# The Value of Public Goods in Altruistic Societies 

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#### Abstract

This thesis is concerned with the valuation of public goods through the elicitation of preferences from citizens in society. This thesis explores theoretically and empirically the impact of different preference sets on the value of a public good, specifically when preferences are extended beyond one's own welfare. Three frames for eliciting preferences are considered: Consumer Frame 1, Consumer Frame 2, and the Citizen Frame. Consumer Frame 1 requires individuals to take a purely self-interested perspective. This represents the standard methodology. Consumer Frame 2 allows for altruistic preferences. The Citizen Frame, a novel methodology, places individuals behind a veil of ignorance based on Harsanyi (1953) to generate impersonal preferences from personal preferences. The frames are assessed for their impact on the societal value, individual values, and their appropriateness for cost-benefit analysis by respecting consumer sovereignty and satisfying the underlying preferences when aggregated.

The contributions made in thesis are split into two parts. Part I models the three frames using a model of optimal provision based on Jones-Lee (1991) to assess the societal value and a model of cost-benefit analysis based on Bergstrom (2006) to assess the individual value and the costbenefit test. In Part II, two laboratory experiments are used to test certain hypotheses from the theoretical chapters. The results of the experiments support the findings of the theory.

The findings of this thesis suggest that the use of Consumer Frame 1 in preference elicitation studies should be used with caution as they potentially violate consumer sovereignty. Whilst Consumer Frame 2 allows altruistic preferences into valuations and therefore respects consumer sovereignty, the values may not be compatible with the cost-benefit test. The Citizen Frame offers a novel methodology for individuals to consider both the distribution of benefits and costs to society and thus include preferences for distributive justice in their valuations.


## Dedication

To those who gave help along the way. Large or small, whether you knew it or not, your support has been much appreciated.

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## Declaration

I declare that this thesis is my own work. Some of the theoretical models presented in Chapters 3 and 4 have been combined with the experiment presented in Chapter 6 to form a paper published with my supervisors Susan Chilton, Hugh Metcalf, and Jytte Seested Nielsen as well as Michael Jones-Lee in the Journal of Risk and Uncertainty. As such the work has benefited from their comments. This has been noted and the paper referenced in the relative subsections in the body of the thesis.

## Table of Contents

Abstract ..... i
Dedication ..... ii
Acknowledgments ..... iii
Declaration ..... iv
Table of Contents ..... vi
List of Tables ..... x
List of Figures ..... xi
List of Abbreviations ..... xii
Chapter 1 Introduction ..... 1
Chapter 2 Background to The Value of Public Goods in Altruistic Societies ..... 7
$2.1 \quad$ Introduction ..... 7
2.2 The problem of public good provision ..... 8
2.3 Valuing public goods through cost-benefit analysis ..... 9
2.3.1 Compensation tests ..... 9
2.3.2 Willingness-to-pay ..... 10
2.3.3 Valuation studies ..... 11
2.4 Principles of welfare economics ..... 11
2.4.1 Proposition 1 ..... 12
2.4.2 Proposition 2 ..... 13
2.4.3 Proposition 3 ..... 13
2.5 Distributive justice ..... 14
2.6 Altruism and other-regarding preferences ..... 17
2.6.1 Theoretical literature ..... 19
2.6.2 Valuation Studies measuring altruism ..... 21
2.6.3 Other-regarding preferences ..... 22
2.7 Incorporating different preference sets into WTP values ..... 23
2.8 Summary and research direction ..... 26
PART I THEORY ..... 29
Chapter 3 Consumer Frames 1 and 2 ..... 33
3.1 Introduction ..... 33
3.2 Optimal provision of public safety expenditure ..... 35
3.2.1 Optimal provision under first-best taxation ..... 36
3.2.2 Optimal provision under uniform taxation ..... 37
3.2.3 Optimal provision under proportional taxation ..... 38
3.2.4 Applying altruism to the general solutions ..... 39
3.2.5 Simulations ..... 44
3.2.6 Summary ..... 56
3.3 Willingness-to-pay ..... 57
3.3.1 The naïve cost-benefit analyst ..... 57
3.3.2 Willingness-to pay with a first-best tax vehicle ..... 59
3.3.3 Willingness-to pay with alternative tax vehicles ..... 60
3.4.2 Safety example ..... 64
3.4 Compensation tests ..... 68
3.4.1 Model ..... 68
3.4.2 Example ..... 69
3.5 Consumer Frames ..... 71
3.5.1 Consumer Frame 1 ..... 71
3.5.2 Consumer Frame 2 ..... 72
3.6 Summary ..... 73
Chapter 4 Citizen Frame ..... 77
4.1 Introduction ..... 77
4.2 Optimal provision of public safety expenditure ..... 78
4.2.1 Optimal provision under first-best taxation ..... 80
4.2.2 Optimal provision under uniform taxation ..... 80
4.2.3 Optimal provision under proportional taxation ..... 81
4.2.4 Applying altruism to the general solutions ..... 82
4.2.5 Simulations ..... 87
4.3 Cost-benefit analysis ..... 97
4.3.1 Model ..... 97
4.3.2 Altruism ..... 99
4.3.3 General Example ..... 100
4.3.4 Safety Example ..... 101
4.4 Citizen Frame ..... 106
4.5 Summary ..... 107
PART II EMPIRICAL ANALYSES ..... 109
Chapter 5 Experiment A: An Experiment Comparing the Value of Group ..... 111 Insurance in the Consumer and Citizen Frames
5.1 Introduction ..... 111
5.2 Random Price Voting Mechanism ..... 112
5.3 Experiment overview ..... 115
5.3.1 Scenarios ..... 116
5.3.2 Altruism ..... 119
5.3.3 Inequality Aversion ..... 124
5.3.4 Predictions ..... 129
5.4 Experimental Design ..... 133
5.4.1 Treatments ..... 133
5.4.2 Estimation strategy ..... 134
5.4.3 Protocol ..... 134
5.5 Results ..... 135
5.5.1 Sample and demographics ..... 135
5.5.2 Altruism and the effect of skewed distributions ..... 138
5.3.3 The VoI effect ..... 139
5.6 Summary ..... 140
Chapter 6 Experiment B: An Experiment Testing the Taxation Hypothesis ..... 143
6.1 Introduction ..... 143
6.2 Model ..... 144
6.2 .1 ..... 144
6.2.2 ..... 145
6.2.3 ..... 146
6.3 Experimental design ..... 150
6.3.1 Tax Extension and protocol ..... 150
6.3.2 Treatments ..... 152
6.3.3 Estimation strategy ..... 153
6.4 Results ..... 156
6.4.1 Sample and demographics ..... 156
6.4.2 In front of a VoI ..... 158
6.4.3 Behind a VoI ..... 158
6.5 Summary ..... 159
Chapter 7 Discussion ..... 163
7.1 Introduction ..... 163
7.2 Part I Summary ..... 165
7.2.1 Optimal Provision ..... 165
7.2.2 Cost-benefit Analysis ..... 168
7.3 Part II Summary ..... 169
7.4 Societal value of the provision of public goods ..... 171
7.5 Individual values for the provision of public goods ..... 171
7.6 Incorporating values into CBA ..... 173
Chapter 8 Conclusion ..... 177
References ..... 180
Appendices ..... 186
Appendix A Solutions for the first-best case of the optimal provision model ..... 186 presented in Chapter 3.
Appendix B The interaction between taxation and altruism on WTP for the middle ..... 188 cases of altruism.
Appendix C Solutions for the first-best case of the optimal provision model ..... 189 presented in Chapter 4.
Appendix D Experiment A materials ..... 190
Appendix E Experiment B materials ..... 207

## List of tables

Table 3.1: Model solutions for optimal provision under first-best, uniform, and proportional wealth taxation with a general utility function, SI, PA, SFA, and WFA preferences.
Table 3.2: Model solutions for WTP for different forms of altruism. ..... 62
Table 4.1: Model solutions for optimal provision behind a VoI under first-best, uniform, and proportional wealth taxation with a general utility ..... 83 function, SI, PA, SFA, and WFA preferences.
Table 4.2: Model solutions for optimal provision for different forms of altruism under different systems of taxation behind a VoI for the two-person case with altruism entering linearly into utility functions.
Table 4.3: Example CBA behind a VoI. ..... 101
Table 5.1: WTP Predictions in front and behind a VoI with SI, PA, MM, and Quasi-MM preferences and risk neutral preferences.
Table 5.2: WTP Predictions in front and behind a VoI with FS-IA and BO-IA preferences and risk neutral preferences.
Table 5.3: Example group types with parameters and $\Delta \mathrm{WTP}$ predictions for PA, MM, Quasi-MM, FS-IA, and BO-IA preferences.
Table 5.4: Treatments. ..... 133
Table 5.5: Sample demographics split by country. ..... 136
Table 5.6: T-test results by question type. ..... 139
Table 6.1: Model predictions for SI, PA, and IA preferences. ..... 150
Table 6.2: Experimental parameter setup. ..... 153
Table 6.3: OLS regression results for WTP from Positions S (Equation 6.7) and L (Equation 6.8), Aggregate WTP (Equation 6.9) and MAC behind a VoI ..... 159 (Equation 6.10).
Table 7.1: Summary of Part I results: the value of a public good relative to the SI level. ..... 167
Table 7.2: Summary of Part II results. ..... 170

## List of figures

Figure 3.1: Optimal provision and the resulting social welfare for SI preferences at $\mathrm{w} 1=9$, $\mathrm{w} 2=11$.

Figure 3.2: Optimal provision for SI preferences at $\mathrm{w} 1=8 \mathrm{w} 2=12$; w $1=9, \mathrm{w} 2=11$; and $\mathrm{w} 1=\mathrm{w} 2=10$.

Figure 3.3: Optimal provision and the resulting social welfare for all preference types at $\mathrm{w} 1=10$, $\mathrm{w} 2=10$.

Figure 3.4: Optimal provision for PA preferences at $\mathrm{w} 1=10, \mathrm{w} 2=10$ with all strengths of altruism.

Figure 3.5: Optimal provision for PA preferences at $\mathrm{w} 1=8 \mathrm{w} 2=12$; $\mathrm{w} 1=9$, $\mathrm{w} 2=11$; and $\mathrm{w} 1=\mathrm{w} 2=10$ with Individual b as the altruist at $\gamma_{b}=0.2$.

Figure 3.6: Optimal provision for SFA preferences at $\mathrm{w} 1=10$, $\mathrm{w} 2=10$ with all strengths of altruism.

Figure 3.7: Optimal provision for SFA preferences at $\mathrm{w} 1=8 \mathrm{w} 2=12$; $\mathrm{w} 1=9$, $\mathrm{w} 2=11$; and $\mathrm{w} 1=\mathrm{w} 2=10$ with Individual b as the altruist at $\gamma_{b}=0.2$.

Figure 3.8: Optimal provision for WFA preferences at $\mathrm{w} 1=10, \mathrm{w} 2=10$ with all strengths of altruism.

Figure 3.9: Optimal provision for WFA preferences at $\mathrm{w} 1=8 \mathrm{w} 2=12$; $\mathrm{w} 1=9$, $\mathrm{w} 2=11$; and $\mathrm{w} 1=\mathrm{w} 2=10$ with Individual b as the altruist at $\gamma_{b}=0.2$.

Figure 3.10: WTP for SI, PA, SFA, and WFA as tax share varies.63

Figure 3.11: WTP for safety improvements with $w_{a}=10, s=5, \gamma=0.2$. 65
Figure 3.12: WTP for safety improvements varying with $\gamma$. 66

Figure 3.13: WTP for safety improvements varying with $s$. 66

Figure 3.14: WTP for safety improvements varying with $w_{a}$.67

Figure 3.15: Aggregate WTP and compensation as tax share varies 70

Figure 4.1: Optimal provision and the resulting social welfare for all preference types with person-based altruism at w1=10, w2=10 both in front and behind a VoI.
Figure 4.2: Optimal provision and the resulting social welfare for all preference ..... 91 types with distributional altruism at $\mathrm{w} 1=10, \mathrm{w} 2=10$ both in front and behind a VoI.
Figure 4.3: Optimal provision and the resulting social welfare for PA preferences with person-based altruism for each wealth combination both in front and behind a VoI.
Figure 4.4: Optimal provision and the resulting social welfare for PA preferences with distributional altruism for each wealth combination both in front and behind a VoI.
Figure 4.5: Optimal provision and the resulting social welfare for SFA ..... 95 preferences with person-based altruism for each wealth combination both in front and behind a VoI.
Figure 4.6: Optimal provision and the resulting social welfare for SFA ..... 95 preferences with distributional altruism a for each wealth combination both in front and behind a VoI.
Figure 4.7: Optimal provision and the resulting social welfare for WFA ..... 96 preferences with person-based altruism for each wealth combination both in front and behind a VoI.
Figure 4.8: Optimal provision and the resulting social welfare for WFA ..... 96 preferences with distributional altruism for each wealth combination both in front and behind a VoI.
Figure 4.9: Aggregate WTP and MAC values for safety improvements with $w_{a}=$ ..... 103 $10, s=5, \gamma=0.2$
Figure 4.10: Aggregate WTP and MAC values for safety improvements varying ..... 104 with $\gamma$.
Figure 4.11: Aggregate WTP and MAC values for safety improvements varying ..... 105 with $s$.
Figure 4.12: Aggregate WTP and MAC values for safety improvements varying ..... 105 with $w_{a}$.
Figure 5.1: Figure 5.1: Histogram of $\triangle W T P$ from Position A ..... 137
Figure 5.2: Histogram of $\triangle W T P$ from Position B ..... 137
Figure 5.3: Histogram of $\triangle W T P$ from Position C ..... 137
Figure 5.4: Histogram of $\triangle W T P$ from behind a VoI ..... 137
Figure 6.1: Net WTP against tax distortion for Position S with fractional- ..... 156 polynomial line of best fit with $95 \%$ confidence intervals
Figure 6.2: Net WTP against tax distortion for Position L with fractional- ..... 157 polynomial line of best fit with $95 \%$ confidence intervals
Figure 6.3: Net MAC against tax distortion for MAC value behind a VoI with ..... 157 fractional-polynomial line of best fit with $95 \%$ confidence intervalsFigure B1: WTP for all cases of pure and paternalistic altruism 186
Figure D1: Experiment A: Example decision screen for respondents in front of a ..... 190 ..... VoI
Figure D2: Experiment A: Example decision screen for respondents behind a VoI ..... 191
Figure E1: Experiment B: Example decision screen for respondents in front of a ..... 207
VoI
Figure E2: Experiment B: Example decision screen for respondents behind a VoI ..... 208

## List of Abbreviations

| BDM | Becker-DeGroot-Marschack |
| :--- | :--- |
| BO-IA | Bolton-Ockenfels Inequality Aversion |
| CBA | Cost-benefit analysis |
| EL | Expected Loss |
| IA | Inequality Aversion |
| FS-IA | Fehr-Schmidt Inequality Aversion |
| MAC | Maximum Acceptable Cost |
| MM | Maxi-min |
| PA | Pure Altruism |
| RPVM | Random Price Voting Mechanism |
| SFA | Pure Safety-Focused Altruism |
| SI | Pure Self-interest |
| WFA | Pure Wealth Focused Altruism |
| WTA | Willingness-to-Accept |
| WTP | Willingness-to-Pay |
| vNM | von Neumann Morgenstern |
| VoI | Veil of Ignorance |
| VSL | Value of a Statistical Life |

## CHAPTER ONE:

## Introduction

This thesis is concerned with the valuation of public goods through the elicitation of preferences from citizens in society. Public goods are goods provided to all members of a society, often by its government. Public goods are defined by two features: non-excludability and non-rivalry. Non-excludability means anyone can consume the public good. Non-rivalry means one individual's consumption of the public good does not preclude another's consumption of it. When provided by the government, public goods are most often funded through coercive taxation. The public decision-maker, acting on behalf of the government, then faces two decisions: choosing the level of provision and choosing the level of taxation best for society.

As provision is chosen for all individuals collectively, individuals cannot necessarily maximise utility as they can with private consumption alone. This results in the sovereignty of the consumer to choose on behalf of themselves coming into question. The choice over levels of provision and taxation results in the potential for trade-offs between different individuals' welfare. This involves decisions over fairness and distributive justice. Those being decisions over which people could reasonably disagree. It is then of great concern to the government that the value of a public good is openly and accurately assessed to ensure a fair and democratic process that leads to efficient outcomes.

One method for assessing the value of a public good is cost-benefit analysis (CBA). Based on the compensation test introduced by Hicks (1939) and Kaldor (1939), CBA is used to identify a set of proposals for changes to public good provision that could improve social welfare. By eliciting the values of individuals in society, the preferences of those individuals can be understood and a measure of changes to societal welfare can be generated. By improving the
measurement of the value individuals in society place on public goods and the process of incorporating those values into the decision-making process, public decision-making becomes more efficient by truly reflecting the wants of society.

Value can be attributed to a public good for a number of reasons. One distinction in the valuation literature is the difference between a consumer and citizen perspective. Individuals could act as pure consumers who value their own consumption of private and public goods. Alternatively, individuals could act as pure citizens who value the welfare of society based on concerns of distributive justice. Sagoff (1988) and Blamey et al. (1995) discuss the difference between consumers and citizens when valuing the environment and Orr (2007) considers the difference between citizens and consumers in relation to CBA. Concerns over distributive justice have been used to criticise the standard consumer focussed approach of CBA. For example, Little (1949) objected to the compensation test as a welfare criterion as it defines efficiency improvements but gives no consideration to equity.

There is also the potential for middle cases between pure consumer and pure citizen for which individuals have preferences over their own welfare and altruistic preferences over others' welfare. Empirical studies measuring the impact of altruistic preferences have shown that given the opportunity respondents will take into account others in their responses. The inclusion of different preferences could feasibly lead to increased (Viscusi et al. (1988); Arana and Leon (2002)) or decreased (Johannesson et al. (1996); de Blaeij et al. (2003); Hultkrantz et al. (2006); Andersson and Lindberg (2009); Svensson and Johansson (2010); Gyrd-Hansen et al. (2016)) values placed on a proposed change to provision relative to self-interested values alone.

It is important to understand how different preference sets impact the societal and individual values placed on public goods. Ultimately, this could lead to different recommendations once used in the CBA process. As values are based upon different preferences, it is important that the process respects the preferences and values of citizens. Further to this, the inclusion of these preferences in the cost-benefit calculation must lead to efficient outcomes. That is the outcome of the decision-making process satisfies the preferences of the individuals in question.

There is some debate as to whether, and how, sympathy towards others should be included in valuations. It has long been recognised in the value of safety literature that preferences over other's safety should be included in values (Mishan (1971), Needleman (1976), Jones-Lee (1980, 1991, 1992), and Bergstrom (1982)). However, other authors such as Milgrom (1993)
disagree on the grounds that the resulting outcomes of a cost-benefit calculation are inefficient when altruistic values are included.

This thesis explores both theoretically and empirically the inclusion of different preference sets into societal-level values, individual values, and into CBA. Three alternate preference sets are identified and studied. The first is a purely self-interested consumer. This is termed Consumer Frame 1 and reflects by the standard methodology. The second is an altruistic individual who acts on preferences over both their own and others' welfare. This is termed Consumer Frame 2. The third is an impartial citizen who acts on preferences over distributive justice. The citizen is operationalised by placing an individual behind a veil of ignorance (VoI), as described by John Harsanyi (1953) and John Rawls (1972), thereby generating impersonal preferences from personal preferences. This is termed the Citizen Frame.

There are three key elements of the decision-making process that require study:
[1] The impact of different preference sets on the societal value.
[2] The impact of different preference sets on individual values.
[3] The use of these values in CBA. Two questions are asked. First, does the method of elicitation respect the individual's sovereignty by respecting their preferences? Second, does the aggregation of these preferences pass a compensation test?

There are two parts to this thesis: Part I presents the theoretical chapters, and Part II presents the empirical chapters. Part I models the three frames which are outlined in Chapter 2: Consumer Frame 1, Consumer Frame 2, and the Citizen Frame. The frames are modelled under two different scenarios. A model of optimal provision based on Jones-Lee (1991) extended to consider the societal value of a public good under different systems of taxation. A model of CBA based on Bergstrom (2006) is used to model individual values under different payment vehicles and the compensation test.

Chapter 3 models the Consumer Frames. Consumer Frame 1 is shown to be efficient for use in a compensation but does not respect consumer sovereignty if respondents are altruistic. Under Consumer Frame 2, optimal provision is shown to either increase or decrease based on the form altruism takes and the tax system in place. This frame does respect consumer sovereignty; however, it is shown to not necessarily be efficient when used in a compensation test. The
problem is that if individuals care for the cost borne by others then elements of the cost-side of the cost-benefit calculation are brought into the benefits-side.

The Citizen Frame is presented as a novel mechanism for eliciting societal values in Chapter 4. The Citizen Frame is shown to respect consumer sovereignty and to remove altruism directed at individuals whilst preferences for the consumption of the public good and over distributive justice remain.

Part II of this thesis tests empirically the respondent frames set out in Part I using laboratory experiments. The findings of two experiments are presented. Hypotheses tested in both experiments are developed from the theoretical contributions of Part I. The first experiment, Experiment A, compares individual values in front and behind a VoI. The second experiment, Experiment B, tests the effect of payment vehicle on individual values in front and behind a VoI.

Experiment A presented in Chapter 5 elicits WTP under uniform taxation in front and behind a VoI. The design extends that of Messer et al. (2013) to include a VoI. Results show that choice in front of a VoI is best represented by pure altruism (PA) which replicates the findings of Messer et al. (2010) and Messer et al. (2013). Behind a VoI, choice is indistinguishable between pure self-interest (SI) and PA as the model finds. These findings confirm the results of the models presented in Chapters 3 and 4.

Experiment B presented in Chapter 6 elicits individual values under first-best, uniform, proportional income and progressive income taxation in front and behind a VoI. The design extends that of Messer et al. (2010) and Experiment A to allow for a range of cost-sharing rules. Results show that choice in front of a VoI that the payment vehicle used has an effect on values as the model in Chapter 3 finds. Behind a VoI, the payment vehicle has no effect on individual values as the model in Chapter 4 finds.

Overall, the empirical findings of this thesis support the results of the theoretical modelling. The findings of the experiments show that individuals do take into account others in their valuations. This suggests that the use of Consumer Frame 1 in preference elicitation studies should be used with caution as they potentially ignore altruistic preferences. Whilst Consumer Frame 2 allows these altruistic preferences into valuations, the theory presented in Chapter 3 show that these values may not be compatible with the compensation test. Experiment B shows that individuals care about both the benefits and costs resulting from a proposal. As such whether, or not, compensation is actually paid matters to the value of the public good.

Individuals may then share the same concerns as Little (1949). The Citizen Frame offers a novel methodology for individuals to consider both the distribution of benefits and costs to society and thus include preferences for distributive justice in their valuations.

## CHAPTER TWO:

## Background to The Value of Public Goods in Altruistic Societies

### 2.1 Introduction

This chapter describes the concepts that underpin this thesis and discusses the existing literature before setting out the research agenda for this thesis. Sections 2.2-2.4 ${ }^{1}$ discuss the process of valuing public goods. Section 2.2 describes public goods and the problem faced by the public decision-maker. Section 2.3 describes the process of decision-making using CBA. Section 2.4 discusses CBA in relation to the principles of welfare economics which underpin CBA and provides the framework to discuss social welfare. Here criticisms of CBA for its limited focus are discussed.

Sections 2.5 and 2.6 discuss determinants of value beyond private consumption. Section 2.5 discusses the concept of distributive justice. Impartiality and the VoI are considered as a framework by which questions of distributive justice can be approached. Section 2.6 discusses the debate over the inclusion of altruistic preferences into values for public goods. This is supported by a review of the theoretical and empirical literature studying the incorporation of altruistic preferences into individual values and CBA.

Sections 2.7 and 2.8 set out the research agenda. Section 2.7 introduces three frameworks through which preferences over public good provision can be elicited to include altruism and distributive justice. Section 2.8 summarises and sets out the research agenda for this thesis.

[^0]
### 2.2 The problem of public good provision

Public goods are goods provided to all members of a society, often by its government. Public goods are defined by two features: non-excludability and non-rivalry. Non-excludable means that when an individual purchases a public good, that individual pays for it, and anyone can consume it. Non-rivalry means one individual's consumption of the good does not limit another's consumption of the good. This leads to the classic problem in public good provision called the free-rider problem where no individual has the incentive to privately provide a public good. Instead every individual relies on others' provision and the public good is not provided. To avoid the free-rider problem, public goods are often provided by the government and funded through coercive taxation. Under a coercive tax system, contributions to funding the public good are then also non-excludable.

When choosing the provision of a public good, the public decision-maker, acting on behalf of the government, faces two decisions. The first is determining the level of provision best for society. Some individuals may value the consumption of the public good highly and therefore prefer relatively high levels of provision, whilst others may not value the consumption of the public good highly and therefore prefer relatively low levels of provision. The second problem is optimally generating funds through a system of taxation. With private consumption, individuals will choose their level of consumption based in their relative preferences for the available goods, the relative prices of the good and their income constraints. When public goods are provided and paid for through taxation the sovereignty of the consumer to choose on behalf of themselves comes into question as neither the quantity consumed nor the price (tax) are under the individual's control. Individuals cannot necessarily maximise utility as they can with private consumption alone.

Choosing a level of provision and taxation results in the potential for required trade-offs between individuals' welfare. Choosing a higher level of provision may satisfy one set of individuals with relatively strong preferences for the public good, but may lead to the dissatisfaction of another set of individuals who have weak preferences for the public good relative to private consumption and would ultimately still bear the cost of increased taxation to fund provision. The objective of the public decision-maker is to maximise the welfare of society. Defining social welfare is a problem as it is an inherently normative question as it involves decisions over fairness and distributive justice. As the problem is a normative one, it is reasonable that different individuals could disagree over the best course of action. It is then
of great concern to the government that the value of a public good is openly and accurately assessed to ensure a fair and democratic process which leads to efficient outcomes.

One method for assessing the value of a public good, CBA, is used to identify a set of proposals for changes to public good provision that improve social welfare. By eliciting the values from individuals in society, the preferences of those individuals can be understood and a measure of changes to societal welfare can be generated.

### 2.3 Valuing public goods through cost-benefit analysis

CBA is a test comparing the benefits of a proposal to the costs. That could be thought of as a proposal to move to a new state of the society, State Y, from an alternate state, State X, which is usually the current state. If the benefits of the proposal exceed the costs, then the proposal is deemed efficient i.e. State $Y$ is preferable from a societal perspective to State $X$, the status quo.

There are three elements of CBA to discuss in detail. [1] The compensation test, which compares the sum of costs to individual and the sum of benefits to individuals. Under this test, individuals either require compensation against losses or can pay compensation from gains associated with a change in the state of the world. This provides the theoretical underpinnings of CBA. [2] Willingness-to-pay (WTP) which provides a practical method for eliciting an individual's preferences as a money measure of benefits and costs. [3] Valuation studies which are used for eliciting WTP values from members of society.

### 2.3.1 Compensation tests

The compensation test, introduced by Kaldor (1939) and Hicks (1939), is essentially a test of efficiency. Kaldor suggested that if the 'gainers' from a proposal can compensate the 'losers' and still be better off, then the proposition is a good one. Hicks (1939) continued this idea looking closer at the trade-off of efficiency and distribution. The compensation test ${ }^{2}$ is based on the Pareto criterion which states a social welfare improvement is achieved if at least one member of society can be made better off without any being made worse off. The compensation

[^1]test turns the Pareto criterion into a potential one, as everyone could potentially be better off, and would be if the compensation is transferred from the 'gainers' to the 'losers'. An improvement based on the Pareto criterion is called a Pareto improvement and an improvement based on the compensation test is then called a potential Pareto improvement.

Scitovsky (1941) identified a paradox where the compensation test could allow for the reversal of compensation so that the move back to the initial state is identified as a good proposal. That is, the compensation test does not resolve indifferences between two states. The reversal paradox causes trouble as the 'losers' could bribe the 'gainers' to not take any action, which in turn leads to transitivity problems between social states. To avoid this issue the Scitovsky criterion requires that for a new State Y to be identified as better than the status quo, State X , the compensation test must be satisfied for the shift from State X to Y but cannot be applied in reverse to show that the shift from State Y to X is also acceptable.

### 2.3.2 Willingness-to-pay

The compensation test relies on utility differences for individuals between states. This creates difficulty when putting the compensation test into practice as utility is not necessarily comparable across individuals. The changes in utility can be measurable by using a numeraire, the most obvious being money. WTP is used as a money measure of the difference in utility levels between two states and is based on the concept of consumer surplus ${ }^{3}$ as defined in Hicks (1943). WTP is the change in wealth required to make an individual as well off in a new state, State Y which all other things being equal is preferred, as the current state, State X , whilst accounting for any changes in prices. ${ }^{4}$ In simple terms the change in wealth required to maintain the same level of utility. This method of measuring benefits is then effective for CBA, as WTP elicited from the society of affected individuals, aggregated, and compared to the cost to complete a compensation test.

[^2]
### 2.3.3 Valuation studies

Valuation studies are used to elicit the WTP values for different levels of provision from a sample of the population. ${ }^{5}$ Valuation studies are most commonly used to value goods which do not have a market, and thus do not have a price. If a market does exist, then preferences for the good are revealed by market demand and thus called revealed preferences. Without a market, preferences must be elicited by respondents stating their choices based on a hypothetical market. These are called stated preferences. Common examples of goods which are valued using stated preferences include mortality risk reductions, environmental risk, and health as these goods do not always have markets.

### 2.4 Principles of welfare economics

CBA is underpinned by the principles of Welfare Economics, and thus analyses of the CBA process: compensations tests, WTP, and valuation studies, should be informed by these principles. Clear definitions of the principles of welfare economics are key to proposing the circumstances in which one allocation of public funds is better than another allocation. The objective of welfare economics is to explore the implications of social choice based on sets of value judgements. That is to describe what is optimal for a society.

To continue the example from the previous section, under what conditions can it be concluded "State X is better than State Y for society"? Say there are two individuals in the society and Individual A is better off in State X than Y and Individual B is better off in State Y than X . To give a ranking over the two states requires a trade-off between the two individuals. It is not clear as to which state is better for society as a value judgement is required. Because in this scenario two reasonable people could come to different conclusions. It cannot be concluded unequivocally that one state is better than the other because individuals have different preferences over distributions.

Welfare economics ranks the states based on different sets of values, and thus lies between positive and normative as a field of study. It is argued that as few value judgements are to be

[^3]made as possible by relying on the preferences of the individuals in society to make any normative prescriptions (Sugden, 1981).

Sugden (1981) describes two approaches to welfare economics:

The Paretian Approach (named after Vilfredo Pareto, but also attributed to Abram Bergson and Paul Samuelson) provides a number of propositions by which states of the world can be compared, and

The Contractual Approach (based on social contract theory whose history begins with Thomas Hobbes, John Locke, and Jean-Jacques Rousseau) considers individuals in society to be contracting parties deciding from an original position.

This thesis adopts the Paretian approach for ranking states of the world, however, addresses some of the criticisms and aims to incorporate features of the Contractual Approach. Mishan (1960) sets out three propositions on which the Paretian approach is founded:
[1] Each individual is the best judge of their own welfare.
[2] Social welfare is dependent only on the welfare of those in society.
[3] A Pareto improvement implies a social welfare improvement.

### 2.4.1 Proposition 1

The first proposition asserts that if an individual prefers State X to State Y then this implies that X is better for the individual than State Y is. That is each individual knows what is good for themselves and thus their preferences should be respected. ${ }^{6}$ The first proposition is linked heavily with the principle of consumer sovereignty. Lerner (1972) describes the idea of consumer sovereignty as arranging

[^4]"for everybody to have what he prefers whenever this does not involve any sacrifice for anybody else."

Lerner (1972, pg. 258)
The requirement of this proposition for CBA is that the measures of benefits and costs must reflect the preferences of the individual.

### 2.4.2 Proposition 2

The second proposition asserts that it is only the preferences of those individuals who make up society that matter to the decisions. This proposition determines the set of individuals who are relevant to social welfare. For CBA, the society would be the set of affected individuals. That is the users and the funders of the public good. As public goods are non-excludable by nature and often funded through taxation, the set of affected individuals will be wide as almost everyone is either a user, or a funder, or both. Based on Proposition 1, any attempt to measure social welfare changes must include the impact on society as a whole. Practically this is difficult which is where valuation studies are used to elicit a representative sample of preferences.

### 2.4.3 Proposition 3

The third proposition argues that if one individual is better off and no other individual is worse off in terms of welfare then society is better off. This value judgement is widely regarded as very weak in the sense that there are few cases or few individuals who would oppose its application. CBA identifies proposals which have the potential to achieve a Pareto improvement. In the case of public good provision, if we assume the good is normal in the sense that consuming more is preferred to less, then increased provision will deliver benefits through consumption to each individual which are distributed across society. CBA asks if there is a feasible distribution of costs such that will achieve a Pareto improvement. If the proposal is delivered under these terms, then a Pareto improvement has been achieved. This signals an improvement in the welfare of society.

Based on these proposals, the CBA process can be evaluated. For WTP values to adhere to Proposition 1 they must respect the consumer's sovereignty by truly reflecting their preferences. To observe Proposition 2, decisions must be based on the preferences of society and therefore
the sample of a valuation study must reflect accurately the affected population. Once this is achieved, the outcome of the aggregation of values through the compensation test must also reflect the preferences of society to respect the first two propositions. The compensation test identifies potential Pareto improvements. While Proposition 3 is not a requirement for a social welfare improvement, decisions based on the compensation test only achieved with certainty if compensation is paid. The problem of deciding how much compensation (if any) should be paid and how should it be distributed does not have a clear solution as it is a matter of fairness.

For this reason, Little (1949) objected to the compensation test as a welfare criterion as it defines efficiency improvements but gives no consideration to equity. Little argued that as compensation is only hypothetical, any change based on the compensation test could be consistent with making the wealthy richer at the expense of the poor. Little argued this would constitute a reduction social welfare due to increasing inequality; a value judgement in itself. So, in the pursuit of a more acceptable welfare test, Little proposed an additional criterion that the new distribution had to be 'better' than the previous. CBA should also be evaluated based on this additional criterion. However as previously discussed, defining 'better' is a question of distributive justice with no clear answer as it requires value judgements which can be a reasonable point of disagreement.

Practically speaking a public decision-maker, likely a government official, will ultimately have to make a decision regarding a proposal. However, to best reflect the preferences of the individuals, the ideal process would have no input from the public decision-maker. Instead their decision should be informed by the evaluation. Ideally as far as possible the preferences of the individuals in society should inform decision-making. This could be extended to any value judgements made, in this case whether compensation should be paid. Whilst the process as described thus far largely treats individuals as self-interested consumers, individuals do have preferences over the welfare of others and distributive justice. The next two sections explore the literature on the incorporation of these preferences into CBA.

### 2.5 Distributive justice

Concerns over distributive justice are concerns over what is morally preferable when comparing different distributions in society. That is the distribution of welfare across the individual
members of society. This is highly relevant to CBA which evaluates the changes in welfare of a proposal. A proposed change to provision, even one that passes a compensation test, will have implications for the distribution for welfare and welfare changes. Whether compensation should be paid and how much should be paid is a question of distributive justice.

Distributive justice allows for sets of principles or values to be formed to compare different distributions. Individuals both have principles they believe in and can reasonably disagree on the correct set of principles. Based on different sets of value judgements, different conclusions could be reached on the question of compensation.

Some philosophers consider impartiality as a standpoint from which what is good, or moral, can be determined. There are a number of conceptions of what an impartial standpoint is. For example, the impartial individual can be an observer such as Adam Smith (1759)'s impartial observer, or the individual can be a stakeholder as an individual is in Rawls (1971)' Original Position.

Adam Smith (1759) introduces his impartial observer in "A theory of moral sentiments". The impartial observer has no stake in decision at hand. This concept does not fit with Proposition 2 of Paretian welfare economics as described in Section 2.4 . 2 which requires social welfare to be determined by affected individuals. The impartial stakeholder that is acting on their own preferences rather than moral reasoning from a detached standpoint is much more in line with welfare economics. ${ }^{7}$

Harsanyi (1953, 1955, 1975, 1977a,b, 1978, 1979) argues for a version of utilitarianism, sometimes called neo-utilitarianism, in which agents are uncertain of their position in society. This version of utilitarianism is defended based on an equi-probability model and the rational choice based on von Neumann Morgenstern (vNM) utility functions as set out by von Neumann \& Morgenstern (1947). The problem Harsanyi poses is as follows: suppose there are $n$ individuals subscripted $i$, each who have vNM preferences for wealth given by $u_{i}$. There is an amount of wealth $W$ to be justly divided among $p$ positions each taken by an individual. Harsanyi has an impartial stakeholder who will be in one of these positions with equal probability. The expected utility of the stakeholder is

$$
\begin{equation*}
E(U)=\frac{1}{n} \sum_{p=1}^{n} u_{i}\left(w_{p}\right) . \tag{2.1}
\end{equation*}
$$

[^5]The impartial observer chooses the distribution of wealth based on her own vNM utility function, maximising expected utility subject to the constraint that $\sum_{p=1}^{n} w_{p}=W$. Harsanyi argues that the expected utility of the individual can then be regarded as her social welfare function.

Rawls (1971) considers a similar approach in 'A theory of Justice'. Rawls' ideas, stemming from the contractarian theories of justice of John Locke (Locke, J. \& Laslett, P. (1689)), JeanJacques Rousseau (1762) and Immanuel Kant (1777), seeks to determine what defining principles for society could be agreed upon for a hypothetical social contact when the contracting parties are impartial. To consider the features of such a contract, Rawls outlines a thought experiment to decide which principles of justice constitute the fairest or most just. These conditions and assumptions are realised in the form of Rawls' 'Original Position'. The idea is that from the original position, any principles that are decided upon by free and equal citizens will constitute a fair system. It should not be seen as an actual historical state, but rather a purely hypothetical scenario that will lead to a conception of what is just by creating an initial choice situation.

The original position thought experiment involves a group of individuals who will represent the larger society will consider principles of justice and come to an agreement on a set of principles which will define a social contract. A key part of the original position is that these individuals are placed behind a VoI, where
> "no one knows his place in society, his class position or social status; nor does he know his fortune in the distribution of natural assets and abilities, his intelligence and strength, and the like."

$$
\text { Rawls, J. (1971) pp. } 137 .
$$

Using this hypothetical scenario, Rawls argues sets out a line of reasoning regarding the distributive justice surrounding the provision of certain rights and goods in society. Rawls concluded that from the original position individuals will always reach unanimous agreement, and that the same principle will be chosen each time. That principle is the 'Difference Principle', or maxi-min (MM) principle, which is to maximise the welfare of the worst-off.

Both authors set out clear reasons for their conclusions, but ultimately each individual would reach their own conclusion based on their own preferences and beliefs. Choice behind a VoI is one of choice under uncertainty. Therefore, risk preferences will partially drive choice. Other
preferences and values such as altruism may come into the decision-making process. Harsanyi's conclusion could lead to a range of outcomes based on the individual's preferences, whilst Rawls' conclusion is only consistent with a strong desire to help the worst-off individual, or as Harsanyi (1975) argues risk aversion so extreme that choice over uncertainty becomes lexicographic over the positions in society rather than continuous as most utility theory would suggest.

A number of empirical studies have considered choice behind a VoI. The original position experiment was replicated in a laboratory setting with results suggesting that individuals maximise expected utility and are generally risk averse (Frohlich et al. (1987a, 1987b)). In fact, almost none chose the MM principle experiment, but rather what Rawls (1972) termed a compound principle which trades-off improving the average and lowest wealth levels. Schildberg-Hörisch (2010) tests the concept of a VoI in a lab experiment by comparing choice in a dictator game in front and behind a VoI. Few respondents chose the MM principle and choice was best characterised as driven by risk and for equal outcomes among subjects.

### 2.6 Altruism and other-regarding preferences

A significant body of research has been undertaken in the last 3 decades on how altruism, or in a wider sense concern for others, impacts upon individuals' choices. . It is clear from the prevalence of charitable giving (Andreoni, J. (1990)), and evidence from experimental and behavioural economics (Bolton \& Ockenfels (2000); Engelmann \& Strobel (2004); Fehr \& Schmidt (1999)) that in one form, or another, the welfare and outcomes of others do enter into the decision-making process of economic agents. The form that regard for others takes is likely to be based on the context and institutions in place.

Evidence suggests that for emotive areas such as mortality risk, health, and environmental quality that concerns for others' consumption of the public good are present (Viscusi et al. (1988); Arana and Leon (2002); Johannesson et al. (1996); de Blaeij et al. (2003); Hultkrantz et al. (2006); Andersson and Lindberg (2009); Svensson and Johansson (2010); Gyrd-Hansen et al. (2016)). These studies are discussed in Section 2.6.2.

As Kagel \& Roth (2015, pg. 218) point out, no single tractable model of concern for others exists, but rather it appears as if a broad range of motivations for regard for others exist. A number of theories of concern for others have been developed and are generally categorised as other-regarding preferences (ORPs). These theories account for, amongst other things, charitable or gift giving, preferences over inequality, reciprocity, cooperation and social norms. These theories are discussed in Section 2.6.3.

Even though individuals have been shown to have been shown to have preferences for others welfare, there is some debate as to whether, and how, sympathy towards others should be included in valuations. Milgrom (1993) opposed the inclusion of anything but private values on two grounds. First, that sympathy towards others does not fit with standard neoclassical theory. Second, he argues that only with private values can a potential Pareto improvement be identified and that the inclusion of another's welfare in WTP leads to a double-counting problem. ${ }^{8}$ The double-counting problem when related to altruism, although not well defined in the literature, occurs when the benefits to one individual are counted more than once due to altruism and thus values become inflated. Milgrom's concern highlights two issues that require addressing. First, does sympathy towards others constitute a meaningful 'want' or measure of welfare? Second, does incorporating sympathy towards others into WTP and CBA lead to efficient outcomes?

In response to the first concern Hanemann (1994), citing Arrow (1963, pg. 17) and Becker (1993, pg. 386), argued that sympathy is as an acceptable motivation for economic value as any another. The argument for the inclusion of sympathetic values is summarised by the first proposition of Paretian welfare economics and the principle of consumer sovereignty. In that regard Lerner (1972), when discussing consumer sovereignty, argues that economics
"must be concerned with the mechanisms for getting people what they want, no matter how these wants were acquired."

Lerner (1972, pg. 258) ${ }^{9}$.

All 'wants' are then valid including the desire for the welfare of others to be improved. The concern is ensuring the mechanisms in place achieve these wants wherever possible.

[^6]The second concern of Milgrom (1993) is then entirely valid as it is important that the mechanisms in place operate correctly when sympathetic values are included. Understanding the forms altruism are likely to take, and the impact they have on valuations, is then important. If the valuation process does not respect the preferences of individuals, then it does not satisfy the propositions of Paretian welfare economics set out in Section 2.4. The fault is with the process not the preferences. The next two subsections cover the theoretical literature modelling the impact of altruism and the empirical studies measuring altruism in public good values.

### 2.6.1 Theoretical literature

Three approaches are taken when considering the theoretical value of a public good: optimal provision, WTP, and compensation tests. The optimal provision approach maximises a social welfare function to find the conditions for optimal provision. From this the standard Samuelson (1954) condition, that social welfare is maximised when the sum of the marginal benefits of public good provision equal the marginal cost, is derived under different altruistic and selfinterested preference sets. The WTP models explore the value individuals place on changes to public good provision. The compensation test models evaluate the use of altruistic values in a cost-benefit calculation.

When considering the value of public safety Bergstrom (1982) considers altruism to take the form of PA, for which altruistic individuals take into account the welfare of others and respect their preferences over public and private consumption. ${ }^{10}$ Bergstrom shows that optimal provision for safety expenditure is equal under both SI and PA preferences. Jones-Lee (1991) considers a wider range of altruistic preferences which include others' consumption of the public and private goods. Jones-Lee shows that optimal provision is highest when individuals are paternalistic ${ }^{11}$ and value others' safety, but not their consumption of the private good. That is, their altruism is pure safety-focused altruism (SFA).

[^7]Jones-Lee (1992) extends the model to that altruism can take infinite number of forms on a spectrum from pure wealth-focused altruism (WFA) to SFA, on which PA sits somewhere as a middle case. Optimal provision is lowest when altruism is WFA and highest when altruism is SFA. Altruism can also vary in intensity, with no intensity representing the corner case of SI. These findings are shown to hold in other domains than safety. McConnell (1997) considers use and non-use values for the environment and altruistic motives considering PA, mixed altruism, paternalism again finding similar implications to Jones-Lee (1991, 1992).

Johansson (1992) and Johansson (1994) considers the implications of Jones-Lee (1991, 1992) for WTP finding similar implications. Both highlight the need for care in survey design. In that regard, Johansson (1994) suggested that respondents should be asked to give values based on other individuals contributing their maximum WTP.

Flores (2002) considers a model of the compensation test in which there is an altruist and a set of beneficiaries. The findings show that a private values cost-benefit test fails to find all socially potential Pareto improving proposals. A case is shown for PA that if the sum of altruistic and private WTP values exceeds the sum of private WTP then there may exist a case in which the cost of the proposal is between the two which is potentially Pareto improving and thus a private values compensation test fails where a social values compensation test would pass.

Bergstrom (2006) formalises the result by Flores (2002) showing that passing a private values compensation test is a sufficient but not necessary condition for a potential Pareto improvement with social values aligning with Flores' findings. Both authors as well as Milgrom (1993) identify a potential problem which Bergstrom (2006) calls the naïve benefit-cost analyst. ${ }^{12}$ This occurs when the question posed to respondents does not lead them to take into account both the benefits and costs to the individuals that they show altruism towards i.e. the recipients of altruism, or the beneficiaries in the parlance of Flores (2002).

[^8]
### 2.6.2 Valuation Studies measuring altruism

A number of WTP surveys have been run to measure preferences for safety and health with some finding positive values for altruism (Viscusi et al. (1988); Arana and Leon (2002)), but many negative (Johannesson et al. (1996); de Blaeij et al. (2003); Hultkrantz et al. (2006); Andersson and Lindberg (2009); Svensson and Johansson (2010); Gyrd-Hansen et al. (2016)).

Viscusi et al. (1988) ask if respondents would be willing to contribute to a program that provides risk reductions to those in their state and then the rest of the USA. Their results suggest that respondents are willing to contribute and that as the affected population increases the additional contribution amount is decreasing. Contributions are not reliant on others paying so this survey. Whilst showing that individuals do indeed value others' safety, the elicited values may in part be impacted by what others later identified as a double-counting or naïve cost-benefit analyst problem. As is shown in Chapter 3 this may lead to an inflation of the values. The positive contributions do suggest that preferences outweigh the free-rider problem that may arise due to the contribution payment vehicle. The free-rider would deflate values.

Arana and Leon (2002) similarly found public valuations for flu vaccinations to be higher than private valuations. In this case the public program would be run if all agreed to pay a certain amount. This result may suggest that respondents are willing to contribute more if they believe others value the program highly and therefore do not want to prevent a program which is beneficial to others.

Johannesson et al. (1996) devise a survey which compares WTP for a private safety device and a public safety program which deliver the same safety improvements. In the case of the public safety program, funds are generated through a uniform taxation. Johannesson et al. (1996) make an important distinction that the payment vehicle is likely to be important to how respondents value a public good if they are altruistic. It is shown that WTP may deviate from the SI level for PA if taxation is uniform and respondents do not place the same value on the safety improvement. Their results suggest that private WTP is greater than public WTP and that respondents tend to perceive their value higher than others. This would suggest they perceive that others are over-paying for the safety improvement and reduce their WTP to reduce the cost put on others.

Hultkrantz et al. (2006), Andersson \& Lindberg (2009) and Svensson \& Johansson (2010) similarly find that private valuations are significantly higher than public valuations when taxation is uniform and coercive. Andersson \& Lindberg (2009) consider also the WTP for
purchasing safety devices on behalf of others (close friends and relatives) and conclude that altruism may be best described as between PA and SFA.

Gyrd-Hansen et al. (2016) compare public valuations under first-best and uniform taxation and find that first-best taxation results in larger WTP values. This comparison does not suffer from the potential biases that some of the others do when comparing a personal safety device to a public safety program which have the potential to be interpreted by respondents as largely different goods even if they provide the same safety reduction. The comparison of the two payment vehicles is complimented by a prediction of others' WTP. Results suggest that respondents believe they personally have larger than average valuations which may explain why valuations are lower under a coercive tax.

### 2.6.3 Other-regarding preferences

A number of models are of particular interest when considering preferences for distributive justice. These models have been developed around distribution games such as the one described in Charness \& Rabin (2002) and fairness games such as the Dictator (Forsythe et al., 1994) and Ultimatum (Guth et al., 1982) games where individuals who have control over a pot of money must decide how much if any to share with other individuals. The IA models of Fehr \& Schmidt (1999) and Bolton \& Ockenfels (2000) focus on the distribution of outcomes as the otherregarding element of an individual's utility function. Under these preferences, individuals compare their own outcome relative to other's outcomes. That is individuals act with a sense of fairness and are said to be averse to inequality i.e. they experience disutility from being either worse off than others, better off than others, or both. A number of hybrid models exist that allow for more than one motivations for concern for others such as the quasi-maximin model of Charness \& Rabin (2002) and the Erlei (2008) model of inequity aversion, social welfare preferences, reciprocity, and heterogeneity. Due to the variation in benefits received from consuming public goods and the costs of provision, different motivations for concern for other will have different implications for the impact upon valuations.

As the reviews of contribution games in public goods experiments by Ledyard (1995) and Chaudhuri (2011) show, models of reciprocity, punishment, and cooperation are often discussed in relation overcoming the free-rider problem in public good provision. In these experiments social norms are replicated through the expectation of reciprocity and/or punishments giving non-outcome-based motivations for the regard for others. In this case
contributions are considered as a method of signalling to others. Similarly, warm glow in the context of charitable giving considers impure altruists on a continuum from pure altruist who values the outcome of the other through total giving to pure egoist who values only their own contribution (Andreoni, J. (1990)). In the context of this thesis, public goods provided through coercive taxation so there is no charitable giving and the cooperation required to overcome the free-rider problem is avoided and. Although the valuation process discussed in this thesis is generally a private decision, social norms may play a role in the expectation of contribution to public goods through higher valuations.

Evidence from stated preference surveys suggests that altruism is present in valuations (Viscusi et al. (1988); Arana and Leon (2002); Johannesson et al. (1996); de Blaeij et al. (2003); Hultkrantz et al. (2006); Andersson and Lindberg (2009); Svensson and Johansson (2010); Gyrd-Hansen et al. (2016)). It is yet unclear the form concern for others takes and how that impacts upon values placed on public good provision. Theory suggests altruism could take a number of forms and that altruism could have either a positive or negative effect depending on the aspects of others welfare the altruist values: their utility, their wealth, or their safety, health, and experience of their environment.

Some theory suggests there are case in which the inclusion of altruistic values in CBA could lead to inefficient outcomes. Evidence from stated preference surveys also suggests that the choice of payment vehicle used in the survey will affect the elicited values. This appears to be linked to the issue of double-counting raised by Milgrom (1993) and discussed by others. The result also suggests that altruistic respondents have preferences over both the costs and benefits to others. This would indicate some middle case of altruism on Jones-Lee (1992)'s spectrum of altruism.

### 2.7 Incorporating different preference sets into WTP values

The potential for the inclusion of different preference sets into values has been discussed in the previous sections. The elicitation of these preferences is highly dependent on the question asked of the respondents. The framing of the question can require a respondent to take differing perspectives. In this thesis this is termed a frame. The perspective a respondent takes will affect the preference set from which they make choices and therefore impact upon the value elicited.

There is the potential for a number of perspectives to exist when considering questions of value. Dolan et al. (2003) and Tsuchiya \& Watson (2017) define a perspective in two dimensions: preference and context. The preference dimension defines the role of the respondent in the valuation process i.e. stakeholder or observer, whilst the context dimension defines the affected set of individuals.

This thesis focuses on the preference dimension, specifically when respondents are stakeholders of which Dolan et al. (2003) identify two: personal and socially inclusive. In both perspectives the respondents are potential users and funders of the good. This aligns with second proposition of the Paretian approach that social welfare is dependent on those in society and thus are regarded as suitable for measuring social welfare.

In the personal perspective, the individual, who is a user and funder of the public good, acts in accordance with their personal preferences. In the socially inclusive perspective, the individual is a member of the group of users and funders and acts in accordance with their personal and social preferences. In this thesis these are termed consumer perspectives and the associated frames termed: Consumer Frame 1 and 2 respectively. These are defined as

Consumer Frame 1: Respondents act in accordance with their preferences over their own outcome.

Consumer Frame 2: Respondents act in accordance with their preferences over their own outcome and the outcomes of others.

In a valuation exercise Consumer Frame 1 requires individuals to ignore their altruistic concern, if any exists, whilst Consumer Frame 2 does not. Consumer Frame 1 reflects the standard methodology for eliciting values. Consumer Frame 2 takes a broader view of what constitutes value and as evidence from the empirical studies discussed Section 2.6 suggests that, given the chance, some respondents to surveys measuring preferences for public safety do value the welfare changes to others. The inclusion of these preferences may lead to different outcomes when included in the CBA process. However, it is important to note that SI is a valid preference set for Consumer Frame 2.

These frames are termed consumer frames due to their requirement of inward-looking perspective. These preferences are described in the literature as personal preferences to distinguish them from impersonal preferences (Sagoff (1986), Orr (2006), Harsanyi (1982)).

Impersonal preferences imply a degree of impartiality to determine what is good for society and come from a broader set of values which do not relate to one's self. Impartiality can come from the positions of an observer or a stakeholder. One concept of impartiality from the position of observer is Smith (1759)'s Impartial Spectator who has no personal stake in the decision at hand and thus is making choices with impersonal preferences. Another concept of impartiality in which individuals are stakeholders is the Original Position as described by Harsanyi (1953) and Rawls (1972). As argued previously, the interest of the valuation process is to incorporate the values of stakeholders to be in line with the welfare propositions.

Harsanyi (1953) and Rawls (1972) present separate versions of a VoI designed to create an impartial standpoint for stakeholders. The impartiality which defines the impersonal stance is created through uncertainty. The VoI then acts as a mechanism for eliciting impersonal preferences from personal preferences. The Citizen Frame which is discussed here uses the VoI as a mechanism to allow individuals to take an impersonal stance by acting under uncertainty with personal preferences. The Citizen Frame is defined as

Citizen Frame: Respondents act in accordance with their preferences for their own outcome and the outcomes of others. Respondents are placed behind a VoI and thus are informed about the distribution of income and public good endowments, but not their own position within the distribution.

Individuals maximise their expected utility based on their personal preferences and by doing so take into account each position in society based on the equi-probability assumption of Harsanyi (1978, 1979). Harsanyi (1953) argues that the expected utility of those in society represents a Utilitarian social welfare function. An individual's expected utility function could be seen as their own social welfare function of which the societal aggregate generates the societal social welfare function. The use of values elicited in the Citizen Frame would reflect the impersonal preferences derived from personal preferences. These values would include preferences for each position in society with each given equal weight and therefore may include both private and altruistic preferences, as well as preferences over distributive justice.

### 2.8 Summary and research direction

Public goods can be provided to a society to achieve large social welfare improvements. To do so, two problems must be overcome. The first, the free-rider problem is resolved by funding public good provision through coercive taxation. This leads to the second problem of identifying the correct level of provision and therefore taxation to fund it. It is then of great concern to public decision-makers to accurately assess the value of a public good to optimally set the level of provision.

One method for evaluating a proposal is CBA which uses a simple test comparing the costs and benefits. This test is underpinned by the guiding principles of welfare economics. One principle is that social welfare is determined only by the welfare of those individuals who make up the society in question. It is therefore important that any decision regarding public good provision reflect the welfare of the individuals in society. Individuals who are sovereign in their right to determine what constitutes changes to their own welfare.

By improving the measurement of societal values, the process of public decision-making can be improved. This requires a fundamental understanding of both the how individuals value public goods and how these values should be incorporated into the decision-making process. This thesis furthers the literature linking these two elements, specifically when individuals have preferences over both their own and others consumption of public goods.

One question that remains unresolved in the literature is which preferences should be included in values. Three possible preference sets are studied in this thesis and the frameworks through which they are elicited:

Consumer Frame 1: Respondents act in accordance with their preferences over their own outcome.

Consumer Frame 2: Respondents act in accordance with their preferences over their own outcome and the outcomes of others.

Citizen Frame: Respondents act in accordance with their preferences for their own outcome and the outcomes of others. Respondents are placed behind a VoI and thus are informed about the distribution of income and public good endowments, but not their own position within the distribution.

These are used as frameworks for considering different preference sets and the practical method of eliciting the values. There are three elements to consider when evaluating each frame:
[1] The impact of different preference sets on the societal value.
[2] The impact of different preference sets on individual values.
[3] The use of these values in CBA. Two questions are asked. First, does the method of elicitation respect the individual's sovereignty by respecting their preferences? Second, does the aggregation of these preferences pass a compensation test?

The proceeding analysis is split into two parts: Part I presents the two theoretical studies modelling the three frames, and Part II presents two laboratory experiments eliciting individual values.

The theoretical part of this thesis uses and extends existing models of optimal provision of public goods based on Bergstrom (1982) and Jones-Lee (1991) and compensation tests based on Bergstrom (2006) to model: the societal value, WTP values, and the CBA process under each frame. The optimal provision model is used to consider the changes in societal value based on different preferences. Modelling WTP under different preferences provides an analysis of the impact of the choice of frame on the individual value. Considering these values in a compensation test allows for the consideration of the principle of consumer sovereignty and efficiency of CBA under each frame.

The empirical part of this thesis elicits individual values for each frame using laboratory experiments. These values are used to test hypotheses generated from the theory presented in Part I by comparing individual values.

Ultimately the findings are used to consider the two elements highlighted in the introduction: the value individuals place on the provision of public goods, and the process of aggregating these preferences through CBA.

## PART I

## THEORY

Part I consists of two chapters presenting the theoretical contributions of this thesis. The results of these models underpin the predictions and hypotheses tested in empirical chapters presented in Part II. Here the three frames described in Chapter 2: Consumer Frame 1, Consumer Frame 2, and the Citizen Frame are modelled. Consumer Frame 1 assumes SI, whilst Consumer Frame 2 allows for altruistic preferences. Although a number of forms of altruism have been identified in the literature, the analysis here is limited to 3 interesting cases of altruism based on preferences for public good provision based on Bergstrom (1982) and Jones-Lee (1991, 1992): PA, SFA, and WFA. The Citizen Frame places individuals behind a VoI. Chapter 3 models the Consumer Frames and Chapter 4 models the Citizen Frame.

The frames are modelled under two different scenarios: a model of optimal provision and a model of CBA. The model of optimal provision is based in the model of optimal safety expenditure presented in Jones-Lee (1991). Safety expenditure is chosen as the public good to be modelled as this is a good focussed on in the literature, however the results are generalisable to other public goods. Optimal provision is modelled under different tax systems because welfare is based on both benefits and costs of provision. If altruists care for others costs as well as benefits, then they will likely have preferences over the system of taxation in place. The solution to the model of optimal provision identifies the condition for social welfare maximisation. This allows for a comparison of the impact of different preference sets on the societal value placed on public good provision.

The model of CBA is based on the models presented in Milgrom (1993) and Bergstrom (2006). First, the model allows for a comparison of the impact of different preference sets on the value an individual places on a proposed change in public good provision. Second the values can be considered for the inclusion into CBA by considering the compensation test. These models are also presented in Beeson et al. (2019).

## CHAPTER THREE:

## Consumer Frames 1 and 2

### 3.1 Introduction

The aim of this chapter is to model the value of a public good under the two consumer frames:

Consumer Frame 1: Respondents act in accordance with their preferences over their own outcome.

Consumer Frame 2: Respondents act in accordance with their preferences over their own outcome and the outcomes of others.

Under Consumer Frame 1 preferences which are used to find optimal provision are SI, even if the respondents have altruistic preferences. Under Consumer Frame 2 these altruistic preferences are included in values and the resulting decision-making process.

Two approaches are taken. First, optimal provision is modelled to find the societal-level value of a public good. Second, the elements of CBA: WTP and the compensation test are modelled to find individual-level values for a public good and assess the use of the values in a standard compensation test. A range of altruistic preferences, based on those defined in Jones-Lee (1991, 1992) are considered within each model. The literature has highlighted the need to consider the role of taxation in achieving optimal provision of a public good. In response, a range of tax systems are also considered to explore the interactions between altruism and taxation.

The model of optimal provision uses public safety expenditure as the public good. The model replicates and extends the model in Jones-Lee (1991). This model allows for the change in optimal provision and therefore value of the public good, in this case the value of a statistical life (VSL), to be modelled. That is the level of safety spending such that the marginal benefit equals the marginal cost i.e. the Samuelson (1954) Condition for optimal public good provision. Jones-Lee (1991) uses the model to determine the changes in the VSL under different altruistic preferences including: SI, PA, and SFA assuming first-best taxation. Jones-Lee (1992) presents an extension to the model which considers altruism as a spectrum from WFA to SFA for which PA is one middle case. The model presented here includes WFA which together with the cases presented in Jones-Lee (1991) of SI, PA, and SFA is sufficient to understand the impacts of altruism on optimal provision.

Both papers by Jones-Lee assume first-best taxation. First-best taxation is not practically feasible and rarely, if ever, seen in practice as it requires tax contributions to be set at the individual level based on the individual's preferences. It is therefore important to understand the impact of different forms of taxation on optimal provision through their interactions with the different forms altruism may take. The model is extended in this chapter to include two alternate taxes: uniform and a proportional wealth tax. Simulations are used to allow for better predictions of the impact of different forms of taxation on provision.

Then a series of models of the cost-benefit framework are presented. First, the two-person naïve cost-benefit analyst model highlights the double-counting problem as described by Milgrom (1993). Next the two-person non-naïve cost-benefit analyst model, based on Bergstrom (2006)'s Alice and Bob model, highlights the correct method to avoid the problem which requires the altruist to take into account that the other individual will also contribute to funding provision.

Two n-person models are then presented: the first a model of WTP responses, and the second models the compensation test, both assuming the different forms altruism examined in the optimal provision model. The WTP model extends non-naïve cost-benefit analyst model to allow for a full range of payment vehicles. By comparing the WTP model to the compensation test model, the efficiency of altruistic values for use in CBA can be evaluated.

Finally, the results of these models are used to assess each of the Consumer Frames. Three questions are answered. What happens to the socially optimal provision value of a public good? What happens to WTP values? Are these values appropriate for CBA by respecting Consumer Sovereignty and achieving a potential Pareto improvement when used in a compensation test?

### 3.2 Optimal provision of public safety expenditure

Following the model assumptions and notation of Jones-Lee (1991), consider a society of $n$ individuals indexed $i=1 \ldots, n$, whose differentiable and well-behaved utility functions, $u_{i}$, reflect preferences for own and others' survival probability $\pi_{1}, \ldots, \pi_{n}$ and wealth $w_{1}, \ldots, w_{n}$

$$
\begin{equation*}
u_{i}=u_{i}\left(\pi_{1}, w_{1}, \ldots, \pi_{n}, w_{n}\right), \quad i=1, \ldots, n \tag{3.1}
\end{equation*}
$$

where $u_{i}$ is strictly increasing in $\pi_{i}$ and $w_{i}$. Survival probabilities are a strictly increasing function of public spending, $s$, with diminishing returns.

Public spending is funded through taxation levied on each of the $n$ individuals with Individual $i$ contributing $t_{i}$. Increasing public spending will increase utility through increased survival probabilities and decrease utility through decreased wealth after tax, $w_{i}-t_{i}$ which is assumed to be spent on private consumption.

The optimal level of safety expenditure is based on a utilitarian social welfare function constrained by public funds:

$$
\begin{equation*}
\max \sum_{i}^{n} a_{i} u_{i}\left(w_{1}-t_{1}, \pi_{1}(s), \ldots, w_{n}-t_{n}, \pi_{n}(s)\right) \quad \text { s.t. } s=\sum_{i}^{n} t_{i}, \tag{3.2}
\end{equation*}
$$

where $a_{1} \ldots a_{n}$ are positive distributional weights.
The condition for optimal provision is found by solving the Lagrangean:

$$
\begin{equation*}
L=\sum_{i}^{n} a_{i} u_{i}\left(w_{1}-t_{1}, \pi_{1}(s), \ldots, w_{n}-t_{n}, \pi_{n}(s)\right)+\lambda\left(s-\sum_{i}^{n} t_{i}\right), \tag{3.3}
\end{equation*}
$$

where $\lambda$ is a Lagrange multiplier. The partial derivatives with respect to $s$ set to zero gives the first-order condition

$$
\begin{equation*}
\frac{\partial L}{\partial s}=\sum_{i}^{n} \sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}} \frac{\partial \pi_{i}}{\partial s}+\lambda=0 . \tag{3.4}
\end{equation*}
$$

Following Jones-Lee (1991), under the assumption that $a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}$ and $\partial \pi_{i} / \partial s$ are uncorrelated across the $n$ individuals. This assumption requires each Individual $j$ 's preference for each Individual $i$ 's safety to be uncorrelated with the cost of improving Individual $i$ 's safety. Under this assumption Equation (3.4) can be rearranged to give

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \frac{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\lambda}=-\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} . \tag{3.5}
\end{equation*}
$$

The assumption used by Jones-Lee (1991) is the natural extension to the assumption used by Dehez and Drèze (1982) and Bergstrom (1982) to cover the altruistic terms as well as the SI terms.

To solve the partial derivative with respect to taxation is dependent on the system of taxation which is assumed. The choice over system of taxation has implications for social welfare and more specifically the distribution of individual welfare as taxation is often correlated with wealth or income. Sub-section 3.2.1 replicates the model of optimal provision under first-best taxation set out in Bergstrom (1982) and Jones-Lee (1991) with some of the extensions from Jones-Lee (1992). As a first-best tax is rarely, if ever, seen in practice, Sub-sections 3.2.2 and 3.2.3 extend the model to consider two second-best taxes: uniform taxation, and proportional wealth taxation.

### 3.2.1 Optimal provision under first-best taxation ${ }^{15}$

Under first-best taxation, individual levels of taxation are set independently to maximise individual welfare. Setting the first partial derivative of the Lagrangean set out in Equation (3.3) with respect to $t_{i}$ to zero gives the first order condition

$$
\begin{equation*}
\frac{\partial L}{\partial t_{i}}=-\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}-\lambda=0 \quad \text { for } i=1, \ldots, n, \tag{3.6}
\end{equation*}
$$

Giving a first order condition for each of the $n$ individuals. Rearranging Equation (3.6) gives both

$$
\begin{align*}
& a_{j}=-\left(\lambda+\sum_{k \neq j}^{n} a_{k} \frac{\partial u_{k}}{\partial w_{i}}\right) \frac{\partial w_{i}}{\partial u_{j}}, \text { and }  \tag{3.7}\\
& \lambda=-\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}, \tag{3.8}
\end{align*}
$$

Substituting Equations (3.7) and (3.8) into Equation (3.5) and rearranging gives

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \frac{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)}, \tag{3.9}
\end{equation*}
$$

the general solution for the condition of optimal provision. Social welfare is maximised when the mean ratio of the distributional weighted sum of each Individual $j$ 's marginal utility for Individual $i$ 's safety to the distributional weighted sum of each Individual $j$ 's marginal utility

[^9]for Individual $i$ 's wealth equals the cost of saving one statistical life. The left side of the equation is simply the ratio of marginal utilities and the right side of the equation the ratio of prices if private consumption has a price of 1 . This gives the standard solution of a consumption model that the ratio of marginal utilities equals the ratio of prices.

### 3.2.2 Optimal provision under uniform taxation

Under uniform taxation each member of society contributes an equal amount such that $t_{i}=$ $T \forall i$ so that public good expenditure $s=n T$. Under proportional wealth taxation each individual is taxed at the same rate $\tau$ so that each individual's tax contribution is $\tau w_{i}$ and public expenditure is $s=\tau \sum_{i}^{n} w_{i}$.

Starting with uniform taxation if the assumption that $t_{i}=T \forall i$ is included then Lagrangean presented in Equation (3.3) becomes

$$
\begin{equation*}
L=\sum_{i}^{n} a_{i} u_{i}\left(w_{1}-T, \pi_{1}(s), \ldots, w_{n}-T, \pi_{n}(s)\right)+\lambda(s-n T), \tag{3.10}
\end{equation*}
$$

The first partial derivative with respect to $s$ remains the same as Equation (3.4) whilst the firstorder condition with respect to $T$ is

$$
\begin{equation*}
\frac{\partial L}{\partial T}=-\sum_{i}^{n} \sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}-n \lambda=0, \tag{3.11}
\end{equation*}
$$

which rearranges to give

$$
\begin{equation*}
n \lambda=-\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u_{j}}{\partial w_{i}} . \tag{3.12}
\end{equation*}
$$

Substituting Equation (3.12) into Equation (3.5) gives

$$
\begin{equation*}
\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u_{j}}{\partial \pi_{i}}=\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u_{j}}{\partial w_{i}} \frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)}, \tag{3.13}
\end{equation*}
$$

which rearranges to

$$
\begin{equation*}
\frac{\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u}{\partial w_{i}}}=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)}, \tag{3.14}
\end{equation*}
$$

the general solution for the case of uniform taxation. That is the ratio of the distributional weighted sum of each of Individual $j$ 's marginal utility for each Individual $i$ 's probability of survival and the distributional weighted sum of each of Individual $j$ 's marginal utility for each

Individual $i$ 's wealth equals the marginal cost of saving one statistical life. That is the cost of reducing the expected number of deaths by one.

### 3.2.3 Optimal provision under proportional taxation

Under proportional wealth taxation each individual is taxed at the same rate $\tau$ so that each individual's tax contribution is $\tau w_{i}$ and public expenditure is $s=\tau \sum_{i}^{n} w_{i}$. Including this assumption in the Lagrangean gives

$$
\begin{equation*}
L=\sum_{i}^{n} a_{i} u_{i}\left(w_{1}(1-\tau), \pi_{1}(s), \ldots, w_{n}(1-\tau), \pi_{n}(s)\right)+\lambda\left(s-\tau \sum_{i}^{n} w_{i}\right) . \tag{3.15}
\end{equation*}
$$

The first order condition with respect to $\tau$ is

$$
\begin{equation*}
\frac{\partial L}{\partial \tau}=-\sum_{j}^{n} a_{j} \sum_{i}^{n} w_{i} \frac{\partial u_{j}}{\partial w_{i}}-\sum_{i}^{n} w_{i} \lambda=0, \tag{3.16}
\end{equation*}
$$

which rearranges to give

$$
\begin{equation*}
\sum_{i}^{n} w_{i} \lambda=-\sum_{j}^{n} a_{j} \sum_{i}^{n} w_{i} \frac{\partial u_{j}}{\partial w_{i}} . \tag{3.17}
\end{equation*}
$$

Equation (3.17) can be re-expressed by substituting mean wealth, $\bar{w}=\frac{\sum_{i}^{n} w_{i}}{n}$, on the left-hand side of the equation and rearranging to give

$$
\begin{equation*}
n \lambda=-\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{w_{i}}{\bar{w}} \frac{\partial u_{j}}{\partial w_{i}} . \tag{3.18}
\end{equation*}
$$

Substituting Equation (3.18) into Equation (3.5) gives

$$
\begin{equation*}
\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u_{j}}{\partial \pi_{i}}=\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{w_{i}}{\bar{w}} \frac{\partial u_{j}}{\partial w_{i}} \frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} . \tag{3.19}
\end{equation*}
$$

which rearranges to

$$
\begin{equation*}
\frac{\sum_{j}^{n} a_{j} \sum_{i}^{n} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{j}^{n} a_{j} \sum_{i}^{n \frac{w_{i}}{\partial} \frac{u_{j}}{\bar{w}} \partial w_{i}}}=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)}, \tag{3.20}
\end{equation*}
$$

the general solution for the case of proportional wealth taxation. That is the ratio of the distributional weighted sum of each of Individual $j$ 's marginal utility for each Individual $i$ 's probability of survival the distributional and the sum of each of Individual $j$ 's marginal utility for each Individual $i$ 's wealth weighted by distributional weights and relative income weights equals the marginal cost of saving one statistical life.

### 3.2.4 Applying altruism to the general solutions

The general solutions to the optimal provision problem can then be simplified based on the different functional forms of altruism. Whilst Jones-Lee (1992) shows there is a spectrum of forms of altruism which are reflected in the general solution; for simplicity, the analysis here is limited to four interesting cases:
[1] $S I$, or a complete lack of regard for others welfare, in which $(\forall j \neq i) \frac{\partial u_{i}}{\partial \pi_{j}}=\frac{\partial u_{i}}{\partial w_{j}}=0$,
[2] $\quad P A$, a regard for others which respects their preferences in that $i$ 's marginal rate of substitution of $w_{j}$ for $\pi_{j}$ is equal to $j$ 's marginal rate of substitution for their own wealth and own safety such that $\frac{\partial u_{i}}{\partial \pi_{j}} / \frac{\partial u_{i}}{\partial w_{j}}=\frac{\partial u_{j}}{\partial \pi_{j}} / \frac{\partial u_{j}}{\partial w_{j}}$,
[3] $S F A$, for which $i$ is concerned with $j$ 's safety, $\frac{\partial u_{j}}{\partial \pi_{j}}>0$, but indifferent to all other determinants of $j$ 's wellbeing, $\frac{\partial u_{i}}{\partial w_{j}}=0$, and
[4] $W F A$, for which $i$ is concerned with $j$ 's consumption of the private good, $\frac{\partial u_{j}}{\partial w_{j}}>0$, but indifferent to all other determinants of $j$ 's wellbeing, $\frac{\partial u_{i}}{\partial \pi_{j}}=$ 0.

These four cases show the range of preferences altruism of this type can take. SI preferences show the corner case of no altruism. SFA and WFA show the extreme cases for which altruism is directed at a single determinant of others' welfare with SFA being entirely focussed on an individual's probability of survival and WFA being entirely focussed on an individual's ability to consume. Whilst it is unlikely that any given individual is entirely altruistic towards an individual's wealth or safety with complete ignorance of the alternate, these cases are necessary to show the extremes of spectrum of altruistic concern in which all middle cases sit. PA is identified as a special middle case.

Table 3.1 tabulates the marginal benefits for each system of taxation and functional form.

Consider first the impact of altruism for the first-best case. For the general solution, optimal provision requires that mean ratio of the distributional weighted sum of each Individual $j$ 's marginal utility for Individual $i$ 's safety to the distributional weighted sum of each Individual $j$ 's marginal utility for Individual $i$ 's wealth equals the cost of saving one statistical life. Under

SI preferences terms within both the numerator and denominator of the general solution cancel to zero. Under SFA preferences, terms within the denominator which represent altruistic preferences for others' wealth cancel to zero. Under WFA preferences, terms within the numerator which represent altruistic preferences for others' safety cancel to zero.

Social welfare with SI preferences under a first-best tax is maximised when the marginal rate of substitution of wealth for survival probability equals the marginal cost of saving one statistical life as shown in Dehez \& Drèze (1982) and Bergstrom (1982). The condition for social welfare maximisation with PA preferences are identical to that for SI preferences as shown by Bergstrom (1982). The reason is that if a PA respects the preferences of the recipient of their altruism then those preferences are satisfied if the individual welfare of the recipient of altruism is maximised as they are under SI preferences with a first-best tax. Based on this finding Bergstrom (1982) concludes that if preferences take the form of PA then it is satisfactory to use SI preferences to determine optimal provision.

Under SFA preferences with first-best taxation, optimal provision occurs when mean ratio of the distributional weighted sum of each Individual $j$ 's marginal utility for an Individual $i$ 's safety to that Individual $i$ 's marginal utility for their own wealth equals the cost of saving one statistical life. This result derived in Jones-Lee (1991) shows that optimal provision is necessarily higher when preferences are safety-focused when compared to SI and PA.

Under WFA preferences with first-best taxation, optimal provision occurs when the mean ratio of Individual $i$ 's marginal utility for their own safety to the distributional weighted sum of each Individual $j$ 's marginal utility for Individual $i$ 's wealth equals the cost of saving one statistical life. This result derived in Jones-Lee (1992) shows that optimal provision is necessarily lower when preferences are wealth-focused when compared to SI and PA.

Jones-Lee (1992) shows that SFA and WFA are only corner cases with a spectrum of altruistic preferences between of which PA is a special case and SI is another corner solution for which the intensity of altruism is 0 . Optimal provision then is continuously increasing between the WFA and SFA cases which provide the minimum and maximum levels of optimal provision respectively. For both the SFA and WFA solutions the distributional weights do not drop out of the solution which means that the setting of theses weights, a value judgement, will impact the exact level of provision.

The tax extension of the models presented in this chapter has a number of results:
[1] Taxation impacts optimal provision for all functional forms.
[2] Regardless of tax structure the directional impact of SFA and WFA in relation to the SI result remains, and
[3] When taxation is no longer first-best, optimal provision is no longer equal under SI and PA preferences.

The choice over system of taxation affects the solution for optimal provision under all preferences. Under first-best taxation the ratio of marginal utilities for each individual's survival probability and wealth are considered separately. The condition for optimal provision is the mean across individuals. This is because the tax contribution for each individual is set independently. Under the other tax systems, the individual contributions are dependent on each other. Under uniform taxation that is a single tax contribution for each individual and under proportional wealth taxation that is the single tax rate term. The marginal rates of substitution for each individuals' wealth and survival probability are no longer aggregated independently. The condition for optimal provision under non-first-best tax systems is a single ratio of the marginal utilities for everyone's safety and the marginal utilities for everyone's wealth.

The only difference between the solutions for uniform and proportional taxation is the inclusion of the relative income weight, $w_{i} / \bar{w}$ on the marginal utilities for wealth. The result is a greater weight placed on the marginal utilities for wealthier individuals who bear a greater proportion of the cost of provision. Social welfare will be lower for both uniform and proportional wealth taxation relative to first-best taxation because individual tax contributions cannot be optimised for each individual separately.

Optimal provision is greatest under SFA preferences regardless of the tax system in place. The altruistic terms on the denominator of the ratios of marginal utility drop out of the condition for optimal provision. This necessarily decreases the size of the denominator when compared to the general solution and therefore increases the left-side of the condition. This means the rightside of the condition is also larger and therefore the higher cost of saving a statistical life, $\partial s / \partial\left(\sum_{i}^{n} \pi_{i}\right)$, due to increased levels of safety expenditure is accepted.

Optimal provision is lowest under WFA preferences regardless of the tax system in place. The altruistic terms on the numerator of the ratios of marginal utility drop out of the condition for optimal provision. This necessarily decreases the size of the numerator when compared to the general solution and therefore decreases the left-side of the condition. This means the right-side
of the condition is also smaller and therefore a lower cost of saving a statistical life, $\partial s / \partial\left(\sum_{i}^{n} \pi_{i}\right)$, due to lower levels of safety expenditure is accepted.

Whilst the ordering for optimal provision remains for SFA and WFA preferences relative to SI preferences, when taxation is no longer first-best the conditions for optimal provision for SI and PA preferences are no longer identical. Under a first-best tax, tax contributions for each individual are set to maximise the individual's utility. As PA preferences respect the individual's preferences what is good for the recipient of altruism is good for the altruist and there is no change in optimal provision. Under alternate tax systems individual preferences can no longer be satisfied independently. PA preferences place greater weight on the utility of the recipient of altruism. The PA effect depends on the relative weight placed on each individual.
Table 3.1: Model solutions for optimal provision under first-best, uniform, and proportional wealth taxation with a general utility function, SI, PA, SFA, and WFA preferences.

| Functional Form | First-best Taxation | Uniform Taxation | Proportional Wealth Taxation |
| :---: | :---: | :---: | :---: |
| General Case | $\frac{1}{n} \sum_{i}^{\sum_{j}^{\sum_{n}^{n}} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}} \sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}$ | $\frac{\sum_{i} \sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{i} \sum_{i} \sum_{j} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}$ | $\frac{\sum_{i} \sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{i} \sum_{j} n_{j} a_{j} \frac{w_{i}}{\bar{w}} \frac{\partial u_{j}}{\partial w_{i}}}$ |
| SI | $\frac{1}{n} \sum_{i}^{2} \frac{\frac{\partial u_{i}}{\partial \pi_{i}}}{\frac{\partial u_{i}}{\partial w_{i}}}$ | $\frac{\sum_{i}^{n} a_{i} \frac{\partial u_{i}}{\partial \pi_{i}}}{\sum_{i n} a_{i} \frac{\partial u_{i}}{\partial w_{i}}}$ | $\frac{\sum_{i}^{n} a_{i} \frac{\partial u_{i}}{\partial \pi_{i}}}{\sum_{i n} a_{i} \frac{w_{i}}{\bar{w}} \frac{\partial u_{i}}{\partial w_{i}}}$ |
| PA | $\frac{1}{n} \sum_{i}^{2} \frac{\frac{\partial u_{i}}{\partial \pi_{i}}}{\frac{\partial u_{i}}{\partial w_{i}}}$ | $\frac{\sum_{i}^{n}\left(a_{i}+\sum_{j \neq i}^{n} a_{j} b_{j}^{i}\right) \frac{\partial u_{i}}{\partial \pi_{i}}}{\sum_{i}^{n}\left(a_{i}+\sum_{j \neq i} a_{j} b_{j}^{i}\right) \frac{\partial u_{i}}{\partial w_{i}}}$ | $\frac{\sum_{i}^{n}\left(a_{i}+\sum_{j \neq i} a_{j} b_{j}^{i}\right) \frac{\partial u_{i}}{\partial \pi_{i}}}{\sum_{i}^{n}\left(a_{i}+\sum_{j \neq i} a_{j} b_{j}^{i}\right) \frac{w_{i}}{\bar{w}} \frac{\partial u_{i}}{\partial w_{i}}}$ |
| SFA | $\frac{1}{n} \sum_{i}^{\sum_{j}^{\sum_{n}} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}} \frac{\partial u_{i}}{\partial w_{i}}$ | $\frac{\sum_{i} \sum_{j} \sum_{j} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{i} a_{i} \frac{\partial u_{i}}{\partial w_{i}}}$ | $\frac{\sum_{i} \sum_{j} \sum_{j} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}{\sum_{i n} a_{i} \frac{w_{i}}{\bar{w}} \frac{\partial u_{i}}{\partial w_{i}}}$ |
| WFA | $\frac{1}{n} \sum_{i} \frac{\frac{\partial u_{i}}{\partial \pi_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}$ | $\frac{\sum_{i} a_{i} \frac{\partial u_{i}}{\partial \pi_{i}}}{\sum_{i} \sum_{j} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}$ | $\frac{\sum_{i}^{n} a_{i} \frac{\partial u_{i}}{\partial w_{i}}}{\sum_{i} \sum_{j}^{n} a_{j} \frac{w_{i}}{\bar{w}} \frac{\partial u_{j}}{\partial w_{i}}}$ |

[^10]
### 3.2.5 Simulations

The solutions presented thus far do not always give an indication of the expected direction of the effects for each functional form. Under certain preferences such as SFA and WFA, the directional impact is clear from changes to the marginal rates of substitution of wealth and safety. However, the impact of taxation is less clear as it will now depend on the distributions of wealth, altruism, and preferences for safety.

This section presents a series of simulations to show how the different functional forms and tax systems will impact optimal provision and the resulting social welfare. Consider a society of two individuals indexed $i=a, b$. Each individual has a differentiable and well-behaved utility function, $u_{i}$, which reflects preferences for own and others' survival probability $\pi_{a}=\pi_{b}$ and wealth $w_{a}, w_{b}$ where $u_{i}$ is strictly increasing in $\pi_{i}$ and $w_{i}$ and non-decreasing in its other arguments. Survival probability is a function of public spending, $s$, taking the form

$$
\begin{equation*}
\pi_{i}(s)=1-s^{-1} \tag{3.21}
\end{equation*}
$$

such that

$$
\begin{equation*}
\frac{d \pi_{i}}{d s}=s^{-2}>0, \text { and } \frac{d^{2} \pi_{i}}{d s^{2}}=-2 s^{-3}<0, \tag{3.22}
\end{equation*}
$$

so that survival probability increases in public spending with diminishing returns to spending. Public spending is funded through taxation. Each individual contributes a share of the total cost of providing the public good. Individual $a$ contributes a share $t \in[0,1]$ and Individual $b$ contributes $1-t$. All possible tax systems that share the cost between the two individuals can then be considered through simulation. The tax systems describe above sit in this range. Firstbest taxation is the tax share that produces the greatest social welfare, uniform tax is always $t=0.5$, and proportional wealth tax is the ratio of the individual's wealth to aggregate wealth. Individuals are assumed to be expected utility maximisers and thus probability of survival enters the utility function linearly. The expected utility function takes the form:

$$
\begin{equation*}
u_{i}=\pi_{i} \ln \left(w_{i}-t_{i} s\right)+\gamma_{i} \pi_{j}^{\alpha} \ln \left(w_{j}-\left(1-t_{i}\right) s\right)^{\beta} . \tag{3.23}
\end{equation*}
$$

Where the individual's private utility is given by the first term of the utility function, the product of their probability of survival and their logged net wealth after tax. The second term of the utility function represents the altruistic concern of the individual. It is weighted by $\gamma_{i}$, representing the strength of the altruism, which takes a value of zero for SI and is greater than
zero otherwise. The individual's altruistic utility is given by the product of the other individual's probability of survival to the power of $\alpha$ and their logged net wealth after tax to the power of $\beta$. The $\alpha$ and $\beta$ terms can take a value of 0 or 1 depending on the form altruism takes. For PA preferences shown in Equation (3.25): $\alpha=1$ and $\beta=1$. For SFA preferences shown in Equation (3.26): $\alpha=1$ and $\beta=0$, causing the wealth part of the function to take a value of 1 . For WFA preferences shown in Equation (3.27): $\alpha=0$ and $\beta=1$, causing the probability of survival part of the function to take a value of 1 . These forms follow the assumptions laid out for the general model presented at the beginning of Section 3.4.2. The individual expected utility functions for SI, PA, SFA, and WFA therefore take the following functional forms:

$$
\begin{array}{ll}
\text { SI: } & \left(1-s^{-1}\right) \ln \left(w_{i}-t_{i} s\right) \\
\text { PA: } & \left(1-s^{-1}\right) \ln \left(w_{i}-t_{i} s\right)+\gamma_{i}\left(1-s^{-1}\right) \ln \left(w_{j}-\left(1-t_{i}\right) s\right) \\
\text { SFA: } & \left(1-s^{-1}\right) \ln \left(w_{i}-t_{i} s\right)+\gamma_{i}\left(1-s^{-1}\right) \\
\text { WFA: } & \left(1-s^{-1}\right) \ln \left(w_{i}-t_{i} s\right)+\gamma_{i} \ln \left(w_{j}-\left(1-t_{i}\right) s\right) \tag{3.27}
\end{array}
$$

Optimal provision is simulated using Matlab for the full range of possible tax shares $(t \in[0,1])$ for different forms of altruism (PA, SFA, WFA), variation in the identity of the altruist (Individual a or b), strength of the altruistic concern ( $\gamma_{i}=0,0.2$ or 0.4 ) and the variation in wealth between individuals ( $w_{a}=10, w_{b}=10 ; w_{a}=9, w_{b}=11$; and $w_{a}=8, w_{b}=12$ ). The optimal level of provision, $\mathrm{S}^{*}$, is found for each tax share. This is the social welfare maximising point, Social welfare is defined as the aggregate utility. Optimal provision, $\mathrm{S}^{*}$, is plotted as a function of tax share, $t$, and is named the optimal provision curve. The point on this curve which gives the highest aggregate utility or social welfare is the first-best point.

Figure 3.1 shows the optimal provision curve and resulting aggregate utility for the SI case with $w_{a}=9, w_{b}=11$. Figure 3.2 illustrates how the SI optimal provision curve shifts as the relative wealth levels between the two individual in the society varies. Figure 3.3 illustrates how the optimal provision curve shifts under different forms of altruism when Individuals have the same endowment, $w_{a}=10, w_{b}=10$. Each form of altruism is then considered in turn. Figures 3.4 and 3.5 illustrates how the optimal provision curves shift under PA preferences when the strength of altruism increases and relative wealth levels vary respectively. Figures 3.6 and 3.7 illustrate the same effects for SFA preferences and Figures 3.8 and 3.9 for WFA preferences.

Figure 3.1 illustrates the optimal provision curve and the resulting aggregate utilities for the SI case with wealth levels $w_{a}=9, w_{b}=11$. The optimal provision curve takes an inverted ushape. At $t=0$, Individual $b$ covers the full cost of provision. At this point, optimal provision is approximately $S^{*} \approx 5.3$. Optimal provision increases as t increases, reaching a maximum of
$S^{*} \approx 5.8$ at $t \approx 0.375$. As t increases further optimal provision decreases to $S^{*} \approx 4.6$ at $t=1$, where Individual $a$ covers the full cost of provision. Survival probabilities are therefore first increasing and then decreasing.

Social welfare is calculated based on the optimal level of provision at each tax share. The resulting social welfare curve, $\mathrm{U}^{*}$ as a function of t , also takes an inverted u -shape. However, $U^{*}$ is not necessarily maximised at the point $S^{*}$ is at a maximum. The highest social welfare is achieved at $t^{*} \approx 0.325$ and is marked on the optimal provision curve by the point. This represents the first-best case as social welfare is at it's maximum and the optimal combinations of provision and cost sharing has been found. As $t$ deviates from $t^{*}$, social welfare necessarily becomes increasingly lower. As the first-best point is not necessarily at the peak of the optimal provision curve, moving from first-best taxation may increase or decrease the optimal level of provision. As the proportional tax share, indicated by the vertical line at $t=0.45$, differs from the first-best tax share social welfare is necessarily lower even though optimal provision is roughly the same. As the tax share for uniform tax, indicated by the vertical line at $t=0.5$, is further from first-best social welfare is even lower.

Figure 3.2 shows optimal provision for SI preferences for each combination of wealth levels. The right-hand curve is the optimal provision curve when $w_{a}=10, w_{b}=10$. As the incomes become less equal the optimal provision curves shift leftward. For each, the point on the optimal provision curve marks the first-best optimal provision point. For $w_{a}=10, w_{b}=10$ that is $t^{*}=$ $0.5, S^{*} \approx 5.8$. At this point the first-best, proportional and uniform tax shares are equal. As the incomes become unequal both the first-best tax share and proportional tax shares become smaller $\left(t^{*}=0.325, t^{\text {prop }}=0.45\right.$ for $w_{a}=9, w_{b}=11$ and $t^{*}=0.15, t^{\text {prop }}=0.4$ for $w_{a}=$ $8, w_{b}=12$ ). The optimal provision level remains around $S^{*} \approx 5.8$, so the individuals are paying the same combined amount, but the wealthier individual contributes increasingly more as $t$ becomes smaller. Both first-best tax shares are less than the proportional tax share and therefore the first-best tax in this case is progressive.

Although Pigou (1947) shows that optimal provision is necessarily highest under first-best taxation, Gaube (2000) shows that this is not necessarily true in a model of heterogeneous agents. The simulation shown in Figure 3.2 is an example of this result as optimal provision is largest at the first-best tax share when the two individual are identical in endowments, each with of $w=10$. When endowments are no longer equal, optimal provision is greater for a range of second-best taxes. That range for both $w_{a}=8, w_{b}=12$, and $w_{a}=9, w_{b}=11$ occurs when the tax share moves past the first-best tax share towards the proportional and uniform taxes.
Figure 3.2: Optimal provision for SI preferences at $w 1=8 \mathbf{w} 2=12$; $w 1=9, w 2=11$; and $w 1=w 2=10$.



Solid lines - Optimal provision curves. Left to right $-w_{a}=8, w_{b}=12$;
$w_{a}=9, w_{b}=11 ; w_{a}=10, w_{b}=10$. Points mark the first-best tax.

Solid line - Optimal provision curve, Dotted line - aggregate utility,
vertical lines (left to right) - first-best, proportion, and uniform tax share.

Figure 3.3 illustrates how different forms of altruism shift the optimal provision curve. Seven societies are depicted each with relative wealth levels: $w_{a}=10, w_{b}=10$. The black line illustrates a society with SI preferences. The blue lines illustrate two different societies each with PA preferences. The difference between the two PA societies occurs because the individuals have different strengths of altruism. The left-hand blue curve shows a society where Individual b is the altruist and Individual a is not such that $\gamma_{a}=0, \gamma_{b}=0.2$. The right-hand blue curve shows a society where Individual a is the altruist and Individual b is not such that $\gamma_{a}=0.2, \gamma_{b}=0$. The green line illustrates two societies with SFA preferences where Individual $a$ is the altruist in one and Individual $b$ is the altruist in the other. These lines are identical. The red lines illustrate two societies with WFA preferences. The difference between the two WFA societies occurs because the individuals have different strengths of altruism. As with PA preferences, the left-hand blue curve shows a society where Individual $b$ is the altruist and the right-hand blue curve shows a society where Individual a is the altruist.

Figure 3.3: Optimal provision and the resulting social welfare for all preference types at $\mathbf{w} 1=\mathbf{1 0}, \mathbf{w} \mathbf{2}=\mathbf{1 0}$.


Black line - SI, Green line - SFA, Red line - WFA (left individual b is the altruist, right individual a is the altruist), Blue line - PA (left individual $b$ is the altruist, right individual $a$ is the altruist)

As Figure 3.3 shows, when Individual $a$ has PA preferences and Individual $b$ has SI preferences, the optimal provision curve (Blue) shifts up and rightward relative to the SI optimal provision curve (Black). When Individual $b$ is the altruist the result is mirrored. The first-best optimal provision point under PA preferences is at the same level of provision as the SI firstbest result. This result is shown in Bergstrom (1982) and Jones-Lee (1991) and replicated here in the model presented in Section 3.2.1. The PA first-best tax shares are not the same as the SI first-best tax share. The PA first-best tax share is less than the SI first-best tax share and when Individual $b$ is the altruist, greater than the SI first-best tax share when Individual $a$ is the altruist. The optimal provision level remains around $S^{*} \approx 5.8$, so individuals pay the same combined amount, but the altruist contributes a greater share.

As the PA optimal provision curve intersects the SI optimal provision at the first-best point, the tax shares between the SI and PA first-best have higher levels of optimal provision than the first-best optimal level of provision. Gaube (2000) notes that Pigou (1947)'s finding is based on maximisation based on a pure efficiency motive of a social planner. Gaube (2000) shows that when a social planner maximises a function with equity concerns for a society with heterogeneous endowments then first-best optimal provision may be lower than second-best optimal provision as the public good can act as a redistribution device. A similar result may be seen in Figure 3.3 as the first-best optimal provision point is not at the peak of the PA optimal provision curve. Pure altruism places greater weight on the recipient of altruism in the social welfare function and therefore the function is no longer maximising the sum of private utilities. Here the equity concerns have entered the social welfare function through the preferences of those individual's in society.

Figure 3.4 illustrates how the optimal provision curves shift under PA preferences when the strength of altruism increases, and Figure 3.5 illustrates how the SI and PA optimal provision curves shift when relative wealth levels vary. In Figure 3.4 the green lines are the optimal provision curves when $\gamma_{i}=0.2$ and the blue lines are the optimal provision curves when $\gamma_{i}=$ 0.4. The two left-hand optimal provision curves depict societies where Individual b is the altruist and the two right-hand optimal provision curves depict societies where Individual a is the altruist. As the strength of altruism increases the PA curves shift further from the SI curve, but the first-best optimal level of provision remains equal to the SI result. The first-best tax shares differ between each curve. As the strength of altruism increases the altruist must pay a larger share of the cost to maximise social welfare.

Figure 3.5 shows the optimal provision curves for SI preferences (solid lines) and PA preferences (dotted lines) when Individual $b$ is the altruist for strengths for each combination of wealth levels (Black: $w_{a}=w_{b}=10$; Red: $w_{a}=9, w_{b}=11$; and Blue: $w_{a}=8, w_{b}=12$ ). As Figure 3.2 shows, optimal provision under the first-best case remains the same for all SI curves although the optimal tax level changes. Each PA curve is shifted left from the SI curve in relates to. As the optimal level of provision remains the same, the result by Bergstrom (1982) and Jones-Lee (1991) is robust to heterogeneity in wealth between individuals.

All PA optimal provision curves intersect the SI optimal provision curve at the SI first-best point and the first-best point under PA preferences is not at the peak of the PA optimal provision curve. This means provision at the first-best tax share under SI preferences is the same for SI and PA preferences. As the first-best tax shares are not the same, social welfare under PA preferences is not maximised if SI preferences were assumed by the decision-maker. In this case Consumer Frame 1 leads to inefficient provision as reaching the social optimal is a matter of selecting both the correct level of provision and tax system. There is also a range of secondbest taxes under PA preferences for which optimal provision is greater than the first-best. The welfare maximising point is therefore not necessarily the safety maximising point.
Figure 3.4: Optimal provision for PA preferences at $\mathbf{w} 1=10, \mathbf{w} \mathbf{2}=10$ with all strengths of altruism.

Figure 3.5: Optimal provision for PA preferences at $\mathbf{w} 1=\mathbf{x} \mathbf{w} \mathbf{2}=\mathbf{1 2}$;
$\mathbf{w} 1=\mathbf{9}, \mathbf{w} \mathbf{2}=11$; and $\mathbf{w} 1=\mathbf{w} 2=10$ with Individual $b$ as the altruist at
$\gamma_{b}=0.2$.

${ }^{4}$ Tax Share
Black line -SI , Green line - PA $\gamma_{i}=0.2$ (left individual b is the altruist, Solid lines - SI Optimal provision curves. Dotted lines - PA Optimal right individual a is the altruist), Blue line - PA $\gamma_{i}=0.4$ (left individual b provision curves. Blue lines $-w_{a}=8, w_{b}=12$; Red lines $-w_{a}=9$, is the altruist, right individual a is the altruist) $\quad w_{b}=11$; Black lines $-w_{a}=10, w_{b}=10$.

Figure 3.3 shows that when either individual has SFA preferences (Green), the optimal provision curve shifts upward relative to the SI (black) optimal provision curve. This is the result shown in Jones-Lee (1991) for the first-best case. The theory presented in Sections 3.2.2 and 3.2 .3 shows this result also holds for the proportional and uniform tax cases. These simulations show that this result holds for all tax shares. The first-best tax share does not deviate from the SI case as the altruistic individual places no additional weight on the other's tax contribution. Figure 3.6 illustrates how the optimal provision curves shift when the strength of altruism increases, and Figure 3.7 illustrates how the optimal provision curves shift when relative wealth levels vary.

Figure 3.6 shows the optimal provision curves for SFA preferences when $\gamma_{i}=0.2$ (Green) and $\gamma_{i}=0.4$ (Blue). As the strength of altruism increases the SFA curves shift further upward from the SI curve. As both individuals benefit equally from additional safety expenditure, the effect of SFA preferences on the optimal provision curve is the same regardless of who the altruist is as long as the strength of altruistic concern, $\gamma_{i}$, is the same. Again, that is because the SFA individual places no additional weight on the other's tax contribution.

Figure 3.7 shows the optimal provision curves for SI preferences (solid lines) and SFA preferences (dotted lines) when Individual $b$ is the altruist for strengths for each combination of wealth levels (Black: $w_{a}=w_{b}=10$; Red: $w_{a}=9, w_{b}=11$; and Blue: $w_{a}=8, w_{b}=12$ ). As the wealth levels become less equal the pairs of curves shift left. For each the SFA optimal provision curve is shifted upwards from the SI curve. As the SI curves shift left due to changes in relative wealth levels, the first-best point shifts along the curve. The same effect is seen on the SFA curves that suggests the welfare maximising point is not necessarily the safety maximising point even when altruism is focussed on safety.
Figure 3.6: Optimal provision for SFA preferences at $w 1=10, w 2=10 \quad$ Figure 3.7: Optimal provision for SFA preferences at $\mathbf{w} 1=8$
with all strengths of altruism.

Black line - SI, Green line - PA $\gamma_{i}=0.2$ Blue line - PA $\gamma_{i}=0.4$

As Figure 3.3 shows, when Individual $a$ has WFA preferences and Individual $b$ has SI preferences, the optimal provision curve (red) shifts down and rightward relative to the SI optimal provision curve (Black) leading to lower levels of provision. When Individual $b$ is the altruist the result is mirrored with a down and leftward shift. This is the result shown in JonesLee (1991) for the first-best case and in Sections 3.2.2 and 3.2.3 for the proportional and uniform tax cases. These simulations show that this result holds for all tax shares except when the altruist contributes the entire cost at which point the curves meet. That is $t=1$ when Individual a is the altruist and $t=0$ when Individual b is the altruist.

The first-best tax share for WFA preferences is greater than the SI case when Individual a is the altruist as more weight has been placed on the net wealth of Individual b relative to Individual a. The shift in the first-best tax share is greater than the shift under PA preferences. This is because the weight placed on Individual b's net wealth relative to safety is greater under WFA than PA preferences. When Individual $b$ is the altruist the result is mirrored. As with PA preferences, the first-best point is not at the peak of the optimal provision curve. The shift down the curve is greater which means there is a larger set of second-best taxes in which optimal provision is greater than at the first-best point. Figure 3.8 illustrates how the optimal provision curves shift when the strength of altruism increases, and Figure 3.9 illustrates how the optimal provision curves shift when relative wealth levels vary.

Figure 3.8 shows the optimal provision curves for WFA preferences when $\gamma_{i}=0.2$ (Green) and $\gamma_{i}=0.4$ (Blue). As the strength of altruism increases, the WFA curves shift further from the SI curve in the same directions. Therefore, the optimal level of provision becomes smaller for a given tax share and the first-best tax share deviates further from the SI result.

Figure 3.9 shows the optimal provision curves for SI preferences (solid lines) and WFA preferences (dotted lines) when Individual $b$ is the altruist for strengths for each combination of wealth levels (Black: $w_{a}=w_{b}=10$; Red: $w_{a}=9, w_{b}=11$; and Blue: $w_{a}=8, w_{b}=12$ ). For each the WFA optimal provision curve is shifted downwards and left from the SI curve. The effect of WFA preferences on the optimal provision curve is therefore consistent when the relative wealth levels of the individuals varies.
Figure 3.9: Optimal provision for WFA preferences at w1=8 $\mathbf{w} 2=12$; $\mathbf{w} 1=9, \mathrm{w} 2=11$; and $\mathbf{w} 1=w 2=10$ with Individual $b$ as the
Figure 3.8: Optimal provision for WFA preferences at $\mathbf{w} \mathbf{1}=\mathbf{1 0}, \mathbf{w} \mathbf{2}=10$ with all strengths of altruism. altruist at $\gamma_{b}=0.2$.


Black line - SI, Green line - PA $\gamma_{i}=0.2$ (left individual $b$ is the altruist, Solid lines - SI Optimal provision curves. Dotted lines - PA Optimal right individual a is the altruist), Blue line - PA $\gamma_{i}=0.4$ (left individual b is provision curves. Blue lines $-w_{a}=0, w_{b}$ the altruist, right individual $a$ is the altruist)

### 3.2.6 Summary

This section presented the solutions to an optimal provision model under different systems of taxation and different forms of altruism. The key result is that there is an interaction between taxation and altruism in determining the societal value. Taxation was shown to affect the condition for optimal provision for all forms of altruism. The simulations show, optimal provision can be found for any set of cost sharing rules, however the social welfare maximising point is a combination of the optimal tax system and optimal spending. This combined optimal depends on the relative incomes of the individuals and the presence of altruism. Altruistic preferences play a role in determining the optimal level of provision with the type of altruism, strength of altruistic concern and the identity of the altruist determining the size and direction of the effect on optimal taxation.

The directional effect of paternalistic altruism shown by Jones-Lee $(1991,1992)$ for the firstbest case is found to hold for all forms of taxation. SFA consistently results in the highest level of provision, and WFA the lowest level of provision. The result shown in Bergstrom (1982) that when altruists are PA then the first-best level of provision does not change relative to the SI result is confirmed, however simulations show the first-best tax share does vary. When taxation is not first-best, optimal provision under SI and PA preferences is no longer equal. Simulations showed that the deviation of the PA result from the SI result depends on the identity of the recipient of altruism and the change in taxation.

When altruism takes into account the impact of provision on an individuals consumption of the private good, the first-best tax share shifts. This occurs with PA and WFA preferences. In these cases, the altruist contributes a greater share of the cost of provision. At the SI first-best tax share optimal provision under both SI and PA preferences are the same. From this point if tax shares shift to favours the recipient of altruism the optimal level of provision first increases and then decreases back to the same level as the SI first-best. This is the PA first-best point. This results in first-best level of provision being less than optimal provision under a range of secondbest taxes. This range of second-best taxes are consistently closer to an equal share than the first-best tax share. A result similar to that shown in Gaube (2000) for societies that are maximising based on equity concerns. As tax shares shift further from the SI first-best point, optimal provision continues to fall.

Simulations suggest that these findings appear robust to changes in relative wealth levels of the individuals in society and that as the strength of altruism increases the effect size increases for all forms of altruism. As endowments become less equal, holding the aggregate endowment
constant, every optimal provision curve shifts so that the less wealthy individual pays a lower share. Unequal endowments lead to a similar result discussed in Gaube (2000) where the firstbest level of provision is not necessarily the highest level of provision. This means adjusting taxes away from first-best can lead to higher levels of safety even under SFA preferences which places specific focus on safety.

### 3.3 Willingness-to-pay ${ }^{17}$

This section models individual values to consider whether elicited values reflect the effects shown in the optimal provision model. Three models of WTP are presented. The first presents the case of the naïve cost-benefit analyst as described by Bergstrom (2006) which highlights the concerns of Milgrom (1993) who argued that including altruistic preferences leads to a form of double-counting. An alternate model is then presented which assumes a first-best tax vehicle so that altruistic respondents takes into account both the costs and benefits to other individuals and therefore avoid the double counting problem as described by Milgrom. Under this model it is assumed that each individual pays their own WTP and therefore will always be equally well off with or without the public good. In reality individuals do not contribute according to their WTP, and in practice alternate payment vehicles are used. A third model is presented which allows for alternate payment vehicles to be assumed.

### 3.3.1 The naïve cost-benefit analyst

Consider an economy of two individuals with standard preferences indexed $i$ and $j$. The individuals, who have incomes of $w_{i}$ and $w_{j}$, have the opportunity to fund a public good which provides separate private benefits on which the individuals place values of $x_{i}$ and $x_{j}$. The provision of the public good comes at an additional cost, requiring the individuals to pay $c_{i}$ and $c_{j}$, where and $c_{i}+c_{j}=C$.

In the case of the naïve cost-benefit analyst each individual values the other's welfare with a weighting of $\alpha_{i}$ and thus has a utility function represented by

[^11]\[

$$
\begin{equation*}
u_{i}=w_{i}+\alpha_{i} w_{j}+z\left(x_{i}+\alpha_{i} x_{j}-c_{i}\right) \tag{3.28}
\end{equation*}
$$

\]

where $z=1$ if the public good is funded, and 0 otherwise.

To complete the cost-benefit calculation, WTP for the public good is elicited from each individual. WTP for Individual $i$ is the maximum cost, $c_{i}$, such that

$$
\begin{equation*}
w_{i}+\alpha_{i} w_{j}=w_{i}+\alpha_{i} w_{j}+x_{i}-c_{i}+\alpha_{i} x_{j} . \tag{3.29}
\end{equation*}
$$

That is her social utility with the public good $(z=1)$ equals her social utility without the public good $(z=0)$. Rearranging gives

$$
\begin{equation*}
W T P_{i}=x_{i}+\alpha_{i} x_{j} . \tag{3.30}
\end{equation*}
$$

If the proposal is accepted at the sum of WTP each individual pays according to their WTP then each individual will have a private utility of

$$
\begin{equation*}
u_{i}=w_{i}+\alpha_{i} w_{j}-\alpha_{i} x_{j}, \tag{3.31}
\end{equation*}
$$

which is worse off than the status quo shown in Equation (3.28) when $z=0$. Both individuals are then privately worse off, which based on the Pareto criterion would suggest that there has unequivocally been a social dis-improvement.

Bergstrom (2006) uses the example of Alice and Bob to show that in this case potential Pareto improvement is only possible when the cost is less than or equal to the sum of the private benefits. ${ }^{18}$ The problem is that WTP has not taken into account that both individuals will pay for the public good. By requiring the altruist to take into account both the benefits and costs to others, the double-counting problem as described by Milgrom (1993) can be avoided. This is shown in the next section.

[^12]
### 3.3.2 Willingness-to pay with a first-best tax vehicle

Take another example where costs are taken into account so where the PA utility function for Individual $i$ is given by

$$
\begin{equation*}
u_{i}=w_{i}+\alpha_{i} w_{j}+z\left(x_{i}-c_{i}+\alpha_{i}\left(x_{j}-c_{j}\right)\right) \tag{3.32}
\end{equation*}
$$

WTP for Individual $i$ is the maximum $\operatorname{cost}, c_{i}$, such that

$$
\begin{equation*}
w_{i}+\alpha_{i} w_{j}=w_{i}+\alpha_{i} w_{j}+x_{i}-c_{i}+\alpha_{i}\left(x_{j}-c_{j}\right) . \tag{3.33}
\end{equation*}
$$

That is her social utility with the public good $(z=1)$ equals her social utility without the public good $(z=0)$. Rearranging Equation (3.33) gives

$$
\begin{equation*}
c_{i}=x_{i}+\alpha_{i} x_{j}-\alpha_{i} c_{j} \tag{3.34}
\end{equation*}
$$

where $c_{i}$ is Individual $i$ 's WTP. Given Individual $i$ knows that Individual $j$ also gives his WTP, $c_{j}$, Individual $i$ knows that Individual $j$ 's WTP response will be

$$
\begin{equation*}
c_{j}=x_{j}+\alpha_{j} x_{i}-\alpha_{j} c_{i}, \tag{3.35}
\end{equation*}
$$

Substituting Equation (3.35) into Equation (3.34) gives

$$
\begin{equation*}
c_{i}=x_{i}+\alpha_{i} x_{j}-\alpha_{i}\left(x_{j}+\alpha_{j} x_{i}-\alpha_{j} c_{i}\right), \tag{3.36}
\end{equation*}
$$

from which it follows that $W T P_{i}=c_{i}=x_{i}$; Individual $i$ 's private benefit. Under this case the sum of the WTP values is equal to the sum of the private benefits. If the total cost paid is equal to $c_{i}+c_{j}$ then the net benefit is zero to both individuals without the need for compensation.

Now the naïve cost-benefit analyst's problem has been avoided as WTP takes into account that both individuals will pay for the public good. This results in WTP being equal to the private benefit regardless of the strength of a respondent's altruistic preferences. Each individual is simply willing to pay up to their own private benefit because the choice of WTP is independent of both the other individual's WTP and tax contribution. The measure of double-counting from the naïve cost-benefit analyst's problem is found by taking the difference between Equations (3.30) and the solution to Equation (3.36). The Individual $i$ 's WTP is inflated by double counting by $\alpha_{i} x_{j}$.

In reality public goods are not funded based on individual WTP but with a system of taxation. Whilst under this model, WTP takes into account the other individual will contribute to funding the provision, the actual cost to the other individual is not correctly incorporated into the value. In this case, if a respondent cares about another's welfare beyond their consumption of the public good, then those preferences are ignored by the assumption of a first-best tax vehicle. The next section considers a model that allows individuals to take into account the costs to others based on a range of tax vehicles which could be used in a valuation study.

### 3.3.3 Willingness-to pay with alternative tax vehicles

Consider an economy of $n$ individuals with standard preferences indexed $i=1, \ldots, n$. The individuals, who have wealth, $w_{i}$, have the opportunity to fund a discrete increase in public good provision which results in separate private benefits on which the individuals place values of $x_{i}$. The provision of the public good comes at an additional cost, $C$, and requires the individuals each to pay a share $t_{i}$, where and $\sum_{i}^{n} t_{i}=1$ to ensure the full cost is covered.

Each individual values the other's welfare so that utility functions, $u_{i}$, reflect both the determinants of own utility and the determinants of others' utility, with $\alpha_{i}^{j}$ measuring the weight placed on Individual $j$ 's consumption of the public good and $\beta_{i}^{j}$ measuring the weight placed on Individual $j$ 's consumption of the private good. The utility function for Individual $i$ is given by

$$
\begin{equation*}
u_{i}=w_{i}-t_{i} C+x_{i}+\sum_{j \neq i}^{n-1}\left[\beta_{i}^{j}\left(w_{j}-t_{j} C\right)+\alpha_{i}^{j} x_{j}\right], \tag{3.37}
\end{equation*}
$$

This functional form allows for SI $(\alpha=0, \beta=0)$, $\operatorname{PA}(\alpha=\beta>0)$, SFA $(\alpha>0, \beta=0){ }^{19}$, and WFA $(\alpha=0, \beta>0)$.

It is agreed that the decision of whether to fund the public good shall be left to the rules of CBA. That is, if the sum of the benefits exceeds the sum of the costs then the public good will be provided. As Equation (3.36) demonstrates, each individual has correctly accounted for both the benefits and costs to the other individual so that the problem of Bergstrom (2006)'s 'naive

[^13]cost-benefit analyst' is avoided and there is no issue of double-counting in the form Milgrom (1993) describes.

To complete the cost-benefit calculation, WTP for the public good is elicited from each individual. WTP for Individual $i$ is the maximum $\operatorname{cost}, t_{i} C^{*}$, such that

$$
\begin{equation*}
w_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j} w_{j}=w_{i}-t_{i} C+x_{i}+\sum_{j \neq i}^{n-1}\left[\beta_{i}^{j}\left(w_{j}-t_{j} C\right)+\alpha_{i}^{j} x_{j}\right] . \tag{3.38}
\end{equation*}
$$

That is social utility with the public good equals social utility without the public good. Rearranging Equation (3.26) gives

$$
\begin{equation*}
W T P_{i}=t_{i} C^{*}=t_{i} \frac{x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}}{t_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j} t_{j}}, \tag{3.39}
\end{equation*}
$$

where $t_{i} C^{*}$ is Individual $i$ 's WTP. This implies the other $n-1$ individuals would share the remainder of the $\operatorname{cost}\left(1-t_{i}\right) C^{*}$. Using the constraint $\sum_{i}^{n} t_{i}=1$ so that $t_{j}=1-t_{i}-\sum_{k \neq i, j}^{n-2} t_{k}$, Equation (3.39) expands to

$$
\begin{equation*}
W T P_{i}=t_{i} C^{*}=t_{i} \frac{x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}}{\left(1-\sum_{j \neq i}^{n-1} \beta_{i}^{j}\right) t_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)} . \tag{3.40}
\end{equation*}
$$

The first differential with respect to own tax share is then

$$
\begin{equation*}
\frac{\partial W T P_{i}}{\partial t_{i}}=\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right) \frac{x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}}{\left[\left(1-\sum_{j \neq i}^{n-1} \beta_{i}^{j} t_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\right]^{2}\right.} . \tag{3.41}
\end{equation*}
$$

The partial derivative of WTP with respect to own tax share is then strictly non-negative and strictly positive if $\beta_{i}^{j}$ is positive for any $j$. That is, if Individual $i$ cares about another other individual's consumption of the private good, then the Individual $i$ 's tax share matters to their own WTP. Two cases can be identified for which $\beta_{i}^{j}=0 \forall j$ : SI, and SFA. For both these cases, WTP will be insensitive to taxation.

This solution simplifies for each of the functional forms: $\mathrm{SI}(\alpha=0, \beta=0), \mathrm{PA}(\alpha=\beta>0)$, SFA $(\alpha>0, \beta=0)$, and WFA $(\alpha=0, \beta>0)$. The resulting solutions for Equations (3.40) and (3.41) are tabulated in Table 3.2.

For the simplest case of SI, respondents are willing to pay up to their private benefit regardless of tax share. Similarly WTP, when preferences take the form of SGFA, is independent of tax share. In this case respondents are willing to pay up to their own private benefit plus an
additional amount according to their altruistic concern for other's benefits and is therefore strictly larger than SI result. WTP increases with benefits to others and the degree of altruistic concern.

For all other cases, individual tax share impacts upon WTP. These cases can be summarised as those in which a respondent values other's private consumption. When preferences take the form of WFA, WTP is necessarily lower than the SI result except in the case where $t_{i}=1$ for which WTP equals the private benefit as there is no cost imposed on the other individuals. When preferences take the form of PA, WTP may be lower or greater than the private benefit.

Table 3.2: Model solutions for WTP for different forms of altruism.

| Functional <br> Form | $\boldsymbol{W T P} \boldsymbol{P}_{\boldsymbol{i}}$ | $\frac{d W T P_{i}}{d t_{i}}$ |
| :---: | :---: | :---: |
| General Case | $t_{i} \frac{x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}}{\left(1-\sum_{j \neq i}^{n-1} \beta_{i}^{j} t_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\right.}$ | $\frac{\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\left(x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}\right)}{\left[\left(1-\sum_{j \neq i}^{n-1} \beta_{i}^{j} t_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\right]^{2}\right.}$ |
| SI | $x_{i}$ | 0 |
| PA | $t_{i} \frac{x_{i}+\sum_{j \neq i}^{n+1} \alpha_{i}^{j} x_{j}}{t_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} t_{j}}$ | $\frac{\sum_{j \neq i}^{n-1} \alpha_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\left(x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}\right)}{\left[\left(1-\sum_{j \neq i}^{n-1} \alpha_{i}^{j}\right)_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\right]^{2}}$ |
| SFA | $x_{i}+\sum_{j \neq i}^{n-1} \alpha_{i}^{j} x_{j}$ | 0 |
| WFA | $t_{i} \frac{x_{i}}{t_{i}+\sum_{j \neq i}^{n-1} \beta_{i}^{j} t_{j}}$ | $\frac{\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right) x_{i}}{\left[\left(1-\sum_{j \neq i}^{n-1} \beta_{i}^{j}\right)_{t}+\sum_{j \neq i}^{n-1} \beta_{i}^{j}\left(1-\sum_{k \neq i, j}^{n-2} t_{k}\right)\right]^{2}}$ |

At the first-best level of taxation for which the tax share is given by $t_{i}^{*}=x_{i} / \sum_{i}^{n} x_{i}$. At this level each individual pays their share of the total benefits. In this case, under PA preferences $W T P_{i}=x_{i}$. As the first derivative of WTP with respect to individual tax share is strictly positive, if $t_{i}<t_{i}{ }^{*}$ then $W T P_{i}<x_{i}$, and if $t_{i}>t_{i}{ }^{*}$ then $W T P_{i}>x_{i}$. This result suggests that a PA individual will offer a WTP greater than their private benefit if they are required to pay more than their first-best share of the cost. That is they will increase their own potential private costs to increase the probability of the public good, which gives net gains to others, being provided. The opposite is true for tax shares less than the first-best share. The PA individual will decrease their WTP below their private benefit to reduce the chance of public good provision to avoid net losses to others.

These findings are represented by an example of the two-person case in Figure 3.10 which depicts the case in which Individual $i$ receives a return of 100 and Individual $j 50$, so that $t_{i}{ }^{*}=$ $2 / 3$ and altruism, where present, is given a weighting of 0.5 . In this example WTP for SI is set at the private benefit for Individual $i$ of 100 , and WTP for SFA is set at 125 , Individual $i$ 's private benefit plus half of Individual $j$ 's benefit. Neither varies with taxation as for both cases Individual $i$ is indifferent to Individual $j$ 's level of private consumption. ${ }^{20}$

For the cases of PA and WFA, WTP is strictly increasing in $t$. Both start at $W T P=0$ where $t=0$ and increase in $t$ at different rates. WTP for the case of WFA is flatter and reaches the private benefit to Individual $i$ at $t=1$. WTP is equal for the SI and WFA results at this level of taxation because Individual $i$ is taking on the whole tax burden and thus the private consumption of Individual $j$ remains unaffected.

Similarly for the case of PA, WTP reaches the level of SFA when $t=1$ because Individual $i$ is taking on the entire tax burden and again the private consumption of Individual $j$ remains unaffected. The middle case of first-best taxation sees WTP for a PA individual equal to their private benefit. The reason being is that Individual $j$ is paying their share of the cost so that if aggregate WTP equals the sum of the private benefits then the cost to each individual is equal to their benefits and thus each individual remains at their initial level of utility.

Figure 3.10: WTP for SI, PA, SFA, and WFA as tax share varies.


[^14]
### 3.4.3 Safety example

This example presents WTP for an increase in safety expenditure for a society of two individuals, $a$ and $b$, over a range of tax shares. The example uses the society set out in the simulations in Section 3.2. The total wealth of the society is 20 and split either $w_{a}=10, w_{b}=$ 10 , or $w_{a}=8, w_{b}=12$. Individuals have a general utility function as shown in Equation (3.23) which simplifies based on their altruistic preferences that are either SI, PA, SFA, or WFA preferences as set out in Equations (3.24) to (2.27). The strength of the altruistic component is either $\gamma_{i}=0,0.2,0.4$ for which all preferences collapse to SI under $\gamma_{i}=0$. In this example the base expenditure is 4 . Based on Equation (3.21), probability of survival is $\pi=0.75$. WTP is found for additional safety expenditure that takes a value of either $s=5$ or $s=6$ for which the probability of survival is then $\pi=0.89$ or $\pi=0.9$ respectively.

Figures 3.11 to 3.14 illustrate WTP for individuals $a$ and $b$ and the resulting aggregate WTP for each set of parameters and preference types: SI (black lines), PA (blue lines), SFA (green lines), and WFA (red lines). Figure 3.11 illustrates WTP for Individual a (dashed lines), Individual b (dot-dash lines) and the resulting aggregate WTP (solid lines) when $w_{a}=10, w_{b}=10, s=$ $5, \gamma=0.2$. Figure 3.12 illustrates the effect of changes in the strength of altruism term $\gamma_{i}$ which takes a value of $0,0.2$, and 0.4 . Figure 3.13 illustrates the effect of changes in the increased expenditure that is valued, $s$, which takes values of 5 and 6 . Figure 3.14 illustrates the effect of changes in the relative levels of wealth of the two individuals. The wealth levels of the individuals either take a value of $w_{a}=10, w_{b}=10$ or $w_{a}=8, w_{b}=12$.

As shown in Figure 3.10 for a general public good, Figure 3.11 shows that WTP under SI (black lines) and SFA (green lines) preferences are constant across the range of taxes. SFA individuals value the safety of others and therefore WTP for a safety improvement is always greater than under SI preferences. WTP under PA (blue lines) and WFA (red lines) preferences are increasing in own tax share. The lines sloping upward for the origin illustrate Individual a's WTP and the downward sloping lines Individual b. The PA lines crosses through the SI line at the first-best point and the WFA line is consistently lower than the SI line. The WTP lines for PA and WFA individuals are concave and the resulting aggregate WTP lines take an inverted U-shape resulting in lower values for the public good when tax shares are less equal. In this example the public good has a cost of $s=5$ shown in the figure by the grey line. Aggregate WTP is greater than the cost under SI and SFA preferences under all tax shares, and under PA and WFA preferences for a range of taxes. This range of taxes is smaller for WFA preferences.

Figure 3.11: WTP for safety improvements with $\boldsymbol{w}_{a}=10, s=5, \gamma=0.2$


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Solid lines are aggregate WTP, long dashed lines are Position a, and dot-dash lines are Position b.

Figure 3.12 shows that as the strength of altruism increases from $\gamma=0.2$ (solid lines) to $\gamma=$ 0.4 (dashed lines) the SFA line shifts upward increasing the value of the public good. The WFA line shifts downward and right for Individual a and downward and left for Individual b resulting in a lower aggregate value for the public good. When the strength of altruism increases ender PA preferences the WTP line tilts around the first-best point such that the public good takes higher values when the tax share redistributes to the other individual (i.e their own tax share is greater than first-best). and a lower value when the public good redistributes to themselves (i.e their own tax share is less than first-best). There is little effect to aggregate WTP under PA preferences as the individuals' altruism acts against one another. These results align with those seen for the same effect in the optimal provision model illustrated in Figures 3.4, 6, and 8.

Figure 3.13 shows that as the underlying value of the public good increases from $s=5$ (solid lines) to $s=6$ (dashed lines) all lines move upward and therefore WTP is sensitive to the underlying value under all preferences.

Figure 3.12: WTP for safety improvements varying with $\gamma$.


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Parameters are $w_{a}=10, s=5$, solid lines are $\gamma=$ 0.2 , dashed line are $\gamma=0.4$.

Figure 3.13: WTP for safety improvements varying with $s$.


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Parameters are $w_{a}=10, \gamma=0.2$, solid lines are $s=$ 5 , dashed line are $s=6$.

Figure 3.14 shows that as the wealth levels change from $w_{a}=10, w_{b}=10$ (solid lines) to $w_{a}=8, w_{b}=12$ (dashed lines) all lines for Individual $a$ shit downward as the individuals wealth decreases and upward for Individual $b$ as wealth increases. This results in a lower firstbest tax share and a leftward shift in the aggregate WTP curve as the optimal provision simulations show.

Figure 3.14: WTP for safety improvements varying with $\boldsymbol{w}_{a}$.


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Parameters are $\gamma=0.2, s=5$, solid lines are $w_{a}=$ 10 , dashed line are $w_{a}=8$.

This section looked at the effect of altruism and taxation on WTP values. These findings suggest that individual values for public good provision are impacted by the tax system in place. These general results apply to the specific example of safety improvement and the results shown for the optimal provision model apply to WTP. As Milgrom (1993) casts doubt on whether altruistic values should be used in CBA, the next section considers the use of the values derived in this section. The question asked is 'do proposals accepted or rejected based on altruistic WTP responses also pass a compensation test leading to potential Pareto improvements?'

### 3.4 Compensation tests

### 3.4.1 Model

Consider a simple compensation test for two individuals. If each individual after a proposal is accepted at cost $C$ then their utility is given by

$$
\begin{equation*}
y_{i}+x_{i}-t_{i} C+\beta_{i} y_{j}+\alpha_{i} x_{j}-\beta_{i}\left(1-t_{i}\right) C \tag{3.42}
\end{equation*}
$$

The compensation payable or required by that individual can be calculated as

$$
\begin{equation*}
y_{i}+x_{i}-t_{i} C+\beta_{i} y_{j}+\alpha_{i} x_{j}-\beta_{i}\left(1-t_{i}\right) C=y_{i}+\beta_{i} y_{j}+\left(1-\beta_{i}\right) m_{i} \tag{3.43}
\end{equation*}
$$

Where $m_{i}$ is the compensation required or available depending on whether the individual is worse or better off after the proposal is put into place. Compensation is weighted by altruistic preferences over the others private consumption as they will benefit directly from compensation paid. Compensation received similarly comes at the cost to another and therefore comes at an additional cost.

Solving for $m_{i}$ gives

$$
\begin{equation*}
m_{i}=\frac{x_{i}-t_{i} C+\alpha_{i} x_{j}-\beta_{i}\left(1-t_{i}\right) C}{1-\beta_{i}} . \tag{3.44}
\end{equation*}
$$

Total compensation for individuals $i$ and $j$ is given by

$$
\begin{equation*}
m_{i}+m_{j}=\frac{x_{i}-t_{i} C+\alpha_{i} x_{j}-\beta_{i}\left(1-t_{i}\right) C}{1-\beta_{i}}+\frac{x_{j}-\left(1-t_{i}\right) C+\alpha_{j} x_{i}-\beta_{j} t_{i} C}{1-\beta_{j}} . \tag{3.45}
\end{equation*}
$$

Given total compensation must sum to 0 to pass a compensation test, the maximum cost such that compensation is payable can be found by setting equation (3.45) to zero giving

$$
\begin{equation*}
\frac{x_{i}-t_{i} C+\alpha_{i} x_{j}-\beta_{i}\left(1-t_{i}\right) C}{1-\beta_{i}}+\frac{x_{j}-\left(1-t_{i}\right) C+\alpha_{j} x_{i}-\beta_{j} t_{i} C}{1-\beta_{j}}=0 . \tag{3.46}
\end{equation*}
$$

Solving for $C$ gives the maximum cost that can be paid and compensation covered is given by

$$
\begin{equation*}
C=\frac{\frac{x_{i}+\alpha_{i} x_{j}}{1-\beta_{i}}+\frac{x_{j}+\alpha_{j} x_{i}}{1-\beta_{j}}}{\frac{t_{i}+\beta_{i}\left(1-t_{i}\right)}{1-\beta_{i}}+\frac{\left(1-t_{i}\right)+\beta_{j} t_{i}}{1-t_{j}}}, \tag{3.47}
\end{equation*}
$$

Which simplifies further to

$$
\begin{equation*}
C=\frac{\left(1-\beta_{j}+\left(1-\beta_{i}\right) \alpha_{j}\right) x_{i}+\left(1-\beta_{i}+\left(1-\beta_{j}\right) \alpha_{i}\right) x_{j}}{1-\beta_{j} \beta_{i}} \tag{3.48}
\end{equation*}
$$

Notice for all functional forms that taxation has no impact on the cost at which compensation is payable. For each functional form this solution can be simplified. For $\operatorname{SI}(\alpha=0, \beta=0)$ and PA $(\alpha=\beta>0)$ this simplifies to

$$
\begin{equation*}
C=x_{i}+x_{j}, \tag{3.49}
\end{equation*}
$$

the sum of the private benefits. For SFA $(\alpha>0, \beta=0)$ to

$$
\begin{equation*}
C=\left(1+\alpha_{j}\right) x_{i}+\left(1+\alpha_{i}\right) x_{j} \tag{3.50}
\end{equation*}
$$

the sum of the private benefits each given an additional weighting based on the altruism felt towards that individual. For WFA $(\alpha=0, \beta>0)$ to

$$
\begin{equation*}
C=\frac{\left(1-\beta_{j}\right) x_{i}+\left(1-\beta_{i}\right) x_{j}}{1-\beta_{j} \beta_{i}} . \tag{3.51}
\end{equation*}
$$

The sum of private benefits weighted down by the preferences for each other's wealth. These findings suggest that the largest cost payable for which compensation is payable is strictly higher for SFA, and lower for WFA, than the SI and PA results. Comparing these findings to the WTP results suggests that for SI and SFA, WTP values are consistent with the rules of CBA, whilst for WTP values which take into account others private consumption may not. For example WTP values elicited under PA preferences may lead to misleading conclusions as they do not consider compensation actually being paid.

### 3.4.2 Example

These findings are represented by an example of the two-person case in Figure 3.15 which depicts the case in which both individuals receive a return of 100 so that $t_{i}{ }^{*}=1 / 2$ and altruism, where present, is given a weighting of 0.25 towards Individual $j$ and 0.5 towards Individual $i$. Aggregate WTP and the maximum compensable amount is found for each individual.

Aggregate WTP equals the compensable amount for SI and SFA but not for PA and WFA. For some tax shares: PA, $\mathrm{t}[\approx 0.25,0.5]$ and WFA, $\mathrm{t}[\approx 0.25, \approx 0.6]$, aggregate WTP is greater than the compensable amount. Proposals accepted in these ranges according to aggregate WTP would lead to costly proposals being accepted. For the remaining ranges aggregate WTP is less
than the compensable amount and potential Pareto improving projects may ultimately be rejected.

If in this example, the richer individual is the stronger altruist then the ranges for which WTP is greater than compensation are consistent with a large range of plausible tax systems which uniform taxation at $t=0.5$ and $t<0.5$ moving towards proportional and ultimately progressive systems of taxation. These results suggest caution should be taken when using WTP values for CBA that may include preferences for other's private good consumption.

Figure 3.15: Aggregate WTP and compensation as tax share varies


Notes: Solid lines show aggregate WTP and dotted lines show the maximum cost for which compensation can be paid. The SI (blue lines) result shows WTP $=$ Comp $=200$. Similarly Comp $=200$ for PA (red lines) whilst WTP does not necessarily equal 200. WTP and Comp are equal for SFA (green lines) and aren't for WFA (orange lines).

### 3.5 Consumer Frames

There are three elements to consider when evaluating each frame:
[1] The impact of different preference sets on the societal value.
[2] The impact of different preference sets on individual values.
[3] The use of these values in CBA. Two questions are asked. First, does the method of elicitation respect the individual's sovereignty by respecting their preferences? Second, does the aggregation of these preferences pass a compensation test?

The findings of the optimal provision model provides the answer to the first question, whilst the remaining questions are answered using the CBA models.

### 3.5.1 Consumer Frame 1

Under Consumer Frame 1, respondents act in accordance with their preferences over their own outcome. The result of the optimal provision model shows that under a first-best tax, social welfare is maximised when the mean marginal rate of substitution of own wealth for own safety is equal to the marginal cost of saving one statistical life which is the ratio of prices of safety and private consumption. Under alternate taxes social welfare is no longer reaches the potential maximum as taxation cannot be set to maximise individual utilities. Instead taxes are set at a societal level through a defined system. Under uniform and proportional taxation, the condition for maximising social welfare is no longer based on the marginal rates of substitution between each individuals wealth and survival probability but the weighted sum of the marginal utilities for survival probability divided by the weighted sum of the marginal utilities for wealth.

Depending on the system of taxation, optimal provision could then be larger or smaller than under the first-best case even though social welfare is by definition lower. The simulation of optimal provision under SI preferences presented in Figure 3.1 shows the case in which optimal provision is greater than the first-best level for the tax shares between $\mathrm{t} \sim(0.33 ; 0.45)$. Thus the value of safety expenditure is increased through this range.

As preferences are limited to SI alone, if individuals in society have preferences over others' welfare then the principle of consumer sovereignty has been violated as individuals are not
preferences are not being respected. The results of the simulations show the potential for incorrect choice over public good provision. For example if individuals in society have SFA preferences, then providing the public good under the assumption of SI preferences will lead to consistent under-provision. The opposite is true if individuals have WFA preferences which would lead to over-provision. If preferences are PA then even assuming SI preferences under a first-best tax may lead to the distribution of costs being inefficient as simulations show optimal taxation varies from the SI case.

The use of WTP values elicited through Consumer Frame 1 are efficient under all tax vehicles and naïve cost-benefit analyst problem is avoided. As such the use of these values could be used within a benefit cost calculation without concern for distorted values. The use of these values still however violate consumer sovereignty and have the potential to inefficiency. For example, if preferences are SFA, which are still immune to naïve cost-benefit analyst, then beneficial proposals may be rejected. Alternatively using SI values in a society with WFA preferences could lead to costly proposals being accepted.

### 3.5.2 Consumer Frame 2

Under Consumer Frame 2, respondents act in accordance with their preferences over their own outcome and the outcomes of others. As respondents can include preferences over others' welfare in their values, the principle of consumer sovereignty is respected. The optimal provision model shows that the form altruism takes impacts the value of the public good with SFA increasing the optimal level of provision and therefore the value and WFA decreasing the optimal level of provision and therefore the value. As with SI preferences the choice over tax system impacts the value of the public good.

Whilst Consumer Frame 2 respects consumer sovereignty, elicited values are not necessarily efficient when used in a compensation test. If altruism is SFA then elicited values are efficient as they are not impacted by the payment vehicle. This leads to a similar conclusion to JonesLee (1991) statement that altruism should be included in values if and only if it is safetyfocused. If a first-best tax vehicle is used then PA values are also efficient but WFA is not. If altruism takes into account costs then tax vehicle matters as there is a positive relationship between tax share and WTP. There is also the problem of assuming a first-best tax vehicle and then funding the public provision under an alternate tax vehicle as respondents potentially have different values for the alternate proposals.

### 3.6 Summary

The aim of this chapter was to evaluate the two Consumer Frames described in the previous section. Three models were used to consider the effect of altruism and taxation on the value of a public good. An optimal provision model was used to model the societal value. A WTP model was used to model individual values. Finally, a model of the compensation test is presented to assess if these WTP would lead to potential Pareto improvements if used in CBA.

The model of optimal provision of public safety expenditure considered SI, PA, SFA, and WFA preferences under first-best, uniform, and proportional wealth taxation. A two-person society was simulated for the full range of cost sharing rules. Optimal provision is highest under SFA preferences and lowest under WFA preferences. This result holds regardless of the tax system in place.

Under first-best taxation optimal provision is consistent under SI and PA preferences, however simulations show that the optimal tax share isn't the same. When taxation deviates from firstbest, optimal provision is different under SI and PA preferences as provision and taxation favours the recipient of altruism. Optimal provision is not necessarily greatest at the first-best which aligns with Gaube (2000)'s result that equity concerns may result in higher levels of optimal provision at second best due to redistribution through public good provision.

The choice of frame will impact the value attached to the public good. Further to this the choice of tax structure will affect the value. It can be argued that Consumer Frame 1 does not respect consumer sovereignty as individuals are limited to their private preferences. Depending on the form of altruism, assuming Consumer Frame 1 could lead to inefficient outcomes due to overor under-provision and inefficient taxation.

Second the frames were modelled under a cost-benefit framework. Two models were presented: the first, a model of WTP responses, and the second models the compensation test. Both considered the different forms altruism examined in the optimal provision model. The WTP model showed that double counting occurs due to responses not correctly accounting for both the costs and benefits to others. A general model was used to show the impact of altruism and taxation when both benefits and costs are taken into account by the individual. At the individual level, values are shown mirror the results of the optimal provision model with values largest for SFA and smallest for WFA.

WTP is equal to the private value for SI preferences and for PA preferences under a first-best tax. When alternate tax vehicles are assumed, PA values deviate from the private value. A positive relationship is shown between WTP and tax share. This result holds true for all
altruistic preferences which take into account the costs borne by others. This result is tested in Chapter 5 and 6.

These values were then compared to a compensation test to evaluate their use in a cost-benefit calculation. Under SI and SFA preferences, WTP values are efficient for use in CBA. Under PA and WFA preferences, WTP values aren't efficient for use in CBA. These preference types take into account the costs to others and therefore become a problem when considering compensation. The sum of WTP varies based on tax share, however there is a single total cost for which compensation is payable regardless of tax structure. Consumer Frame 2 is therefore inefficient under these preferences.

In conclusion, under Consumer Frame 1 elicited values are efficient but the principle of citizen sovereignty is violated. Under Consumer Frame 2, the principle of consumer sovereignty is respected but values may be inefficient for use in CBA. These findings are discussed in Chapter 7 alongside the findings of the next chapter.

## CHAPTER FOUR <br> Citizen Frame

### 4.1 Introduction

The aim of this chapter is to model the value of a public good under a novel method for eliciting values for public goods, the Citizen Frame:

Citizen Frame: Respondents act in accordance with their preferences for their own outcome and the outcomes of others. Respondents are placed behind a VoI and thus are informed about the distribution of income and public good endowments, but not their own position within the distribution.

Under the Citizen Frame, individuals are assumed to have the same preferences as Consumer Frame 2, however they are uncertain of their position in society due to the addition of a VoI. Based on the equi-probability model of Harsanyi $(1978,1979)$, each individual has an equal probability of being in each position and thus, a priori, each position is given equal weight. To reflect Chapter 3, the same two approaches are taken.

First, the optimal provision presented in Section 3.2 is extended modelled to find the societallevel value of a public good for a Citizen Frame. The same altruistic preferences, based on those defined in Jones-Lee $(1991,1992)$ and the tax extension presented in Chapter 3 remain. The extension presented in this chapter adds a VoI which makes hidden each individual's wealth and survival probability. Behind a VoI, the social welfare function which is maximised to find optimal provision, becomes the sum of the expected utilities across individuals. Optimal provision is found by maximising the expected social welfare constrained by the tax system.

Second, individual-level values for a public good are modelled. Individuals from which values are elicited are assumed to face the same problem as presented in Section 3.3.3 but are now placed behind a VoI. Again each individual is uncertain of their position in society and could be in any with equal probability. A position is defined by its level of wealth, return from public good provision, and a tax share defining their share of the contribution. For consistency, rather than eliciting a WTP for the public good, a single societal level value is elicited from the individual which implies each position would contribute their pre-defined share of the cost.

Finally, the results of these models are used to assess the Citizen Frame and compared back to the Consumer Frames. The same three questions are considered. What happens to the socially optimal provision value of a public good? What happens to WTP values? Are these values appropriate for CBA by respecting Consumer Sovereignty and achieving a potential Pareto improvement when used in a compensation test?

### 4.2 Optimal provision

Extending the model from Section 3.2, based on Jones-Lee (1991), now consider a society of $n$ positions, indexed $p=1 \ldots n$, with each of the $n$ individuals, indexed $i=1 \ldots n$, being assigned to a position. In order to determine optimal provision with impartiality as described by Harsanyi $(1953,1955)$ individuals are placed behind a VoI which makes hidden their position in society. Wealth and survival probability is then assigned to a position rather than an individual.

Each individual $i$ has a state-dependent differentiable and well-behaved utility function, $u_{p}^{i}$, which reflects their preferences over their own and others' survival probabilities $\pi_{1}, \ldots, \pi_{n}$ and levels of wealth $w_{1}, \ldots, w_{n}$ from each position $p$. Each individual, as an expected utility maximiser in the form of a vNM utility function, expects to realise each of $n$ positions in society with probability of $1 / n$. Thus expected utility behind a veil of ignorance is given by $U^{i}$ such that

$$
\begin{equation*}
U^{i}=\frac{1}{n} \sum_{p}^{n} u_{p}^{i}\left(\pi_{1}, w_{1}, \ldots, \pi_{n}, w_{n}\right) . \tag{4.1}
\end{equation*}
$$

where $u_{p}^{i}$ is strictly increasing in $w_{p}$ and $\pi_{p}$ and non-decreasing in its other arguments. This functional form is sufficient to represent a broad range of forms of pure and paternalistic altruism including those discussed in Chapters 2 and 3.

The Utilitarian social welfare function in expectation is given by

$$
\begin{equation*}
\max _{s} \frac{1}{n} \sum_{p}^{n} a_{p} \sum_{i}^{n} u_{p}^{i}\left(\pi_{1}, w_{1}-t_{1}, \ldots, \pi_{n}, w_{n}-t_{n}\right) \quad \text { s.t. } s=\sum_{p}^{n} t_{p} \tag{4.2}
\end{equation*}
$$

which is the weighted aggregation of individual state-dependent utilities. Weights are applied by position not individual as they are most commonly based on wealth level which in this model is dependent on position not individual. The public good consumed by the individual in each position is differentiable, increasing and strictly concave in public good expenditure, $s$, financed through tax, $t_{p}$, levied on the $n$ positions. The system of taxation in place is defined by the assumptions over the relationship between the $t_{p}$ terms.

For the general case, the Lagrangean is set out as

$$
\begin{equation*}
L=\frac{1}{n} \sum_{p}^{n} a_{p} \sum_{i}^{n} u_{p}^{i}\left(w_{1}-t_{1}, \pi_{1}(s), \ldots, w_{n}-t_{n}, \pi_{n}(s)\right)+\lambda\left(s-\sum_{p}^{n} t_{p}\right), \tag{4.3}
\end{equation*}
$$

where $\lambda$ is a Lagrange multiplier. The partial derivative with respect so public expenditure is identical for each tax structure and when set to zero gives the following first-order condition

$$
\begin{equation*}
\frac{\partial L}{\partial s}=\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}} \frac{\partial \pi_{p}}{\partial s}+\lambda=0 . \tag{4.4}
\end{equation*}
$$

Following Jones-Lee (1991), under the assumption that $a_{q} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}$ and $\frac{\partial \pi_{p}}{\partial s}$ are uncorrelated across the $p$ positions are uncorrelated across the $n$ individuals, Equation (4.4) can be rearranged to give

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n} \frac{a_{q}}{n \lambda} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{4.5}
\end{equation*}
$$

This assumption is the natural counterpart to those applied in Chapter 3 and states that individuals' preferences for the safety of Position $p$ from Position $q$ is uncorrelated with the marginal cost of increasing Position $p$ 's survival probability.

Whilst for each system of taxation the partial derivative with respect to public safety expenditure, $s$, is the same, the partial derivative with respect to tax levels differs based on the system of taxation.

### 4.2.1 Optimal provision under first-best taxation

Starting with the case of first-best lump-sum taxation for which $t_{1}, \ldots, t_{n}$ are each independent. Thus no further adjustment is needed to the Lagrangean presented in Equation (4.3). The partial derivative and with respect to $t_{p}$ when set to zero gives the first-order condition

$$
\begin{equation*}
\frac{\partial L}{\partial t_{p}}=-\frac{1}{n} \sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}-\lambda=0 \quad \text { for } p=1, \ldots, n \tag{4.6}
\end{equation*}
$$

Giving a total of $n+1$ first-order conditions. Rearranging Equation (4.6) gives

$$
\begin{align*}
& a_{q}=-\left(n \lambda+\sum_{r \neq q}^{n} a_{r} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)\left(\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)^{-1}, \text { and }  \tag{4.7}\\
& n \lambda=-\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}, \tag{4.8}
\end{align*}
$$

By substituting Equation (4.7) and (4.8) into Equation (4.5) it follows that

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \frac{\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}}=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} \tag{4.9}
\end{equation*}
$$

The public good is provided up to the point where the marginal cost of providing the public good equals the marginal benefit. The marginal benefit is defined as the sum across positions $p$ the ratio of the weighted sum marginal utilities for that position's survival probability and the weighted sum marginal utilities for that positions consumption of the private good.

### 4.2.2 Optimal provision uniform taxation

Under uniform taxation each an equal amount is contributed from each position such that $t_{p}=$ $T \forall p$ so that public safety expenditure $s=n T$. Including this assumption in the Lagrangean gives

$$
\begin{equation*}
L=\frac{1}{n} \sum_{p}^{n} a_{p} \sum_{i}^{n} u_{p}^{i}\left(w_{1}-T, \pi_{1}(s), \ldots, w_{n}-T, \pi_{n}(s)\right)+\lambda(s-n T), \tag{4.10}
\end{equation*}
$$

The first order condition with respect to $T$ is

$$
\begin{equation*}
\frac{\partial L}{\partial T}=-\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}-n \lambda=0, \tag{4.11}
\end{equation*}
$$

which rearranges to give

$$
\begin{equation*}
n^{2} \lambda=-\sum_{q}^{n} a_{q} \sum_{p}^{n} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}, \tag{4.12}
\end{equation*}
$$

Substituting Equation (4.12) into Equation (4.5) and rearranging gives

$$
\begin{equation*}
\frac{\sum_{p}^{n} \sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{p}^{n} \sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}}=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} \tag{4.13}
\end{equation*}
$$

the a general solution for the case of uniform taxation. The marginal benefit is defined as the ratio of the sum across positions $p$ of the weighted sum marginal utilities for that position's survival probability and the sum across positions $p$ of the weighted sum marginal utilities for that position's consumption of the private good.

### 4.2.3 Optimal provision proportional taxation

Under proportional wealth taxation each position is taxed at the same rate $\tau$ so that each position's tax contribution is $\tau w_{p}$ and public safety expenditure is $s=\tau \sum_{p}^{n} w_{p}$. Including this assumption in the Lagrangean gives

$$
\begin{align*}
L=\frac{1}{n} \sum_{p}^{n} a_{p} \sum_{i}^{n} u_{p}^{i}\left(w_{1}(1-\tau), \pi_{1}(s), \ldots,(1-\tau),\right. & \left.\pi_{n}(s)\right)  \tag{4.14}\\
& \left.+\lambda\left(s-\tau \sum_{p}^{n} w_{p}\right)\right) .
\end{align*}
$$

The partial derivative with respect to $\tau$ when set to zero gives the first-order condition

$$
\begin{equation*}
\frac{\partial L}{\partial \tau}=-\frac{1}{n} \sum_{q}^{n} a_{q} \sum_{p}^{n} w_{p} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}-\sum_{i}^{n} w_{i} \lambda=0, \tag{4.15}
\end{equation*}
$$

which rearranges to give

$$
\begin{equation*}
n \sum_{i}^{n} w_{i} \lambda=-\sum_{q}^{n} a_{q} \sum_{p}^{n} w_{p} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}} . \tag{4.16}
\end{equation*}
$$

Equation (4.16) can be re-expressed by including mean wealth, $\bar{w}=\frac{\sum_{i}^{n} w_{i}}{n}$, giving

$$
\begin{equation*}
n^{2} \lambda=-\sum_{q}^{n} a_{q} \sum_{p}^{n} \frac{w_{p}}{\bar{w}} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}} . \tag{4.17}
\end{equation*}
$$

Substituting Equation (4.17) into Equation (4.5) and rearranging gives

$$
\begin{equation*}
\frac{\sum_{p}^{n} \sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{p}^{n} \sum_{q}^{n} a_{q} \frac{w_{p}}{\bar{w}} \sum_{i}^{\sum_{i}} \frac{\partial u_{q}^{i}}{\partial w_{p}}}=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{4.18}
\end{equation*}
$$

the general solution for the case of proportional wealth taxation. The marginal benefit is defined as the ratio of the sum across positions $p$ of the weighted sum marginal utilities for that position's survival probability and the sum across positions $p$ of the sum marginal utilities for that position's consumption of the private good weighted by distributional weights and wealth of the positions relative to the societal mean.

### 4.2.4 Applying altruism to the general solutions

The solutions for optimal provision derived in the previous section can be simplified further by applying the assumptions over functional form for each form of altruism as in Chapter 3:
[1] SI is defined as $\frac{\partial u_{i}}{\partial \pi_{j}}=\frac{\partial u_{i}}{\partial w_{j}}=0 \forall j \neq i$,
[2] PA is defined as $\frac{\partial u_{i}}{\partial \pi_{j}} / \frac{\partial u_{i}}{\partial w_{j}}=\frac{\partial u_{j}}{\partial \pi_{j}} / \frac{\partial u_{j}}{\partial w_{j}}$,
[3] SFA is defined as $\frac{\partial u_{i}}{\partial w_{j}}=0 \forall j \neq i$, and
[4] WFA is defined as $\frac{\partial u_{i}}{\partial \pi_{j}}=0 \forall j \neq i$.
Table 4.1 tabulates the marginal benefits for each system of taxation and each functional form.
Table 4.1: Model solutions for optimal provision behind a VoI under first-best, uniform, and proportional wealth taxation with a general utility function, SI, PA, SFA, and WFA preferences.

| Functional Form | First-best Taxation | Uniform Taxation | Proportional Wealth Taxation |
| :---: | :---: | :---: | :---: |
| General | $\frac{1}{n} \sum_{p}^{\sum_{q}^{2}} \frac{\sum_{q} a_{q} \sum_{i} \sum_{j} \frac{\partial u_{q}^{i}}{\partial \pi_{p}^{j}}}{\sum_{n} \sum_{i} \sum_{i} \sum_{j} \frac{\partial u_{q}^{i}}{\partial w_{p}^{j}}}$ | $\frac{\sum_{p} \sum_{q}^{n} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{p} \sum_{q} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p} \sum_{q}^{n} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{p} \sum_{q} a_{q} \frac{w_{p}}{\bar{w}} \sum_{i} \frac{\partial u_{q}^{i}}{\partial w_{p}}}$ |
| SI | $\frac{1}{n} \sum_{p}^{\sum_{i} \frac{\partial u_{p}^{i}}{\partial \pi_{p}}} \frac{\sum_{i} \frac{\partial u_{p}^{i}}{\partial w_{p}}}{\sum_{n}}$ | $\frac{\sum_{p}^{n} a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{n} a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p}^{n} a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{p} a_{p} \frac{w_{p}}{\bar{w}} \sum_{i} \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ |
| PA | $\frac{1}{n} \sum_{p}^{\sum_{i}} \frac{\sum_{i}\left[a_{p}+\sum_{\substack{n-1 \\ q \neq p}} a_{q} \sum_{\substack{n-1 \\ j \neq i}} b_{p}^{j}\right] \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{n}\left[a_{p}+\sum_{\substack{n-1 \\ q \neq p}} a_{q} \sum_{\substack{n-1 \\ j \neq i}} b_{p}^{j}\right] \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p} \sum_{i}\left[a_{p}+\sum_{\substack{n-1 \\ q \neq p}} a_{q} \sum_{\substack{n-1 \\ j \neq i}} b_{p}^{j}\right] \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{i} \sum_{i}\left[a_{p}+\sum_{\substack{n-1 \\ q \neq p}} a_{q} \sum_{\substack{n-1 \\ j \neq i}} b_{p}^{j}\right] \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p} \sum_{i}\left[a_{p}+\sum_{\substack{n-1 \\ q \neq p}} \sum_{q} \sum_{\substack{n-1 \\ j \neq i}} b_{p}^{j}\right] \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{p} \frac{w_{p}}{\bar{w}} \sum_{p} \sum_{i}\left[a_{p}+\sum_{\substack{n-1 \\ q \neq p}} \sum_{q} \sum_{n-1}{ }_{j \neq i}^{j} b_{p}^{j}\right] \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ |
| SFA | $\frac{1}{n} \sum_{p}^{\sum_{q}^{\sum_{n}} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}} \frac{a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial w_{p}}}{1}$ | $\frac{\sum_{p} \sum_{q}^{n} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{p} a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p} \sum_{q}^{n} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{p} a_{p} \frac{w_{p}}{\bar{w}} \sum_{i} \frac{\partial u_{p}^{i}}{\partial w_{p}}}$ |
| WFA | $\frac{1}{n} \sum_{p}^{i} \frac{a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{q} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p}^{n} a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{p} \sum_{q} a_{q} \sum_{i} \frac{\partial u_{q}^{i}}{\partial w_{p}}}$ | $\frac{\sum_{p}^{n} a_{p} \sum_{i} \frac{\partial u_{p}^{i}}{\partial \pi_{p}}}{\sum_{p} \sum_{q} a_{q} \frac{w_{p}}{\bar{w}} \sum_{i} \frac{\partial u_{q}^{i}}{\partial w_{p}}}$ |

Optimal provision is impacted by taxation in a similar manner for the Citizen Frame as Consumer Frame 2 (see Table 3.1 for comparison). Under a first-best tax, the condition for optimal provision is averaged across positions because taxation is set independently for each position. Under the alternate tax systems, the condition for optimal provision is given by a single term which is the ratio of all weighted marginal utilities for survival probability over all weighted marginal utilities for wealth.

A key difference is that the Citizen Frame takes into account each individuals preference for each position, whilst Consumer Frame 2 only takes into account the preference of the individual in that position. This will result in the impact of outlying preferences are reduced for the Citizen Frame. For example, in the context of VSL if one individual has a strong preference for safety this value will be applied across the whole society increasing the value of safety for each position.

Optimal provision for SFA is larger than the SI result for all systems of taxation. Similarly, optimal provision is lower for WFA than the SI result. This suggests that paternalistic preferences remain with the addition of a VoI. Again, the preferences of an individual are spread across the positions of society so that the 'average' preference is applied to each position. Assuming an individual's altruistic preferences remains consistent across positions then their paternalistic preferences will apply across the positions. Depending on the type of altruism this will increase or decrease the value of the public good.

The findings of Chapter 3 showed that for PA optimal provision is equal to SI for the first-best tax rate but deviated from the SI result when taxation was no longer first-best. Simulations suggested the difference depended on the movement from the first best tax rate and the distribution of altruistic preferences. Under a Citizen Frame, this does not necessarily hold true. Table 4.1 shows the PA result when other's utility enters the altruist's linearly with a weighting of $b_{p}^{i}$, where the superscript represents the altruist and the subscript is the position to which the altruism is directed. Under a first-best tax, the PA result approaches the SI result if $\sum_{q \neq p}^{n-1} a_{q} \sum_{\substack{j \neq i}}^{n-1} b_{p}^{j}$ is becomes consistent across all individuals. That is, each individual is the recipient of equal altruistic concern. This concept can be considered further by differentiating between different distributions of altruistic concern.

For each functional form an individual's altruistic concern is then spread across positions. It is possible that an individual could be an altruist from one position but not another. It is also possible that an individual's altruistic concern could be person-based or distributional. Here
person-based altruism is defined as altruism which is directed at an individual rather than a position. For example, an Individual $a$ being altruistic towards an Individual $b$ regardless of the position Individual $b$ draws when the VoI is lifted. Distributional altruism is defined as altruism directed towards a position rather than an individual. For example, an Individual $a$ being altruistic towards Individual $b$ if Individual $b$ draws Position 1 but not if they Individual $b$ draws Position 2 when the VoI is lifted.

Table 4.2 demonstrates this for a general functional form with two individuals, $a$ and $b$, where other's utility enters the altruist's utility function linearly. Four forms of altruism are considered: person-based altruism when the recipient of altruism is Individual $a$, person-based altruism when the recipient of altruism is Individual $b$, distributional altruism when the recipient of altruism is Position 1, and distributional altruism when the recipient of altruism is Position 2. When altruism is directed from Individual $a$ to Individual $b$, Individual $b$ 's utility is given a weighting of $b_{b}^{a}$. When altruism is directed from Individual $a$ to Position 1, Individual $b$ 's utility when in Position 1 is given a weighting of $b_{1}^{a}$. Each form of taxation is considered.

Under a first-best tax, when Individual $a$ is the recipient of altruism, the marginal utilities of $a$ are given additional weighting. This holds across all positions. The opposite is true when Individual $b$ is the recipient of altruism, for which the marginal utilities of $b$ are given additional weighting. When Position 1 is the recipient of altruism, the marginal utilities of $a$ and $b$ when in Position 1 are given additional weighting. When Position 2 is the recipient of altruism, the marginal utilities of $a$ and $b$ when in Position 2 are given additional weighting. Under personbased altruism, each position is given equal weight with the recipient of altruism's preferences in each position given more weight. This reduces the impact of altruism. Under distributional altruism, a position is given additional weight regardless of which individual is in the position. In this case the impact of altruism remains.

This result holds for the other forms of taxation. For both, distributional altruism gives additional weighting to a given position and therefore the condition for optimal provision favours the position. For person-based altruism, altruistic concern is spread across positions evenly and thus has a reduced impact across positions. The impact is to increase weight on the particular preferences of the recipient of altruism. For example, if the individual values their own survival probability highly then the survival probability will be given additional weight relative to wealth across all positions.
Table 4.2: Model solutions for optimal provision for different forms of altruism under different systems of taxation behind a VoI for the two-person case with
altruism entering linearly into utility functions.
Proportional Wealth Taxation
$\left(w_{1}+w_{2}\right) \frac{\frac{\left(a_{1}+a_{2} b_{A}^{\mathrm{B}}\right) \partial u_{1}^{A}+a_{1} \partial u_{1}^{\mathrm{B}}}{}+\frac{\left(a_{2}+a_{1} b_{1}^{\mathrm{B}}\right) \partial u_{2}^{A}+a_{2} \partial u_{2}^{\mathrm{B}}}{\partial \pi_{1}}}{w_{1} \frac{\left(a_{1}+a_{2} b_{A}^{A} \partial w_{1}^{A}+a_{1} \partial u_{1}^{\mathrm{B}}\right.}{\partial w_{1}}+w_{2} \frac{\left(a_{2}+a_{1} b_{A}^{2}\right) \partial u_{2}^{A}+a_{2} \partial u_{2}^{\mathrm{B}}}{\partial w_{2}}}$
$\left(w_{1}+w_{2}\right) \frac{\frac{a_{1} \partial u_{1}^{A}+\left(a_{1}+a_{2} b_{B}^{A}\right) \partial u_{1}^{B}}{\partial \sigma_{1}}+\frac{a_{2} \partial u_{2}^{A}+\left(a_{2}+a_{1} b_{B}^{A}\right) \partial u_{2}^{B}}{\partial \pi_{1}}}{w_{1} \frac{a_{1} \partial u_{1}^{A}+\left(a_{1}+a_{2} b_{B}^{A} \partial u_{1}^{B}\right.}{\partial w_{1}}+w_{2} \frac{a_{2} \partial u_{2}^{A}+\left(a_{2}+a_{1} b_{B}^{A}\right.}{\partial w_{2}^{B}}}$
$\left.\left.\left(w_{1}+w_{2}\right) \frac{\left(a_{1}+a_{2} b_{1}^{B}\right) \partial u_{1}^{A}+\left(a_{1}+a_{2} b_{1}^{A}\right) \partial u_{1}^{B}}{\left.\left.w_{1} \frac{\left(a_{1}+a_{2}\left(\partial u_{2}^{A}+\partial u_{1}^{B}\right)\right.}{a_{1}^{A}}\right) u_{1}^{A}+a_{1}+a_{1}+a_{2} b_{1}^{A}\right) \partial u_{1}^{B}}\right)+w_{2} \frac{a_{2} \partial u_{2}^{A}+\partial u_{2}^{B}}{\partial w_{1}}\right)$


$\underline{ }$
First-Best Taxation




 | Person-Based |
| :--- |
| Altruism, |
| Individual a |
| receives altruism |
|  |
| Person-Based |
| Altruism, |
| Individual b |
| receives altruism |
|  |
| Distributional |
| Altruism, |
| Position 1 |
| receives altruism |
|  |
| Distributional |
| Altruism, |
| Position 2 |
| receives altruism |

### 4.2.5 Simulations

Section 3.2.5 presented simulations to illustrate the effects of altruism and taxation on optimal provision in front of a VoI that were not clear from the general model. Those simulations showed that optimal provision could be found for any set of cost sharing rules, however the social welfare maximising point is a combination of the optimal tax system and optimal spending. This combined optimal depends on the relative incomes of the individuals and the presence of altruism. The type of altruism, strength of altruistic concern, and the identity of the altruist determined the size and direction of the effect on optimal taxation. The simulations presented here show how optimal provision behind a VoI varies with type of altruism, strength of altruistic concern, and the source of altruistic concern. That is whether the altruistic concern is directed at an individual or a position in society.

Consider the same society as presented in Chapter 3 consisting of two individuals $i=a, b$ each in one of two positions indexed $p=1,2$. Each position is allocated an initial wealth with possible pairings: $w_{1}=10, w_{2}=10 ; w_{1}=9, w_{2}=11$; and $w_{1}=8, w_{2}=12$. Each individual has a differentiable and well-behaved utility function, $u_{i}$, which reflects preferences for own and others' survival probability $\pi_{a}=\pi_{b}$ and wealth $w_{a}, w_{b}$ where $u_{i}$ is strictly increasing in $\pi_{i}$ and $w_{i}$ and non-decreasing in its other arguments. Survival probability is a strictly increasing function of public spending, $s$, with diminishing returns which takes the form

$$
\begin{equation*}
\pi_{i}(s)=1-s^{-1}, \tag{4.19}
\end{equation*}
$$

such that

$$
\begin{equation*}
\frac{\partial \pi_{i}}{\partial s}=s^{-2}>0, \text { and } \frac{\partial^{2} \pi_{i}}{\partial s^{2}}=-2 s^{-3}<0, \tag{4.20}
\end{equation*}
$$

so that survival probability increases in public spending with diminishing returns to spending. Public spending is funded through taxation.

Each individual contributes a share of the total cost of providing the public good. The contribution is determined by the position with the individual in Position 1 contributing a share $t \in[0,1]$ and the individual in Position 2 contributing $1-t$. Individuals are assumed to be expected utility maximisers and thus probability of survival enters the utility function linearly. As in Chapter 3, utility functions takes the form:

$$
\begin{equation*}
u_{i}=\pi_{i} \ln \left(w_{i}-t_{i} s\right)+\gamma_{i} \pi_{j}^{\alpha} \ln \left(w_{j}-\left(1-t_{i}\right) s\right)^{\beta} . \tag{4.21}
\end{equation*}
$$

The individual utility functions for SI, PA, SFA, and WFA are considered by altering the $\alpha$ and $\beta$ terms to take a value of 0 or 1 depending on the form altruism takes. For PA preferences: $\alpha=1$ and $\beta=1$. For SFA preferences: $\alpha=1$ and $\beta=0$. For WFA: $\alpha=0$ and $\beta=1$.

As individuals are behind a VoI and therefore are uncertain of their position (1 or 2) their expected utility function is the probability weighted sum of utilities across positions. Equation (4.22) gives the SI expected utility function.

$$
\begin{equation*}
\frac{1}{2} \pi\left[\ln \left(w_{1}-t s\right)+\ln \left(w_{2}-(1-t) s\right)\right] \tag{4.22}
\end{equation*}
$$

As in Chapter 3, optimal provision is simulated using Matlab. For each simulation, the aggregate utility maximising level of provision through the full range of tax shares is found. This curve shows the societal optimum. This is called the optimal provision curve or optimal provision, $S^{*}$, as a function of tax share, $t$. Through this range the point which gives the highest aggregate utility or social welfare is the first-best point. Simulations are generated for each different forms of altruism (PA, SFA, and WFA). The strength of the altruistic concern is varied ( $\gamma_{i}=0$ or $\gamma_{i}=0.4$ ), and the relative levels of wealth between positions is varied ( $w_{1}=10$, $w_{2}=10 ; w_{1}=9, w_{2}=11 ;$ and $w_{1}=8, w_{2}=12$ ).

Altruism may be directed at an individual or a position. That is an individual may be altruistic towards another for another reason unrelated to their position in society (i.e. a family member or friend), or an individual may be altruistic towards another because of their position in society (i.e. a maximin preferences focusses on the worst-off member in society. In front of a VoI an individual is not distinguishable from their position in society, but behind a VoI they are. When considering altruism behind a VoI it is necessary to categorise whether altruism is person-based or distributional. Equation (4.23) gives an example of person-based altruism where Individual $a$ is altruistic towards Individual $b$ regardless of the position they draw when the VoI is lifted. Equation (4.24) gives an example of distributional altruism where Individual $a$ being altruistic towards Individual $b$ if they draw Position 1 but not if they draw Position 2 when the VoI is lifted. Equation (4.25) gives the alternate example for distributional altruism where Individual $a$ being altruistic towards Individual $b$ if they draw Position 2 but not if they draw Position 1.

$$
\begin{gather*}
\frac{1}{2}\left[\pi\left[\ln \left(w_{1}-t s\right)+\ln \left(w_{2}-(1-t) s\right)\right]\right.  \tag{4.23}\\
\left.\left.+\gamma_{a} \pi_{j}^{\alpha}\left[\ln \left(w_{1}-t s\right)^{\beta}+\ln \left(w_{2}-(1-t) s\right)\right)^{\beta}\right]\right] \\
\frac{1}{2}\left[\pi\left[\ln \left(w_{1}-t s\right)+\ln \left(w_{2}-(1-t) s\right)\right]+\gamma_{a} \pi_{j}^{\alpha} \ln \left(w_{1}-t s\right)^{\beta}\right]  \tag{4.24}\\
\left.\frac{1}{2}\left[\pi\left[\ln \left(w_{1}-t s\right)+\ln \left(w_{2}-(1-t) s\right)\right]+\gamma_{a} \pi_{j}^{\alpha} \ln \left(w_{2}-(1-t) s\right)\right)^{\beta}\right] \tag{4.25}
\end{gather*}
$$

Figures 4.1 and 4.2 show the optimal provision curves for SI (Black line), PA (Blue line), SFA (Green line), and WFA (Red line) preferences for the relative wealth levels: $w_{1}=w_{2}=10$ both in front and behind a VoI. The strength of altruism is always $\gamma=0.4$. Dotted lines represent the optimal provision curves in front of a VoI and are the same for both figures. The solid lines are the optimal provision curves behind a VoI. Figure 4.1 illustrates person-based altruism and Figure 4.2 illustrates distributional altruism.

The dotted lines in both Figures 4.1 and 4.2 show the basic findings of the simulations presented in Chapter 3 shown in Figure 3.3. The solid lines illustrate the optimal provision curves behind a VoI. As the optimal provision curve for SI preferences is the same in front and behind a VoI, the solid black line illustrates optimal provision both in front and behind a VoI. Relative to the SI result, the PA optimal provision in front of a VoI (Green dotted lines) curve shifts left or right depending on the recipient of altruism. The left-hand green dotted line is the optimal provision curve under PA preferences when the individual in Position 2 is the altruist. The righthand green dotted line is the optimal provision curve under PA preferences when the individual in Position 1 is the altruist. First-best optimal provision under PA preferences is the same as the SI preferences, but the tax share to reach this optimum differs. Under PA preferences the altruist pays a greater share of the cost relative to the SI result.

The SFA optimal provision curve in front of a VoI (solid blue line in Figure 4.1, dotted blue line in Figure 4.2) is consistently greater than under SI preferences and the first-best tax shares are the same. Relative to the SI result, the WFA in front of a VoI optimal provision curve (Red dotted lines) shifts downward and either left or right depending on the recipient of altruism. The left-hand red dotted line is the optimal provision curve under WFA preferences when the individual in Position 2 is the altruist. The right-hand red dotted line is the optimal provision curve under WFA preferences when the individual in Position 1 is the altruist.

The solid lines in Figure 4.1 illustrate the optimal provision curves behind the VoI when altruism is person-based. As noted above the SI optimal provision curve is identical in front and behind a VoI. When altruism is person-based the VoI shifts all of the optimal provision curves in a vertical line with the SI result such that the optimal tax share is the same under all preferences. The PA optimal provision curve behind a VoI is the same as the SI curve regardless whether altruist is Individual a or b, so that optimal provision under PA and SI preferences is the same. This is the result shown for person-based altruism in the two-person model shown in Table 4.2 above.

The SFA optimal provision curves are the same both in front and behind a VoI. This means thee VoI has no effect when SFA preferences are person-based. The WFA optimal provision curve behind a VoI is directly below the SI optimal provision curve. The optimal level of provision is the same as WFA preferences in front of the VoI, but the first-best tax share is equal to the SI result. The VoI has reduced the tax effect of WFA, but not the reduction in the value of the public good. Behind a VoI the tax effect of altruism is reduced but the increased or decreased value due to paternalism remains. Figures 4.3, 4.5, and 4.7 explore the result of person-based altruism further for PA, SFA, and WFA preferences respectively.

The solid lines in Figure 4.2 illustrate the optimal provision curves for distributional altruism behind the VoI. Behind the VoI the effect of altruism on the optimal provision curves remain but are reduced. This holds for all forms of altruism. When an individual has distributional PA preferences for Position 1 and the other has SI preferences, the PA optimal provision curve behind a VoI (Green solid line) sits between the SI optimal provision curve (Black line) and the PA optimal provision curve in front of a VoI (Green dashed line). Similar to in front of a VoI, the leftward shift of the optimal provision curve results in the first-best level of provision being the same as the SI result and the first-best tax share differs.

When an individual has distributional SFA preferences for either position, the SFA optimal provision curve behind a VoI (Blue solid line) is between the SI optimal provision curve (Black line) and the SFA optimal provision curve in front of a VoI (Blue dash line). When an individual has distributional WFA preferences, the WFA optimal provision curve behind a VoI (Red solid line) is between the SI optimal provision curve (Black line) and the WFA optimal provision curve in front of a VoI (Red dash line). Just as in front of the VoI, if the distributional altruism is directed at Position 1 then the curve shifts down and left and if the distributional altruism is directed at Position 2 then the curve shifts down and right. Figures 4.4, 4.6, and 4.8 explore the result of distributional altruism further for PA, SFA, and WFA preferences respectively.
Figure 4.1: Optimal provision and the resulting social welfare for all Figure 4.2: Optimal provision and the resulting social welfare for all
preference types with person-based altruism at $w 1=10, w 2=10$ both in preference types with distributional altruism at $w 1=10, w 2=10$ both front and behind a VoI.

Black - SI, Blue - SFA, Green - PA, Red - WFA. Solid lines - in front of a VoI, Dotted lines - behind a VoI.

Figures 4.1 and 4.2 illustrate that behind a VoI under person-based PA preferences the optimal provision curve is identical to the SI optimal provision curve and that under distributional PA the optimal provision curve is between SI optimal provision curve and the PA optimal provision curve in front of a VoI.

If altruism is person-based, the optimal level of provision and taxation is the same as the SI result. As the altruist cannot identify the position of the recipient of their altruism, the effect of altruism is spread equally across all positions and cancels out. Figure 4.3 shows that this effect holds as the relative wealth levels (Black $-w_{a}=10, w_{b}=10$; Red $-w_{a}=9, w_{b}=11$; and Blue - $w_{a}=8, w_{b}=12$ ) change. ${ }^{21}$

In the case of distributional altruism, the optimal level of provision is the same as the SI result, but the first-best tax share is not. When the distributional altruism is directed at Position 1, the PA first-best tax share behind a VoI is less than the SI tax share, but greater than the PA firstbest tax share in front of a VoI. The individual in Position 1 therefore pays a lesser share of the cost of provision. The opposite is true when distributional altruism is directed at Position 2. Whilst the effect has the same direction both in front and behind the VoI, the VoI reduces the size of the effect. As with PA preferences in front of a VoI, under distributional altruism behind a VoI there is a range of second-best taxes which results in a higher optimal level of provision than the first-best result. This range is between the PA first-best and SI first-best tax shares. As the VoI reduces the effect of PA preferences the range of second-best taxes is reduced behind a VoI relative to in front of the VoI. Figure 4.4 shows that this effect holds as the relative wealth levels (Black $-w_{a}=10, w_{b}=10 ;$ Red $-w_{a}=9, w_{b}=11$; and Blue $-w_{a}=8, w_{b}=12$ ) change.

[^15]Figure 4.4: Optimal provision and the resulting social welfare for PA
preferences with distributional altruism for each wealth combination both in front and behind a VoI.


Solid lines - SI, Dotted lines - PA in front of a VoI. Dashed lines - PA behind a VoI. Black $-w_{a}=10, w_{b}=10$; Red $-w_{a}=9, w_{b}=11$; and Blue
$w_{a}=8, w_{b}=12$

Figures 4.1 and 4.2 show that under SFA preferences the VoI has no effect on person-based altruism but has a reducing effect on distributional altruism. Under the full range of tax shares optimal provision remains greater than SI optimal provision. Figures 4.5 and 4.6 show that these results hold when the wealth distribution changes.

Figures 4.1 and 4.2 illustrate that behind a VoI under person-based WFA preferences the optimal provision curve (Figure 4.1: Red solid line) is directly below the SI optimal provision curve (Black line) and under distributional WFA preferences the optimal provision curve (Figure 4.2: Red solid lines) are between the SI optimal provision curve and the WFA optimal provision curve in front of a VoI.

In the case of person-based altruism, the optimal level of provision is the same as the WFA result in front of the VoI and optimal taxation is the same as the SI result. As the altruist cannot identify the position of the recipient of their altruism, the downwards effect of altruism on the value of the public good remains but the tax effect is and cancelled out. This removes the range of second-best taxes which result in greater levels of provision than the first-best result caused by altruism. Figure 4.7 shows that this effect holds as the relative wealth levels (Black $-w_{a}=$ $10, w_{b}=10 ;$ Red $-w_{a}=9, w_{b}=11$; and Blue $-w_{a}=8, w_{b}=12$ ) change.

In the case of distributional altruism, the optimal level of provision is the same as the WFA result, but the first-best tax share is not. When distributional altruism is directed at Position 1, the WFA optimal level of provision and first-best tax share behind a VoI are less than the SI optimal level of provision and tax share, but greater than the PA optimal level of provision and first-best tax share in front of a VoI. The society pays a lesser amount for the provision of the public good and the individual in Position 1 pays a lesser share of the cost of provision relative to the SI result. The opposite is true when distributional altruism is directed at Position 2.

As with PA preferences in front of a VoI, under distributional altruism behind a VoI there is a range of second-best taxes which results in a higher optimal level of provision than the firstbest result. This range is between the PA first-best and SI first-best tax shares. As the VoI reduces the effect of PA preferences the range of second-best taxes is reduced behind a VoI relative to in front of the VoI. Figure 4.8 shows that this effect holds as the relative wealth levels (Black $-w_{a}=10, w_{b}=10$; Red $-w_{a}=9, w_{b}=11$; and Blue $-w_{a}=8, w_{b}=12$ ) change.
Figure 4.6: Optimal provision and the resulting social welfare for SFA
preferences with distributional altruism a for each wealth combination both in front and behind a VoI.

 Figure 4.5: Optimal provision and the resulting social welfare for SFA preferences with person-based altruism for each wealth combination both
in front and behind a VoI.

Solid lines - SI, Dotted lines - PA in front of a VoI. Dashed lines - PA behind a VoI. Black $-w_{a}=10, w_{b}=10 ; \operatorname{Red}-w_{a}=9, w_{b}=11$; and Blue $-w_{a}=$
$8, w_{b}=12$
Figure 4.8: Optimal provision and the resulting social welfare for
preferences with person-based altruism for each wealth combination both WFA preferences with distributional altruism for each wealth
combination both in front and behind a VoI.
in front and behind a VoI.

$8, w_{b}=12$

Overall the results of the optimal provision model and simulations suggest that the inclusion of a VoI will reduce the tax effect of person-based altruism whether that be PA or WFA whilst the directional effects of paternalistic altruism and distributional altruism remain. The magnitude of the tax effect depends on the form altruism takes. When altruism is safety-focused there is no tax effect as no preference is given for the costs borne by others. When altruism is wealthfocused the tax effect is largest as preference is only given for the costs borne by others. Under PA, altruistic concern is based on both others' safety and the cost borne. The tax effect is the greater than the SFA and less than the WFA results suggesting that as greater preference is given to the costs borne by others the greater the impact on optimal taxation. The next step is to consider the impact of a VoI on individual values.

### 4.3 Cost-benefit analysis ${ }^{22}$

### 4.3.1 Model

Extending the model presented in Section 3.5.1, consider a society of $n$ positions, indexed $p=$ $1 \ldots n$, with each of the $n$ individuals, indexed $i=1 \ldots n$, being assigned to a position. The levels of wealth and the private benefits from a discrete change in public good are now attached to a position rather than individual and therefore denoted $w_{p}$ and $x_{p}$ respectively. The provision of the public good comes at an additional cost, $C$, and requires the individual in a given position, $p$, to pay a share $t_{p}$, where and $\sum_{p}^{n} t_{p}=1$ to ensure the full cost is covered.

Each individual values the other's welfare so that utility functions, $u_{i}$, reflect both the determinants of own utility and the determinants of others' utility, with $\alpha_{p}^{i}$ measuring the weight placed on the individual in Position $p$ 's consumption of the public good and $\beta_{p}^{i}$ measuring the weight placed on the individual in Position $p$ 's consumption of the private good. Individuals are placed behind a VoI and therefore expects to realise each of $n$ positions in society with probability of $1 / n$. Thus expected utility behind a veil of ignorance is given for a risk neutral individual by $U^{i}$ such that

$$
\begin{equation*}
U^{i}=\frac{1}{n} \sum_{p=1}^{n}\left[w_{p}+\beta_{p}^{i} \sum_{q \neq p}^{n-1} w_{q}+z\left(x_{p}-t_{p} C+\alpha_{p}^{i} \sum_{q \neq p}^{n-1} x_{q}-\beta_{p}^{i} \sum_{q \neq p}^{n-1} t_{q} C\right)\right] \tag{4.26}
\end{equation*}
$$

[^16]This functional form allows for $\operatorname{SI}(\alpha=0, \beta=0)$, $\operatorname{PA}(\alpha=\beta>0)$, $\operatorname{SFA}(\alpha>0, \beta=0)$, and WFA ( $\alpha=0, \beta>0$ ).

To consider the value of the discrete change in public good provision the individual equates expected utility with and without the public good provided. Individual $i$ 's problem can be expressed as:

$$
\begin{align*}
& \frac{1}{n} \sum_{p=1}^{n}\left[w_{p}+\beta_{p}^{i} \sum_{q \neq p}^{n-1} w_{q}\right]=  \tag{4.27}\\
& \quad \frac{1}{n} \sum_{p=1}^{n}\left[w_{p}+\beta_{p}^{i} \sum_{q \neq p}^{n-1} w_{q}+x_{p}-t_{p} C+\alpha_{p}^{i} \sum_{q \neq p}^{n-1} x_{q}-\beta_{p}^{i} \sum_{q \neq p}^{n-1} t_{q} C\right] .
\end{align*}
$$

Solving for $C$ we get:

$$
\begin{equation*}
C_{i}^{*}=\frac{\sum_{p=1}^{n}\left(1+(n-1) \alpha_{p}^{i}\right) x_{p}}{\sum_{p=1}^{n}\left(1+(n-1) \beta_{p}^{i}\right) t_{p}} . \tag{4.28}
\end{equation*}
$$

Where $C_{i}^{*}$ is the upper threshold total cost Individual $i$ would prefer the group to not surpass, termed a MAC value. Behind a VoI, the MAC value implies a separate WTP value for each position, generated by multiplying the individual's MAC value by a given position's tax share. The MAC value acts as the natural extension to Bergstrom (2006)'s suggested question of
"what is the largest tax increase you would be willing to accept for you and those like you"?
for avoiding the naïve cost-benefit analyst problem by requiring individuals to take into account fully the costs and benefits to each position in society.

The mean MAC across individuals is then given by

$$
\begin{equation*}
\bar{C}=\frac{1}{n} \sum_{i}^{n} \frac{\sum_{p=1}^{n}\left(1+(n-1) \alpha_{p}^{i}\right) x_{p}}{\sum_{p=1}^{n}\left(1+(n-1) \beta_{p}^{i}\right) t_{p}} . \tag{4.29}
\end{equation*}
$$

Which simplifies to

$$
\begin{equation*}
\bar{C}=\frac{\sum_{p=1}^{n}\left(1+\frac{n-1}{n} \sum_{i}^{n} \alpha_{p}^{i}\right) x_{p}}{\sum_{p=1}^{n}\left(1+\frac{n-1}{n} \sum_{i}^{n} \beta_{p}^{i}\right) t_{p}} . \tag{4.30}
\end{equation*}
$$

The ratio of the sum of altruism weighted benefits over the sum of altruism weighted tax shares.

As in the optimal provision model, two simplifying assumptions can be made for person-based altruism and distributional altruism. For personal altruism the individual is consistently altruistic across positions so that the individual MAC simplifies to

$$
\begin{equation*}
C_{i}^{*}=\frac{1+(n-1) i^{i}}{1+(n-1) \beta^{i}} \sum_{p=1}^{n} x_{p} . \tag{4.31}
\end{equation*}
$$

And the mean MAC to

$$
\begin{equation*}
\bar{C}=\frac{1}{n} \frac{1+(n-1) \sum_{i}^{n} \alpha^{i}}{1+(n-1) \sum_{i}^{n} \beta^{i}} \sum_{p=1}^{n} x_{p} . \tag{4.32}
\end{equation*}
$$

So that taxation no longer plays a role in the valuation and the value is adjusted up or down according to the form of altruism.

For distributional altruism the altruistic terms are directed towards positions so that the individual and mean MACs simplify to

$$
\begin{equation*}
C_{i}^{*}=\bar{C}=\frac{\sum_{p=1}^{n}\left(1+(n-1) \alpha_{p}\right) x_{p}}{\sum_{p=1}^{n}\left(1+(n-1) \beta_{p}\right) t_{p}} . \tag{4.33}
\end{equation*}
$$

### 4.3.2 Altruism

For each functional form this solution can be simplified. For SI both individual and mean MAC simplify to

$$
\begin{equation*}
C_{i}^{*}=\bar{C}=\sum_{p=1}^{n} x_{p} . \tag{4.34}
\end{equation*}
$$

The sum of the private values.

For PA MACs simplify to the sum of the private values for person-based altruism but for distributional altruism to

$$
\begin{equation*}
C_{i}^{*}=\bar{C}=\frac{\sum_{p=1}^{n}\left(1+(n-1) \alpha_{p}\right) x_{p}}{\sum_{p=1}^{n}\left(1+(n-1) \alpha_{p}\right) t_{p}} . \tag{4.35}
\end{equation*}
$$

For pure public good focussed the individual MAC simplifies to

$$
\begin{equation*}
C_{i}^{*}=\sum_{p=1}^{n}\left(1+(n-1) \alpha_{p}^{i}\right) x_{p} . \tag{4.36}
\end{equation*}
$$

And the mean MAC to

$$
\begin{equation*}
\bar{C}=\sum_{p=1}^{n}\left(1+\frac{n-1}{n} \sum_{i}^{n} \alpha_{p}^{i}\right) x_{p} . \tag{4.37}
\end{equation*}
$$

So that neither are dependent on taxation.

For pure wealth focussed altruism the individual MAC simplifies to

$$
\begin{equation*}
C_{i}^{*}=\frac{\sum_{p=1}^{n} x_{p}}{\sum_{p=1}^{n}\left(1+(n-1) \beta_{p}^{i}\right) t_{p}} . \tag{4.38}
\end{equation*}
$$

And mean to

$$
\begin{equation*}
\bar{C}=\frac{\sum_{p=1}^{n} x_{p}}{\sum_{p=1}^{n}\left(1+\frac{n-1}{n} \sum_{i}^{n} \beta_{p}^{i}\right) t_{p}} . \tag{4.39}
\end{equation*}
$$

For the person-based altruism case this simplifies further to

$$
\begin{equation*}
C_{i}^{*}=\frac{\sum_{p=1}^{n} x_{p}}{\left(1+(n-1) \beta^{i}\right)} . \tag{4.40}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{C}=\frac{\sum_{p=1}^{n} x_{p}}{1+\frac{n-1}{n} \sum_{i}^{n} \beta^{i}} . \tag{4.41}
\end{equation*}
$$

So that taxation has no impact on the value.

### 4.3.3 General Example

Continuing the example from Chapter 3, Table 4.3 shows an example of how individuals value public goods behind a VoI. In this example, both positions have a benefit of $x_{1}=x_{2}=100$ and Position A contributes $t_{A}=0.4$ and Position B contributes $t_{B}=0.6$. Individual A , when altruistic has a coefficient of altruism of 0.25 , whilst Individual B a coefficient of altruism of 0.5 .

Table 4.3: Example CBA behind a VoI

| Position | 1 | 2 | Total |
| :---: | :---: | :---: | :---: |
| Xi | 100 | 100 | 200 |
| Ti | 0.4 | 0.6 | 1 |
| Individual | A | B | Mean |
| SI | 200 | 200 | 200 |
| Personal PA | 200 | 200 | 200 |
| Distributive PA towards Position 1 | 204.55 | 208.33 | 206.44 |
| Distributive PA towards Position 2 | 195.65 | 192.31 | 193.98 |
| SFA | 250 | 300 | 275 |
| WFA | 160 | 133.33 | 146.67 |

MAC values for each of SI, SFA, WFA and PA when person-based are constant over $t$. As this is the case, compensation is payable and values elicited are consistent with cost-benefit rules. Distributional preferences remain and are distorted by the choice of tax vehicle and therefore may not be consistent with CBA rules, however the values do include important information of individuals regarding their impartial preferences over distribution.

### 4.3.4 Safety example

This example presents the VoI extension to the example presented in Section 3.4.4. In this example aggregate WTP and MAC values for an increase in safety expenditure for a society of two individuals, $a$ and $b$, are simulated over a range of tax shares. The example uses the society set out in the simulations in Sections 3.2 and 4.2. The total wealth of the society is 20 and split across two positions, 1 and 2 , such that $w_{1}=10, w_{2}=10$, or $w_{1}=8, w_{2}=12$. Individuals a and $b$ are uncertain of their positions and have a general utility function as shown in Equation
(4.21) which simplifies based on their altruistic preferences that are either SI, PA, SFA, or WFA preferences for person-based altruism and distributional altruism directed at Position 1 as shown in Equations (4.23) and (4.24) respectively. The strength of the altruistic component is either $\gamma_{i}=0,0.2,0.4$. As in the example presented in Chapter 3 , the base expenditure is 4 resulting a probability of survival of $\pi=0.75$. WTP and MAC values are found for additional safety expenditure that takes a value of either $s=5$ for which the probability of survival is then $\pi=0.89$, or $s=6$ for which the probability of survival is then $\pi=0.9$.

Figures 4.9 to 4.12 illustrate aggregate WTP and MAC values for each set of parameters and preference types: SI (black lines), PA (blue lines), SFA (green lines), and WFA (red lines). Figure 4.9 illustrates aggregate WTP (solid lines) based on Chapter 3, MAC values with personbased preferences (dash lines) and MAC values with distributional preferences directed at Position 1 (dot-dash lines) when $w_{1}=10, w_{2}=10, s=5, \gamma=0.2$. Figure 4.10 illustrates the effect of changes in the strength of altruism term $\gamma_{i}$ which takes a value of $0,0.2$, or 0.4 . Figure 4.11 illustrates the effect of changes in the increased expenditure that is valued, $s$, which takes values of 5 or 6 . Figure 4.12 illustrates the effect of changes in the relative levels of wealth of the two individuals. The wealth levels of the individuals either take a value of $w_{1}=10, w_{2}=$ 10 or $w_{1}=8, w_{2}=12$.

As the example in Chapter 3 showed, aggregate WTP is unaffected by tax share for SI and SFA preferences as no altruistic weight is placed on others' wealth. The value of the public good is greater under SFA preferences than SI preferences. Figure 4.9 shows that MAC values for SI and SFA preferences vary with tax share as individuals are risk averse. The lines take an inverted U-shape as less equal tax shares lead to greater variation in outcomes when the VoI is lifted and the individual's position is revealed. Under SFA preferences, MAC values are between the SI result behind a VoI and the aggregate WTP result for both person-based altruism and distributional altruism.

The example in Chapter 3 showed that aggregate WTP is affected by tax share for PA and WFA preferences as altruistic weight is placed on others' wealth. Figure 4.9 shows that this holds true for MAC values. Under person-based altruism with PA preferences the MAC value is identical to the value under SI preferences. Under distributional altruism the MAC curve shifts upward and right. The value of the public good is then greater than the SI result for a range of tax shares that favour the individual in Position 1. When the tax share becomes sufficiently unequal the value of the public good falls below the SI result. The VoI therefore removes person-based PA preferences and leaves distributional preferences in place as the results for the optimal provision
and CBA models show. Under WFA preferences is flatter behind a VoI suggesting the tax effect is reduced by the VoI. Under distributional preferences the MAC curve shifts up and right as with PA preferences suggesting the VoI does not impact upon distributional preferences.

Figure 4.9: Aggregate WTP and MAC values for safety improvements with $w_{a}=10, s=5, \gamma=0.2$


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Solid lines are aggregate WTP, long dashed lines are Position a, and dot-dash lines are Position b.

Figure 4.10 shows that as the strength of altruism increases from $\gamma=0.2$ (solid lines) to $\gamma=$ 0.4 (dashed lines) the PA MAC curve remains unchanged under person-based preferences and shifts further upward and left under distributional preferences. The SFA MAC curve shifts further upward for both types of altruism. The WFA MAC curve shifts downward under personbased preferences and downward and right under distributional preferences. The increase in the strength of altruism therefore simply increases the effects under each preference type.

Figure 4.10: Aggregate WTP and MAC values for safety improvements varying with $\gamma$.


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Parameters are $w_{a}=10, s=5$, solid lines are $\gamma=$ 0.2 , dashed line are $\gamma=0.4$.

Figure 4.11 shows that as the underlying value of the public good increases from $s=5$ (solid lines) to $s=6$ (dashed lines) all lines move upward and therefore both WTP and MAC values are sensitive to the underlying value under all preferences. Figure 4.12 shows that as the wealth levels change from $w_{1}=10, w_{2}=10$ (solid lines) to $w_{1}=8, w_{2}=12$ (dashed lines) all lines shift upward and left. This results in greater values under lower tax shares.

Overall, the results align with those shown in the optimal provision and CBA models. The VoI reduces the effects of altruism, most notably the effect of taxation and altruism. Under person-based preferences the tax effect is completely reduced such that PA and SI preferences result in the same value under all tax shares, whilst the effects of distributional and paternalistic altruism remain.

Figure 4.11: Aggregate WTP and MAC values for safety improvements varying with $\boldsymbol{s}$.

(b) Veil - Person-based altruism

(c) Veil - Distributional altruism


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Parameters are $w_{a}=10, \gamma=0.2$, solid lines are $s=$ 5 , dashed line are $s=6$.

Figure 4.12: Aggregate WTP and MAC values for safety improvements varying with $\boldsymbol{w}_{\boldsymbol{a}}$.


Notes: Black - SI, Blue - PA, Green - SFA, Red, WFA. Parameters are $\gamma=0.2, s=5$, solid lines are $w_{a}=$ 10 , dashed line are $w_{a}=8$.

### 4.4 Citizen Frame

There are three elements to consider when evaluating each frame:
[1] The impact of different preference sets on the societal value.
[2] The impact of different preference sets on individual values.
[3] The use of these values in CBA. Two questions are asked. First, does the method of elicitation respect the individual's sovereignty by respecting their preferences? Second, does the aggregation of these preferences pass a compensation test?

The findings of the optimal provision model provide the answer to the first question, whilst the remaining questions are answered using the CBA models.

The results of the optimal provision model suggest that behind a VoI, the societal value is highest when altruists are safety-focused and lowest when altruists are wealth-focused. This mirrors the result of Consumer Frame 2. For PA preferences, optimal provision is not equal to the SI results. Altruism is differentiated into two forms: person-based and distributional. Person-based altruism is directed at individuals, whilst distributional altruism represents concerns over distributive justice. The results show that behind a VoI, person-based altruism is reduced whilst distributive altruism remains.

Individual values reflect the optimal provision result with values largest for SFA and lowest for WFA. Under a first-best tax and risk neutrality, the PA value is equal to the SI value. Behind a VoI, if altruism is person-based then MAC values are independent of the tax vehicle. However, if preferences are distributional then the cost to each position impacts upon the MAC value.

Eliciting citizen preferences through MAC values behind a VoI allows for respondents to consider a full proposal. That is the costs and benefits to each individual in society. This allows for preferences over distributive justice to be incorporated into individual values. These individual values respect the principle of consumer sovereignty. These values require additional study to consider their impact in CBA. If values are independent of taxation, then they are consistent with the concept of compensation. Because the values are based on both the benefits and costs across society, the values already take into account net benefits to each position. The MAC value takes into account that in expectation and individual is as well off with the proposal as without.

### 4.5 Summary

The aim of this chapter was to evaluate the set out and evaluate the use of a Citizen Frame described in Chapter 2. The Citizen Frame is operationalised by placing individuals behind a VoI. Two models were considered to model the effect of a VoI on the value of a public good. An optimal provision model was used to model the societal value and a WTP model was used to model individual values. Choice behind a VoI is considered as a choice under uncertainty and expected utility under an expected utility framework.

First, a model of optimal provision of public safety expenditure under SI, PA, SFA, and WFA preferences for first-best, uniform, and proportional wealth taxation. A two-person society was simulated for the full range of cost sharing rules. As with in front of a VoI, optimal provision is highest under SFA preferences and lowest under WFA preferences. This result holds regardless of the tax system in place.

Under first-best taxation optimal provision is consistent under SI and PA preferences, however simulations show that the optimal tax share isn't the same. When taxation deviates from firstbest, optimal provision is different under SI and PA preferences as provision if altruism is distributional but not if altruism is person-based. This holds true for WFA leading to the conclusion that a VoI reduces the tax effect observed in Consumer Frame 2 for person-based altruism whilst paternalistic and distributional altruism remains.

Second a Citizen Frame was modelled under a cost-benefit framework. The results are similar to the optimal provision model. The results show that the tax effect shown in Consumer Frame 2 disappears if altruistic preferences are person-based. This result is explored further in Part II which aims to empirically test the frames and is the first attempt at operationalising the Citizen Frame. These findings are discussed in Chapter 7 alongside the findings of the previous chapter and the findings of Part II.

## PART II

## EMPIRICAL ANALYSES

## CHAPTER FIVE

## Experiment A: An Experiment Comparing the Value of Group Insurance in the Consumer and Citizen Frames

### 5.1 Introduction

Two key results of the CBA models presented in Chapters 3 and 4 are that in front of a VoI PA preferences can impact upon individual values when the tax vehicle is not first-best. Behind a VoI, PA preferences will only impact upon individual values if altruism is based on distributional preferences. Experiment A, described in this chapter, is designed to elicit WTP under uniform taxation in front and behind a VoI. This chapter presents the experimental design and the results. The findings are discussed in Chapter 7.

First, the mechanism used to elicit WTP values in a group setting, the RPVM, is introduced. This was first introduced in Messer et al. (2010) and Messer et al. (2013). The experiment presented in this chapter extends the Messer et al. (2013) design in a number of ways. One key extension examines alternate distributions of the benefits of public good provision and variation in endowments across group members. The second key extension was to introduce a VoI. Model predictions are made for SI, PA, and MM preferences as described previously, the Efficiency \& Maximin (Quasi-MM) based on Charness \& Rabin (2002), and models of inequality aversion based on Bolton and Ockenfels (2000) (BO-IA) and Fehr \& Schmidt (1999) (FS-BO). In front of a VoI, regard for other will affect values in different ways depending on the form it takes. Behind a VoI, distributional altruism and IA will affect values, whilst PA preferences has no effect as the model presented in Chapter 4 shows.

The experiment is designed to measure the effect of regard for others by measuring the differences in WTP for private and social values. Based on predictions presented in Section 5.3, mean differences in WTP values elicited from subjects are tested using t-tests to measure the impact of altruism. By doing so the dominant form that regard for others takes is determined. Choice in front and behind a VoI is then compared to determine if preferences remain consistent in front and behind a VoI. The results show that altruism is best characterised as PA in front of a VoI based on the model predictions presented in Chapter 3 and in this chapter for the experimental scenario. Behind a VoI values are not distinguishable from SI preferences following the predictions of the model presented in in Chapter 4 and the predictions presented in this chapter. These results indicate that the VoI is operating as expected based on Chapter 4.

### 5.2 Random Price Voting Mechanism

To test the Consumer and Citizen Frames, an experimental framework is required to elicit WTP in a public goods setting. Public goods have been studied extensively in the experimental literature. As such there is a large literature and many experimental designs aimed at testing different elements of public good provision and altruism. A large proportion of the public goods games come under the category of contribution games which focus on the free-rider problem (see Croson (2010)).

This thesis is concerned with the value of a public good funded through taxation for which contributions are theoretically mandatory. As such, the free-rider problem is not an issue to focus on. A contribution game would therefore only introduce free-rider strategies and act as a confounder. Similarly, the experimental designs aimed at testing for altruism, or social preferences in the wider sense, focus on games of giving, receiving, gift exchange etc. These designs have less similarities to public good provision and therefore were also not suitable. As we wish to measure WTP for a good, in this case a laboratory public good, consider instead the BDM mechanism (Becker et al. (1964)).

The BDM mechanism is designed to elicit WTP from individuals. Respondents bid against a randomly generated competing bid for an item. The mechanism, based on the second-price auction design, is incentive compatible as the optimal response is the individual's true value for the item. As shown by Vickrey (1961), overbidding may lead to paying a price higher than the
value, whilst underbidding may lead to not buying an item. The item used could be a physical object for replicating a market situation, or a non-physical object such as the purchase, or avoidance, of a lottery.

The good to be valued must have the characteristics of a public good and exist in a group decision-making context. Whilst the BDM mechanism has an individual focus, there is an extension called the Random Price Voting Mechanism (RPVM). This extension, presented in Messer et al. (2010), allows for WTP elicitation in a group context. The RPVM transforms an individual's bid into a vote. The vote in turn is used in conjunction with group members' bids, and resulting votes, to determine whether the item is bought for the group using a majority voting rule. The mechanism retains the incentive compatibility of the BDM mechanism whilst allowing for a public good context.

Messer et al. (2013) uses the RPVM to elicit WTP for the case of social insurance. In the experiment, subjects are placed into groups with each group member receiving an endowment and facing a negative lottery which may lead to the loss of some or all of their endowment. The public good is a group insurance which avoids the lotteries to each group member, and is by design non-excludable and non-rival. The RPVM is used to elicit WTP values for the public good. This mechanism provides the ideal design to base the experiments on.

The RPVM procedure is as follows. At the beginning of a round, subjects are randomly placed into groups. A group is made up of positions defined by their endowment $w_{i}$, and a negative lottery ${ }^{23}$ which in turn is defined by a loss amount, $l_{i}$, with probability of loss, $p_{i}$. The group has the opportunity fund a public good which insures against all lotteries. The public good, group insurance, comes at a cost, $C$, to each group member. The group decides whether to accept the public good at cost $C$ using the following stages of the RPVM procedure:

1. Information is presented on group members': endowment $w_{i}$, and lottery defined by a loss amount, $l_{i}$, and probability of loss, $p_{i}$.
2. Subjects enter their maximum WTP.
3. The individual cost, $C$, is randomly drawn.

[^17]4. Each WTP is compared to the randomly drawn cost to generate votes using the following rule:
a. The subject votes yes to buying group insurance if their WTP $\geq C$.
b. The subject votes no to buying group insurance if their $\mathrm{WTP}<C$.
5. The number of YES votes is counted to determine if group insurance is bought with the majority voting decision rule.
a. With group insurance, each group member receives a payoff of $w_{i}-c$.
b. Without group insurance, each group member independently faces their personal lottery. Each individual receives a payoff of $w_{i}-l_{i}$ with probability $p_{i}$, and receives a payoff of $w_{i}$ with probability $1-p_{i}$.

This procedure mirrors the second-price auction procedure of Vickrey (1961) which underpins the BDM mechanism by eliciting WTP before the cost is known to the subjects. By doing so, the individual's weakly dominant strategy is to truthfully offer their maximum WTP.

Messer et al. (2010) and Messer et al. (2013) use the RPVM mechanism to measure the impact of social preferences on WTP and WTA values for public goods. Both papers consider groups of three individuals and public goods which deliver both gains and losses. Messer et al. (2010) measures WTP/WTA for absolute gains and losses and Messer et al. (2013) measures WTP/WTA for positive and negative lotteries. Both papers compare WTP values for a private good and a public good holding the individual's private induced value constant.

In both papers WTP values were compared for three scenarios:
Private: Group size of 1 . The RPVM reduces to the BDM mechanism.

Homogenous: Group size of 3. Each group member has the same endowment and lottery.

Heterogeneous: Group size of 3. Each group member sees a different lottery.

WTP values elicited for each scenario are compared to measure the impact of altruism. Tests hold an individual's endowment and lottery constant for each scenario.

Relative to the private scenario, the homogenous scenario adds group members identical in endowment and lottery. As there is no difference in the private value and the values of other group members, both Messer et al. (2010) and Messer et al. (2013) predict no difference between Private and Homogenous WTP values. The results of, both Messer et al. (2010) and Messer et al. (2013) show no difference between mean WTP responses for the two scenarios.

The heterogeneous scenario changes the group members' lotteries, so they are no longer identical. The difference in the individual's WTP for each scenario is used as a measure of the impact of altruism and compared to predictions for different forms of altruism. Messer et al. (2010) make predictions based on models of Social Efficiency (PA), Maximin (MM) and Efficiency \& Maximin (Quasi-MM) based on Charness \& Rabin (2002), and models of inequality aversion based on Bolton and Ockenfels (2000) (BO-IA) and Fehr \& Schmidt (1999) (FS-BO). The results show that in a Heterogeneous group, WTP responses are significantly closer to the group optimal response than the equivalent private for homogenous scenario responses. That is WTP responses were lower (higher) for the Heterogeneous Scenario than the Private and Homogenous Scenarios when Heterogeneous group member had the highest (lowest) private value. Messer et al. (2013) conclude based on model predictions that behaviour best represents a group efficiency motive consistent with Charness \& Rabin (2002) and Johannesson et al. (1996) described as person-based PA motive discussed in Chapter 4.

### 5.3 Experiment overview

The experiment described here replicates the homogenous and heterogeneous scenarios of Messer et al. (2013). The primary extension to the design by Messer et al. (2013) used in this experiment applies a VoI to the Homogenous and Heterogeneous scenarios. This generates four scenarios to be compared defined by whether the group is homogenous or heterogeneous and in front or behind a VoI. The experiment is designed to measure the difference between WTP values for heterogeneous and homogenous groups at the individual level. To do so, each subject completes two of the four scenarios: either the Homogenous (no veil) and Heterogenous (no veil) scenarios, or the Homogenous (veil) and Heterogenous (veil) scenarios. These are described in detail in the next section.

Subjects in both Messer et al. (2010) and Messer et al. (2013) were in groups of three and when in heterogenous groups had equal endowments but different lotteries. There was one group member with a low, medium and high private value. The secondary extension to the design by Messer et al. (2013) considers heterogeneity in endowments among group members and unequal numbers of group members with low, medium and high private values. To do so group sizes of four are used. Groups are described by the endowments and negative lotteries faced by the group members. For every heterogeneous group there are three distinct endowment and negative lottery pairings labelled positions. ${ }^{24}$ Based on the relative expected losses (EL) of the negative lotteries the positions are labelled: Position A (Low EL), Position B (Mid EL) and Position C (High EL). As there are four group members this means two group members must have the same endowment and expected loss. An example heterogenous group may be defined as ABBC where there is one Position A (Low EL), two Position Bs (Mid EL) and one Position C (High EL). The four group types that are used in this experiment are $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC where endowments are homogenous among group members and ABBC (INC) where endowments are heterogeneous among group members.

### 5.3.1 Scenarios

At the beginning of each round, group members draw a position. In homogenous groups, each individual draws the same position. In heterogeneous groups, each individual draws a different position. In front of the VoI, group members are informed of their position before they offer their WTP. Behind the VoI, group members are uncertain of their position until after they offer their WTP. The four scenarios used in this experiment are defined as:

Homogenous (No Veil): The homogenous scenario from Messer et al. (2013).
Each group member draws the same position. Each group member either draws Position A, Position B, or Position C. The group draws their position before entering their WTP.

Heterogeneous (No Veil): The heterogeneous scenario from Messer et al. (2013).
Each group member draws a different position. Some group members draw Position A, some Position B, and some Position C. The number in each position

[^18]is dependent on the group type $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC of which the group members are informed. Group members draw their positions before entering their WTP.

Homogenous (Veil): An equivalent to homogenous scenario behind a VoI. Each group member draws the same position. Group members are therefore uncertain if they are in Position A, Position B, or Position C, but know each group member has drawn the same position. The group draws their position after entering their WTP.

Heterogeneous (Veil): The heterogeneous scenario behind a VoI. Each group member draws a different position. Group members are therefore uncertain if they are in Position A, Position B, or Position C, but know some group members have drawn Position A, some Position B, and some Position C. The number in each position is dependent on the group type $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC of which the group members are informed. Group members draw their positions after entering their WTP.

In Homogenous (No Veil) each group member has the same endowment and negative lottery, replicating the homogenous group treatments in Messer et al. (2010) and Messer et al. (2013). Assuming an individual's risk preferences are equal to the group average, the individual's private optimal response is indistinguishable from the group optimal response. This should result in respondents offering WTP responses equal to private WTP responses. Both previous studies confirm this result, finding that responses in homogenous groups are insignificantly different from responses in a private game which exactly represents a Consumer Frame 1. Based on these findings, Homogenous (No Veil) is used to measure private preferences and replicate Consumer Frame 1.

Heterogeneous (No Veil) places subjects within Consumer Frame 2. As group members have heterogeneous negative lotteries, private optimal responses are not necessarily equal to group optimal responses. If respondents are altruistic then their WTP response for Heterogeneous (No Veil) does not necessarily equal their private optimal response. Both Messer et al. (2010) and Messer et al. (2013) show that mean WTP responses vary when comparing subjects' WTP for homogenous and heterogeneous groups when the individual's endowment and benefit of the public good is held constant.

Following the method of analysis used in Messer et al. (2013), responses are compared between the Heterogeneous (No Veil) and Homogenous (No Veil) scenarios is used to test for the effect of altruistic preferences over others' payoffs. This study considers differences in WTP at the individual level by holding the individual's endowment and negative lottery constant. In Homogenous (No Veil), WTP responses are driven by risk preferences. In Heterogeneous (No Veil), choice is driven by both risk and altruistic preferences. By taking the difference between WTP responses for the Heterogeneous (No Veil) and Homogenous (No Veil) scenarios, risk preferences are controlled for and the effect of any altruistic preferences remains. An example of this is comparing the WTP response of an individual in Position A in the Heterogeneous (No Veil) group ABCC and the same individual's response in the Homogenous (No Veil) group AAAA where every group member has drawn Position A. The difference in WTP responses is driven by the change in the endowment and negative lotteries of their group members.

The Homogenous (Veil) is designed to act as an equivalent to the Homogenous (No Veil) behind a VoI. Individuals are uncertain over their endowment and lottery; however, they know that each other group member will draw the same endowment and lottery. Assuming risk preferences are consistent across group members, their private optimal WTP responses are identical and therefore altruism and inequality aversion have no impact on responses. The Homogenous (Veil) acts as a measure of private values to be compared against the Heterogeneous (Veil).

The Heterogeneous (Veil) is designed to replicate the Citizen Frame described in Chapter 4 and is the equivalent of Heterogeneous (No Veil) with uncertainty over position within the group. Each individual is identical prior to the VoI being lifted as they are all equally uncertain of their positions. However, altruism and inequality aversion can impact upon WTP because group members will have different endowment and lotteries once the VoI is lifted. Responses are compared at the individual level for the Heterogeneous (Veil) and Homogenous (Veil) scenarios to measure the effect of altruism in Heterogeneous (Veil) using the same reasoning as the difference between the Heterogeneous (No Veil) and Homogenous (No Veil) scenarios.

Predictions for WTP responses are generated under the assumption of risk neutrality for a general altruistic utility function and two models of inequality aversion based on FS-IA and BO-IA. The general altruistic utility function models altruism as including others' weighted payoff in one's own utility function. This allows for multiple forms of altruism and is simplified
to SI, PA, MM, and quasi-MM preferences. Under PA preferences altruism is directed equally to all group members. Under MM preferences altruism is directed towards the worst of group member. Under quasi-MM preferences altruism is directed to the group with additional weight placed on the worst-off group member. Under FS-IA preferences the individual is averse to being better or worse off than other individuals. Under BO-IA preferences the individual is averse to being better or worse off than group mean outcome. Predictions are first presented in a general form in sections 5.3.2 and 5.3.3 for altruism and inequality aversion respectively, and then again based on the different positions: A (Low EL), B (Mid EL), and C (High EL) and the different group types: $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC in section 5.3.4. It is these predictions that are tested in the experiment.

### 5.3.2 Altruism

Starting with the SI case, the optimal WTP response is the maximum the individual would pay such that expected utility with insurance is equal to expected utility without insurance. For an individual $i$ with SI preferences the problem faced is

$$
\begin{equation*}
w_{i}-w t p_{i}=p_{i}\left(w_{i}-l_{i}\right)+\left(1-p_{i}\right) w_{i} . \tag{5.1}
\end{equation*}
$$

Where $w_{i}$ is the individual's endowment, $p_{i}$ their probability of loss without the insurance, and $l_{i}$ their loss amount without insurance. Solving for $w t p_{i}$ gives

$$
\begin{equation*}
w t p_{i}=p_{i} l_{i} \tag{5.2}
\end{equation*}
$$

The private SI WTP response is equal to the individual's expected loss, their probability of loss multiplied by the loss amount.

Under a general altruistic utility function for which the altruist, Individual $i$, weights each group member's, subscripted $j$, payoff by $\alpha_{j}$ in their own utility function. The problem faced by the individual for this general utility function is

$$
\begin{align*}
& w_{i}-w t p_{i}+\sum_{j \neq i}^{n-1} \alpha_{j}\left(w_{j}-w t p_{i}\right) \\
& \quad=p_{i}\left(w_{i}-l_{i}\right)+\left(1-p_{i}\right) w_{i}+\sum_{j \neq i}^{n-1} \alpha_{j}\left(p_{j}\left(w_{j}-l_{j}\right)+\left(1-p_{j}\right) w_{j}\right) \tag{5.3}
\end{align*}
$$

For Homogenous (No Veil), $w_{j}=w_{i}, l_{j}=l_{i}$, and $p_{j}=p_{i}$. Equation (5.3) simplifies to

$$
\begin{equation*}
\left(1+\sum_{j \neq i}^{n-1} \alpha_{j}\right)\left(w_{i}-w t p_{i}\right)=\left(1+\sum_{j \neq i}^{n-1} \alpha_{j}\right)\left(p_{i}\left(w_{i}-l_{i}\right)+\left(1-p_{i}\right) w_{i}\right) . \tag{5.4}
\end{equation*}
$$

The altruistic terms then cancel down to Equation (5.1) and again solves to $w t p_{i}=p_{i} l_{i}$. For Homogenous (No Veil), $w t p_{i}=p_{i} l_{i}$ under all altruistic preferences.

For Heterogeneous (No Veil), Equation (5.3) solves for $w t p_{i}$ to give

$$
\begin{equation*}
w t p_{i}=\frac{p_{i} l_{i}+\sum_{j \neq i}^{3} \alpha_{j} p_{j} l_{j}}{1+\sum_{j \neq i}^{3} \alpha_{j}} . \tag{5.5}
\end{equation*}
$$

Taking the difference between WTP for Heterogeneous (No Veil) and Homogenous (No Veil) gives

$$
\begin{equation*}
\Delta w t p_{i}=\frac{\sum_{l \neq i}^{n-1} \alpha_{j}\left(p_{p} l_{j}-p_{p i l} l_{i}\right)}{1+\sum_{j \neq i}^{j} \alpha_{j}}, \tag{5.6}
\end{equation*}
$$

the sum of the altruism weighted differences in expected losses across positions.

For Homogenous (Veil), an altruist can only be altruistic towards their identical group members, therefore there cannot be any distributional altruism. ${ }^{25}$ The problem faced by an altruistic individual can then be expressed as

$$
\begin{align*}
& \frac{1}{n} \sum_{p}^{n}(1+(n-1) \alpha)\left(w_{p}-w t p_{i}\right) \\
& \quad=\frac{1}{n} \sum_{p}^{n}(1+(n-1) \alpha)\left(p_{p}\left(w_{p}-l_{p}\right)+\left(1-p_{p}\right) w_{p}\right) \tag{5.7}
\end{align*}
$$

which simplifies to

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n}\left(w_{p}-w t p_{i}\right)=\frac{1}{n} \sum_{p}^{n}\left(p_{p}\left(w_{p}-l_{p}\right)+\left(1-p_{p}\right) w_{p}\right) \tag{5.8}
\end{equation*}
$$

Solving for $w t p_{i}$ gives

[^19]\[

$$
\begin{equation*}
w t p_{i}=\frac{1}{n} \sum_{p}^{n} p_{p} l_{p} \tag{5.9}
\end{equation*}
$$

\]

the mean expected loss.

For Heterogeneous (Veil), the problem faced by an altruistic individual is given by

$$
\begin{align*}
\frac{1}{n} \sum_{p}^{n}\left(w_{p}-w t p_{i}\right. & \left.+\sum_{p \neq q}^{n-1} \alpha_{q}\left(w_{q}-w t p_{i}\right)\right) \\
& =\frac{1}{n} \sum_{p}^{n}\left(\left(p_{p}\left(w_{p}-l_{p}\right)+\left(1-p_{p}\right) w_{p}\right)+\sum_{p \neq q}^{n-1} \alpha_{q}\left(p_{q}\left(w_{q}-l_{q}\right)+\left(1-p_{q}\right) w_{q}\right)\right) \tag{5.10}
\end{align*}
$$

Solving for $w t p_{i}$ gives

$$
\begin{equation*}
w t p_{i}=\frac{\sum_{p}^{n}\left(1+(n-1) \alpha_{p}\right) p_{p} l_{p}}{n+\sum_{p=1}^{n}(n-1) \alpha_{p}} \tag{5.11}
\end{equation*}
$$

the weighted average across positions of the expected loss when the individual is in the position plus the altruism felt toward the position from every other position.

Taking the difference between WTP for Heterogeneous (Veil) and gives

$$
\begin{equation*}
\Delta w t p_{i}=\frac{(n-1) \sum_{p}^{n} \alpha_{p}\left(p_{p} l_{p}-\overline{p l}\right)}{n+\sum_{p=1}^{n}(n-1) \alpha_{p}} \tag{5.12}
\end{equation*}
$$

the weighted average across positions of the altruism weighted differences between the expected loss for the position and the mean expected loss.

Table 5.1: WTP Predictions in front and behind a VoI with SI, PA, MM, and Quasi-MM preferences and risk neutral preferences.

| Preference Type | $\boldsymbol{W T P} \boldsymbol{P}_{i}$ (Heterogeneous) | $\Delta W T P_{i}$ (Heterogeneous - Homogenous) | Relative Private values | $\begin{gathered} \hline \hline \Delta W T P_{i} \\ \text { Predicted } \\ \text { Sign } \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \hline \text { SI } \\ \text { (No Veil) } \end{gathered}$ | $p_{i} l_{i}$ | 0 |  | = 0 |
| $\begin{gathered} \text { SI } \\ \text { (Veil) } \end{gathered}$ | $\frac{1}{n} \sum_{p}^{n} p_{p} l_{p}$ | 0 |  | $=0$ |
| $\begin{gathered} \text { PA } \\ \text { (No Veil) } \end{gathered}$ | $\frac{p_{i} l_{i}+\alpha \sum_{j \neq i}^{n-1} p_{j} l_{j}}{1+(n-1) \alpha}$ | $\frac{\alpha \sum_{j \neq i}^{3}\left(p_{j} l_{j}-p_{i} l_{i}\right)}{1+(n-1) \alpha}$ | $\begin{aligned} & \overline{p l}>p_{i} l_{i} \\ & \bar{p} l=p_{i} l_{i} \\ & \bar{p} l<p_{i} l_{i} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |
| $\begin{gathered} \text { PA } \\ \text { (Veil) } \end{gathered}$ | $\frac{1}{n} \sum_{p}^{n} p_{p} l_{p}$ | 0 |  | $=0$ |
| $\begin{gathered} \text { MM } \\ \text { (No Veil) } \end{gathered}$ | $\frac{p_{i} l_{i}+\alpha p_{w} l_{w}}{1+\alpha}$ | $\frac{\alpha\left(p_{w} l_{w}-p_{i} l_{i}\right)}{1+\alpha}$ | $\begin{aligned} & p_{w} l_{w}>p_{i} l_{i} \\ & p_{w} l_{w}=p_{i} l_{i} \\ & p_{w} l_{w}<p_{i} l_{i} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |
| MM <br> (Veil) | $\frac{\sum_{i}^{n} p_{i} l_{i}+(n-1) \alpha_{w} p_{w} l_{w}}{n+(n-1) \alpha_{w}}$ | $\frac{(n-1) \alpha_{w} \sum_{i}^{n}\left(p_{w} l_{w}-\overline{p l}\right)}{n+(n-1) \alpha_{w}}$ | $\begin{aligned} & x_{w}>\bar{x} \\ & x_{w}=\bar{x} \\ & x_{w}<\bar{x} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |
| Quasi-MM <br> (No Veil) | $\frac{p_{i} l_{i}+\sum_{j \neq i}^{3} \alpha p_{j} l_{j}+\alpha_{w} p_{w} l_{w}}{1+(n-1) \alpha+\alpha_{w}}$ | $\frac{\sum_{j \neq i}^{3} \alpha_{j}\left(p_{j} l_{j}-p_{i} l_{i}\right)+\alpha_{w}\left(p_{w} l_{w}-p_{i} l_{i}\right)}{1+\sum_{j \neq i}^{3} \alpha_{j}+\alpha_{j}}$ | $\begin{gathered} p_{w} l_{w}>p_{i} l_{i} \\ \bar{p} l>p_{i} l_{i}=p_{w} l_{w} \\ \bar{p} l<p_{i} l_{i}=p_{w} l_{w} \\ p_{w} l_{w}<p_{i} l_{i} \end{gathered}$ | $\begin{aligned} & >0 \\ & >0 \\ & <0 \\ & <0 \\ & <0 \end{aligned}$ |
| Quasi-MM (Veil) | $\frac{(1+(n-1) \alpha) \sum_{p}^{n} p_{p} l_{p}+(n-1) \alpha_{w} p_{w} l_{w}}{n+(n-1) \sum_{p=1}^{n} \alpha_{p}+(n-1) \alpha_{w}}$ | $\frac{(n-1) \alpha_{w} \sum_{i}^{n}\left(p_{w} l_{w}-\overline{p l}\right)}{n+(n-1)\left(n \alpha+\alpha_{w}\right)}$ | $\begin{aligned} & x_{w}>\bar{x} \\ & x_{w}=\bar{x} \\ & x_{w}<\bar{x} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |

[^20]The general functional form is simplified for $\mathrm{SI}, \mathrm{PA}, \mathrm{MM}$, and quasi-MM preferences. Table 5.1 shows Heterogenous WTP values for each preference type in front of the VoI based on Equation (5.5) and behind the VoI based on Equation (5.11). The difference between WTP values for Homogenous and Heterogeneous scenarios, $\Delta w t p_{i}$, is also presented in the table based on Equation (5.6) for preferences in front of the VoI and Equation (5.12) behind the VoI.

For SI preferences $\alpha_{j}=0$ and $\Delta w t p_{i}=0$ both in front and behind the VoI.

For PA preferences altruism is directed equally toward all group members such that $\alpha_{j}=\alpha$ for all $j$. Heterogeneous (No Veil) WTP values are the weighted sum of the individuals expected loss and the other group members expected losses. The value of $\Delta w t p_{i}$ for Heterogeneous (No Veil) and Homogenous (No Veil) is determined by the difference between the individual's expected loss and the group average expected loss. PA individuals with expected losses greater (less) than their group members' average expected loss will have a negative (positive) $\Delta w t p_{i}$ value.

WTP values for Heterogeneous (Veil) and Homogenous (Veil) are both equal to the private SI response both and therefore $\Delta w t p_{i}=0$. This is the result shown in Section 4.3.2 that WTP with person-based altruism equals the private WTP behind a VoI.

For MM preferences altruism is directed towards the worst-off group member such that $\alpha_{j}=0$ for all $j$ except for the worst-off group member (subscripted $w$ ) for which $\alpha_{w}=\alpha$. Heterogeneous (No Veil) WTP values are the weighted sum of expected losses for themselves and the worst of individual weighted by $\alpha$. The value of $\Delta w t p_{i}$ for Heterogeneous (No Veil) and Homogenous (No Veil) is determined by the difference between the individual's expected loss and the worst-off individual's expected loss. If the individual's expected loss is greater (less) than the worst-off individual's expected loss then $\Delta w t p_{i}$ will be negative (positive). If they are the worst-off group member there is no change.

Heterogeneous (Veil) WTP with MM preferences equals the average of the expected losses for themselves and the worst of individual weighted by $\alpha$. Behind the VoI $\Delta w t p_{i}$ is determined by the difference between the group mean expected loss and the expected loss of the worst-off position. If the mean expected loss is greater (less) than the expected loss of the worst-off position then $\Delta w t p_{i}$ will be negative (positive).

Quasi-MM preferences are a combination of PA and MM preferences. For quasi-MM preferences altruism is directed towards each member of the group equally such that $\alpha_{j}=\alpha$ for all $j$ except the worst-off group member (subscripted $w$ ) for which $\alpha_{j}=\alpha+\alpha_{w}$ giving the worst-off member additional weight. Heterogeneous (No Veil) WTP values are the weighted sum of expected loss for themselves, their group members, and an additional term for the worst of individual. The difference between WTP for Heterogeneous (No Veil) and Homogenous (No Veil) is determined in part by the difference between the individual's expected loss and the group mean expected loss and in part by the difference between the individual's loss and the worst of individual's expected loss. If their expected loss is greater (less) than the worst-off individual's expected loss then $\Delta W T P_{i}$ will be negative (positive). If they are the worst-off group member WTP becomes closer to the group mean private value.

Heterogeneous (Veil) WTP values are the weighted mean expected loss across positions with the worst-off position having additional weight relative to the other positions. Similar to the MM result, the difference in WTP for Heterogeneous (Veil) and Homogenous (Veil) is determined by difference between the group mean expected loss and the expected loss of the worst-off position. The two $\Delta W T P_{i}$ terms have the same numerator but the denominator for quasi-MM is larger and therefore the effect is smaller.

### 5.3.3 Inequality Aversion

Next consider the following inequality averse utility function based on the FS-IA and BO-IA models The model results are shown in Table 5.2.

Under FS-IA preferences, individuals receive disutility based on the difference between their expected outcome and the expected outcome of each other group member:

$$
\begin{equation*}
u_{i}=w_{i}-\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[w_{j}-w_{i}, 0\right]-\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[w_{i}-w_{j}, 0\right] \tag{5.13}
\end{equation*}
$$

Where $\alpha \geq \beta \geq 0$ such that unequal outcomes reduce utility. Being worse off than other individuals results in equal or greater disutility than being better off than other individuals for the same level of inequality. Starting with the WTP problem in front of a VoI, utility is equalised with and without the public good such that:

$$
\begin{aligned}
& w_{i}-w t p_{i}-\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{j}-w t p_{i}\right)-\left(w_{i}-w t p_{i}\right), 0\right] \\
&-\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{i}-w t p_{i}\right)-\left(w_{j}-w t p_{i}\right), 0\right] \\
&=w_{i}-p_{i} l_{i}-\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{j}-p_{j} l_{j}\right)-\left(w_{i}-p_{i} l_{i}\right), 0\right] \\
&-\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{i}-p_{i} l_{i}\right)-\left(w_{j}-p_{j} l_{j}\right), 0\right]
\end{aligned}
$$

Solving for WTP gives:

$$
\begin{align*}
w t p_{i}=p_{i} l_{i}+ & \frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{j}-w_{i}\right)-\left(p_{j} l_{j}-p_{i} l_{i}\right), 0\right]-\max \left[w_{j}-w_{i}, 0\right]  \tag{5.15}\\
& +\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{i}-w_{j}\right)-\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]-\max \left[w_{i}-w_{j}, 0\right]
\end{align*}
$$

In the following experiment, the relative ordering group members payoffs is maintained with and without the public good such that the worst-off individual is always the worst-off and the best-off individual is always the best-off. This means the maximisation terms can be differenced and Equ. (5.15) then simplifies to

$$
\begin{equation*}
w t p_{i}=p_{i} l_{i}+\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]+\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(p_{j} l_{j}-p_{i} l_{i}\right), 0\right] \tag{5.16}
\end{equation*}
$$

As Table 5.1 shows, when the public good reduces inequality i.e. the expected loss of the bestoff individual, $p_{b} l_{b}$, is greater than the expected loss of the worst-off individual, $p_{w} l_{w}$, the inequality averse individual is more likely to want the public good provided even if the group mean expected payoff is lower. When the public good maintains inequality i.e. the expected loss of the best-off individual, $p_{b} l_{b}$, is less than the expected loss of the worst-off individual, $p_{w} l_{w}$, the inequality averse individual is less likely to want the public good provided even if the group mean expected payoff is higher.

Behind a VoI expected utility is equalised with and without the public good such that:

$$
\begin{aligned}
\frac{1}{n} \sum_{i}^{n}\left[w_{i}-w t p_{i}\right. & -\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{j}-w t p_{i}\right)-\left(w_{i}-w t p_{i}\right), 0\right] \\
& \left.-\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{i}-w t p_{i}\right)-\left(w_{j}-w t p_{i}\right), 0\right]\right] \\
& =\frac{1}{n} \sum_{i}^{n}\left[w_{i}-p_{i} l_{i}-\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{j}-p_{j} l_{j}\right)-\left(w_{i}-p_{i} l_{i}\right), 0\right]\right. \\
& \left.-\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{i}-p_{i} l_{i}\right)-\left(w_{j}-p_{j} l_{j}\right), 0\right]\right]
\end{aligned}
$$

Solving for WTP gives:

$$
\begin{aligned}
& n w t p_{i}=\frac{1}{n}\left[\sum_{i}^{n} p_{i} l_{i}\right. \\
& \\
& \quad+\sum_{i}^{n}\left[\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{j}-p_{j} l_{j}\right)-\left(w_{i}-p_{i} l_{i}\right), 0\right]\right. \\
& \\
& \quad-\max \left[w_{j}-w_{i}, 0\right] \\
& \\
& \left.\left.\quad+\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(w_{i}-p_{i} l_{i}\right)-\left(w_{j}-p_{j} l_{j}\right), 0\right]-\max \left[w_{i}-w_{j}, 0\right]\right]\right]
\end{aligned}
$$

Again, because relative ordering group members payoffs is maintained with and without the public good the maximisation terms can be differenced and Equ. (5.18) simplifies to

$$
\begin{aligned}
w t p_{i}=\frac{1}{n} \sum_{i}^{n} p_{i} l_{i} & \\
& +\frac{1}{n} \sum_{i}^{n}\left[\frac{\alpha}{n-1} \sum_{j \neq i}^{n-1} \max \left[\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]\right. \\
& \left.+\frac{\beta}{n-1} \sum_{j \neq i}^{n-1} \max \left[p_{j} l_{j}-p_{i} l_{i}, 0\right]\right]
\end{aligned}
$$

As Table 5.1 shows, just as in front of a VoI when the public good reduces inequality i.e. the expected loss of the best-off individual, $p_{b} l_{b}$, is greater than the expected loss of the worst-off individual, $p_{w} l_{w}$, the inequality averse individual is more likely to want the public good provided even if the group mean expected payoff is lower. When the public good maintains inequality i.e. the expected loss of the best-off individual, $p_{b} l_{b}$, is less than the expected loss of
the worst-off individual, $p_{w} l_{w}$, the inequality averse individual is less likely to want the public good provided even if the group mean expected payoff is higher.

Now consider the Bolton \& Ockenfels (2000) inequality aversion utility function such that individuals prefer their expected outcome to be equal to the group expected outcome.

$$
\begin{equation*}
u_{i}=w_{i}-\alpha\left|w_{i}-\frac{1}{n} \sum_{j}^{n} w_{j}\right| \tag{5.20}
\end{equation*}
$$

Where $\alpha \geq 0$ such that being better or worse off than the group average provides disutility. The WTP problem in front of a VoI requires equalised utility with and without the public good such that:

$$
\begin{equation*}
w_{i}-w t p_{i}-\alpha\left|w_{i}-w t p_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-w t p_{i}\right)\right|=w_{i}-p_{i} l_{i}-\alpha\left|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right| \tag{5.21}
\end{equation*}
$$

Simplifying terms gives:

$$
\begin{equation*}
w t p_{i}=p_{i} l_{i}+\alpha\left[\left|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right|-\left|w_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}\right)\right|\right] \tag{5.22}
\end{equation*}
$$

In the homogenous setting each individual has the same outcome equal to the group average and WTP equals the private SI result. As in the heterogenous setting WTP will be greater than the SI result if the difference between the individuals payoff and the group average payoff is less with the public good than without. That is, the public good becomes more valuable if it reduces inequality for the individual. WTP will be less than the SI result if the difference between the individuals payoff and the group average payoff is greater with the public good than without. That is, the public good becomes less valuable if it increases inequality for the individual. If the individual's difference in payoff relative to the group average is maintained with and without the public good then there is no effect on WTP. If the individual has a private value and endowment equal to the group mean, then WTP is always equal to the SI result. These predictions are summarised in Table 5.1.

Behind a VoI equalising expected utility with and without the public good gives:

$$
\begin{align*}
& \frac{1}{n}\left[\sum_{i}^{n} w_{i}-n \cdot w t p_{i}-\alpha \sum_{i}^{n}\left|w_{i}-w t p_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-w t p_{i}\right)\right|\right] \\
& =\frac{1}{n}\left[\sum_{i}^{n}\left(w_{i}-p_{i} l_{i}\right)-\alpha \sum_{i}^{n}\left|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right|\right] \tag{5.23}
\end{align*}
$$

Which simplified gives

$$
\begin{equation*}
w t p_{i}=\frac{\sum_{i}^{n} p_{i} l_{i}}{n}+\frac{\alpha}{n}\left[\sum_{i}^{n}\left|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right|-\sum_{i}^{n}\left|w_{i}-\frac{1}{n} \sum_{j}^{n} w_{j}\right|\right] \tag{5.24}
\end{equation*}
$$

In the homogenous setting behind a VoI, each individual faces the same expected loss once the veil has been lifted and therefore WTP equals the private SI result. In the heterogeneous setting behind a VoI, WTP will be greater than the SI result if the total inequality across all positions is less with the public good than without. As with in front of a VoI, the public good is more valuable if it reduces inequality. Conversely, WTP will be less than the SI result if the total inequality across all positions is greater with the public good than without. As with in front of a VoI, the public good is less valuable if it reduces inequality.

Table 5.2: WTP Predictions in front and behind a VoI with FS-IA and BO-IA preferences and risk neutral preferences.

| Preference Type | $W T P_{i}$ (Heterogeneous) | $\Delta W T P_{i}$ (Heterogeneous - Homogenous) | Relative Private values | $\Delta W T P_{i}$ Predicted Sign |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { FS IA } \\ \text { (No Veil) } \end{gathered}$ | $p_{i} l_{i}+\frac{\alpha}{3} \sum_{j \neq i}^{n-1} \max \left[\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]+\frac{\beta}{3} \sum_{j \neq i}^{n-1} \max \left[\left(p_{j} l_{j}-p_{i} l_{i}\right), 0\right]$ | $\frac{\alpha}{3} \sum_{j \neq i}^{n-1} \max \left[\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]+\frac{\beta}{3} \sum_{j \neq i}^{n-1} \max \left[\left(p_{j} l_{j}-p_{i} l_{i}\right), 0\right]$ | $\begin{aligned} & p_{b} l_{b}>p_{w} l_{w} \\ & p_{b} l_{b}=p_{w} l_{w} \\ & p_{b} l_{b}<p_{w} l_{w} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |
| $\begin{aligned} & \text { FS IA } \\ & \text { (Veil) } \end{aligned}$ | $\begin{aligned} & \frac{1}{4} \sum_{i}^{4}\left[p_{i} l_{i}+\frac{\alpha}{3} \sum_{j \neq i}^{3} \max \left[\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]\right. \\ & \\ & \left.\quad+\frac{\beta}{3} \sum_{j \neq i}^{3} \max \left[\left(p_{j} l_{j}-p_{i} l_{i}\right), 0\right]\right] \end{aligned}$ | $\begin{aligned} & \frac{1}{4} \sum_{i}^{4}\left[\frac{\alpha}{3} \sum_{j \neq i}^{3} \max \left[\left(p_{i} l_{i}-p_{j} l_{j}\right), 0\right]\right. \\ & \\ & \left.\quad+\frac{\beta}{3} \sum_{j \neq i}^{3} \max \left[\left(p_{j} l_{j}-p_{i} l_{i}\right), 0\right]\right] \end{aligned}$ | $\begin{aligned} & p_{b} l_{b}>p_{w} l_{w} \\ & p_{b} l_{b}=p_{w} l_{w} \\ & p_{b} l_{b}<p_{w} l_{w} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |
| $\begin{gathered} \text { BO IA } \\ \text { (No Veil) } \end{gathered}$ | $\begin{array}{r} p_{i} l_{i}+\alpha\left[\left\|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right\|\right. \\ \left.-\left\|w_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}\right)\right\|\right] \end{array}$ | $\alpha\left[\left\|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right\|-\left\|w_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}\right)\right\|\right]$ | $\begin{aligned} & \overline{p l}>p_{i} l_{i} \\ & \bar{p} l=p_{i} l_{i} \\ & \overline{p l}<p_{i} l_{i} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |
| $\begin{aligned} & \text { BO IA } \\ & \text { (Veil) } \end{aligned}$ | $\begin{gathered} \frac{\sum_{i}^{n} p_{i} l_{i}}{n}+\frac{\alpha}{n}\left[\sum_{i}^{n}\left\|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right\|\right. \\ \left.-\sum_{i}^{n}\left\|w_{i}-\frac{1}{n} \sum_{j}^{n} w_{j}\right\|\right] \end{gathered}$ | $\frac{\alpha}{n}\left[\sum_{i}^{n}\left\|w_{i}-p_{i} l_{i}-\frac{1}{n} \sum_{j}^{n}\left(w_{j}-p_{j} l_{j}\right)\right\|-\sum_{i}^{n}\left\|w_{i}-\frac{1}{n} \sum_{j}^{n} w_{j}\right\|\right]$ | $\begin{aligned} & \overline{p l}>p_{i} l_{i} \\ & \overline{p l}=p_{i} l_{i} \\ & \overline{p l}<p_{i} l_{i} \end{aligned}$ | $\begin{aligned} & >0 \\ & =0 \\ & <0 \end{aligned}$ |

[^21]
### 5.3.4 Predictions

Table 5.3 gives an example group for group types: $\mathrm{ABBC}, \mathrm{ABBC}$ (INC), AABC , and ABCC . The examples used here were used in the experiment with other questions where the parameters (w, 1, and p) varied. Predictions for $\Delta W T P_{i}$ are presented for PA, MM, Quasi-MM, FS-IA, and BO-IA by position: A, B and C in front of a VoI and a single prediction behind a VoI.

For SI preferences, the prediction for the difference in WTP values is always $\triangle W T P=0$ regardless of the position and group type. Behind a VoI the individual's expected utility is consistent for the Homogenous (Veil) and Heterogeneous (Veil) scenarios and therefore $\Delta W T P=0$. These predictions are consistent across the four group types.

The PA individual will adjust his/her WTP to be between his/her private optimal response and the group optimal response. This means a subject in Position A (Low EL) will increase his/her WTP from Homogenous (No Veil) to Heterogeneous (No Veil) as they change his/her WTP to be closer to the group optimal response. Similarly, the subject in Position C (High EL) will decrease his/her WTP from Homogenous (No Veil) to Heterogeneous (No Veil) again to align more closely with the group optimal response. The group parameters (endowments and expected losses) are set up such that Position B (Mid EL) is always equal to the group mean expected loss. This means the PA response is to offer the same WTP response in both Homogenous (No Veil) and Heterogeneous (No Veil) regardless of the strength of altruistic preferences. These predictions hold for all four group types. The size of the effect on $\triangle W T P$ is dependent on the mean difference between the individual's expected loss and the expected losses of the other group members. Assuming a consistent $\alpha$ term, $\triangle W T P$ will be equal in absolute terms for Position A (Low EL) and Position $C$ (High EL) in group ABBC and ABBC (INC), larger for Position C (High EL) than Position A (Low EL) in group AABC, and larger for Position A (Low EL) than Position C (High EL) in group ABCC. For PA preferences the difference between WTP for Heterogeneous (Veil) and Homogenous (Veil) is zero. This is because the private and group optimal responses are aligned.

If preferences are MM, $\triangle W T P$ is determined by the size of the difference between the EL of the altruist and the EL of the worst-off individual. The response from the worst-off position is to not change his/her WTP response from the private value. The response for the other two Positions is to shift his/her WTP towards the optimal response of the worst-off group member. The worst-off group member is the group member in Position C (High EL) for groups ABBC, AABC , and ABCC . This means the individuals with MM preferences in the Low EL and Mid

EL positions will increase his/her WTP in Heterogeneous (No Veil) relative to Homogenous (No Veil). Because $E L_{\text {Low }}-E L_{\text {High }}>E L_{\text {Mid }}-E L_{\text {High }}$ the difference in WTP responses is predicted to be greater for the Position A (Low EL) than Position B (Mid EL). For the group ABBC (INC) the opposite is true as the worst-off group member is in the Position A (Low EL). This means the individuals with MM preferences in Position B (Mid EL) and Position C (High EL) will decrease his/her WTP in Heterogeneous (No Veil) relative to Homogenous (No Veil). For MM behind a VoI, $\triangle W T P$ has the same directional effect as in front of the VoI. This is because the MM individual seeks to improve the outcome of the individual who is allocated to the worst-off position which does not happen in Homogenous (Veil) as all group members are allocated to the same position and there is no worst-off individual. This results in a positive effect for groups $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC and negative effect for ABBC (INC).

Quasi-MM is a mix of PA and MM. The key difference to differentiate these preferences from MM is that the individual in the worst-off position will move his/her WTP closer to the group average. The PA part of the preferences are removed by the VoI, whilst the distributional MM part remains. This results in a prediction for Quasi-MM behind a VoI similar to that of MM.

The prediction for $\triangle W T P$ under FS-IA preferences is positive if the public good decreases inequality as in groups $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC and negative if the public good increases inequality as in group ABBC (INC). This prediction holds for each position $\mathrm{A}, \mathrm{B}$, and C and behind the VoI. The same is true under BO-IA preferences except that because Position B has an expected outcome equal to the group expected outcome the predicted $\triangle W T P$ is zero. Behind a VoI the positive/negative effect remains with the same direction as in front of a VoI.

Table 5.3: Example group types with parameters and $\triangle W T P$ predictions for PA, MM, Quasi-MM, FSIA, and BO-IA preferences.

| Group <br> Type | Position | M | X | P | EL | PA | MM | Quasi-MM | FS-IA | BO-IA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A (Low <br> EL) | 600 | 240 | 0.75 | 180 | $\frac{240 \alpha}{1+3 \alpha}$ | $\frac{120 \beta}{1+\beta}$ | $\frac{240 \alpha+120 \beta}{1+3 \alpha+\beta}$ | $80 \beta$ | $60 \alpha$ |
|  | B (Mid <br> EL) | 600 | 320 | 0.75 | 240 | 0 | $\frac{60 \beta}{1+\beta}$ | $\frac{60 \beta}{1+3 \alpha+\beta}$ | $20 \alpha$ | $+20 \beta$ |

Notes: M represents the endowment. L represents the loss amount, P the probability of loss, and EL the expected loss. Note that whilst parameters varied across rounds the general patterns $\mathrm{ABBC}, \mathrm{AABC}$, and ABCC remained and Position B always has an expected loss equal to the group average.

### 5.4 Experimental design

### 5.4.1 Treatments

The experimental design consists of four treatments described in Table 5.4. The treatments are set up in a $2 \times 2$ design. Each treatment was designed to have a sample size of 80 , however due to low turnout in one experimental session Treatment 3 has a lower sample size of 72 .

As the primary purpose of the experiment is to test the impact of a VoI, one dimension of the treatment design places subjects either in front (T1 \& T2) or behind (T3 \& T4) a VoI. For Treatments 1 and 2 experimental subjects are in front of a VoI and therefore respond to Homogenous (no veil) and Heterogenous (no veil). For Treatments 3 and 4 experimental subjects are behind a VoI and therefore respond to Homogenous (veil) and Heterogenous (veil).

The secondary dimension of the design alters the distribution of private values of the public good to act as a robustness check to test for further distributional preferences. In one pair of treatments (T1 \& T3) the group types used are ABBC and ABBC (INC) for which there are equal number of high private value and low private value group members. In the other pair of treatments (T2 \& T4) the group types used are AABC and ABCC for which there are more low private value and high private value group members respectively.

Table 5.4: Treatments

| Treatment | No. of <br> Subjects | VoI | ABBC | ABBC <br> (INC) | AABC | ABCC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 80 | No | Yes | Yes | No | No |
| 2 | 80 | No | No | No | Yes | Yes |
| 3 | 72 | Yes | Yes | Yes | No | No |
| 4 | 80 | Yes | No | No | Yes | Yes |

This design replicates the Messer et al. (2010) and Messer et al. (2013) designs closely with Treatment 1, and extends the design in the two dimensions described above: adding a VoI to generate a Citizen Frame, and altering the distribution of negative lotteries to act as a robustness check testing for further distributional preferences. With this design the within subject comparison between scenarios measures the impact of altruism in whichever form it takes, whilst the between subject comparison allows for the impact of different distributions of endowments and negative lotteries were observed as a robustness check on the results as different distributions could allow for preferences over the distribution of payoffs to impact on WTP decisions.

### 5.4.2 Estimation strategy

The analysis is split over two sections. First, the impact of altruism on individual WTP values in front of a VoI is considered by testing the model predictions for $\triangle W T P$ presented in Table 5.3. Second, these findings are used to explain behaviour behind a VoI, again based on the model predictions for $\triangle W T P$ presented in Table 5.3. T-tests are used to test the sample mean $\Delta W T P$ value for significance based on a null hypothesis that $\triangle W T P=0$. Differences in WTP are split by position within the group $(\mathrm{A}, \mathrm{B}, \mathrm{C}$, and behind a VoI) and by the group type ( ABBC , $\mathrm{ABBC}(\mathrm{INC}), \mathrm{AABC}$, and ABCC ).

### 5.4.3 Protocol

At the beginning of each experimental session ${ }^{29}$, subjects ( $n=312,152 \mathrm{UK}, 160$ Australia) were informed that during the session they could earn tokens, and that their earnings would be dependent on both the choices made by themselves and the choices of others during the session. Tokens, worth $£ 0.01$ ( $\$ 0.02$ ) each, were converted into cash at the end of the session. ${ }^{30}$ Subjects were split into groups of 8 , with each group being randomly allocated to a treatment. Instructions were read out loud to subjects who could follow the instructions by reading the instruction booklet. ${ }^{31}$

In the first part of the session, in accordance with the Holt and Laury (2002) procedure, subjects chose between 2 lotteries for 11 rounds to collect data on risk preferences. ${ }^{32}$ One round was chosen randomly at the end of the session to be played out using the subjects' choice for that round. The outcome of that lottery determined part of their earnings. Experimental subjects were aware that their choices would determine part of their final earnings but were not aware

[^22]of the chosen round or outcome until after they had completed the main task, thus avoiding any wealth effects.

Subjects were then given instructions explaining the task and RPVM, and then completed 6 practice rounds: 4 rounds that were generic with the purpose of explaining the RPVM, and 2 rounds specific to their particular treatment. The main task of the experiment followed, with each subject completing 16 rounds split across two different scenarios dependent on their treatment. The order of the scenarios and the rounds within them were randomised to control for any order effects. A within subject comparison of choice between the two scenarios is used to generate the data to test model predictions. ${ }^{33}$

Subjects completed a questionnaire to collect demographic data before feedback was given on the two rounds which determined their payoff: one round from the Holt-Laury task, and one round from the main task. Subjects were paid their session earnings at the end of the session as they left. Earnings had a guaranteed minimum of $£ 5.00(\$ 10.00)$ and theoretical maximum of $£ 20.00$ ( $\$ 40.00$ ). On average subjects earned $£ 12.70$ and $\$ 23.75$ for the UK and Australia respectively.

### 5.5 Results

### 5.5.1 Sample and demographics

Table 5.5 summarises the demographics of the subjects by country. The treatment sub-samples each have similar pools of subjects as the demographic makeup of each sub-sample is not significantly different from any other at the $95 \%$ confidence level.

The two countries have significantly different sample means for gender, age and mathematics education, with the UK having more female subjects, a higher average age and a lower level of mathematics education. ${ }^{34}$ The difference in the mean ages is driven by the number of 17 year

[^23]olds in the Australian sample which do not appear in the UK sample, and the longer right tail in the UK sample. Subjects were asked their level of mathematical education from the options of prior to the end of secondary education, to the end of secondary education, or as a component of their degree. Subjects from both countries are most likely to have completed secondary school maths education. The differences in mathematics education is likely due to the education policies, and the university entrance requirements in Australia, so that almost no subjects had a level of mathematics education less than the full secondary school. The Australian subjects are also more likely to come from degree programs that have mathematical backgrounds.

Table 5.5: Sample demographics split by country.

|  | Total | UK | Australia | $\begin{gathered} \text { t-stat } \\ \text { (p-value) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| UK Dummy | $\begin{gathered} 0.49 \\ (0.50) \end{gathered}$ |  |  |  |
| Male Dummy | $\begin{gathered} 0.47 \\ (0.03) \end{gathered}$ | $\begin{gathered} 0.41 \\ (0.04) \end{gathered}$ | $\begin{gathered} 0.52 \\ (0.04) \end{gathered}$ | $\begin{aligned} & 1.91^{*} \\ & (0.06) \end{aligned}$ |
| Age | $\begin{aligned} & 22.38 \\ & (0.29) \end{aligned}$ | $\begin{aligned} & 23.26 \\ & (0.48) \end{aligned}$ | $\begin{aligned} & 21.56 \\ & (0.33) \end{aligned}$ | $\begin{gathered} -2.94 * * * \\ (0.00) \end{gathered}$ |
| Age22 | $\begin{gathered} 0.42 \\ (0.03) \end{gathered}$ | $\begin{gathered} 0.49 \\ (0.04) \end{gathered}$ | $\begin{gathered} 0.35 \\ (0.04) \end{gathered}$ | $\begin{gathered} -2.47 * * * \\ (0.00) \end{gathered}$ |
| Maths | $\begin{gathered} 1.17 \\ (0.04) \end{gathered}$ | $\begin{gathered} 0.91 \\ (0.06) \end{gathered}$ | $\begin{gathered} 1.41 \\ (0.05) \end{gathered}$ | $\begin{gathered} 6.89 * * * \\ (0.00) \end{gathered}$ |
| Uni Dummy | $\begin{gathered} 0.36 \\ (0.03) \end{gathered}$ | $\begin{gathered} 0.22 \\ (0.03) \end{gathered}$ | $\begin{gathered} 0.48 \\ (0.04) \end{gathered}$ | $\begin{gathered} 4.92 * * * \\ (0.00) \end{gathered}$ |

Note: Means with standard errors in parentheses. Two-tailed T-tests compare sample means between Australia and the UK with ${ }^{*}$, ${ }^{* *}$, and ${ }^{* * *}$ denoting significance at the $10 \%, 5 \%$, and $1 \%$ significance levels respectively.

Figure 5.1-5.4 show histograms of the dependent variable to be tested using t-tests. The data is pooled across group types and split by Position: A (Figure 5.1), B (Figure 5.2) and C (Figure 5.3), and the VoI responses (Figure 5.4). Each has a large spike around zero suggesting a large proportion of experimental subjects follow the SI prediction. As Position A (C) has the lowest (highest) EL, positive (negative) values represent a shift towards the group optimal response. Position B has the middle EL and behind a VoI the group mean EL. Therefore, from Position B or behind the VoI, a negative value is a shift towards the Position A optimal WTP response and a positive value is a shift towards the Position C optimal WTP response. Table 5.6 presents the mean difference in WTP by position and skewness discussed below. The associated $p$-values are the p -value for a 2 -tail t -test of the mean value equalling 0 .

Figure 5.1: Histogram of $\triangle W T P$ from Position A


Figure 5.2: Histogram of $\triangle W T P$ from Position B


Figure 5.3: Histogram of $\triangle W T P$ from Position C


Figure 5.4: Histogram of $\triangle W T P$ from behind a VoI


### 5.5.2 Altruism and the effect of skewed distributions

For the ABBC groups, the mean differences were significant at the $95 \%$ confidence interval and positive with a mean 18.77** for Position A (Low EL), insignificant at the $90 \%$ confidence interval with a mean of -6.86 for Position B (Mid EL), and significant at the $99 \%$ confidence interval and negative with a mean of -31.62*** for Position C (High EL).

For the AABC groups, the mean differences were significant at the $95 \%$ confidence interval and positive with a mean 20.5** for Position A (Low EL), insignificant at the $90 \%$ confidence interval with a mean of -0.78 for Position B (Mid EL), and significant at the $99 \%$ confidence interval and negative with a mean of -165.31*** for Position C (High EL).

For the ABCC groups, the mean differences were significant at the $99 \%$ confidence interval and positive with a mean $38.34 * *$ for Position A (Low EL), insignificant at the $90 \%$ confidence interval with a mean of -3.54 for Position B (Mid EL), and significant at the $90 \%$ confidence interval and negative with a mean of -12.29*** for Position C (High EL).

Each of these results aligns with the predictions for PA preferences from Table 5.2. Furthermore, this result is robust to skews in the distribution of private benefit of the public good. Based on these results the expected differences between WTP for Homogenous (Veil) and Heterogeneous (Veil) is zero.

For the ABBC (INC) groups, the mean differences were insignificantly different from zero at the $90 \%$ confidence interval for all positions. Each of these results aligns with SI preferences.

Table 5.6: T-test results by question type.

| Group Type | VoI | Treatment | Position | Mean $\triangle W T P$ | P-Value | Effect <br> Direction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ABBC |  |  | A (Low EL) | 18.77** | 0.02 | $>0$ |
|  | No | T1 | B (Mid EL) | -6.86 | 0.26 | $=0$ |
|  |  |  | C (High EL) | $-31.62 * * *$ | 0.00 | $<0$ |
|  | Yes | T3 |  | -8.64 | 0.14 | $=0$ |
| AABC |  |  | A (Low EL) | 20.5** | 0.01 | $>0$ |
|  | No | T2 | B (Mid EL) | -0.78 | 0.94 | $=0$ |
|  |  |  | C (High EL) | -165.31*** | 0.00 | $<0$ |
|  | Yes | T4 |  | -14.5 | 0.10 | $=0$ |
| ABCC |  |  | A (Low EL) | 38.34*** | 0.00 | $>0$ |
|  | No | T2 | B (Mid EL) | -3.54 | 0.68 | $=0$ |
|  |  |  | C (High EL) | -12.29* | 0.05 | $<0$ |
|  | Yes | T4 |  | -6.98 | 0.28 | $=0$ |
| $\begin{gathered} \mathrm{ABBC} \\ (\mathrm{INC}) \end{gathered}$ |  |  | A (Low Inc/EL) | -4.92 | 0.49 | $=0$ |
|  | No | T1 | B (Mid Inc/EL) | -6.39 | 0.31 | $=0$ |
|  |  |  | C (High Inc/EL) | 1.44 | 0.93 | $=0$ |
|  | Yes | T3 |  | -6.17 | 0.13 | $=0$ |

### 5.3.3 The VoI effect

Behind a VoI, the mean difference between WTP for Homogenous (Veil) and Heterogeneous (Veil) are all insignificant at the $90 \%$ confidence interval with means of $-8.64,-14.5,-6.98$, and -6.17 for $\mathrm{ABBC}, \mathrm{AABC}, \mathrm{ABCC}$, and ABBC (INC) respectively. This aligns with both the SI and PA model predictions and is robust to the distribution of negative lotteries. Based on the No VoI treatments (T1 \& T2) in which PA preferences are observed for groups ABBC, AABC, and ABCC , this suggests the VoI working as expected to reduce the impact of PA preferences.

### 5.5 Summary and discussion

The purpose of this experiment was to compare the Consumer Frames and the Citizen Frame. The experiment extended the design of Messer et al. (2013) to include variation in the distribution of induced values from public good provision and to allow for a VoI. The frames were evaluated by first comparing the Consumer Frames to characterise any altruistic or inequality averse concern. Six types of preferences were considered: SI, PA, MM, Quasi-MM, FS-IA, and BO-IA preferences. Choice under the Citizen Frame was then compared to predictions based on choice in front of a VoI.

The Homogenous (No Veil) scenario was designed to replicate Consumer Frame 1 by keeping each group member's negative lottery the same. This result is shown in Messer et al. (2010) and Messer et al. (2013). Heterogeneous (No Veil) was designed to replicate Consumer Frame 2 for which group members vary in their negative lotteries. The impact of altruism is measured by holding an individual's lottery constant and comparing their WTP when their group members have the same lottery and when their group members have different lotteries. The results show that respondents are acting with PA. This result replicated the findings of Messer et al. (2010) and Messer et al. (2013).

Heterogeneous (Veil) was designed to replicate a Citizen Frame by placing respondents behind a VoI. By comparing WTP in Heterogeneous (Veil) to the private values equivalent in Homogenous (Veil) allowed for the impact of altruism to be measured. There was no significant difference between mean responses Homogenous (Veil) and Heterogeneous (Veil) suggesting altruism had no impact on values. Under the assumption that the underlying preferences of those responding in front of the VoI, the Citizen Frame is working as the model presented in Chapter 4 predicted by reducing altruism from values.

The robustness of these results was checked by varying the distribution of the groups expected losses introducing groups (AABC, ABCC and $\mathrm{ABBC}(\mathrm{INC})$ ) with alternate distributions of negative lotteries and heterogeneous endowments. This allowed for more distributive altruism to enter into individuals' decision-making. The results were found to be robust to the these alternate distributions, but the introduction of heterogeneous endowments resulted in the average response to be that of an SI individual both in front and behind the VoI.

## CHAPTER SIX

## Experiment B: An Experiment Testing the Taxation Hypothesis ${ }^{35}$

### 6.1 Introduction

Experiment B is designed to test the impact of taxation on individual valuations. This chapter presents the experimental design and the results. The findings are discussed in Chapter 7. The experiment extends the Messer et al. (2010) design to incorporate different cost-sharing rules, whilst maintaining the VoI extension used in Experiment A. This experiment also provides the first operationalisation of the concept of a MAC which is presented theoretically in Chapter 4.

Two hypotheses from the theoretical models presented in Chapters 3 and 4 are tested. The WTP model presented in Chapter 3 predicts that if respondents are purely altruistic then his/her WTP response will be increasing with their own tax share. The CBA model presented in Chapter 4 suggests that if respondents are consistently purely altruistic across positions then their MAC response behind a VoI will remain unaffected by the share of costs each position must pay. However, if preferences are distributional then MAC values will vary with taxation. An alternate hypothesis is tested that individual's are inequality averse based on a Bolton \& Ockenfels (2000) model of Inequality Aversion and a Fehr \& Schmidt (1999) model with equal disutility for being better or worse off than others.

[^24]
### 6.2 Model

Consider an economy of 2 individuals indexed $S$ for small and $L$ for large who have wealth, $w_{S}=100$ and $w_{L}=150$ respectively. The individuals have the opportunity to fund a discrete increase in public good provision which results in separate private benefits on which the individuals place values of $x_{S}$ and $x_{L}$. The provision of the public good comes at an additional $\operatorname{cost}, C$, and requires the individuals each to pay a share $t_{S}$ and $t_{L}=1-t_{S}$, to ensure the full cost is covered. WTP is elicited from each individual to decide whether to fund the public good. The problem faced by the individual deciding on a WTP is to equalise utility with and without the public good. The following section presents models of WTP, first-derivative with respect to tax share and WTP at the first-best tax share are found for SI, PA and IA preferences. The model predictions are summarised in Table 6.1.

### 6.2.1 Self-interest

Under SI preferences the problem faced in deciding maximum WTP in front of a VoI for either individual indexed $i$ is

$$
\begin{equation*}
w_{i}=w_{i}+x_{i}-t_{i} C . \tag{6.1}
\end{equation*}
$$

Rearranging gives

$$
\begin{equation*}
W T P=t_{i} C^{*}=x_{i} . \tag{6.2}
\end{equation*}
$$

where $t_{i} C^{*}$ is Individual $i$ 's WTP, such that WTP equals the private value.
Behind a VoI the problem with risk neutral preferences is

$$
\begin{equation*}
\frac{1}{2}\left(w_{S}+w_{L}\right)=\frac{1}{2}\left(w_{S}+x_{S}-t_{S} C+w_{L}+x_{L}-\left(1-t_{S}\right) C\right) . \tag{6.3}
\end{equation*}
$$

Rearranging gives

$$
\begin{equation*}
M A C=C^{*}=x_{S}+x_{L} . \tag{6.3}
\end{equation*}
$$

Such that the MAC value equals the sum of the private values. Neither WTP in front of the VoI or MAC behind a VoI vary with tax share. This prediction is shown in Table 6.1.

### 6.2.2 Altruism

An altruist includes others weighted payoff in their utility function as described in Chapter 5. The problem faced by an altruistic individual in front of a VoI for either individual indexed $i$ is

$$
\begin{equation*}
w_{i}+\alpha_{i} w_{j}=w_{i}+x_{i}-t_{i} C+\alpha_{i}\left(w_{j}+x_{j}-\left(1-t_{i}\right) C\right) . \tag{6.4}
\end{equation*}
$$

Where $\alpha_{i}$ is the weight Individual $i$ places on the other individual indexed $j$ 's utility. That is social utility with the public good equals social utility without the public good. Rearranging gives

$$
\begin{equation*}
W T P_{i}=t_{i} C^{*}=t_{i} \frac{x_{i}+\alpha_{i} x_{j}}{t_{i}+\alpha_{i}\left(1-t_{i}\right)}, \tag{6.5}
\end{equation*}
$$

where $t_{i} C^{*}$ is Individual $i$ 's WTP. The first differential with respect to own tax share is then

$$
\begin{equation*}
\frac{\partial W T P_{i}}{\partial t_{i}}=\frac{\alpha_{i}\left(x_{i}+\alpha_{i} x_{j}\right)}{\left(t_{i}+\alpha_{i}\left(1-t_{i}\right)\right)^{2}}>0 . \tag{6.6}
\end{equation*}
$$

Such that WTP is increasing in one's own tax share and therefore decreasing with the other individuals tax share. That is $\frac{\partial W T P_{S}}{\partial t_{S}}>0$ and $\frac{\partial W T P_{S}}{\partial t_{S}}<0$. The greater the strength of altruism $\alpha_{i}$ the derivative becomes larger in absolute terms. If the tax is first-best, that is the tax share equals the individuals share of private returns $t_{S}=\frac{x_{S}}{x_{S}+x_{L}}, W T P_{i}=x_{i}$. This is the prediction shown in Chapter 3 Table 3.2 simplified for the two-person case.

The problem faced by the altruistic individual behind a VoI is

$$
\begin{align*}
\frac{1}{2}\left(\left(1+\alpha_{L}\right) w_{S}\right. & \left.+\left(1+\alpha_{S}\right) w_{L}\right) \\
& =\frac{1}{2}\left(\left(1+\alpha_{L}\right)\left(w_{S}+x_{S}-t_{S} C\right)+\left(1+\alpha_{S}\right)\left(w_{L}+x_{L}\right.\right.  \tag{6.7}\\
& \left.\left.-\left(1-t_{S}\right) C\right)\right)
\end{align*}
$$

This simplifies to

$$
\begin{equation*}
M A C=C^{*}=\frac{\left(1+\alpha_{L}\right) x_{S}+\left(1+\alpha_{S}\right) x_{L}}{1+\alpha_{S}+t_{S}\left(\alpha_{L}-\alpha_{S}\right)} . \tag{6.8}
\end{equation*}
$$

Differentiating with respect to $t_{i}$ gives

$$
\begin{equation*}
\frac{\partial c_{i}^{*}}{\partial t_{S}}=-\left(\alpha_{L}-\alpha_{S}\right) \frac{\left(1+\alpha_{L}\right) x_{S}+\left(1+\alpha_{S}\right) x_{L}}{\left[\left(1+\alpha_{S}\right)+\left(\alpha_{L}-\alpha_{S}\right) t_{S}\right]^{2}} . \tag{6.9}
\end{equation*}
$$

This is the prediction shown in Chapter 4 Equation (4.33) simplified for the two-person case. Predictions are summarised in Table 6.1 for PA preferences where $\frac{\partial W T P_{i}}{\partial t_{i}}>0$ in front of the VoI. Behind the VoI the direction of the effect depends on the relative sizes of $\alpha_{L}$ and $\alpha_{S}$. If $\alpha_{L}=\alpha_{S}$ then $\frac{\partial C_{i}^{*}}{\partial t_{S}}=0$. If altruism favours Position S then $\alpha_{L}>\alpha_{S}$ and $\frac{\partial C_{i}^{*}}{\partial t_{S}}<0$. If altruism favours Position L then $\alpha_{L}<\alpha_{S}$ and $\frac{\partial C_{i}^{*}}{\partial t_{S}}>0$.

### 6.2.3 Inequality Aversion

As described in Chapter 5, a BO-IA receives disutility when their payoff is different from the group mean payoff. The problem faced by the inequality averse individual indexed $i$ is as follows

$$
\begin{align*}
w_{i}-\alpha \mid w_{L}- & \left.\frac{w_{S}+w_{L}}{2} \right\rvert\, \\
& =w_{i}+x_{i}-t_{i} C \\
& -\alpha \mid w_{S}+x_{S}-t_{S} C  \tag{6.10}\\
& \left.-\frac{w_{S}+x_{S}-t_{S} C+w_{L}+x_{L}-\left(1-t_{S}\right) C}{2} \right\rvert\,
\end{align*}
$$

So that utility without the public good equals utility with the public good. The sign of the two absolute terms depends on the relative payoffs of the two individuals with and without the public good. Without the public good, the individual in Position L is always better off as $w_{S}<$ $w_{L}$. This means Equation (6.10) simplifies to

$$
\begin{equation*}
-\frac{\alpha}{2}\left(w_{L}-w_{S}\right)=x_{i}-t_{i} C-\frac{\alpha}{2}\left|w_{S}+x_{S}-w_{L}-x_{L}-\left(1-2 t_{S}\right) C\right| \tag{6.11}
\end{equation*}
$$

The sign of the remaining absolute term depends on whether Individual L remains better-off or if there is redistribution through the public good provision that means Position S becomes better off. Individual L remains better-off if $w_{S}+x_{S}-t_{S} C<w_{L}+x_{L}-\left(1-t_{S}\right) C$. This occurs when the tax shares are more equal and the relative differences in return from the public good are small. Individual S becomes better-off if $w_{S}+x_{S}-t_{S} C>w_{L}+x_{L}-\left(1-t_{S}\right) C$. This could occur when the tax shares favour Position S i.e. less than the first-best rate $\left(t_{S}<\frac{x_{S}}{x_{S}+x_{L}}\right)$ and/or the return from the public good favours Individual $\mathrm{S}\left(x_{S}>x_{L}\right)$.

Individual L remains better-off then Equation (6.11) simplifies for Individual $S$ to

$$
\begin{equation*}
W T P_{S}=t_{S} C^{*}=t_{S} \frac{x_{S}-\frac{\alpha}{2}\left(x_{L}-x_{S}\right)}{t_{S}-\frac{\alpha}{2}\left(1-2 t_{S}\right)} \tag{6.12}
\end{equation*}
$$

which collapses to $x_{S}$ at the first-best tax share where $t_{S}=\frac{x_{S}}{x_{S}+x_{L}}$. Differentiating with respect to $t_{S}$ gives

$$
\begin{equation*}
\frac{d W T P_{S}}{d t_{S}}=-\frac{\alpha}{2} \frac{x_{S}-\frac{\alpha}{2}\left(x_{L}-x_{S}\right)}{\left(t_{S}-\frac{\alpha}{2}\left(1-2 t_{S}\right)\right)^{2}} \tag{6.13}
\end{equation*}
$$

Such that $\frac{d W T P_{S}}{d t_{S}}<0$ unless $x_{S}<\frac{\alpha}{2}\left(x_{L}-x_{S}\right)$, that is the benefit to Individual $S$ is outweighed by the negative impact of increasing inequality. ${ }^{36}$ Inequality is captured in the WTP values for both positions by the differences in return in the numerator and differences in tax share in the denominator.

Individual L remains better-off then Equation (6.11) simplifies for Individual $L$ to

$$
\begin{equation*}
W T P_{L}=\left(1-t_{S}\right) C^{*}=\left(1-t_{S}\right) \frac{x_{L}-\frac{\alpha}{2}\left(x_{L}-x_{S}\right)}{\left(1-t_{S}\right)-\frac{\alpha}{2}\left(1-2 t_{S}\right)} \tag{6.14}
\end{equation*}
$$

which collapses to $x_{L}$ at the first-best tax share where $t_{S}=\frac{x_{S}}{x_{S}+x_{L}}$. Differentiating with respect to $t_{s}$ gives

$$
\begin{equation*}
\frac{d W T P_{L}}{d t_{S}}=-\frac{\alpha}{2} \frac{x_{L}-\frac{\alpha}{2}\left(x_{L}-x_{S}\right)}{\left(\left(1-t_{S}\right)-\frac{\alpha}{2}\left(1-2 t_{S}\right)\right)^{2}}<0 \tag{6.15}
\end{equation*}
$$

For both Individuals, at lower levels of $t_{S}$ the public good tends to be more redistributive towards individual $S$ and therefore Inequality Averse individuals offer higher WTP as the public good becomes more redistributive.

Individual $S$ becomes better-off then Equation (6.11) simplifies for Individual $S$ to

$$
\begin{equation*}
W T P_{S}=t_{S} C^{*}=t_{S} \frac{x_{S}+\frac{\alpha}{2}\left(2 w_{L}-2 w_{S}+x_{L}-x_{S}\right)}{t_{S}+\frac{\alpha}{2}\left(1-2 t_{S}\right)} \tag{6.16}
\end{equation*}
$$

[^25]Differentiating with respect to $t_{S}$ gives

$$
\begin{equation*}
\frac{d t_{S} C}{d t_{S}}=\frac{\alpha}{2} \frac{x_{S}+\frac{\alpha}{2}\left(2 w_{L}-2 w_{S}+x_{L}-x_{S}\right)}{\left(t_{S}+\frac{\alpha}{2}\left(1-2 t_{S}\right)\right)^{2}}>0 \tag{6.17}
\end{equation*}
$$

Individual S becomes better-off then Equation (6.11) simplifies for Individual $L$ to

$$
\begin{equation*}
W T P_{L}=\left(1-t_{S}\right) C^{*}=\left(1-t_{S}\right) \frac{x_{L}-\frac{\alpha}{2}\left(2 w_{S}+x_{S}-2 w_{L}-x_{L}\right)}{\left(1-t_{S}\right)+\frac{\alpha}{2}\left(1-2 t_{S}\right)} \tag{6.16}
\end{equation*}
$$

which collapses to $x_{L}$ at the first-best tax share where $t_{S}=\frac{x_{S}}{x_{S}+x_{L}}$. Differentiating with respect to $t_{s}$ gives

$$
\begin{equation*}
\frac{d\left(1-t_{S}\right) C}{d t_{S}}=\frac{\alpha}{2} \frac{x_{L}+\frac{\alpha}{2}\left(2 w_{L}-2 w_{S}+x_{L}-x_{S}\right)}{\left(\left(1-t_{S}\right)+\frac{\alpha}{2}\left(1-2 t_{S}\right)\right)^{2}} \tag{6.17}
\end{equation*}
$$

Which is positive unless $x_{L}+\frac{\alpha}{2}\left(2 w_{L}-2 w_{S}+x_{L}-x_{S}\right)<0$, that is the redistribution towards Individual $S$ is large enough that S is better off and inequality is increased. ${ }^{37}$ Inequality is captured in the WTP values for both positions by the differences in endowment and return in the numerator and differences in tax share in the denominator.

At lower levels of $t_{S}$ the public good tends to be more redistributive towards individual $S$. As the public good is already making Individual $S$ better off than Individual $L$ any decreases in $t_{S}$ increase inequality and therefore Individual $L$ will reduce his/her WTP and Individual $S$ will increase his/her WTP.

The problem faced by an IA individual behind a VoI is

[^26]\[

$$
\begin{aligned}
\frac{1}{2}\left(w_{S}+w_{L}\right)- & \alpha\left|w_{L}-\frac{w_{S}+w_{L}}{2}\right| \\
& =\frac{1}{2}\left(\left(w_{S}+x_{S}-t_{S} C+w_{L}+x_{L}-\left(1-t_{S}\right) C\right)\right. \\
& -\alpha \mid w_{S}+x_{S}-t_{S} C \\
& \left.-\frac{w_{S}+x_{S}-t_{S} C+w_{L}+x_{L}-\left(1-t_{S}\right) C}{2} \right\rvert\,
\end{aligned}
$$
\]

Which simplifies to

$$
\begin{align*}
\left((1+\alpha) w_{S}+\right. & \left.(1-\alpha) w_{L}\right) \\
& =\left(w_{S}+x_{S}+w_{L}+x_{L}-C\right)  \tag{6.19}\\
& -\alpha\left|w_{S}+x_{S}-w_{L}-x_{L}+\left(1-2 t_{S}\right) C\right|
\end{align*}
$$

If Individual L remains better-off then Equation (6.19) simplifies to

$$
\begin{equation*}
M A C=C^{*}=\frac{x_{S}+x_{L}-\alpha\left(x_{L}-x_{S}\right)}{1-\alpha\left(1-2 t_{S}\right)} \tag{6.20}
\end{equation*}
$$

which collapses to $x_{S}+x_{L}$ at the first-best tax share where $t_{S}=\frac{x_{S}}{x_{S}+x_{L}}$. Taking the first derivative with respect to $t_{S}$ gives

$$
\begin{equation*}
\frac{d C^{*}}{d t_{S}}=-2 \alpha \frac{x_{S}+x_{L}-\alpha\left(x_{L}-x_{S}\right)}{\left(1-\alpha\left(1-2 t_{S}\right)\right)^{2}}<0 \tag{6.21}
\end{equation*}
$$

Over this range of values, inequality between the individuals decreases as $t_{S}$ decreases so the value of the public good increases.

If Individual $S$ becomes better-off then Equation (6.19) simplifies to

$$
\begin{equation*}
M A C=C^{*}=\frac{x_{S}+x_{L}-\alpha\left(2 w_{S}-2 w_{L}+x_{S}-x_{L}\right)}{1+\alpha\left(1-2 t_{S}\right)} \tag{6.22}
\end{equation*}
$$

Taking the first derivative with respect to $t_{S}$ gives

$$
\begin{equation*}
\frac{d C^{*}}{d t_{S}}=2 \alpha \frac{x_{S}+x_{L}-\alpha\left(2 w_{S}-2 w_{L}+x_{S}-x_{L}\right)}{\left(1+\alpha\left(1-2 t_{S}\right)\right)^{2}}>0 \tag{6.23}
\end{equation*}
$$

Over this range of values, inequality between the individuals decreases as $t_{S}$ increases so the value of the public good increases with $t_{s}$. Inequality is captured in the MC values by the
differences in return in the numerator and differences in tax share in the denominator. When there is redistribution through the public good then differences in endowment also enter the numerator.

Table 6.1: Model predictions for SI, PA, and IA preferences.

| Preference | Position S <br> $\frac{\boldsymbol{d} \boldsymbol{t}_{\boldsymbol{S}} \boldsymbol{C}^{*}}{\boldsymbol{d} \boldsymbol{t}_{\boldsymbol{S}}}, \boldsymbol{W T P}\left(\boldsymbol{t}^{*}\right)$ | Position L <br> $\frac{\boldsymbol{d}\left(\mathbf{1}-\boldsymbol{t}_{\boldsymbol{S}}\right) \boldsymbol{C}^{*}}{\boldsymbol{d} \boldsymbol{t}_{\boldsymbol{S}}}, \boldsymbol{W T P}\left(\boldsymbol{t}^{*}\right)$ | $\boldsymbol{d} \boldsymbol{C}^{*}$ <br> $\boldsymbol{d} \boldsymbol{t}_{\boldsymbol{S}}$, $\boldsymbol{M} \boldsymbol{A C}\left(\boldsymbol{t}^{*}\right)$ |
| :---: | :---: | :---: | :---: |
| SI | $0, x_{S}$ | $0, x_{L}$ | $0, x_{S}+x_{L}$ |
| PA | $>0, x_{S}$ | $<0, x_{L}$ | $0, x_{S}+x_{L}$ |
| IA | $<0$ if $U_{L}>U_{S}, x_{S}$ | $<0$ if $U_{L}>U_{S}, x_{L}$ | $<0$ if $U_{L}>U_{S}, x_{S}+x_{L}$ |
| 0 if $U_{L}<U_{S}$ | $>0$ if $U_{L}<U_{S}$ | $>0$ if $U_{L}<U_{S}$ |  |

Notes: $U_{L} \lessgtr U_{S}$ when $w_{L}+x_{L}-\left(1-t_{S}\right) C \lessgtr w_{S}+x_{S}-t_{S} C ., \boldsymbol{W T P}\left(\boldsymbol{t}^{*}\right)$ and, $\boldsymbol{M A C}\left(\boldsymbol{t}^{*}\right)$

### 6.3 Experimental Design

### 6.3.1 Tax extension and protocol

At the beginning of each experimental session ${ }^{38}$ subjects $(n=204)$ were informed that during the session they could earn tokens and that their earnings would be dependent on choices made by themselves and the choices of others during the session. Tokens, worth $£ 0.05$ each, were converted into cash at the end of the session. Instructions were read out loud to subjects who completed 4 practice rounds before completing 22 actual rounds. ${ }^{39}$

[^27]Subjects then completed a questionnaire to collect demographic data. Following standard experimental procedures, a round was randomly selected to be played out for real and a cost was drawn at random for this round. ${ }^{40}$ The randomly selected round was played out following the procedure described below. Subjects were paid at the end of the session and they were guaranteed a minimum of $£ 5.00$ and on average earned $£ 12.07$.

At the start of each round subjects were randomly placed into groups of 2 and assigned one of 2 positions: Position S, or Position L. Position S had a smaller endowment of $w_{S}=100$ Tokens, and Position L had a larger endowment of $w_{L}=150$ Tokens. Groups of 2 provide the simplest design to unambiguously test the distortionary effect of tax. ${ }^{41}$ Each group had the opportunity to fund a public good which provided positive returns of $x_{S}$ and $x_{L}$ respectively with certainty. Subjects were given information on endowments, private returns from the public good, and the share of the cost each Position would pay if the public good was funded.

The group decision rule is similar to that presented in Chapter 5. The group decides whether to accept the public good at cost $C$ to the group using the following stages of the RPVM procedure:

1. Information is presented on group members': endowment, benefit, and the share of the cost each individual will pay.
2. Subjects enter their MAC. ${ }^{42}$
3. The group cost, C, is randomly drawn.
4. Each MAC is compared to the randomly drawn cost to generate votes using the following rule:
a. YES vote if the MAC $\geq \mathrm{C}$.
b. NO vote if the MAC $<\mathrm{C}$.

[^28]5. The number of YES votes is counted to determine the outcome with the decision rule:
a. If majority of votes are YES then each subject pays their share of the cost from their endowment and receives their benefit from the public good.
b. If majority of votes are NO then each subject is left with their endowment.

In front of a VoI subjects knew which position they were assigned to, whilst behind a VoI they did not. The weakly dominant strategy for a subject, as in the BDM mechanism, was to enter her true MAC as, in that case, a subject would always vote yes for cost she was willing to pay and vote no otherwise.

### 6.3.2 Treatments

Table 6.2 describes the private returns for 4 different Public Goods. For each public good, subjects offered MAC responses both in front of and behind a VoI from both Position S (smaller endowment) and Position L (larger endowment). For every Public Good each subject was randomly assigned to a tax system and this was maintained for valuations made in front and behind a VoI. This allows for a direct comparison, whilst ensuring that differences in valuations between tax types were not an artefact of treatment assignment.

Four different tax systems were considered to generate different tax shares: first-best tax uniform tax, proportional endowment tax, and progressive endowment tax. The first-best tax, $t_{i}^{*}$ was calculated as a ratio between an individual's return and the group total return

$$
\begin{equation*}
t_{S}^{*}=\frac{x_{S}}{x_{S}+x_{L}}=0.4 \tag{6.24}
\end{equation*}
$$

and therefore varied across public goods with different relative private benefits. For the uniform tax the cost was split equally among group members so that tax share, $t_{s}^{U n i}=0.5$. The proportional endowment tax for Position $\mathrm{S}, t_{s}^{\text {Prop }}$ was calculated using:

$$
\begin{equation*}
t_{S}^{\text {Prop }}=\frac{w_{S}}{w_{S}+w_{L}}=0.4 \tag{6.25}
\end{equation*}
$$

i.e. Position $S$ 's endowment as a ratio of the group total endowment. The tax share for the progressive endowment tax Position $\mathrm{S}, t_{s}^{\text {Prog }}$ was calculated using:

$$
\begin{equation*}
t_{S}^{\text {Prog }}=\frac{w_{S}^{2}}{w_{S}^{2}+w_{L}^{2}} \approx 0.3 \tag{6.26}
\end{equation*}
$$

i.e. Position $S$ 's endowment squared as a ratio of the sum of the squared endowments of the group, resulting in the most sensitivity to relative endowments.

Table 6.2: Experimental parameter setup

| Public <br> Good <br> setting | Endowment | Return | T1: <br> First-Best | T2: <br> Uniform | T3: <br> Proportional | T4: <br> Progressive |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $(100,150)$ | $(40,60)$ | $(0.4,0.6)$ | $(0.5,0.5)$ | $(0.4,0.6)$ | $(0.3,0.7)$ |
| 2 | $(100,150)$ | $(50,50)$ | $(0.5,0.5)$ | $(0.5,0.5)$ | $(0.4,0.6)$ | $(0.3,0.7)$ |
| 3 | $(100,150)$ | $(60,40)$ | $(0.6,0.4)$ | $(0.5,0.5)$ | $(0.4,0.6)$ | $(0.3,0.7)^{*}$ |
| 4 | $(100,150)$ | $(50,00)$ | $(1.0,0.0)$ | $(0.5,0.5)$ | $\left.(0.4,0.6)^{*}\right)$ | $(0.3,0.7)^{*}$ |

Notes: Endowments were fixed at 100 and 150 tokens which means the tax shares for Uniform, Proportional, and Progressive taxes do not vary across public goods. The returns do vary across Pubic Goods causing the first-best rate to vary and thus the size of the tax distortion.
*If the public good is provided at the sum of private values then the public good redistributes such that Individual S is better off than Individual L .

### 6.3.3 Estimation strategy

The predicted signs presented in Table 6.1 of the first derivatives of WTP values in front of a VoI and MAC values behind a VoI with respect to $t_{S}$ are tested using OLS regression. The data is generated from the 204 subjects who each valued each of the 4 Public Goods described in Table 6.2 for a randomly assigned tax structure. This generated a sample consisting of 816 observations for Position S, Position L and behind a VoI.

To test the effect of distortionary taxation on valuations in front and behind a VoI, four dependent variables are considered: WTP from Positions $\mathrm{S}\left(W T P_{S}\right)$ and $\mathrm{L}\left(W T P_{L}\right)$, the sum of the individual's WTP $\left(\Sigma W T P=W T P_{S}+W T P_{L}\right)$, and MAC given from behind a $\operatorname{VoI}\left(M A C_{v}\right)$. WTP values are calculated by multiplying the appropriate MAC by the individual's tax share. $W T P_{S}$ and $W T P_{L}$ are used to test for preferences in front of the VoI and $M A C_{v}$ is used to test preferences behind the VoI. $\sum W T P$ allows for additional comparisons of aggregate WTP and MAC behind a VoI.

In the analyses, variation in WTP and MAC values are explained by three independent variables: the private return to each group member ( $x_{S}$ and $x_{L}$ ) and the size of the tax distortion for Position S. The private returns capture differences in underlying values between Public Goods. Tax distortion is calculated as:

$$
\begin{equation*}
\Delta t_{S, i}=t_{S, i}-\frac{x_{S, i}}{x_{S, i}+x_{L, i}}, \tag{6.27}
\end{equation*}
$$

i.e. the difference between the tax share contributed by Position $S$ and the first-best tax share for Position S. Tax Distortion ( $\Delta t_{\text {small }, i}$ ) can take a positive or negative value dependent of whether Position $S$ is paying more or less than the first-best share. Tax distortion is used rather than tax share as it accounts for variation in the first-best tax share between the 4 Public Goods. The following models ${ }^{43}$ are estimated using OLS regression with standard errors clustered at the subject level for each of the dependent variables described above:

$$
\begin{equation*}
W T P_{S}=\beta_{0}+\beta_{1} x_{S, i}+\beta_{2} x_{L, i}+\beta_{3} \Delta t_{S, i}+\varepsilon_{i} . \tag{6.28}
\end{equation*}
$$

$$
\begin{equation*}
M A C_{V}=\beta_{0}+\beta_{1} x_{S, i}+\beta_{2} x_{L, i}+\beta_{3} \Delta t_{s, i}+\varepsilon_{i} . \tag{6.31}
\end{equation*}
$$

[^29]To test the predictions presented in Table 6.1, the coefficient on the $\Delta t_{s, i}$ term $\left(\beta_{3}\right)$ is tested for significance based on the following hypothesis:

$$
\begin{array}{ll}
H_{o}: & \beta_{3}=0, \\
H_{A}: & \beta_{3} \neq 0 .
\end{array}
$$

For the model with $W T P_{S}$ presented in Equation (6.28) as the dependent variable a rejection of the null with a positive coefficient for $\beta_{3}$ would indicate subjects are acting in accordance with PA predictions by offering a larger WTP to increase the probability the public good is provided. For the model with $W T P_{L}$ presented in Equation (6.29) as the dependent variable a rejection of the null with a negative coefficient for $\beta_{3}$ would indicate that experimental subjects are acting in accordance with PA predictions. For the model with MAC ( $C_{i}^{*}$ ) presented in Equation (6.31) the interpretation of the $\beta_{3}$ term depends on the preferences identified in front of a VoI. If preferences are PA, a non-rejection of the null hypothesis would suggest the VoI mechanism is working as expected to reduce altruism. A rejection of the null with a positive effect would indicate greater altruistic weighting placed on Position 1, whilst a negative effect would indicate greater altruistic weighting placed on Position 2.

For the model of $W T P_{S}$ presented in Equation (6.28) and $W T P_{L}$ presented in Equation (6.29), a rejection of the null with a negative coefficient for $\beta_{3}$ using data from treatments where the public good reduces inequality would indicate that experimental subjects are responding to tax distortion in accordance with IA predictions. If preferences are identified as IA in front of a VoI, a rejection of the null with a negative effect for the model with MAC $\left(C_{i}^{*}\right)$ presented in Equation (6.31) would indicate preferences are consistent in front and behind a VoI.

The model of $\sum W T P$ presented in Equation (6.30) will measure the net effect of $W T P_{S}$ and $W T P_{L}$ and therefore measure the net effect of distortionary taxation on the value of the public good. The coefficients for the $\sum W T P$ model will be the sum of the coefficients for Equations (6.7) and (6.8). The $\beta_{3}$ coefficient could take either a positive or negative sign depending on the relative sizes of the $\beta_{3}$ terms for Positions S and L .

### 6.4 Results

### 6.4.1 Sample and demographics

Mean subject age was 22.2 years and $45 \%$ of subjects were male respondents. $51 \%$ of subjects were home students, and $31 \%$ postgraduate students.

Figures 6.1-3 plot tax distortion against the difference between social WTP and the private WTP values for WTP from Positions $\mathrm{S}\left(W T P_{S}\right)$ and $\mathrm{L}\left(W T P_{L}\right)$, and MAC given from behind a VoI $\left(M A C_{v}\right)$ respectively. The line of best fit is upward sloping for $W T P_{S}$ in Figure 6.1, downward sloping for $W T P_{L}$ in Figure 6.2, and flat for $M A C_{v}$ in Figure 6.3, which based on the predictions presented in Table 6.1 each align with PA preferences.

Figure 6.1: Net WTP against tax distortion for Position $S$ with fractional-polynomial line of best fit with $95 \%$ confidence intervals


Figure 6.2: Net WTP against tax distortion for Position L with fractional-polynomial line of best fit with $95 \%$ confidence intervals


Figure 6.3: Net MAC against tax distortion for MAC value behind a VoI with fractionalpolynomial line of best fit with $95 \%$ confidence intervals


### 6.4.2 In front of a VoI

To test for PA and IA preferences in front of a VoI, the regression results for $W T P_{S}$ and $W T P_{L}$ are used. Tax distortion has a positive significant impact on mean WTP for $W T P_{S}$. This means that WTP increases when the difference between the actual tax share paid by S and first-best tax share for S increases. This positive $\left(\beta_{3}=12.4\right)$ and highly significant coefficient leads to the rejection of the null hypothesis $(p=0.01)$ for $W T P_{S}$. The positive coefficient follows the PA prediction. Likewise, the regression results for $W T P_{L}$ are considered. Tax distortion has a negative ( $\beta_{3}=-29.7$ ) significant impact on WTP. This means that the null hypothesis ( $p=$ 0.00 ) is also rejected for $W T P_{L}$. The negative coefficient could indicate either a PA or IA preference.

Rejecting the null hypotheses with a positive slope for $W T P_{S}$ and negative slope for $W T P_{L}$ suggests that subjects are responding to changes in distortionary taxation as the model predicts for a PA individual and the SI and IA models are rejected. The direction and significance of these results hold when the data is limited to public goods that do not cause complete redistribution and are therefore the rejection of the IA model is robust.

In this experiment, the distortionary tax effect is stronger when subjects are in the larger endowment position. This results in a significant $(p=0.03)$ net effect, measured by $\sum W T P$, having tax distortion effect of $\beta_{3}=-17.3$. Overall, the value of public goods with a distortionary tax that favours redistribution towards Position S i.e. $t_{s}<t_{s}{ }^{*}$ have significantly higher total values than at the first-best rate.

### 6.4.3 Behind a VoI

If respondents are PA , as the results in front of the VoI suggest, and risk neutral the model predicts that $\frac{\partial C_{i}^{*}}{\partial t_{1}}=0$. To test this prediction we use the regression results for $M A C_{V}$ and find an insignificant impact of tax distortion. This insignificant coefficient ( $\beta_{3}=10.7$ ) implies that the null hypothesis cannot be rejected ( $p=0.15$ ). This result suggests that behind a VoI valuations are not impacted significantly by distortionary taxation even though the results from in front of a VoI suggest that subjects were acting purely altruistically towards their partners. This result follows the prediction of the model for risk neutral PA individuals and suggests that by applying a VoI, distortion of valuations through taxation and altruism has been reduced to the point of insignificance.

Table 6.3: OLS regression results for WTP from Positions $S$ (Equation 6.7) and $L$ (Equation 6.8), Aggregate WTP (Equation 6.9) and MAC behind a VoI (Equation 6.10).

|  | $W^{\text {W }} \mathrm{P}_{S}$ | $\boldsymbol{W T P} \boldsymbol{P}_{L}$ | $\sum W T P$ | $M A C_{V}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Coeff. (s.e.) | Coeff. (s.e.) | Coeff. (s.e.) | Coeff. (s.e.) |
| $x_{S}$ | 0.440*** | -0.009 | $0.431^{* * *}$ | 0.501*** |
|  | (0.070) | (0.095) | $(0.120)$ | (0.137) |
| $x_{L}$ | -0.046 | 0.819*** | $0.773^{* * *}$ | $0.727^{* * *}$ |
|  | (0.046) | (0.038) | (0.060) | (0.058) |
| $\Delta t_{s}$ | 12.406** | -29.729*** | -17.324** | 10.656 |
|  | (4.869) | (5.325) | (7.867) | (7.349) |
| $\beta_{0}$ | 14.578*** | -0.601 | 13.977* | 13.620* |
|  | (4.764) | (5.214) | (7.386) | (8.213) |
| $R^{2}$ | 0.035 | 0.257 | 0.142 | 0.189 |
| n | 816 | 816 | 816 | 816 |

Notes: OLS regressions with standard errors clustered by subject. Significance is denoted by *, **, and *** for the $90 \%, 95 \%$, and $99 \%$ confidence levels respectively. The independent variables are $x_{S}$ (return to Position S), $x_{L}$ (return to Position L ), $\Delta t_{s}$ (the measure of tax distortion expressed in Equation 16), and $\beta_{0}$, the constant term.

### 6.5 Summary

The purpose of this chapter and the experiment it describes was to test the impact of taxation on valuations both in front and behind a VoI. Predictions of behaviour were generated from the models presented in Chapters 3 and 4. The expectation in front a VoI was that if respondents were purely altruistic and correctly took into account the cost imposed on others, his/her WTP would be increasing with the share of contribution they would make if the public good was funded. The expectation behind a VoI was that if respondents were consistently purely altruistic and risk neutral that MAC values would be insensitive to the share of the contributions assigned to each position. If respondents were inequality averse then the value of the public good would
increase when inequality decreases and decrease when inequality increases. This would hold true both in front and behind a VoI.

The results for choice in front of a VoI indicates that own tax share has a positive effect on WTP for both the large and small endowment Positions as predicted for PA preferences. This is shown by the positive (negative) and significant coefficient on tax distortion for Position S (L). This result suggests that for the range of tax systems valued in the experiment that as the tax share for Position S pays increases relative to the first-best rate, respondents in Position S will increase his/her WTP and respondents in Position L will decrease his/her WTP. The altruistic motivation for Position S is to increase his/her WTP as their tax burden increases to increase the probability of the public good going ahead. The altruistic motivation for Position L is to decrease the probability the public good is provided as Position S is required to take a larger proportion of the cost.

The tax effect is greater for Position L than Position S which means that aggregate WTP has a negative tax effect suggesting that for the range of tax distortion studied the public good takes on a higher value when the tax system favours the small endowment Position. Based on the model presented in Chapter 3, this effect is unlikely to be consistent for the full range of tax shares as the tax effect is decreasing as tax share increases for both individuals giving aggregate WTP an inverted-U shape as depicted in Figure 3.15.

Behind a VoI, tax distortion doesn't have a significant effect on MAC responses behind a VoI. This suggests that a VoI could be used to reduce the distortionary effect of taxation on valuations.

## CHAPTER SEVEN:

## Discussion

### 7.1 Introduction

This chapter restates the problem posed in this thesis and discusses the contributions in relation to the existing literature. Public goods can be provided to improve society if they are provided at the correct level and funded appropriately through taxation. This thesis is concerned with the cost-benefit analysis of proposed changes to provision with the aim of identifying proposals which would improve society. Three key elements of the decision-making process are studied:
[1] The impact of different preference sets on the societal value.
[2] The impact of different preference sets on individual values.
[3] The use of these values in CBA. Two questions are asked. First, does the method of elicitation respect the individual's sovereignty by respecting their preferences? Second, does the aggregation of these preferences pass a compensation test?

As public decision-making should operate as a part of the democratic process there is a need to understand individuals' values for public goods. It is not possible to satisfy every individual's preferences for public good provision as they have heterogeneous preferences. Instead the ideal is social welfare maximisation. There is not a single measurement of social welfare as tradeoffs are often required between individuals. This requires a value judgement, meaning different individuals could reasonably come to different conclusions. Welfare economics sets out propositions for the measurement of social welfare designed to be such weak value judgements that everyone would agree to them. One proposition states that only the values of the individuals
who will be affected matter when considering the welfare of society. Another proposition states that each individual is the best judge of their own welfare. To achieve the best for society, a public decision-maker needs to elicit the values of those affected. It is therefore important that these values truly and accurate reflect the preferences of the individual.

It is recognised in Dolan et al. (2003) and Tsuchiya \& Watson (2017) that individuals in society can take a number of perspectives when valuing a proposal. The perspective an individual will take may impact their valuation of a proposal as the individual acts on different preferences e.g. preferences for self, preferences over others, and preferences over distributive justice. To elicit different preferences requires different elicitation procedure which in this thesis is termed a frame. Three frames are discussed here:

Consumer Frame 1: Respondents act in accordance with their preferences over their own outcome.

Consumer Frame 2: Respondents act in accordance with their preferences over their own outcome and the outcomes of others.

Citizen Frame: Respondents act in accordance with their preferences for their own outcome and the outcomes of others. Respondents are placed behind a VoI and thus are informed about the distribution of income and public good endowments, but not their own position within the distribution.

Once the values of individuals have been elicited, the problem of aggregating these values to achieve an improvement in society still remains. There is therefore a need to understand the process of aggregation of preferences and how that impacts the outcome. This thesis focusses on CBA as a method of aggregating WTP values to evaluate proposals. However there remains choices such as whether and how much compensation should be awarded to individuals made worse-off by a proposal. They are questions of distributive justice which as mentioned require value judgements to answer.

This chapter first summarises the results of Parts I and II of this thesis in Sections 7.2 and 7.3. The following three Sections 7.4-6 then discuss the three elements of the decision-making process identified at the beginning of this thesis and re-expressed at the beginning of this chapter.

### 7.2 Part I Summary

Part I models the three frames described in Chapter 2: Consumer Frame 1, Consumer Frame 2, and the Citizen Frame. Two models presented: a model of optimal provision and a model of cost-benefit analysis. The main findings are tabulated in Table 7.1.

### 7.2.1 Optimal Provision

The model of optimal provision was based on the model of optimal safety expenditure presented in Jones-Lee (1991) from which VSL is derived. Safety expenditure is chosen as the public good to be modelled as this is the good the literature often focusses on, however the results hold for a broad set of public goods. Optimal provision is modelled under different tax systems because welfare is based on both benefits and costs of provision. If altruists care for others costs as well as benefits, then they will likely have preferences over taxation. The solution to the model of optimal provision identifies the condition for social welfare maximisation. This allows for a comparison of the impact of different preference sets on the societal value placed on public good provision. Simulations were also presented to support the findings of the model. The results from the simulations indicate the size and direction of the impact of different forms of altruism.

Chapter 3 first presents model for a general utility function for three systems of taxation: firstbest, uniform and proportional wealth taxation. Solutions for optimal provision are found by maximising a social welfare function constrained by the system of taxation. The result shows the condition of marginal benefit of provision equalling the marginal cost of provision, the Samuelson Condition (1954). These solutions are simplified based on the assumptions of the SI, PA, SFA, and WFA functional forms.

The results for the first-best case replicate the results of Bergstrom (1982), and Jones-Lee (1991, 1992). Under a first-best tax, optimal provision is equal for SI and PA preferences as Bergstrom (1982) shows. Under PA preferences, the altruist respects the preferences of the recipient of altruism. Therefore, under a first-best tax by satisfying the preferences of the recipient of altruism, the preferences of the altruist are also satisfied. Simulations confirm this result but also show that the optimal tax share for two individuals differs for PA and SI preferences. In that case taxation favours the recipient of altruism.

The tax extension presented in Chapter 3 shows that when taxation is no longer first-best, optimal provision is no longer equal for SI and PA preferences. Under alternate systems of taxation, such as the uniform and proportional wealth cases shown, tax contributions cannot be set independently. This means each individual's welfare cannot be maximised by optimising their contribution based on their wealth and preferences. Because the utility of the recipient of altruism cannot be maximised by optimising their individual tax contribution, the PA individual's preferences for their welfare cannot be satisfied in the same manner as in the firstbest case. This leads to deviations from the SI result as greater weight is placed on the outcome of the recipient of altruism.

Optimal provision under paternalistic preferences are shown to either increase or decrease depending on the focus of altruism. Jones-Lee (1991) showed that optimal provision is maximised when altruism is safety-focused. That is the altruist cares only for others' safety. Jones-Lee (1992) showed that optimal provision is maximised when altruism is wealth-focused. That is the altruist cares only for others' wealth. Simulations show that the first-best tax share is equal to the SI result for SFA preferences, but not for WFA preferences. The deviations from SI result are larger for WFA than PA preferences. This suggests that as greater focus is given to the wealth of others relative to their safety, the greater the impact on the first-best optimal tax share.

Jones-Lee (1992) considers altruism as a spectrum with SFA and WFA as the extreme cases and SI as the common case for which the intensity of altruism is zero. As altruism moves from SFA to WFA the altruist places greater relative weight on wealth relative to safety. This results in a smooth decrease in optimal provision. The results presented in Chapter 3 suggest that this result would hold true for all forms of taxation. The tax effect shown by simulations may also hold resulting in increased variation in the optimal tax share as altruism becomes increasingly wealth focussed. A further middle case is also identified by Jones-Lee (1992), pure paternalism in which an altruist projects their preferences for own safety and own wealth on another. An extension to the model would be required to identify the impacts of this form of altruism.

Chapter 4 presents the VoI extension. Each individual is placed behind a VoI and therefore has an equal probability of ending in each position once the VoI has been lifted. This represents Harsanyi (1953, 1955)'s conception of the VoI. The condition for optimal provision considers each individual's preferences for each position in society. Behind a VoI, the effect of paternalistic altruism remain. Optimal provision is still highest for SFA and lowest for WFA. The result for PA preferences does not necessarily equal the SI result.

To consider these results further, altruism is considered in a different manner. Two motivations for altruism are considered: person-based altruism and distributional altruism. Results show that if preferences are person-based then the altruistic concern is spread across all positions and thereby largely reduced. Simulations show that under PA preferences the result reduces to the SI result for optimal provision and taxation. For all forms of altruism the first-best tax share aligns with the SI result. Under distributional preferences, this is not the case. The effect of altruism then depends of the position altruism is directed towards.

Overall this suggests that by adding a VoI, if altruism is person-based then only paternalistic altruism remains; either increasing or decreasing optimal provision. If altruism is based on concerns for distributive justice then these preferences also remain. It is possible that an individual has both person-based and distributional preferences. This could be considered further by extending the model of Jones-Lee (1992) which includes preferences over family and others. Assuming a family unit remains behind a VoI, person-based and distributional altruism could be separated to consider the competing effects.

Table 7.1: Summary of Part I results: the value of a public good relative to the SI level.

| Preference | VoI | Individual |
| :---: | :---: | :--- |
| PA | No | $\begin{array}{l}\text { Same if tax is first-best, otherwise it could } \\ \text { be higher or lower depending on the tax } \\ \text { system. }\end{array}$ | \(\left.\begin{array}{l}Same if tax is first-best, otherwise it could <br>

be higher or lower depending on the tax <br>
system and the identity of the altruist.\end{array}\right]\).

Notes: The individual value is based on the CBA models and the societal level is based on the optimal provision model. Aggregating the individual values would also come to similar conclusions as the optimal provision model.

### 7.2.2 Cost-benefit Analysis

Chapter 3 first presents a model of WTP based on the Naïve cost-benefit analyst as shown by Bergstrom (2006) to highlight the concerns over the efficiency of CBA with altruistic preferences of Milgrom (1993). The Naïve cost-benefit analyst 'forgets' to take into account that each individual contributes to the provision of the public good and therefore inflates WTP values. Bergstrom highlights the need to take into account the costs others in altruistic valuations. The findings of the optimal provision model suggest that the cost borne by others does impact the societal value of a public good so it also should effect the individual values.

The model presented in Chapter 3 considers WTP under the full range of cost sharing rules. The results show that paternalistic altruism affects individual values as the optimal provision model suggests. WTP is largest for SFA and lowest for WFA. Under a first-best tax share, WTP under PA preferences is equal to the private value, the SI result. WTP for SI and SFA preferences is shown to be independent of taxation, whilst WFA and PA are not. For both, there is a positive relationship between tax share and WTP. That is, as an individual is required to contribute a greater proportion of the costs they are willing to contribute more. By doing so they increase the chance a proposal that is beneficial to others is undertaken. The opposite must hold true as one individual is required to contribute more, another must be required to pay less. This results in decreased WTP for the individual required to pay less, if that individual is also an altruist, causing an opposing effect. These opposing effects do not always cancel causing variation in aggregate WTP depending on the assumed tax vehicle.

An alternate model of CBA is presented in Chapter 3 based on the compensation test. This model finds the largest cost that could be paid for a proposal such that there is just enough compensation for the 'winners' to compensate the 'losers'. This model again shows that the highest cost payable occurs when preferences are SFA and lowest when preferences are WFA. The key result of this model is that the largest compensable amount is independent of taxation. This is because if compensation is paid then the initial tax vehicle becomes irrelevant. This result does not align with the WTP model which suggests that WTP under certain preferences with certain tax vehicles may not be compatible with CBA.

Chapter 4 explores an alternate method for eliciting value by placing respondents behind a VoI. As with the optimal provision model, each individual has an equal probability of ending in each position once the VoI has been lifted. To do so a different measure of value must be considered, the MAC value. That is the maximum cost to society under a set cost-sharing rule the individual
is willing to incur. Based on the cost share to each position, a WTP value can be calculated for each position by multiplying the MAC value by the cost share. The sum of these WTP values is the MAC value. The results show that under the assumption of risk neutrality, the MAC value is largest under SFA and lowest under WFA. If altruism is person-based then the MAC value is independent of the tax vehicle. However, if the respondent has preferences over distributive justice then the distributional preferences impact the value. This reflects the results seen in the optimal provision model.

As eliciting preferences behind a VoI allows for respondents to consider a full proposal (distributions of costs and benefits), preferences over altruism and distributive justice can be incorporated fully into individual values. These values correctly include preferences over costs and benefits and therefore may be more appropriate than the values collected in front of a VoI which are shown to be inefficient due to not correctly incorporating preferences over costs and benefits. This requires further study by eliciting preferences in front and behind a VoI. This was done in Part II.

### 7.3 Part 2 Summary

Part II of this thesis tested empirically the respondent frames set out in Part I using laboratory experiments. The experiment designs extend those of Messer et al. (2010) and Messer et al. (2013) to allow for a VoI and alternate cost-sharing rules. The main findings are presented in Table 7.2

Chapter 5 replicated the result of Messer et al. (2013) that individuals trade-off their own best interest and the best interest of the group. This leads to individual's altruistic responses moving towards the group best response from their own private value. The robustness of these results to distribution and income variation were tested. This result was found to be robust to the distribution of the benefit of the public good. The robustness to distribution suggested that preferences were not distributional and that the VoI would operate as expected to reduce the impact of altruism on values.

Choice was found to be consistent both in front and behind a VoI. As the model predicted, if individuals act with PA, then his/her WTP value will be indifferent from the SI result behind a VoI. This was suggested by the results.

Table 7.2: Summary of Part II experiment results.

| Experiment | Test | VoI? | Hypothesis | Result | Conclusion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | T-test of mean differences between social and private WTP. | No | There will be a difference between social and private values if individuals are altruistic. | Mean WTP varies between private and social values. | PA best characterises altruistic behaviour. |
|  |  | Yes | There will be no difference between social and private values for PA but will for MM. | Mean WTP varies between private and social values. | PA best characterises altruistic behaviour. |
| B | OLS <br> regression <br> with a t-test <br> on the tax <br> share <br> coefficient. | No | There will be a positive relationship between WTP and tax share. | A significant positive relationship is found. | Tax affects WTP as Chapter 3 suggests. |
|  |  | Yes | There will no relationship between MAC and tax share. | No significant relationship is found. | Tax doesn't affects WTP as Chapter 4 suggests. |

Chapter 6 tested the WTP tax hypotheses presented in Chapters 3 and 4. That is, in front of a VoI, under PA preferences there is a positive relationship between an individual's tax share and WTP. However behind a VoI, MAC values are independent of tax shares if PA is not distributional. These were presented as two testable hypotheses: Hypothesis 1 and Hypothesis 2 respectively. To do so a further extension was required to Messer et al. (2010) and the experiment presented in Chapter 5 which allowed for variation in the size of the contribution each group member made. The findings rejected Hypothesis 1 and could not reject Hypothesis 2. WTP is responsive to tax in the suggested direction but not behind the VoI.

The results of Experiment A also support these findings. As the model in Chapter 3 shows, depicted in Figure 3.10, WTP for PA individuals will not differ from their private value if they contribute the first-best tax share in front of a VoI. The result of the experiment from Chapter 5 showed no difference between WTP for private WTP values and social WTP values for the Mid EL position for which the tax share is the first-best level by design. The model in Chapter 3 also shows that individuals with tax shares greater (less) than the first-best rate are expected to have WTP values greater (less) than their private values (see Figure 3.10). This prediction is also reflected in the results from Chapter 5 which showed individuals in the Low (High) EL increase (decrease) their WTP compared to their private value.

### 7.4 Societal value of the provision of public goods

Under Consumer Frame 1 respondents act only on their preferences for their own welfare. In the models presented in this thesis, that is acting with SI preferences. The model based on JonesLee (1991) presented in Section 3.2 shows that when a Utilitarian SWF is maximised under the assumption of first-best taxation optimal provision occurs when the mean MRS of wealth for public good consumption equals the ratio of prices. In the example of public safety expenditure that is the mean MRS of wealth for survival probability equals the marginal cost of saving one statistical life. This is the standard optimal public good provision result of the Samuelson Condition (Samuelson, 1954). The tax extension shows that the choice over system of taxation impacts the condition for optimality in Consumer Frame 1.

Under Consumer Frame 2 respondents act on their preferences for their own welfare and the welfare of others. This altruism has the potential to manifest in many ways. The models presented in this thesis, suggest that the impact of altruism is dependent on the form altruism takes. There is also an interaction between the form of altruism and the system of taxation in place. This is due to certain types of altruism which result in altruists having preferences over both the benefits of public good provision and the costs.

Under the Citizen Frame respondents act on their preferences for their own welfare and the welfare of others, however they are made unaware of their own position in society. This level of uncertainty generated by a VoI requires respondents to evaluate a proposal in its entirety as the respondent is potentially in any one of the positions which make up society. Under the Citizen Frame each individual's preferences for each positions are taken into account when identifying optimal provision. Behind a VoI it is shown that altruism directed towards individuals has a reduced impact on provision. Instead concerns over the underlying value of the good are represented through paternalistic preferences, whilst concerns over fairness are represented by distributional preferences.

### 7.5 Individual values for the provision of public goods

Under Consumer Frame 1, individual values are equal to private values alone. This potentially violates the principle of consumer sovereignty as it does not respect the preferences of respondents. The case when Consumer Frame 1 is appropriate is when respondents are SI,
however based on the findings from Experiment 1 and 2, as well as evidence from surveys, this is not the case in reality.

In comparison, Consumer Frame 2 allows respondents to act on preferences beyond their selfinterest. The WTP model in Chapter 3 shows that depending on the type of altruism the value of the public good could increase or decrease. The ORP models presented in Chapter 5 and 6 also shows that values can increase and decrease. This is seen in the literature with some studies finding concern for others to result in individual valuations either greater than (Viscusi et al. 1988, Arana \& Leon, 2002) or less than (Johannesson et al., 1996; de Blaeij et al., 2003; Hultkrantz et al. 2006; Andersson \& Lindberg, 2009; Svensson \& Johansson, 2010; GyrdHansen et al., 2016) private valuations.

The model presented in Chapter 3 shows that not only the type of altruism impacts individual values but also the assumed payment vehicle. When cost sharing rules are first-best, WTP is equal for PA and SI preferences. The model in Chapter 6 of IA shows that this result holds true as inequality remains unaffected. When taxation that is not first-best is considered, WTP under both PA and WFA preferences varies. The results presented in Chapter 3 show that both the WTP solution for PA and WFA is a function of tax share. The model in Chapter 6 shows that values under IA preferences vary with taxation with higher values of the public good associated with reduced inequality.

The findings of the PA model are supported by the results of Experiment A which shows that under a uniform tax, individuals will change their WTP to be closer to the group optimum. Similar findings are reported in Johannesson et al. (1996) and Gyrd-Hansen et al. (2016) which compare payment vehicles. It is difficult in these studies to separate the impact of taxation and paternalistic altruism. Experiment B helps to support the findings of these papers by showing a positive relationship between WTP and tax share. The benefit of using a laboratory experiment is that induced values are under the control of the experimenter to clearly identify effects. Another benefit is that the laboratory experiment only uses one good, money. This removes the effects of paternalistic altruism to leave only the tax effect.

When the Citizen Frame is applied to individual valuations, the results of the model presented in Chapter 4 suggest that MAC values can take into account both altruism and distributive concerns including ORPs. Experiment A, presented in Chapter 5, tests for differences between values for SI and altruistic preferences. The results find that in front of a VoI individuals are acting altruistically. Behind a VoI, no altruism is detected suggesting that preferences were
person-based rather than distributional or ORP. Experiment B, presented in Chapter 6, tests for the relationship between MAC values and tax distortion. The results find no relationship, again suggesting that preferences are best categorised as person-based altruism. Both suggest the VoI is successfully reducing the impact of pure altruism which is shown to lead to inefficient decision-making in Chapter 3

The results of these experiments suggest that respondents are acting as expected utility maximisers rather than based on a MM rule such as the one proposed by Rawls (1972). As individuals are acting as expected utility maximisers, the behaviour of individuals is better represented by the models of Harsanyi $(1953,1955)$ than Rawls (1972) and that the modelling choices for behaviour behind a VoI presented in Chapter 4 are reasonable. Overall, these findings suggest that by introducing a VoI, the impact of altruism will be limited to paternalistic preferences, distributional preferences, and ORPs alone. Eliciting values behind a VoI provides an opportunity to control in part for the distortion of altruism.

### 7.6 Incorporating values into CBA

The compensation test as described by Kaldor (1939) and Hicks (1939) suggests that if the sum of the WTP values is greater than the sum of the costs, then a proposal achieves a potential Pareto improvement. Under Consumer Frame 1, WTP is equal to the private benefit regardless of the tax vehicle in place. The sum of WTP values is therefore the sum of the private benefits. As long as the cost is less than the sum of the private benefits then a Pareto improvement could be achieved.

The problem with values elicited under Consumer Frame 1 is that it potentially violates the principle of consumer sovereignty as it does not respect the preferences of respondents beyond those for one's own welfare. This is reflected in Little's (1949) criticism of CBA that decisions are purely efficiency based and highlights this point. Little suggested that an additional criterion must be passed that the new distribution had to be 'better' than the previous. Defining 'better' requires a value judgement and becomes problematic as it requires a value judgement. These judgements should be made by individuals in society as the welfare propositions suggest. It is therefore important that values take into account other preferences.

Consumer Frame 2 allows for altruistic preferences to be included in valuations regardless of the form they take. However the resulting WTP values do not necessarily reach efficient outcomes when used in a cost-benefit calculation. The problem is caused by the choice of payment vehicle which impacts valuations when respondents care about others private consumption.

One option is to use first-best taxation as payment vehicle which assumes each individual pays up to their private value and thus the decision of one individual has no impact on others. The problem is that decisions based on values collected under the assumption of first-best tax do not necessarily represent the welfare changes that would occur if a different tax system is used to fund provision.

Another option is to use the actual payment vehicle, however the cost side of the calculation becomes included in the measure of benefits, WTP. This allows values to include preferences over the net benefits to individuals, however they are not necessarily useful for CBA. Consumer Frame 1 and 2 could be used in conjunction to evaluate the welfare implications of a proposal with full understanding of the implications of using them. This problem could be solved in part by eliciting MAC values in front of a VoI.

This concept, introduced in Chapter 4, requires eliciting the maximum increase in societal spending based on a distribution of costs defined by the tax system to provide the provision of the public good, the MAC value. As described in the theory section, WTP and MAC values are inextricably linked, but MAC values could provide a better framework for the elicitation of values for a complete proposal.

The novel Citizen Frame has a potential use in survey design in conjunction with Consumer Frames in order to elicit a series of values. When designing a stated preference survey to elicit values from the public a VoI could be incorporated to mitigate the problems observed with Consumer Frame 2 and, if successful, might indicate that once behind a VoI choice of a particular tax system is less important than in front of a VoI. By collecting citizen preferences, proposals can be evaluated using individuals' impersonal preferences as well as the personal preferences elicited in the Consumer Frames. These values can be used with values elicited in Consumer Frames to answer normative questions like those posed by Little (1949) to avoid relying on the value judgments of public decision-makers where possible.

## CHAPTER EIGHT:

## Conclusion

This thesis studied the impact of different preference sets and methodologies for eliciting the values of individual in society for public goods. Three key elements of the decision-making process were studied:
[1] The impact of different preference sets on the societal value.
[2] The impact of different preference sets on individual values.
[3] The use of these values in CBA. Two questions are asked. First, does the method of elicitation respect the individual's sovereignty by respecting their preferences? Second, does the aggregation of these preferences pass a compensation test?

This was achieved through a combination of theoretical work and empirical study. Part I presented the theoretical work of this thesis. Two types of model were presented: optimal provision and CBA which considered individual values and the process of CBA. Part II presented the empirical work of this thesis. Two experiments are presented which elicit individual values under different scenarios.

Three specific methodologies were studied. Consumer Frame 1 which requires SI preferences. Consumer Frame 1 represents the standard methodology and is shown to be consistent with CBA rules but violate consumer sovereignty. Consumer Frame 2 allows for altruistic preferences. Consumer Frame 2 is shown to respect consumer sovereignty, but is shown to be inefficient when included in a cost-benefit calculations due to the cost and benefit sides of the
calculation no longer being independent. The Citizen Frame is presented as a novel mechanism for eliciting societal values. This frame places individuals behind a VoI to generate impersonal preferences from personal preferences thereby creating an impartial stakeholder. The Citizen frame is shown to respect individual values by allowing paternal altruism and concerns over distributive justice into valuations whilst reducing the impact of other forms of altruism, notably PA. The results of the experiments support these findings suggesting that under Consumer Frame 2, individual values are impacted by the tax vehicle in place whilst behind a VoI they are not.

Based on these results further research is warranted into the Citizen Frame as well as further understanding into the effects of altruism on the Consumer Frames. The next step would be to consider the impact of the frames by eliciting values through a survey designed to value realworld public goods such as safety, health and the environment. By doing so, the impact of paternalistic altruism and the interactions with tax vehicle can be studied in greater detail.

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## Appendices

## Appendix A: Solutions for the first-best case of the optimal provision model presented in

 Chapter 3.Start with Equation 3.5, 3.7, and 3.8.

$$
\begin{align*}
& \frac{1}{n} \sum_{i}^{n} \frac{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\lambda}=-\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} .  \tag{3.5}\\
& a_{j}=-\left(\lambda+\sum_{k \neq j}^{n} a_{k} \frac{\partial u_{k}}{\partial w_{i}}\right) \frac{\partial w_{i}}{\partial u_{j}}, \text { and }  \tag{3.7}\\
& \lambda=-\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}, \tag{3.8}
\end{align*}
$$

Substitute 3.7 into 3.5 gives

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \sum_{j}^{n}\left[\frac{\left(\lambda+\sum_{k \neq j}^{n} a_{k} \frac{\partial u_{k}}{\partial w_{i}}\right)}{\lambda} \frac{\partial w_{i}}{\partial u_{j}} \frac{\partial u_{j}}{\partial \pi_{i}}\right]=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} . \tag{A1}
\end{equation*}
$$

3.8 into A1 gives

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \sum_{j}^{n}\left(\frac{-\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}+\sum_{k \neq j}^{n} a_{k} \frac{\partial u_{k}}{\partial w_{i}}}{-\sum_{j}^{n} a_{j} \frac{\partial w_{j}}{\partial w_{i}}} \frac{\partial u_{j}}{\partial u_{j}} \frac{\partial s}{\partial \pi_{i}}\right)=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} \tag{A2}
\end{equation*}
$$

Which simplifies to

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \sum_{j}^{n}\left(\frac{a_{j} \frac{\partial u_{j}}{\partial w_{i}}}{\sum_{j}^{n} a_{j}^{\frac{\partial u_{j}}{\partial w_{i}}}} \frac{\partial w_{i}}{\partial u_{j}} \frac{\partial u_{j}}{\partial \pi_{i}}\right)=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)^{\prime}} . \tag{A3}
\end{equation*}
$$

For the general solution, the $\frac{\partial u_{j}}{\partial w_{i}}$ and $\frac{\partial w_{i}}{\partial u_{j}}$ terms cancel giving

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \sum_{j}^{n}\left(\frac{a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}\right)=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} \tag{A4}
\end{equation*}
$$

As the denominator of the brackets is the sum across $j$, it can be brought in front of the summation across $j$ of the brackets giving

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \frac{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial \pi_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} . \tag{A5}
\end{equation*}
$$

The solutions for SI, SFA, and WFA follow by restricting the altruistic marginal utilities to zero as is described for each case.

For the PA solution, using the assumption that $\frac{\partial w_{i}}{\partial u_{j}} \frac{\partial u_{j}}{\partial \pi_{i}}=\frac{\partial w_{i}}{\partial u_{i}} \frac{\partial u_{i}}{\partial \pi_{i}}$, A3 becomes

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \sum_{j}^{n}\left(\frac{a_{j} \frac{\partial u_{j}}{\partial w_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}} \frac{\partial w_{i}}{\partial u_{i}} \frac{\partial u_{i}}{\partial \pi_{i}}\right)=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} . \tag{A6}
\end{equation*}
$$

As $\frac{\partial w_{i}}{\partial u_{i}} \frac{\partial u_{i}}{\partial \pi_{i}}$ is independent of $j$ it can be moved in front of the summation over $j$ giving
(A7) $\quad \frac{1}{n} \sum_{i}^{n} \frac{\partial w_{i}}{\partial u_{i}} \frac{\partial u_{i}}{\partial \pi_{i}} \sum_{j}^{n}\left(\frac{a_{j} \frac{\partial u_{j}}{\partial w_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}\right)=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)}$.
Again as the denominator of the brackets is the sum across $j$, it can be brought in front of the summation across $j$ of the brackets giving

$$
\begin{equation*}
\frac{1}{n} \sum_{i}^{n} \frac{\frac{\partial w_{i} \partial u_{i}}{\partial u_{i} \partial \pi_{i}} \sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}{\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}}=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)} . \tag{A8}
\end{equation*}
$$

The $\sum_{j}^{n} a_{j} \frac{\partial u_{j}}{\partial w_{i}}$ terms cancel leaving
(A9) $\frac{1}{n} \sum_{i}^{n} \frac{\partial w_{i}}{\partial u_{i}} \frac{\partial u_{i}}{\partial \pi_{i}}=\frac{\partial s}{\partial\left(\sum_{i}^{n} \pi_{i}\right)}$.

## Appendix B: The interaction between taxation and altruism on WTP for the middle cases of altruism.

Figure B1: WTP for all cases of pure and paternalistic altruism


Notes: The 3 planes each show WTP varying across $\alpha$ and $\beta$ for 3 different tax shares. The top plane shows $\mathrm{t}=0.2$, the middle $\mathrm{t}=0.5$ which is the first-best rate, and the bottom plane $\mathrm{t}=0.2$.

There are many cases between SFA and WFA which set the upper and lower bounds for WTP under altruistic preferences. These limits and each of the middle cases are dependent on the relative magnitudes of the weights placed on others public and private good consumption represented by $\alpha$ and $\beta$ respectively. Figure B 1 shows WTP for the same example as depicted in Figure 3.10 over the full range of altruistic preferences for three different levels of taxation: $t=1 / 3, t=2 / 3=t^{*}$, and $t=1$.

WTP is strictly increasing along the $\alpha$ axis as preferences for other's public good consumption increases and strictly decreasing along the $\beta$ axis as preferences for other's private good consumption increases. That is as a respondent becomes more altruistic the impact on their WTP increases. Along the middle 45 degree line on which $\alpha=\beta$, preferences are characterised as PA. Along this line WTP: increases for the top plane for which $t=1$ and therefore $t>t^{*}$; decreases for the bottom plane for which $t=1 / 3$ and therefore $t<t^{*}$; and, remains flat at the level of the private benefit for the middle plane for which $t=2 / 3$ and therefore $t=t^{*}$, representing the first-best case. Notice then that WTP varies with taxation for all forms of altruism except those represented along the axis where $\beta=0$ and $\alpha \geq 0$. That is when preferences take the form of SFA for which SI is a special corner solution where $\beta=0$ and $\alpha=0$.

## Appendix C: Solutions for the first-best case of the optimal provision model presented in

## Chapter 4.

Start with Equation 4.5, 4.7, and 4.8.

$$
\begin{align*}
& \frac{1}{n} \sum_{p}^{n} \sum_{q}^{n}\left[\frac{a_{q}}{-n \lambda} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}\right]=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} .  \tag{4.5}\\
& a_{q}=-\left(n \lambda+\sum_{r \neq q}^{n} a_{r} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)\left(\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)^{-1}, \text { and }  \tag{4.7}\\
& n \lambda=-\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}, \tag{4.8}
\end{align*}
$$

Substituting 4.7 into 4.5 gives

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n}\left[\frac{\left(n \lambda+\sum_{r \neq q}^{n} a_{r} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)}{n \lambda}\left(\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)^{-1} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}\right]=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{C1}
\end{equation*}
$$

4.8 into Cl gives

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n}\left[\frac{\left(-\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\frac{w_{p}}{p}}+\sum_{r \neq q}^{n} a_{r} \sum_{i}^{n} \frac{\partial \partial_{q}^{i}}{\partial w_{p}}\right)}{-\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial \partial_{q}^{i}}{\partial w_{p}}}\left(\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)^{-1} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}\right]=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{C2}
\end{equation*}
$$

Simplifying gives

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n}\left[\frac{a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}}{\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}}\left(\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)^{-1} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}\right]=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{C3}
\end{equation*}
$$

Cancelling the $\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}$ and $\left(\sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}\right)^{-1}$ terms gives

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \sum_{q}^{n}\left[\frac{a_{q} \sum_{i}^{\frac{\partial u_{q}^{i}}{\partial \pi_{p}}}}{\sum_{q}^{n} a_{q} \sum_{i}^{\sum_{i}} \frac{\partial u_{q}^{i}}{\partial w_{p}}}\right]=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{C4}
\end{equation*}
$$

As the denominator of the brackets is the sum across $q$, it can be brought in front of the summation across $q$ of the brackets giving

$$
\begin{equation*}
\frac{1}{n} \sum_{p}^{n} \frac{\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial \pi_{p}}}{\sum_{q}^{n} a_{q} \sum_{i}^{n} \frac{\partial u_{q}^{i}}{\partial w_{p}}}=\frac{\partial s}{\partial\left(\sum_{p}^{n} \pi_{p}\right)} . \tag{C5}
\end{equation*}
$$

## Appendix D: Experiment A materials

Figure D1: Experiment A: Example decision screen for respondents in front of a VoI

| Question 1 |
| :---: |
| In this question, the members of your group have: |
| the same starting tokens, |
| different loss amounts, |
| and the same probability of loss. |
| Click OK to see the information in the table. |


|  | Starting Tokens | Loss | Probability (\%) |
| :---: | :---: | :---: | :---: |
| Person W | 600 | 240 | $75 \%$ |
| Person X | 600 | 320 | $75 \%$ |
| Person Y | 600 | 320 | $75 \%$ |
| Person Z | 600 | 480 | $75 \%$ |

You are Person X
Step 1:
You need to decide how much you think the insurance is worth to you.
Refer to pages 11 and 12 of your booklet when making your decision.
What is the most you would personally be willing to pay in tokens for the insurance? (1Token =
2c)

Figure D2: Experiment A: Example decision screen for respondents in behind a VoI

| Question 1 |
| :---: |
| In this question, the members of your group have: |
| the same starting tokens, |
| different loss amounts, |
| and the same probability of loss. |
| Click OK to see the information in the table. |


|  | Starting Tokens | Loss | Probability (\%) |
| :---: | :---: | :---: | :---: |
| Person M | 600 | 240 | $75 \%$ |
| Person N | 600 | 320 | $75 \%$ |
| Person O | 600 | 320 | $75 \%$ |
| Person P | 600 | 480 | $75 \%$ |

You will draw a letter after you have made an offer for insurance.

## Step 1:

You need to decide how much you think the insurance is worth to you
Refer to pages 11 and 12 of your booklet when making your decision
What is the most you would personally be willing to pay in tokens for the insurance? (1Token = 2c)

## Participant Booklet

## Welcome

The purpose of this study is to look at how people make decisions in different areas. Today's session is concerned with choices over lotteries. I would like to stress that there are no right or wrong answers - what is the right choice for you is the wrong choice for someone else. You will see quite a lot of information today, you don't need to memorise it as we will give you the information as you need it.

During the session you will have the opportunity to earn some tokens. At the end of the session, these tokens will be converted into money at a rate of $1 p$ per token. Where relevant there will be a reminder in red in the top left corner of your screens. In some of the tasks you will be participating as a group, so the amount each of you earn will be based on your own choices and the choices of others. Whatever the outcome, it is important to stress that you are guaranteed $£ 5$ towards your expenses.

Throughout the session please do not talk to anyone else as we are interested in your own choices. If you have questions, just raise your hand and I will come over to answer your question. Furthermore, please do not use the computer for any other purpose than participating in this study and please put your mobiles to silent mode.

At the moment you will see a welcome screen. During the session you will need to click the OK button to move to the next screen. At some points you will not be able to move on without a code that you will need to enter into the box next to the OK button.

## Lottery Task

In this task you will need to make decisions between a series of lotteries. If you look at your screen you will see a table. For each row of the table you will need to decide whether you would prefer Lottery A, or Lottery B. Each lottery is made of a high payoff and a low payoff.

For example if you look at Row 3 on your screens:

- Lottery A gives a $20 \%$ chance of paying out the higher payoff of 200 tokens and an $80 \%$ chance of paying out the lower payoff of 160 tokens.
- Lottery B gives a $20 \%$ chance of paying out the higher payoff of 385 tokens and an $80 \%$ chance of paying out the lower payoff of 10 tokens.

As you look down the rows, note that the payoffs for lotteries $A$ and $B$ remain the same but the probabilities vary.

At the end of today's session, one of the eleven rows will be chosen at random for each of you. You will play your chosen lottery for that row and the outcome of that lottery will determine your earnings for this task. Remember the exchange rate is $1 p$ per token.

On the next screen you will need to click to choose between Lottery A and Lottery B. First you will need to choose for Row 1 and Row 11 and then click OK and answer the remaining rows. Once you are happy with your choices please click OK.

## Practice Buying Task

Now I want you to think about buying insurance. Let's say you've just moved into a flat with three of your friends. You all have valuables, but none of them are insured. You personally have a laptop and phone worth about $£ 800$ and you have £1000 spending money.

You talk with your flatmates, and the group decides it might be a good idea to look into buying insurance in case your items get damaged, lost or stolen. This is full insurance that covers everyone in the flat, so it is decided that if the group buys the insurance; everyone will split the cost equally. This means either:

- The group buys insurance, everyone is insured, and everyone pays an equal share of the cost, or
- The group doesn't buy insurance, no one is insured, and nobody pays.

From your point of view, if your laptop and phone went missing:

- With insurance, the insurance company would pay to replace them and you've only had to pay your share of the cost of insurance.
- Without insurance, if you chose to replace your items you would need to pay out of your own money. This would cost $£ 800$.

The problem is that not everyone is willing to pay the same price for the insurance. Depending on the price, some of your flatmates may want to buy insurance and others may not. To solve this it is decided that if the majority of the group want to buy insurance, then the group will buy.

You are given the task of ringing up the insurance company to get a quote for how much the insurance will cost per person. To decide whether the group
should buy insurance you will need to know what the maximum price each of your flatmates are willing to pay for the insurance. You go to each of your flatmates and ask them to state the most they would individually be willing to pay for one year's insurance. Let's call that their offers.

If the price per person of the insurance is less than or equal to someone's offer, then they are willing to pay for the insurance. So when you call up the insurance company to get the price:

- If three or more flatmates would pay for the insurance then the group will buy. So each flatmate pays the price of the insurance, not their offer.
- If one or no flatmates would pay for the insurance then the group won't buy. In that case you all pay nothing, but your items are not insured.
- If there's a tie and two flatmates would pay for the insurance and two wouldn't, a random draw will decide.

Because it is a group decision each flatmate is affected by the choices of the other flatmates. This also means not everyone in the flat is always satisfied with the outcome of the group decision, as whatever the group decides everyone has to go along with.

I want you to think about how much a year's insurance for your laptop and phone would be worth to you. In a moment I will give you the code to move to the next screen. First you will be asked to enter an offer. This offer should be the most you would be willing to pay for the insurance in pounds. When you click OK, you'll see your flatmates offers, which have been generated by the computer. Then, you will need click a red button to find out the price and then a series of questions for you to answer will follow.

In that example the price was $£ 91$. Many of you will have made offers lower than the price of $£ 91$, meaning you are unwilling to buy the insurance at that price. This would have meant your group didn't buy the insurance in Step 2. You will have seen in Step 3 that without insurance one of two things may happen:

- You lose your items, costing you money to replace them, or
- You don't lose your items, meaning you keep all of your money.

I want you to think what would have happened if someone had convinced you to make a higher offer say $£ 100$. You would now be counted as being willing to pay for insurance at a price of $£ 91$, even though you think the insurance is worth less. By offering more than your true willingness to pay you may end up being counted as willing to pay for insurance at a price you are not willing to pay. This will increase the chance that your group buys insurance at a price you do not want to pay.

In the previous example you saw that without insurance you cannot be sure of how much money you will be left with. In this example you have seen that with insurance, you will always have your starting money minus the price of the insurance.

I want you to think what would have happened if someone had convinced you to make a lower offer say 50p. You would now be counted as not being willing to buy at a price of $£ 1$, even though you think the insurance is worth $£ 1$ or more. By offering less than your true willingness to pay you may end up being counted as not willing to buy insurance. This will increase the chance that your group doesn't buy insurance at a price you would be willing to pay.

So the best offer you can make is the most you would personally be willing to pay for insurance:

- If you make a higher offer, there's a chance you are counted as willing to pay for the insurance at a price you aren't willing to pay.
- If you make a lower offer, there's a chance you are counted as not willing to pay for insurance at a price you are willing to pay.

In that task you could lose your phone and laptop, but we didn't talk about how likely it would have been that you lost those items. To consider this we're going to do another task, but this time we are using our lab currency, tokens. Just as before, these tokens are worth 1 p, but for now we are just practising so this won't affect your final payment.

This next example is going to be different from before, but you'll still be making group decisions about buying insurance. So remember the rules for making offers and deciding within a group, because you will be using those again.

## Practice Rounds

You are in a group of four. Each group member has a number of starting tokens and with some probability faces losing a number of those tokens. Your group is given the opportunity to buy insurance against all losses. As it is group insurance either:

- the group chooses to buy insurance, everybody pays the price from their starting tokens and nobody faces their own loss.
- or the group chooses not to buy insurance, nobody pays, and each member faces the possibility of losing some of their tokens.

To find out if you lose tokens, you will draw a random number between 0 and 100:

- If the number is less than or equal to your probability of loss you will lose tokens, leaving you with your starting tokens less your loss amount.
- If the number is greater than your probability of loss you will not lose tokens, leaving you with your starting tokens.


## Reminder - How does the group decide?

Step 1: Each group member is individually asked to make an offer for the group insurance. This offer should represent the most you are personally willing to pay for the group insurance.

Step 2: The price of the insurance is revealed. If the price of insurance is less than or equal to someone's offer, then they are willing to pay for the insurance. So one of three things will happen:

- If three or more group members would pay for the insurance then the group will buy.
- If one or no group members would pay for the insurance then the group won't buy.
- If there's a tie, and two group members would pay for the insurance and two wouldn't, a random draw will decide.

There are four practice rounds for you to complete. On your screens now you will see the question box and table for the first round. The table gives information on your group which is made up of four people labelled: Person A, Person B, Person C and Person D. Each member of your group will be assigned to a person. For now the computer will generate the offers for the other three members of your group.

If you click OK you will see that each Person has 50 starting tokens.
Click OK again and you will see that each person faces a loss of 40 tokens.
Click OK again and you will see that each person will lose those 40 tokens with a probability of $80 \%$.

So from your point of view without insurance there is an:

- $80 \%$ chance you will lose 40 tokens, leaving 10 tokens, and a
- $20 \%$ chance you won't lose any tokens leaving 50 tokens.

The other members of your group would also face the possibility of losing tokens.

If your group were to buy insurance, each member of your group would be left with their 50 starting tokens less the group's agreed price for the insurance.

Click OK again and you will see Step 1. In a moment I will tell you the code to move on. Once you begin, please read the text on your screen and then enter your offer and continue with Steps 2 and 3. When making your offers consider the information on pages 11 and 12 of your booklets. If you have any questions
as you go just raise your hand. When you have finished the first round you may move straight onto the second.

There are two more practice rounds for you to do. These rounds are to practice for your next task in which each of you will complete two types of question. There will be one practice round for each type of question. An information screen will appear before each round to explain the task. Please read the information screen carefully and work through the task.

Again, if you have any questions as you go through just raise your hand.

## Real Rounds

In this part of the session you can earn tokens that will be added to your final payment. You will complete 16 rounds similar to those in the last task, however for these rounds you will only have to make your choice of how much to offer.

For each of these rounds you will be randomly placed into groups of four with other people in the room. The groups will change for each round, so you won't always be with the same people.

At the end of the session one round will be chosen at random for your group to play out using the group's offers for that round. The payoff for each group member will be added to their final payment, so it is in your interest to answer each question as if it is the paid one. Remember the exchange rate is $1 p$ per token.

Remember not all the questions will be the same, so make sure you read everything carefully. All the information you need will be on your screens and on pages 11 and 12 of your booklet.

Before you begin there will be an information screen telling you about the questions you will be answering. Please read the screen carefully and then click OK to begin.

## Questionnaire

Before you get feedback on your choices I'm going to ask you to fill out a questionnaire about yourself to help us understand your answers.

## Feedback

The remaining part of today's session is for you to get feedback on your choices. Two questions have been chosen at random for each of you - one from the first task and another from the task you have just completed. You can now move through the screens to play out your chosen rounds. The earnings from these two rounds will be added to the $£ 5$ show-up fee to give your total earnings.

Once you have reached the page with your total earnings, I will ask each of you to fill out a receipt for our record keeping. You will be able to collect your earnings as you leave at the back of the room.

Thank you for participating in today's session.

## Appendix E: Experiment B materials

Figure E1: Experiment B: Example decision screen for respondents in front of a VoI


Notes: To enter a MAC subjects moved a slider on the screen to the amount they wanted to select. The slider, which started in a random position, had a minimum of 0 and the maximum amount was set such that no group member could go bankrupt. As the subject moved the slider along, a table underneath showed the number of tokens each group member would contribute and their net payoff if the public good was funded at the selected amount. By doing so, subjects could see the cost the other group members would pay, allowing them to take this into account in their decision and thereby avoiding any double counting.

Figure E2: Experiment B: Example decision screen for respondents behind a VoI

You will not know which position you are in until after you enter your Maximum Group Investment. Your position will be determined by a random draw.

The other members of your group will be assigned to the remaining positions.

| Information on the investment is shown in the table below. |  |  |  |
| :---: | :---: | :---: | :---: |
| Position | A | B | Total |
| Starting <br> Balance | 100 | 150 | 250 |
| Payoff | 50 | 50 | 100 |
| Required <br> Contribution | $40 \%$ | $60 \%$ | $100 \%$ |
| Contribution <br> Amount | 40.00 | 140.00 | 100.00 |
| Final Balance | 110.00 | 250.00 |  |

Step 1: Use the slider to set your Maximum Group Investment.
Your Maximum Group Investment should be the most you believe the group should invest in order to receive their payoffs.


The Contribution Amount row shows how much each group member would contribute if the group invested your chosen Maximum Group Investment The Final Balance row shows each group member's final balance if the group invested your chosen Maximum Group Investment.

When you are satisfied with your decision click the Confirm Amount button to enter your Maximum Group Investment

# Participant Booklet 

October 2017

## Instructions

In today's session, you will complete a number of rounds where you will be making investments in groups.

- At the beginning of the round, the computer will randomly assign you to groups made up of either 2 or 4 members.
- You will not know who else in your group.
- You will then be given information on your group and the investment on offer.
- Each group member will be given a starting balance of tokens.
- Not everyone in your group will necessarily have the same starting balance.
- The investment gives each group member a guaranteed payoff of tokens.
- Not everyone in your group will necessarily get the same payoff from the investment.
- If the investment goes ahead, the group will be required to invest a certain amount of tokens which will not be refunded.
- This is the cost of the investment.
- Each group member will have to contribute to cover the cost of the investment.
o The share of the cost that each group member will pay is called their required contribution.
- Not everyone in your group will necessarily have the same required contribution.

Once everyone has had an opportunity to look at this information:

- Each group member will be asked to enter into the computer the maximum amount they believe the group should invest in order for each group member to get their payoff.
o This amount is called their Maximum Group Investment, or MGI.
- Next, the cost of the investment to the group is randomly drawn by the computer.
- Each group member's Maximum Group Investment is compared to the Cost of the Investment.
- Group members with an Maximum Group Investment greater than or equal to the cost of the investment are counted as wanting to invest.
- Group members with an Maximum Group Investment less than the cost of the investment are counted as not wanting to invest.
- The number of group member's wanting to invest is then counted.
- If the majority of the group wants the investment to go ahead then the group will invest.
- In that case, each group member will contribute their share of the cost of the investment.
- Contribution $=$ Cost of the Investment $\times$ Required Contribution.
- Each group member receives their payoff.
- Final Balance $=$ Starting Balance - Contribution + Payoff
- Alternatively if the majority of the group doesn't want the investment to go ahead then the group won't invest.
- In this case no one contributes and no one receives their payoff.
- The final balance for each group member is the same as their starting balance.
- Tied decisions, where equal group members are for and against investing, are resolved randomly by the computer.
- Whatever the outcome of the group decision everyone has to go along with it. So not everyone in the group will always get what they want.


## Practice Rounds

You should all see a box on your screens now with some text and a table. In this round you are in a group made up of four positions labelled: A, B, C, and D. In this round you are in Position A. The other positions are filled by your group members. Because this is just practice, the other members of your group will be played by the computer and this won't affect your final payment.

I said before that at the start of the round each member of the group is given a starting balance of tokens. The Starting Balance row in the table shows that in this round each member of the group has 100 starting tokens. The total column tells us that the group has a total of 400 starting tokens.

The payoff row shows how much each group member would get if the group makes the investment. For this round each group member would receive 40 tokens, giving a total payoff of 160 tokens to the group.

I explained earlier that if the investment goes ahead, the group will have to invest a certain amount of tokens - that's the cost of the investment. To cover that cost, each group member would have to personally contribute some tokens from their starting balance. The bottom row of the table shows the share of the cost of the investment that each group member would pay. In this case that's 25\% each.

Once you've considered this information, you will need to enter your Maximum Group Investment - that's the maximum you believe the group should invest. You can do that on the next screen.

On your screens will have appeared two more rows to the table, some text and a slider. I will explain each of these now. You can set your Maximum Group Investment by setting the slider to the point you want. Underneath the slider the screen says Maximum Group Investment with a number next to it. That's the amount the slider is set to.

The slider is moved by either clicking or dragging the bar, or you can adjust by 1 using the plus and minus buttons. Just take a few seconds now; move the slider up and down and you will be the number changes.

You may have noticed that the numbers in the bottom 2 rows of the table changing too. These two rows show what would happen to each group member if the investment went ahead at the amount the slider is set to. The Contribution Amount row shows how much each group member would have to contribute and the Final Balance row shows what each group members' final balance would be.

I want you all to set your sliders to a Maximum Group Investment of 80 tokens. You can see that the total column of the contribution amount row now says 80 tokens - the total amount invested by the group.

Let's take a look at column A - that's you. The required contribution row shows that you would have to contribute $25 \%$ of the total amount invested by the group. So for a total investment of 80 tokens you would contribute 20 tokens. You would contribute these 20 tokens from your starting balance and received your payoff of 40 tokens. This would give you a final balance of 120 tokens that's your 100 starting tokens minus your contribution of 20 tokens plus your payoff of 40 tokens. So if your group invested a total of 80 tokens, your balance would increase by 20 tokens.

I want you all to set your slider to 200 tokens. Now the Contribution Amount row shows each group member contributing 50 tokens - that's $25 \%$ of the total of 200 tokens. In the final balance each member of the group has 90 tokens - that's their 100 starting tokens minus their contribution of 50 tokens plus their payoff of 40 tokens. So if your group invested a total of 200 tokens, your balance would decrease by 10 tokens.

When compared with the cost of the investment, the amount you choose for your Maximum Group Investment (MGI) will determine if you are counted as wanting the investment to go ahead or not.

- If your MGI is greater than or equal to the cost of the investment then you will be counted as wanting to invest, and
- if your MGI is less than the cost of the investment then you will be counted as not wanting to invest.

If the group decides to invest then each group member will contribute their share of the cost of the investment not their Maximum Group Investment.

With this being the case, the best amount you can enter as your Maximum Group Investment is the maximum amount you believe the group should invest in order to receive the payoffs. This is what you think the investment is truly worth, or your true value of the investment.

If the cost of the investment is drawn and it is higher than your true MGI that means that you do not think the investment is worthwhile.

But what if instead of your true MGI; you had entered a higher MGI, one that is greater than your true value. Let's call this a High MGI. What might happen?

- If the cost of the investment is more than the High MGI you entered; then it doesn't matter as you will not be counted as wanting to invest at that cost.
- However, if the cost of the investment is less than the High MGI you entered; then you would be counted as wanting to make that investment. If this cost was greater than your true MGI then the investment may go ahead at a cost you aren't willing to pay, but had you put in your true value it may not gone ahead.

You can never make yourself better off by entering a MGI greater than your true value, and you may make yourself worse by increasing the chance your group makes an investment that you don't think is worthwhile.

On the other hand, if the cost of the investment is drawn and it is lower than your true MGI that means that you do think the investment is worthwhile.

But what if instead of your true MGI; you had entered a lower MGI, one that is less than your true value. Let's call this a Low MGI. What might happen?

- If the cost of the investment is less than the Low MGI you entered; then it doesn't matter as you will be counted as wanting to invest at that cost.
- However, if the cost of the investment is more than the Low MGI you entered; then you would be counted as wanting to make that investment. If this cost was less than your true MGI then the investment may not go ahead at a cost you are willing to pay, but had you put in your true value it may have gone ahead.

You can never make yourself better off by entering a MGI greater than your true value, and you may make yourself worse by increasing the chance your group makes an investment that you don't think is worthwhile.

So whatever you think the cost of the investment might be, the best amount you can enter as your MGI is most you think the investment is worth.

You now have the opportunity to enter your own Maximum Group Investment and to play the rest of this round out. In a moment I will give you the code to move on. Once you have entered that code an enter amount button will appear. You can set the slider to the maximum amount you believe the group should invest and then click the confirm amount button. This will take you to next screen where you can complete the rest of the round. If you have any questions as you go through just raise your hand.

In that round the cost of the investment was 20 tokens. You will have seen that each of your group member's wanted the investment to go ahead. Because at least three members of the group wanted to invest, the investment went ahead. This left you with a final balance of 135 tokens - that's your starting balance of 100 tokens minus your contribution of 5 tokens plus your payoff of 40 tokens.

I want you to think what would have happened if someone had convinced you to enter a Maximum Group Investment that was less than 20 tokens. You would no longer be counted as wanting to make the investment at a cost of 20 tokens even though you may think it's worth more. By doing so you increase the chance that your group does not make the investment, even though it was at a cost you would have been willing to pay.

This time you will have seen that none of your group member's wanted the investment to go ahead. Because at least three members of the group didn't want to invest the group didn't. So nobody contributed and nobody received their payoff, leaving you with a final balance of 50 tokens - the same as your starting balance.

I want you to think what would have happened if someone had convinced you to enter a Maximum Group Investment that was more than 108 tokens. You would now be counted as wanting to make the investment at a cost of 108 tokens even though you may think it's worth less. By doing so you increase the chance that your group makes the investment, even though it would have required you to pay cost you would not have been willing to pay.

## Paid Rounds

In this part of the session you can earn tokens that will be added to your final payment. There will be three different scenarios. You will complete 22 rounds in total, split into three sections; one for each type of scenario. An information screen will appear before each round begins to explain the scenario for that round, so make sure you read that carefully.

For each round, you will be randomly placed into groups of two or four with other people in the room. The groups will change between rounds, so you won't always be with the same people. In each round you will only have to enter your chosen Maximum Group Investment. At the end of the session, one round will be chosen at random for your group to play out, using your group's choices for that round. The Final Balance for each group member will be added to their $£ 5$ show-up fee to give their final payment, so it is in your interest to answer each question as if it is the paid one. Remember the exchange rate is $1 p$ per token. Also not all the questions will be the same, so make sure you read everything carefully.

## Questionnaire \& Feedback

Before you get feedback on your choices I'm going to ask you to fill out a questionnaire about yourself to help us understand your answers. Please answer the questions and click OK.

The remaining part of today's session is for you to get feedback on your choices. One round has been chosen at random to be played out using your choices for that round. You can now move through the screens to play out your chosen rounds. The earnings from this round will be added to the $£ 5$ show-up fee to give your total earnings. Once you have reached the page with your total earnings, I will ask each of you to fill out a receipt for our record keeping. You will be able to collect your earnings as you leave.


[^0]:    ${ }^{1}$ Throughout the thesis when referring to another section the following format is used: Section 3.2 would refer to Chapter 3 Section 2, and Section 3.2.1 would refer to Chapter 3 Section 2 Subsection 1.

[^1]:    ${ }^{2}$ This compensation test is also referred to as the Kaldor-Hicks criterion.

[^2]:    ${ }^{3}$ Two measures of consumer surplus are often considered within WTP: Compensating Variation and Equivalent Variation. See Hicks (1943) for discussion.
    ${ }^{4}$ Willingness-to-accept (WTA) is the natural opposite to WTP. WTA is the increase wealth required to compensate a loss leaving the individual with the same utility as opposed to WTP which is the decrease in wealth that would leave the individual with the same utility after a gain. This thesis looks at WTP, however the results would equally be seen in the WTA domain.

[^3]:    ${ }^{5}$ See Carson, R. T. (2012) for a discussion of the contingent valuation method.

[^4]:    ${ }^{6}$ There are arguments for paternalistic propositions, however in the case of a democratic society based on individuals to which most welfare statements are made, this proposition is quite reasonable. One strong argument against this proposition is the case where the individual in question has less relevant information than the paternalistic party. See Sugden (1981) for discussion.

[^5]:    ${ }^{7}$ The contractarian approach take the same position as it also accepts Proposition 2.

[^6]:    ${ }^{8}$ The double-counting problem is defined as counting either and element of the benefits or costs side of the calculation twice, or more than twice, thereby inflating that element. Mishan (1988) pg. 74-82 gives a number of examples.
    ${ }^{9}$ Emphasis by Lerner.

[^7]:    ${ }^{10}$ This thesis follows Jones-Lee (1991)'s definition of "pure altruism", that if an individual has an altruistic concern for another then they respect their preferences in the sense that, for example, Individual A's marginal rate of substitution of Individual B's wealth for Individual B's safety is the same as Individual B's marginal rate of substitution of her own wealth for her own safety. This is equivalent to "benevolence" as described by Bergstrom (1982). For the sake of consistency, the term "pure altruism" shall be used throughout.
    ${ }^{11}$ Continuing the example from Footnote 8, a paternalistic altruist is one who values the determinants of another's welfare (safety and wealth) but the paternalistic altruist, Individual B's marginal rate of substitution of Individual

[^8]:    A's wealth for Individual A's safety is not the same as Individual A's marginal rate of substitution of his own wealth for his own safety.
    ${ }^{12}$ From here on referred to as naïve cost-benefit analyst for consistency with the rest of the terminology used in the thesis.

[^9]:    ${ }^{15} \mathrm{~A}$ fuller derivation of this solution is presented in Appendix A.

[^10]:    Notes: [1] For optimal provision each solution for the marginal benefit must equal $\left.\frac{\partial s}{\partial\left(2 n_{i}^{n} \pi_{i}\right.}\right)$, the marginal cost of providing the public good. [2] For PA, as $\frac{\partial u_{j}}{\partial \pi_{i}} / \frac{\partial u_{j}}{\partial w_{i}}=\frac{\partial u_{i} / \partial u_{i}}{\partial \pi_{i}} \frac{\partial w_{i}}{\partial w_{i}}$, must also be true that $\frac{\partial u_{j}}{\partial \pi_{i}} \frac{\partial u_{j}}{\partial w_{i}}=b_{j}^{i} \frac{\partial u_{i}}{\partial \pi_{i}} / b_{j}^{i} \frac{\partial u_{i}}{\partial w_{i}}$, where $b_{j}^{i}$ is the weight Individual $j$ places on Individual $i$, s marginal utilities which must be equal for both survival probability and private good consumption.

[^11]:    ${ }^{17}$ A version of the models presented in this section appear in Beeson et al. (2019).

[^12]:    ${ }^{18}$ Bergstrom (2006) concludes from a general model that under purely altruistic preferences that using private values in a compensation test is a sufficient but not necessary condition for a potential Pareto improvement. There are some cases in which a Pareto improvement was possible at costs above the sum of the private values, however the identification of these cases is not clear.

[^13]:    ${ }^{19}$ This model is focussed on a general proposal for a change in public good provision. The term SFA is used here to refer to altruism based purely on benefits associated with consumption of a public good. This form of altruism could alternatively be termed public good-focused altruism and is equivalent to SFA for a generic public good. Whilst the term SFA is used, these terms are interchangeable when considering the findings of this model.

[^14]:    ${ }^{20}$ As Jones-Lee (1992) highlights, there is the potential for a distribution of middles cases of altruism between SFA and WFA. Appendix B presents a graphical representation for 3 distinct tax shares.

[^15]:    ${ }^{21}$ The case of $w_{a}=8, w_{b}=12$ results in a corner solution in front of a VoI and the first-best point is therefore at $\mathrm{t}=0$ and a greater level of provision than the other simulations.

[^16]:    ${ }^{22}$ A version of the models presented in this section appear in Beeson et al. (2019).

[^17]:    ${ }^{23}$ Here the single context of the avoidance of negative lottery is used. Messer et al. (2013) also considered positive lotteries. Messer et al. (2010) considered certain gains and losses.

[^18]:    ${ }^{24}$ This also holds true for homogenous groups behind a VoI before the VoI has been lifted. This is not true for homogenous groups in front of a VoI who are identical in endowment and lottery.

[^19]:    ${ }^{25}$ A form of state-dependent altruism could be considered here for which an individual is more or less altruistic towards their identical group members depending on the position drawn. Under this form of preferences greater weight would be placed on the position for which the individual is more altruistic and thus impact upon WTP.

[^20]:    Notes: $\Delta W T P_{i}=W T P^{S I, P A, M M, I A}-W T P^{S I}, \overline{p l}$ is the group mean private value, $p_{w} l_{w}$ is the worst-off group member's private value, and $p_{b} l_{b}$ is the best-off group members private value.

[^21]:    Notes: $\Delta W T P_{i}=W T P^{S I, P A, M M, I A}-W T P^{S I}, \overline{p l}$ is the group mean private value, $p_{w} l_{w}$ is the worst-off group member's private value, and $p_{b} l_{b}$ is the best-off group members private value.

[^22]:    ${ }^{29}$ Sessions were conducted using Z-Tree (Fischbacher, 2007) at the Behavioural and Experimental Northeast Cluster (BENC) experimental laboratory at Newcastle University in February 2017 and at the Monash Laboratory for Experimental Economics (MonLEE) in April 2017 for which students were invited to participate using the respective laboratories' recruitment systems. 18 sessions were run in total, each taking approximately 90 minutes. Sessions were of size 24 or 16 with 1 session of size 12 due to no-shows. The experiment was piloted in Newcastle, UK in November 2016 and January 2017.
    ${ }^{30}$ The exchange rate is based on cost of living comparisons (OECD, 2018) and payments aligned with the averages for the respective labs.
    ${ }^{31}$ See Appendix D for instruction booklet.
    ${ }^{32}$ See Appendix D for decision screens.

[^23]:    ${ }^{33}$ The groups changed between rounds so that subjects were not with the same group members in consecutive rounds and remained anonymous to the other subjects. The order of rounds was randomised to control for order effects.
    ${ }^{34}$ Given that each treatment has equal numbers of subjects from each country and the subjects in each treatment sub-sample are not statistically treatment the data is pooled for the analysis.

[^24]:    ${ }^{35}$ The experimental design and results presented in this Chapter appear in Beeson et al. (2019).

[^25]:    ${ }^{36}$ This does not occur in the selected parameters for the experiment.

[^26]:    ${ }^{37}{ }^{37}$ This does not occur in the selected parameters for the experiment.

[^27]:    ${ }^{38}$ Sessions were conducted using Z-Tree (Fischbacher 2007) at the Behavioural and Experimental Northeast Cluster (BENC) experimental laboratory at Newcastle University for which students were invited to participate using the laboratory's recruitment system. 11 sessions were run in total, each taking approximately 80 minutes. Nine sessions had 20 participants, with one session consisting of 8 subjects and one of 16 subjects due to no-shows for a total of 204 subjects.
    ${ }^{39}$ The groups changed between rounds so that subjects were not with the same group members in consecutive rounds and remained anonymous to the other subjects. The order of rounds was randomised to control for order effects.

[^28]:    ${ }^{40}$ This took place at the end of the session to avoid wealth effects between rounds.
    ${ }^{41}$ Extensions to our design could consider groups of larger numbers if experimenters wished to introduce asymmetries in endowments and payoffs.
    ${ }^{42}$ See Appendix F for the instruction booklet and Figures F1 and F2 showing an example decision screens.

[^29]:    ${ }^{43}$ A general to specific setup was used to test for non-linear relationships. Squared and cubic terms were incorporated for all variables and were found to be insignificant and therefore a model of linear relationships was opted for.

