varepsilon_{abc}, \quad \gamma_7 u_\pm = \pm u_\pm . \end{aligned} \tag{B.15}$$

$$\begin{aligned} u_\pm^\dagger \gamma^{ab\bar{c}\bar{d}} u_\pm &= \frac{1}{6}(g^{a\bar{c}} g^{b\bar{d}} - g^{a\bar{d}} g^{b\bar{c}}), \quad u_\mp^\dagger \gamma^{ab\bar{c}\bar{d}} u_\pm = 0, \\ u_\pm^\dagger \gamma_{a\bar{b}} u_\pm &= \mp g_{a\bar{b}}, \quad \gamma^a u_+ = 0, \\ \gamma^{a\bar{b}} u_\pm &= \pm g^{a\bar{b}} u_\pm, \gamma^{\bar{a}\bar{b}\bar{c}} u_+ = i\varepsilon^{\bar{a}\bar{b}\bar{c}} u_-, \quad \gamma^{abc} u_- = i\varepsilon^{abc} u_+, \\ \gamma^{abc} u_+ &= 0, \quad \gamma^{\bar{a}\bar{b}\bar{c}} u_- = 0, \\ \gamma^{a\bar{b}\bar{c}} u_+ &= 2g^{a[\bar{b}} \gamma^{\bar{c}]} u_+, \quad \gamma^a \gamma_a = 6P_+, \quad \gamma^{\bar{a}} \gamma_{\bar{a}} = 6P_-, \\ P_\pm u_\pm &= u_\pm, \quad P_\pm u_\mp = 0, \\ I u_\pm &= u_\mp, \quad J u_\pm = \mp i u_\mp, \quad K u_\pm = \pm u_\pm. \end{aligned} \tag{B.16}$$

where

$$\varepsilon_{\alpha\beta\gamma\delta\epsilon} \gamma^{\alpha\beta\gamma\delta\epsilon} = i5!, \quad \varepsilon_{\alpha\beta\gamma\delta\epsilon} \gamma^{\alpha\beta\gamma\delta} = i4! \gamma_\epsilon. \tag{B.17}$$

B.5 Five dimensional identities

A useful anticommutation relation for the 5-dimensional Γ^5 where $\Gamma^5 = N\gamma^5$ and N is 5D normal,

$$\begin{aligned} \{\not{N}, \not{\nabla}\} &= \Gamma^A \Gamma^B N_A \nabla_B + \Gamma^B \Gamma^A N_A \nabla_B \\ &= -2\left(\nabla_N + \frac{K}{2}\right). \end{aligned} \tag{B.18}$$

C. GEOMETRICAL CONVENTIONS

C.1 Differential Forms

A differential form of order r is a totally antisymmetric tensor of type $(0, r)$. If v is a $p-$form and w is a $q-$form, then

$$v = \frac{1}{p!} v_{a_1 \ldots a_p} dx^{a_1} \wedge \ldots dx^{a_p}. \tag{C.1}$$

The wedge product is defined as

$$(v \wedge w)_{a_1 \ldots a_b b_1 \ldots b_q} = \frac{(p+q)!}{p!q!} v_{[a_1 \ldots a_b} w_{b_1 \ldots b_q]}. \tag{C.2}$$

This implies

$$(v \wedge w) = (-1)^{pq} w \wedge p. \tag{C.3}$$

The exterior derivative operator d is defined as

$$d = dx^a \wedge \partial_a \tag{C.4}$$

Then

$$(dv)_{a_1 \ldots a_{p+1}} = (p+1) \partial_{[a_1} v_{a_2 \ldots a_{p+1}]} \tag{C.5}$$

$$d(v \wedge w) = dv \wedge w + (-1)^p v \wedge dw \tag{C.6}$$

A $p-$form ω is closed if $d\omega = 0$ and exact if $\omega = d\alpha = 0$ for some globally defined $p-1$ form α.

C.1.1 Cartan equations and the curvature tensor form

The curvature components of the metric (3.15) has been calculated using Cartan's structure equations which are

$$T^a = d\theta^a + \omega^a_b \wedge \theta^b \tag{C.7}$$

$$\Omega^a_b = d\omega^a_b + \omega^a_c \wedge \omega^c_b \tag{C.8}$$

For the torsion and curvature 2-form respectively. For the metric

$$ds^2 = e^{-2\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + d\tilde{z}^2 \tag{C.9}$$

we get

$$\begin{aligned} R_{5\mu5\nu} &= (\sigma^{''} - (\sigma^{'})^2)g_{\mu\nu}, \\ R_{\mu\nu} &= (\sigma^{''} - 4(\sigma^{'})^2)g_{\mu\nu}, \\ R_{55} &= 4(\sigma^{''} - (\sigma^{'})^2), \\ R &= 8\sigma^{''} - 20(\sigma^{'})^2. \end{aligned} \tag{C.10}$$

C.2 A review of complex manifolds and Kähler geometry

In analogy to the notion of a real $2k$-dimensional manifold M which is defined as a set of points that behaves locally like R^{2k}, such that $2k$ real parameters $(x^1, ..., x^\alpha, ..., x^{2k})$ are coordinates on M [34], we can define a complex q- dimensional manifold as a set of points that behaves locally like C^q. A complex manifold always admits a hermitian metric [118]. A Hermitian manifold is a complex manifold with a preferred coordinate systems such that

$$g_{ab} = g_{\bar{a}\bar{b}} = 0. \tag{C.11}$$

The line element then becomes

$$ds^2 = 2g_{a\bar{b}}dz^a dz^{\bar{b}}. \tag{C.12}$$

On any Hermitian manifold, a real 2-form can be defined such that

$$\omega = ig_{a\bar{b}}dz^a \wedge dz^{\bar{b}}. \tag{C.13}$$

where ω^α is defined to be a set of $2k$ complex coordinates where the index runs through the k holomorphic (unbarred) indices, then through the antiholomorphic (barred) indices. Now we can come to the definition of a Kähler manifold which is a Hermitian manifold whose 2-form is closed, i.e. $d\omega = 0$. In this case ω is called the Kähler form. That leads to the condition

$$\partial_a g_{b\bar{\rho}} - \partial_b g_{a\bar{\rho}} = 0, \tag{C.14}$$

which is used also to define a Kähler manifold. Now, suppose that a Hermitian metric $g_{\mu\bar{\nu}}$ is given in terms of a scalar function K by

$$g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K \tag{C.15}$$

This metric clearly satisfies the condition (C.14) and hence it is Kähler. It can be shown that any Kähler metric is locally expressed as (C.15). The scalar function K is called the Kähler potential of a Kähler metric.
Now, given a Ricci tensor $R_{\mu\overline{\nu}}$ of a Kähler manifold, the Ricci form is defined by

$$\Re = R_{a\bar{b}} dz^a \wedge dz^{\bar{b}} \tag{C.16}$$

The Ricci form is closed and defines a non-trivial element $c_1(M) \equiv \Re/2\pi$ which is called the first Chern class. A compact Kähler manifold with vanishing first Chern class is called a Calabi-Yau manifold. Equivalently, Calabi-Yau manifold is a Kähler manifold with Ricci flat metric.

C.2.1 Calabi-Yau space and Hodge numbers

Calabi-Yau manifolds have a cohomology groups structure that may be summed up by the so called Hodge diamond

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{1,2} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{C.17}$$

The Hodge numbers $h^{p,q}$ are the equivalent to Betti numbers for a real manifold. Formally, they are the dimensions of the respective cohomology groups the manifold admits, i.e.

$$h^{p,q} = \dim\ H^{p,q}. \tag{C.18}$$

So this diamond simply says that for a Calabi-Yau manifold we have:

• A single $(3,0)$ Hodge number $h^{3,0} = \dim\ H^{3,0} = 1$, This is the holomorphic volume form Ω, and $h^{3,0} = h^{0,3} = h^{0,0} = h^{3,3} = 1$.

• $h^{1,0} = h^{0,1} = h^{0,2} = h^{2,0} = h^{2,3} = h^{3,2} = h^{3,1} = h^{1,3} = 0$.

• The values of the remaining Hodge numbers $h^{1,1}$ and $h^{2,1}$ depends on the particular choice of the Calabi-Yau manifold.

C.3 The tetrad formalism

The description of gravity in terms of a metric tensor $g_{\mu\nu}$ is sufficient when the matter fields, to which gravity is coupled, are restricted to scalars, vectors and tensors. But when gravity is coupled to spinor fields, then the tetrad formulation of gravity is more convenient. The tetrad $e^{\widehat{a}}{}_{\mu}$ is connected to the metric by

$$g_{\mu\nu} = \eta_{\widehat{a}\widehat{b}}\ e^{\widehat{a}}{}_{\mu}\ e^{\widehat{b}}{}_{\nu} \tag{C.19}$$

Where the indices μ, ν,... label general coordinates with basis dx^{μ} and $\widehat{a}$, $\widehat{b}$,... label coordinates in a locally inertial coordinate system which we take as orthonormal frame. The Lorentz metric $\eta_{\widehat{a}\widehat{b}} = diag(+1,+1,...,-1)$. We have then orthonormal basis $\left\{e^{\widehat{a}} = e^{\widehat{a}}{}_{\mu}\ dx^{\mu}\right\}$ constructed by the vielbein field. The vielbein dual $e^{\mu}{}_{\widehat{a}}$ is its inverse so that

$$e^{\widehat{a}}{}_{\mu}\ e^{\mu}{}_{\widehat{b}} = \delta^{\widehat{a}}{}_{\widehat{b}}, \tag{C.20}$$

$$e^{\widehat{a}}{}_{\mu}\ e^{\nu}{}_{\widehat{a}} = \delta^{\nu}{}_{\mu}. \tag{C.21}$$

For the Calabi Yau metric in (5.9), we have

$$g_{a\bar{b}} = V^{1/3}\widetilde{g}_{a\bar{b}}. \tag{C.22}$$

Hence

$$e_a{}^{\hat{a}} = V^{1/6}\ \tilde{e}_a{}^{\hat{a}}, \quad e^a{}_{\hat{a}} = V^{-1/6}\ \tilde{e}^a_{\hat{a}}. \tag{C.23}$$

Since,

$$\Gamma_a = e_a{}^{\hat{a}}\ \Gamma_{\hat{a}}. \tag{C.24}$$

we arrive at Eqs. (5.38) and (5.39) for the factors of V in the reduction formulae for the gamma matrices.

C.4 Embedding hypersurfaces and ADM $(3+1)$ formalism in a nutshell.

For the sake of completeness, we summarize here the mathematical basics of the embedding hypersurfaces.

A hypersurface is an $(n-1)$ dimensional (co-dimension one) submanifold Σ of an n dimensional manifold M. In the ADM (Arnowitt, Deser and Misner) formalism, spacetime is decomposed into layers of three-dimensional space-like hypersurfaces (slices), threaded by a time-like normal

$$n^\mu = \frac{(1, -\beta^\mu)}{\alpha}. \tag{C.25}$$

where α and β^μ are the lapse function (defines the proper time between consecutive layers of spatial hypersurfaces) and shift vector (propagates the coordinate system from 3-surface to 3-surface) respectively. The general spacetime metric is written as

$$ds^2 = (-\alpha^2 + \beta_\mu\beta^\mu)dt^2 + 2\beta_\mu dx^\mu dt + \gamma_{\mu\nu}dx^\mu dx^\nu. \tag{C.26}$$

With $\gamma_{\mu\nu}$ is the induced spatial 3-metric on the hypersurface. It is related to the 4-metric via $\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$. Another concept that is closely related to the induced metric is called the projection tensor $\perp^\mu_\nu$ and defined as

$$g^{\mu\rho}\gamma_{\rho\nu} = \delta^\mu_\nu + n^\mu n_\nu \equiv \perp^\mu_\nu. \tag{C.27}$$

Given any vector $V^{\mu} \in T_P(M)$, the projection tensor can project it tangent to the hypersurface (that means orthogonal to n^{ν}):

$$\left(\perp_{\mu\nu} V^{\mu}\right) n^{\nu} = 0. \tag{C.28}$$

Using n^{μ},and assuming that the integral curves of n^{μ} are not geodesics ,we can define a quantitiy called 'the acceleration' as

$$a^{\mu} = n^{\nu} \nabla_{\nu} n^{\mu}. \tag{C.29}$$

Another quantity can be defined using n^{μ} which is the extrinsic curvature $K_{\mu\nu}$. If the embedded slide is bent, the normal vector n^{μ} changes along each coordinate. This is expressed by the non-vanishing of the covariant four derivative $\nabla_{\mu} n_{\nu}$. Then, the projection of this derivative is the change of the normal vector for an infinitesimal displacement within the surface and defines the extrinsic curvature tensor

$$K_{\mu\nu} = -\perp_{\mu}^{\alpha} \perp_{\nu}^{\beta} n_{\alpha;\beta} \tag{C.30}$$

Projecting all indices of the 4D Riemann tensor onto the slice gives the Gauss equation ($\perp$ denotes projection over all free indices)

$$\perp R^{(n+1)}_{\mu\nu\alpha\beta} = R^{n}_{\mu\nu\alpha\beta} + K_{\mu\alpha} K_{\nu\beta} - K_{\mu\beta} K_{\nu\alpha} \tag{C.31}$$

Contracting of one index with the normal vector and then subsequent projection of the remaining indices gives the Codacci equation

$$\perp R^{(n+1)}_{\mu\nu\alpha\beta} = D_{\nu} K_{\mu\alpha} - D_{\mu} K_{\nu\alpha} \tag{C.32}$$

Finally, Einstein equations could be written as Hamiltonian and momentum constraints:

$$R^{(n)} + K^2 - K_{\mu\nu} K^{\mu\nu} = 16\pi\rho \qquad \text{Hamiltonian constraint} \tag{C.33}$$

$$\nabla_{\mu} \left(K^{\mu\nu} - \gamma_{\mu\nu} K\right) = 8\pi j^{a} \qquad \text{momentum constraint} \tag{C.34}$$

Where ρ and j^{a} are matter terms given by projections of the stress energy tensor $T_{\alpha\beta}$.

Glossary of Terms

(Anti)de Sitter (AdS) a constant-curvature spacetime with maximal symmetry describing a positive (negative) cosmological constant.

AdS/CFT the conjecture of the equivalence between the gravity (string theory) on an AdS space and a CFT on its boundary.

Axion the RR scalar field of type IIB string theory that combines with the dilaton into a complex scalar controlling the $Sl(2, R)$ symmetry of the theory.

β-function a function giving the running of the coupling constant with the scale of the theory.

BPS solution a special type of supersymmetric solution.

Braneworld scenarios models in which matter fields are confined to a hypersurface within a higher-dimensional geometry.

BRST transformations (Becchi-Rouet-Stora-Tyutin) a fermionic invariance of the extended action. It is usually represented by a differential s.

Calabi-Yau a geometrical space with special properties (ie, a complex structure and vanishing Ricci tensor) normally used for compactification of string/M-theory down to four/five dimensions.

CFT (Conformal Field Theory) a conformally-invariant field theory.

Chern-Simons forms arise in gauge theories, although they are not themselves gauge invariant.

Compactification a procedure to reduce the number of dimensions by considering some of them to be compact and very small.

Conformal symmetry the group of transformations that leaves angles invariant.

$D = 11$ SUGRA eleven-dimensional supergravity theory considered as low-energy limit of M-theory.

D-brane a special case of a p-brane on which open strings can end.

Dilaton a scalar field in string theory whose vacuum expectation value controls the string coupling constant g_S.

Domain wall topological defect of co-dimension one, ie, an object separating the space (along one coordinate) into two disjoint regions.

Duality the property of two (apparently) different theories which describe the same physics for different values of their parameters.

Electroweak theory a theory unifies the electromagnetism and the weak interactions. The the unification energy is of order of 10^2 GeV above which they merge into a single electroweak force. Its gauge group is $SU(2) \times U(1)$.

Fixed-point solution SUGRA solution with constant scalars.

Gaugino the superpartner of the gauge boson.

Gaugino condensate Non-zero vacuum expectation value of the gaugino.

Gauged SUGRA theory of SUGRA containing (at least) some gauge vectors that serve to gauge some rigid symmetry of the ungauged version.

Gauge fixing procedure followed when eliminating undesired gauge degrees of freedom from a theory.

Ghost commutative fermion or spin $\frac{1}{2}$ boson.

Grand unification theory (GUT) theory that would incorporate the strong and electroweak force within on single theory.

Hadrons strong interacting particles (e.g., quarks, protons, neutrons, etc.).

Heterotic string consistent closed string theory supporting 16 supercharges and gauge group $SO(32)$ or $E_8 \times E_8$.

Hidden brane the brane at which SUSY breaking happens.

IR region (infrared) describes the behavior of a theory at large distances (small energies).

Israel Junction condition the discontinuity in the extrinsic curvature across a hypersurface is related to the energy momentum tensor on that hypersurface.

M2-brane fundamental object of M-theory extended in two spatial directions.

M5-brane the magnetic dual of a M2-brane.

Moduli space the space parametrized by the scalars (moduli) of the theory.

Modulus stabilization getting a minimum for the modulus potential.

Majorana spinors spinors constrained by a reality condition.

Majorana-Weyl spinors spinors with both Majorana and Weyl properties.

M-theory a quantum theory believed to describe all five string theories and $D = 11$ SUGRA as different limits.

Orbifold The resultant quotient space $\Gamma \equiv M/G$ with M is a manifold and G is a discrete group acts on M. The resultant space Γ has some singular points at which we locate the brane with matter (recall israel junction condition).

p-form a field described by a skew-symmetric tensor of rank p.

QCD (Quantum Chromodynamics) quantum field theory of the strong interactions, based on the gauge group $SU(3)$.

QED (Quantum Electrodynamics) unifying theory of weak and electromagnetic interactions, based on the gauge group $SU(2) \times U(1)$.

R-symmetry automorphism group of extended SUSY that rotates supercharges into each other.

RS scenario (Randall-Sundrum) a particular realization of braneworlds with one (or two) 3-brane(s) embedded in a five-dimensional space.

S-duality a duality relating the strong coupling regime of a theory with the weak coupling description of another, or the same, theory.

Self-duality property of some p-forms of having self-dual (under Hodge duality) field strength, realized in $D = 2$, $D = 6$ and $D = 10$ (for spaces with Minkowski signature).

Standard Model (still incomplete) a theory unifying all non-gravitational forces (strong and electro-weak). Its symmetry group is $U_1 \times SU(2) \times SU(3)$ and it is still incomplete.

String theory a theory of elementary particles where the fundamental constituents (e.g., the electron, the photon, etc.) are described as different vibration modes of a fundamental string.

Supergravity a supersymmetric version of general relativity (local supersymmetry includes gravity).

Superpotential function whose square and derivative squared determines the potential of a theory.

Supersymmetry a symmetry connecting bosons to fermions and vice versa. It implies the existence of a superpartner for each known elementary particle.

Susy breaking a necessary step from which a non-supersymmetric theory is obtained from a supersymmetric theory.

Type I string string theory of closed and open strings supporting 16 supercharges.

Type IIA string string theory of closed strings containing $N = 2$ MW spinors (32 supercharges) of opposite handedness.

Type IIB string string theory of closed strings containing $N = 2$ MW spinors (32 supercharges) with the same handedness.

UV region (ultraviolet) describes the behavior of a theory at small distances (large energies).

Visible brane The brane on which we are living, also called the TeV brane.

weak nuclear force one of the four fundamental forces, best known for mediating radioactive decay.

Weyl spinors spinors restricted via a chirality projection.

Yang-Mills theory Non-abelian Gauge theory based on the $SU(N)$ group. In other words, if the gauge group of the theory is non-commutative then the gauge theory is called Yang-Mills theory.

Notation

Action S, S_{EH}, S_{YM}, etc.

Antisymmetric rank-3 field $C_{\mu\nu\rho}$.

Bessel functions J_ν, Y_ν, I_ν and K_ν.

Beta function $\beta(g)$.

BRST differential s.

Calabi-Yau metric Ω_{AB} or $g_{a\bar{b}}$.

Calabi-Yau volumes V, V_1 and V_2.

chirality operator $P_\pm$ (untwisted) and $P_{\pm\chi}$ (twisted).

Condensate scale Λ.

Cosmological constant Λ.

Coupling constants g, g_s, α_G (for GUT) and λ(in RS S_{vis})

Covariant derivative ∇_μ, D_μ.

Covariantly constant spinors $u_\pm$, $A^\pm$ and $B^\pm$.

Dilaton ϕ.

Dirac operator D.

Energy-momentum tensor $T_{\mu\nu}$.

Einstein tensor $G_{\mu\nu}$

Extrinsic curvature $K_{\mu\nu}$.

Field strength $G_{\alpha\mu\nu\rho}$

Gamma matrices Γ and γ.

SUSY breaking parameters θ and χ.

Ghosts c and η.

Gravitino λ_α and ψ_α.

Graviton e^a_μ, $g_{\mu\nu}$, $h_{\mu\nu}$.

Kähler metric K_{IJ}.

Kähler potential K.

Lagrangian density L.

electromagnetic vector A_μ.

Planck

length $l_p \sim 10^{-33}$ *cm*

mass $M_p \sim 10^{19}$ *GeV*

Radion σ.

Renormalization scale μ_R.

Ricci scalar $\mathcal{R}$.

Scalar potential $V(\phi)$.

Superpotential W.

Superfields S and T.

Vacuum energy $V_C(0)$, $V_C(\theta)$ ΔV_c and ΔV_η.

Vielbeins $e_\mu{}^a$.

Wave function Ψ and ψ.

Zeta function ζ.

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