Integrating GeoGebra into a primary mathematics teaching intervention: impact on students' learning processes and outcomes

A Thesis Submitted for the Degree of Doctor of Education in Education

EdD in Education and Communication

by

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December 2022

Abstract

This thesis aims to investigate the impact of integrating GeoGebra into teaching intervention on students' geometrical learning process and outcomes. This includes geometric performance, sustainable learning, spatial thinking, students' views of learning using GeoGebra and attitudes towards learning mathematics for Year Five students in Saudi Arabia. Students’ attitudes towards mathematics covers mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics. Besides, the present research examined the correlation between students geometric performance and spatial thinking, attitudes towards learning mathematics, and their ability to sustain their learning for a long time. This research goes deeper to explore pairs' patterns of interaction and the association between pairs' interaction patterns and their geometric performance, spatial thinking skill, mathematics academic self-concept, enjoyment of mathematics, perceived value of mathematics, and sustainable learning.

To do so, I adopted pre and post-test quasi-experiment non-equivalent group research design based on control and experimental groups. This research employed mixed methods, including the use of geometric performance test, delayed test, spatial thinking test, GeoGebra visual questionnaire, visual questionnaire of students' attitudes towards mathematics, and video data to explore pair's patterns of interaction.

The findings show that the teaching intervention with GeoGebra significantly improves students’ geometric performance, spatial thinking skills, mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics more than teaching intervention with hands-on and traditional teaching. Besides, students show a steady positive change in their view of learning using GeoGebra over time. The results explored six patterns of interaction collaborative, dominant/dominant, cooperative, dominant/passive, passive/passive, and expert/novice. Where collaborative students consistently performed better than other students, while passive/passive students were the lower achievers. Overarching these conclusions has gradually developed my understanding of the nature of learning. The learning activity cannot be designed (Goodyear and Carvalho, 2014a) but can be guided by learning tasks. Although a social setting can be designed, it cannot ensure that students work collaboratively throughout the learning tasks. In short, teaching should be learner-centered and pay more attention to encouraging students to adopt collaborative interaction pattern.

Acknowledgement

In the beginning and before anything, I thank Allah Almighty for giving me the insight, power and the will to finish this thesis.

Next, I would extend express my sincere gratitude to my supervisors Dr Pamela Woolner and Dr Helen Burns, for the endless support of my EdD course and related research, for their patience, motivation, and immense knowledge. Their friendly guidance and professional advice have been invaluable throughout all stages of the research project.

Besides, I would like to thank my Panelists, Dr Steve Humble and Dr James Stanfield, for their constructive critical suggestions on my research.

I also wish to thank the Ministry of Education and the Al-Manarat primary school headteacher for allowing me to conduct my research experiment at his school, all the participants in the research, and my friends and colleagues and the other people who contributed to this research directly and indirectly, specially, Professor. Mohamed Alnatheer, Dr. Mamun Albnaa, Dr. Ibrahim Khalil, Dr. Thamer Bin Marzouq Alzubaidi, Dr. Saad Albogami, Areeb Alshoshan, Abdulrahman Binrsheed, Rashed Alqufaydi, and Mr. Micheal Henderson.

Last but not least, my heartfelt gratefulness and gratitude go to my family, particularly my mother, my father who passed away few years ago, my sisters and brothers in the Kingdom of Saudi Arabia, for their encouragement and cooperation; and to my wife for being with me and her patience and support during my doctoral degree journey, and my children, Raad and Rozan.

Thank you all.

Author's Declaration

I certify that, to best of my knowledge, all the material in this thesis represents my own work and that no material is included which has been submitted for any other award or qualification.

Signature: ....

Date: ....

List of Publications

1. Khormi, S., Woolner, P. (2019) ‘Development of Saudi Mathematics Curriculum between Hope and Reality’, *International Journal of Management and Applied Science (IJMAS)*, 5(12), pp. 26-36.
2. Khormi, S. (2020) ‘Can Developed Countries Make History of Poverty?’, *Archives of Business Review*, 8(7), pp. 85-66.
3. Khormi, S., Alshoshan, A. and Binrsheed, A. (2021) ‘THE IMPACT OF USING LOGO PROGRAMMING LANGUAGE ON THE DEVELOPMENT OF SPATIAL VISUALISATION ABILITY AMONG ELEMENTARY SCHOOL STUDENTS’, *International Journal of Research-GRANTHAALAYAH*, *9*(3), pp.118-127.
4. Khormi, S., (2021) ‘Unstructured Interview and Biography: which one is more beneficial to investigate the Process of Teaching and Learning Mathematics’, *Journal of Research & Method in Education*, 11(06), pp. 17-21.

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Chapter 1. Study context[1]

1.1 Introduction

King Abdul Aziz bin Abdul Rahman Al-Saud established the Kingdom of Saudi Arabia (KSA) in 1936 in the Arabian Peninsula. KSA is bordered by Kuwait, Bahrain, the Arabian Gulf, the United Arab Emirates and Qatar to the east, the Red Sea to the west, Jordan and Iraq to the north, and Oman and Yemen to the south. It covers an area of roughly 2149700 Km^2, including thirteen administrative regions; specifically, Al-Riyadh, Al-Madinah Al-Monawrah, Al-Qaseem, Makkah Al-Mokaramah, the Eastern Region, the Northern Borders, Jazan, Najran, Al-Baha, Asir, Tabuk, Hail, and Al-Jouf. Each administrative region is divided into a number of governorates. Moreover, each governorate is divided into a number of sub-governorates. Each administrative region has its own geographical and ethnographical features, which are different from other administrative regions (see figure 1.1).

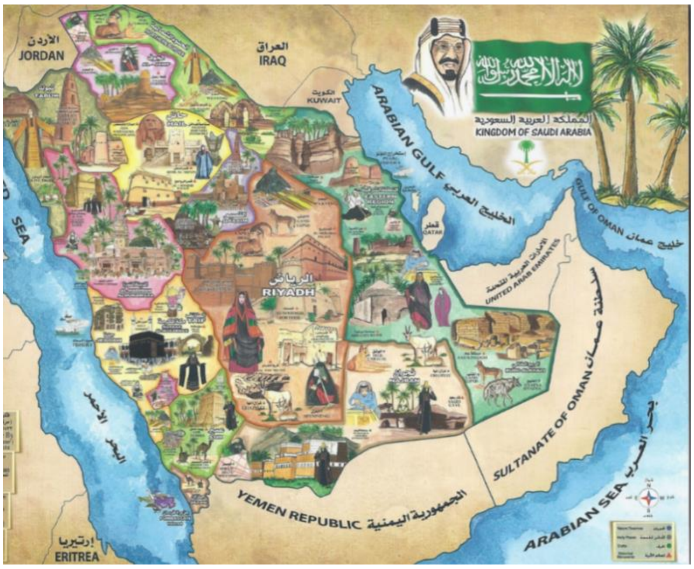


Figure 1.1 Map of cultural differences in Saudi Arabia

KSA is home to two of the holiest cities for Muslims; in particular, Makkah Al-Mokaramah and Medina Al-Monawrah, where Islam originated over 1400 years ago and began to spread around the world (Al-Raqiba, 1999). Hence, Islam is the main religion of the country. The official language of Saudi Arabia is Arabic. The country’s population has increased sharply

[1] This chapter is part of a paper titled “Development of Saudi Mathematics Curriculum between Hope and Reality”, published in 2019 in the International Journal of Management and Applied Science

from almost, 8 million in 1974, to approximately, 35 in 2020, with the majority being young people (The General Authority for Statistics Kingdom of Saudi Arabia, 2021).

In addition to its religious standing, KSA is gaining economic importance. It is one of the biggest economies in the Middle East and moreover, one of the largest exporters of oil and petrochemicals globally (Al-Saleh, 1999). Likewise, it is a member of the G20 countries, which is a global policy forum, consisting of twenty representatives from the world's largest economies (Al-Saleh, 1999, Szczepański and Bassot, 2015). Recently, KSA has begun to diversify its economy and increase the wellbeing of its people. Consequently, the Crown Prince announced Vision 2030, which is a road map and campaign to develop the country in the coming years. It is built on three distinctive themes (Saudi Arabia Government, 2016):

1. An energetic society, which is vital to achieving this Vision, and a strong foundation for economic prosperity
2. A prosperous economy, which will provide opportunities for all by building an education system, answering the needs of the job market.
3. An ambitious country, which is built on an operative, transparent, accountable, empowering, and high-performing government. Additionally, the correct environment will be prepared for the nation, private sector, and non-profit sectors to take responsibility and action in facing challenges and seizing opportunities

The aim of Vision 2030 is to build a successful country and be a model of excellence on all fronts. In addition to this Vision, the Saudi government planned the National Transformation Programme 2020 across 24 governmental organisations, including the Ministry of Education, working on the economic and development sectors, in order to create the ability and criteria required to achieve the ambitious aims pertaining to Vision 2030 (Saudi Arabia Government, 2016).

1.2 Overview of the Saudi Education System

The KSA has attached importance to education since it was established in 1936. After discovering oil in 1938, the Saudi government considered education to be more significant than before, as to maintain its substantial economic and social development (Alharbi, 2017). Currently however, the Saudi education system is in its infancy compared with the education systems in developed countries. In 1951, the Ministry of Education (MoE) was established to oversee every aspect of general education within its three levels (Primary, Intermediate, and Secondary. Later, the Kindergarten level was included as well), such as teachers' salaries, construction of schools, professional development, and paying for pensions and education books. The MoE divided the country into 42 general directorates of education for implementing policy for state and private schools. In 2015, the MoE began to supervise both state and private universities. The education system in KSA remains segregated by gender from year four in primary schools until post-graduate studies. However, there are two optional systems from years one to three: co-education with female teachers and single-sex education where female teachers teach girls, and male teachers teach boys.

Each of the educational levels (Primary, Intermediate, Secondary and University) in KSA are free of charge for residents (Abdulatif, 2008). In the last few decades, due to the increase in oil prices, the number of Saudi universities has increased from eight to twenty-nine, whilst the number of schools has risen from roughly 28,100 in 2010 to approximately 36,300 in 2015 (MoE, 2019a).). Besides, the private sector has been participating in educational services in private schools, international schools, colleges, and universities under MoE supervision. More specifically, state and private schools have taught the same curricula throughout the country, whereas the international schools have their own curricula, depending on the country they originate in. Furthermore, education in KSA is compulsory from primary school until secondary school (from year 1 to year 12). It must be noted that pre-school, which is for children under six years old, is optional (MoE, 2019c).

The Saudi education system has been influenced by geographical features, the significance of religion, the increase in the country's population, as well as the economy and industry. The education policy in KSA emerged from Islam, which highlights the importance of education and learning, besides manners. As a result, Saudi education policy states the following:

- Believe in Allah, as God, Islam as religion, and Mohammad, as the Prophet (PBUH)
- Islamic vision of humans and life so that each individual will conduct his/her tasks without any interruptions from external sources
- Teach the Muslim how to depend on his faith for productivity, creativity, and to guide his immortal life
- Believe that Islamic civilization requires both wisdom [derived from faith] and human constructions to achieve glory on earth
- Follow the highest example that Islam has brought to human civilization through Prophet Mohammad's model to achieve glory on earth and happiness in the afterlife

- Assert that education is a core component of every individual in Islam and spreading education at each stage of life is the duty of the nation
- Assert that Islamic Scientific courses are a core component of every stage of the curriculum in Primary, Intermediate, and Secondary Schools. Similarly, Islamic culture is a core component in higher education
- Articulate comprehensive harmony between science and religion under Islam
- Encourage and develop scientific research by means of enhancing observation, contemplation, and opening our minds (MoE, 1970).

The KSA has made a significant effort to develop the nation by focusing on education. Since KSA was established by King Abdul Aziz, the MoE has sought to develop the national curriculum. Recently, with the announcement of Vision 2030, the Ministry of Education stated eight strategic objectives in the National Transformation Programme 2020 (Saudi Arabia's Vision 2030, 2016):

1- Deliver education services for all student levels
2- Develop the recruitment, training, and development of teachers
3- Develop the learning environment to stimulate creativity and innovation
4- Develop curricula and teaching methods
5- Develop students' values and core skills
6- Boost the educational system's ability to address national development requirements and to meet labour market demands
7- Improve creative financing methods and develop the educational system's financial efficiency
8- Increase private sector participation in the education sector.

Additionally, the MoE in the National Transformation Programme (2020) highlights the significance of developing the national curriculum generally and the mathematics curriculum specifically. Likewise, they emphasise the significance of mathematical performance improvements in the international TIMSS test. It must be noted that TIMSS is American, as is the mathematics curriculum. Table (1.1) below illustrates the key performance indicators in relation to the TIMSS test.

Key performance indicator	Baseline	2020 Target	Regional benchmark	International benchmark
Average student results in international TIMSS tests (Year Eight: Maths)	394	450	452	611
Average student results in international TIMSS tests (Year Four: Maths)	410	460	469	606

Table 1.1 Key performance indicators for Saudi Students in the TIMSS test

KSA has been participating in the international TIMSS test from 2003. Since that time, the results have demonstrated poor performance of Saudi students regarding mathematics. On that note, Al-Ewasheq and Rafea (2010), pointed out the weak output of mathematics education in the kingdom in contrast to several countries in the developing world and developed countries. This was illustrated by national and international studies and evidenced by the findings of a study into international trends in mathematics and science TIMSS in 2003, when KSA was second from bottom on the list. The poor achievement of Saudi students in mathematics continued in TIMSS tests in 2007, 2011, and 2015. However, the best result for Saudi students was in 2011, when year eight attained a score of 394 and year four obtained 410. This score was mentioned as a baseline in Table 1 above. Furthermore, Saudi society sees the TIMSS results as an indicator of the quality of the mathematics curriculum and teaching practices. It can be said that improving the performance of Saudi students in the international test has become a fundamental requirement with respect to achieving Vision 2030 (Albalawy, 2019, Aldwsary, 2016, Alharbi, 2009, Alnatheer, 2009, Al-TAlb, 2018, Bakhit, 2017).

1.3 Overview of the Mathematics Curriculum in Saudi Arabia

At its early stages, the mathematics curriculum was divided into three textbooks: algebra, geometry, and accounting. Between the 1970s and 1980s, the MoE made several developments to the national curriculum in general, and to the mathematics, in particular (Batterjee, 2011). In 1976, the first mathematics curriculum in Saudi Arabia was an extension of the effort by the Educational Centre of Mathematics and Science at the American University in Beirut. This effort focussed on primary and intermediate levels, whereas the secondary school mathematics curriculum was designed by a group of professors from King Saud University, who specialised in pure mathematics. This led to several issues in presenting the textbook content, which tended to be abstract, given that information from university references is transferred without any understanding of the educational aspect (AL-AGLA, 1985, Al-Makoshi, 1984, Al-Qadi, 1994, Al-Makoshi, 1996, Al-Faleh, 1988, MoE, 2019b).

In spite of the fact that considerable effort has been made to develop this curriculum, the following issues remain noticeable (Al-Hian, 2006): The dominance of dictation and poor attention to building the mental abilities and scientific skills that students need, such as critical analysis, problem solving, decision-making, and deduction; the limited use of modern trends and theories in mathematics in the construction and organisation of the curriculum and design of educational tools to support student learning; a lack of educational materials that support teachers and students in the education process. For example, Al-Makoshi (1996) and Al-Mowayshir (2000), ascertained that the majority of the suggested educational tools in the teachers' guidebooks are not available at schools. Similarly, Al-Dhash (1994) reported that there is a lack of professional development for teachers. In addition, as mentioned earlier, the poor outcome of education in mathematics, in contrast to many countries in the developing world and developed countries was illustrated by research, evidenced by TIMSS results in 2003, 2007, 2011, 2015, when KSA came near the bottom of the list.

Moving towards economic matters, Jenkins (2008), claimed that KSA has established numerous economic initiatives to diversify the country's income to move away from relying heavily on oil production. On that note, acknowledgement of the role that education plays in preparing Saudi people for the competitive worldwide market created the implementation of various educational reforms from 2003 to meet the needs of the labour market and society's values, as well as 21st century skills; therefore, generating a positive generation able to solve both its own and national problems (Al-Shaya and Abdulhamid, 2011, Jenkins, 2008). Likewise, the MoE (2004) stated that the economics of knowledge and power of ever-renewing sciences govern the world, continuing to say, we endure a world with complex relationships and communications and those who have the knowledge and skills will join the march of human progress. KSA always sees the development of science and mathematics curricula as a fundamental factor in improving its economy and society.

Due to these reasons, the MoE made its decision to develop the mathematics curriculum on international experiences which have proved effective in improving education. Therefore, decision-makers selected McGraw-Hill Education and its representative in KSA, Al-Obekan Education to undertake the task of developing the mathematics curriculum according to the standards and principles of the US National Council of Teachers of Mathematics, known as 'NCTM 2000', for all general education levels (MoE, 2006). In addition, to attain success with this project, the task of translating and harmonising the McGraw-Hill Educational Series for Mathematics was undertaken by McGraw-Hill and Al-Obekan. Meanwhile, the MoE was responsible for creating an appropriate educational environment in which to implement this curriculum (MoE, 2006).

This curriculum was built based on the following ten philosophical principles: ‘learner centred learning, thinking skills development, collaborative learning, technology usage such as multimedia, learning through multiple inputs, exchange of knowledge and communication and representation in multiple ways, active learning based on exploration, development of decision-making skills, development of learners’ abilities to deliver planned initiatives, and linking the learner with real life contexts’ (MoE, 2006, p.18).

Moreover, this project aimed to develop the mathematics curriculum and supportive educational instruments (textbook, teacher guide, exercise book, assessment book, learning resources book, educational tools) similar to developed countries, particularly the US. It also wanted to attain access to the latest scientific research institutions and standards centres, plus evaluate studies in the field of mathematics development. For that matter, benefitting from international expertise and specialisation in producing educational tools to support students and educators was paramount. The project aimed to employ and integrate technology and its applications in the mathematics curriculum. It should also be mentioned that this project aimed to assist the professional development of teachers, educational leaders, and curriculum experts in KSA by means of the continuous support and growth of specialised international expertise. This was to be accomplished via training based on international standards and philosophy, on which the mathematics curriculum was built, as well as teaching methods, evaluation, classroom management, and by integrating technology in education. It aspired to enhance students’ learning in accordance with the principles of active learning and self-learning, along with promoting access to and the construction of knowledge (MoE, 2006).

Several aspects have been improved in regard to developing the mathematics curriculum. It focusses on problem solving by using George Polya’s four-step process for problem solving, and, paying attention to higher order thinking skills by including one question or more in each lesson to train students as to improve their higher thinking. In addition, it builds links between mathematics lessons and real life by incorporating a question or task in each lesson that is integrated with other sciences (Al-Ewasheq and Rafea, 2011). For instance, there is a section in every lesson, titled ‘real life problems’. Moreover, it concentrates on improving mathematical communication skills by encouraging students to write explanations of their investigations and discuss their thoughts (Al-Ewasheq and Rafea, 2011). As an illustration, each lesson contains a task that requires debate or an explanation. This curriculum concentrates on differentiating instructions, as it considers individual differences. Hence, it is evident that there are a variety of questions that are appropriate for students' levels. In addition to this, meaningful assessment occurs in this curriculum to enhance the learning process and offer students the opportunity to recognise their development and discover what steps they can take to improve. This is done via continuous assessment, which includes several types, such as diagnostic, formative, summative, and fixing common mistakes (Al-Ewasheq and Rafea, 2011 , Travis, 1996).

Development of the mathematics curriculum has covered several aspects, such as curriculum content, teaching methods, educational technology, and cognitive research. Firstly, curriculum content considers modernity and linking other cognitive aspects, as well as diversity and focusing on critical thinking and problem-solving skills that are linked to real life (scientific application, real contexts in relation to problems and exploration, etc). Next, teaching methods considers the harmony between teaching methods and the nature of mathematics and its teaching objectives, plus considering individual differences, besides students' needs and abilities which are essential. Thirdly, educational technology considers technical and educational levels, inclusiveness, and diversity. Finally, it emphasises cognitive research results that concentrate on learning styles and methods, differentiating learners and constructivism theory, active learning/effective learning, focussing on basic concepts and skills and the practice of meta-cognitive thinking (Al-Ewasheq and Rafea, 2010).

It should be acknowledged that the mathematics curriculum emphasises that the learner will achieve the knowledge and skills that he/she desires based on development of thinking skills, problem-solving, real-life applications, consideration of individual differences, communication skills, employing technology to improve the learning process, in addition to communication between families and societies and self-learning. In addition, it adheres to the principles and standards of mathematics related to the NCTM (2006) and the focal points which reflect the significance of mathematics topics for each stage. According to the NCTM (2006), these documents aim to provide one possible response to the question of how to organise curriculum standards within a coherent, concentrated curriculum by showing how to construct significant mathematical content and the connections identified for each stage. Each of these was translated into a series of textbooks to support teachers for assisting students to become mathematically proficient, according to standards set by the NCTM. Moreover, the purpose of creating the Mathematics Curriculum Series was to reflect on the results obtained from key research on mathematics instruction, instructional best practices, and curricular focal points (McGraw-Hill, 2012).

It is important to mention that the mathematics content covers the key mathematics branches. Mathematics textbooks include Numbers and Operations, Geometry, Algebra, Measurement and Data analysis, and Probability. Each of the education stages have the same curriculum content with different weights. For example, the curriculum in primary schools focuses on numbers and operations more than algebra, whereas, the mathematics curriculum in secondary schools concentrates on algebra (see Figure 1.1). Additionally, this series concentrates on the skills and subjects that students face difficulties in; for instance, problem solving, which is extremely challenging and taught at each stage (see Table 1.2).

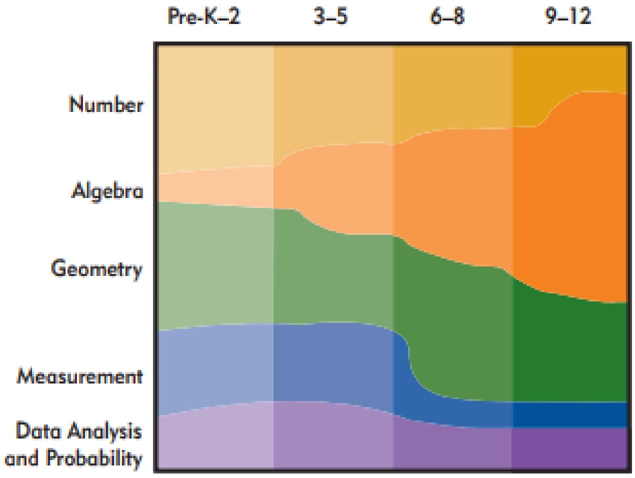


Figure 1.2 Curriculum content focuses on this theme across the stages (MoE, 2010)

Years 1-2	**Years 3-5**
Problem solving	Problem solving
Money	Ordinary Fractions
Time	Measurement
Measurement	Decimal Fractions
Ordinary Fractions	Time
Accounts	Algebra
Year 6-8	**Year 9-12**
Ordinary Fractions	Problem solving
Problem Solving	Fractions
Measurement	Algebra
Algebra	Geometry
Accounts	Accounts
	Statistics

Table 1.2 Skills and subjects in the mathematics series based on stages (Ministry of Education, 2010)

This curriculum is created to be vertically integrated from Year One to Year Twelve. This association involves the following three dimensions (Al-Shaya and Abdulhamid, 2011, Tang et al., 2010):

1. Vertically integrated in content design, which is important to help students in verifying the exact sequence of content and follow it from stage to stage. In addition, this helps to fill the gap and prevent unjustified repetition which enables the teacher to direct his/her teaching and suit it to the students.
2. Vertically integrated in relation to teaching design which can assist the passing of students throughout their education and make it more straightforward. The curriculum uses terms or vocabulary in each lesson and defines them, plus, offering technology, toolkits, and lesson plans, as well as methods in the teachers' guidebook. Each one of these can reduce difficulties and distractions.
3. Vertically integrated in visual design so that the series pages contain consistent visual designs from year to year. Therefore, students can be transferred smoothly and are more motivated to learn and be successful when they are familiar with the layout and content of the textbooks.

In support of teaching, this series includes a guidebook for teachers which is designed to be used as a core element. This guidebook includes a time plan, lesson plan, alternative lesson plan, enriching information, teaching methods, in addition to common mistakes and assessment methods for each lesson. Therefore, mathematics teachers only need to be prepared mentally and follow the lesson structure.

The MoE adopted a four-phase plan to begin teaching the mathematics series. The first phase is for Years One, Four and Seven. The second phase for Years Two, Eight and Ten. Phase Three is for Years Six, Nine and Eleven, while phase four is for Year Twelve. Each phase is divided into levels. The first level is the experimental stage which covered 110 schools in sixteen cities. At this level, they investigate the integrity and clarity of the sentences and its suitability for students plus checking whether the teaching suggestions and assessment methods are appropriate. In addition to this, teachers are trained and participate in training courses. In the next level, which was in 2012, the mathematics curriculum was generalised for all schools, whether state or private. This research is located in the generalised level in addition to the improvement in teaching practices.

1.4 Issues with the Developed Mathematics Curriculum

Since the implementation of the developed mathematics curriculum in 2010, many issues have been raised by teachers, mathematics education leaders, parents, students, and scholars. The issues related to developed mathematics curricula come from different aspects such as curriculum content, teaching methods, educational tools, and students' performance. These issues will be discussed based on the researcher's experience.

The students' mathematics textbook is the main element that expresses the curriculum content in KSA. The developed mathematics textbook offers a wide range of selected vocabulary. The diverse use of words to explain mathematical concepts and present ideas can help in developing students' mathematical communication and thinking skills. Likewise, there are questions in their textbook which ask them to explain by talking or writing how they solve the problem (Abedi and Lord, 2001, Alsalim, 2018, O'Keeffe and O'Donoghue, 2015). Nevertheless, the reading levels in the developed mathematics curriculum are higher than the students' level. Many teachers and parents have raised this issue since the curriculum began to be taught; In particular Year One where the students start to learn how to read. Due to the fact that kindergarten is not compulsory, students, parents and teachers discovered that the curriculum is difficult to read, which can create dissatisfaction amongst students with regards to mathematics. Furthermore, it is widely believed that problem solving is a core component of the mathematics curriculum that can support students to develop their thinking skills. When students engage in problem solving, they can improve their critical thinking and creativity, and thus, they gain skills for the 21^{st} century (Crimbricz et al., 2015). However, many teachers have been avoiding teaching problem solving or ignoring it for the reason that students have difficulty in reading and understanding the questions. Consequently, students may miss one of the significant skills in this curriculum.

Next, the lack of professional development for mathematics teachers is one of the fundamental issues related to the developed mathematics curriculum (Ali and Abdul Hakeem, 2013). Although developing teacher training programmes to train teachers in the developed mathematics curriculum is one core element of the mathematics curriculum, the way the teachers were trained was ineffective (Ali and Abdul Hakeem, 2013). One mathematical leader from each General Directorate of Education was trained by the Ministry of Education. After this individual has been trained, he/she subsequently trains every mathematical leader in his General Directorate of Education. Once their training is completed the leaders then train all teachers in their area. Experience suggests that there is an educational loss between each phase of teacher training. Additionally, the training programme was theoretical and did not give a clear picture of how to deal with the curriculum. Unfortunately, three years later the training programme was discontinued rather than developed, with the knowledge that a large number of teachers were not trained. This fact was mentioned in 'an informal conversation' I had with a representative of Al-Obekan Educational Company. As a result, the MoE and Al-Obekan failed to improve teachers' abilities and skills to implement the developed mathematics curriculum as it should have been.

It is important to say that the MoE failed to provide the developed mathematics curriculum resources. They failed to deliver the teacher guidebook, which is an extremely significant part of teaching the curriculum, besides other books that support mathematics teachers to teach and support the students' learning process. Additionally, some educational tools are not available, while the school environment was not prepared. For instance, several lessons require access to the internet which certain schools do not have, whilst computers are not available in every class. Generally, the shortage of learning resources in schools is one of the fundamental issues that accompanied the implementation of the developed mathematics curriculum (Ali and Abdul Hakeem, 2013, Hassan and Hamid, 2014, Ezz Al-Deen and Subahi, 2014). Moreover, the developed mathematics curriculum did not correspond to the exact time of the mathematics classes. Many of the teachers raised this issue, in particular, in relation to when they should teach all the lessons in the textbook.

Additionally, the same methods are constantly used to present the lessons, whereas it should be diverse from stage to stage. In fact, one of the weaknesses of the new curriculum is that it took a typical form and is always in the same teaching style for all school stages. For example, Years 1, 2 and 3 have the same teaching style and Years 4 and 5 have their teaching style, which means that the following three years also have their teaching style. Consequently, mathematics lessons can be uninteresting.

It is worthwhile mentioning that teachers taught the new curriculum using the same approach that was used to teach the old curriculum. Moreover, the teaching practices to implement the curriculum in the mathematics classrooms are not consistent with the philosophy and directions of the developed curriculum. In general, the research on teaching practices in the mathematics classroom is weak to average usually. It is especially obvious in geometry, which is the subject that enhances high order thinking and activity learning supporting the learning process and students' performance (Al-Dgain, 2013, Al-Eid, 2014, Al-Harbi, 2013, Al-Ony, 2011, Al-Rwais et al., 2013, Al-Shaya, 2013, Al-Yami, 2012, Kashan et al., 2013, Khalil and Al-Rwais, 2014, Alsalim, 2018). According to my experience as a mathematics teacher and teacher trainer on the developed mathematics curriculum many teachers find teaching geometry challenging, especially in primary school. This is because of the difficulty in providing good examples to make the geometric concepts easy to understand. Similarly, teachers continue teaching geometry using lecture methods without using tangible examples and tools to make the geometric concept visual for students and easy to understand.

Besides, one of the main reasons for changing the mathematics curriculum in KSA is to increase students' performance. The issue of poor performances in mathematics in KSA and the result of the TIMSS Test in 2003 is clear evidence of underachievement in relation to this subject. This low-level performance continued in 2007. However, four years later with the new mathematics curriculum in place, the results of the TIMSS Test in 2011 confirmed an improvement in students' performances regarding mathematics, when they Year Four obtained 410. Nonetheless, in 2015 the TIMSS results decreased to 383 along with 381 in geometry, which is lower than the benchmark (TIMSS, 2016). It is believed that the low score in geometric performance can be related to the teaching methods, since many teachers have found teaching geometry complicated, and they think it is higher than the students' level. Furthermore, students found it difficult given that teachers do not use the appropriate educational tools to explain the concepts, which in turn, makes them difficult to recognise.

1.5 Conclusion

This chapter has discussed the educational system in the KSA and its efforts to make it compatible with the Islamic religion, its geographical location, and its economic position, as well as its ambitions to make the country successful and a model of excellence on all fronts, according to Vision 2030. This has led to educational reform and students being prepared for universal competition. It also attempts to meet the needs of the labour market and society's values, besides equipping people with the skills required for the 21st century and thus, building a positive generation able to solve personal and national problems (Jenkins, 2008; Alshaya and Abdulhamid, 2011). Moreover, the international TIMSS (2003, 2007, 2015) showed the poor performance of Saudi students in mathematics.

Consequently, the Ministry of Education established a project to develop the national curriculum, in general, and mathematics curriculum, in particular. Developing the mathematics curriculum was based on international experiences that have proved effective in developing education. This curriculum designed by McGraw-Hill and the Al-Obekan Company depends on the NCTM 2000 standards and principles which cover each of the general education stages. The developed mathematics curriculum requires a more active role and additional engagement from students in the lessons than the previous curriculum. In addition, the improvement of the learning environment to stimulate creativity and innovation, improve teaching methods and improve students' values and skills are objectives that the MoE stated in the National Transformation Programme 2020 (p. 60).

Furthermore, educational technology is a significant feature of the developed mathematics curriculum. NCTM (2000) stated that technology is vital in teaching and learning mathematics; it affects mathematics, which is taught and enhances student learning. Therefore, each lesson in the developed curriculum has been linked to the curriculum website. Technically speaking, Dynamic Mathematics Software (DMS) such as 3D Capri, Sketchpad, and GeoGebra were intended to be part of the new curriculum; however, they were not employed appropriately when the curriculum was implemented because teachers were not trained. Hence, the MoE has recently adopted a project to train all mathematics teachers on using DMS and GeoGebra, especially in the mathematics classroom.

Nevertheless, mathematics teachers and students are still facing difficulties in a number of topics relating to the developed mathematics curriculum, especially in geometry. The TIMSS tests conducted in 2011 and 2015 are clear evidence of the continued poor performance of Saudi students in mathematics and geometry. This is for the reason that mathematics teachers continue teaching the developed curriculum in the same way as the previous one, in addition to poor usage of educational technology in the mathematics classroom (Ali and Abdul Hakeem, 2013). Al-Shmrany (2009) studied the 2007 TIMSS results and established that students who use a computer, either in school or at home, obtained better scores in the TIMSS than those who did not use a computer. Therefore, the current research aims to improve mathematics teaching methods which can help to improve students' performance, thinking skills, enhance the learning process and sustainable learning by integrating Information and Communication Technology (ICT) in the classroom. Accordingly, this research investigates the impact of using dynamic software (GeoGebra) to enhance the learning process and improve geometric performance, spatial thinking, and sustainable learning among primary school students.

The following chapter presents a review of the literature on the use of ICTs in education, both generally and in mathematics, as well as spatial thinking and its relation to mathematics learning.

Chapter 2. Literature Review

2.1 Introduction

This chapter will discuss the research underlying of the nature of learning and the constructivist learning theory concerning Piaget, Vygotsky, and Van Hiele theory. Then, the literature will continue by discussing the application of constructivism in teaching and learning mathematics. This is followed by discussing interaction patterns and using ICT in education from a constructivist preceptive. The discussion will ground the way for explaining students' attitudes towards learning mathematics and the possibility of using ICT to develop students' attitudes. The following section is going to discuss spatial thinking and its importance in mathematics education, as well as the different viewpoints for developing spatial thinking using ICT. The last section will overview ICT history and use ICT in education, generally, and mathematics education concerning dynamic mathematics and GeoGebra. In the end, this chapter will identify the research gap and the need for conducting this research.

2.2 Research View of the Nature of Leaning

This research aims to explore the impact of integrating GeoGebra as DMS into teaching interventions in relation to the learning process and outcomes. Moreover, it concentrates on the learning activities and the students' interactions with the physical and social environment. To do so, an understanding of the nature of learning is required. The current research has foundations in the understanding that learning is a complex concept, consisting of many elements that have an impact on each other, specifically physical, social, epistemic, activity, and outcome. These elements have been described by Peter Goodyear and Lucila Carvalho (2013, 2014a, 2014b, 2021) in Activity Centred Analysis and Design (ACAD), which is a meta-theoretical framework for understanding and improving complex learning situations (see Figure 2.1). The ACAD framework has been employed in several studies with learners of different ages. For example, ACAD has been used to explore the learning environment with ICT, in conjunction with university students (Ellis and Goodyear, 2016, Goodyear, 2000, Sun, 2018, Susan and Peter, 2020, Yeoman and Wilson, 2019, Sun and Goodyear, 2020). Likewise, it has been employed to understand learning process activity in a digital learning environment with primary school students (Thibaut et al., 2015, Yeoman, 2015, Yeoman, 2018, Yeoman and Carvalho, 2014). Therefore, the following sections will explain how this thesis regards the nature of learning, in light of the confidence in ACAD.

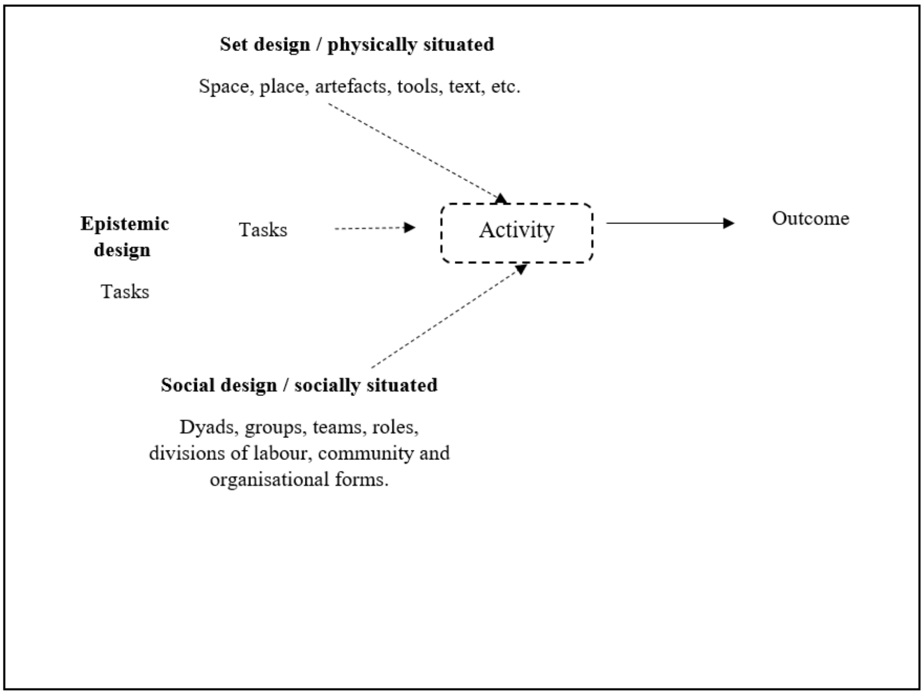


Figure 2.1 The ACAD framework (Goodyear and Carvalho, 2014a)

2.2.1 My Research's Understanding of the Nature of Learning

ACAD is an approach employed to design and understand learning situations, in which being activity-centred is the core, with the aim of understanding the nature of learning (Goodyear and Carvalho, 2014a, Goodyear and Carvalho, 2014b). According to ACAD, understanding the nature of learning and designing learning situations requires more consideration to be given to what students do mentally and physically, how they use the tools and resources, and how they interact with the social environment that develops in their activity, given that there is no learning experience without activity (Goodyear and Carvalho, 2014a, Goodyear and Carvalho, 2014b). Muñoz-Cristóbal et al. (2018) asserted that ACAD is an idea created to support understanding learning activity within complex learning situations and to forge connections between learning activities and design tasks. Goodyear and Carvalho (2014b) emphasised that ACAD's belief in the nature of learning helps study the relationships between learning outcomes and learning tasks, relationships between tools and resources and results, and between social relationships and learning outcomes. In other words, it is appropriate for studying both the process and learning outcomes.

Furthermore, considering the ACAD framework, the current research has foundations in the understanding that students perform learning activities to be dynamic and interactive physically, epistemically, and socially, which means the learning activity itself cannot be designed. Nevertheless, design can affect activity by means of proposed tasks and by framing the physical and social contexts in which the activities have to be performed (Goodyear and Carvalho, 2014a, 2014b; Muñoz-Cristóbal et al., 2018). Therefore, students construct their knowledge through learning activities and their engagement with the physical and social environment. Put differently, understanding learning requires that additional consideration should be given to observing what students do through learning activities and their engagement with the physical and social environment (Yeoman, 2015).

The current research supposes that the nature of a learning situation includes four principal components identified by Goodyear and Carvalho (2014a); specifically the structure (set design), task (epistemic design), social organisation (social design), and activity (see Figure 2.1). The ACAD framework distinguishes between these elements, which can be designed, together with the aim of producing particular emergent activity. Physical and social situated plus tasks are the elements that can be designed, whereas activity is an emergent entity that cannot be designed, although it can be influenced by students' interaction with physical and social design as well as learning tasks (Carvalho et al., 2016).

As the purpose of education is to help students to sustain their learning for an extended period and future development, it is believed that learning outcomes are divided into immediate outcomes and delayed outcomes. Furthermore, to make the ACAD framework fit more with the present research's belief in the nature of learning and the general purpose of education, the learning outcome was divided into two phases: the immediate outcome and the delayed outcome (see Figure 2.2). The immediate outcome refers to the assessment after each lesson and at the end of teaching the selected unit from the mathematics curriculum, whilst the delayed outcome refers to the late test that students perform in order to examine to what extent students can maintain their knowledge for a considerable time. Hence, approximately two months after the pre-test, students performed the sustainable learning test (see section 3.10.4).

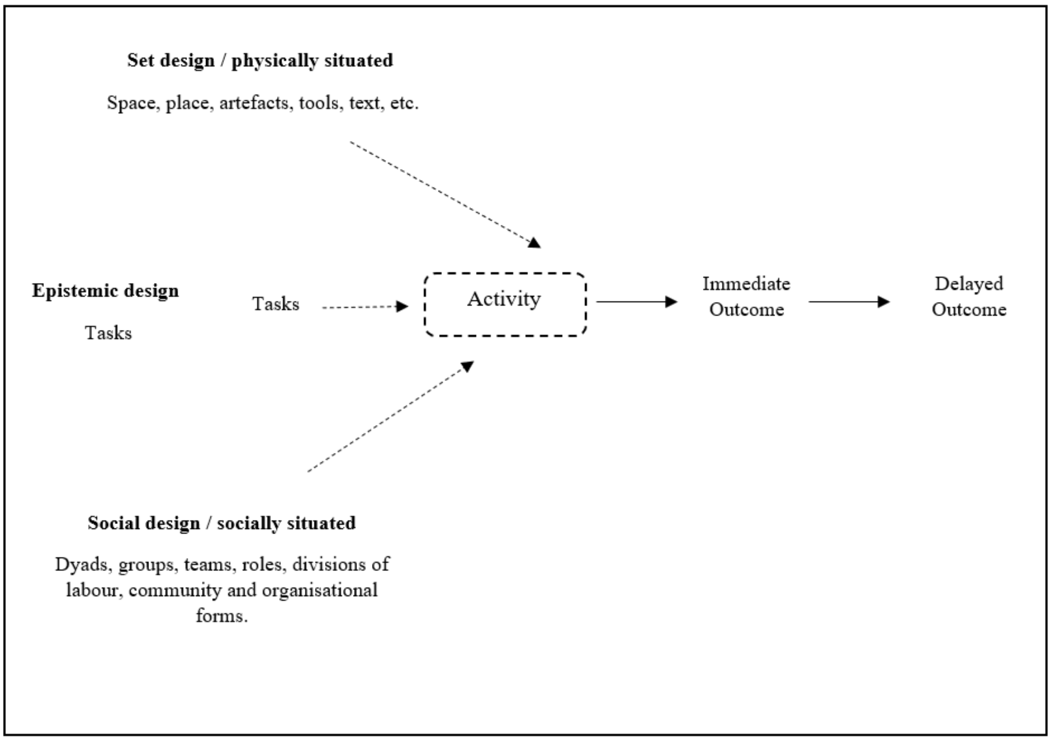


Figure 2.2 Developed ACAD framework

The following sections will present a description of the three elements of the learning situation set design, tasks, and social design for the present research.

2.2.1.1 Set Design

The current research assumes that set design or physical situated refers to all physical elements in the educational environment. This implies that set design may include a school building or a classroom in a school. It may include physical objects inside the classroom, including tools which come to hand, furniture (chairs, desks, tables, whiteboard), computers, books, notebook, pen, hands-on material, texts, a webpage, a word processor, e-print, along with software (Goodyear and Carvalho, 2014b).

In practice for the current research experiment, the structure of the set design means preparing the physical elements required to implement this research. Consequently, the set design comprises a school that is appropriate to implement the experiment with an IT room, a substantial number of computers, as well as furniture chairs and desks. Similarly, it includes DMS, namely GeoGebra, which had to be installed in the IT room computers. Simultaneously, the classroom environment of the other class participating in the research experiment required reorganising to be appropriate for learning in pairs and preparing the necessary hands-on materials required to perform the learning tasks. The third class, which is a traditional teaching group, does not need further preparation for the physical element as they studied in their normal daily mathematics classroom.

2.2.1.2 Epistemic Design

Epistemic design or task in the viewpoint of this thesis, refers to a recommendation on doing something that is worthwhile (Goodyear and Carvalho, 2014b). The tasks are divided into teaching and learning tasks. Learning task refers to recommendations of things to perform that the teacher often presents to students. Epistemic design, in relation to learning tasks, can include determining how to deliver knowledge, its selection, pacing, and sequencing, which can result in instructions in regard to doing something meaningful (Muñoz-Cristóbal et al., 2018).

Concerning the implementation of the present research experiment, the tasks are divided into teaching tasks that the teacher should have completed and learning tasks that students performed. Teaching tasks relate to teaching interventions with both GeoGebra and teaching with hands-on material. These tasks in the teachers' guidebooks are presented to both groups, while the learning tasks were introduced in the learning tasks textbook. Both the teacher's guidebook and learning tasks were prepared considering the geometric units of the Year Five Saudi mathematics curriculum in the school year 2019 – 2020 (see section 3.11); while traditional group learning task differed from GeoGebra and hands-on groups learning task.

2.2.1.3 Social Design

In the view of the current research, social design or social situation refers to an idea for a group arrangement. It can include factors similar to roles, dyads, groups, teams, divisions of labour, community, organisational forms, etc. (Carvalho et al., 2016, Goodyear and Carvalho, 2014a, Goodyear and Carvalho, 2014b). Put differently, it contains concerns about how students are socially formed when performing learning tasks and whether they are working in pairs, groups, or following scripted roles (Muñoz-Cristóbal et al., 2018).

Concerning the implementation of the current research experiment, in the initial stage of the teaching intervention, students worked in pairs in the GeoGebra group, sharing a computer and performing other learning tasks (see section 3.7). However, in the second stage, they were performing the paper tasks collaboratively and individually at times. Next, the teacher discussed the students' answers on the learning tasks to help students construct their understanding and move on to the next teaching stage, whilst in the last stage, students performed the learning task in each of the class group discussions with their teacher. Hence, the social situation diversified between pairs, individuals, and group discussions.

The hands-on group had the same social design as the GeoGebra group. Students in the hands-on group worked in pairs sharing manipulative materials and performing learning tasks in the first phase of the teaching intervention. After that, they performed the paper tasks collaboratively and individually at times. Similar to the GeoGebra group, the last phase of the hands-on intervention involved a classroom discussion with their teacher. However, the traditional group worked separately and was taught according to their usual mathematics classroom, as they were also different in terms of task/epistemic design.

2.2.1.4 Activity

Activity is the fourth dimension of the ACAD framework, where all the entities that have been designed are connected to what students do. Activity, in regards to this present research, refers to "what students are actually doing – mentally, physically and emotionally – during a period of time in which they are meant to be learning something (a learning episode or at learn-time)" (Goodyear et al., 2021p. 446). ACAD gives the activity a central position, focusing on what students really do in the learning situation. Carvalho et al. (2017) assert that the central location of activity in ACAD makes learning activity easy to observe, capture, and understand. This is because each student in the learning situation is doing something: experiencing, seeing, hearing, thinking, talking and reflecting. These activities are related to what they learn, which is influenced by the designed material, tasks, and social situation. It is important to note that through the diverse interaction between students and the physical, epistemic, and social input, as well as their response to the situations they encounter, we could learn something from them (Biesta, 2007). Therefore, the current research concentrates on what students had been doing during activities and how they use technology to learn, combined with student interaction and engagement during learning activities.

Thus, the researcher observed all the sessions in the experimental group (GeoGebra group), focusing on the learning process and students' activities in terms of the way they use GeoGebra to learn geometric concepts and skills and train their spatial thinking skills, as well as the students interaction. The researcher also monitored any improvements in the geometric performance and spatial thinking skills, session to session, regarding their learning processes and interaction with technology. This is for the reason that students learn by means of their activity, including thinking, making, discussion, writing, and reflecting. Furthermore, the learning outcomes are diverse and include the skills, knowledge and understanding of the concepts that are explained in class and improve abilities (Carvalho and Goodyear, 2018).

2.2.2 Conclusion

This section aimed to illustrate the underpinnings and foundations of the present research in the nature of learning. The reason behind this is that it provides conceptual ground to explore the learning process by examining the relationship between learning activities and the design structures within which it occurs. This belief helps connect observations in the classroom to teaching and learning processes using GeoGebra to learn geometric concepts. This confidence guided this thesis and its procedures, framing current research activity, and assisted the author of this study to achieve its objectives. Specifically, the developed ACAD was employed to guide the present research process from reviewing the related literature concerning constructivism, as an underpinning learning theory, to designing the teaching intervention and understanding the learning. Along with identifying the research gap, it focused on what students do and how they achieve learning aims in a complex learning situation. Subsequently, the author of this research started designing the research materials and preparing each of the elements necessary to conduct the research experiment, based on the three designable components of the developed ACAD: physical design, task and social design.

2.3 Constructivist Learning Theory

The process of learning and how it is interpreted has occupied educational and psychological scholars thought for a long time. Therefore, several theories have emerged to explain the process of learning. This began with Behaviourist Learning Theory in the early 20th century, based on the belief that learning occurs as a response to certain stimuli. However, behaviourism was unable to explain the most obviously language learning (Harasim, 2012). Therefore, the Cognitive Theory emerged as an extension of and reaction to Behaviourist Theory. Cognitivist researchers and psychologists conducted scientific studies to find out the power of the mind to influence or make decisions that are not directly related to a stimulus. This means they are concerned about what comes between the stimulus and response. They sought to understand the processes of mind which are rejected by the behaviourists. Cognitivism is concerned with mental processes, or in other words, modelling the psychological structure and the processes which operate in mind to explain behaviour (Pritchard, 2017, Harasim, 2012).

Nonetheless, Cognitive Theory came under criticism, and educational researchers began to reject the idea that humans always respond to material in the same way. Furthermore, the social reforming and civil rights movements around the 1970s had its impact on education. Then, Constructivism emerged to refer to a group of theories about learning that believed learners are much more active and involved in the learning activities with teachers and peers in creating (constructing) knowledge (Bélanger, 2011, Harasim, 2012, Pritchard, 2017). Constructivism, as a philosophy, concentrates on issues regarding the origins of human knowledge, as well as the development of individual understanding (Wilding-Martin, 2009).

Constructivism proposed that learners must actively construct their knowledge and understanding of the world by experiencing the world and reflecting on those experiences. This means learning is a process of creating meaning and how learners make sense of their experience. For example, when learners encounter new ideas, new things, and new perspectives, they have to reconcile the latest with their previous understanding and experience; does it fit with their prior knowledge? and if not, they maybe change what they believe, or discard the new idea as irrelevant, or integrate it into existing beliefs. Therefore, learners are active in constructing and creating their understanding (Bada and Olusegun, 2015, Bélanger, 2011, Harasim, 2012). Constructivism is based on the expectation that student learning is an interdependent process in which only the learner can actively construct personal meaning of the knowledge being acquired based on his or her cognitive developmental stages and his or her socio-cultural experiences (Piaget, 1971, Vygotsky, 1978a).

In Constructivism, learners construct knowledge through their interaction with society and the environment. Hence, learning is viewed as dynamic and changing, construed and negotiated socially (Bada and Olusegun, 2015, Harasim, 2012). It can be stated that it usually aims to encourage learners to be active and use active methods (experiments, real-life problem solving) to build their knowledge and then, to reflect on and talk about activity and how their understanding has changed (Bada and Olusegun, 2015). Constructivist pedagogy concentrates on creating situations and activities where students are encouraged and guided to construct meaning for themselves using such methods such as exploration and inquiry (Van De Walle, 2004).

Constructivism emphasizes two primary principles. The first is that learning is not passively received but actively built up by cognizing the subject. Learning is a process that requires active participation rather than passive observation. Learners deal with their understanding in light of what they observe in a new learning situation. Whether what learners encounter is or is not consistent with their current understanding, their understanding can be modified to accommodate new experiences. The second is that cognition plays an adaptive role and serves the organisation of the empirical world, not the discovery of ontological reality. Learners build a new understanding by using what they know. Learners come to learn new situations with information gained from prior experience, and that previous information affects what new or modified information they will build on for new learning experiences (Von Glasersfeld, 2013, Phillips, 1995).

According to Constructivism, learning is an active, continuous, and purpose-oriented process. The constructivist perspective is the most suitable learning paradigm when learners experience a problem related to their real-life situations. This learning process consists of reconstructing knowledge or an individual's knowledge in a social interaction. In this case, prior learning or experience is a prerequisite for meaningful learning. Therefore, learners construct meaning or understanding by creating relationships between new concepts and the other concepts that are part of the same existing framework of previous knowledge. Hence, learning is a dynamic process of making understanding or meaning and a life-long process (PÁYER SÁNCHEZ, 2005, Bada and Olusegun, 2015). Constructivism refers to a set of values about how students learn actively and are self-learners (Phillips, 2000).

The concept of Constructivism can be described from different perspectives. Samara and Al-Adili (2008) defined Constructivism Theory as a theory of knowledge in which individuals have their own understanding on the entities around them by combining their prior knowledge and beliefs with what they face about phenomena they observe. According to Phillips (2000), the concept of Constructivism, from an educational point of view, includes an interest in learners to have an active role in learning and the opportunity to redefine or discover new meanings for the things they interact with. Koohang et al. (2009) and Richardson (2003) define it as the theory of active learning and constructing new knowledge based on the student's previous experience.

Constructivism is a theory of knowledge that has roots in philosophy, psychology, sociology, cybernetics, and education. It can be described as a theory for philosophy of learning, and its concepts are not new, but go back to psychology and science in the eighteenth century. When Giambarrisa Vico published his treatise on the construction of knowledge, he expressed the idea that the human mind constructs expertise and knows only what it constructed itself. In other words, people know nothing that they have not made (Bada and Olusegun, 2015, Ernest, 1994, Ernest et al., 2016, Von Glasersfeld, 2013). Nevertheless, Piaget, who is known as the founder of the Cognitive Constructivism Theory, considers that all knowledge resulting from a psychological and biological related structure leads to the continuous creation of new knowledge; in fact, the individual learner understands the world, in terms of biological, developmental stages (Sillamy, 1983, Bada and Olusegun, 2015). On the other hand, Vygotsky is known as the founder of the Social Constructivism Theory, who believes learning and understanding occurs in a social environment (Bada and Olusegun, 2015).

Papert, somewhat similar to Vygotsky, contributed to the development of the Constructivist Educational Theory. He believes that knowledge remains fundamentally grounded in situations and formed by uses. The use of external aids and mediation is also crucial to expand the human mind's abilities at any level of people growth. Papert concentrates on the art of learning to learn and on the importance of making things in learning. He is concerned with how students engage in discussion with others and how this discussion boosts self-directed learning and helps in constructing new knowledge at the end. Papert emphasizes the significance of tools, technology, media, and context in human development (Ackermann, 2001).

No discussion of Constructivism would be complete without acknowledging the influences of Piaget and Vygotsky. In the following sections, a proper discussion of Piagetian and Vygotskian theories will be presented.

2.3.1 Piaget’s Theory

Jean Piaget is considered to be one of the most influential proponents of the Constructivist Theory of Learning. Piaget, in his theory, described the learning process as an assimilation, in which learners add new knowledge to their current structure, and accommodation, in which new knowledge causes cognitive conflict, resulting in reorganisation of knowledge structures (Huitt and Hummel, 2003). Piaget's studies focused on cognitive development and knowledge creation. They led to conclude that knowledge growth is the product of individual constructs produced by the learner, according to Huitt and Hummel (2003). Piaget developed his genetic epistemology of learning through a series of rigorous clinical case studies that focused on the individual learner and his/her cognitive development (Huitt and Hummel, 2003, Piaget et al., 1969). Piaget discovered structural changes in the production of knowledge and beliefs via his observations and documentation.

Consequently, Piaget identified four phases of learners’ cognitive development: sensorimotor, preoperational, concrete operational, and formal operational. Each stage is characterised by how people understand the world through observing and discovering the environment around them like ‘little scientists’ (Cherry, 2015). During the Sensorimotor stage

(0 – 2 years), infants develop their mental and cognitive attributes from birth to language emergence. In this stage, children construct their meanings by manipulating the world around them through using their five senses: hearing, touching, smelling, seeing, and tasting. Piaget divided this stage into six substages, having a specific time for each substage with a texture of actions such as hearing, visualisation, investigation, seeing, motor, or physical practice (Ghazi et al., 2014). This stage is characterised by the gradual development in acquisition of object permanence, in which children become able to find objects after they have been taken away out of their sight, even if the objects have been displaced from their field of vision. In addition to this, children at this stage have the ability to link numbers to objects; three dogs, one cat, four birds, for example. Hence, if children act in an open environment (but safe), this can help them start constructing their mathematical understanding. On this point, Fuson (2012) stated that evidence suggested that children at this stage have some understanding of the number and counting concepts. Therefore, nursery staff or parents should lay a solid mathematical foundation by providing activities involving counting and improving children's conceptual understanding of numbers (Ojose, 2008).

In the preoperational stage (2 – 7 years), children continue to increase their language ability, symbolic thought, logical reasoning, memory, and imagination. However, their thinking is still nonlogical, in a non-reversible manner, and from an egocentrically perspective (Huitt and Hummel, 2003; Ojose, 2008). Piaget explained that children at this stage acquire knowledge through imaginary play when they engage with problem-solving tasks by using available materials, such as blocks, sand, and water, and use their language to get other peoples' opinions (Ojose, 2008; Ghazi et al., 2014). The verbalisation of children and their actions on the materials can give a foundation that allow teachers to infer the mechanisms of their learning processes. However, children in this stage cannot think abstractly and in concrete physical situations. Their observations are generally restricted to one dimension or aspect of an object at the expense of the other elements.

On that issue, Johnson et al. (2016) pointed out that learners' understanding of numbers and geometry starts with concrete objects and interactions with peers and adults. Thus, effective questioning about classifying objects should be employed to teach children in this stage. For instance, when learners investigate geometric shapes, the teacher can ask them to put them in groups, according to similar features. After the investigation stage, they can ask questions, including "How did you make your decision as to where each shape fits in? Are there other ways to put the shapes in groups together?" When students engage in discussion or interactions with other students, it can provoke them to discover many ways to classify objects; and then, help students think about the quantities in novel ways (Ojose, 2008).

During the concrete operational stage (7 – 11 years), children start to think logically, organise thoughts coherently, and make an effort with abstract and theoretical thought (Huitt and Hummel, 2003; Ghazi et al., 2014). This stage is featured by loss of egocentric thinking and remarkable cognitive growth when children develop their language and dramatically acquire basic skills. They use their senses in order to understand and build their meaning. In this stage, children can consider two or three dimensions or aspects simultaneously rather than successively. In addition, they grow their ability to order objects according to length, weight, or volume. Also, their ability to classify objects in groups based on a common characteristic improves (Huitt and Hummel, 2003; Ojose, 2008). Huitt and Hummel (2003) addressed that children, in this stage, can perform concrete problem-solving and begin to understand reversibility. Johnson et al. (2016) stated that students master the underlying structure of numbers, geometry, and measurement. Using concrete objects is the foundation for developing mathematical understanding represented with pictures, symbols, and mental images. They learn to consider parts and wholes needed for infractions and division. Manipulations of objects and visual representation develops into mental images and operations as they internalise those actions. Therefore, using physical materials or visual representation in mathematics activities provides children with the opportunity to make abstract concepts concrete, allowing them to use these concepts, which can be useful tools for problem-solving. Using physical and visual materials helps learners acquire experiences that lay the foundation for more advanced mathematical thinking and constructs their mathematical confidence by giving them a method to examine and confirm their reasoning. This can let them construct meaningful understanding (Ojose, 2008).

It must not be forgotten that, the development of visual representation and mental images and operations led students to develop their ability to think spatially (Gray, 1999). Newcombe and Stieff (2012) and Cole et al. (2018) addressed that students begin to develop their spatial thinking through visualisations, develop their ability to understand topological representations, plus enhance their competencies in understanding projective and Euclidean representations since childhood. Piaget determined a number of spatial skills that improve over childhood to the concrete operational stage, including "the ability to use categorical (e.g., near and far) and metric spatial representations to describe spatial extent; facility at shifting between egocentric (viewer-dependent) and allocentric (viewer-independent) frames of visual reference, and skill at using symbolic spatial representations, including, maps, diagrams, and sketches"

(Cole et al., 2018, p. 3). This raises the importance of visualisation being most effective earlier in instruction because it supports concrete operational students learning abstract concepts (Newcombe and Stieff, 2012).

Accordingly, it is claimed that visualisation is critical as it provides concrete visual representations of mathematical concepts that assist students to construct their understanding. The main point, nevertheless, is that the association between chronological or developmental age and the capability to represent spatial relationships is complicated, and that there is often a way to provide young students with spatial materials in a manner that they will find helpful and that will prompt them to engage their attention while also developing their spatial skills (Newcombe and Stieff, 2012). Therefore, the current research aims to develop teaching intervention integrated with GeoGebra to help students aged 9 – 11 years visualise the mathematical concepts to construct their understanding and develop their spatial thinking skills.

In line with Piaget's cognitive developmental stages, people from 12 years onwards and through adulthood will be in the formal operation stage. Students at this stage are characterised by formulating hypotheses and systematically examining them to solve a problem. Cherry (2015) asserted that this stage includes growth in logic or sense, people at this stage are capable of exercising a deductive approach of thinking and an understanding of conceptual thoughts. On that matter, Ghazi et al. (2014) stated that learners in this cognitive stage expand their ability to understand and reflect upon abstract concepts and build up their ability to think logically, reasoning deductively and arranging systematically. Children at this stage can form hypotheses and deducing possible results, allowing them to construct their own mathematics, and develop abstract thought patterns where interpretation is performed using pure symbols without the need for sensitive data (Decano, 2017, Ojose, 2008). People at this stage think sophisticatedly about mathematics, involving proportional reasoning, and correlational reasoning. This begins and continues to develop during the teenage years and into adulthood. They consider all factors of a problem when they think, conclude, and examine hypotheses in order to solve them (Johnson et al., 2016). Thinking skills at this stage refer to the mental process included in generalising and evaluating logical arguments and involves clarification, inference, evaluation, and application (Ojose, 2008).

Despite the fact that Piaget proposed that, without exception, all individuals go through the four different stages of cognitive development, recent literature has shown that not all individuals reach the formal operational phase (Babakr et al., 2019, Martin, 2019). Cacioppo et al. (2021) stated that biological psychologists have proposed that young people, until the age of

20, cannot deal with complex calculations properly due to the limitation of their brain activity. Besides, studies demonstrated half of the learners in some societies approaching the formal operational stage owing to the lack of educational background and not focusing on critical thinking, which is essential to approach a formal operational stage (Babakr et al., 2019, Cole, 1990). On top of that, Adey et al. (2006) point out that many students enrolling in secondary school performed in mathematics problems well below what may be expected from their Piagetian cognitive development stages. This is ascribed to a failure in instructions in their primary schools where drilling in the "four operations" takes precedence over standing out of a problem and considering which kind of operation will be most profitable there. Adey et al. (2006) mention that the difference is between mathematics, as a descriptive language (concrete operational) for which primary schools well prepare their learners, and the act of thinking on the laws of that language, or on which mathematical model could be suitable for usage. Furthermore, sometimes learners can illustrate formal operational skills in a single area. For instance, a learner who is an excellent engineer can logically think about this specific field; however, at the same time, it is possible to have difficulty thinking logically about poetry (Martin, 2019).

Piaget believed that understanding and gaining new experience does not simply emerge from sensory knowledge; some initial structure is required to make sense of the world. Therefore, he defined four distinctive terminologies to how children proceed through the cognitive process: schemas, assimilation, accommodation, and equilibrium. According to Piaget and Cook (1952), schemas is: "a cohesive, repeatable action sequence possessing component actions that are tightly interconnected and governed by a core meaning." He described it as the fundamental constructing block of thinking and behaviour and the methods of organising units of knowledge, including action, abstract objects, or thought that children build to make sense of their interactions with the environment. Schemas can be thought of as files in which children store knowledge. Thus, each schema deals with all objects and events in the same way (Wadsworth, 1996). It is a set of joined mental representations of the world that children utilise to understand and respond to situations. These mental representations are stored to be used when needed. Piaget views thinking as an internalised action. People interact with and make sense of their surroundings, and this physical interaction becomes internalised to create thinking (McLeod, 2018).

Assimilation, on the other hand, is the cognitive process of integrating new information into current cognitive schemas, beliefs, and understanding. Assimilation is the procedure of incorporating new knowledge into current schemas or reacting to the environment using previously established patterns of behaviour or schemas (McLeod, 2018, Siegler et al., 2003). Accommodation refers to an organism's attempt to adjust or modify an activity or capacity in order to accommodate new knowledge or respond to the environment in a novel way if previously learned patterns of behaviour or schemas are insufficient. When a person's perspective of the world fits into pre-existing schemas, he or she is said to be in equilibrium. It is a continuous state of action in which a person compensates for system disruptions. There is disequilibrium when established schemas are unable to handle new experiences.

Piaget believed that learning is a lifelong process of assimilation and accommodation. People go through the cognitive stages and interact with objects, events, and other people in the real world, constructing their meaning for new experiences concerning previous experience and knowledge. The complexity of mental structures, or schemas, are represented by the unique understanding of how this world is working. While new experience is assimilated or taken into the mental framework, they are compared to existing schemas. If they do not resemble each other, they produce a state of disequilibrium. Disequilibrium ends when students reconcile new experiences through accommodation or by modifying their understanding. Embarrassment and making mistakes in the process of assimilation and accommodation are natural and fundamental parts of building a new schema (Johnson et al., 2016, Roberts et al., 2008). The construction of knowledge and the instruments to construct new knowledge are produced among the integrated networks, or cognitive schemas, as identified by Piaget. As students learn, networks within the brain are reorganised, added to, developed, or modified by reflective, deliberate thought so that learners can improve their current understanding (Fogarty, 1999, Huitt and Hummel, 2003, Lerman, 2014).

2.3.2 Vygotsky’s Theory

Lev Vygotsky’s works began in the 1920s and contributed to and complemented the beliefs of Piaget (Fogarty, 1999), Vygotsky believes that knowledge acquisition is a process of continuous self-construction. Individuals develop understanding through their actions and pass through phases of assimilation, accommodation, and equilibrium in the cognitive construction process. Piaget concentrates more clearly on cognitive constructivism and proposes that teachers should play a limited role in students’ learning. However, Vygotsky’s works confirmed the importance of social interaction through the cognitive learning process. Vygotsky’s theory, commonly called Social Constructivism or Sociocultural Constructivism, suggested that learning is an active process that involves a teacher or peer during the learning process (Wilding-Martin, 2009, Amineh and Asl, 2015). Hence, Piaget views the development of knowledge as the mental organisation of learner experience, while Vygotsky views the development of knowledge as social experience.

Activities of individuals play a crucial role in cognitive development, whether activities take place in a collective manner, or in a situation where the subject deals directly with the surrounding environment of objects. Besides, cultural mediation and grounding of understanding in activity presume the context specificity of mental processes (Cole, 2013). Vygotsky (1978b) believes that the mind is not a complicated network of general abilities. Nevertheless, it is a set of specific abilities, and learning is the attainment of many specialised capabilities for thinking. Furthermore, language plays an essential role in cognitive development; not just does communication and language afford the means for social interaction, but moreover they are tools for learning (Vygotsky, 1978b). In other words, language teaches individuals how they are supposed to act and provides them with the tools to formulate understanding and build their conceptual thoughts (Wilding-Martin, 2009).

Vygotsky views learning and cognitive development as collaborative activities in which learners and individuals develop their own cognitive skills via mediation and interaction between teacher and peer. Vygotsky (1978) established the Zone of Proximal Development (ZPD) on the belief that learners maintain an area within their brain for future learning. ZPD was defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development determined (Vygotsky, 1978, p.86). In accordance with Vygotsky, a learner must be able to be autonomous with a skill once he/she has been guided and instructed through the process before his/her autonomy. The ZPD theory stresses the need for guidance or tools through the learning process, mainly when students learn a new concept. This guidance can be a teacher or more capable peer who assists students in advancing their personal zone of learning, because they are challenged to think by a more advanced peer. This is the reason why it is assumed that "more competent peers" may be the best source of further help (Davydov, 1995, Bodner and Elmas, 2020, Treisman, 1992, Tudge, 1992).

Therefore, the interaction between teacher and students and between peers is an effective method to develop students' skills and strategies (Vygotsky, 1978b). Teachers use collaborative learning methods in which the teacher mediates the student's learning within ZPD. This means that Students require social interaction, scaffolding teaching, and the opportunity to engage with a more evolved learner, according to the social constructivist approach to Vygotsky's ZPD. Educators might scaffold instruction and learning to encourage collaborative processes that enhance and support students' cognitive development using this social constructivist approach to learning. A wide range of valuable mathematical connections is created through joint efforts and discussion, allowing students to make beneficial connections and structures within their mathematical learning (Ernest, 1998). Because of the connections created through their cognitive assimilations and accommodations, students develop a process of cognitive, social, and emotional interchange as they learn within their ZPD (Hausfather, 1996, Vygotsky, 1978b). When students are permitted to participate in this social process as part of their mathematical education, they encounter a sense of rationalisation and respect, which encourages them to learn more (Davydov, 1995).

Vygotsky suggests that when a student is solving a learning problem within the ZPD, it will provide suitable help enhancing the student's skill to succeed in the task. In other words, students will achieve a better quality of learning when the teacher mediates it than when they learn individually. Vygotsky's theory draws three principal conclusions. Firstly, social interaction plays a crucial role in the cognitive development process. According to Vygotsky (1978), individuals' higher functions emerge through genuine human connections. Children's development occurs on two levels: on a social level, between people, and, on an individual level. In other words, individuals learn initially through social interactions and subsequently, through an internalisation process that leads to profound knowledge on their own. Because of the links they discover with their own levels of development, children (and humans in general) grow in their learning through social interactions and connections with other people (Hausfather, 1996, Vygotsky, 1978b). This explains how social activities play a vital role in children's cognitive development. Secondly, although the significant other is typically someone with more information than the learner, such as a teacher or trainer, it can also be a peer, a young person, or anybody else who can help the learners improve their knowledge. Thirdly, the final conclusion is that learning occurs between the students' ability to perform a task under adult guidance or peer collaboration and their capability to solve the problem autonomously (Vygotsky, 1978b).

Vygotsky (1978b) believed humans are active, energetic participants in their existence and that at each phase of development, children learn how they can skilfully influence their surroundings and themselves. Vygotsky thought that children's playing was an essential component in conceptual understanding and that, when playing with others, children imitated adult actions and roles, which helped them acquire abilities for future purposes (Davydov, 1995). Vygotsky suggested that children's playing in the educational context did not disappear but rather reappeared during other learning, laying the groundwork for future knowledge and beliefs (Vygotsky, 1978b).

Vygotsky (1978b) stressed the significance of communication and speech, as part of this development. He stated that language not only helps children manipulate tools successfully, but also regulates the child's own behaviour. This development provides children with the ability to build connections through communication. As per Dangel and Guyton (2004), schools must form interactive classrooms that promote discussion and collaboration. Leaners' development relies upon the occasions to interact, communicate, and collaborate; hence, collaborative learning environments must promote social discussion with others to share, justify, and respect ideas and thoughts (Hausfather, 1996).

2.3.2.1 The Implications of Vygotsky and Piaget for Mathematics

It is essentially Vygotsky’s view that culture plays a significant role in the learning process because learners utilise tools established in a sociocultural environment. These tools assist students in developing higher levels of understanding and thinking skills. Vygotsky's theory demonstrates the significance of active participants who interact and discuss their thoughts, employing the available tools in their physical setting to develop their understanding and thus, enhance their learning process. In other words, learners construct their own understanding via collaboration with teachers or classmates who have better or more experience in a social learning setting. Frobisher (1999) and Hartshorn and Boren (1990) suggest that the use of available tools, either manipulatives or ICT tools in a mathematical classroom, can help build a social environment in which discussing mathematical ideas and sharing knowledge with each other would be easier, as well as providing them with opportunities to improve their social skills and increase their confidence.

Currently, in Saudi Arabia, teaching instruction is teacher-centred rather than students-centred, and students are almost passive. The current research explores the impact of integrating dynamic mathematics software into teaching intervention among primary school students. Teaching structure is learner-centred by active participation in their learning, through interaction with pair under the teacher's guidance, and with their teachers. The research sample of this research was Year Five students in Saudi Arabia aged between 10 to 11, which means they are in their concrete operational stage, according to Piaget’s cognitive development stages. Learners in this stage can use physical and visual materials in mathematics learning activities to make abstract concepts concrete.

Furthermore, according to Vygotsky's theory, the use of tools, whether physical or visual materials available in the mathematics classroom, help construct a social setting in which students interact with each other. Therefore, the present research intervention aims to employ dynamic mathematics software to help students visualise geometric concepts in a social environment where pairs discuss and negotiate their thoughts to construct their own geometric understanding. Since this thesis aims to improve students' understanding of geometric concepts, the following section will discuss Van Hiele's Theory, as Constructivist Theory that describes geometric thinking levels.

2.3.3 Van Hiele Geometric Thinking Levels

Van Hiele geometric thinking levels is another Constructivist Learning Theory that has gained acceptance in the field of mathematics education (George, 2017). According to this theory, learners construct their geometrical understanding through rearranging existing experiences that can be developed using appropriate teacher questioning (Sharp and Zachary, 2004). The theory assumes five levels of developing geometric understanding, which learners in sequence progress. Several identifiable learner actions characterise each level. This theory does not relate to chronological age, like Piaget's theory. Furthermore, Van Hiele concentrates on the didactic experiment to raise learners' thought levels, while Piaget created the scheme and psychological principles (Ma et al., 2015).

Van Hiele suggested a model that learners might learn geometry throughout a structure for reasoning. This model concentrates on the language and structure of simple axioms for primary and secondary school mathematics. It is built hierarchically and reflects five levels of understanding of geometrical concepts, which learners move through them to construct their knowledge (Burger and Shaughnessy, 1986, Duroisin and Demeuse, 2015, Sharp and Zachary, 2004, Decano, 2017, Wang, 2016). Vojkuvkova (2012, p. 72) asserted that the Van Hiele theory has three aspects: "the existence of levels, the properties of the levels, and the progress from one level to the next level". Decano (2017) and Vojkuvkova (2012) mentioned that there are different numbering methods noticed in the literature; however, Van Hiele talked of levels 0 through 4, where Level 0 is visual, Level 1 descriptive, Level 2 theoretical, Level 3 formal logic, and Level 4 is the nature of logical laws (Van Hiele, 1986).

Students learn at level 0 through *visualisation*. Van Hiele (1986) and Van Hiele (1999) stated that shapes are judged by their appearance. Students recognise a rectangle by its form, and a rectangle looks dissimilar to him than a square. Learners at this level can identify and distinguish figures and other geometric parts (e.g., angels, lines, grids, etc.) by their universal appearance. Learners can say square, rectangle, cube, triangle, etc., but they cannot explicitly identify the properties of the shapes (Ma et al., 2015). Learners at this level recognise the concept of geometric shape as a whole without respect to the properties of its components, which means they start to learn and understand the geometric shapes in general. Students at this level can put in groups the objects with similar forms (Decano, 2017, Fitriani et al., 2018). To sum up, students achieve this level when they are able to recognise shapes from their appearance; and they do not have to list the properties of the presented geometric shape. Therefore, mathematics teachers should provide students with an opportunity to classify shapes according to visual differences.

At the *descriptive* (analysis) level, students begin analysing and naming properties of geometric shapes. The geometric shapes are analysed empirically based on their components and properties (Vojkuvkova, 2012; Decano, 2017). They may make a list of all properties they perceive about the shape and use them to solve problems. However, they are not able to recognise relationships between properties and consider that all properties are crucial. They do not realise a need for proof of facts discovered empirically (Decano, 2017, Fitriani et al., 2018, Ma et al., 2015, Wijaya et al., 2019). Wang (2016) addressed that learner at this level order properties and deduce one from another; definitions of shape come into play, but they do not understand the meaning of deduction.

Moreover, they learn to use appropriate words correlated to properties but cannot link figures and their properties. Students at this level are able to measure, fold, and cut paper, and use geometric software (Decano, 2017). Learners must have the ability to think spatially about and analyse embedded components of geometric shapes before being able to make informal deductions, which is described temporarily. It is crucial to notice that the word analysis, according to van Hiele theory, differs from the common use of the word, which usually includes making conclusions or deductions (Sharp and Zachary, 2004). Duroisin and Demeuse (2015) pointed out that students reach this level when they succeed in classifying and abstracting some of the properties of geometrical shapes, although without establishing logical connections between them.

At the *theoretical level or informal deductions*, learners deduce some facts about the geometrical shapes due to prior experience about it. They cease to depend on visualisation (level 0), a property list, or empirical evidence (level 1). They use connections to conclude, stating outcomes without offering proof (Sharp and Zachary, 2004). Students order properties and deduce one from another at this level; one property leads or follows another property. At this level, the inherent meaning of reasoning is not understood by learners. They recognise a square as being a rectangle because, at this level, descriptions of shape come into play (Hiele, 1986). At the *informal deductions level*, the learner perceives relationships between properties and shapes. He/she creates meaningful definitions. He/she is able to give simple arguments to justify his/her reasoning. Learners at this phase can use grid paper, sketches, and geometric software to construct their understanding of the geometric concepts and build correlations between them (Vojkuvkova, 2012). To conclude, learners achieve this level when they are able to launch logical relationships between various properties of one or more forms.

When pupils can understand what a theorem is or construct a proof, it means they reach the level of *formal deduction*. At this level, they can draw deductive conclusions from general to more specific. They have the ability to distinguish between essential and sufficient conditions. They can identify which properties are inferred by others. Here, learners also start using the axioms or postulates to prove many things. However, they still do not comprehend why it is a postulate or a theorem (Vojkuvkova, 2012, Fitriyani et al., 2018, Wijaya et al., 2019).

At Level 4, the nature of logical laws or rigour, learners understand the approach of how mathematical systems are founded and would be enrolled in tertiary education in geometry. They can use all forms of evidence. They understand Euclidean and non-Euclidean geometry. They have the ability to describe the impact of adding or removing an axiom on a given geometric system (Vojkuvkova, 2012, Wijaya et al., 2019). Fitriyani et al. (2018) stated that rigour level is the highest level of geometric thinking levels, problematic, and complicated. Consequently, it is not surprising that learners still have not reached this level of thinking, even if they are already in high school or even college.

Mason (2009) and Watan (2018) conclude that there are five phases of learning that students should progress through each level of thought, according to Van Hiele theory, namely: inquiry/information, directed orientation, explication, free orientation, and phase of integration. In the inquiry/information phase, teachers make discussions with students in order to identify what they already know about the new concept which should be learnt. This activity aims to determine the students' existing knowledge of the idea discussed and then decide which instructions should be taken in the next lesson. Students become oriented towards the new concept or experience. Then, they move to the directed orientation phase when teachers design instructions to make students explore objects or problems in structuring tasks such as rotating, measuring, folding, or drawing. The teachers have to make sure that students explore implicit concepts. In the third learning phase, explication, they use their language to describe what they learn about the new idea while their teacher introduces the relevant mathematical concepts. In the following phase, which is free orientation, the teacher designs instructions to explore complex tasks for discovering relationships to solve a problem and more open-ended tasks. At the integration phase, which is the last stage, students summarise and integrate what they have learned through all the lessons, then reflect upon them to construct new knowledge.

According to the National Council of Teachers of Mathematics (NCTM), students in primary school are expected to deal with visualisation and detection (Level 0) and then be taught the identification of parts (Level 1). Students are expected to recognise shapes based on their appearance; for instance, a student is able to recognise a triangle, rectangle, circle, or other shapes based on their look. Students in higher primary school level should be able to deal with recognition and description (Level 2), and explain that a triangle has three sides, a square has four sides, and so on. They are also expected to deal primarily with an accurate description, utilising manipulatives or dynamic software to assist them in perceiving some mathematical concept by manipulating shapes and making causal inferences about transformation rules and solving issues (Level 2). Students are required to be able to explain how certain transformations occur. A student may, for example, list and compare the features of two figures, noting that the properties of a square are present in a rectangle and that a square is a rectangle, despite the fact that a rectangle is not a square. Students must obtain a Van Hiele's Level 1 understanding of geometry in primary school in order to achieve and understand geometry in intermediate secondary school (Crowley, 1987, Knight, 2006, Noh and Abdullah, 2016). This means that finding strategies to ensure that children grasp geometric ideas is critical to their understanding and success in mathematics.

Based on the above discussion and in light of the Saudi mathematics curriculum, it can be said that Constructivism Theory should be employed in teaching mathematics generally, and in geometry specifically. When a new mathematical concept is going to be taught, teachers should start from students' prior knowledge and concretise the new concept using available tools, and then gradually progress in teaching from concrete to abstract. Besides, encouraging students to collaborate and work together in small groups, providing them with a social environment where they interact with classmates and exchange their opinions with them is beneficial. In order to obtain that, teachers have to design learning activities in a way that encourages learners to solve problems, discover, think, and discuss, in which they accept and present students' views and conclusions, whether true or false, direct them to correct their mistakes, and reach the correct answers based on what they have learned from new experiences.

Furthermore, teachers have to choose teaching strategies that allow students to reconstruct the mathematical content for themselves and by themselves and explore the interdependence between the different mathematical branches. This also provides learners with learning activities that allow them to use the new experiences gained in a new situation by using manipulative materials and appropriate ICTs, promoting the employment of Constructivism Theory in teaching mathematics.

2.3.4 Assumptions of Constructivist Theory

In spite of the fact that there are different points of view for Constructivism, most Constructivist Theories share certain assumptions that reflect the features of Constructivism in detail as a learning theory (Loyens et al., 2007, Vrasidas, 2000, Brown, 1998, Carwile, 2007). The above discussion and reviewing the literature concluded the following assumptions of the constructivist theory that this research draws upon.

2.3.4.1 Knowledge is constructed

Constructivism emphasizes that knowledge is constructed, not transferred. Therefore, learning is not merely acquiring information by the learner. It implies a deep understanding of the subject matter. Knowledge is constructed within the learner's head rather than being transferred and copied there by someone else. Learning is not only about gaining new experiences and knowledge. It involves raising awareness and sensitivity of the learner to the way in which his or her activities and ideas can contribute to creating a more flexible and adaptive knowledge structure to the world. That is, knowledge is constructed with experience; therefore, learning is a constructive process in which the learner constructs an internal display of expertise, in which, a deep understanding of the learner is achieved rather than the acquisition and accumulation of knowledge. Learning, thus, becomes a continuous innovation process in which the learner reorganizes his or her experiences to reach a deep understanding (Jonassen et al., 1995, Vrasidas, 2000, Loyens et al., 2007).

2.3.4.2 Learning is an active process

Learning is an active process in which learners make mental or physical effort to discover knowledge themselves. Learners participate and interact with the surrounding (physical or social) environment to construct their meaning and understanding. That is when learners encounter their knowledge in light of what they meet in the new learning situation. If what learners meet is incompatible with their understanding, they can change their understanding in order to accommodate new experiences (Bada and Olusegun, 2015, Jonassen et al., 1995, Bélanger, 2011). Hein (1991) pointed out that learning is an active process in which learners use different sensory inputs and create meaning out of them, and involves the learners engaging with the real world. Constructivists engage the learners so that the knowledge they construct is not inert, but can be usable in other situations (Jonassen et al., 1995). Learners remain active in the learning process, wherein, they use their prior knowledge to construct their meaning and understanding of the new learning situation. Thus, learners actively seek, construct, discuss, and adapt their knowledge, skills, strategies, and beliefs (Schunk, 2018).

2.3.4.3 Collaborative Learning

One of the significant assumptions of the Constructivist Learning Theory is collaborative learning. This learning involves learners reconstructing their knowledge through negotiation and interaction with others (Loyens et al., 2007). Cooperative learning can lead to stronger social solidarity, enhancing mental processing like cognitive elaboration. Furthermore, reflective response and collaborative construction can be fostered by collaborative learning (Huang, 2002). Also, cooperative learning can help learners to improve their social skills, and interpersonal skills, if instruction uses it properly. Moreover, according to Zhan (2008), collaborative learning can enhance learners to participate, interact, and work together to achieve the learning objective and increase the satisfaction level and feelings of connection and society.

Learners in collaborative activities have the opportunity to think and reflect on what is happening and being learned. Through sharing the new knowledge with other learners in their group, they can interact and get feedback from others. With the new experience, the learners can participate in constructing new knowledge based on previous knowledge (Merriam, 2020). Therefore, learners' participation in collaborative learning and towards becoming active learners is heavily influenced by learners' prior knowledge, values, beliefs, rewards, and physical and social elements that form a learning environment (Brown, 1998). Students' interactions, plus the interactions between the new knowledge and learners' prior knowledge, are vital components of meaningful learning. Because these interactions can help learners construct deep understanding and a meaningful knowledge system, the previous knowledge may serve as a bridge over which new experience crosses the learners' mind and may be an obstacle to the passage of new knowledge into their minds (Driscoll, 2005, Bada and Olusegun, 2015, McIlveen and Schultheiss, 2012, Jeffery-Clay, 1998, Amineh and Asl, 2015).

2.3.4.4 Using authentic problem enhance constructing meaningful learning

The best learning situation that can promote meaningful learning occurs when learners face a problem, or a task related to their real world. Constructivists believe that meaningful learning can be promoted by using authentic problems related to real-life that present the natural complexity of the real world (Applefield et al., 2000). The learning environment invites learners to interact with others to engage in problem solving and learning inquiry. Combining complex real-world problems and social context can encourage learners to discover new meaning or revise previous meaning to construct their knowledge and obtain a deeper understanding of the concepts (Applefield et al., 2000, Mattar, 2018). When a learner is confronted with real problems or tasks, this creates the best conditions for learning, because it helps the student to build a sense of what he/she has learned, develops confidence in his/her ability to solve the problems he/she is facing, and earns him/her viable strategies. The learner is observing and negotiating by using his/her prior knowledge related to the new experience in order to construct his/her meaning and understanding (Symeonidis and Schwarz, 2016). Hence, this can encourage learners to take responsibility for their learning and metaphoric thinking, learn about their metacognitive processing, and comprehend the complexity of their thinking (Meyers and Feeney, 2016).

To conclude, recalling previous knowledge is the foundation of the new learning experience. Learners recall their prior knowledge when they confront learning activities or problems that are related to their real life. Learners collaborate, negotiate, and use the physical and social environment to find a solution for the problem. According to the ACAD framework (see section 2.2), students performing learning task activities, in a way, have to be dynamic and interactive, employing physical, epistemic, and social settings to reach a solution for the learning task problem (Goodyear and Carvalho, 2014a, 2014b; Yeoman, 2015). Hence, each learner constructs his/her meaning and understanding, which means knowledge is constructed in an individual’s mind, not transferred by others. Thus, learning is a continuous active process in which prior learning experience is the basis for the new learning experience. The following section will discuss the implications of Constructivism in mathematics education.

2.3.5 Constructivism and Mathematics Education

There is a strong relationship between mathematics and Constructivism, given the nature of mathematical knowledge with the cumulative associated structure. Simon (1995) and Wilson (1994) emphasise that mathematics is more consistent with Constructivism, and it is consistent with mathematics more than other learning theories. As a result, many empirical and theoretical works in mathematics education that have been contributing to influencing mathematics reform efforts, are based in Constructivism. For instance, the NCTM standards and principles (1989; 2000), which form many mathematics curricula in the USA, is grounded in Constructivism. In the UK, many educators believe that Constructivism has considerable implications for mathematics education (Jaworski, 2002). Furthermore, in this respect, Constructivism has had a considerable usage in KSA since implementing the Saudi developed mathematics curriculum, which was borrowed from USA and grounded in Constructivist Theory. It is indeed, globally considered as a theory that has much to offer mathematics education.

Mathematics is linked to a philosophy that supports active and meaningful learning, ensuring learners' interaction and visualisation to obtain deep understanding. Learners construct mathematical knowledge based on meaningful experience, leading to strength in their mathematical thinking (Díaz, 2017). Additionally, students construct new mathematical knowledge by reflecting upon their mental, visual, and physical actions. Concepts are built or made meaningful when learners engage in mathematical activities related to their real-world (Clements and Battista, 1990). Thus, learners can believe that they own the mathematics they learn when constructing their own mathematical understanding (Ellerton and Clements, 1992).

Cobb (1988) claimed that despite Constructivism having many different interpretations, there are two principal purposes for using its implications in mathematics instruction. Firstly, mathematical structures which are more abstract, complex, and powerful should be developed by learners so that they are increasingly capable of finding solutions for a wide variety of meaningful problems. Secondly, learners ought to become independent and self-motivated in their mathematical activities. Thus, such learners consider that mathematics is a way of thinking about problems. They believe that they are able to get their mathematical knowledge from their explorations, thinking, and participation in discussions, rather than receiving it from their teacher. Therefore, they become responsible for their learning in the mathematics classroom and making sense of, and communicating about mathematics, not just so much as completing given tasks. Such autonomous learners feel dominant in constructing mathematics (Clements and Battista, 1990). Consequently, Constructivist mathematics instruction gives learners the opportunity to be actively engaged in the teaching and learning process and takes into account the individual differences between learners.

Mathematics can be seen as a social construction. Ernest et al. (2016) argue the social nature of mathematical knowledge in which learners use their language, rules, and agreements, which play a vital role in explaining, discussing, and justifying mathematical concepts and understanding. Thus, interpersonal social processes are essential to turn learners' subjective mathematical knowledge into accepted objective mathematical knowledge. Accordingly, constructing a new mathematical concept begins from subjective knowledge built by the learner, via reforming personal information to objective knowledge by negotiating, visualising, and collaborating in the mathematics learning activities. Objective knowledge is reconstructed during learning mathematics by an individual learner to become subjective knowledge and reusable for a new learning experience. Mathematical knowledge is constructed through conjectures and refutations in a social learning environment, employing the physical set to perform learning tasks (see Chapter 3 ACAD framework).

Additionally, the Constructive approach also directed mathematics teachers to listen to their students, speak less than their students, and create and organise attitudes that allow students to build understanding and mathematical knowledge. Therefore, learners become more active and engage in constructing their mathematical knowledge. They investigate and crystallise their mathematical knowledge more deeply when they produce their questions, representations, and problems, and they are better than when given ready-made facts (Fried, 2006). In Constructive approaches, learners are encouraged to refine their own methods to solve problems through interacting with mathematical tasks and other learners and visualising their thought instead of copying others' thinking. The learners' conjectural mathematical thinking, increasingly, become more abstract and meaningful this way (Bhowmik, 2015). Hanson and Sinclair (2008) stated that Constructivist teaching methods aid learners to build a deeper understanding of mathematical concepts, which are better connected with practical experience, and support them to improve their skills in performing the routine problem-solving tasks of their lives. In addition to this, Constructivist approaches help learners develop their knowledge constructing capacity, competencies, and dispositions for engaging in collaborative problem-based inquiry.

2.3.5.1 Collaborative learning and Mathematics

Collaborative learning has become popular in mathematics education since the 1980s. Collaborative learning is founded on the belief that knowledge is constructed culturally while people communicate and exchange experiences, thoughts, and information (Lahann and Lambdin, 2020, Dillenbourg, 1999). Collaborative learning is defined as the situation where two or more students learn something together (Dillenbourg, 1999). It comprises a group of students who learn by sharing ideas, solving problems, or achieving common objectives. The fundamental assumption of collaborative learning is that participation in learning task activities is crucial to learning; therefore, the more learners participate in task activities, negotiations, and discussions, the more they are anticipated to learn (Leidner and Jarvenpaa, 1995). From a social view, collaborative learning is preferred to individualistic learning since it allows learners to develop interpersonal attitudes positively, encourages active participation, and a feeling of society (Grabinger et al., 2007, Milrad, 2002). Furthermore, collaborative learning activities offer occasions to explore various viewpoints and improve communication skills.

On the other hand, the cognitive perspective of collaborative learning is connected with enhancing personal achievement. Learners are more likely to develop critical thinking through assessing, reflecting, clarifying, and discussing for or against different perspectives. In addition, learners tend to illustrate a higher level of thinking when they are actively learning in groups more than when they are learning individually (Dillenbourg, 1999).

Researchers in mathematics education have recognized the potential of collaborative settings for learning and development in classroom settings. Collaborative learning has been demonstrated in research to benefit students learning mathematics. Collaborative learning improves students' thought by expanding their range of thinking through shared understanding and ideas (Davidson and Worsham, 1992). Johnson et al. (1981) discovered that collaborative learning encourages using higher quality thinking strategies and constructing new ideas and resolutions in a meta-analysis of 122 studies on the effectiveness of collaborative learning in mathematics classrooms at multiple grade levels. They also discovered that the mathematical knowledge acquired in the group translated effectively to tasks completed by the students individually. Furthermore, Slavin (1991) conducted a meta-analysis of 99 research, using collaborative learning in mathematics classes at different proficiency levels and concluded that it successfully increased student achievement.

Whicker et al. (1997) also investigated secondary mathematics students, and the results showed that students who participated in collaborative learning groups performed increasingly better on exams than those who learnt independently. Webb (1982) added to this knowledge, discovering that collaborative learning promotes higher-level thinking skills in mathematics, while Hagman and Hayes (1986) noticed that it fosters high success in mathematics and class attendance. Davidson and Kroll (1991) state that there is evidence that collaborative learning in mathematics increases self-esteem, increased attempts to accomplish, improved psychological health, and the capacity to accept another's point of view. On the same front, Barham (2002) found that collaborative learning strategies transfer focus from teacher-centred to the student-centred learning context, enriching cognitive, competitive, and social interaction and developing affective, motivational, and social outcomes. Results also demonstrated that collaborative learning developed students' interaction, communication, social skills, and built more positive attitudes towards learning than traditional methods.

Furthermore, in a meta-analysis conducted t by Ginsburg-Block et al. (2006), the results indicated that other advantages to collaborative small group learning involve social, self-concept, and behavioural outcomes. The same investigation also demonstrated a positive relationship between enhanced social and self-concept outcomes and improved student performance. Moreover, a study carried out by Moreno-Guerrero et al. (2020) showed that collaborative learning causes developments in the attitudes towards learning mathematics and improvement in geometry performance. On that note, Moreno-Guerrero et al. (2020) state that collaborative learning improves interactions and communications among students, encourages peer communication, and improves attitude, motivation, and feeling of community. It also encourages the students' state of mind and independence, actively engaging them in their learning process. In the field of mathematics, collaborative learning prompts students to improve their attitude towards the teaching and learning process and their ability in problem solving and study skills (Moreno-Guerrero et al., 2020).

Other research has found that collaborative situations might actually be detrimental to learning. Barron (2003), for instance, looked at small groups of students working on mathematics problems and found that in collaborative situations, smart students frequently fail to coordinate their activities and solve the issues. Following Good et al. (1992), merely putting students in groups will not enhance student learning; rather, the quality of group activity is crucial. Cobb and Bauersfeld (2012) and Cobb et al. (2001) demonstrated that the small group setting is critical for embedding social or socio-mathematical standards and developing mathematical thinking. However, the collaborative setting is not adequate in and of itself, and a body of research shows that for group discussions to be fruitful, students need to share their thoughts and justify them, exploring various points of view, and resolving them to establish group consensus (Waite, 2012, Howe, 2009, Mercer and Howe, 2012, Mercer and Littleton, 2007). Light et al. (1994) emphasise the significance of the quality of group interactions and creating a shared thought through discussion.

2.3.5.2 *Mathematical Dialogue*

The importance of interaction within the mathematics classroom is a fundamental principle of many Constructivist learning theories. Social interactions among learners, and, between learners and the physical set in the classroom, play a crucial role in understanding development. Students learn through their interactions with their classmates, and also teachers. Language is one of the essential tools available for meaning-making and constructing mathematical knowledge (Ernest et al., 2016, Staarman and Mercer, 2010). Over the last few decades, education policy has prompted the development of dialogue as a teaching and learning tool (Jones, 2017). Wilfred (1982) argues that mathematics instruction at every level should include occasions for negotiation between, teacher and students, and among students themselves. Ofsted. (2008) affirms that most classes in mathematics emphasise insufficient speaking, and therefore, children struggle to articulate and improve their thinking. Likewise, the Department of Education's 2014 National Curriculum Framework for Mathematics (DfE, 2014) states that instructors should see that students develop sustained foundations through discussions to investigate and correct their misunderstanding. Furthermore, the Saudi mathematics curriculum includes a section at each lesson (talk, write, discuss, or explain) to encourage students to interact with teachers and peers.

Dialogic talk in education strives to improve knowledge and transform understanding via interaction and reflection, particularly, through the participation of multiple views and discussions (Staarman and Mercer, 2010). The employment of discussion in a mathematical classroom, as a learning tool, provides many opportunities to improve mathematical learning, such as affording a critical method for learners to share their thought and knowledge. At the same time, it helps teachers evaluate their students' understanding and facilitate the group discussion of meaning(Cobb and Bauersfeld, 1995). On that regard, Sfard et al. (1998) differentiate between three types of mathematical dialogue: mathematical dialogues in the process of creating mathematics, mathematical dialogues between a teacher and student, and mathematical dialogues between children.

The mathematical dialogues in the process of creating mathematics, referring to this type of talk, are frequently held in journals, conferences, etc. The mathematical community accepts this particular style of speech. Mathematicians have agreed on the norms of argument and are unique to the official mathematical language. In contrast, from the Constructivist standpoint, the mathematical dialogues between a teacher and students, offer a main route for instructors to learn and understand the thinking of the students and have a real discussion. These dialogues are between students in which the learners seek to justify their methods. This is in some ways parallel to the discussion of the mathematicians. In the process of education, teachers could benefit from this type of communication, if the rules and structures of the learning tasks were explicit. Teachers may enable students to understand effective argument in mathematics and how it is comparable to an ethical or aesthetic argument, or distinct from it.

Furthermore, in a study conducted to explore students' collaborative activities with computers, Mercer (1995) and Wegerif and Mercer (1996) distinguish between three types of talk and activity that can be found in the classroom: disputational, cumulative, and exploratory. Disputational activity is distinguished by disagreement and individualised decision making. There are few efforts to combine resources or to give constructive criticism of ideas. Disputational talk also has some distinctive dialogic features; short exchanges consisting of confirmations and challenges, or counter confirmations. Cumulative dialogue, however, involves more collaborative activity of constructing a 'common knowledge' in which students work together to construct positively, but uncritically via repetitions, confirmations, and elaborations of what the other has said. The last type of dialogical interaction is the exploratory discourse, in which learners engage with each other's ideas in a critically constructive way. In exploratory collaborative activities, students offer statements and suggestions for joint consideration. In contrast to the other two dialogic categories, knowledge is made more publicly accountable in exploratory talk, and reasoning is more evident in the discussion. Then, improvement emerges from the final collective agreement reached (Mercer, 1995). Solomon and Black (2008) conclude that accessing ‘exploratory dialogue’ really makes a difference to the improvement of active mathematics learners compared to the sort of learner identity we may otherwise observe in many of the ‘conventional classrooms’.

Despite teachers usually dominating the classroom discussion, studies suggest that learning through talk can positively impact students' learning and enhance their motivation for learning (Myhill, 2006, Chapin and O’Connor, 2007, Obrycki et al., 2009, Jansen and Middleton, 2011). However, Mercer and Sams (2006) found that conversation that occurs in primary schools, when students work collectively, can often be uncollaborative, unbalanced, off-task, and sequentially unproductive. Notwithstanding the overwhelming evidence that proves dialogue has a vital role in primary school classrooms, it has been indicated that this is not necessarily an effective learning approach (Kutnick and Manson, 2002, Williams, 2008). This might especially be the case if students are not invited to engage in meaningful discourse by the nature of the given task. This demonstrates the importance of structuring learning tasks that support collaborative learning and direct students’ interaction, which also require designing physical and social sets in a way to help students engage in learning actives (see section 2.2.1 ACAD framework).

Stein et al. (2008) and Rabel and Wooldridge (2013) point out the significance of careful and purposeful structured learning tasks to guide classroom dialogue for helping students engage in meaningful discussion and maintain collective activities. Chapin and O'Connor (2007) emphasise that mathematical dialogue must be academically productive, which supports learners developing their reasoning and abilities to discuss their ideas clearly (Chapin and O'Connor, 2007). Fawcett and Garton (2005) found that students with lower skills working with higher skills had the main advantages of collaborative interaction. However, Light et al. (1994) found that all children can take advantage of group discussion opportunities, provided they work in equally able partnerships since these groups ensure that the standing of power is more even and that children have more chances to learn from each other.

Several interventions in the USA, the UK, and Europe have shown that it is possible to improve effective mathematics classroom discussion, promoting student action. However, reviewing the literature could not find studies exploring students' collaborative interaction in a mathematics classroom in KSA. Therefore, this research investigates the impact of teaching intervention on pair interaction patterns in a Saudi mathematics classroom.

2.3.6 Interaction Patterns

Despite the fact that collaboration is usually valuable for promoting learning and performance outcomes, several factors can influence the extent to which collaboration demonstrates fruitful results. The nature of interactions amongst group members is one of the crucial factors. On this line, dialogue of the whole classroom or small group learners is a fundamental social situation that allows learners to engage in different types of interactions. Several studies have revealed various distinct patterns of interaction among collaborators in group situations, with certain patterns being more conducive to learning than others (Li and Zhu, 2013, Liu and Tsai, 2008, Brooks, 1990, Ahmadian and Tajabadi, 2017). For example, the analysis conducted by Damon and Phelps (1989) identifies three categories of peer interaction: peer tutoring, cooperative learning, and peer collaboration. These three types of interaction contrasted with one another along the dimensions of equality and mutuality of engagement. They found that peer tutoring produces conversations with a low level of equality and a wide range of mutuality; cooperative learning encourages discourses with a high level of equality and a low to moderate level of mutuality, and peer collaboration fosters dialogues with a high level of both. Equality refers to a setting where "both parties in an engagement take direction from one another rather than one party submitting to a unilateral flow of direction from the other", and mutuality refers to a situation where "the discourse in the engagement is extensive, intimate, and connected" (Damon and Phelps,1989, p. 10). In other words, equality describes students' participation in the learning tasks, with high equality revealing more or less equal contribution by each member of the group and low equality revealing unequal participation.

Mutuality demonstrates the dialogue in the engagement, with high mutuality showing extensive interaction and intimate dialogue and low mutuality showing limited, disconnected dialogue where each student could not publicly make known his/her thoughts (Tao and Gunstone, 1999). Teasley and Roschelle (1993) state that collaborative interaction consists of two parallel actions, addressing the problem jointly and creating a Joint Problem Space (JPS). Collaborators' shared knowledge framework assists problem solving by combining objectives, descriptions of the present problem status, awareness of the problem-solving action available and related goals, characteristics of the current problem status, and actions accessible. These activities inevitably co-exist. The process of constructing and maintaining a JPS is the dialogue in the context of problem solving. The JPS is the structure that allows meaningful discussions on problem solving to occur simultaneously. Soller (2001) distinguishes three patterns of interactive conversation, active learning, and creative conflict, that enhance the effectiveness of peer performance.

Storch (2002) carried out one of the first investigations based on the work of Damon and Phelps (1989) to explore the nature of the connections between students when working in pairs. This longitudinal research was classroom-based among university ESL students and focused on the data of ten pairs over an entire semester. Storch (2002) found that the ten pairs of students in her study developed different kinds of relationships. Such relationships, when established, tended to continue, despite tasks or the passage of time. Significantly, the relationships students made affected the language learning opportunities that collaborative learning tasks provided. Storch (2002), qualitatively analysing the data of the pair talk, developed a model of pair patterns for interaction. This model distinguishes between four patterns of pair interaction: collaborative, dominant/dominant, dominant/passive, and expert/novice. These four patterns are represented by four quadrants formed by two intersecting axes: equality and mutuality. Storch (2002) explains that the two axes should be seen as a continuum for each. The horizontal axis of equality ranges from low to high. The vertical axis represents mutuality, ranging from low to high reciprocity. In addition, the point of intersection describes a moderate level, rather than a zero, since both axes represent continuum (Figure 2.3).

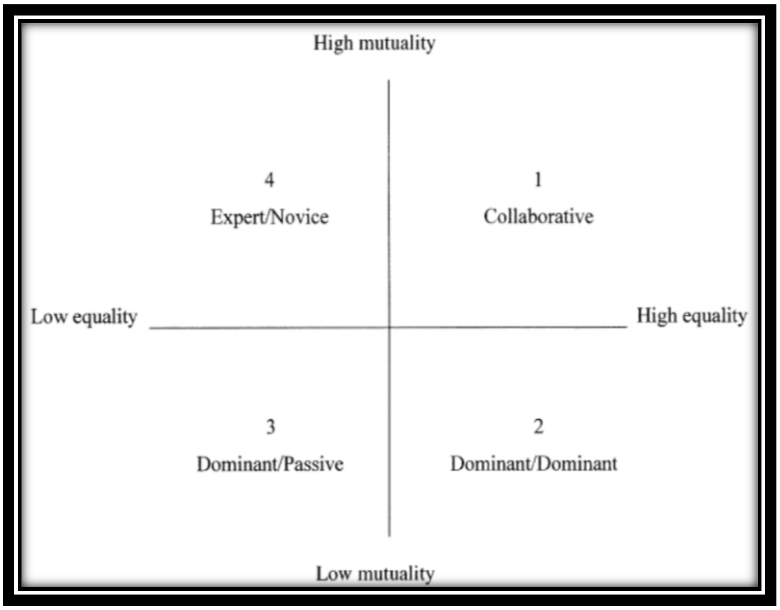


Figure 2.3 Storch's (2002) Model of Dyadic Interaction

The first quadrant represents a collaborative pattern of dyadic interaction where there is moderate to high equality and moderate to high mutuality. The collaborative pattern describes a pair working jointly on all parts of the learning task. Students are able to give and engage with each other's thoughts. Pairs discuss, negotiate, and offer alternative views, leading to resolutions acceptable to both participants (Storch, 2002). Quadrant 2 represents a dominant/dominant pattern of interaction, where there may be moderate to high equality, but a moderate to low level of mutuality. Here, both partners contribute to the learning task, but they are unwilling or could not fully engage with each other's participation. The dialogue of this interaction pattern is characterised by a high level of disagreement and incompetence in reaching an agreement (Storch, 2002). Quadrant 3 represents a dominant/passive pattern of interaction where the level of equality and mutuality are both moderate to low. The dominant student takes control of the learning task with little attempt to involve and encourage the passive student who maintains a subservient role. There is little discussion in this interaction pattern because few contributions or challenges are forthcoming from the more passive student (Storch, 2002).

The last interaction pattern is expert/novice, represented in Quadrant 4, where there is moderate to low equality, but moderate to high mutuality. In this interaction pattern, one student (expert) participates more in the learning task, but actively encourages the other student (novice) to participate (Storch, 2002). Furthermore, Storch (2002) found that pairs who adopted collaborative and expert/novice interaction patterns performed better than those who adopted dominant/dominant and dominant/passive interaction patterns. Likewise, Ahmadian and Tajabadi (2017) investigated the dyadic interaction in an EFL course for young learners at the pre-elementary school level using Storch's (2002) framework. The analysis outcome revealed the same interaction patterns that Storch (2002) found, with evidence that collaborative and expert/novice were associated with better learning outcomes.

Several studies have drawn on Storch’s (2002) framework which found different additional patterns. For example, Watanabe and Swain (2007) studied the relationship between patterns of interaction and frequency of LREs, and between patterns of interaction and post-test outcomes among 12 Japanese ESL students. Their analysis found a new pattern, expert/passive, added to the previous four patterns. Similar to Storch (2002), they found post-test scores showing that participants whose role was either, collaborative or expert/novice patterns, improved their results more than other patterns. These findings indicate that group interaction patterns, where group members work together throughout task activities and engage with each other's thoughts, can positively affect a student's learning and performance.

In addition, a study conducted by Tan et al. (2010) found an additional pattern of interaction, termed cooperative, which employs pair learning using Computer-Mediated Communication (CMC). The cooperative pattern is distinguished by a division of work such that group members participate equally in the task, but their participation comes as a result of each member performing their part of the learning task. Thus, group members engage little with each other’s contributions. Similarly, Todd and Toscano (2020) found the same patterns of interaction as Tan et al. (2010) in a study investigating patterns of interaction in completing online mathematics tasks among middle school and high school students. They found that the collaborative interaction pattern positively affects students’ performance. Likewise, Cardimona (2011) indicates that the interaction pattern has a positive effect on mathematics learning.

Moreover, Zheng (2012) added a fifth interaction pattern to Sorch's (2002) framework, namely passive/passive, in which partners demonstrate their failure or inability to solve a problem. Nevertheless, Andrews et al. (2017) could not find dominant/passive and expert/novice patterns of interaction in their analysis. However, they found a new pattern of interaction, namely fake collaboration, which is characterised by a mismatch between pairs’ discussion and practice in revising their individual responses such that pairs seemed to collaborate publicly and agree with their peers but privately sustained their own views.

Reviewing the literature suggests that the majority of studies that are interested in exploring the nature of interaction are conducted in foreign language learning. However, only a few studies pay more attention to thoroughly exploring the interaction patterns in the mathematics classroom. These studies show there is insufficient talk in the mathematics classroom; besides, primary school students often work uncollaboratively, inequitably, off-task, and sequentially unproductive, when performing collaborative learning tasks (Mercer and Sams, 2006; Ofsted, 2008). Furthermore, reviewing the literature suggested the significance of designing and structuring mathematics teaching and learning in a way that encourages students to interact with each other and engage collectively to construct their knowledge and improve their thinking. Additionally, there has been increasing research, examining interaction patterns in the context of ICT based learning in a regular classroom environment (Taar, 2013, Ben-Dor and Heyd-Metzuyanim, 2021). Therefore, the current research aims to develop teaching intervention integrated with GeoGebra in a way to facilitate collaboration and prompt students to actively engage with each other in performing learning tasks.

2.3.7 Constructivism and ICT

Constructivists believe learners are active participants in the learning process. This means that learning is an active process which requires an environment prepared with tools, ensuring students can engage in meaningful interactions with physical and social environments to construct understanding based on their own experiences. Such learning environments can exist by facilitating the classroom with ICTs. This is why constructivists have worked to integrate ICT into the process of teaching and learning. Therefore, the use of ICT, especially computers, in education is one aspect of enhancing the applications of Constructivist theory. ICTs, particularly associated with Constructivist teaching and learning, were often described as a computer-based or open-ended learning environment which enables and requires users to input and manipulate action and agency. They are predominantly related to computer-based software and web-based environments (Ben-Dor and Heyd-Metzuyanim, 2021, Harasim, 2017, Jonassen and Rohrer-Murphy, 1999).

ICT and Constructivist theory support each other to remake the concept of challenges of learning. The practical part of their applications to education produces the computer metaphor of the mind, as an information processor that must be seen rather than be organising data. Nevertheless, students have to wield the data flexibly during the learning process, make hypotheses, test tentative interpretation processing, etc. Indeed, ICTs, generally, and computers, especially, provide teachers with practical tools which encourage and support students to interact with each other and construct their understanding (Perkins, 2009).

Constructivist pedagogical strategies employ ICT in the learning activities, which help in constructing knowledge and meaningful understanding. The use of ICT in mathematics classrooms can enhance meaningful understanding, active learning, and problem-based learning, which challenges students to learn how to learn. Learners seek to find solutions to real-world issues, which, depending on an ICT framework, are used to engage their curiosity and begin learning, which leads to critical and analytical thinking (Alimisis, 2007). In addition, Lynch (1997) pointed out that ICT works on applying the principles of Constructivism theory in different learning situations and can support the two vital processes for cognitive construction: link between the previous experience and the new experience, and social interaction. Consequently, many constructivist designers have developed applications, tools, and software that ensure schools offer students several opportunities to interact with their environment and to help them engage in critical and higher-order thinking about the content.

In this case, students are responsible for organising and recognising information processes. At the same time, the computer or software performs the calculation, stores and recovers the necessary data to create and reflect on the process (Juniu, 2006). In this way, students learn with ICTs to construct their mathematical understanding actively by integrating ICT into the mathematics classroom, promoting them to interact with each other. In addition, computer software such as Logo, Spreadsheet, Sketchpad, 3D Capri, and GeoGebra, can help and benefit students in their mathematical thinking and make the abstract mathematical concepts easier and tangible in which learners can connect them to their existing knowledge.

2.3.8 Summary

The above argument illustrates that learners construct new knowledge and build meaningful understanding based on their previous experience. In addition to this, most knowledge being an interpretation of personal experiences, is also social in nature: knowledge is collectively constructed in interaction. From a constructivist perspective, learners have to be collaborative meaning-makers among a group defined by standard practices, communication, use of available tools, values, ideas, etc. Such collaborative learning should be designed to integrate ICTs in a way that encourages students to discuss, negotiate, share knowledge, visualise their thoughts, and interact with each other to construct meaningful understanding. ICTs in constructivist classrooms can play a crucial role in encouraging students to interact with each other and help them visualise mathematical concepts.

In the following section, I will give an overview on attitudes towards mathematics and spatial thinking, their significance concerning mathematics teaching and learning, and discuss their malleability and how ICTs affect them.

2.4 Students' Attitude Towards Mathematics

The learning process occurs in the mathematics classroom when communication between students and teachers happens and employs physical elements efficiently to achieve learning objectives. As discussed in the above section, the constructive mathematics teaching approach that allows students to discuss, communicate, and visualise their thoughts can help students develop a positive attitude to learning mathematics. Attitude is an individual mental process that governs individuals' real and potential behaviours in a social context. Since an attitude constantly aims at an object, it may be described as the individual's state of mind towards a value (Allport, 1935). Singh et al. (2002) point out that attitudinal aspects have emerged as important variables affecting mathematical performance and perseverance. This means that negative mathematical attitudes may impede the process of learning. This is why research repeatedly suggested that attitude towards mathematics is a crucial construct related to learning (Singh et al., 2002). Therefore, several studies have been carried out focusing on attitude towards educational matters in the last few decades (e.g., Davadas and Lay, 2020, Hyde et al., 1990, Maio et al., 2010, Singh et al., 2002, Triandis, 1971, Vandecandelaere et al., 2012, Aiken, 1970). The following section will discuss attitude definition and attitude towards mathematics.

2.4.1 Attitude Definition

The attitude concept has been considered difficult to define because it is recognised as multidimensional (Di Martino and Zan, 2010, Tapia and Marsh II, 2004, Vandecandelaere et al., 2012). Attitude is described as mental sets, which are a group of preconditions to assess a task, circumstance, institution, or item (Lewis, 1981). Halloran (1967) defines attitude as a permanent system of positive or negative assessment, emotional feeling, and disposition for actions concerning a social purpose. Baron et al. (1997) believe that attitude is a relatively enduring cluster of beliefs, feelings and behaviour inclinations directed towards certain people, notions, objects, or groups.

On a similar note, Triandis (1971) describes attitude as an emotional concept predisposed to a particular social class. However, as Vandecandelaere et al. (2012) mention, the commonly quoted definition is the one offered by Zimbardo and Leippe (1991). They define attitude as ‘‘an evaluative disposition towards some object based upon cognitions, affective reactions, behavioural intentions, and past behaviour that can influence cognitions, affective responses, and future intentions and behaviours' (p. 32). Overall, attitude is categorised into three interconnected components: cognition, behavioural intentions and affective responses (Ajzen, 1988, Di Martino and Zan, 2010, Gomez-Chacon, 2000, Ruffell et al., 1998, Triandis, 1971). The cognitive component is made up of a student's perceptions and thoughts about the situation, item, or person. The affective responses component includes a student's assessment of a situation, item, or person, as well as the degree to which the student enjoys it and his or her emotional reaction to it. Finally, the behavioural intention component encompasses the tendency and plan to act in a certain manner. Nevertheless, the behavioural component is not usually viewed as an attitude dimension (Daskalogianni and Simpson, 2000).

2.4.2 Attitude in Mathematics Education

The various definitions of attitude lead to different outlooks of attitude towards mathematics. Di Martino and Zan (2001) distinguish two fundamental methods to characterise attitude toward mathematics, concentrating primarily on the emotional aspect (first method) and the second method, integrating affective, cognitive, and behavioural dimensions. The first approach considers attitude as a basic description of an emotional disposition toward mathematics that is more likely to be about liking or disliking it. Aiken (1970), for example, describes attitude toward mathematics as an acquired propensity or inclination on the part of an individual to respond favourably or adversely to an item, situation, concept, or another person. Hart (1989) analyses attitudes toward mathematics from an emotional standpoint as well. He defines attitude toward mathematics, as a proclivity to respond positively or negatively towards it. This first method focuses on the emotional dimension of attitude while ignoring the cognitive domain, assuming that attitude is distinct from beliefs (Ma and Kishor, 1997).

The second method is based on the idea that attitude involves emotional responses, cognition, and behaviour (Ajzen, 1988, Di Martino and Zan, 2010, Gomez-Chacon, 2000, Ruffell et al., 1998, Triandis, 1971). Several studies have used this approach to describe the attitude towards mathematics. Neale (1969), for instance, defines students’ attitudes towards mathematics concerning their beliefs which, depends on whether they are good or bad in mathematics, they like or dislike mathematics, they believe mathematics is valuable or worthless, and if they are inclined to participate in or avoid mathematical activities. In addition, Kay (1993) and Ma and Kishor (1997) classified the three areas of mathematical attitude that comprise the cognitive component, which refers to beliefs about mathematics, the emotional component, which refers to the feeling that a learner likes or dislikes mathematics, and the behavioural component, which refers to predisposition to engage in or avoid mathematical activities. Watt (2000) discusses that attitude towards maths comprises perceived skill, predicted success, the effort needed, difficulty, interest, and usefulness. Hannula (2002) , on the other hand, identifies four evaluative processes: emotions evoked by circumstance, emotions connected with stimulus, predicted outcomes, and the act of connecting the event to personal beliefs.

Furthermore, Meelissen and Doornekamp (2004) identified four components: self-esteem, value, enjoyment and support. On the same issue, Tapia and Marsh II (2004) believe that critical components of the mathematics attitude include self-confidence, the usefulness of mathematics, enjoyment of mathematics, and motivation. In addition, within works by Di Martino and Zan (2010) and Di Martino and Zan (2011), the student's vision, perceived competence, and emotional dimension were presented as the three-dimensional model of mathematics attitude. Their models take explicit account of ideas and emotions (about themselves and mathematics) and their interactions. Attitude towards mathematics should include affective, emotional, and behavioural reactions to liking or disliking mathematics, perceptions of the ease or difficulty of learning mathematics, and beliefs about the contribution of mathematics to students' educational performance and career, according to Kadijevich (2008).

Concerning the same matter, Vandecandelaere et al. (2012) identify three crucial components of the structure of students' attitudes towards mathematics. These components are mathematics academic self-concept, enjoyment of mathematics and perceived value of mathematics. The mathematics academic self-concept is concerned with the student's perceptions about his/her ability to comprehend the subject matter and do well in mathematics. The enjoyment of mathematics considers how much the student enjoys mathematics classes, as well as the subject matter itself. Finally, the perceived value of mathematics refers to the students' beliefs regarding the significance of mathematics in everyday life and the future. The mathematics academic self-concept and the perceived value of mathematics represent the cognitive components of attitude, especially beliefs about the subject matter and ideas about an individual's own subject-related ability. The enjoyment of mathematics represents the affective component of attitude (Vandecandelaere et al., 2012). These components of attitude towards mathematics are also found in the TIMSS attitude test.

The multifaceted structure of attitude to mathematics has led scholars to carry out studies to investigate attitude towards mathematics from different perspectives, such as enhancing and modifying students' attitude, and association with other variables; for instance, outcome, gender, language, country of origin, study section, mean cognitive abilities of the class group, socioeconomic factors, and parents’ education level. In terms of gender, for example, numerous studies have demonstrated that grils show a slightly more negative attitude towards mathematics (Berger et al., 2020, Chouinard et al., 1999, Frost et al., 1994, Leder, 1995, Meelissen and Doornekamp, 2004, Van Damme et al., 2004). Meelissen and Doornekamp (2004) indicated that eighth-grade boys had a more positive attitude about their self-confidence and the perceived value of mathematics. However, they found that there was no statistically significant difference in their enjoyment of mathematics. Besides, other studies have examined the correlation between cognitive abilities and attitude towards mathematics. The results have shown that students with greater cognitive abilities illustrate a more positive attitude towards mathematics than students with lower cognitive abilities (Ma, 1997, Moenikia and Zahed-Babelan, 2010, Van Damme et al., 2004, Yee, 2010). Van Damme et al. (2004) reported that students in the experimental class tend to show a more negative mathematics attitude than students in the traditional class. It also has been shown that the students’ cognitive abilities and the average cognitive abilities of the group affect their academic self-concept. They also reported that constructivist mathematics teaching methods could influence students' attitudes toward mathematics.

2.4.3 Enhancing Students’ Attitude Towards Mathematics

This study is interested in the degree to which integration of ICT into teaching intervention is associated with attitude towards mathematics. This entails the assumption that attitude to mathematics can be affected, and subsequently, calls for an investigation on the development and change of attitude. In fact, attitude is directly and indirectly improved by curriculum, teaching instruction, situational stimuli, and/or learning environment, including physical, social, and epistemological settings (see section 2.2 Research View of Nature of Learning (ACAD framework)) (Vandecandelaere et al., 2012, Maushak and Simonson, 2001, Zimbardo and Leippe, 1991). Therefore, several studies have attempted to find out if/how curriculum, teaching methods, and practices in the classroom can improve and alter attitudinal aspects. Papanastasiou (2008) found a positive correlation between a clear, well-organised teaching strategy that keeps students active and involved in learning activities, and students’ attitudes towards mathematics. Papanastasiou's (2008) finding shows a positive correlation between students' mathematics performance and their attitude towards mathematics.

On a similar, yet different issue, Sanchal and Sharma (2017) investigated the impact of teaching mathematics in a sports context, examining students' attitudes towards mathematics. The findings show that students' attitudes towards mathematics (self-confidence, value of mathematics, enjoyment of mathematics) and engagement improved due to the research experiment. They also found that self-confidence is associated positively with developing mathematical thinking. Furthermore, Ifamuyiwa and Akinsola (2008) investigated the effects of two instructional strategies (self and cooperative) on students' secondary school mathematics achievements. The results revealed that the treatments have a more significant impact in improving attitudes towards mathematics, than the conventional method. Adding to that outlook, Maushak and Simonson (2001) suggested that authentic circumstances, relevant to students, and the experience of conscious emotional participation are likely to influence attitude in the direction in which the situation is promoted. Students are motivated and react favourably when the learning environment involves discovering relevant new information about a topic. In addition, post-instruction activities, including following conversations, have been identified as an effective technique to bring about changes in attitude (Maushak and Simonson, 2001).

On another, yet related, front, Zakaria and Syamaun (2017) conducted a study to discover the effect of the Realistic Mathematics Education Approach (RMEA) on mathematics performance and student attitudes towards mathematics. They found that using RMEA enhanced students' mathematics achievement, but not attitudes towards mathematics. They also found that RMEA encourages students to engage actively in the teaching and learning of mathematics. Prior to that investigation, Vandecandelaere et al. (2012) investigated the association between students' views of the learning environment and three components of their attitude towards mathematics (mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics). The outcomes revealed that the learning environment has a significant impact on the students' enjoyment of mathematics. At the same time, the mathematics academic self-concept and the perceived value of mathematics were not influenced by the learning environment.

2.4.4 ICTs and Improving Student's Attitude Towards Mathematics

Several scholars have been exploring the improvement of students' attitudes towards mathematics when ICTs are used in mathematics teaching interventions. However, the available literature shows uncertain results, based on the ICT used, the teaching approach involved, and the length of intervention (Fabian et al., 2018, Li and Ma, 2010). In other words, some studies have reported a positive development in students' attitudes (Afari et al., 2013, Cai et al., 2020, Eyyam and Yaratan, 2014, Mavridis et al., 2017, Pierce et al., 2007), while others found no significant improvement in attitudes towards mathematics (Dalton and Hannafin, 1988, Fabian et al., 2018, Kebritchi et al., 2010, Larkin and Jorgensen, 2016, Afari et al., 2013, Cai et al., 2020, Eyyam and Yaratan, 2014, Mavridis et al., 2017, Pierce et al., 2007). For example, regarding the usage of ICTs in teaching and learning mathematics, Pierce et al. (2007) reported that ICTs positively impact students' attitudes towards mathematics. In their quasi-experimental study, Yang and Yi Fang (2010) investigated the effect of integrating technology into mathematics teaching on students' number sense and their learning attitudes among primary students. They found a positive impact on students' performance and attitude towards learning mathematics.

Furthermore, Eyyam and Yaratan (2014) conducted a quasi-experiment to investigate the impact of technology-based instruction use in mathematics classes on improving students' academic achievement and attitude. The results showed a significant positive impact of using technology on students' performance, attitude, and perception of learning mathematics using technology. Another research by, Dalton and Hannafin (1988) exploring the effects of computer-assisted instructional strategies, designed to promote computation mastery, found significant impact on students' performance, but no significant impact on students' attitudes. Moreover, Olsen and Chernobilsky (2016) carried out a research to explore the impact of using a web-based interactive mathematics lesson intervention for ten weeks. The outcome indicated that students' attitude changes before and after the web-based learning activities was not statistically significant. However, students' perceptions about using this technology in learning mathematics tended to be negative, with only 9% of the sample having positive attitudes towards it. Besides, there was no link found between technology use and students' attitudes towards mathematics.

More researchers have examined the impact of employing educational games in teaching and learning mathematics on students' attitudes towards it. The outcomes were varying some of them found a positive impact and others reported a negative effect. For example, Ritzhaupt et al. (2011) investigated the effects of educational game playing on middle school students' attitudes towards mathematics, mathematics self-efficacy, and mathematics achievement (during 16 weeks of game intervention). The findings discovered significant positive developments in students' attitudes towards mathematics and mathematics self-efficacy. However, there was no significant improvement in students' mathematics performance. A further study regarding game usage was carried out by Afari et al. (2013), exploring a game-based mathematics classroom intervention for six weeks among college students aged 18 to 35 years. They found that students who used mathematics games had significantly more positive attitudes toward mathematics, but the impact size was small. The results also revealed that students' enjoyment of mathematics was higher in classrooms with more teacher assistance, cooperation, and perceived value of mathematics.

Further, Mavridis et al. (2017) investigated the impacts of utilising an online flexible educational game on secondary school students' attitudes towards mathematics, compared to the conventional method of solving mathematical problems, for 14 weeks. The analysis outcome showed that the game approach effectively improved students' attitudes towards mathematics. It also indicated better learning outcomes in the treatment group. Similarly, Kebritchi et al. (2010) conducted a game-based learning intervention for 18-weeks. The analysis found no significant development in students' motivation to study mathematics but found significant differences in motivation scores, depending on where they were played. Besides, student interviews revealed that they had positive perceptions about the game-based learning environment. This study emphasises that favourable student perceptions of the intervention do not always correlate with positive increases in student attitudes toward mathematics.

Concerning research that utilises mobile technologies and iPad, different findings in terms of attitude improvement was reported. For example, Cai et al. (2020) examined the effect of mobile AR-based learning applications on junior students' learning performances and attitudes. The results show that mobile AR-based applications positively impact students' performance and attitudes towards learning mathematics. Ursini et al. (2007) found that students exhibited a more positive attitude towards mathematics, when learning using ICT; they also felt positive towards computer-based mathematics. Likewise, Fabian et al. (2018) conducted a quasi-experimental mixed-method design to investigate the effects of mobile technologies in collaborative learning activities on students' attitudes and performance. The results indicated that using mobile technologies elicits positive responses from students regarding how they perceive mobile activities and improve their performance. However, it does not affect students' attitudes towards mathematics. Besides, Larkin and Jorgensen (2016) used iPads and a video diary to explore students' attitudes towards mathematics. The result revealed negative attitudes and emotions and suggested that these negative attitudes are well formed by the end of the early years of schooling.

Over time, different researchers have conducted examinations to determine the impact of ICTs on students' attitudes towards mathematics. For example, Ursini and Sánchez (2008) carried out a longitudinal study to explore computer-based mathematics' influence on girls' and boys' attitudes toward mathematics and self-confidence in the subject. The findings revealed gender differences in the changes in students' attitudes and self-confidence over three years. The use of technology had no significant impact on students' self-confidence. The usage of technology had no beneficial effect on pupils' self-confidence. Regardless of whether they used computers or not, both boys' and girls' self-confidence in mathematics decreased from grades 9 to 7. Similarly, in a longitudinal study, Bakker (2014) investigated the impact of using online mini-games on primary school students' multiplication and division performance and their attitude towards mathematics. The result revealed a positive impact on student's performance, as well as a correlation between students' performance and attitude towards mathematics. However, students' mathematics attitudes declined over time. In contrast, Hilton (2018), in her two-year longitudinal research of utilising iPads in the mathematics classroom, found a positive impact on students' engagement and attitudes towards mathematics. Moreover, the teaching intervention contributed positively to these outcomes.

The above discussion has shown different outcomes of the studies examining the effects of ICT on students' attitudes towards mathematics. These different results depended on the research intervention, the type of ICTs used, the study length, and the school level of participants. Based on an intensive meta-analysis Higgins et al. (2019) reported a significant overall ICT influence on student performance, motivation, and attitude; however, outcomes vary according to the various elements of the intervention. This can explain why some studies have reported negative impacts on students' mathematics attitudes when using ICTs in teaching the subject.

2.4.5 Summary

This section has reviewed students' attitudes towards mathematics. The review found disagreements between scholars on the definition of attitude, which led to disagreement on the definition of 'attitude to mathematics', since they define it based on different measurement tools used. This is why researchers have studied different components of students' attitudes from different perspectives. Attitudes towards mathematics play a significant role in the processes of teaching and learning this subject and have been consistently studied. However, improving students' attitudes to mathematics as a result of ICT use is limited. As such, studies on improving primary school students' attitudes to mathematics and ICT are even more limited; sometimes, with contrasting results. Studies frequently examined gender differences in attitude and the impact of using ICT on one or two components of mathematics attitude, rather than integrating ICT into teaching mathematics to improve attitudes towards it. More specifically, the literature review could not find a study (to the best of the author's search) investigating the impact of integrating ICT into teaching and learning mathematics on students' academic self-concept, enjoyment of mathematics, and perceived value of mathematics. Therefore, this research investigated the impact of integrating GeoGebra into teaching interventions to improve primary school students' attitudes towards mathematics in the light of Vandecandelaere et al. (2012) definition.

The following section will discuss spatial thinking, as another factor that can improve by using technology, and influence the learning process and outcomes.

2.5 Spatial Thinking

Spatial thinking is one of the principle components of intelligence that is used in our everyday lives and is important for human adaptation and modern living (Uttal et al., 2013). For instance, people employ their spatial thinking skills while reading street maps, giving directions, completing puzzles, constructing self-assembly furniture, and reorganising home furniture (Turgut and Uygan, 2015).

The nineteenth century saw the first scientific study related to spatial thinking when modern psychology began (Wai and Kell, 2017). Since Galton's systematic psychological inquiries, spatial thinking has been the theme of research interests in various subjects such as sciences, mathematics, engineering, and psychology (Quintero et al., 2015, Baki et al., 2011). Khine (2017) suggested that past and present research has detected important relationships between spatial thinking and success in Science, Technology, Engineering, and Mathematics (STEM), recognising talented students, encouraging them to follow STEM correlated careers and function well in a techno-centric world. Spatial thinking is one factor that plays a vital role in the overall development of students' understanding of mathematics and predicts students' success in higher levels of mathematics, such as proportional thinking and algebraic thinking.

The National Research Council report urges instructors in all areas of mathematics to recognise the significance of improving spatial thinking skills with students (Rich and Brendefur, 2018). In addition, various countries, such as Saudi Arabia and USA, include spatial thinking in their programmes, particularly mathematics, as it is an essential skill for studying different disciplines and to achieve success (NCTM, 2006). Accordingly, many researchers have suggested that spatial ability is flexible and can be developed with intervention enrichment and training activities within a formal classroom setting or outside the classroom as a special training course related to spatial thinking. Nevertheless, there are others who disagree, as they believe development of spatial thinking needs long term course training (Uttal et al., 2013, Baki et al., 2011, Khine, 2017, Saha et al., 2010, NRC, 2006).

2.5.1 Spatial Thinking Definition

There has been much debate concerning the definition of spatial thinking. There is no common agreement with respect to the term. This is because several terms are used to refer to spatial thinking, such as spatial skills, spatial ability, spatial perception, spatial reasoning, spatial sense, visual thinking, spatial visualisation thinking, and visualisation thinking (Baki et al., 2011, Quintero et al., 2015, Metoyer et al., 2015, Strong and Smith, 2001, Yüksel, 2017). Spatial thinking was defined by McGee (1979) as the mental operation of shape, whereas, Linn and Petersen (1985) claimed that a common idea regarding spatial thinking is the use of skills in representing, transforming, generating, and recalling symbolic and non-verbal information. Clements and Battista (1992), under the term spatial visualisation, described it as understanding the performance of imagined movements of objects in 2D and 3D space. However, the NRC (2006) believed that thinking spatially is a collection of cognitive skills, consisting of declarative and perceptual forms of knowledge, and several cognitive operations, which can be used to transform, combine, or otherwise operate this knowledge by using a constructive combination of three elements: concepts of space, tools of representation, and reasoning processes.

Relating to the aforementioned issue, Uttal et al. (2013) described it as the mental procedure of representing, analysing, and drawing conclusions from spatial relations. Additionally, Ramful et al. (2015) suggested that it is a method where individuals generate and manipulate mental images in reflecting objects on a line of symmetry. Despite the fact that descriptions vary, spatial thinking is generally defined as the complex ability to produce, maintain, recover, and transform well-structured visual images. Furthermore, other definitions include references to physical interactions with the environment; for instance, navigational skills, along with dealing with forming and manipulating visual-spatial mental images to connect the perceived and constructive 3D world (Hawes et al., 2017, Nagy-Kondor, 2017).

It is worth mentioning that components of spatial thinking are different in the relevant literature due to various definitions (Yurt and Tünkler, 2016). As an illustration, McGee (1973, p.893) suggested that the two main factors concerning spatial thinking are spatial visualisation, which was explained as “the ability to mentally manipulate, rotate, twist or invert a pictorially presented stimulus", and spatial orientation, which was defined as "the comprehension of the arrangement of elements within a visual stimulus pattern and the aptitude to remain unconfused by the changing orientation in which a spatial configuration may be presented". Tartre (1990), classified spatial thinking components based on mental processes into two distinct skills: Firstly, spatial visualisation, includes the mental movement of an object. This can be further divided into two parts; namely, mental rotation, where the entire object is transformed by turning in space and mental transformation, where part of the object is transformed in some way. Secondly, spatial orientation comprises being able to mentally move your viewpoint while the object remains fixed in space (see figure 2.4).

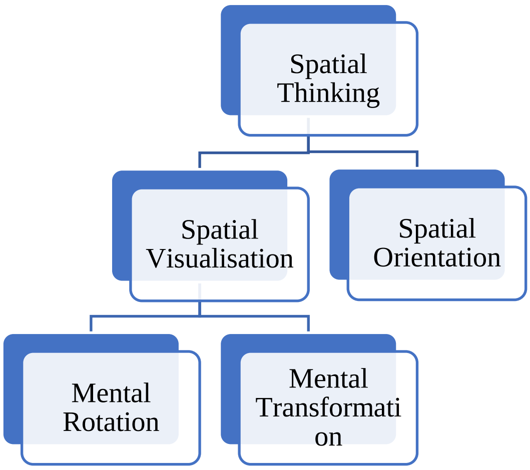


Figure 2.4 Classification of spatial thinking

Maier (1998), categorised five components related to spatial skills based on numerous theories of intelligence, meta- analyses, and several studies conducted on spatial ability. These are as follows:

- Spatial perception is the vertical and horizontal fixation of direction, irrespective of disrupting information
- Spatial visualisation is the capability to describe situations when the components are moving compared to each other
- Mental rotation is the capacity to alternate three dimensional shapes mentally
- Spatial relations are the ability to distinguish the relationship between the parts of a shape
- Spatial orientation is the ability to react to a given spatial situation

Wang (2017) separated spatial thinking into three subcategories: Spatial perception, which is the ability to define spatial relationships relative to the orientation of a figure; mental rotation (or spatial relations), which is the ability to achieve single-step mental transformation/rotation of two- or three-dimensional figures precisely in the mind; Spatial visualisation, which is the potential to achieve complex and multistep mental transformation/rotation. Maier's and Wang's categories are practically similar; however, Wang (2017) combined five components in three elements, besides changing the employment of spatial visualisation from Maier's (1998) description. Nonetheless, Maier's (1998) frame can be used in other subjects as he created it to be more suitable in relation to designing training courses for spatial thinking in a technological era (Maier, 1998).

2.5.2 Spatial Thinking and Mathematics Education

Spatial thinking is one of the principal elements of the mathematics curriculum and is a core component of teaching and learning geometry. For example, Saudi Arabia includes spatial thinking in the mathematics curriculum in general education (primary, elementary, and secondary), and it is also one of the basic principles in the standards and principles of mathematics teachers. It is a component of the national general aptitude test, which must be taken by every student who wishes to study at university. In addition, spatial visualisation is the principle phase in geometric thinking, which plays a significant role in learning graphics and shapes (Vojkuvkova, 2012). Furthermore, students have used spatial thinking to imagine and manipulate visual data during geometric modelling to solve problems and learn mathematical concepts (NCTM, 2006). Several researchers have also illustrated that spatial thinking is correlated positively to problem-solving skills in mathematics and geometry (Baki et al., 2011, Hawes et al., 2017, Idris, 2005, Nagy-Kondor, 2017, Idris, 2007). The research also found a significant correlation between spatial thinking and geometry, predicting how effectively students can complete geometric problem-solving tasks (Clements and Battista, 1992, Clements and Sarama, 2004, Pittalis and Christou, 2010). Turgut (2017) concluded that it is obviously in the related literature that spatial thinking predicts mathematics achievement; hence, development of spatial ability can contribute to the improvement of mathematics achievement.

Problem-solving tasks regarding spatial skills such as orientation, transformation, and movement of shapes generate an opportunity for students and teachers to engage actively in the mathematical discussion. As students explain their thought, they will use their hands or the available tools to visualise their ideas while explaining their views surrounding the task. Spatial thinking allows students to explain the visual imagery inside their heads while working on specific problem-solving tasks (Ehrlich et al., 2006). For example, students' gestures describe the transformation movement and produce an avenue for their thinking to develop through the discussion. Alibali and Nathan (2012) determined gestures to be a valuable tool for teaching students how to solve spatial transformation tasks by emphasising moving the pieces without the actual physical movement. Students used their hands to visualise what their mind was creating and thus explain their mathematical understanding. The capacity to gesture what the mind is thinking depends on students' ability to visualize mathematical transformations (Education, 2014, Rich and Brendefur, 2018).

Furthermore, spatial thinking is highly associated with mathematical performance (Battista, 1981, Clements and Sarama, 2007, Gustafsson and Undheim, 1996), in which students with higher spatial performance also do better in mathematics exams (Cheng and Mix, 2014, Geary et al., 2007, Lowrie et al., 2017). In fact, researchers have demonstrated a strong relationship between spatial thinking and academic performance in many disciplines and, more remarkably, mathematics (Hawes et al., 2017). For example, in their longitudinal investigations, Lauer and Lourenco (2016) along with Gunderson et al. (2012) correlated young children's spatial thinking with their spatial and mathematical performance at a later age, particularly mental rotation and visuospatial memory. Also, Cheng and Mix (2014) investigated whether spatial thinking training can improve mathematics performance among children aged 6 to 8 years. The results revealed that children in the spatial training group improved significantly on calculation performance and largely improved in missing term problems. Cheng and Mix (2014) point out that spatial thinking training can transfer mathematics performance, which is necessary for developing mathematical knowledge. In fact, spatial thinking skills and mathematical competency are directly connected (Battista, 1990, Casey et al., 1997, Reuhkala, 2001, Rohde and Thompson, 2007). Learning with particular spatial thinking tasks enhances students' ability in mathematics and other disciplines (Newcombe and Frick, 2010, Uttal et al., 2013).

Besides, additional research highlighted the significance of using spatial thinking to find an enhanced approach to teach mathematics and geometry, in contrast to present techniques, which do not pay adequate attention to spatial thinking skills (Olkun et al., 2005). Therefore, teachers can use spatial thinking to trigger learning and teaching activities in mathematics and geometry, where it is crucial for students to develop effective ways of thinking. In light of this notion, spatial thinking skills can be improved by integrating and supporting practice throughout mathematics instruction (Verdin, 2014). The result of the study undertaken by Hawes et al. (2017) emphasised the possible significance of attending to and improving young children's spatial thinking as part of early mathematics education and suggesting that it has a positive impact on learning numeracy.

Verdine et al. (2014) stated that children's spatial thinking skills could predict their overall mathematical performance from the time students reach kindergarten. That is one of the reasons why NCTM (2006) emphasised that the mathematics curriculum should include spatial thinking skills across all the school stages as spatial thinking is improved by mathematics and daily school activities. For instance, students start gaining spatial thinking skills through games such as football, marbles, painting, and puzzles, in addition to those which require visual imagery such as computer games, interactive software on iPad or Smartphone, plus involvement in mathematics/science classes. These scholars suggested that developing spatial thinking skills is associated with learning and teaching mathematics.

2.5.3 Developing Spatial Thinking

To date, several research have attempted to identify methods that can help develop spatial thinking skills. This research has increased because of technology and its effect on people's everyday lives. Moreover, improvements in spatial thinking are a significant factor as students must understand and improve their knowledge of geometry and mathematics, in agreement with theoretical knowledge and spatial abilities (Nagy-Kondor, 2017). Nagy-Kondor (2017), stressed that spatial thinking skills can be improved by employing appropriate teaching methods in the classroom by means of using ICT. This has received considerable attention in the reviewed literature, especially with regards to Dynamic Geometry Systems, interactive animation, and virtual figures that are promising methods for developing spatial thinking. Moreover, Martín-Gutiérrez and González (2017) suggested that specific training can improve spatial thinking by using different methodologies that rely on the field of application. In fact, an intensive meta-analysis review of 217 studies over 25 years suggests that through a diversity of training methods, people of all ages can develop their ability to think spatially by means of pen and paper sketches, isometric sketching, multi-media platforms, on-line platforms, video games, virtual reality, augmented reality, specific software, and physical materials, considerably (Uttal et al., 2013).

Conversely, several studies have claimed that spatial thinking cannot be developed via typical instructional approaches, but can be developed by way of life experience (Strong and Smith, 2001). In addition, Baki et al. (2011) explained that there is no evidence to confirm the positive impact of instruction on spatial thinking. For example, Sexton (1992) found that there was no improvement when he used 3D wireframes and Zavotka (1987) did not ascertain any change in spatial visualisation. Based on their results, certain researchers have claimed that people's spatial thinking is hereditary and cannot be improved by training or teaching.

Numerous research have indicated that spatial thinking develops over the long-term and by means of real world experience (Robichaux and Guarino, 2000), while others have implied that the positive impact of experimental training regarding spatial thinking does not mean more concrete evidence on the effectiveness of short-term training. In addition, some classes, such as art, the sciences, and mathematics can play a significant role in enhancing spatial thinking (Baki et al., 2011, Wai and Kell, 2017).

In recent years, questions related to whether spatial thinking can be improved remain unanswered; nevertheless, all the research that strives to explore how spatial thinking can be improved through life experience, either in or outside classroom, has contradicted a substantial number of findings in both previous research and recent studies (Strong and Smith, 2001). One of the most significant areas pertaining to research into the development of spatial thinking is the application of dynamic instruments in mathematics classes to improve spatial thinking. Many researchers have found that dynamic geometry software such as Cabri, Geometry Sketchpad, and Geogebra, could be influential instruments for teaching mathematics and geometry, especially with respect to developing spatial thinking (Baki et al., 2011, Nagy-Kondor, 2017). Christou et al. (2006) stressed that 3D dynamic applications would improve students' dynamic visualisation ability and empower them to obtain a greater understanding of 3D mathematical and spatial concepts. In fact, these applications can provide an environment where geometric relationships make and test conjectures which can be explored and can construct geometrical objects plus specify relationships between them (Baki et al., 2011). Owing to these features, using 3D dynamic software to teach geometry could facilitate dynamic visualisation of 3D shapes and subsequently develop spatial thinking (Kösa, 2016).

It should be mentioned that Cabri geometry is one of the dynamic geometry softwares that has been employed to improve spatial thinking in mathematics classes, as it is presumed to revolutionise computer-assisted spatial thinking skills in 3D geometry (Baki et al., 2011). The primary purpose of Cabri is to assist students to improve spatial visualisation skills essential to continue complex geometric manipulations and abstract projections mentally (Leischner, 2002). In that regard, Prianta (2017) stated that students who were taught with 3D Cabri were better than students who received expository learning, either in general, or rely on their early mathematical abilities. Moreover, Hartiana et al. (2017) found that learning with 3D Cabri has a significant impact on developing students' spatial thinking skills, making it easier for them to understand concept lines and angles, plus causing students to be more enthusiastic. Consequently, teachers can use it to develop student's spatial thinking skills in mathematics. For instance, when I use it to teach Year Six 3D geometric shapes, such as such as cubes, cuboids, pyramids, cylinders, and prisms, I can include a variety of activities where students can use 3D Cabri to analyse and elaborate shapes and divide a cube (unfold Cube). Similarly, they can rotate it to see the shape from different viewpoints. Hence, it may improve their spatial thinking. As a spatial thinking task is unfolding, the use of 3D Cabri can support the construction of connections between spatial and geometrical thinking by allowing students to use graphical space via playing with didactic variables. Additionally, it offers feedback with respect to validation (Soury-Lavergne and Maschietto, 2015).

Currently, GeoGebra is one of the commonly used dynamic geometry software programs. It has been used extensively in mathematics to teach a variety of lessons (see section 2.6.1). GeoGebra has more traits than Cabri 3D and other dynamic mathematics software since it can explore the concepts of algebra, 2D geometry, 3D geometry, calculus, matrices, vectors, and statistics (Rahadyan, 2019), while, for example, Cabri 3D can be used only to explore the concepts of 2D geometry and 3D geometry (Rahadyan, 2019). Concerning this, Nagy-Kondor (2017) suggested that Geogebra can be used within mathematics lessons to develop spatial thinking skills, given that it has the ability to provide a definition of the concept of planer and spatial objects, geometric transformations in the plane and space, and the application of angle functions in 2D and 3D. Additionally, Saha et al. (2010), Farrajallah (2016) determined that Geogebra has a significant impact on the development of spatial thinking among high school students. Moreover, Ismail and Abd Rahman (2017) found that GeoGebra significantly impacts Year Two students' spatial visualisation thinking. However, research on using GeoGebra in learning and teaching mathematics is limited and even more limited, among primary school students (more detail is section 2.6.1).

2.5.4 Summary

This section gives an overview of spatial thinking and how it can be developed in mathematics classes with the concern of using ICT software. Spatial thinking is a critical component of mathematics curricula and an essential element in teaching and learning geometry. The literature review illustrates that there has been no consensus on spatial thinking terminology since it is commonly used in different disciplines due to its importance for students' performance and success. This disagreement extends to the research on the development of spatial thinking skills. Some scholars believe that spatial thinking improves through life experience in long-term training, while others believe that spatial thinking can improve in short-term training intervention (Strong and Smith, 2002; Baki et al. 2013). Notwithstanding, research has demonstrated that spatial thinking can be developed using appropriate mathematical instructional strategies, consisting of ICTs, especially DMS, in short-term intervention. However, the research investigating the impact of integrating DMS into teaching intervention to improve students' spatial thinking skills is limited, particularly with the use of GeoGebra among primary school students.

2.6 Dynamic Mathematics Software

Dynamic Mathematics Software (DMS) or Dynamic Geometry Software (DGS) is a computer programme for producing and manipulating mathematical structures which enables students to understand unseen mathematical concepts and helps teachers reveal the links between mathematical concepts to their students (Wang et al., 2017). Since the 1990s, as a modern way of teaching, mathematics teachers have integrated DMS; for instance, bringing Cabri 3D, Geometer's Sketchpad, and GeoGebra into teaching and learning mathematics (Guven, 2012, Botana et al., 2015). The reasoning behind these developments is that DMS provides an open environment with visual properties permitting students to perceive unseen things, which helps them to visualise mathematical structures, concretise the abstract nature, and construct links between algebra and geometry, which can potentially positively affect students' mathematical and geometrical understanding (Ocal, 2017, Crompton et al., 2018, Battista, 2009, Dvir and Tabach, 2017). Hence, they can attain a better understanding of mathematical concepts in general, and geometric concepts, in particular. Furthermore, Leung and Lee (2013) believed that computers and DMS have an ability that allows students to learn many complex mathematical tasks to be accomplished efficiently. Thus, students in the technological era have the possibility of learning more mathematical forms in effective ways than before. They also believed that the most significant effect of technology on mathematics education is changing the nature of mathematics that has been taught, learned, and assessed. According to (Sanders, 1998), the appropriate use of DMS can improve mathematics teaching, conceptual development, visualisation, besides laying the foundation for deductive reasoning.

Furthermore, Battista (2009) asserts that although DGS enabled students to drag and manipulate geometric shapes, students do not identify the properties of shapes. They recognise the shape as a whole, illustrating students' geometric thinking at stage one of the Van Hiele levels. As students continue to interact utilising the features of DGS, they begin to notice geometric shape properties; then, distinguish between the geometric shapes by using these features; afterwards, they construct the concept of the geometric shape. This can encourage and create exploration and mathematical exploration. It also helps students progress through the Van Hiele levels.

Several studies have examined the effect of using DMS, generally, and DGS, specifically (Cabri 3D, Geometer's Sketchpad, and GeoGebra, which are the most commonly used in schools) (Moyer-Packenham et al., 2005). DMS has been established to have a positive impact on students' performance and attitudes (Tutkun and Ozturk, 2013, Pierce and Stacey, 2008, Healy and Hoyles, 2002, Souter, 2002, Laborde and Laborde, 2011, Sarama and Clements, 2009, Hannafin et al., 2008). Mammana et al. (2012) conducted a study investigating DGS (3 D Cabri). They found that the use of 3D Cabri was enjoyable and allowed students to examine and form assumptions, plus verify them through proof, as well as finding that the use of DGS encouraged an exploration environment in the mathematics classroom. Leong (2013) conducted a study to determine the effects of using Geometer's Sketchpad (GSP) in teaching and learning graph functions. The findings show a positive impact of using GSP on students' achievement and attitude towards learning graph functions.

On that note, Hannafin et al. (2008) conducted a study to explore the influence of using Sketchpad application on sixth-grade students (11 – 12 years old). They found that students learning with Sketchpad understood most geometry concepts much better, and positively impacted spatial thinking skills. In addition, a review study was also completed by Rahim (2002) to look at the impact of using technological tools in general, including dynamic geometry software applications in classrooms over the last decades. The study explored the impact of such applications on teacher attitudes and the overall academic system, with no reference to student performance, specifically. It found that teachers would use software to teach geometry, algebra, and trigonometry. Another review has recently been completed by Üstün (2021) indicating that DGS has a positive impact on providing a constructivist classroom learning environment. However, using DGS solely cannot create a learning environment shaped by constructivism; it should take into account students' prior knowledge, skills, needs, time for group discussions, plus appropriate design of DGS tasks and worksheets.

Conversely, some studies displayed the insignificant impact of utilising DGS on teaching and learning. Brovey and Null (2004) conducted a study to examine the effects of using Geometer's Sketchpad on students' achievement on traditional assessments and students' attitudes towards geometry among sixty-eight ninth-grade geometry students. The results showed no significant impact of Geometer's Sketchpad on students' achievement and did not improve students' attitudes. Moreover, Chan and Zhou (2020) explored the effects of cooperative learning with DMS on students' performance, attitudes, and views of learning using DMS. The results showed a significant impact of using DMS with cooperative learning. However, the findings found no change in students' attitudes towards mathematics and no negative views of integrating DMS with cooperative learning. In a meta-analysis study, Chan and Leung (2014) found evidence on the insignificant impact of using DGS on students' engagement when they work in pairs. Furthermore, they found a significant impact of using DGS on students' performance, with a preference for short-term instruction with DGS significantly developing the mathematical performance of elementary school students. On that matter, Niess (2006) highlighted that DGS could transform mathematical concepts into an understandable form for teachers and students. This is why Dynamic Geometry Software (DGS) has been used widely, and many softwares have been innovated to be used in mathematics classrooms. The current one that has been an increasing number of its users in mathematics education is GeoGebra.

2.6.1 GeoGebra

One of the most common dynamic softwares is GeoGebra, which was created by Mark Hohenwater in 2001. It was designed to combine geometry, algebra, and calculus in one dynamic environment. GeoGebra brings together geometry, graphing, spreadsheets, calculus, and statistics in one easy to use application. It is open-source and permits users to create mathematical objects and interact with them. Both teachers and students can use it to explain, discover, and represent mathematical concepts and their relationships. Its principal goal is assisting users to create various representations and visualisations of mathematical concepts. Additionally, GeoGebra helps users produce activities integrating several representations of mathematical concepts linked dynamically (Hohenwarter and Jones, 2007, Zengin et al., 2012, GeoGebra.com).

Geogebra has been used widely. This is because it is available in many different languages, and there are several online lessons available in those languages. Moreover, it is free software to download and can be installed on many different types of devices: computers (e.g., Windows, Mac OS act.), tablets, iPads, and mobile phones. Geogebra is available in various versions as well (e.g., Graphing Calculator, Classic GeoGebra, Geometry, 3D Calculator, and GeoGebra AR). Every object in GeoGebra is dynamic, allowing a student to see how the shape changes when changing the problem's parameters ((Majerek, 2014).

Furthermore, students can save their projects in multiple formats, and the teachers can share and publish their work on the GeoGebra website. It has several display options of mathematical concepts, such as symbolic to graphic. Žilinskiene and Demirbilek (2015) stated that GeoGebra has the ability to demonstrate, visualise, and clarify mathematical concepts during the process of learning and problem-solving, which has always been very important to understanding mathematics. Also, they mentioned that GeoGebra is a construction software with all the capabilities required from a suitable drawing and designing program, which is very significant for constructive teaching of geometry. GeoGebra allows students to discover new patterns, explore and test conjectures, and manipulate various geometric shapes among the diverse activities that students can perform by creating and designing their drawings (Belgheis and Kamalludeen, 2018, Stols and Kriek, 2011). This is why many schools have been using GeoGebra in the mathematics classroom.

As a result, several studies have been conducted to examine the use of GeoGebra in the mathematics classroom from different perspectives. Many scholars have been investigating the impact of teaching using GeoGebra on students' performance. For example, Adelabu et al. (2019) examine the impact of using GeoGebra on secondary school students' geometric performance. They found that GeoGebra improves students' performance. Tay and Wonkyi (2018) investigated the effect of using GeoGebra on senior high school students' performance in circle theorems. The results indicated a significant positive impact for the students who used GeoGebra to learn circle theorems. Likewise, the GeoGebra method made the lessons more interesting, practical and easy to understand.

Moreover, Reisa (2010) conducted a case study to investigate the impact of computer-supported mathematics with GeoGebra, compared with traditional teaching. The findings showed that teaching with Geogebra improved students' performance and helped them to retain their understanding, more than traditional teaching. In addition to this, using Geogebra to teach mathematics made students more engaged in learning, and more sense organs were attracted. A study was conducted by Agreeing with the aforementioned, Onaifoh and Ekwueme (2017) to examine innovative strategies on teaching plane geometry using GeoGebra software in secondary schools in delta state. The results showed significant differences between the mean performances of students' when taught plane geometry using Geogebra software and problem-based learning. Furthermore, Shadaan and Leong (2013) investigated students' understanding in learning circles using GeoGebra. The results showed a significant impact of learning circles using GeoGebra on students' performance and a positive perception of using GeoGebra in learning circles. Arbain and Shukor (2015) also found a positive impact of using GeoGebra on students' performance and perception towards learning mathematics. In contrast, Martinez (2017) found that there were no significant differences between teaching using GeoGebra and traditional teaching on students' performance. Agreeing with the aforementioned, Ocal (2017) found there was no significant impact of using GeoGebra on students' procedural knowledge.

Other researchers have studied the impact of integrating GeoGebra into teaching mathematics. For instance, Zengin et al. (2012) studied the impact of integrating GeoGebra into flipped classroom approach by using a mixed research method. The research found that students understood the mathematical concepts much better by integrating GeoGebra into flipped learning method. Besides, the results showed that integrating GeoGebra with flipped learning approach helped students visualise the concepts, retain the information, and promote more accessible learning concepts. It also enhanced students learning and increased their motivation.

Meanwhile, Priatna (2017) investigated learning models and teaching materials by employing the principles of brain-based learning assisted by GeoGebra to enhance junior high school students' mathematical representation skills. The findings revealed that the increase in the mathematical representation skills of students who were treated with mathematics instruction, applying the brain-based learning principles supported by GeoGebra, was better than the increase of the students given conventional instruction. In addition, Zengin and Tatar (2017) conducted a mixed research study to integrate GeoGebra into cooperative learning environments in mathematics among high school students, aged 16 – 17 years. The findings showed that integrating GeoGebra into teaching and learning quadratic functions positively impacts students' achievement. Further, the students' views identified that the model enabled them to understand better, visualise and concretise the course, and create a pleasant and enjoyable learning environment. Similarly, Tutkun and Ozturk (2013) conducted research to determine the impact of using GeoGebra on the students' academic success and Van Hiele geometrical thinking level in teaching 'trigonometry and slope' subjects in 8th grade maths. They found that GeoGebra has a positive impact on overall academic success. There was no significant difference between the Van Hiele geometrical thinking levels of the students who use GeoGebra and the control group. In addition, utilising GeoGebra in Mathematics teaching affected on retention of learning positively. It can be concluded that the majority of research has been found to demonstrate the positive impact of utilising GeoGebra on students' performance, retaining learning, and enhance the process of learning and teaching mathematics.

Additionally, some other researchers go to examine the effect of utilising GeoGebra on the students' performance and spatial thinking skills, such as Saha et al. (2010), Hannafin et al. (2008), and Farrajallah (2016). Saha et al. (2010) conducted a study to explore the impact of using Geogebra application on secondary school students. The results showed a significant difference between the means of students' achievement favouring the GeoGebra group, plus the classroom instruction with GeoGebra being more effective than the traditional classroom.

Furthermore, the findings showed that there was no significant difference in high spatial visualisation ability. However, they found that GeoGebra has had a positive impact on spatial visualisation ability. Farrajallah (2016) investigated the impact of the employment of Geogebra software in acquiring some visual thinking skills and on the academic achievement among 8th-grade students aged between 12 – 14 years. The results showed a significant statistical impact of using Geogebra on visual thinking skills and mathematics achievement. In addition, Hassan and Abdullah (2016) examined the effectiveness of using the GeoGebra program on acquiring geometric transformation concepts and developing visual thinking and mathematical self-concept for Saudi middle school pupils. The results showed a significant statistical impact of GeoGebra on students' geometric transformation concepts, visual thinking skills, and the mathematics self-concept. Somewhat varying from the main direction of previous findings, Hannafin et al. (2008) concluded that there is a relation between spatial thinking and achievement in mathematics, only in some cases; also, the use of dynamic software can positively impact spatial thinking skills, especially with students who have higher cognitive abilities.

Some researchers have investigated the impact of using GeoGebra on the learning process and students' attitudes. Adegoke (2016) and Horzum and Ünlü (2017) conclude that GeoGebra can improve students' attitudes towards mathematics and facilitate the teaching and learning process. This conclusion is supported Murni et al. (2017) study that examined the effect of using GeoGebra in the discovery learning model on mathematical problem-solving ability and students' attitude toward mathematics. The findings showed a positive impact of using GeoGebra on problem-solving ability and attitudes towards mathematics. Celen (2020) conducted a case study approach and focused group interview to explore Year 7 students' opinions on learning mathematics using Geogebra. The results concluded that GeoGebra makes mathematics learning processes enjoyable, and assists students in concretising abstract concepts. Besides, students with low computer literacy found difficulty in performing GeoGebra activities.

Additionally, Zulnaidi et al. (2020) determined the impact of using GeoGebra, as a teaching aid on students' performance. The findings indicate a significant impact of using GeoGebra on students' performance concerning the lessons of functions and limit functions. They also found that using GeoGebra is time-consuming. Teaching using GeoGebra can make the learning process more active, allowing active interaction between teachers and students. To sum up, most of the research implies that GeoGebra positively affects student performance, retention learning, students' attitude, and spatial thinking. However, the present research integrated GeoGebra into a new way of teaching as a fundamental element of the teaching and learning processes. Hence, this research is concerned with the learning process, not just outcomes.

2.6.2 Summary

This section has given an overview of Dynamic Mathematics Software (DMS) and its use in mathematics education. The literature shows the potential benefits of using DMS in teaching and learning mathematics on students' performance, thinking skills, and attitude. The related research to the use of DMS, especially GeoGebra, frequently shows the positive impact on students' learning outcomes and processes in terms of performance, spatial thinking skills, and attitude. However, there are some negative effects which have been reported. Although students' interaction is crucial in solving problems, sharing ideas, making connections, and developing their understanding and thinking skills, the research discovered that pairs' interaction patterns in collaborative mathematics classrooms using GeoGebra is limited, particularly with primary school students, and could not find investigations in a mathematics classroom in KSA. In addition, reviewing the literature concerning using GeoGebra to improve students' attitudes towards mathematics could not find a study investigating these three components of students' attitudes together: academic self-concept, enjoyment of mathematics, and perceived value of mathematics.

Furthermore, concerning the development of the Saudi mathematics curriculum, which requires constructivist teaching methods, teachers continue the same approach that was used to teach the old curriculum, which is teacher-centred rather than learner-centred. The research on teaching practices with ICTs in the Saudi mathematics classrooms to enhance thinking skills, activity learning, and developing students' performance is still limited and even more regarding geometry (MoE, 2013; Al-Shaya, 2013; Kashan et al., 2013; Al-Yami, 2012; Al-Ony, 2011; Al- Harbi, 2013; Al-Rwais et al., 2013; Al-Dgain, 2013; Al-Eid, 2014; Khalil and Al-Rwais, 2014; Alsalim, 2018). What must not be forgotten regarding the modern Saudi Arabia is that the Saudi Vision 2030 aims to develop teaching methods and improve students' performance and thinking skills (see section 1.2). These illustrate the real need in KSA to develop a teaching strategy integrated with DMS to improve students' learning processes and outcomes to fulfil the need in mathematics education and the aims of the Saudis Vision 2030. Therefore, this research is aimed at examining the impact of integrating GeoGebra into teaching interventions on students' geometric performance, spatial thinking, and attitudes, including academic self-concept, enjoyment of mathematics, and perceived value of mathematics, in addition to exploring pair patterns of interaction.

Chapter 3. Research Methodology

3.1 Introduction

This chapter will discuss the research objectives, research questions, research methodology, research procedure, and research sample. Additionally, it will discuss the research methods and the process to build them, as well as the verification of their clarity, validity, and reliability. It will also explain the research intervention, and then, the way the data will be analysed and discussed, including the statistical methods that the researcher relied on in processing data and analysing the results.

3.2 Research Objectives

As has been stated in the previous chapters, this research aims to investigate the impact of using GeoGebra on the learning process and learning outcomes. In this research, the researcher seeks to explore the effect of integrating GeoGebra into a teaching intervention on students' patterns of interaction, attitude towards mathematics, geometric performance, spatial thinking skills, and sustainability of learning among primary school students. Additionally, this research seeks to investigate the relationship between geometric performance and spatial thinking, the association between spatial thinking and sustainable learning, the relationship between student performance and sustainable learning, in addition to the impact of utilising GeoGebra on the learning process.

It seems, therefore, what is required is conducting an experimental study, comparing the selected classes: one taught using GeoGebra with teaching intervention, the other one taught utilising the teaching intervention with hands-on, and the third group taught traditionally with no teaching intervention, nor technology. So, designing a teaching guidebook and learning tasks using GeoGebra, and an alternative one without technology are required for investigating what impact is resulted by teaching intervention or happens as a result of the integrating GeoGebra into the teaching intervention. Consequently, in order to examine the process and the outcome of learning, the relationship between spatial thinking and geometric performance, the relationship between geometric performance and the sustainability of learning, and the relationship between spatial thinking and sustainable learning, a collection of qualitative and quantitative data is required during the experiment.

3.3 Research Questions

This research aims to investigate the impact of a teaching intervention on the learning process and outcome, and it attempts to answer the following questions:

1. What is the impact of teaching intervention using GeoGebra on
    - Geometric performance for Year Five students?
    - Spatial thinking for Year Five students?
    - Sustaining knowledge for more extended periods of time for Year Five students
    - Students' attitude towards Mathematics in terms of Mathematics academic self-concept, enjoyment of mathematics, and the Perceived value of mathematics?
2. What are the students' views on learning using GeoGebra over time?
3. What are the relationships between geometry performance and spatial thinking, sustainable learning, and students' attitude towards Mathematics?
4. What patterns of dyadic interaction can be found in a primary mathematics classroom while learning using GeoGebra?
5. Do differences in the nature of dyadic interaction result in different outcomes in terms of,
    - Students' attitude towards Mathematics regrading mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics?
    - Spatial thinking?
    - Geometric performance?
    - Sustainable learning?

3.4 Research Stance

This section will present a foundation for choosing quantitative and qualitative data in this investigation. This will include some critical assumptions and perspectives associated with ontology and epistemology, considering how these assumptions and perspectives have influenced the research methodology for the present research. When the researcher designs a classroom research examination, the research should make several crucial considerations and choices. The most vital ones are those related to the research sample, research participants, data collection methods, data analysis, and tasks employed. Nevertheless, these crucial considerations and selections are fundamentally based on how the researcher views the world, looks at the knowledge, and the relationship between the environment and human beings. Therefore, the researcher's philosophical assumptions and views directly impact his/her research purpose and what he or she deems is a valuable contribution to knowledge, selection of a theory, the research design, its implementation, and the interpretation of outcomes. Thus, the researcher's decision can be to follow quantitative research methodology, according to which, social reality can be broken down into multiple variables to be examined. Alternatively, the researcher's might be inclined towards qualitative research, according to which, reality is complex and can only be studied within its social set consideration, or to choose mixed-method research, which combines qualitative and quantitative methodologies (McKay, 2005, Kos, 2017, Popkewitz, 1984).

As this proposed research aims to investigate the impact of the integration of GeoGebra into teaching intervention on students' attitudes towards mathematics (in terms of mathematics self-concept, enjoyment of mathematics, and perceived value of mathematics), students' views on mathematics learning activities with GeoGebra, pairs' interaction patterns, and learning outcomes, I will discuss the research stance concerning teaching and learning process and learning outcome accordingly. This research will draw on the ontologies and epistemologies of constructivism, which believes learning is a social process and occurs in a social situation through learners' interaction (Lantolf, 2006, Kim, 2001, Yimer and Feza, 2019, Amineh and Asl, 2015, Kos, 2017). Also, teaching and learning activities can be designed in a way to make learner-centred learning instead of instructor-centred (Adom et al., 2016). The assumptions underlying constructivism are based on the conceptualisations of reality, knowledge, and learning (Kim, 2001). Constructivism does not view reality as something out there; instead, it can be constructed through human activity (Orey, 2010, Amineh and Asl, 2015). Therefore, individual learners are responsible for constructing their own unique understanding through discussion or interaction and collaboration with each other and the surrounding environment (Kim, 2001, Creswell, 2014). This suggests that learning occurs when learners interact with each other, with the teacher as a facilitator, the learning materials and environment.

Thus, meaningful learning can develop via engaging learners in social activities; for instance, interaction and collaboration (Amineh and Asl, 2015). Hence, experience is the outcome of human activities which are constructed in the social and cultural context. This means learning needs to be considered as a social process that ought to be conducted by learners who actively engage using the available factors through interaction and collaboration in the surrounding environment. This means learning activities according to the constructivism paradigm tends to pave the way for knowledge development (Orey, 2010). Additionally, students' views of their learning will be considered due to the fact that this research focus on the actual processes of learning and development. Thorne (2005) precisely addressed that if the researcher's aim is focused on the concrete processes of learning and improvement that take the learner's opinion into account, then a concentration on activity is essential and desirable. Creswell (2014) stated that constructivism's perspective is typically seen as a qualitative research approach in which it obtains data through observing participants' behaviours during their engagement in activities to explore how they develop and share patterns of behaviour over time. Therefore, this research will employ video data to explore pairs' patterns of interaction to observe participant interaction.

On the other hand, post-positivism believes that causes (possibly) determine effects or consequences (i.e., the social construction of parts of reality that exist out there in the world). Hence, the phenomena that have been studied by this paradigm investigate and evaluate the causes that impact outcomes by controlling variables. The understanding and predictions that develop through the post-positivist paradigm relies on rigorous measurement and observation of the objective truth that exists out there in the world. Therefore, developing quantitative instruments of observation and investigating the behaviour of individuals becomes paramount for a post-positivist (Creswell, 2014, Tashakkori and Teddlie, 2010, Ponterotto, 2005). This paradigm permits precise measurements, manipulation, and control of variables, allowing researchers to test critical hypotheses and infer causal relationships between variables (Piki, 2010). The post-positivist epistemology is manifested by pre- and post-test quasi-experimental research designs that employ treatment, outcome assessments, and experimental units, but do not apply random assignment to compare which treatment let the change happen. The post-positivism paradigm is wider than comparing mean scores. However, it depends on non-equivalent groups who are different from each other in many ways other than the presence of the treatment whose impacts are being examined (Taylor and Medina, 2011, DePoy, 2016). According to Kankam (2019) and Scotland (2012) the post-positivist paradigm is a suitable approach to investigate the behaviour of individuals and explore participants' views. Therefore, this research will employ some quantitative methods to investigate how the research intervention helps students improve their geometric performance, spatial thinking skills, sustainable learning, as well as student attitude towards learning mathematics via questionnaires and GeoGebra visual questionnaires to attain learners' point views over time of the research experiment.

This research views the learning process as socially constructed and sees learning occurring when learners interact with each other, their teacher, learning materials, and the environment around them, while learning outcome is constructed within individuals through discussion, interaction, and collaboration. In other words, students develop their understanding in a social environment (classroom) via interacting and collaborating to perform learning tasks.

Thus, learning activities are social processes, but the learners' understanding of the concept he/she learned is different and exists individually. Hence, the learning process is a social operation, but learning outcomes are developed in individuals' minds separately, which means each person has his/her understanding of the concept he/she learned. This paves the way to adopt both constructivist and post-positivist stances and therefore, aims to explore pair patterns of interaction while performing mathematical tasks using GeoGebra, and see to which extent GeoGebra helps in enhancing collaborative learning. It also aims to investigate the impact of research intervention on participants' learning outcomes and spatial thinking skills, taking into account students' attitudes and views on their learning and then, seeking plausible causes (Cohen et al., 2007).

Notwithstanding the different schools of thought in social research, which were historically presented as largely mutually exclusive, and despite the apparent tension found between proponents of the paradigms, multiparadigms can be used, indicating how different approaches have characteristics that can be harmoniously encapsulated within a consistent research design (Weaver and Olson, 2006, Schultz and Hatch, 1996, Brewer and Hunter, 2006, Creswell, 2014, Tashakkori and Teddlie, 2010, Taylor and Medina, 2011). Multiparadigm is beneficial and desirable in light of predictions about diversity in modern society. Because the acceptance of a diversity of paradigms can offer a system that benefits from the advantages of different paradigms, it can generate a deep understanding of the phenomenon under examination (Schultz and Hatch, 1996). This is why there has been increasing evidence that proves the overall legitimacy of mixed methods, as a distinct methodological approach, different from purely quantitative or qualitative methods. Particularly, in social sciences and education, mixed methods and multiple research paradigms are increasingly being used as an alternative to single paradigm and mono-method ways of conducting investigations (Brewer and Hunter, 2006, Creswell, 2014, Piki, 2010, Tashakkori and Teddlie, 2010, Teddlie, 2009, Piki, 2011). Consequently, this research employed quantitative and qualitative mixed research methods.

3.5 Research Methodology

Many researchers have highlighted the importance of using quantitative and qualitative research to study the use of ICT applications to better understand its impact on students' performance and why a specific outcome is realised (Agyei and Voogt, 2011). Gathering both quantitative and qualitative data and integrating them for drawing interpretations based on the strengths of both sets of data to understand research problems in the social, behavioural, and health sciences is termed mixed methods research (Creswell, 2014). Johnson and Onwuegbuzie (2004) defined mixed methods research as the kind of research where the researcher mixes elements of qualitative and quantitative methods, such as using qualitative and quantitative perspectives, data collection, analysis, and inference approaches to obtain depth and breadth of understanding regarding the research problem. Mixed method research design assists the researcher to go for inductive and deductive rationalisation to obtain more accurate answers to the research questions that cannot be answered solely by way of qualitative or quantitative research (Denzin and Lincoln 2000). It can bridge the gap between qualitative and quantitative views by focusing on the usefulness of both approaches and how they can be used together in one piece of research to benefit from their advantages and minimise their disadvantages (Doyle et al., 2009, Johnson and Onwuegbuzie, 2004).

Mixed methods research has two distinctive features. Firstly, it includes collecting and analysing qualitative and quantitative data in rigorous techniques, which means gathering and analysing data thoroughly and based on a prearranged and tested systems. Secondly, it includes integrating qualitative and quantitative data in approaches that underline the benefits of using both research methods to illustrate and expand the understanding of the research problem (Watkins and Gioia, 2015). Nevertheless, there has been difficulty integrating the data and the type and level of data integration subjected to the researcher. However, the research questions and research idea can guide quantitative and qualitative data integration throughout the data collection and data analysis phases (Tashakkori and Creswell, 2007, Watkins and Gioia, 2015).

Mixed methods research has been used in the field of education to examine many complex educational phenomena. Ponce and Pagán-Maldonado (2015) found, based on a literature review, that the majority of educational scholars have used mixed methods research to study teaching and learning in the school. For example, Miranda (2012) employed a complementary mixed research method to examine the impact of a virtual laboratory on students' academic performance in ninth grade in a biology course. In the quantitative stage, a pre- and post-test was utilized in two groups (experimental and control). The qualitative stage involved focus groups with the study participants and observations executed by the teachers during the experiment. This is while Chévere (2012) used a complementary mixed method design to evaluate the influence of self-monitoring approaches on students' performance in year four, concerning to sum skill in regrouping up to a million and discovering their learning experience with the teaching method. In the quantitative phase, Chévere (2012) used an experimental design (pre- and post-test). In the qualitative stage, interviews were conducted to understand the point of view of students towards self-monitoring strategies. To sum up, mixed methods design gives the researchers an opportunity to describe and explain the complexity of the process of teaching and learning as educational phenomena. The quantitative elements assess the impact of teaching strategies on the students' performance by using experimental treatments, whereas the qualitative factors allow researchers to understand how students identify the teaching approaches, what occurred in their minds, and what components will enable them to learn (Pence and Pagán-Maldonado, 2015).

3.6 Research Design

There are many different designs of mixed methods research. The four major types of mixed methods research designs are as follows: triangulation design, embedded design, explanatory design, and exploratory design (Creswell and Plano Clark, 2017). Concerning this research, the researcher employs an embedded design in which quantitative and qualitative data were gathered and analysed concurrently, and via the research process (Creswell, 2012, Matthews, 2010). This design is suitable for examining the learning process, students' performance, and the correlation with different types of data (Asensio-Pérez et al., 2017, Creswell, 2012, Mertens, 2019, Ponce and Pagán-Maldonado, 2015, Xie et al., 2017). Therefore, to examine the research questions, the researcher conducted a pre- and post-test quasi-experiment non-equivalent group, based on control and experimental groups (see table 3.1 experimental design). During the experiment, the researcher observed each session in the GeoGebra experimental group. In addition, students in the GeoGebra experimental group answered on visual questionnaires, as well as the researcher recording all the GeoGebra lessons to investigate pairs' patterns of interaction. At the end of the experiment, students performed the post-test in relation to geometric performance and spatial thinking to examine the impact of using GeoGebra on students' performance and spatial thinking skills, and students' attitude towards mathematics questionnaires, to find out if the research intervention helped learners develop positive attitudes towards mathematics. After two months, students performed the delayed test to examine sustainable learning

	Pre-test	Treatment	Post-test	Sustainable test
Group 1	X	O	X	D
Group 2	X	T	X	D
Group 3	X		X	D

Table 3.1 Experiment Design

X: Performing geometric test, spatial thinking test, students' attitudes towards mathematics questionnaire, GeoGebra visual questionnaire

O: Teaching intervention with GeoGebra

T: Teaching intervention with hands-on

D: Performing delayed test

This research design is one of the most frequent designs used in educational research. It allows researchers to investigate the educational outcomes and explore the learning process within established groups. Furthermore, the main limitation of this research design is the difficulty regarding generalisation, which is related to external validity due to the lack of randomisation and sample size (Dugard and Todman, 1995, De Vaus and de Vaus, 2013, Cohen, 2018, Hawes et al., 2017).

3.7 Research Community and Research Sample

The research community for this study is all Year Five students aged 9 – 11 in the general directorate of education in Jeddah city in the school year 2019/2020, with a total of 24423 students. The research sample was selected based on the school environment, the acceptance of school management staff, and the teacher participating in the research experiment. The researcher selected Al-Manarat primary school in Jeddah city since it has all the facilities required for this research, such as an IT room. Besides, the headteacher accepted implementing this research experiment in his school, and the mathematics teacher agreed to participate in this research because he is interested in using technology to teach mathematics. The students' parents and guardians also agreed that their children participate in this research by signing the consent form. The research sample in a total of 79 students was divided into three classes. In addition, the researcher randomly selected the experiment groups for the reason that which classroom does not have conflict in mathematics timetable with ICT timetable.

Two students were excluded from the research experiment since one of them had exam anxiety and the other one had a health condition, which made him miss four sessions. The two students remained attending their class as their typical school day. As a result, the GeoGebra experiment group has 25 students, the hands-on experiment group has 27 students, and the control group has 27 students. They were grouped based on their scores obtained on pre-test geometric performance, where they were categorised into two levels of high achievers and low achievers. Therefore, each pair consisted of one high achiever student and one low achiever student in the GeoGebra group and hands-on groups.

Furthermore, the same teacher teaches all three groups following the school day timetable in a typical school day as to reduce any negative impact on the setting which comes from the researcher or another teacher teaching them (also avoids researcher bias). This is also ensuring equality in the aspect of teacher skills and experience for all research groups.

3.8 Data Collection process

The process of data collection (Figure 3.1) started with selecting the appropriate school with the facilities and equipment required to implement the current research experiment. This is why the research sample was selected by mean from the Jeddah city schools. The selected school was Al-Manarat primary school which has four classes in Year Five. Three of them were taghut by the same teacher, and a different teacher taught the other class. Therefore, the three classes taught by the same teacher were selected to participate in the research experiment. These three groups of fifth-grade students were formed with at least 25 students in each by means of random demographic distribution. One was the control group, and the others were the experimental groups, namely, the GeoGebra experimental group and the hands-on experimental group. Students were involved in this experiment during their customarily scheduled mathematics periods. Students use GeoGebra as an open-source learning environment based on the teaching intervention to explore whether it enhances pairs' pattern of interaction, geometric performance, spatial thinking skills, students' attitude towards mathematics, and students' views on learning using GeoGebra. While the hands-on group followed the same teaching process as the GeoGebra group but it differs in performing the learning tasks using hands-on materials instead of using GeoGebra. The traditional teaching group were taught as they studied mathematics in their typical school day where the teacher explained, and they listened. After specifying the research group, students performed pre-test for the Geometric test, spatial thinking test and students' attitude towards mathematics on different separate days to avoid any negative impact on students. In addition, the GeoGebra group performed GeoGebra visual questionnaire before having the training sessions on GeoGebra and at the end of every week over the research experiment.

Furthermore, the geometric performance pre-test results were used to group GeoGebra and hands-on groups students in pairs, where possible based on their performance level (low and high). Hence, students in the GeoGebra and hands-on groups performed learning tasks in pairs according to the teaching intervention, whereas students in the traditional teaching group performed learning tasks individually. All research groups were taught by the same teacher, as explained earlier.

During the research experiment, the researcher observed the GeoGebra experimental group to ensure the teaching process was going on as planned and support the participated teacher if needed. In addition to this, to obtain the qualitative data, all the learning activities in the GeoGebra group were recorded using 12 mini cameras over the research course. At the end of the research experiment, students in all research groups perform the post-test for the Geometric test, spatial thinking test, and attitude towards mathematics. The sustainable learning test was performed after six weeks of the end of the research experiment. Furthermore, post-tests were used to ascertain details concerning the relationship between pairs patterns of interaction and pairs' level of geometric performance, spatial thinking, sustainable learning, students' attitudes toward mathematics, students' views towards GeoGebra. It should be mentioned that both qualitative and quantitative data were integrated to interpret the result of the experiment.

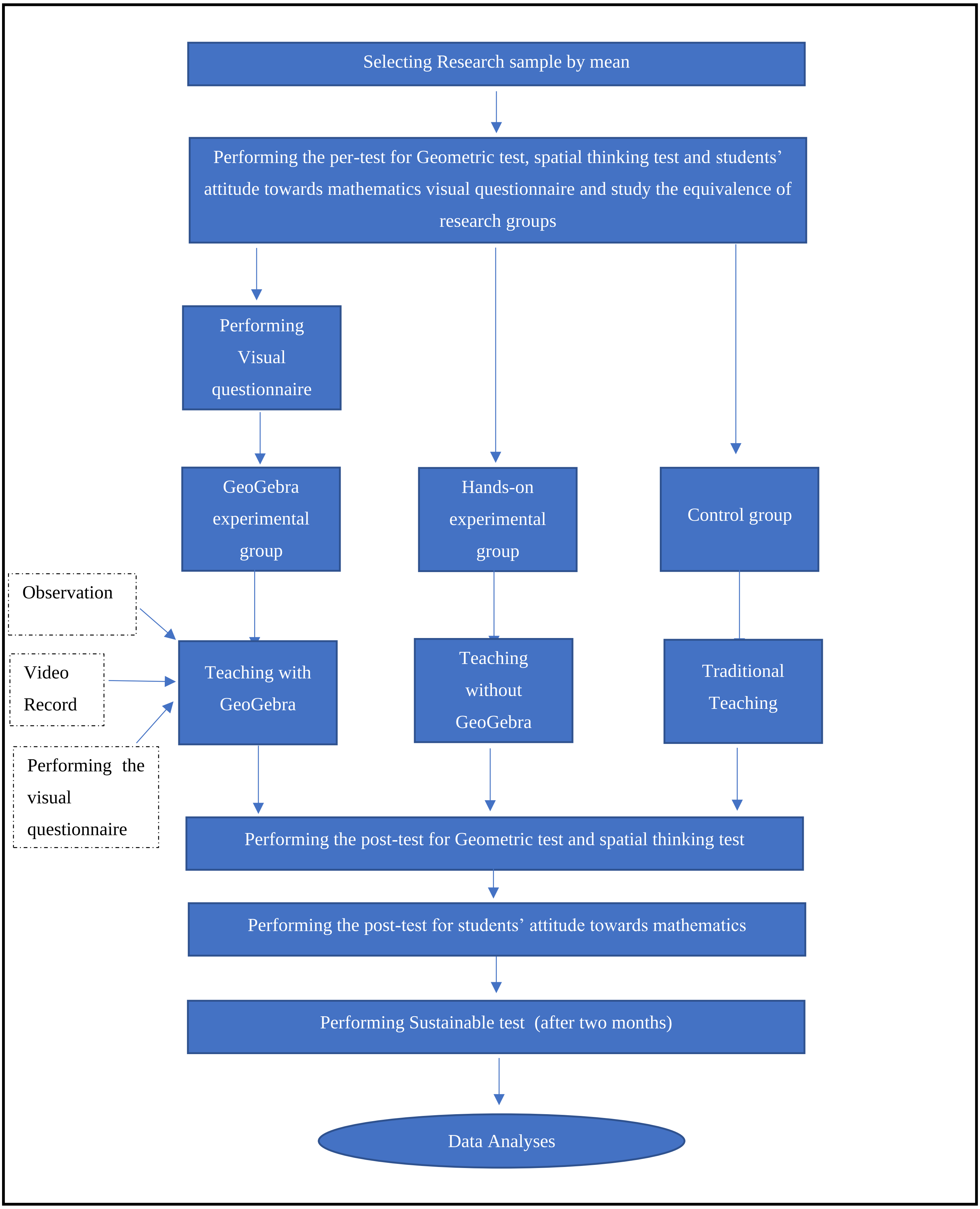


Figure 3.1 Data Collection Process

3.9 Research Procedure

To accomplish this examination, the researcher analysed the content of the geometric unit in the Year Five textbook concerning geometric concepts, generalisations and skills, and subsequently identified learning objectives. Thus, the researcher designed a guidebook of the teaching intervention for teachers and designed a series of learning tasks for this experiment. This was followed by creating a performance test and making equivalent copies of it for examining geometric performance and sustainable learning. It is crucial to point out that the teacher's guidebook, series of learning tasks, and the performance test copies were reviewed by mathematics experts, teachers, mathematics educational leadership, and PhD students. Thus, the teacher's guidebook, series of learning tasks, and performance test copies were developed in light of referees' experiences and suggestions. After that, the reliability and validity of all copies of the performance test were examined via a different experimental sample to the research sample two months before implementing the research experiment. This followed by selecting spatial thinking tests appropriate for primary school students. Furthermore, the researcher designed a visual questionnaire to obtain the students' views and reflect on their learning process. This questionnaire was piloted to test its clarity, reliability, and validity.

In addition, the researcher visited the selected school, which has all the facilities required for the experiment. During this visit, the researcher met the participating teacher to ensure he had his training course on GeoGebra since the Ministry of Education in Saudi Arabia established a plan to train all mathematics teachers on using GeoGebra. However, the teacher was not trained. That is why the researcher trained him on how to use GeoGebra and asked the IT department in the selected school to install GeoGebra on IT computers. Importantly, the participating students had three training sessions on using GeoGebra to familiarise them as to avoid any distraction throughout the experiment implementation. During the experiment, the researcher observed all experimental group sessions to ensure the teaching process was performed as planned and recording videos of the student's activities and interactions was done appropriately. It is good to mention that the ACAD framework guided the research process and implementation, as shown in table (3.2).

Physical design	Epistemological design	Social design
Selecting school with IT room	Content analysis for geometric unit and then, identifying learning aims	Pair learning in the first stage of the teaching intervention
Preparing hands-on material	Teaching intervention guidebook for GeoGebra and hands-on groups	Pair learning in the second stage of teaching intervention and sometimes individual
Installing GeoGebra on the PCs	Developing learning tasks for GeoGebra and hands-on groups	Individual learning in the last phase of teaching intervention Group discussion with teaching
Organising hands-on group classroom for pair learning	Training participating teacher and students on GeoGebra	
Organising IT for pair learning		
Using mini cameras for recording each pair activities		

Table 3.2 ACAD Guidance for Research Process

All the appropriate documents and papers from the educational authority, school, and parents' consent forms were obtained before implementing the experiment to ensure the learners' consent and confidentiality. After the experiment, students carried out the post-test concerning geometric performance and spatial thinking on consecutive days, and after two months, the participants performed the delayed test. However, participants performed the post-tests online due to Covid-19 and schools closing.

In the following section, more details on the research process and data collection methods are offered:

3.10 Research Methods and Instruments

Using data obtained by adopting different methods increases the reliability and validity of the research results (Mertens, 2014; Bryman, 2016). Therefore, both qualitative and quantitative data collection methods, specifically, video recording, GeoGebra visual questionnaire, performance test, spatial thinking test, sustainable test and students' attitude towards mathematics test were used in the research and analysed. In order to conduct the experiment of this research, the researcher started with analysing the content of the geometric unit in the year five mathematics textbook to identify the mathematical concepts in this unit and then indicate and blueprint the learning aims. Accordingly, designing the learning tasks, planning the teaching process, developing the selected unit in light of research intervention, designing geometric performance tests, and delayed tests were conducted.

3.10.1 Content Analysis of Geometric Unit

Planning a lesson for teaching requires the teacher analysing the content which will be taught to know the components of that knowledge, skill, and emotional content (Abu Libdeh et al., 1996). Knowing the content is vital to being creative in designing worthwhile tasks for learning that take learners' experience, curiosity, and needs into account. Knowing the content knowledge is also crucial to help in preparing the valuable resources and challenges of a diverse classroom that take individual differences into account for making the learning situation enjoyable (Ball, 2000). In that regard, Johnston-Wilder and Johnston-Wild (1999) assert that analysing content is a significant part of planning mathematics lessons. This is because it helps teachers develop knowledge and understanding of the complex set of mathematics that will be taught, including how students learn mathematics, their previous experience of students, knowing skills and concepts, and knowing the best ways to teach individual lessons. They believe that content analysis is an essential step in planning sequences of lessons and designing learning tasks that help take account of students' differences, select and prepare the most appropriate resources, including ICTs, and build assessment tools that fit with the content.

Content analysis in mathematics education refers to the fragmentation of scientific material into its components, namely facts, concepts, principles, formulas, generalisations, theories, etc. (Obeidat et al., 1992). Alternatively, Al-Sawa'i (2004) defined it as identifying the components of knowledge included in the lesson or textbook. Complementing both views, Jones and Edwards (2017) defined content analysis as identifying the mathematical themes, mathematical reasoning, and problem solving to be taught. In order to design the learning tasks of this research, planning sequences lessons in the light of the teaching intervention, designing the geometric performance test and delayed test, the mathematical content analysis in this research aims to identify and highlight concepts, generalisations, and skills included in the textbook (Abu Zina and Ababneh, 2010). Mathematical content is divided into three components, namely:

- **Mathematical concepts** refer to defining the basic characteristics that give a term a mathematical meaning (Badawi, 2003).
- **Mathematical generalisation** is a mathematical phrase or formula which defines the relationship between two or more mathematical concepts (Abu Zina and Ababneh, 2010).

- **Mathematical skills** are the abilities to prove a formula or rule, draw a shape, demonstrate an exercise, or solve a problem at a high level of mastery through understanding, and with minimal effort in the least time possible (Khalifa, 1999).

After specifying the definitions for the components of the mathematical content adopted for this research, the researcher read the content of the Geometric Shapes unit of year five mathematics textbook edition 2019/2020 meticulously and analysed it in light of the above definitions. This unit includes seven lessons, specifically, geometric concepts, quadrilateral shapes, geometry ordered pairs, algebra function representation, translation at a coordinate plane, the reflection at a coordinate plane, and the rotation at a coordinate plane. This unit was selected because it is the first-time students study geometric transformation and geometric concepts according to the Saudi mathematics curriculum (see Appendix 3.1 for reliability and validity of the content analysis).

3.10.2 Learning Objectives

In light of the result of the content analysis, the researcher wrote the learning objectives according to an expanded model taxonomy which was developed by James W. Wilson in 1968. He developed his cognitive taxonomy based upon Bloom's taxonomy to fit mathematics in order to help mathematics teachers and mathematicians working on the curriculum build a test that effectively assesses students. The expanded model taxonomy was divided into four levels: Computation or remembering, understanding or comprehension, application, and analysis which includes the top three levels of Bloom's taxonomy levels as shown in figure (3.2) (Nayef et al., 2013, Wilson, 1968 cited in Al-Makoshi, 2001):

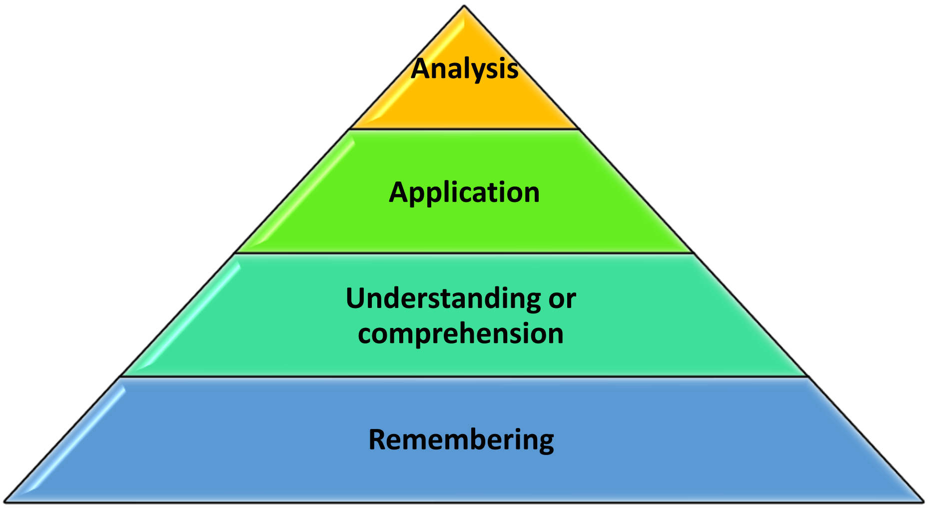


Figure 3.2 Wilson's expanded model taxonomy

After defining the expanded model taxonomy of Wilson (1968), the researcher identified the learning objectives of the Geometrics Shapes units in light of the results of the content analysis and according to Wilson's expanded model taxonomy (Appendix 3.2). Each learning aim describes one learning outcome of the geometric unit that should have been measured to determine the impact of the research teaching intervention. Therefore, a question was written for each learning objective using the blueprint table to measure what students have learned. The following section will describe the geometric performance test.

3.10.3 Geometric Performance Test

Student performance was measured through a test at the end of the fifteenth session to examine students' understanding of the various instructed concepts. The test was developed in light of the content analyses of the geometric shapes unit in the year five textbook, and the learning objectives of this unit according to Wilson's expanded model taxonomy. After specifying the learning objectives, the researcher created the exam blueprint, which is defined as a table that associates learning outcomes with the relative weight to each outcome that is allocated on the exam with the level of performance of the used cognitive taxonomy (Young et al., 2019). The use of test blueprint helps in building exam questions matching both the content and the cognitive level with the right weight for each level of learning objectives, as well as constructing exams covering all the aspects of the content (Young et al., 2019). Al-Rafi'i and Sabri (2003) described the test blueprint table as a two-dimensional table; one of which represents unit content (topics), and the other, represents learning outcomes (aims) associated with this content. Therefore, after the researcher determined the learning aims according to the levels of the expanded model for Wilson, he prepared a blueprint table for the learning objectives to guide developing the geometric performance test questions, as follows:

Object Level / Content	Remembering	Comprehension	Application	Analysis	Total	Weight
Geometric Concepts	0	18	0	0	18	32%
Quadrilaterals Shapes	1	5	1	9	16	29%
Geometry: Ordered Pairs	3	1	2	0	6	11%
Algebra and Geometry: Function Representation	0	0	3	0	3	5%
Translation at Coordinate Plane	0	3	1	0	4	7%
The Reflection at Coordinate Plane	0	2	1	2	5	9%
The Rotation at Coordinate Plane	0	1	1	2	4	7%
Total	4	30	9	13	56	100%
Weight	7%	54%	16%	23%	100%	

Table 3.3 Blueprint table of Geometric Shapes unit

In the light of the test blueprint table, the researcher prepared the geometric performance test in its original form and equivalent form of the geometric performance test. Each consists of 18 questions that cover all aspects of the Geometric Shapes unit. The test questions were distributed according to the expanded model for Wilson, as shown in the above Table (3.3). Each of their forms was divided into two sections: The first section is multiple-choice questions which consist of 13 items, and the second section is an open question which consists of two questions with five sub-questions. The researcher took into account the weight of Wilson's cognitive levels while designing the geometric performance test and the equivalent.

3.10.3.1 Piloting Geometric Performance Test and Equivalent Form

The purpose of the pilot study is to examine validity and reliability, identify the appropriate duration for performing the test, assess possible difficulties of the test questions and the test language. The pilot study took three phases in order to examine the geometric performance test.

3.10.3.1.1 Phase one: Language Check

The purpose of this phase was to examine the language of both forms of the geometric test by offering them to five students between 10 to 11 years old. The researcher revised the test's words with each one of them in terms of clarity and readability of the test questions. After that, he revised the language of the test with them in a group. In this step, one issue in test clarity was raised, which was in question 12 from the multiple-choice section. This question is repeated in both forms of the test. This question gives three correct items and one wrong, and asks the participants to choose the incorrect answer. As a result, the researcher developed the language of the geometric test and its equivalent form based on the students' suggestions by making the confusing word in this question in bold (which sentence in the following **is incorrect?**). After developing the test language in light of the children's feedback, the researcher moved to the next phase to examine the validation of the geometric test (see Appendix 3.3 & 3.4).

3.10.3.1.2 Phase two: Validity

Validity is defined as the extent to which any assessing method measures what it is proposed to assess (Carmines and Zeller, 1979). There are many types of validity, such as face validity, criterion-related validity, construct validity, content validity, etc. The most common type of validity used in social science and education is content validity. It has been playing a key role in developing and evaluating various types of tests in education. Content validity refers to the level in which an assessment tool is related to, and representative of, the targeted build it is intended to measure (Maggino and SpringerLink, 2020). It depends upon the degree to which an experimental assessment reflects a specific domain of content (Carmines and Zeller, 1979). So, the researcher constructed both forms of the geometric performance test in light of the content analysis and learning objectives, according to Wilson's expanded model for the Geometric Shapes unit. He then prepared a form of this test to link each question with the geometrical concepts, the learning objective, and its level in order to send them to mathematical education experts (university staff, researchers, PhD students, mathematics education leadership, and mathematics teachers).

Consequently, the researcher received feedback from the experts concerning ‘match the questions to the objective learning level’ and ‘type of question 12’. However, they agreed that each question matched the concepts and the learning objectives and agreed on the equivalent of the test forms. As a result, the researcher developed both forms of the geometric performance test in light of experts' feedback and kept question 12. Since this type of question has been used widely in the year five textbook, it is familiar to year five students, and there was no comment on this question during the pilot study from the participated students, the researcher became confident to continue keeping this question in the geometric performance test (see Appendix 3.5 & 3.6).

3.10.3.1.3 Phase three: Reliability

The purpose of this stage is to examine the reliability of the geometric performance test and its equivalent form, the appropriate time for answering the test, and the difficulties of the test questions. The researcher employed many different techniques to assess test reliability and the other statistical elements. To do so, the researcher identified the score of each question in both forms of the performance test. He gave 1 score for each question in the multiple-choice section, and for the open question section, 1 score was given for each element in question 14, as well as 4 scores for question 15 (1 score for reflecting the shape and 3 scores for writing the order pairs for the new shape after the reflection). The suggested score and the type of test's questions help to increase objectivity while marking the performance test.

One of the crucial aspects of the assessment is reflecting upon students' ability and cognitive skills. Hence, the reliability of the geometric performance test must be measured. Reliability refers to the consistency of an exam result (Akib and Ghafar, 2015). In other words, it describes the extent to which the results produced by the assessment procedure are reproducible (John and Benet-Martínez, 2014). For examining reliability, there are three main ways: test-retest, paralleled or equivalence forms, and split half (John and Benet-Martínez, 2014, Meyer, 2010). As a result, the researcher conducted a pilot study for examining the reliability of geometric performance test and its equivalent form. To do so, he employed different methods in order to explore the geometric performance reliability.

The first technique employed in the pilot study was equivalent forms or alternate forms. In this technique, the researcher had to administer both forms of the test to the same participants with a short time between administrations (Wiersma, 2000, Henchy, 2013). Therefore, the geometric performance test and its equivalent form were administrated to the same participating students in the following two days. For calculating the reliability of the parallel forms, the researcher used SPSS to calculate the Pearson correlation coefficient and Alpha Cronbach to determine reliability, as shown in table (3.4).

Correlation	Results
Pearson	0.717
Cronbach's Alpha	0.832

Table 3.4 Parallel reliability performance test

As shown in the above table (3.4), the Pearson reliability coefficient is (0.717), and Cronbach's alpha is (0.832), which indicates that the geometric performance test and its equivalent form have a high degree of reliability (Mukaka, 2012).

In addition, the researcher employed the test-retest method the examine the reliability of the geometric performance test and the equivalent form of it. The rationale behind this step is to find out the stability of both structures regarding the time. In other words, this technique examines the reliability in the aspect of changing students' responses over time and the change in the test situation (John and Benet-Martinez, 2014). Another aim of this process is to identify the appropriate time duration for answering each form of the geometric test. Consequently, the geometric performance test and the equivalent were administrated twice to different groups; at the beginning and end of the pilot study, two different times just over four weeks apart. Hence, by using SPSS, the researcher measured the Pearson correlation coefficient and Cronbach's Alpha between students' results to find out the reliability of the geometric performance test. Therefore, the parallel reliability result in table (3.5 and 3.6) indicates that both forms of the performance test are equal. While the test-retest reliability results reveal that the geometric performance test and the equivalent form are stable regarding the change occur over time, these results evidence the suitability of using them to achieve this research aim.

Correlation	Results
Pearson	0.721
Cronbach's Alpha	0.829

Table 3.5 Geometric Performance Test Reliability Coefficient

Correlation	Results
Pearson	0.723
Cronbach's Alpha	0.836

Table 3.6 Delayed Test (Equivalent Form) Reliability Coefficient

3.10.3.1.4 Required Time for Answering the Test

For identifying the time required for answering the test, the researcher measured the test time by calculating the average time taken by the first and last student who finished answering the test in the pilot study. The time first student took to answer the test questions was (15) minutes, while the last student took (25) minutes. Hence, by calculating the average time for the first and last students, the researcher found the time necessary for answering the geometric performance test (20 minutes). Concerning the equivalent form of the performance test, the first student took 17 minutes, and the last student took 28 minutes. Thus, the average time between them was almost 23 minutes.

3.10.3.1.5 Test Difficulty Coefficient

The measurement of test difficulty coefficient for each test item contributes to judging the validity and suitability for measurement purposes. Test questions' difficulty is referred to the percentage of students who answer a question correctly. In other words, it can be described as the frequency of which students give a correct response to a question (McCowan and McCowan, 1999). Thus, the test difficulty range is between 0 to 1, which means the lowest value of test difficulty is 0.00, and the highest value is 1.00. Therefore, the hard questions have their difficulty coefficient value approaching 0.00, while very easy questions have their difficulty coefficient value approaching 1.00. To calculate the test item difficulty, the researcher used the following formula (McCowan and McCowan, 1999):

$$\text{difficulty coefficient} = \frac{\text{the number of students who aswered a question correctly}}{\text{the total number of students who perfromed the test}} \times 100$$

The result of calculating the difficulty coefficient of the geometric performance test in Appendix (3.7) shows that the difficulty coefficient for the geometric performance test items is considered and that the overall difficulty coefficient for the test is (0.52), which is close to (0.50). Consequently, this indicates the appropriateness of the test items. As Sheikh et al. (2009) and Musa et al. (2018) mention the range of the difficulty coefficient for the performance test is between (0.10 – 0.90) and the best of it was (50,0). It is also explicit from the Appendix (3.7) that the difficulty coefficient of geometric performance test questions ranges from 0.19 to 0.85, which indicates the suitability and acceptability of all test questions.

Meanwhile, the difficulty coefficient of the equivalent form questions is shown in Appendix (3.8) to be between 0.19 and 0.85, and the difficulty coefficient for the whole test is 0.51, which is close to (0.50). As a result, the difficulty coefficient results evidence the appropriateness of the test items, as well as all the test items being considered, since their difficulty coefficients are between 0.10 and 0.90 and the best difficulty coefficients performance test is 0.50 (Sheikh et al.,2009; Musa et al., 2018).

3.10.3.1.6 Discrimination Coefficient

One of the significant characteristics that should be present in a performance test is the discrimination feature. The discrimination coefficient is referred to the extent in which the test can measure individual differences. In other words, it refers to test's capability to distinguish between students' abilities (Musa et al., 2018). For calculating the discrimination coefficient for both forms of the geometric performance test, the researcher used the following formula (F, 1987):

$$\text{Discrimination coefficient} = \text{difficulty coefficient} \times \text{ease coefficient}$$

By applying the above formula to compute the discrimination coefficient for geometric performance test and equivalent form, the results were:

$$\text{Geometric Performance Test Discrimination coefficient} = 0.52 \times 0.48 = 0.2496$$

$$\text{Equivalent form of Geometric Performance Test Discrimination coefficient} = 0.51 \times 0.49 = 0.2499$$

The above results demonstrate that the discrimination coefficient for the geometric performance test is (0.2496), and the discrimination coefficient for equivalent form is (0.2499). According to Patock (2004), suitable discrimination coefficients range between 0.10 and 0.30. Hence, these results indicate that both forms of the geometric performance test can distinguish between students.

3.10.4 Delayed Test

One of the critical goals of this research is to measure how the integration of GeoGebra into teaching intervention can improve sustainable learning. Edelman et al. (2010) defined sustainable learning as the ability to maintain experience and learning outcomes that make recalling or recognising the information possible. In other words, sustainable knowledge means the capability of students to retain and retrieve the learned materials and skills in subsequent classes. Therefore, the researcher calculated such a metric by monitoring each student’s performance in the geometric shapes unit. A preliminary indicator of this metric was measured by way of a delayed exam, which is an equivalent test of the geometric post-test done six weeks after the research experiment to determine students' abilities to retrieve the knowledge and thus, compare their results with the results they attained in the post-test.

Consequently, the delayed test or equivalent test was designed in light of the geometric performance test. The researcher followed the same procedure to construct a delayed test and pilot study as geometric performance test (see section 3.10.3).

3.10.5 Spatial Thinking Test

As explained previously, this research defines spatial thinking as the collection of cognitive skills, consisting of declarative and perceptual forms of knowledge and many cognitive operations that can be used to transform, combine, or otherwise operate this knowledge by using a constructive combination of three elements: specifically, concepts of space, tools of representation, and processes of reasoning (National Research Council, 2006). Spatial thinking skills are vital skills that can play a crucial role in learning geometry and can be developed in geometry classes. This research expected that such thinking skills could be improved due to the integration of GeoGebra into the research intervention. Hence, the researcher selected Raven's Coloured Progressive Matrices to examine spatial thinking skills.

Raven's Coloured Progressive Matrices is one of the non-verbal intelligence tests which was developed for children from 5.5 to 11 years old. Coloured Progressive matrices are free of cultural influence on a high degree (Cotton et al., 2005, Abu Hammad, 2011). This measurement consists of 36 non-representational items incomplete in the right bottom corner from the end was divided into three sections. Each question is given six alternative figures to choose from as to which one is the best to complete the pattern (Abu Hammad, 2011, Basso et al., 1987) (see Appendix 3.9). The guideline of the intelligence test and other studies stated that answering the test questions depends on spatial skills (Schweizer et al., 2007, Abu Hammad, 2011, Muniz et al., 2016). In addition to this, some evidence suggested that the act of completing Raven’s Coloured Progressive Matrices relies on the spatial ability, to a significant degree. Besides, some scholars confirmed that Raven's Coloured Progressive Matrices could be served as a measurement of spatial thinking (Risberg et al., 1977, Abu Al-Nile, 1988, Zmzmi, 1999, Hammad, 2012, Carpenter et al., 1990, Newman et al., 1995). Consequently, Raven's Coloured Progressive Matrices was employed in this research to assess if there is an improvement in spatial thinking skills.

Many scholars have examined Raven's Coloured Progressive Matrices to assess its validity and reliability. The guideline of intelligence test pointed out that Raven's Coloured Progressive Matrices has had a good level of validity and reliability, resulting in several studies confirming it via different techniques (Abu Hammad, 2011). such as internal consistency, split-half reliability, and test-retest reliability. Methods employed to measure the internal reliability of Raven's Coloured Progressive Matrices involve Kuder Richardson Formula 20 (K–R20), Cronbach's alpha, and item analysis. The estimations of internal reliability using the K–R20 and Cronbach's alpha have been in the region of about 0.85. A little higher assessment of internal reliability has been found reliability coefficient from item analysis of 0.89 (Abu Hammad, 2011, Cantwell, 1967, Green and Kluever, 1991, Simoes, 1989). Moreover, Cotton et al. (2005) found that Raven's Coloured Progressive Matrices proved a good split-half reliability and inter-item reliability. These results denote that Raven's Coloured Progressive Matrices has acceptable inter-item stability and reliability, and thus, it is suitable to examine spatial thinking skills.

3.10.6 GeoGebra Visual Questionnaire

To obtain the students' views and reflect on their learning process in the mathematics classroom using GeoGebra, a visual questionnaire was designed. During the research experiment, GeoGebra visual questionnaire was distributed to the experimental group at the end of the last session of each week of the research experiment and collected in the same session. For the purpose of designing the GeoGebra visual questionnaire, the related literature and research instruments have been reviewed, particularly literature on learning mathematics using GeoGebra and the process of learning with ICTs. Hart (2018) asserts that a useful literature review seeks to weigh up the contribution that specific theories, opinions, or methods have yielded to the subject matter. This emphasises the significance of the critical purpose of literature reviews, which were respected in this research when designing GeoGebra visual questionnaire. Thus, a review was conducted of the available research and instruments on the process of learning (e.g. Agyei and Voogt, 2011, Avidov-Ungar and Amir, 2018, Caeli and Bundsgaard, 2020, Ghavifekr and Rosdy, 2015, Mazana et al., 2019, Khan et al., 2011). As a result of the literature review, the GeoGebra visual questionnaire was developed based on the work of Mehdiyev (2009), Pamungkas et al. (2020), Orcos et al. (2019), Dunn and Kennedy (2019), Arbain and Shukor (2015), and Shadaan and Leong (2013).

Consequently, the visual questionnaire was designed in its preliminary form, consisting of 23 items written in positive sentences (Appendix 3.10) that reflect upon students' perception and opinion on their learning process using GeoGebra from different perspectives such as enjoyment, confidence, motivation, engagement collaboration. The questionnaire was scaled using Likert scale five points: a set of statements presented for a natural or theoretical situation under investigation, where applicants are asked to express their degree of agreement from strongly agree to strongly disagree (Joshi et al., 2015). This questionnaire replaced the verbal statements with emojis to describing participants' level of agreement and disagreement. The use of emojis to visualise the questionnaire is because it is commonly used these days by children and can add some enjoyment to respond to the questionnaire's items.

3.10.6.1 Pilot Study

As per Shaughnessy et al. (2000), a pilot study should be conducted to examines the questionnaire before carrying out data collection for the main research. Besides, Creswell and Plano Clark (2017) emphasised that the main principles of a pilot study are to achieve the following: to examine the questionnaire to prevent any potential issues that can be raised throughout answering the questionnaire's questions by the candidates, to prevent problems while entering data, and come to comprehensive estimation in terms of validity and reliability. In addition, the pilot study aims to examine the language clarity and readability of the questionnaire and language suitability for participants' age. In order to ensure the language clarity, readability, validity, and reliability of the visual questionnaire, a pilot study took three phases to accomplish its goal.

3.10.6.1.1 Phase one: language Check

This phase aimed to check the language of the questionnaire by offering it to five children between 10 to 11 years old. The researcher reviewed the test words with each of them regarding the questionnaire’s clarity and readability. After that, the questionnaire was revised with them in a group. As a result of this revision, the majority of the questionnaire items' language was developed in light of children's feedback. In addition to this, the researcher took children's opinions on the clarity of the emojis in the GeoGebra visual questionnaire they agreed on. After developing the questionnaire in light of the children's opinions, the researcher moved to the next stage to assess the validation of the visual questionnaire (see Appendix 3.11).

3.10.6.1.2 Phase two: Validity

As mentioned earlier, validity is the extent to which any assessing method measures what it is proposed to assess (Carmines and Zeller, 1979). Questionnaire validity is "the amount of systematic or built-in error in the questionnaire" (Bolarinwa, 2015, pp. 196). Hence, investigating the validity of the questionnaire can be launched using a group of experts to explore theoretical constructs. Those experts test how fit the notion of a theoretical construct is represented in the questionnaire. Bolarinwa (2015) called this method of validity a translational or representational validity, which combines two sub-methods of validity belonging to this type precisely: face validity and content validity. Face validity refers to the degree to which assessment appears relevant to a particular construct (Taherdoost, 2016); therefore, when the experts look at the questionnaire's items and then agree that the questionnaire is a valid measurement of the concept being assessed just on its face (Bolarinwa, 2015, Taherdoost, 2016).

On the other hand, content validity is described as the level to which an assessment tool is related to, and representative of, the targeted build it is intended to measure (Ariely et al., 2014). Hence, the experts review all the questionnaire items in terms of clarity, readability, and breadth, reaching some level of consensus as to which items ought to be contained in the final draft of the questionnaire (Ariely et al., 2014, Bolarinwa, 2015). Accordingly, the researcher considered both face validity and content validity, which are referred to as translational or representational validity, to assess the validity of the visual questionnaire. Thus, the draft has been sent to educational experts generally, and mathematics education experts specifically, to test the questionnaire regarding its relevance to the concept of the learning process of mathematics and items' readability and clarity.

Accordingly, the visual questionnaire has been developed in light of experts' feedback. The majority of the experts' feedback was related to the questionnaire's language. Moreover, there was an agreement on including all the questionnaire items. Except for questions number 10 and 23, there was agreement on removing these questions because question 10 was repetitive, and question 23 was not related to the aim of the questionnaire and the concept of learning mathematics. Therefore, the final draft of the visual questionnaire consists of 21 items that reflect students' learning process using GeoGebra (see Appendix 3.12).

3.10.6.1.3 Phase three: Reliability

The visual questionnaire was administered to a group of 27 students from Year Five. This is to examine its reliability and appropriate time for answering the questionnaire. The reliability of a questionnaire refers to the level at which obtained results by an assessment and procedure can be replicated. One of the critical factors of reliability is increasing the validity of the questionnaire. As a result, I began by marking the participants' responses on the GeoGebra visual questionnaire in which each item was marked, given a score from 5 to 1 (strongly agree = 5, agree = 4, neither agree nor do not agree = 3, disagree = 2, strongly disagree = 1) in order to test questionnaire reliability. Thus, the researcher employed Cronbach's alpha and Spilt Halves, which are the common ways of testing questionnaire reliability in a single test (Meyer, 2010, Bolarinwa, 2015). Hence, by using SPSS, the researcher measures Split Halves correlation coefficient and Cronbach's Alpha between students' results to determine the reliability of the visual questionnaire. The following table (3.7) demonstrates that Cronbach's Alpha was (0.829) and Split Halves was (0.887), which indicate that the visual questionnaire of GeoGebra has a high degree of reliability as Mukaka (2012) stated the correlation coefficient is high if it is between 0.70 and 0.89.

Correlation	Results
Cronbach's Alpha	0.897
Split Halves	0.887

Table 3.7 Reliability of GeoGebra Visual Questionnaire

Additionally, to identify the time required for answering the GeoGebra visual questionnaire, the researcher measured the questionnaire time by calculating the average time taken by the first student to finish answering the questionnaire and the last student as well within the pilot study. The time the first student took to answer the questionnaire questions was (18) minutes, while the last student took (26) minutes. Hence, by calculating the average time for the first student and the last student, the researcher found that the time necessary for answering the geometric performance test is (22) minutes.

3.10.7 Students' Attitude Towards Mathematics

In order to investigate the impact of the research treatment on students' attitude towards mathematics, the researcher adopted the metric of attitude towards mathematics which was designed by Vandecandelaere et al. (2012). This measurement consists of 24 items, divided into three sections: mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics. Some items of students' attitudes towards mathematics were written in a positive way, and the others were negative. This measurement was developed based on international assessment TIMSS 2003 and Flemish work to be answered on a five-point Likert scale (from strongly agree to disagree strongly) (see Appendix 3.13). For the purpose of this research, this measurement has been translated into Arabic language and modified by including emojis to express the viable scale to be more suitable for primary school students. The translation was revised and improved by two PhD students in applied linguistics who have experience in translation.

Students' attitudes towards mathematics measurement consist of two types of items, positive and negative. Positive items were given from 5 to 1 (strongly agree = 5, agree = 4, neither agree or do not agree = 3, disagree = 2, strongly disagree = 1) with the score reversed to the negative items from 1 to 5 (strongly disagree = 5, disagree = 4, neither agree nor do not agree = 3, agree = 2, strongly agree = 1). Vandecandelaere et al. (2012) state that high scores obtained on the measurement denoted a higher degree of positive attitude towards mathematics. The internal consistency coefficients Cronbach's Alpha of mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics were 0.92, 0.93, and 0.83 respectively.

3.10.8 Video Recording

Video recording is a qualitative research method that entails catching moving pictures, with or without sound, to examine the visual details of behaviour and interaction (Given, 2008). Jewitt (2012) argued that the purpose of using video is recording the people's interaction in the natural environment, and record the aspects of the environment that structure the interaction, and undertake casual ongoing research treatment with contributors. In other words, the use of video in research helps in collecting data that is naturally occurring. On that issue, Cohen et al. (2018, p. 556) stated:

> *" Video recording can overcome the partialness of the observer's view of a single event (a video can be shared by several researchers) and can overcome the tendency towards only recording the frequently occurring events. Video recording can offer a more 'unfiltered' observational record of natural human behaviour in real-time, and it maintains the sequence of the event. The video record can be viewed several times; it is not a 'once-and-for-all observation. Video data have the capacity for completeness of analysis and comprehensiveness of material, reducing the dependence on prior interpretations by the researcher and enabling the researcher to scrutinize data".*

In addition, Garcez et al. (2011) contended that video recording of the classroom is a useful method to study the process of teaching and learning in detail through capturing aspects that might go unseen when other resources are used. Such aspects are body language, facial expressions, dialogue used in teaching and learning activities, students' interactions when performing proposed learning tasks in groups or individually, etc. This is why the author of the current research employed video recording to study pairs' patterns of interaction while learning geometry using GeoGebra. As a result, 12 mini cameras were installed on each computer in the IT room, where the GeoGebra group members studied mathematics lessons for the duration of the experiment. The reasoning behind using the mini cameras is recording learning activities for each group, due to the difficulty of observing students while they are performing the learning tasks using GeoGebra. This is because the existing partition between each PC impedes personal observation and making notes.

Accordingly, during the experiment process, videos recorded the learning activities in every session for each group in GeoGebra experimental group, given that the videos can help to interpret students' interaction with GeoGebra and their peers in the collaborative learning task. Thus, they can help to offer an insight to interpret the learning process when the quantitative data and video data are integrated. Nevertheless, to avoid any ethical issues, the researcher obtained the participants' consent to be recorded (see section 3.15 Ethical Considerations).

3.11 Intervention

As mentioned earlier, this research aims to examine the integration of GeoGebra, as a dynamic software into the suggested teaching intervention in which students deal with technology and collaborate to discover geometric concepts. This intervention includes three phases: In the first phase, the students complete the activities of learning tasks using GeoGebra concerning the main concept of that lesson. These activities include discussions and collaborative learning, and a teacher plays a crucial role to ensure the lesson is conducted appropriately; For the second phase, the teacher discusses the findings with the students, gives them feedback, and observes them when they undertake paper and pen activities. In the third and final phase, the teacher asks students to describe the concept and movement of geometric shape in space, such as the rotation of a triangle in 2D by using their spatial thinking skills, without using technology and paper and pen, which can help to improve their spatial thinking skills.

However, although, the hands-on group follows the same teaching process, the only difference between the GeoGebra group and the hands-on group is that the hands-on group uses hands-on or manipulative materials instead of GeoGebra. In this teaching intervention, the students are the fulcrum of the teaching process. It allows them to utilise their mathematical communication skills in collaborative and descriptive tasks when using their language to describe the geometric shape and its movement in the space (figure 3.3). On the other hand, the traditional teaching group was taught using the lecture method, which is teacher-centred. Details explaining the teaching intervention stages are as follows:

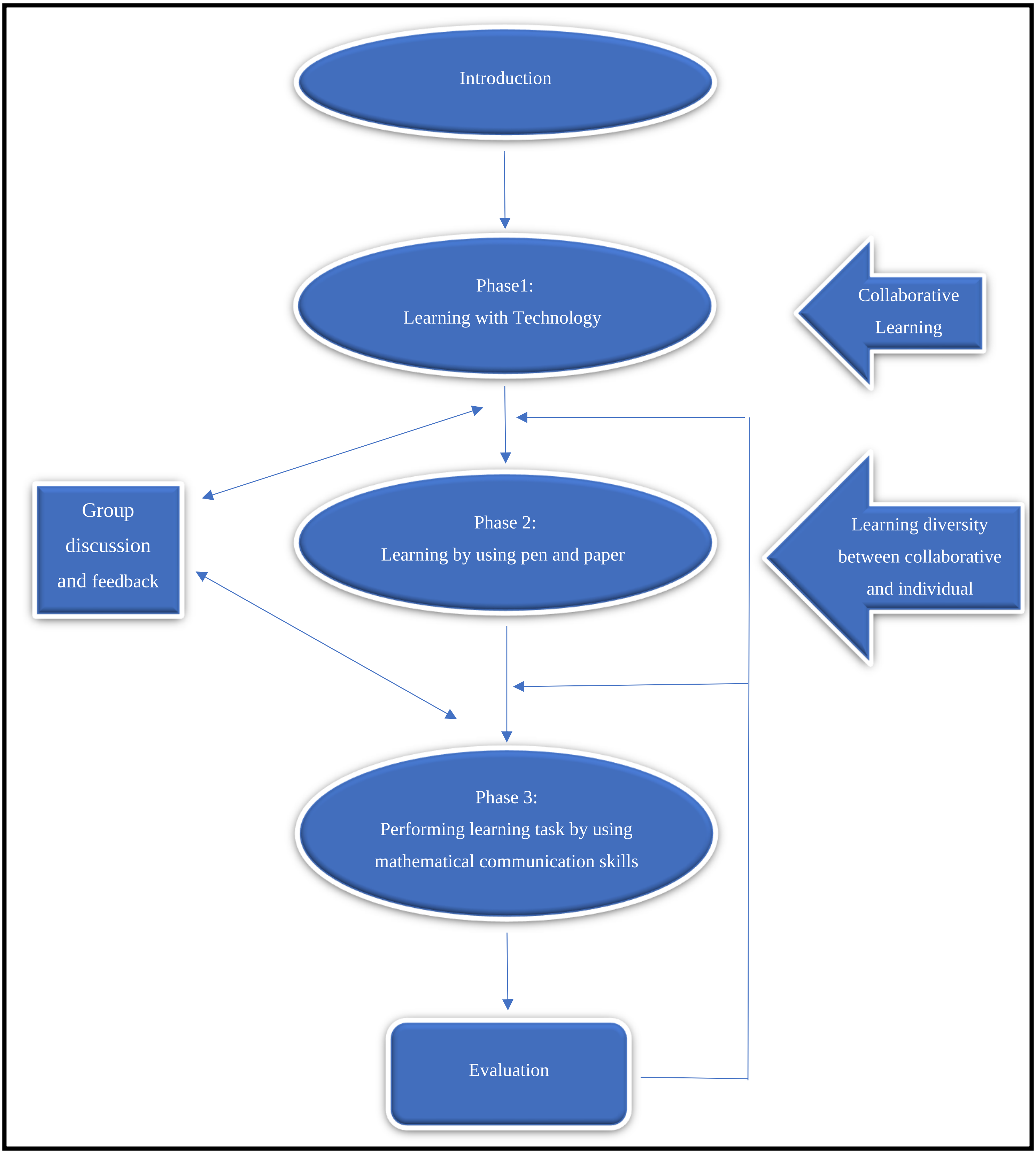


Figure 33. The Model of Teaching Intervention

Stage one: Technology phase

At this stage, students learn by using technology and implementing learning activities by using Geogebra as DMS. The process of teaching and learning for new experiences starts with students. For practical reasons, students perform the learning tasks and activities in groups of two because the number of computers in several Saudi primary schools does not match the number of students, which is often double the number of computers. This is why students in this stage learn collaboratively. Furthermore, at this stage, technology helps convert abstract, invisible mathematical concepts into visual concepts that students can see and create visual images of. Students then used their visual and spatial abilities to analyse and construct their understanding of mathematical concepts in greater depth and promote meaningful learning. After the students perform the learning activities at this stage, the teacher discusses students with their conclusions and provides feedback to ensure that students are on the right path towards achieving learning goals and building the correct understanding of the mathematical concept.

- **Stage two: Pen and paper phase**

In the second stage, students perform learning activities according to the learning task using pen and paper, as to move from the use of technology towards learning by using pen and paper for transferring what they learned in the previous stage to use in different ways. Hence, this stage aims to shift the technically learned experiences to manual experiences and work to develop application skills using paper and pen. In this stage, students start transferring their experiences that have been acquired by using technology or visual aids to be used by pen and paper, which reinforces the idea of constructivist learning that the building of new knowledge is based on previous experiences. However, if students show vague understanding, they go back to stage one to make sure and build a correct understanding of the concepts. If the students express a clear understanding of the concepts, they can move to the following teaching and learning stage. At the end of this stage, students have discussions about their conclusions and provided with feedback by teacher, ensuring that they continue on the right path to achieve the learning goals. It should be noted here that after this stage, the teacher can assess his students.

- **Stage three: Communication skill phase**

Here, students move from using technology and pen and paper, to using their mental abilities and thinking skills in general, and spatial thinking specifically, to implement the final learning activity. In other words, students use their thinking and spatial thinking skills to provide solutions to mathematical problems or provide a description of mathematical operations mentally. They use their language, giving explanations and examples of mathematical concepts learned in the previous stages, without using technology or pen and paper. This stage aims to train students to develop their higher-order thinking skills and train their spatial-thinking skills, which may positively impact the sustainability of learning, maintaining the effects of education, and developing various learning skills. In addition, this can help students to link the mathematical concept to their real-life, when they give actual examples and use body language to describe mathematical concepts. After completing this stage, students carry out the evaluation activities, and the teacher has the option to advance forward.

Accordingly, the research intervention involves developing lessons in light of using GeoGebra, developing lessons based on using hands-on materials, and teaching plans for both types of developing lessons; GeoGebra and hands-on materials.

3.11.1 Lessons Intervention

Since this research aims to integrate technology into learning and teaching mathematics, the researcher developed the Geometric Shapes Unit from Year Five Mathematics Curriculum with the intention of DMS, as a fundamental part of the learning process and in a manner consistent with the research idea. Besides, the development of this unit was made in light of the content analysis. So, each lesson of this unit was designed to consist of the lesson's learning objectives, mathematical concepts, introduction, GeoGebra tasks, pen and paper tasks, summary of learning concepts, mathematical knowledge, and exercise which was called make sure. The unit was designed to have, almost, the exact structure of the original unit so that students are familiar with it which allows them to engage with research material easier than if the structure was different than what they were used to (see Appendix 3.14).

Moreover, the researcher designed hands-on lessons. This form of lesson intervention was intended to be taught by using hands-on materials. The hands-on lessons follow the same structure as GeoGebra lessons. The only differences were in some learning tasks' questions for them to be consistent with the learning process using hands-on material (see Appendix 3.15). Both forms of the lesson intervention have to be taught in light of the teaching intervention of this research.

3.11.2 Teaching Guidebook

To teach both forms of the lessons intervention in light of this research’s teaching intervention, the researcher created teaching plans to structure the teaching process according to the current teaching intervention for each lesson. The teaching guidebook helps the participating teacher to teach lesson interventions to achieve the research aim. The researcher designed two forms of teaching guides: one for GeoGebra lesson interventions and the other one for the hands-on lesson intervention. Both forms of teaching guidebooks were divided into two sections: introduction, giving brief information about the teaching intervention and research concepts, and teaching plan. Each teaching plan was created according to the content analysis, learning objectives, (see sections 3.10.1 and 3.10.2) and the lessons interventions (see section 3.11.1). The teaching plan included lesson title, proposed teaching time, learning objectives, mathematical concepts, teaching process based on teaching intervention, assessment, and homework (see Appendix 3.16 & 3.17).

3.12 Research Implementation

This research was implemented in semester two from the school year of 2019-2020 in Al-Manrat primary school. The preparation for implementing this research project started from the year before. The researcher visited the selected school and agreed with the headteacher and the mathematics teacher, who agreed to participate in this research. Afterwards, ethical approval was obtained from the educational authority in Saudi Arabia and Newcastle University. Two months before research implementation, the researcher visited the participating school to sign the consent forms by the participating teachers and school.

In contrast, the students' consent forms were sent by the selected school to participate parents to be signed and returned to the researcher. At the same time, the researcher and the school's IT technician installed GeoGebra on all computers in the IT room. Besides, arranging with the headteacher to organise mathematics classes timetable for the GeoGebra classroom to be taken in the IT room in semester 2.

At the beginning of the second semester, the researcher attended the participating school to start the research implementation. At the end of the first week, he briefly introduced the research project for the research groups. Hence, in the second and third week of the semester, two students performed the pre-test: geometric performance test, spatial thinking test, GeoGebra visual questionnaire, and students' attitude towards mathematics metrics. As a result, the researcher grouped students in the GeoGebra group and hands-on group according to their geometric performance pre-test score in pairs. Therefore, the GeoGebra group (25 students) was formed into 12 groups; one of them was a group of three students. This was while the hands-on group (27 students) was formed into 13 groups; one of them was a group of three students. During these two weeks, the researcher had met the participating teacher to explain to him how to use GeoGebra and perform the teaching intervention, and the researcher explained to the GeoGebra group how to use GeoGebra. However, on the last day in the third week, a technical issue occurred and resulted in a delay in the research implementation for two weeks. Within these two weeks, the researcher seized the opportunity to give GeoGebra students more time to use GeoGebra in order to make them more familiar with the GeoGebra software. Therefore, they had three sessions more to use GeoGebra and do some activities using GeoGebra.

This was followed by three weeks with a total of 14 sessions for implementing the research project. During the research implementation, the researcher observed all the sessions for GeoGebra experimental and hands-on experimental groups to ensure the teaching process was performed as planned. Besides, each group activities in the GeoGebra experimental group was video recorded using mini cameras; moreover, all the class activities were recorded using a video camera. Furthermore, some photos were taken for the hands-on experimental group while performing learning tasks using hands-on material.

Unfortunately, in the last week of the research implementation, Covid-19 affected Saudi Arabia. As a result, the Ministry of Education decided on schools closing and education turned into e-learning and distance learning. This caused us to miss the last two sessions from the research implementation and resulted in missing the opportunity to collect post-test data from the school. Therefore, the discussion was made between the researcher and supervisors on the Covid-19 circumstance. Thus, the research instruments were turned to electronic form to overcome Covid-19 effects. The e-forms of the research instruments were created by using Google forms. The e-forms of the research instruments were sent to the participating teacher, who sent them to the participating students via the school's e-learning system. The post-test data collection took two weeks, and after six weeks, participating students performed the delayed test. It is believed that turning the research instruments into e-forms was the best solution with the 24 hours lockdown, which caused difficulties in sending and receiving the research instruments via post.

3.13 Quantitative Data Analysis

Quantitative data analysis was required to investigate the impact of the research intervention on geometric performance, spatial thinking, students' attitudes towards mathematics, and students' views on using GeoGebra. To do so, I employed a descriptive data analysis using IBM SPSS 24. The quantitative analysis for geometric performance test, spatial thinking test, and attitude towards mathematics went through two stages. In the first stage, one-way ANOVA was utilised to analyse post-test results for comparing and determining the impact of the research treatment on research groups. (Bowerman et al., 2015) stated that the purpose of using one-way ANOVA is to estimate and compare the effects of the different research treatments on the research phenomenon. Such a model is robust to depart from parametric assumptions; however, the homogeneity variance assumption was monitored using Levene's test, and if the homogeneity was violated, Welch test was used to overcome not meeting the homogeneity variance assumption and to see if the ANOVA outcome is still reliable (Mendeş and Akkartal, 2010, Liu, 2015, Berg, 2020). Besides, Tukey's post hoc test was employed when applicable and where significant findings were observed. Moreover, the effect size is also considered to discover to what extend the research treatment experiment caused the improvement (Howell, 2012, Field, 2013).

In the second stage, the analysis of covariance ANCOVA was used to remove the impact of the previous knowledge on these research findings. This helps the researcher determine if the improvement occurred by the research experiment or previous knowledge played a role to influence students' performance improvement. In other words, neutralizing the impact of students' previous knowledge was necessary to see whether student achievement growth occurred due to the research experiment, and that students' prior experience had no role in improving students' outcomes. Besides, in experimental research that involves random selection of the research groups, the covariate, compared to the response variable, decreases the error variance occurring in increased statistical power and higher accuracy in estimating group impacts (Keselman et al., 1998). It is useful to mention that ANCOVA is one type of ANOVA which means the ANCOVA has the same assumptions of ANOVA in addition to two other assumptions which are homogeneity of regression slopes and linearity (Davis, 2013, Berg, 2020). Therefore, conducting ANCOVA analysis happened in two phases. The first phase was to examine if the data met the ANCOVA assumptions. If the data were satisfied with the assumptions, during the second phase, the researcher could run actual ANCOVA. It is essential to remind the reader that taking into account these phases and maintaining excellent strategy for the entire analysis helps obtaining a reliable result (Davis, 2013, Berg, 2020).

Furthermore, students' views in the GeoGebra group were examined throughout the research experiment. Therefore, students performed pre- and post-questionnaire; besides, their views were explored at the end of each week of the research experiment in order to investigate to what extent they changed their opinions on their learning using GeoGebra during the research experiment. Therefore, the ANOVA repeated measured was employed to analyse the GeoGebra visual questionnaire for determining how students in the GeoGebra group change their opinions and ideas about learning mathematics via GeoGebra. Besides, the multivariate linear regression was applied to find out the relationship between the research variables and the extent to which one variable can be predicted based on other research variables.

3.14 Qualitative Data Analysis

Video recordings were used to gain all the possible data for learning activities in the GeoGebra group classroom. This is because video records have been increasingly relied upon to analyse the processes of teaching and learning. This type of video research called video-based fieldwork, which involves collecting data naturally occurred in the mathematics classroom and commonly followed onto social interactional research (Jewitt, 2012). The video data offers both depth (a richly detailed, moment-to-moment interactional record) and breadth (footage that spans each pair's activities in the GeoGebra group). To do video analysis, the researcher started by reviewing several research guides on how to analyse video data (Flewitt, 2006, Derry et al., 2010, Lefstein and Snell, 2011, Jewitt, 2012, Higgins et al., 2012, Clark, 2013, Blikstad-Balas, 2017, Mercier et al., 2014). There are vivid debates on methods to progressively refine hypotheses, conceptualise the epistemology of video data descriptions, and represent video data in satisfactory ethnographical ways. However, there is disagreement on theoretical and practical guidelines for processing the analysis of video data between scholars (Engle et al., 2007, Goldman, 2007, Ramey et al., 2016). Nonetheless, Derry et al. (2010), in their recent article on conducting video research in learning science, discuss the challenges that face researchers when dealing with video data collected from a complex learning environment. Furthermore, they provide guidance for researchers in selecting, capturing, and representing video data in the study of teaching and learning.

Consequently, the researcher began the process of analysing video data by watching all videos to obtain a general sense of the data and organise them by the time of recording. This helped the researcher to observe the development of pairs' interactions while they were using GeoGebra, day by day. Next, the researcher watched the video data again concerning the pair learning activities while performing learning tasks. During watching videos at this phase, the researcher created quotations that include one learning task, whether GeoGebra task or pen and paper task. After that, those quotations were watching concerning pair interaction. At the same time, the literature related to pair interaction patterns was reviewed in order to select the appropriate framework for analysing the video data in light of the research questions (Andrews et al., 2017, Ives, 2004, Kim and McDonough, 2008, Storch, 2002, Storch, 2004, Storch and Aldosari, 2013, Zheng, 2012). It found that several dyadic patterns of interaction have been reported in different situations, and the majority of them are related to teaching and learning language studies. For instance, in his study, Ives (2004) found three patterns of interaction in a primary L2 class in which grade 6 pupils were paired with native English-speaking students. These patterns of interaction are collaborative, expert/novice, and expert/passive.

Furthermore, Storch and Aldosari (2013) found more evidence of collaboration in dyads of equal proficiency (high-high and low-low) than in dyads when students had different proficiency (high-low). Complementing the aforementioned study, Kim and McDonough (2008) conducted a study with students of South Korea who studied English, as a second language; they also found different pair patterns of interaction, depending on whether the student worked with a speaker of the same or higher L2 proficiency. In addition to this, these investigations illustrate that simply assigning learners to work in pairs does not guarantee collaboration.

However, Storch (2002) was one of the first investigators who considered the nature of students' relationships when working in pairs. This longitudinal research was classroom-based. She conducted her study among ESL university students focused on the data of ten pairs over an entire semester. Storch (2002) found that the ten pairs of students in her study developed different kinds of relationships. Such relationships, when established, tended to continue, despite tasks or the passage of time. Significantly, the relationships students made affected the language learning opportunities that collaborative learning tasks provided. Storch (2002), qualitatively analysing the data of the pair talk, developed a model of pair patterns of interaction. This model distinguishes between four patterns of pair interaction: collaborative, dominant/dominant, dominant/passive, and expert/novice. She identified the criteria and features for each pattern of interaction (Appendix 3.18; section 2.3.6).

Storch’s (2002) model of pair patterns of interaction has been increasingly used to explore learners’ interaction in the classroom (Ahmadian and Tajabadi, 2017, Andrews et al., 2017, Cardimona, 2011, Todd and Toscano, 2020, Tan et al., 2010, Zheng, 2012, Watanabe and Swain, 2007). Reviewing the literature found many studies conducted to explore learners’ patterns of interaction in teaching and learning English language among different educational levels in a language classroom or using computer-mediated communication. At the same time, some studies used it to explore learners’ interaction patterns while using technology. Nevertheless, the literature review could not find research explore pair patterns of interaction in the field of mathematics education in Arabic literature, particularly in KSA. Also, few studies in the English literature used Storch’s (2002) model for pair interaction to analyse students' interaction in the mathematics classroom (Todd and Toscano, 2020), which can increase the originality of this research and fill the gap in the literature.

In addition to this, while watching the research data videos concerning interaction patterns, students seemed to have shared control over the learning task activities (equality) and engaged with each other's thoughts (mutuality). Therefore, I found that the earlier identified features in Appendix (3.18) captured how participants in this research controlled learning tasks and engaged together. Consequently, I adopted the pair interaction model (Storch, 2002) to analyse my video data and explore the pairs' interaction patterns in the GeoGebra classroom.

3.14.1 Phase one: Exploring Pairs Patterns of Interaction

In this stage, I started analysing video data using the identified criteria in the appendix (3.18) to explore pair patterns of interaction while learning using GeoGebra. At the same time, paying attention for any emerging interaction pattern or other criteria related to the four patterns of interaction in Storch's model (2002) will be considered. In other words, this research employed both deductive and inductive approaches in analysing the video data. The deductive analysis refers to using an organising framework consisting of themes for the coding process. The framework often referred to a start list employed in the analysis in anticipation that some core concepts exist in the data (Bradley et al., 2007, Derry et al., 2010, Braun and Clarke, 2006, Thomas, 2006, Azungah, 2018). According to Dörnyei (2007), one of the great benefits of deductive analysis is that having a list of set categories makes it possible to deal with the original coding in a concentrated and time-efficient way, establishing connections between extracts from various accounts earlier in the process.

In contrast, inductive analysis refers to methods that mainly use detailed raw data readings or careful observation for visual data to determine concepts and themes. In light of this research, it requires going through video data second by second, systematically and assigning codes to video footage as concepts unfold concerning research questions (Azungah, 2018, Bradley et al., 2007, Curry et al., 2009, Derry et al., 2010, Thomas, 2006). It is a recursive process that entails going back and forth between data analysis and the literature to develop meaning out of emerging concepts and obtaining the most grounded empirical and interesting theoretical factors (Azungah, 2018, Neeley and Dumas, 2016, Schüssler et al., 2014). In the inductive analysis, the results arise directly from the analysis of the raw data, not from prior expectations or models. However, the results are influenced by the evaluation objectives or questions outlined by the researcher (Thomas, 2006). Though, Azungah, (2018) emphasises the effectiveness of combining the deductive and inductive ways of analysis.

Therefore, the categories in Appendix (3.18) were imposed on the data and further analysed. During this step, each GeoGebra learning task was watched several times and assigned to one of the above mentioned patterns of interaction. Adopted from Storch's (2002) patterns of the interaction model to explore pairs patterns of interaction using the features of the patterns as codes and the name of the interaction patterns as themes. At the same time, a great deal of attention was paid to details to bring out the unique nature of the interactions, and then comments on them were made. The analysis procedure was coding the videos and commenting on the emerging aspect in my data without transcribing pair talk, considering the verbal and nonverbal behaviour presented while pairs were performing learning tasks. This means the deductive and inductive approaches went together concurrently through a complete video data analysis (see figure 3.4). Furthermore, the examples on the analysis findings were transcribed and presented with screenshots to presents the pairs' nonverbal behaviour (see section 4.2.1). Then concluded this stage by investigating the development of learners' patterns of interaction throughout the research experiment.

Once this stage was completed, the inter-rater reliability was checked by giving the data to two raters. Each rater was given the video data for five GeoGebra tasks selected randomly (60 videos representing 50% of the data set came from Geogebra tasks) and asked to label the patterns of interaction depicted on each video according to the descriptive types established by the researcher. Disagreement only arose over six videos (representing 10% of the selected videos and 5% of the total video data), and this was solved, with agreement reached via discussion. Then, two of the original codes were changed (representing 1.7% of the total video data). Thereafter, the inter-rater reliability was computed using Miles's (1994, p. 64) formula with the result showing the inter-rater reliability being 90%, which is considered to be an acceptable level of reliability (Miles, 1994)

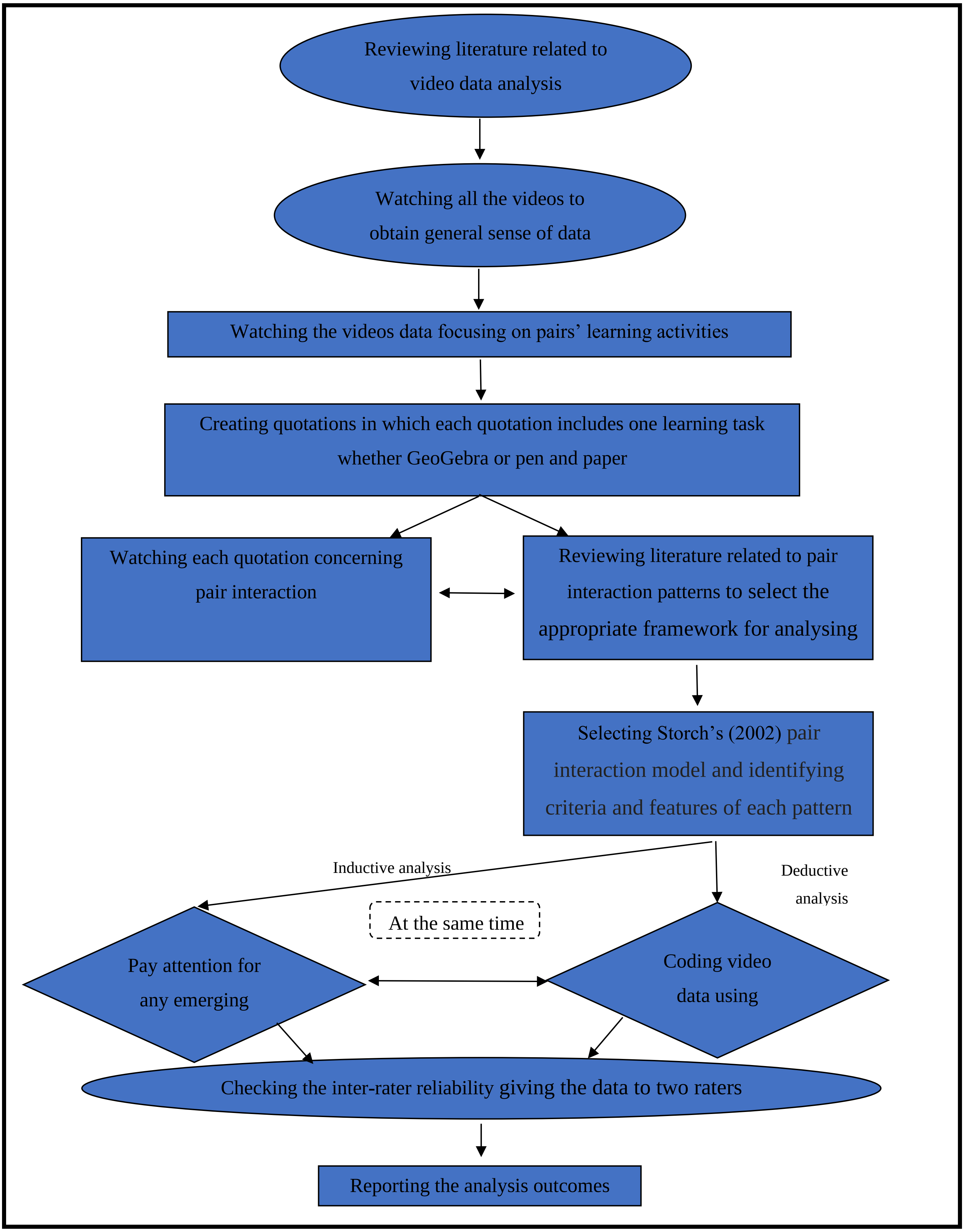


Figure 3.4 The process of analysis of the video data

3.14.2 Phase two: Investigating the association between different patterns of interaction and students' outcomes

The data analysis in this phase aimed to explore the relationships between different patterns of interaction and the research variables. To conduct the analysis in this stage, the researcher analysed video data for the pen and paper learning task using the criteria found in the analysis of the first phase of qualitative analysis. This allowed the researcher to categorise pairs based on their patterns of interaction in complete learning activities during the research experiment. Hence, the data analysis in this phase tries to figure out the correlation between different learners' patterns of interaction and the other research variables, relying on learners' outcomes on the post-test and students' interaction patterns. To do so, the researcher began by identifying the overall pattern for each pair in both types of learning tasks, GeoGebra, and pen and paper, to investigate the association between pair patterns of interaction and their learning outcomes. Hence, the patterns of interaction presented with the highest percentage in pair activities was nominated as the overall pattern.

Nevertheless, as mentioned earlier, it should be noted that students were sorted in 12 groups according to their geometric performance pre-test score (see section 4.10). The high achiever students were put with low achiever students (see section 4.11 research implementation for more details). For instance, among the sixteen tasks (ten GeoGebra tasks and six pen and paper tasks) between the participants in Pair 4, 56.25% were marked as dominant/passive, 18.75% as collaborative, 6.25% as dominant/dominant, 6.25% as cooperative, and 12.5% as passive/passive. Thus, the overall pattern of interaction in Pair 4 across all the research sessions was classified as dominant/passive. This stage was followed by computing the mean scores of the post-intervention geometry test, spatial thinking, students' attitude towards mathematics (including mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics) and students' views on GeoGebra. Pairs' mean scores and patterns of interaction were labelled in tables to ease the comparison process and explore the relationship between their pattern of interaction and the other research variables. This was followed by producing a narrative description of the findings of this analysis.

3.15 Ethical Considerations

The research was subjected to the ethical procedures of Newcastle University. To obtain ethical approval, the researcher prepared the information sheet and a consent form for the school headteacher, participating teacher, children's parents, and guardians with a simple statement to demonstrate the aim of the research and detail the procedures in respect of collecting the data. Also, the consent form (which was signed) stated that they are all willing to participate and are free to withdraw from the research experiment at any time, without needing to provide a reason. Once these documents were ready, the application for obtaining the ethical approval was submitted to the Ethics Committee at the Faculty of Humanities and Social Sciences (HASS). In that manner, the ethical approval had been reviewed and approved by the HASS Ethics Committee (see appendix 3.19). In addition, permission and ethical approval were obtained from the General Directorate of Education in Jeddah City at KSA, in order to implement the research project (see appendix 3.20).

All the data collected has been maintained strictly confidential. The school headteacher, teacher, children, and their parents were informed that all GeoGebra sessions would be recorded and gave their consent for video recording to be used. They were also informed that there would be no use for their personal data, and their images were anonymised. They were notified that when the research has achieved its purpose, the data would be destroyed.

The school headteacher and the participating teacher also signed the consent form. The information sheet and consent forms were handed out to participating students to be signed by their parents and guardians. This was done after I provided them with a plain language explanation on my research project as to ensure that they are fully aware of what the research is about, how they will take part and be assessed, why the researcher will observe GeoGebra and Hands-on groups, and how long the implementation of the research experiment will last. The school headteacher and the participating teacher were informed that the research activities were designed to fit smoothly into the curriculum lessons and be fun. Furthermore, students were informed they are free to withdraw at any time without giving reasons, and this will not affect them negatively.

3.16 Summary

This chapter has presented the approach of the investigation undertaken in this research. The research implementation and data collection have been done within the guidelines of the ethical procedures of Newcastle University and the General Directorate of Education in Jeddah City. Mixed research methods were employed to obtain the required data to answer the research questions. The research sample, participants, research instruments, teaching and learning intervention, and research procedures were described in detail. The data collection methods were piloted, and the results showed they are valid and reliable to be used. Statistical tests were selected to investigate the learners' performance and attitude, and the qualitative analysis procedures used to explore learners' patterns of interaction. These were fully described in this chapter. Subsequently, the next chapter will present the analysis of the results from this research.

Chapter 4. Data Analysis

4.1 Introduction

The previous chapter discussed the research methodology, which was used in this research. The analysis in this chapter is divided into two sections. The first section looks at the analysis of the quantitative data which came from geometric performance test, delayed test, spatial thinking test, Geogebra visual questionnaire, and students' attitude towards the mathematics visual questionnaire. In addition to this, this section presents an analysis of the relationship between the research variables. The second section of the analysis looks at the analysis of qualitative data that came from video recordings and classroom observation. Thus, the results from both sections are integrated to help readers comprehensively understand to what extent GeoGebra can enhance the process and outcome of learning mathematics.

4.2 Section 1: Quantitative Data Analysis

This section is going to present the findings of the quantitative data analysis. IBM SPSS 24 was employed to analyse the data of geometric performance test, spatial thinking test, and attitude towards mathematics at a 5% ($p = 0.05$) level of significance. All assessment outcome variables are presented as summary statistics. One-way ANOVA was applied to check the equality of the research groups' students using pre-intervention tests. It also was utilised to analyse post-test results to compare and determine the impact of the research treatment on the research group (Bowerman et al., 2015). The analysis of covariance ANCOVA was utilised to investigate the impact of research treatment, controlling participants' previous experience. Tukey's post hoc test was used when appropriate and when significant results were found (Howell, 2012; Field, 2013). Also, the Effect Size [Partial Eta Squared (η^2)] was reported. Furthermore, the change in students' views on using GeoGebra throughout the research experiment was analysed by applying ANOVA repeated measures. Besides, the multiple linear regression was applied to determine the relationship between the research variables, and explore if the research intervention improves the lower achiever students' performance more than higher achiever students by adding a geometric students' performance level (low and high) as a dummy variable.

In the following sections, I will present data analysis results of the learning outcomes in the first section: geometric performance, spatial thinking, and sustainable learning. Next, I will present the data analysis of students' views on the learning process: attitude towards mathematics and students' views towards GeoGebra. Afterwards, analysis outcomes of the correlation between research variables will be presented.

4.2.1 Examination of Effectiveness of Research Treatment

This section will present data analysis results from the impact of the research intervention on learning outcomes and processes.

4.2.1.1 Geometric Performance Outcomes

Pre and post-tests were employed to explore the effects of the research intervention using GeoGebra on students' geometric performance, spatial thinking, sustainable learning, and attitude towards learning mathematics in three domains (mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics). Three groups were assigned with the same teacher to be part of this research. Two groups were taught using the research teaching intervention; one instructed using GeoGebra, and the other, using hands-on material, while the third group was traditionally taught (see section 3.6).

Therefore, the geometric performance was examined by using geometric performance test before and after the research treatment. Therefore, One-way ANOVA was used on pre-test of geometric performance to ensure that all students across the groups have the same conditions before research treatment.

The results in the table (4.1) show that there were no statistically significant differences in the pre-test of geometric performance between groups ($F = 0.172$, $p = 0.842 > 0.05$) (see Appendix 4.1). As a result, all research groups were equal in terms of geometric performance for implementation of the treatment.

Group	N	Mean	SD	F	Sig.
GeoGebra	25	5.16	1.86	0.172	0.842
Hands-on	26	5.23	2.23		
Traditional Teaching	26	5.46	1.63		

Table 4.1 Summary of the Descriptive statistics for Geometric performance Pre-test

After three weeks of the research treatment, students across research groups completed post-test geometric performance. Consequently, one-way ANOVA was applied to compare the impact of teaching intervention with GeoGebra, teaching intervention with hands-on, and traditional teaching on students' geometric performance. The results in appendix (4.2) shows that there was a significant impact from teaching intervention on students' geometric performance at the ($p < 0.001$) level for the three conditions, [$F(2,74) = 13.663$, $p = .000$].

Group	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	15.6400	2.73679	.54736	14.5103	16.7697	11.00	21.00
Hands-on	26	12.9231	3.58801	.70367	11.4738	14.3723	5.00	20.00
Traditional teaching	26	10.6538	3.78357	.74202	9.1256	12.1821	4.00	19.00

Table 4.2 Summary of the Descriptive statistics for geometric performance post-test

Since one-way ANOVA does not tell which group has a statistically significant difference, post hoc using Tukey HSD was conducted. Therefore, post hoc comparisons using the Tukey HSD test indicated that the mean score for the teaching intervention with GeoGebra condition was significantly different than the hands-on condition, and the traditional teaching condition (see Appendix 4.2; table 4.2). Notably, the teaching intervention with the hands-on condition significantly differed from the traditional teaching condition (see appendix 4.2; table 4.2). Taken together, these results suggest that the teaching intervention really does have an impact on students' geometric performance. Specifically, the results indicate that when students learn geometry with teaching intervention, they achieve better than learning with traditional teaching. However, it should be noted that students performed better when they were taught by using teaching intervention with GeoGebra.

Moving on, after using one-way ANOVA, I employed the analysis of covariance ANCOVA to compare the effectiveness of three teaching methods whilst controlling for prior experience using the pre-test result. The results in Appendix (4.3) show that there is a statistically significant difference [$F(2,73) = 13.432$, $p = .000$] between teaching methods, whilst adjusted for prior experience. The partial Eta Squared value ($\eta 2 = .269$) indicates the impact size of the research treatment, which means about 27% of the improvement in students' geometric performance is resulted by the research treatment. According to the Cohen guideline, the effect size of the teaching intervention on the geometric performance is medium (Cohen, 1992).

Moreover, the Post hoc test (Pairwise Comparisons) in Appendix (4.3) indicated that there was a significant difference between teaching intervention with GeoGebra and teaching intervention with hands-on ($p = .006 < .05$); also, between teaching intervention with GeoGebra and traditional method ($p = .000 < .05$). In addition to this, there was a significant difference between teaching intervention with hands-on and traditional teaching ($p = .021 < .05$).

Dependent Variable: Geometric Performance Post-test				
Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	15.640[a]	.686	14.272	17.008
Hands-on	12.923[a]	.676	11.576	14.271
Traditional teaching	10.654[a]	.676	9.306	12.001
a. Covariates appearing in the model are evaluated at the following values: Geometric Performance Pre-test = 5.1558.				

Table 4.3 Estimates marginal means

Comparing the estimated marginal means (Table 4.3) showed that the highest geometric performance improvement on the teaching intervention was with GeoGebra ($M = 15.640$), compared with teaching intervention with hands-on and traditional teaching ($M = 12.923$, 10.654, respectively). Therefore, it can be said that GeoGebra group made better improvement than the other two groups. This means teaching intervention with GeoGebra played a key role in helping students to improve their geometric performance better than other teaching ways without GeoGebra.

To sum up, the above results from the one-way ANOVA and ANCOVA have shown that teaching intervention helps students to improve their geometric performance and achieve better than the traditional teaching. Nevertheless, students who studied geometry with GeoGebra performed better than students in the hands-on group. In conclusion, teaching intervention with GeoGebra helps students to obtain greater improvement in their geometric performance than other teaching strategy by 27%. Therefore, this section has answered the research question "*What is the impact of teaching intervention using GeoGebra on Geometric performance for Year Five students?*".

4.2.1.2 Spatial Thinking Outcomes

Spatial thinking was tested by using Raven's Coloured Progressive Matrices before and after the research treatment. Thus, One-way ANOVA had been used on pre-test of spatial thinking to ensure that all students across the groups are similar before the treatment.

The results in Table (4.4) demonstrate that there were no statistically significant differences in the spatial thinking pre-test between groups ($F = 0.098$, $p = 0.907 > 0.05$) (see Appendix 4.4). Accordingly, all research groups were equal, in terms of spatial thinking for implementation of the treatment.

Group	N	Mean	SD	F	Sig.
GeoGebra	25	27.68	5.03	0.098	0.907
Hands-on	26	27.73	4.66		
Traditional Teaching	26	27.23	3.66		

Table 4.4 Summary of the Descriptive statistics for Spatial Thinking Pre-test

After the research treatment, students across research groups performed post-test of spatial thinking. One-way ANOVA was employed to compare the effect of teaching intervention with GeoGebra, hands-on, and traditional conditions on students' spatial thinking. The outcome in Appendix (4.5) reveals that there was a significant impact of teaching intervention on students' spatial thinking at the ($p < 0.001$) level for the three conditions [$F(2,74) = 7.849$, $p = .001$].

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	32.2000	3.70810	.74162	30.6694	33.7306	17.00	35.00
Hands-on	26	29.4615	4.56273	.89482	27.6186	31.3045	18.00	35.00
Traditional teaching	26	27.9231	3.30966	.64908	26.5863	29.2599	16.00	32.00

Table 4.5 Summary of the Descriptive statistics for Spatial Thinking post-test

Importantly, post hoc comparisons using Tukey HSD were utilised to indicate which group has a statistically significant difference. The results showed that mean score for the teaching intervention with GeoGebra condition was significantly different from the hands-on condition, and the traditional teaching condition (see Appendix 4.5 and Table 4.5). However, the teaching intervention with hands-on material condition was not significantly different from the traditional teaching condition (see Appendix 4.5 and Table 4.5). Despite that, there was no significant difference between the hands-on condition and traditional teaching condition. Students in the teaching intervention with hands-on group performed better than students in traditional groups (Table 4.5). Taken together, these findings indicate that the teaching intervention has an impact on students' spatial thinking. Specifically, the results indicate that when students learn geometry with teaching intervention using GeoGebra, they performed better in spatial thinking test than learning with hands-on and traditional teaching. In other words, learning geometry using the teaching intervention of this research with GeoGebra helps students in improving their spatial thinking skills.

After using one-way ANOVA, I conducted the analysis of covariance ANCOVA to compare the effectiveness of the three teaching methods of this research whilst controlling for prior experience using spatial thinking pre-test. The results in Appendix (4.6) indicate that there were statistically significant differences [$F(2,73) = 16.457$, $p = .000 < 0.001$] between teaching methods, whilst adjusted for previous experience. The partial Eta Squared value ($\eta 2 = 0.311$) reveals the impact size of the research treatment, which means about 31% of the improvement in students' spatial thinking skills resulted from the research treatment. As per the Cohen guideline, the effect size of the teaching intervention on spatial thinking is medium (Cohen, 1992).

Furthermore, the Post hoc test (Pairwise Comparisons) in Appendix (4.6) showed that there was a significant difference between teaching intervention with GeoGebra and the other teaching methods in this experiment [teaching intervention with hands-on and traditional method] ($p = .000 < .05$). However, there was no significant difference between teaching intervention with hands-on and traditional teaching ($P= .09 > .05$).

Dependent Variable: Spatial thinking Post-test				
Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	32.111[a]	.506	31.101	33.120
Hands-on	29.338[a]	.497	28.348	30.328
Traditional teaching	28.132[a]	.497	27.142	29.123
a. Covariates appearing in the model are evaluated at the following values: Spatial thinking Pre-test = 27.5455.				

Table 4.6 Estimates marginal means

The above estimated marginal means Table (4.6) demonstrated that the highest spatial thinking improvement on teaching intervention was with GeoGebra (M = 32.111), compared to teaching intervention with hands-on and traditional teaching (M = 29.338, 28.132, respectively). It can be said that GeoGebra group made better improvement than the other two groups. This means teaching intervention with GeoGebra plays a crucial role in helping students improve their spatial thinking skills more than other teaching ways without GeoGebra.

Taken together, the above results from the one-way ANOVA and ANCOVA have revealed that teaching intervention helps students improve their spatial thinking skills better than traditional teaching. Nevertheless, students who studied geometry with GeoGebra improved their spatial thinking skills better than students in hands-on group. Using GeoGebra with teaching intervention of this research helps students obtain more significant improvement in their spatial thinking than hands-on and traditional teaching conditions, by 31%. In comparison to the impact of the teaching intervention on geometric performance and spatial thinking, it is found that the teaching intervention with both conditions, GeoGebra and hands-on, has a significant impact on geometric performance. In contrast, teaching intervention with GeoGebra improved students' spatial thinking skills. This means the teaching intervention with the GeoGebra condition helped students improve their geometric performance and spatial thinking skills. In conclusion, this section has answered the research question “*What is the impact of teaching intervention using GeoGebra on Spatial Thinking for Year Five students?*”

4.2.1.3 Sustainable Learning

Sustainable learning was tested using the delayed test, which is an equivalent form of geometric performance test after six weeks of the research treatment. Again, One-way ANOVA was employed on the delayed test of geometric performance to examine the extent to which the research treatment encourages students to retain their learning.

The result in Appendix (4.7) shows that there was a statistically significant impact from teaching intervention on students' sustainable learning at ($P < 0.001$) level for the three conditions [$F(2,74) = 12.071$, $p = .000$].

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	11.92	3.04029	.60806	10.6650	13.1750	6.00	18.00
Hands-on	26	9.923	3.24867	.63712	8.6109	11.2352	4.00	17.00
Traditional teaching	26	7.885	2.45482	.48143	6.8931	8.8761	2.00	13.00
Total	77	9.883	3.33235	.37976	9.1268	10.6395	2.00	18.00

Table 4.7 Summary of the Descriptive statistics for Delayed test

Next, Post hoc comparisons using the Tukey HSD were performed and the outcomes showed that the mean score for the teaching intervention with the GeoGebra condition was significantly different than hands-on condition, and the traditional teaching condition (see Appendix 4.7 and Table 4.7). Furthermore, the teaching intervention with hands-on condition significantly differed from the traditional teaching condition (see Appendix 4.7 and Table 4.7). These findings are similar to the impact of teaching intervention on the geometric performance, since the teaching intervention with both conditions, GeoGebra and hands-on, impacted significantly on geometric performance and sustainable learning. In contrast, this outcome differed from spatial thinking in which teaching intervention with GeoGebra improved students' spatial thinking skills and there was no significant impact from the teaching intervention with hands-on.

Taken together, these results suggest that the teaching intervention really does have an impact on students' sustainable learning. Specifically, the results indicate that when students learn geometry with teaching intervention with both conditions, GeoGebra and hands-on, they retain and sustain their experience better in geometry than learning with traditional teaching. However, it should be noted that students retain better geometric understanding when they were taught by using teaching intervention with GeoGebra. Consequently, this section answered the research question "*What is the impact of teaching intervention using GeoGebra on Sustaining knowledge for more extended periods of time for Year Five students?*".

4.2.1.4 Summary

The above sections illustrated the results from the analysis of the learning outcomes data. These results demonstrated that the teaching intervention with both conditions, GeoGebra and hands-on, has a significant impact on geometric performance and sustainable learning. In contrast, the teaching intervention has a significant impact on spatial thinking, only in the GeoGebra condition. These results indicated that the teaching intervention with GeoGebra helps students to improve their learning outcomes better than the other hands-on and traditional teaching conditions. Therefore, these findings are consistent in showing the effectiveness of the teaching intervention with GeoGebra on learning outcomes. Consequently, this section has answered the research question "*what the impact of teaching intervention using GeoGebra on Year Five students' learning outcomes in terms of Geometric performance, Spatial thinking and Sustainable learning?*" As a reminder, the aim of this study was examining the impact of the research intervention on both, the learning process and outcomes. Subsequently, the following sections will present the findings from analysing students' views on their learning.

4.2.2 Examination of Effectiveness of Research Treatment on Learning Process

This section will present outcomes of the data analysis considering the impact of the research intervention on learning and processes.

4.2.2.1 Attitude Towards Learning Mathematics

The attitude towards learning mathematics across the three groups of students was examined by using the attitude towards mathematics metric designed by (Vandecandelaere et al., 2012) before and after the research treatment. Three domains were selected to assess the students' attitude towards learning mathematics. As pointed to many times, these are mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics. In this section, the one-way ANOVA and ANCOVA were used to analyse each domain separately.

4.2.2.1.1 Mathematics Academic Self-concept

Once more, one-way ANOVA was used on pre-test of mathematics academic self-concept to ensure that all students across the groups were in same conditions before the treatment. Thus, the results in Table (4.8) show that there was no statistically significant difference in the pre-test of mathematics academic self-concept between groups (F = 1.454, p

= 0.240 > 0.05) (see Appendix 4.8). Accordingly, all research groups were the same in terms of mathematics academic self-concept for implementation of the treatment.

Group	N	Mean	SD	F	Sig.
GeoGebra	25	34.72	4.83	1.454	0.240
Hands-on	26	37.27	6.12		
Traditional Teaching	26	36.73	5.77		

Table 4.8 Summary of the Descriptive statistics for Mathematics Academic Self-concept Pre-test

After three weeks of the research treatment, students across research groups performed post-test mathematics academic self-concept at the end of the research experiment. Thus, one-way ANOVA was employed to compare the impact of teaching intervention with GeoGebra, teaching intervention with hands-on, and traditional teaching on mathematics academic self-concept. The results in Appendix (4.9) indicate that there was a significant impact from the teaching intervention on students' mathematics academic self-concept at the ($p < 0.05$) level for the three conditions [$F(2,74) = 4.336$, $p = 0.017$].

Group	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	41.9600	5.59375	1.11875	39.6510	44.2690	32.00	50.00
Hands-on	26	37.4231	7.28930	1.42955	34.4789	40.3673	23.00	50.00
Traditional teaching	26	36.9231	7.07063	1.38667	34.0672	39.7790	23.00	48.00

Table 4.9 Summary of the Descriptive statistics for mathematics academic self-concept post-test

Afterward, post hoc comparisons using the Tukey HSD test were applied and the results showed that the mean score for the teaching intervention with the GeoGebra condition was significantly different than the hands-no, and the traditional teaching conditions (see Appendix 4.9 and Table 4.9). However, the teaching intervention with hands-on condition was not significantly different from the traditional teaching condition (see Appendix 4.9 and Table 4.9). Compared to findings previously found in this research, the teaching intervention with GeoGebra significantly impacted mathematics academic self-concept, similar to geometric performance, spatial thinking, and sustainable learning. However, this result slightly differed from geometric performance and sustainable learning, which were improved by the teaching intervention with the hands-on condition. While the teaching intervention with the hands-on condition had no significant impact on the mathematics academic self-concept and spatial thinking.

Taken together, these results imply that the teaching intervention with GeoGebra really does have an impact on students' mathematics academic self-concept. Particularly, the results reveal that when students learn geometry with teaching intervention using GeoGebra, they improve their mathematics academic self-concept better than students who studied geometry with traditional teaching. However, it ought to be noted that teaching intervention with Geogebra helps students to develop better mathematics academic self-concept.

In the second phase of analysing mathematics academic self-concept, I utilised covariance ANCOVA analysis to compare the effectiveness of three teaching methods' whilst controlling for prior experience. The findings in Appendix (4.10) reveal that there were statically significant differences [F (2,73) = 6.761, p = 0.002 < 0.05] between teaching methods, whilst adjusted for previous experience. The partial Eta Squared value ($\eta 2 = 0.156$) suggests the impact size of the research treatment, which means about 16% of the improvement in students' mathematics academic self-concept, resulted from the research treatment. Regarding Cohen guidelines, the effect size of the teaching intervention on the mathematics academic self-concept is small (Cohen, 1992).

The Post hoc test (Pairwise Comparisons table) in Appendix (4.10) showed that there were significant differences between teaching intervention with GeoGebra and teaching intervention with hands-on (p = 0.003 < 0.05), and between teaching intervention with GeoGebra and traditional method (p = 0.002 < 0.05). Yet, the results showed that there were no significant differences between teaching intervention with hands-on and traditional teaching (p = .0877 > 0.05).

Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	42.611[a]	1.279	40.062	45.161
Hands-on	36.996[a]	1.246	34.513	39.479
Traditional teaching	36.724[a]	1.240	34.252	39.196
a. Covariates appearing in the model are evaluated at the following values: Mathematics Academic Post-test = 36.2597.				

Table 4.10 Estimates marginal means

The above Table (4.10) showed that the most mathematics academic self-concept improvement resulted from teaching intervention with GeoGebra (M = 42.611), compared to teaching intervention with hands-on and traditional teaching (M = 36.996, 36.724, respectively). Hence, it can be said that the GeoGebra group made better improvements than the other two groups. This means teaching intervention with GeoGebra can play a crucial role to help students improve their mathematics academic self-concept compared to hands-on and traditional teaching condition. In other words, learning geometry using GeoGebra has a positive impact on students' mathematics academic self-concept.

In conclusion, the above findings from the one-way ANOVA, and ANCOVA have shown that teaching intervention with GeoGebra helps students to improve their mathematics academic self-concept more than teaching intervention with hands-on and traditional teaching. Specifically, students who learned geometry using GeoGebra improve their mathematics academic self-concept more than other students taught by other teaching methods in this research. In a nutshell, teaching intervention with GeoGebra can help students improve their mathematics academic self-concept better than other teaching strategies by 16%. Thus, this section has answered the research question, "*What is the impact of teaching intervention using GeoGebra on students' mathematics academic self-concept?*".

4.2.2.1.2 Enjoyment of Mathematics

One more time, one-way ANOVA has been used on pre-test for enjoyment of mathematics to ensure that all students across the groups are not different before the treatment. Consequently, the results in Table (4.11) reveal that there were no statistically significant differences in the pre-test for enjoyment of mathematics between groups (F = 0.15, p = 0.861 > 0.05), as shown in the Appendix (4.11). Accordingly, all research groups were equal in terms of enjoyment of mathematics for implementation of the treatment.

Group	N	Mean	SD	F	Sig.
GeoGebra	25	27.80	5.39	0.150	0.861
Hands-on	26	28.54	5.32		
Traditional Teaching	26	28.65	7.17		

Table 4.11 Summary of the Descriptive statistics for Enjoyment of Mathematics Pre-test

Three weeks later, at the end of the research treatment, students across research groups performed post-test enjoyment of mathematics. Thus, one-way ANOVA was applied to compare the impact of teaching intervention with GeoGebra, teaching intervention with hands-on, and traditional teaching on students' enjoyment of mathematics. The results in Table (4.12) show that there was a significant impact of teaching intervention on students' enjoyment of mathematics at the ($p < 0.05$) level for the three conditions [$F(2,74) = 4.256$, $p = 0.018$].

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	33.7600	5.71022	1.14204	31.4029	36.1171	20.00	40.00
Hands-on	26	29.7308	5.45203	1.06923	27.5286	31.9329	18.00	37.00
Traditional teaching	26	29.5000	6.31348	1.23817	26.9499	32.0501	8.00	36.00

Table 4.12 Summary of the Descriptive statistics for Enjoyment of Mathematics post-test

As one-way ANOVA does not reveal which group has a statistically significant difference, post hoc using Tukey HSD was employed. Therefore, post hoc comparisons using the Tukey HSD test showed that the mean score for the teaching intervention with GeoGebra condition was significantly different than the hands-on condition, and the traditional teaching condition (see Appendix 4.12 and Table 4.12). Nevertheless, the teaching intervention with hands-on condition was not significantly different from the traditional teaching condition (see Appendix 4.12 and Table 4.12). Taken together, these outcomes indicate that the teaching intervention with GeoGebra condition has an impact on students' enjoyment of mathematics. Specifically, the findings show that when students learn geometry with teaching intervention using GeoGebra, they enjoy learning mathematics more than students who studied geometry with hands-on and traditional teaching conditions. However, it should be noted that teaching intervention with Geogebra made students enjoy more in learning mathematics than using hands-on materials to learn geometry, which means using GeoGebra raises students' enjoyment of mathematics

In the second stage of analysing enjoyment of mathematics data, I employed covariance ANCOVA analysis to compare the impact of the three teaching strategies in this research whilst controlling for past experience. The results in Appendix (4.13) show that there was a statistically significant difference [$F(2,73) = 6.243$, $p = 0.003 < 0.05$] between teaching methods, whilst adjusted for previous experience. The partial Eta Squared value ($\eta 2 = 0.146$) indicates the impact size of the research treatment, which means roughly 15% of the increase in students' enjoyment of mathematics resulted from the research treatment. Concerning Cohen guideline, the effect size of the teaching intervention on the enjoyment of mathematics is small (Cohen, 1992).

The post hoc test (Pairwise Comparisons) in Appendix (4.13) proved that there were significant differences between teaching intervention with GeoGebra and teaching intervention with hands-on ($p = 0.004 < 0.05$); also, between teaching intervention with GeoGebra and traditional method ($p = 0.002 < 0.05$). However, there were insignificant differences between teaching intervention with hands-on and traditional teaching ($p = .0846 > 0.05$).

Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	33.998[a]	1.047	31.912	36.084
Hands-on	29.642[a]	1.025	27.598	31.685
Traditional teaching	29.360[a]	1.026	27.316	31.404
a. Covariates appearing in the model are evaluated at the following values: Enjoyment of Mathematics Pre-test = 28.3377.				

Table 4.13 Estimates marginal means

Considering the above Table (4.13), comparing the estimated marginal means, revealed that the most enjoyment of mathematics improvement on teaching intervention was with GeoGebra ($M = 33.998$), compared to teaching intervention with hands-on and traditional teaching ($M = 29.642, 29.360$, respectively). These results suggested that teaching intervention with GeoGebra helps students to increase their enjoyment of mathematics. In other words, learning geometry using GeoGebra makes students enjoy learning mathematics more. In contrast, these results are similar to mathematics academic self-concept, spatial thinking, geometric performance, and sustainable learning, regarding the significant impact of the teaching intervention with GeoGebra condition on them. These results differ from the geometric performance and sustainable learning, since the teaching intervention with the hands-on condition had a significant impact on them, whereas the teaching intervention with the hands-on condition had no significant impact on the enjoyment of mathematics, mathematics academic self-concept, and spatial thinking.

To conclude, the above outcomes from the one-way ANOVA and ANCOVA have indicated that teaching intervention with GeoGebra supports students to enhance their enjoyment of mathematics more than hands-on and traditional teaching conditions. Students who learned geometry using GeoGebra increase their enjoyment of mathematics. To sum up, teaching intervention with GeoGebra helps students increase the enjoyment of mathematics better than other teaching conditions in this research by about 15%. Hence, this section has answered the research question, "*What is the impact of teaching intervention using GeoGebra on students' enjoyment of mathematics?*".

4.2.2.1.3 Perceived Value of Mathematics

Once again, one-way ANOVA had been used on pre-test of perceived value of mathematics to ensure that all students across the groups are not different before the treatment. Therefore, the results in Table (4.14) demonstrate that there were no statistically significant differences in the pre-test of perceived value of mathematics between groups ($F = 2.068$, $p = 0.134 > 0.05$). Accordingly, all research groups were equal in terms of perceived value of mathematics for implementation of the treatment.

Group	N	Mean	SD	F	Sig.
GeoGebra	25	33.68	7.91	2.068	0.134
Hands-on	26	36.19	5.69		
Traditional Teaching	26	37.42	6.31		

Table 4.14 Summary of the Descriptive statistics for Perceived Value of Mathematics Pre-test

Three weeks later, students across research groups performed post-test of perceived value of mathematics. Consequently, one-way ANOVA was employed to compare the impact of teaching intervention with GeoGebra, teaching intervention with hands-on, and traditional teaching on students' perceived value of mathematics. The results in Table (4.15) showed that there was a statistically significant impact from teaching intervention on students' perceived value of mathematics at ($p < 0.05$) level for the three conditions [$F(2,74) = 6.312$, $p = 0.003$].

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	41.7200	5.71198	1.14240	39.3622	44.0778	22.00	47.00
Hands-on	26	37.7308	4.82956	.94715	35.7801	39.6815	28.00	45.00
Traditional teaching	26	36.4615	5.90749	1.15855	34.0755	38.8476	17.00	43.00

Table 4.15 Summary of the Descriptive statistics for Perceived Value of Mathematics post-test

Next, post hoc comparisons using the Tukey HSD test were employed to identify which group has a statistically significant difference. The results indicated that the mean score for the teaching intervention with GeoGebra condition was significantly different than the hands-on condition, and the traditional teaching condition (see Appendix 4.15 and Table 4.15). However, the teaching intervention with hands-on condition was not significantly different from the traditional teaching condition (see Appendix 4.15 and Table 4.15). Taken together, these outcomes suggest that the teaching intervention with GeoGebra has an impact on students' perceived value of mathematics. Specifically, the findings demonstrate that when students learn geometry with teaching intervention using GeoGebra, they improve their perceived value of mathematics better than other students who studied geometry with hands-on and traditional teaching. This means that teaching intervention with Geogebra helps student to develop better perceived value of mathematics. In comparison to the previous results found in this research, these results are similar to mathematics academic self-concept, enjoyment of mathematics, spatial thinking, geometric performance, and sustainable learning, concerning the significant impact of the teaching intervention with GeoGebra condition on them. At the same time, the teaching intervention with the hands-on condition had no significant impact on the perceived value of mathematics, enjoyment of mathematics, mathematics academic self-concept, and spatial thinking. Importantly, the geometric performance and sustainable learning improved by using the teaching intervention with the hands-on condition.

Next, I applied covariance ANCOVA analysis to compare the effectiveness of three teaching methods whilst controlling for past experience. The results in Appendix (4.16) illustrate that there were statically significant differences [$F (2,73) = 8.094$, $p = 0.001 < 0.05$] between teaching methods, whilst adjusted for previous experience. The partial Eta Squared value ($\eta2 = 0.182$) denotes the effect size of the research treatment, which means approximately 18% of the growth in students' perceived value of mathematics resulted from the research treatment. According to the Cohen guideline, the effect size of the teaching intervention on the perceived value of mathematics is small (Cohen, 1992).

The post hoc test (Pairwise Comparisons table) in Appendix (4.16) demonstrated that there were statistically significant differences between teaching intervention with GeoGebra and teaching intervention with hands-on ($p = 0.004 < 0.05$); also, between teaching intervention with GeoGebra and traditional method ($p = 0.000 < 0.05$). However, there was no significant difference between teaching intervention with hands-on and traditional teaching ($p = .315 > 0.05$).

Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1.00	42.138[a]	1.093	39.959	44.316
2.00	37.652[a]	1.055	35.549	39.754
3.00	36.139[a]	1.065	34.016	38.262
a. Covariates appearing in the model are evaluated at the following values: Perceived Value of Mathematics Post-test = 35.7922.				

Table 4.16 Estimates marginal means

The above Table (4.16) compares the estimated marginal means, showing that the most perceived value of mathematics improvement resulted from the teaching intervention with GeoGebra (M = 42.138), compared with teaching intervention with hands-on and traditional teaching (M = 37.652, 36.139 respectively). Therefore, it can be said that GeoGebra group made better improvement than the other two groups. This means teaching intervention with GeoGebra enhances students to improve their perceived value of mathematics better than other teaching ways without GeoGebra. In other words, these results indicated that teaching intervention with GeoGebra enhances students to improve their perceived value of mathematics.

In conclusion, the above outcomes from the one-way ANOVA and ANCOVA have shown that teaching intervention using GeoGebra helps students to enhance their perceived value of mathematics more than hands-on and traditional teaching. Particularly, students who learned geometry using GeoGebra improve their perceived value of mathematics better than other students. To sum up, teaching intervention with GeoGebra supports students to increase their perceived value of mathematics better than hands-on and traditional teaching conditions by about 18%. Overall, the above outcomes from mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics indicated that teaching intervention with GeoGebra enhances students to develop a positive attitude towards learning mathematics. Therefore, this section has answered the research question, "*What is the impact of teaching intervention using GeoGebra on students' enjoyment of mathematics?*".

4.2.2.1.4 Summary

To sum up, the above sections explored the research question *What is the impact of teaching intervention using GeoGebra on students' attitudes towards learning mathematics, including three domains (mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics)? And which were improved due to the research treatment?*. The findings showed that teaching intervention with the GeoGebra condition significantly impacted these three domains. Therefore, students who learned geometry using GeoGebra had more significant positive changes in their attitude towards learning mathematics than those who learned geometry using teaching intervention with the hands-on condition and traditional teaching.

These results are similar to the above findings (sections 4.2.1; 4.2.2.1) on the impact of the research teaching intervention with GeoGebra on geometric performance, spatial thinking, and sustainable learning. It can be said that teaching intervention with GeoGebra helps students develop a positive attitude towards mathematics and improve students' spatial thinking skills, which thus, helped them improve their geometric performance and sustainable learning. Hence, it is worth investigating how the GeoGebra student group had developed their views on learning geometry using GeoGebra over time through the intervention. Therefore, the following section will explore how the GeoGebra group changed their opinions on using GeoGebra throughout the research experiment.

4.2.2.2 GeoGebra Learning Process

The students' views of learning geometry using GeoGebra was explored by using GeoGebra visual questionnaire before and at the end of each week of the research experiment, and after the research experiment finished. Therefore, a one-way repeated measure ANOVA was conducted to compare the effect of the time of using GeoGebra on the students' views on their learning process using GeoGebra; before, during, and after the research treatment.

(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Before	Week1	-7.120	3.863	.466	-18.227	3.987
	Week2	-17.840*	3.742	.000	-28.600	-7.080
	After	-26.160*	3.742	.000	-36.917	-15.403
Week1	Before	7.120	3.863	.466	-3.987	18.227
	Week2	-10.720*	2.482	.001	-17.857	-3.583
	After	-19.040*	2.879	.000	-27.317	-10.763
Week2	Before	17.840*	3.742	.000	7.080	28.600
	Week1	10.720*	2.482	.001	3.583	17.857
	After	-8.320*	1.695	.000	-13.193	-3.447
After	Before	26.160*	3.742	.000	15.403	36.917
	Week1	19.040*	2.879	.000	10.763	27.317
	Week2	8.320*	1.695	.000	3.447	13.193
Based on estimated marginal means						
*. The mean difference is significant at the .05 level.						
b. Adjustment for multiple comparisons: Bonferroni.						

Table 4.17 Paired Samples T-test s with a Bonferroni Correction

The results from Mauchly's Test of Sphericity (Appendix 5.17) show that the data has not met the assumption of sphericity, as Mauchly's test was statistically significant ($p = 0.002 < 0.05$). As the sphericity was violated, the Greenhouse-Geisser test was used, which adjusts the degrees of freedom of the one-way repeated measures ANOVA. Further, tests of Within-Subjects Effects (Appendix 5.17) indicate that Greenhouse-Geisser test was statistically significant [$F (2.099, 50.383) = 26.555$, $p = .000 < .05$]. This means that there was a difference between times of using GeoGebra on students' views on learning geometry using GeoGebra. Post hoc using Bonferroni correction (Table. 4.17) revealed that there was no statistically significant difference between students' views on using GeoGebra before the research treatment ($M = 64.72$, $SD = 21.71$) and at the end of the first week of the research experiment ($M = 71.84$, $SD = 17.32$, ($p = 0.466 > 0.05$). However, the mean scores of students' views of GeoGebra positively improved after weeks of the research experiment ($M = 82.56$, $M = 90.88$, respectively), since the finding shows statistically significant differences from pre-treatment and first week. Therefore, it can be said that students developed a positive view of GeoGebra over time, but this is not before week one of studying using GeoGebra.

	Mean	Std. Deviation	N
Before Treatment	64.7200	21.71121	25
Week 1	71.8400	17.32455	25
Week 2	82.5600	10.61477	25
After Treatment	90.8800	9.44863	25

Table 4.18 Summary of the Descriptive statistics

To conclude, participants used GeoGebra for three weeks. Their views on their learning process using GeoGebra were measured before the research experiment, during the research treatment and after the research experiment. Testing normality of data was carried out on the residuals, which were approximately normally distributed. Repeated measures ANOVA with a Greenhouse-Geisser correction indicated that mean of students' views on using GeoGebra differed significantly between time points [$F\ (2.099, 50.383) = 26.555$, $p = .000 < .05$]. In addition, post hoc tests applying the Bonferroni correction revealed that students tend to make positive views on learning geometry using GeoGebra. As shown in Figure (5.1), there was a steady growth in students' views using GeoGebra, and then, no sign of flattening off at least over the time of the research experiment, which means students tend to prefer learning geometry using GeoGebra the more they use it. Consequently, this section has answered the research question, *"What are the students' views on learning using GeoGebra over time?"*.

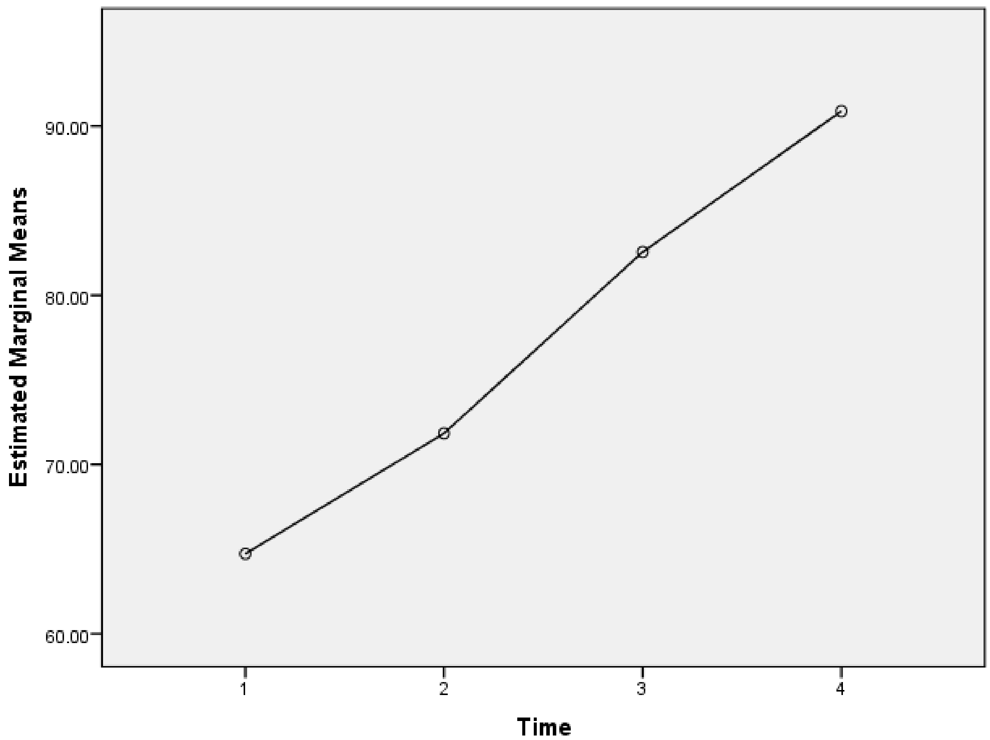


Figure 4.1 Summary of the Descriptive statistics for Students' Impression on Using GeoGebra over the time of Research Experiment

4.2.2.3 Summary

Pre- and post-tests were employed to explore the effects of the research intervention using GeoGebra on students’ geometric performance, spatial thinking, sustainable learning, and attitude towards learning mathematics in three domains mentioned before. Three groups were assigned with the same teacher to be part of this research. Two groups were taught using the research teaching intervention; one instructed using GeoGebra, and the other using hands-on material, while the third group was traditionally taught. The results showed significant differences between research groups in favour of the GeoGebra group, in terms of geometric performance, spatial thinking, sustainable learning, and attitude towards learning mathematics.

In addition, teaching intervention with the hands-on group was significantly different from the traditional teaching group in terms of geometric performance and sustainable learning. This indicates that the outcomes of implementing teaching intervention in the classroom, whether using GeoGebra or hands-on material, is better than traditional methods in improving students’ geometric performance and sustainable learning, with a preference of using GeoGebra. Besides, teaching intervention with GeoGebra improves students’ spatial thinking and attitude towards learning mathematics more than hands-on and traditional teaching conditions. Overall, it can be said that teaching intervention with GeoGebra plays a pivotal role to help students develop a positive attitude towards learning mathematics and improve their spatial thinking skills, and then, improve students' geometric performance and sustainable learning.

Within the GeoGebra class, students’ views of their learning using GeoGebra over time showed steady growth, where they became more positive over the time of the research experiment as they used it more. Accordingly, the following section will examine the relationships between research variables. The reason behind this is to investigate if the improvement of the learning process variables that occurred due to the research treatment led to developing learning outcome variables. Furthermore, this process can help explore how developing students' attitudes towards learning mathematics and improving spatial thinking skills can enhance students' geometric performance.

4.2.3 Correlations Examination

This section will present the result of the correlations between the geometric performance and other variables of this research, using simple linear regression and multivariate linear regression. In the beginning, I examined the relationship between geometric performance and the other research variables separately, using simple linear regression. After that, the correlation between geometric performance and other research variables, including students' performance level (high achiever and low achiever) and research treatment, were investigated using multivariate regression. The findings in this section can help in explaining how each variable impacts the other variables. Besides, it can elucidate that if there is an improvement in one variable, to what extent that can explain the improvement in the other variables.

4.2.3.1 The Relationship between Geometric Performance and Spatial Thinking

The simple linear regression was employed to test the relationship between geometric performance and spatial thinking. The result in the Appendix (4.18) indicated that there was a positive correlation between the two variables [$r = 0.483$, $N = 77$, $p = 0.000 < 0.001$]. According to Evans (1996), the correlation between geometric performance and spatial thinking was medium.

As a result, a simple linear regression was computed to predict geometric performance based on spatial thinking. The outcomes in Appendix (4.18) showed a significant regression equation was found [$F(1,75) = 22.803$, $p = 0.000 < 0.001$], with an R^2 of 0.233. Therefore, the model containing only spatial thinking can explain 23.3% of the variation in geometric performance score. Participants' predicted geometric performance is equal to 0.347 + 0.449 * (Spatial thinking) (Appendix 4.18). This means students' geometric performance score increased 0.347, for each score of spatial thinking. In other words, the slope coefficient for spatial thinking skills score was 0.347; so, the score of geometric performance increased by 0.347 for each extra score of the spatial thinking skills test. To sum up, improving students' spatial thinking skills leads to refining the geometric performance. Therefore, this section has answered the research question, "*What is the relationship between geometry performance and spatial thinking?*".

4.2.3.2 The Relationship between Geometric Performance and Mathematics Academic Self-concept

The simple linear regression was used to test the relationship between geometric performance and mathematics academic self-concept. The result in Appendix (4.19, Table 1) revealed that there was a positive correlation between the two variables [$r = 0.494$, $N = 77$, $p = 0.000 < 0.001$]. As per Evan (1996), the correlation between geometric performance and mathematics academic self-concept was medium.

Therefore, a simple linear regression was calculated to predict geometric performance based on mathematics academic self-concept. The outcomes in Appendix 4.19, Tables 2 and 3 showed that a significant regression equation was found [F (1,75) = 24.167, p = 0.000 < 0.001], with an R^2 of 0.244. Consequently, the model containing mathematics academic self-concept can explain 24.4% of the geometric performance score variation. Students' predicted geometric performance is equal to 2.285 + 0.278 * (mathematics academic self-concept) (Appendix 4.19, Table 4). This means students' geometric performance score improved 0.278 for each score of the mathematics academic self-concept. In other words, the slope coefficient for mathematics academic self-concept score was 0.278; so, the score of geometric performance improved by 0.278 for each extra score of the mathematics academic self-concept test. To conclude, enhancing students' mathematics academic self-concept leads to improving geometric performance. Hence, this section has answered the research question, "*What is the relationship between geometry performance and Mathematics academic self-concept?*".

4.2.3.3 The Relationship between Geometric Performance and Enjoyment of Mathematics

The simple linear regression was used to test the relationship between geometric performance and enjoyment of mathematics. The result in Appendix 4.20, Table 1 revealed that there was positive correlation between the two variables [r = 0.256, N = 77, p = 0.01 < 0.05]. According to Evan (1996), the correlation between geometric performance and enjoyment of mathematics was weak.

Consequently, a simple linear regression was calculated to predict geometric performance based on enjoyment of learning mathematics. The outcomes in Appendix 4.20, Tables 2 and 3) showed a significant regression equation was found [F(1,75) = 5.244, p = 0.025 < 0.05], with an R^2 of 0.065. Consequently, the model containing enjoyment of mathematics can explain 6.5% of the geometric performance score variation. Students' predicted geometric performance is equal to 7.919 + 0.165 * (enjoyment of mathematics) (Appendix 4.20, Table 4). This means students' geometric performance score improved by 0.278 for each score of the mathematics academic self-concept. In other words, the slope coefficient for mathematics academic self-concept score was 0.165; so, the score of geometric performance improved by 0.165 for each extra score of the enjoyment of mathematics test. To conclude, enhancing students' enjoyment of mathematics leads to improving geometric performance. Thus, this section has answered the research question, "*What is the relationship between geometry performance and enjoyment of mathematics?*".

4.2.3.4 The Relationship between Geometric Performance and Perceived Value of Mathematics

Once more, the simple linear regression was applied to test the correlation between geometric performance and perceived value of mathematics. The findings in Appendix 4.21, Table 1 proved that there was a positive correlation between the two variables [r = 0.503, N = 77, p = 0.000 < 0.001]. As per Evan (1996), the correlation between geometric performance and perceived value of mathematics was moderate.

Hence, a simple linear regression was calculated to predict geometric performance based on perceived value of mathematics. The results in Appendix (4.21, Tables 2 and 3) indicated that a significant regression equation to predict geometric performance based on the perceived value of mathematics was discovered [F(1,75) = 25.393, p = 0.000 < 0.001], with an R^2 of 0.253. Therefore, the model containing the perceived value of mathematics can explain 25.3% of the geometric performance score variation. Students' predicted geometric performance is equal to 0.03 + 0.337 * (perceived value of mathematics) (Appendix 4.21, Table 4). This means students' geometric performance score improved by 0.337 for each score of the perceived value of mathematics. In other words, the slope coefficient for the perceived value of mathematics score was 0.337. Thus, the score of geometric performance improved by 0.337 for each extra score of the perceived value of mathematics test. In conclusion, enhancing students' perceived value of mathematics leads to developing geometric performance. Then, this section has answered the research question, "*What is the relationship between geometry performance and perceived value of mathematics?*".

4.2.3.5 The Relationship between Geometric Performance and Sustainable Learning

The simple linear regression was utilized to test the correlation between geometric performance and sustainable learning. The outcome in Appendix 4.22, Table 1 revealed that there was a positive correlation between the two variables [r = 0.644, N = 77, p = 0.000 < 0.001]. In line with Evan (1996), the correlation between geometric performance and sustainable learning was strong.

Thus, again, the simple linear regression was employed to predict sustainable learning based on geometric performance. The outcomes in Appendix (4.22, Tables 2 and 3) proved a significant regression equation was found [F(1,75) = 53.016, p = 0.000 < 0.001], with an R^2 of 0.414. Hence, the model containing geometric performance can explain 41.4% of the variation in sustainable learning score. Students' predicted sustainable learning is equal to 2.777 + 0.545

* (geometric performance) (Appendix 4.22, Table 4). This means students' sustainable learning score improved by 0.545 for each score of geometric performance. In other words, the slope coefficient for the geometric performance score was 0.545; therefore, the score of sustainable learning increases by 0.545 for each extra score of the geometric performance test. To conclude, improving students' geometric performance leads to better sustainable learning, and they can maintain their experience as long as possible. Therefore, this section has answered the research question, *"What is the relationship between geometry performance and sustainable learning?"*.

4.2.3.6 The Relationship between Geometric Performance, spatial thinking and attitude towards mathematics

Since learning is a complex operation, and many variables impact learning outcomes, they cannot be separated. This suggests that the correlation between students' performance and the other research variables should be tested together using a multivariate regression test. Therefore, multivariate regression was employed to explore the relationship between geometric performance and spatial thinking attitude towards mathematics. This will also explore if the research intervention improves the lower achiever students' performance more than higher achievers by adding a geometric performance level (low and high), as a dummy variable (see section 4.6 for grouping scheme).

Hence, a multivariate regression was carried out to investigate the relationship between geometric performance and the other research variables. The result in the correlation table in Appendix (4.23) shows there was no statistically significant different relationship between geometric performance and students' performance level. However, the result also indicated that there was a significant positive correlation between geometric performance and research treatment [$r = 0.519$, $N = 77$, $p = 0.000 < 0.001$], geometric performance and spatial thinking [$r = 0.488$, $N = 77$, $p = 0.000 < 0.001$], and geometric performance and students' attitude towards mathematics [$r = 0.496$, $N = 77$, $p = 0.000 < 0.001$]. According to Evan (1996), the correlations between geometric performance and spatial thinking and students' attitude towards mathematics were medium.

The outcome in the ANOVA table in Appendix (4.23) indicated that the multivariate regression model with all predictor variables was significant and produced the equation ($F (5,71) = 10.27$, $p = .000$), with an $R2$ of 0.42. Hence, this correlation explains 42% of geometric performance post-test. Furthermore, participants' predicted geometric performance post intervention is equal to 47.138 + 1.487 (research treatment) – 0.038 (students' performance level) – 0.183 (geometric performance pre-test) + 0.226 (spatial thinking) + 0.068 (attitude towards mathematics), where research treatment was coded as GeoGebra = 3, Hands-on = 2, traditional teaching = 1, and the students' performance level based on geometric performance pre-test score was coded as high achiever = 1, low achiever = 0.

As shown in the coefficient table in Appendix (4.23), research treatment, spatial thinking, and students' attitude towards mathematics had significant positive regression and thus, they were significant predictors ($p = 0.004 < 0.05$, $p = 0.029 < 0.05$, $p = 0.011 < 0.05$, respectively). Therefore, participants' geometric performance post-test increased by 0.226 scores, for each score of spatial thinking post-test, and increased 0.068 scores, for each score of students' attitudes towards mathematics post-test. Besides, studying geometry using teaching intervention with GeoGebra improved participants' geometric performance post-test by 1.487 scores for each session. Furthermore, the GeoGebra group (high score pre) is showing a significant improvement in geometric post-test over low pre-test. As geometric performance pre-test is not significant, then this result is true and unrelated to their pre-test score. But it is related to having higher scores in spatial thinking ($B = 0.226$, $p<0.05$) and greater values in total attitude ($B = 0.068$, $p<0.05$). Consequently, this section has answered the research question, "*What are the relationships between geometry performance and spatial thinking, sustainable learning, and students' attitude towards Mathematics*?". At the same time, it has explored the impact of teaching intervention using GeoGebra on geometric performance controlling spatial thinking, students' attitudes towards mathematics, students' performance level (low and high), and students' prior geometric experience.

4.2.4 Conclusion

The above sections have demonstrated the quantitative analysis of the research data. The first section presented the outcome from the impact of the research treatment, and the second section explored the relationship between the research variables. The outcomes of the first part of this analysis show that there was a significant impact from the research treatment in favour of integrating GeoGebra into teaching intervention, in terms of geometric performance, spatial thinking, sustainable learning, students' attitude towards mathematics (including mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics). This is while the results of the second part indicated that there was a moderate positive relationship between geometric performance and spatial thinking, mathematics academic self-concept, and the perceived value of mathematics, and a weak positive relationship with the enjoyment of mathematics. Moreover, the multivariate regression revealed that there was a moderate correlation between geometrics performance and spatial thinking and total students' attitude towards mathematics. It can be said that the research treatment improves students' spatial thinking, mathematics academic self-concept, enjoyment of mathematics, and the perceived value of mathematics geometric performance, which likely helps students to improve their geometric performance.

Nevertheless, all the above indications show both immediate and delayed outcomes of the research implementation, as explained in section (3.12). Since the learning environment includes a social setting and the research participants were performing the research task in pairs, it is worth investigating their interaction patterns while performing GeoGebra tasks to determine what interaction patterns can help students improve their achievement.

4.3 Section 2: Qualitative Data Analysis

This section is going to introduce qualitative data analysis. The data in this section tries to explore the pairs' interactions while learning using GeoGebra. The pairs were grouped based on their geometric performance pre-test score in 12 groups. The high achiever students were put with low achiever students and the moderate achievers were grouped together (see section 3.7 and 3.12 for more details).

The data analysis consisted of two phases. Firstly, pair activity data were analysed for the patterns of pair interaction and the salient characteristics that distinguish these patterns based on Storch's dyadic patterns of interaction (2002). To do so, the ATLAS.ti qualitative data analysis software edition 9 was employed to perform video data analysis. In the second phase, the data analysis tries to determine the effect of pair interaction on geometric achievement and attitude towards mathematics. Therefore, the pair interaction patterns will be associated with their geometric performance test score, spatial thinking, sustainable learning, and attitude towards learning mathematics scores (Figure 4.2). the following sections will present qualitative data analysis process of both phases.

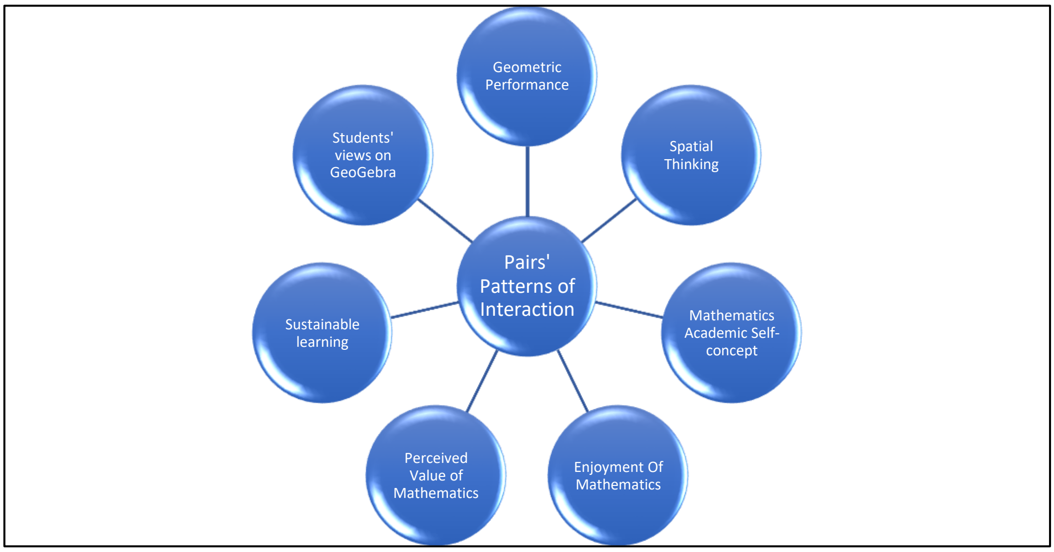


Figure 4.2 Content of The Second Phase of The Qualitative Data Analysis

4.3.1 Phase 1: Patterns of Pair Interaction

The data analysis in this stage was based on the dyadic interaction model found by Storch (2002). This model distinguishes between four patterns of pair interaction: collaborative, dominant/dominant, dominant/passive, and expert/novice. She identified these patterns on the basis of equality and mutuality (see section 3.14.1). The criteria of each pattern as identified by Storch (2002) was applied on each GeoGebra task. In addition, other interaction patterns emerged during the analysis, promoting the creation of additional criteria. As a result, video data analysis began with four interaction patterns, namely: collaborative, dominant/dominant, dominant/passive, and expert/novice. Thereafter, it ended up with six patterns of interaction: collaborative pattern in the quadrant 1, dominant/dominant, cooperative, and passive/passive in quadrant 2, dominant/passive in quadrant 3, and expert/novice in quadrant 4 (Figure 4.3).

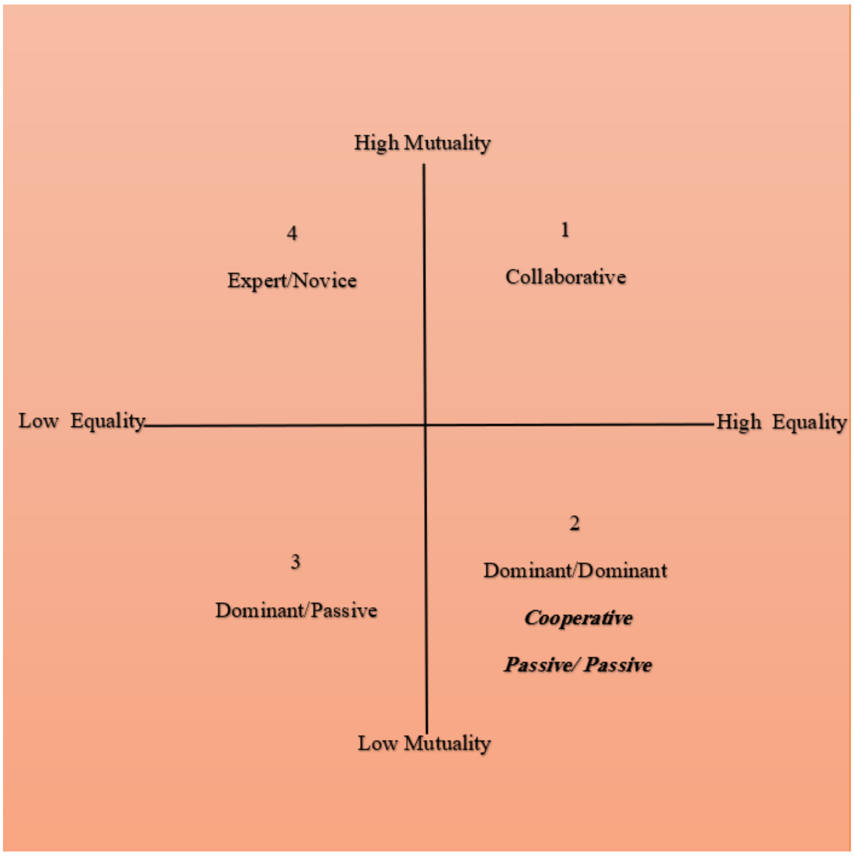


Figure 4.3 Model of Dyads Patterns of Interaction

Therefore, the pattern in Quadrant 1 represents moderate to high level of equality and mutuality and is labelled 'collaborative'. The term collaborative explains dyadic working together on all the task activity parts and engaging with each other's thoughts in which they retain their concentration on the task aim. Hence, participants create and maintain Joint Problem Space (JPS) (JPS) (Roschelle and Teasley, 1995). During task activity, pairs discuss different ideas which lead to resolutions that seem acceptable to both.

Quadrant 2 is labelled 'dominant/dominant', 'cooperative', and 'Passive/Passive'. Firstly, the dominant/dominant pattern of interaction represents moderate to high equality, and medium to low level of mutuality. Both participants contribute to accomplishing the task's aim, but they cannot fully engage with each other's thoughts. This interaction pattern is featured by a high degree of disagreements, and they find difficulty reaching an agreement, and both pairs contribute to the task. Still, they seem to compete to control the task's activity. While the second pattern in the quadrant is labelled 'cooperative'; pairs contribute to the task activity with moderate to high equality, and low mutuality (Tan et al., 2010). The third pattern of interaction of this quadrant is labelled 'passive/passive'; both participants contribute to the task activity with high equality and low mutuality. Both participants show a weak level of participation and concentrate on the aim of the task.

Quadrant 3 is labelled ‘dominant/passive’. The first pattern of interaction in this quadrant represents medium to low equality and mutuality. In this pattern, one of the pairs takes the task's role and controls the activities, while the other member seems to be more passive. Little efforts are made to encourage the passive member to engage in the task’s activities. Besides, this pattern of interaction is marked by little negotiation as there are few contributions from the more passive participant. Quadrant 4 is labelled ‘expert/novice’. In this quadrant, the pattern of interaction represents moderate to low equality, and moderate to high mutuality. In this pattern, one participant contributes to the task’s activity more than the other member. However, unlike the dominant/passive pattern, the active participant acts as an expert who actively encourages the inactive member to participate in the task.

It is crucial to note that pair interaction on each of the tasks was characterised independently of interaction on other tasks. Then, it was located in the quadrant that best describes the prevailing pattern manifest in the pair activity on the task. The use of terms such as low, moderate, and high to explain equality and mutuality levels helped in the process of categorization. Besides, it had been noticed that during classroom observation and video data analysis, participants improved their ability to use GeoGebra throughout the research experiment. Hence, their level patterns of interaction were enhanced day by day during the research experiment. After identifying the six patterns of pair interaction, the interrater reliability was checked by giving the data to two raters (see section 3.14.1). The interrater reliability was calculated using Miles’s formula, and the result shows that the interrater reliability was 90%, which is considered an acceptable level of reliability (Miles, 1994).

The following Table (4.19) summarises examples that relate to each of the criteria of each pattern of interaction which were identified by Storch (2002) and found in this analysis.

Collaborative Pattern of Interaction	
Criteria	Example
Pair Evaluates and discusses each other's ideas and fixing their error	21 S: Look at the image shape, what happened to the image after translation? 22 S: The shape has changed 23 J: No, nothing happened, nothing changed 24 S: look at the shape has changed 25 J: It is true, the shape has changed its location, but the shape remains unchanged. 26 S: this is true, see what happened to the order pairs when the image was shifted four units to the right?
Pair gives explanations to correct each other mistakes	5 S: The shape has two vertices. 6 J: No, it has four vertices, look 1,2,3,4 (referring to Geogebra as shown in screenshot 4.1) Screenshot (5.1): J refer to GeoGebra to Explain his Idea's  7 S: True
Pair builds on each other suggestion	8 J: We want to shift the image; the image's shape will change? 9 S: shifting the image! I don't know how, do you know? *Both of them silent and thing looking at the PC (see screenshot 5.2)* Screenshot (4.2): Pair Looking at the PC

	 10 J: Yes, translate 11 S: translate, how can we do it? 12 J: from here (He is pointing with his hand at the transformation icon in Geogebra, see screenshot (4.3) Screenshot (4.3): J showing S GeoGebra transformation icon  13 S: Yes, from here, then from here, then we do like this
Pair gives negative feedback	**<u>Example 1</u>** 6 J: No, it has four vertices, look 1,2,3,4 (referring to Geogebra as shown in screenshot 4.1) Screenshot (4.1): J refer to GeoGebra to Explain his Idea's  **<u>Example 2</u>** 16 S: What you are doing is wrong, you can translate the image from here .

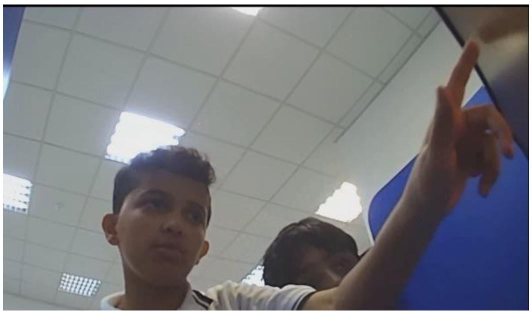
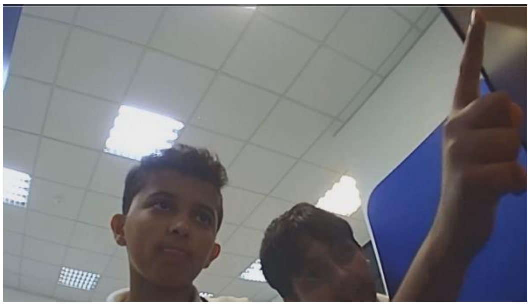
	Example 3 23 J: No, nothing happened, nothing changed
Pair provides positive feedback	**Example 1** 2 S: all right. **Example 2** 15 S: ok.
Pair gives confirmation	**Example 1** 13 S: Yes, from here, then from here, then we do like this **Example 2** 26 S: this is true, see what happened to the order pairs when the image was shifted four units to the right? **Example 3** 31 J: Aha, yes, this is right.
Pair asks and answers on each other questions	**Example 1** 9 S: shifting the image! I don't know how, do you know? 10 J: Yes, translate. **Example 2** 11 S: translate! how can we do it? **Example 3** 21 S: Look at the image shape, what happened to the image after translation? **Example 2** 12 J: from here (He is pointing with his hand at the transformation icon in Geogebra, see screenshot 4.3) Screenshot (4.3): J showing S GeoGebra transformation icon  **Example 4**

	26 S: this is true, see what happened to the order pairs when the image was shifted four units to the right?
Pair engages critically and constructively with each other's thoughts following task structure.	8 J: We want to shift the image; the image's shape will change? 9 S: shifting the image! I don't know how, do you know? *Both of them silent and thing looking at the PC (see screenshot 4.2)* Screenshot (4.2): Pair Looking at the PC  10 J: Yes, translate 11 S: translate! how can we do it? 12 J: from here (He is pointing with his hand at the transformation icon in Geogebra, see screenshot 5.3) Screenshot (4.3): J showing S GeoGebra transformation icon  13 S: Yes, from here, then from here, then we do like this 14 J: Not like this. Let me do it. 15 S: ok. 16 S: What you are doing is wrong, you can translate the image from here 17 J: No.

<table>
<tr><td></td><td>18 S: translation from here (points to Geogebra)

19 J: Yes, I did it. (his clap his hands, and both were laughing)
</td></tr>
<tr><td>Pair often reaches resolutions via explaining ideas.</td><td><u>Example 1</u>
5 S: The shape has two vertices.
6 J: No, it has four vertices, look 1,2,3,4 (referring to Geogebra as shown in screenshot (4.1)
Screenshot (4.1): J refer to GeoGebra to Explain his Idea's

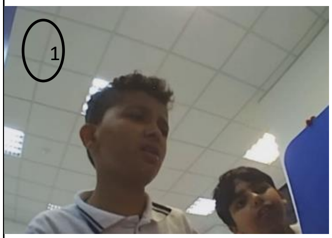
7 S: True
<u>Example 2</u>
24 S: look at the shape has changed
25 J: It is true, the shape has changed its location, but the shape remains unchanged.</td></tr>
<tr><td colspan="2">Dominant/Dominant Pattern of Interaction</td></tr>
</table>

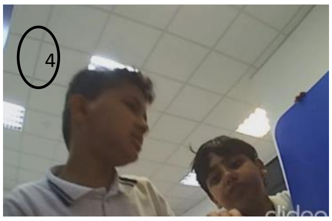
<table>
<tr><th>Criteria</th><th>Example</th></tr>
<tr><td>Pair contribute to the task, but it is not a shared construction.</td><td>3 M: You will write the answers on the activity sheet, and I will do GeoGebra
4 H: No, I will not write. I told you today that I am the king, I will carry out the activity on Geogebra
5 M: I will not write either.
6 M: Do whatever you want. Start doing the task on Geogebra</td></tr>
<tr><td>The engagement level with each other's ideas is via fixing their mistakes, which are not always accepted by each participant.</td><td><u>Example 1</u>
14 M: Where will they be located?
15 H: here, then here, then here

16 M: No, the first point will be here, the other one is here, and then here

17 M: I am telling you the point is here

<u>Example 2</u></td></tr>
</table>

	24 M: Wrong. 3 points are required, not 4 25 H: 3 points only! 26 M: Yes, I told you that. You were wrong. **Example 3** 29 M: Ok, so what is the point represented by the ordered pair (2,4)? 30 H: It's P 31 M: No, it's S. 32 H: How to be S? Look here it's P. X = 2 and Y = 4 33 M: You are wrong. Look. in the first X which is equal 4 and Y equal 2 
High level of disagreement and inability or difficulty to reach consensus.	3 M: You will write the answers on the activity sheet, and I will do GeoGebra 4 H: No, I will not write. I told you today that I am the king, I will carry out the activity on Geogebra 5 M: I will not write either. 6 M: Do whatever you want. Start doing the task on Geogebra (complains to the teacher) 7 H: Today, I am the king. I will carry out GeoGebra 8 M: Represent the points on Geogebra. 9 M: give me the mouse I want to do it

10 H: No, leave it.





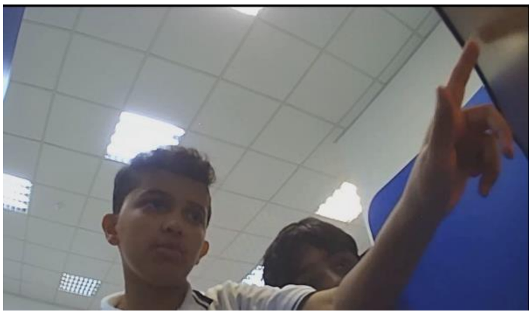
11 M: Say, what the first thing you will do?

12 M: How many points will you represent?

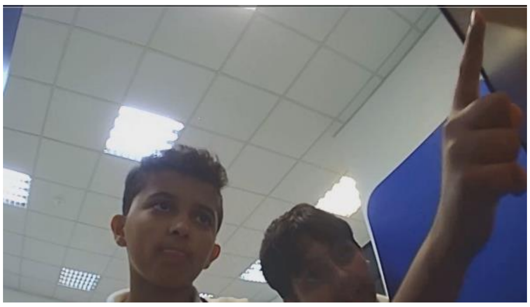
13 H: 3 points, not difficult

14 M: Where will they be located?

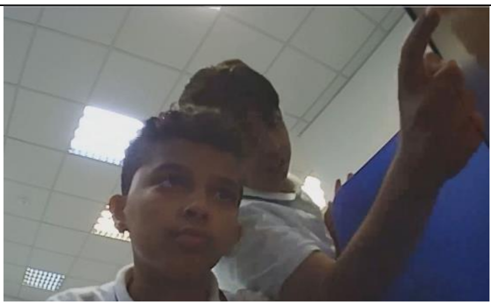
15 H: here, then here, then here



16 M: No, the first point will be here, the other one is here, and then here

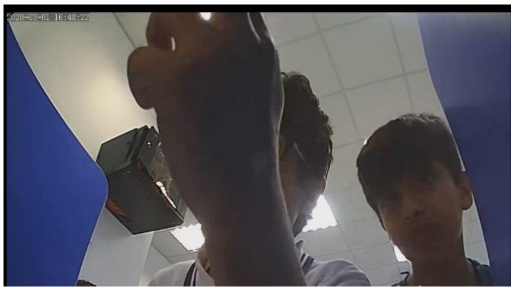
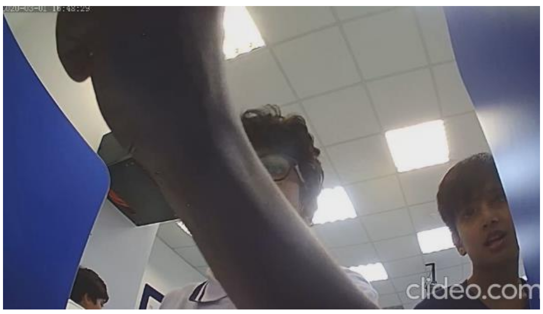


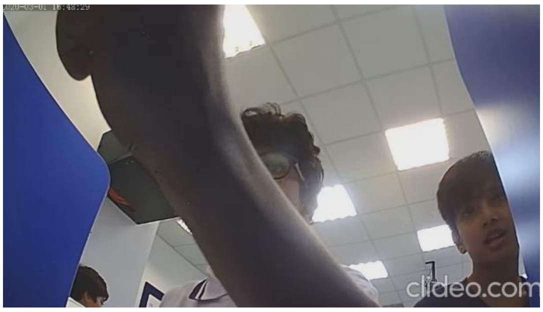
17 M: I am telling you the point is here

<table>
<tr><td></td><td>
<u>Example 2</u>
29 M: Ok, so what is the point represented by the ordered pair (2,4)?
30 H: It's P
31 M: No, it's S.
32 H: How to be S? Look here it's P. X = 2 and Y = 4
33 M: You are wrong. Look. in the first X which is equal 4 and Y equal 2

34 H: Wait, ask the teacher to see who his answer is correct</td></tr>
<tr><td>One member dominates the majority space front of the PC (e.g., lines 6 – 8 and screenshots).</td><td>7 H: Today, I am the king. I will carry out GeoGebra
8 M: Represent the points on Geogebra.
9 M: give me the mouse I want to do it</td></tr>
</table>

	10 H: No, leave it. 
Difficulty to reach a resolution that both could accept.	29 M: Ok, so what is the point represented by the ordered pair (2,4)? 30 H: It's P 31 M: No, it's S. 32 H: How to be S? Look here it's P. X = 2 and Y = 4 33 M: You are wrong. Look. in the first X which is equal 4 and Y equal 2  34 H: Wait, ask the teacher to see who his answer is correct
Pair do not use the plural pronoun, and they highlight the error in the other member's way of thinking.	**Example 1** 33 M: You are wrong. Look. in the first X which is equal 4 and Y equal 2 **Example2** 22 M: I'm telling you to represent the point, and you don't know what to do
Dominant/Passive Pattern of Interaction	

Criteria	Example
One learner dominate the task activity and appropriates the task and contribute more.	**Example 1** 4 Y: This is the square 5 Y: What is the coordinate? 6 Y: Coordinate! They meant coordinates of the figure 7 Y: these are the coordinates. (writing the results)   **Example 2** 9 Y: How do I find the coordinates of head B? 10 Y: like this, see here, we look at the number below the point on the X axis and then look at the number on the Y axis 
Dominant member asks self-directed questions rather than trying to involve the other to	**Example 1** 5 Y: What is the coordinate? **Example 2** 9 Y: How do I find the coordinates of head B?

<table>
<tr><td>contribute to the task activity.</td><td></td></tr>
<tr><td>Dominant pair use self-directed questions to guide his behaviour throughout the task.</td><td>10 Y: like this, see here, we look at the number below
the point on the X axis and then look at the
number on the Y axis


11 S: Yes</td></tr>
<tr><td>One learner appears limited or passive as he follows what dominant proposed or suggest. His participation is sort of agreeing or confirming dominant’s ideas.</td><td><u>Example 1</u>
3 S: Right, start drawing
<u>Example 2</u>
8 S: Right
<u>Example 3</u>
11 S: Yes</td></tr>
<tr><td>Passive learner does not give many suggestion, and this suggestion is a type of referring to dominant learner’s ideas.</td><td>13 S: We can use the same method that you suggested</td></tr>
<tr><td colspan="2" align="center">Expert/Novice Pattern of Interaction</td></tr>
<tr><td align="center">Criteria</td><td align="center">Example</td></tr>
<tr><td>Expert learner repaired his pair’s error and did not impose his opinion but give explanations.</td><td><u>Example 1</u>
3 O: Here 2
4 J: No, 9 and 4
5 O: This point A.
6 J: No, this is point C; look, did you see
7 O: yes.</td></tr>
</table>

	Example 2 12 J: Ok, what are the new order pairs? 13 O: A (2,5) 14 J: No, see here 4, point D.  15 J: D (4, - 4) See minus!
Expert learner asks question to engage other pair in the task and encourage him to learn from the interaction	**Example 1** 9 J: Now the triangle is ready for the translation, how can we do it? 10 O: From here (pointing with his hand at GeoGebra)  **Example 2** 12 J: Ok, what are the new order pairs? 13 O: A (2,5)
Expert provide positive feedback and negative feedback.	**Positive feedback** 11 J: Yes, from here 1, 2, 3, we made the translation 26 J: Correct, point F is the last point; see it is lucky (0,4) **Negative feedback** 4 J: No, 9 and 4 6 J: No, this is point C; look, did you see 14 J: No, see here 4, point D. 21 J: No, see point E.

Novice learner answers on the expert learner's questions.	**Example 1** 9 J: Now the triangle is ready for the translation, how can we do it? 10 O: From here (pointing with his hand at GeoGebra)  **Example 2** 19 J: What do you think? 20 O: It seems that what we are doing is wrong **Example 3** 24 J: Now, what is the last point? 25 O: Point F

Table 4.19 Summary Findings of Pairs' Patterns of Interaction

The following excerpts and screenshots from the video-recording data of GeoGebra tasks present description for the collaborative, cooperative, and passive/passive patterns of interaction and some of their notable characteristics. For the Dominant/Dominant, Dominant/Passive, and Expert/Novice patterns of interaction, see Appendices (4.25, 4.26, and 4.27) for more details and full explanations.

4.3.1.1 Collaborative Pattern of Interaction

Excerpt 1 in Appendix (4.24) is an example of the type of interaction pattern found in collaborative pairs. It comes from the pair activity of Jasser and Suhail interacting in doing GeoGebra task (16) (see Appendix 3.14). The two participants remain to concentrate on the task aim throughout the task process. They contribute jointly to task structure and engage with each other's ideas. Their discussion can be described as a constructive conversation. They assessed and negotiated each other's ideas and corrected each other's mistakes [e.g., lines 5 – 6, 21 – 26, 26 – 31]. These include some explanations for fixing each other's mistakes by visualising their demonstration through referencing GeoGebra [e.g., lines 5 – 7 and screenshot 5.1]. Both participants build on each other's suggestions [e.g., lines 8 – 13], give negative feedback [e.g., line 6, 16, 23], as well as positive feedback [e.g., line 2, 15], and provide confirmation [e.g., line 13, 26, 31]. The excerpt also illustrates evidence of the pair asking each other questions [e.g., line 9, 11, 21, 26], and giving information [e.g., line 10, 12 and screenshot 5.3].

This type of activity in this pattern can be described as 'exploratory activity' (Wegerif and Mercer, 1996; Sfard et al., 1998). The pair engages critically and constructively with each other's thoughts following task structure. This can be seen, for instance, in line 8 – 19 concerning the way to perform translation using GeoGebra, and in line 20 – 31, concerning the task structure exploring translation features. In addition, often resolutions are reached via the process of explaining and visualising their ideas with the aid of GeoGebra [e.g., lines 5 – 7, 24 – 25]. Therefore, a high level of equality and mutuality was revealed in the activity of this pattern of interaction.

Excerpts 1

1 J: Let us see task sheet first

2 S: all right

3 J: (reading task question) In cooperation with your group, bring a picture from the picture file to GeoGebra …..

4 S: (complete reading the task questions) Write the ordered pairs that are the vertices.

5 S: The shape has two vertices.

6 J: No, it has four vertices, look 1,2,3,4 (referring to Geogebra as shown in screenshot 4.1)

Screenshot (4.1): J refer to GeoGebra to Explain his ideas

7 S: True

8 J: We want to shift the image; the image's shape will change?

9 S: shifting the image! I don't know how, do you know?

Both of them silent and thing looking at the PC (see screenshot 4.2)

Screenshot (4.2): Pair Looking at the PC

10 J: Yes, translate

11 S: translate, how can we do it?

12 J: from here (He is pointing with his hand at the transformation icon in Geogebra, see screenshot 4.3)

Screenshot (4.3): J showing S GeoGebra transformation icon

13 S: Yes, from here, then from here, then we do like this

14 J: Not like this. Let me do it.

15 S: ok.

16 S: What you are doing is wrong, you can translate the image from here

17 J: No.

18 S: translation from here (points to Geogebra)

19 J: Yes, I did it. (he claps his hands, and both were laughing)

20 J: Now, what should we do?

21 S: Look at the image shape, what happened to the image after translation?

22 S: The shape has changed

23 J: No, nothing happened, nothing changed

24 S: look at the shape has changed

25 J: It is true that the shape has changed its location, but the shape remains unchanged.

26 S: this is true, see what happened to the order pairs when the image was shifted four units to the right?

27 J: Nothing happened

28 S: Nothing happened! are you sure?

29 J: Yes.

30 S: No, they were changed. We add four to the number of the ordered pairs.

31 J: Aha, yes, this is right.

(Jasser and Suhail, Geometric Translation, Task 16)

Moreover, during data analysis, two new patterns of interaction emerged and were added to the model of pairs' patterns of interaction. The following excerpts and screenshots from video data demonstrate these two new pairs' interaction patterns and some of their notable characteristics.

4.3.1.2 Cooperative Pattern of Interaction

The first new interaction pattern which emerged from data analysis was cooperative pattern. Excerpt 5 (Appendix 4.28) is an example that illustrates the cooperative interaction pattern. It comes from Omar1 and Walid's pair activity, interacting in doing the GeoGebra task (1). Both participants contributed to the task activity, were able to divide the task's labour, take turns to perform the task on GeoGebra, and write their finding on the task sheet. In a cooperative interaction pattern, both participants focus on the task aim. In this interaction pattern, the pair is able to engage with each other to a degree, and there is no disagreement between them. In this pattern, participants give attention to each other's mistakes and correct them (e.g., lines 9 – 10; 15 – 17; 21). There were a few questions, but there were no answers as the other participant did not give attention or ignored his pair's question (e.g., lines 4; 15 – 16). Here, unlike the dominant/dominant interaction pattern, the pair does not show any kind of negative emotions such as exasperation, resentment, and anger (e.g., line 7 and screenshot). Due to equality of contribution appearing moderate to high and the mutuality being low, thus, this interaction of activity was labelled cooperative.

4.3.1.3 Passive/Passive Pattern of Interaction

The passive/passive pattern of interaction was the second new interaction pattern found in this analysis. Excerpt 6 (Appendix 4.29) is an example that illustrates the passive/passive interaction pattern. The data comes from Abdullah2 and Fayez's pair activity, interacting in performing the GeoGebra task (11). Here, in this pattern, the pair does not focus on the task aim. As shown in the excerpt, there is no discussion between them or any kind of interaction related to the task aim (e.g., lines 3 – 18), even though they started by reading the task question (e.g., lines 1 – 2). There is also evidence that Fayez attempts to perform the task, but Abdullah2 did not take it seriously, and then, they continue doing something not related to the task activity (e.g., lines 13 – 18). Both of them were distracted and looking around at what their classmates were doing (e.g., lines 11 – 12 and screenshots). Despite the fact that the teacher gave them attention and encouraged them to work on the task, they remained interacting in doing non-useful things (e.g., lines 10 – 12). Therefore, the pair showed a weak level of participation and concentrated on the aim of the task. Thus, a high level of equality and low mutuality was revealed in the activity of this pattern of interaction.

Consequently, this section (4.3.1) has answered the research question, "*What patterns of dyadic interaction can be found in a primary mathematics classroom while learning using GeoGebra?*".

4.3.1.4 Conclusion

This section has investigated students' patterns of interaction while they were using GeoGebra. The process of data analysis began deductively using Storch's dyadic patterns of interaction model, but I also worked inductively and found that there were two additional interaction patterns. The findings of this investigation showed that there are six patterns of interaction which were adopted by students. These patterns are collaborative, dominant/dominant, cooperative, dominant/passive, passive/passive, and expert/novice. Students were fully engaged with each other when they adopted a collaborative pattern. In contrast, the dominant/dominant students found it challenging to engage with each other. Besides, the patterns of interaction can be associated with the level of concentration on the learning aim.

Collaborative, expert/novice, and cooperative remained focused on the learning aim more than the rest of the patterns of interaction. Students were discussing and negotiating constructively, supporting each other. The dominant/dominant focus on the learning aim is lesser than collaborative, expert/novice, and cooperative, due to both pairs struggling to control the learning activity throughout learning tasks, which leads to the weakening of their focus on the learning objective. However, in the dominant/passive interaction pattern, the dominant student focuses on the learning aim more than the passive member. This is while the passive/passive pattern of interaction is the lower pattern in terms of focusing on the learning aim and performing the learning task.

Therefore, the analysis in this stage found that the collaborative pattern of interaction was the frequent pattern due to students adopting it 31 times throughout the research experiment. In more detail, the collaborative interaction pattern was adopted two times in the first two days, and the number of the groups who adopted this pattern increased gradually from 2 to 6 (16.7% - 50%). In comparison, the dominant/dominant and cooperative interaction patterns were the second patterns controlling students' interaction patterns, which were adopted 22 times throughout the experiment. However, the dominant/dominant interaction pattern dominated the majority of students' interaction at the beginning of the research experiment by 25%. It then decreased steadily to be at the end of the experiment to 8.3% of the total interaction patterns students adopted.

In contrast, the cooperative pattern of interaction was adopted by few pairs at the beginning of the research experiment, and the number of pairs who adopted the cooperative interaction pattern was growing progressively from 16.7% - 25%. At the same time, the dominant/passive pattern of interaction was adopted 20 times during the research experiment since the number of pairs who adopted dominant/passive reduced gradually from about 20% in the first week to roughly 12% at the end of the experiment. The expert/novice was the second lower pattern which was adopted throughout the experiment by 19 times, and the passive/passive pattern of interaction was adopted 6 times. Hence, it can be said that during the experiment, students became familiar with the use of GeoGebra and the process of learning activities of the research intervention. Therefore, GeoGebra helps students to enhance their patterns of interaction, their ability to collaborate with their pairs, discuss their thoughts, and visualise their ideas and explanations. These points can have an impact on students' learning outcomes and process. As a result, the next section will explore the relationship between the patterns of interaction and learning process and learning outcomes.

4.3.2 Phase 2: The association between different patterns of interaction and students' outcomes

The data analysis in this phase aimed to investigate the correlation between different patterns of interaction and the research variables. The focus was on exploring the association between the different outcomes on the post-test and student's pattern of interaction. To do so, I identified the overall pattern for each pair in both types of tasks, GeoGebra and pen and paper, in the beginning, in order to investigate such an association. Hence, the pattern of interaction that was present with the highest percentage in the pair activities was nominated as the overall pattern.

Nevertheless, it should be mentioned that students were grouped in 12 groups based on their geometric performance pre-test score. The high achiever students were put with low achiever students (see sections 3.7 and 3.12 for more details). For instance, among the sixteen tasks, ten GeoGebra tasks and six pen and paper tasks, between the participants in Pair 4, 56.25% were marked as dominant/passive, 18.75% as collaborative, 6.25% as dominant/dominant, 6.25% as cooperative, and 12.5% as passive/passive. Thus, the overall pattern of interaction in Pair 4 across all the research sessions was classified as dominant/passive. In addition, the mean score was computed for the post-intervention geometry test score for each pair to compare between all pairs to explore the pattern of interaction of students who achieved better.

The following sections will investigate the association between students' patterns of interaction and their post-test outcome of geometric performance, spatial thinking, sustainable learning, and attitude towards mathematics. It will take into account the above quantitative and qualitative findings together to explain such a relationship between the pattern of interaction and other variables of this research.

4.3.2.1 The Association between Patterns of Interaction and Students' Views on GeoGebra

Interaction Pattern of Group	Participants	Students' views score on learning using GeoGebra	Average	Overall Average
Collaborative	Essa	105.00	97	99.17
	Saeed	89.00		
	Talal	93.00	96	
	Mohammed 1	97.00		
	Abdulwahab	98.00		
	Jasser	104.00	104.5	
	Suhil	105.00		
Expert/Novice	Jawad	84.00	94	94.25
	Omar 2	104.00		
	Abdullah 1	101.00	94.5	
	Mohammed 2	88.00		
Cooperative	Adnan	94.00	90.5	91.5
	Abdullah 3	87.00		
	Omar 1	90.00	92.5	
	Walid	95.00		
Dominant/Dominant	Muhnnad	104.00	92	87.75
	Hamad	80.00		
	Ibrahim	85.00	83.5	
	Thamer	82.00		
Dominant/Passive	Yousuf	80.00	85	82.5
	Sulayman	90.00		
	Saad	76.00	80	
	Mohammed 3	84.00		
Passive/Passive	Abdullah 2	77.00	78.5	78.5
	Fayez	80.00		

Table 4.20 Patterns of interaction for each pair (overall) and Students' Views on GeoGebra post-test scores

The outcomes in the above Table (4.20) show that the three pairs who adopted a collaborative pattern revealed the highest level of students' views on learning using GeoGebra since they attained the highest overall mean score. The second highest overall mean level was expert/novice pairs, whilst the cooperative pairs' views on learning using GeoGebra were lower than expert/novice. In contrast, the level for dominant/dominant pairs' views was higher than the dominant/passive pairs, which achieved the second lowest overall mean score. In comparison, the passive/passive pair gained the lowest mean score of 78.5 (Figure 4.4). Therefore, it seems that there is a relationship between pair interaction patterns and their views on learning using GeoGebra regarding the pattern of interaction and the positive view on using GeoGebra. This could be because the collaborative, expert/novice, and cooperative patterns of interaction remain focusing on the task aim throughout the task activity. Besides, the use of GeoGebra help them to collaborate with their pair and interact with him to solve the learning task problem through discussing the result and ideas with others.

Nevertheless, on some occasions, the dominant/dominant students can develop positive views on GeoGebra, higher than his peer with the same interaction pattern, and similar to collaborative pairs level. It can be seen in Table (4.20) that Muhnnad had one of the highest student levels in terms of students' views on GeoGebra as he obtained 104 scores in the relevant metrics. Furthermore, the expert/novice students can develop positive views on GeoGebra lower than their peer with the same interaction pattern, comparable to dominant/dominant and dominant/passive pairs. This can be seen in the Table (4.20) as Jawad achieved 84 scores in the students' views on learning using GeoGebra measurement. This is because he disagreed with the grouping system used in this research. His preference was the self-organisation system; as Jawad highlighted this issue in his answers on the GeoGebra questionnaire, he said: "Why we do not choose our pair?". However, students found that Geogebra and learning activities help them collaborate and interact with each other (table 4.20). Therefore, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms of students' views on using GeoGebra?*".

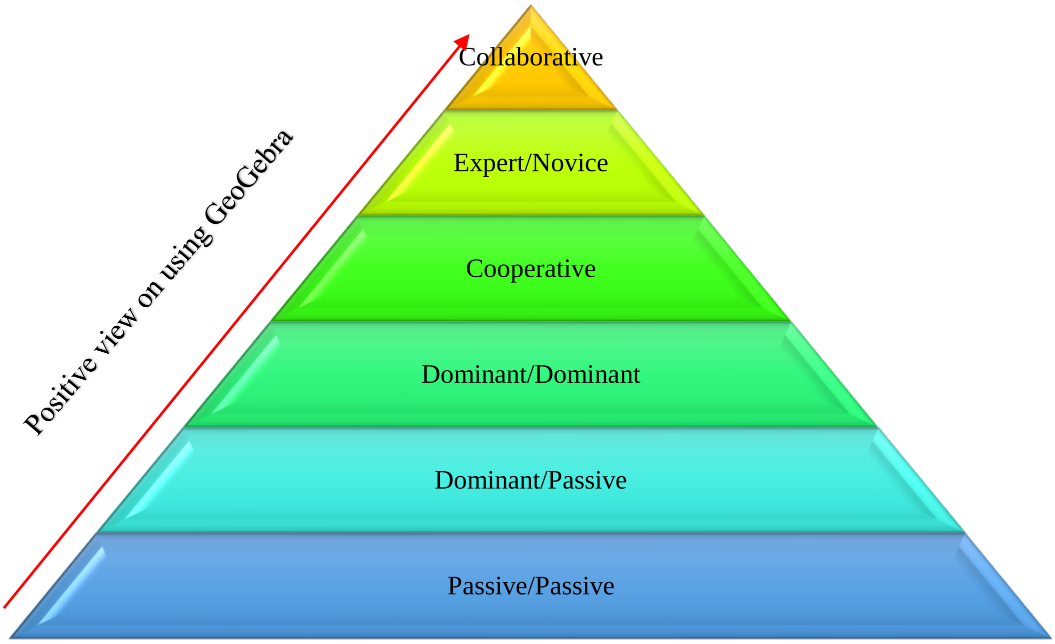


Figure 4.4 The Relationship Between Patterns of Interaction and Students' Views on Using GeoGebra

4.3.2.2 The Association between Patterns of Interaction and Mathematics Academic self-concept.

Interaction Pattern of Group	Participants	Mathematics Academic Self-concept Pre-test score	Pre-test average score	Mathematics Academic Self-concept Post-test score	Post-test average score	Overall Average
Collaborative	Essa	38	34	45	44.5	45.33
	Saeed	30		44		
	Talal	41	34.33	44	46	
	Mohammed 1	32		45		
	Abdulwahab	30		49		
	Jasser	33	34	49	45.5	
	Suhil	35		42		
Cooperative	Adnan	37	33.5	44	45	43.5
	Abdullah 3	30		46		
	Omar 1	34	35	36	42	
	Walid	36		48		
Expert/Novice	Jawad	32	38	38	41	

	Omar 2	44		44		41.5
	Abdullah 1	34	33.5	42	42	
	Mohammed 2	33		42		
Dominant/Dominant	Muhnnad	38	34	50	41	40.25
	Hamad	30		32		
	Ibrahim	36	36.5	32	39.5	
	Thamer	37		47		
Dominant/Passive	Yousuf	41	35.5	44	40.5	40.25
	Sulayman	30		37		
	Saad	46	39.5	34	40	
	Mohammed 3	33		46		
Passive/Passive	Abdullah 2	25	29	33	34.5	34.5
	Fayez	33		36		

Table 4.21 Patterns of interaction for each pair (overall) and Mathematics Academic Self-concept post-test scores

As shown in Table (4.21), the highest pairs' levels of mathematics academic self-concept were collaborative pairs since they obtained the highest mean scores on mathematics academic self-concept post-test. The second highest mean level was cooperative pairs. In contrast, the expert/novice pairs achieved lower than cooperative as they attained overall mean scores of 41.5 on mathematics academic self-concept post-test. However, the dominant/dominant pairs and dominant/passive pairs were at the same level of mathematics academic self-concept as they achieved an overall mean score of 40.25. Moreover, the only passive/passive pair gained the lowest mean score of 34.5 (Figure 4.5). Consequently, it seems that there is a relationship between interaction pattern and the level of mathematics academic self-concept concerning the pattern of interaction and the concentration on the learning aim. As the collaborative, cooperative, and expert/novice patterns of interaction remain focused on the task aim throughout the task activity. It can be extrapolated that the discussion and the explanation of their ideas can likely help them understand mathematical concepts and then, help them improve their perception of their ability to master the subject matter and do well in mathematics; therefore, developing their mathematics academic self-concept.

Nevertheless, on some occasions, the dominant/dominant student can achieve higher than his group member with the same interaction and other patterns. It can be seen in Table (4.21) that Muhnnad had the highest student level of mathematics academic self-concept as he obtained 50 scores in the mathematics academic self-concept metric. At the same time, Ibrahim was a dominant/dominant student; his mathematics academic self-concept post-test score decreased, compared to the pre-test score. Furthermore, on some occasions, the novice student in expert/novice pairs' pattern of interaction does not improve as his peer with the same interaction pattern and other patterns. For example, Omar2 was an expert/novice student obtained the same score in the mathematics academic self-concept pre and post-test (table 4.21). Despite this, it can be said that the pattern of interaction students adopt can help to improve their mathematics academic self-concept level. Thus, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms of mathematics academic self-concept*?".

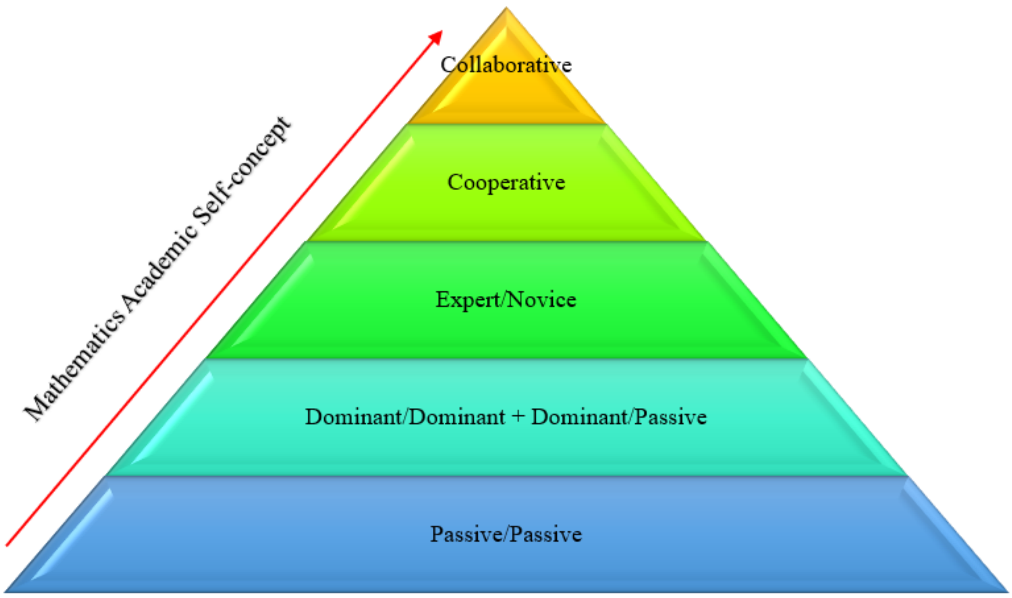


Figure 4.5 The Relationship Between Patterns of Interaction and The Leve of Mathematics Academic Self-concept

4.3.2.3 The Association between Patterns of Interaction and Enjoyment of Mathematics.

Interaction Pattern of Group	Participants	Enjoyment of Mathematics Pre-test score	Average score of Enjoyment of Mathematics Pre-test	Enjoyment of Mathematics Post-test score	Average score of Enjoyment of Mathematics Post-test	Overall Average
Collaborative	Essa	35	34.5	39	38	38
	Saeed	34		37		
	Talal	22		40	38	
	Mohammed 1	33	28	37		
	Abdulwahab	29		37		
	Jasser	31	28.5	40	38	
	Suhil	26		36		
Expert/Novice	Jawad	26	29.5	38	36	36.75
	Omar 2	33		34		
	Abdullah 1	30	33	38	37.5	
	Mohammed 2	36		37		
Cooperative	Adnan	25	25	33	34.5	33.25
	Abdullah 3	25		36		
	Omar 1	18	24	25	32	
	Walid	30		39		
Dominant/Dominant	Muhnnad	29	27.5	33	31.5	31
	Hamad	26		28		
	Ibrahim	22	27	26	30.5	
	Thamer	32		37		
Dominant/Passive	Yousuf	34	35	38	31.5	29.75
	Sulayman	27		25		
	Saad	26	21	36	28	
	Mohammed 3	16		20		
Passive/Passive	Abdullah 2	25	28	30	27.5	27.5
	Fayez	31		25		

Table 4.22 Patterns of interaction for each pair (overall) and Enjoyment of Mathematics post-test scores

Table (4.22) demonstrates that the three pairs who adopted a collaborative pattern revealed the highest level of enjoyment of mathematics since they obtained the highest overall mean score on the enjoyment of mathematics post-test. The second highest overall mean level was expert/novice pairs, while the cooperative pairs illustrated a lower level of enjoyment of mathematics than expert/novice, as they achieved an overall average score of 33.25 on the enjoyment of mathematics post-test. In comparison, the dominant/dominant pairs attained the overall mean score of 31 higher than the dominant/passive pairs which achieved the second lowest overall mean score. Continuing, the passive/passive pair gained the lowest mean score of 27.5 (figure 4.6). Hence, it can be said that there is a relationship between patterns of interaction and the level of enjoyment of mathematics. Therefore, the type of pattern students adopted can have its impact on improving their level of enjoyment of mathematics.

It is beneficial to mention that in the dominant/passive interaction pattern, the dominant student enjoys learning mathematics more than the passive one. Besides, in the expert/novice interaction pattern, the expert students enjoy a little more than the novice students. This is because the dominant and expert students led the learning tasks activities and used GeoGebra more than their group member. The expert students also played the role of a tutor who explained for his pair and encouraged him to work. At the same time, the passive/passive students did not show improvement in their enjoyment of mathematics. Then, this section has answered the research question, *"Do differences in the nature of dyadic interaction result in different outcomes in terms of enjoyment of mathematics?*".

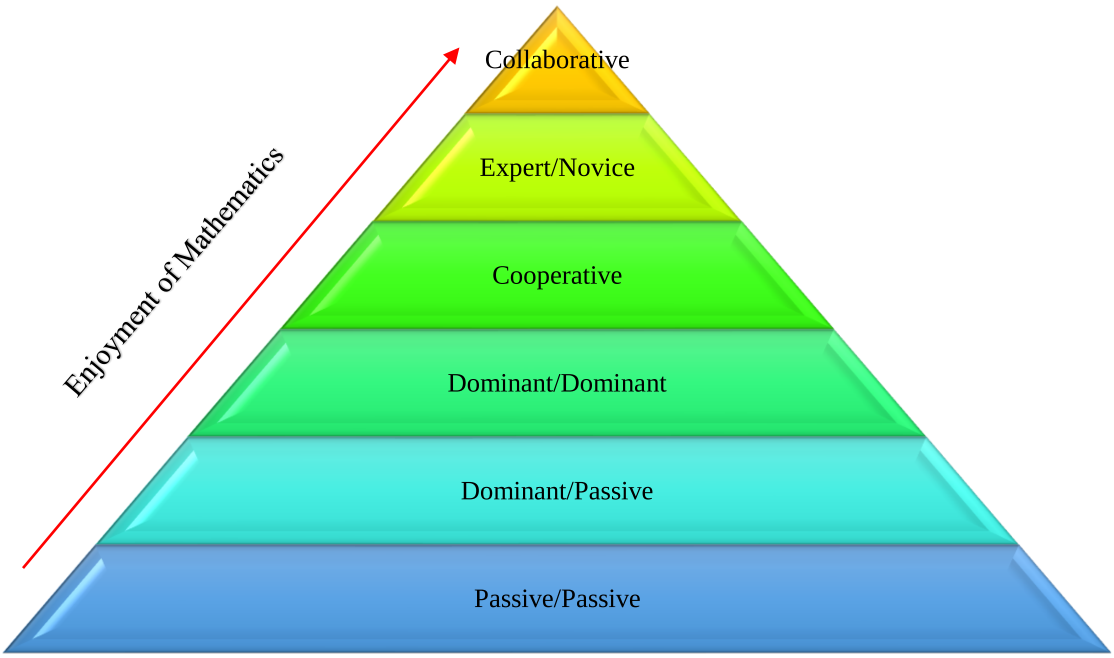


Figure 4.6 The Relationship Between Patterns of Interaction and The Leve of Enjoyment of Mathematics

4.3.2.4 The Association between Patterns of Interaction and Perceived Value of Mathematics.

Interaction Pattern of Group	Participants	Perceived Value of Mathematics Pre-test score	Average score of Pre-test	Perceived Value of Mathematics Post-test score	Average score of Post-test	Overall Average
Collaborative	Essa	33	25.5	46	45.5	45.67
	Saeed	18		45		
	Talal	34	37	46	68.5	
	Mohammed 1	37		47		
	Abdulwahab	40		44		
	Jasser	32	31	45	45	
	Suhil	30		45		
Expert/Novice	Jawad	30	33	46	44.5	44.25
	Omar 2	36		43		
	Abdullah 1	38	32	47	44	
	Mohammed 2	26		41		
Cooperative	Adnan	34	27.5	39	41.5	43
	Abdullah 3	21		44		
	Omar 1	45	43.5	44	44.5	
	Walid	42		45		
Dominant/Dominant	Muhnnad	41	37	45	41	40.5
	Hamad	33		35		
	Ibrahim	45	41	38	40	
	Thamer	37		44		
Dominant/Passive	Yousuf	44	38.5	44	39.5	39.75
	Sulayman	33		35		
	Saad	33	36.5	42	40	
	Mohammed 3	40		38		
Passive/Passive	Abdullah 2	23	20	22	27.5	27.5
	Fayez	17		33		

Table 4.23 Patterns of interaction for each pair (overall) and Perceived Value of Mathematics post-test scores

The above Table (4.23) revealed that the three pairs who adopted a collaborative pattern demonstrated the highest level of perceived value of mathematics since they attained the highest mean scores on the perceived value of mathematics post-test with an overall average score of 45.39. The second highest overall mean level was expert/novice pairs. In contrast, the cooperative pairs showed a lower level of perceived value of mathematics than expert/novice, as they achieved an overall average score of 43 on the perceived value of mathematics post-test. In comparison, the dominant/dominant pairs accomplished the overall mean score of 40.5. while the dominant/passive pairs achieved the second lowest overall mean scores, and the passive/passive pair gained the lowest mean score (figure 4.7). Consequently, it seems that there is a relationship between pairs' interaction pattern and their level of perceived value of mathematics.

Moreover, by comparing students' scores on the perceived value of mathematics, pre-test and their scores in the post-test, it was noticed that collaborative, cooperative, and expert/novice pairs improve their level of the perceived value of mathematics better compared to dominant/dominant, dominant/passive, passive/passive pairs. In addition to this, it was found that two students' perceived value of mathematics was decreased. One of them was dominant/dominant, and the other one was the passive pair in the dominant/passive interaction pattern. Therefore, pairs who discussed, negotiated, explained, and visualised their thoughts and were active throughout learning tasks, improve their level of the perceived value of mathematics. Furthermore, it can be suggested that the patterns of interaction that students adopt can help them change their beliefs on mathematics and then develop their level of the perceived value of mathematics. Consequently, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms perceived value of mathematics?*".

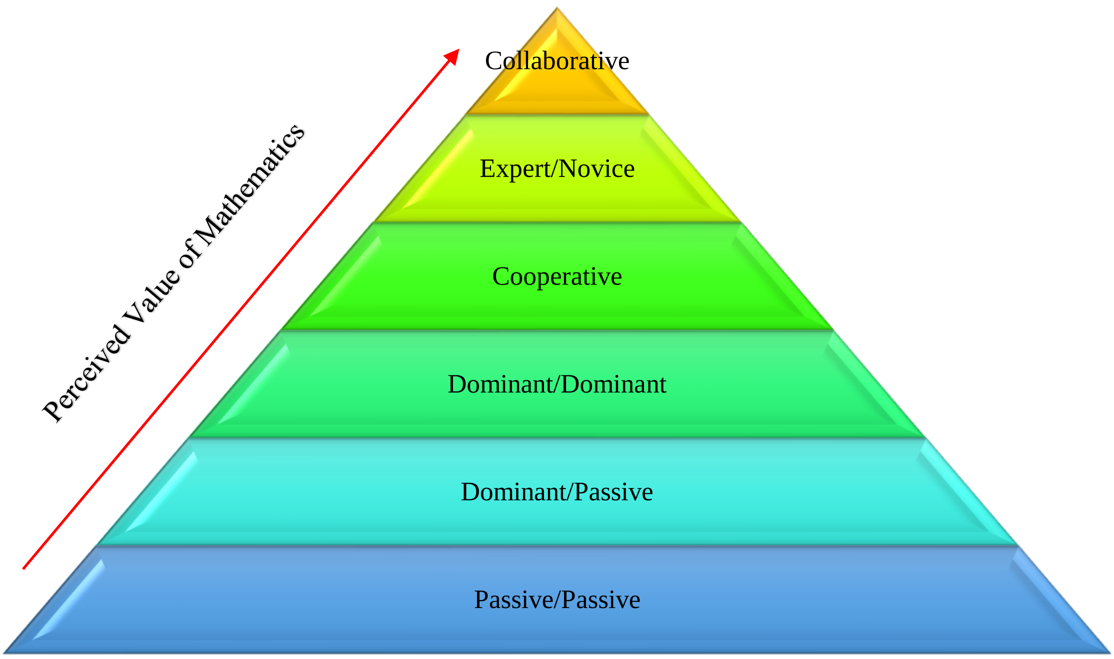


Figure 4.7 The Relationship Between Patterns of Interaction and The Leve of Perceived Value of Mathematics

4.3.2.5 The Association between Pattern of Interaction and Spatial Thinking.

Interaction Pattern of Group	Participants	Spatial Thinking Pre-test score	Pre-test average score	Spatial Thinking Post-test score	Post-test average score	Overall Average
Collaborative	Essa	32	26.5	34	33.5	34.11
	Saeed	21		33		
	Talal	25	29.67	33	34.33	
	Mohammed 1	32		35		
	Abdulwahab	32		35		
	Jasser	30	32	34	34.5	
	Suhil	34		35		
Expert/Novice	Jawad	33	31	34	33.5	33.25
	Omar 2	29		33		
	Abdullah 1	30	27	34	33	
	Mohammed 2	24		32		
Cooperative	Adnan	29	30	30	32	32.5
	Abdullah 3	31		34		
	Omar 1	23	25.5	33	33	
	Walid	28		33		

Dominant/Passive	Yousuf	31	23	33	31.5	32.5
	Sulayman	15		30		
	Saad	31	29	35	32.5	
	Mohammed 3	27		30		
Dominant/Dominant	Muhnnad	27	27	33	32	30.25
	Hamad	27		31		
	Ibrahim	20	25	27	30.5	
	Thamer	30		34		
Passive/Passive	Abdullah 2	18	25.5	17	25	25
	Fayez	33		33		

Table 4.24 Patterns of interaction for each pair (overall) and Spatial Thinking post-test scores

As shown in Table (4.24), the three pairs who adopted a collaborative pattern proved the highest spatial thinking level since they achieved the highest overall mean score on spatial thinking post-test. The second highest overall mean level was expert/novice pairs. In contrast, the cooperative pairs illustrate a lower level of spatial thinking than expert/novice as they attained overall mean score of 32.5 on the spatial thinking post-test. In comparison, there was no considerable differences in spatial thinking level between cooperative pairs and the dominant/passive pairs who accomplished the overall mean score of 32.5. This is while the dominant/dominant pairs accomplished the second lowest overall mean score of 30.25, and the only passive/passive pair gained the lowest mean score of 25 (Figure 4.8). Consequently, it can be said there is a relationship between the patterns of interaction and the development of spatial thinking skills.

By looking at each pair score, all the pairs improved their spatial thinking except the passive/passive pair. This is because they had been less active than their classmate, and they did not use GeoGebra as their friend in the different interaction patterns. Additionally, expert students improved their spatial thinking skills better in comparison to the novice students. In the dominant/passive pattern of interaction, the dominant students also developed their spatial thinking skills better than the passive students because expert and dominant students use GeoGebra more than their peers, which help them improve their spatial thinking. This happened since the collaborative, cooperative, expert/novice, dominant/passive, and dominant/dominant students concentrated on performing the learning task on GeoGebra and spent their time exercising their spatial thinking while doing the learning task and visualizing their ideas. Therefore, the type of pattern students adopt can impact students to improve their spatial thinking skills. Hence, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms of spatial thinking*?".

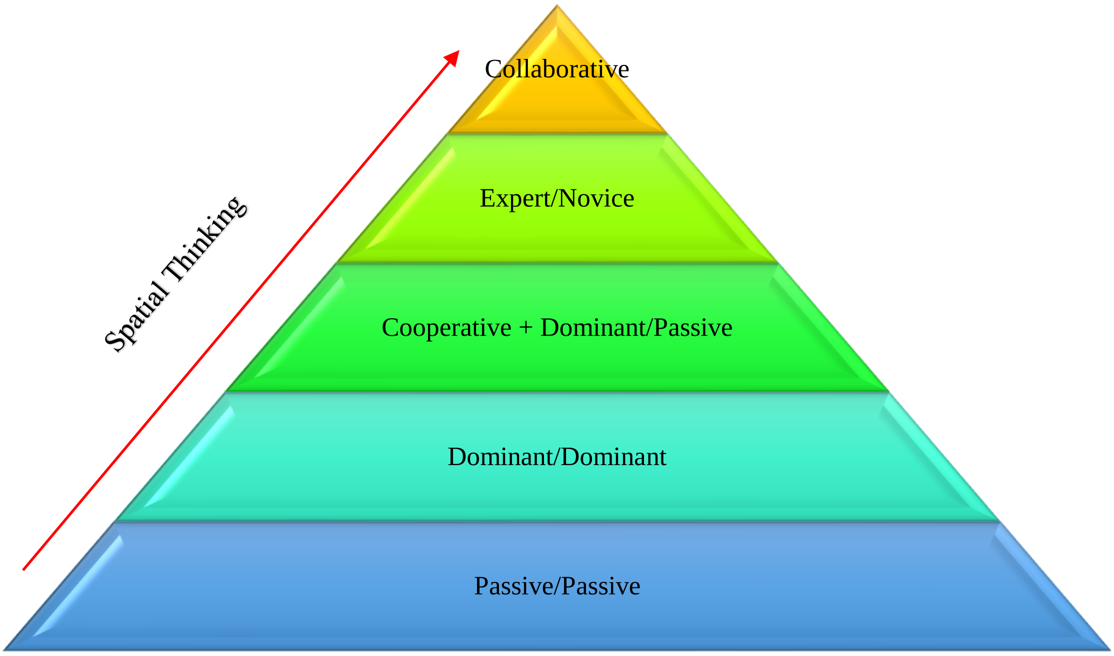


Figure 4.8 The Relationship Between Patterns of Interaction and Spatial Thinking

4.3.2.6 The Association between Pattern of Interaction and Geometric Performance.

Interaction Pattern of Group	Participants	Geometric Performance Pre-test score	Pre-test average score	Geometric Performance Post-test score	Post-test average score	Overall Average
Collaborative	Essa	7	4.5	18	18.5	18.17
	Saeed	2		19		
	Talal	6	6.33	16	17	
	Mohammed 1	4		17		
	Abdulwahab	9		18		
	Jasser	5	4.5	21	19	
	Suhil	4		17		
Expert/Novice	Jawad	8	6.5	16	16	16
	Omar 2	5		16		
	Abdullah 1	7	5.5	15	16	
	Mohammed 2	4		17		
Cooperative	Adnan	6	5.5	14	15	15.75
	Abdullah 3	5		16		
	Omar 1	8	6.5	18	16.5	

	Walid	5		15		
Dominant/Dominant	Muhnnad	6	6	21	17	14.75
	Hamad	6		13		
	Ibrahim	5	4.5	14	12.5	
	Thamer	4		11		
Dominant/Passive	Yousuf	3	4.5	11	13	13.75
	Sulayman	6		15		
	Saad	5	4	14	14.5	
	Mohammed 3	3		15		
Passive/Passive	Abdullah 2	1	3	11	12	12
	Fayez	5		13		

Table 4.25 Patterns of interaction for each pair (overall) and Geometric Performance post-test scores

Table (4.25) revealed that the highest achiever pairs were collaborative pairs since they achieved the highest overall mean score on geometric performance post-test. The second highest mean score was expert/novice pairs with an overall average score of 16, while the cooperative pairs achieved slightly lower than expert/novice, as they achieved the overall mean score of 15.75 on geometric performance post-test. In comparison, the dominant/dominant pairs gained the overall mean score of 14.75, and the dominant/passive pairs achieved the second lowest mean scores with an overall mean score of 13.75, and the only passive/passive pair gained the lowest mean score of 12. Consequently, it seems that there is a relationship between pair interaction patterns and pairs' geometric performance post-test score. However, on some occasions, the dominant/dominant student can achieve higher than his group member with the same interaction pattern. It can be seen in the above Table (4.25) that Muhnnad was one of the highest achiever students as he obtained 21 scores in the geometric performance test. In comparing Muhnnad with collaborative students, he has the same achievement level as collaborative students.

Another instance was found among cooperative students where Omar 2 is also one of the higher achiever students. He obtained 18 scores in the geometric performance test, and this score is higher than his peers in the same interaction patterns and equal to his peers in the collaborative students. Despite this, it can be said that the type of pattern students adopted can help them improve their performance level. Therefore, the patterns of interaction can be placed in a hierarchy, according to their impact on students' performance as follows: collaborative, expert/novice, cooperative, dominant/dominant, dominant/passive, passive/passive (Figure 4.9). Therefore, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms of geometric performance*?".

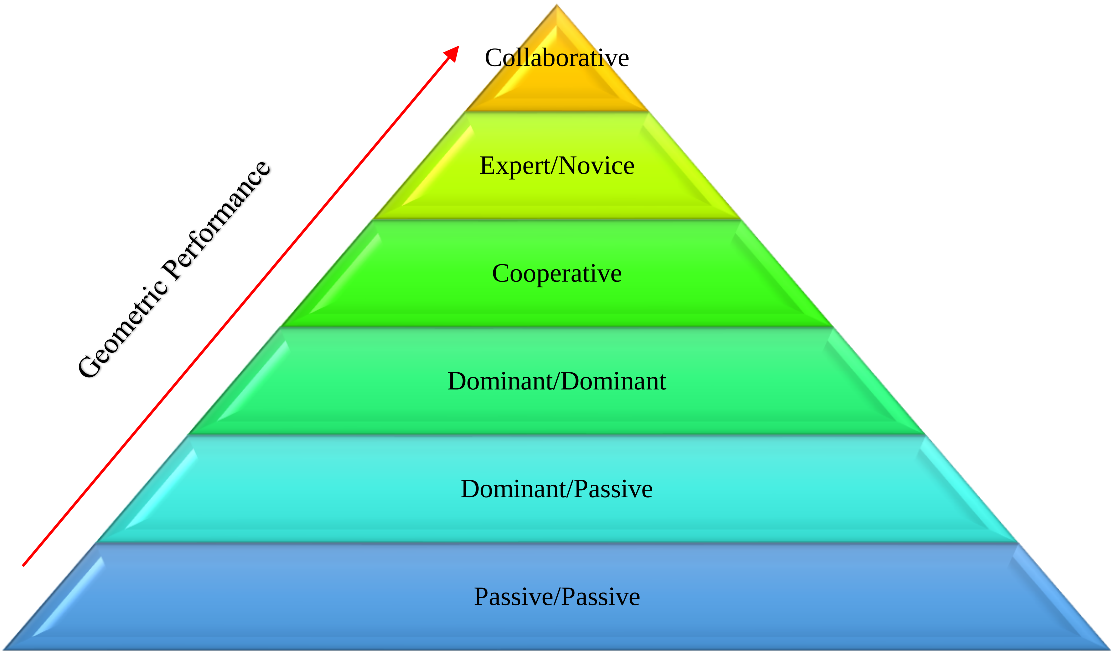


Figure 4.9 The Relationship Between Patterns of Interaction and Geometric Performance

4.3.2.7 The Association between Pattern of Interaction and Sustainable Learning.

Interaction Pattern of Group	Participants	Sustainable Learning Post-test score	Average	Overall Average
Collaborative	Essa	13	15	14.44
	Saeed	17		
	Talal	12	13.33	
	Mohammed 1	14		
	Abdulwahab	14		
	Jasser	16	15	
	Suhil	14		
Expert/Novice	Jawad	11	12	12
	Omar 2	13		
	Abdullah 1	11	12	
	Mohammed 2	13		
Cooperative	Adnan	9	11	12
	Abdullah 3	13		
	Omar 1	15	13	
	Walid	11		
Dominant/Dominant	Muhnnad	18	12.5	11
	Hamad	7		

	Ibrahim	10	9.5	
	Thamer	9		
Dominant/Passive	Yousuf	6	9.5	10.5
	Sulayman	13		
	Saad	12	11.5	
	Mohammed 3	11		
Passive/Passive	Abdullah 2	8	8	8
	Fayez	8		

Table 4.26 Patterns of interaction for each pair (overall) and Sustainable Learning post-test scores

As demonstrated in Table (4.26), the three collaborative pairs achieved the highest score on the sustainable learning test since they obtained the highest overall mean score. The second highest mean score was expert/novice pairs and cooperative pairs since they attained the same overall average score of 12. In contrast, the dominant/passive pairs achieved the second lowest overall mean score of 10.5 and were slightly lower than the dominant/dominant pairs who accomplished the overall mean score of 11. The only passive/passive pair gained the lowest mean score of 8. Therefore, there is perhaps a correlation between students' patterns of interaction and their ability to sustain their learning (Figure 4.10).

However, on some occasions, dominant/dominant students can achieve higher than their peers with the same interaction pattern. It can be seen in the Table (4.26) that Muhnnad was a dominant/dominant student who achieved 18 scores in the sustainable test, which means he attained the highest score in the sustainable learning test. The reason behind this can be his high achievement in the geometric performance test since he was one of the highest achiever students. In addition to this, there is a relationship between students' performance and their sustainable learning (see section 4.1.5.5), which can explain why Muhnnad retained his experience more than his classmates. Consequently, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms of sustainable learning?*".

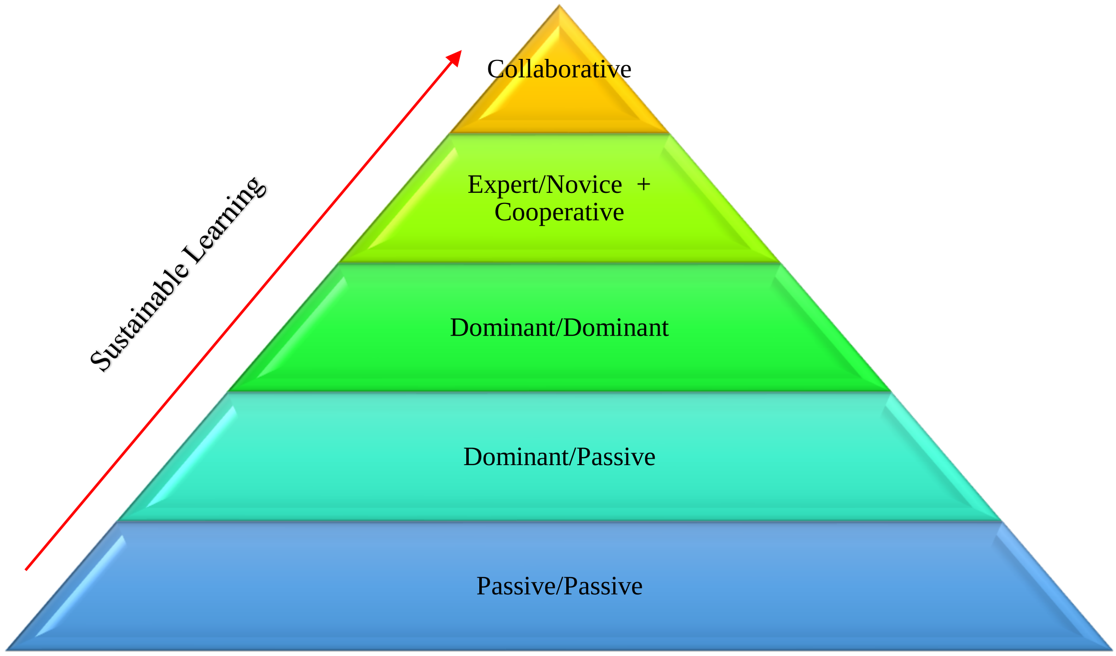


Figure 4.10 The Relationship Between Patterns of Interaction and Sustainable Learning

4.3.2.8 Conclusion

The above sections have correlated students' patterns of interaction and geometric performance, spatial thinking, sustainable learning, mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics. The reasoning behind this is to explore to what extent the patterns of interaction influence students' learning outcomes and processes. Therefore, based on the above discussion, the patterns of interaction can be divided into two categories: high patterns of interaction, which includes collaborative, expert/novice, and cooperative, while the other category is low patterns of interaction which includes dominant/dominant, dominant/passive, and passive/passive.

Additionally, the findings indicated a relationship between students' attitude towards learning mathematics, in terms of mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics, and patterns of interaction, since results showed that students' scores of the three components of attitude towards learning mathematics increased when they adopted the high patterns of interaction. In other words, students with high levels of the three aforementioned parameters adopted a high level of interaction patterns. It can be said that the active students who interact with their pairs by discussing, negotiating, explaining their ideas, and concentrating on learning tasks' aim improve their attitude towards learning mathematics level.

Moreover, the result revealed that there is a correlation between spatial thinking and the patterns of interaction. Students who adopted a high level of interaction patterns and were active in doing tasks improve their spatial thinking skills better than the passive students. Likewise, geometric performance and sustainable learning have a relationship with students' interaction pattern since the outcomes indicated that the level of geometric performance and sustainable learning increased with raising the level of students' interaction pattern. Thus, this section has answered the research question, "*Do differences in the nature of dyadic interaction result in different outcomes in terms of students' views on using GeoGebra, students' attitude towards mathematics (including mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics), spatial thinking, geometric performance, and sustainable learning?*".

4.3.3 Integrating Qualitative and Quantitative Outcomes

In this section, I will present the quantitative findings in relation to qualitative findings. I will start by presenting the findings of students' attitude towards mathematics. Next, the outcomes of spatial thinking, geometric performance, and sustainable learning will be presented. So, the results of the process of learning will be presented in the beginning and the findings of learning outcomes.

4.3.3.1 Students' Attitude Towards Mathematics

It has been established that the research teaching intervention with GeoGebra improves students' attitudes towards mathematics. This may be due to the fact that they become accustomed to using GeoGebra and discuss their thoughts in a collaborative social environment. Overall, these lead to enhance learners' patterns of interaction. It also leads them to begin actively performing learning tasks, as well as instilling in them, the positive view on learning mathematics using GeoGebra, not to forget, the improvement of spatial thinking skills, which results from the research intervention. Thereafter, it helps students improve their level of enjoyment in learning geometry and develops the level of mathematics academic concept and perceived value of mathematics. Therefore, they develop a positive attitude towards mathematics.

4.3.3.2 Spatial Thinking

The integration of GeoGebra into the current teaching intervention helps students improve their spatial thinking skills better than other teaching strategies. In addition to this, qualitative data shows that students who actively worked with their group member improve their spatial thinking skills. In contrast, the inactive students could not develop their spatial thinking skill. This is because the active students concentrate on performing the learning task on GeoGebra. They spent more time exercising their spatial thinking skills while doing the learning task and visualizing their ideas and referring to GeoGebra to explain their ideas thoughtfully.

4.3.3.3 Geometric Performance and Sustainable Learning

The results show that the integration of GeoGebra into the research teaching intervention helps students in improving their geometric performance better than other teaching strategies. This may be due to the students' improvement of attitude toward learning mathematics and spatial thinking skills. Moreover, the collaborative social environment that allowed students to discuss, explain, and visualise their thoughts, and then reach an agreement to solve the mathematical problem, leads to improved geometric performance, especially when students adopt a high-level pattern of interaction. By considering the correlations between geometric performance and the other variables, it can be said that the improvement of spatial thinking, mathematics academic self-concept, enjoyment of mathematics, the perceived value of mathematics, positive views on learning using GeoGebra, and collaborative social activity using GeoGebra, which possibly enhanced students' pattern of interaction, caused the improvement of geometric performance. Afterwards, the aforementioned helped students sustain their learning for a long time.

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Chapter 5. Discussion and Recommendation

5.1 Introduction

This research aimed to investigate the effectiveness of integrating GeoGebra into teaching interventions on primary school students' learning process and outcomes in a Saudi Arabian school, compared to traditional teaching methods. Moreover, it explored the interaction patterns of pairs while using GeoGebra and the association between students' interaction patterns and their geometric performance, spatial thinking, sustainable learning and attitudes towards mathematics.

In this research concerning a specific teaching intervention, students learn actively via three phases of the teaching process, by using GeoGebra in the GeoGebra group and using manipulative materials in the hands-on group, in which teaching was learner-centred. However, in the traditional teaching group, students passively received the knowledge from their teachers, where the learning was teacher-centred. Based on the theoretical learning differences between the research teaching intervention and traditional teaching, students' learning process and outcomes can be influenced. Learning outcomes include geometric performance, spatial thinking, and sustainable learning, while learning process includes students' views on using GeoGebra and students' attitudes towards mathematics, covering mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics.

Students' results regarding these variables can be associated with several factors, including the pairs' interaction patterns. Therefore, this research also explored pairs' interaction patterns while learning using GeoGebra, and the correlation between the interaction patterns of pairs with geometric performance, spatial thinking, sustainable learning, mathematics academic self-concept, enjoyment of mathematics, in addition to perceived value of mathematics. Therefore, in this chapter, the major research findings pertaining to the research questions conducted in Saudi Arabia for Year Five students will be discussed in light of previous research and theory. Furthermore, the research implications and recommendations will be contemplated. This chapter also considers the limitations of this research and makes a number of recommendations for further investigation.

5.2 Answering the Research Questions

Having investigated the data, what remains is to make correlations between the emergent results from the present research and the existing literature to discuss how and why these findings have eventuated due to the research teaching intervention. On that note, the following sections answer the research questions:

5.2.1 What is the impact of teaching intervention using GeoGebra on Geometric performance, Spatial thinking, sustaining knowledge for more extended periods, and students' attitude towards Mathematics (including mathematics academic self-concept, enjoyment of mathematics, and the Perceived value of mathematics) for Year Five students?

This research question investigates how integrating GeoGebra into teaching intervention affects students' learning outcomes and processes. The results show that teaching intervention with GeoGebra improved students' geometric performance, spatial thinking, sustainable learning, and attitudes towards mathematics regarding three domains: mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics. Students who learned with GeoGebra showed better improvement than those who learned with teaching intervention with hands-on and traditional teaching.

5.2.2 What are the students' views on learning using GeoGebra over time?

This research question explores how students change their opinions on learning geometry using GeoGebra. The research findings revealed that students' views of their learning mathematics using GeoGebra were subject to a positive shift. Students' responses documented steady developments in their views regarding using GeoGebra to study geometry throughout the first measure of the research experiment, and this remained constant throughout the next three weeks.

5.2.3 What are the relationships between geometry performance and spatial thinking, sustainable learning, and students' attitude towards Mathematics?

This research question aims to find out correlations between the present research variables and how they affect students' learning outcomes. The findings show a positive correlation between geometric performance and spatial thinking and students' attitudes to mathematics (including mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics). This means that using GeoGebra helps students improve their spatial thinking skills and develop positive attitudes toward mathematics, helping them improve their geometric performance and sustain their learning for a long time.

5.2.4 What patterns of dyadic interaction can be found in a primary mathematics classroom while learning using GeoGebra?

This research questions explore pairs interaction patterns while they learning geometry using GeoGebra. The results revealed that, students adopted six interaction patterns during performing GeoGebra learning tasks. These patterns are collaborative, dominant/dominant, cooperative, passive/passive, dominant/passive, and expert/novice. They improved their interaction patterns to work collaboratively throughout the course of the research since they became more familiar with using GeoGebra and gradually developed their positive view of learning with GeoGebra. This suggests that pairs' interaction became increasingly focused on discussions and constructively contributed to performing learning tasks throughout the research experiment. It implies that pairs have increased opportunities to engage critically and constructively with each other's thoughts following task structure and the consequent development of attitudes towards mathematics, spatial thinking, and geometric performance.

5.2.5 Do differences in the nature of dyadic interaction result in different outcomes in terms of, Students' attitude towards Mathematics regrading mathematics academic self-concept, enjoyment of mathematics, and perceived value of mathematics, Spatial thinking, Geometric performance, Sustainable learning?

This research question aimed to investigate the correlation between different patterns of interaction and the research variables. The focus was on exploring the association between the different outcomes on the post-test and student's pattern of interaction. Therefore, exploring the correlation between pairs' patterns of interaction and their outcomes suggests that as pairs developed a positive view of learning using GeoGebra over time, they became increasingly focused on the learning aims and turned to adopting a collaborative pattern of interaction, suggesting that pairs shifted away from unrelated discussions to concentrate more on the learning task and constructive dialogue, plus spending more of their time performing learning tasks using GeoGebra. It indicates that collaborative group work contributed positively towards progress, attitudes towards mathematics, and sustainable learning.

The present research interprets the data by paying attention to the purpose of this research. This thesis agrees with Hammersley's (2003) definition of practical science; it firmly believes that any knowledge attained as a result of the current research is, to some extent, subject to the context in which it was produced. Consequently, this research strives to be informative and provide information that can be related to the audience without any sense of accountability or the right, to attempt to control how individuals derive practical or policy implications from the knowledge provided, or to attempt to control what individuals do based on it (Hammersley, 2003). As a teacher-researcher, I would expect some similarity and differences in outcome if another teacher used this current research teaching intervention. This is because students are different, and teachers are different, each one with his style of instruction, working within a different set of circumstances. Therefore, the results will always be different, which is not considered a serious limitation of the present research.

The following section will focus on a synthesised discussion of the key findings of this research in relation to the previous literature, in the context of Saudi Arabia, and its contribution to the knowledge in this field.

5.3 Using ACAD to Understand the Research Results

The current research explored the effects of integrating GeoGebra into teaching intervention on students' geometric performance, spatial thinking, attitudes toward mathematics, and students' ability to sustain their learning for a long time. The data consistently shows the significant impact of the teaching intervention with GeoGebra condition on students' geometric performance, spatial thinking, attitudes towards mathematics and sustainable learning.

Remarkably, the findings of the present research suggest different trends to the belief that the development of spatial thinking requires long-term training (Saha et al., 2010; Uttal et al., 2013; Baki et al., 2011; Khine, 2017; NRC, 2006; Robichaux and Guarino, 2000). Moreover, the findings of the current research disagreed with the previous studies that suggest spatial thinking cannot be developed via typical instructional approaches but can be developed by way of life experience (Zavotka, 1987; Sexton, 1992; Strong and Smith, 2001; Baki et al., 2011). However, it is clear that teaching intervention with GeoGebra condition was more effective than the research teaching intervention with the hands-on condition and traditional teaching. Further explanations of why teaching intervention with GeoGebra is effective is needed. Since the ACAD is an approach to understand and improve complex learning situations (Goodyear et al., 2021). The following sections will therefore endeavour to investigate why the teaching intervention with GeoGebra was indeed particularly effective based on ACAD framework, giving distinction between "learn time" and "design time" in relation to the Piaget's theory, Vygotsky's theory, Van Hiele geometric thinking theory.

5.3.1 Learn Time

During learning time, students' actual activity emerges in response to a range of non-deterministic influences. According to the ACAD, students' activity is epistemically, physically and socially situated. Put differently, what students do is substantially affected by the tasks they were set, the tools and other resources that were provided for them, and the interaction between teacher and students, and pairs (Goodyear et al., 2021). Therefore, the following three sections: physical set, epistemic set, and socially set, will explain this research findings in light of Piaget's theory, Vygotsky's theory, and Van Hiele's theory.

5.3.1.1 Set Design (Physical Situated)

According to the ACAD framework, set design or physical situated refers to all physical elements in the learning environment (see section 2.2.1.1). The physical setting for the current research experiment was prepared in light of Piaget's views, who believes that the interaction between students and physical elements helps them to develop their thinking skills and to construct meaningful understanding (see section 2.3.1). This is why the present teaching intervention provides students with GeoGebra as a crucial element during learning time. Using GeoGebra provides students with a visual representation of geometric concepts during learning activities. This can permit students to make abstract concepts concrete, which in turn, can help them acquire experiences and construct their understanding; therefore, develop their geometric performance. Since the visual representation makes objects concrete and helps students to create mental images, it provides students with a foundation to develop their understanding of the mathematical concepts (Johnson et al., 2016). Using physical materials or visual representations to perform mathematics tasks provide students with the opportunity to make abstract concepts concrete, letting them use these concepts, allowing them to construct their knowledge, develop their mathematical performance, and enhance their thinking skills (Ojose, 2008). It enables them to enjoy learning mathematics because the abstract aspect of the concept has been removed. This is supported by Celen (2020) findings who found GeoGebra makes the process of learning enjoyable and assists students to concretise abstract concepts and acquire better geometrical understanding. Therefore, it can be said that the current teaching intervention with GeoGebra condition help students improve their geometric performance and enjoy their learning of mathematics.

Moreover, the teaching intervention with GeoGebra enables students to exercise their spatial thinking skills, since they began their learning process using GeoGebra by visualising the geometric shapes and concepts. Put differently, the first stage of learning geometry, according to this teaching intervention, is using GeoGebra to create a visual representation. When students see the shape on the GeoGebra screen, they can create a mental image of it (Johnson et al., 2016). They are able to recognise its location in the coordinate plane and the direction of the shape. This task frequently occurred throughout the research experiment, indicates that students exercise their spatial perception skills on a daily basis.

Furthermore, while performing the GeoGebra learning tasks, students can manipulate the shapes. Hence, the mental image they create moves to other levels and begins to operate and develop as they internalise these actions. These actions include discovering and describing the relationship between geometric concepts, such as the relationship between a square and rectangle, the difference between parallel lines and intersecting lines, as well as distinguishing the relationship between the parts of a shape. This function provides students with an opportunity to train their spatial relations, which can also help students to construct meaningful understanding of the geometric relationships (Johnson et al., 2016, Ojose, 2008). Hence, the teaching intervention with GeoGebra supports students to develop their spatial thinking skills and improve geometric performance.

Likewise, when applying GeoGebra, students are able to manipulate the shape and play with it. It allows them to see the shape from different angles. Besides, in this research experiment, students studied geometric transformation. Consequently, their spatial rotation and spatial visualisation skills are directly exercised when they study geometric transformation lessons and indirectly as well, when they manipulate the shape and drag it, during other lessons. In addition to this, students work out their spatial orientation when they describe the movement of the shape in space, when performing geometric transformation tasks. Remarkably, GeoGebra helped students to improve their spatial thinking skills better than hands-on material. I think this is because students could find it difficult to draw or create geometric shapes using hands-on material and then perform learning tasks, which is time consuming. This is while GeoGebra is easy to use and accurate where students can draw the shape easier compared to using hand-on and geometric set. This gave students in the GeoGebra group more time to practice and exercise their spatial thinking skills.

Moreover, this could be related to the nature of Saudi students, who are familiar with the use of technology, since the majority of Saudi children owned mobile phones, iPads, or tablets. Therefore, it can be stated that the present research teaching intervention with GeoGebra provides students with an opportunity to train and develop their spatial thinking skills. This is supported by the previous literature that reported spatial thinking could be enhanced by means of teaching mathematics and geometry (Olkun et al., 2005; Hawes et al., 2017; Verdine et al., 2014), in addition to daily mathematics learning activities (NCTM, 2006). Furthermore, from a constructivist perspective, Ojose (2008) stated that using technology helps learners construct their mathematical understanding and attitudes towards mathematics by giving them a method to examine and confirm their reasoning.

5.3.1.2 Epistemic (Task) Design

Based on the ACAD framework, Epistemic design or task refers to a recommendation on doing something worthwhile (see section 2.2.1.2) (Goodyear & Carvalho, 2014b). This includes the role of knowledge-laden task specifications in giving students directions about the concept they learn and how they perform the task (Goodyear et al., 2021). Therefore, GeoGebra and hands-on learning tasks were constructed based on Van Hiele's geometric thinking levels to meet students' thinking processes while learning geometric concepts (see section 2.3.3). In order to make the intervention learning tasks worth performing by students and make them confident when they find the learning process meets their thinking level or the process of thinking about a geometric concept. Consequently, the probable reason behind this finding could be that the teaching intervention enables students in the GeoGebra group to visualise the geometrical concepts, given that the teaching intervention starts with using technology (Zengin et al., 2012, Zengin and Tatar, 2017, Priatna, 2017). In the first phase of the teaching intervention students start learning mathematics with visualising the concepts using GeoGebra which helps them to see the abstract concepts. Students, therefore, can concretise mathematical concepts. Notably, starting to learn the geometric concept by way of visualisation, means that this teaching intervention meets the first level of Van Heile's geometric thinking levels (see section 2.3.3). This is evident in some students' activities while performing GeoGebra learning tasks, where they refer to GeoGebra to visualise their thoughts and explanations to their group member or correcting their error (Figure 5.1).

Figure 5.1 Students referred to GeoGebra to give explanations to correct each other mistakes

When students performed the learning task in pairs in first stage of the current teaching intervention, they discussed and negotiated the mathematical concept. This can help distinguish figures and other geometric parts, identify the shapes' properties, and then move to the *descriptive* (analysis) level of Van Heile's thinking levels. In addition, they use their previous experience to explain their ideas and refer to GeoGebra to visualise their thoughts. For example, excerpt 1 [lines 5 – 7] (see section 4.2.1.1) shows pairs visualising their ideas and describing the shape properties. They distinguished the shape from its appearance and use their language to describe and analyse the shape's parts. Thus, this process can help students move to the next level of Van Heile geometric thinking levels, particularly in the following two phases of this research teaching intervention, specifically teaching intervention and using their spatial thinking skills. Therefore, the teaching intervention with GeoGebra can support learners to use their higher geometric thinking level.

Furthermore, the current research findings revealed that students developed positive attitudes towards mathematics, besides significantly improving geometric performance and spatial thinking. The teaching intervention can also be the possible reason for improving positive attitude towards mathematics, because the teaching process fits with students' geometric thinking levels, according to Van Hiele's geometric thinking levels, as explained earlier, students found the teaching process suitable for their cognitive processes to construct geometrical knowledge, making it easier for them to construct better meaningful understanding and develop their spatial thinking skills. Therefore, students' improvement of mathematical understanding and spatial thinking skills made them feel confident about the subjects they studied. Thus, they developed positive perceptions about their abilities to comprehend the subject matter, do well in mathematics and enjoy learning it. These led students to develop their beliefs regarding the significance of mathematics in everyday life and their future. In short, the current teaching intervention with GeoGebra helped students improve their geometric performance and spatial thinking skills, leading them to improve their mathematics academic self-concept, enjoyment of mathematics, and then their perceived value of mathematics. This agreed with previous research that found a positive impact from the teaching intervention based on Van Hiele geometric thinking levels on students' geometric performance and attitudes towards mathematics (Al-ebous, 2016, Duatepe, 2004).

5.3.1.3 Social Design

According to the ACAD framework, social design or social situation refers to an idea for a group arrangement, how students are socially formed when performing learning tasks and whether they are working in pairs, groups, or following scripted roles (see section 2.2.1.3). This idea is linked to Vygotsky's theory, which believes that learning is an active process that involves a teacher or peer during the learning process. Furthermore, Vygotsky views learning and cognitive development as collaborative activities in which learners and individuals develop their cognitive skills via mediation and interaction between teacher and peer (see section 2.3.2). Therefore, a key reason for improving students' learning processes and outcomes is a collaborative learning environment. The present research teaching intervention with GeoGebra provides students with a collaborative social environment that enables students to interact with each other, discuss, explain, visualise their thoughts, and then, reach an agreement to solve the mathematical problem, leading to an improved geometric performance, spatial thinking skills and attitudes to mathematics. Collaborative learning allows learners to develop positive attitudes, encourages active participation, and a sense of others (Grabinger et al., 2007; Milrad, 2002). Besides, performing learning tasks in a collaborative environment can encourage students to help each other by giving explanations and providing positive or negative feedback (see Table 4.19). It can be stated that in the collaborative learning setting, pairs more likely help each other for making progress, to understand when feeling stuck, and to feel more confident. Therefore, students can develop positive views of learning using GeoGebra and enjoy their learning more. This is supported by Vandecandelaere's et al. (2012) findings, who found that students' attitudes towards mathematics, particularly the enjoyment of mathematics, is sensitive to influence from the learning environment and can be enhanced in a relatively short period. Research also provides extensive evidence of GeoGebra having a positive motivational impact on participation in classroom activities, increased concentration in the classroom, enjoyment while performing learning tasks, besides increasing self-confidence (BECTA, 2013).

The present teaching intervention with GeoGebra provides students with a collaborative learning environment where they can enjoy, feel confident and develop their mathematics academic self-concept. When students perform learning tasks with pairs using GeoGebra, they are able to fix each other mistakes through discussions and positive and negative feedback.

Students can feel relaxed when they discuss their ideas with their peers without threatening or negative feelings, which can cause an adverse effect on students' learning. This assumption is supported by Ke and Grabowski (2007), who found that collaborative learning helps students remove frustration and offers a support network. Furthermore, Jansen (2008) indicates that talks in a small group may be less threatening than whole-class discussions. Some students who mentioned feeling threatened during class discussions also reported a reduced feeling of threat when discussing with particular students. This indicates that by allowing students to work more regularly in pairs, students may feel more confident in sharing their opinions and asking questions to further their understanding and then construct meaningful insight. Hence, this can lead students to enjoy learning mathematic and develop to their academic self-concept, and then improve their attitudes towards mathematics.

I believe that the fundamental aspect towards success in collaboration is raising students' concentration on the learning objectives of the task, which leads to increased opportunities for discussion, and is what teaching intervention with GeoGebra provides. The significance of discussion is emphasised prominently in the literature related to teaching and learning mathematics. Several scholars, including Cobb and Bauersfeld (1995), Wegerif and Mercer (1996), Watson (2001), Leat and Higgins (2002), Nichols (2006), Ke and Grabowski (2007), Hu et al. (2011), McGrane and Lofthouse (2010), Staarman and Mercer (2010), Mulholland Shipley (2016) and Ernest et al. (2016) stress the importance of opportunities for dialogue and collaboration in the development of understanding and thinking. This suggests that by verbalising their thinking, students accept reasoning at a higher level than they start with (Hu et al., 2010: p. 5).

Likewise, Jansen (2008) and Boaler (2006) emphasise the positive effect which discussion is likely to have upon the improvement of mathematical understanding. Furthermore, discussion can be described as a vehicle for meaning-making that constructs mathematical knowledge and develops thinking skills (Ernest et al., 2016, Staarman and Mercer, 2010), where learners can engage and offer feedback which has the potential to improve their mathematical performance and thinking skills. Connecting to Hattie and Timperley (2007) conclusion that feedback has an average impact size of 0.79 (twice the average effect), putting it in the top 5 to 10 most significant influences on progress, particularly feedback about a task and how to do it more effectively. During performing learning tasks, students provide their peers with negative and positive feedback directed to accomplish the learning task. The negative feedback helps students correct misunderstandings of the concepts of another member in their group. In contrast, positive feedback assists students to express their agreement on a solution or each other ideas and confirming correct knowledge, encouraging their peers to continue working. Therefore, both negative and positive feedback can help students maintain their concentration on tasks learning aim, thus assisting them in engaging in the learning activities and being more active, which leads them to improve their understanding and skills.

Simply, I believe that, due to increased opportunities for collaboration, and hence, discussion during performing learning tasks, students engaged more frequently in conversations, as is evident in the increased collaboration between pairs from about 17%, at the beginning of the current research experiment, to 50% before the experiment was stopped due Covid-19 (see section 4.2.1.4). This is because students become accustomed to using GeoGebra and become more familiar with the way to carry out the learning tasks, allowing them to engage with each other actively, visualise, discuss, and negotiate their thoughts to construct meaningful understanding. Put differently, when students became familiar with using GeoGebra, they could visualise the concepts and discuss their ideas with their peers and thus construct a better understanding. This led them to feel more confident to perform learning tasks jointly and increase their level of collaboration Therefore, I believe students developed a shared understanding of geometrical concepts, consequently prompting them to foster positive views of their learning using GeoGebra, as is evident in the GeoGebra visual questionnaire data, which measuring students' opinions on their learning using GeoGebra (see section 4.3.1). I believe this is especially critical, given the likely relationship between students views and students' performance (Sammons et al., 2008, Gilbert et al., 2014).

This suggests that students become familiar with using GeoGebra to perform learning activities in a collaborative setting and focus more on achieving tasks' learning aims. Therefore, students spend their time performing learning tasks using GeoGebra, and discussion became more focused, leading them to construct their understanding. This indicates that students during the process of performing learning tasks exercised their spatial thinking skills, felt confident in their abilities as mathematicians, and enjoyed learning geometry. Thus, they approached work with more positive attitudes to mathematics and were more successful when achieving learning tasks; thereby, leading them to improve their geometric performance and spatial thinking skills, as evident in the geometric performance test and spatial thinking test completed at the end of the research experiment (see sections 4.1.1.1 and 4.1.1.2). Consequently, it can be said that students' dialogue seems to have had a substantial influence on the successful improvement of learning outcomes and process of the teaching intervention with GeoGebra.

Pairs' patterns of interaction make me wonder if there is an association between this interaction and their learning outcomes and processes. Significantly, this thesis explored a correlation between certain interaction patterns and students' learning outcomes and processes (see section 4.2.2). The result indicates that students who adopted collaborative, expert/novice, and cooperative interaction patterns developed better geometric performance, spatial thinking, attitudes towards mathematics, and sustainable learning than the other students. Similarly, Storch (2002) and Ahmadian and Tajabadi (2017) found that in EFL classes, pairs who adopted collaborative and expert/novice interaction patterns performed better than those who adopted dominant/dominant and dominant/passive interaction patterns.

Concerning mathematics teaching and learning, I believe it is imperative to understand how higher-level interaction patterns support students to achieve better. I believe this could be because they are focused more on the learning task and spend more time working on GeoGebra than other students with different interaction patterns. This implies that they exercised their spatial thinking skills more than others. Besides, while students performed the learning tasks in this intervention, they engaged actively in the mathematical discussion, negotiated alternative ideas, engaged with each other's views, and then enjoyed their learning (Figure 5.2). During tasks, students explained their thoughts and referred to GeoGebra to visualise their ideas to solve the problem and achieve an acceptable resolution. They also use their hands and body language to visualise their ideas while explaining their understanding (Figure 5. 3), enhancing students' insight; thus, improve their geometric performance. However, it would be suggested that further research and closer analysis of the students' collaboration process can reveal more precisely what is happening, and how it is correlated to their earning outcomes (see section 5.6).

Further, students can use visual imagery or the mental image and combine this with their body language to describe geometric concepts or geometric transformation. This can therefore prompt them to improve their self-confidence and perceived value of mathematics.

This explanation agrees with the previous literature which determined that visualisation and active engagement to perform mathematical tasks allow students to develop their spatial thinking skills and attitudes to learn mathematics (Ehrlich et al., 2006; Alibali and Nathan, 2012; DfE, 2014; Rich and Brendefur, 2018, Celen, 2020, Zengin and Tatar, 2017). Consequently, it can be emphasised that adopting a higher level of interaction patterns leads students to concentrate their activity more on the learning aims, and train their spatial thinking skills; thus, developing positive attitudes towards mathematics and, hence, developing their mathematical performance.

Figure 5.2a Students Look Happy while Visualizing their thoughts

Figure 5.3b Students Look Happy while Visualizing their thoughts

Figure 5.4Student uses his hands to describe the intersecting lines

Additionally, the findings in section (4.2.2) showed that the collaborative pairs always performed better than other students with other interaction patterns. This could be because learners actively engaged via the process of learning activity. Besides, a collaborative interaction pattern allows students to engage critically and constructively with each other in an exploratory activity to construct their meaningful understanding (Mercer,1995; Wegerif and Mercer, 1996), which improves students' performance and engagement in learning mathematics (Rabel and Wooldridge, 2013, Solomon and Black, 2008). Students have opportunities to their benefit by engaging in asking questions and explaining their thoughts, which resulted in their mathematical knowledge being meaningfully understood. They shared this knowledge while using GeoGebra to perform learning tasks via giving feedback on each other's ideas.

The focus on the learning aims and discussing the geometric concept presented on the GeoGebra screen could make pairs co-construct knowledge. Thus, collaborative pairs could create JPS via a shared knowledge framework to assist problem-solving. This is completed by combining objectives, descriptions, visualisation of the present problem, awareness of problem-solving action, characteristics of the current problem status, and accessible actions. The collaborative negotiation that occurs when pairs collaborate to discover a solution to a problem or acceptable answer for both of them in the GeoGebra task allows them to obtain meaningful discussion on problem-solving, to understand the concept and also promote their knowledge, as witnessed in their post-test geometric performance. Therefore, they felt confident and enjoyed learning geometry and then, developed their mathematics self-concept and perceived value of mathematics.

According to ACAD's view, students' activity is crucial. It is the heart of the learning process. It should be understood as real in that it exists regardless of designers' or teachers' assumptions about what is or should be occurring, and it has direct implications for what students learn. Student activity during learn-time is likewise emergent in the sense that it is influenced but not determined by epistemic, physical, and social circumstances. Instead, the activity comes through collaboration processes in which students use GeoGebra to visualise the geometric concept, discuss their thought and understanding of it, and then construct their meaningful understanding (see Figure 5.5).

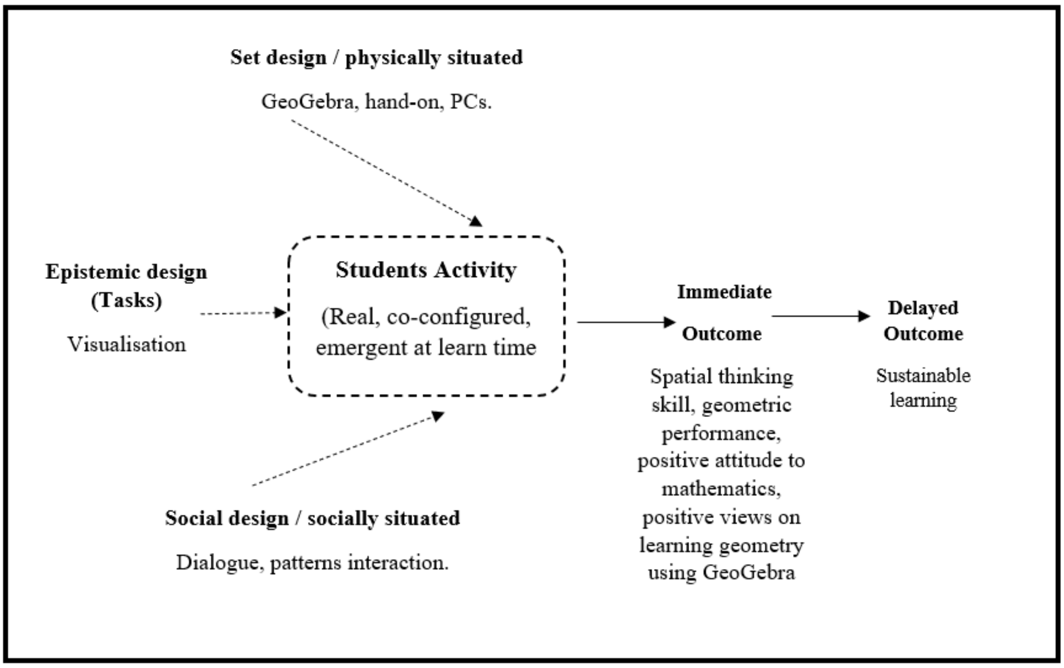

Figure 5.5 ACAD at learn time: understanding the outcomes of teaching intervention with GeoGebra

5.3.2 Design Time

Figure 5.6 demonstrates the developed ACAD at design time. Developed ACAD's understanding of designing the current teaching intervention reflects the teaching process at learn-time. The developed ACAD uses three designable components: epistemic (task), physical (set) and social (set). Each one of these components was designed based on the theory that fits with the present research aims and beliefs. In the following, I will discuss the current research outcomes in the light of ACAD components at design time stage, specifically, what was planned and what was achieved, along with what can be developed and implemented in the future.

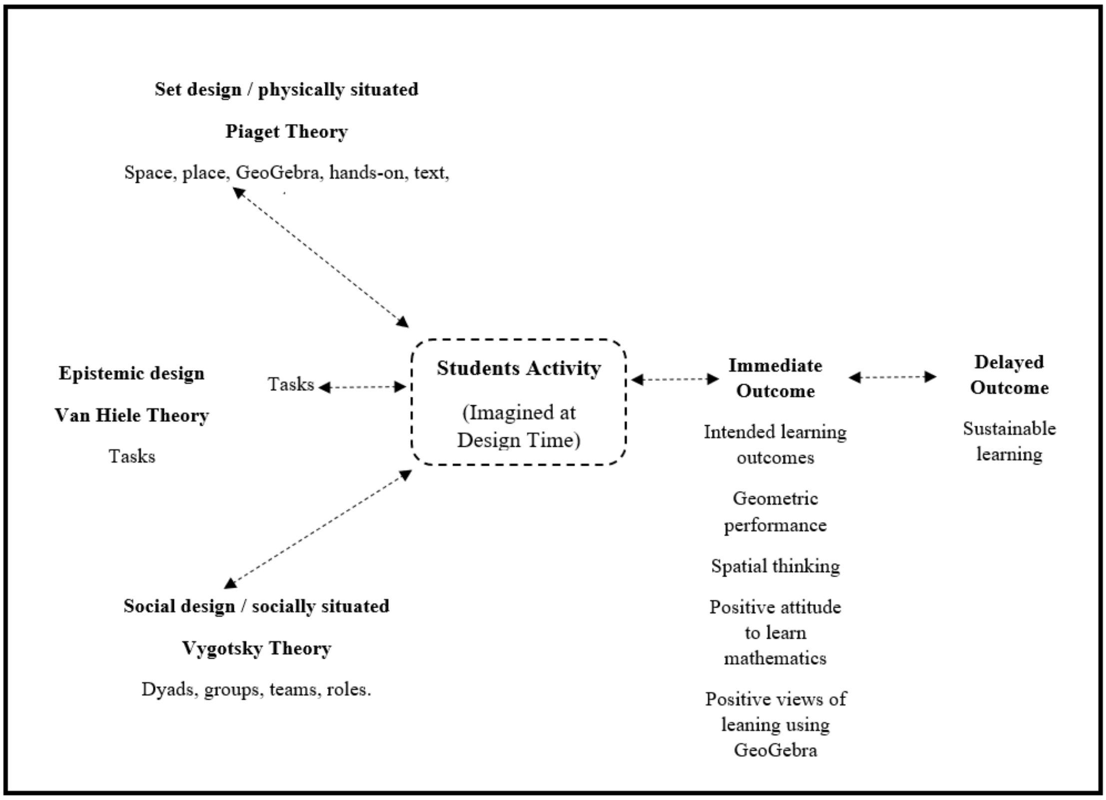

Figure 5.6 Developed ACAD framework at design time: conceptualising the design of the current research teaching intervention

5.3.2.1 Set Design (Physically Situated)

According to the ACAD framework, the set design or physical setting in design time refers to the "resources that may be useful in carrying out the suggested tasks: including material, digital and hybrid tools and other artefacts, learning spaces, etc." (Goodyear et al., 2021, p. 449). This suggests that the structure of the set design requires preparing the physical elements needed to implement the teaching intervention in a way that enables students to perform learning tasks to achieve the intended learning outcomes. This is associated with Piaget's belief that using physical materials or visual representation in mathematics activities allows students to make abstract concepts concrete, allowing them to use these concepts, which can be useful tools for problem-solving. Using physical and visual materials helps learners acquire experiences that lay the foundation for more advanced mathematical thinking and builds their mathematical confidence by giving them a way to examine and confirm their reasoning. This can enable them to construct a better understanding (Ojose, 2008).

Therefore, I carefully prepared the physical design, which allows students to visualise and concretise the geometric concepts, to help them achieve the intended learning outcomes. During the design time, it was ascertained that GeoGebra plays this role and can help students visualise geometric concepts, train their spatial thinking skills, and provide them with opportunities to enjoy learning mathematics and develop positive attitudes towards it. However, during learning time, it was observed that students performed the GeoGebra learning task in a shorter time than was suggested. Similarly, they drew figures with heightened features, which were better than those proposed in the learning tasks, such as drawing a house using geometric shapes. This implies that they train their spatial thinking and do something enjoyable using the geometric concepts they have learnt in previous lessons. This could be the probable reason for achieving the intended learning outcome in relation to improving their geometric performance, spatial thinking skills, positive attitude towards learning mathematics and positive views of using GeoGebra. This highlights the significance of selecting and preparing the physical elements that help learners to improve their learning processes and outcomes. Additionally, it is suggested that teachers should be more aware when preparing and selecting the physical elements while planning their teaching or preparing lessons that help students achieve the intended learning aims and hence, improve their learning processes and outcomes.

Furthermore, at design time, the use of GeoGebra and the physical environment aimed to encourage students to be more active during the learning process. Using physical elements would allow students to build relationships, develop communication skills and use higher-order thinking skills. More importantly, using GeoGebra, to manipulate the geometric concepts and communicating with others to encourage students to connect the new experience to prior knowledge and construct meaningful learning, can facilitate establishing lifelong learning patterns. This suggests that using GeoGebra and technology can modify learning expectations via more active learning and make the teaching process learner-centred rather than teacher-centred (Wolff, 2003).

However, using physical elements does not always proceed as planned during design time, since some students did not use GeoGebra as planned or were passive, as shown in the passive/passive pair (see section 4.3.1.3). Put differently, during learning activities using physical elements does not always go as suggested at design time. Therefore, teachers should pay more attention to students when they perform learning tasks. Likewise, the students should be monitored to ensure that they follow what was planned, with the intention of helping them to attain the intended learning aims and thus improve their learning processes and outcomes.

5.3.2.2 Epistemic Design (Task)

Epistemic design or the task at design time refers to "suggestions of good things to do, and knowledge on which to draw, in the service of learning" (Goodyear et al., 2021, p. 449). The suggestions related to doing something in the classroom have to fit with students' thinking levels and the process of understanding geometric concepts. Learners improve their geometrical understanding by rearranging existing experiences that can be developed via performing the learning task that match students' thinking levels. This correlates with Van Hiele's idea that students learn geometry through a structure for reasoning, concentrating on the language and the structure of simple axioms. Van Hiele believes that students move through five levels (Level 0: visual, Level 1: descriptive, Level 2: theoretical, Level 3: formal logic, and Level 4: the nature of logical laws) to construct their experience and to understand geometric concepts (see section 2.3.3). Therefore, during the task design time, these stages remained in the researcher's mind and the learning tasks were designed in light of Van Hiele's theory relating to students' geometric thinking levels**.** This is because designing a task helps to facilitate harmony between teaching and learning processes, leading to effective learning. This can allow students to enjoy learning processes and feel confident when they perform learning tasks that match their thinking level and the process of understanding geometric concepts. Likewise, it also helps them to develop a positive attitude towards learning mathematics and then improving their thinking skills and geometric performance. This can explain why participating students achieve the intended learning outcomes.

Task design plays a crucial role in encouraging students to perform activities that meet learning objectives. Designing a learning a task helps facilitate the teaching and learning process and enables students to continue performing learning activities according to the learning tasks, so as to achieve the learning aims. According to Khairunnisa (2018), a mathematics learning task is a type of exercise problem that assists understanding mathematical concepts and improves students' thinking skills and communication. Therefore, creating a learning task enables the teacher to perform a learner-centred teaching process; instead of being teacher-centred and involves students in the learning process by using GeoGebra to meet their geometric thinking process. The current research findings determined that students commonly perform learning tasks as planned, given that most students (collaborative, cooperative, expert, novice, dominant/dominant, dominant/passive) followed the instructions for the learning tasks. This was except for two students (passive/passive) who did not follow the learning task instructions or did not perform the learning task and did something entirely different. Therefore, planning learning helps to make the teaching process learner-centred rather than teacher-centred and assists students to accomplish the learning aims and in turn, achieve the intended learning outcomes.

5.3.2.3 Social Design

According to the ACAD framework, social design at design time refers to “proposals about ways students may work with their peers: groupings, roles, divisions of labour, learning networks and communities etc.” (Goodyear et al., 2021, p. 450). Specifically, the social situation involves the preparation of how students will perform learning tasks, whether they are working individually or collaboratively following the learning tasks' instructions. But learning activities are emergent and cannot be designed (Carvalho et al., 2016; Goodyear and Carvalho, 2014a; Goodyear and Carvalho, 2014b; Muñoz-Cristóbal et al., 2018). ACAD believes in the social situation in the classroom as part of the learning environment. This is in agreement with Vygotsky's suggestion that learning is an active process that involves a teacher or peer (Wilding-Martin, 2009, Amineh and Asl, 2015); learning and cognitive development as collaborative activities in which learners and individuals develop their own cognitive skills via discussions and interaction between teacher and peer. This idea laid the foundation to design the social setting for the present teaching intervention. The participating students were grouped in pairs or groups of three students in one group, depending on the number of students in the classroom. GeoGebra and hands-on learning tasks were designed to be performed collaboratively.

However, collaboration is not always the case in mathematics classrooms since some students do not collectively engage with their group members, whilst others remain passive (Li and Zhu, 2013; Liu and Tsai, 2008; Brooks, 1990; Ahmadian and Tajabadi, 2017). The evidence of this view is the findings of data analysis of the video, which ascertained that pairs adopted six interaction patterns: collaborative, dominant/dominant, cooperative, passive/passive, dominant/passive and expert/novice (see Section 4.2.1). This finding is supported by Storch (2002), Ahmadian and Tajabadi (2017), Watanabe and Swain (2007), Tan et al. (2010), Zheng (2012), and Andrews et al. (2017), who found that students adopted different interaction patterns in ELT collaborative classrooms. Furthermore, Todd and Toscano (2020) noted that students did not always work collaboratively while performing online mathematical tasks and that they adopted different interaction patterns. It should be mentioned that the physical setting, tasks and social setting were carefully prepared as regards the current research teaching intervention. Despite this, students were not always fully engaged with each other in regard to performing the learning tasks. This implies that designing a collaborative social set is not easy and does not always guarantee that students will work together to complete learning tasks. This is because even if learning tasks direct students to work collaboratively have been well prepared, their learning activities can be developed and cannot be pre-designed in a fixed way. This view aligns with ACAD's belief that learning activities cannot be designed.

5.4 Research Implication and Recommendation

This research aimed to evaluate the effectiveness of integrating GeoGebra into teaching intervention on primary school students' learning process and outcomes. The rationale behind conducting this research was to provide evidence on whether or not the utilisation of GeoGebra inside Saudi mathematics classrooms is beneficial. This is because the MOE in the country adopted a project in 2017 to train all mathematics teachers in Saudi Arabia on the use of GeoGebra to teach mathematics. Therefore, the present research findings can provide the MOE with evidence on the potential positive effect of GeoGebra on the learning processes and outcomes.

The findings of this research do not just exhibit the effectiveness of GeoGebra but also shows the educational policymakers, researchers and mathematics teachers how GeoGebra can be used effectively. The conclusions revealed that integrating GeoGebra with teaching intervention encourages students to develop positive attitudes towards learning mathematics and supports them to actively engage in the learning process. The research findings also confirm that the teaching intervention with GeoGebra enables students to improve their spatial thinking skills and explore geometric concepts to construct their own meaningful understanding via visualisation and interaction within pairs. This suggests that teacher professional development on the use of ICTs in mathematics classrooms should include practical sessions to train teachers on how to design teaching strategies integrated with ICTs in light of learning theories and research evidence and suggestions to see what if it is beneficial or not. This implies that if research findings revealed the effectiveness of the training course's subject matter, it should be continued and developed in light of research suggestions and recommendations for better practice. However, suppose the research results showed that the training course topic is less effective or invalid. In that case, hence, it has to be stopped and then benefit from research suggestions and recommendations. This would help MoE, and policymakers increase financial efficiency and not waste time and money on something that is ineffective or impractical to help teachers improve their students' learning processes and outcomes. Besides, an effective training courses can be developed for better practice to increase teaching quality in light of research findings and suggestions. This can also capture the Saudi National Transformation Program 2020 and Saudi Vision 2030, which aims to improve teachers' recruitment, training and development, and increase the financial efficiency of the educational system (Saudi Arabia's Vision 2030, 2016).

The present research results indicate the effectiveness of the research teaching intervention and the developed geometric unit that developed in a way include GeoGebra as a principal element in teaching and learning geometry. This suggested that Saudi mathematics textbooks need to be developed in which incorporate GeoGebra or DMS to improve teaching and learning. However, the other education systems that textbooks are not central in the teaching and learning process can benefit from the current teaching intervention to include GeoGebra as a principal part of the lessons instructions and teaching process. Teachers can follow the teaching intervention, including GeoGebra or DMS and prepare to learn tasks to fit with their national curriculum. Furthermore, it is suggested that either the GeoGebra or DMS learning tasks should be designed by experts, or teachers should receive the appropriate training in how to design learning integrated with GeoGebra or DMS; not just train them on the use of DMS. Furthermore, the DMS learning tasks should be designed to fit into students' geometric thinking and cognitive development levels and structured to prompt them to interact with each other, visualise their ideas to construct their understanding, leading them to improve their academic self-concept and perceived value of mathematics. Spatial thinking skills should also be trained and developed through the mathematics curriculum on a daily basis. The present constructivist teaching intervention provides teachers and researchers with a teaching method that integrates GeoGebra, as a principal part of the teaching and learning process in a socially collaborative environment in which the teaching process is learner-centred. Moreover, it also allows students to enthusiastically perform learning tasks and discuss and negotiate their thoughts within pairs with the aim of achieving an acceptable solution for the problem they are seeking to solve. This can give insight to the Saudi National Transformation Program 2020 and Saudi Vision 2030, which aim to improve curricula and teaching methods and improve students' value and skills (Saudi Arabia's Vision 2030, 2016).

Particularly, in light of the Saudi National Transformation Program 2020, it can be suggested that the Geometric Shapes unit developed in this research, which integrates GeoGebra as a fundamental element in this particular unit, can be included in the Year Five textbook. Furthermore, the mathematics textbooks for other school levels in Saudi Arabia can be developed to integrate GeoGebra or other DMS as a crucial aspect of the geometric content or other mathematical branches. It is vital that DMS be integrated into the mathematics curriculum and teaching to be manipulated by students instead of teachers, using it as a tool. Integrating DMS into the teaching process concretises mathematical concepts via visualisation and can facilitate constructing students' meaningful understanding and develop their attitudes and thinking skills.

It should be noted that materials for lessons based on the current teaching intervention can be used in teaching and learning geometry and mathematics. The materials highlighted in this research were designed according to the Saudi mathematics curriculum. Therefore, the Year Five Saudi mathematics teachers can use them to teach the Geometric Shapes unit, while the mathematics teachers for other school levels in Saudi Arabia and other countries can prepare learning tasks according to this teaching intervention, considering the content they aim to teach. Teachers should pay attention while designing learning tasks and consider how students construct geometric and mathematical understanding to ensure that teaching and learning processes fit with their cognitive and thinking levels. This means that teachers should have a good understanding of cognitive development and thinking levels.

Furthermore, teachers should be exposed to the knowledge of students' interaction patterns and mathematical dialogue to encourage them to adopt the dialogical interaction patterns that help them to construct their understanding by using ICTs, as a principal element of their teaching process, in students' hands not in teachers' hand. This is because when teachers plan collaborative learning tasks, there is a risk that students can be inactive and adopt a passive interaction pattern. Therefore, teachers have to pay considerable attention to students' interaction patterns since students may adopt passive/passive, dominant/passive, and dominant/dominant interaction patterns, which could make the teaching and learning process ineffective (see section 4.2.2). Specifically, teachers should encourage students to be more active and collaboratively perform learning tasks. This could be through orally directing passive students and less active students to work jointly with their peers, and rewarding collaborative, and active group members.

The present research believes that students' engagement is a crucial element of the learning process. Student engagement is essentially the driving force that can develop mechanisms for promoting learning. Therefore, this research intervention grouped students in pairs based on their pre-test score (low-achiever and high achiever). The reason for this was to ensure high achiever students support low achiever students and that the teaching and learning process move smoothly, instead of putting low achievers together and maintaining their struggle through the teaching and learning process. From a social constructivist viewpoint, the high achiever acts as an expert and knowledgeable person who assists the other student to construct his/her understanding. This points to the importance of testing students' previous knowledge and placing them in groups so that they work in a collaborative setting. Moreover, this suggestion may help achieve the Saudi National Transformation Program 2020, which aims to develop the learning environment to stimulate learning and deliver education services for all student levels (Saudi Arabia's Vision 2030, 2016).

From a theoretical perspective, learners' mathematical engagement level can be influenced by a multi-agent process, whereby multiple agents (students' performance level, interaction patterns, personal issues, students' view of using ICT, mathematics academic self-concepts, enjoyment of mathematics, and perceived value of mathematics), act collectively as a driving force which enables learners to engage in performing learning tasks. The present research findings revealed that the interaction pattern of a pair is associated with their level of attitude towards learning mathematics, spatial thinking, and performance level. This illustrates that the interaction pattern of a pair can enable or hinder learners from engaging in the learning process and performing learning tasks.

Although the value of collaborative learning has been advocated for decades, students' interaction patterns in collaborative learning practices, when these are mediated by technology must be addressed. The findings of this research contribute to the literature on interaction patterns by adding two new interaction patterns to those identified by Storch (2002), namely cooperative and passive/passive. Furthermore, it emphasises the possible relationship between the pair's interaction patterns and their geometric performance, spatial thinking, attitude to mathematics, and views of using technology to learn mathematics that could play a role in establishing engagement. The relationship between interaction patterns and other research factors also has theoretical implications since it determines that learner engagement is not confined to the individual learner; rather, it is complex and influenced by multi-agents. These factors that can influence students' interaction and engagement have practical implications for designing the curriculum and teaching strategies, since they isolate and incorporate those aspects that educators can regulate to positively influence students' engagement in collaborative learning.

This study indicates that integrating ICT into teaching and learning mathematics, as a fundamental element, positively affected students' interaction patterns, attitudes towards mathematics, and spatial thinking skills. Hence, it can assist in constructing their knowledge and improve their mathematics performance in sustainable learning.

5.5 Limitation of this Research

Several limitations have been noted for this research. Firstly, the selected sample of Year Five male students from Jeddah City in Saudi Arabia was not, to any degree, representative of the entire population, and limited to male students due to gender segregation system in Saudi Arabia. Besides, the small sample size consists of 77 students, divided into three groups (GeoGebra group: 25 students, Hands-on group: 27 students, and traditional teaching group: 27 students). This was a convenience sample, but not a representative sample. Therefore, the generalisation of the research findings is limited to similar populations in Saudi Arabia. In addition, the sample sizes for investigating the pairs' interaction patterns while using GeoGebra was small. Yet again, as with video research, it is challenging to generalise the findings. Furthermore, the results of this thesis cannot be generalised to other school levels due to individual differences.

Secondly, the size of the sample for teachers was small, with only one teacher. The same teacher taught students throughout the research experiment. Thus, the findings of this experiment are limited to the teacher's experience and belief in the benefits of using ICTs to teach mathematics, as the results might be altered if other teachers taught the same content with the belief that ICTs are not valuable for teaching mathematics for primary school students.

Thirdly, the effectiveness of this research experiment has been affected by Covid-19. In fact, the research intervention had to be discontinued due to the closure of the school. Consequently, the post-test was collected using e-forms which have the same weaknesses as an online assessment, such as a lack of access to the internet. Additionally, students were concerned about touching and sitting close to their peers, since they were afraid of being infected by Covid-19, particularly in the last three sessions ahead of school closing. Therefore, these research findings have been affected by the pandemic, making the results difficult to generalise.

5.6 Further Research

In light of the present research findings and limitations, the author of this research can provide academics with recommendations for further research in the field of mathematics education. Given that the present research sample consisted of male students and teachers, thus the gender of the participants could be a significant suggestion for further research. Therefore, expanding research to involve female students and teachers would enrich the issue under investigation and reveal other interesting outcomes concerning students' views of using

GeoGebra, attitude towards learning mathematics, spatial thinking, geometric performance and the interaction patterns of pairs.

Subsequently, it can be recommended that examinations be conducted with various school levels with a larger sample size and different mathematics topics. Expanding the investigation of the current teaching intervention with GeoGebra or other DMS among other participants of different ages would provide supportive results to confirm or reject the present research findings and provide recommendations to improve the current teaching intervention.

The characteristics of the learning tasks to be performed using GeoGebra or DMS seem to play an essential role in students' engagement in the learning activities, their mathematics achievement, and attitudes towards mathematics. This area of research requires more investigation into the effects of task design on students' learning process and outcomes. Task design issues may cover difficulty, curiosity, length, interest, and connection of task to real-life issues, along with the effectiveness of the selected technology to perform learning tasks. Besides, measuring students' perspectives regarding their learning using technology could be helpful.

I would be highly interested in further exploring the students' collaboration process. Specifically, how students visualise and use their boy language to explain their thoughts and the types of dialogue that hold the most possibility for increasing students' self-concept, enjoyment, levels of engagement, and achievement. Besides, it is recommended for close analysis for students' interaction patterns during collaborative learning tasks and how the patterns of interaction are associated with students' learning outcomes.

It is recommended that for future research, a semi-structured interview research method be involved to obtain students' views before and after the research experiment concerning their learning processes and outcomes. This would provide further interesting findings and could also increase the validity and reliability of the research findings.

Finally, further longitudinal research is recommended to examine students' experiences of being exposed to the present teaching intervention with GeoGebra or other DMS over a long time. The purpose of such research could be to examine more reliable experiences, since the current research exposed the students over a short time. Similarly, this would discover other interaction patterns and how technology enhances students' interaction patterns.

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Appendices

Appendix (3.1) Validity and Reliability of Geometric Content Analysis

1. Reliability of Content Analysis

The conventional technique to examine the reliability of content analyses is reanalysis the content, which takes one of the two ways. The researcher performs the analysis of the content twice, separated by a period of time. Alternatively, two researchers conduct the analysis in which they agree from beginning on the criteria and foundation of the analysis, and then they analyse the content individually (To'eima,1987; Lombard et al., 2010). Hence, to compute the reliability coefficient of the content analysis, the author of this research adopted the first technique, where the researcher analysed the content of the Geometric Shapes unit and then re-analysed it after one month. In both times, the researcher adhered to the previous definitions of Mathematical concepts, mathematical generalisations and Mathematical skills.

Table (1): Outcomes of the content analysis of Geometric Shapes unit

Concepts			Generalisations			Skills		
First analysis	Second analysis	agreement	First analysis	Second analysis	agreement	First analysis	Second analysis	agreement
21	20	20	20	18	18	15	18	15

In order to compute the content reliability coefficient, the researcher used the Holsti formula (1969, p137):

$$RC = \frac{2A}{N1 + N2}$$

Where:

A is the total number of decisions agree on in the first and second analysis.

N1 is the number of decisions in the first analysis

N2 is the number of decisions in the second analysis

The following table demonstrates the reliability coefficient of the content analysis of the Geometric shapes unit:

Table (2): The outcomes of reliability coefficient for content analysis

Mathematical learning aspects	**Reliability Coefficient**
Concepts	0.98
Generalisation	0.95
Skills	0.91

Total	0.95

The above table has illustrated that the reliability coefficient of the mathematical components is evident in the high degree of the reliability of the content analysis. Wimmer and Dominick (1997 cited in Witcher 2000) mention that the minimum reliability coefficient is 90% or above when using Holsti's formula. While Abu Libdeh et al. (1996) indicate that the reliability coefficient is acceptable if it exceeds (80%). Furthermore, To'eima (1987) specified a criterion for the coefficient of analysis that the reliability is low analysis if less than (70%) and that the high-reliability coefficient is more than (80%). This gave the researcher the confidence to write the learning objectives and progress to design teacher guidebook, learning tasks, performance test, and delayed test.

2. Validity of Content Analysis

The validity of content analysis refers to the extent to which the analysis is valid for translating the phenomenon analysed regarding the analysis rules (To'eima,1987). Krippendorff (2018, pp. 361) asserts that content analysis is "valid if the inferences drawn from the available texts withstand the test of the independently available evidence, of separate observations, of competing theories or interpretations, or of being able to inform successful actions". Therefore, to ensure the validity of the content analysis, the researcher offered the outcome of content analysis in its initial form to some experts in mathematics education. The feedback of most of them confirmed the validity of the analysis concerning the mathematical components.

As a result, the reliability and validity of the content analysis allowed the researcher to progress to identify learning objectives of the Geometric shapes units' lessons to design learning tasks, teacher guidebook, geometric performance test, and delayed test.

Appendix (3.2) Learning Objectives

المعرفة الرياضية **Mathematica content**	المستوى المعرفي **Cognitive level**	الهدف	الدرس
		التعرف على مفردات هندسية: Learn about geometric concepts	**مفردات هندسية Geometric concepts**
مفهوم Concept	فهم understanding	1. التعرف على مفهوم النقطة Find out the concept of the point	
مهارة skill	تطبيق Application	2. تسمية النقطة Point label	
مفهوم Concept	فهم understanding	3. التعرف على مفهوم المستقيم Find out the concept of the line	
مهارة skill	تطبيق Application	4. تسمية الخط المستقيم Straight line label	
مفهوم Concept	فهم understanding	5. التعرف على مفهوم نصف المستقيم Find out the concept of half line	
مهارة skill	تطبيق Application	6. تسمية نصف المستقيم Half line label	
مفهوم Concept	فهم understanding	7. التعرف على مفهوم القطعة المستقيمة Find out the concept of segment line	
مهارة skill	تطبيق Application	8. تسمية القطعة المستقيمة Segment line label	
مفهوم Concept	فهم understanding	9. التعرف على مفهوم المستوى Find out the concept of plane	
مهارة skill	تطبيق Application	10. تسمية المستوى Plane label	
تعميم generalisation	فهم understanding	11. التعرف على التقاطع Explore the concept of intersection	
مهارة skill	تطبيق Application	12. التعبير عن حالة المستقيمات المتقاطعة Identify the state of intersecting lines by mathematical symbols	
تعميم generalisation	فهم understanding	13. التعرف على التعامد Explore the concept of perpendicular	
مهارة skill	تطبيق Application	14. وصف المستقيمات المتعامدة باستعمال الرموز Description of perpendicular lines using symbols	
تعميم generalisation	فهم understanding	15. التعرف على التوازي Identify parallelism	
مهارة skill	تطبيق Application	16. التعبير عن المستقيمات المتوازية Expression of parallel lines using mathematical symbols	
تعميم generalisation	فهم understanding	17. التعرف على تطابق المستقيمات Identify congruent lines	

مهارة skill	تطبيق Application	18. التعبير عن التطابق بالرموز Expression of congruent lines using mathematical symbols	
مفهوم Concept	تذكر Remembering	التعرف على خصائص الاشكال الرباعية: Learn about the characteristics of quadrants: 1. التعرف على الشكل الرباعي Recognize the quadrant shape	**الأشكال الرباعية** **Quadrant shapes**
مفهوم Concept	فهم understanding	2. التعرف على المستطيل Identify the rectangle	
مفهوم Concept	فهم understanding	3. التعرف على المربع Identify the square	
مفهوم Concept	فهم understanding	4. التعرف على متوازي الأضلاع Identify the Parallelogram	
مفهوم Concept	فهم understanding	5. التعرف على شبه المنحرف Identify the trapezium	
مفهوم Concept	فهم understanding	6. التعرف على المعين Identify the rhombus	
تعميم Generalisation	تحليل analysis	7. استنتاج العلاقة بين متوازي الأضلاع والمربع، المستطيل، المعين Discover the relationship between parallelogram, rectangle, square, and rhombus	
مهارة skill	تطبيق Application	8. تصنيف الاشكال الرباعية Classification of quadrant shapes	
تعميم Generalisation	تحليل Analysis	9. تحديد خصائص المربع Identify the square's properties	
تعميم Generalisation	تحليل Analysis	10. تحديد خصائص المستطيل Determining the characteristics of the rectangle	
تعميم Generalisation	تحليل Analysis	11. تحديد خصائص متوازي الأضلاع Determining the characteristics of the parallelogram	
تعميم Generalisation	تحليل Analysis	12. تحديد خصائص المعين Identify the rhombus's properties	
تعميم Generalisation	تحليل Analysis	13. تحديد خصائص شبه المنحرف Identify the trapezium's properties	
تعميم Generalisation	تحليل Analysis	14. يميز نوع الزاوية في الشكل الرباعي Distinguishes angle type in quadrant shape	
تعميم Generalisation	تحليل Analysis	15. تحديد الأضلاع المتطابقة في الشكل الرباعي وتحديد العلاقة بينهما (توازي، تعامد) Identify congruent sides in the quadrant shape and determine	

		the relationship between them (parallel, perpendicular)	
تعميم Generalisation	تحليل Analysis	16. استخدام اللغة اللفظية للتفريق بين الأشكال الرباعية Distinguishing between quadrants shapes using verbal language	
مفهوم Concept	فهم understanding	تسمية النقاط في المستوى الاحداثي: Name points at the coordinate level: 1. التعرف على المستوى الاحداثي Get to know the coordinate plane	**الهندسة الزواج المرتبة** **Geometry: ordered pair**
مفهوم Concept	تذكر Remembering	2. التعرف على نقطة الأصل Identify the origin point	
مفهوم Concept	تذكر Remembering	3. التعرف على الزوج المرتب Get to know the ordered pair	
مفهوم Concept	تذكر Remembering	4. تمييز الاحداثي السيني والصادي Distinguish the x and y coordinates	
مهارة skill	تطبيق Application	5. تحديد موقع النقطة في المستوى الاحداثي باستعمال الأزواج المرتبة Determine the position of the point in the coordinate plane using ordered pairs	
مهارة skill	تطبيق Application	6. تسمية النقطة التي يمثلها الزوج مرتب على المستوى الاحداثي Label the point represented by the ordered pair on the coordinate plane	
مهارة skill	تطبيق Application	تمثيل النقاط في المستوى الاحداثي: Representation of points in the coordinate plane: 1. تمثيل الزوج المرتب في المستوى الاحداثي Represent the ordered pair in the coordinate plane	**الجبر والهندسة: تمثيل الدوال** **Algebra and geometry: representation of functions**
مهارة skill	تطبيق Application	2. تمثيل الدالة باستعمال الأزواج المرتبة Represent the function using ordered pairs	
مهارة skill	تطبيق Application	3. تمثيل الدالة على المستوى الاحداث Representing the function on the coordinate plane	
مفهوم Concept	فهم understanding	رسم صورة شكل بالانسحاب على المستوى الاحداثي: Draw copy of a shape due to geometric translation in coordinate plane 1. التعرف على مفهوم التحويل الهندسي Understand the concept of geometric transformation	**الانسحاب في المستوى الاحداثي** **Translation on the coordinate plane**
مفهوم Concept	فهم understanding	2. التعرف على صورة الشكل	

		Recognize the copy of the shape	
مفهوم Concept	فهم understanding	3 . التعرف على الانسحاب Understand the geometric translation	
مهارة skill	تطبيق Application	4 . تمثيل الانسحاب Perform geometric translation	
مفهوم Concept	فهم understanding	رسم صورة شكل بالانعكاس على المستوى الاحداثي: Draw a copy of a shape by reflection on the coordinate plane: 1 . التعرف على مفهوم الانعكاس Understand the concept of reflection	**الانعكاس في المستوى الاحداثي** **Reflection in the coordinate plane**
مفهوم Concept	فهم understanding	2 . التعرف على محور الانعكاس Recognize the axis of reflection	
مهارة skill	تطبيق Application	3 . تمثيل الانعكاس perform geometric reflection	
تعميم Generalisation	تحليل Analysis	4 . العلاقة بين الرؤوس المتناظرة ومحور الانعكاس The relationship between symmetrical vertices and the axis of reflection	
تعميم Generalisation	تحليل Analysis	5 . العلاقة بين الشكل وصورته بعد الانعكاس The relationship between a shape and its copy after reflection	
مفهوم Concept	فهم understanding	رسم صورة شكل بالدوران على المستوى الاحداثي: Draw a copy of a rotating shape on the coordinate plane 1 . التعرف على مفهوم الدوران Understand the concept of rotation	**الدوران في المستوى الاحداثي** **Rotation in the coordinate plane**
تعميم Generalisation	تحليل Analysis	2 . تحديد اتجاه الدوران (عقارب الساعة، عكس عقارب الساعة) Identify rotation direction (clockwise, counterclockwise)	
مهارة skill	تطبيق Application	3 . تمثيل الدوران Perform geometric rotation	
تعميم Generalisation	تحليل Analysis	4 . الفرق بين الدوران والانعكاس Explore the difference between rotation and reflection	

Appendix (3. 3) Geometric performance Test Initial Copy

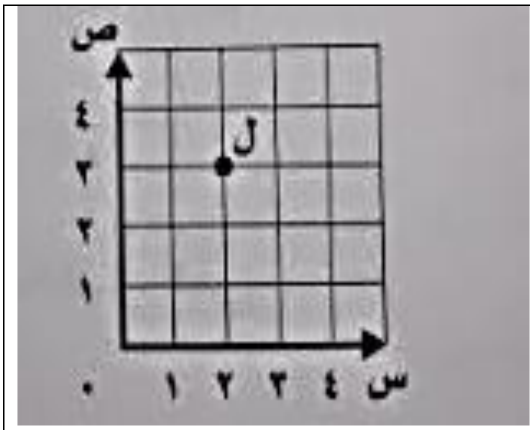
اختبار التحصيل الهندسي

Geometric Performance Test

الاسمName/الصف...:Class/..........:

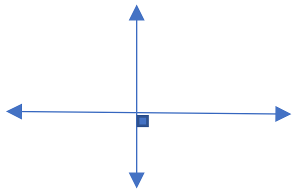
القسم الأول: اختر الاجابة الصحيحة فيما يلي:Section One: In following questions select the correct answer:

مستقيمان بينهما مسافة ثابتة و لا يلتقيان مهما امتدا: Two lines are always in the same distance and never touch				(1
د) متطابقان congruent	ج) متعامدان perpendicular	متقاطعان (ب intersect	أ) متوازيان Parallel	
مجموعة من النقاط تشكل مستقيماً يمتد في الاتجاهين بلا نهاية: A straight set of points that extended from both sides endlessly.				(2
د) المستوى Plane	ج) نصف المستقيم Half line	المستقيم (ب line	أ) القطعة المستقيمة Segment of line	
شكل رباعي فيه كل ضلعين متقابلين متطابقين وجميع زواياه قائمة: A quadrilateral with four right angles and opposite sides that are parallel				(3
د) معين Rhombus	ج) مستطيل Rectangle	شبه منحرف (ب Trapezium	أ) متوازي أضلاع Parallelogram	
تدوير الشكل حول نقطة دون تغيير قياساته أو نوعه: Turning a shape around a point without changing its measurement and type				(4
د) دوران Rotation	ج) تكبير Enlargement	انعكاس (ب Reflection	أ) انسحاب Translation	
الشكل المجاور هو: The following shape is: 				(5
د) شبه منحرف Trapezium	ج) معين Rhombus	متوازي أضلاع (ب parallelogram	أ) مستطيل Rectangle	
ما الزوج المرتب الذي يمثل النقطة (ل) في الشكل أدناه: What is the ordered pair that represent (L) in the following figure?				(6



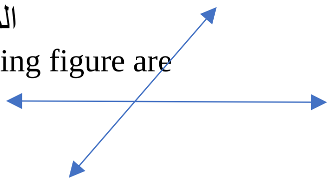
(1،3) (د	(3،1) (ج	ب) (3،2)	أ) (2،3)

(7 المستقيمان في الشكل المجاور: The two lines in the following figure are



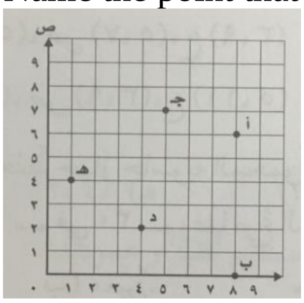
د) متقاطعان Intersect	ج) متوازيان Parallel	ب) متطابقان congruent	أ) متعامدان perpendicular

(8 المستقيمان في الشكل المجاور:
The two lines in the following figure are



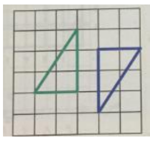
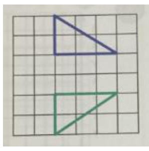
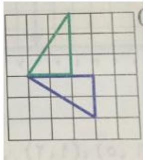
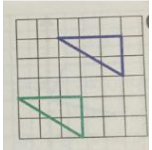
د) متطابقان Congruent	ت) متعامدان perpendicular	ب) متوازيان parallel	أ) متقاطعان intersect

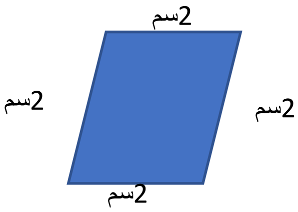
(9 سم النقطة التي يمثلها الزوج المرتب (5،7)
Name the point that is represented by ordered pair (5,7)



د) النقطة ب Point B	ج) النقطة د Point D	ب) النقطة أ Point A	أ) النقطة جـ Point C

(10 ما التحويل الذي يمثل انسحاباً؟
Which translation represent a translation?

د) 	 ج)	 ب)	 أ)

أفضل تسمية للشكل المجاور هي: what is the best discerption of the following shape? 	(11

د) متوازي أضلاع Parallelogram	ج) مستطيل Rectangle	ب) معين Rhombus	أ) مربع Square

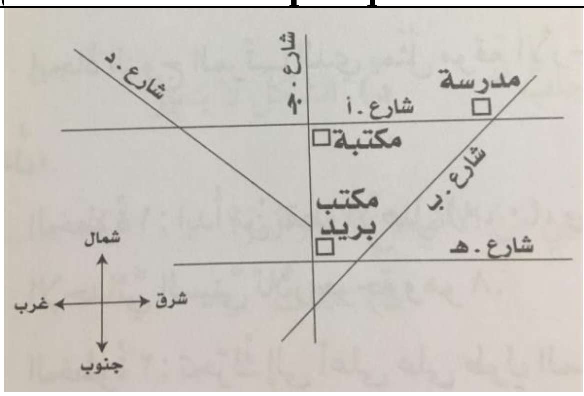
أي الجمل التالية ليست صحيحة؟ which sentence is not correct?	(12

المعين شكل رباعي (ب The Rhombus is quadrilateral	أ) كل مربع مستطيل Every square is rectangle
د) المثلث متطابق الأضلاع مثلث حاد الزوايا The equilateral triangle is an acute triangle	ج) شبه المنحرف هو متوازي أضلاع Trapezium is parallelogram

يريد علي أن يحفظ جهاز حاسوبه المحمول في صندوق. إذا كانت قياسات الحاسوب 29سم في 34سم، فما نوع الشكل الرباعي الأفضل لحفظ الحاسوب؟ Ali wants to pack his lap top in a box. If the dimensions size of his lap top are 29cm x 34cm, what is the best quadrilateral box fit with Ali's lap top?	(13

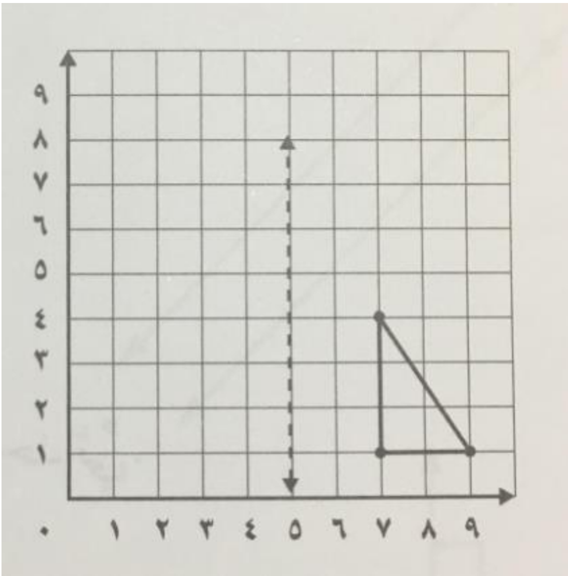
د) شبه منحرف Trapezium	ج) مربع Square	ب) معين rhombus	أ) مستطيل rectangle

القسم الثاني: إجابة مطولة Section two: open questions

 في الشكل أعلاه: In the above figure	(14

....	حدد العلاقة بين الشارع جـ والشارع هـ؟ Identify the relationship between street C and street H?	أ)

....	حدد العلاقة بين الشارع ب والشارع أ ؟ Identify the relationship between street B and street A?	ب)
....	إذا مشيت من المدرسة إلى المكتبة ثم إلى مكتب البريد، فما نوع الزاوية التي تمر بها؟ If you walked from school to library then to post office, what would the type of angle that you make?	ج)
....	حدد العلاقة بين الشارع ب والشارع جـ؟ Identify the relationship between street C and street B?	د)
ارسم صورة الشكل بالانعكاس حول محور، ثم اكتب الأزواج المرتبة للرؤوس الجديدة؟ Reflect the shape around line, and name the new ordered pairs?		15)



Appendix (3.4) Delayed Test Initial Copy

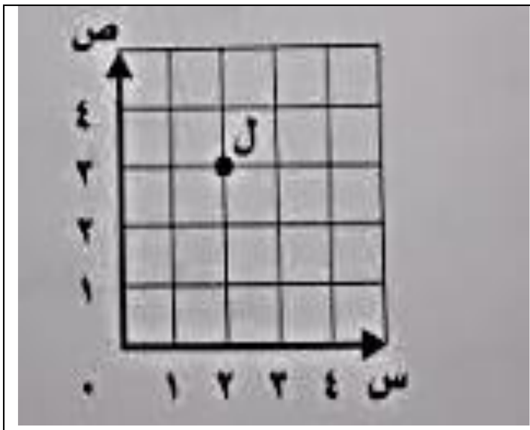
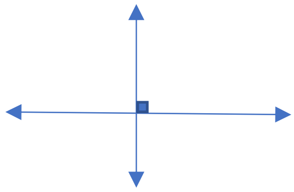
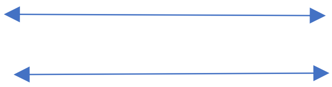
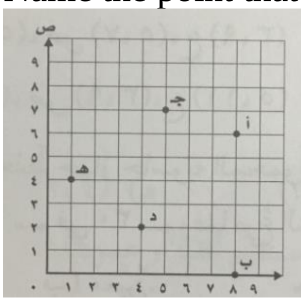
اختبار التحصيل الهندسي المؤجل

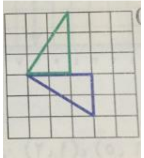
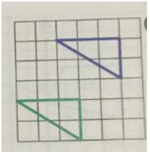
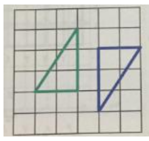
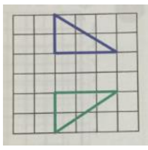
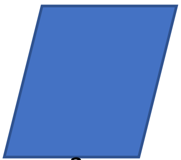
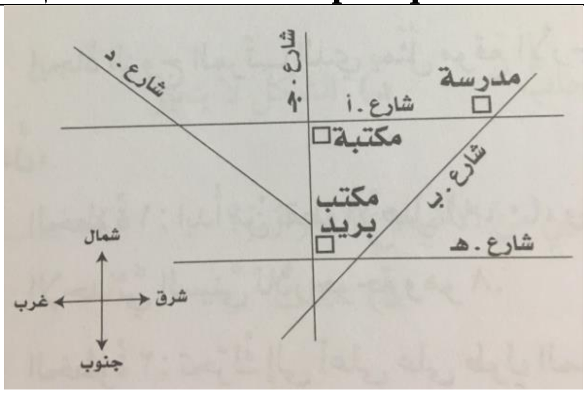
Delayed test

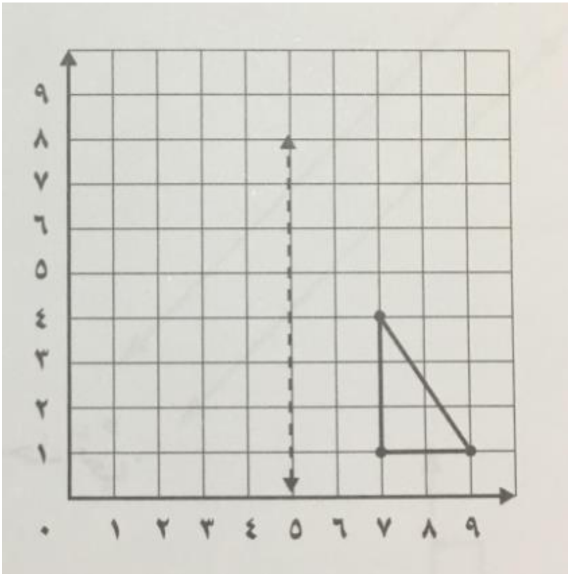
الاسمName/الصف...:Class/..........:

القسم الأول: اختر الاجابة الصحيحة فيما يلي: Section One: In following questions select the correct answer:

مستقيمان بينهما مسافة ثابتة ولا يلتقيان مهما امتدا: Two lines are always in the same distance and never touch				(1
د) متطابقان congruent	ج) متوازيان Parallel	متقاطعان (ب intersect	متعامدان أ) Perpendicular	
تدوير الشكل حول نقطة دون تغيير قياساته أو نوعه: Turning a shape around a point without changing its measurement and type				(2
انعكاس (ث Reflection	تكبير (ت Enlargement	انسحاب (ب Translation	دوران (أ Rotation	
الشكل المجاور هو: The following shape is: 				(3
د) معين Rhombus	ج) مستطيل Rectangle	شبه منحرف (ب Trapezium	متوازي أضلاع (أ Parallelogram	
مجموعة من النقاط تشكل مستقيماً يمتد في الاتجاهين بلا نهاية: A straight set of points that extended from both sides endlessly.				(4
د) المستوى Plane	ج) نصف المستقيم Half line	المستقيم (ب Line	القطعة المستقيمة (أ Segment of line	
شكل رباعي فيه كل ضلعين متقابلين متطابقين وجميع زواياه قائمة: A quadrilateral with four right angles and opposite sides that are parallel				(5
د) شبه منحرف Trapezium	ج) معين Rhombus	متوازي أضلاع (ب parallelogram	مستطيل (أ Rectangle	
ما الزوج المرتب الذي يمثل النقطة (ل) في الشكل أدناه: What is the ordered pair that represent (L) in the following figure?				(6

				
(2،3) (د	(3،1) (ج	ب) (3،2)	أ) (1،3)	
:المستقيمان في الشكل المجاورThe two lines in the following figure are 				(7
د) متوازيان Parallel	ج) متقاطعان Intersect	ب) متعامدان perpendicular	أ) متطابقان congruent	
:المستقيمان في الشكل المجاور The two lines in the following figure are 				(8
د) متطابقان Congruent	ج) متقاطعان Intersect	أ) متوازيان parallel	ت) متعامدان perpendicular	
سم النقطة التي يمثلها الزوج المرتب(8,6) Name the point that is represented by ordered pair (8,6) 				(9
د) النقطة ب Point B	ج) النقطة د Point D	ب) النقطة أ Point A	أ) النقطة جـ Point C	
ما التحويل الذي يمثل انسحاباً؟				(10

Which translation represent a translation?				
د) 	ج) 	ب) 	أ) 	
يريد علي أن يحفظ جهاز حاسوبه المحمول في صندوق. إذا كانت قياسات الحاسوب 29سم في 34سم، فما نوع الشكل الرباعي الأفضل لحفظ الحاسوب؟ Ali wants to pack his lap top in a box. If the dimensions size of his lap top are 29cm x 34cm, what is the best quadrilateral box fit with Ali's lap top?				(11
د) شبه منحرف Trapezium	ج) مستطيل Rectangle	ب) معين Rhombus	أ) مربع Square	
أي الجمل التالية ليست صحيحة؟ which sentence is not correct?				(12
ب) المعين شكل رباعي The Rhombus is quadrilateral		أ) كل مربع مستطيل Every square is rectangle		
د) المثلث متطابق الأضلاع مثلث حاد الزوايا The equilateral triangle is an acute triangle		ج) شبه المنحرف هو متوازي أضلاع Trapezium is parallelogram		
أفضل تسمية للشكل المجاور هي: what is the best discerption of the following shape? 2سم 2سم 2سم  2سم				(13
د) متوازي أضلاع Parallelogram	ج) مربع Square	ب) معين rhombus	أ) مستطيل rectangle	
القسم الثاني: إجابة مطولة Section two: open questions				
 في الشكل أعلاه: In the above figure				(14
....	حدد العلاقة بين الشارع أ و الشارع هـ؟			(أ

	Identify the relationship between street C and street B?	
....	حدد العلاقة بين الشارع ب والشارع أ ؟ Identify the relationship between street B and street A?	ب)
....	إذا مشيت من المدرسة إلى المكتبة ثم إلى مكتب البريد، فما نوع الزاوية التي تمر بها؟ If you walked from school to library then to post office, what would the type of angle that you make?	ج)
....	حدد العلاقة بين الشارع جـ والشارع هـ؟ Identify the relationship between street C and street H?	د)
ارسم صورة الشكل بالانعكاس حول محور، ثم اكتب الأزواج المرتبة للرؤوس الجديدة؟ Rotate the shape around line, and name the new ordered pairs?		15)
		

Appendix (3.5) Geometric performance Test Final Copy

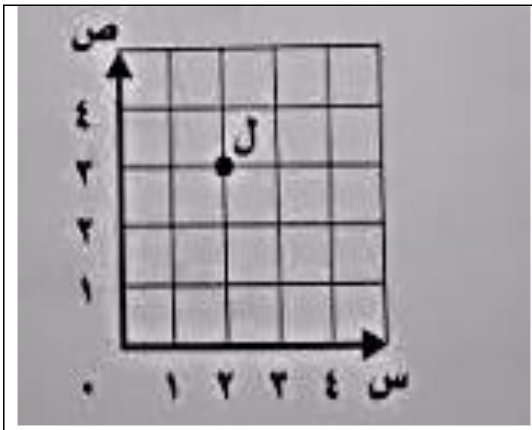
اختبار التحصيل الهندسي

Geometric Performance Test

الاسمName/الصف...:Class/..........:

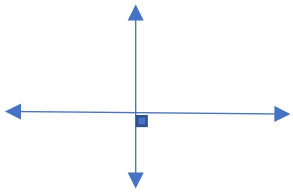
القسم الأول: اختر الاجابة الصحيحة فيما يلي: Section One: In following questions select the correct answer:

مستقيمان بينهما مسافة ثابتة و لا يلتقيان مهما امتدا: Two lines are always in the same distance and never touch				(1
د) متطابقان congruent	ج) متعامدان perpendicular	ث) متقاطعان intersect	ت) متوازيان Parallel	
مجموعة من النقاط تشكل مستقيماً يمتد في الاتجاهين بلا نهاية: A straight set of points that extended from both sides endlessly.				(2
د) المستوى Plane	ج) نصف المستقيم Half line	ث) المستقيم line	ت) القطعة المستقيمة Segment of line	
شكل رباعي فيه كل ضلعين متقابلين متطابقين وجميع زواياه قائمة: A quadrilateral with four right angles and opposite sides that are parallel				(3
د) معين Rhombus	ج) مستطيل Rectangle	ث) شبه منحرف Trapezium	ت) متوازي أضلاع Parallelogram	
تدوير الشكل حول نقطة دون تغيير قياساته أو نوعه: Turning a shape around a point without changing its measurement and type				(4
د) دوران Rotation	ج) تكبير Enlargement	ث) انعكاس Reflection	ت) انسحاب Translation	
الشكل المجاور هو: The following shape is: 				(5
د) شبه منحرف Trapezium	ج) معين Rhombus	ث) متوازي أضلاع parallelogram	ت) مستطيل Rectangle	
ما الزوج المرتب الذي يمثل النقطة (ل) في الشكل أدناه: What is the ordered pair that represent (L) in the following figure?				(6



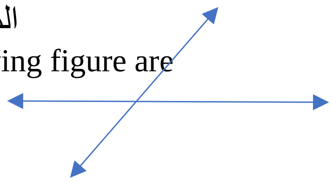
د) (1،3)	ج) (3،1)	ث) (3،2)	ت) (2،3)

(7 المستقيمان في الشكل المجاور: The two lines in the following figure are



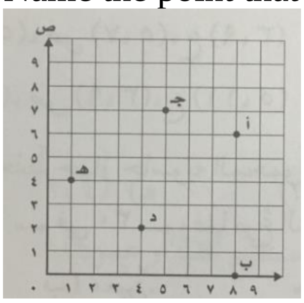
د) متقاطعان Intersect	ج) متوازيان Parallel	ج) متطابقان congruent	ث) متعامدان perpendicular

(8 المستقيمان في الشكل المجاور:
The two lines in the following figure are



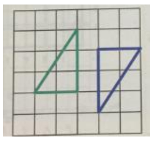
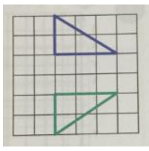
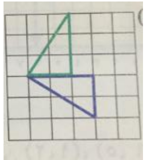
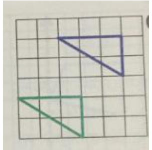
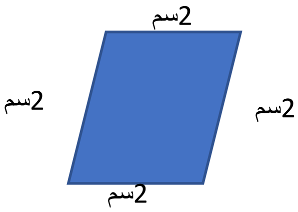
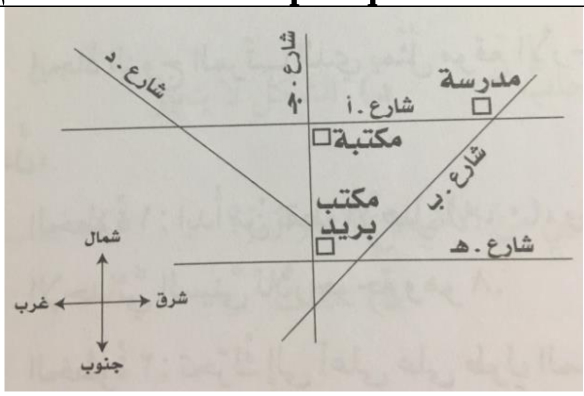
د) متطابقان Congruent	ح) متعامدان perpendicular	ت) متوازيان parallel	ب) متقاطعان intersect

(9 سم النقطة التي يمثلها الزوج المرتب (7،5)
Name the point that is represented by ordered pair (5,7)

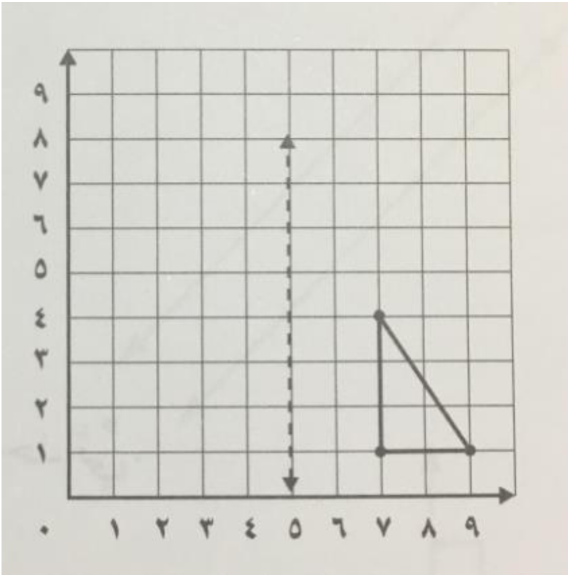


د) النقطة ب Point B	ج) النقطة د Point D	ث) النقطة أ Point A	ت) النقطة جـ Point C

(10 ما التحويل الذي يمثل انسحاباً؟
Which translation represent a translation?

د) 	ج) 	ث) 	ت) 	
أفضل تسمية للشكل المجاور هي: what is the best discerption of the following shape? 				(11
د) متوازي أضلاع Parallelogram	ج) مستطيل Rectangle	ث) معين Rhombus	ت) مربع Square	
أي الجمل التالية **<u>ليست صحيحة</u>؟** which sentence **<u>is not correct</u>**?				(12
ث) المعين شكل رباعي The Rhombus is quadrilateral		ت) كل مربع مستطيل Every square is rectangle		
د) المثلث متطابق الأضلاع مثلث حاد الزوايا The equilateral triangle is an acute triangle		ج) شبه المنحرف هو متوازي أضلاع Trapezium is parallelogram		
يريد علي أن يحفظ جهاز حاسوبه المحمول في صندوق. إذا كانت قياسات الحاسوب 29سم في 34سم، فما نوع الشكل الرباعي الأفضل لحفظ الحاسوب؟ Ali wants to pack his lap top in a box. If the dimensions size of his lap top are 29cm x 34cm, what is the best quadrilateral box fit with Ali's lap top?				(13
د) شبه منحرف Trapezium	ج) مربع Square	ث) معين rhombus	ت) مستطيل rectangle	
القسم الثاني: إجابة مطولة Section two: open questions				
 في الشكل أعلاه: In the above figure				(14
....	حدد العلاقة بين الشارع جـ والشارع هـ؟ Identify the relationship between street C and street H?			أ)

....	حدد العلاقة بين الشارع ب والشارع أ ؟ Identify the relationship between street B and street A?	ب)
....	إذا مشيت من المدرسة إلى المكتبة ثم إلى مكتب البريد، فما نوع الزاوية التي تمر بها؟ If you walked from school to library then to post office, what would the type of angle that you make?	ج)
....	حدد العلاقة بين الشارع ب والشارع جـ؟ Identify the relationship between street C and street B?	د)
ارسم صورة الشكل بالانعكاس حول محور، ثم اكتب الأزواج المرتبة للرؤوس الجديدة؟ Reflect the shape around line, and name the new ordered pairs?		15)



Appendix (3.6) Delayed Test Final Copy

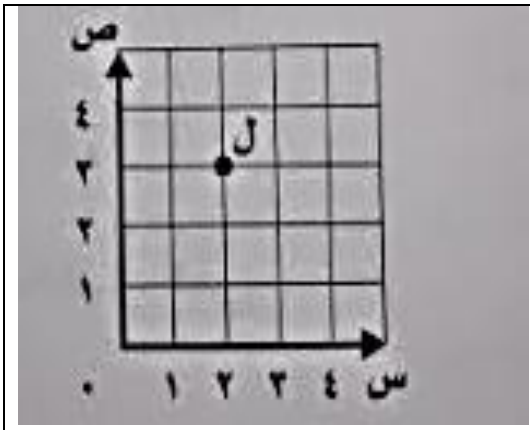
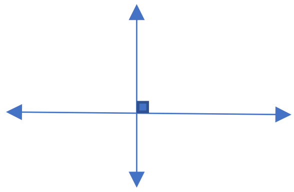
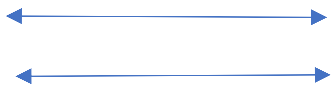
اختبار التحصيل الهندسي المؤجل

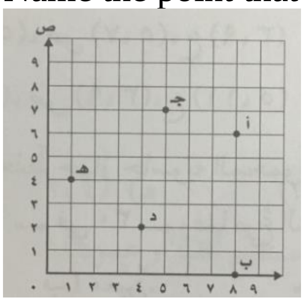
Delayed test

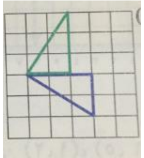
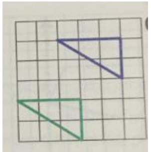
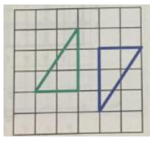
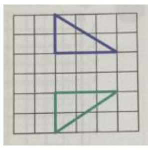
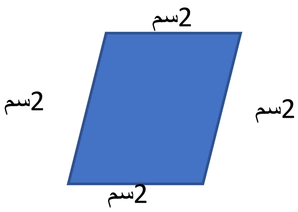
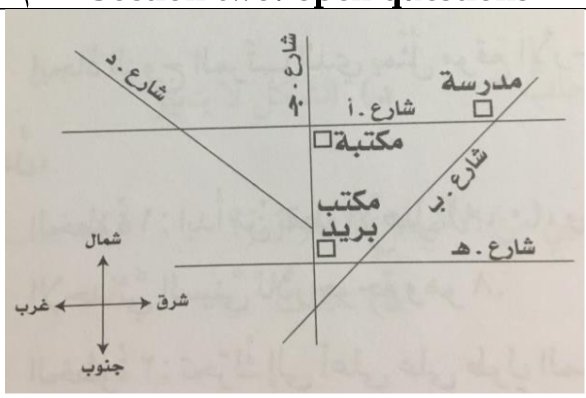
الاسمName/الصف..:Class/..........:

القسم الأول: اختر الاجابة الصحيحة فيما يلي: Section One: In following questions select the correct answer:

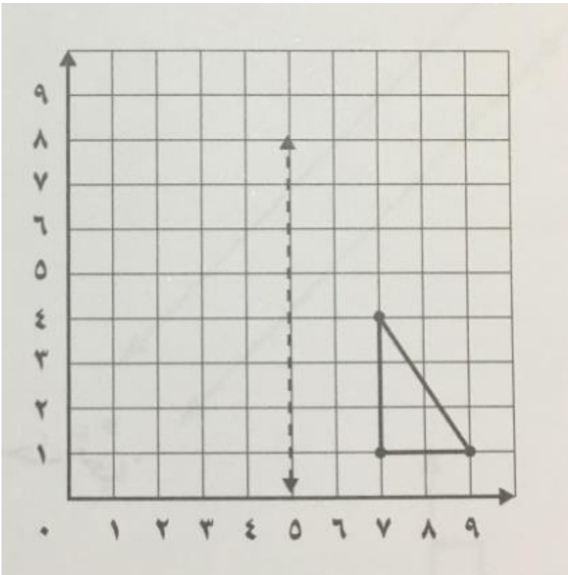
<table>
<tr><td colspan="4">مستقيمان بينهما مسافة ثابتة ولا يلتقيان مهما امتدا:
Two lines are always in the same distance and never touch</td><td>(1</td></tr>
<tr><td>د) متطابقان
congruent</td><td>ج) متوازيان
Parallel</td><td>متقاطعان ح)
intersect</td><td colspan="2">متعامدان
ج) Perpendicular</td></tr>
<tr><td colspan="4">تدوير الشكل حول نقطة دون تغيير قياساته أو نوعه:
Turning a shape around a point without changing its measurement and type</td><td>(2</td></tr>
<tr><td>انعكاس ذ)
Reflection</td><td>تكبير د)
Enlargement</td><td>انسحاب خ)
Translation</td><td colspan="2">دوران ح)
Rotation</td></tr>
<tr><td colspan="4">الشكل المجاور هو:
The following shape is:
</td><td>(3</td></tr>
<tr><td>د) معين
Rhombus</td><td>ج) مستطيل
Rectangle</td><td>شبه منحرف ح)
Trapezium</td><td colspan="2">متوازي أضلاع ج)
Parallelogram</td></tr>
<tr><td colspan="4">مجموعة من النقاط تشكل مستقيماً يمتد في الاتجاهين بلا نهاية:
A straight set of points that extended from both sides endlessly.</td><td>(4</td></tr>
<tr><td>د) المستوى
Plane</td><td>ج) نصف المستقيم
Half line</td><td>المستقيم ح)
Line</td><td colspan="2">القطعة المستقيمة ج)
Segment of line</td></tr>
<tr><td colspan="4">شكل رباعي فيه كل ضلعين متقابلين متطابقين وجميع زواياه قائمة:
A quadrilateral with four right angles and opposite sides that are parallel</td><td>(5</td></tr>
<tr><td>د) شبه منحرف
Trapezium</td><td>ج) معين
Rhombus</td><td>متوازي أضلاع ح)
parallelogram</td><td colspan="2">مستطيل ج)
Rectangle</td></tr>
<tr><td colspan="4">ما الزوج المرتب الذي يمثل النقطة (ل) في الشكل أدناه:
What is the ordered pair that represent (L) in the following figure?</td><td>(6</td></tr>
</table>

<table>
<tr><td colspan="4">
</td><td></td></tr>
<tr><td>(2،3) د)</td><td>(3،1) ج)</td><td>ح) (3،2)</td><td>ج) (1،3)</td><td></td></tr>
<tr><td colspan="4">المستقيمان في الشكل المجاور: The two lines in the following figure are
</td><td>(7</td></tr>
<tr><td>د) متوازيان
Parallel</td><td>ج) متقاطعان
Intersect</td><td>د) متعامدان
perpendicular</td><td>خ) متطابقان
congruent</td><td></td></tr>
<tr><td colspan="4">المستقيمان في الشكل المجاور:
The two lines in the following figure are
</td><td>(8</td></tr>
<tr><td>د) متطابقان
Congruent</td><td>ج) متقاطعان
Intersect</td><td>ث) متوازيان
parallel</td><td>ذ) متعامدان
perpendicular</td><td></td></tr>
<tr><td colspan="4">سم النقطة التي يمثلها الزوج المرتب (8,6)

Name the point that is represented by ordered pair (8,6)
</td><td>(9</td></tr>
<tr><td>د) النقطة ب
Point B</td><td>ج) النقطة د
Point D</td><td>ح) النقطة أ
Point A</td><td>ج) النقطة جـ
Point C</td><td></td></tr>
<tr><td colspan="4">ما التحويل الذي يمثل انسحاباً؟</td><td>(10</td></tr>
</table>

Which translation represent a translation?				
 د)	ر) 	ح) 	ج) 	
يريد علي أن يحفظ جهاز حاسوبه المحمول في صندوق. إذا كانت قياسات الحاسوب 29سم في 34سم، فما نوع الشكل الرباعي الأفضل لحفظ الحاسوب؟ Ali wants to pack his lap top in a box. If the dimensions size of his lap top are 29cm x 34cm, what is the best quadrilateral box fit with Ali's lap top?				(11
د) شبه منحرف Trapezium	ج) مستطيل Rectangle	ح) معين Rhombus	ج) مربع Square	
أي الجمل التالية **ليست صحيحة**؟ which sentence **is not correct**?				(12
ح) المعين شكل رباعي The Rhombus is quadrilateral		ج) كل مربع مستطيل Every square is rectangle		
د) المثلث متطابق الأضلاع مثلث حاد الزوايا The equilateral triangle is an acute triangle		ج) شبه المنحرف هو متوازي أضلاع Trapezium is parallelogram		
أفضل تسمية للشكل المجاور هي: what is the best discerption of the following shape? 				(13
د) متوازي أضلاع Parallelogram	ج) مربع Square	ح) معين rhombus	ج) مستطيل rectangle	
القسم الثاني: إجابة مطولة Section two: open questions				
 في الشكل أعلاه: In the above figure				(14
....	حدد العلاقة بين الشارع أ والشارع هـ؟ Identify the relationship between street C and street B?			أ)

....	حدد العلاقة بين الشارع ب والشارع أ ؟ Identify the relationship between street B and street A?	ب)
....	إذا مشيت من المدرسة إلى المكتبة ثم إلى مكتب البريد، فما نوع الزاوية التي تمر بها؟ If you walked from school to library then to post office, what would the type of angle that you make?	ج)
....	حدد العلاقة بين الشارع جـ والشارع هـ؟ Identify the relationship between street C and street H?	د)
ارسم صورة الشكل بالانعكاس حول محور، ثم اكتب الأزواج المرتبة للرؤوس الجديدة؟ Rotate the shape around line, and name the new ordered pairs?		15)



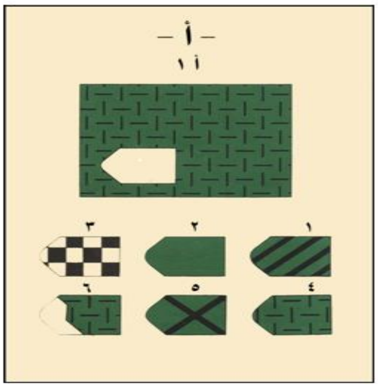
Appendix (3.7): Difficulty Coefficient of The Geometric Performance Test

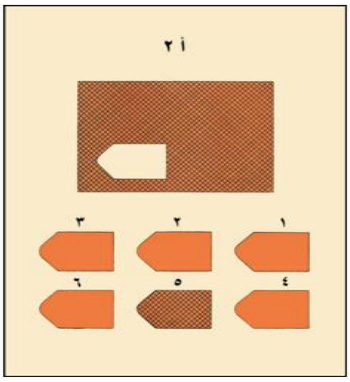
Question Number	Number of Correct Answer	Number of Incorrect Answer	Difficulty Coefficient
1	23	4	0.85
2	19	8	0.70
3	15	12	0.56
4	18	9	0.67
5	19	8	0.70
6	22	5	0.81
7	16	11	0.59
8	18	9	0.67
9	16	11	0.59
10	15	12	0.56
11	12	15	0.44
12	11	18	0.40
13	13	15	0.48
14 A	8	19	0.30
14 B	5	22	0.19
14 C	9	18	0.33
14 D	7	20	0.26
15	6	21	0.22
Total	252	234	0.52

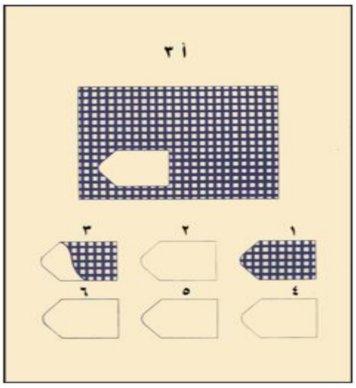
Appendix (3.8): Difficulty Coefficient of The Equivalent Form (Delayed Test)

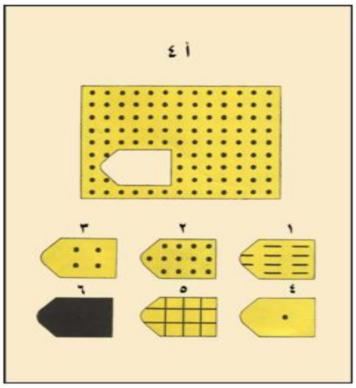
Question Number	**Number of Correct Answer**	**Number of Incorrect Answer**	**Difficulty Coefficient**
1	17	10	0.63
2	11	16	0.40
3	17	10	0.63
4	17	10	0.63
5	16	11	0.59
6	23	4	0.85
7	20	7	0.74
8	18	9	0.67
9	20	7	0.74
10	11	16	0.41
11	13	14	0.48
12	10	17	0.37
13	17	10	0.63
14 A	6	21	0.22
14 B	8	19	0.30
14 C	12	15	0.44
14 D	5	22	0.19
15	5	20	0.19
Total	246	240	0.51

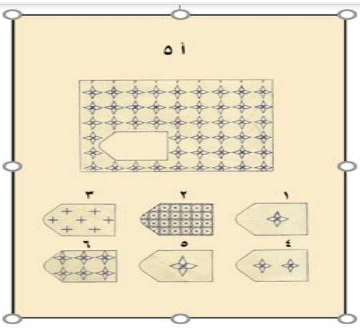
Appendix (3.9) Spatial Thinking Test

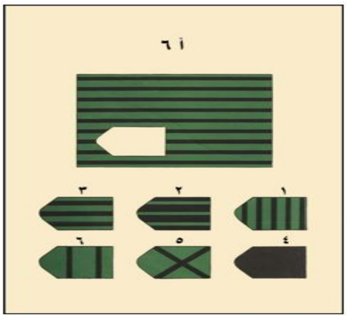


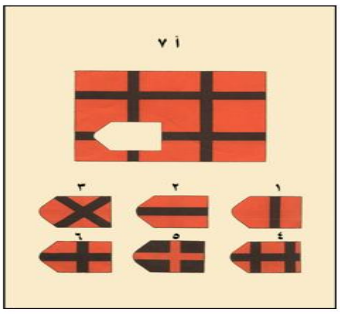


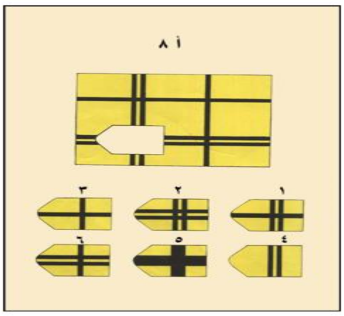


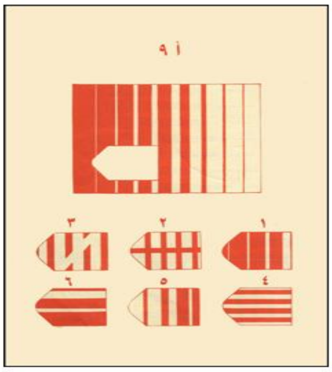








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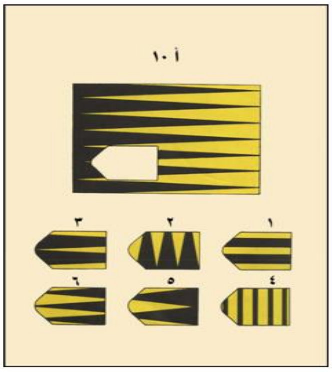
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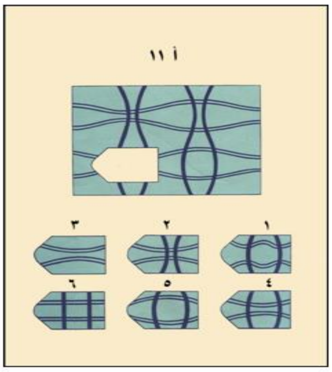
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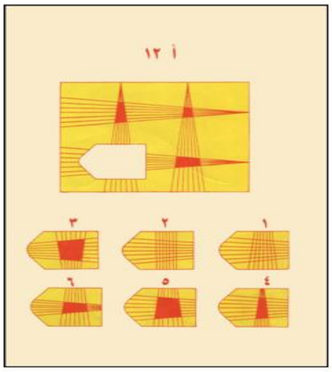
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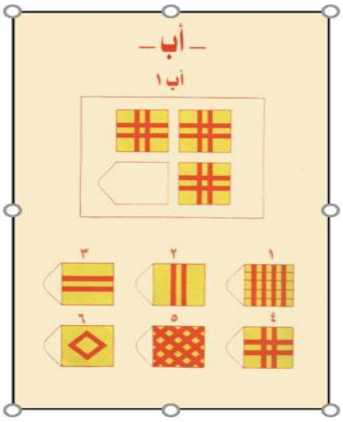
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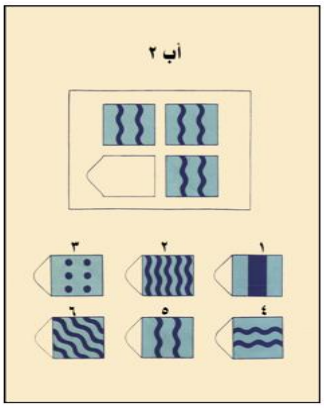
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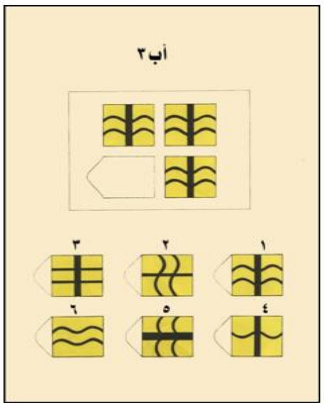
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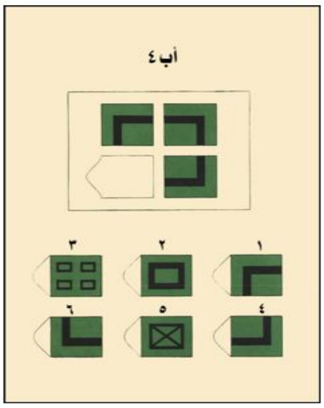
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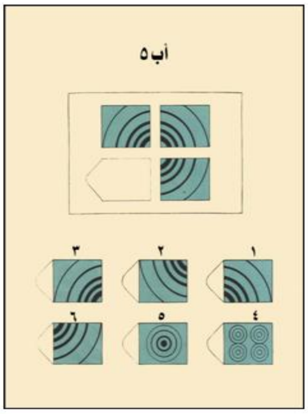
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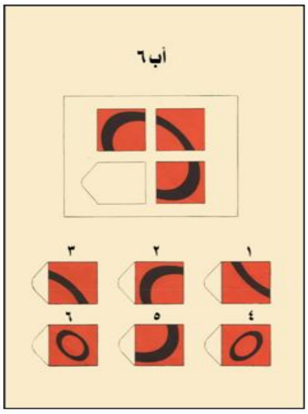
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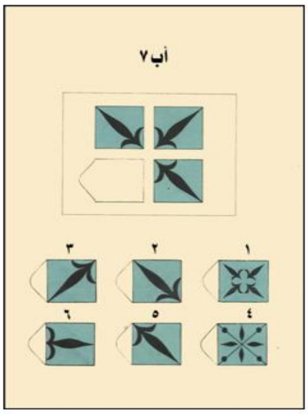
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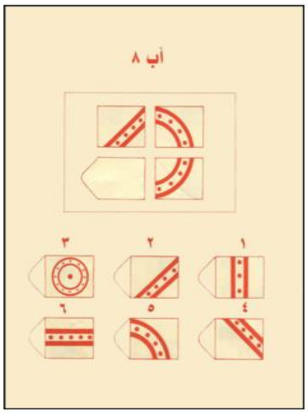
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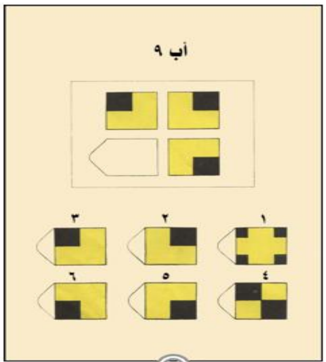
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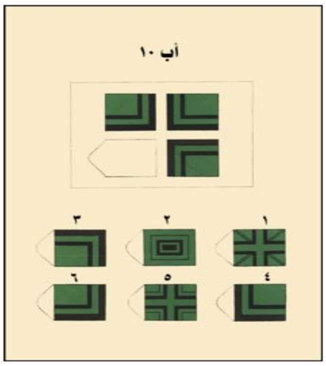
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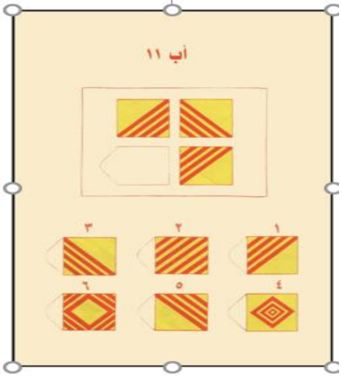
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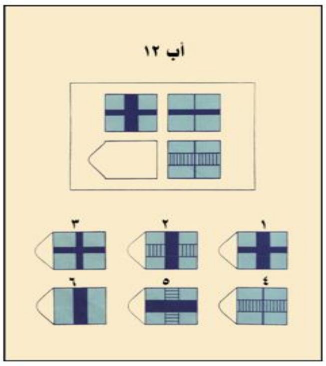
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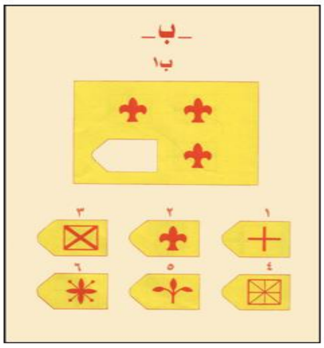
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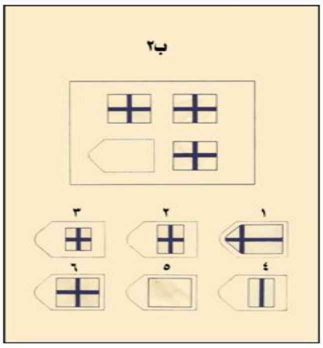
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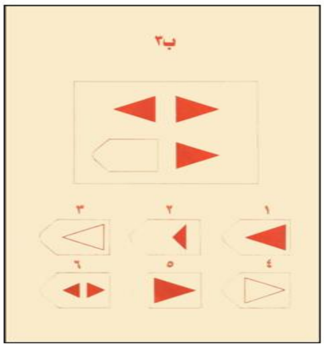
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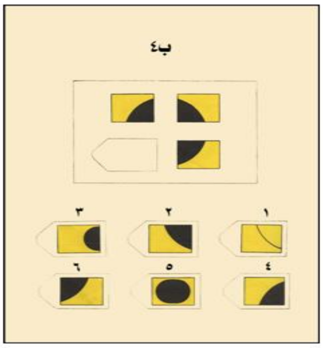
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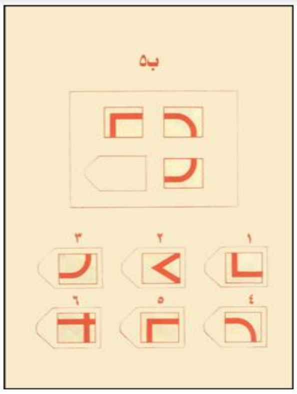
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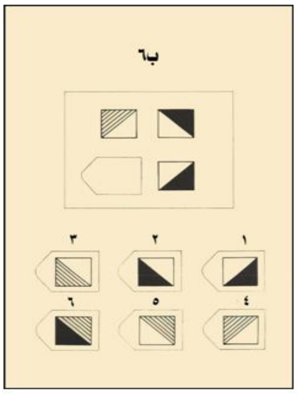
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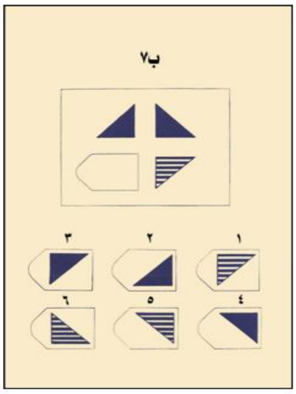
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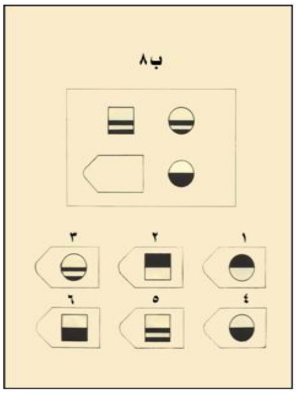
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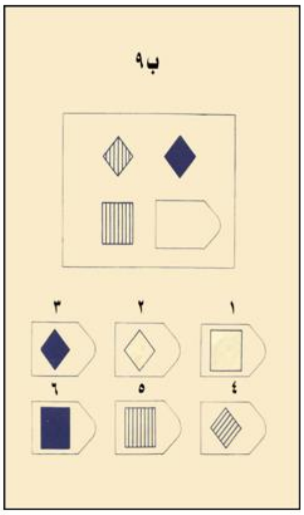
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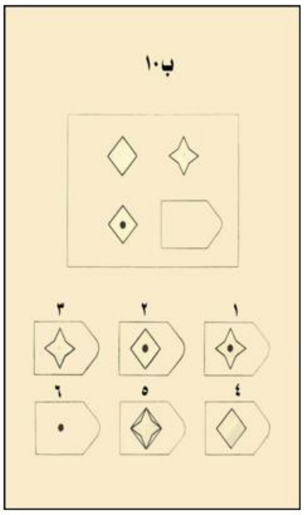
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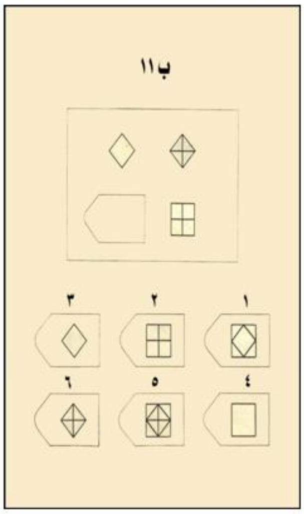
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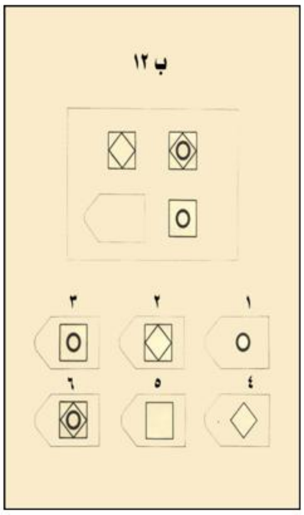
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ورقة إجابة اختبار المصفوفات المتتابعة الملونة لجون رافن Answer Sheet

الاسم	
الصف	
الرقم	

المجموعة أ						
	1	2	3	4	5	6
أ1						
أ2						
أ3						
أ4						
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أ6						
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أ12						

المجموعة أب						
	1	2	3	4	5	6
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أب2						
أب3						
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أب5						
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أب8						
أب9						
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أب11						
أب12						

المجموعة ب						
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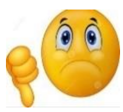
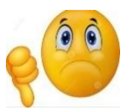
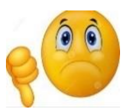
مجموع أ	مجموع أب	مجموع ب	الدرجة الكليه

Appendix (3.10) GeoGebra Visual Questionnaire Initial form

No.	Statement	Strongly agree	Agree	Neither agree nor disagree	Disagree	Strongly disagree
1	احب ان ادرس الهندسة باستعمال جيوجبرا I like studying geometry lessons with GeoGebra					
2	جيوجبرا ساعدني على تعلم المفاهيم الهندسية Geogebra helps me learn geometric concepts which were taught this week					
3	أفضل ان أتعلم باستعمال جيوجبرا اكثر من التعلم باستعمال الكتاب المدرسي I prefer lessons with Geogebra more than having to learn traditionally through text book					
4	ارغب في تعلم كل دروس الهندسة باستعمال جيوجبرا I would like to learn all geometry concept with GeoGebra					
5	دافعيتي للتعلم ارتفعت Geogebra has boosted my motivation					
6	استمتع في دروس الرياضيات عند استعمال جيوجبرا I enjoy in math lessons through GeoGebra					

7	اشعر بالثقة عند استعمال جيوجبرا اثناء أنشطة التعلم I feel confident when using GeoGebra to carry out the learning activities					
8	استعمال جيوجبرا جعلني أتعاون مع زملائي بسهولة Geogebra has enabled me to collaborate with classmate more easily					
9	جيوجبرا ساعدني على تكوين صور بصرية للإجابة على أسئلة أنشطة التعلم Geogebra has been fitted me in visualising possible answers needed to complete the required learning activities.					
10	جيوجبرا ساعدني للاندماج في أنشطة عملية التعلم Geogebra has engaged me in the learning process					
11	احب طريقة التدريس باستعمال جيوجبرا اكثر من طرق التدريس التقليدية I like how we are taught through GeoGebra more than traditional methods					
12	أنشطة التعلم ساعدتني على التفاعل مع زملائي The learning activities have helped me to interact with others more effectively					

13	استعمال جيوجبرا ساعدني على رفع مستواي الدراسي Using GeoGebra can help me to increase my performance of mathematics					
14	اركز اكثر داخل الصف عند استعمال جيوجبرا I concentrate better in the class when GeoGebra is used					
15	أنشطة التعلم باستعمال جيوجبرا جعلتني نشطا اكثر Learning activities with GeoGebra has made me more active in the class					
16	أنشطة التعلم باستعمال جيوجبرا ساعدتني على التعبير عن افكاري بشكل أفضل Learning activities with GeoGebra helped me to express my thoughts and ideas more effectively					
17	أنشطة التعلم باستعمال جيوجبرا ساعدتني على التفاعل مع زملائي والمعلم Learning activities with GeoGebra help me to interact with my classmates and my teacher more effectively					
18	أنشطة التعلم ساعدتني على مناقشة نواتج أنشطة التعلم مع الآخرين Learning activities help discuss the results of group work with the others					

19	اسأل معلمي عندما لا افهم شيء ما One I do not understand any questions, I am happy to ask the teacher					
20	ساعدتني على باستعمال جيوجبرا أنشطة التعلم العمل أكثر learning through GeoGebra has helped me become more active					
21	التعليم باستعمال جيوجبرا قد يساعدني على الاحتفاظ بمعلوماتي أكثر Using GeoGebra can help me to better retain the content that I have learned by such teaching method					
22	أنشطة التعلم باستعمال جيوجبرا ساعدتني على ربط الدروس السابقة مع الدروس الحديدة Learning activities with GeoGebra has helped in making connection between new learning and previous learning					
23	حملت برنامج جيوجبرا على جهازي في المنزل I have installed GeoGebra on my device					

Appendix (3.11) GeoGebra Visual Questionnaire After Developing Its language

غير موافق بشدة	غير موافق	لا اعرف	موافق	موافق بشدة	السؤال	رقم السؤال
					احب ان ادرس الهندسة باستعمال جيوجبرا I liked studying geometry lessons with GeoGebra	1
					جيوجبرا ساعدني على تعلم المفاهيم الهندسية Geogebra helps me to learn geometric concepts taught	2
					أفضل ان أتعلم باستعمال جيوجبرا اكثر من التعلم باستعمال الكتاب المدرسي I prefer lessons with Geogebra, not only with text book	3..
					ارغب في تعلم كل دروس الهندسة باستعمال جيوجبرا I want to learn all geometry concept with GeoGebra	4.
					....دافعيتي للتعلم ارتفعت رغبتي في التعلم ارتفعت My motivation has increased	5…
					استمتع في دروس الرياضيات عند استعمال جيوجبرا I enjoy in the lessons with GeoGebra	6
					اشعر بالثقة عند استعمال جيوجبرا اثناء أنشطة التعلم اشعر بالثقة عند استعمال جيوجبرا في حصة الرياضيات I feel confident when do the activities using GeoGebra	7…

					استعمال جيوجبرا جعلني أتعاون في الحل مع زملائي بسهولة Geogebra allows me collaborate with other easily	8
					جيوجبرا ساعدني على تكوين صور بصرية للإجابة على أسئلة أنشطة التعلم جيوجبرا ساعدني على التخيل والإجابة على أسئلة أنشطة التعلم Geogebra helps me to visualise and answering questions after each activity	9
					جيوجبرا ساعدني للاندماج في أنشطة عملية التعلم Geogebra helps me to engage in the learning process	10
					احب طريقة التدريس باستعمال جيوجبرا اكثر من طرق التدريس التقليدية I like this methods more than traditional method	11
					أنشطة التعلم ساعدتني على التفاعل مع زملائي Learning activities help me to interact with others	12
					استعمال جيوجبرا ساعدني على رفع مستواي الدراسي Using GeoGebra can help me to increase my performance in mathematics	13
					اركز اكثر داخل الصف عند استعمال جيوجبرا I concentrate better in the class when GeoGebra is used	14
					أنشطة التعلم باستعمال جيوجبرا جعلتني نشطا اكثر Learning activities with GeoGebra makes me more active	15

					أنشطة التعلم باستعمال جيوجبرا ساعدتني على التعبير عن افكاري بشكل أفضل Learning activities with GeoGebra helps me to express my thoughts and ideas better	16
					أنشطة التعلم باستعمال جيوجبرا ساعدتني على التفاعل مع زملائي والمعلم Learning activities with GeoGebra help me to interact with my classmates and my teacher	17
					أنشطة التعلم ساعدتني على مناقشة نواتج أنشطة التعلم مع الآخرين Learning activities help discuss the results of group work with others	18
					اسأل معلمي عندما لا افهم شيء ما I am happy to ask my teacher questions when I did not understand something	19
					ساعدتني على باستعمال جيوجبرا أنشطة التعلم العمل اكثر Learning activities with GeoGebra help me to work a lot	20
					التعليم باستعمال جيوجبرا قد يساعدني على الاحتفاظ بمعلوماتي اكثر Using GeoGebra can help me to retain better the content I have learned by using this teaching method	21
					أنشطة التعلم باستعمال جيوجبرا ساعدتني على ربط الدروس السابقة مع الدروس الحديدة Learning activities with GeoGebra help me to connect new learning and previous learning	22
					حملت برنامج جيوجبرا على جهازي في المنزل I installed GeoGebra on my device	23

Appendix (3.12) GeoGebra Visual Questionnaire Final Form

استبانة حول عملية التعلم باستعمال جيوجبرا

الاسم:...

الصف:..

التاريخ:...

عزيزي الطالب تهدف هذا الاستبانة الى التعرف على رأيك حول دراستك للهندسة باستعمال جيوجبرا.

ارجو منك قراءة التعليمات التالية قبل البدء في الاجابة على أسئلة الاستبانة:

- اقرأ كل فقرة جيدا ثم الاجابة باختيار الفيس الذي يعبر عن رأيك.
- تدل الفيسات على

✓ موافق بشدة 

✓ موافق 

✓ لا اعرف 

✓ غير موافق 

✓ غير موافق بشدة 

- اجب على اسئلة الاستبانة كما في المثال التالي:

السؤال	موافق بشدة	موافق	لا اعرف	غير موافق	غير موافق بشدة
أحب حصة الرياضيات	 √				

- يمكنك الان البدء بالإجابة على الاستبانة.

رقم السؤال	السؤال	موافق بشدة	موافق	لا اعرف	غير موافق	غير موافق بشدة
1	أحب أن ادرس الهندسة باستعمال جيوجبرا I like studying geometry lessons with GeoGebra					
2	ساعدني جيوجبرا على تعلم المفردات الهندسية Geogebra helps me learn geometric concepts					
3	أفضّل أن أحل التمارين باستعمال جيوجبرا أكثر من الحل باستعمال القلم والورقة I prefer lessons with Geogebra more than learn by pen and paper					
4	ارغب في تعلم كل دروس الهندسة باستعمال جيوجبرا I would like to learn all geometry concept with GeoGebra					
5	رغبتي في التعلم زادت Geogebra has boosted my motivation					
6	استمتع في دروس الرياضيات عند استعمال جيوجبرا I enjoy in math lessons when using GeoGebra					
7	اشعر بالثقة عند استعمال جيوجبرا في حصة الرياضيات I feel confident when using GeoGebra to carry out the learning activities					
8	استعمال جيوجبرا جعلني أتعاون في الحل مع زملائي بسهولة Geogebra has enabled me to collaborate with classmate more easily					
9	جيوجبرا ساعدني على التخيّل والإجابة على أسئلة أنشطة التعلم Geogebra help me to visualise the concept and answers learning task questions.					
10	أفضل اتعلم باستعمال جيوجبرا أكثر من طرق التدريس العادية I prefer learn mathematics using GeoGebra more than traditional teaching.					
11	أنشطة التعلم ساعدتني على المشاركة مع زملائي Learning tasks helped me to collaborate with me classmates.					

21	استعمال جيوجبرا ساعدني على زيادة مستواي الدراسي Using GeoGebra helped me to increase my performance					
31	أركز أكثر داخل الصف عند استعمال جيوجبرا I concentrate inside my class when I use GeoGebra					
14	أنشطة التعلم باستعمال جيوجبرا رفعت من نشاطي Learning tasks using Geogebra has made me more active in the class					
15	أنشطة التعلم باستعمال جيوجبرا ساعدتني على التعبير عن افكاري بشكل جيد Learning tasks with GeoGebra helped me to express my thoughts and ideas more effectively					
16	أنشطة التعلم باستعمال جيوجبرا ساعدتني على المشاركة مع زملائي والمعلم Learning tasks with GeoGebra help me to interact with my classmates and my teacher more effectively					
17	أنشطة التعلم ساعدتني على مناقشة نتائج التمارين والمسائل مع الآخرين Learning tasks help discuss the results of group work with the others					
18	اسأل معلمي عندما لا افهم شيئاً ما One I do not understand any questions, I am happy to ask the teacher					
19	ساعدتني أنشطة التعلم باستعمال جيوجبرا على العمل أكثر learning through GeoGebra has helped me become more active					
20	التعليم باستعمال جيوجبرا قد يساعدني على حفظ معلوماتي لمدة أطول Using GeoGebra can help me to better retain the content that I have learned by such teaching method					
21	أنشطة التعلم باستعمال جيوجبرا ساعدتني على ربط الدروس السابقة مع الدروس الجديدة Learning tasks with GeoGebra has helped in making connection between new learning and previous learning					

مع خالص تمنياتي لك بالتوفيق

Appendix (3.13) The Measurement of The Process of Learning Mathematics

No.	Question					
1	In general, I'm good at mathematics. بشكل عام أنا جيد في الرياضيات					
2	I'm pretty good at mathematics. أنا جيدجداً في الرياضيات					
3	I usually do well in mathematics. أنا عادة ما أحسن الأداء في الرياضيات.					
4	Not everyone can be gifted for every school subject. For mathematics, I'm not really gifted لا يمكن أن يكون الجميع موهوبين في كل المواد دراسية. بالنسبة للرياضيات، أنا لست موهوبًا حقًا.					
5	For some reason, I can't master mathematics لسبب ما، لا أستطيع إتقان مادة الرياضيات.					
6	Mathematics is not one of my strengths الرياضيات ليست من نقاط قوتي					
7	Mathematics is more difficult for me than for many of my classmates الرياضيات صعبة بالنسبة لي مقارنة بزملائي في الفصل					
8	I learn things quickly in mathematics اتعلم الاشياء بسرعة في مادة الرياضيات					
9	I would like a job that involved using mathematics اود الحصول على وظيفة تعتمد على استعمال الرياضيات					

10	Sometimes, when I do not initially understand a new topic in mathematics, I know that I will never really understand في بعض الأحيان، عندما لا أفهم الموضوع الجديد في مادة الرياضيات منذ البداية، أعلم أنني لن أفهمه لاحقا.					
11	Mathematics is boring. الرياضيات مادة مملة					
12	I find mathematics a pleasant school subject أجد الرياضيات مادة دراسية ممتعة					
13	Our lessons in mathematics are mostly fascinating and interesting معظم دروس الرياضيات رائعة ومثيرة للاهتمام.					
14	I'm sick of mathematics سئمت من مادة الرياضيات					
15	Especially for mathematics, I'm happy when class is over اشعر بالسعادة عندما تنتهي حصة الرياضيات خصوصاً					
16	I enjoy learning mathematics استمتع عند تعلم الرياضيات					
17	I would like to take more mathematics in school أود دراسة المزيد من الرياضيات في المدرسة					
18	I think for most occupations, mathematics is not useful الرياضيات ليست مفيدة، وكذلك التفكير في معظم المهن					
19	I believe mathematics has little use. أعتقد أن الرياضيات لها فائدة بسيطة					
20	Most of mathematics can be useful later on معظم دروس الرياضيات يمكن الاستفادة منها في المستقبل					

21	I think learning mathematics will help me in my daily life. اعتقد أن تعلم الرياضيات يساعدني في حياتي اليومية					
22	For a lot of things, occurring daily, mathematics is useful. تعتبر الرياضيات مفيدة بالنسبة لكثير من الاشياء التي تحدث يوميا.					
23	I need mathematics to learn other school subjects أحتاج إلى الرياضيات لتعلم المواد الدراسية الأخرى.					
24	I need to do well in mathematics to get the job I want يجب أن يكون مستواي في مادة الرياضيات مرتفعاً حتى احصل على الوظيفة التي أرديها					
25	I need to do well in mathematics to get into the university of my choice يجب أن يكون مستواي في مادة الرياضيات مرتفعاً حتى اتمكن الالتحاق بالجامعة التي اريدها					
26	To be good at mathematics, is a case of luck لتكون جيدا في الرياضيات، يجب أن تكون محظوظاً					

Appendix (3.14) GeoGebra Intervention Lessons

التعلم باستعمال جيوجبرا

Learning by using GeoGebra

مفردات هندسية(Geometric Concepts)

نشاط 1 (Task 1)

بالتعاون مع مجموعتك استعمل برنامج جيوجبرا لتنفيذ ما يلي:

- مثّل النقطة، ثم صفها؟

....

....

- ارسم خطاً مستقيماً، ثم صفه؟

....

....

- ارسم نصف مستقيم، ثم صفه؟

....

....

- ارسم قطعة مستقيمة، ثم صفها؟

....

....

- ما الفرق بين المستقيم، نصف المستقيم، القطعة المستقيمة؟

....

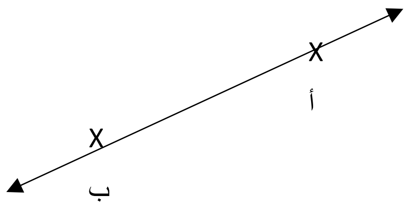
....

....

نشاط 2 (Task 2)

بالتعاون مع مجموعتك سمِ الأشكال التالية:

....



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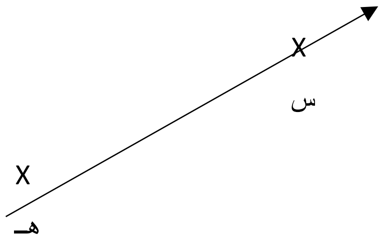

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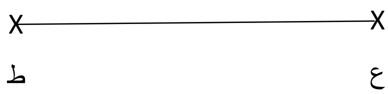

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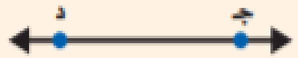
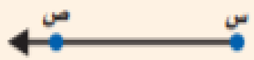
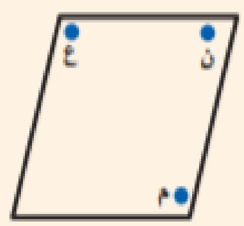
المفردات الهندسية

مفهوم أساسي

التَّعريفُ	النموذجُ
النُّقطةُ مَوقعٌ مُحدَّدٌ في الفضَاءِ وتُمثِّلها نُقطةٌ بالقَلمِ.	 **التعبيرُ اللفظيُّ:** النقطةُ أ
المُستقيمُ مَجموعةُ نُقَطٍ تُشكِّلُ مَسارًا مُستقيمًا يَمتدُّ في الاتجاهينِ دونَ نِهايةٍ.	 **التعبيرُ اللفظيُّ:** المُستقيم دجـ أو المُستقيم جـ د **بالرُّموزِ:** $\overleftrightarrow{\text{دجـ}}$ أو $\overleftrightarrow{\text{جـ د}}$
نصفُ المستقيمِ جُزءٌ من مُستقيمٍ له نُقطةُ بدايةٍ يَمتدُّ في أحد الاتجاهين دونَ نهايةٍ.	 **التعبيرُ اللفظيُّ:** نصف المستقيم س ص **بالرُّموزِ:** $\overleftarrow{\text{س ص}}$
القِطعةُ المُستقيمةُ جُزءٌ من مُستقيمٍ، لها نُقطةُ بِدايةٍ، ولها نُقطةُ نِهايةٍ.	 **التعبيرُ اللفظيُّ:** القطعة المستقيمة أب أو القطعة المستقيمة ب أ **بالرُّموزِ:** $\overline{\text{أب}}$ أو $\overline{\text{ب أ}}$
المُستوى هُو سَطحٌ مُنبسِطٌ يمتدُّ في جَميعِ الاتِّجاهاتِ دونَ نِهايةٍ.	 **التعبيرُ اللفظيُّ:** المُستوى ن م ع

النشاط 3 (Task 3) العلاقة بين المستقيمات

بالتعاون مع مجموعتك استعمل برنامج جيوجبرا ارسم ما يلي:

☜ مستقيمات تتقاطع في نقطة واحدة

☜ مستقيمات تشكل زاوية قائمة عند تقاطعها

☜ مستقيمات لا تتقاطع ابدا مهما امتدا.

☜ قطع مستقيمة لها نفس الطول

ثم أجب على ما يلي:

- ماذا يمكنك أن تسمي المستقيمات التي تتقاطع في نقطة واحدة؟

....

....

- ماذا يمكنك أن تسمي المستقيمات التي تتقاطع وتكون زاوية قائمة؟

....

....

- ماذا يمكنك أن تسمي المستقيمان اللذان لا تلتقيان أبدا مهما امتدا؟

....

....

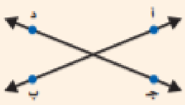
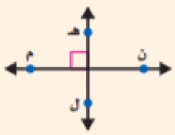
- ماذا تسمى القطع المستقيمة التي لها نفس الطول؟

....

....

يُمكنُ أن يَرتبطَ أَيُّ مُستقيمينِ مختلفينِ في المُستوى بإحدى ثلاثِ عَلاقاتٍ: التَّقاطعِ أو التَّعامدِ أو التَّوازي

أزواج المستقيمات
مفهوم أساسي

التَّعريفُ	النموذجُ
المُستقيمانِ المُتَقاطِعانِ مُستقيمانِ يَلتقِيانِ أو يَتَقاطَعانِ عندَ نُقطةٍ واحدةٍ فقط.	 **التعبيرُ اللفظيُّ:** المستقيمُ أ ب يتقاطع مع المستقيمِ جـ د **بالرُّموز:** $\overleftrightarrow{\text{أب}}$ يتقاطع مع $\overleftrightarrow{\text{جـد}}$
المُستقيمانِ المُتعامِدانِ مُستقيمانِ يَلتقِيانِ، فَيقطَعُ أَحدُهُما الآخرَ مُشكِّلًا زاويةً قائمةً.	 **التعبيرُ اللفظيُّ:** المستقيمُ هـ ل عمودي على المستقيمِ م ن **بالرُّموز:** $\overleftrightarrow{\text{هـل}} \perp \overleftrightarrow{\text{من}}$
المُستقيمانِ المُتوازِيانِ مُستقيمانِ بَينَهُما مَسافةٌ ثابتةٌ لا تساوي صفرًا ولا يَلتقِيانِ أو يَتقَاطَعانِ مَهما امتدَّا.	 **التعبيرُ اللفظيُّ:** المستقيمُ س ص يوازِي المستقيمَ ع ل **بالرُّموز:** $\overleftrightarrow{\text{سص}} \parallel \overleftrightarrow{\text{عل}}$

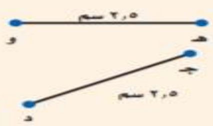
وتسمى المستقيمات التي لها نفس الطول

القطع المستقيمة المتطابقة
مفهوم أساسي

تُسمَّى القِطعُ المُستقيمةُ المُتساويةُ في طُولِها **قِطَعًا مُستقيمةً متطابقةً**.

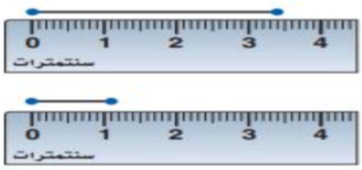
بالكلمات: $\overline{\text{هـو}}$ تطابق $\overline{\text{جـد}}$

بالرُّموز: $\overline{\text{هـو}} \cong \overline{\text{جـد}}$



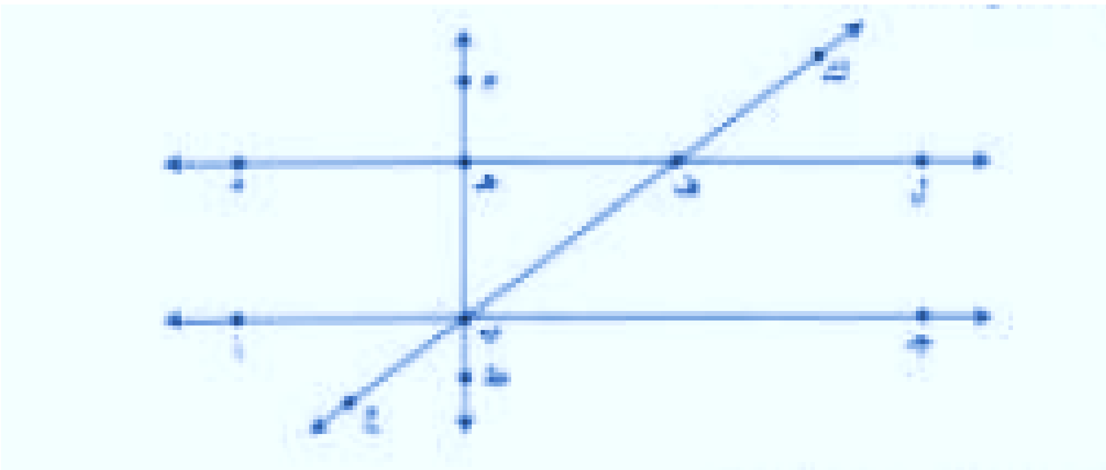
مثال: تعرُّف القطع المستقيمة المتطابقة

٣ **القياسُ:** بيِّنْ ما إذا كانت القِطْعتانِ المُستقيمتانِ في الشَّكلِ المُجاورِ مُتَطابِقتينِ أم لا.



بما أنَّ القِطعتينِ المستقيمتينِ غيرُ مُتساويتينِ في الطُّولِ، فهُما غيرُ مُتطابِقتينِ.

نشاط 4 (Task 4) فردي



استعمل الشكل أعلاه في حل ما يلي:

- سم زوجاً من المستقيمات المتوازية؟

....

- سم زوجاً من المستقيمات المتقاطعة؟

....

- سم زوجاً من المستقيمات المتعامدة؟

....

نشاط 5 (task 5) نشاط جماعي

اعط امثلة على العلاقات بين المستقيمات من واقع الحياة اليومية؟

تأكَّـدْ

سمِّ كُلَّ شَكْلٍ فيما يأتي، ثم عَبِّرْ عنه بالرُّموزِ: مثال ١

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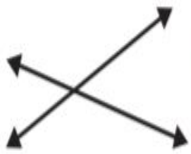
الدرس ١١-١: م

بَيِّنْ ما إذا كانَ المُستقيمانِ مُتقَاطِعينِ أو مُتعامِدينِ أو مُتوازيينِ فيما يأتي: مثال ٢

٤ 

٥ 

قِسْ طولَ كُلِّ قِطعةٍ مُستقيمةٍ، ثم بَيِّنْ ما إذا كانَت القِطعتانِ المُستَقيمتانِ مُتَطابِقتينِ أم لا. اكتبْ نعم أو لا: مثال ٣

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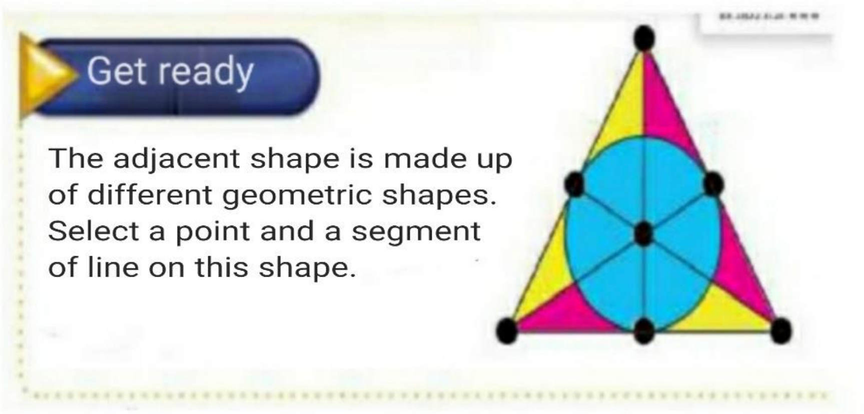
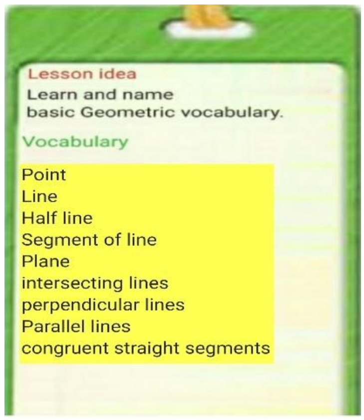
٨ ما نَوعُ الخَطَّينِ المُزْدوجينِ الظَّاهِرينِ في صورةِ الطَّريقِ؟ فَسِّرْ إجابَتَكَ.

٩ **تَحَدَّثْ** وَضِّح الفرقَ بينَ نِصْفِ المستقيمِ والمُستقيمِ.

(Geometric Concepts)





Tasks 1

Together with your group, use GeoGebra to do the following:

• Represents a point, then describe it?

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• Draw a line, then describe it?

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………………………………………………………………………………………

• Draw a half line, then describe it?

………………………………………………………………………………………

………………………………………………………………………………………

• Draw a line segment, then describe it?

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• What is the difference between a line, half a line, and a line segment?

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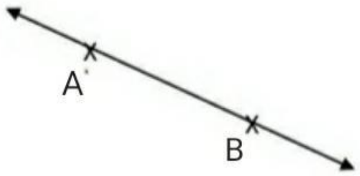
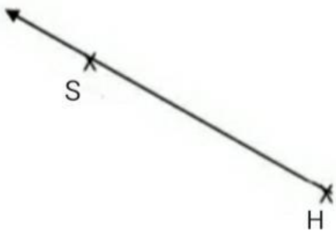
Tasks 2

Together with your group, name the following shapes:

....



....



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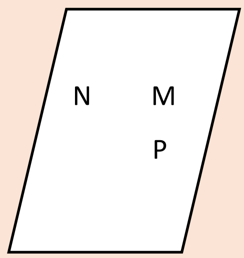
Geometric Vocabulary	Basic Concept
The definition	Sample
A point is a specific location in space and is represented by a point by a pen.	**A** verbal expression, point A
A line is a set of points that form a straight path that extends in both directions without end.	D C <--•-----------------•--> Verbal expression, line D C or straight line C D With symbols <-----> <-----> d c c d
A half line is a part of a straight line that has a starting point and extends in one direction without end.	X Y ---•----------------•--> Verbal expression, half- line x y With symbols, -----> X Y
A line segment is part of a line that has a starting point and an end point.	A B •--------------------• Verbal expression, line segment A B or line segment B A With symbols, ------ ------ A B or B A
A plane is a flat surface that extends in all directions without end.	Verbal expression Plane N M P 

Task 3 The relationship between the lines

Together with your group, use the Geogebra program to draw the following:

° Lines intersect at one point.

° Lines that form a right angle at their intersection.

° Lines never intersect, no matter how extended they are.

° Straight segments of the same length.

Then answer the following:

• What can you call the lines that intersect at one point?

……………………………………………………………………………………

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• What can you call the lines that intersect and form a right angle?

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• What can you name the two lines that never meet, no matter how extended they are?

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• What are straight segments of the same length called?

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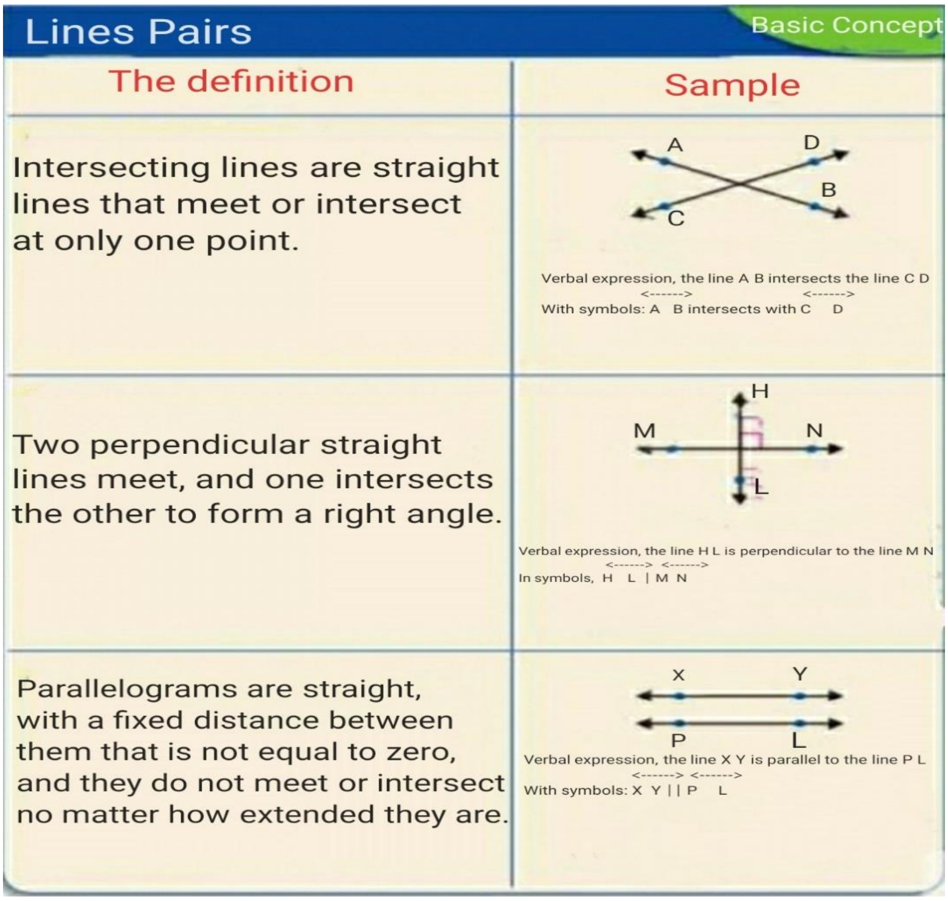
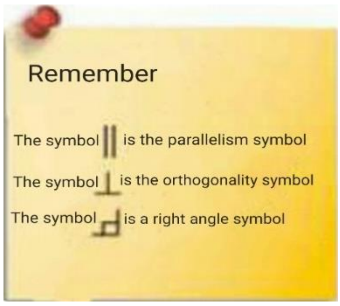
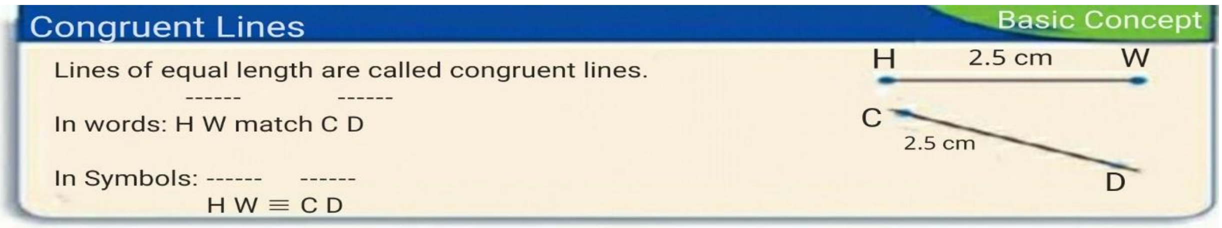
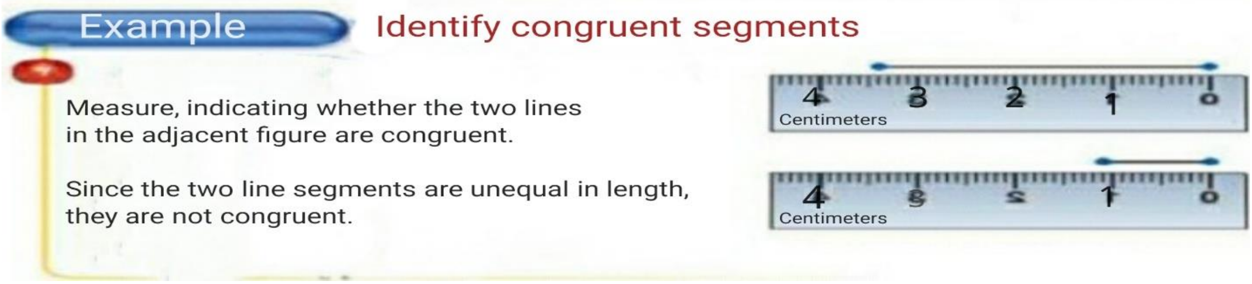
Any two straight lines in a plane can have one of three relationships: intersection, perpendicular, or parallelism



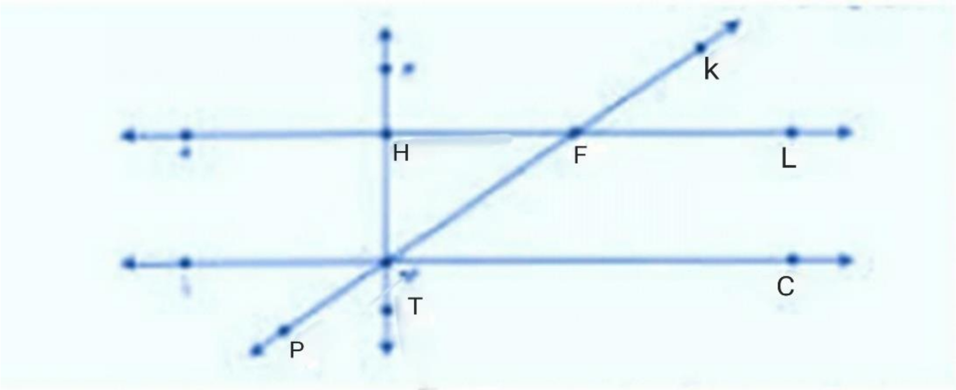


Lines with the same length called





Task 4 (Individual Task)



Use the above figure to solve the following:

- **Name a pair of parallel lines?**

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- **Name a pair of intersecting lines?**

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……………………………………………………………………………………

- **Name a pair of perpendicular lines?**

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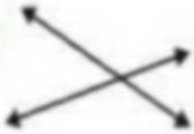
Task 5 (Whole class discussion)

Provide examples of relationships between lines from your real life?

BE SURE

Name each shape as follows, then express it with symbols:

Determine whether the lines are intersecting, perpendicular or parallel in the following:



Measure the length of each line segment, and then indicate whether the two line segments are congruent. Write yes or no:

What kind of double lines shown in the road picture? Explain your answer.

Speak Explain the difference between half line and a line.

الاشكال الرباعية

فِكْرَةُ الدَّرْسِ

أتعرفُ خصائصَ الأشكالِ الرُّباعيَّةِ.

المُفْرَدَاتُ

الشكلُ الرُّباعيُّ

المُستطيلُ

المُربعُ

مُتوازي الأضلاعِ

شِبهُ المُنحرِفِ

المَعيَّنُ

اسْتَعِدَّ

يَحتوي الشَّكلُ المُجاورُ على مُربَّعاتٍ ومُستطيلاتٍ، وتُعَدُّ المُربَّعاتُ والمُستطيلاتُ منَ الأشكالِ الرباعيَّةِ.

نشاط 6 (Task 6)

بالتعاون مع مجموعتك ارسم باستعمال جيوجبرا ثلاثة أشكال تمثل متوازي أضلاع وثلاثة أشكال لا تمثل متوازي أضلاع.

ثم أجب على الاسئلة التالية:

- كيف تتشابه الأشكال التي رسمتها؟

....

....

- كيف تختلف الأشكال التي رسمتها؟

....

....

- صف الزوايا في كل شكل؟

....

....

- تأمل الأشكال التي رسمتها، ثم اكتب تعريفاً للأشكال الرباعية؟

....

....

- تأمل الأشكال التي رسمتها، ثم فكر بطريقة لتصنيف الأشكال الرباعية؟

....

....

- ما الخاصية التي تنطبق على جميع متوازيات الاضلاع، ولا تنطبق على الأشكال الرباعية الأخرى؟

....

....

تعريف الشكل الرباعي:

الشَّكلُ الرُّباعِيُّ هو مُضَلَّعٌ له أَربعةُ أَضلاعٍ وأَربعُ زَوايا.

تصنيف الاشكال الرباعية:

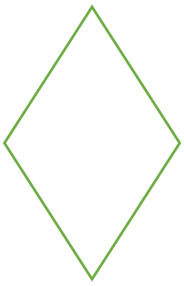
يُمكنُ تَصنيفُ الأَشكالِ الرُّباعيةِ وَفْقًا لِواحدةٍ أو أَكثرَ من الخَصائصِ التاليةِ:

- تَطابُقِ الأَضلاعِ
- تَوازي الأَضلاعِ
- تَعامُدِ الأَضلاعِ

نشاط 7

بالتعاون مع مجموعتك حدد أي الأشكال الرباعية التالية تمثل متوازيات أضلاع:

....



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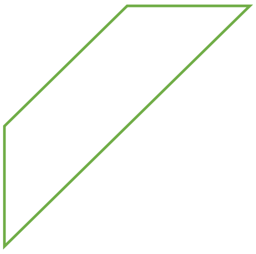

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نشاط 8

بالتعاون مع مجموعتك ارسم باستعمال جيوجبرا مربعا ومستطيلا. ثم أجب على الاسئلة التالية:

- استعمل ما تعرفه عن الزوايا والأضلاع لوصف كل شكل.
المربع:..
المستطيل:..
- ما الفرق بين المربع والمستطيل؟
....
....
- ما العلاقة بين المربع والمستطيل؟
....
....
- الآن، ارسم معيناً ثم استعمل ما تعرفه عن الزوايا والأضلاع لوصفه.
....
....
- ما الفرق بين المربع والمعين؟
....
....
- ما العلاقة بين المربع والمعين؟
....
....
- الآن، ارسم متوازي أضلاع ثم استعمل ما تعرفه عن الزوايا والأضلاع لوصفه.
....
....
- كيف تتشابه الأشكال الأربعة؟
....
....
....
....
- الآن، ارسم شبه منحرف ثم استعمل ما تعرفه عن الزوايا والأضلاع لوصفه.
....
....
- قارن بين المربع، المستطيل، المعين، متوازي الأضلاع، وشبه المنحرف؟
....
....
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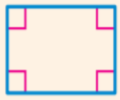
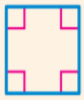
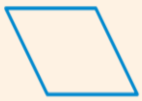
....
....

تصنيف الأشكال الرباعية

مفهوم أساسي

الشكل الرباعي	مثال	الخَصائِصُ
مُستطيلٌ		• كُلُّ ضِلعينِ مُتَقابِلينِ مُتَطابِقانِ. • جميعُ الزَّوايا قائِمةٌ. • كُلُّ ضِلْعينِ مُتَقابِلينِ مُتوازِيانِ.
مُربَّعٌ		• جَميعُ أَضلاعِه مُتَطابِقةٌ. • جَميعُ الزَّوايا قائِمةٌ. • كُلُّ ضِلْعينِ مُتَقابِلينِ مُتوازِيانِ.
مُتَوازي أَضلاعٍ		• كُلُّ ضِلْعينِ مُتقابِلينِ مُتطابِقانِ. • كُلُّ ضِلْعينِ مُتَقابِلينِ مُتوازِيانِ.
مَعينٌ		• جَميعُ أَضلاعِه مُتَطابِقةٌ. • كُلُّ ضِلْعينِ مُتَقابِلينِ مُتوازِيانِ.
شِبهُ مُنحرِفٍ		• ضِلْعانِ فَقطْ مِنْ أَضلاعِهِ المُتَقابلةِ مُتَوازِيانِ.

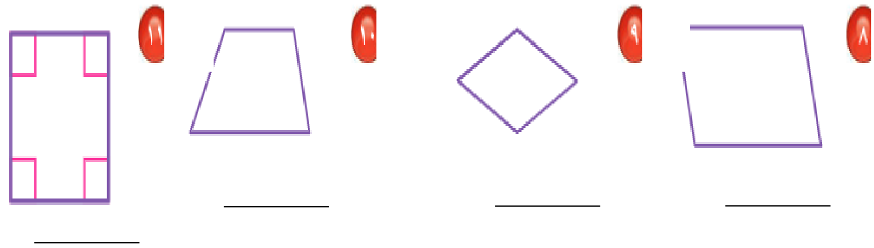
تَذَكَّر

إشارةُ المربعِ الصغيرةِ في زاويةِ الشكلِ تدلُّ على أنَّ الزاويةَ قائمةٌ.

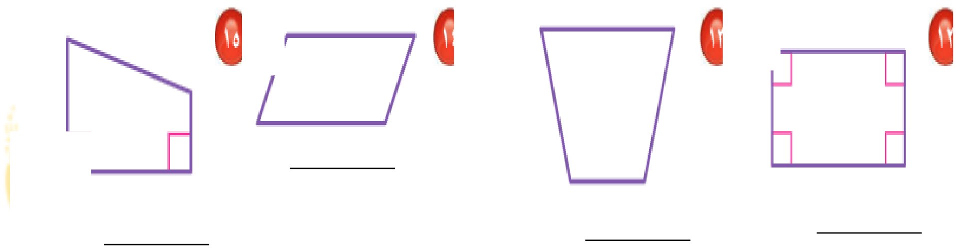
نشاط فردي

نشاط 9

صِفِ الأَضلاعَ التي تَبدو مُتطابِقةً في كُلِّ شَكْلٍ رُباعِيٍّ مِمَّا يأتي، ثم اذكُرْ ما إذا كانَ أَيٌّ من أَضلاعِها تَبدو مُتَوازِيَةً أو مُتعامِداً ل ١

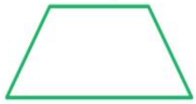


أَوجِدْ عَددَ الزَّوايا الحادَّةِ في كُلِّ شكلٍ رُباعِيٍّ مِمَّا يأ ٢



تأكَّـدْ

صِفِ الأَضلاعَ التي تَبدُو مُتَطابِقَةً في كُلِّ شكلٍ رُباعِيٍّ مِمَّا يأتي، ثم اذكُرْ ما إذا كانَ أَيٌّ مِن أَضلاعِه تَبدو مُتوازِيَةً أَو مُتَعامِدَةً:

١ 

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أَوجِدْ عَددَ الزَّوايا الحادَّةِ في كُلِّ شكلٍ رُباعِيٍّ مِمَّا يأتي:

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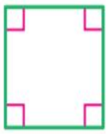
٥ 

٦ أَوجِدْ عَددَ الزَّوايا المُنْفَرِجةِ في كُلِّ شكلٍ مِمَّا يأتي:

٧ **تَحَدَّثْ** ما الفَرقُ بينَ المعيَّنِ وشِبهِ المُنْحرِفِ؟

الهندسة: الأزواج المرتبة


فِكرةُ الدَّرس
أُسمي النقاط في المستوى الإحداثيّ.
المُفرَدات
المُستوى الإحداثيُّ
نُقطةُ الأصلِ
الزوجُ المُرتَّبُ
الإحداثيُّ السينيُّ
الإحداثيُّ الصاديُّ

اسْتعِدَّ
عندما يَعودُ عبدُ الملكِ من المَدرسةِ إلى البيتِ، فإنَّهُ يمشي ٣ وحداتٍ إلى اليَمينِ و٥ وحداتٍ إلى أعلى، كيفَ يمشي عبدُ الملكِ من المدرسةِ إلى المَكتبةِ؟ وكيفَ يمشي إلى الحَديقةِ؟

نشاط 10

بالتعاون مع مجموعتك ارسم شكلاً رباعياً رؤوسه أ، **ب**، **جـ**، **د** مستعملا جيوجبرا، ثم اجب على الأسئلة التالية:

- ما هو الإحداثي؟

....

....

- كيف يمكنك إيجاد إحداثيات الرأس ب؟

....

....

- ما هي إحداثيات الرؤوس التي الشكل الذي رسمته؟

أ ....

ب ....

جـ ....

د ....

نشاط 11

بالتعاون مع مجموعتك مستعملا جيوجبرا مثل النقاط التالية على المستوى الإحداثي:

أ (6، 7)

ب (3،2)

جـ (0،5)

د (5،0)

- ما هو الاحداثي السيني للنقطة أ؟

....

- ما هو الإحداثي الصادي للنقطة ب؟

....

- هل النقطة جـ والنقطة د في نفس الموقع؟ ولماذا؟

....

....

- صف المستوى الإحداثي؟

....

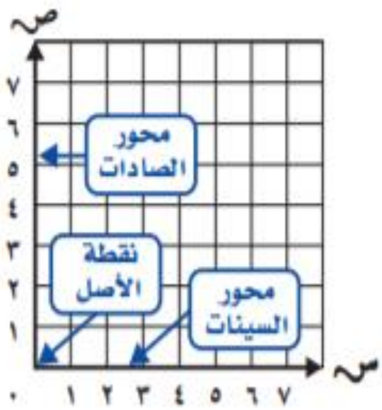
....

- صف الزوج المرتب؟

....

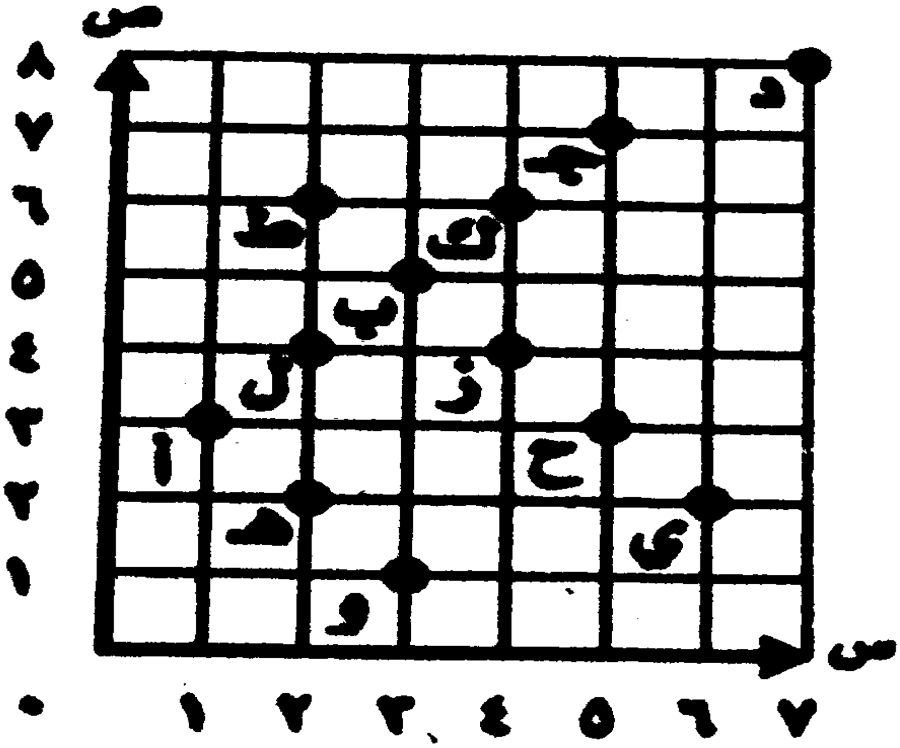
....

يتشكَّلُ **المستوى الإحداثيُّ** عندَ تَقاطعِ خطي أَعدادٍ. وتكونُ أعدادُ أحدِ خَطَّي الأَعدادِ على طُولِ المِحورِ الأُفقيِّ (محور السينات)، وتكونُ أَعدادُ الخَطِّ الثاني على طُولِ المِحورِ الرأْسِيِّ (محور الصادات)، أَما نُقطةُ التقاءِ المِحورينِ فَهيَ **نُقطةُ الأصلِ**.



الزوجُ المُرتَّبُ هو زَوجٌ من الأَعدادِ يُستعملُ لِتسميةِ نقطةٍ في المستوى الإِحداثيِّ.

نشاط 12



بالتعاون مع مجموعتك اجب عما يلي:

- اكتب الزوج المرتب لكل نقطة مما يأتي:

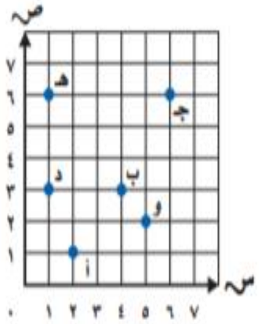
1	أ	4	د ....
2	ب	5	هـ ....
3	جـ	6	و.............

- سم النقطة التي يمثلها كل زوج مرتب مما يلي:

1	(3،5)	4	(6،4) ....
2	(4،4)	5	(4،2) ....
3	(6،2)	6	(2،6) ....

تأكَّدْ

سمِّ الزَّوجَ المُرتَّبَ لكلِّ نقطةٍ ممَّا يأتي:



١ أ	٢ جـ	٣ د

سمِّ النقطةَ التي يمثِّلُها الزوجُ المرتَّبُ:

٤ (٤،٣)	٥ (١،٦)	٦ (٥،٢)

٧ ارجعْ إلى المِثالِ ٣، واكتبِ الزوجَ المرتَّبَ الذي يُمثِّلُ موقعَ الخَاتمِ في المستوى الإحداثيِّ.

٨ تَحَدَّثْ هل تقعُ النُّقطتانِ (٣ ، ٨) ، (٨ ، ٣) في المَوقعِ نفسِهِ؟ برِّرْ إجابتَكَ.

فِكْرَةُ الدَّرْسِ

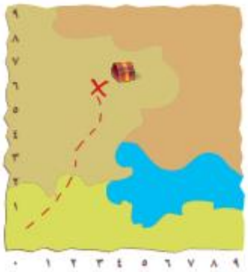
أُمثِّلُ نقاطًا في المستوى الإحداثيِّ.

المُفْرَدَاتُ

التَّمثيلُ

اسْتَعِدَّ

أَرادَ رائدٌ أنْ يصنعَ خريطةَ كنْزٍ لِلعْبةٍ كان يَلعبُها معَ أختِهِ، وقدْ قرَّرَ أنْ يكونَ الكنزُ على بُعدِ ٣ وحداتٍ يَمينًا وَ ٦ وحْداتٍ إلى أعلى، فَوضعَ علامةَ × عندَ تلكَ النقطةِ.



عِنْدَ **تمثيل** نقطةٍ في المستوى الإحداثيِّ نضعُ علامةً عندَ النقطةِ التي يُمثِّلُها الزوجُ المرتَّبُ المطلوبُ تمثيلُهُ.

نشاط 13

بالتعاون مع مجموعتك مستعملا جيوجبرا عين النقاط التالية على المستوى الإحداثي:

س (2،4)

ص (4،2)

ر (5،5)

- ما هي النقطة التي يمثلها الزوج المرتب (5،5)؟

....

- ما هي النقطة التي يمثلها الزوج المرتب (2،4)؟

....

يمكن كتابة مدخلات ومخرجات الدالة على صورة أزواج مرتبة

نشاط 14

بالتعاون مع مجموعتك مستعملا جيوجبرا أجب على الأسئلة التالية:

- إذا كان محمد أكبر من ياسر بـ 3 أعوام، استعمل قاعدة الدالة س + 3 لإيجاد عمر محمد عندما يصبح عمر ياسر 6، 7، 8، 9، 10. (استعمل جدول الدالة للحل)

- باستعمال قاعدة الدالة س + 3، ما عمر محمد عندما يصبح عمر ياسر 10 سنوات؟

- كيف أوجدت عمر محمد؟

- كيف ستكون الازواج المرتبة من خلال جدول الدالة؟

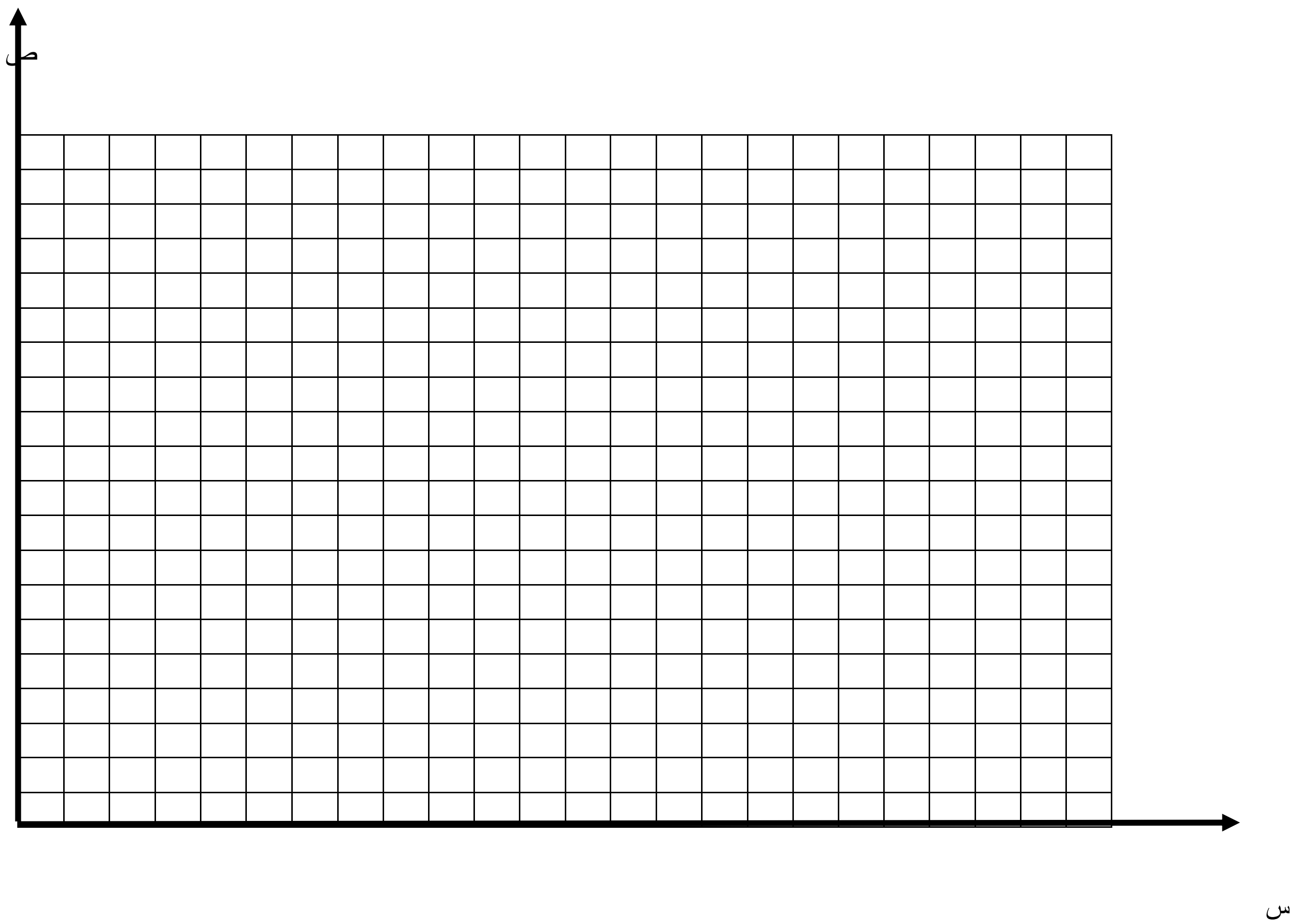
- اكتب الازواج المرتبة التي تمثل العلاقة بين عمر محمد وياسر؟

- مثل الدالة س+3 على المستوى الاحداثي؟

نشاط فردي

نشاط 15

استعمل قاعدة الدالة 9 – ع، وأوجد القيم عندما تكون قيمة ع: 2، 4، 6. كوّن جدولا للدالة، ثم مثلها على المستوى الإحداثي.



تأكد

مثّل كلَّ نقطةٍ ممّا يأتي في المستوى الإحداثيِّ، ثم سمِّها: مثال ١

١ ع (٢،٢)　　٢ س (٤، ٠)　　٣ ص (٥، ٦)

٤ جـ (٠، ٤)　　٥ ل (٧، ٦)　　٦ ب (٣، ٧)

٧ كيسُ حُبوبٍ وزْنُهُ ٥ كيلوجراماتٍ. استعملْ قاعدةَ الدالَّةِ ٥ح لإيجادِ مَجموعِ الأوزانِ في حالاتِ عددِ الأكياسِ: ٠، ١، ٢، ٣. مثال ٢

٨ تَحَدَّثْ وضِّحْ كيفَ تمثِّلُ النقطةَ ك (١٠، ٧) في المستوى الإحداثيِّ.

الانسحاب في المستوى الإحداثي

نشاط 16

بالتعاون مع مجموعتك اجلب صورة من ملف الصور إلى جيوجبرا، ثم اجب على الأسئلة التالية:

- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة؟
- استخدم امر الانسحاب في برنامج جيوجبرا لتحريك الصورة أربع وحدات إلى اليمين؟
- ماذا يحدث للصورة عند إزاحتها أربع وحدات إلى اليمين؟
- هل تغير شكل الصورة؟
- ماذا حدث للإحداثيات عند انسحاب الشكل أربع وحدات إلى اليمين؟
- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة بعد الانسحاب.
- حرك الصورة باستخدام الفأرة، ماذا تلاحظ؟

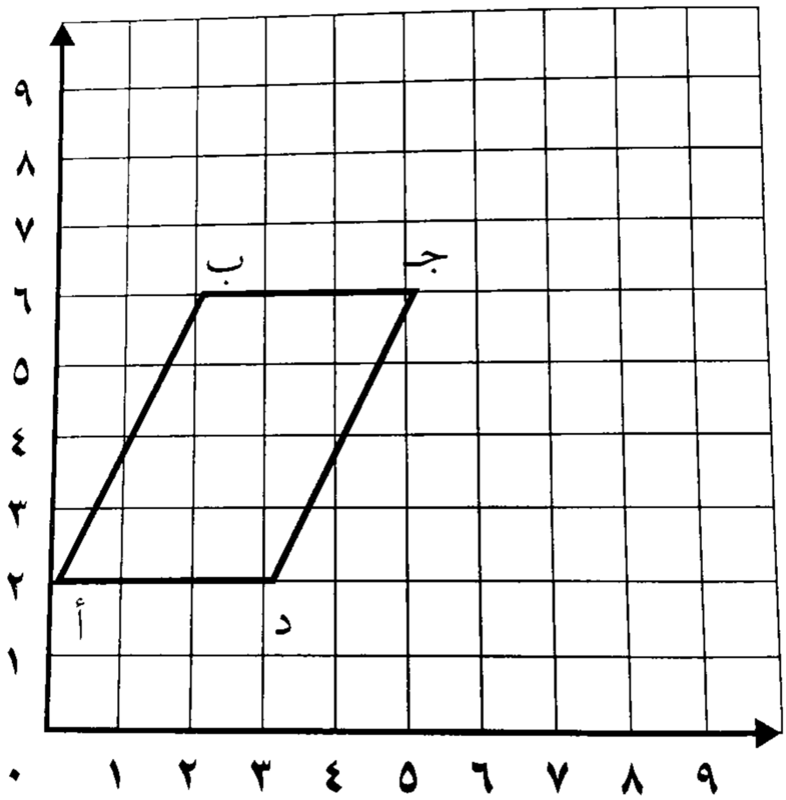
نشاط 17

بالتعاون مع مجموعتك ارسم المثلث أ (2،5)، ب (6،7)، جـ (4،9) مستعملاً جيوجبرا، ثم ارسم صورة انسحابه 3 وحدات إلى اليسار، واكتب الأزواج المرتبة لرؤوس الصورة.

نشاط فردي

نشاط 18

ارسم صورة الرباعي أ (0،2)، ب (2،6)، جـ (5،6)، د (3،2) بانسحاب 4 وحدات إلى اليمين، واكتب الأزواج المرتبة لرؤوس الصورة.



نشاط 19

ناقش مع مجموعتك ما يلي، ثم اكتب إجابتك في المكان المخصص:

- التحويل الهندسي:

....

....

....

....

....

- الانسحاب

....

....

....

....

....

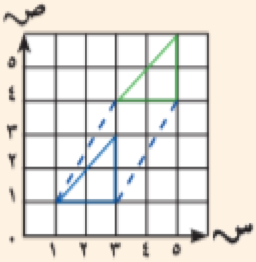
ملخص الدرس:

تُسمَّى حَركةُ الشَكلِ الهَنْدِسيِّ **تَحويلًا هَندسيًّا**، ويُسمَّى الشكلُ الناتجُ عن هذِه الحَركةِ **صُورةَ الشكلِ**. والانسِحابُ أحدُ أنواعِ التحويلاتِ الهَنْدسيَّةِ.

الانسحاب

مفهوم أساسي

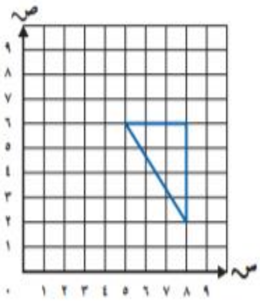
الانسِحابُ هو إزاحةُ شَكلٍ دونَ تَدويرِه، ولا يَنتجُ عنْ ذلكَ تَغيرٌ في قِياساتِه أو شكلِه.



لكَيْ تُجرِيَ انسحابًا لشكلٍ، حَرِّكْ جَميعَ رؤوسِهِ مَسافةً مُتساويةً في اتِّجاهٍ واحِدٍ.

تأكد

ارْسُمِ المُثلَّثَ بعدَ كُلِّ انسِحابٍ مِمَّا يأتي، ثم اكتُبِ الأَزواجَ المُرتبةَ لِرُؤوسِ الصورةِ: مثال ١



١ ٣ وحداتٍ إلى اليَسارِ.

٢ ٤ وحداتٍ إلى أَعلى.

٣ ٥ وحداتٍ إلى اليَسارِ ووحدتان إلى أَسفَلَ.

لحَلِّ المَسألتينِ ٤ ، ٥، ارسُمِ الشكلَ وصُورتَه بالانسِحابِ، وأكتُبِ الأَزواجَ المُرتَّبَةَ لِرُؤوسِ الصورةِ: مثال ١

٤ الشكلُ الرُّباعِيُّ أ (١،٥)، ب (٢،٨)، جـ (٤،٨)، د (٣، ٥) ؛ انسِحابُ ٥ وحداتٍ إلى اليَمينِ.

٥ المُثلثُ هـ (٧، ٢)، ل (٨، ٦)، ز (٩، ٣) ؛ انسِحابُ ٦ وحداتٍ إلى اليَسارِ وَوحدةٍ واحدةٍ إلى أَعلى.

٦ مَشتْ نجلاءُ ٦ أَمتارٍ غَربًا و ٤ أَمتارٍ شَمالًا. صِفْ هذا التَّحويلَ.

٧ **تَحَدَّثْ** وضِّحْ سَببَ تَسميةِ الانسِحابِ أَحيانًا بالإِزاحةِ.

الانعكاس في المستوى الإحداثي

نشاط 20

بالتعاون مع مجموعتك اجلب صورة من ملف الصور إلى جيوجبرا، ثم اجب على الأسئلة التالية:

- ارسم مستقيما على بعد ثلاث وحدات إلى يسار الصورة؟
- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة؟
- استخدم امر الانعكاس (تناظر محوري) في برنامج جيوجبرا لرسم الصورة بعد انعكاسها؟
- ماذا يحدث للصورة عند انعكاسها؟
- هل تغير شكل الصورة؟
- ماذا حدث للإحداثيات عند انعكاس الشكل؟

اكتب الأزواج المرتبة التي تمثل رؤوس الصورة بعد الانعكاس.

- هل الصورة المنعكسة متطابقة مع الصورة الأولى؟ ولماذا؟
- ماذا تلاحظ على المسافة بين الصورتين والخط المستقيم؟
- حرك الصورة باستخدام الفأرة، ماذا تلاحظ؟

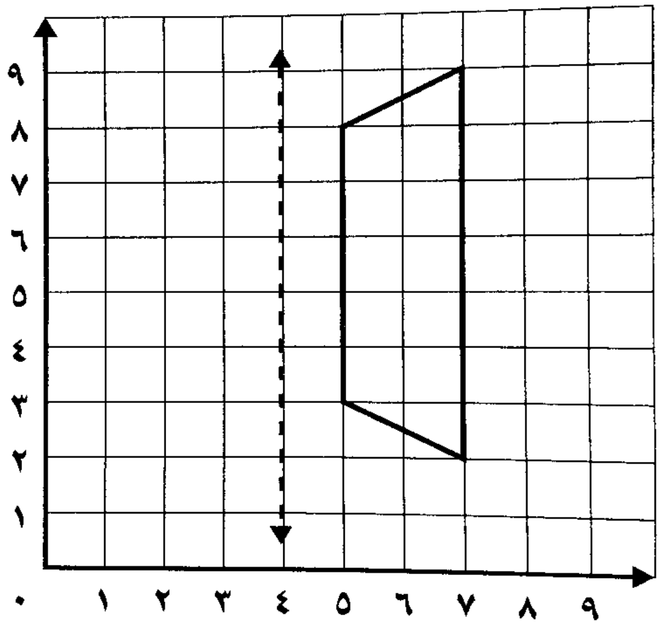
نشاط 21

مستعملاً جيوجبرا وبالتعاون مع مجموعتك ارسم صورة المثلث أ (2،5)، ب (6،7)، جـ (4،9) بالانعكاس حول المستقيم الرأسي الذي يمر بالنقطة 5 على المحور السيني، واكتب الأزواج المرتبة لرؤوس الصورة.

نشاط فردي

نشاط 22

ارسم صورة كل شكل مما يأتي بالانعكاس حول المحور، ثم اكتب الأزواج المرتبة للرؤوس الجديدة



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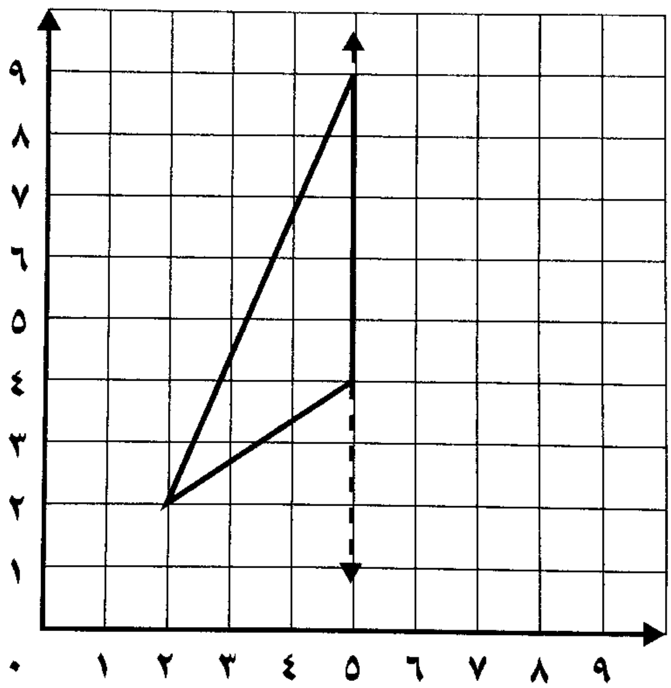


....

....

....

نشاط 23

ناقش مع مجموعتك ما يلي، ثم اكتب إجابتك في المكان المخصص:

- الانعكاس (Reflection):

....

....

....

....

....

- محور الانعكاس (Reflection axis):

....

....

....

....

....

- الفرق بين الانسحاب والانعكاس (Different between Translation and Reflection)

....

....

....

....

....

ملخص الدرس:

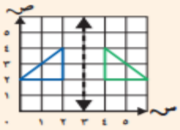
تَذكَّر

في الانعكاس، يُقلَبُ الشكلُ من مكانٍ إلى آخرَ دون تدويره. الانعكاسُ يُسمَّى أحيانًا قَلْبَ الشكلِ.

الانعِكاسُ هوَ تَحويلٌ هَنْدسِيٌّ آخَرُ لا يُغَيِّرُ من قِياساتِ الشكلِ أو نَوعِه.

الانعكاس — مفهوم أساسي

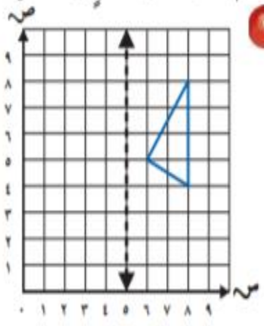
يُسمَّى قَلْبُ شَكلٍ هَندسِيٍّ حَولَ مُستقيم والحُصولُ على صُورةِ مِرآةٍ لهذا الشَّكلِ **انعِكاسًا**، ويُسمَّى المُستقيمُ **مِحورَ الانعِكاسِ**.

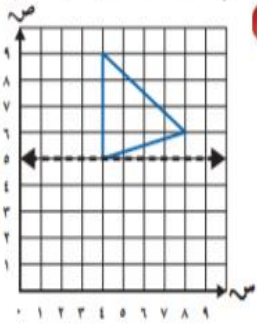


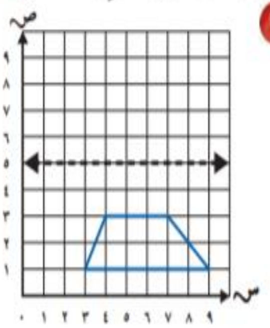
عندَ انعِكاسِ شَكلٍ حَولَ مُستقيمٍ تَكونُ الرُّؤوسُ المُتَناظِرةُ على مَسافةٍ مُتَساويةٍ مِن مِحورِ الانعِكاسِ.

تأكَّدْ

ارسُمْ صورةَ كُلِّ شَكلٍ مِمَّا يأتي بالانعكاسِ حَولَ المِحورِ، ثم اكْتُبِ الأَزواجَ المُرتَّبَةَ لِلرُّؤوسِ الجَديدةِ: مثال ١

١


٢


٣


٤ اذكرْ رقمًا لا يتغيرُ انعكاسهُ حولَ محورٍ عموديٍّ.

٥ تَحَدَّثْ ما أوجُهُ الشبهِ والاختِلافِ بينَ الانسِحابِ والانعِكاسِ؟

<u>**الدوران في المستوى الإحداثي**</u>

نشاط 24

بالتعاون مع مجموعتك اجلب صورة من ملف الصور إلى جيوجبرا، ثم اجب على الأسئلة التالية:

- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة؟
- استخدم امر الدوران في برنامج جيوجبرا لتدوير الصورة 90° باتجاه عقارب الساعة؟
- ماذا يحدث للصورة عند دورانها 90° باتجاه عقارب الساعة؟
- هل تغير شكل الصورة؟
- ماذا حدث للإحداثيات عند دوران الشكل 90° باتجاه عقارب الساعة؟
- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة بعد الدوران.
- حرك الصورة باستخدام الفأرة، ماذا تلاحظ؟

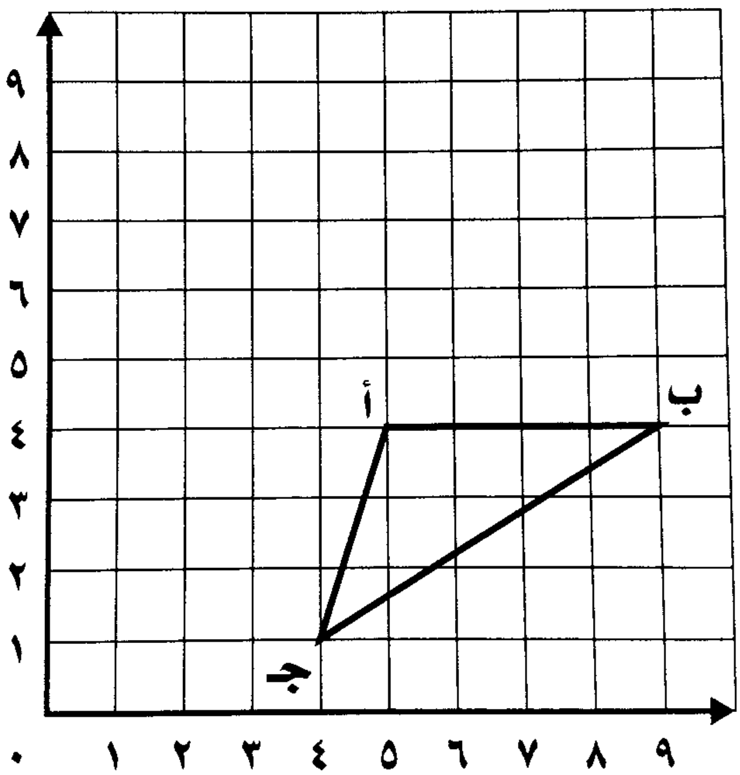
نشاط 25

بالتعاون مع مجموعتك ارسم المثلث أ (2،5)، ب (6،7)، جـ (4،9) مستعملاً جيوجبرا، ثم ارسم صورته بدوران 180° حول النقطة ب وباتجاه عكس عقارب الساعة، واكتب الأزواج المرتبة لرؤوس الصورة.

نشاط فردي

نشاط 26

ارسم صورة المثلث أ ب جـ بالدوران حول أ عكس عقارب الساعة، ثم اكتب الأزواج المرتبة للرؤوس الجديدة:



نشاط 27

ناقش مع مجموعتك ما يلي، ثم اكتب إجابتك في المكان المخصص:

- الدوران:

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- الفرق بين الدوران والانعكاس:

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- الفرق بين الانسحاب والدوران

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- العلاقة بين التحويل الهندسي والانسحاب، الانعكاس، الدوران

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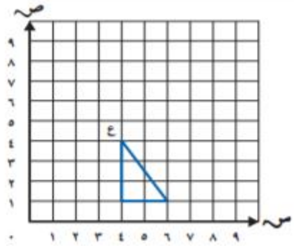
تأكَّدْ



ارسُمْ صورةَ المُثلَّثِ بالدورانِ حَولَ النقطةِ ع في كُلٍّ من الحالاتِ الآتيةِ، ثم اكتُبِ الأَزواجَ المُرتبةَ لِلرُّؤوسِ الجَديدةِ: مثال ١

١ ٩٠° باتجاهِ عَقاربِ الساعةِ.

٢ ١٨٠° بِعكسِ اتِّجاهِ عَقاربِ الساعةِ.

ارْسُمِ المُثلَّثَ المُعطاةَ رُؤوسُه، ثم ارسُمْ صورَتَه بالدورانِ المعْطَى في كلٍّ مما يأتي، ثم اكْتُبِ الأَزواجَ المُرتَّبةَ لِلرُّؤوسِ الجَديدةِ: مثال ١

٣ ك(٥،٥)، ل(٥، ٢)، م(١، ٥)؛ ٩٠° بِعكْسِ اتِّجاهِ عَقاربِ الساعةِ حَولَ النقطةِ ك.

٤ أ(٦، ٥)، ب(٦، ٩) جـ(٩، ٨)؛ ١٨٠° باتِّجاهِ عَقاربِ الساعةِ حَولَ النقطةِ أ.

٥ اذكُرْ رَقمينِ يُمثِّلُ كُلٌّ مِنهُما صورةَ الآخرِ بتحويلٍ هندسيٍّ، ثم سمِّ هذا التحويلَ.

٦ **تَحَدَّثْ** ما الفَرقُ بينَ الدَّوَرانِ والانعِكاسِ؟

Appendix (3.15) Hands-on Intervention Lessons

باستعمال اليدويات التعلم

Learning by using Hands-on Materials

مفردات هندسية(Geometric Concepts)

فِكْرَةُ الدَّرْسِ

أتعرفُ مُفرداتٍ هندسيةً أساسيةً وأسميها.

المُفرَدات

النقطة

المستقيم

نصف المستقيم

القطعة المستقيمة

المُستوى

المُستقيماتُ المُتقاطعةُ

المُستقيماتُ المُتعامِدةُ

المُستقيماتُ المُتوازيةُ

القِطَعُ المُستقيمةُ المُتطابِقةُ

اسْتعِدَّ

يتكونُ الشكلُ المُجاورُ من أَشكالٍ هَندسيَّةٍ مُختلفةٍ. حَدّدْ نُقطةً وقِطعةً مُستقيمةً على هذا الشكلِ.

نشاط 1 (Task 1)

بالتعاون مع مجموعتك استعمل المواد المتاحة أمامك لتنفيذ ما يلي:

- مثّل النقطة، ثم صفها؟

....

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- ارسم خطاً مستقيماً، ثم صفه؟

....

....

- ارسم نصف مستقيم، ثم صفه؟

....

....

- ارسم قطعة مستقيمة، ثم صفها؟

....

....

- ما الفرق بين المستقيم، نصف المستقيم، القطعة المستقيمة؟

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نشاط 2 (Task 2)

بالتعاون مع مجموعتك سمِ الأشكال التالية:

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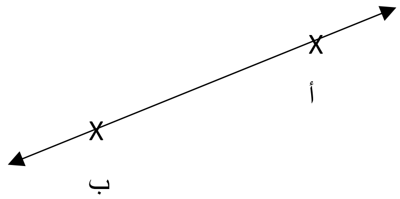

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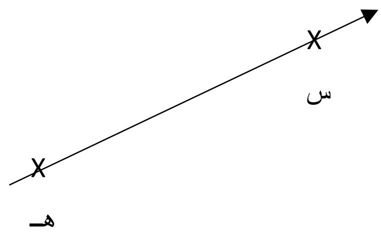

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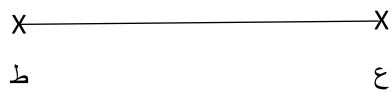
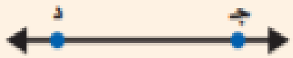
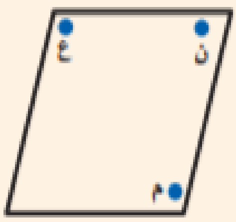


....

المفردات الهندسية

مفهوم أساسي

التَّعريفُ	النموذجُ
النُّقطةُ مَوقعٌ مُحدَّدٌ في الفضَاءِ وتُمثِّلها نُقطةٌ بالقَلمِ.	 **التعبيرُ اللفظيُّ:** النقطةُ أ
المُستقيمُ مَجموعةُ نُقَطٍ تُشكِّلُ مَسارًا مُستقيمًا يَمتدُّ في الاتجاهينِ دونَ نِهايةٍ.	 **التعبيرُ اللفظيُّ:** المُستقيم دجـ أو المُستقيم جـ د **بالرُّموزِ:** $\overleftrightarrow{\text{دجـ}}$ أو $\overleftrightarrow{\text{جـ د}}$
نصفُ المستقيمِ جُزءٌ من مُستقيمٍ له نُقطةُ بدايةٍ يَمتدُّ في أحدِ الاتجاهينِ دونَ نهايةٍ.	 **التعبيرُ اللفظيُّ:** نصف المستقيم س ص **بالرُّموزِ:** $\overleftarrow{\text{س ص}}$
القِطعةُ المُستقيمةُ جُزءٌ من مُستقيمٍ، لها نُقطةُ بِدايةٍ، ولها نُقطةُ نِهايةٍ.	 **التعبيرُ اللفظيُّ:** القطعة المستقيمة أب أو القطعة المستقيمة ب أ **بالرُّموزِ:** $\overline{\text{أب}}$ أو $\overline{\text{ب أ}}$
المُستوى هُو سَطحٌ مُنبسِطٌ يمتدُّ في جَميعِ الاتِّجاهاتِ دونَ نِهايةٍ.	 **التعبيرُ اللفظيُّ:** المُستوى ن م ع

النشاط 3 (Task 3) العلاقة بين المستقيمات

بالتعاون مع مجموعتك استعمل المواد المتاحة أمامك ارسم ما يلي:

☜ مستقيمات تتقاطع في نقطة واحدة

☜ مستقيمات تشكل زاوية قائمة عند تقاطعها

☜ مستقيمات لا تتقاطع ابدا مهما امتدا.

☜ قطع مستقيمة لها نفس الطول

ثم أجب على ما يلي:

- ماذا يمكنك أن تسمي المستقيمات التي تتقاطع في نقطة واحدة؟

....

....

- ماذا يمكنك أن تسمي المستقيمات التي تتقاطع وتكون زاوية قائمة؟

....

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- ماذا يمكنك أن تسمي المستقيمان اللذان لا تلتقيان أبدا مهما امتدا؟

....

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- ماذا تسمى القطع المستقيمة التي لها نفس الطول؟

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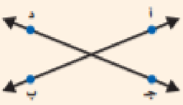
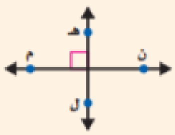
يُمكنُ أن يَرتبطَ أَيُّ مُستقيمينِ مختلفينِ في المُستوى بإحدى ثلاثِ عَلاقاتٍ: التَّقاطعِ أو التَّعامدِ أو التَّوازِي

أزواج المستقيمات

مفهوم أساسي

التَّعريفُ	النموذجُ
المُستقيمانِ المُتَقاطِعانِ مُستقيمانِ يَلتقيانِ أو يَتَقاطَعانِ عندَ نُقطةٍ واحدةٍ فقط.	 **التعبيرُ اللفظيُّ:** المستقيمُ أ ب يتقاطع مع المستقيم جـ د **بالرُّموز:** $\overleftrightarrow{\text{أب}}$ يتقاطع مع $\overleftrightarrow{\text{جـد}}$
المُستقيمانِ المُتعامِدانِ مُستقيمانِ يَلتقيانِ، فيقطَعُ أحدُهُما الآخرَ مُشكِّلًا زاويةً قائمةً.	 **التعبيرُ اللفظيُّ:** المستقيمُ هـ ل عمودي على المستقيم م ن **بالرُّموز:** $\overleftrightarrow{\text{هـل}} \perp \overleftrightarrow{\text{م ن}}$
المُستقيمانِ المُتوازِيانِ مُستقيمانِ بَينَهُما مَسافةٌ ثابتةٌ لا تساوي صفرًا ولا يَلتقيانِ أو يَتقاطعانِ مَهما امتدّا.	 **التعبيرُ اللفظيُّ:** المستقيمُ س ص يوازِي المستقيم ع ل **بالرُّموز:** $\overleftrightarrow{\text{س ص}} \parallel \overleftrightarrow{\text{ع ل}}$

تذكّر

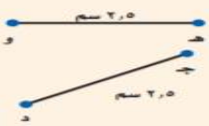
الرمز $\parallel$ هوَ رمزُ التوازي.
الرمزُ $\perp$ هوَ رمزُ التعامُد.
الرمزُ $\llcorner$ هو رمزُ زاويةٍ قائمة.

وتسمى المستقيمات التي لها نفس الطول

القطع المستقيمة المتطابقة

مفهوم أساسي

تُسمّى القِطعُ المُستقيمةُ المُتساويةُ في طُولِها **قِطَعًا مُستقيمةً متطابقةً.**

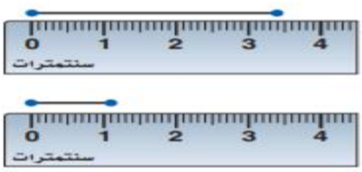


بالكلمات: $\overline{\text{هـو}}$ تطابق $\overline{\text{جـد}}$

بالرُّموز: $\overline{\text{هـو}} \cong \overline{\text{جـد}}$

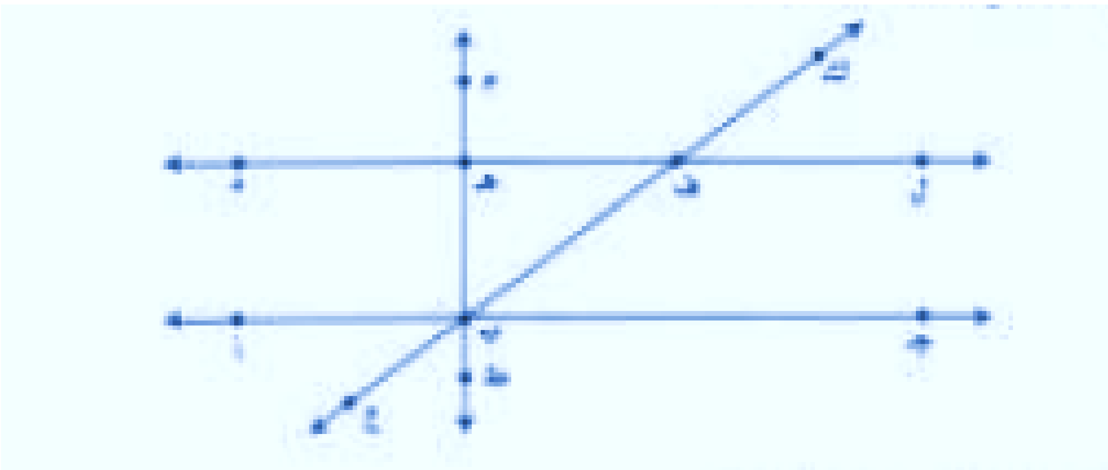
مثال: تعرُّفُ القطعِ المستقيمةِ المتطابقةِ

٣ **القياسُ:** بيِّنْ ما إذا كانت القِطْعتانِ المُستقيمتانِ في الشَّكلِ المُجاورِ مُتَطابِقتينِ أم لا.



بما أنَّ القِطعتينِ المستقيمتينِ غيرُ مُتساويتينِ في الطُّولِ، فهُما غيرُ مُتطابِقتينِ.

نشاط 4 (Task 4) فردي



استعمل الشكل أعلاه في حل ما يلي:

- سم زوجاً من المستقيمات المتوازية؟

....

- سم زوجاً من المستقيمات المتقاطعة؟

....

- سم زوجاً من المستقيمات المتعامدة؟

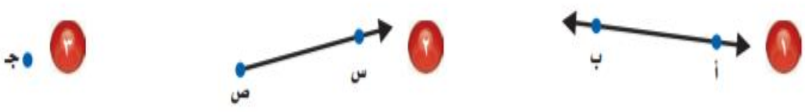
....

نشاط 5 (task 5) نشاط جماعي

اعط امثلة على العلاقات بين المستقيمات من واقع الحياة اليومية؟

تأكَّـدْ

سمِّ كُلَّ شَكْلٍ فيما يأتي، ثم عَبِّرْ عنه بالرُّموز: مثال ١



الدرس ١١-١: م

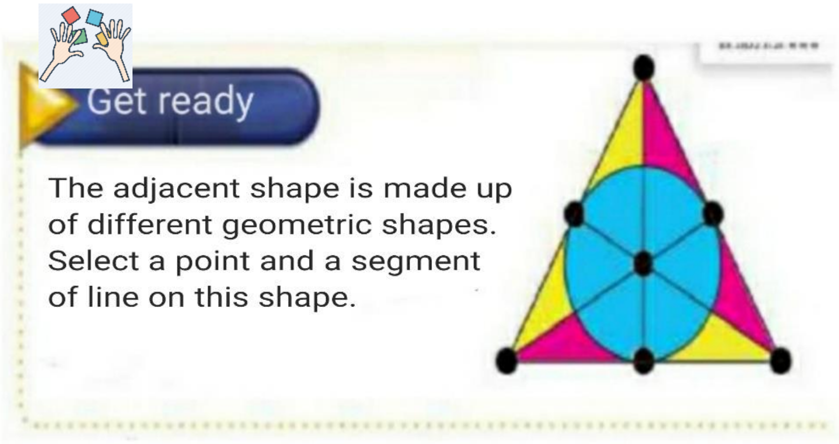
بَيِّنْ ما إذا كانَ المُستقيمانِ مُتقَاطِعينِ أو مُتعامِدينِ أو مُتوازيينِ فيما يأتي: مثال ٢

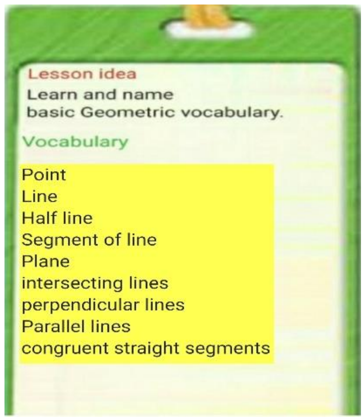
قِسْ طولَ كُلِّ قِطعةٍ مُستقيمةٍ، ثم بَيِّنْ ما إذا كانَت القِطعتانِ المُستَقيمتانِ مُتَطابِقتينِ أم لا. اكتبْ نعم أو لا: مثال ٣

٨ ما نَوعُ الخَطَّينِ المُزْدَوجينِ الظَّاهِرينِ في صورةِ الطَّريقِ؟ فَسِّرْ إجابَتَك.

٩ **تَحَدَّثْ** وَضِّح الفرقَ بينَ نِصْفِ المستقيمِ والمُستقيمِ.

(Geometric Concepts)





Tasks 1

Together with your group, use the available materials to do the following:

• Represents a point, then describe it?

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• Draw a line, then describe it?

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• Draw a half line, then describe it?

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• Draw a line segment, then describe it?

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• What is the difference between a line, half a line, and a line segment?

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Tasks 2

Together with your group, name the following shapes:

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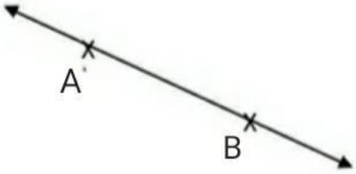

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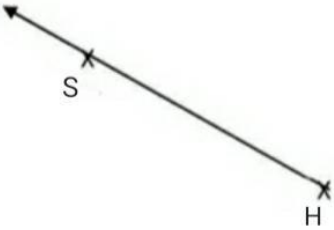

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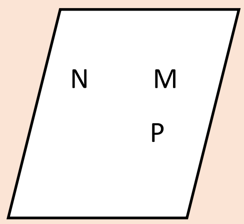
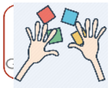
Geometric Vocabulary	Basic Concept
The definition	Sample
A point is a specific location in space and is represented by a point by a pen.	**A** verbal expression, point A
A line is a set of points that form a straight path that extends in both directions without end.	D C <--•-----------------•--> Verbal expression, line D C or straight line C D With symbols <-----> <-----> d c c d
A half line is a part of a straight line that has a starting point and extends in one direction without end.	X Y ---•---------------•--> Verbal expression, half- line x y With symbols, -----> X Y
A line segment is part of a line that has a starting point and an end point.	A B •--------------------• Verbal expression, line segment A B or line segment B A With symbols, ------ ------ A B or B A
A plane is a flat surface that extends in all directions without end.	Verbal expression Plane N M P 



Task 3 The relationship between the lines

Together with your group, use the available materials to draw the following:

° Lines intersect at one point.

° Lines that form a right angle at their intersection.

° Lines never intersect, no matter how extended they are.

° Straight segments of the same length.

Then answer the following:

• What can you call the lines that intersect at one point?

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• What can you call the lines that intersect and form a right angle?

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• What can you name the two lines that never meet, no matter how extended they are?

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• What are straight segments of the same length called?

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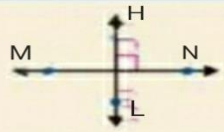
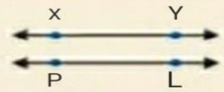
Any two straight lines in a plane can have one of three relationships: intersection, perpendicular, or parallelism

Lines Pairs

Basic Concept

The definition	Sample
Intersecting lines are straight lines that meet or intersect at only one point.	 Verbal expression, the line A B intersects the line C D With symbols: $\overleftrightarrow{AB}$ intersects with $\overleftrightarrow{CD}$
Two perpendicular straight lines meet, and one intersects the other to form a right angle.	 Verbal expression, the line H L is perpendicular to the line M N In symbols, $\overleftrightarrow{HL} \perp \overleftrightarrow{MN}$
Parallelograms are straight, with a fixed distance between them that is not equal to zero, and they do not meet or intersect no matter how extended they are.	 Verbal expression, the line X Y is parallel to the line P L With symbols: $\overleftrightarrow{XY} \parallel \overleftrightarrow{PL}$

Remember

The symbol $\parallel$ is the parallelism symbol

The symbol $\perp$ is the orthogonality symbol

The symbol ∟ is a right angle symbol

Lines with the same length called

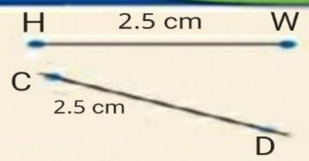
Congruent Lines

Basic Concept

Lines of equal length are called congruent lines.

In words: $\overline{HW}$ match $\overline{CD}$

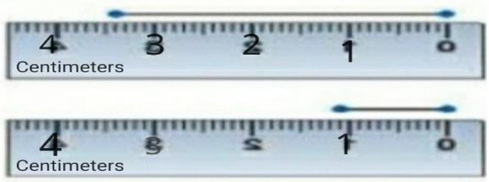
In Symbols: $\overline{HW} \equiv \overline{CD}$



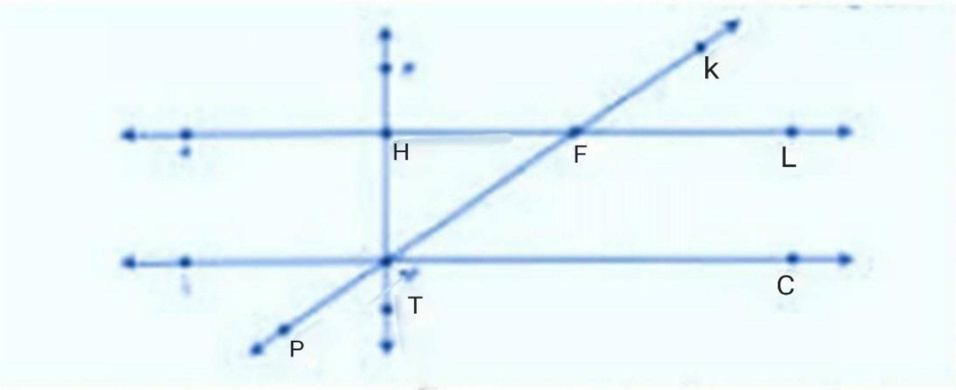
Example Identify congruent segments

Measure, indicating whether the two lines in the adjacent figure are congruent.

Since the two line segments are unequal in length, they are not congruent.



Task 4 (Individual Task)



Use the above figure to solve the following:

- **Name a pair of parallel lines?**

……………………………………………………………………………………

……………………………………………………………………………………

…………………………………………………………………………………

- **Name a pair of intersecting lines?**

……………………………………………………………………………………

……………………………………………………………………………………

…………………………………………………………………………………

- **Name a pair of perpendicular lines?**

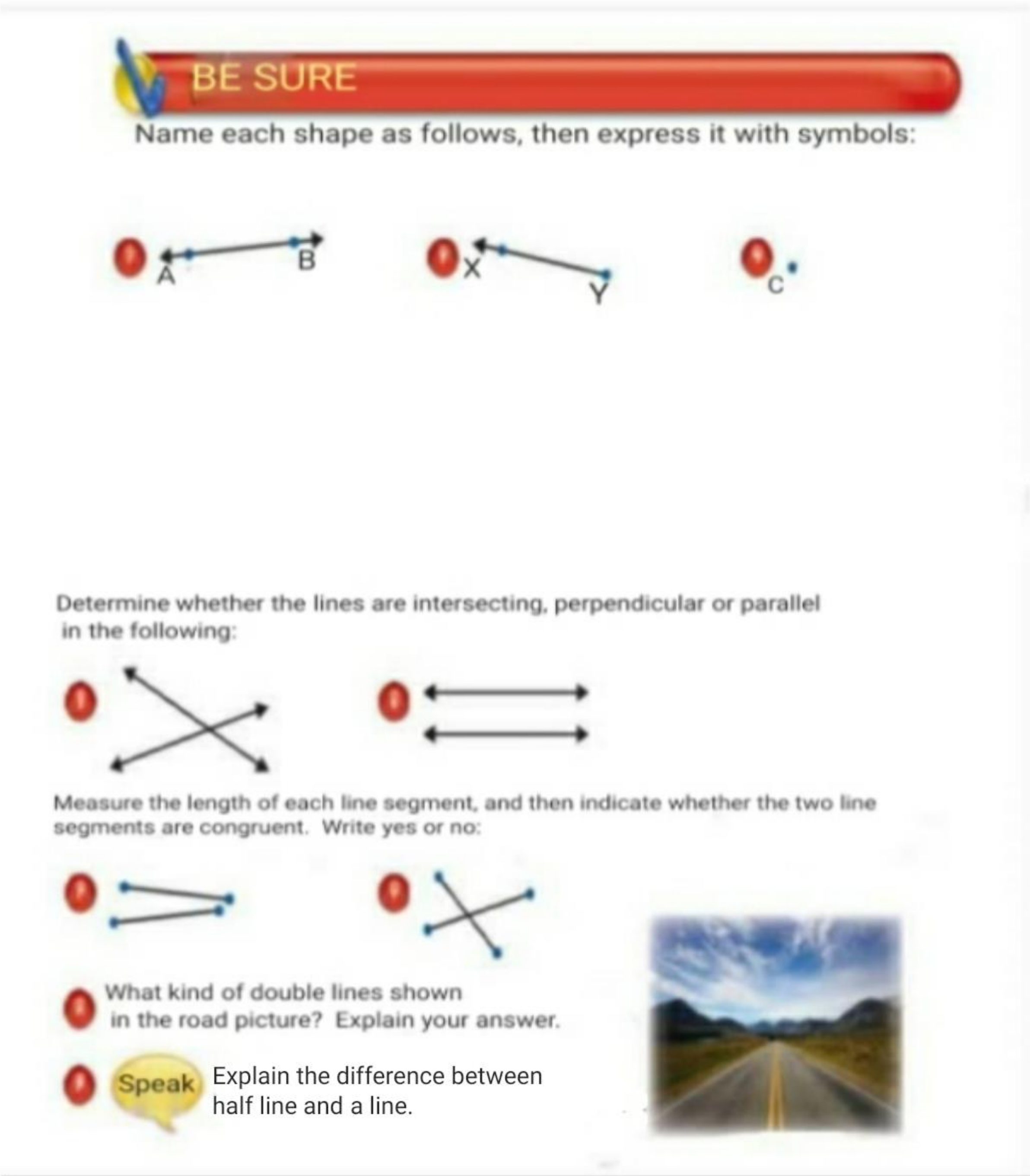
……………………………………………………………………………………

…………………………………………………………………………………

…………………………………………………………………………………

Task 5 (Whole class discussion)

Provide examples of relationships between lines from your real life?



BE SURE

Name each shape as follows, then express it with symbols:

A B

X Y

C

Determine whether the lines are intersecting, perpendicular or parallel in the following:

Measure the length of each line segment, and then indicate whether the two line segments are congruent. Write yes or no:

What kind of double lines shown in the road picture? Explain your answer.

Speak Explain the difference between half line and a line.

نشاط 6 (Task 6)

باستعمال الورق الملون والادوات المتاحة لديك وبالتعاون مع مجموعتك مثّل ثلاثة أشكال تمثل متوازي أضلاع وثلاثة أشكال لا تمثل متوازي أضلاع. ثم أجب على الاسئلة التالية:

- كيف تتشابه الأشكال التي رسمتها؟

....

....

- كيف تختلف الأشكال التي رسمتها؟

....

....

- صف الزوايا في كل شكل؟

....

....

- تأمل الأشكال التي رسمتها، ثم اكتب تعريفاً للأشكال الرباعية؟

....

....

- تأمل الأشكال التي رسمتها، ثم فكر بطريقة لتصنيف الأشكال الرباعية؟

....

....

- ما الخاصية التي تنطبق على جميع متوازيات الاضلاع، ولا تنطبق على الأشكال الرباعية الأخرى؟

....

....

تعريف الشكل الرباعي:

الشَّكلُ الرُّباعِيُّ هو مُضَلَّعٌ له أَربعةُ أَضلاعٍ وأَربعُ زَوايا.

تصنيف الاشكال الرباعية:

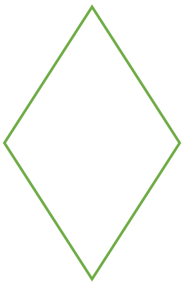
يُمكنُ تَصنيفُ الأَشكالِ الرُّباعيةِ وَفْقًا لِواحدةٍ أو أَكثرَ من الخَصائصِ التاليةِ:

- تطابُقِ الأَضلاعِ
- توازي الأَضلاعِ
- تَعامُدِ الأَضلاعِ

نشاط 7

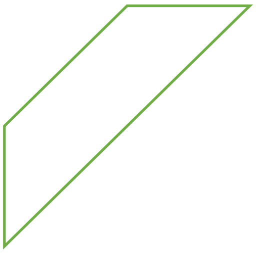
بالتعاون مع مجموعتك حدد أي الأشكال الرباعية التالية تمثل متوازيات أضلاع:

……………..……………



……………………………….

………………………



……………………………..

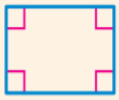
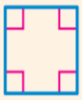
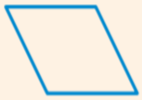
نشاط 8

بالتعاون مع مجموعتك صمّم باستعمال الأدوات المتاحة لديك مربعا ومستطيلا. ثم أجب على الاسئلة التالية:

- استعمل ما تعرفه عن الزوايا والأضلاع لوصف كل شكل.
  المربع:..
  المستطيل:..
- ما الفرق بين المربع والمستطيل؟
  ....
  ....
- ما العلاقة بين المربع والمستطيل؟
  ....
  ....
- الآن، ارسم معيناً ثم استعمل ما تعرفه عن الزوايا والأضلاع لوصفه.
  ....
  ....
- ما الفرق بين المربع والمعين؟
  ....
  ....
- ما العلاقة بين المربع والمعين؟
  ....
  ....
- الآن، ارسم متوازي أضلاع ثم استعمل ما تعرفه عن الزوايا والأضلاع لوصفه.
  ....
  ....
- كيف تتشابه الأشكال الأربعة؟
  ....
  ....
  ....
  ....
- الآن، ارسم شبه منحرف ثم استعمل ما تعرفه عن الزوايا والأضلاع لوصفه.
  ....
  ....
- قارن بين المربع، المستطيل، المعين، متوازي الأضلاع، وشبه المنحرف؟
  ....
  ....
  ....
  ....
  ....
  ....

تصنيف الأشكال الرباعية

مفهوم أساسي

الشكل الرباعي	مثال	الخَصائِصُ
مُستطيلٌ		• كُلُّ ضِلعين مُتَقابِلين مُتَطابِقانِ. • جميعُ الزَّوايا قائِمةٌ. • كُلُّ ضِلْعين مُتَقابِلين مُتَوازِيانِ.
مُربَّعٌ		• جَميعُ أضلاعِهِ مُتَطابِقةٌ. • جَميعُ الزَّوايا قائِمةٌ. • كُلُّ ضِلْعين مُتَقابِلين مُتَوازِيانِ.
مُتَوازي أَضلاعٍ		• كُلُّ ضِلْعين مُتقابِلين مُتطابِقانِ. • كُلُّ ضِلْعين مُتقابِلين مُتَوازِيانِ.
معينٌ		• جَميعُ أَضلاعِهِ مُتَطابِقةٌ. • كُلُّ ضِلْعين مُتَقابِلين مُتَوازِيانِ.
شِبهُ مُنحرِفٍ		• ضِلْعانِ فَقطْ مِنْ أَضلاعِهِ المُتَقابِلةِ مُتَوازِيانِ.

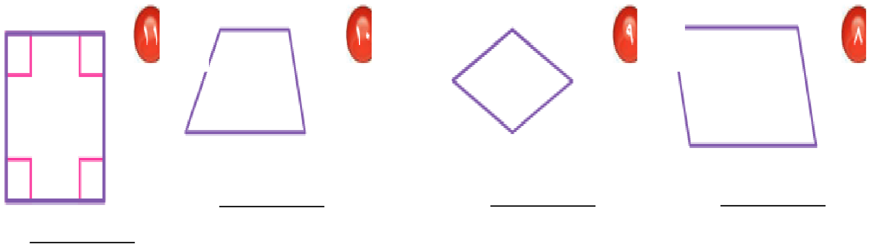
تَذَكَّر

إشارةُ المربعِ الصغيرةِ في زاويـةِ الشكلِ تـدلُّ على أنَّ الزاويةَ قائمةٌ.

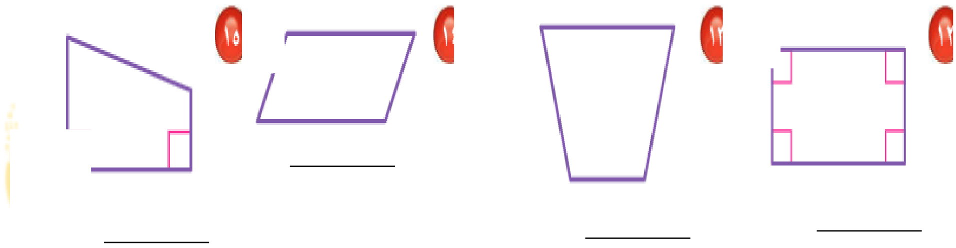
نشاط فردي

نشاط 9

صفِ الأَضلاعَ التي تَبدو مُتطابِقةً في كُلِّ شَكْلٍ رُباعِيٍّ مِمَّا يأتي، ثم اذكُرْ ما إذا كانَ أَيٌّ من أَضلاعِها تَبدو مُتَوازيَةً أو مُتعامِداً ل١



أَوجِدْ عَددَ الزَّوايا الحادَّةِ في كُلِّ شكلٍ رُباعِيٍّ مِمَّا يأ



تأكَّـدْ

صِفِ الأَضلاعَ التي تَبدُو مُتَطابِقَّة في كُلِّ شكلٍ رُباعِيٍّ مِمَّا يأتي، ثم اذكُرْ ما إذا كانَ أَيٌّ مِن أَضلاعِه تَبدو مُتوازِيَة أَو مُتَعامِدَّة:

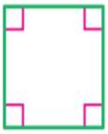
١ 

٢ 

أَوجِدْ عَددَ الزَّوايا الحادَّةِ في كُلِّ شكلٍ رُباعِيٍّ مِمَّا يأتي:

٣ 

٤ 

٥ 

٦ أَوجِدْ عَددَ الزَّوايا المُنْفَرِجةِ في كُلِّ شكلٍ مِمَّا يأتي:

٧ تَحَدَّثْ ما الفَرقُ بينَ المعيَّنِ وشِبهِ المُنْحرِفِ؟

الهندسة: الأزواج المرتبة


فِكْرَةُ الدَّرْسِ
أُسمي النقاط في المستوى الإحداثيّ.
الْمُفْرَدَاتُ
الْمُستوى الإحداثيُّ
نُقطةُ الأَصلِ
الزوجُ المُرتَّبُ
الإحداثيُّ السينيُّ
الإحداثيُّ الصاديُّ

اِسْتَعِدَّ
عِندما يَعودُ عبدُ الملِكِ من المَدرسةِ إلى البيتِ، فإنَّهُ يمشي ٣ وحداتٍ إلى اليَمينِ و٥ وحداتٍ إلى أعلى، كيفَ يمشي عبدُ الملِكِ من المدرسةِ إلى المَكتبةِ؟ وكيفَ يمشي إلى الحَديقةِ؟

نشاط 10

بالتعاون مع مجموعتك ارسم شكلاً رباعياً رؤوسه أ، **ب**، **جـ**، **د** مستعملا الورق الملون وشبكة التربيع، ثم اجب على الأسئلة التالية:

- ما هو الإحداثي؟

....

....

- كيف يمكنك إيجاد إحداثيات الرأس ب؟

....

....

- ما هي إحداثيات الرؤوس التي الشكل الذي رسمته؟

أ ....

ب ....

جـ ....

د ....

نشاط 11

بالتعاون مع مجموعتك مستعملا التربيع والادوات المتاحة أمامك مثل النقاط التالية على المستوى الإحداثي:

أ (6، 7)
ب (3،2)
جـ (0،5)
د (5،0)

- ما هو الاحداثي السيني للنقطة أ؟

....

- ما هو الإحداثي الصادي للنقطة ب؟

....

- هل النقطة جـ والنقطة د في نفس الموقع؟ ولماذا؟

....

....

- صف المستوى الإحداثي؟

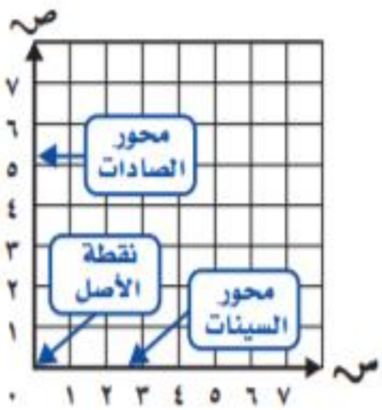
....

....

- صف الزوج المرتب؟

....

....

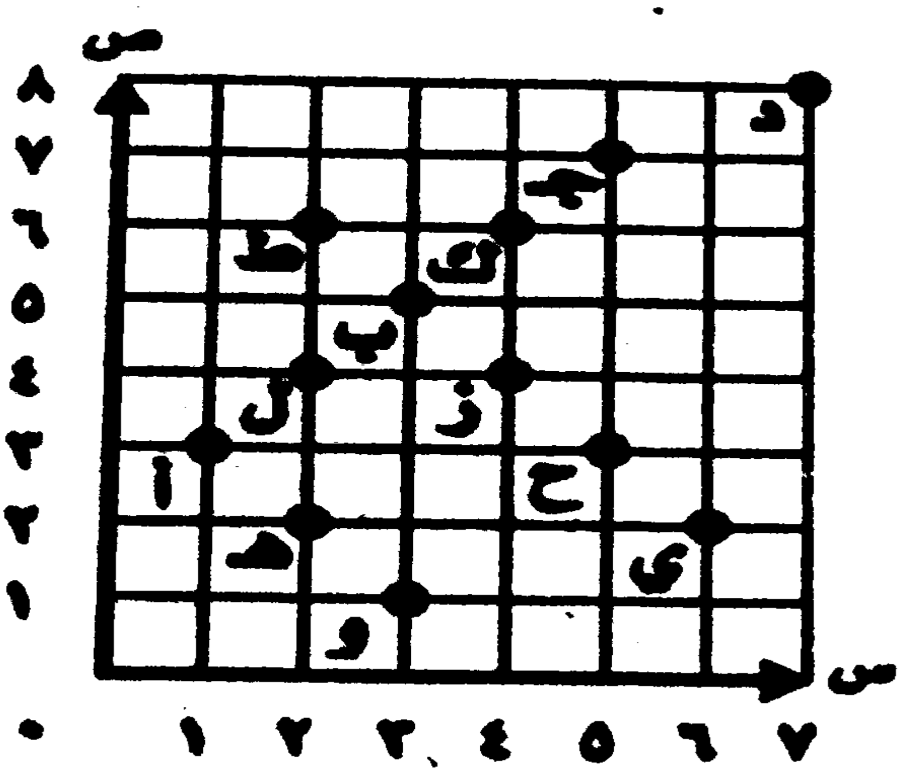


يتشكَّلُ **المستوى الإحداثيُّ** عندَ تَقاطعِ خطي أَعدادٍ. وتكونُ أعدادُ أحدِ خَطَّي الأَعدادِ على طُولِ المِحورِ الأُفقيِّ (محور السينات)، وتكونُ أَعدادُ الخَطِّ الثاني على طُولِ المِحورِ الرأْسِيِّ (محور الصادات)، أَما نُقطةُ التقاءِ المِحورينِ فَهيَ **نُقطةُ الأصلِ**.

الزوجُ المُرتَّبُ هو زَوجٌ من الأَعدادِ يُستعملُ لِتسميةِ نقطةٍ في المستوى الإِحداثيِّ.

الإحداثي السيني (٣ ، ٢) **الإحداثي الصادي**

نشاط 12



بالتعاون مع مجموعتك اجب عما يلي:

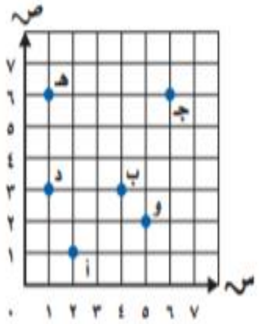
- اكتب الزوج المرتب لكل نقطة مما يأتي:

1	أ	4	د ....
2	ب	5	هـ ....
3	جـ	6	و..............

- سم النقطة التي يمثلها كل زوج مرتب مما يلي:

1	(3،5)	4	(6،4) ....
2	(4،4)	5	(4،2) ....
3	(6،2)	6	(2،6) ....

تأكَّد



سمِّ الزَّوجَ المُرتَّبَ لكلِّ نقطةٍ ممَّا يأتي:

١ أ
٢ جـ
٣ د

سمِّ النقطةَ التي يمثِّلُها الزوجُ المرتَّبُ:

٤ (٣،٤)
٥ (٦،١)
٦ (٢،٥)

٧ ارجعْ إلى المِثالِ ٣، واكتبِ الزوجَ المرتَّبَ الذي يُمثِّلُ موقعَ الخَاتمِ في المستوى الإحداثيِّ.

٨ **تَحَدَّثْ** هل تقعُ النُّقطتانِ (٣ ، ٨) ، (٨ ، ٣) في المَوقعِ نفسِهِ؟ برِّرْ إجابتَكَ.

فِكْرَةُ الدَّرْسِ

أُمثِّلُ نقاطًا في المستوى الإحداثي.

الْمُفْرَدَاتُ

التَّمْثِيلُ

اسْتَعِدَّ

أَرادَ رائدٌ أَنْ يصنعَ خَريطةَ كنْزٍ لِلعبَةٍ كان يَلعبُها معَ أختِه، وقدْ قرَّرَ أنْ يكونَ الكنزُ على بُعدِ ٣ وحداتٍ يَمينًا وَ ٦ وحْداتٍ إلى أَعلى، فَوضعَ علامةَ × عندَ تلكَ النقطةِ.

عِنْدَ **تمثيل** نقطةٍ في المستوى الإحداثيِّ نضعُ علامةً عندَ النقطةِ التي يُمثِّلُها الزوجُ المرتَّبُ المطلوبُ تمثيلُهُ.

نشاط 13

بالتعاون مع مجموعتك مستعملا شبكة التربيع والأدوات المتاحة عين النقاط التالية على المستوى الإحداثي:

س (2،4)

ص (4،2)

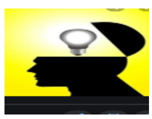
ر (5،5)

- ما هي النقطة التي يمثلها الزوج المرتب (5،5)؟

....

- ما هي النقطة التي يمثلها الزوج المرتب (2،4)؟

....



يمكن كتابة مدخلات ومخرجات الدالة على صورة أزواج مرتبة

نشاط 14

بالتعاون مع مجموعتك مستعملا شبكة التربيع والأدوات المتاحة لديك أجب على الأسئلة التالية:

- إذا كان محمد أكبر من ياسر بــ 3 أعوام، استعمل قاعدة الدالة   س + 3 لإيجاد عمر محمد عندما يصبح عمر ياسر 6، 7، 8، 9، 10. (استعمل جدول الدالة للحل)

- باستعمال قاعدة الدالة س + 3، ما عمر محمد عندما يصبح عمر ياسر 10 سنوات؟

- كيف أوجدت عمر محمد؟

- كيف ستكون الازواج المرتبة من خلال جدول الدالة؟

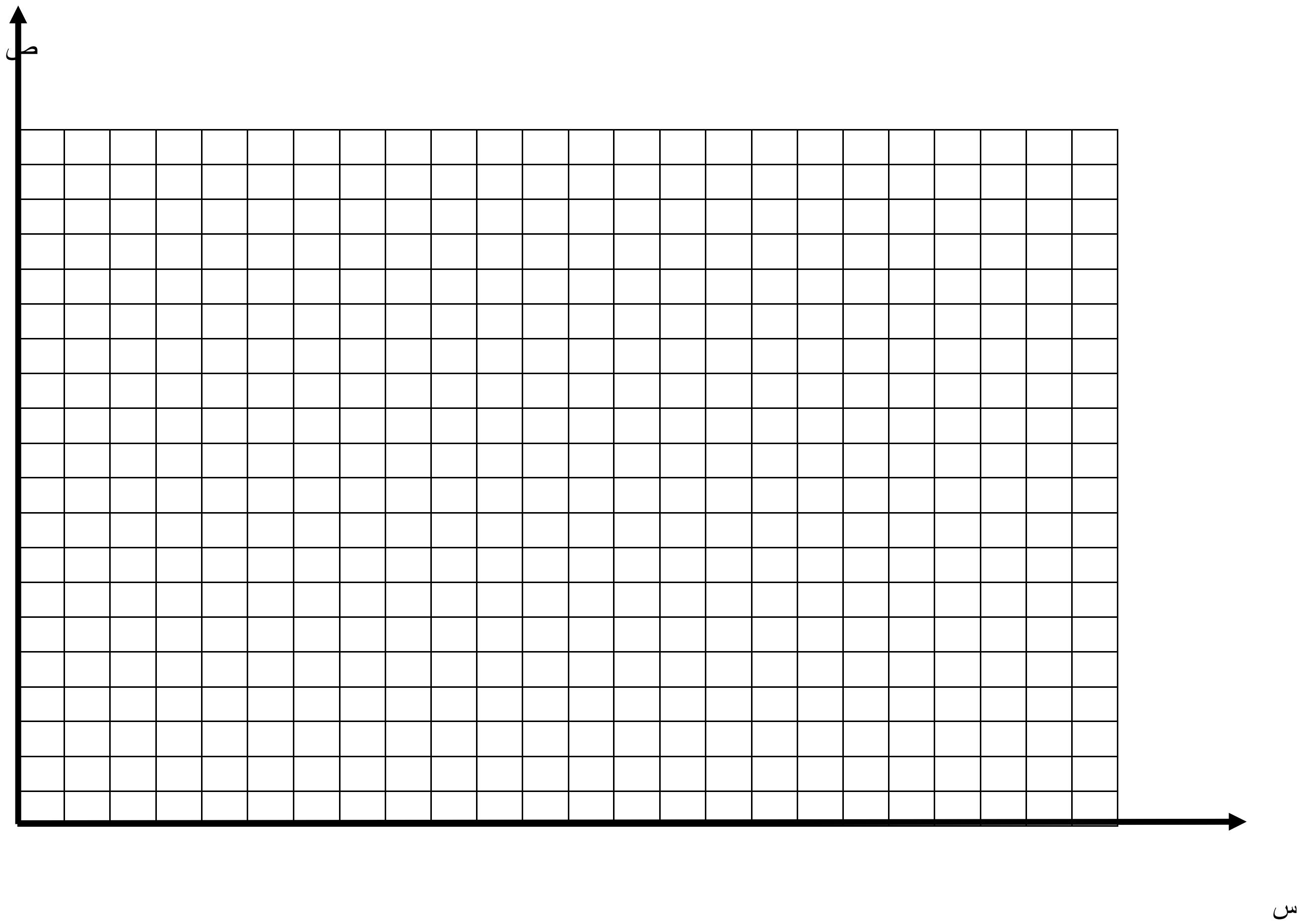
- اكتب الازواج المرتبة التي تمثل العلاقة بين عمر محمد وياسر؟

- مثل الدالة س+3 على المستوى الاحداثي؟

نشاط فردي

نشاط 15

استعمل قاعدة الدالة 9 – ع، وأوجد القيم عندما تكون قيمة ع: 2، 4، 6. كوّن جدولا للدالة، ثم مثلها على المستوى الإحداثي.



تأكد

مثِّل كلَّ نقطةٍ ممَّا يأتي في المستوى الإحداثيِّ، ثم سمِّها: مثال ١

١ ع (٢،٢)　　٢ س (٤، ٠)　　٣ ص (٥، ٦)

٤ جـ (٠، ٤)　　٥ ل (٧، ٦)　　٦ ب (٣، ٧)

٧ كيسُ حُبوبٍ وزنُهُ ٥ كيلوجراماتٍ. استعملْ قاعدةَ الدالَّةِ ٥ح لإيجادِ مَجموعِ الأوزانِ في حالاتِ عددِ الأكياسِ: ٠، ١، ٢، ٣. مثال ٢

٨ **تَحَدَّثْ** وضِّحْ كيفَ تمثِّلُ النقطةَ ك (١٠، ٧) في المستوى الإحداثيِّ.

الانسحاب في المستوى الإحداثي

نشاط 16

بالتعاون مع مجموعتك وباستعمال الادوات كوّن مربعين متطابقين ثم ضع واحداً منهما على شبكة التربيع المتاحة بحيث يكون أحد رؤوسه النقطة (2، 1) أمامك، ثم أجب على الأسئلة التالية:

- اكتب الأزواج المرتبة التي تمثل رؤوس المربع؟
- نفذ عملية انسحاب لتحريك الصورة أربع وحدات إلى اليمين؟
- ماذا يحدث للصورة عند إزاحتها أربع وحدات إلى اليمين؟
- هل تغير شكل المربع بعد الانسحاب؟
- ماذا حدث للإحداثيات عند انسحاب الشكل أربع وحدات إلى اليمين؟
- اكتب الأزواج المرتبة التي تمثل رؤوس المربع بعد الانسحاب.

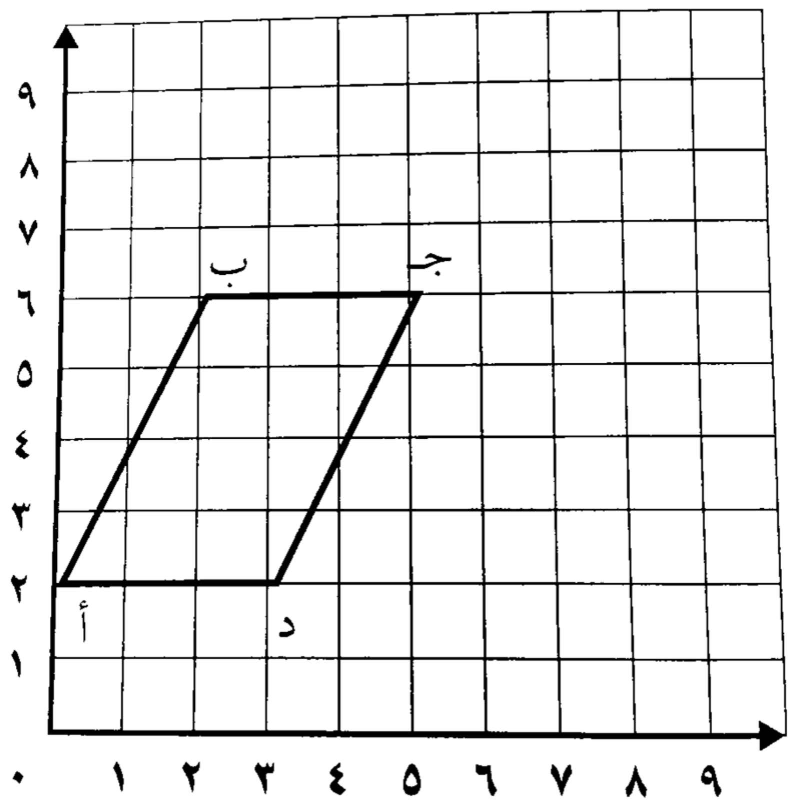
نشاط 17

بالتعاون مع مجموعتك كوّن المثلث أ (2،5)، ب (6،7)، جـ (4،9) مستعملاً شبكة التربيع والأدوات المتاحة أمامك، ثم نفذ عملية انسحاب للمثلث 3 وحدات إلى اليسار، واكتب الأزواج المرتبة لرؤوس الصورة.

نشاط فردي

نشاط 18

ارسم صورة الرباعي أ (0،2)، ب (2،6)، جـ (5،6)، د (3،2) بانسحاب 4 وحدات إلى اليمين، واكتب الأزواج المرتبة لرؤوس الصورة.



نشاط 19

ناقش مع مجموعتك ما يلي، ثم اكتب إجابتك في المكان المخصص:

- التحويل الهندسي:

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- الانسحاب

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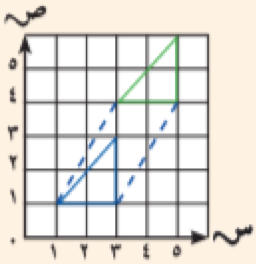
ملخص الدرس:

تُسمّى حَركةُ الشَكلِ الهَنْدسيِّ **تَحويلًا هَندسيًّا**، ويُسمّى الشكلُ الناتجُ عن هذِه الحَركةِ **صُورةَ الشكلِ**. والانسِحابُ أحدُ أنواعِ التحويلاتِ الهَنْدسيَّةِ.

الانسحاب

مفهوم أساسي

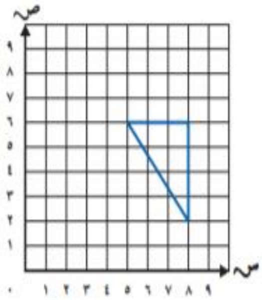
الانسِحابُ هو إزاحةُ شَكلٍ دونَ تَدويرِه، ولا يَنتجُ عنْ ذلكَ تَغيرٌ في قِياساتِه أو شكلِه.



لكَيْ تُجرِيَ انسحابًا لشكلٍ، حَرِّكْ جَميعَ رؤوسِه مَسافةً مُتساويةً في اتِّجاهٍ واحِدٍ.

تأكد

ارْسُمِ المُثلَّثَ بعدَ كُلِّ انسِحابٍ مِمَّا يأتي، ثم اكتُبِ الأَزواجَ المُرتبةَ لِرُؤوسِ الصورةِ: مثال ١



١ ٣ وحداتٍ إلى اليَسارِ.

٢ ٤ وحداتٍ إلى أَعلى.

٣ ٥ وحداتٍ إلى اليَسارِ ووحدتان إلى أَسفَلَ.

لحَلِّ المَسألتينِ ٤ ، ٥، ارسُمِ الشكلَ وصُورتَه بالانسِحابِ، وأكتُبِ الأَزواجَ المُرتَّبةَ لِرُؤوسِ الصورةِ: مثال ١

٤ الشكلُ الرُّباعيُّ أ (١،٥)، ب (٢،٨)، جـ (٤،٨)، د (٣،٥)؛ انسِحابُ ٥ وحداتٍ إلى اليَمينِ.

٥ المُثلثُ هـ (٧،٢)، ل (٨،٦)، ز (٩،٣)؛ انسِحابُ ٦ وحداتٍ إلى اليَسارِ وَوحدةٍ واحدةٍ إلى أَعلى.

٦ مَشتْ نجلاءُ ٦ أَمتارٍ غَربًا و٤ أَمتارٍ شَمالًا. صِفْ هذا التَّحويلَ.

٧ **تحدث** وضِّحْ سَببَ تَسميةِ الانسحابِ أَحيانًا بالإزاحةِ.

<u>**الانعكاس في المستوى الإحداثي**</u>

نشاط 20

بالتعاون مع مجموعتك وباستعمال الأدوات كوّن مربعين متطابقين ثم ضع واحداً منهما على شبكة التربيع المتاحة أمامك بحيث يكون أحد رؤوسه (9، 1)، ثم اجب على الأسئلة التالية:

- ارسم مستقيما على بعد ثلاث وحدات إلى يسار المربع؟
- اكتب الأزواج المرتبة التي تمثل رؤوس المربع؟
- قم بعملية انعكاس للمربع حول الخط المستقيم الذي رسمته؟
- ماذا يحدث للمربع عند انعكاسها؟
- هل تغير شكل المربع؟
- ماذا حدث للإحداثيات عند انعكاس الشكل؟
- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة بعد الانعكاس.
- هل المربع المنعكس متطابق مع المربع الأول؟ ولماذا؟
- ماذا تلاحظ على المسافة بين المربعين والخط المستقيم؟

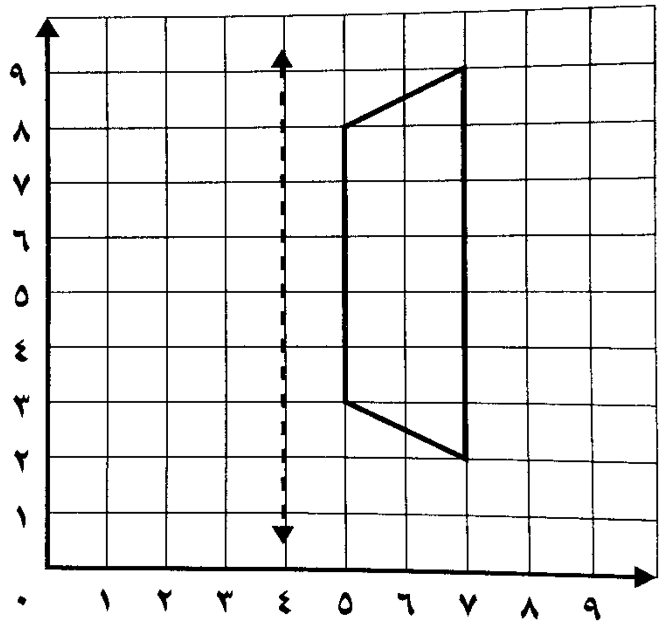
نشاط 21

بالتعاون مع مجموعتك مثل المثلث أ (2،5)، ب (6،7)، جـ (4،9) مستعملاً الأدوات المتاحة لديك، ثم ارسم مستقيما رأسيا يمر بالنقطة 5 على المحور السيني، ثم مثل الشكل عد انعكاسه واكتب الأزواج المرتبة لرؤوس الصورة.

نشاط فردي

نشاط 22

ارسم صورة كل شكل مما يأتي بالانعكاس حول المحور، ثم اكتب الأزواج المرتبة للرؤوس الجديدة:



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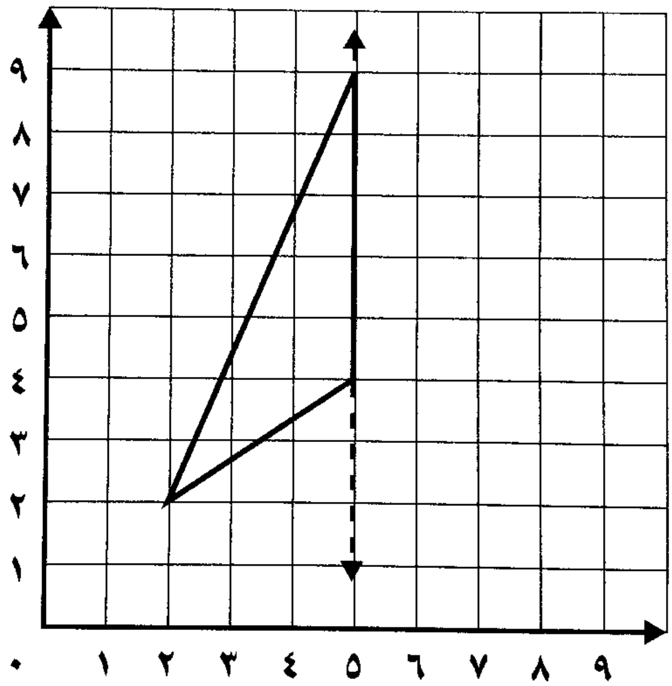

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نشاط 23

ناقش مع مجموعتك ما يلي، ثم اكتب إجابتك في المكان المخصص:

- الانعكاس (Reflection):

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- محور الانعكاس (Reflection axis):

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- الفرق بين الانسحاب والانعكاس (Different between Translation and Reflection)

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ملخص الدرس:

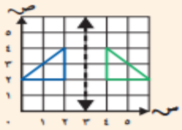
تذكّر

في الانعكاس، يُقلَبُ الشكلُ من مكانٍ إلى آخرَ دون تدويرِه. الانعكاسُ يُسمى أحيانًا قلبَ الشكلِ.

الانعِكاسُ هوَ تَحويلٌ هَنْدسِيٌّ آخَرُ لا يُغَيِّرُ من قِياساتِ الشكلِ أَو نَوعِهِ.

الانعكاس — مفهوم أساسي

يُسمَّى قَلْبُ شَكلٍ هَندسِيٍّ حَولَ مُستقيمٍ والحُصولُ على صُورةِ مِرآةٍ لهذا الشَّكلِ **انعِكاسًا**، ويُسمَّى المُستقيمُ **مِحورَ الانعِكاسِ**.



عندَ انعِكاسِ شَكلٍ حَولَ مُستقيمٍ تَكونُ الرُّؤوسُ المُتَناظِرةُ على مَسافةٍ مُتَساويةٍ مِن مِحورِ الانعِكاسِ.

تأكَّد

ارسُمْ صورةَ كُلِّ شَكلٍ مِمَّا يأتي بالانعكاسِ حَولَ المِحورِ، ثم اكْتُبِ الأَزواجَ المُرتَّبةَ لِلرُّؤوسِ الجَديدةِ: مثال ١

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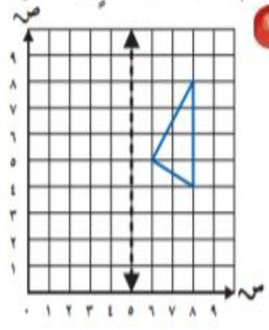

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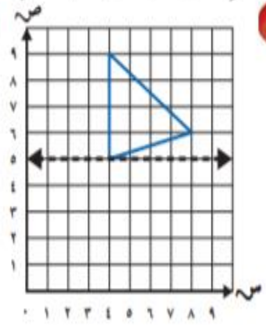

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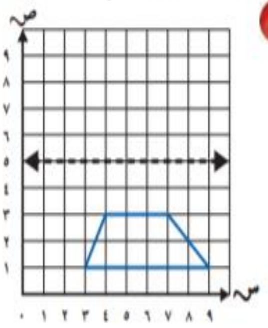

٤ اذكرْ رقمًا لا يتغيرُ انعكاسهُ حولَ محورٍ عموديٍّ.

٥ تَحَدَّثْ ما أَوجُهُ الشبهِ والاختِلافِ بينَ الانسِحابِ والانعِكاسِ؟

<u>**الدوران في المستوى الإحداثي**</u>

نشاط 24

بالتعاون مع مجموعتك بالتعاون وباستعمال الأدوات المتاحة لديك كوّن مربعين متطابقين ثم ضع واحداً منهما على شبكة التربيع المتاحة أمامك بحيث يكون أحد رؤوسه النقطة (3، 2)، ثم اجب على الأسئلة التالية:

- اكتب الأزواج المرتبة التي تمثل رؤوس الشكل؟
- نفذ عملية الدوران في للمربع بزاوية 90° باتجاه عقارب الساعة؟
- ماذا يحدث للصورة عند دورانها 90° باتجاه عقارب الساعة؟
- هل تغير شكل الصورة؟
- ماذا حدث للإحداثيات عند دوران الشكل 90° باتجاه عقارب الساعة؟
- اكتب الأزواج المرتبة التي تمثل رؤوس الصورة بعد الدوران.

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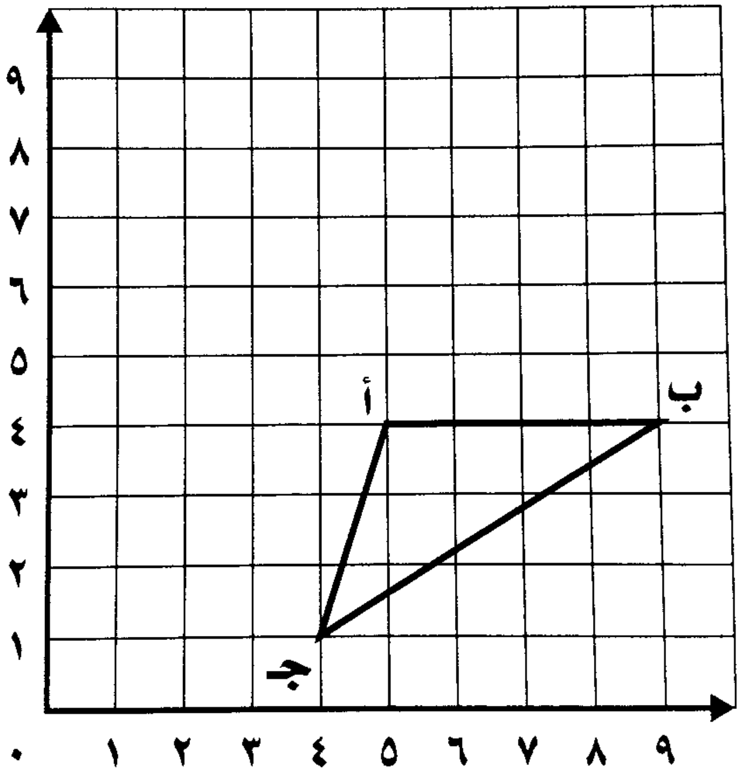
نشاط 25

بالتعاون مع مجموعتك ارسم المثلث أ (2،5)، ب (6،7)، جـ (4،9) مستعملاً الأدوات المتاحة لديك، ثم مثّل صورته بدوران 180° حول النقطة ب وباتجاه عكس عقارب الساعة، واكتب الأزواج المرتبة لرؤوس الصورة.

نشاط فردي

نشاط 26

ارسم صورة المثلث أ ب جـ بالدوران حول أ عكس عقارب الساعة، ثم اكتب الأزواج المرتبة للرؤوس الجديدة:



نشاط 27

ناقش مع مجموعتك ما يلي، ثم اكتب إجابتك في المكان المخصص:

- الدوران:

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- الفرق بين الدوران والانعكاس:

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- الفرق بين الانسحاب والدوران

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- العلاقة بين التحويل الهندسي والانسحاب، الانعكاس، الدوران

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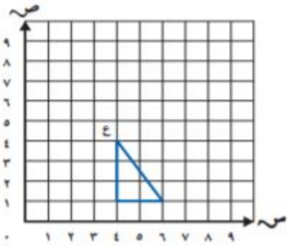
تأكد



ارسُمْ صورةَ المُثلَّثِ بالدورانِ حَولَ النقطةِ ع في كُلٍّ من الحالاتِ الآتيةِ، ثم اكتُبِ الأَزواجَ المُرتبةَ لِلرُّؤوسِ الجَديدةِ: مثال ١

١ ٩٠° باتجاهِ عَقاربِ الساعةِ.

٢ ١٨٠° بِعكسِ اتِّجاهِ عَقاربِ الساعةِ.

ارْسُمِ المُثلَّثَ المُعطاةَ رُؤوسُه، ثم ارسُمْ صورَتَه بالدورانِ المعْطَى في كلٍّ مما يأتي، ثم اكْتُبِ الأَزواجَ المُرتَّبَةَ لِلرُّؤوسِ الجَديدةِ: مثال ١

٣ ك(٥،٥)، ل(٥، ٢)، م(١، ٥)؛ ٩٠° بِعكْسِ اتِّجاهِ عَقاربِ الساعةِ حَولَ النقطةِ ك.

٤ أ(٦، ٥)، ب(٦، ٩) جـ(٩، ٨)؛ ١٨٠° باتِّجاهِ عَقاربِ الساعةِ حَولَ النقطةِ أ.

٥ اذكُرْ رَقمينِ يُمثِّلُ كُلٌّ مِنهُما صورةَ الآخرِ بتحويلٍ هندسيٍّ، ثم سمِّ هذا التحويلَ.

٦ تَحَدَّث ما الفَرقُ بينَ الدَّوَرانِ والانعِكاسِ؟

Appendix (3.16) GeoGebra Teacher Guidebook

أثر التدريس باستعمال البرامج التفاعلية في تعزيز عملية التعلم وتنمية التحصيل الهندسي والقدرة على التفكير المكاني وبقاء أثر التعلم لدى طلاب الصف الخامس الابتدائي

رسالة دكتوراه في الآداب تخصص طرق تدريس رياضيات **بجامعة** نيوكاسل

دليل المعلم

إعداد الطالب

سمير علي حسين خرمي

إشراف

د. باميلا وولنر

د. هيلن بيرنز

مقدمة:

أخي المعلم:

يقوم الباحث بإجراء بحث علمي بعنوان "**أثر التدريس باستعمال البرامج التفاعلية في تعزيز عملية التعلم وتنمية التحصيل الهندسي والقدرة على التفكير المكاني وبقاء أثر التعلم لدى طلاب الصف الخامس الابتدائي**". ضمن متطلبات الحصول على درجة الدكتوراه في المناهج وطرق تدريس الرياضيات، ويهدف البحث الحالي إلى تقصي أثر تكامل برنامج جيوجبرا مع نموذج تدريس مقترح على تعزيز عملية التعلم وتنمية التحصيل الهندسي والقدرة على التفكير المكاني وبقاء أثر التعلم على عينة من طلاب الصف الخامس بمدينة جده.

ولذا فقد وقع اختيار الباحث على وحدة الأشكال الهندسية من كتاب الصف الخامس الابتدائي ليكون موضوع التجربة، وقد أعد الباحث هذا الدليل ليكون مرشداً لك لتدريس هذه الوحدة وفق نموذج التدريس المقترح الذي يستند على فكرة التعلم المتمركز حول الطالب، وباستعمال جيوجبرا التي تعد من أحدث الوسائل التقنية المستندة على أفكار النظرية البنائية.

كما أن عليك أخي الفاضل مسؤولية إدارة البيئة التعليمة داخل الصف بما يثير انتباه الطلاب ويشوقهم لتعلم مواضيع الرياضيات المختلفة بطرق تدريسية تساعد على جعلهم يبنون معرفتهم الرياضية باستعمال الوسائل التعليمية المناسبة ولاسيما الوسائل التقنية، مما يؤدي إلى متانة البنية الرياضية لدى الطالب؛ لذا يسرني أن أضع بين يديك دليل المعلم للمجموعة التجريبية (جيوجبرا) والذي يقوم فيه المعلم بتدريس المجموعة التجريبية وفق نموذج التدريس المقترح وباستعمال جيوجبرا؛ ليكون عونا لك على تحقيق الأهداف المرجوة من موضوع البحث الحالي.

أولاً: نموذج التدريس: (Teaching Intervention as explained in section 3.11)

من منطلقات النظرية البنائية التأكيد على أن التعلم عملية نشطة مستمرة محورها الرئيس هو الطالب الذي يقوم ببناء معرفته الجديدة بناءً على خبراته السابقة. فالطالب خلال عملية التعلم البنائية عنصر نشط متفاعل مع البيئة الاجتماعية والمادية داخل الصف. وبما أن النظرية البنائية تؤكد على أهمية استخدام التقنية في عملية التدريس والتعلم قام الباحث باقتراح نموذج تدريسي يقوم على ادماج التقنية في عملية تعلم وتعليم الرياضيات. يتكون هذا النموذج التدريسي من ثلاث مراحل تدريسية يكون فيها الطالب عنصراً نشطاً تارة متعلماً بشكل تعاوني وتارة يتعلم فردياً. بحيث يتدرج الطالب في التعلم من التعلم باستعمال التقنية إلى استخدام الورقة والقلم، ثم إلى استخدام قدراته العقلية ومهارات التفكير العليا للتحليل وتقديم شروحات لفظية رابطاً ذلك بأمثله من واقعه المعاش الامر الذي يعزز من تنمية مهارة التواصل الرياضي. فيما يلي وصف لمراحل التدريس الثلاث:

المرحلة الأولى:

في هذه المرحلة يقوم الطالب بالتعلم باستخدام التقنية وتنفيذ أنشطة التعلم مستخدماً برنامج جيوجبرا. بحيث تبدأ عملية التعليم والتعلم من الطالب. وفي هذه المرحلة تساعد التقنية على تحويل المفاهيم الرياضية المجردة غير المرئية إلى مفاهيم مرئية يمكن للطالب رؤيتها وتكوين صور بصرية لها. ثم استخدام قدراته البصرية والمكانية لتحليل وبناء فهمه للمفاهيم الرياضية بشكل أعمق وتعزيز التعلم ذو المعنى. بعد أن يؤدي الطالب أنشطة التعلم في هذه المرحلة يقوم المعلم بمناقشة طلابه في استنتاجاتهم ويقدم التغذية الراجعة. للتأكد من أن الطلاب يسيرون في الطريق الصحيح نحو تحقيق أهداف التعلم وبناء الفهم الصحيح للمفهوم الرياضي.

المرحلة الثانية:

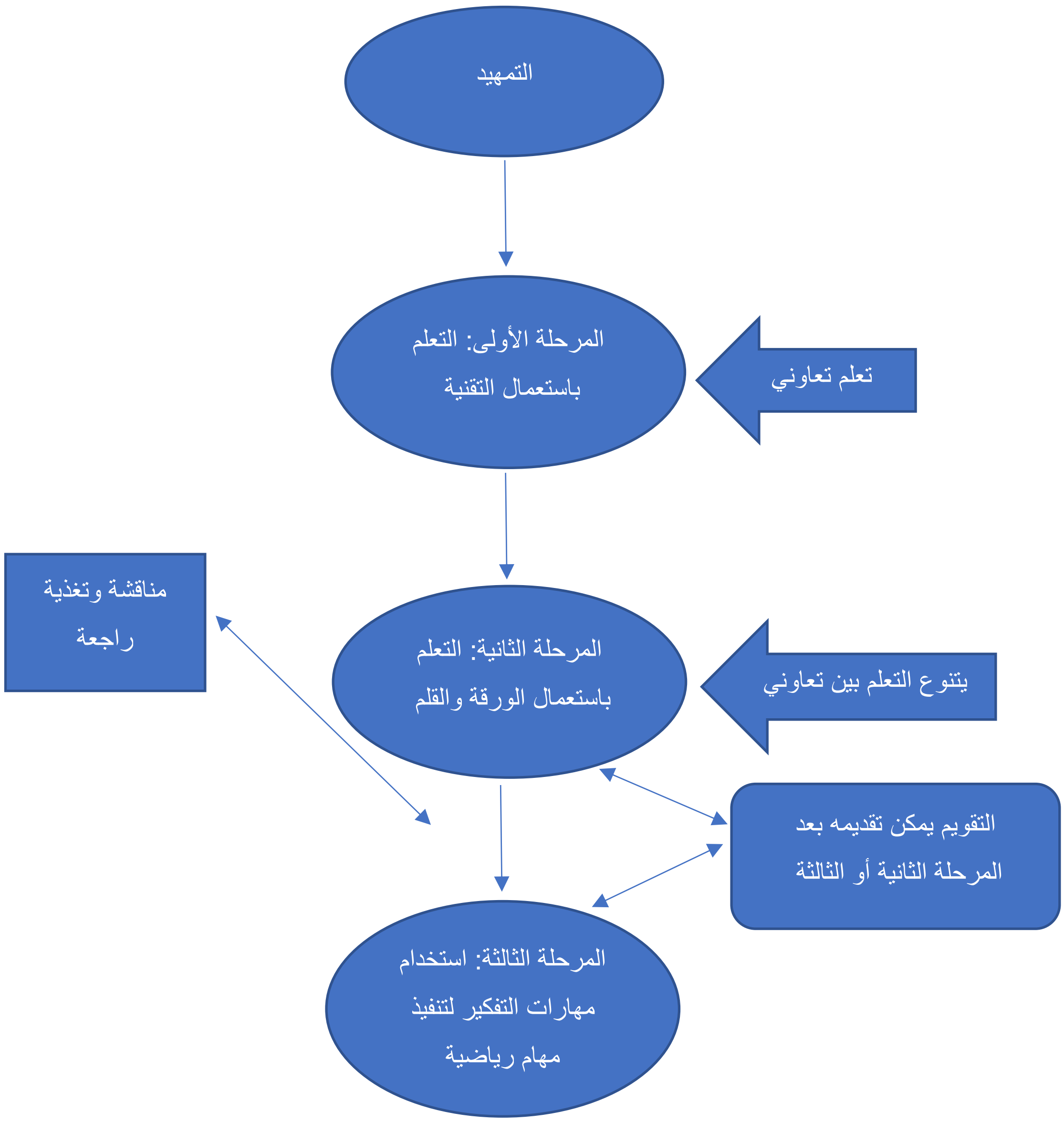
يقوم الطالب في هذه المرحلة بتنفيذ الانشطة التعليمية باستخدام الورقة والقلم، والانتقال من استخدام التقنية للتعلم إلى نقل خبراته التي تعلمها في المرحلة السابقة إلى استخدامها بالورقة والقلم. وتهدف هذه المرحلة إلى نقل الخبرات التي يتم تعلمها تقنياً إلى خبرات

يدوية والعمل على تنمية مهارات التطبيق باستخدام الورقة والقلم. وهنا تظهر قدرة الطالب على نقل خبراته التي يتعلمها باستخدام التقنية أو الوسائل البصرية إلى استخدامها بواسطة الورقة والقلم وبدون مما يعزز فكرة التعلم البنائي على أن بناء المعرفة الجديدة قائم على الخبرات السابقة. وفي ختام هذه المرحلة يتم مناقشة الطلاب في استنتاجاتهم وتقديم التغذية الراجعة لهم مما يضمن استمرار الطلاب على الطريق الصحيح لتحقيق أهداف التعلم. تجدر الإشارة هنا إلى أنه بعد نهاية هذه يمكن للمعلم أن يقيم طلابه.

المرحلة الثالثة:

هنا ينتقل الطالب من إلى استخدام قدراته العقلية ومهارات التفكير عموما والتفكير المكاني خصوصاً لتنفيذ نشاط التعلم الختامي. حيث يقوم الطالب باستخدام مهارات التفكير المكاني عموما والتصور البصري المكاني خصوصا لتقديم حلول للمشكلات الرياضية، أو تقديم وصف للعمليات الرياضية ذهنيا وباستخدام لغته، وإعطاء أمثلة على المفاهيم الرياضية التي تعلمها دون الاستعانة بالتقنية أو الورقة والقلم. والهدف من هذه المرحلة تدريب الطلاب على تنمية مهارات التفكير العليا وتدريب مهارات التفكير المكاني. مما قد يكون له أثر إيجابي في تعزيز استدامة التعلم وبقاء أثر التعلم وتنمية مهارات التعلم المختلفة. بالإضافة إلى ربط المفهوم الرياضي بالحياة الواقعية للطالب. وفي ختام هذه المرحلة يقوم الطلاب بتنفيذ أنشطة التقويم ولدى المعلم الخيار بتقدمها بعد نهاية المرحلة السابقة.

[اجذب انتباه القارئ باستخدام اقتباس كبير من المستند أو استخدام هذه المساحة للتأكيد على نقطة أساسية. لوضع مربع النص هذا في أي مكان بالصفحة، فقط قم بسحبه.]



الدرس الأول	مفردات هندسية	الوسائل التعليمة	أجهزة الحاسب الآلي (عدد 15) – جيوجبرا- أدوات هندسية.
الزمن	حصتان	التمهيد	يقوم المعلم بتوزيع الطلاب في مجموعات مكونة من طالبين في كل مجموعة، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا. وبعد ذلك يسأل المعلم عن أهمية الهندسة في الحياة. ثم يسأل الطلاب عن الاشكال الهندسية للتعرف على خلفيتهم الهندسية.
التاريخ		الصف	3/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يتعرف على المفاهيم الهندسية الأساسية الشكل (القطة، المستقيم، نصف المستقيم، القطعة المستقيمة، المستوى).	النقطة: هي موقع محدد في الفضاء وتمثلها نقطة بالقلم. المستقيم: مجموع نقط تمثل مساراً مستقيما يمتد في كلا الاتجاهين بلا نهاية نصف المستقيم: جزء من مستقيم له نقطة بداية يمتد في أحد الاتجاهين دون نهاية. القطعة المستقيمة: جزء من مستقيم لها نقطة بداية ولها نقطة نهاية. المستوى: هو سطح منبسط يمتد في جميع الاتجاهات دون نهاية.	1- يوزع المعلم على طلابه ورقة النشاط 1 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (7 -10 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويتأكد من أنهم ينفذون النشاط بشكل تعاوني. 2- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الأول ويوضح لهم كيفية استخدام النقاط لتسمية النقطة، المستقيم، نصف المستقيم، القطعة المستقيمة، والمستوى. (5 دقائق). 3- يوزع المعلم ورقة النشاط 2 ويطلب من طلابه تنفيذه بشكل تعاوني ويقوم بالإشراف عليهم ويوجه من يحتاج إلى مساعدة (3-5 دقائق). 4- يناقش المعلم طلابه في إجاباتهم على أسئلة النشاط الثاني. ثم بعد ذلك يناقشهم في الأخطاء التي وقعوا فيها (5 دقائق). 5- يقوم الطلاب بحل التمارين 1، 2، 3. ويقوم المعلم بالإشراف على طلابه ويقيمهم (5 دقائق). 6- يسأل المعلم طلابه عن المفاهيم الهندسية وهنا يجب عليهم أن يستخدموا قدرتهم على التخيل والتفكير المكاني للتعبير عن المفاهيم الهندسية التي تعلموها وذلك بدون النظر إلى المواد التعليمية المتاحة أمامهم ودون استخدام التقنية أو الورقة والقلم (5 دقائق).
يحدد العلاقة بين المستقيمات.	التقاطع: مستقيمان يلتقيان أو يتقاطعان عند نقطة واحدة. التعامد: مستقيمان يلتقيان فيقطع أحدهما الآخر مكونا زاوية قائمة. التوازي: مستقيمان بينهما مسافة ثابتة لا تساوي صفراً ولا يلتقيان أو يتقاطعان أبداً مهما امتدا.	1- يوزع المعلم على طلابه ورقة النشاط 3 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (10 دقائق) بشكل تعاوني. بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويتأكد من سير العمل كما هو محدد. 2- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الأول (7 دقائق). 3- يوزع المعلم ورقة النشاط 4 ويطلب من طلابه تنفيذه بشكل فردي مستعملين أدواتهم الهندسية والورقة والقلم (10 دقائق).

	التطابق: قطع مستقيمة متساوية في الطول.	4- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الرابع. ثم بعد ذلك يناقشهم في طريقة التعبير الرياضي للمفاهيم الهندسية التي توصلوا لها (8 دقائق). 5- يقوم الطلاب بحل التمارين 4، 5، 6، 7. ويقوم المعلم بالإشراف على طلابه ويقيمهم (5 دقائق). 6- وفقًا للنشاط ٥ يسأل المعلم طلابه عن العلاقة بين المستقيمات مستخدمين قدرتهم على التفكير المكاني للتعبير عن المفاهيم الهندسية التي تعلموها دون استخدام التقنية أو الورقة والقلم، على أن يربطوا بين ما تعلموه وبين حياتهم الواقعية (5 دقائق).
التقويم	حل السؤال ١، ٢، ٣، ٤، ٥، ٦، ٧ وبعد أن ينتهي الطلاب من حل التمارين يقوم المعلم بتصحيح الحل ومناقشة طلابه في حلهم.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 10، 11، 12، 13، 14، 15، 16، 17، 18 من كتاب الطالب الصفحة 128	

Lesson 1	Geometric Concepts	Teaching tools	Computers (Number 15) - GeoGebra – geometry set.
Time	Two Classes	Introduction	The teacher group students in pairs where is possible, so that each group has a computer equipped with GeoGebra software. The teacher then asks about the importance of geometry in life. After that, Students are asked about geometry forms to learn about their geometry background.
Date		Class	5/3

Aim	Content	Teaching strategy
The student should: define the basic geometric concepts (cat, line, half line, segment line, plane).	Point: is a specific location in space and is represented by a point by a pen A line: is a set of points that form a straight path that extends in both directions without end. Half-line: is a part of a line that has a starting point and extends in one direction without end A line segment is part of a line that has a starting point and an end point Plane: is a flat surface that extends in all directions without end.	1. The teacher hands out learning task 1 to his students and asks them to implement it on the Geogebra program and answer all the task questions within (7-10 minutes). 2. The teacher discusses his students' conclusions on the questions of the first learning task and explains to them how to use the point to name line, half line, segment line, plane. 3. The teacher distributes learning task 2 and asks his students to implement it collaboratively and supervises them and directs those who need help (3-5 minutes). 4. The teacher discusses his students in their answers to the questions of the second learning task. The teacher also should discuss with his students the errors they made. 5. Students solve 1, 2, 3 exercises. The teacher supervises and evaluates his students (5 minutes). 6. The teacher asks his students about geometric concepts. Here students should use their ability to imagine and think spatially to express the geometric concepts they have learned without looking at the educational materials available to them and without using GeoGebra or paper and pen (5 minutes).

<table>
<tr>
<td>explore the relationships between the lines.</td>
<td>Intersection: Two straight lines that meet or intersect at one point.
Perpendicular: Two straight lines meet, and one intersect the other to form right angle.
Parallel: Two straight straights with fixed distance between them that is not equal to zero and never meet or intersect no matter how extended they are.
Congruent: lines with the same length.</td>
<td>7- The teacher hands out learning task 3 to his students and asks them to implement it in pairs on the Geogebra program and answer all the learning task 3 questions within (1 0 minutes). While the students are carrying out the learning activity, the teacher supervises his students and makes sure that work is going on as planned.

8- The teacher discusses with his students their conclusions on the questions of the first activity (7 minutes).

9- The teacher distributes learning task 4 and asks his students to do it individually using their geometry set, paper and pen (10 minutes).

10- The teacher discusses with his students their conclusions on the questions of the fourth learning task. Then he discusses with them the way of the mathematical expression of the geometric concepts that they reached (8 minutes).

11- Students perform exercises 4, 5, 6, 7. The teacher supervises and evaluates his students (5 minutes).

12- According to learning task 5, the teacher asks his students about the relationship between the lines, using their spatial thinking ability to express the geometric concepts they have learned without using technology or paper and pen, in which they link what they have learned with their real life (5 minutes).</td>
</tr>
<tr>
<td>Assessment</td>
<td colspan="2">Solve questions 1, 2, 3, 4, 5, 6, 7, and after the students have finished solving the exercises, the teacher marks their answers and discusses his students in their solutions.</td>
</tr>
<tr>
<td>Homework</td>
<td colspan="2">The teacher asks his students to answer the following questions 10, 11, 12, 13, 14, 15, 16, 17, 18 of the student's book page 128</td>
</tr>
</table>

الدرس الثاني	الأشكال الرباعية	الوسائل التعليمية	أجهزة الحاسب الآلي (عدد 15) – جيوجبرا- أدوات هندسية.
الزمن	حصتان	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يسأل الطلاب عن المفاهيم الهندسية وأنواع الزوايا ومفهوم الشكل الرباعي.
التاريخ		الصف	3/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يتعرف على مفهوم الشكل الرباعي وكيفية تصنيفها.	الشكل الرباعي: هي شكل رباعي له أربع أضلاع وأربع زوايا. تصنف الأشكال الرباعية وفقا لواحدة أو أكثر من الخصائص التالية: – تطابق الأضلاع. – توازي الأضلاع. – تعامد الأضلاع.	1- يوزع المعلم على طلابه ورقة النشاط 6 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (15 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويلفت انتباه الطلاب إلى التركيز على العلاقة بين الأضلاع والزوايا. 2- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط السادس. ثم يدون مفهوم الشكل الرباعي، وكيفية تصنيف الأشكال الرباعية على السبورة (10 دقائق). 3- يوزع المعلم على طلابه ورقة النشاط 7 ويطلب منهم تنفيذه باستخدام الورقة والقلم. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (10 دقائق). 4- يطلب المعلم من طلابه تقديم أمثلة من واقع الحياة لأشكال رباعية مبررين سبب كون هذا الشكل رباعي (5 دقائق).
يتعرف خصائص الأشكال الرباعية (المستطيل، المربع، المعين، متوازي الأضلاع، شبه المنحرف).	<u>خصائص المستطيل:</u> – كل ضلعين متقابلين متطابقين – جميع زواياه قائمة – كل ضلعين متقابلين متوازيان <u>خصائص المربع:</u> – جميع أضلاعه متطابقة – جميع زواياه قائمة – كل ضلعين متقابلين متوازيان	1- يوزع المعلم على طلابه ورقة النشاط 8 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (10 دقائق) بشكل تعاوني. بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويتأكد من سير العمل كما هو محدد ملفتاً انتباه الطلاب إلى أنواع الزوايا في كل شكل، والاضلاع المتطابقة والمتوازية. 2- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الثامن (5 دقائق). 3- يوزع المعلم ورقة النشاط 9 ويطلب من طلابه تنفيذه بشكل فردي مستعملين أدواتهم الهندسية والورقة والقلم (10 دقائق). 4- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط التاسع. ويسجل استنتاجاتهم على السبورة (5 دقائق). 5- يطلب المعلم من طلابه تقديم أمثلة من واقع الحياة لشكلين رباعيين مختلفين موضحا الفرق بينهما (5 دقائق).

	<u>خصائص متوازي الأضلاع:</u> - كل ضلعين متقابلين متطابقان - كل ضلعين متقابلين متوازيان <u>خصائص المعين:</u> - جميع أضلاعه متطابقة - كل ضلعين متقابلين متطابقان <u>خصائص شبه المنحرف:</u> - فيه ضلعان فقط من أضلاعه المتقابلة متوازيان.	**6- يطلب المعلم من طلابه حل التمارين 1-9. ويقوم المعلم بالإشراف على طلابه ويقيمهم (10 دقائق).**
التقويم	**حل الاسئلة ١، ٢، ٣، ٤، ٥، ٦، ٧، 8، 9 وبعد أن ينتهي الطلاب من حل التمارين يقوم المعلم بتصحيح الحل ومناقشة طلابه في حلهم.**	
الواجب المنزلي	**يطلب المعلم من طلابه الإجابة على الأسئلة التالية 10، 11، 12، 13، 14، 15 من كتاب الطالب الصفحة 134**	

<table dir="rtl">
<tr><td>الدرس الثالث</td><td>الهندسة: الأزواج المرتبة</td><td>الوسائل التعليمة</td><td>أجهزة الحاسب الآلي (عدد 15) – جيوجبرا.</td></tr>
<tr><td>الزمن</td><td>حصة واحده</td><td>التمهيد</td><td>يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا.
يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه.
ثم يسأل الطلاب عن مفهوم الاشكال الرباعية، المربع، المستطيل، المعين، متوازي الاضلاع، شبه المنحرف؟</td></tr>
<tr><td>التاريخ</td><td></td><td>الصف</td><td>3/5</td></tr>
</table>

<table dir="rtl">
<tr><th>الهدف</th><th>المحتوى</th><th>إستراتيجية التدريس</th></tr>
<tr><td>ينبغي من الطالب أن: يسمي النقاط في المستوى الاحداثي</td><td>– المستوى الاحداثي: يتكون عند قاطع خطي أعداد حيث يكون أحدهما أفقي وهو المحور السيني والآخر عمودي مشكلا المحور الصادي.
– نقطة الأصل: هي نقطة التقاء المحور السيني والصادي.
– الزوج المرتب: هو زوج من الأعداد يستعمل لتسمية نقطة في المستوى الإحداثي.
– الاحداثي السيني
– الاحداثي الصادي</td><td>1. يوزع المعلم على طلابه ورقة النشاط 10 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (8 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة.
2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 10. ثم يدون ملاحظات الطلاب واستنتاجاتهم على السبورة (5 دقائق).
3. يوزع المعلم على طلابه ورقة النشاط 11 ويطلب منهم تنفيذه باستخدام جيوجبرا. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (8 دقائق).
4. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 11 (5 دقائق).
5. يوزع المعلم على طلابه ورقة النشاط 12 ويطلب منهم تنفيذه باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (10 دقائق).
6. يطلب المعلم من طلابه تقديم أمثلة من واقع الحياة لاستخدام المستوى الاحداثي والزوج (5 دقائق).</td></tr>
<tr><td>التقويم</td><td colspan="2">حل الاسئلة ١، ٢، ٣، ٤، ٥، ٦ وبعد أن ينتهي الطلاب من حل التمارين يقوم المعلم بتصحيح الحل ومناقشة طلابه في حلهم.</td></tr>
<tr><td>الواجب المنزلي</td><td colspan="2">يطلب المعلم من طلابه الإجابة على الأسئلة التالية 21، 22، 23، 24 من كتاب الطالب الصفحة 138</td></tr>
</table>

<table dir="rtl">
<tr><td>الدرس الرابع</td><td>الجبر والهندسة: تمثيل الدوال</td><td>الوسائل التعليمية</td><td colspan="2">أجهزة الحاسب الآلي (عدد 15) – جيوجبرا.</td></tr>
<tr><td>الزمن</td><td>حصة واحده</td><td>التمهيد</td><td colspan="2">يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا.
يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه.
ثم يسأل الطلاب عن مفهوم المستوى الاحداثي، والزوج المرتب؟</td></tr>
<tr><td>التاريخ</td><td></td><td>الصف</td><td colspan="2">3/5</td></tr>
<tr><td>الهدف</td><td colspan="2">المحتوى</td><td colspan="2">إستراتيجية التدريس</td></tr>
<tr><td>ينبغي من الطالب أن: يمثل نقاطاً في المستوى الاحداثي</td><td colspan="2">– التمثيل: تمثل النقطة في المستوى الاحداثي بوضع نقطة عند الزوج المرتب الذي يمثلها</td><td colspan="2">1- يوزع المعلم على طلابه ورقة النشاط 13 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (8 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه.
2- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 13. ثم يدون ملاحظات الطلاب واستنتاجاتهم على السبورة (5 دقائق).
3- يوزع المعلم على طلابه ورقة النشاط 14 ويطلب منهم تنفيذه باستخدام جيوجبرا. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه وقدم التوجيه والارشاد عند الحاجة (10 دقائق).
4- يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 14 (7 دقائق).
5- يوزع المعلم على طلابه ورقة النشاط 15 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (10 دقائق).
6- يسأل المعلم طلابه عن كيفية تمثيل الدالة في المستوى الاحداثي (5 دقائق).</td></tr>
<tr><td>التقويم</td><td colspan="4">حل النشاط 15.</td></tr>
<tr><td>الواجب المنزلي</td><td colspan="4">يطلب المعلم من طلابه الإجابة على الأسئلة التالية 1، 2، 3، 4، 5، 6، 7 من كتاب الطالب الصفحة 140</td></tr>
</table>

الدرس الخامس	الانسحاب في المستوى الاحداثي	الوسائل التعليمة	أجهزة الحاسب الآلي (عدد 15) – جيوجبرا – أدوات هندسية.
الزمن	حصتان	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يطلب من طلابه تمثيل النقط التالية على المستوى الاحداثي باستعمال جيوجبرا ط (3، 4)، ع (2، 6) ، ص (6، 2)
التاريخ		الصف	3/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يرسم صورة شكل بالانسحاب على المستوى الاحداثي	– التحويل الهندسي: تسمى حركة الشكل الهندسي في الفضاء تحويلا هندسياً – صورة الشكل: الصورة الناتجة عن حركة الشكل الهندسي في الفضاء – الانسحاب: هو إزاحة شكل دون تدويره ولا ينتج عن ذلك تغير في قياساته أو شكله.	1. يوزع المعلم على طلابه ورقة النشاط 16 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (15 دقيقة). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة. 2. يقوم المعلم بتقديم مثال مشابه للنشاط ويناقش الطلاب في استنتاجاتهم على أسئلة النشاط 16. (10 دقائق). 3. يوزع المعلم على طلابه ورقة النشاط 17 ويطلب منهم تنفيذه باستخدام جيوجبرا. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة (10 دقائق). 4. يوزع المعلم على طلابه ورقة النشاط 18 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (15 دقيقة). 5. بعد ذلك يقوم الطلاب بتنفيذ النشاط 19. (5 دقائق) 6. بعد انتهاء الطلاب من النشاط 19 يقوم المعلم بمناقشتهم في استنتاجاتهم. (5 دقائق) 7. الان يطلب المعلم من طلابه حل التمرين رقم 1 في كتاب الطالب (7 دقائق). 8. يرسم المعلم مثلثاً ويطلب من طلابه أن يجروا عليه عملية انسحاب 3 وحدات لليسار ذهنيا ثم يصفوا حركة الشكل باستعمال مخيلتهم وقدرتهم على التفكير المكاني ويصفوا عملية الانسحاب وماذا يحدث للشكل عند انسحابه (8 دقائق). 9. إذا تبقى وقت يترك المعلم المجال للطلاب لتنفيذ عمليات انسحاب باستعمال جيوجبرا
التقويم	حل التمرين 1.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 3 من كتاب الطالب الصفحة 143	

الدرس السادس	الانعكاس في المستوى الاحداثي	الوسائل التعليمة	أجهزة الحاسب الآلي (عدد 15) – جيوجبرا – أدوات هندسية.
الزمن	حصتان	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يسأل عن مفهوم التحويل الهندسي والانسحاب.
التاريخ		الصف	3/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يرسم صورة شكل بالانعكاس في المستوى الاحداثي	– الانعكاس: يسمى قلب شكل هندسي حول مستقيم والحصول على صورة لهذا الشكل – محور الانعكاس: هو الخط المستقيم الذي يتم حوله الانعكاس	1) يوزع المعلم على طلابه ورقة النشاط 20 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (15 دقيقة). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة. 2) يقوم المعلم بتقديم مثال مشابه للنشاط ويناقش الطلاب في استنتاجاتهم على أسئلة النشاط 20. (10 دقائق). 3) يوزع المعلم على طلابه ورقة النشاط 21 ويطلب منهم تنفيذه باستخدام جيوجبرا. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة (10 دقائق). 4) يوزع المعلم على طلابه ورقة النشاط 22 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (15 دقيقة). 5) بعد ذلك يقوم الطلاب بتنفيذ النشاط 23. (5 دقائق) 6) بعد انتهاء الطلاب من النشاط 23 يقوم المعلم بمناقشتهم في استنتاجاتهم. (5 دقائق). 7) الان يطلب المعلم من طلابه حل التمرين رقم 1 في كتاب الطالب (7 دقائق). 8) يرسم المعلم صورة إلى جيوجبرا ويطلب من طلابه أن يجروا عليه عملية انعكاس حول محور السينات ذهنيا ثم يصفوا حركة الشكل باستعمال مخيلتهم وقدرتهم على التفكير المكاني ويصفوا عملية الانعكاس وماذا يحدث للشكل عند انعكاسه (8 دقائق). 10. إذا تبقى وقت يترك المعلم المجال للطلاب لتنفيذ عمليات انعكاس باستعمال جيوجبرا
التقويم	حل التمرين 1.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 2، 3 من كتاب الطالب الصفحة 146	

الدرس السابع	الدوران في المستوى الاحداثي	الوسائل التعليمة	أجهزة الحاسب الآلي (عدد 15) – جيوجبرا – أدوات هندسية.
الزمن	حصتان	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة جهاز حاسب مزود ببرنامج جيوجبرا. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يسأل عن مفهوم التحويل الهندسي، والانسحاب، والانعكاس، والفرق بين الانسحاب والانعكاس. بعد ذلك يسأل طلابه عن الحركة باتجاه عقارب الساعة وكذلك الحركة في عكس عقارب الساعة
التاريخ		الصف	3/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يرسم صورة شكل بالدوران في المستوى الاحداثي	– الدوران: يسمى تدوير الشكل الهندسي حول نقطة دوراناً.	1. يوزع المعلم على طلابه ورقة النشاط 24 ويطلب منهم تنفيذه على برنامج جيوجبرا والإجابة على جميع أسئلة النشاط خلال (15 دقيقة). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة. 2. يقوم المعلم بتقديم مثال مشابه للنشاط ويناقش الطلاب في استنتاجاتهم على أسئلة النشاط 24. (10 دقائق). 3. يوزع المعلم على طلابه ورقة النشاط 25 ويطلب منهم تنفيذه باستخدام جيوجبرا. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة (10 دقائق). 4. يوزع المعلم على طلابه ورقة النشاط 26 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (15 دقيقة). 5. بعد ذلك يقوم الطلاب بتنفيذ النشاط 27. (5 دقائق) 6. بعد انتهاء الطلاب من النشاط 27 يقوم المعلم بمناقشتهم في استنتاجاتهم. (5 دقائق) 7. الان يطلب المعلم من طلابه حل التمرين رقم 1 في كتاب الطالب (7 دقائق). 8. يرسم المعلم صورة إلى جيوجبرا ويطلب من طلابه أن يجروا عليه عملية دوران باتجاه عقارب الساعة بزاوية °90 ثم يصفوا حركة الشكل باستعمال مخيلتهم وقدرتهم على التفكير المكاني ويصفوا عملية الدوران وماذا يحدث للشكل عند دورانه (8 دقائق). 9. إذا تبقى وقت يترك المعلم المجال للطلاب لتنفيذ عمليات دوران باتجاه عقارب الساعة وكذلك عكس عقارب الساعة بزوايا مختلفة في القياسات باستعمال جيوجبرا
التقويم	حل التمرين 1.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 2 من كتاب الطالب الصفحة 149	

أثر التدريس باستعمال البرامج التفاعلية في تعزيز عملية التعلم وتنمية التحصيل الهندسي والقدرة على التفكير المكاني وبقاء أثر التعلم لدى طلاب الصف الخامس الابتدائي

رسالة دكتوراه في الآداب تخصص طرق تدريس رياضيات **بجامعة** نيوكاسل

دليل المعلم

إعداد الطالب

سمير علي حسين خرمي

إشراف

د. باميلا وولنر

د. هيلن بيرنز

مقدمة:

أخي المعلم:

يقوم الباحث بإجراء بحث علمي بعنوان "**أثر التدريس باستعمال البرامج التفاعلية في تعزيز عملية التعلم وتنمية التحصيل الهندسي والقدرة على التفكير المكاني وبقاء أثر التعلم لدى طلاب الصف الخامس الابتدائي**". ضمن متطلبات الحصول على درجة الدكتوراه في المناهج وطرق تدريس الرياضيات، ويهدف البحث الحالي إلى تقصي أثر تكامل برنامج جيوجبرا مع نموذج تدريس مقترح على تعزيز عملية التعلم وتنمية التحصيل الهندسي والقدرة على التفكير المكاني وبقاء أثر التعلم على عينة من طلاب الصف الخامس بمدينة جده.

ولذا فقد وقع اختيار الباحث على وحدة الأشكال الهندسية من كتاب الصف الخامس الابتدائي ليكون موضوع التجربة، وقد أعد الباحث هذا الدليل ليكون مرشداً لك لتدريس هذه الوحدة وفق نموذج التدريس المقترح الذي يستند على فكرة التعلم المتمركز حول الطالب المستندة على أفكار النظرية البنائية.

كما أن عليك أخي الفاضل مسؤولية إدارة البيئة التعليمية داخل الصف بما يثير انتباه الطلاب ويشوقهم لتعلم مواضيع الرياضيات المختلفة بطرق تدريسية تساعد على جعلهم يبنون معرفتهم الرياضية باستعمال الوسائل التعليمية المناسبة ولاسيما الوسائل التقنية، مما يؤدي إلى متانة البنية الرياضية لدى الطالب؛ لذا يسرني أن أضع بين يديك دليل المعلم للمجموعة التجريبية (نموذج التدريس دون استخدام التقنية) والذي يقوم فيه المعلم بتدريس المجموعة التجريبية وفق نموذج التدريس المقترح وباستعمال أدوات غير تكنولوجية؛ ليكون عونا لك على تحقيق الأهداف المرجوة من موضوع البحث الحالي.

الدرس الأول	مفردات هندسية	الوسائل التعليمة	خيوط صوف- مقص- دبابيس- لوح – شبكة تربيع - أدوات هندسية.
الزمن	حصتان	التمهيد	يقوم المعلم بتوزيع الطلاب في مجموعات مكونة من طالبين في كل مجموعة، بحيث يكون لدى كل مجموعة من الورق الملون مقص. وبعد ذلك يسأل المعلم عن أهمية الهندسة في الحياة. ثم يسأل الطلاب عن الاشكال الهندسية للتعرف على خلفيتهم الهندسية.
التاريخ		الصف	1/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يتعرف على المفاهيم الهندسية الأساسية (القطة، الشكل المستقيم، نصف المستقيم، القطعة المستقيمة، المستوى).	النقطة: هي موقع محدد في الفضاء وتمثلها نقطة بالقلم. المستقيم: مجموع نقط تمثل مساراً مستقيما يمتد في كلا الاتجاهين بلا نهاية نصف المستقيم: جزء من مستقيم له نقطة بداية يمتد في أحد الاتجاهين دون نهاية. القطعة المستقيمة: جزء من مستقيم لها نقطة بداية ولها نقطة نهاية. المستوى: هو سطح منبسط يمتد في جميع الاتجاهات دون نهاية.	1. يوزع المعلم على طلابه ورقة النشاط 1 ويطلب منهم تنفيذه مستعينين بالمواد المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (7-10 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويتأكد من أنهم ينفذون النشاط بشكل تعاوني. 2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الأول ويوضح لهم كيفية تسمية المفاهيم الهندسية باستخدام الرموز (5 دقائق). 3. يوزع المعلم ورقة النشاط 2 ويطلب من طلابه تنفيذه بشكل تعاوني ويقدم التوجيه والارشاد لمن يحتاج المساعدة (3-5 دقائق). 4. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الثاني. ثم بعد ذلك يناقشهم الأخطاء التي وقعوا فيها (5 دقائق). 5. يقوم الطلاب بحل التمارين 1، 2، 3. ويقوم المعلم بالإشراف على طلابه ويقيمهم (5 دقائق). 6. يسأل المعلم طلابه عن المفاهيم الهندسية وهنا يجب عليهم أن يستخدموا قدرتهم على التخيل والتفكير المكاني للتعبير عن المفاهيم الهندسية التي تعلموها وذلك بدون النظر إلى المواد التعليمية المتاحة أمامهم ودون استخدام أدوات مساعدة أو الورقة والقلم (5 دقائق).
يحدد العلاقة بين المستقيمات.	التقاطع: مستقيمان يلتقيان أو يتقاطعان عند نقطة واحدة. التعامد: مستقيمان يلتقيان فيقطع أحدهما الآخر مكوناً زاوية قائمة. التوازي: مستقيمان بينهما مسافة ثابتة لا تساوي صفراً ولا يلتقيان أو يتقاطعان أبداً مهما امتدا. التطابق: قطع مستقيمة متساوية في الطول.	1. يوزع المعلم على طلابه ورقة النشاط 3 ويطلب منهم تنفيذه باستعمال الادوات المساعدة المتوفرة أمامهم والإجابة على جميع أسئلة النشاط خلال (10 دقائق) بشكل تعاوني. بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويتأكد من سير العمل كما هو محدد. 2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الأول (7 دقائق). 3. يوزع المعلم ورقة النشاط 4 ويطلب من طلابه تنفيذه بشكل فردي مستعملين أدواتهم الهندسية والورقة والقلم (10 دقائق). 4. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الرابع. ثم بعد ذلك يناقشهم في طريقة التعبير الرياضي للمفاهيم الهندسية التي توصلوا لها (8 دقائق). 5. يقوم الطلاب بحل التمارين 4، 5، 6، 7. ويقوم المعلم بالإشراف على طلابه ويقيمهم (5 دقائق). 6. وفقًا للنشاط ٥ يسأل المعلم طلابه عن العلاقة بين المستقيمات مستخدمين قدرتهم على التفكير المكاني

		للتعبير عن المفاهيم الهندسية التي تعلموها دون استخدام التقنية أو الورقة والقلم، على أن يربطوا بين ما تعلموه وبين حياتهم الواقعية (5 دقائق).
التقويم	حل السؤال ١، ٢، ٣، ٤، ٥، ٦، ٧ وبعد أن ينتهي الطلاب من حل التمارين يقوم المعلم بتصحيح الحل ومناقشة طلابه في حلهم.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 10، 11، 12، 13، 14، 15، 16، 17، 18 من كتاب الطالب الصفحة 128	

Lesson 1	Geometric Concepts	Teaching tools	Wool threads - scissors - pins - crok board - coordinate sheet - geometry set.
Time	Two Classes	Introduction	The teacher groups students in pairs, where possible, and give each group coloured paper and scissors. After that, the teacher asks students about the importance of geometry in life. Then he asks students about geometric shapes to know about their geometric background.
Date		Class	5/1

Aims	Content	Teaching strategy
The student should: define the basic geometric concepts (cat, line, half line, segment line, plane).	Point: is a specific location in space and is represented by a point by a pen A line: is a set of points that form a straight path that extends in both directions without end. Half-line: is a part of a line that has a starting point and extends in one direction without end A line segment: is part of a line that has a starting point and an end point Plane: is a flat surface that extends in all directions without end.	1- The teacher hands out learning task 1 to his students and asks them to implement it using the available materials and answer all the questions of the learning task within (7-10 minutes). While the students are carrying out the learning activity, the teacher supervises his students and makes sure that they carry out the activity collaboratively. 2- The teacher discusses with his students their conclusions on the questions of the first learning task and shows them how to name geometric concepts using symbols (5 minutes). 3- The teacher hands out the sheet of learning task 2, asks his students to implement it collaboratively, and provides direction and guidance to those who need help (3-5 minutes). 4- The teacher discusses with his students their conclusions on the questions of the second learning task. Then he discusses with them the mistakes they made (5 minutes). 5- Students solve exercises 1, 2, 3. The teacher supervises and evaluates his students (5 minutes). 6- The teacher asks his students about geometric concepts. Here, they should use their ability to imagine and spatial thinking

		to express the geometric concepts they have learned, without looking at the educational materials available to them and without using aid tools or paper and pen (5 minutes).
explore the relationships between the straight lines.	Intersection: Two straight lines that meet or intersect at one point. Perpendicular: Two straight lines meet, and one intersect the other to form right angle. Parallel: Two straight straights with fixed distance between them that is not equal to zero and never meet or intersect no matter how extended they are. Congruent: lines with the same length.	1- The teacher hands out to his students the sheet of learning task 3 and asks them to implement it using the available aided tools to answer all the questions of the learning task within (10 minutes) collaboratively. While the students are carrying out the learning activity, the teacher supervises his students and makes sure that work is going on as planned. 2- The teacher discusses with his students their conclusions on the questions of the third learning task (7 minutes). 3- The teacher hands out learning task sheet 4 and asks his students to perform it individually using their geometry set, paper and pen (10 minutes). 4- The teacher discusses with his students their conclusions on the questions of the fourth learning task. Then he discusses with them the method of mathematical expression of the geometric concepts that they reached (8 minutes). 5- Students solve exercises 4, 5, 6, 7. The teacher supervises and evaluates his students (5 minutes). 6- According to learning task 5, the teacher asks his students about the relationship between the lines, using their spatial thinking ability to express the geometric concepts they have learned without using aided tools or paper and pen, in which they provided examples that link what they have learned with their real life (5 minutes).
Assessment	Solve questions 1, 2, 3, 4, 5, 6, 7, and after the students have finished solving the exercises, the teacher marks their answers and discusses his students in their solutions.	
Homework	The teacher asks his students to answer the following questions 10, 11, 12, 13, 14, 15, 16, 17, 18 of the student's book page 128	

<table>
<tr><td>الدرس الثاني</td><td>الأشكال الرباعية</td><td>الوسائل التعليمة</td><td>ورق ملون – مقص - أدوات هندسية.</td></tr>
<tr><td>الزمن</td><td>حصتان</td><td>التمهيد</td><td>يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة الادوات المطلوبة.
يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يسأل الطلاب عن المفاهيم الهندسية وأنواع الزوايا ومفهوم الشكل الرباعي.</td></tr>
<tr><td>التاريخ</td><td></td><td>الصف</td><td>1/5</td></tr>
</table>

<table>
<tr><th>الهدف</th><th>المحتوى</th><th>إستراتيجية التدريس</th></tr>
<tr><td>ينبغي من الطالب أن: يتعرف على مفهوم الشكل الرباعي وكيفية تصنيفها.</td><td>الشكل الرباعي: هي شكل رباعي له أربع أضلاع وأربع زوايا. تصنف الأشكال الرباعية وفقا لواحدة أو أكثر من الخصائص التالية:
– تطابق الأضلاع.
– توازي الأضلاع.
– تعامد الأضلاع.</td><td>1. يوزع المعلم على طلابه ورقة النشاط 6 ويطلب منهم تنفيذه مستعملين الادوات المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (15 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويلفت انتباه الطلاب إلى التركيز على العلاقة بين الأضلاع والزوايا.
2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط السادس. ثم يدون مفهوم الشكل الرباعي، وكيفية تصنيف الأشكال الرباعية على السبورة (10 دقائق).
3. يوزع المعلم على طلابه ورقة النشاط 7 ويطلب منهم تنفيذه باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (10 دقائق).
4. يطلب المعلم من طلابه تقديم أمثلة من واقع الحياة لأشكال رباعية مبررين سبب كون هذا الشكل رباعي (5 دقائق).</td></tr>
<tr><td>يتعرف خصائص الأشكال الرباعية (المستطيل، المربع، المعين، متوازي الأضلاع، شبه المنحرف).</td><td><u>خصائص المستطيل:</u>
– كل ضلعين متقابلين متطابقين
– جميع زواياه قائمة
– كل ضلعين متقابلين متوازيان
<u>خصائص المربع:</u>
– جميع أضلاعه متطابقة
– جميع زواياه قائمة</td><td>1. يوزع المعلم على طلابه ورقة النشاط 8 ويطلب منهم تنفيذه مستعملين الأدوات المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (10 دقائق) بشكل تعاوني. بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويتأكد من سير العمل كما هو محدد ملفتاً انتباه الطلاب إلى أنواع الزوايا في كل شكل، والاضلاع المتطابقة والمتوازية.
2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط الثامن (5 دقائق).
3. يوزع المعلم ورقة النشاط 9 ويطلب من طلابه تنفيذه بشكل فردي مستعملين أدواتهم الهندسية والورقة والقلم (10 دقائق).
4. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط التاسع. ويسجل استنتاجاتهم على السبورة (5 دقائق).</td></tr>
</table>

	– كل ضلعين متقابلين متوازيان خصائص متوازي الأضلاع: – كل ضلعين متقابلين متطابقان – كل ضلعين متقابلين متوازيان خصائص المعين: – جميع أضلاعه متطابقة – كل ضلعين متقابلين متطابقان خصائص شبه المنحرف: – فيه ضلعان فقط من أضلاعه المتقابلة متوازيان.	5. يطلب المعلم من طلابه تقديم أمثلة من واقع الحياة لشكلين رباعيين مختلفين موضحين الفرق بينهما (5 دقائق). 6. يطلب المعلم من طلابه حل التمارين 1-9. ويقوم المعلم بالإشراف على طلابه ويقيمهم (10 دقائق).
التقويم	حل الاسئلة ١، ٢، ٣، ٤، ٥، ٦، ٧، 8، 9 وبعد أن ينتهي الطلاب من حل التمارين يقوم المعلم بتصحيح الحل ومناقشة طلابه في حلهم.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 10، 11، 12، 13، 14، 15 من كتاب الطالب الصفحة 134	

<table>
<tr><td>الدرس الثالث</td><td>الهندسة: الأزواج المرتبة</td><td>الوسائل التعليمة</td><td>شبكة تربيع - ورق ملون لاصق – ايموجي لاصق- مقص.</td></tr>
<tr><td>الزمن</td><td>حصة واحده</td><td>التمهيد</td><td>يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة الادوات المطلوبة لتنفيذ أنشطة التعلم.
يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه.
ثم يسأل الطلاب عن مفهوم الاشكال الرباعية، المربع، المستطيل، المعين، متوازي الاضلاع، شبه المنحرف؟</td></tr>
<tr><td>التاريخ</td><td></td><td>الصف</td><td>1/5</td></tr>
</table>

<table>
<tr><th>الهدف</th><th>المحتوى</th><th>إستراتيجية التدريس</th></tr>
<tr><td>ينبغي من الطالب أن:
يسمي النقاط في المستوى الاحداثي</td><td>– المستوى الاحداثي: يتكون عند قاطع خطي أعداد حيث يكون أحدهما أفقي وهو المحور السيني والآخر عمودي مشكلا المحور الصادي.
– نقطة الأصل: هي نقطة التقاء المحور السيني والصادي.
– الزوج المرتب: هو زوج من الأعداد يستعمل لتسمية نقطة في المستوى الإحداثي.
– الاحداثي السيني
– الاحداثي الصادي</td><td>1. يوزع المعلم على طلابه ورقة النشاط 10 ويطلب منهم تنفيذه باستعمال شبكة التربيع والادوات المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (8 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة.
2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 10. ثم يدون ملاحظات الطلاب واستنتاجاتهم على السبورة (5 دقائق).
3. يوزع المعلم على طلابه ورقة النشاط 11 ويطلب منهم تنفيذه باستخدام المتاحة لديهم. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (8 دقائق).
4. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 11 (5 دقائق).
5. يوزع المعلم على طلابه ورقة النشاط 12 ويطلب منهم تنفيذه باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (10 دقائق).
6. يطلب المعلم من طلابه تقديم أمثلة من واقع الحياة لاستخدام المستوى الاحداثي والزوج (5 دقائق).</td></tr>
<tr><td>التقويم</td><td colspan="2">حل الاسئلة ١، ٢، ٣، ٤، ٥، ٦ وبعد أن ينتهي الطلاب من حل التمارين يقوم المعلم بتصحيح الحل ومناقشة طلابه في حلهم.</td></tr>
<tr><td>الواجب المنزلي</td><td colspan="2">يطلب المعلم من طلابه الإجابة على الأسئلة التالية 21، 22، 23، 24 من كتاب الطالب الصفحة 138</td></tr>
</table>

الدرس الرابع	الجبر والهندسة: تمثيل الدوال	الوسائل التعليمية	شبكة التربيع – ايموجي لاصق.
الزمن	حصة واحده	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة الأدوات المتاحة لتنفيذ أنشطة التعلم. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يسأل الطلاب عن مفهوم المستوى الاحداثي، والزوج المرتب؟
التاريخ		الصف	1/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يمثل نقاطاً في المستوى الاحداثي	– التمثيل: تمثل النقطة في المستوى الاحداثي بوضع نقطة عند الزوج المرتب الذي يمثلها	1. يوزع المعلم على طلابه ورقة النشاط 13 ويطلب منهم تنفيذه باستعمال الأدوات المتاحة والإجابة على جميع أسئلة النشاط خلال (8 دقائق). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه. 2. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 13. ثم يدون ملاحظات الطلاب واستنتاجاتهم على السبورة (5 دقائق). 3. يوزع المعلم على طلابه ورقة النشاط 14 ويطلب منهم تنفيذه باستخدام الأدوات المتاحة. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه وقدم التوجيه والارشاد عند الحاجة (10 دقائق). 4. يناقش المعلم طلابه في استنتاجاتهم على أسئلة النشاط 14 (7 دقائق). 5. يوزع المعلم على طلابه ورقة النشاط 15 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (10 دقائق). 6. يسأل المعلم طلابه عن كيفية تمثيل الدالة في المستوى الاحداثي (5 دقائق).
التقويم	حل النشاط 15.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 1، 2، 3، 4، 5، 6، 7 من كتاب الطالب الصفحة 140	

الدرس الخامس	الانسحاب في المستوى الاحداثي	الوسائل التعليمية	شبكة تربيع – ورق ملون لاصق – مقص – أدوات هندسية.
الزمن	حصتان	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة الأدوات المتاحة لتنفيذ أنشطة التعلم. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يطلب من طلابه تمثيل النقط التالية على المستوى الاحداثي ط (3، 4)، ع (2، 6) ، ص (6، 2)
التاريخ		الصف	1/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يرسم صورة شكل بالانسحاب على المستوى الاحداثي	– التحويل الهندسي: تسمى حركة الشكل الهندسي في الفضاء تحويلا هندسياً – صورة الشكل: الصورة الناتجة عن حركة الشكل الهندسي في الفضاء – الانسحاب: هو إزاحة شكل دون تدويره ولا ينتج عن ذلك تغير في قياساته أو شكله.	1. يوزع المعلم على طلابه ورقة النشاط 16 ويطلب منهم تنفيذه مستعملين شبكة التربيع والأدوات المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (15 دقيقة). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة. 2. يقوم المعلم بتقديم مثال مشابه للنشاط ويناقش الطلاب في استنتاجاتهم على أسئلة النشاط 16. (10 دقائق). 3. يوزع المعلم على طلابه ورقة النشاط 17 ويطلب منهم تنفيذه باستخدام الأدوات المتاحة لديهم. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة (10 دقائق). 4. يوزع المعلم على طلابه ورقة النشاط 18 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (15 دقيقة). 5. بعد ذلك يقوم الطلاب بتنفيذ النشاط 19. (5 دقائق) 6. بعد انتهاء الطلاب من النشاط 19 يقوم المعلم بمناقشتهم في استنتاجاتهم. (5 دقائق) 7. الان يطلب المعلم من طلابه حل التمرين رقم 1 في كتاب الطالب (7 دقائق). 8. يرسم المعلم مثلثاً ويطلب من طلابه أن يجروا عليه عملية انسحاب 3 وحدات لليسار ذهنيا ثم يصفوا حركة الشكل باستعمال مخيلتهم وقدرتهم على التفكير المكاني ويصفوا عملية الانسحاب وماذا يحدث للشكل عند انسحابه (8 دقائق). 9. إذا تبقى وقت يترك المعلم المجال للطلاب لتنفيذ عمليات انسحاب باستعمال الأدوات المتاحة لديهم
التقويم	حل التمرين 1.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 3 من كتاب الطالب الصفحة 143	

<table>
<tr><td>الدرس السادس</td><td>الانعكاس في المستوى الاحداثي</td><td>الوسائل التعليمة</td><td>شبكة تربيع – ورق ملون لاصق – مقص – أدوات هندسية.</td></tr>
<tr><td>الزمن</td><td>حصتان</td><td>التمهيد</td><td>يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة الأدوات المتاحة لتنفيذ أنشطة التعلم.
يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه.
ثم يسأل عن مفهوم التحويل الهندسي والانسحاب.</td></tr>
<tr><td>التاريخ</td><td></td><td>الصف</td><td>1/5</td></tr>
</table>

<table>
<tr><th>الهدف</th><th>المحتوى</th><th>إستراتيجية التدريس</th></tr>
<tr><td>ينبغي من الطالب أن: يرسم صورة شكل بالانعكاس في المستوى الاحداثي</td><td>– الانعكاس: يسمى قلب شكل هندسي حول مستقيم والحصول على صورة لهذا الشكل

– محور الانعكاس: هو الخط المستقيم الذي يتم حوله الانعكاس</td><td>1. يوزع المعلم على طلابه ورقة النشاط 20 ويطلب منهم تنفيذه مستعملين شبكة التربيع والأدوات المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (15 دقيقة). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة.
2. يقوم المعلم بتقديم مثال مشابه للنشاط ويناقش الطلاب في استنتاجاتهم على أسئلة النشاط 20. (10 دقائق).
3. يوزع المعلم على طلابه ورقة النشاط 21 ويطلب منهم تنفيذه باستخدام الأدوات المتاحة لديهم. وخلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة (10 دقائق).
4. يوزع المعلم على طلابه ورقة النشاط 22 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (15 دقيقة).
5. بعد ذلك يقوم الطلاب بتنفيذ النشاط 23. (5 دقائق)
6. بعد انتهاء الطلاب من النشاط 23 يقوم المعلم بمناقشتهم في استنتاجاتهم. (5 دقائق)
7. الان يطلب المعلم من طلابه حل التمرين رقم 1 في كتاب الطالب (7 دقائق).
8. يرسم المعلم شكلا هندسياً أو يضع صورة على شبكة التربيع ويطلب من طلابه أن يجروا عليه عملية انعكاس حول محور الانعكاس ذهنيا ثم يصفوا حركة الشكل باستعمال مخيلتهم وقدرتهم على التفكير المكاني ويصفوا عملية الانعكاس وماذا يحدث للشكل عند انعكاسه (8 دقائق).
9. إذا تبقى وقت يترك المعلم المجال للطلاب لتنفيذ عمليات انعكاس باستعمال الأدوات المتاحة لديهم</td></tr>
<tr><td>التقويم</td><td colspan="2">حل التمرين 1.</td></tr>
<tr><td>الواجب المنزلي</td><td colspan="2">يطلب المعلم من طلابه الإجابة على الأسئلة التالية 2، 3 من كتاب الطالب الصفحة 146</td></tr>
</table>

الدرس السابع	الدوران في المستوى الاحداثي	الوسائل التعليمة	شبكة تربيع – ورق ملون لاصق – مقص – ساعة – دبابيس - أدوات هندسية.
الزمن	حصتان	التمهيد	يجلس الطلاب في مجموعاتهم، بحيث يكون لدى كل مجموعة الأدوات المتاحة لتنفيذ أنشطة التعلم. يناقش المعلم مع طلابه حل الواجب ويقوم بتصحيحه. ثم يسأل عن مفهوم التحويل الهندسي، والانسحاب، والانعكاس، والفرق بين الانسحاب والانعكاس. بعد ذلك يسأل طلابه عن الحركة باتجاه عقارب الساعة وكذلك الحركة في عكس عقارب الساعة
التاريخ		الصف	3/5

الهدف	المحتوى	إستراتيجية التدريس
ينبغي من الطالب أن: يرسم صورة شكل بالدوران في المستوى الاحداثي	– الدوران: يسمى تدوير الشكل الهندسي حول نقطة دوراناً.	1. يوزع المعلم على طلابه ورقة النشاط 24 ويطلب منهم تنفيذه مستعملين شبكة التربيع والأدوات المتاحة لديهم والإجابة على جميع أسئلة النشاط خلال (15 دقيقة). بينما يقوم الطلاب بتنفيذ النشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة. 2. يقوم المعلم بتقديم مثال مشابه للنشاط ويناقش الطلاب في استنتاجاتهم على أسئلة النشاط 24. (10 دقائق). 3. يوزع المعلم على طلابه ورقة النشاط 25 ويطلب منهم تنفيذه باستخدام الأدوات المتاحة. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويقدم التوجيه والارشاد عند الحاجة (10 دقائق). 4. يوزع المعلم على طلابه ورقة النشاط 26 ويطلب من طلابه تنفيذه بشكل فردي باستخدام الورقة والقلم والادوات الهندسية. خلال تنفيذ الطلاب للنشاط يقوم المعلم بالإشراف على طلابه ويناقشهم في إجاباتهم (15 دقيقة). 5. بعد ذلك يقوم الطلاب بتنفيذ النشاط 27. (5 دقائق) 6. بعد انتهاء الطلاب من النشاط 27 يقوم المعلم بمناقشتهم في استنتاجاتهم. (5 دقائق) 7. الان يطلب المعلم من طلابه حل التمرين رقم 1 في كتاب الطالب (7 دقائق). 8. يرسم المعلم شكلاً هندسياً أو يضع صورة على شبكة التربيع ويطلب من طلابه أن يجروا عليه عملية دوران باتجاه عقارب الساعة بزاوية 90° ثم يصفوا حركة الشكل باستعمال مخيلتهم وقدرتهم على التفكير المكاني ويصفوا عملية الدوران وماذا يحدث للشكل عند دورانه (8 دقائق). 9. إذا تبقى وقت يترك المعلم المجال للطلاب لتنفيذ عمليات دوران باتجاه عقارب الساعة وكذلك عكس عقارب الساعة بزوايا مختلفة في القياسات باستعمال الأدوات المتاحة لديهم.
التقويم	حل التمرين 1.	
الواجب المنزلي	يطلب المعلم من طلابه الإجابة على الأسئلة التالية 2 من كتاب الطالب الصفحة 149	

Appendix (3.18) Characteristics and Features of Storch's (2002) Pair Pattern of Interaction

Pattern of Interaction	Characteristic	Feature
Collaborative	• Moderate to high equality • Moderate to high mutuality	• Pairs incorporate or repeat each other's utterances and extend on them. • Pairs provide negative or corrective feedback in the form of explicit peer repair, as well as positive feedback in the form of confirmations • Pairs provide positive feedback in the form of confirmations • Pairs make many requests and provision of information • Pairs engage critically but constructively with each other's suggestions • participants often reach resolutions via a process of pooling resources.
Dominant/Dominant	• Moderate to high equality • Moderate to low mutuality	• Pair participate in the task, but it is not a joint construction • The level of engagement with each other's thoughts is via fixing their mistakes, which each participant does not always accept. • High level of disagreement and inability to discuss and reach agreement • Difficulty to reach a resolution that both could accept. • Pair used singular pronouns and each one of them tries to emphasise his own opinion. and

		highlight the error in the other participant's way of thinking • Pairs' voices were often raised, and emotions such as exasperation, anger, and indignation expressed
Dominant/Passive	• Moderate to low equality • Moderate to low mutuality	• One student dominates the interaction through task activity and appropriates the task and contribute more. • Dominant member asks self-directed questions rather than trying to involve the other to contribute to the task activity. • Dominant pair use self-directed questions to guide his thinking throughout the task. • One student appears limited or passive as he follows what dominant student proposed or suggest, and his participation is sort of agreeing or confirming dominant student's ideas.
Expert/Novice	• Moderate to low equality • Moderate to high mutuality	• Expert student fixed novice student's error and did not impose his opinion but give explanations. • Expert student asks question to seek to involve novice student in the task and encourage him to learn from the interaction. • Novice student confirm and repeat the suggestions made by the expert student.

Appendix (3.19) Ethical Approval from Newcastle University

Ethical Approval

Wendy Davison [redacted]
Tue 16/07/2019 13:40
To: Sameer Khormi (PGR) [redacted]
Cc: Pamela Woolner [redacted]

Dear Sameer

Thank you for your application for ethical approval of your project "*The Impact of Using Dynamic Software on Learning Process and Improving Geometry Performance, Spatial Thinking and Achieving Sustainable Learning among Primary School Students*". I confirm that Dr Simon Woods has approved it on behalf of the Faculty of Humanities and Social Sciences Ethics Committee.

Please note that this approval applies to the project protocol as stated in your application - if any amendments are made to this during the course of the project, please submit the revisions to the Ethics Committee in order for them to be reviewed and approved.

Kind regards,

Wendy

Wendy Davison
PA to Professor Matthew Grenby, Dean of Research and Innovation
Mrs Lorna Taylor, Faculty Research Manager
and Ms Louise Kempton, Associate Dean of Research and Innovation
Faculty of Humanities and Social Sciences
Great North House
Sandyford Road
Newcastle upon Tyne, NE1 8ND
https://goo.gl/maps/2K6SeRCuVY42

Telephone: 0191 208 6349
E mail: Wendy.Davison@ncl.ac.uk

Newcastle University UK | Malaysia | Singapore   Athena SWAN Silver Award   TEF Gold

Appendix (3.20) Ethical Approval from Saudi Ministry of Education

المملكة العربية السعودية
وزارة التعليم
(٢٨٠)
الإدارة العامة للتعليم بمحافظة جدة
ادارة التخطيط والمعلومات – البحوث و الدراسات

وزارة التعليم
Ministry of Education

الرقـم : ٦٨٠٤٠٧٧١٨١٠ح
التاريخ : ١٨/٥/ هـ
المرفقات :

رؤيتنا : متعلم .. معتز بدينه .. منتم لوطنه .. منتج للمعرفة .. منافس عالمياً .

" تطبيق البحث "

الأسم	سمير علي حسين خرمي			السجل المدني	١٠١٩٨٢٩٣٠٦
الجوال	٠٥٩٩٩٩٤٤٥٤		البريد الإلكتروني	skhoormy@yahoo.com	
الملحقية الثقافية	بلندن		التخصص	طرق تدريس رياضيات	
الدرجة العلمية	دكتوراه		عينة الدراسة	طلاب المرحلة الابتدائية	
عنوان الدراسة	أثر التدريس باستخدام البرامج التفاعلية على تعزيز عملية التعلم و تنمية التحصيل الهندسي و القدرة على الفكير المكاني وبقاء أثر التعلم لدى طلاب المرحلة الابتدائية.				
الموضوع بشأن	بتطبيق أدوات بحثه في تعليم جدة .				

إلـــى : سعادة الملحق الثقافي بلندن.

مـــــن : مدير عام التعليم بمحافظة جدة .

السلام عليكم ورحمة الله وبركاته

بناء على إفادتكم (الموضح بياناته أعلاه)، واستجابة لرغبته في تزويدكم بالموافقة بتطبيق بحثه و جمع البيانات المتعلقة بعينة الدراسة .

نفيدكم أنه لا مانع لدينا بالموافقة بعد دراسة أداة البحث ؛ و ذلك تشجيعاً للبحث العلمي وبما يعود على الوطن بمثل هذه البحوث الميدانية ؛ شاكرين ومقدرين تعاونكم واهتمامكم.

والسلام عليكم ورحمة الله وبركاته

مدير عام التعليم بمحافظة جدة

عبدالله بن أحمد الثقفي

هاتف ٦٤٤٤٣٠٥ – فاكس ٦٤٣٤٠٤٠ – الرمز البريدي : ٢١١٥٨

Appendix (4.1) ANOVA Outcome for Geometric Performance Pre-test

Descriptives								
Pre-test								
Group	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	5.1600	1.86369	.37274	4.3907	5.9293	1.00	9.00
Hands-on	26	5.2308	2.23263	.43785	4.3290	6.1325	2.00	10.00
Traditional teaching	26	5.4615	1.63048	.31976	4.8030	6.1201	2.00	8.00
Total	77	5.2857	1.90468	.21706	4.8534	5.7180	1.00	10.00

Test of Homogeneity of Variances			
Pre-test			
Levene Statistic	df1	df2	Sig.
1.768	2	74	.178

ANOVA					
Pre-test					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1.277	2	.639	.172	.842
Within Groups	274.437	74	3.709		
Total	275.714	76			

Robust Tests of Equality of Means				
Pre-test				
	Statistic[a]	df1	df2	Sig.
Welch	.207	2	48.533	.814
a. Asymptotically F distributed.				

Multiple Comparisons
Dependent Variable: pre-test
Tukey HSD

(I) group	(J) group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	-.07077	.53943	.991	-1.3610	1.2194
	Traditional teaching	-.30154	.53943	.842	-1.5917	.9886
Hands-on	Geogebra	.07077	.53943	.991	-1.2194	1.3610
	Traditional teaching	-.23077	.53411	.902	-1.5082	1.0467
Traditional teaching	Geogebra	.30154	.53943	.842	-.9886	1.5917
	Hands-on	.23077	.53411	.902	-1.0467	1.5082

Homogeneous Subsets

Pre-test		
Tukey HSD[a,b]		
group	N	Subset for alpha = 0.05
		1
GeoGebra	25	5.1600
Hands-on	26	5.2308
Traditional teaching	26	5.4615
Sig.		.841
Means for groups in homogeneous subsets are displayed.		
a. Uses Harmonic Mean Sample Size = 25.658.		
b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.		

Appendix (4.2) ANOVA Outcome for Geometric Performance Post-test

Descriptives

Post-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	15.640	2.73679	.54736	14.5103	16.7697	11.00	21.00
Hands-on	26	12.923	3.58801	.70367	11.4738	14.3723	5.00	20.00
Traditional teaching	26	10.653	3.78357	.74202	9.1256	12.1821	4.00	19.00
Total	77	13.039	3.93514	.44845	12.1458	13.9321	4.00	21.00

Test of Homogeneity of Variances

Post-test

Levene Statistic	df1	df2	Sig.
1.544	2	74	.220

ANOVA

Post-test

	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	317.392	2	158.696	13.663	.000
Within Groups	859.491	74	11.615		
Total	1176.883	76			

Robust Tests of Equality of Means

Post-test

	Statistic[a]	df1	df2	Sig.
Welch	15.083	2	48.599	.000

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Post-test

Tukey HSD

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	2.71692*	.95463	.016	.4337	5.0002
	Traditional teaching	4.98615*	.95463	.000	2.7029	7.2694
Hands-on	Geogebra	-2.71692*	.95463	.016	-5.0002	-.4337
	Traditional teaching	2.26923*	.94522	.049	.0085	4.5300
Traditional teaching	Geogebra	-4.98615*	.95463	.000	-7.2694	-2.7029
	Hands-on	-2.26923*	.94522	.049	-4.5300	-.0085

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Post-test

Tukey HSD[a,b]

Group	N	Subset for alpha = 0.05	
		1	2
Traditional Teaching	26	10.6538	
Hands-on	26	12.9231	
GeoGebra	25		15.6400
Sig.		.051	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 25.658.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Appendix (4.3) Univariate Analysis of Variance of Geometric Performance ANCOVA

Between-Subjects Factors		
		N
Group	Geogebra	25
	Hands-on	26
	Traditional teaching	26

Tests of Between-Subjects Effects					
Dependent Variable: Geometric Performance Pre-test					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4.924[a]	2	2.462	.662	.519
Intercept	2046.213	1	2046.213	550.205	.000
Group	4.924	2	2.462	.662	.519
Error	275.206	74	3.719		
Total	2327.000	77			
Corrected Total	280.130	76			
a. R Squared = .018 (Adjusted R Squared = -.009)					

Levene's Test of Equality of Error Variances[a]			
F	df1	df2	Sig.
1.546	2	74	.220
Tests the null hypothesis that the error variance of the dependent variable is equal across groups.			
a. Design: Intercept + Geometric Performance Pre-test + Teaching Method			

Univariate Analysis of Variance

Between-Subjects Factors		
		N
Group	Geogebra	25
	Hands-on	26
	Traditional teaching	26

Descriptive Statistics			
Dependent Variable: Geometric Performance Post-test			
Group	Mean	Std. Deviation	N
Geogebra	15.6400	2.73679	25
Hands-on	12.9231	3.58801	26
Traditional teaching	10.6538	3.78357	26
Total	13.0390	3.93514	77

Levene's Test of Equality of Error Variances[a]			
Dependent Variable: Geometric Performance Post-test			
F	df1	df2	Sig.
1.546	2	74	.220
Tests the null hypothesis that the error variance of the dependent variable is equal across groups.			
a. Design: Intercept + Geometric Performance Pre-test + Teaching Method			

Tests of Between-Subjects Effects						
Dependent Variable: Geometric Performance Post-test						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	317.393[a]	3	105.798	8.986	.000	.270
Intercept	1558.099	1	1558.099	132.336	.000	.644
GP_before	.000	1	.000	.000	.996	.000
Group	316.303	2	158.152	13.432	.000	.269
Error	859.490	73	11.774			
Total	14268.000	77				
Corrected Total	1176.883	76				
a. R Squared = .270 (Adjusted R Squared = .240)						

Estimated Marginal Means

1. Grand Mean			
Dependent Variable: Geometric Performance Post-test			
Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
13.072[a]	.391	12.293	13.852
a. Covariates appearing in the model are evaluated at the following values: Geometric Performance Pre-test = 5.1558.			

2. Group

Estimates				
Dependent Variable: Geometric Performance Post-test				
Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	15.640[a]	.686	14.272	17.008
Hands-on	12.923[a]	.676	11.576	14.271
Traditional teaching	10.654[a]	.676	9.306	12.001
a. Covariates appearing in the model are evaluated at the following values: Geometric Performance Pre-test = 5.1558.				

Pairwise Comparisons						
Dependent Variable: Geometric Performance Post-test						
(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Geogebra	Hands-on	2.717*	.963	.019	.356	5.077
	Traditional teaching	4.986*	.963	.000	2.626	7.347
Hands-on	Geogebra	-2.717*	.963	.019	-5.077	-.356
	Traditional teaching	2.270	.960	.062	-.083	4.623
Traditional teaching	Geogebra	-4.986*	.963	.000	-7.347	-2.626
	Hands-on	-2.270	.960	.062	-4.623	.083
Based on estimated marginal means						
*. The mean difference is significant at the .05 level.						
b. Adjustment for multiple comparisons: Bonferroni.						

Univariate Tests						
Dependent Variable: Geometric Performance Post-test						
	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Contrast	316.303	2	158.152	13.432	.000	.269
Error	859.490	73	11.774			
The F tests the effect of Group. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.						

Appendix (4.4) ANOVA Outcome for Spatial Thinking Pre-test

Descriptives

Spatial thinking Pre-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	27.6800	5.03090	1.00618	25.6033	29.7567	15.00	34.00
Hands-on	26	27.7308	4.66097	.91409	25.8482	29.6134	16.00	34.00
Traditional teaching	26	27.2308	3.65850	.71749	25.7531	28.7085	14.00	35.00
Total	77	27.5455	4.42643	.50444	26.5408	28.5501	14.00	35.00

Test of Homogeneity of Variances

Spatial thinking Pre-test

Levene Statistic	df1	df2	Sig.
2.461	2	74	.092

ANOVA

Spatial thinking Pre-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3.920	2	1.960	.098	.907
Within Groups	1485.171	74	20.070		
Total	1489.091	76			

Robust Tests of Equality of Means

Spatial thinking Pre-test

	Statistic[a]	df1	df2	Sig.
Welch	.116	2	48.065	.891

a. Asymptotically F distributed.

Appendix (4.5) ANOVA Outcome for Spatial Thinking Post-test

Descriptives

Spatial thinking Post-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	32.2000	3.70810	.74162	30.6694	33.7306	17.00	35.00
Hands-on	26	29.4615	4.56273	.89482	27.6186	31.3045	18.00	35.00
Traditional teaching	26	27.9231	3.30966	.64908	26.5863	29.2599	16.00	32.00
Total	77	29.8312	4.23458	.48257	28.8700	30.7923	16.00	35.00

Test of Homogeneity of Variances

Spatial thinking Post-test

Levene Statistic	df1	df2	Sig.
1.693	2	74	.191

ANOVA

Spatial thinking Post-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	238.498	2	119.249	7.849	.001
Within Groups	1124.308	74	15.193		
Total	1362.805	76			

Robust Tests of Equality of Means

Spatial thinking Post-test

	Statistic[a]	df1	df2	Sig.
Welch	9.336	2	48.549	.000

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons						
Dependent Variable: Spatial thinking Post-test						
Tukey HSD						
(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	2.73846*	1.09183	.038	.1271	5.3499
	Traditional teaching	4.27692*	1.09183	.001	1.6655	6.8883
Hands-on	Geogebra	-2.73846*	1.09183	.038	-5.3499	-.1271
	Traditional teaching	1.53846	1.08107	.334	-1.0472	4.1241
Traditional teaching	Geogebra	-4.27692*	1.09183	.001	-6.8883	-1.6655
	Hands-on	-1.53846	1.08107	.334	-4.1241	1.0472
*. The mean difference is significant at the 0.05 level.						

Homogeneous Subsets

SP			
Tukey HSD[a,b]			
Group	N	Subset for alpha = 0.05	
		1	2
Traditional Teaching	26	27.9231	
Hands-on	26	29.4615	
GeoGebra	25		32.2000
Sig.		.339	1.000
Means for groups in homogeneous subsets are displayed.			
a. Uses Harmonic Mean Sample Size = 25.658.			
b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.			

Appendix (4.6) Univariate Analysis of Variance of Spatial Thinking

ANCOVA

Between-Subjects Factors		
		N
Group	Geogebra	25
	Hands-on	26
	Traditional teaching	26

Tests of Between-Subjects Effects					
Dependent Variable: Spatial thinking Pre-test					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	925.827[a]	5	185.165	30.086	.000
Intercept	217.860	1	217.860	35.398	.000
Method	58.815	2	29.407	4.778	.011
Spatial thinking Pre-test	651.372	1	651.372	105.835	.000
Method * Spatial thinking Pre-test	31.010	2	15.505	2.519	.088
Error	436.978	71	6.155		
Total	69885.000	77			
Corrected Total	1362.805	76			
a. R Squared = .679 (Adjusted R Squared = .657)					

Levene's Test of Equality of Error Variances[a]			
F	df1	df2	Sig.
1.207	2	74	.305
Tests the null hypothesis that the error variance of the dependent variable is equal across groups.			
a. Design: Intercept + Spatial thinking Pre-test + Teaching Methods			

Univariate Analysis of Variance

Between-Subjects Factors		
		N
Methods	Geogebra	25
	Hands-on	26

	Traditional teaching	26

Descriptive Statistics			
Dependent Variable: Spatial thinking Post-test			
Methods	Mean	Std. Deviation	N
Geogebra	32.2000	3.70810	25
Hands-on	29.4615	4.56273	26
Traditional teaching	27.9231	3.30966	26
Total	29.8312	4.23458	77

Levene's Test of Equality of Error Variances[a]			
Dependent Variable: Spatial thinking Post-test			
F	df1	df2	Sig.
1.207	2	74	.305
Tests the null hypothesis that the error variance of the dependent variable is equal across groups.			
a. Design: Intercept + Spatial thinking Pre-test + Teaching Methods			

Tests of Between-Subjects Effects						
Dependent Variable: Spatial thinking Post-test						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	894.817[a]	3	298.272	46.527	.000	.657
Intercept	254.573	1	254.573	39.710	.000	.352
Spatial thinking Pre-test	656.319	1	656.319	102.377	.000	.584
Methods	211.001	2	105.500	16.457	.000	.311
Error	467.988	73	6.411			
Total	69885.000	77				
Corrected Total	1362.805	76				
a. R Squared = .657 (Adjusted R Squared = .642)						

Estimated Marginal Means

1. Grand Mean			
Dependent Variable: Spatial thinking Post-test			
Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
29.860[a]	.289	29.285	30.436

a. Covariates appearing in the model are evaluated at the following values: Spatial thinking Pre-test = 27.5455.

2. Methods

Estimates

Dependent Variable: Spatial thinking Post-test

Methods	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	32.111[a]	.506	31.101	33.120
Hands-on	29.338[a]	.497	28.348	30.328
Traditional teaching	28.132[a]	.497	27.142	29.123

a. Covariates appearing in the model are evaluated at the following values: Spatial thinking Pre-test = 27.5455.

Pairwise Comparisons

Dependent Variable: Spatial thinking Post-test

(I) Methods	(J) Methods	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Geogebra	Hands-on	2.772*	.709	.000	1.359	4.186
	Traditional teaching	3.978*	.710	.000	2.564	5.393
Hands-on	Geogebra	-2.772*	.709	.000	-4.186	-1.359
	Traditional teaching	1.206	.703	.090	-.195	2.607
Traditional teaching	Geogebra	-3.978*	.710	.000	-5.393	-2.564
	Hands-on	-1.206	.703	.090	-2.607	.195

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: Spatial thinking Pre-test

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Contrast	211.001	2	105.500	16.457	.000	.311
Error	467.988	73	6.411			

The F tests the effect of Methods. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Appendix (4.7) ANOVA Outcome for Delayed Test

Descriptives

Delayed test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	11.920	3.04029	.60806	10.6650	13.1750	6.00	18.00
Hands-on	26	9.9231	3.24867	.63712	8.6109	11.2352	4.00	17.00
Traditional teaching	26	7.8846	2.45482	.48143	6.8931	8.8761	2.00	13.00
Total	77	9.8831	3.33235	.37976	9.1268	10.6395	2.00	18.00

Test of Homogeneity of Variances

Delayed test

Levene Statistic	df1	df2	Sig.
.788	2	74	.459

ANOVA

Delayed test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	207.608	2	103.804	12.071	.000
Within Groups	636.340	74	8.599		
Total	843.948	76			

Robust Tests of Equality of Means

Delayed

	Statistic[a]	df1	df2	Sig.
Welch	13.563	2	48.409	.000

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons						
Dependent Variable: Delayed test						
Tukey HSD						
(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	1.99692*	.82140	.045	.0323	3.9615
	Traditional teaching	4.03538*	.82140	.000	2.0708	6.0000
Hands-on	Geogebra	-1.99692*	.82140	.045	-3.9615	-.0323
	Traditional teaching	2.03846*	.81331	.038	.0932	3.9837
Traditional teaching	Geogebra	-4.03538*	.82140	.000	-6.0000	-2.0708
	Hands-on	-2.03846*	.81331	.038	-3.9837	-.0932
*. The mean difference is significant at the 0.05 level.						

Homogeneous Subsets

Delayed				
Tukey HSD[a,b]				
Group	N	Subset for alpha = 0.05		
		1	2	3
Traditional Teaching	26	7.8846		
Hands-on	26		9.9231	
GeoGebra	25			11.9200
Sig.		1.000	1.000	1.000
Means for groups in homogeneous subsets are displayed.				
a. Uses Harmonic Mean Sample Size = 25.658.				
b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.				

Appendix (4.8) ANOVA Outcome for Mathematics Academic Self-concept Pre-test

Descriptives

Math academic self-concept pre-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	34.7200	4.83494	.96699	32.7242	36.7158	25.00	46.00
Hands-on	26	37.2692	6.11593	1.19943	34.7990	39.7395	25.00	48.00
Traditional teaching	26	36.7308	5.77275	1.13213	34.3991	39.0624	20.00	46.00
Total	77	36.2597	5.64382	.64317	34.9788	37.5407	20.00	48.00

Test of Homogeneity of Variances

Math academic self-concept pre-test

Levene Statistic	df1	df2	Sig.
.477	2	74	.623

ANOVA

Math academic self-concept pre-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	91.534	2	45.767	1.454	.240
Within Groups	2329.271	74	31.477		
Total	2420.805	76			

Robust Tests of Equality of Means

Math academic self-concept pre-test

	Statistic[a]	df1	df2	Sig.
Welch	1.621	2	49.047	.208

a. Asymptotically F distributed.

Appendix (4.9) ANOVA Outcome for Mathematics Academic Self-concept Post-test

Descriptives

Math academic self-concept post-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	41.9600	5.59375	1.11875	39.6510	44.2690	32.00	50.00
Hands-on	26	37.4231	7.28930	1.42955	34.4789	40.3673	23.00	50.00
Traditional teaching	26	36.9231	7.07063	1.38667	34.0672	39.7790	23.00	48.00
Total	77	38.7273	6.99556	.79722	37.1395	40.3151	23.00	50.00

Test of Homogeneity of Variances

Math academic self-concept post-test

Levene Statistic	df1	df2	Sig.
.930	2	74	.399

ANOVA

Math academic self-concept post-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	390.120	2	195.060	4.336	.017
Within Groups	3329.152	74	44.989		
Total	3719.273	76			

Robust Tests of Equality of Means

Math academic self-concept post-test

	Statistic[a]	df1	df2	Sig.
Welch	5.088	2	48.872	.010

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Math academic self-concept post-test

Tukey HSD

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	4.53692*	1.87879	.047	.0433	9.0306
	Traditional teaching	5.03692*	1.87879	.024	.5433	9.5306
Hands-on	Geogebra	-4.53692*	1.87879	.047	-9.0306	-.0433
	Traditional teaching	.50000	1.86028	.961	-3.9494	4.9494
Traditional teaching	Geogebra	-5.03692*	1.87879	.024	-9.5306	-.5433
	Hands-on	-.50000	1.86028	.961	-4.9494	3.9494

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Math academic self-concept post-test

Tukey HSD[a,b]

Group	N	Subset for alpha = 0.05	
		1	2
Traditional Teaching	26	36.9231	
Hands-on	26	37.4231	
GeoGebra	25		41.9600
Sig.		.961	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 25.658.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Appendix (4.10) Univariate Analysis of Variance of Mathematics Academic Self-concept ANCOVA

Between-Subjects Factors

		N
Group	Geogebra	25
	Hands-on	26
	Traditional teaching	26

Tests of Between-Subjects Effects

Dependent Variable: Math academic self-concept post-test

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	873.274[a]	5	174.655	4.357	.002	.235
Intercept	985.953	1	985.953	24.597	.000	.257
Group	122.744	2	61.372	1.531	.223	.041
Math academic self-concept pre-test	348.766	1	348.766	8.701	.004	.109
Group * Math academic self-concept pre-test	66.432	2	33.216	.829	.441	.023
Error	2845.998	71	40.084			
Total	119204.000	77				
Corrected Total	3719.273	76				

a. R Squared = .235 (Adjusted R Squared = .181)

Univariate Analysis of Variance

Descriptive Statistics

Dependent Variable: Math academic self-concept post-test

Group	Mean	Std. Deviation	N
Geogebra	41.9600	5.59375	25
Hands-on	37.4231	7.28930	26
Traditional teaching	36.9231	7.07063	26
Total	38.7273	6.99556	77

Levene's Test of Equality of Error Variances[a]			
Dependent Variable: Math academic self-concept post-test			
F	df1	df2	Sig.
.308	2	74	.736
Tests the null hypothesis that the error variance of the dependent variable is equal across groups.			
a. Design: Intercept + Math academic self-concept pre-test + Teaching Method			

Tests of Between-Subjects Effects

Dependent Variable: Math academic self-concept post-test

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	806.842[a]	3	268.947	6.741	.000	.217
Intercept	952.514	1	952.514	23.875	.000	.246
Math academic self-concept pre-test	416.722	1	416.722	10.445	.002	.125
Group	539.515	2	269.758	6.761	.002	.156
Error	2912.430	73	39.896			
Total	119204.000	77				
Corrected Total	3719.273	76				

a. R Squared = .217 (Adjusted R Squared = .185)

Estimated Marginal Means

1. Grand Mean

Dependent Variable: Math academic self-concept post-test

Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
38.777[a]	.720	37.342	40.212

a. Covariates appearing in the model are evaluated at the following values: Math academic self-concept pre-test = 36.2597.

2. Group

Estimates

Dependent Variable: Math academic self-concept post-test

Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound

Geogebra	42.611[a]	1.279	40.062	45.161
Hands-on	36.996[a]	1.246	34.513	39.479
Traditional teaching	36.724[a]	1.240	34.252	39.196
a. Covariates appearing in the model are evaluated at the following values: Math academic self-concept pre-test = 36.2597.				

Pairwise Comparisons

Dependent Variable: Math academic self-concept post-test

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Geogebra	Hands-on	5.615*	1.800	.008	1.203	10.027
	Traditional teaching	5.887*	1.789	.005	1.504	10.271
Hands-on	Geogebra	-5.615*	1.800	.008	-10.027	-1.203
	Traditional teaching	.272	1.753	1.000	-4.024	4.568
Traditional teaching	Geogebra	-5.887*	1.789	.005	-10.271	-1.504
	Hands-on	-.272	1.753	1.000	-4.568	4.024

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Bonferroni.

Univariate Tests

Dependent Variable: Math academic self-concept post-test

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Contrast	539.515	2	269.758	6.761	.002	.156	13.523	.907
Error	2912.430	73	39.896					

The F tests the effect of Group. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Computed using alpha = .05

Appendix (4.11) ANOVA Outcome for Enjoyment of Mathematics Pre-test

Descriptives

Enjoyment of Math pre-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	27.8000	5.39290	1.07858	25.5739	30.0261	16.00	36.00
Hands-on	26	28.5385	5.31587	1.04253	26.3913	30.6856	16.00	36.00
Traditional teaching	26	28.6538	7.16627	1.40542	25.7593	31.5484	9.00	35.00
Total	77	28.3377	5.95954	.67915	26.9850	29.6903	9.00	36.00

Test of Homogeneity of Variances

Enjoyment of Math pre-test

Levene Statistic	df1	df2	Sig.
2.477	2	74	.091

ANOVA

Enjoyment of Math pre-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	10.875	2	5.437	.150	.861
Within Groups	2688.346	74	36.329		
Total	2699.221	76			

Robust Tests of Equality of Means

Enjoyment of Math pre-test

	Statistic[a]	df1	df2	Sig.
Welch	.163	2	48.709	.850

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons						
Dependent Variable: Enjoyment of Math pre-test						
Tukey HSD						
(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	-.73846	1.68832	.900	-4.7765	3.2996
	Traditional teaching	-.85385	1.68832	.869	-4.8919	3.1842
Hands-on	Geogebra	.73846	1.68832	.900	-3.2996	4.7765
	Traditional teaching	-.11538	1.67169	.997	-4.1137	3.8829
Traditional teaching	Geogebra	.85385	1.68832	.869	-3.1842	4.8919
	Hands-on	.11538	1.67169	.997	-3.8829	4.1137

Enjoyment of Math pre-test		
Tukey HSD[a,b]		
Group	N	Subset for alpha = 0.05
		1
Geogebra	25	27.8000
Hands-on	26	28.5385
Traditional teaching	26	28.6538
Sig.		.868
Means for groups in homogeneous subsets are displayed.		
a. Uses Harmonic Mean Sample Size = 25.658.		
b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.		

Appendix (4.12) ANOVA Outcome for Enjoyment of Mathematics Post-test

Descriptives

Enjoyment of Math Post-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	33.760	5.71022	1.1420	31.4029	36.1171	20.00	40.00
Hands-on	26	29.731	5.45203	1.0692	27.5286	31.9329	18.00	37.00
Traditional teaching	26	29.500	6.31348	1.2382	26.9499	32.0501	8.00	36.00
Total	77	30.961	6.08372	.6933	29.5802	32.3419	8.00	40.00

Test of Homogeneity of Variances

score

Levene Statistic	df1	df2	Sig.
.053	2	74	.949

ANOVA

Enjoyment of Math Post-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	290.708	2	145.354	4.265	.018
Within Groups	2522.175	74	34.083		
Total	2812.883	76			

Robust Tests of Equality of Means

Enjoyment of Math Post-test

	Statistic[a]	df1	df2	Sig.
Welch	4.300	2	49.146	.019

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Enjoyment of Math Post-test

Tukey HSD

(I) group	(J) group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	4.02923*	1.63531	.042	.1180	7.9405
	Traditional teaching	4.26000*	1.63531	.030	.3487	8.1713
Hands-on	Geogebra	-4.02923*	1.63531	.042	-7.9405	-.1180
	Traditional teaching	.23077	1.61920	.989	-3.6420	4.1035
Traditional teaching	Geogebra	-4.26000*	1.63531	.030	-8.1713	-.3487
	Hands-on	-.23077	1.61920	.989	-4.1035	3.6420

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Enjoyment of Math Post-test

Tukey HSD[a,b]

group	N	Subset for alpha = 0.05	
		1	2
Traditional Teaching	26	29.5000	
Hands-on	26	29.7308	
GeoGebra	25		33.7600
Sig.		.989	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 25.658.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Appendix (4.13) Univariate Analysis of Variance of Enjoyment of Mathematics ANCOVA

Between-Subjects Factors		
		N
Group	Geogebra	25
	Hands-on	26
	Traditional teaching	26

Descriptive Statistics			
Dependent Variable: Enjoyment of Math Pre-test			
Group	Mean	Std. Deviation	N
Geogebra	27.8000	5.39290	25
Hands-on	28.5385	5.31587	26
Traditional teaching	28.6538	7.16627	26
Total	28.3377	5.95954	77

Tests of Between-Subjects Effects						
Dependent Variable: Enjoyment of Math Pre-test						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	10.875[a]	2	5.437	.150	.861	.004
Intercept	61781.579	1	61781.579	1700.613	.000	.958
Group	10.875	2	5.437	.150	.861	.004
Error	2688.346	74	36.329			
Total	64532.000	77				
Corrected Total	2699.221	76				
a. R Squared = .004 (Adjusted R Squared = -.023)						

Univariate Analysis of Variance

Descriptive Statistics

Dependent Variable: Enjoyment of Math Post-test

Group	Mean	Std. Deviation	N
Geogebra	33.7600	5.71022	25
Hands-on	29.7308	5.45203	26
Traditional teaching	29.5000	6.31348	26
Total	30.9610	6.08372	77

Levene's Test of Equality of Error Variances[a]

Dependent Variable: Enjoyment of Math Post-test

F	df1	df2	Sig.
.281	2	74	.756

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Enjoyment of Math Pre-test + Teaching Method

Tests of Between-Subjects Effects

Dependent Variable: Enjoyment of Math Post-test

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	818.410[a]	3	272.803	9.985	.000	.291
Intercept	1092.013	1	1092.013	39.969	.000	.354
Enjoyment of Math Pre-test	527.702	1	527.702	19.314	.000	.209
Group	341.141	2	170.570	6.243	.003	.146
Error	1994.473	73	27.322			
Total	76624.000	77				
Corrected Total	2812.883	76				

a. R Squared = .291 (Adjusted R Squared = .262)

Estimated Marginal Means

1. Grand Mean			
Dependent Variable: Enjoyment of Math Post-test			
Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
31.000[a]	.596	29.813	32.187
a. Covariates appearing in the model are evaluated at the following values: Enjoyment of Math Pre-test = 28.3377.			

2. Group

Estimates				
Dependent Variable Enjoyment of Math Post-test				
Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	33.998[a]	1.047	31.912	36.084
Hands-on	29.642[a]	1.025	27.598	31.685
Traditional teaching	29.360[a]	1.026	27.316	31.404
a. Covariates appearing in the model are evaluated at the following values: Enjoyment of Math Pre-test = 28.3377.				

Pairwise Comparisons						
Dependent Variable: Enjoyment of Math Post-test						
(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Geogebra	Hands-on	4.356*	1.466	.004	1.435	7.278
	Traditional teaching	4.638*	1.467	.002	1.715	7.561
Hands-on	Geogebra	-4.356*	1.466	.004	-7.278	-1.435
	Traditional teaching	.282	1.450	.846	-2.607	3.171
Traditional teaching	Geogebra	-4.638*	1.467	.002	-7.561	-1.715
	Hands-on	-.282	1.450	.846	-3.171	2.607
Based on estimated marginal means						
*. The mean difference is significant at the .05 level.						
b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).						

Univariate Tests					
Dependent Variable: Enjoyment of Math Post-test					
	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Contrast	341.141	2	170.570	6.243	.003	.146
Error	1994.473	73	27.322			
The F tests the effect of Group. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.						

Appendix (4.14) ANOVA Outcome for Perceived Value of Mathematics Pre-test

Descriptives

Perceived Value of Math Pre-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	33.6800	7.90949	1.58190	30.4151	36.9449	17.00	45.00
Hands-on	26	36.1923	5.69223	1.11634	33.8932	38.4915	21.00	45.00
Traditional teaching	26	37.4231	6.31299	1.23808	34.8732	39.9729	23.00	45.00
Total	77	35.7922	6.77910	.77255	34.2535	37.3309	17.00	45.00

Test of Homogeneity of Variances

Perceived Value of Math Pre-test

Levene Statistic	df1	df2	Sig.
.993	2	74	.375

ANOVA

Perceived Value of Math Pre-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	184.851	2	92.425	2.068	.134
Within Groups	3307.825	74	44.700		
Total	3492.675	76			

Robust Tests of Equality of Means

Perceived Value of Math Pre-test

	Statistic[a]	df1	df2	Sig.
Welch	1.721	2	48.202	.190

a. Asymptotically F distributed.

Multiple Comparisons

Dependent Variable: Perceived Value of Math Pre-test

Tukey HSD

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	-2.51231	1.87277	.377	-6.9915	1.9669
	Traditional teaching	-3.74308	1.87277	.120	-8.2223	.7361
Hands-on	Geogebra	2.51231	1.87277	.377	-1.9669	6.9915
	Traditional teaching	-1.23077	1.85432	.785	-5.6659	3.2043
Traditional teaching	Geogebra	3.74308	1.87277	.120	-.7361	8.2223
	Hands-on	1.23077	1.85432	.785	-3.2043	5.6659

Appendix (4.15) ANOVA Outcome for Perceived Value of Mathematics Post-test

Descriptives

Perceived Value of Math Post-test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Geogebra	25	41.720	5.71198	1.14240	39.3622	44.0778	22.00	47.00
Hands-on	26	37.731	4.82956	.94715	35.7801	39.6815	28.00	45.00
Traditional teaching	26	36.462	5.90749	1.15855	34.0755	38.8476	17.00	43.00
Total	77	38.597	5.87201	.66918	37.2646	39.9302	17.00	47.00

Test of Homogeneity of Variances

Perceived Value of Math Post-test

Levene Statistic	df1	df2	Sig.
.140	2	74	.870

ANOVA

Perceived Value of Math Post-test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	381.903	2	190.951	6.312	.003
Within Groups	2238.617	74	30.252		
Total	2620.519	76			

Robust Tests of Equality of Means

Perceived Value of Math Post-test

	Statistic[a]	df1	df2	Sig.
Welch	5.770	2	48.778	.006

a. Asymptotically F distributed.

Post Hoc Tests

Multiple Comparisons

Dependent Variable: Perceived Value of Math Post-test

Tukey HSD

(I) group	(J) group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Geogebra	Hands-on	3.98923*	1.54064	.031	.3044	7.6741
	Traditional teaching	5.25846*	1.54064	.003	1.5736	8.9433
Hands-on	Geogebra	-3.98923*	1.54064	.031	-7.6741	-.3044
	Traditional teaching	1.26923	1.52547	.684	-2.3793	4.9178
Traditional teaching	Geogebra	-5.25846*	1.54064	.003	-8.9433	-1.5736
	Hands-on	-1.26923	1.52547	.684	-4.9178	2.3793

*. The mean difference is significant at the 0.05 level.

Homogeneous Subsets

Perceived Value of Math Post-test

Tukey HSD[a,b]

group	N	Subset for alpha = 0.05	
		1	2
Traditional teaching	26	36.4615	
Hands-on	26	37.7308	
GeoGebra	25		41.7200
Sig.		.688	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 25.658.

b. The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Appendix (4.16) Univariate Analysis of Variance of Perceived Value of Mathematics ANCOVA

Between-Subjects Factors		
		N
Group	Geogebra	25
	Hands-on	26
	Traditional teaching	26

Tests of Between-Subjects Effects						
Dependent Variable: Perceived Value of Math Post-test						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	525.098[a]	5	105.020	3.558	.006	.200
Intercept	2331.047	1	2331.047	78.984	.000	.527
Group	10.312	2	5.156	.175	.840	.005
Perceived Value of Math pre-test	116.991	1	116.991	3.964	.050	.053
Group * Perceived Value of Math pre-test	13.880	2	6.940	.235	.791	.007
Error	2095.421	71	29.513			
Total	117332.000	77				
Corrected Total	2620.519	76				
a. R Squared = .200 (Adjusted R Squared = .144)						

Univariate Analysis of Variance

Descriptive Statistics			
Dependent Variable: Perceived Value of Math Post-test			
Group	Mean	Std. Deviation	N
Geogebra	41.7200	5.71198	25
Hands-on	37.7308	4.82956	26
Traditional teaching	36.4615	5.90749	26
Total	38.5974	5.87201	77

Levene's Test of Equality of Error Variances[a]			
Dependent Variable: Perceived Value of Math Post-test			
F	df1	df2	Sig.
.124	2	74	.884
Tests the null hypothesis that the error variance of the dependent variable is equal across groups.			
a. Design: Intercept + Perceived Value of Math Pre-test + teaching method			

Tests of Between-Subjects Effects						
Dependent Variable: Perceived Value of Math Post-test						
Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	511.218[a]	3	170.406	5.898	.001	.195
Intercept	2492.920	1	2492.920	86.277	.000	.542
Math_value_before	129.316	1	129.316	4.475	.038	.058
Group	467.769	2	233.884	8.094	.001	.182
Error	2109.301	73	28.895			
Total	117332.000	77				
Corrected Total	2620.519	76				
a. R Squared = .195 (Adjusted R Squared = .162)						

Estimated Marginal Means

1. Grand Mean			
Dependent Variable: Perceived Value of Math Post-test			
Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
38.643[a]	.613	37.422	39.864
a. Covariates appearing in the model are evaluated at the following values: Perceived Value of Math Pre-test = 35.7922.			

2. Group

Dependent Variable: Perceived Value of Math Post-test

Group	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Geogebra	42.138[a]	1.093	39.959	44.316
Hands-on	37.652[a]	1.055	35.549	39.754
Traditional teaching	36.139[a]	1.065	34.016	38.262

a. Covariates appearing in the model are evaluated at the following values: Perceived Value of Math Pre-test = 35.7922.

Pairwise Comparisons

Dependent Variable: Perceived Value of Math Post-test

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Geogebra	Hands-on	4.486*	1.524	.004	1.449	7.523
	Traditional teaching	5.999*	1.546	.000	2.918	9.079
Hands-on	Geogebra	-4.486*	1.524	.004	-7.523	-1.449
	Traditional teaching	1.513	1.495	.315	-1.468	4.493
Traditional teaching	Geogebra	-5.999*	1.546	.000	-9.079	-2.918
	Hands-on	-1.513	1.495	.315	-4.493	1.468

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

Univariate Tests

Dependent Variable: Perceived Value of Math Post-test

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Contrast	467.769	2	233.884	8.094	.001	.182
Error	2109.301	73	28.895			

The F tests the effect of Group. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Appendix (4.17) One-way Repeated Measure ANOVA

Mauchly's Test of Sphericity							
Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[b]		
					Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Time	.427	19.331	5	.002	.700	.768	.333
Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.							
a. Design: Intercept Within Subjects Design: Time							
b. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.							

Tests of Within-Subjects Effects of Time							
Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	Sphericity Assumed	9999.800	3	3333.267	26.555	.000	.525
	Greenhouse-Geisser	9999.800	2.099	4763.462	26.555	.000	.525
	Huynh-Feldt	9999.800	2.305	4338.256	26.555	.000	.525
	Lower-bound	9999.800	1.000	9999.800	26.555	.000	.525
Error(Time)	Sphericity Assumed	9037.700	72	125.524			
	Greenhouse-Geisser	9037.700	50.383	179.382			
	Huynh-Feldt	9037.700	55.321	163.369			
	Lower-bound	9037.700	24.000	376.571			

Paired Samples T-test s with a Bonferroni Correction						
(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Before	Week1	-7.120	3.863	.466	-18.227	3.987
	Week2	-17.840*	3.742	.000	-28.600	-7.080

	After	-26.160*	3.742	.000	-36.917	-15.403
Week1	Before	7.120	3.863	.466	-3.987	18.227
	Week2	-10.720*	2.482	.001	-17.857	-3.583
	After	-19.040*	2.879	.000	-27.317	-10.763
Week2	Before	17.840*	3.742	.000	7.080	28.600
	Week1	10.720*	2.482	.001	3.583	17.857
	After	-8.320*	1.695	.000	-13.193	-3.447
After	Before	26.160*	3.742	.000	15.403	36.917
	Week1	19.040*	2.879	.000	10.763	27.317
	Week2	8.320*	1.695	.000	3.447	13.193
Based on estimated marginal means						
*. The mean difference is significant at the .05 level.						
b. Adjustment for multiple comparisons: Bonferroni.						

Summary of the Descriptive statistics			
	Mean	Std. Deviation	N
Before	64.7200	21.71121	25
Week 1	71.8400	17.32455	25
Week 2	82.5600	10.61477	25
After	90.8800	9.44863	25

Appendix (4.18) Simple linear regression between Geometric Performance and Spatial Thinking

Table (1): Person Correlations Coefficient between Geometric Performance and Spatial Thinking

		Geometric Performance	Spatial Thinking
Pearson Correlation	Geometric Performance	1.000	.483
	Spatial Thinking	.483	1.000
Sig. (1-tailed)	Geometric Performance	.	.000
	Spatial Thinking	.000	.
N	Geometric Performance	77	77
	Spatial Thinking	77	77

Table (2): Outcome of ANOVA for Linear Regression between Geometric Performance and Spatial Thinking

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	274.390	1	274.390	22.803	.000[b]
	Residual	902.493	75	12.033		
	Total	1176.883	76			

a. Dependent Variable: Geometric Performance

b. Predictors: (Constant), Spatial thinking

Table (3): The Result of R^2 for Linear Regression between Geometric Performance and Spatial Thinking

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.483[a]	.233	.223	3.46890	.233	22.803	1	75	.000

a. Predictors: (Constant), Spatial thinking

b. Dependent Variable: Geometric Performance

Table (4): Prediction of Geometric Performance Based on Spatial Thinking											
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	.347	2.831		.122	.903	-5.986	5.293			
	Spatial thinking	.449	.094	.483	4.775	.000	.262	.636	.483	.483	.483
a. Dependent Variable: Geometric Performance											

Appendix (4.19) Simple linear regression between Geometric Performance and Mathematics Academic Self-concept

Table (1): Person Correlations Coefficient between Geometric Performance and Mathematics Academic Self-concept			
		GP	Mathematics Academic Self-concept
Pearson Correlation	Geometric Performance	1.000	.494
	Mathematics Academic Self-concept	.494	1.000
Sig. (1-tailed)	Geometric Performance	.	.000
	Mathematics Academic Self-concept	.000	.
N	Geometric Performance	77	77
	Mathematics Academic Self-concept	77	77

Table (2): Outcome of ANOVA for Linear Regression between Geometric Performance and Mathematics Academic Self-concept						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	286.807	1	286.807	24.167	.000[b]
	Residual	890.076	75	11.868		
	Total	1176.883	76			
a. Dependent Variable: Geometric Performance						
b. Predictors: (Constant), Mathematics Academic Self-concept						

Table (3): The Result of R^2 for Linear Regression between Geometric Performance and Mathematics Academic Self-concept									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.494[a]	.244	.234	3.44495	.244	24.167	1	75	.000
a. Predictors: (Constant), Mathematics Academic Self-concept									
b. Dependent Variable: Geometric Performance									

Table (4): Prediction of Geometric Performance Based on Mathematics Academic Self-concept											
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	2.285	2.223		1.028	.307	-2.143	6.712			
	Mathematics Academic Self-concept	.278	.056	.494	4.916	.000	.165	.390	.494	.494	.494
a. Dependent Variable: Geometric Performance											

Appendix (4.20) Simple linear regression between Geometric Performance and Enjoyment of Mathematics

Table (1): Person Correlations Coefficient between Geometric Performance and Mathematics Academic Self-concept

		GP	Mathematics Academic Self-concept
Pearson Correlation	Geometric Performance	1.000	.256
	Enjoyment of Mathematics	.256	1.000
Sig. (1-tailed)	Geometric Performance	.	.012
	Enjoyment of Mathematics	.012	.
N	Geometric Performance	77	77
	Enjoyment of Mathematics	77	77

Table (2): Outcome of ANOVA for Linear Regression between Geometric Performance and Enjoyment of Mathematics

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	76.908	1	76.908	5.244	.025[b]
	Residual	1099.975	75	14.666		
	Total	1176.883	76			

a. Dependent Variable: Geometric Performance

b. Predictors: (Constant), Enjoyment of Mathematics

Table (3): The Result of R^2 for Linear Regression between Geometric Performance and Enjoyment of Mathematics

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.256[a]	.065	.053	3.82966	.065	5.244	1	75	.025

a. Predictors: (Constant), Enjoyment of Mathematics

b. Dependent Variable: Geometric Performance

Table (4): Prediction of Geometric Performance Based on Enjoyment of Mathematics

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	7.919	2.278		3.477	.001	3.382	12.457			
	Enjoyment of Mathematics	.165	.072	.256	2.290	.025	.022	.309	.256	.256	.256

a. Dependent Variable: Geometric Performance

Appendix (4.21) Simple linear regression between Geometric Performance and Perceived Value of Mathematics

Table (1): Person Correlations Coefficient between Geometric Performance and Perceived Value of Mathematics

		Geometric Performance	Perceived Value of Mathematics
Pearson Correlation	Geometric Performance	1.000	.503
	Perceived Value of Mathematics	.503	1.000
Sig. (1-tailed)	Geometric Performance	.	.000
	Perceived Value of Mathematics	.000	.
N	Geometric Performance	77	77
	Perceived Value of Mathematics	77	77

Table (2): Outcome of ANOVA for Linear Regression between Geometric Performance and Perceived Value of Mathematics

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	297.672	1	297.672	25.393	.000[b]
	Residual	879.211	75	11.723		
	Total	1176.883	76			

a. Dependent Variable: Geometric Performance

b. Predictors: (Constant), Perceived Value of Mathematics

Table (3): The Result of R^2 for Linear Regression between Geometric Performance and Perceived Value of Mathematics

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.503[a]	.253	.243	3.42386	.253	25.393	1	75	.000

a. Predictors: (Constant), Perceived Value of Mathematics

b. Dependent Variable: Geometric Performance

Table (4): Prediction of Geometric Performance Based on Perceived Value of Mathematics

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	.030	2.611		.012	.991	-5.171	5.231			
	Perceived Value of Mathematics	.337	.067	.503	5.039	.000	.204	.470	.503	.503	.503

a. Dependent Variable: Geometric Performance

Appendix (4.22) Simple linear regression between Geometric Performance and Sustainable Learning

Table (1): Person Correlations Coefficient between Geometric Performance and Sustainable Learning

		Sustainable Learning	Geometric Performance
Pearson Correlation	Sustainable Learning	1.000	.644
	Geometric Performance	.644	1.000
Sig. (1-tailed)	Sustainable Learning	.	.000
	Geometric Performance	.000	.
N	Sustainable Learning	77	77
	Geometric Performance	77	77

Table (2): Outcome of ANOVA for Linear Regression between Geometric Performance and Sustainable Learning

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	349.509	1	349.509	53.016	.000[b]
	Residual	494.440	75	6.593		
	Total	843.948	76			

a. Dependent Variable: Sustainable Learning

b. Predictors: (Constant), Geometric Performance

Table (3): The Result of R^2 for Linear Regression between Geometric Performance and Sustainable Learning

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.644[a]	.414	.406	2.56759	.414	53.016	1	75	.000

a. Predictors: (Constant), Geometric Performance

b. Dependent Variable: Sustainable Learning

Table (4): Prediction of Sustainable Learning Based on Geometric Performance											
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	2.777	1.019		2.726	.008	.748	4.807			
	Geometric Performance	.545	.075	.644	7.281	.000	.396	.694	.644	.644	.644
a. Dependent Variable: Sustainable Learning											

Appendix (4.23) Multivariate Regression

Descriptive Statistics			
	Mean	Std. Deviation	N
Geometric Performance post-test	13.0390	3.93514	77
research group	.9870	.81907	77
students' performance level	.6364	.48420	77
Geometric Performance pre-test	5.1558	1.91987	77
Spatial thinking post-test	29.8312	4.23458	77
Total score of attitude to Math post-test	108.2857	16.06472	77

Correlations							
		Geometric Performance post-test	research group	students' performance level	Geometric Performance pre-test	Spatial thinking post-test	Total score of attitude to Math post-test
Pearson Correlation	Geometric Performance post-test	1.000	.519	-.034	-.030	.488	.496
	research group	.519	1.000	-.045	-.066	.413	.369
	students' performance level	-.034	-.045	1.000	.812	.137	.086
	Geometric Performance pre-test	-.030	-.066	.812	1.000	.178	.142
	Spatial thinking post-test	.488	.413	.137	.178	1.000	.479
	Total score of attitude to Math post-test	.496	.369	.086	.142	.479	1.000
Sig. (1-tailed)	Geometric Performance post-test	.	.000	.385	.396	.000	.000
	research group	.000	.	.348	.285	.000	.000

	students' performance level	.385	.348	.	.000	.118	.228
	Geometric Performance pre-test	.396	.285	.000	.	.061	.108
	Spatial thinking post-test	.000	.000	.118	.061	.	.000
	Total score of attitude to Math post-test	.000	.000	.228	.108	.000	.
N	Geometric Performance post-test	77	77	77	77	77	77
	research group	77	77	77	77	77	77
	students' performance level	77	77	77	77	77	77
	Geometric Performance pre-test	77	77	77	77	77	77
	Spatial thinking post-test	77	77	77	77	77	77
	Total score of attitude to Math post-test	77	77	77	77	77	77

Variables Entered/Removed[a]

Model	Variables Entered	Variables Removed	Method
1	Total score of attitude to Math post-test, students' performance level , research group , Spatial thinking post-test, Geometric Performance pre-test[b]	.	Enter

a. Dependent Variable: Geometric Performance post-test

b. All requested variables entered.

Model Summary[b]									
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.648[a]	.420	.379	3.10144	.420	10.270	5	71	.000
a. Predictors: (Constant), Total score of attitude to Math post-test, students' performance level , research group , Spatial thinking post-test, Geometric Performance pre-test									
b. Dependent Variable: Geometric Performance post-test									

ANOVA[a]						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	493.937	5	98.787	10.270	.000[b]
	Residual	682.946	71	9.619		
	Total	1176.883	76			
a. Dependent Variable: Geometric Performance post-test						
b. Predictors: (Constant), Total score of attitude to Math post-test, students' performance level , research group , Spatial thinking post-test, Geometric Performance pre-test						

Coefficients[a]												
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		Correlations		
		B	Std. Error	Beta			Lower Bound	Upper Bound	Zero-order	Partial	Part
1	(Constant)	-1.575	3.075		-.512	.610	-7.707	4.556			
	research group	1.487	.496	.310	2.997	.004	.498	2.476	.519	.335	.271
	students' performance level	-.038	1.261	-.005	-.030	.976	-2.553	2.477	-.034	-.004	-.003
	Geometric Performance pre-test	-.183	.323	-.089	-.565	.574	-.828	.462	-.030	-.067	-.051
	Spatial thinking post-test	.226	.102	.243	2.226	.029	.024	.428	.488	.255	.201
	Total score of attitude to Math post-test	.068	.026	.278	2.617	.011	.016	.120	.496	.297	.237

a. Dependent Variable: Geometric Performance post-test

Residuals Statistics[a]					
	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	7.3944	17.3590	13.0390	2.54935	77
Residual	-6.19922	7.64316	.00000	2.99769	77
Std. Predicted Value	-2.214	1.695	.000	1.000	77
Std. Residual	-1.999	2.464	.000	.967	77

a. Dependent Variable: Geometric Performance post-test

Appendix (4.24) Dominant/Dominant Pattern of Interaction

Excerpt 2 is an example of the interactions in the dominant/dominant pattern of interaction. The excerpt comes from Muhannad and Hamad's pair activity interacting in performing GeoGebra task (13). Although the two dyads concentrate on the task aim and contribute, it is not a shared construction. There was an unsuccessful attempt to distribute task role between them (e.g., lines 3 – 6). Besides, the engagement level with each other's ideas is via fixing their mistakes (e.g., lines 14 – 17; 24 – 26; 29 – 33), which are not always accepted by each participant. The significant characteristic of this pattern of pair interaction is the high level of disagreement and inability or difficulty to reach consensus (e.g., lines 3 – 17; 29 – 34), and one member dominates the majority space front of the PC (e.g., lines 7 – 10 and screenshots). For an instant, in lines 29 – 34, it is clear that the two participants found it challenging to reach a resolution that both could accept. The two participants focus on finding the point that its coordinates is (2,4). Muhannad (line 29) asked Hamad what the point represented by order pair (2,4) is. In line 30 and 32, Hamad gave his answered and justified his thought. Muhannad, however, rejected Hamad's ideas and explained why his answer incorrect (line 31 and 33), thus, Hamad did not accept Muhannad opinion and said: "wait for the teacher to decide which of our answers is correct". Such activity interaction can be referred to Wegerif and Mercer's (1996) "disputational activity".

Furthermore, the classroom observation and video recording reference that voices were often raised in this pair, and emotions such as exasperation, resentment, and anger were expressed by their facial expression or talk. Both participants often complained to their teacher about who should perform GeoGebra. In some cases, they almost were fighting each other on using the PC. Besides, they do not use the plural pronoun, and they highlight the error in the other member's way of thinking (line 24 and 33 "you are wrong" and line 22 "you don't know what to do"). Therefore, the pattern of interaction dominant/ dominant is moderate to high equality but moderate to low mutuality.

Excerpt 2

1 H: Leave it. Today, I am the king...

2 H: Give me the task sheet

3 M: You will write the answers on the activity sheet, and I will do GeoGebra

4 H: No, I will not write. I told you today that I am the king, I will carry out the activity on Geogebra

5 M: I will not write either.

6 M: Do whatever you want. Start doing the task on Geogebra

(complains to the teacher)

7 H: Today, I am the king. I will carry out GeoGebra

8 M: Represent the points on Geogebra.

9 M: give me the mouse I want to do it

10 H: No, leave it.

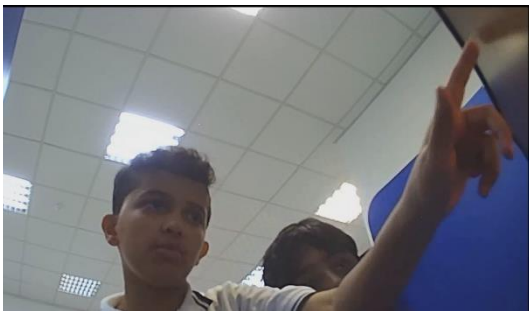
11 M: Say, what the first thing you will do?

12 M: How many points will you represent?

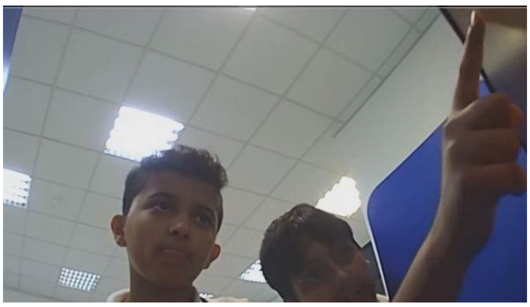
13 H: 3 points, not difficult

14 M: Where will they be located?

15 H: here, then here, then here



16 M: No, the first point will be here, the other one is here, and then here



17 M: I am telling you the point is here

18 H: Oh, do not mess up

19 M: Off, perform the task quickly, it is a point, not difficult to take all this time

20 H: Leave me, (their voice rises)

21 M: The point is here, the point is here …..

22 M: I'm telling you to represent the point, and you don't know what to do

23 H: Be quiet! …. teacher, I did it

24 M: Wrong. 3 points are required, not 4

25 H: 3 points only!

26 M: Yes, I told you that. You were wrong.

27 M: I told you, but you don't hear me

28 H: Be quiet, teacher I did it

29 M: Ok, so what is the point represented by the ordered pair (2,4)?

30 H: It's P

31 M: No, it's S.

32 H: How to be S? Look here it's P. X = 2 and Y = 4

33 M: You are wrong. Look. in the first X which is equal 4 and Y equal 2

34 H: Wait, ask the teacher to see who his answer is correct

(Muhannad and Hamad, Algebra and Geometry: The Representation of Functions, Task 13)

Appendix (4.25) Dominant/Passive Pattern of Interaction

Excerpt (3) is an example of dominant/passive interaction pattern. It comes from the pair activity of Youssef and Sulayman in doing the GeoGebra task (10). In this interaction pattern, one learner took control of the task and dominate the use of GeoGebra and writing the group's findings throughout the task activity. The other learner remains watching his groupmate performing the task on GeoGebra and give confirmation. As shown in excerpt 4, Youssef appropriates the task and contribute more (e.g., lines 2, 4 – 7, 9 – 10, 12, 14) in which he performs the task activity on GeoGebra, reads the task questions, decides on how to perform the task and what they should draw. Despite asking questions, he made them self-directed rather than trying to involve Sulayman to take part and contribute to the task activity (e.g., line 5, 9). It seems he used the self-directed questions to guide his thought and direct his mental activity to perform the task, especially when he finds difficulties in exploring new concept (e.g., lines 10 – 11). Whereas Sulayman appears limited or passive as he follows what Youssef proposed or suggest. His participation is kind of agreeing or confirming Youssef's ideas (e.g., lines 3, 8, 11). He does not give any suggestion unless on one occasion when attempt and make a suggestion (e.g., line 13), and this suggestion is a type of referring to Youssef's explanation in line 10. It should be noted, here, unlike the dominant/dominant pattern, the dominant learner gives the other one little space to see and watch what he does on the GeoGebra. Thus, there is little assist sought or suggested in this pair interaction pattern, and the dominant participant produced the majority of the activity.

Excerpt 3

1 S: What is required to be done?

2 Y: We have to draw a square ABCD. The square has four sides

3 S: Right, start drawing

4 Y: This is the square

5 Y: What is the coordinate?

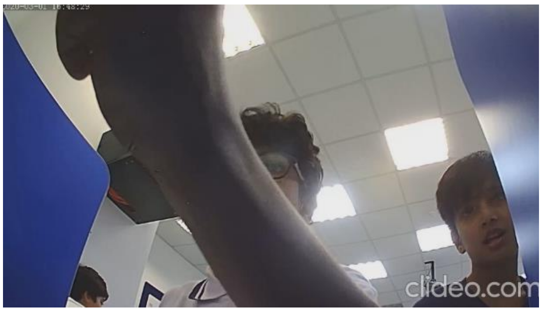
6 Y: Coordinate! They meant coordinates of the figure

7 Y: these are the coordinates. (writing the results)

8 S: Right

9 Y: How do I find the coordinates of head B?

10 Y: like this, see here, we look at the number below the point on the X axis and then look at the number on the Y axis



11 S: Yes

12 Y: What are the coordinates of the vertices of the square?

13 S: We can use the same method that you suggested

14 Y: A (2,4) B (4, 8) C (6, 8) D (6, 2)

15 Y: Oh, they are here on GeoGebra.

16 S: Yes (both were laughing)

(Youssef and Sulayman, Geometry: ordered pairs, task 10)

Appendix (4.26) Expert/Novice Pattern of Interaction

Excerpt (4) is an example of the expert/novice pattern of interaction. The data comes from Omar2 and Jawad's pair activity interacting in performing the GeoGebra task (17). Here in this interaction pattern, one participant takes the role of the task activity and leads the task as an expert. Unlike the dominant/passive pattern interaction, the expert learner encourages and involves the novice learner in the task activities and affords assistance that can help the novice engage with him and learn through the interactions in task activities. Therefore, excerpt 6 shows evidence that Jawad contributed to the task more than Omar2. He was performing translating the geometric shape using GeoGebra and writing the group's findings on the task sheet throughout the task activity. He repaired Omar2's error and did not impose his opinion but give explanations (e.g., lines 3 – 7, 12 – 15). Also, he asked to engage Omar2 in the task and encouraged him to learn (e.g., lines 9 – 10, 12 -13, 9 – 23), and provided positive feedback (e.g., line 11, 26) and negative feedback (e.g., lines 4, 6, 14, 21). While, Omar2, who was the novice learner, answering on the expert learner's questions (e.g., lines 9 – 10, 19 – 20, 24 – 25). It should be noted, novice participant contributes less than the expert learner, but he remains to concentrate on the task aim and keep looking on the screen watching what the expert has done (see the screenshots excerpt 6). Consequently, moderate to low level of equality and medium to high mutuality was revealed in this interaction pattern.

Excerpt 4

1 J: What is required of us is to draw a triangle, then translated to the left 3 units, and after that, we will write the new vertices

2 J: The points are A (2,5), B (6,7), C (4,9). We are starting with the first point

3 O: Here 2

4 J: No, 9 and 4

5 O: This point A.

6 J: No, this is point C; look, did you see

7 O: yes.

8 J: Wait

9 J: Now the triangle is ready for the translation, how can we do it?

10 O: From here (pointing with his hand at Geogbra)

11 J: Yes, from here 1, 2, 3, we made the translation

12 J: Ok, what are the new order pairs?

13 O: A (2,5)

14 J: No, see here 4, point D.

15 J: D (4, - 4) See minus!

16 J: Also - 4

17 J: Now, point E

18 O: Are you sure?

19 J: What do you think?

20 O: It seems that what we are doing is wrong

21 J: No, see point E.

22 O: What about it

23 J: -2 look, E (-2, 6), did you see

24 J: Now, what is the last point?

25 O: Point F

26 J: Correct, point F is the last point; see it is lucky (0, 4)

27 O: Right

28 J: Teacher, we finished

(Jawad and Omar$_2$, Geometric Translation, Task 17)

Appendix (4.28) Excerpt 5

1 O: The question says to draw a straight line

2 W: drawing a straight line!

3 O: Ok, I'll draw a straight line

4 O: A straight line, like what?

5 O: I drew a straight line

6 W: We want to describe it.

7 O: give me. (He took the activity sheet to write the straight line definition).

8 O: the straight line has no beginning and end and is long

9 W: This is not a straight line

10 W: We drew a straight line, now.

11 O: All groups have finished the task

12 W: let's draw a half of a straight line

13 W: half straight

14 O: Not straight line, half straight line. What is required now is to draw a half-line, which means we do not do this but do like this.

15 W: Why did you draw a triangle? This is an error.

16 O: A half straight line is a curved line, not a straight line

17 W: Moment, this is a half straight line

18 O: Draw a line segment

19 W: We haven't resolved anything yet

20 W: Draw a line segment

21 O: Give me the task sheet. You are writing everything wrong

22 W: Ok, quickly write

23 O: A line segment has a beginning and an end

24 O: What is the difference between a straight line, a half-line, and a straight segment?

25 W: The straight line is very long and extends from both sides, and the half of the straight line, you see, starts from here and extends and does not stop. As for the line segment, small and delimited, it starts from here and ends here.

(Omar1 and Walid, Geometric Concepts, Task, 1)

Appendix (4.29) Excerpt 6

1 F: Read the task questions

2 A: In cooperation with your group using Geogebra, such as the following points on the plan ...

3 P: Eye

4 A: Eye! A (6, 7), B (2,3), C (5,0), D (0,5)

5 F: Look, look here, Eye. M. A. S.

6 A: Listen, if you want my full name: M. A. S. M. K.

7 F: this is your full name

8 A: Yes, my full name

9 F: There is no grandfather, great grandfather, great great grandfather.

10 A: My grandfather is Mohamed,

The teacher task Fayez and Abdullah to focus on the task

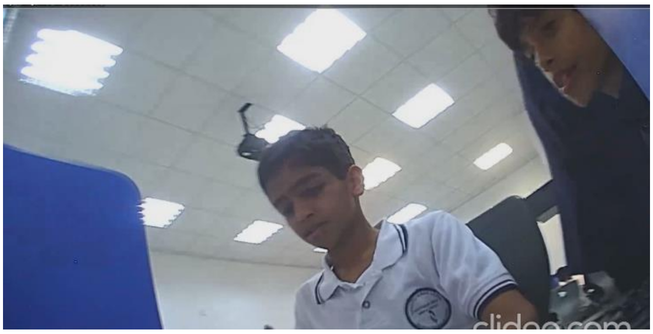
11 A: See M. S. how he works

(laughing)

12 A: Oooooh. See Walid and Omar working close to each other

13 F: We want to perform the task

14 A: ok, Search on Google



15 A: Wow, oh

16 F: Wow

Laughing

17 A: Aww

18 F: Hahaha

(Fayez and Abdullah2, Geometry: ordered pairs, task 11)