A COMPUTER-AIDED DESIGN OF MULTI-STOREY STEEL FRAMES

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CONTENTS

CHAPTER 1

CHAPTER 2

CHAPTER 3

CHAPTER 4

CHAPTER 5

CHAPTER 6

CHAPTER 7

RESULTS

CHAPTER 8

CONCLUSIONS

APPENDIX

FIGURES

Figure No.

SUMMARY

This thesis describes the application of computer aided optimum design techniques to the design of braced multi-storey steel frameworks.

Optimization methods which have been used successfully in structural engineering are described and classified. Applications of the methods to structural design are reviewed and conclusions are drawn concerning the current state of the structural optimization art.

The development of a model which assigns a cost to a typical multi-storey framework is described. A computer programme which comprises this model and structural design routines, allowing the interactive cost comparison of alternative designs is also described.

The thesis goes on to describe the development of suitable problem orientated optimization algorithms which can be combined with the cost model and structural design routines. A design strategy is then developed which, if followed will produce economic designs. The results of using the design system are presented and conclusions are drawn concerning optimum structural design. Finally avenues of possible further research, which this research indicates will prove profitable are indicated.

CHAPTER 1

INTRODUCTION

1.1 SUMMARY

This chapter describes the structure of multi-storey buildings and frameworks. It tells why braced multi-storey rigid steel frames were chosen for investigation. Current commercial design and execution practice is examined, and ways in which computer aided design techniques can improve on current methods are discussed.

1.2 STRUCTURE SELECTION

Multi-storey structures were chosen for investigation because they represent a substantial proportion of commercial building development and in their simplest form they are relatively simple to design.

The structure chosen comprises a three-dimensional rectangular steel framework, cladding, flooring, stabilizing elements and foundations. The steel framework comprises the main vertical-load carrying components of the structure, the framework selected is described in the next section. The cladding forms a weatherproof skin around the sides of the structure, it may consist of facing elements which are either supported on the framework or self-supporting on mullions. The horizontal wind loads which act on the cladding are transferred to the floors which act as rigid diaphragms transferring these loads to the stabilising elements. The floors may span in one or two directions. The stabilising elements comprise either lift cores, shear walls or diagonally braced steel frames. The whole structure is supported on a foundation which varies in its construction to suit the ground and site conditions.

1.3 FRAMEWORK SELECTION

The possible types of frameworks which can be used in a multi-storey building are many and varied. The three-dimensional braced rigid frame that has been selected was chosen because of three reasons. Firstly this form of construction is often used in multi-storey construction. Secondly three-dimensional frames involve considerable problems with the fitting together of steel sections with suitable connections. This results in the selection of sections because of geometric and stress considerations. Thirdly the absence of wind loading on the framework renders optimization of the framework possible. If sway frames had been used the elastic response of the structure to sway loading would be complex, resulting in an optimization problem that would be impractical using present day computers.

The steel framework is that part of the structure which carries vertical loading from the floors, roof and possibly the cladding. This framework consists of major axis beams, minor axis beams and stanchions. The stanchions are placed vertically on a rectangular grid. At each floor level, the major axis beams span between the stanchion flanges and the minor axis beams span between the stanchion webs. Connections occur at each node point of the framework. Stanchion splices may be used just above any floor level. The types of connections are all based on shop welding and site bolting which is current practice in Great Britain. A typical framework is shown in figure 1.1, in which each element is shown. A design can be considered complete when available steel sections can be assigned to each member and bolts, welds and plates can be assigned to each connection.

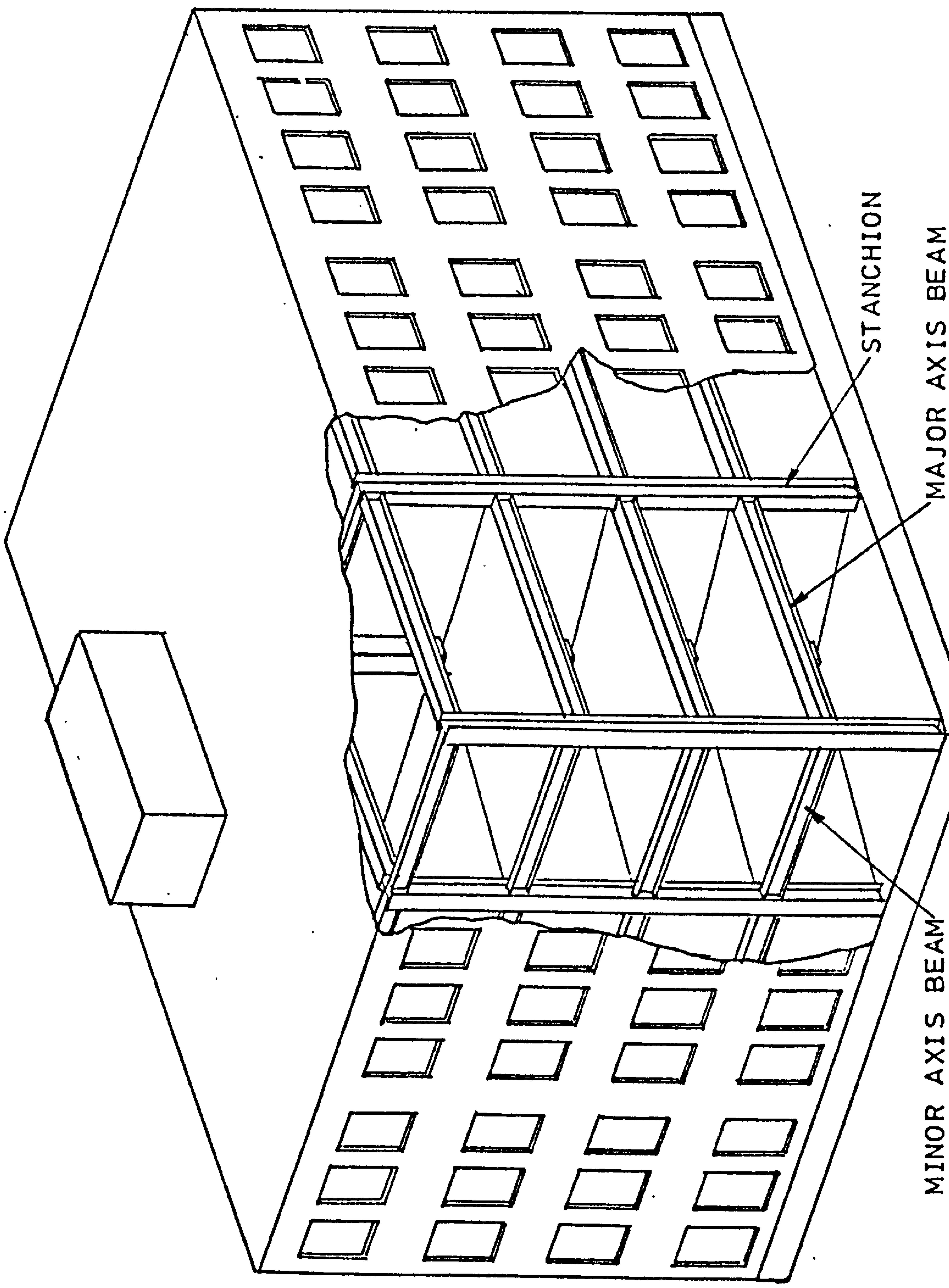

FIGURE 1.1 SKELETAL ARRANGEMENT

1.4 EXAMINATION OF CURRENT PRACTICE

When a multi-storey structure is to be constructed, the developer approaches an architect. The architect, acting as the developer's agent, will decide on the form of the structure. A Consulting Engineer will then either design the structural elements, complying with the constraints set by the architect or he will allow the fabricator to perform the design.

The fabricator will be given sufficient information for him to tender a price for designing, supplying and erecting the framework. The fabricator having decided to tender a price, hands the information to his designer, who determines the structural sections required. The designer's results are passed to be fabricator's estimator who calculates a tender price. The tender is returned to the Consulting Engineer who compares it with those submitted by other fabricators and so selects one to undertake the work.

The decisions that are taken during this process affect the final cost, these decisions will now be considered. Once it has been decided to develop a site, the general shape of the structure is determined by the architect, taking into account the shape of the site, planning considerations and the use of the structure. Within the shell of the structure the lift shafts, stairs, walls and stanchions have to be located. The architect ideally requires large unobstructed areas with shallow beams and small stanchions. In practice this is not generally possible and a comprise solution has to be developed with the 2 4
Consulting Engineer. The decisions taken at this stage have the greatest effect on the cost of the final structure. The structural arrangement is constrained by aesthetic, habitability, heating, ventilating and structural criteria. An arrangement which satisfies all the criteria is unlikely to be found and therefore compromises have to be evolved. This decision process cannot be readily defined in a mathematical sense and therefore mathematical optimization techniques cannot be used for its solution. The next decision that has to be taken is the selection of available sections for the beams and stanchions. These decisions are taken by the designer who uses his intuition and his

previous experience of estimating. The designer is expected to produce details in the minimum time possible, this effectively prohibits the costing of alternative solutions. The estimator may take decisions on how a component is to be manufactured, however he is bound by the details provided by the designer.

Thus an examination of current practice shows that there are two decision processes which permit reductions in cost. Firstly the structural arrangement is devised, any reductions in cost are in this case passed directly to the developer. Secondly, available sections are allocated to the members of the framework, any reductions in cost may be passed to the developer, via lower tender prices or alternatively they may be absorbed by the fabricator as additional profit. It is therefore apparent that rapid estimation of costs of various structural arrangements may aid the first decision process and that the selection of optimum steel sections may aid the second decision process.

1.5 IMPROVEMENT OF CURRENT PRACTICE USING COMPUTER TECHNIQUES

This thesis explains the development of a computer aided design method, which selects a set of economic steel sections for a framework and then systematically improves this set of sections.

Before this design method could be developed, research was necessary into several topics. Firstly a design method which had been "codified" had to be selected, therefore a survey of design methods was undertaken. Secondly an optimisation algorithm had to be selected, a survey of the algorithms used by previous investigators was therefore undertaken. These two surveys form a two part literature review which follows this chapter.

The optimization techniques were found to require a means by which a chosen set of sections could be tested for satisfaction of the structural constraints. The description of this process is the subject of Chapter 4. The optimisation algorithms also require the evaluation of the objective function which, in this case, is the cost. The cost model that has been developed is described in Chapter 5. This model was developed in cooperation with members of the structural steelwork industry and it includes all the major factors that influence the cost of the framework.

Once a means of checking the validity and determining the cost of a set of steel sections was available, it then became possible to develop suitable optimisation algorithms. These algorithms take advantage of the particular properties of the problem to facilitate a solution. These algorithms and their development are described in Chapter 6.

A design strategy was then developed which could be applied to various frameworks. This strategy was then used on a number of frameworks to test its efficiency. This strategy, together with the results of using the design programmes for various investigations, are given in Chapter 7. In the final chapter, conclusions are drawn concerning the structure, the optimisation algorithms developed and the applicability of computer aided design to a structure of this type.

CHAPTER 2

Braced Rigid Frame Design Methods

2.1 INTRODUCTION

The scope of this review is confined to the consideration of rigidly jointed, rectangular space frames, braced against sidesway by a system of bracing. The design of the bracing system, which may be steel bracing, lift and service cores or shear walls, will not be considered further.

The design of steel frames in the U.K. and elsewhere is currently regulated by the provisions of BS449[4]. The design philosophy of the standard is based on either:-

(a) A semi-empirical design method, based on assuming pin-jointed connections when finding moments, but rigid joints when assessing the stability of the columns.

or

(b) Using the same allowable stresses as the simple elastic design method (a) and performing a complete elastic analysis.

These methods do not take account of the magnification of minor axis moments due to the effect of axial load. Therefore the load factors which occur in practice when these methods are used can be very variable[10]. The second method was devised for use with any type of structural frame and therefore cannot exploit the particular characteristics of the braced rigidly jointed frame. For these reasons the frame design method provisions of BS449 will not be considered further for the design of members. 5

The different types of braced frames which have been considered in the literature have been classified by HORNE[12]. The classification is based on the type of restraint afforded by the beams to the columns. The types of restraint which can occur are:-

(a) Elastic (E): the beam concerned remains fully elastic up to the point of failure of the column.

(b) Plastic (P): the beam concerned always develops a plastic hinge at the beam/column connection before the point of failure of the column.

(c) Pinned (O): the joints between the beam and column can rotate without providing restraint to the column.

(d) Elastic-Plastic (E-P): the beam concerned may or may not have developed a plastic hinge at the beam/column connection at the point of failure of the column. The moment/rotation characteristics of the beam are perfectly elasto-plastic.

The last type of restraint has been included in order to help in the description of some of the design methods to be reviewed. The type of restraint is also classified by HORNE[12] on the basis of the axis of the stanchion about which it occurs, e.g. P_x denotes plastic restraint about the x axis. The design methods to be presented are classified according to Hornes classification in Table 2.1.

$y \downarrow$ \ $x \rightarrow$	O_x	E_x	P_x	$E-P_x$
O_y	(6) (7)	(1)	(2), (6) (7)	(3)
E_y	N.P	(1)	(4), (5)	N.P
P_y	(2), (6) (7)	N.P	(2), (6) (7)	N.P
$E-P_y$	N.P	N.P		

(1) = Steel Structures Research Committee 1936

(2) = Horne 1955

(3) = Lehigh Unv. 1965

(4) = Joint Committee report 1971

(5) = Wood's Method (BRS) 1974

(6) = Youngs Method 1973

(7) = American Inst. Steel Construction and CRC guide 1977

NP denotes this case is not practical

Table 2.1 - Classification of Design Methods

No significant gaps appear in the range of practical cases which could occur. The design methods included above are described within this chapter. The design methods to be reviewed have common assumptions which are now described:-

2.1.1 Limited Frames

All the design methods are based on the concept of a limited frame or subassemblage. For instance, when considering the design of a beam, only those members which are in the plane of the beam and are directly joined to it are considered. For example, see figure 2.1. Depending on the design method the remote ends i.e. the perimeter nodes of the subassemblage, of the numbers may be assumed to deform in various ways. The validity of the limited frame concept can be verified theoretically, by showing that the loads on other parts of the frame have little effect on the moments within the limited frame. This is also true for columns for which a more complex limited frame is used, see figure 2.2. This approach allows the consideration of

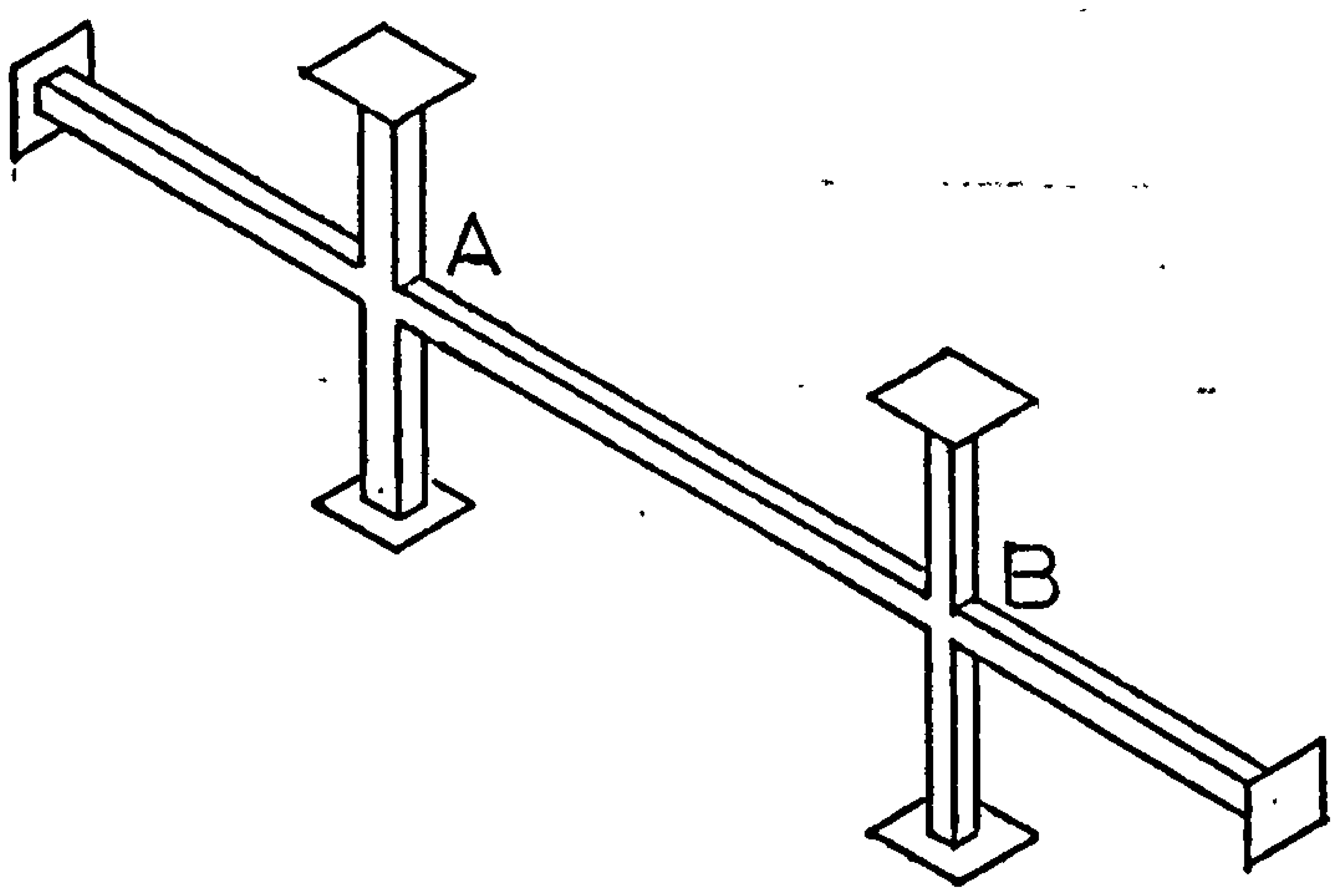

Figure 2.1
Limited Frame for Beam A.B

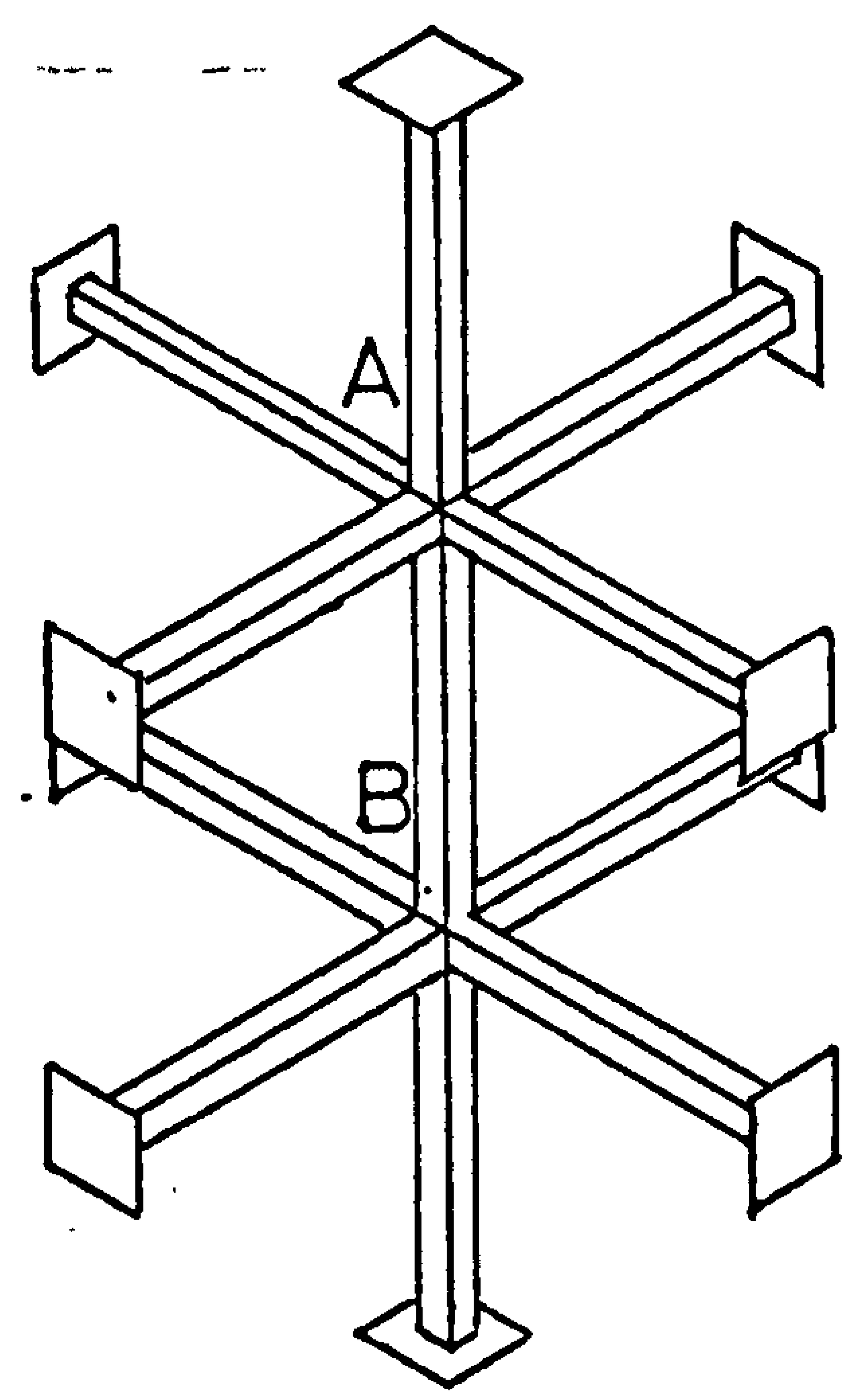

Figure 2.2
Limited Frame for Column A.B

all the load cases which affect a particular member without a full scale frame analysis. Because of the degree of repetition implicit in the use of limited frames, the limited frame concept is eminently suitable for computer aided design.

2.1.2 Local Buckling

In all the design methods, limitations are put on the ratios of breadth of section divided by flange thickness and depth of section divided by web thickness. This is the case both for beams and columns. These are invariably found from an elastic buckling analysis, the differences reflecting the safety factors in use by the design methods.

2.1.3 Plastically Designed Beams

For the design methods which use plastically designed beams, the procedure is to find a three hinged mechanism for the beam being designed. The resulting moments are then compared with the full plastic moment of the section. The plastic moment of the section may be modified to include the effects of shear, if this is significant. Deflection under working load is also checked.

2.1.4 Elastically Designed Beams

In the design methods which use elastically designed beams, the procedure is to analyse the limited frame for 3 load cases which give the maximum moments at the two supports and at midspan respectively. The maximum of these moments is compared with the full elastic moment of the section. The deflections under working load are also checked.

2.1.5 Column Behaviour

With these points in mind it can be seen that the problem becomes one of the design of a single column. Numerous investigations of column strength have been undertaken and many computer models of columns have been proposed. An excellent review of this work is provided by the C.R.C. guide[8]. The behaviour of columns can be predicted accurately, but all of the models are far too complex for design office use. Therefore design methods which consider the behaviour of a column in a simplified way have to be used. There are many variables which affect the strength of columns in braced frames. The effects of some of the main variables will now be considered:-

2.1.6 Initial Imperfections

The main imperfections in columns are due to initial curvature, initial twist and residual stresses. American practice tends to favour the use of residual stresses to define the strength of columns. British practice tends to use initial curvature. Residual stresses decrease the stiffness of the column under load which magnifies deflections and causes failure by Euler buckling or excessive bending. Initial curvature causes an increase in the central bending moment in the column causing the development of plastic zones. The mode of failure is similar to that caused by residual stress.

2.1.7 Restraint

By rotationally restraining a stanchion at its ends about its minor axis the failure load can be increased. This is due to the fact that the stiffness of the beams contributes to the stiffness of the column by reducing rotation of the ends. This is related to the commonly used "effective length" concept. If a stanchion is unrestrained it can never carry a load greater than the Euler load for its length. However restraint allows the use of loads greater than the Euler load. The restraint provided by beams which frame into the minor axis and have a plastic hinge within the beam is negligible. Some design methods are based on directional restraint provided about the minor axis of the stanchion within the height of the stanchion. This constrains buckling to occur within the plane of the major axis beams, resulting in higher collapse loads, but requiring inconvenient bracing.

The design methods previously listed represent all of the main design methods proposed for the design of braced rigid jointed steel frames. These design methods will now be described in detail.

2.2 THE STEEL STRUCTURES RESEARCH COMMITTEE 1936[24]

The steel structures research committee was set up in 1929, to investigate the application of modern theory to the design of all forms of steel structures. The first seven years were spent in an investigation of multi-storey steel building frames. The results of this work were published as a final report[24] in 1936. Since that time interest in the design method has been revived by the inclusion in BS449[5] of the recommendations for design of the final report.

The design method was an attempt to provide a rational basis for the design of braced multi-storey frames in which the major and minor axis beams were designed elastically. The development of the design method is described by BAKER, HORNE and HEYMAN[2].

The design method for beams is based on elastic design taking into account the effect on the restraining moments of flexible
connections. An effort was made to make the design of beams independant 18
of the stanchions by making conservative assumptions, as to the stiffness of the stanchions, and the second moment of area of the beams. The method consists of choosing a type of connection from a set of standard bolted connection types. A trial beam section is chosen and
by the use of a chart which is dependant on the type of connection, the 13
end restraining moments are found. These charts are based on the moment rotation characteristics found from tests. The section is then checked to ensure that the elastic stress produced is not greater than 9tons/in^2 (145N/mm^2).

The design method for stanchions again makes a number of simplifying conservative assumptions. The allowable bending stresses are based on a pinned ended stanchion bent about the y-axis in either S/C or partial D/C curvature, taking into account stresses due to initial curvature. The allowable bending stress is read from a chart which depends on the slenderness ratio and axial stress in the stanchion. The bending moments at the ends of the stanchion are devised for the worst cases that may occur and are read from a table.

The design method gained little favour from its inception because the large number of simplifying assumptions produced an expensive frame. The method was recently revived in BS449[5] as a semi-rigid elastic design method. As far as the author knows the design method has not been used for any actual building design. The committee found that it was difficult to produce a rational totally elastic design method for this type of frame and in order to solve the problem made a large number of conservative simplifying assumptions.

2.3 HORNE'S DESIGN METHOD 1955[12]

This design method is applicable to frames in which the beams are assumed to be failing plastically at the point of failure of the column. The minor axis beams may be pin jointed to the columns as an extreme case. The definition of failure used is the attainment of first yield in any fibre of the column.

The theory behind the design method was developed by Horne and his co-workers[3, 11, 12] in the early 1950's at Cambridge University. The method has gained widespread acceptance within the British Steelwork industry because of the prominence given to it by the British Constructional Steelwork Association[6, 14]. A summary of the development is given in a report of the investigations at Cambridge[3]. The design method was chosen at that time because:-

(a) The inelastic failure of restrained stanchions was too difficult to predict by means of simple formulae.

and (b) The choice of a pinned ended elastic stanchion was not thought to have a large reserve of strength, especially as redistribution does not in this case occur theoretically.

The design method for minor and major axis beams is based on plastic analysis. The effect of shear on the plastic moment of resistance is taken into account.

The design method for columns is based on satisfying the following interaction equations:-

$$\frac{P}{A} + \frac{N_x M_x C_{mx}}{Z_x} + \frac{N_y M_y C_{my}}{Z_y} + fi \leqslant fy$$

for the column as a whole.

and

$$\frac{P}{A} + \frac{M_x}{Z_x} + \frac{M_y}{Z_y} \leqslant fy$$

at each end of the column.

where

P = direct load

A = sectional area

M_x = maximum moment about the x axis

M_y = maximum moment about the y axis

N_x = magnification factor about the x axis

N_y = magnification factor about the y axis

C_{mx} = equivalent moment factor for x axis moments

C_{my} = equivalent moment factor for y axis moments

Z_x = major axis section modulus

fi = stress due to initial curvature

fy = yield stress

This is checked for each of eight possible S/C and D/C load cases.

The analysis of the stanchion is based on elastic analysis allowing for initial curvature and torsional buckling. The analysis is done for single curvature bending and the result is modified using the equivalent moment factors C_{mx} and C_{my} so that it can be used for any type of bending.

The magnification factors N_x and N_y express the effect of axial load and twisting on the bending moments within the stanchion.

As the axial load approaches the Euler load of the stanchion N_y approaches infinity.

The equivalent moment factors C_{mx} and C_{my} are based on the elastic theory, for bending about one axis combined with direct load, for a pinned end stanchion[11].

The stress due to initial curvature is also based on a non linear analysis assuming the column is pinned ended. The stress is derived from the bending moment produced by the axial load not acting at the centroid of the section. This bending moment is modified to include the effect of twisting. The initial curvature of the stanchion is given in terms of the minor axis radius of gyration and the distance from the centroid of the section to the extreme fibre in the y direction. This approximation allows the minor axis section modulus to be removed from the formulas.

The method has been extended by HORNE[13] to take in the case where a plastic hinge may occur at one end of the stanchion before failure. However minor axis moments must not occur. This method is primarily suited to the checking of stanchions in single storey structures.

The design method was first applied to two structures[15, 16] both at Cambridge University. Design examples are given in both these papers. The design method is also allowed by the Joint Committee Report Design Method (see section 2.6) as an alternative design method.

This design method provides a simple conservative technique for the design of unrestrained stanchions. Despite the very 17
conservative nature of the design method it is perhaps the most accepted design method in British Industry for 3-dimensional braced frames with plastically designed beams.

2.4 LEHIGH UNIVERSITY DESIGN METHOD 1965[19]

In 1965 Lehigh University attempted to collect all of the results of experimental and theoretical investigations done by them into a coherent plastic design method for multi-storey frames. The design method is described in a set of lecture notes[19] provided at the resulting conference. Design methods were presented for braced and unbraced frames. The design method was the first method proposed which dealt with elasto-plastic design.

The design method is applicable to the design of beams and columns in braced frames which are rigidly connected to the major axis beams, and pin connected to the minor axis beams. The design method takes account of residual stress.

The design of beams is done by plastic theory assuming a three hinged mechanism. The effect of local buckling, bracing spacing and shear are also considered.

The design of the columns is done by the consideration of the moment-rotation characteristics of the column as it is loaded up. The column in question is isolated into a limited frame or subassemblage as shown in figure 2.3. Assumptions are made as to the rotation of the remote ends of the beams, in accordance with the load condition being considered. The relationship between the applied moment and the rotation of the members in the subframe are then found from curves. If the moment rotation characteristics of the beam and column are added, the resulting moment rotation characteristic is that of the subassemblage. The maximum moment from this curve must be greater than the unbalanced joint moment for the frame to be stable. In order to render the problem suitable for hand computation, it is usual to assume that the bending of the column corresponds to one of three cases. The design method is strictly only applicable to columns braced in the y-direction. However a modification is given which, by ensuring that the column moment is never greater than the elastic critical moment, will be applicable to members which are unbraced in the y-direction.

The design method has been verified by tests[21, 38]. The first set of tests was done on a 3-storey, 2-bay plane frame with welded connections. The general observations were that:-

(a) The design method was satisfactory in that all the columns failed at loads greater than the theoretical maximum load.

(b) Beams and columns not directly connected to a member had little effect on that member.

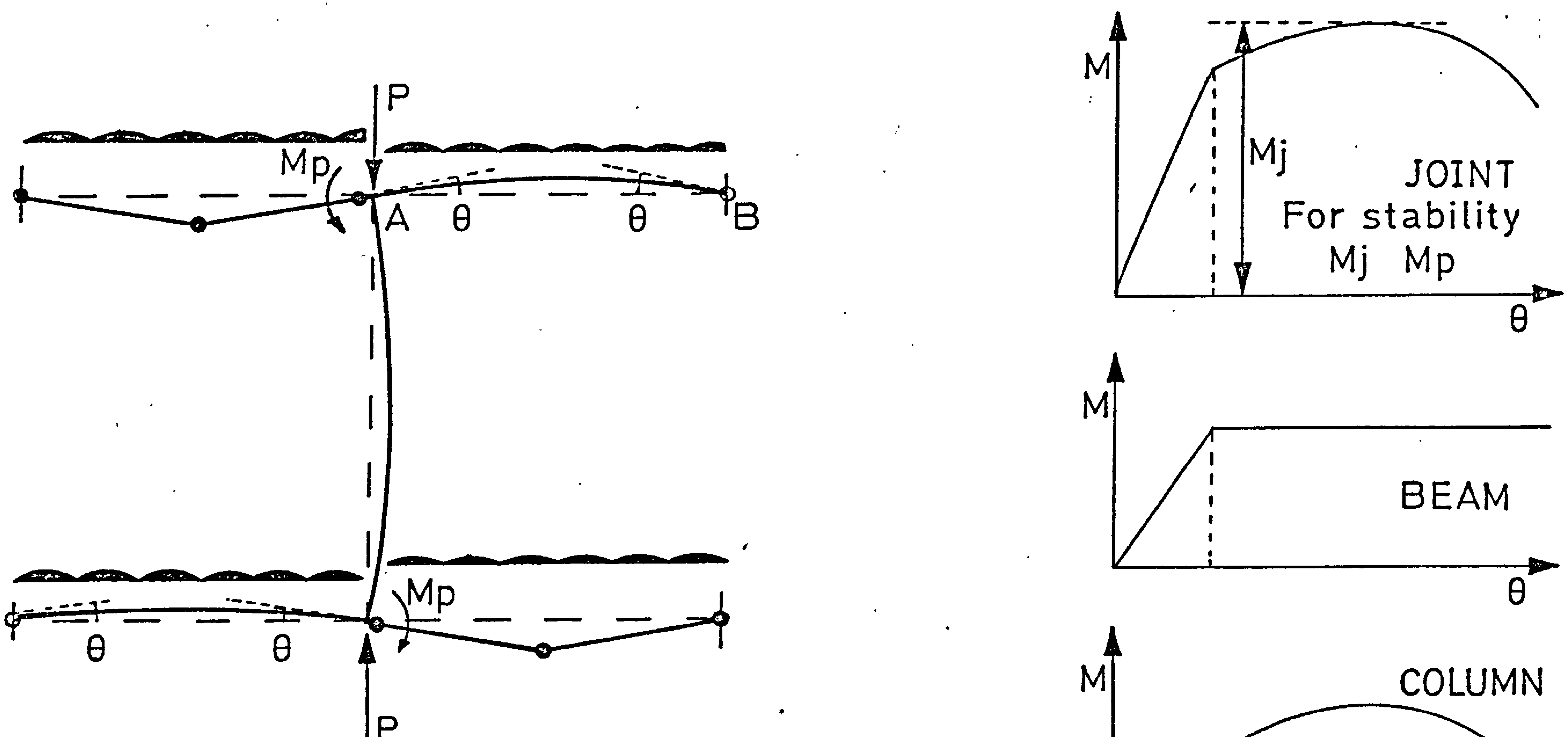

FIG. 2.3 TYPICAL SUBASSEMBLAGE & M−θ RELATIONSHIPS USED IN THE LEHIGH UNIVERSITY DESIGN METHOD

(c) The bracing used took almost all the lateral load and the lateral load had little effect on the behaviour of the columns.

The second set of tests were performed on subassemblages which tested the validity of adding moment rotation curves. It was also found that stanchions bent in D/C had a large reserve of strength because they strain harden.

The design method has been used in at least three tall framed structures[1, 9]. These are all in Maryland, U.S.A. However the method is difficult to apply manually, and recourse was taken to using a large number of computer produced tables in order to simplify the design calculations[1].

This design method represents an attempt to apply elasto-plastic design principles to braced frames. The method has two main failings:

(1) Minor axis restraint is not allowed for.
(2) The data required for design is only available for American sections.

2.5 COLUMN RESEARCH COUNCIL[8]

The column research council was founded in 1944 in order to foster research on the behaviour of compressive components of metal structures. The finding of the council are periodically published in the form of a guide, the most recent of which was published in 1976. The guide provides a review of research and design methods which have been proposed. Those design methods which show good correlation with experiment are described in detail. These design methods represent current American practice.

The guide concentrates on biaxially loaded columns, without restraint at the ends. The design methods reviewed are based on interaction formulae. Two sets of interaction equations are proposed for the case where the column is inelastic.

The first set of interaction equations were proposed a number of years ago[7] and are of the form:-

$$\frac{P}{P_u} + \frac{C_{mx}M_x}{M_{ux}(1-{}^P/P_{ex})} + \frac{C_{my}M_y}{M_{uy}(1-{}^P/P_{ey})} \leqslant 1.0$$

for the column as a whole and

$$\frac{P}{P_y} + \frac{0.85\ M_x}{M_{px}} + \frac{0.6\ M_y}{M_{py}} \leqslant 1$$

$$\frac{M_x}{M_{px}} + \frac{M_y}{M_{py}} < 1$$

for each end of the column.

where

P = applied axial load

P_u = ultimate load for the centrally loaded column

M_x = maximum applied moment about x axis neglecting the effect of axial load

C_{mx} = moment reduction factor dependant on the shape of the bending moment diagram

M_{ux} = ultimate bending moment in the absence of axial load taking account of out of plane bending

P_{ex} = elastic critical load for buckling about the x axis

P_y = yield load of the column

M_{px} = plastic moment of column about the x axis.

These equations are similar to those derived from HORNE[12] but they are applied empirically to inelastic failure. The method applies to an isolated column which forms a plastic hinge at one or both ends. Such a formulation has some of the disadvantages of the elastic limiting stress approach in that in certain cases it can still be very conservative.

The second set of interaction equations is of the form:-

$$\left(\frac{M_x}{M_{pcx}}\right)^{\xi} + \left(\frac{M_y}{M_{pcy}}\right)^{\xi} \leqslant 1.0$$

at the column ends.

$$\left(\frac{C_{mx}M_x}{M_{ucx}}\right)^{\eta} + \left(\frac{C_{my}M_y}{M_{ucy}}\right)^{\eta} \leq 1.0$$

for the column as a whole.

where

M_{pcx} = Plastic moment about axis x taking into account the reduction due to axial load

M_{ucx} = the maximum uniform single curvature moment, which can be resisted by the member about the x axis in the presence of axial load and no other moment.

ξ & η = variables depending on the shape of the section and the axial load.

These equations were proposed by TEBEDGE and CHEN[25] in 1974 and are less conservative than the previous equations. The guide concludes that these formulae should prove useful in the future for inclusion in design codes.

Where restraint exists about either axis the guide recommends that an effective column length should be used, instead of the true length of the column, in the stability equations. The effective length used is taken from charts.

The above design equations are simple to apply and accurate. Both sets of interaction equations have the disadvantage that some of the terms in the equations are based on steel sections which are available only in America.

2.6 THE JOINT COMMITTEE REPORT 1971[17, 18]

This design method is applicable to frames in which the minor axis of the stanchions are elastically restrained by the minor axis beams, the major axis beams being designed by plastic theory.

The theory behind the report was a rationalization of work undertaken during the 1950's at the Building Research Station by WOOD and his co-workers[26, 27, 32]. A summary of the development of the design method is given by PARTRIDGE[20]. The design method for major axis beams is based on plastic analysis, using a three hinge mechanism under the maximum load. The effect of shear force on the plastic section modulus is taken into account by using a Von-Misės yield criterion. The deflection of the beam at working load is also checked.

The design method for minor axis beams is based on elastic analysis, three loading conditions being chosen for each beam which cause maximum moments at each of the supports and midspan. A maximum value of shear stress is given. Deflections are checked at working load.

The design method for columns is based on satisfying the following interaction equation:-

$$fa + m\ fy + fx + fic \leqslant Fy$$

where

fa = the stress due to axial load

fy = maximum bending stress about the minor axis neglecting axial load effects

fx = maximum bending stress about the major axis neglecting axial load effects

m = magnification factor which represents the effect of axial load on the minor axis moment

fic = stress due to initial curvature of the stanchion.

The magnification factor is dependant on the ratio of stanchion end moments, the elastic critical load and the axial load on the stanchion. The value of m is derived from a theoretical elastic analysis of the stanchion, which takes account of the effects of axial load[26, 27]. The bending moment diagram, when axial load effects are included, is part of a sine wave. Theoretical equations can be found which locate the point of maximum moment, and then the bending moment can be found at this point. WOOD[28] shows that the relationships given in graphical form, within the report, are approximations to the analytical equations which exhibit a small amount of scatter about the relationship chosen. This procedure makes the evaluation of m a simple process. In order to determine m it is however necessary to evaluate the elastic critical load of the stanchion. The elastic critical load can be read from charts similar to those given by WOOD[27]. These charts give the elastic critical load as a function of the stiffness of the adjoining members.

The stress due to initial curvature is also based on an elastic theory which takes axial load into account[26]. The member is assumed to be initially bent in the shape of a sine wave. The stress due to initial curvature is found from a chart which requires knowledge of the stiffness of the adjacent members. The analysis uses a simplified limited frame which is symetrical. A simple mathematical expression gives the value of the stress in terms of the axial load and member stiffnesses. The use of initial curvature is the only way in which imperfections are taken into account in the design method.

The interaction equation is evaluated for four load cases for each stanchion. The load cases are such that the load case which causes the largest component of single curvature bending will always be among them. The use of single curvature bending as a criterion of failure neglects the fact that a double curvature bending load case may cause plasticity in the ends of the stanchion. WOOD[28] says that this was allowed by the committee in order to mitigate the conservation of pure elastic design. The design method is described[28] as "conservative partial-plastic design of restrained columns".

The effect of lateral torsional buckling is included by the use of semi-empirical charts. The derivation of the charts is described by WOOD[28]. Analytical equations do not exist for the relationship given in the charts[31].

The design method allows the reduction of support moments for major axis beams at joints with unbalanced moments due to applied loads. This mechanism allows the choice of columns which would not otherwise be suitable. The end moments can be reduced within limits which ensure that the resulting stanchion is not too slender.

The load factor recommended by the committee is 1.5. PARTRIDGE[20] has stated that the accuracy of the design method allows the use of a constant load factor. This is contrasted with the provisions of BS449, which uses a higher variable load factor with a less accurate design method.

The design method has been verified by two large scale frame tests[22, 33]. The frame which was tested first had no internal columns, and the joints were of unconventional design in order to produce full rigidity. The results of the test can be summarized:-

(a) The beam deflections were lower than predicted.
(b) The end moments in the minor axis beams were approximately 10% in error.
(c) The design method overestimated the column stresses by up to 23%.
(d) A large reserve of strength occurs in the stanchions above the elastic limiting load.
(e) The use of S/C load cases is safe even when D/C load cases produce plasticity in the columns.

WOOD[33] concluded that the accuracy of the design method in estimating elastic stresses was excellent. However he believed that the failure criterion could be improved.

The second set of frame tests was performed on a frame with one internal column , high yield steel and conventional end plate joints. The results of these tests can be summarized:-

(a) Beam deflections were slightly increased by the use of conventional joints.
(b) The minor axis support moments were about 10% lower than predicted by the design method.
(c) For an internal column the loads outside the limited frame have a significant effect on the column moments.
(d) The major axis beams failed at loads greater than the load given by the design method despite torsional failure in some cases.
(e) The perimeter columns showed good agreement with elastic stresses.
(f) The columns failed at substantially higher loads than the elastic limiting load without any torsional buckling.

The conclusions were similar to those made by WOOD[33]:

The Joint Committee Report design method represents an attempt to provide a simple design method which can be applied to frames with plastically designed major axis beams, elastically designed minor axis beams and elastically designed stanchions.

2.7 YOUNG'S DESIGN METHOD 1973[34]

Recently B W Young of Cambridge University proposed a design method for stanchions which are connected to plastically designed beams.

The design method for beams is similar to other plastic design methods. The design method for stanchions consists of satisfying an interaction formula of the form:-

$$M_x \leqslant K_x K_t K_y K_h M_{px}$$

where

M_x = maximum major axis moment on the stanchion

M_{px} = major axis plastic moment of the stanchion section

K_x, K_t, K_y, K_h = reduction factors.

Satisfaction of this equation ensures that the maximum moment on the stanchion is less than an allowable moment. The reduction factors are dependant on:-

(a) Flexural buckling under axial load and major axis moment.

(b) Flexural torsional buckling under major axis moment only.

(c) Flexural bückling under axial load and minor axis moment.

(d) The interaction of minor axis moment, major axis moment and axial load when buckling is neglected.

These reduction factors are found from charts and formulae. The analysis on which the charts are based takes into account residual stress and initial curvature, modified to reflect test results. The design method has been verified by tests.

The interaction equation used has no theoretical basis and therfore tends to become innaccurate where the axial load or major axis moment are low[35]. Another problem which occurs is that of the estimation of minor axis moments to be used for design. This is especially difficult when the stanchion has large plastic regions[35].

The design method is described in detail in two publications[35, 37]. The major disadvantages of the method are:-

(1) A large number of complex charts are required in order to use the design method.
(2) Much of the data contained in the charts is not based on simply analytical equations, and therefore it cannot be easily automated.

This design method has not yet gained widespread acceptance within the industry and has not as yet been used in the design of a building.

2.8 THE BUILDING RESEARCH STATION DESIGN METHOD 1974[30]

The full scale frame tests which were performed to verify the Joint Committee Report Design Method showed a large reserve of strength in the stanchions. During the tests a new design method was developed in order to try to reduce the difference between predicted and observed loads. The design method is described in a recent report[30]. The concept of the design method is based on the fact that failure of a stanchion occurs when the stiffness of the stanchion reduces to zero, i.e. the deformation becomes very large.

The design method consists of using the requirement of the Joint Committee Report for the design of major and minor axis beams, except that in the case of minor axis beams the stanchions are assumed to have no stiffness. The design of stanchions involves the use of an intuitive approach. The minor axis stiffness of the stanchion alone is assumed to be affected by:-

(a) The relationship of maximum moment about the x-axis to the allowable moment at the ends.
(b) The shape of the major axis bending moment diagram.
(c) The ratio of axial load to "squash load", i.e. area

times yield stress.

(d) The unbalanced moment and member stiffness about the minor axis.

The minor axis stiffness of the stanchion is found. Then the elastic critical load of the stanchion is found using charts, taking into account of the stiffness of the adjoining beams. The applied axial load should then be less than the critical load. In order to ensure that all restraint is not lost at the ends of the stanchion, it is necessary to check that the major axis moments are less than the fully plastic moment.

The relationships which define the reduction in stiffness are semi-empirical, and are made to reflect test results. The portions of the stanchion which are assumed to be plastic at failure have been observed from tests. When major axis S/C bending is considered the whole of the compression flange is assumed to become plastic at failure, resulting in a true stiffness of 40% of that of the elastic stanchion. When considering major axis D/C bending, less of the stanchion becomes plastic, resulting in a true stiffness of 80% of that of the elastic stanchion. The effect of axial load and major axis moment is based on simple linear and quadratic functions.

It is necessary also to assess the effect of minor axis bending on the value of the reduced stiffness. Curves which give the reduced stiffness as a function of the beam and column stiffnesses, yield moment and the unbalanced moment at the joints have been derived from tests. These curves show close agreement to S/C bending and a large spread for D/C bending. The principle effect of minor axis bending is to increase the central deflection of the stanchion and so increase the eccentricity of the direct load, resulting in the spread of the plastic regions. The minor axis stiffness charts are based on the relative rotation of the stanchion and adjoining beams under minor axis moments. The actual minor axis moments in the stanchion are not calculated because they would require an involved non-linear analysis. The reduced stiffness which is used is the minimum of the minor axis stiffness produced using major and minor axis bending cases.

The elastic critical load of the stanchion with reduced stiffness is found from charts derived by WOOD[29]. These charts are based on the determinant of the stiffness matrix of the structure, taking into account axial load, being zero at the elastic critical load. This reduced elastic critical load must be greater than the applied axial load for the stanchion to be stable. This procedure is repeated for each of four S/C load cases.

This design method has been simplified by the use of direct design charts for particular cases. The method is extremely quick to use in comparison with other design methods, and tests have shown it to be accurate. The design method has been used in the design of two buildings.

2.9 CONCLUSIONS

The design methods described range from conservative applications, which are simple to use, to complex analytical methods which are tedious to use. It can be seen that none of the design methods are universally applicable and that there is no unifying theme which runs through them all. For the purposes of this investigation it was necessary to choose one or more design methods for use in the cost optimization study. Three design methods were chosen:- 20

(a) Hornes method: because it has gained widespread use within British industry, it is simple to apply and is based on sound (if conservative) theory.

(b) The Joint Committee Report method: because it is an analytical method which allows consideration of axial loads greater than the Eurler load. It has been verified by tests and represents the opinions of a committee taken from industry. It will form the basis of one design method in the forthcoming revision of BS449.

(c) The Building Research Station Design Method: because it is a semi-empirical design method which is rapidly gaining acceptance in industry. It will also form the basis of a design method in the forthcoming revision of BS449.

The other methods were rejected because of one or more of the following reasons:-

(a) Non acceptance by industry.

(b) Difficult to apply to British Steel sections.

(c) Insufficient practical useage.

The design methods chosen cover the range of restraint conditions and each is currently being used or developed for use in Britain and elsewhere.

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CHAPTER 3

Optimization Techniques and Their Application to Structural Engineering

3.1 INTRODUCTION

The use of computer orientated optimization techniques has developed considerably in recent years. Therefore, in presenting this review of previous work, it has been necessary to confine the scope to those algorithms which have been applied within structural engineering.

The aim of an optimization algorithm is to solve a problem which can be defined mathematically as:-

$$\text{Minimize (or Maximize)} \qquad Z = f\left(\underset{\sim}{x}\right) \qquad - (1)$$

Subject to conditions that

$$g_i\left(\underset{\sim}{x}\right) \leqslant b_i \qquad i = 1, \ldots\ldots\ldots\ldots k \qquad - (2)$$

$$\text{and} \quad e_j\left(\underset{\sim}{x}\right) = c_j \qquad j = 1, \ldots\ldots\ldots\ldots m \qquad - (3)$$

where

$Z = f\left(\underset{\sim}{x}\right)$ = The objective or target function which may be of any form, e.g. linear or non linear.

$\left(\underset{\sim}{x}\right)$ = The variables which may be continuous or discrete.

b_i & c_j = Constants.

Equation (2) represents a set of inequality constraints and equation (3) represents a set of equality constraints.

The objective function may represent such quantities as cost (which has to be minimized), profit (which has to be maximized), or any other criterion which allows the rational comparison of combinations of the values of the variables. The variables represent those quantities which may be varied, in order to minimize the objective function. The variables may be split into dependant and independant variables. Independant variables are not affected by changes in other variables. Dependant variables are affected by changes in other variables. The variables represent such factors as the forces in a structure, the size of the sections in a structure, the geometry of a structure or any other set of factors which will fully define the objective function and constraints.

The constraints represent a set of conditions which ensure that the final set of variables found by the optimization algorithm will be satisfactory. These constraints represent such conditions as:-

(a) The deflection of a structure at a certain point must be less than a given amount.

(b) The total stress at a point in a structure must be less than the allowable stress.

(c) The size of two sections within a structure must be the same.

Any set of values of the variables which satisfy the constraints is termed feasible.

Many optimization algorithms have been developed in recent years, and a number of these algorithms have been used successfully for structural engineering problems. This chapter will consider some of these applications. The usual formulation of structural design problems is examined first, and a simple classification of optimization problems is then described. Examples of problems in each category are given, with the algorithms which have been used to solve them. Finally algorithms which take advantage of a particular characteristic of the problem are examined.

3.2. THE OPTIMUM STRUCTURAL DESIGN PROBLEM FOR PRISMATIC FRAMED STRUCTURES

The optimum structural design problem has been considered by many authors, the formulation of the problem is very similar in almost all cases. Therefore the usual statement of the problem will now be given. The problem can be stated mathematically as:-

$$\text{Minimize} \quad Z = \sum_{i=1}^{nm} C_i A_i L_i \qquad - (4)$$

Subject to

$g_s \leqslant 0$ Stress constraints

$g_d \leqslant 0$ Deflection constraints

$g_b \leqslant 0$ Upper and lower bounds on variables

$e_c = 0$ Compatibility constraints

where

C_i = Cost factor for section used for member i

A_i = Weight of section used for member i

L_i = Length of member i

nm = number of members in the frame.

In the non idealized problem, the variables associated with the objective and constraint functions are shown in Table 3.1. It can be seen that no single set of variables can be used to fully define all of these functions. In order to formulate the problem in a continuous form, it is necessary to use relationships between the variables. Such relationships allow the objective function and all the constraints to be defined in terms of one variable. Many such relationships have been proposed. For British Universal Beam Sections, TEMPLEMAN[70] has proposed the following non linear relationships:-

$$A = 0.78\, Z_e^{2/3} \qquad - (5)$$

$$A = 0.56\, I_x^{1/2} \qquad - (6)$$

Other investigators[37, 57] have cast the problem in a linear form by using the relationship:-

$$A \propto Z_p \qquad - (7)$$

Such approximations involve a certain amount of error, but are necessary

Type of Function	Variables Normally Associated With This Type of Function
Objective Function	The cost/unit weight of the section or the crossectional area of the section used for each member.
Stress Constraints	Forces within the members, Sectional Area of the Members, Section Modulus of the Members, Radius of Gyration of the Members.
Deflection Constraints	Moment of Inertia and/or sectional area of each member.
Compatability Constraints	Moment of Inertial and/or sectional area of each member, forces within the members.
Upper and Lower Bound Constraints	Section depth, section area, width of section for member.

Table 3.1

if the problem is to be solved by continuous variable optimization techniques. TOAKLEY[73] and MAJID[38] used discrete variable techniques in order to avoid using the above relationships. For this type of application the objective function may be of the form:-

$$Z = \sum_{i=1}^{nm} L_i \, (A_i \, S_{i1} + A_2 \, S_{i2} \, ... \, A_n \, S_{in})$$

subject to the additional constraints.

$$S_{ij} = 0,1 \text{ and } \sum_{j=1}^{n} S_{ij} = 1$$

where

A_1 to A_n = The areas of the n sections from which the section to be used must be chosen.

A constraint such as:-

Depth of member i > Depth of member j

would be of the form:-

$$\sum_{k=1}^{n} D_i \, S_{ik} > \sum_{k=1}^{n} D_j \, S_{jk}$$

This approach involves no errors but it makes the problem more difficult to solve because of the large number of variables which are required.

The objective function is usually taken as the weight of the frame. The use of weight has many advantages:-

(a) The weight of the frame is easily related to the section properties, in a continuous variable formulation.

(b) The cost of the material in a frame, based on the weight of the frame, represents a major part of the cost of the frame.

(c) Little information is freely available on the cost of fabrication of structures.

(d) When fabrication costs are available they are difficult to relate to the chosen variables.

(e) For a frame in which dead weight is a major factor, the resulting solution will be close to minimum weight.

The major disadvantages of an objective function based on weight, is that costs not related to weight may invalidate the use of minimum weight as a criterion for design.

When more than one material is used in the structure (e.g. reinforced concrete), it is usual to have an objective function based on the weight of each material comprizing the structure, with a different unit cost for each material[6,7].

The most common formulation of the optimal structural design problem is characterised by two attributes:-

(a) The use of functionalized relationships between section properties.

(b) The use of a minimum weight objective function.

A term which is often used in literature on optimum structural design is a "fully stressed design". Such a design occurs at the intersection of as may stress constraints as there are variables. There may be more than one fully stressed design, and the minimum weight design may not be fully stressed. CORNELL, REINSCHMIDT and BROTCHIE[12] give examples of the relationship between minimum weight and fully stressed designs. Figure 3.1 shows the design space for a tied cantilever. It can be seen that in this case the optimum and fully stressed designs do not coincide.

The designer performing a manual design generally attempts to produce a fully stressed design, in that he attempts to find sections in which the stress is as close to the allowable stress as possible. Such a design will often be close to the optimal design.

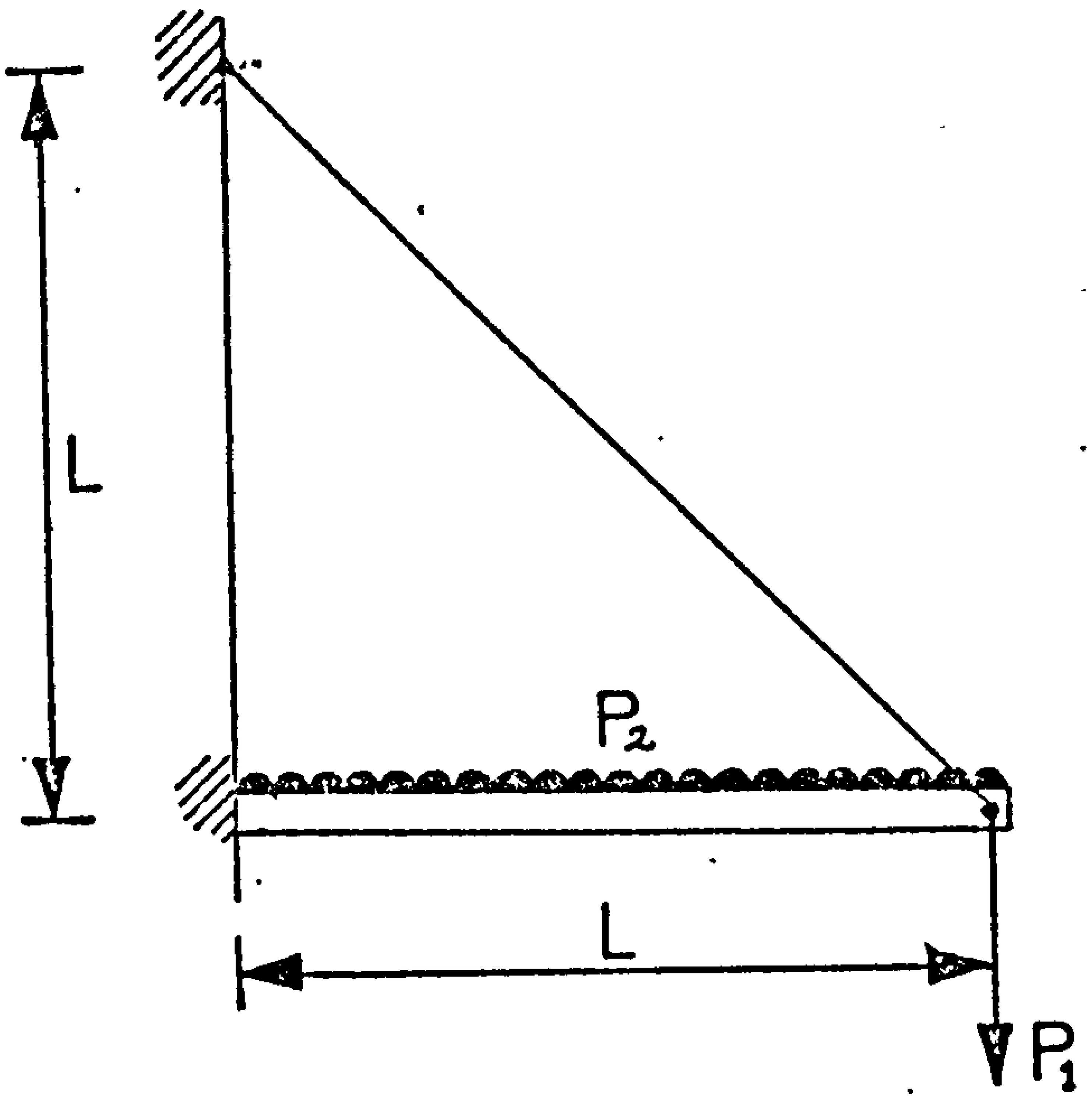
3.3 A CLASSIFICATION OF OPTIMIZATION PROBLEMS

Optimization problems may be classified on the basis of the type of variables, objective function and constraints which occur in the formulation of the problem. Such a classification will now be given.

(a) Analytical. The objective function and constraints are in a form which can be differentiated analytically. The number of variables is small and they are defined continuously. Solution is by one of the classical methods of optimization.

(b) Continous Linear. The objective function and constraints are linear. The number of variables may be large and they are defined continuously. Solution is by the methods of linear programming.

(c) Continuous Non Linear. The objective function and constraints may be linear or non linear. The number of



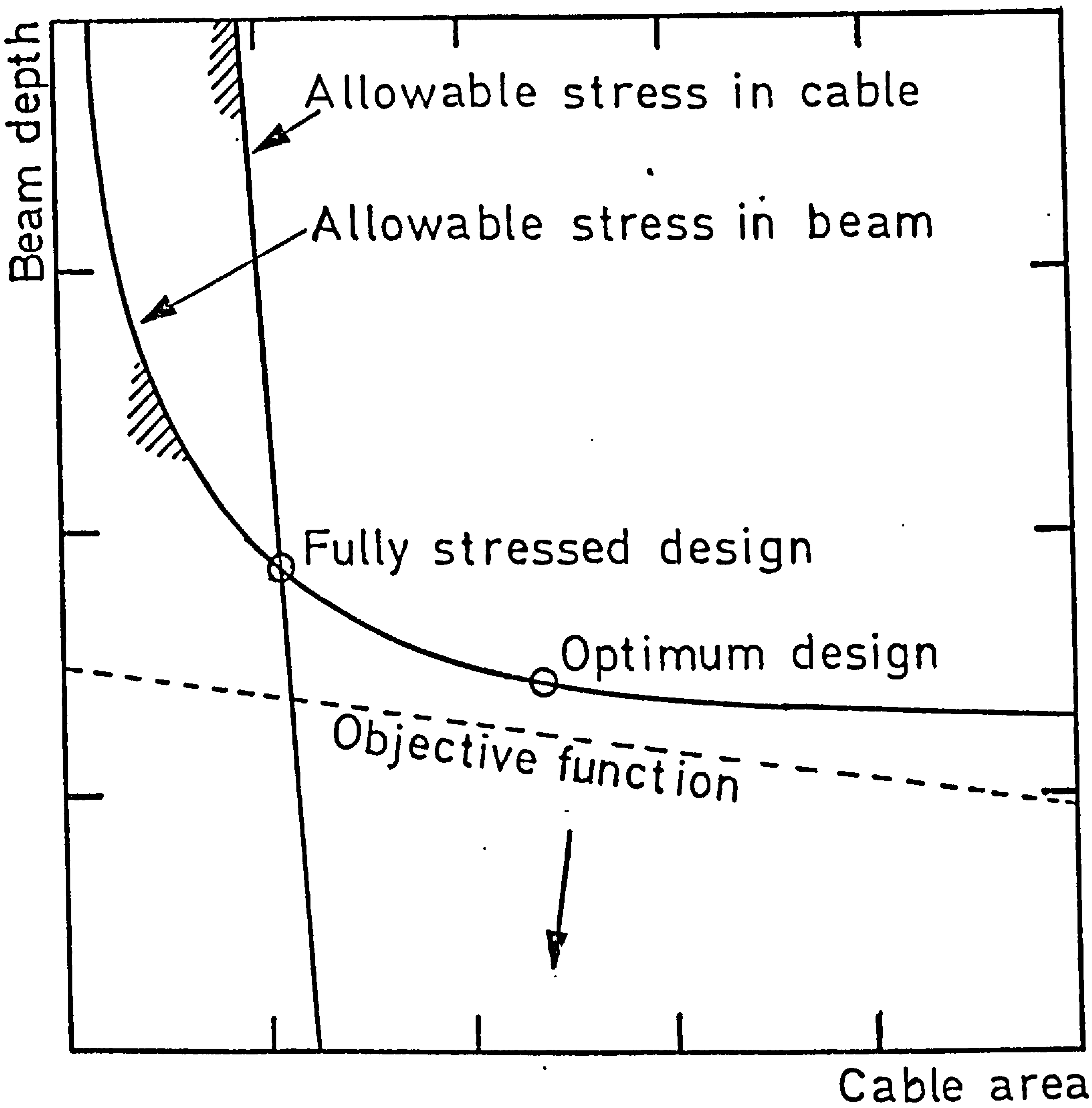


FIG. 3.1 COMPARISON OF FULLY STRESSED AND OPTIMUM DESIGNS (AFTER CORNELL)

variables depends on the characteristics of the problem and the variables are defined continuously. The method of solution is usually by the application of a search strategy. This class of problem also includes those which are discretized artificially, i.e. where a continuous formulation exists but for some reason only discrete values of the design variables are chosen.

(d) Discrete or Mixed. The objective function and constraints may be linear or non linear. Some or all of the variables are discrete (i.e. they are defined only at certain points). The number of variables is usually small. The most common solution method is some form of search strategy which embodies a means of reducing the size of the feasible region. This class of problems represent the most difficult type to solve.

Various techniques which take account of particular aspects of the problem have also been proposed.

When discussing structural optimization techniques two problems arise. Firstly, many applications use more than one optimization algorithm. Secondly almost all investigators report success with the algorithms which they have chosen. Therefore objective comparisons are difficult to make.

Examples of the structural engineering applications within these classifications will now be discussed.

3.4 ANALYTICAL ALGORITHMS

3.4.1 Differentiation

The algorithms within this class form the classical methods of optimization, they rely on the constraints and objective function being continuous and differentiable. The most simple problem is that of unconstrained optimization, in which case the normal methods of differential calculus apply, i.e. satisfying the conditions that:-

$$\frac{\partial z}{\delta x_1} = \frac{\partial z}{\partial x_2} = \frac{\partial z}{\partial x_3} \,\cdots\cdots\cdots\, \frac{\partial z}{\partial x_n} = 0$$

Clearly most structural problems are constrained and therefore this algorithm has limited application.

The simplicity of this approach has led to a number of methods of removing constraints THAKKAR and BULSARI[71] optimized the dimensions of prestressed concrete telegraph poles by examining the active constraints. The active constraints are combined as equalities with the objective function. The resulting function is then differentiated to find the constrained optimum. This approach has been used by many authors[58, 63, 67].

Another method of solving this type of problem is to modify the objective function in such a way that the constraints are removed. Such an approach has been used in the design of structures in which reliability is considered. MAU and SEXSMITH[40] define the cost of a structure as:-

Expected cost = Initial cost + (Cost of failure) x (Probability of failure)

This has the effect of rendering the problem unconstrained, because violation of a stress constraint gives a high probability of failure, this increases the value of the objective function. This approach has been applied to the design of determinate trusses, by making simplifying assumptions about the relationship of failure probability to the area of the members. The expected cost may be found as a simply analytical function, which may be differentiated to find the minimum cost design.

3.4.2 Lagrangian Multipliers

The method of Lagrangian multipliers takes account of equality constraints, which by the use of slack variables may be turned into inequality constraints. This algorithm states that the minimum of a function $f\left(\underset{\sim}{x}\right)$ subject to equality constraints:-

$$e_j(\underset{\sim}{x}) = 0 \qquad j = 1 \,..........\, s$$

is to be found among the turning points of:-

$$\phi(\underset{\sim}{x}, \underset{\sim}{\lambda}) = f(\underset{\sim}{x}) + \sum_{j=1}^{s} \lambda_j e_j(\underset{\sim}{x})$$

where $\underset{\sim}{\lambda}$ are known as Lagrange multipliers. This requires values for $\underset{\sim}{\lambda}$ and $\underset{\sim}{x}$ to be found from the equations:-

$$e_j(\underset{\sim}{x}) = 0 \qquad j = 1 \,..........\, s$$

and

$$\frac{\partial f}{\partial x_i} + \sum_{j=1}^{s} \lambda_j \frac{\partial e_j}{\partial x_i} = 0 \qquad \text{for } i = 1 \,..........\, m$$

Clearly, in all but small problems these equations can become unweilding because of their non linear nature.

KICHER[27] has examined the relationship between fully stressed and minimum weight designs by the use of Lagrange multipliers. The study was confined to linear elastic structures which are simple in concept. By using Lagrange multipliers and the Kuhn Tucker Conditions (which state that at a constrained minimum, the gradient of the objective function is a linear combination of the gradients of the active constraint functions) a set of equations can be derived which define the minimum weight design. Examination of the Lagrange multipliers corresponding to each constraint shows whether that constraint is active therefore if:-

$\lambda_k \neq 0 \quad e_k(\underset{\sim}{x})$ must be active and if

$\lambda_l = 0 \quad e_l(\underset{\sim}{x})$ is not active

By examination rather than solution of the equations a relationship was found between minimum weight and fully stressed designs. Examples were given of the application of the technique to the design of ribbed plates and determinate trusses. From this study the following conclusions were drawn:-

(a) Minimum Weight designs are fully stressed for a large class of structures under single load systems.
(b) The minimum weight design for a determinate stress limited truss will occur when each member is fully stressed under at least one load condition.
(c) Minimum weight designs for structures with many load cases are indeterminate and will not in general be fully stressed.

3.5 LINEAR ALGORITHMS

The methods of linear programming can be applied to linear objective functions with linear constraints. The available algorithms are well developed and only two will be discussed.

3.5.1 The Simplex Algorithm

The simplex algorithm, developed by DANTZIG[13] in 1947, has in a variety of versions the widest application. The algorithm will be explained with the aid of figure 3.2. The algorithm proceeds as follows:-

(a) The constraints are made into equalities by the addition of slack variables.
(b) The constraint and objective functions are tabulated in a standard form.
(c) The values of the slack variables are modified systematically in order to reduce the objective function, If the simplex starts at point A, depending on the details of the approach the search would follow the route A:F:E:D. This in effect consists of searching each vertex either explicitly as in the case of A, F, E and D or implicitly as in the case of B and C.

A number of variations of this algorithm exist which allow the consideration of negative variables or which reduce computation. The benefits of this algorithm lie in the

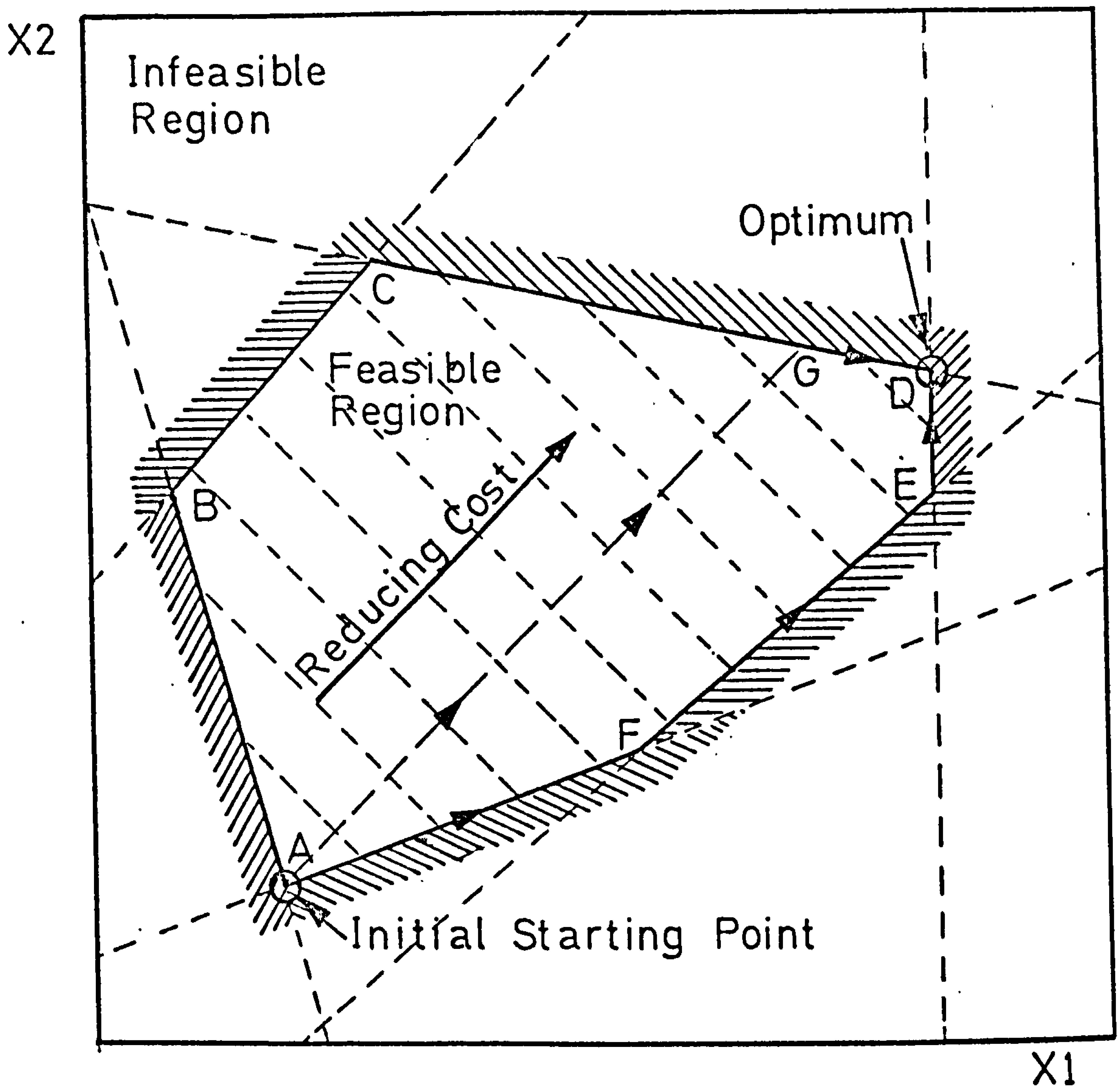


FIG. 3.2 TYPICAL LINEAR PROGRAMMING PROBLEM

fact that standard computer programmes exist which can easily be applied to a problem.

3.5.2 The Gradient Algorithm

The gradient algorithm is a particular case of the non linear gradient algorithm. This algorithm proceeds in the following manner:-

(a) An initial feasible solution is required say point A in figure 3.2.

(b) Movement is made in the direction of reducing cost, the distance and direction to be moved can be found from the geometry of the problem by using simple rules. This corresponds to moving from A to G.

(c) When a constraint is met the direction of movement which is to be followed is determined by considering a smaller linear programming problem. The direction of movement is along any active constraint in the direction of decreasing cost. The distance of movement is again determined by considering the geometry of the problem. This corresponds to moving from G to D.

(d) Step (c) is repeated until no further reduction in cost can be achieved.

Due to the differences in these two approaches the gradient algorithm tends to show economies over the simplex algorithm for problems with large numbers of constraints.

LIVESLEY[37] has applied linear programming to the plastic analysis and design of framed structures. The formulation of the plastic analysis of structures is involved and will only be touched here. The required information is the load factor at collapse. The load factor can be expressed as a linear equality constraint function of the loads, internal moments and geometry of the structure. The condition that plastic hinges occur at a limiting moment can be expressed as linear inequality constraints on the internal moments in the structure. The problem then becomes (in matrix notation):

Maximise $Z = \left(\frac{1}{\ell}, \frac{0}{\ell}\right)\begin{pmatrix}\lambda \\ r\end{pmatrix}$

Subject to $0 = \lambda P + C_r$ Equilibrium

and
$$\left.\begin{array}{r} r \leqslant r_u \\ -r \leqslant r_u \end{array}\right] \text{Moment not greater than the plastic moment.}$$

where

λ = load factor

P = vector of applied loads

r = internal moments

r_u = plastic moment of resistance of the section.

This problem can be solved by either of the algorithms presented. When using the simplex method there is a large increase in the number of variables because the variables have to be made positive in order to solve the problem. Livesley shows how the basic ideas used in the simplex method may be employed in the solution of this problem with a significant reduction in computer storage. The formulation of the plastic design of structures using a linear objective function as previously outlined is also given by Livesley.

The linear programming formulation of the design problem has three main shortcomings. Firstly the use of a linear objective function assumes a continuous range of sections, therefore it is necessary to round up to available sections. Secondly the effect of axial loads on the fully plastic moment in the stanchions cannot be included. Thirdly the effects of frame instability cannot be included.

HORNE and MORRIS[24] and MORRIS[42] describe a computer programme (based on earlier work by TOAKLEY[74]) for the automatic design of sway frames. This programme is intended for practical use and overcomes some of the above shortcomings. The steps that the algorithm takes are:-

(a) Solve the problem using the formulation given by Livesley.
(b) Select discrete sections with a greater plastic modulus than that given by stage (a).
(c) Using thediscrete sections chosen find the reduction in plastic moments due to axial load.
(d) An elastic analysis of the structure is performed to determine the deflection at each storey.
(e) The problem is reformulated including the effects of axial load and the change of geometry due to deflection.
(f) The reformulated problem is then solved and a return is made to stage (b), unless convergence has occured.

The method of solution used is a variation of the simplex method. The results of this iterative algorithm are not claimed to be optimal, however they produce a design which is both practical and efficient.

KIRSCH[29] has considered the elastic design of continuous beams and trusses. By introducing lack of fit and sinking supports it is unnecessary to consider compatibility and the problem reduces to a linear form. The solution consists of a two stage process, the first stage finds an optimal structure ignoring compatibility (as in plastic design), the second stage finds the lact of fit and "controlled" forces which are required to bring about compatibility. The solution of each stage was by the simplex algorithm. The solution method breaks down if there is not a single set of controlled forces which will satisfy all load cases. This algorithm produces prestressed trusses which have a lower weight than conventionally designed elastic trusses.

ARORA[3] has formulated "the inverse problem of structural optimization". He states this problem to be "given a structure, how much load can this structure support safely".

The applied forces are the variables and the sum of these forces is the objective function. The nodal displacements, stresses and applied loads are related through the structure and member stiffness equations, manipulation of these equations results in a set of linear constraints in terms of the applied forces. This problem can then be solved by a linear programming algorithm.

3.6 NON-LINEAR CONTINUOUS ALGORITHMS

Algorithms in this class can be applied to optimization problems in which some of the constraints or the objection function are non linear. The solution is complicated by the need to prove that the problem is convex, in order to ensure the selection of a global optimum. In practice it is often impossible to prove convexity, resulting in the need to start the algorithm at a number of starting points. The global optimum is assumed to be the most optimal of the optima found.

The available algorithms can be classified according to the manner in which they facilitate solution:-

(a) Methods which simplify or transform the problem in such a way that the problem is either reduced in complexity or is solved as a series of smaller problems. This transformed problem may be solved by using another algorithm.

(b) Methods which attempt to solve the problem without simplification.

A description will now be given of some of the algorithms in these classes which have been applied to structural design.

3.6.1 Algorithms which Simplify the Problem

3.6.1.1 Sequence of Linear Programmes (S.L.P)

This class of algorithms (also termed the cutting plane method) takes advantage of the well tried algorithms of linear programming. This feature has made it extremely popular among investigators in structural design. The basic method is to devise linearized forms of the constraints and objective function in the region of the current design point either by using the low order terms of a Taylor Series expansion, piecewise linearization or numerical approximations. To ensure that the approximation used remains accurate,limits are set on the permissable change in design variables at each stage. The resulting linear problem is then solved by the algorithms of linear programming. This results in a new design point about which the approximations are again made until convergence occurs. Convergence is assured by reducing the size of the moves in a predetermined fashion. Convergence on a local optimum may occur and therefore a portion of the feasible design space will not be considered in the solution. DOUTY and CROCKER[17] have used this approach for the design of cold formed truss purlins to an American design code. This design problem is characterised by a large number of non linear, discontinuous design constraints and many variables. The constraints were linearized about the current design point using a Taylor series expansion, producing the following formulation:-

Minimise $\quad Z = (\underset{\sim}{c}^{o})(\underset{\sim}{x})$

Subject to $\quad (\underset{\sim}{a}^{o})(\underset{\sim}{x}) \leqslant -(\underset{\sim}{g}^{o}) + (\underset{\sim}{a}^{o})(\underset{\sim}{x}^{o})$

in which $\quad c_j = \dfrac{\partial f(\underset{\sim}{x})}{xj}$ and $a_{ij} = \dfrac{\partial gi(\underset{\sim}{x})}{xj}$

The supercript ^{o} refers to the values of the variables at the current design point. After a linear programming cycle, these equations are reformulated at the new current design point. In this application there are forty-one constraints, including stress constraints for each load case, buckling constraints on members or the frame and upper and lower limits on section sizes. These constraints must be differentiated with respect to the twenty-four design variables, resulting in a large number of formulae. It was found that only five or six variables could be optimized at a time, otherwise the computing costs became very high. Although the objective function considered material weight only the results of the programme have been used in industry.

MOSES[43] used a similar approach for the design of plane frames and trusses using the usual formulation of the optimum structural design problem. He used a method of accelerating convergence which consists of altering the design variables until the non linear constraints are satisfied after each cycle.

JOHNSON and BROTTON[25] have applied this approach to the design of elastic trusses subject to stress constraints. They considered three different formulations of the problem which are summarised in Table 3.2. The first formulation is the most common formulation used. The second formulation reduces linearization errors because stress and the reciprocal of section area are essentially linearly related. The third formulation also reduces linearization errors because the permissable force is linearly related to section area. It was shown that the third formulation involved the smallest linearization errors and therefore produces the fastest convergence. This investigation shows how essential it is to devise the most efficient formulation of any problem.

DAVIES and WANG[14] have produced a hybrid algorithm which makes use of the cutting plane method. The

Formulation Number	Objective Function in Terms of	Type of Objective Function	Constraint Functions in Terms of	Type of Constraint Function
1	Section Area	Linear	Stress	Non Linear
2	Reciprocal of Section area	Non Linear	Stress	Non Linear
3	Section area	Linear	Force	Non Linear

Table 3.2

algorithm is applied to the design of railway carriage underframes using the usual formulation previously described. The constraints and objective function are linearized about the current design point, this linearized problem is then solved. The next stage is to analyse the structure to determine the stresses and deflections at the current design point. Using this information it is possible to move closer to the constraints by interpolation. The constraints are again linearized and the process repeated. During each iteration the linearized constraints which do not encroach on the feasible design space are accumulated to provide a composite feasible region made up of linear segments. This procedure allows data from previous iterations to influence the current iteration, however in practice the problem becomes complex and requires a steadily increasing amount of computer storage. This algorithm has been compared with simple constraint accumulation and has been found to be more efficient where the optimum occurs at the intersection of constraints.

An application of S.L.P. which uses linearization determined numerically has been described by ANDERSON and SALTER[1]. This application applies to structural frames subject only to deflection constraints. The variables chosen are the moment of inertia of the members. Each variable is incremented in turn and the structure analysed and, by using linear interpolation, a set of linear equations are formed. The results of the algorithm are claimed to be efficient and not necessarily optimal.

ZEINKIEWICZ and CAMPBELL[78] present the formulation of S.L.P. to the optimization of the shape of structural continua using the finite element method. The applications discussed include the design of gravity dams, arch dams and rotating discs. A typical application is shown in figure 3.3. This type of problem is characterised by large numbers of variables (in this case the node coordinates) and large numbers of continuous constraints (in this case stress and displacement at each node). In order to make the problem manageable, stress and deflection constraints are only considered at a few critical locations within the structure. The linearization is performed using an analytical derivation. Solution proceeds by the use of a standard linear programming algorithm coupled with move limits and the accumulation of constraints.

3.6.1.2 Geometric Programming

This algorithm can be applied to objective and constraint functions which are polynomials and have positive coefficients. The objective function is of the form:-

$$Z = \sum_{j=1}^{T} C_j \prod_{i=1}^{N} x_j^{aij}$$

where $\prod_{i=1}^{N}$ indicates repeated multiplication N times.

FIG. 3.3 SHAPE OPTIMIZATION USING S.L.P.
(AFTER ZIENKIEWICZ & CAMPBELL)

The M constraints are of the form:-

$$\sum_{j=1}^{T_m} C_{mj} \prod_{i=1}^{N} x_i^{amji} < 1 \qquad m = 1 \ldots\ldots\ldots\ldots M$$

A summary of the solution method will now be given.

The above problem is termed the primal problem. This problem may be replaced by a dual problem which, has a maximum with the same numerical value as the minimum of the primal problem. It then becomes necessary to maximise the objective function of the dual problem with respect to the dual variables, subject to constraints on the dual variables. The ease by which the problem may be solved depends on "the degree of difficulty", this is defined as:-

(number of terms in the objective function) + (number of variables) - 1

If the degree of difficulty is zero the solution is trivial, if it is greater than zero the dual problem may be linear or non linear. With a large degree of difficulty the dual problem may be more difficult to solve than the primal problem. TEMPLEMAN[70] claims that it is often simpler to solve the dual problem for two reasons. Firstly linear constraints may occur in the dual problem which can be exploited by special algorithms. Secondly the dual problem will give a global optimum because it is always convex. TEMPLEMAN[70, 69] has been the main proponent of the algorithm (applied to structural engineering). He gives a description of the algorithm, and modifications necessary to deal with negative terms in the polynomial equations. He applies the algorithm to the design of a floor system composed of concrete slabs, supported by encased steel beams. The section properties of the available beam sections were approximated by using equations 5 and 6. The problem had ten variables and fifteen constraints. The objective function included the costs of concrete, reinforcing steel, universal beams and formwork taken from published "measured rates". The solution was found to be very sensitive to the cost of the beams which caused accuracy problems when selecting discrete sections.

BRADLEY, BROWN and FEENEY[6] have applied the geometric programming algorithm to the approximate optimization of factory type buildings. The objective function, which consisted of polynomial functions, included elements of the total building cost such as the steel frame, purlins, cladding, foundations, floor slab and fixings. The types of structural framing considered, included portal frames, trusses and grids. The constraints consisted of upper and lower bound constraints on variables, stress and deflection constraints. The final costs were shown to be similar irrespective of the framing system and were very insensitive to any change other than span.

HALL[23] formulated the design of redundant trusses. The primal problem corresponds to the principle of minimum complementary energy and the dual problem was shown to correspond to the principle of minimum potential energy thus showing, in this case, a physical meaning for the dual problem.

3.6.1.3 Penalty Functions

This class of algorithm is also termed the sequential unconstrained minimization technique (S.U.M.T.) or the created response surface technique. As the name implies the violation of a constraint causes a penalty to be added to the objective function. This has the effect of rendering the problem unconstrained.

The solution procedure varies according to the type of penalty function chosen. There are two general types of function which will now be described.

Interior Penalty Functions:- Using this type of function the objective function is modified, using a typical formulation, from f(x) to say:-

$$\phi\ (x,r_k) = f(x) + r_k \Sigma \frac{1}{g_i(x)}$$

Where the m constraints are of the form $g_i(x) \geqslant 0$ and r_k is a constraint (termed the response factor) which is varied during the solution. An example of how such a penalty function modifies the objective function is given in figure 3.4. It can be seen that successive reduction of r_k produces a surface which more closely models the true objective function and constraints. The solution procedure starts with an initial feasible design and a large value for r_k. The resulting problem is then solved using a direct search or gradient technique, relating the step length to r_k so that a step cannot be taken beyond a constraint. This process is then repeated, using the final point of the previous iteration as the next starting point, with a smaller value of r_k, until a low value of r_k is reached. Therefore a constrained problem is replaced by a series of simpler unconstrained problems.

Exterior Penalty Functions:- The objective function is again modified using a typical formulation from f(x) to:-

$$\phi(x,r_k) = f(x) + r_k \Sigma\ (\min\ (0,g_i(x)))^2$$

An example of how this type of penalty function modifies the objective function is given in figure 3.5. In this case successive increases in r_k result in a more closely modelled surface. The solution procedure

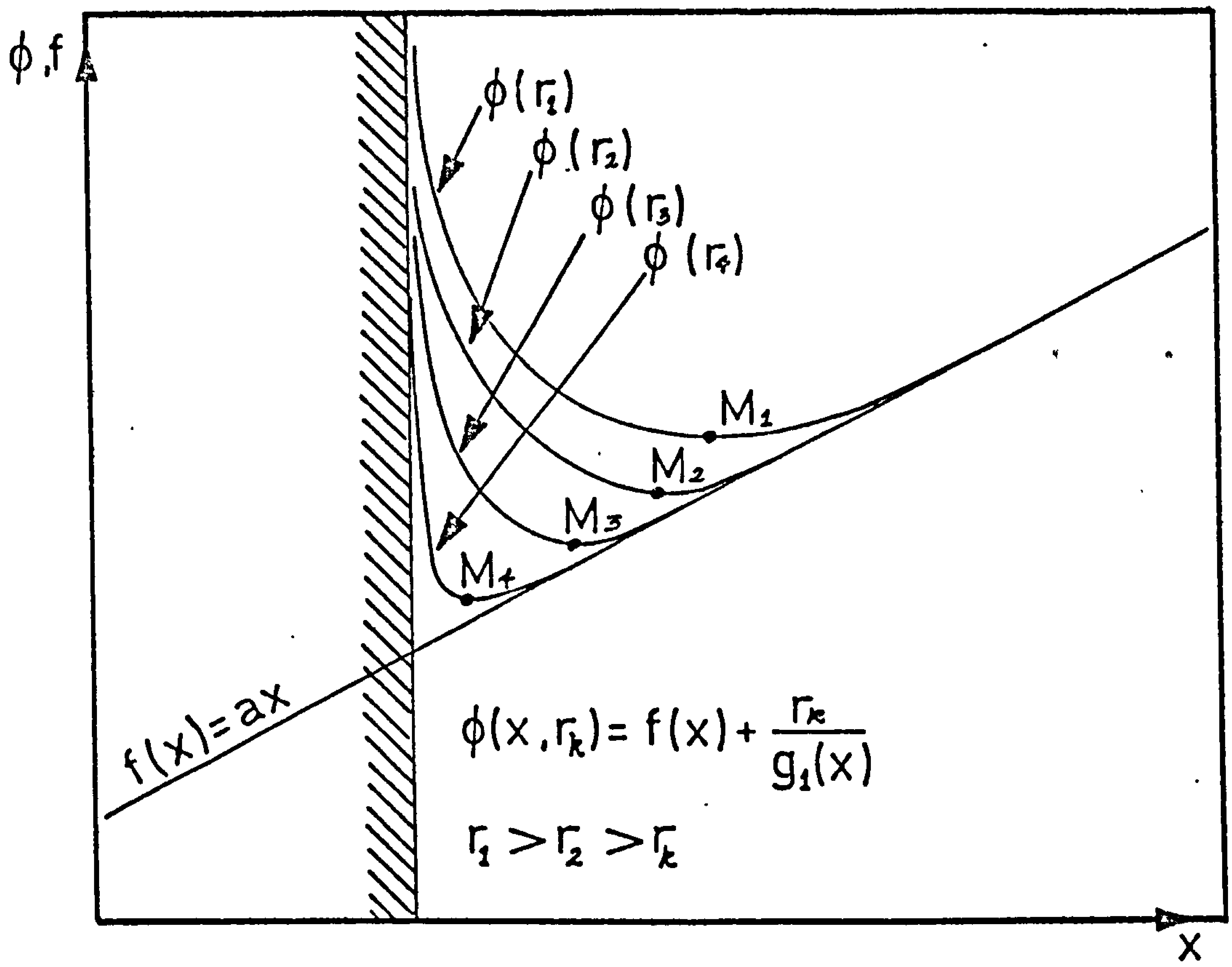


FIG. 3.4 INTERIOR PENALTY FUNCTION

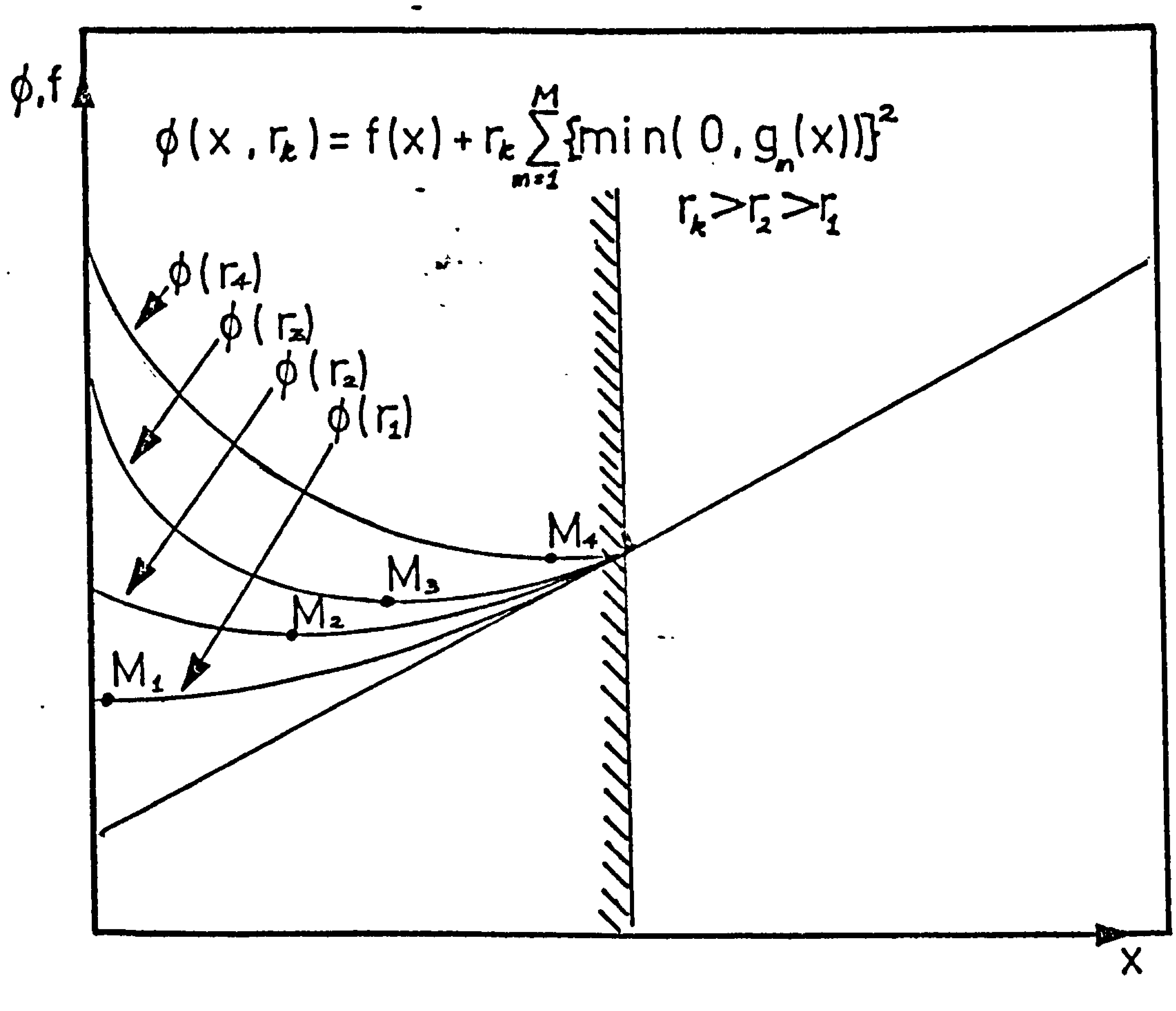


FIG. 3.5 EXTERIOR PENALTY FUNCTION

is similar to that of the interior penalty function except that an initial feasible point need not be known.

Numerous penalty functions have been proposed, as have combinations (termed extended penalty functions) of the two types. The lagrangian multiplier (section 3.4.2) is related to penalty functions in that it modifies the objective function in a similar fashion.

MOE[41] gives two formulations of the optimum structural design problem which can be solved with the aid of penalty functions. The first formulation consists of formulating the equilibrium equations relating the section sizes (independant design variables) and the stresses (dependant design variables) and introducing these relationships into the inequality constraints. This results in a small number of variables and all the constraints being inequalities. The second formulation relates the section sizes and stresses by equality constraints using the equilibrium equations. The stress constraints remain as inequalities. This formulation is less attractive in that the penalty function method is not suited to the use of equality constraints. It is however often essential because the equilibrium equations cannot be solved analytically (e.g. in large plane frames). The second formulation uses a modified penalty function method which consists of minimising the amount by which the equality constraints are violated while also considering the inequality constraints.

KARLIE and MOE[26] have applied the penalty function method to the elastic design of framed structures. A simple grillage (previously examined by MOSES and ONODA[46]) was examined (see figure 3.6). Three

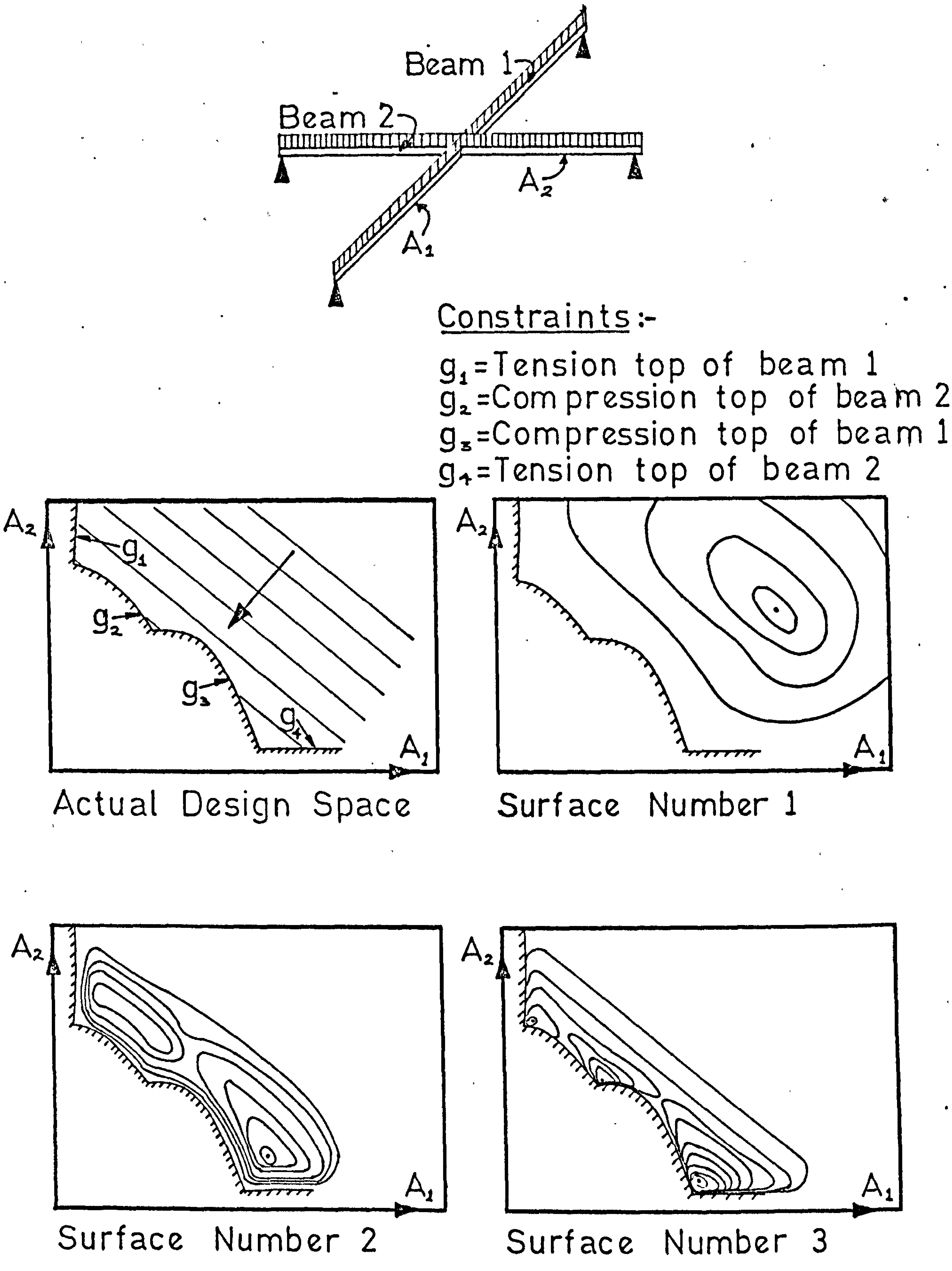


FIG. 3.6 INTERIOR PENALTY FUNCTION RESPONSE SURFACES (AFTER KAVLIE & MOE)

successive response surfaces are shown. It can be seen that there are three local optima in the final solution. An important property of the penalty function method can be seen, the penalty function smooths over the true surface and the successive optima found are likely to correspond to the global optimum. The method was also applied to a more complex grillage and a tanker bulkhead. The large numbers of constraints and variables result in the use of large amounts of computer time. Therefore Karlie and Moe used different approaches. Firstly complete optimization was performed using relationships between the variables to reduce the number of independant variables. Secondly only section shapes were optimized, with analysis at the end of each stage. Thirdly combinations of section shapes and frame variables were optimised. All of these approaches resulted in drastic reductions in computation time.

BOND[5] presents the design of prestressed concrete bridge decks using an interior penalty function. The variables included section dimensions, concrete strengths, size and area of reinforcement. The objective function included material costs, formwork costs and falsework costs which in contrast to steelwork design can all be easily related to the variables. The constraints included, stress, deflection, tendon cover, section size and accomodation of reinforcing bars. Upper and lower bound onstraints were included on all variables and 30
these were included within the penalty terms. The search algorithm used consisted of line searches in the direction of steepest descent with a single parabolic interpolation once the minimum was bracketed. In order to cut the amount of computing time, an approximate method of analysis was used with full analysis periodically.

SHAMIE and SCHMIT[62] describe the design of planar frames which include constraints on the first mode natural frequency, which can be used to prevent frame instability. In contrast to many applications the dimensions of the members are allowed to vary in a continuous fashion. The objective function is the weight of the structure, which can be determined as a continuous function of the variables. An extended interior penalty function was used which takes on a different form when the constraint is satisfied, to when the constraint is not satisfied. The problem was solved using a gradient method because analytical expressions were available for the objective function and constraints.

In a similar vèin CASSIS and SCHMIT[8] investigated the design of plane frames subject to earthquake loading with constraints on natural frequency and dynamic response of the structure. An exterior penalty function in which a penalty was only included when a constraint is violated was used. The exterior penalty function was used because the design is disjoint and therefore if a feasible design is known it may not be in the optimum region. The constraints were modelled using a Taylor series expansion in order to reduce computing effort.

3.6.2 Algorithms Which Do Not Simplify The Problem

3.6.2.1 Direct Search Algorithm

This class of algorithm consist of selecting a direction in which to search which will reduce the objective function. A large number of algorithms exist within this class and so only a few will be described together with their application to structural optimization.

3.6.2.2 Univariate Search

This is the simplest non linear algorithm which has been developed, the algorithm consists of altering each variable in turn until either a constraint is reached or a minimum is found. The algorithm is slow to converge when the path of descent is not parallel to one of the coordinate axis. The slow convergence for certain problems and simplicity of the algorithm have resulted in few practical applications being reported.

LANE[33] has used this algorithm for the design of a reinforced concrete column bases. The independant - 12
variables were the plan dimensions and the thickness of the base. The objective function consisted of the cost of reinforcing steel and concrete. The constraints consisted of stress constraints for both the soil and the concrete and minimum base dimensions. The approach used consisted of varying the ratio length to breadth by increments and each time selecting an optimum length. When this shows little improvement the algorithm then varies the length directly until an optimum is found. The thickness is then varied until the optimum is found. Lane argues that such an approach can show substantial savings if it is taylored to the requirements of the problem by previous investigation of the design space.

3.6.2.3 Multi-Variable Search Algorithms

This class of algorithms seeks to orientate the search direction along the axis of the objective function. Typically a univariate search is made in each variable and the resulting direction of movement is extrapolated and a minimum found along this direction. The process is then repeated from this point until convergence occurs. This method is more efficient than univariate search because it can negotiate valleys in the objective function which are not parallel to a coordinate axis. Algorithms of this class vary in detail and because of their availability as "black box" computer programmes. They are often used in conjunction with the penalty function method.

SURTEES and TORDOFF[68] have used an algorithm of this type combined with an exterior penalty function for the design of steel box girder bridge structures. The variables consist of section dimensions and plate thicknesses, and, although they are defined continuously, they are restricted to discrete values. The objective function consists of the cost of fabrication and material costs and may be discontinuous. The constraints are taken from relevant Codes of Practice and are very large in number. The algorithm consists of using two types of move, exploratory and pattern moves. The exploratory moves consist of moving a step length about each axis in turn, moving the current point each time. The pattern moves consist of travelling along the line of total progress of the previous exploratory moves in search of a further reduction in the objective function. The ability of this algorithm to move between discrete points is in this case very useful, however the variables could have been treated as continuous.

LANE[33] used a multivariate search algorithm for the design of a column base (see section 3.6.2.2). This algorithm consists of searching in each of n orthonormal directions successively using variable step lengths until a successful move has been followed by an unsuccessful move in each direction.

A new set of orthonormal vectors is then generated, the first of which lies in the direction of total recent progress of the search. Lane compared this algorithm with the problem orientated univariate search algorithm previously described and found it to be less attractive. This algorithm has the advantage that it is available as a "black box" subroutine and that it is usually more attractive than simple univariate search.

3.6.2.4 Powell's Direct Search Algorithm

Algorithms which can guarantee to reach the minimum of a quadratic function in a specified number of steps are said to be quadratically convergent. Such algorithms are attractive because many objective functions approximate to a quadratic in the region of the optimum. The usual procedure for minimising such functions is to set up a set of conjugate directions which, if followed, will result in convergence in a set number of steps. POWELL'S algorithm[51] seeks these directions but in order to ensure that, for non quadratic functions, a false result is not achieved modifications are made which affect the quadratic convergence. The procedure consists of:-

(a) Starting with an initial set of directions (e.g. the coordinate axes), line searches are made in each direction in turn resulting in the design x_1.

(b) A further step is then made in the direction of total progress in this iteration, to design x_2. One of two procedures may then happen:-

(i) If the direction of total progress of the last iteration will not help to bring about conjugacy, which can be defined mathematically, when it is included in the current set of directions, the search is restarted at x_1 or x_2.

(ii) If the direction of total progress of the last iteration will help to bring about conjugacy it replaces one of the current set of directions.

(c) A return is then made to step (a) using the current set of directions.

This algorithm is very efficient because it uses information from previous iterations to direct the search in future iterations, resulting in rapid convergence. This algorithm is available as a "black box" subroutine and is therefore often used. Applications of this algorithm used in conjunction with penalty functions have been made by BOND[4] and MOE[41], these applications being similar to those previously described in section 3.6.1.3. MOSES[44] has also used a penalty function combined with Powell's algorithm for the reliability based design of reinforced concrete beams.

3.6.2.5 <u>The Non Linear Simplex Algorithm</u>

This algorithm is essentially a gradient algorithm which does not explicitly calculate gradients. Essentially the simplex algorithm consists of the selection of (n + 1) points in the design space of n variables. The procedure consists of reflecting the point with the largest objective function value through the centroid of all the points in the simplex. This is illustrated in figure 3.7. If the new point has the largest function value in the new simplex, the next largest point is reflected. Failure to proceed results in the simplex being "shrunk" about the lowest point. The procedure is repeated until it has shrunk to a predetermined size.

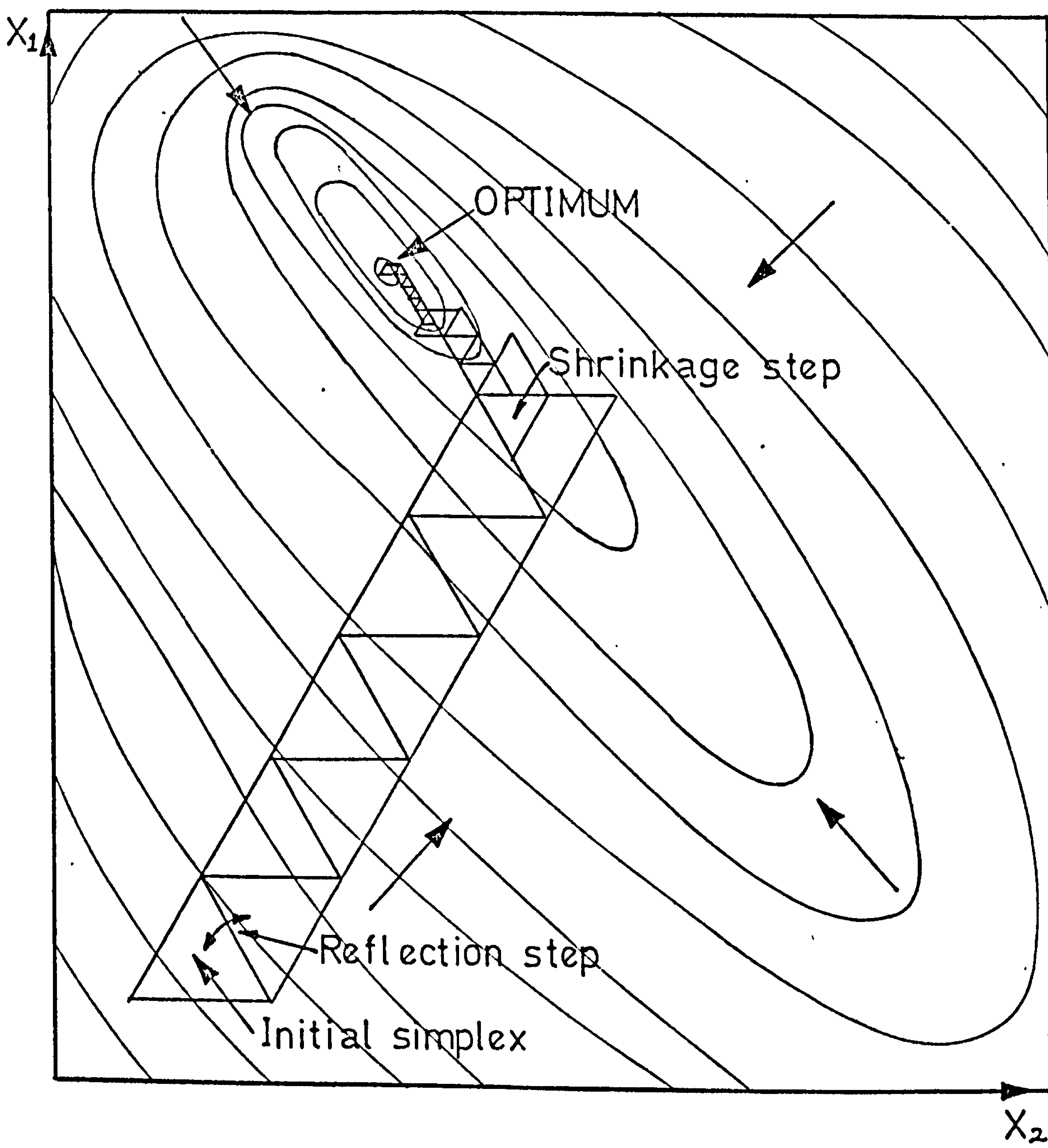


FIG. 3.7 TYPICAL NON-LINEAR SIMPLEX APPLICATION

The shape of the simplex is usually equilateral, however the shape is a function of the scaling of the variables and therefore this shape need not be maintained. For this reason automatic scaling procedures have been suggested so that under unfavourable conditions the simplex may scale itself to the shape of the objective function. A variation of the simplex algorithm (termed the complex algorithm) is also available which makes use of more points and can be used for problems with convex constraints.

KNAPTON[31] and LEE and KNAPTON[34] have used the simplex algorithm for the design of industrial buildings for minimum cost. The algorithm was tried as a method of selecting discrete sections for the design of pitched portal frames, the results were disappointing because the large number of local optima contrived to halt the algorithm at unsuitable designs. A procedure based on the method of selecting sections which is used in industry was then used and the simplex was used to optimize the geometry of the building including such variables as plan dimensions, roof pitch and frame spacing. Despite the non linear, discontinuous nature of the objective function, the algorithm rapidly found local optima close to the global optimum using a number of starting points. The cost model, which defined the objective function, is described in Chapter 5.

LIPSON and RUSSELL[36] have examined the cost optimization of a structural roof system using the complex algorithm. The variables were divided into two sets. The first subset consisted of the independant variables which defined the geometry of the roof system and included, truss spacing, truss depth, truss panel width and purlin spacing. The second subset consisted of the member sizes which were dependant variables. The objective function consisted of the cost of material, fabrication and sheeting. For a given set of independant variables the dependant variables were determined by producing a fully stressed design and using discrete sections. The frame was then costed using these sections. This procedure makes the objective function discontinuous, however the complex algorithm can still operate on such a surface as long as the surface does not consist of plateaus and there is a trend which the complex can follow. Stress constraints were avoided by separating the variables and using fully stressed design principles. The only constraints considered were upper and lower bound constraints on the independant variables. The operation of the complex algorithm is essentially similar to that of the simplex algorithm, however there are two major differences. Firstly the complex contains a surplus of points above the minimum required, this prevents the complex flattenting against the first constraint met. Secondly when generating new designs the complex tests for feasibility, if a constraint is violated the distance of the new point from the centroid is halved until the constraint is satisfied. This procedure will always find a feasible point if the design space is convex. The algorithm was found to be efficient, despite the discontinuous nature of the objective function. Starting the algorithm from different points resulted in substantially similar final designs and the results were shown to be close to optimal by the the use of an exhaustive search.

LIPSON and AGRAWAL[35] in an extension of the above work have used the complex algorithm for the minimum weight design of pin jointed trusses for multiple load cases. The independant variables consisted of the geometry and topology of the frame, the dependant variables were the section sizes which were again discrete. The selection of discrete sections was performed by selecting, from a table of sections and allowable forces for those sections, the minimum weight section which could carry the applied force. When using the complex algorithm an initial complex was generated using a random number generator. When the algorithm was started a rapid improvement occured for thirty to fify iterations, ending with the complex either tightly clustered around its centroid or flattened against a constraint. A new complex was generated around the current best point and the algorithm was restarted. This procedure resulted in an improvement if repeated one or two times. The algorithm terminated if the complex showed no improvement after a certain number of iterations or if a new feasible point could not be devised. A number of examples of the use of the algorithm were given including a transmission tower investigated by other authors. The algorithm produced a design with a similar weight but a different geometry.

3.6.2.6 Gradient Algorithms

This class of algorithms are used if the gradients of the objective function can be determined analytically. This information can be used to move in the direction of steepest descent. The procedure when a constraint is met varies but it can be simplified by a knowledge of the constraint gradients. Occas~~s~~ionally investigators have determined gradients L 9
by the use of finite difference techniques, however this is generally inefficient as it requires at least three function evaluations for each gradient evaluation.

In describing the available algorithms it will be assumed that the gradients of both the objective function and constraints are available in an analytical form. The procedure followed consists of determining a direction of steepest descent from the gradients of the objective function, this is a simple mathematical process and will not be enlarged upon here. Movement is then made in this direction until one or more constraints are violated. The procedure then followed depends on the particular algorithm, however only two main strategies are used. The first strategy consists of finding the point at which the constraint is exactly satisfied by interpolation and then finding a new direction of movement, which reduces the objective function and minimises constraint violations, by solving a small linear programming problem. Movement along this direction tends to violate constraints in a convex design space, and it becomes necessary to return to the feasible region, this is performed using the second strategy. The second strategy tries to return to the feasible region in an "optimal fashion". The return to the feasible region may either, be made along a contour of constant objective function such that constraint violations decrease, or a return may be made along the direction of steepest descent of the constraint function.

3.6.2.7 The Method of Alternate Base Planes or Alternate Steps

This algorithm consists of using either the gradient method or any other non linear algorithm until a constraint is violated, at this point the condition of constant weight is substituted into the constraint equation and movement is made along the resulting plane in a direction which ensures reduction in the violation of the constraint. Movement is stopped when the current point has moved well into the feasible region. The whole procedure is then repeated.

The final point is reached when it is not possible to reduce weight without violating the constraints.

One of the first investigations of non linear optimization to a structural design problem was made SCHMIT[59, 60] using this algorithm. Schmit looked at the classical problem of the three bar truss, trying to produce a minimum weight design with constraints on stress, buckling and member size. The variables were the member areas. For this problem it was simple to find analytical expressions for the objective function and constraint functions. When a constraint was violated a set of random directions of search were found using a random number generator, the distances along these directions to each of the side constraints were found analytically. A set of random distances were then found and proposed new designs were given by using the random distances and directions together with the condition that the structure weight should remain constant. This condition is usually linear and so is easily satisfied. The proposed designs are checked for feasibility. If a feasible design is not found, new random designs are generated and the procedure repeated. When a feasible design is found the algorithm continues by moving in the direction of steepest descent. DE SILVA[15] reports that this algorithm has two major disadvantages. Firstly, for realistic problems, the cost of searching a large number of random (and often not beneficial) directions becomes prohibitive and secondly the algorithm will not operate with large numbers of variables.

DE SILVA[15] has proposed a simpler alternate step procedure, consider figure 3.8. The points A, B and C represent successive designs each with reducing cost and found using the gradient algorithm. Point C is a boundary point. The procedure when point C is reached is to examine point D. If point D is infeasible an alternate step is taken along the direction DC, away from D, otherwise the direction is reversed. The algorithm can be modified to consider multiple constraints. This algorithm was compared with the previous algorithm and was found to be far superior and easier to programme for the general case.

ZARGHAMEE[77] has used the method of alternate steps for the minimum weight design of tall truss towers. The variables are the sectional areas. The constraints which were considered included member stress and buckling criterior and frame instability. The gradients of each constraint could be found analytically at any point. The algorithm consisted of travelling in the direction of steepest descent until a constraint is met, then a movement is made tangentially to the constraint (producing an infeasible design in a convex space). An alternate step is then taken along the constant weight contour until feasibility is achieved, then the process is repeated.

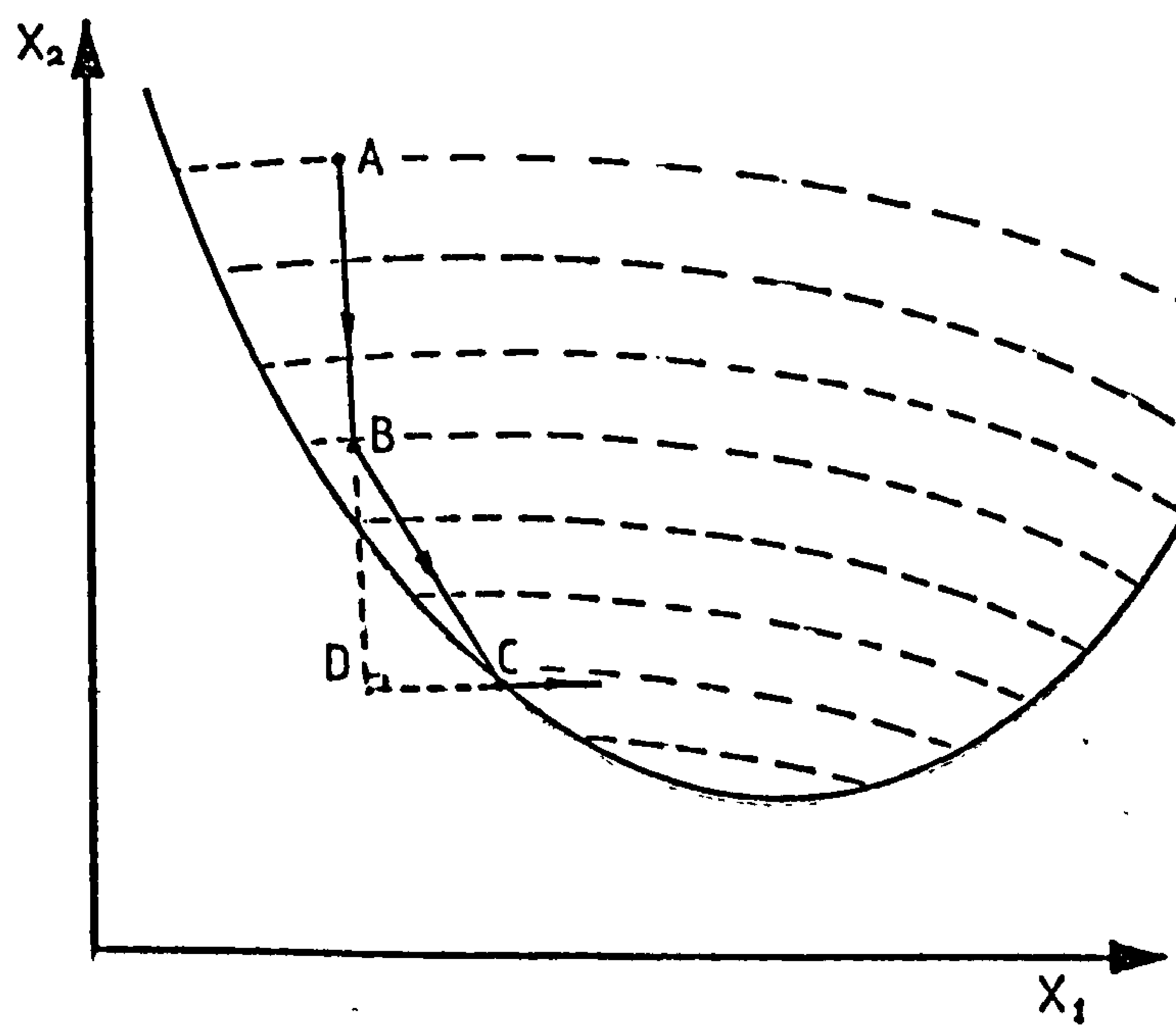


Figure 3.8 Alternate Step Algorithm (after De Silva)

In a similar investigation ZARGHAMEE[76] used the same algorithm for the design of radio antennas. The constraints include a relative deflection constraint which ensure that the deflections of the bowl approximate to a paraboloid, stress constraints were satisfied implicitly by changing the lower bound size of each member.

3.6.2.8 Gradient Projection Algorithms

This class of algorithms chooses the best direction of movement when a constraint is met. The direction chosen allows movement with a minimum of constraint violation and with a decreasing weight. A correction is then made back to the feasible region by movement along the gradient of the constraint functions which are violated.

BROWN and ANG[7] have used an algorithm of this type, developed by ROSEN[56], for the design of plane frames using the formulation of the structural design problem outlined in Section 3.2. The procedure used by the algorithm was to move from an initial feasible starting point along the direction of steepest descent until a constraint is met. The planes which are tangential to the active constraints at this point are then found. The gradient of the objective function is then projected onto the intersection of these hyper-planes. A step is then taken along this direction. The new point will be on the boundary of the feasible region, a step is then taken normal to the projected gradient and in the direction which reduces constraint violation. This process is repeated until cycling occurs between two points, movement is then made along the intersection of the two projected gradients towards the feasible region. The process is repeated with a smaller step length until the minimum is found within a given accuracy. This algorithm has three major advantages over the S.L.P. algorithm with accumulation of constraints. Firstly the number of constraints remains constant. Secondly it is necessary to linearize only active constraints and finally it can easily deal with non vertex solutions. The final sections found by the algorithm were taken from a list of discrete sections, selecting those which were close to the continuous solution.

SEABURGH and SALMON[61] used a similar gradient projection algorithm for the minimum weight design of light gauge members, in accordance with American Codes of Practice. The variables were divided into independant variables (the section dimensions) and dependant variables (the section thickness). The section thickness was made a dependant variable by rearranging the stress constraint equations such that for any set of section dimensions a minimum section thickness could be found. Fabrication constraints relating several independant variables were introduced. A gradient projection algorithm similar to that discussed above was used. The gradients where either found analytically or, where the function which determines thickness was complex, they were found by finite difference techniques.

In an investigation of the design of trusses with variable geometry VANDERPLAATS and MOSES[75] used a gradient projection algorithm. The design procedure consisted of two parts which were repeated:-

(a) Varying member areas for a given geometry, in the "area design space".
(b) Varying joint locations for a given set of areas, in the "coordinate design space".

Investigations in the area design space were performed using the "fully iterative design technique" (see section 3.6.2.9). Step (b) is performed using a gradient projection algorithm which considers the optimum direction of movement at a boundary point by solving a small linear programming problem involving the gradients of the objective function and constraints. It was found that a large saving in weight could be achieved by varing the geometry of a truss.

RIDHA and WRIGHT[55] used a gradient projection algorithm for the minimum cost design of plane frames with welded joints. The variables consisted of the independant variables (i.e. the section sizes) and the dependant variables (i.e. the forces in the joints). The objective function contained the material costs (based on weight) and the connection costs (based on the joint forces). The objective function was made continuous by using a continuous idealization of the true costs. The algorithm found the best direction to move when a constraint was violated and return to the feasible region was in the direction of steepest descent of the constraint function. It was found that a noticeable difference existed between minimum weight designs and minimum cost designs for plane rectangular frames.

3.6.2.9 Iterative Design Algorithms

Iterative design algorithms make use of approximations to the gradients of the objective function and constraints, and so can be classed as gradient algorithms, indeed in their most elegant form they are little different to a gradient algorithm.

The basic procedure is to set up an iterative relationship which, ensures that as many constraints as variables are satisfied exactly. This may not produce an optimum design because either there may be more than one vertex, one of which may be more desireable than that found or the optimum design may not occur at a vertex. It is usual to start the algorithm at a number of widely spaced points and to select the most optimal solution. The problem of a vertex optimum may be overcome by the use of special computational approaches to the problem (e.g. the addition of extra "contrived" constraints).

The algorithm is most easily applied to the design of pin jointed trusses because simple iterative relationships may be derived for such structures. A fully stressed design will have been found in such a structure when each member is fully stressed in at least one load condition.

GALLAGHER[20] examined two fully stressed iterative design algorithms for use in the design of trusses subject only to stress constraints.

The first algorithm, termed the stress ratio method, uses an iteration formula of the form:-

$$A_i^{n+1} = A_i^n \times \frac{\sigma_i^n}{\sigma_{ia}}$$

where A_i^J = The area of member i in iteration j

σ_i^j = The stress in member i in iteration j

σ_{ia} = The allowable stress in member i

This algorithm is essentially similar to the approach taken by a designer and therefore has an intuitive appeal. This algorithm has the disadvantage that it may not reach convergence and therefore requires additional criteria, such as move limits, to bring about convergence.

The second algorithm, termed the complex stress ratio method, accounts for the influence of all member sizes on the stress in a given member. The iteration formula in this case is more complex, the change in stress in a member is estimated by using the formula:-

$$\frac{\partial \sigma_i}{\partial A_j} = F_i \frac{\partial C_i}{\partial A_j} + C_i \frac{\partial F_i}{\partial A_j}$$

where F_i = The force in member i

C_i = The compliance of member i, which may be calculated.

The gradients found are those of the stress constraint functions. The iteration relationship is similar in form to that used in Newton's Algorithm (i.e. linear extrapolation) and is therefore subject to all the disadvantages associated with that algorithm. When flexural members are included the design variables must be the section moduli, because the use of non linear area-modulus relationship will cause convergence diffuculties.

Gallagher proposed that these algorithms should be used in a two stage process. Firstly the simple stress ratio method is used to produce rapid improvement and secondly the complex stress ratio method is used because it is more likely to result in convergence. The quantities calculated in the complex stress ratio algorithm can be used to determine whether the fully stressed design is optimal. It was found that in general the optimal design closely approximated to the lowest weight fully stressed design. It was also found that, in contrast to other algorithms, the number of cycles required to reach convergence was insensitive to the number of variables. He concluded that the use of fully stressed design algorithms provides an efficient and economical method of calculating low weight design proportions for complex structures.

In a previous investigation CORNELL, REINSCHMIDT and

BROTCHIE[12] developed four algorithms for iterative design. The first algorithm was the simple stress ratio algorithm described previously, however to improve convergence the areas of the members were slightly over-estimated to compensate for force redistribution. The second algorithm included differential terms within the iteration formula to take account of changes in the area of the member considered only (i.e. only the dominant term of the Taylor series expansion was taken). For certain problems both of these algorithms failed to converge. This led to the development of the third algorithm which was essentially the complex stress ratio algorithm. Extension of the third algorithm to include consideration of the objective function resulted in the fourth algorithm which was in fact the cutting plane algorithm. In the development of the algorithms it was found that the correct choice of variables was the most important decision, it was found that the selection of a formulation which results in linear constraints, even at the expense of a non linear objective function, is advantageous. This investigation showed the relationship of iterative desing algorithms to familiar non linear programming algorithms. - 14

FRIND and WRIGHT[19] developed two iterative design algorithms for the design of plane frames taking into account stress, displacement and side constraints. The first algorithm, termed the method of incomplete gradients, starts from an infeasible point and moves towards an intersection of constraints. Only the dominant term of a Taylor series expansion of the stress constraint equations was considered, resulting in the linearized stress constraints being perpendicular to the coordinate axes. For displacement constraints a similar analysis was performed, however no assumptions could be made in this case as to the dominance of terms. The technique consisted of moving to the intersection of stress constraints, then each stress constraint was removed in turn and replaced by a deflection constraint, the solution to the resulting equations was a vertex. This was repeated until all vertices were found, the vertex with the lowest cost was taken as the next design point. The second algorithm, termed the method of auxiliary gradients, is essentially similar but more terms in the Taylor series were used, resulting in more computation and better convergence properties. Both algorithms were found to converge rapidly on the optimum design and the rate of convergence was found to be independant of the number of variables.

GELLATLY and BERKE[21] considered the iterative design of trusses using a different approach. The variables are partitioned into two subsets, the passive variables which are not allowed to change during a design cycle and the active variables which are allowed to change during a design cycle. Considering deflection constraints, the total deflection (Δ) is made up of the deflection due to passive members (Δp) and the deflection due to active members (Δa) and the structure weight (W) is made up of the weight of the passive members (Wp) and the weight of the active members (Wa). If the allowable deflection is Δ^* then using a lagrangian multiplier (λ)

$$Z = Wp + Wa + \lambda(\Delta p + \Delta a - \Delta^*)$$

by differentiating this equation and making assumptions regarding the dominance of terms, the required area for each active member can be determined. The resulting equation is exact for determinate frames and is used iteratively for indeterminate frames.

The passive members are chosen each iteration on the basis that either their areas are determined by stress or side constraints, or that their increase will not reduce deflections. The design resulting from the use of this algorithm will be at the intersection of stress, side and a single deflection constraint. The convergence of the algorithm was found to be rapid for large structures.

3.7 DISCRETE VARIABLE ALGORITHMS

Discrete variable optimization problems consist of finding the minimum of a function of variables which are only defined at discrete points. The function may be linear, non linear or discontinuous. The nature of the problem is such that in many cases exhaustive enumeration of all cases is the only solution method, clearly for even small problems this approach is impractical due to the large number of function evaluations which have to be carried out. The types of problems which are encountered can be classified into discretized problems and truly discrete problems. Discretized problems consist of those problems for which a continuous formulation exists but which for some reason uses discrete variables, this type of problem comprizes the majority of discrete variable structural design investigations. Truly discrete problems consist of those problems for which a continuous formulation does not exist.

The addition of linear, non linear or discontinuous constraints causes the problem to become even more complex especially when there are a large number of constraints. CELLA[9] has examined the properties of discretized optima in structural optimization. It was found that the amount of computation required to prove a particular solution optimal would be prohibitive and that it was necessary to define the solution as a local optimum, called in this instance a terminal. A terminal was defined as:-

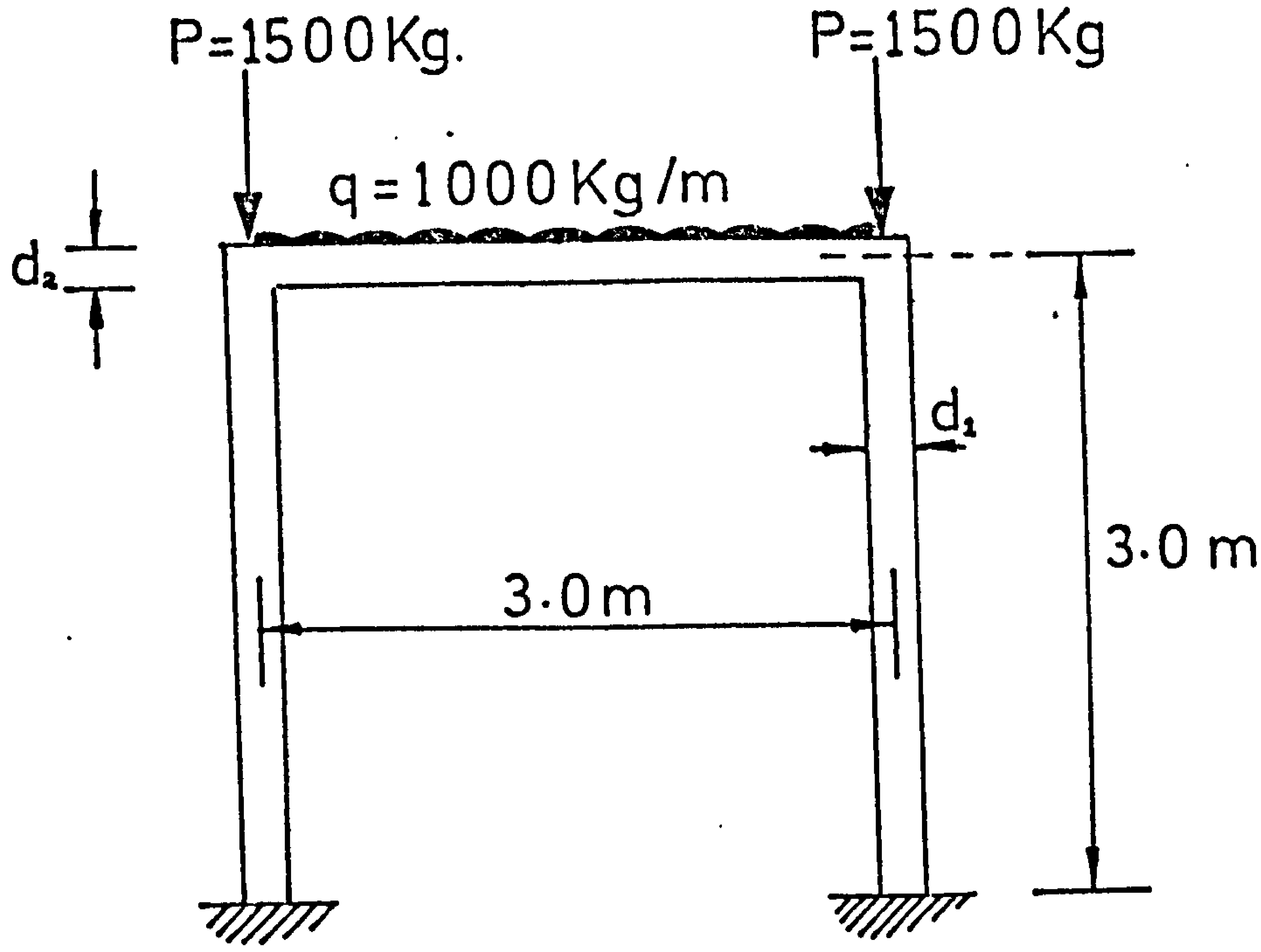
$$x_t \text{ where } f(x_t) \leqslant f(x) \text{ for all } x \text{ in } S_q$$

where S_q is the set of all designs which have been evaluated either implicitly or explicitly. A comparison was made between both continuous and discretized design spaces for a small frame (see figure 3.9). It can be seen that local discrete optima occur at points L, K, Y, X, T and at the global optimum Z. The distances between these "terminals" are termed their radius. The terminal with the largest radius (Z) has a radius of three, all of the other local optima have a radius of one or two. If a problem has n variables it is necessary to evaluate 3^n designs, clearly with large numbers of variables the use of any discrete algorithm becomes prohibitive. Cella concludes that there is hardly any relationship among the local optima except where the lattice is so fine that it closely models the continuous function. A description of some of the major algorithms that have been applied to discrete structural optimization problems will now be given.

3.7.1 Random Algorithms

These types of algorithms are usually applied to truly discrete problems, however they can be applied to discretized problems. The procedure consists of selecting combinations of the design variables at random and evaluating each design selected. This process is continued indefinitely because there is no guarantee that the global optimum will be found. When the process is halted the set of variables which produced the lowest cost design up to that time is chosen as the optimum. The algorithm is very inefficient and is generally only used for problems where there is no trend or mathematical relationship between variables.

SMOLENSKI and KROKOSKY[64] have used an algorithm of this type for the design of structural sandwich panels. The distribution of random designs which was used had a peak at the current design. This distribution of designs results in designs far from the current design being less likely to be chosen as a trial design, than designs close to the current design. The algorithm showed a steady but slow rate of improvement until the final design was chosen.



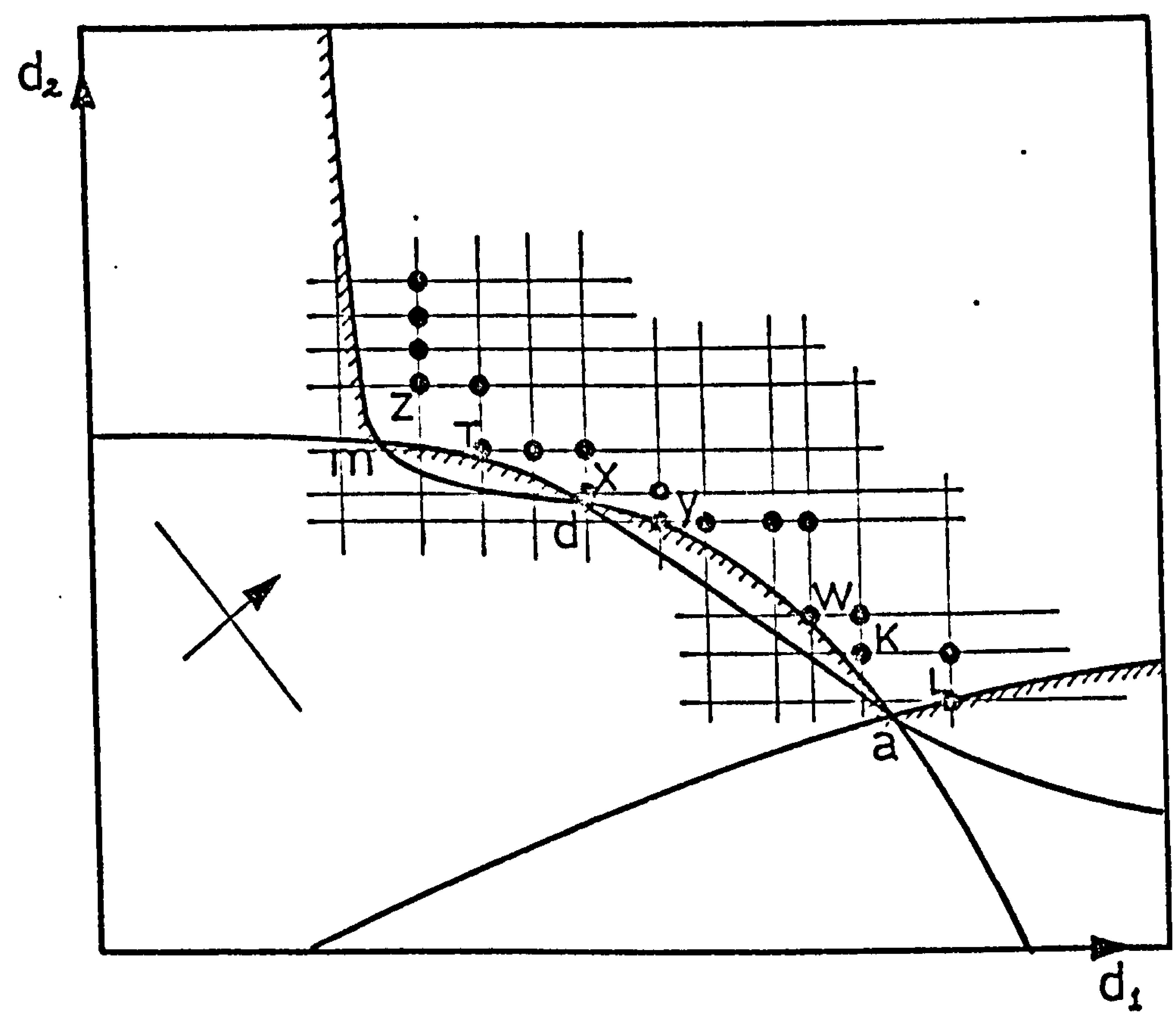


FIG. 3.9 PROPERTIES OF DISCRETE OPTIMA
(AFTER CELLA)

3.7.2 Integer Linear Programming

When a discretized discrete variable problem can be cast in the form of a linear objective function and linear constraints, with variables which take on integer values, it is possible to use adaptions of continuous linear programming algorithms. The simple rounding of continuous linear programming solutions will not in general result in an optimal solution, therefore it is necessary to consider the problem in more detail. Two main algorithms exist for the solution of this problem, they both use the fact that an integer linear programming problem is a continuous variable problem further constrained. The solution is by adding extra constraints which reduce the design space and cause convergence on an integer design.

The branch and bound algorithm first finds the continuous variable solution. It then systematically divides the feasible solutions into sub sets and divides each sub set until the solution is found. Most of the feasible solutions are eliminated implicitly and only a few are eliminated explicitly. This algorithm can also be used on mixed integer/continuous problems.

GOMORY'S cutting plane algorithm first finds the continuous variable solution. It then systematically adds constraints, continuously narrowing the feasible region so that the integer solution occurs at the vertex. This algorithm suffers convergence problems and is inefficient for large problems.

TOAKLEY[73] has used Gomory's algorithm for the plastic design of plane frames using available section. The application of the algorithm is as follows:-

By choosing the redundants R_q the bending moment at any section i can be expressed as:-

$$m_i = \sum_{q=1}^{e} a_{iq} R_q + m_{oi}$$

where a_{iq} = parameters based on geometry of the frame only

m_{oi} = the free bending moments due to the external loads.

suppose that for each group of members there are v discrete sections possible (v may vary between groups). If Mg is the plastic moment of resistance of group g and Wg is the weight of sections in group g then

$$Mg = Mg_1 \delta g_1 + Mg_2 \delta g_2 + \ldots\ldots\ldots\ldots Mg_v \delta g_v$$

$$Wg = Lg\ (Wg_1 \delta g_1 + Wg_2 \delta g_2 + \ldots\ldots\ldots\ldots Wg_v \delta g_v)$$

$$\sum_{i=1}^{v} \delta g_i = 1 \quad \text{and} \quad \delta g_i = 0 \text{ or } 1$$

$$\text{structure weight} = \sum_{g=1}^{G} Wg$$

where Mg_i = The plastic moment of resistance of section i

Lg = The length of members in group g

Wg_i = The weight of section i

Any statically admissable moment system for which the plastic moment of no member is exceeded is a safe design thus:-

$$m_i \geqslant -m_j \qquad m_i \leqslant m_j$$

where j is the group of the member at point i.

This formulation can be solved using Gomory's algorithm. It was found that if there were more than approximately eight integer variables the algorithm produced disappointing convergence. Toakley produced modifications which caused better convergence. Firstly, a lower bound on the true solution can be found, this lower bound is the solution to any stage of Gomory's algorithm as long as 4vG linear optimizations have been performed. Secondly, an upper bound can be found by using the fact that for any admissable moment field, if sections are chosen with plastic moments greater than the admissable moments the design is safe.

Upper bounds may be found at each stage of the optimization process. Thirdly, when the algorithm refuses to converge all designs between the upper and lower bounds are evaluated resulting in the optimum design. TOAKLEY[72] has also used this approach for the design of triangulated frameworks, however in this case it is more difficult to find a suitable upper bound. In both cases the upper and lower bounds before the final search were close to each other.

3.7.3 Zero-One Programming

A zero-one programming algorithm can be used to solve any linear problem and some non linear problems with variables which may only take on the values of zero or one. There are few available algorithms (see PLANE and McMILLAN[50]). The available algorithms are termed implicit enumeration algorithms. These algorithms systematically search through the design space, eliminating designs either explicitly or implicitly. Implicit enumeration of a set of designs is carried out by using the constraints to deduce that certain combinations of design variables are not feasible. The majority of designs are enumerated implicitly with only a small number enumerated explicitly.

The modifications made by TOAKLEY[73] to Gomory's algorithm, mentioned previously, in effect used an implicit enumeration algorithm by eliminating designs outside the upper and lower bounds.

REINSCHMIDT[53] has used an implicit enumeration algorithm for the plastic design of plane frames. The formulation of the problem was based on a mechanism approach rather than an equilibrium approach resulting in a pure integer formulation. This algorithm was found to be more efficient than that used by Toakley and resulted in less programming effort. This application was extended to the elastic design of trusses with non linear constraints with less success because convergence difficulties were encountered due to the use of successive linearization of the constraints. ANNAMALAI, GOLDBERG and LEWIS[2] have used an implicit enumeration algorithm for the optimum cost design of welded plate girders. The algorithm, termed backtrack programming, systematically examines the constraints and objective function to implicitly enumerate designs. Typically the systematic examination consists of the following procedure. If the cost of the girder is given by $f(x_1, x_2)$ where x_1 and x_2 are positive and $f(x_1, x_2)$ increases with increasing values of x_1 and x_2. If the current minimum cost is C_m then if $f(x_m, x_n) > C_m$ with x_n taking any allowable value, then all cases with x_1 greater than x_m are disregarded. This algorithm can also be used with certain types of constraints. Designs may also be rejected because they do not fullfill approximate, conservative constraint functions, rather than continuing through the whole design system each time. For the example given there were six variables and approximately twenty-four thousand possible design combinations. The algorithm was shown to produce optimum designs at very low cost.

3.7.4 Dynamic Programming

This algorithm has been used more than any other discrete variable algorithm for structural design problems. The algorithm is applicable to problems which can be cut into stages. The optimum is found using a recurrence relationship of the form:-

$$f_i\ (Y_i) = \underset{Y_{i-1}}{\text{Minimum over}} \left[g_i\ (Y_{(i-1)}\ Y_{(i)} + f_{i-1}\ (Y_{i-1}) \right]$$

where $f_i(Y_i)$ = The minimum cost up to stage i

$g_i(Y_{i-1}, Y_i)$ = The cost of moving from stage i-1 to stage i.

DOUTY[16] has studied the use of the dynamic programming algorithm applied to the design of a floor system. The floor system considered has the advantage that it is determinate and the decision taken at any stage only affects subsequent stages. Referring to figure 3.10 the

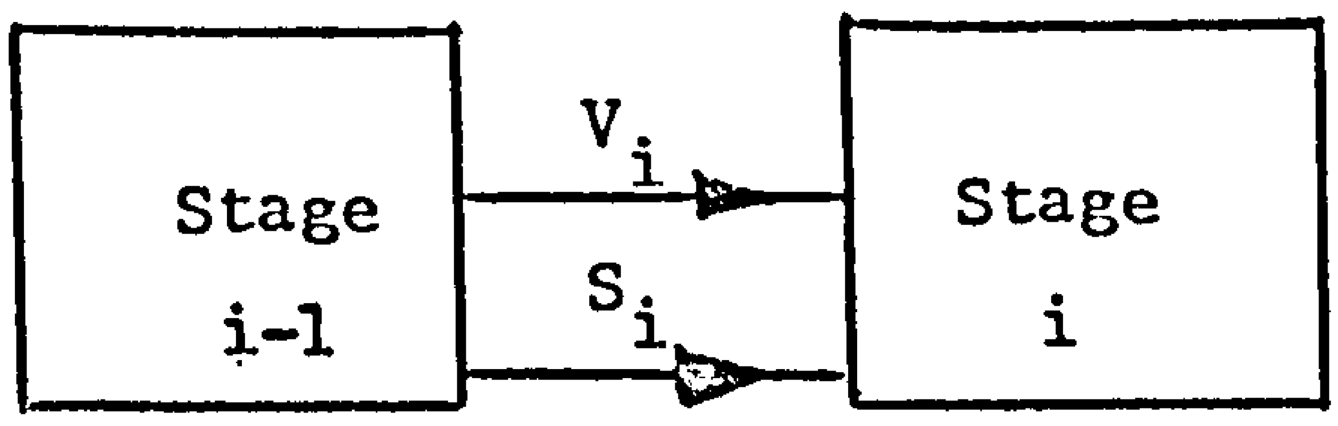


Figure 3.10

flow of data between stages consists of two types of data, state data (S_i) and design variables (V_i). The state data, which defines the general problem, is set up at the beginning of the algorithm and cannot be altered. The design variables are those quantities which can be changed at this stage and represent such quantities as beam sections, number of beams etc. To find the optimal value of the objective function at stage i it is only necessary to look at possible combinations of V_i (these variables must be in discrete form). Each design can thus be made a line entry in a table corresponding to the stage in question. Such an entry contains the values of V_i and the cost up to stage i. This table is termed a "utility table" and can be used to reduce computer storage requirements. Entries in the table are linked according to the optimal policy. At the end of the process the optimal policy is followed back through the tables. The solution starts with the design of the main girders, the maximum and minimum values of the floor beam reaction are estimated and the range is cut up into ten divisions. Then for all combinations of floor beam reaction, number of floor beams, number of stringers and yield stress, a table is set up including the cost of this component for each combination. A similar process is used for the design of the floor beams. The two tables must be linked with due regard to the values of the variables being the same in the linkage. The tables are finally searched back through for the optimal policy. Douty showed that most structural design problems which are indeterminate cannot be solved by the dynamic programming algorithm. This because a change in any variable will change the constraints on any other variable, this does not allow the problem to be stated in terms of stages.

PALMER[47, 48] and PALMER and SHEPHERD[49] have used the dynamic programming algorithm for the design of trusses, continuous beams and plane frames. The application of this algorithm to the design of the layout of cantilever and cross braced and N girder trusses will now be described. The cost of the truss is defined as:-

Σ (force x length) over all members.

This can, with reference to figure 3.11, be rearranged to:-

Σ (Σ (force x length) over all members in a panel) over all panels

Therefore, by considering each panel in turn the optimal design can be found by using dynamic programming. It is shown that by considering the horizontal dimensions of the truss as well, the amount of computation is considerably increased. This increase is due to the fact that as the number of variables need to define the connection between stages increases, the number of calculations required increases exponentially. Palmer terms this effect "the curse of dimensionality".

The plastic design of continuous beams was shown to be a similar problem. The variables this time were the support moments at each stage. The allowable support moments were selected from the plastic moment capacities of available beam sections. It was shown that if there are p spans and q possible sections, the number of designs to be evaluated by dynamic programming is $(p - 1)(q + 1)^2 + q + 1$. This compares favourably with the pq^p designs required by a total enumeration. Extension of this approach to the design of

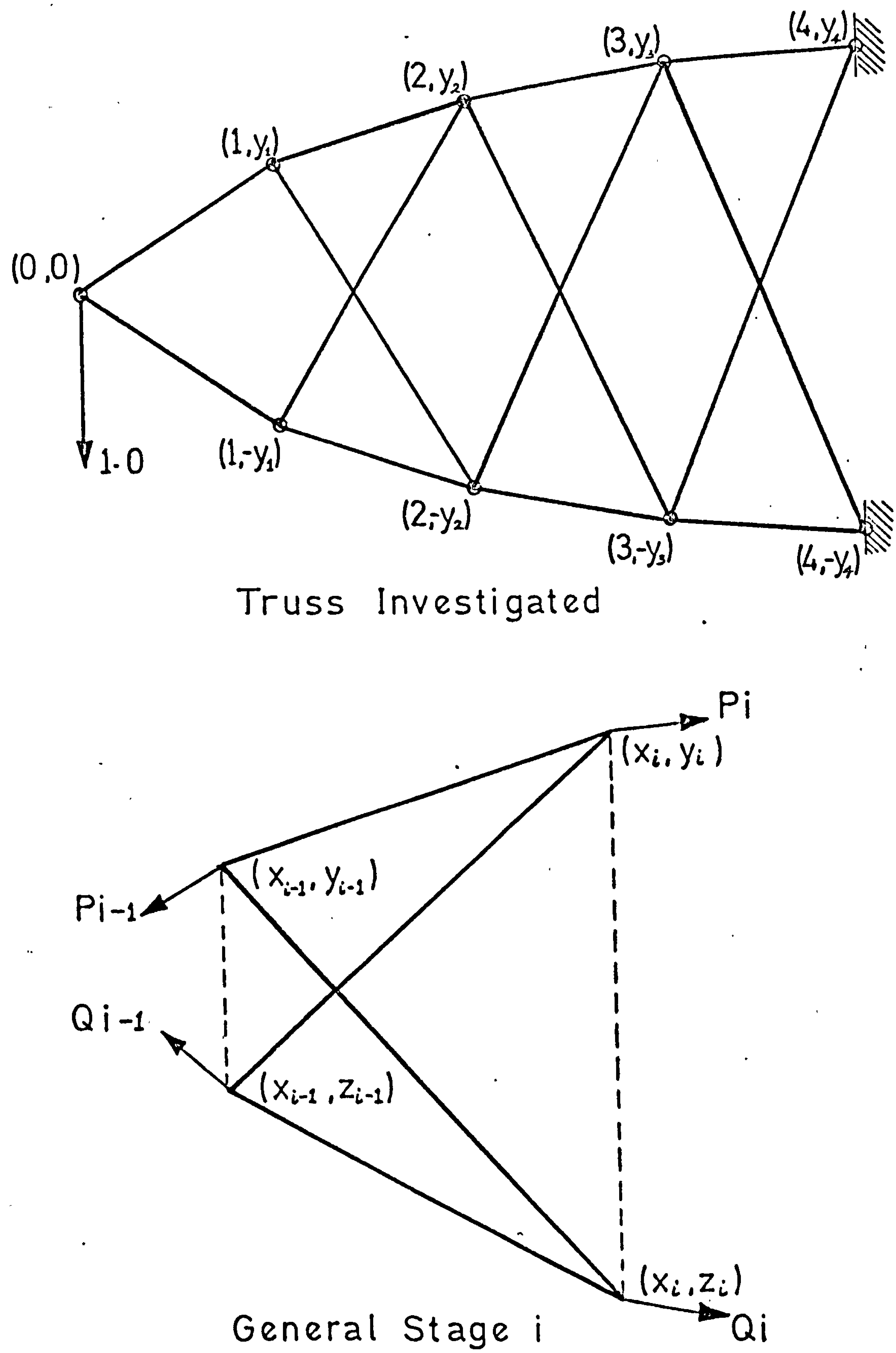


FIG. 3.11 DESIGN OF CANTILEVER TRUSSES USING DYNAMIC PROGRAMMING (AFTER PALMER)

plane frames proved to be inefficient, due to the need to define the connection between stages (stories in this case) by a large number of variables.

The "curse of dimensionality" restricts dynamic programming to the design of long thin structures. Palmer suggested five methods of alleviating this problem:-

(a) Direct Iteration - a coarse initial grid is used, the next grid is centred over the optimal policy of the first grid.

(b) Successive Approximation - A trial optimal policy is assumed. This policy is modified by carrying out dynamic programming with only one set of variables at a time.

(c) State Increment Dynamic Programming - changes between variables in each stage are limited to a small range.

(d) Polynomial Approximation - facilitates minimization at each stage.

(e) Lagrangian Multipliers - may be used to reduce the number of state variables as in non linear continuous problems.

GOBLE and DE SANTIS[22] applied the dynamic programming algorithm to the design of flanges for plate girders. The variables were the plate thicknesses, and the objective function included fabrication and materials. The constraints included stress constraints, which effectively produced a minimum plate thickness at each point in the girder. The flange can be cut into sections (or stages). If two stages have identical flange plates then the cost of splicing them is zero. The recurrence relationship is made up as follows:-

Optimal cost to stage k = (the cost of stage k + the cost of joining stage k to stage k - 1 + the optimal cost up to stage k - 1).

This function is minimised over the flange thickness of stage k. The flange thicknesses which resulted were taken from a table of discrete sizes and therefore produced a practical design.

MOSES and GOBLE[45] used the dynamic programming algorithm for the selection of beam sections for buildings. The investigation attempts to solve the conflict between selecting many different beam sections and incurring a high fabrication cost, and selecting only a few beam sections which saves fabrication cost due to mass production. The problem can be cast in the form of dynamic programming by selecting as the variables the number of members required for each beam section. Starting with the minimum cost beam section, a set of cost functions was devised which was used recursively to find the optimum number of members to be fabricated from each section. The example given used assumed cost savings and was purely hypothetical.

3.7.5 Direct Search Algorithms

Algorithms of this type try to apply the approaches of non linear direct search algorithms to discretized discrete problems. Due to the discrete nature of the design space it is not possible to be sure of reaching the global optimum, however a systematic reduction in the objective function can be achieved.

LAI and ACHENBACH[32] have proposed an algorithm for use in structural design problems. The algorithm consists of two stages:-

(a) The complex algorithm - this algorithm, modified for use with discrete variables, provides initial acceleration towards the optimum design. Providing the initial complex is large enough the complex will follow the trend of the design space.

(b) A patterned search - this provides final examination of the area around the current best design, a local optimum will, in general, be found.

The algorithm is started from a number of points to reach an eventual "optimum" design. For each optimization thirty to forty iterations are required for a problem with two variables. If the number of variables is very large the patterned search takes too much comptation. The algorithm was applied to the design of a portal frame and a tied cantilever.

CELLA and SOOSAR[11] and CELLA and LOGCHER[10] have developed an algorithm for the discrete variable optimization of plane frames. Restrictions are put on the type of design problem:-

(a) The structure is linear elastic.
(b) Behaviour constraints are defined accurately and are convex..
(c) Section properties are ordered in terms of increasing stiffness.
(d) The objective function is linear or almost convex in the design variables.

These assumptions ensure that the optimum design is close to or on a constraint. The algorithm starts from an initial infeasible design. Variations are made in the design variables such that each constraint is satisfied with a minimum of cost increase. This procedure is similar to iterative design. The algorithm then searches a limited feasible band close to the constraints using a technique termed "diagonal enumeration". The band to be searched is totally enumerated so that eventually the optimum design will be found. A typical sequence of enumerations is shown in figure 3.12.

RAZANI and GOBLE[52] have used a pattern search algorithm as part of a two stage optimization procedure for the design of plate girders. The variables considered were plate thicknesses and widths. The profile of the flange plates was determined using a dynamic programming type of algorithm. When designing in the space defined by flange width and beam depth, the algorithm used consisted of searching an area

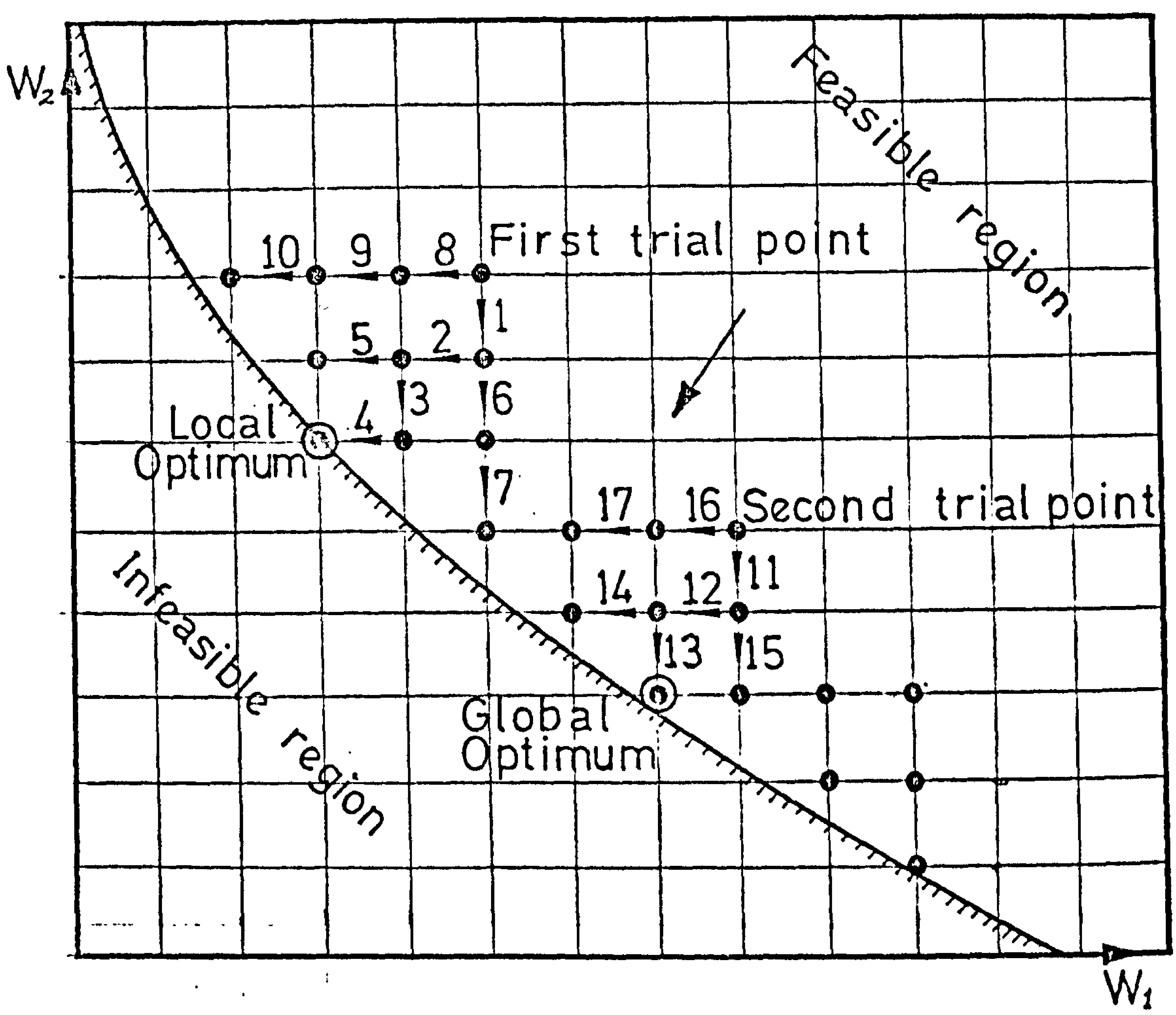


FIG. 3.12 DIAGONAL ENUMERATION EXAMPLE (AFTER CELLA)

defined by twenty-five points on a grid. The grid was centred around the current best design point. When a new "best design" was found the pattern was moved to that point. When the algorithm reached a local optimum the pattern was shrunk around this point and the process repeated. This algorithm was found to work well despite the large numbers of local optima on the design surface.

3.8 PROBLEM ORIENTATED ALGORITHMS

Algorithms in this class are specifically taylored to the problem being solved. The algorithms make use of theoretical statements of the problem, or specific properties of the problem. Three classes of problem orientated algorithms can be easily identified, these will now be described.

3.8.1 Iteractive Satisfaction of Compatibility

Algorithms of this type seek to find an optimum solution neglecting compatibility of deflections and to then further constrain the problem by progressively ensuring compatability. Compatibility is essentially a set of equality constraints which are progressively satisfied.

REINSCHMIDT and NORABHOOMPIPAT[54] have applied an algorithm, which they term "equilibrium linear programming", to the design of grillages. The algorithm starts by solving the linear problem which results from neglecting compatibility and using Taylor series expansions of the constraints. When a solution is found,additional constraints are added to ensure that compatability will be achieved. This algorithm is useful because the initial design space is convex and so a global optimum is found. Addition of concave constraints produces movement from this optimum to a new optimum which is likely to be the true global optimum for the final problem. A typical example is shown in figure 3.13.

A similar application has been used by FARSHI and SCHMIDT[18]

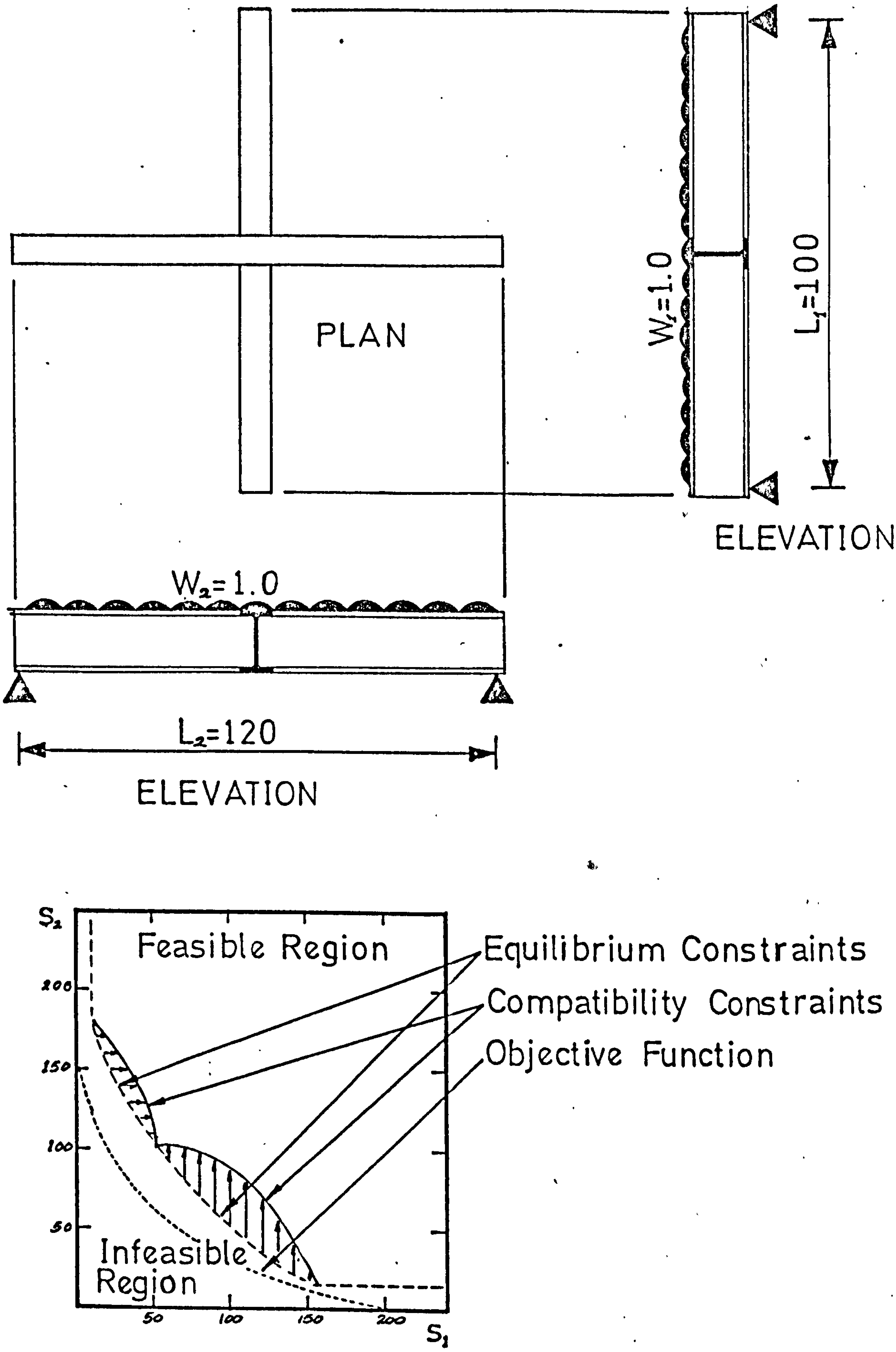


FIG. 3.13 EQUILIBRIUM LINEAR PROGRAMMING
(After Reinschmidt and Norabhoompipat)

for the design of trusses. The procedure in this case is to set up the equations to solve the conditon of equilibrium alone. The deflections at the joints will not then, in general, be compatible. Approximate relationships are then set up between nodal deflections and the member forces. These relationships are then added as constraints to the original linear programming problem, successive use of these relationships results in a compatible design. The algorithm was found to converge rapidly when a suitable choice of redundants was used.

3.8.2 Substructuring

The principle of substructuring is that a frame is partitioned into substructures and each substructure is optimized, assuming that only those constraints and variables which refer to the current substructure are considered. This process is repeated until convergence occurs.

KIRSCH, REISS and SHAMIR[30] have used this type of algorithm for the design of continuous beams and beam grids. The subproblems were solved using a direct search algorithm combined with an exterior penalty function. The algorithm was applied to the design of six span continuous beams. Two choices of substructures were used and the beam was also designed without substructures. The final solution was found to be the same in each case (a fully stressed design). It was found that the use of substructures which overlapped led to better convergence. The algorithm was also applied to a beam grid with similar results. It was concluded that, large scale structures can be optimised using this technique and that convergence is generally rapid.

In a similar investigation KIRSCH[28] described two algorithms which use substructures. The first algorithm termed "the model coordination method", makes use of coordination variables, which are the forces between the substructures.

The coordination variables are fixed and each substructure is solved. An analysis is then performed and new values for the coordination variables are found, the process is repeated until convergence is achieved. The second algorithm, termed the "goal coordination method", allows the coordination variables to vary at each side of the cut. The differences in these variables were included as a penalty on the objective function. This procedure was applied iteratively until no difference occurred. These algorithms were applied to trusses and reinforced concrete frames.

3.8.3 Truss Applications

Few applications of the design of truss geometry and topology have been investigated, however such investigations are likely to show a large reduction in the cost of trusses. Two such applications will now be described.

SPILLERS[65] and SPILLERS and FRIEDLAND[66] have investigated the design of both geometry and topology of trusses. The algorithm proposed consisted of starting with a simple truss and adding one member at a time, until an optimal design was found. Consider figure 3.14. The basic truss is shown in figure 3.14a. This truss can have members added as shown in figures 3.14b, c, d. The procedure used was to optimize the geometry of these three trusses to give the trusses in figures 3.14e, f, g. The cheapest of these trusses was then considered for additions of members. This heuristic procedure is continued until an optimal design is found. The geometrical optimization was performed using linearized relationships between the objective function and the node coordinates, a gradient algorithm was used for this stage. The results were found to be similar to Mitchell trusses, which represent the absolute minimum ie weight design for trusses.

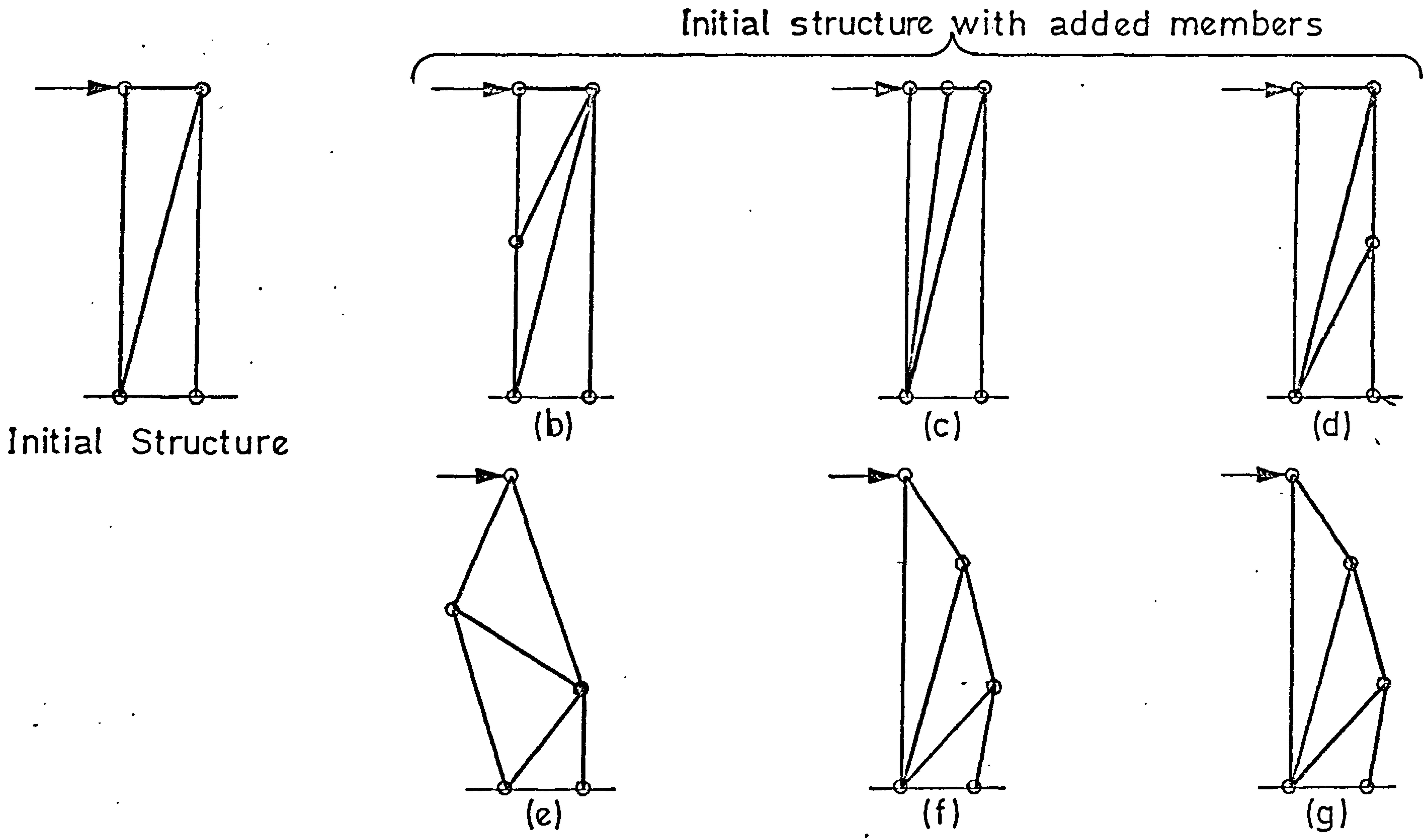


FIG. 3.14 OPTIMIZATION OF GEOMETRY AND TOPOLOGY
(After Spillers and Friedland)

MAJID and ELLIOT[39] have used an "opposite" approach for the topological design of trusses. The algorithm consists of starting with an initial "ground structure" made up of members joining every point on a grid of acceptable node points. A minimum weight design is then found for this structure using an algorithm similar in concept to fully stressed iterative design. Each member is removed, and the effect on the objective function of removing that member is estimated. The member which reduces the objective function by the largest amount is then removed and the cost noted. This procedure is repeated until a determinate solution is found. For design with several load cases the minimum weight design is an indeterminate structure. The inclusion of the self weight of the structure was found to produce faster convergence.

3.9 CONCLUSIONS

Applications of optimization algorithms to structural design are many and varied. Applications with small numbers of continuous variables are well developed and the majority of continuous problems can be solved with a reasonable chance of finding the global optimum.

The nature of structural design problems which select sections for members are however discrete in nature. Discretized discrete variable applications are confined to a few particular types of problems utilizing only a small number of constraints and variables. The chance of finding the optimum design for discretized discrete variable problems is generally low except for certain types of problems and at best when using a search algorithm a local optimum can only be expected. Truly discrete applications are confined to a few problems which can be cast in the form of dynamic programming and generally result in the optimum design. Efficient truly discrete algorithms, applicable to general problems, are not available and recourse has to be made to exploiting the particular aspects of a problem.

Problem orientated algorithms provide useful techniques which can be applied both to continuous and discrete problems and which can assist in the solution of optimization problems.

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CHAPTER 4

Structural Design

4.1 INTRODUCTION

A structural design computer programme was developed for the design of multi-storey braced steel frames. The organisation of the programme is such that each of three separate parts access a common data bank, which describes the building being designed. The three parts to the system include:-

(a) The frame generation and modification section.
(b) The frame design checking section.
(c) The frame optimization section, described in Chapter 6.

The data bank, which is stored on magnetic disc, may be modified or accessed by each section. The organisation of the bank is shown diagrammatically in figure 4.1. The parts of the data bank are as follows:-

(a) Costing data:- A complete set of fabrication and materials costs are stored within this part. This data is outlined in Chapter 5.

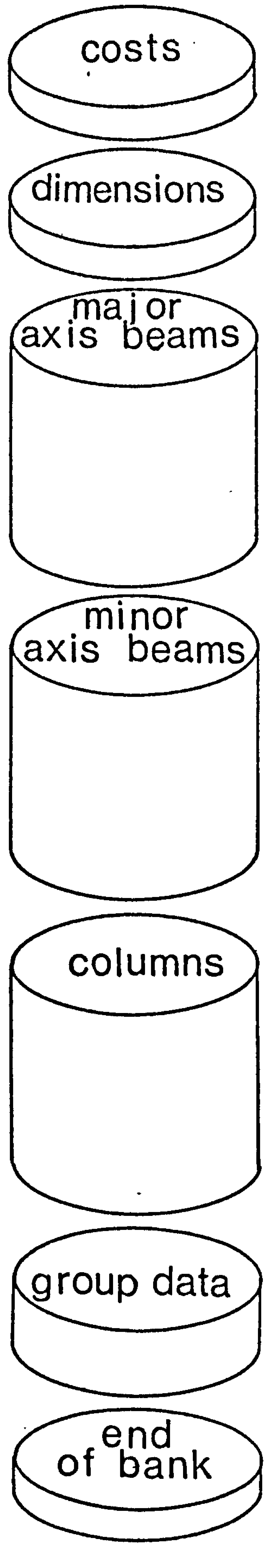


FIG. 4.1 DATA BANK

(b) Frame dimensions:- The geometry and topology of the frame is stored in this part.

(c) Major Axis Beam Records:- All the necessary information required to design the limited frame, which includes each major axis beam is stored within this part.

(d) Minor Axis Beam Records:- The information stored in this part is substantially similar to the major axis beam records.

(e) Column records:- All the necessary information required to design the three dimensional limited frame, which includes each stanchion is stored within this part.

(f) Group records:- The properties of the steel section being considered for each group of members are stored within this part.

(g) End of bank record:- This record defines the end of the data bank and contains no data.

Each part of the design programme will be described, paying particular attention to the requirements of each design method. The design of the connections is also briefly considered.

4.2 FRAME GENERATION AND MODIFICATION

This section of the design system when used in conjunction with the data bank allows the generation of building descriptions within the data bank and the modification of those descriptions. The programme allows certain operations to be performed, these will now be described:-

(a) Set up data bank for a new building:- This operation generates a description of a new building within the bank.

(b) Apply floor loading:- This operation applies uniformlly distributed dead and live loads to areas of floor. These loads are distributed to the supporting beams assuming either one or two way spanning slabs.

(c) Group members together:- This operation groups members together so that they will have the same section during design.

(d) Initialize groups:- This operation allocates a section from a table of available sections to each group. Sections may be designated as variable or constant.

(e) Initialize costs:- This operation selects a set of costs for fabrication and material from a library of costs and places this in the bank.

(f) Distribute loadings:- The direct loads on each column, allowing for reduction in live load due to the number of floors supported, and shears from each beam are evaluated using statics.

Some of these operations may be repeated during the design and optimization stages.

4.3 FRAME DESIGN CHECKING

The frame design checking section is organised so that different design methods may be easily included. This is achieved by loading a library of subroutines for a design method along with the main programme. The operation of the programme is such that a set of sections is postulated for the frame and these are checked against the requirements of the design code in use. The programme then returns a set of "stress ratios" which show how highly stressed each member is. The process may include checking all or some members depending on the application. A description of the main features of each design method will now be given.

Henceforth each design method will be referred to by the use of initials. Horne's design method will be termed the B.C.S.A. method, referring to the publisher's of the design method. The joint committee report design method will be referred to as the J.C.R. method. The Building Research Station design method will be referred to as the B.R.S. method.

4.3.1. The Design of Major Axis Beams (all design methods) and Minor Axis Beams (BCSA method)

The checking of major axis beams for bending strength is in accordance with the plastic theory, taking account of the effect of shear stresses and assuming continuous restraint to the compression flange. When the whole of the web yields in shear the design is not allowed.

The deflection of beams under live load is limited to span divided by three hundred and sixty and is determined using an elastic analysis of the limited frame.

Local buckling of the beam flange and web are controlled by limiting the breadth to thickness ratio as a function of the yield stress.

4.3.2 The Design of Minor Axis Beams (J.C.R. and B.R.S. Design Methods)

The checking of minor axis beams for bending strength is in accordance with the elastic theory, which ensures minor axis restraint to the stanchions. The limited frame is analysed for three load cases which produce maximum moments at the supports and midspan, the stiffness of the columns is neglected in the B.R.S. design method. The maximum bending stress under factored loading is taken as the yield stress.

The average shear stress in the web is limited to a function of the yield stress. Deflection and local buckling are treated in the same way as the major axis beams.

4.3.3 The Design of Stanchions Using the B.C.S.A. Design Method

The background to this design method has already been examined in Chapter 2. Briefly the procedure for checking a section for each of eight possible load cases is as follows:

(a) The limited frame is analysed assuming all fully loaded beams have zero stiffness.

(b) The elastic stresses produced by axial load and bending about both axes are calculated at each end of the stanchion and compared to the yield stress.

(c) The elastic stability of the stanchion is checked by summation of the stresses due to axial load, bending, twisting, initial curvature and bending moment magnification at mid height. This stress is compared to the yield stress.

The section is also checked for local buckling by limiting the flange breadth to thickness ratio to a function of the yield stress.

The design equations were given by HORNE[4], they are simple to programme, simple to use and provide a conservative method of design for braced steel frames.

4.3.4 The Design of Stanchions Using the J.C.R. Design Method

The background to this design method was also examined in Chapter 2. The application of the method by computer was found to be difficult and guidance was sought from Dr R H WOOD[6], one of the committee members. The design method as published makes use of charts for the determination of elastic critical loads, stress due to initial curvature, magnification of bending moments and torsional buckling. It was necessary to replace these charts with either exact analytical solutions or suitable approximations.

The determination of the elastic critical load is performed by finding the axial load which reduces the limited frame stiffness to zero, this involves the iterative solution of non linear simultaneous equations.

The calculation of stress due to initial curvature can be derived as a simple analytical expression.

The magnification of bending moments due to axial load was presented as a set of curves, which had been derived by WOOD[7] as safe approximations to the theoretical magnifications. The approach used was to calculate the theoretical magnification from analytical expressions and to use this magnification.

The control of torsional buckling was achieved by fitting polynomial approximations to the curves given. These curves were derived as a safe lower bound on numerous computer results for which an analytical solution was not available.

The procedure for checking a section for each of four load cases is as follows:

(a) The limited frame is analysed assuming all fully loaded major axis beams to have zero stiffness.

(b) The elastic critical load is found and compared to the axial load.

(c) The overall stability of the member is assessed by finding the stress conditions as mid height, including stresses due to initial curvature, magnification of minor axis moments, axial load and bending about both axes. The total stress is compared to the yield stress.

(d) The effect of torsional buckling is checked.

The section is checked for local buckling by limiting the flange breadth to thickness ratio to a function of the yield stress.

The design equations were difficult to derive and difficult to use manually without charts. The design method provides a reasonably conservative method of design for braced steel frames.

4.3.5 The Design of Stanchions Using the B.R.S. Method

The background to this design method was also described in Chapter 2. The application of the method was found to be difficult because of the use of charts which express non analytical relationships. Charts are given for the determination of elastic critical loads, reduction of minor axis stiffness due to axial load and moment, and torsional buckling. The evaluation of elastic critical loads and torsional buckling are the same as the Joint Committee Report Method.

The reduction of minor axis stiffness is determined from curves which are safe lower bounds determined from test results. These curves were approximated by polynomial functions. The procedure for checking a given section for each of four load cases is as follows:

(a) The major axis limited frame is analysed assuming all fully loaded beams to have zero stiffness.

(b) The stresses due to axial load and major axis moments are found at each end of the member. These are combined to ensure that a plastic hinge does not fully occur.

(c) The reduced stiffness of the member is found using a simple interaction formula for major axis moments and direct loads.

(d) The reduced stiffness of the member considering minor axis bending is determined from polynomial curves.

(e) Using the smallest stiffness found for the stanchion, the elastic critical load is determined.

(f) The collapse load can be found and compared to the axial load.

(g) The effect of torsional buckling is checked.

The design method was easier to adapt to computer application than the Joint Committee Report Method and is simple to use. It provides the least conservative design method for braced steel frames.

4.4 CONNECTION DESIGN

Connection design was carried out in accordance with current industrial practice, determined in conjunction with local steelwork fabricators. The connections use steel plate and welds of the same grade as the sections to be joined and general grade high strength friction grip bolts to BS4604[2]. Allowable stresses are in accordance with BS449[1]. Where necessary, reference was made to a comprehensive review of the behaviour and design of all types of connections[2].

Initially the problem of connection design was tackled by developing a library of suitable connections for each set of possible sections. This approach was found to have three major disadvantages:

(a) The access time required to reference each connection was prohibitive.

(b) The connection chosen was only an approximation because a large range of forces may have to be carried.

(c) The connection costs were found to be insensitive to the bolt diameter, resulting in widely different bolt arrangements for similar load conditions.

These observations resulted in the decision to design each connection individually using a "design policy", which ensures practical connection details. The same policy is used throughout, essentially the policy consists of selecting the connection which uses the least number of bolts. This has two advantages, firstly small bolts are used for lightly loaded connections and large bolts are used for heavily loaded connections. Secondly the cost of erection is related to the number of bolts therefore erection costs will be minimised. The policy does not produce the theoretically cheapest connection but it gives a realistic indication of the true cost of a connection.

The design of each connection is affected by certain dimensions which are related to the bolt size used and the section dimensions:

(a) The bolt centres (Z) are two and a half times their diameter, unless a web or flange lies between the bolts.

(b) The backmark distances (L) are standardized but are not in accordance with the standard backmarks used in industry, because these were found to be too restrictive.

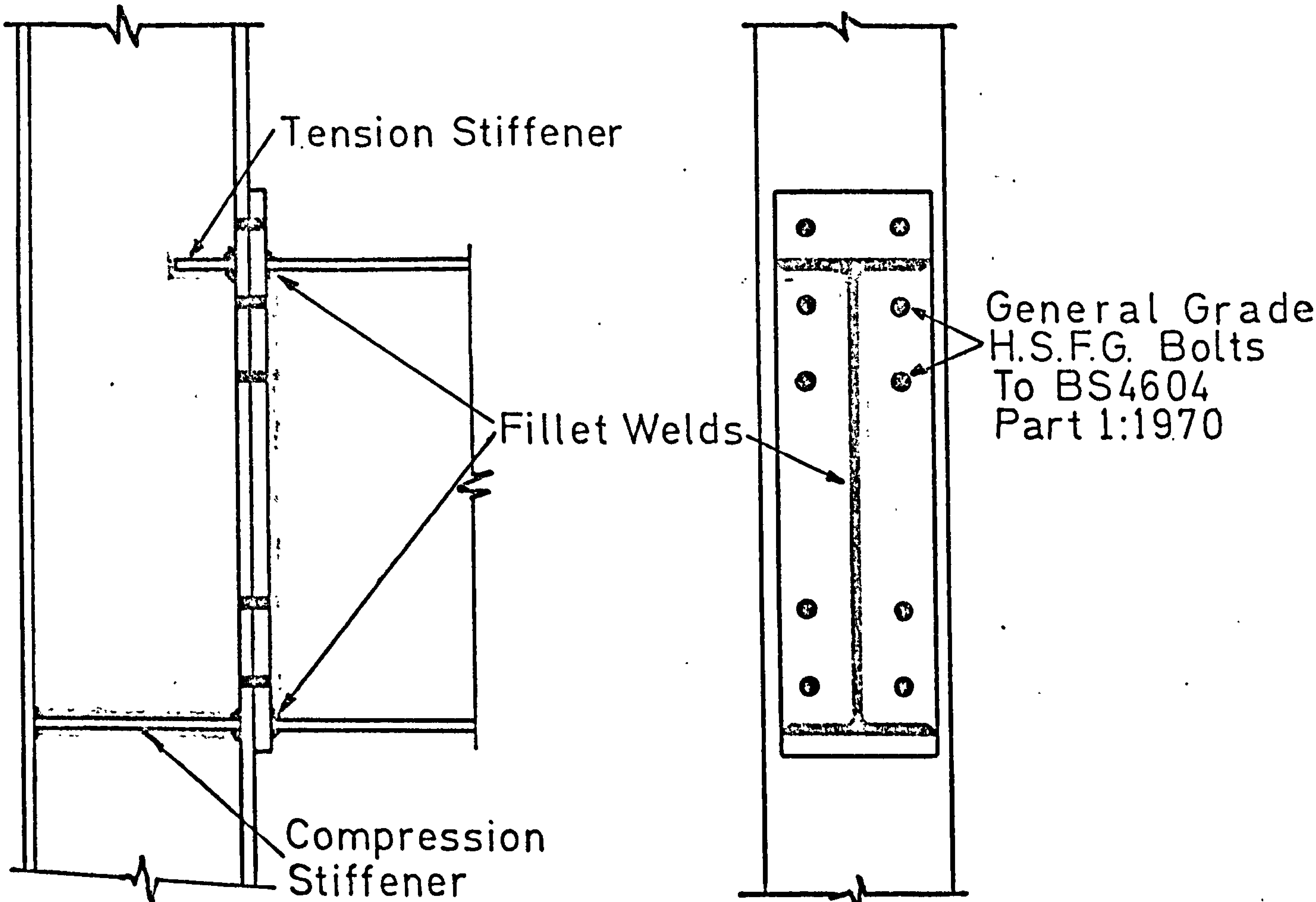
(c) The bolt edge distances are in accordance with BS449.

The thickness or size of bolts, plates and welds are restricted to a set of available sizes.

A description of the design of each type of connection will now be given.

4.4.1 Major Axis Beam/Column Connections

A typical connection detail is shown in figure 4.2. The design of the connection is in accordance with an investigation



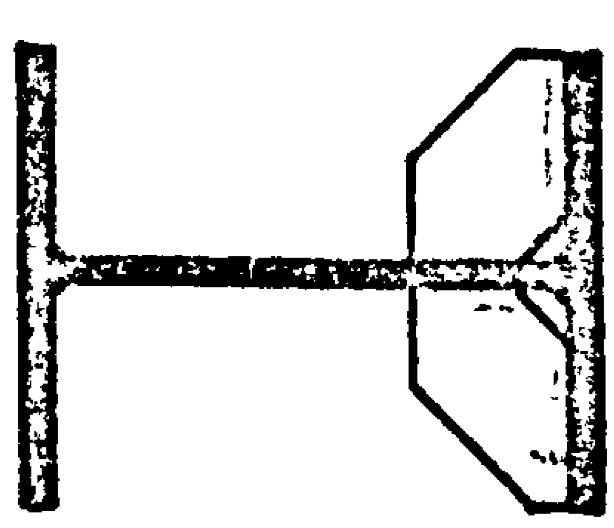
SIDE ELEVATION

SECTION

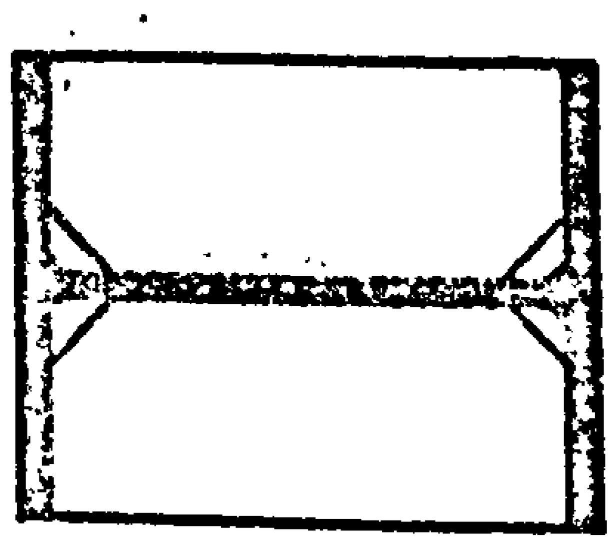
NOTES

1) All dimensions, number of bolts etc. are dependant on actual sections and loads

2) All plates to be of the same grade of steel as, the beam and column sections



TENSION STIFFENER



COMPRESSION STIFFENER

FIG. 4.2
BEAM TO COLUMN MAJOR AXIS CONNECTION DETAIL

by SHERBOURNE[5]. The application of the design method will now be described. The main design variables are shown in figure 4.3. The main assumptions made are:-

(a) The centre of compression is at the centre of the bottom flange of the beam.

(b) The upper group of bolts carries all the forces produced by the bending moment and, if necessary, shear forces.

(c) The lower group of bolts takes shear only.

(d) All loads are factored and permissible stresses are based on BS449, assuming a safety factor of 1.7, or the yield stress as appropriate.

(e) The tension/shear interaction equations for high strength friction grip bolts are as given in BS4604.

(f) In accordance with the assumptions of plastic design all bolts above the neutral axis are equally stressed.

The top group of bolts are proportioned to take all the tensile loads resulting from bending and the bottom group of bolts is proportioned to take the shear force which cannot be carried by the top bolts. This results in the applied moments and shears lying within the bolt group interaction diagram, which is shown in figure 4.4. The geometry coefficient depends entirely on the geometry of the bolt group.

The design of welds was performed using a geometrical determination. It can be shown that the use of welds of the same size as the web or flange thickness of the beam always produces a safe result.

The end plate is designed as a fixed ended beam, spanning either vertically between the top two rows of bolts or horizontally between the lower rows of bolts and, subject to a point load produced by the beam flange or web. The design of the plate thickness uses a plastic analysis which takes account of shear.

The design of compression stiffeners to the column web considers two cases, direct bearing at the root of the

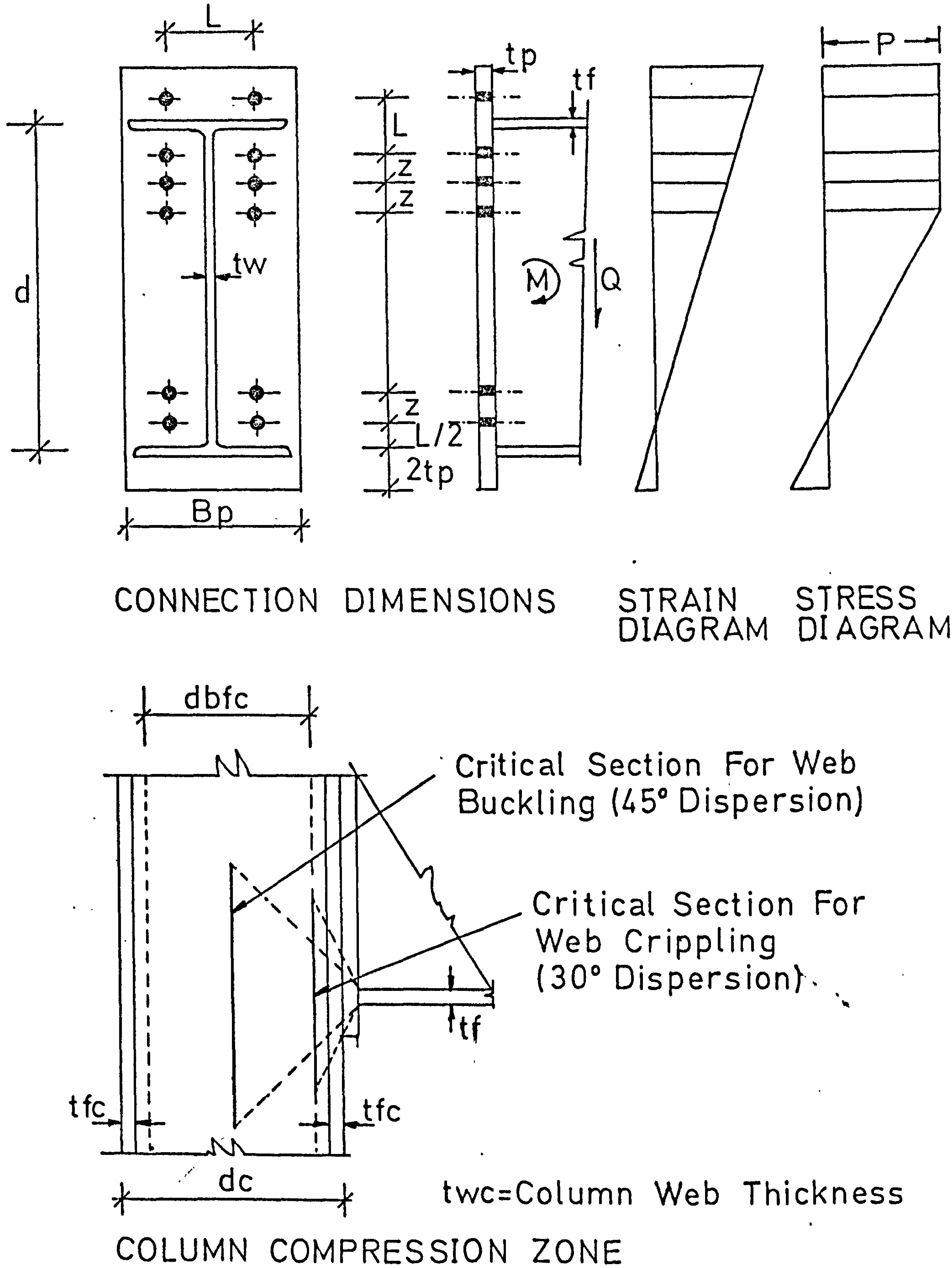
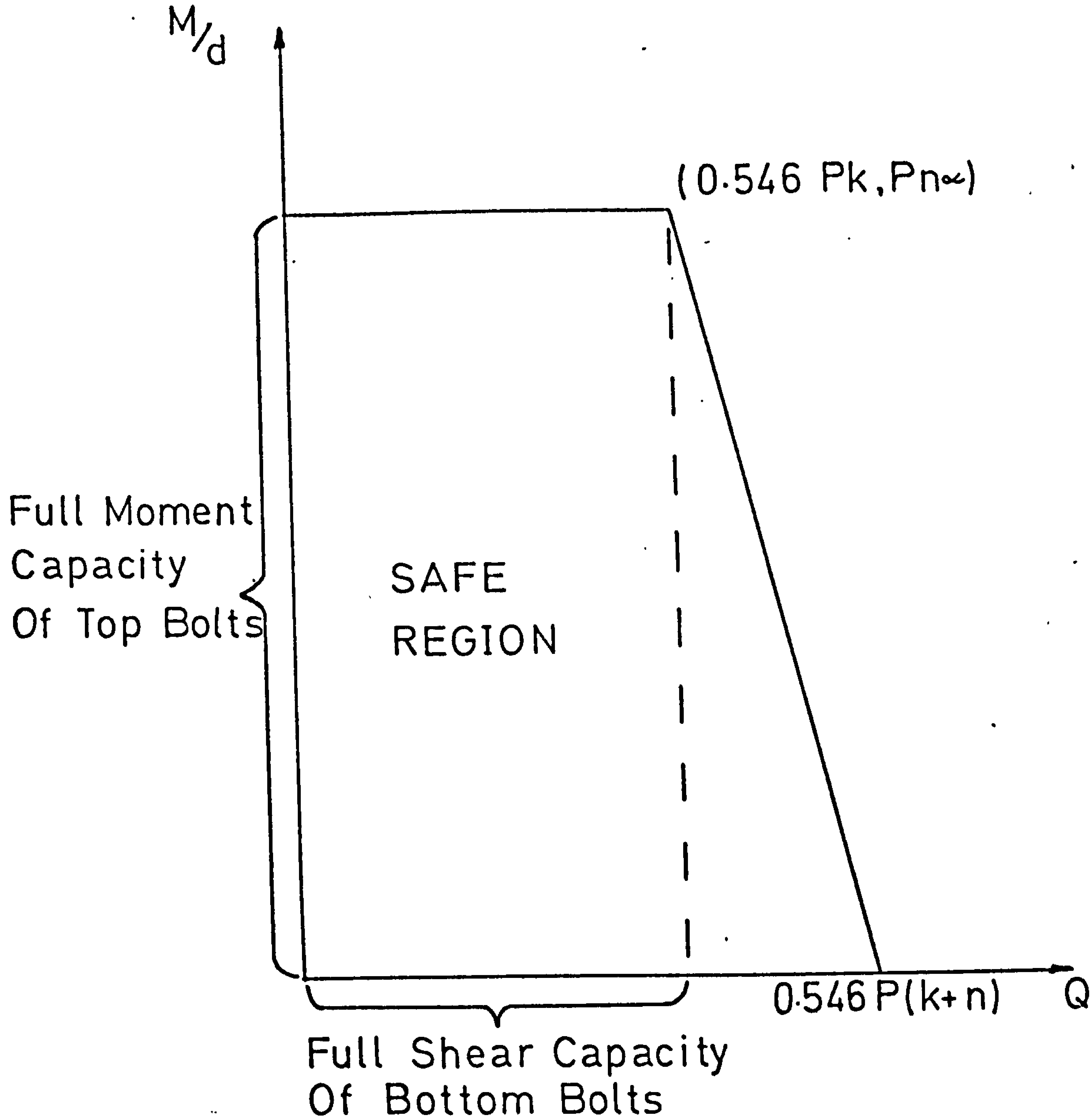


FIG. 4.3 MAJOR AXIS BEAM/COLUMN CONNECTION



Where:-

M = Applied Moment
Q = Applied Shear
P = Bolt Proof Load
d = Section Depth
n = Number of Top Bolts
k = Number of Bottom Bolts
α = Geometry Coefficient

FIG. 4.4 TYPICAL INTERACTION CURVE FOR PLASTICALLY DESIGNED BOLTED JOINTS stanchion web fillet and web buckling. This problem is illustrated in figure 4.3. The web is checked using the requirements of BS449, with all stresses factored by 1.7. If stiffeners are required they are designed to carry the whole flange force.

Where the column flange is thinner than the end plate tension stiffeners are required to control the bending of the column flanges. The triangular stiffeners are taken as either 6mm or the compression stiffener thickness.

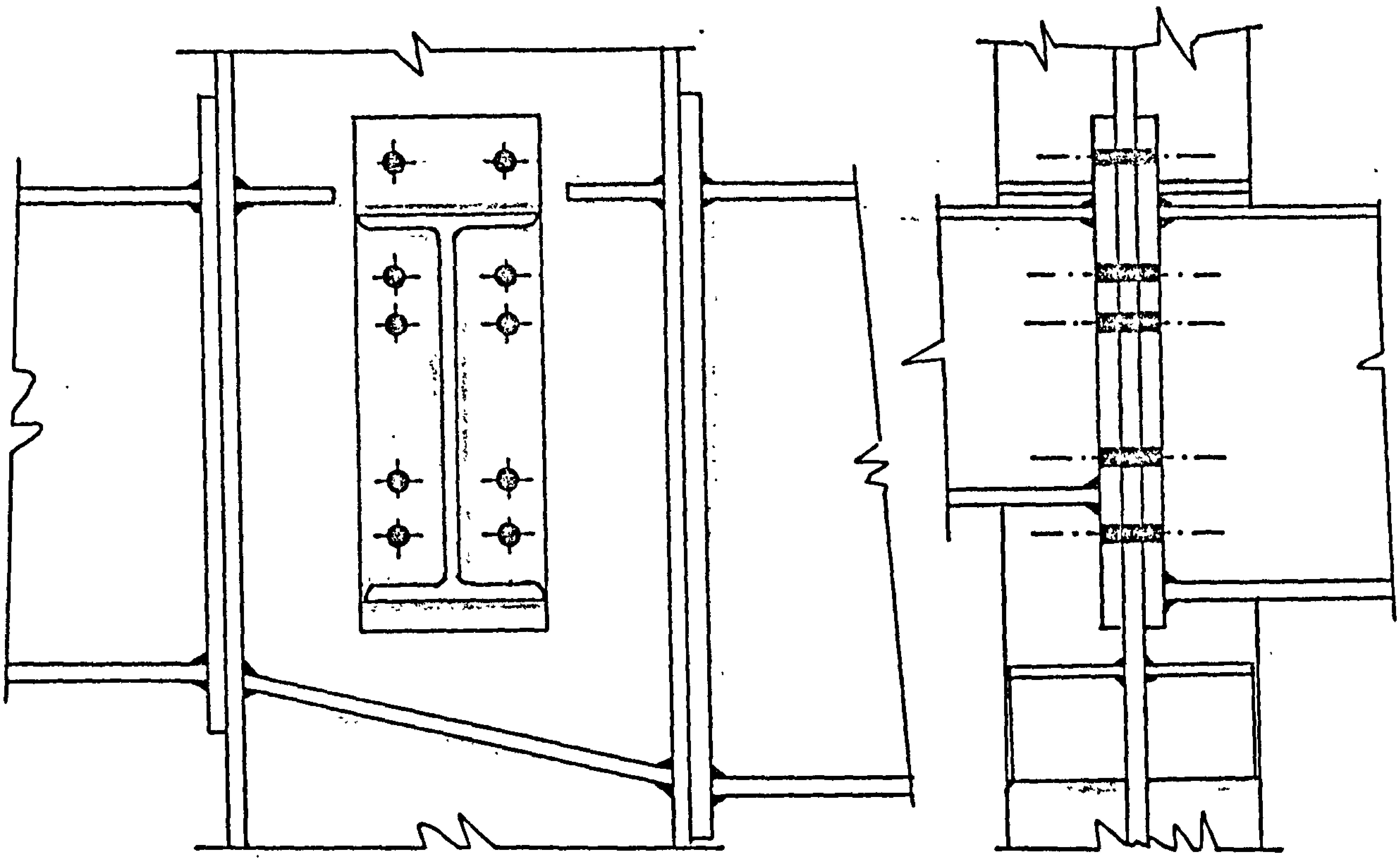
Geometrical constraints are applied to the layout of the connection. Firstly the beam must be deep enough to allow all the bolts required to be fitted between the flanges. Secondly the stanchion flange width must be large enough to allow the bolts to be fitted. Thirdly the column flange must overhang the end plate, which must overhang the beam flange.

The design of the connection consisted of proportioning the connection for each bolt size. The bolt size which needed the least number of bolts was used. In the event of two or more bolt sizes using the same number of bolts, the smaller bolts were used. All other items can be designed once the number of bolts and their size is known. Failure to produce a connection which satisfies all of the criteria results in a very high connection cost being used.

4.4.2 Minor Axis Beam/Column Connections

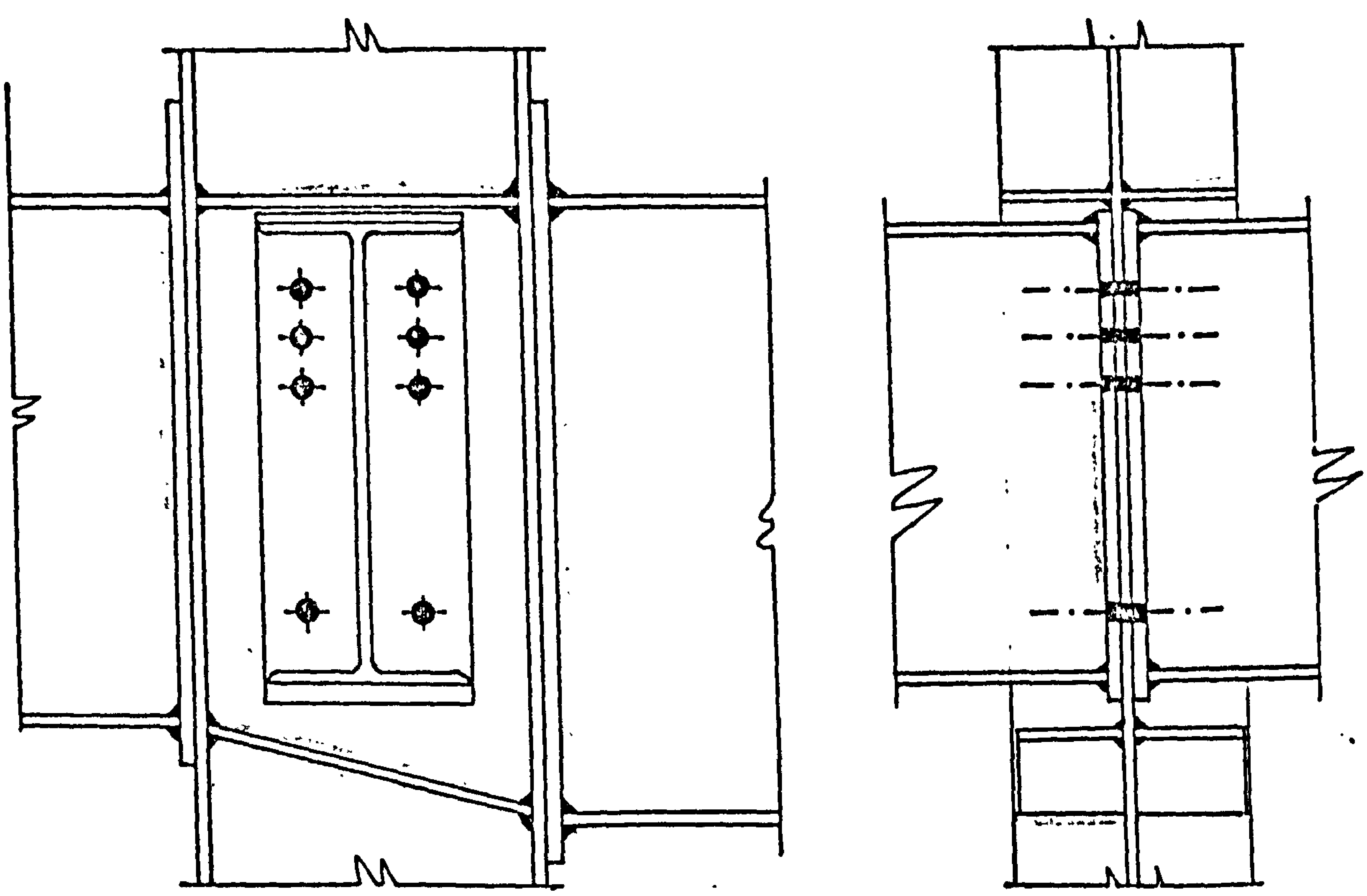
A typical set of connection details for this type of connection is shown in figure 4.5. The design of the connection is carried out using either elastic theory (where the beams are designed elastically) or plastic theory (where the beams are designed plastically).

The application of the design method, which is similar in many respects to the design of major axis beam/column connections, will now be described. The main design variables



SIDE ELEVATION SECTION

EXTENDED END PLATE CONNECTION



SIDE ELEVATION SECTION

NON EXTENDED END PLATE CONNECTION

FIG.4.5 MINOR AXIS BEAM TO COLUMN CONNECTION DETAIL are shown in figure 4.6. The main assumptions are similar to those for major axis connections, except that for elastically designed connections the final assumption becomes:-

(f) The bolt group is designed according to the assumptions of elastic design. The point of zero strain is assumed to lie at the centre of the lowest row of bolts and the highest row of bolts is assumed to be fully stressed.

The design approach consists of determining, by reference to the major axis connection, whether or not an extended end plate can be used.

The design of the bolt group is almost exactly the same as for the major axis connections, however the geometry coefficient takes on a different value.

The design of the end plate and welds is similar to the major axis connection, except that, for elastically designed connections, the lower load in the bolts below the top flange of the beam is taken into account.

Stiffening is provided to the column web where a beam only frames into one side of the stanchion. This stiffening consists of 10mm stiffeners between the stanchion webs. These stiffeners are only used when the major axis connection design does not require a stiffener.

Geometrical constraints are applied to the layout of the connection. Firstly the beam must be deep enough to allow all the bolts required to be fitted between the flanges. Secondly the end plate must be able to fit between any tension or compression stiffeners, taking due regard for levels. Finally the end plate must be narrow enough to fit in the flat area of the stanchion web and it must be wider than the beam.

The design of the connection with particular loads was complicated by the continuity through the stanchion web. Where a beam only exists on one side of the stanchion the design procedure is the same as for the major axis connection.

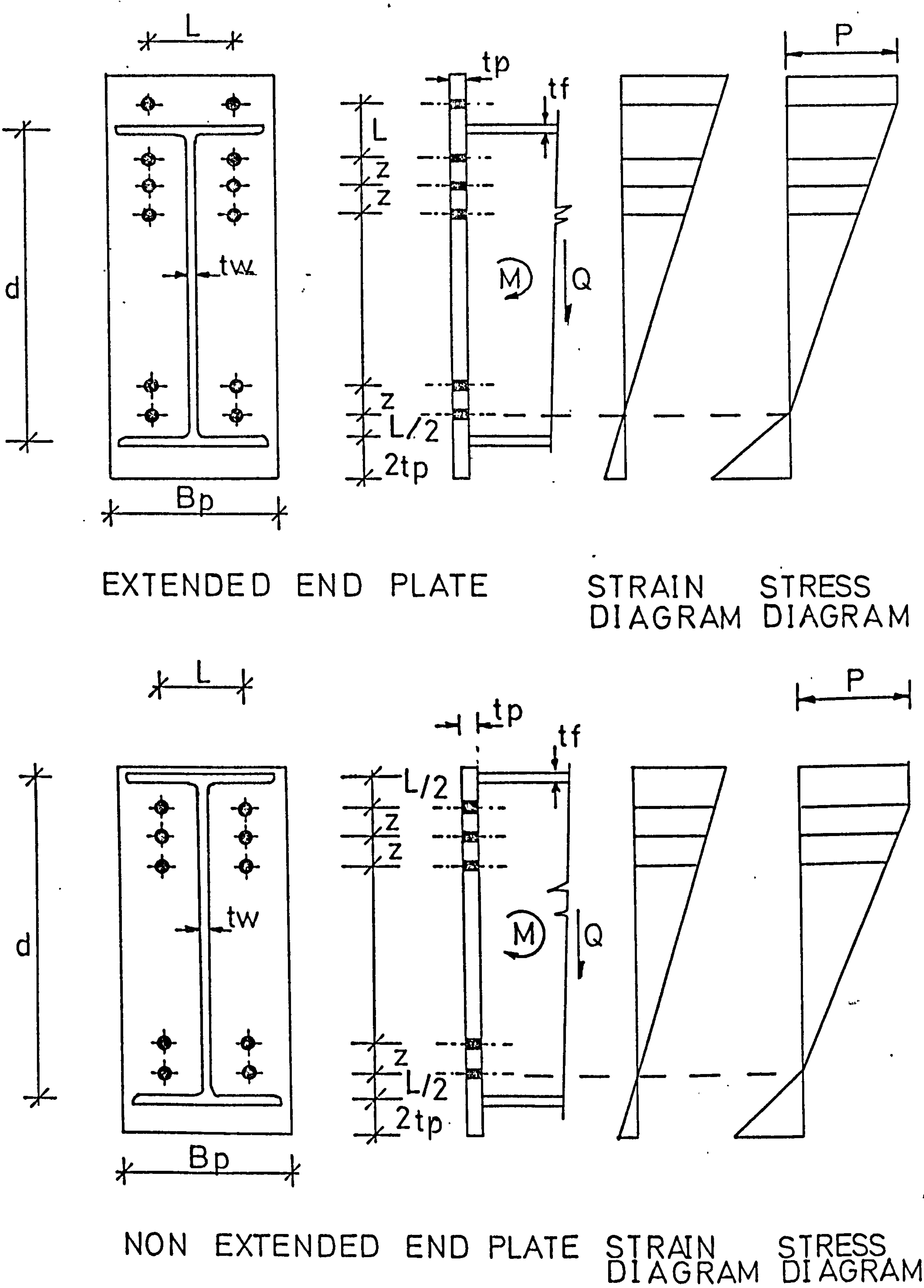


FIG. 4.6 ELASTICALLY DESIGNED MINOR AXIS BEAM/COLUMN CONNECTIONS

Where beams exist on both sides of the stanchion, the connection is designed on each side of the web, and the connection with the highest cost is used on both sides of the web. This approach avoids the complex problem of arranging the bolts to be in the same position on both sides of the stanchion web. Failure to produce a connection which satisfies all of the relevant criteria results in a very high connection cost being used.

4.4.3 Stanchion Splice Connections

A typical detail for this connection is shown in figure 4.7. The design of the connection is in accordance with the elastic theory throughout.

The application of the design method will now be described. The relevant design variables are shown in figure 4.8. The main assumptions made are:-

(a) The bolt group transmits all the loads between the stanchions.

(b) The cover plates transmit all the loads between the bolt groups.

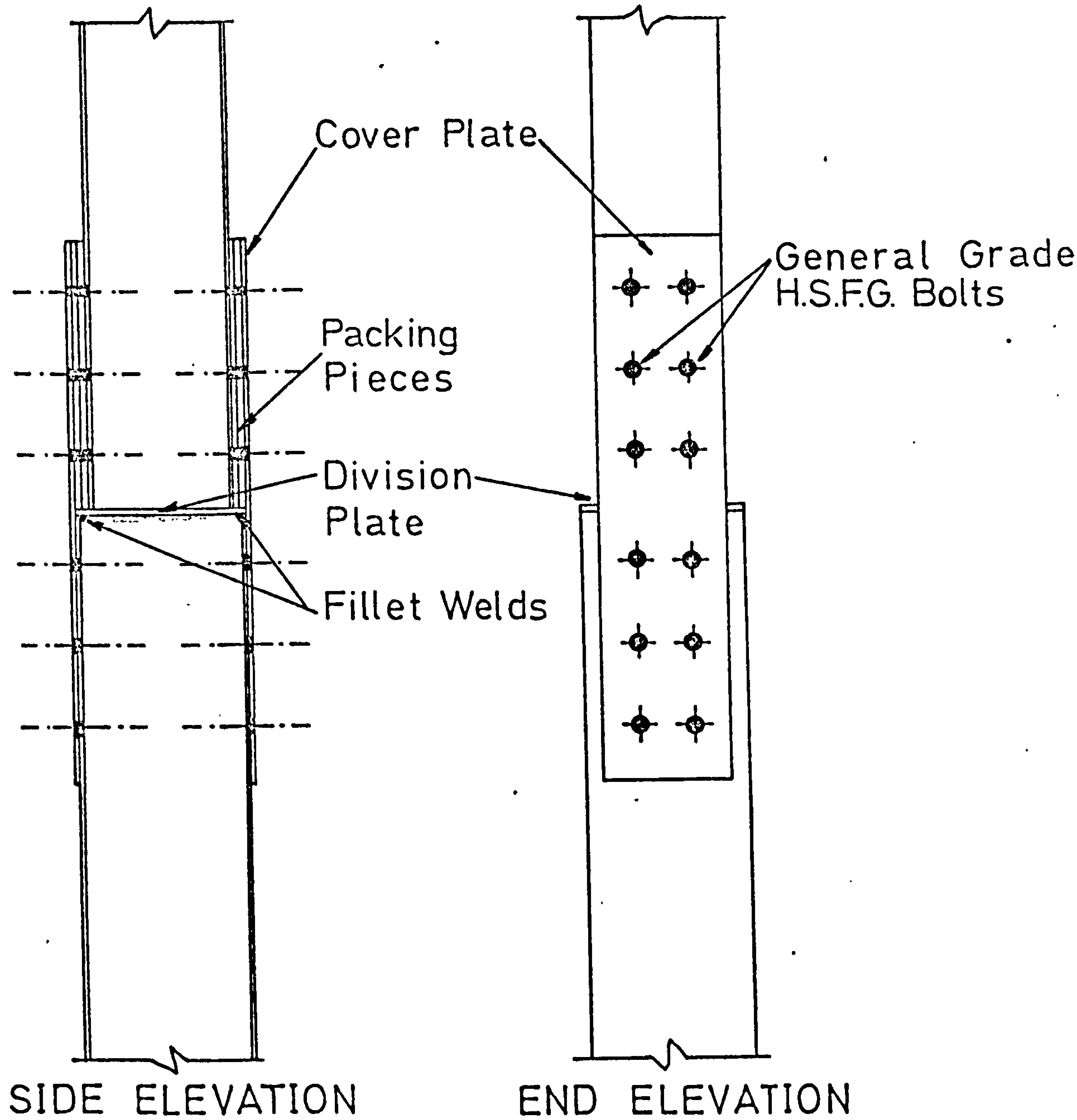
(c) The shear forces in each plane are negligible.

(d) The division plate takes no load.

(e) The ends of the stanchion are assumed to carry no load in face bearing.

(f) All loads are assumed to be concentrated in the column flanges.

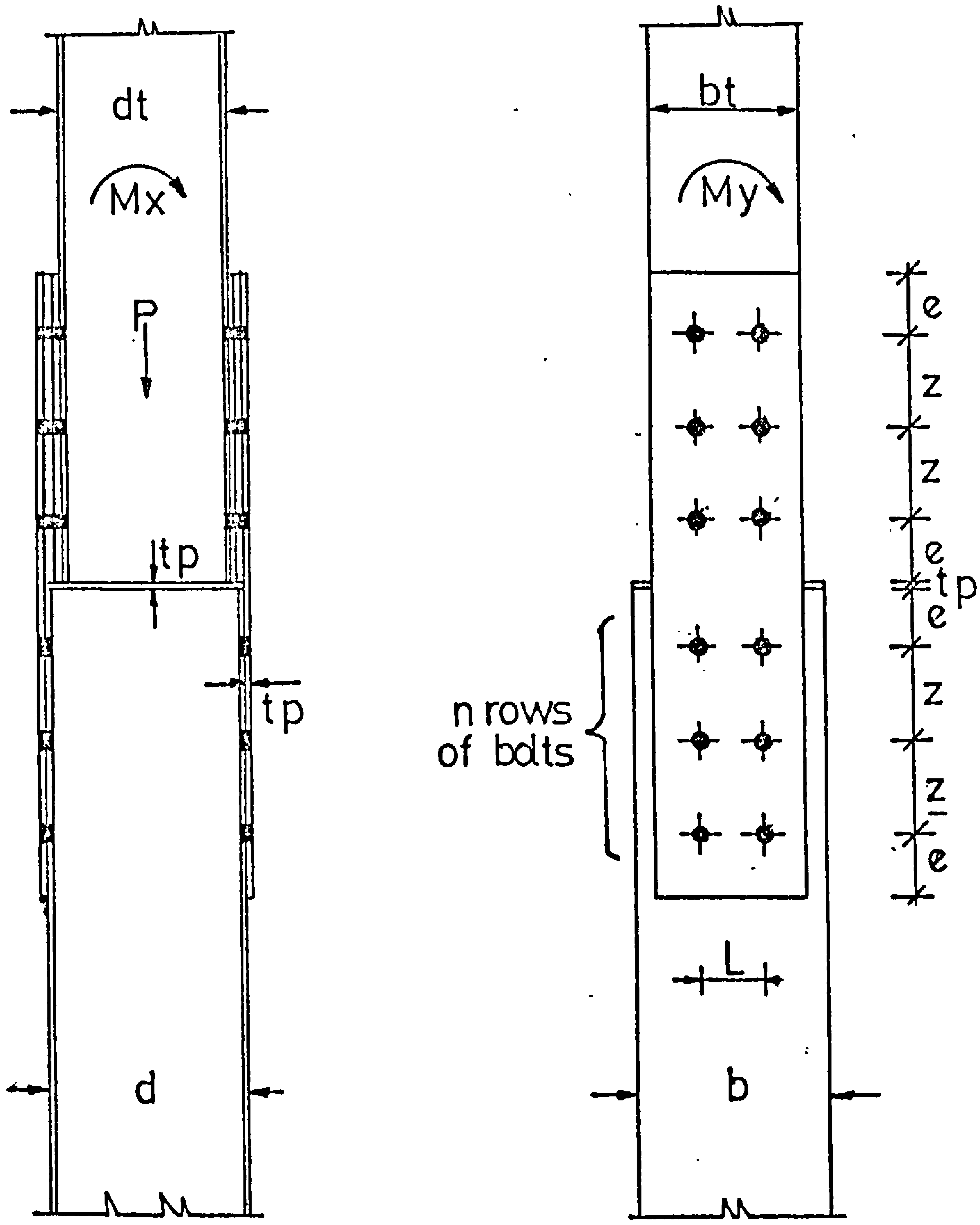
The design of the connection requires a set of axial loads and moments applied to the connection. There may be up to eight load cases for each splice depending on the arrangement of live load on the beams in the floors above and below the splice. These forces, which consist of axial loads, major axis moments and minor axis moments, will now be described. For each design method, the axial load and major axis moments are taken from the first order analysis of the limited frame. The minor axis moments used depend on the design



NOTES

1) All dimensions, number of bolts etc. are dependant on actual sections and loads
2) All plates to be of the same grade of steel as the stanchion sections

FIG. 4.7 STANCHION SPLICE DETAIL



e=edge distance
z=2½x bolt diameter

FIG. 4.8 STANCHION SPLICE DIMENSIONS AND LOADING method. When considering the B.C.S.A. and J.C.R. design methods, the effects of magnification of the minor axis moment and stress due to initial curvature are included. When considering the B.R.S. design method, the minor axis moment that is used is taken as the largest minor axis moment that can occur with the applied direct load and major axis moment. This approach is taken because minor axis moments are not calculated in the B.R.S. design method.

The bolt group is designed assuming all loads are carried through the bolts. The procedure followed consists of determining numbers of bolts required for each bolt size. This is performed by incrementing the number of bolts until all applied load combinations fall within the bolt group interaction diagram. The loads in the cover plate, which are also the loads carried by the bolt group are:-

A direct load $W = {}^{P}/_{2} + {}^{Mx}/_{d}$

and a moment of $M = \frac{My}{2}$

where P = the axial load on the stanchion.

Mx = the major axis moment on the stanchion.

My = the minor axis moment on the stanchion.

d = the depth of the lower stanchion.

The interaction curve of a bolt group is given by:-

$$\frac{W^2}{W_o^{\,2}} \quad \frac{MW\alpha}{M_oW_o} \quad \frac{M^2}{M_o^{\,2}} < 1$$

where W_o = The capacity of the bolt group with no applied moment.

M_o = The capacity of the bolt group with no direct load.

α = A factor dependant only on the bolt group dimensions.

Such an interaction curve is shown in figure 4.9. The bolt size which requires the smallest number of bolts is chosen.

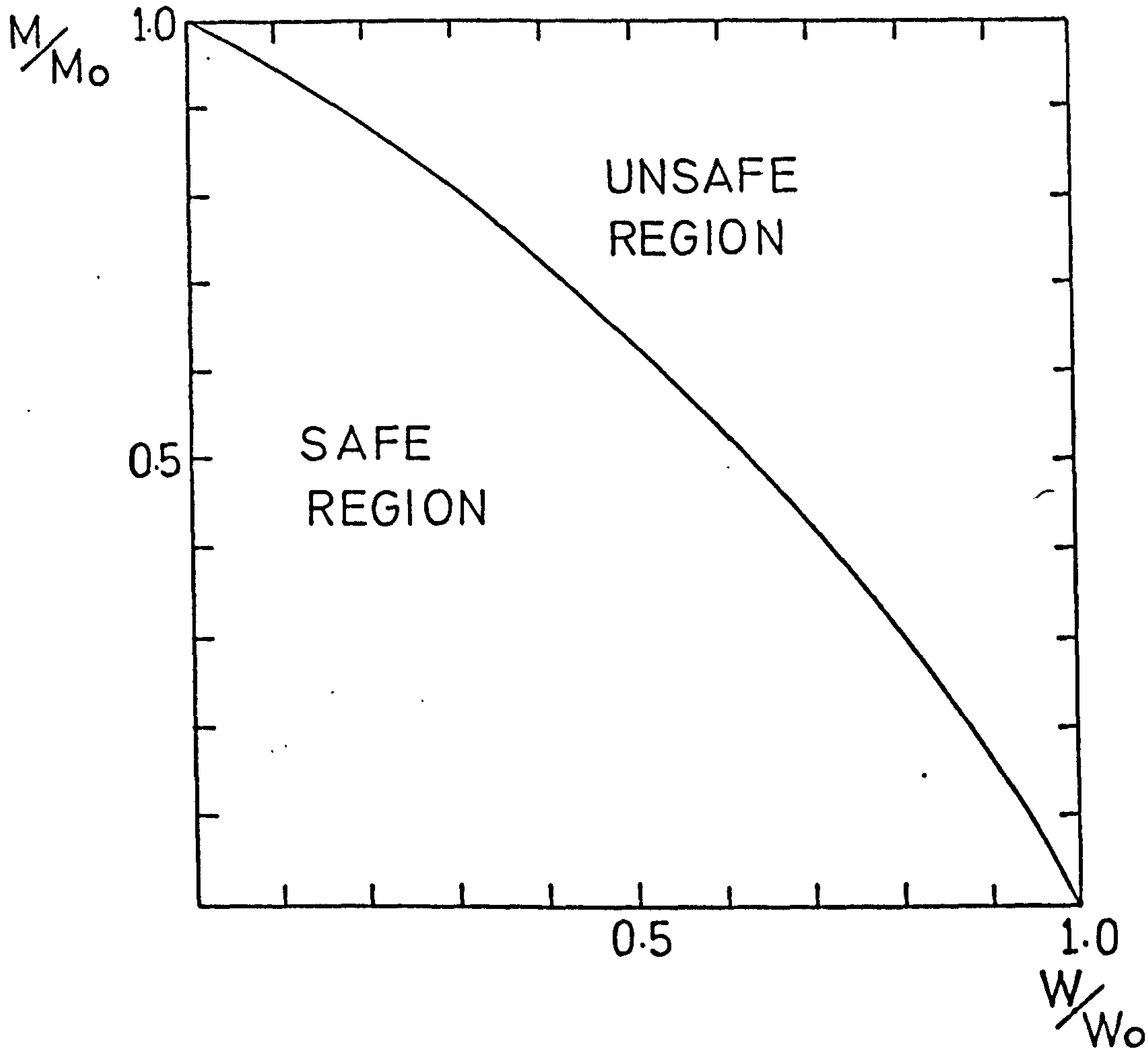


FIG.4.9 STANCHION SPLICE BOLT GROUP INTERACTION DIAGRAM

The design of the cover plates is in accordance with elastic design, using the yield stress as the limiting stress. The cover plate thickness is taken from a standard set of thicknesses.

The packing plates are not designed structurally and are made up of successively thinner plates. The division plate is made the same thickness as the cover plate and is welded to the lower stanchion with a 6mm fillet weld.

Geometrical constraints are applied to the layout of the connection. Firstly the upper stanchion must be shallower than the lower stanchion. Secondly the bolts used must fit into the stanchion flange with the required spacing and edge distances.

The design method chooses the connection which uses the smallest number of bolts. Failure to produce a connection which satisfies all the relevant criteria results in a very high connection cost being used.

4.4.4 Base Plate Design

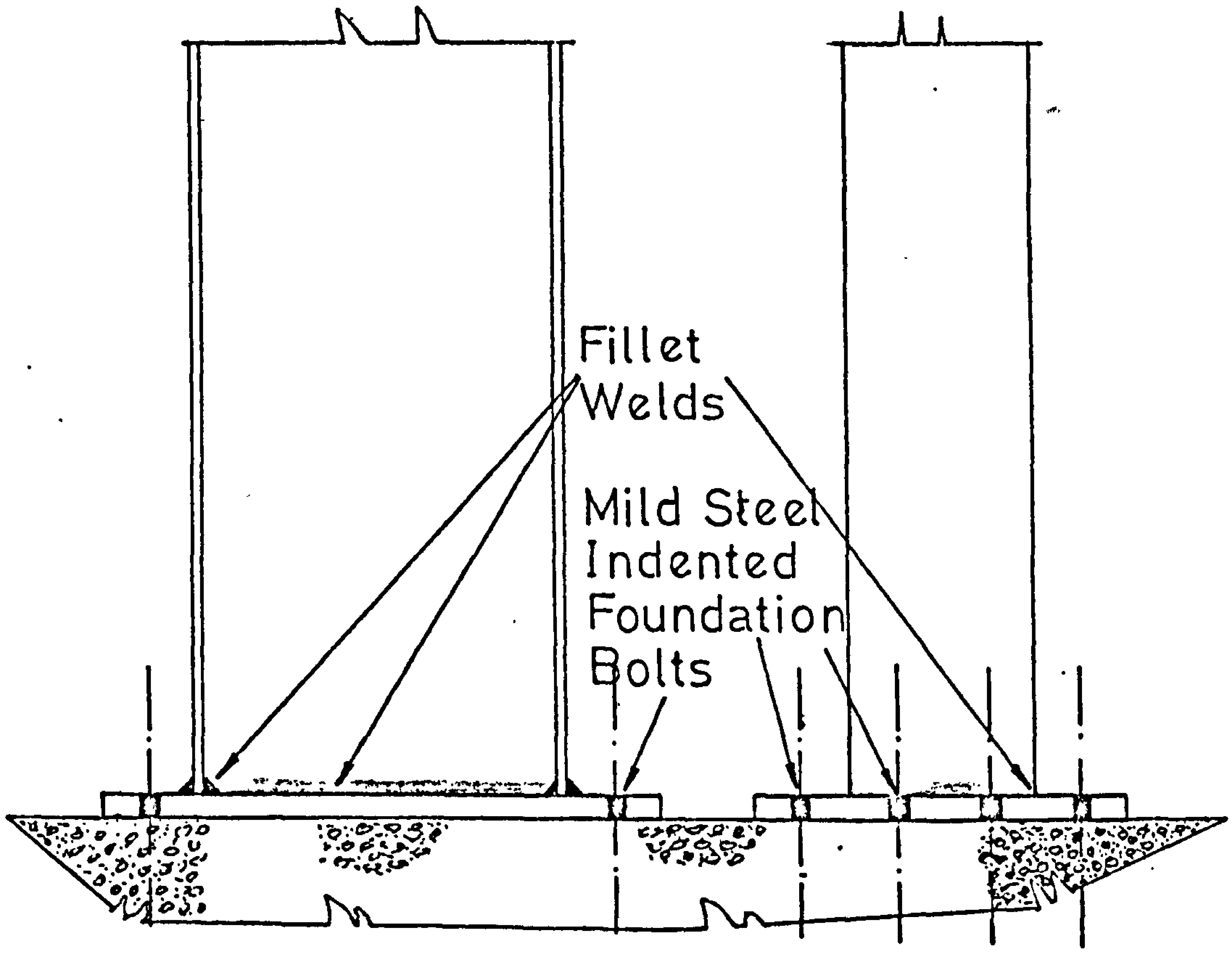
A typical connection detail is shown in figure 4.10. The design of the connection is in accordance with plastic analysis. The application of the design method will now be described. The main design variables are shown in figure 4.11. The main assumptions that are made include:-

(a) The bolts are assumed to be 24mm diameter black indented foundation bolts.

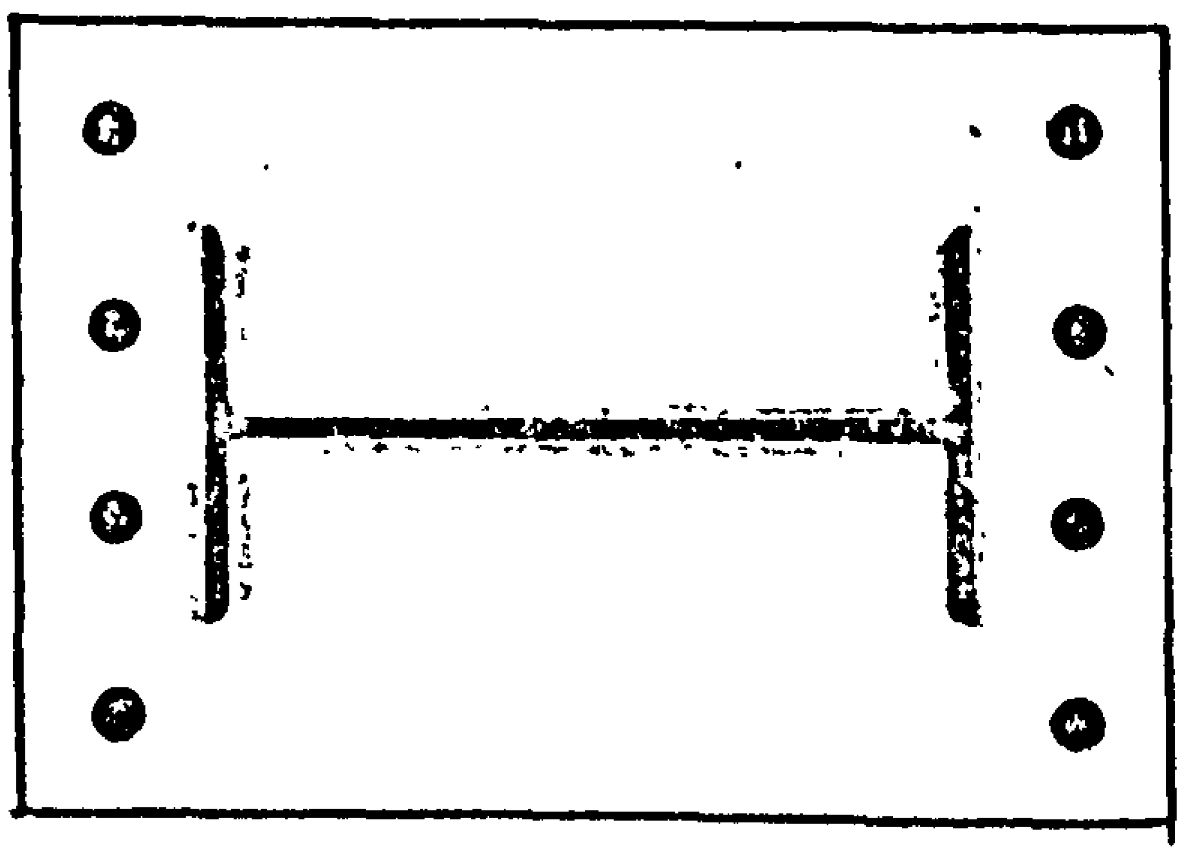
(b) The yield stress in the bolt is assumed to be attained when the bolt strain is greater than the maximum concrete strain.

(c) The ultimate bearing strength of the concrete is 40% of the cube strength and is assumed to have a rectangular distribution.

(d) The base plate can be designed plastically using a yield line analysis.



ELEVATIONS



PLAN

NOTES

1)All dimensions,number of bolts etc. are depe--ndent on actual loads and sections

2)All plates to be of the same grade of steel as the column section

FIG. 4.10 COLUMN BASE CONNECTION DETAIL

The design approach for this type of connection is different to that of the other connections. The reason for this difference is that, due to the biaxial bending, a simple method of positioning the bolts is not available. If an arrangement of bolts is postulated the thickness of the end plate can be easily determined.

The design strategy for this type of connection cannot be easily defined. Therefore it was decided to institue a small optimization procedure for the design of the connection. The problem can be defined as follows:- 8

Given — The number of bolts

Minimise — $f(x)$ = cost of base plate and bolts

Subject to:-

(a) The plate being thinner than 70mm.

(b) The bolt group being satisfactory for all load cases.

(c) x being greater than the minimum dimensions required to fit the bolt in.

(d) y being greater than zero.

The dimensions x and y are shown in figure 4.11. It can be shown that, if the number of bolts remains constant, an increase in x or y will in no circumstances reduce the cost. The design space tends to be discontinuous due to changes in the plate thickness. The procedure used to solve the problem consists of:-

(a) Setting $y = 0$ and x = its lower limit.

(b) Increment x in 10 millimetre steps.

(c) If the resulting design is not feasible goto step (b).

(d) Store the cost and increment y by 10mm until the upper limit is reached set x to its lower limit and return to step (b).

(e) The optimum design is the cheapest design found.

The operation of this algorithm can be demonstrated with reference to figure 4.12. It can be seen that the design

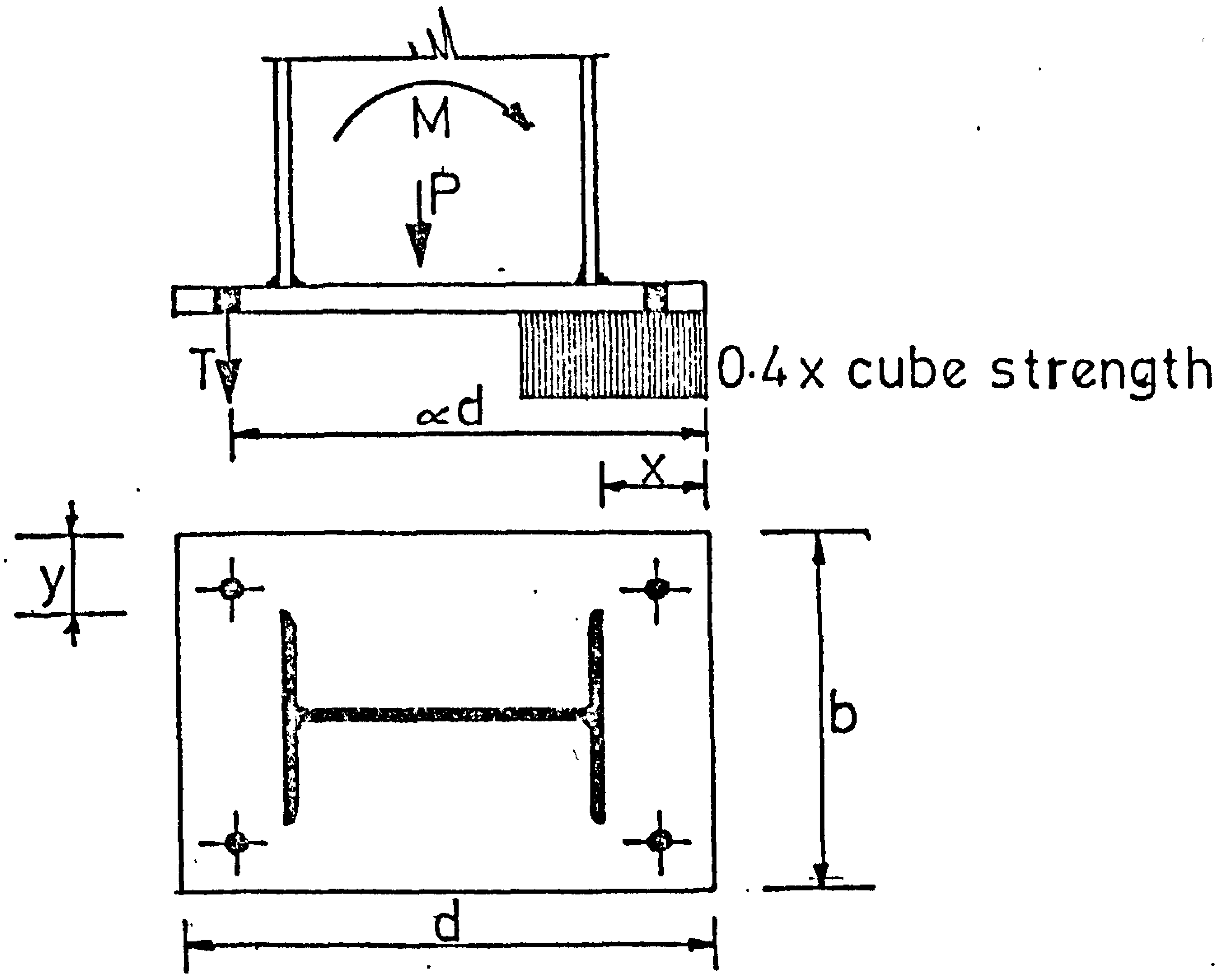


FIG. 4.11 DESIGN PARAMETERS FOR A COLUMN BASE PLATE

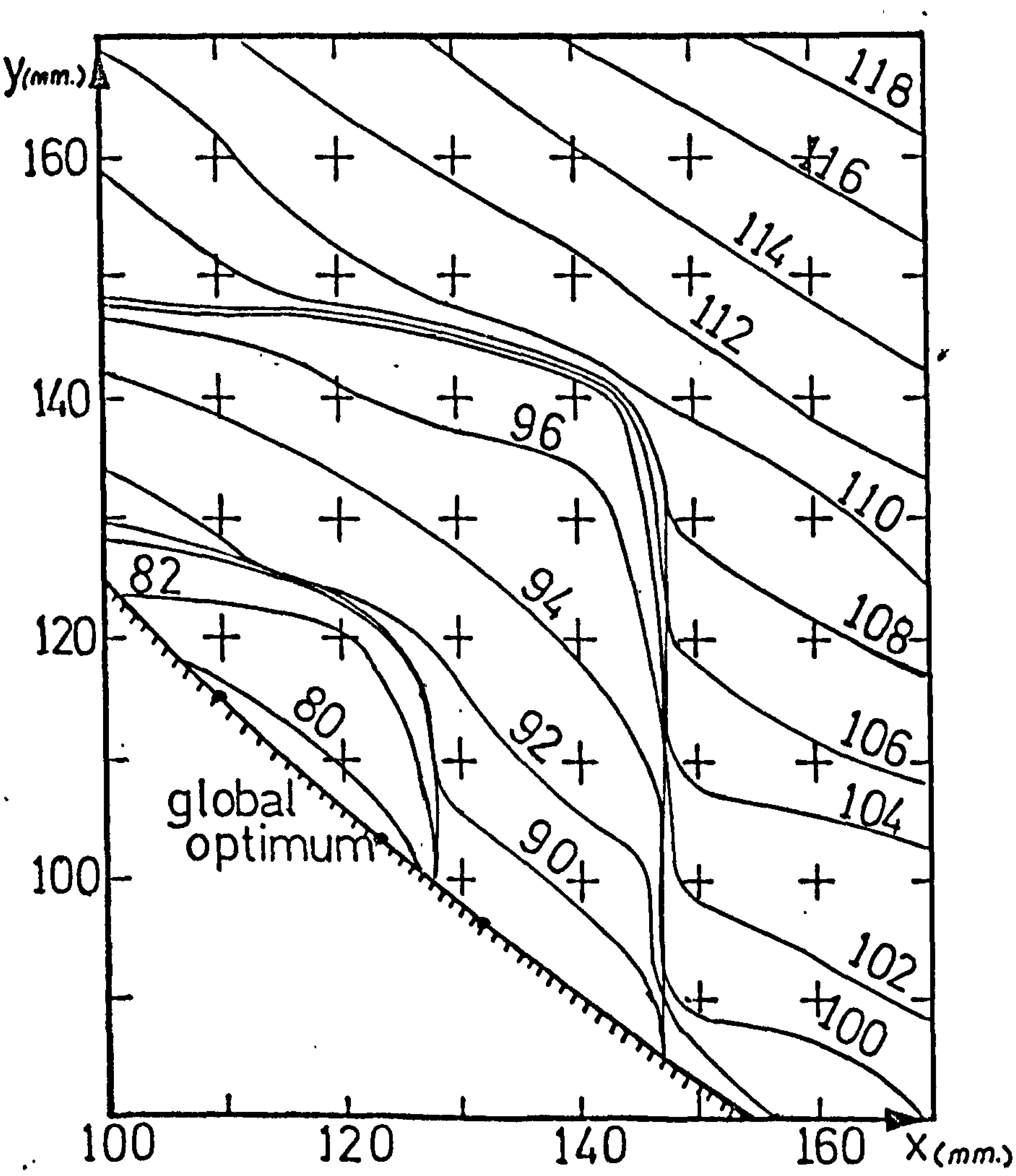


FIG 4.12 TYPICAL DESIGN SPACE FOR A COLUMN BASE PLATE

space contains three optima and that the algorithm finds the global optima, evaluating a significant number of designs implicitly.

The overall design procedure consists of selecting arrangements of bolts with increasing numbers of bolts and using the above procedure until a feasible minimum cost design is found.

The design of the holding down bolts is in accordance with elastic theory when the concrete stress block covers more than half the base plate area and in accordance with the plastic theory in all other cases.

The base plate is designed by using yield line analysis. The modes of failure include distributed loads (under the concrete stress block) and point loads (from the bolts). The thickness of plate required for each failure mode can be determined and the largest taken for use in the connection.

The fillet welds between the stanchion and base plate are designed geometrically using a weld with a leg length equal to the stanchion flange thickness.

The type of base chosen has been used in preference to a gusseted base. The reason that a base plate is used is that a gusseted base is more difficult to design and it is difficult to devise a design strategy for a gusseted base. The cost of each of these types of base is similar because a base plate uses a large amount of material with a small amount of fabrication and a gusseted bases uses a large amount of labour with a small amount of material.

The design strategy seeks to find a design which uses the smallest number of foundation bolts and also satisfies all of the above requirements. Failure to find a suitable connection results in a very high connection cost.

4.5 CONCLUSIONS

A description of the design programme has been given. This programme will generate a building description and check a given set of sections against stress contraints defined by three different design methods. A set of connections may also be designed and costed. The programme provides a complete design checking system for the type of frame being considered.

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CHAPTER 5

A Cost Model

5.1 INTRODUCTION

In order to use mathematical optimization in structural steelwork design it is necessary to have an accurate cost model. The cost model defines the objective function which is to be minimized. The model is a formal description of the values of individual costs incurred in the fabrication of the frame. The model described is an attempt to provide such costs based on the estimating practice of five fabrication companies.

In this chapter the current practices of steelwork estimating are examined. They are shown to be based to a great extent on the experience of the estimator. It is questionable whether such an approach can provide a rational basis for the comparison of designs. Computer estimating systems are also examined and are seen to be generally applicable to estimating the cost of structural steelwork. An examination of computer cost models which have been used as the objective function in the optimum design of steel structures is made, each model is shown to be either inexact or applicable only to a particular type of structure. The cost model and the costing data collected are then described and finally the costing of individual components is examined.

Before proceeding further it is necessary to define two terms, price and cost. The price of a structural steelwork job is the price paid by the client for the structure fabricated, erected and painted. The cost of a structural steelwork job is the cost to the fabricator of providing the structure fabricated, erected and painted. Price and cost are related by profit as shown in figure 5.1.

All costs quoted in this chapter are those which were current in August 1976 in the United Kingdom. Where transport costs are quoted they refer to a fabricator located close to Newcastle-Upon-Tyne.

5.2 STRUCTURAL STEELWORK ESTIMATING PRACTICE

The methods used for estimating the cost of structural steelwork should be seen against the background of the system used in the process of design and construction of steel framed buildings. Estimating practice should not be considered in isolation from design and construction, however current practice tends to discourage such consideration. Generally two cases occur, firstly when a consulting engineer is employed by the client to undertake the design and secondly when the fabricator undertakes his own design.

In the first case, it is usual for the selection of the steel sections to be undertaken by the consultant. A bill of quantities is prepared by the consultant and the contract is put out to tender. The successful company will be retained to perform the fabrication. Some fabricators prefer to design the connections themselves once the tender is accepted, while others prefer this operation to be left to the consultant. The method of payment will vary but will usually be in accordance with the bill of quantities. In this case it is necessary for each fabricator involved at the tender stage to estimate expediently the cost of the job. The method of estimating used has the conflicting requirements of speed and reasonable accuracy. Speed is required because the fabricator will only be successful occasionally, accuracy is required so that the fabricator can be competitive but does not risk a loss if the contract is awarded to him. The principle method used for the estimation is to take the weight of the main steel sections and to multiply this by a "tonnage rate" whose value depends upon the type of construction and commercial factors such as labour cost and the state of the order book. Little if any attention is paid to the structural details such as connections. The value of the "tonnage rate" is determined subjectively by the estimator. The "tonnage rate" for the type of construction being considered in this research varies between £380 and £400 per tonne for grade 43 steel. Tonnage rates include for profit, erection and overheads as well as the cost of sections. Clearly this is a very approximate method of determining the cost of a structure, but it is ideally suited to the circumstances in which it is used. This method of estimating is not suitable for use in defining an objective function in optimization because it reduces to using a minimum weight objective function, which does not satisfactorily deal with fabrication costs. A computer could be used to estimate costs at the tender stage to improve the above method. The improvement would be in terms of time and cost. It becomes feasible to undertake a detailed cost analysis of much of the structural detail even though the work may frequently be abortive. Only when such a detailed cost analysis is undertaken can alternative structural arrangements be meaningfully compared.

In the second case, when a fabricator designs and constructs the frame for the client, it is necessary for him to examine the cost of fabrication more closely in order to maximize his profit and to determine a realistic price for the client. There are two methods used in this case. In the first method it is necessary to calculate the total weight of material including bolts, end plates, stiffeners, splice plates, sections, weld material etc. The resulting weights are each multiplied by an appropriate "tonnage rate", this rate varies for each material. The basic cost is the summation of all the individual costs multiplied by a factor which expresses an approximate relationship between material weight and workmanship costs

$$\text{Cost} = \Sigma \text{ Material Weight x Tonnage Rate x Workmanship Factor.}$$

This method considers the cost of a detail more satisfactorily than the previous method, however it requires periodic modification of the factors in order to keep pace with variations in labour and material costs. The factors are usually based on experience or standard designs and are therefore subject to error. The basic cost is then modified to take account of surface treatment, profit, erection and overheads. The method also provides an inventory of material for ordering. Descriptions of the use of this method and of its application to various structures are described by SAUNDERS[21].

In the second method, which is the method adopted in the cost model, the amount of time taken to perform each of the fabrication 8
operations is evaluated from the results of work study. The resulting times are multiplied by the current labour rate, adjusted to take into account overheads. This cost is then added to the cost of the material, and the total cost is then modified on a tonnage basis to include profit and erection. This method of estimating may also be used to evaluate the factors for use in the previous method.

5.2.1 Applications of manual estimating

A number of structural steelwork applications of the latter type of estimating are now described. CORKER[5-14] examines a number of the fabrication operations involved in the construction of tanks, pressure vessels and plate girders. The fabrication process is examined in great detail, every operation being broken down to its basic constituent parts. Clearly it is necessary to rationalize the data thus produced and to this end Corker has provided a number of useful but intricate charts. The data provides some useful comparisons with the data reported in this cost model, this is discussed later. DONNELLY[15] has produced times for the deposition of weld material for welds of various types and sizes. The quoted times have all been related to the time taken to deposit a 6mm fillet weld. These relative times were determined by time study methods and so can be useful for making comparisons of the costs of various details. It is not possible to use this data within the cost model because comparisons with other fabrication operations were not included. SAUNDERS[21] examines American estimating practice in great detail and shows that it is very similar to British estimating practice. He recommends the use of the second method of estimating described above, the factors being based on the results of time study. Saunders included examples of estimating using time study information and inventories of raw materials, the use of actual times was avoided.

5.2.2 Applications of Computer Estimating

Computers have been used for the purposes of cost estimation and material ordering. Such applications require interactive operation and cost data confidential to the fabricator. The application is essentially an accounting system. WRIGHT AND DAY[22] describe a system which, in conjunction with computer detailing programmes for plane frames, can be used to estimate the cost of steel frames. Their approach is to separate the total cost into individual fabrication operation costs. The fabricator is allowed to assign suitable costs to each operation. The derivation, values and organization of the costs within the computer are confidential to the fabricator, therefore the model could be used only for comparison purposes if costs were readily available. During the estimation process the estimator remained in complete control and could perform comparisons of various designs interactively. The type of programme envisaged by Wright and Day was to be flexible in that an estimating system used by any particular fabricator could be implemented. The computer was to be used essentially for data storage and for simple arithmetic calculations. Under the estimator's direction, the programme would refer to data taken from manuals of standard times and material costs, the estimator would direct the programme interactively, so maintaining control over the estimating process. One example of the need for interaction was described, i.e. the sawing of sections. If each section is sawn separately the total time taken is two minutes per section. If the sections are first fastened together and then sawn, the time taken is one minute per section plus fifteen minutes. In such a case interaction is useful in that it allows the estimator to base his estimate upon the fabricators own workshop practice. Although Wright and Day's work is valuable in providing a rational structure, it has little application to optimum cost design since the degree of interaction required makes the time taken to optimize a structure prohibitively long.

5.3 COST MODELS USED BY PREVIOUS INVESTIGATORS

Applications of cost models to optimization have been used frequently but many of these have either been derived for a particular type of structure or they do not cover the complete fabrication process. Many investigators have used minimum section weight as a cost model and this has many advantages such as:-

(a) Minimum weight can be approximately related to the primary variables such as crossectional area and elastic modulus producing continuous functions. Which permit mathematical solution.

(b) Minimum weight requires no backup data and is therefore attractive to users who do not have access to such data such as consultants.

(c) Minimum weight is readily understood by everyone in industry.

(d) Minimum weight is suitable for use in vehicle design and is essential for use in aeroplane design due to the penalties associated with additional weight.

However a minimum weight objective function neglects significant factors such as the cost of connecting members and additional costs due to ordering small quantities of material. The accuracy required in a cost model can mean the difference between profit and loss, for example consider figure 5.1 this shows the relationship between price cost and profit. The price is fixed at the estimating stage and therefore if an innaccurate cost model is used at this stage a reduction in anticipated profit may result when the fabrication work is performed.

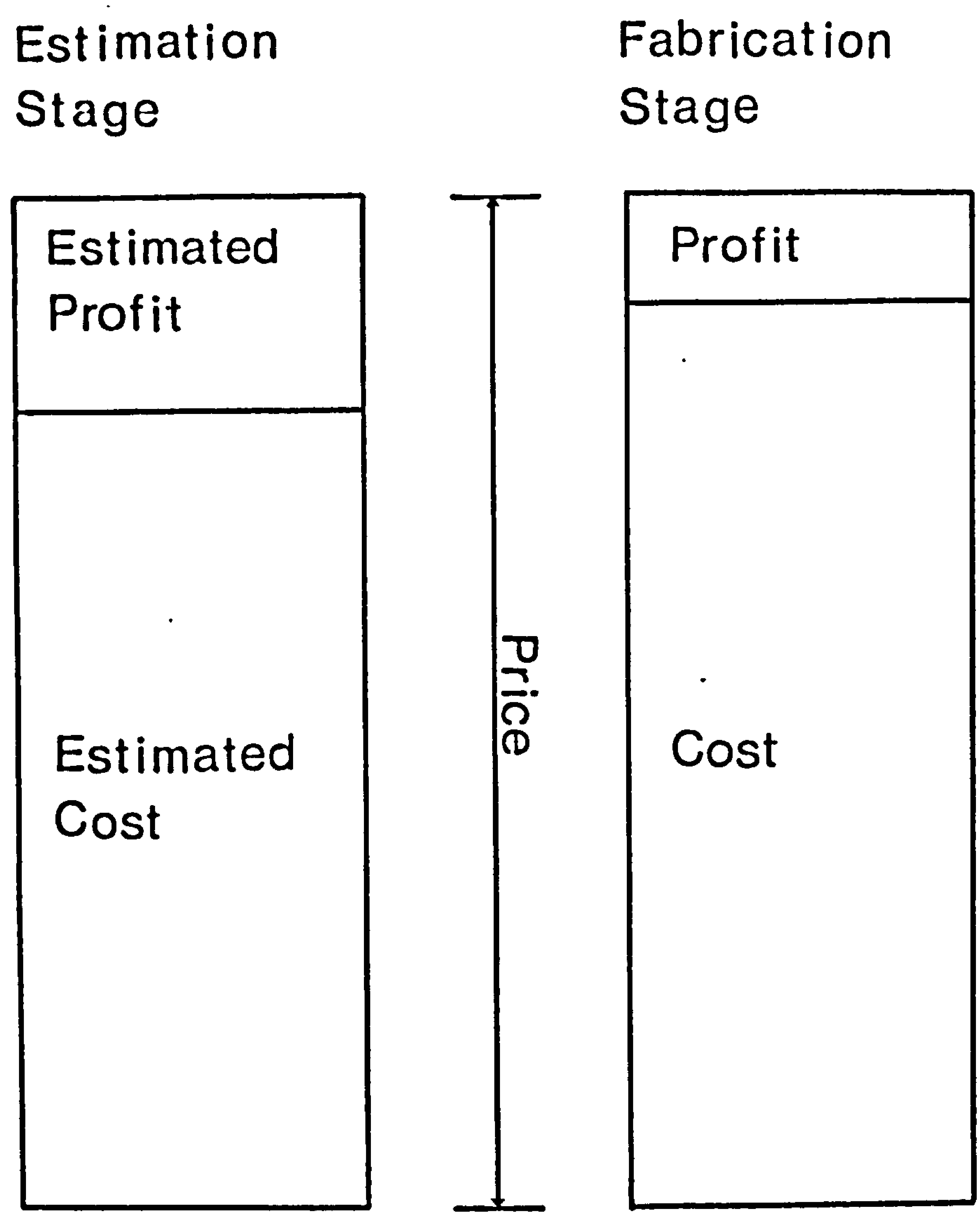


FIG. 5.1 THE RELATIONSHIP BETWEEN PRICE AND COST

For these reasons minimum weight will no longer be considered as a valid criterion.

KNAPTON[18] has considered a cost model for the design of portal frame structures, in which use was made of time study data for the estimation of fabrication operations. Knapton considered welding, cutting, drilling and surface preparation. The fabrication times used by the model were rationalized for use with a set of standard connections. Thus the model has the benefit of simplicity but the data produced was not applicable to non standard connections.

ZWANZIG[23] describes the elements of a cost model for use with multi-storey plane frames. The fabrication costs are based on a works employing capital intensive equipment such as numerically controlled machines. The cost model is very detailed, the fabrication items considered include drilling, welding, straightening welded parts, and surface preparation. Material costs are included and so obscure the costs associated with ordering small quantities of material. This model is used in conjunction with standard details and reflects German practice. No indication of the costs used were given by Zwanzig.

GOBLE AND DE SANTIS[17] and RAZANI AND GOBLE[20] have used a cost model based on the loads at a section and the size and thickness of plates for the optimum design of welded plate girders. The functions which relate these variables to the cost of the girder are provided by the user as an approximation to his estimating system. Clearly this requires work on the part of the model's user in order to find suitable, linear, non linear or step functions to model the cost. A summary of the assumed relationships are given in Table 5.1. The model does not consider small quantity extras but it gives a good estimate of cost for the type of structure considered, providing the functions are made to accurately reflect the true production processes.

Cost Item	Cost assumed to be a function of:-
Steel Plates	Breadth and thickness of plate.
Flange or Web Splice Welding	Breadth and thickness of smaller plate or the moment at the point of splicing plus a constant for each weld.
Welding Flange to Web	Thickness of web plate or shear plus a constant for each weld.
Longitudinal Stiffeners	Volume of longitudinal stiffeners.
Transverse Stiffeners	Number of transverse stiffeners, height of stiffener and area of stiffener.
Supporting Structure	Height of web and flange width.

TABLE 5.1

BRINKERHOFF[1] gives details of a cost model which includes welding, bolting, plate fabrication and steel section material costs. Brinkerhoff abstracted the welding costs from DONNELLY[15] based on a cost which he determined for a 6mm fillet weld. Bolting and plate costs were based on the weight of the material used. Steel section material costs were based on the basic cost to the fabricator of steel, plus extras for the type of section, quality of steel and quantity of section bought. All other costs were assumed to be related to section weight. This model does not consider each of the production processes and in consequence the assumed fabrication costs may be incorrectly related to their true values. The primary reason for using weight related factors in Brinkerhoff's study was his lack of cost information.

LIPSON AND RUSSELL[19] present a cost model for use in the optimum design of trusses. Material costs are based on the weight of the material with no consideration of extra costs, which vary with the section chosen and the quantity. Labour costs are assumed to be a function of the number of members only; one cost for member end preparation and another cost for welding the member end to other members. This proceedure is justified in that all the connections are similar in Lipson's structure irrespective of the size of the members meeting at the connection. Overheads and other such items were included on a tonnage basis. This cost model was derived purely for application to truss type structures.

5.3.1 Summary of work by previous investigators

By critically appraising each of the foregoing models the requirements for a suitable cost model can now be determined. To do this the model must be capable of reflecting accurately the influence of all variables in the production process. Firstly the resulting costs from the cost model should represent as far as practicable the true cost of fabrication and materials for the type of construction being considered. Secondly as little standardization of structural detail as possible should be allowed. The model should in no way constrain the designer, forcing him to abandon a detail because it cannot be costed. Estimating should be subservient to design, but the designer should respect the influence which his decisions have on cost.

5.4 THE COST MODEL DEVELOPED

The cost model which will now be described was developed with all of the above features in mind. Consideration was given to those production processes which are used in the fabrication of multi-storey steel frames and which can be easily quantified. The model seeks to provide a reasonable estimate of production costs of a fabricator, given the time study information relevant to the fabricator. An effort has also been made to establish a reasonable set of fabrication costs to use in the cost model in the absence of particular information, and consideration is given of the variations of these costs between fabricators. For each production process considered, relationships between the fabrication costs and suitable variables, such as plate thickness, which have been derived from time study data are given.

5.5 MATERIAL COSTS

The material costs used in the cost codel have been found from manufacturers' price lists[2,3,4,16] which are readily available. The quantities of materials actually ordered by companies were collected from four sources. These companies also cooperated in the study of fabrication costs and further details are given in Section 5.6.

5.5.1 Steel Section Costs

Steel sections are available from either the British Steel Corporation (B.S.C.) or from steel stockholders. The purchasing policy of the companies which were approached during the course of the research consists of ordering all the steel sections for a given job from the B.S.C., unless very small quantities are required, in which case stockholders are used. It is not common practice to hold stocks of steel sections, because in a period of high interest rates the amount of capital tied up in stock, which may not be required immediately, may affect the profitability of the company. Steel stockholders order large quantities of sections from the B.S.C. and use the resulting discounts as part of their profit. Therefore, for small quantities, steel sections may cost less if bought from a stockholder. For larger quantities the sections may cost more than sections ordered from the B.S.C., however there may be an advantage in that they may be readily available from the stockholder. In the cost model described it has been assumed that all sections will be bought direct from the B.S.C. irrespective of quantity. The particular ordering policy of a given fabricator is beyond the scope of this study. It would be a simple matter to ammend the model to deal with 29
other purchasing policies as long as the policy can be defined as a series of rational decisions.

The cost of steel sections from the B.S.C. is made up of a section "basis cost" plus "section extra costs". The

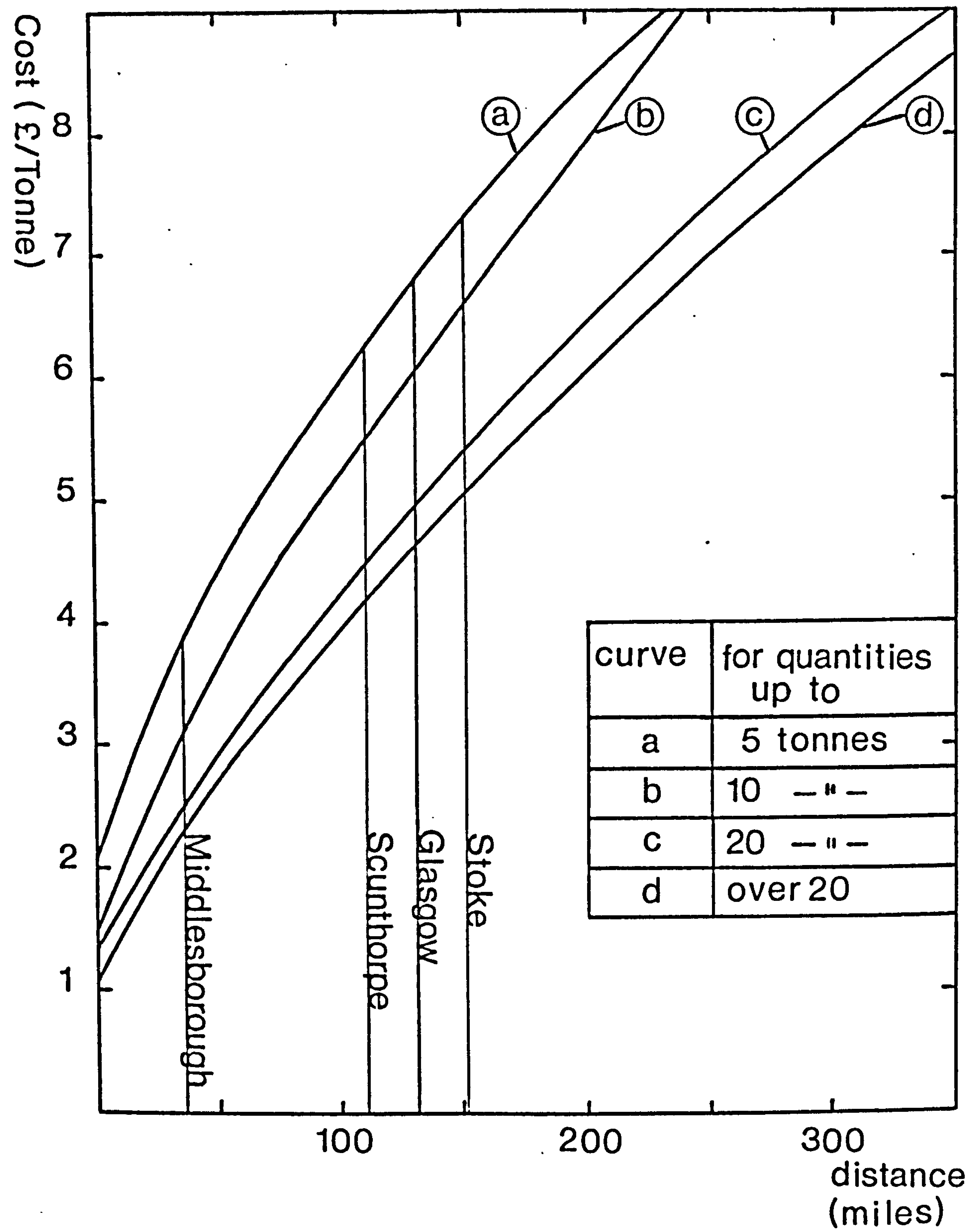


FIG. 5.2 TRANSPORTATION COSTS

Section Type	Cost £/tonne for grade 43 steel		
	Minimum	Mean	Maximum
Universal Beams	156.20	165.47	173.40
Universal Columns	161.30	165.53	170.70
Joists	160.20	169.93	190.00
Add 12£/tonne extra cost for grade 50 steel Add 18£/tonne extra cost for grade 55 steel			

TABLE 5.2 - Material Cost of Sections

Hot rolled beam sections are manufactured at Glasgow, Scunthorpe, Stoke and Manchester, some sizes being manufactured at more than one location.

The above extra costs can be combined into a table for each grade of steel and this table can be used to rapidly determine extra costs. Table 5.3 shows the costs for a fabricator in Newcastle-Upon-Tyne. It can be seen that the extra costs dependant on the quantity ordered have a significant effect on the true cost of a section. Other extra costs may be applied, for lengths over fifteen metres, non standard lengths, other qualities, cold sawing to length and for shot blasting. These are not considered because they are either not applicable or are considered as a fabrication operation.

Steel Quality	quantity (tonnes) / Rolling Mill	1 1≤w≤2 fixed extra £	1≤w≤2 £/tonne	2≤w≤5 £/tonne	5≤w≤10 £/tonne	10≤w≤20 £/tonne	20≤w £/tonne
Grade 43	Middlesborough	7.60	15.00	18.80	8.05	4.50	2.30
	Scunthorpe	12.30	15.00	21.15	10.50	6.40	4.15
	Glasgow	13.30	15.00	21.65	11.10	6.90	4.60
	Stoke	14.30	15.00	22.15	11.65	7.35	5.00
Grade 50	Middlesborough	7.60	27.00	30.80	20.05	16.50	14.30
	Scunthorpe	12.30	27.00	33.15	22.50	18.40	16.15
	Glasgow	13.30	27.00	33.65	23.10	18.90	16.60
	Stoke	14.30	27.00	34.15	23.65	19.35	17.00
Grade 55	Middlesborough	7.60	33.00	36.80	26.05	22.50	20.30
	Scunthorpe	12.30	33.00	39.15	28.50	24.40	22.15
	Glasgow	13.30	33.00	39.65	29.10	24.90	22.60
	Stoke	14.30	33.00	40.15	29.65	25.35	23.00

TABLE 5.3 - Section Extra Costs

5.5.2 Steel Plate Costs

Steel plates are available from the British Steel Corporation or from stockholders in the same way as steel sections. It is common practice to purchase steel plates periodically and not to order plates for each job. The reason for this is that there are only a few thicknesses of plates required and also that plates are used in all types of steel structures resulting in continuous use. For the purpose of this investigation, and because of the above, it is assumed that all steel plate is ordered in quantities of twenty tonnes or over, direct from the B.S.C..

The price of steel plates from the B.S.C. is made up of a "basis cost" plus extra costs in the same manner as steel sections. The prices used are in accordance with the price list for heavy steel plates[2]. In terms of cost per tonne the cheapest plates are between 15mm and 30mm, outside this range the cost may increase considerably. The steel grades considered are 43A, 43C, 43D, 50B, 50C, 55C and 55E in the thicknesses allowed by BS449[24]. The increase in yield stress is not in proportion to the increase in cost, favouring the use of high strength steels. The basis prices refer to "quantities of 20 tonnes and over of one width, one thickness, one specification, for delivery from one works at one time, delivered to one destination."

The extra costs which are applied in the cost model are:-

(a) for thickness of the plate

(b) for transport of the plates to the fabricator

(c) for size of plate.

Other extra costs may be incurred for:-

(a) other qualities

(b) quantity

(c) cutting to close tolerances

(d) surface preparation

(e) marking

(f) testing.

These costs have not been considered because they are dealt with later as part of the fabrication process or because they are not applicable to the type of structure considered. Extra costs for size and thickness of plate are dependant on the width, length and thickness of the plate as ordered. Plates between 4m and 8m long and between 2.25m and 2.5m wide incur the lowest size extra cost. Plates within these size ranges with a thickness of 15mm to 20mm incur no size extra cost at all. The assumption is made in the cost model that plates ordered are 8m long and 2.5m wide irrespective of thickness, resulting in the lowest size extra cost. The size of plates ordered by a fabricator cannot be rationally predetermined from the design of a particular job.

Transport extra costs are determined in the same way as for steel sections. All the thicknesses and sizes of plates required are available at Middlesborough.

The above extra costs can be combined to give a cost per square metre of plate, these are shown in Table 5.4 for steel grades 43, 50 and 55.

Quantity extra costs are determined in the same way as for steel sections and therefore if the quantities of plate for each thickness usually ordered is known, the costs of plates could be readily modified.

It is usual in the type of work being considered to allow a rate for wastage of raw material of 10%. This is low in comparison with other types of construction which can waste up to 35% where large but intricate shapes are fabricated. Wastage occurs when the endplates, baseplates and splice plates are cut from large sheets. The plates used are generally rectangular and are cut from rectangular sheets, thus the amount of waste will be small in relation to the size of the sheet.

Plate Thickness	Grade 43 Steel	Grade 50 Steel	Grade 55 Steel
10	12.03	12.68	17.02
12.5	14.65	15.68	20.59
15	17.23	18.46	24.35
20	22.97	24.62	33.49
25	29.30	31.36	42.45
30	35.16	39.04	50.94
35	41.84	46.38	60.25
40	47.82	53.00	68.86
45	55.21	61.04	80.64
50	61.35	67.82	89.61
60	73.61	81.39	104.53
70	86.98	96.05	126.55

Table 5.4 - Cost per square metre of steel plate

5.5.3 Bolt Costs

High Strength Friction Grip Bolts may be supplied by local stockists or by the manufacturers. The cost of bolts is determined from price lists[16], which are substantially similar for all manufacturers. The price of a bolt, complete with nut and load indicating washer, is dependant on whether it is a stock item, the length of the bolt, the diameter and the quantity ordered. In order to model the cost of bolts in a satisfactory manner, it is necessary to rationalize some of these variables. The basic cost of a bolt of a given diameter is dependant on the length of the bolt. Up to a certain length of bolt the relationship is substantially linear, above this length the relationship is linear but the cost rises more rapidly. Typical relationships between bolt length and cost are shown in figure 5.3. The cost of a bolt with a given grip length can be found from linear relationships such as those shown in figure 5.3. One set of relationships can be found for each bolt diameter. Lengths of bolts within the special region are usually required for column splices only.

extra costs for the ordering of small quantities, qualities other than grade 43 and transport. Section basis costs for all the sections which are currently available are given in the B.S.C. price list[3]. The basis costs vary considerably depending on the section, this variation can be seen by examining Table 5.2, this table gives the range of cost per tonne of each grade of steel for each type of section considered within the cost model.

Extra costs may be incurred because of the quality of the steel being ordered, in the cost model steel of grades 43A, 50C and 55C is considered. Other grade designations exist and will be used elsewhere within this chapter. The extra costs may be determined from Table 5.2. These extra costs are not in proportion to the increase in yield stress achieved, favouring the use of the higher strength steels. Extra costs are also incurred when purchasing small quantities of a given section. The section basis prices are applied where more than twenty tonnes of one section, from one works, in one quality are ordered. For other quantities an extra price is applied. The cost model assumes a minimum order of one tonne per section. These extra costs have been combined with those for transport and are shown in Table 5.3.

Extra costs are applied for the delivery of sections from the British Steel Corporation's Works to the fabricator's works. These costs are given in the "British Iron and Steel Carriage Tarrif Schedule"[4]. The costs depend on the distance and quantity of sections to be transported, with a minimum charge equivalent to that for two tonnes. The relationship between the cost per tonne and distance is non linear and is shown in figure 5.2. In order to calculate this extra cost it is necessary to know from which rolling mill the section will be bought (this has been taken as the mill nearest to the fabricator at which the section is available), the quantity required and the distance from the rolling mill to the fabricator's works.

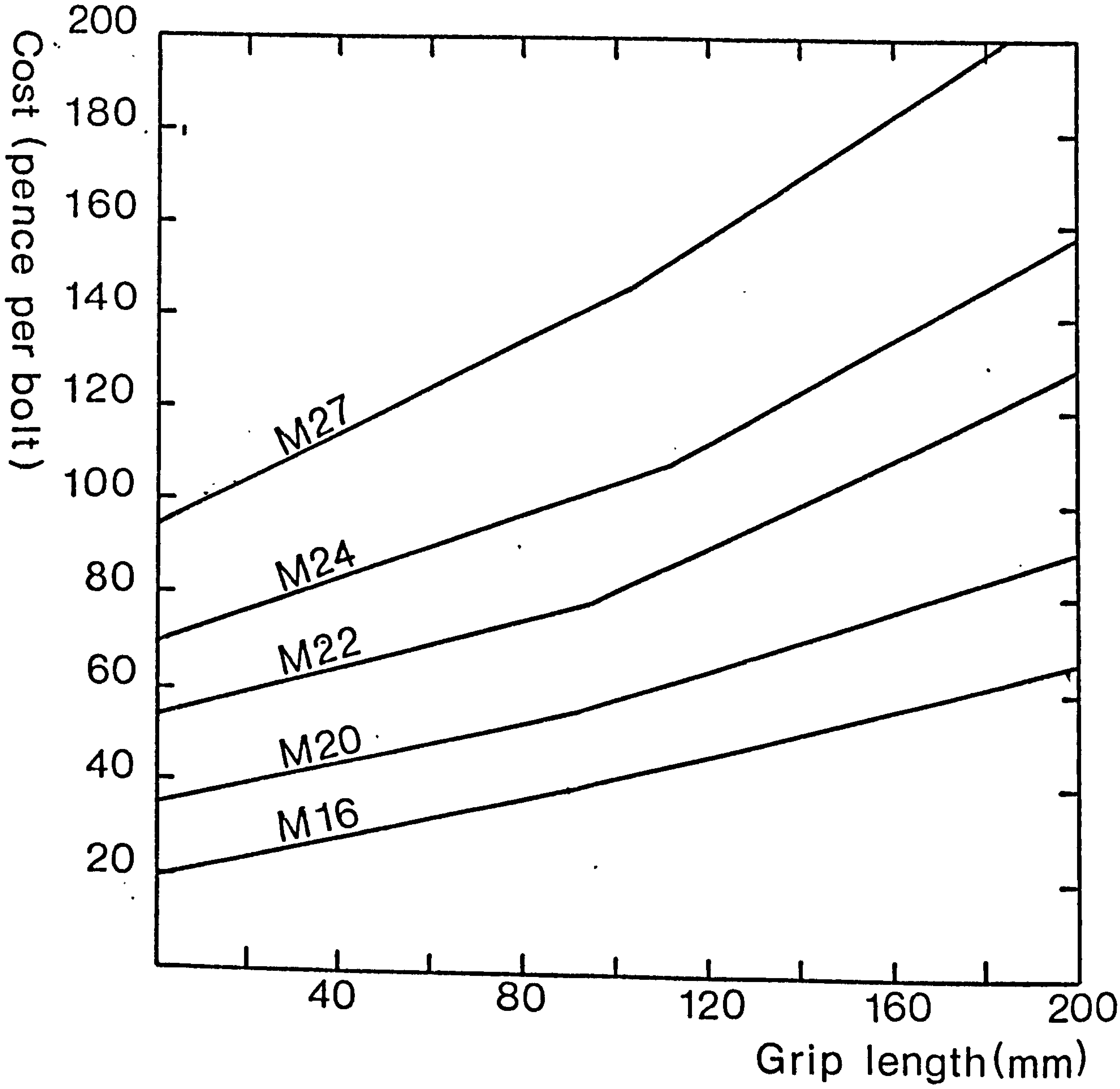


FIG. 5.3 BOLT COSTS

It has been assumed that bolts are available in increments of 10mm in length and that they are bought in the quantities which incur a 75% excess charge for quantity. These quantities vary for each bolt diameter and whether the bolt is a stock item or not. The above model may not predict exactly the true cost to the fabricator of bolts, however it is thought to give a reasonable estimation of the cost involved. A summary of the cost relationships used and quantities assumed are given in Table 5.5. The practice of the fabricators which were approached was to order the bolts for a given job and so small quantity extra costs were very variable.

Bolt Diameter	Cost (£/100 bolts) = minimum of $\begin{cases} m_1 \times \text{grip} + c_1 \\ m_2 \times \text{grip} + c_2 \end{cases}$				Quantity Ordered for stock items	Quantity ordered for non stock items
	m_1	c_1	m_2	c_2		
16mm	0.213	20.05	0.241	17.73	120-239	150-749
20mm	0.239	34.78	0.313	26.96	50-99	50-249
22mm	0.258	54.74	0.482	32.51	40-79	50-249
24mm	0.351	69.16	0.555	46.01	30-59	30-99
27mm	0.500	95.11	0.663	77.65	15-29	30-99

Table 5.5 - Bolt Costs

5.6 FABRICATION COSTS

5.6.1 General Comments

Fabrication consists of changing the raw material into the completed steel frame. The method adopted in this cost model for estimating fabrication costs is to estimate the time taken for the operatives to perform the various fabrication operations. The usual method of finding these times is to use the techniques of work study. This method of estimating has an advantage in that many companies also wish to set up bonus incentive schemes. The resulting times can be used both for estimating and for bonus payment.

The work study data is usually presented in the form of standard minute values for each operation or each part of an operation. A standard minute is the number of "real" minutes taken to perform a given operation, multiplied by a factor which provides an allowance for fatigue, rest allowances etc. For the type of work being considered, which requires neither working in cramped conditions nor working with intricate parts, an allowance of 25% for fatigue, rest allowance etc is usually given. A further allowance of 5% is applied for contingencies. The standard time for a process is then summed and multiplied by the labour rate for the particular process.

The labour rate is the cost of employing the operative who performs the operation. This cost is greater than the cost of wages paid to the operative because such items as national insurance, pension contributions and other related costs may be included. It is usual to use a labour rate which includes a factor to take account of plant overheads. Plant overheads represent the cost of running the fabrication shop, design offices, estimating department, depreciation, capital investments and other related costs. The application of the overhead factor varies between companies in accordance with the accountancy practice of the company concerned, the overhead factor represents a measure of the efficiency of the company. For this reason the overhead factor is confidential to the company concerned.

Another system of applying overheads is to use the labour rate not modified for overheads, and to add the overheads on a tonnage basis. A combination of the two systems is also feasible.

The first system is the one used within the cost model. This system is justified by the following argument. Consider a frame of weight w tonnes, if little or no fabrication is required then the amount of fabrication shop capacity used is very small and so only a small overhead cost should be applied. Alternatively, for the same frame, if a large amount of fabrication is performed then the fabrication shop capacity used will be greater and so the overhead cost applied to the job should be greater. With the second system the same amount of overhead cost would be applied because the tonnage is the same in both cases. The amount of time expended on the fabrication is proportioned to the amount of capacity used and therefore a factor based on the time taken is realistic. In the cost model a value of three is used - 23
for the overhead factor on labour. The overhead factors quoted as reasonable by the companies approached varied between two and four times the labour rate. Some of these companies used tonnage overhead rates for certain overheads in addition to overhead factors.

5.6.2 Source of Data

A large number of steel fabrication companies in the North of England were approached for information on fabrication costs, of these all but five were unwilling to help. The five companies which cooperated varied in size and were concerned with a large variety of type of fabrication. A summary of the information given by each company is given in Table 5.6. The resulting data will now be described in the following order:-

(a) Marking out
(b) Welding
(c) Machine flame cutting
(d) Hand flame cutting
(e) Cold sawing
(f) Drilling
(g) Handling within a works
(h) Painting and Shot blasting.

Each operation will be described, the data collected will be described and then the rationalization of the data for use in the cost model is described.

5.6.3 Marking Out

At the beginning of the fabrication process it is necessary to mark out the raw plates and sections, in order to show the other operatives where holes should be drilled, and where plates should be cut. The usual method of marking out a hole is to use a centre punch to mark the centre of the hole to be drilled. For cutting the method used is to mark the line with a scriber. Various other markings are also required for identification purposes. Due to the large variability in, the complexity of the fabrication required, the thought required to interpret drawings and the skill required for marking out, it is difficult to derive times for marking out in terms of quantities of items marked out. Therefore any standard time derived for marking out will be very approximate. It is necessary to have two operatives working on marking out at one time. In the past where an intricate item was to be marked out a template was produced for use in the works, this process has virtually disappeared because of the large amount of labour required when making templates.

One set of data was collected. This information was

	Company A	Company B	Company C	Company D	Company E
SIZE	Large	Medium	Medium	Medium	Medium
Type of work	All types of fabrication including Multi-Storey frames	Heavy fabrication North Sea Oil Work	Light industrial Buildings	All types of light fabrication including occasional Multi-Storey frames	Petro Chemical Industrial Fabrication
Bonus Scheme Currently in use	No	Yes (except for welding)	No	No	Yes
Times derived from a bonus Scheme	Yes	Yes	No	Yes	Yes
Times used for estimating	For small jobs with large labour content only	Yes	Yes	For small jobs	Yes
Overhead System	3 to 4 times labour rate	2 times labour rate + tonnage rate	Not available	3 times labour rate + tonnage rate	Not available

Table 5.6 - Part I

	Company A	Company B	Company C	Company D	Company E
SIZE	Large	Medium	Medium	Medium	Medium
Marking Out	Yes	No	No	No	No
Fillet Welding	Yes	Yes	Yes	Yes	No
Machine Flame Cutting	Yes	Yes	Yes	Yes	Yes
Hand Flame Cutting	Yes	Yes	Yes	Yes	Yes
Cold Sawing	Yes	No	Yes	Yes	Yes
Drilling	Yes	Yes	Yes	No	Yes
Handling	Yes	No	Yes	No	No
Painting	Yes	Not applicable	No	No	Not applicable

Table 5.6 - Part II - Summary of Data

provided by company "A". Data for marking out was not available in any other company, in these cases an allowance was included in the drilling and cutting times. The data collected consisted of:-

(a) for placing plate or section on bench and referring to drawings. 0.50 minutes per component per operative

(b) for marking out a hole for drilling 0.50 minutes per hole per operative

(c) for marking out a line to be cut 0.33 minutes per metre per operative.

The above values must be multiplied by the number of operatives engaged on marking out. Clearly the time for placing the plate can be consolidated within handling costs. Marking out holes for drilling can be included in the set up time for drilling. Marking out for cutting can be included in the times per metre for cutting. Thus it is not necessary to include marking out as a separate item. The times given above were consolidated within the handling, drilling and cutting costs.

CORKER[6] has examined the operational elements involved in marking out plated structures. The times given are similar to those collected but are given in greater detail and so it is therefore not possible to make an exact comparison between Corker's data and that obtained from company "A". Corker gives factors by which to multiply the total fabrication time to give an approximate time for marking out. This is justified by the fact that the amount of marking is proportional to the amount of fabrication required. He suggests a factor of 8 percent for the type of work being considered.

5.6.4 Hand Fillet Welding

Fillet welding is used to make right angle connections between steel components, and for the applications being considered it is usual to use electric arc welding.

Refinements to the arc welding process and other methods of welding are more time consuming and thus more costly, this tends to outweigh the advantages of using these methods. Preparation for welding consists of clamping (tightly together) the components to be joined, to avoid waste of weld metal, and then the junction is wire brushed. The process consists of striking a low voltage and high current electric arc between the metal to be joined and an electrode. The electrode metal melts and so effects the join, in order to minimise oxidation the electrode is coated with a chemical which vapourizes when hot and forms a shield of gas around the molten metal. Weld metal cannot be deposited in runs with a size greater than 12mm thickness, and so it is necessary in order to construct larger welds to deposit multiple runs (usually of 6mm thickness). After laying each run any slag, which forms when impurities melt, must be removed. Welds are deposited in the downhand position (with the welder looking down on the weld) if possible because it is more costly and dangerous to work in other positions. For the type of work being considered it is common practice and better to manipulate the components into the downhand position.

The speed of fabrication is related primarily to the amount of weld which can be deposited by the electrode with the power set, therefore the times needed to deposit welds can be expected to be related in some way to their volume. Times for depositing small welds are not proportional to the volume of the weld because only one run is needed and the error in the depositing of the weld is large in proportion to the thickness of the weld.

Four sets of times were collected for the complete downhand welding process, including allowances for preparation, electrode size, clamping, changing electrodes and deslagging. 33
Two sets of the times collected were incomplete. In order to investigate the relationship between weld size and labour content, the data for company "A" was taken as the norm, and

a scalar multiplier was found for each of the other sets of data which made each set fit the data of company "A", using a least squares fit. This process allows all the data to be plotted to the same scale on a single graph. The differences in the scalar multipliers represent the differences in the fabrication and accounting process between firms. A quadratic function has been fitted to the data and this function has been incorporated within the cost model. The function for fillet welding time is:-

$$T = K\ (0.186 \times leg^2 + 0.072 \times leg + 4.47)$$

where:-

T = standard time in minutes to lay one metre of weld
K = company multiplication factor given in figure 5.4
leg = fillet weld leg length in millimetres.

This function and the data collected are shown plotted in figure 5.4 to a uniform scale.

DONNELLY[15] gives relative times for fillet welding and these are plotted to the same scale as figure 5.4 in figure 5.5. Examination of figure 5.5 shows that the data collected and Donnelly's data are very similar throughout the range of weld sizes.

The other major component of fillet welding cost is the electrode material cost. This cost is highly dependant on the quantity, size and quality of electrodes employed. Company "A" and company "C" usually buy electrodes in quantities which result in a cost of £0.10 (allowing 20% for waste) for one metre of six millimetre weld. Material costs for other weld sizes can be determined in proportion to the volume of the weld assuming the same size of electrode is always used. If process overheads are included, electrode material costs represent between ten and twenty percent of the labour costs.

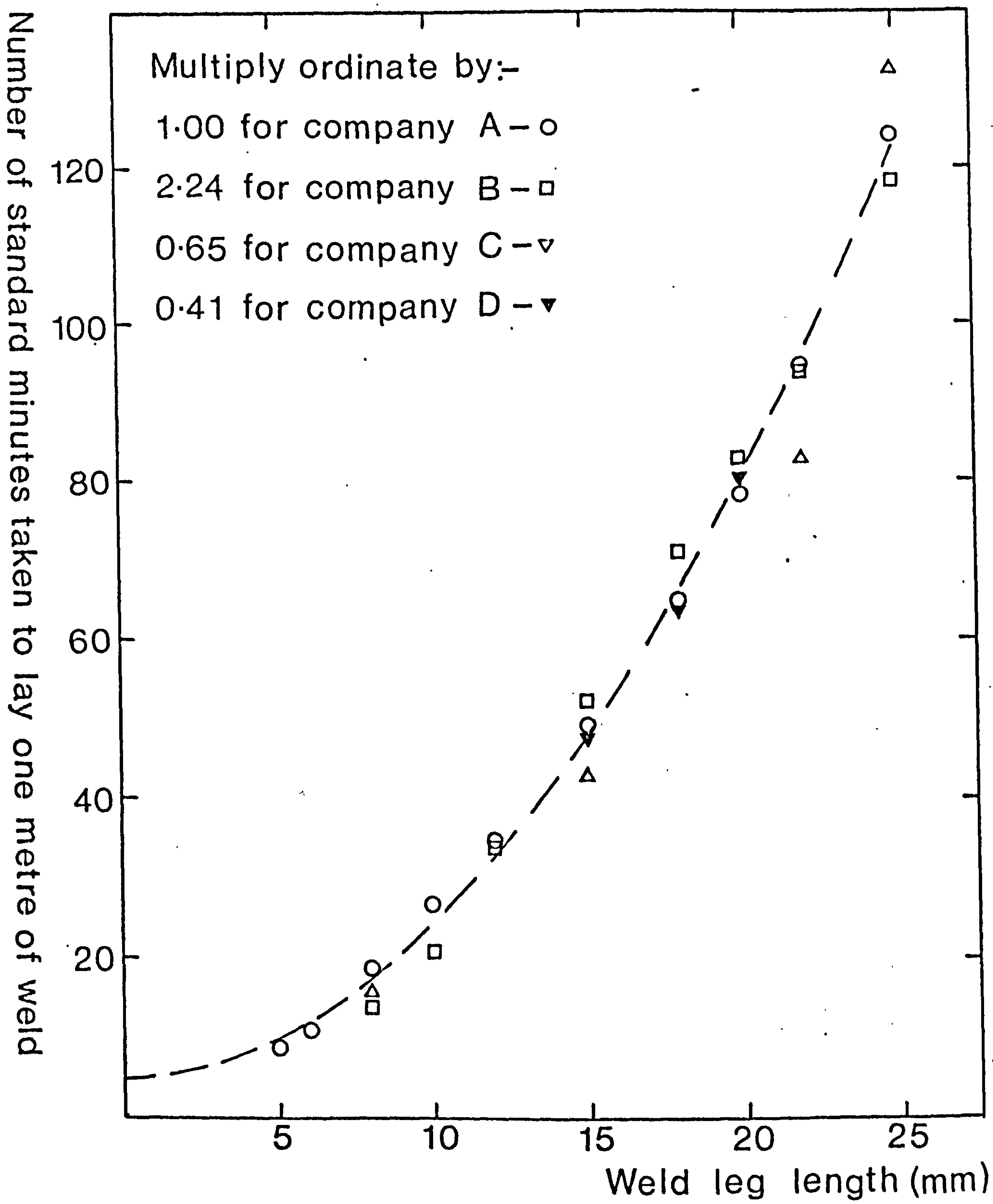


FIG. 5.4 TIME v FILLET WELD SIZE

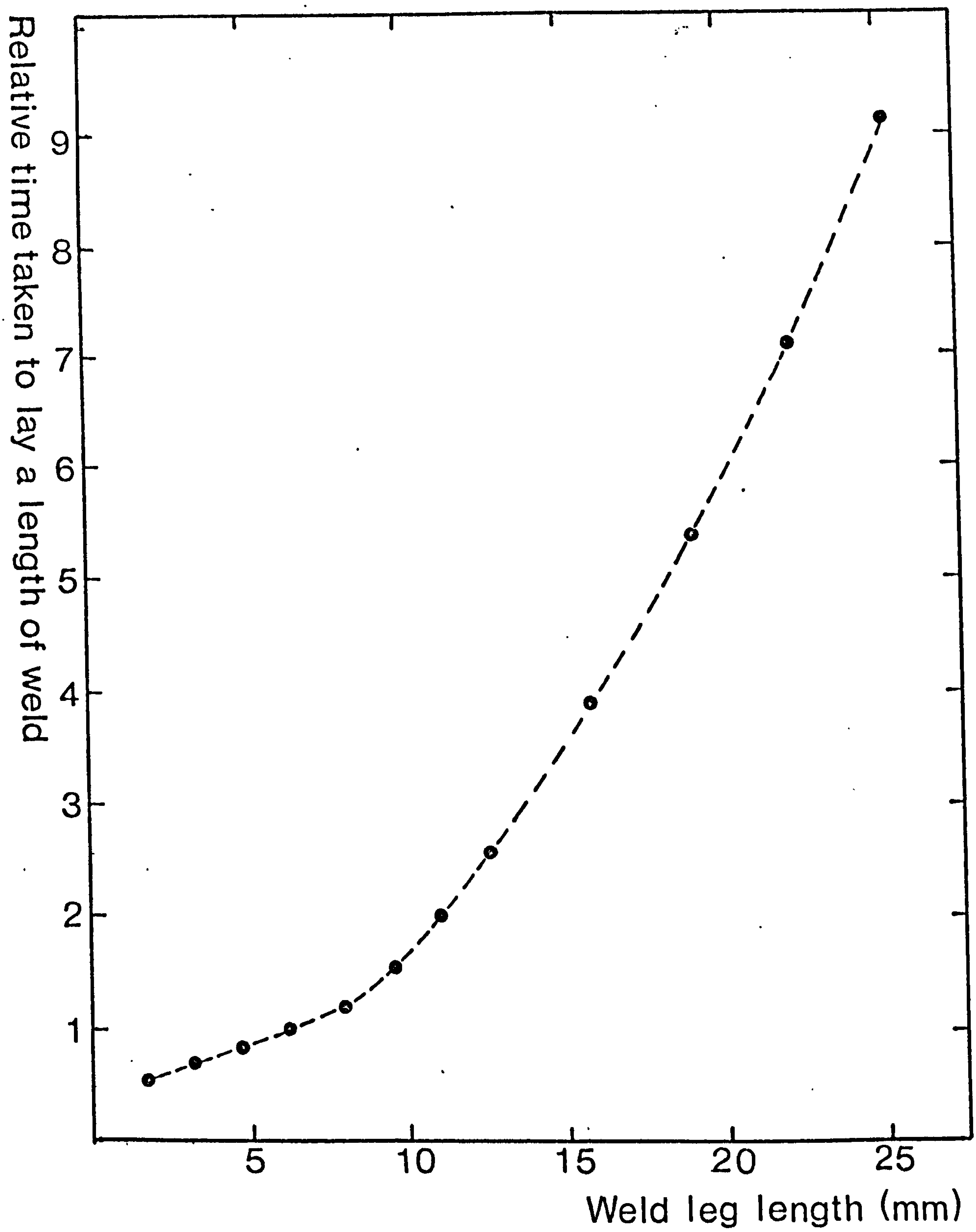


FIG. 5.5 RELATIVE TIME v FILLET WELD SIZE FROM DONNELLY-1968

5.6.5 Hand Flame Cutting

The process of hand flame cutting is used to cut sections and plates which require an intricate cut, or which are small or which do not require a high degree of accuracy in the cutting process. There are a number of techniques which can be used, including oxy-acetelyne and oxy-propane cutting. It is usual to use this method for plates thicker than 10mm. Depending on the practice of the particular fabricator cropping machines may be used for thicknesses less than 10mm. The process consists of using a high temperature flame to preheat the plate to be cut, to above 900^{o}C, then oxygen is passed under pressure through the centre of the flame in order to oxydize the metal under the torch. The amount of time taken depends on the design of the torch, the size of the torch and the thickness of the plate.

Five sets of times were collected for the complete hand flame cutting process including allowances for positioning the components, positioning the torch and preheating the plate. Two of these sets of times were incomplete. In order to investigate the relationship between plate thickness and labour content, the data for company "A" was used as the norm and the data was processed in the same manner as for hand fillet welding. A linear function was fitted to the data and the resulting times were used for costing. The function derived for hand flame cutting is:-

$$T = K\ (0.296 \times \text{plate thickness} + 7.9313)$$

where:-

T = Standard time in minutes to cut one metre of plate
K = company multiplication factor given in figure 5.6.

Examination of figure 5.6 shows that the relationship between labour and plate thickness varies between fabricators and is non-linear. The non-linearity is due to factors others than the plate thickness which affect the labour content such as, the type of equipment being used, the variations between operatives and the differing times taken

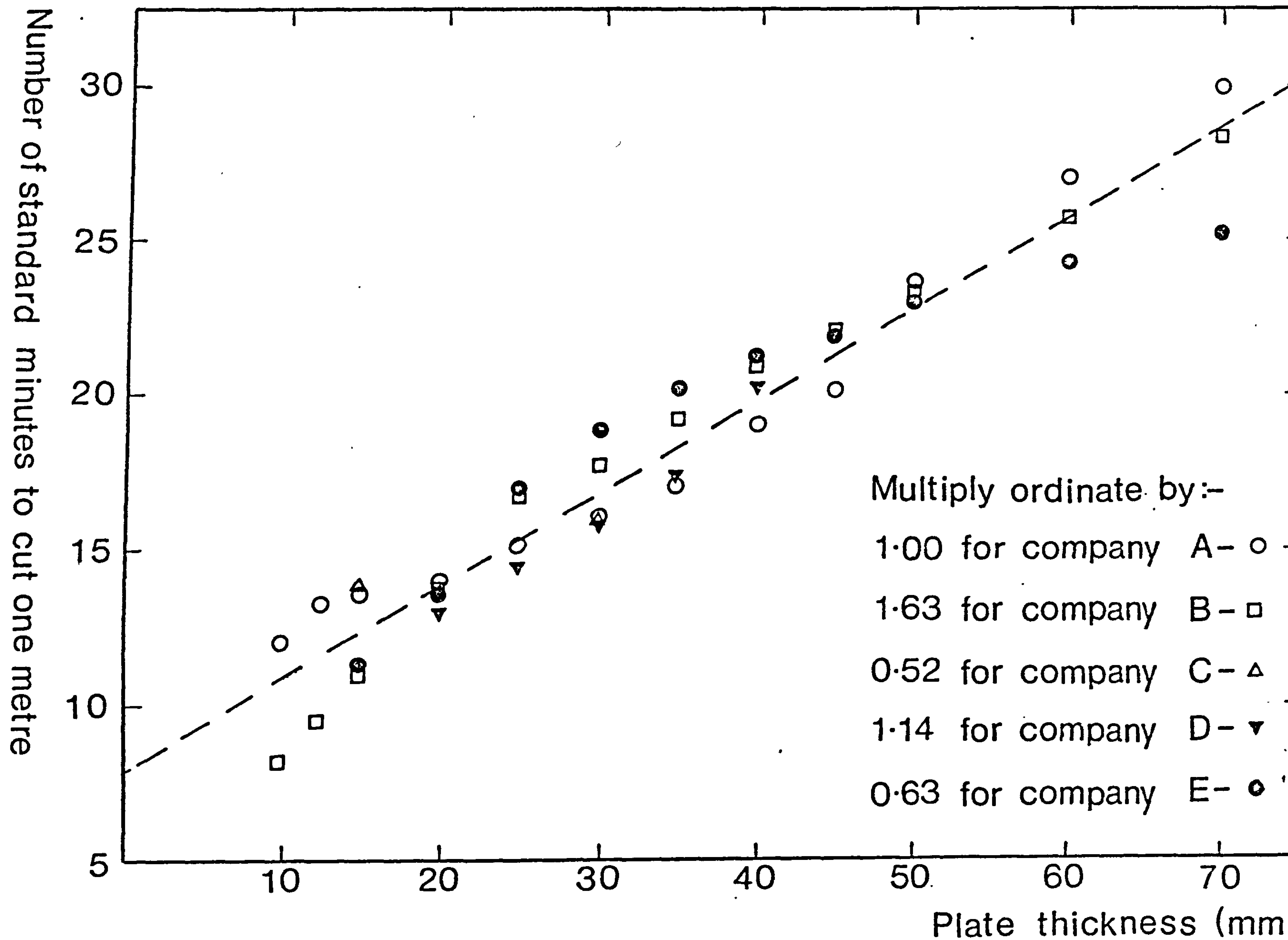


FIG. 5.6 HAND FLAME CUTTING TIME v PLATE THICKNESS

to preheat the plate. The data for company "A" is substantially linear, thus the time taken to cut the plate is proportional to the thickness of the plate. The data for company "B" and company "E" is convex, thus for these companies it is less costly per unit of cut to cut a thick plate than a thin plate. A linear approximation to the data collected can be seen to give reasonable agreement to all the sets of data.

CORKER[10] examines the operational elements in hand flame cutting but does not seek to examine the relationship of labour content to plate thickness, the example given involves the cutting of angles and includes a disproportionate amount of handling and therefore cannot be compared with the above results.

5.6.6 Machine Flame Cutting

Machine flame cutting is used to cut plates where either a very long cut is required or the part is to be cut from a large plate. A large variety of equipment is available for machine flame cutting, this ranges from simple single nozzle track mounted cutters to sophisticated profile burning machines with optical or magnetic control. This section deals with the type of equipment used by most structural steelwork fabricators to cut components from large plates. The type of equipment used includes a bed, usually about half a metre above ground level, on which the plate to be cut is laid; a carriage carrying the nozzles spans the bed and is free to move on tracks which run the length of the bed. This equipment is shown diagramatically in figure 5.7.

The process consits of accurately setting up, on the bed, 30
the plate to be cut, preheating the plate along the line of the cut, and finally cutting through the plate in the same manner as for hand flame cutting. The speed of cutting is sometimes regulated mechanically but often the operator moves the carriage manually. This process requires

two operators, one skilled and one unskilled.

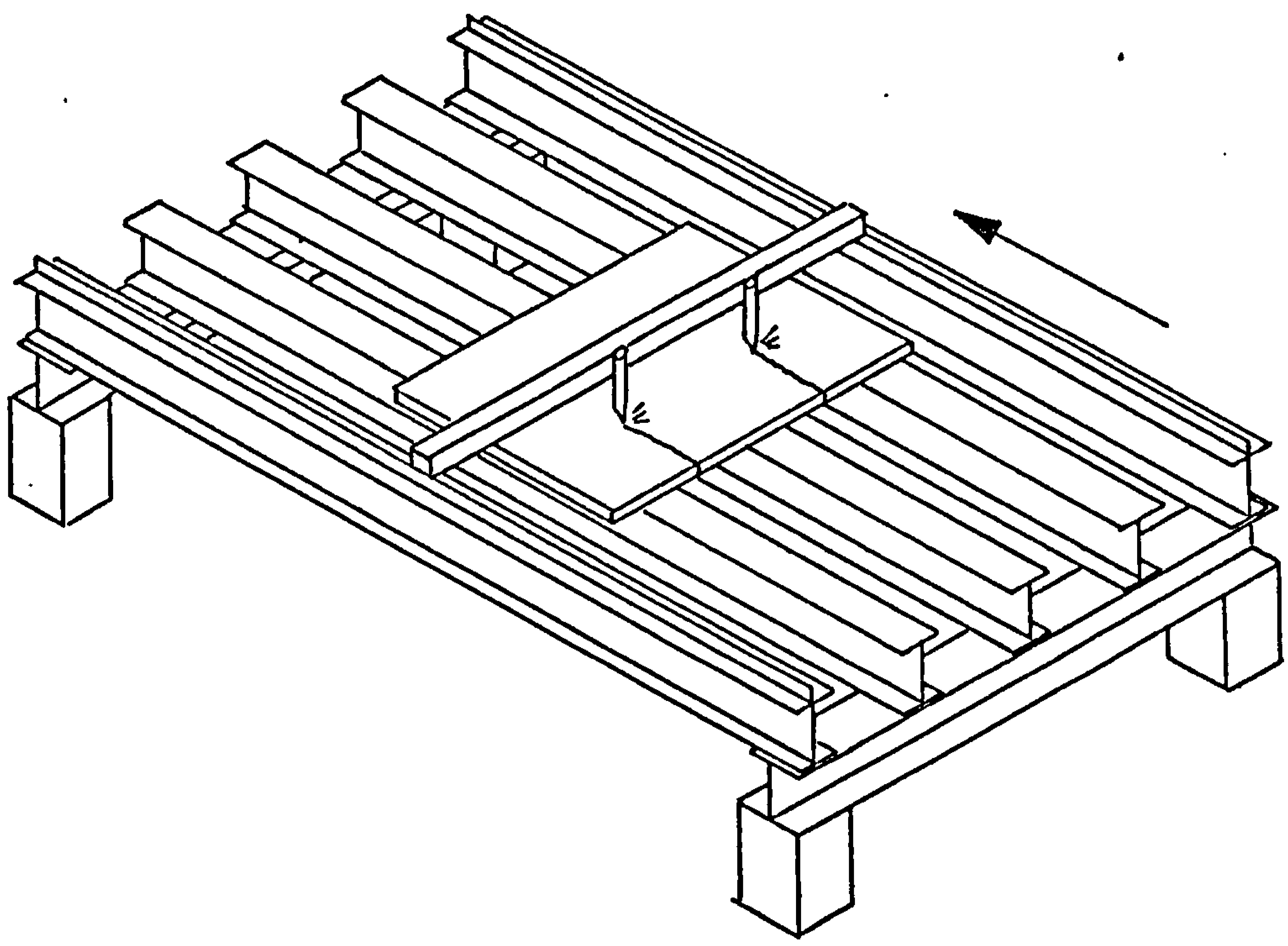


Figure 5.7 - Machine Flame Cutting

When it is necessary to be very accurate it is necessary to heat the sides of the plate parallel to the cut, in order to minimise warping. Multiple thicknesses of plate may be cut providing that no plate is thicker than 18mm, because the plates require clamping together, the setting up and cutting time is longer than for a single plate, but a reduction in the amount of labour required per plate is achieved. Multiple parallel cuts may be made, by using multiple burners on the carriage, with little increase in setting up time and no increase in cutting time. Portable plate cutting equipment is also often used, in this case the nozzle moves along a single track which is set close and parallel to the line of the cut. The amount of labour required in both cases is similar.

The labour content of machine flame cutting is dependant on a large number of factors including, the thickness of the plate, the type of equipment used by the fabricator and the experience of the operator. The development of standard times for the labour content is highly dependant on the practice and equipment of the individual fabricator. This is clearly shown by examining the four sets of times collected. Two of these sets of times were incomplete. Company "A" was again taken as the norm and the data was processed in the same way as for hand fillet welding. The resulting relationship between labour content and plate thickness is shown in figure 5.8. A linear function was fitted to this data and the function derived was:-

$$T = K\ (0.122 \times \text{plate thickness} + 6.51)$$

where:-

T = standard time in minutes to cut one metre of plate
K = company multiplication factor given in figure 5.7.

The times include allowances for positioning the component, positioning the nozzles and preheating the plate. The times are for a single cut in a single plate.

Examination of the data for all the companies shows that the relationship between plate thickness and standard time is substantially linear, though the set up time and cutting time vary between the different companies. A linear approximation to the data can be seen to give reasonable agreement and thus a suitable basis for standardization of the labour requirement.

CORKER[11] gives a range of total burning times for plates, the values are given as a range which are shown in figure 5.9. Corker's range corresponds reasonably well with the range of the data collected from the four companies.

A comparison of hand flame cutting data and machine flame cutting data shows that the set up time is very similar but the cutting time for machine cutting is approximately half that for hand flame cutting. Therefore it always requires less labour to use machine flame cutting and this advantage is more pronounced for the thicker plates. The company

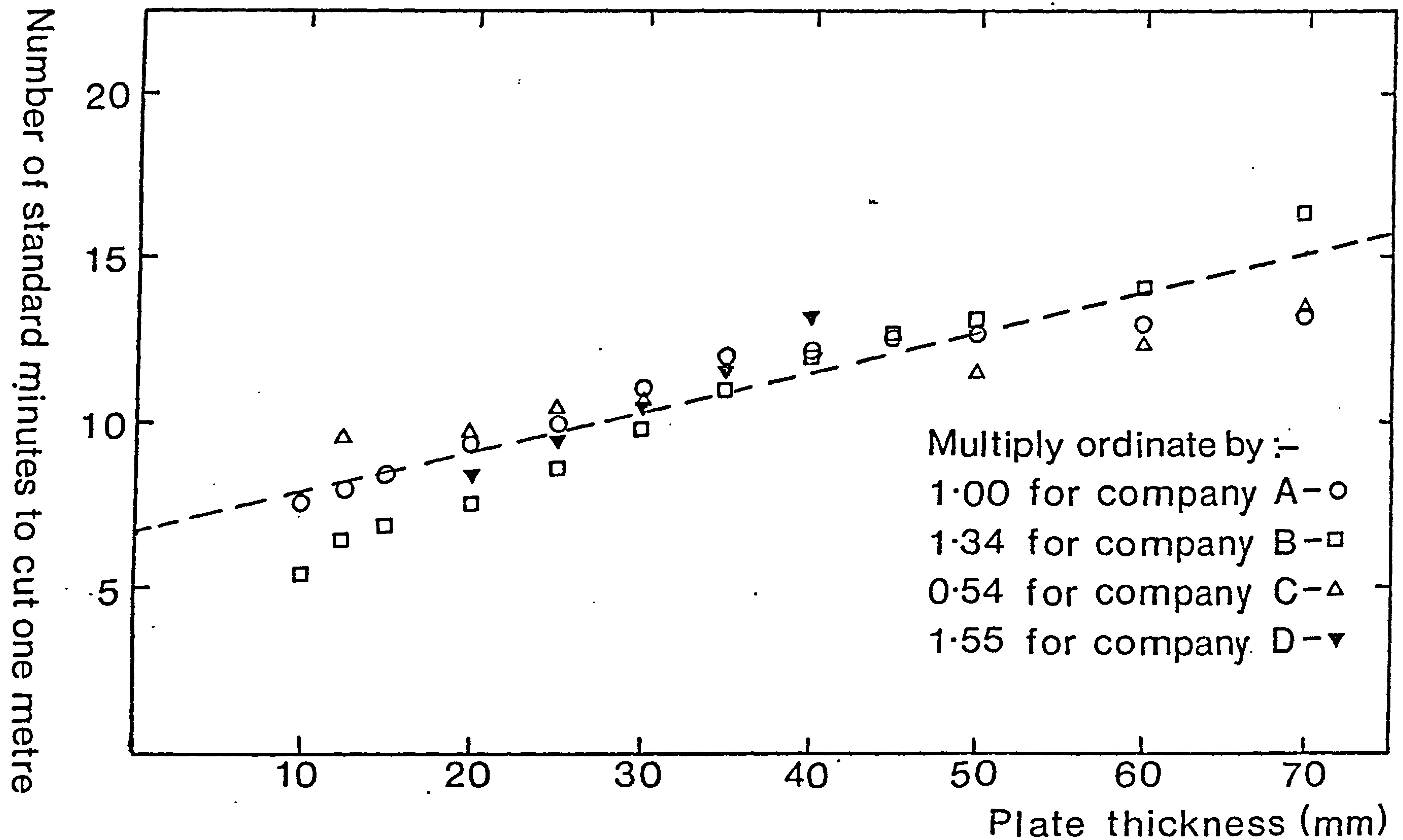


FIG. 5.8 MACHINE FLAME CUTTING TIME v PLATE THICKNESS

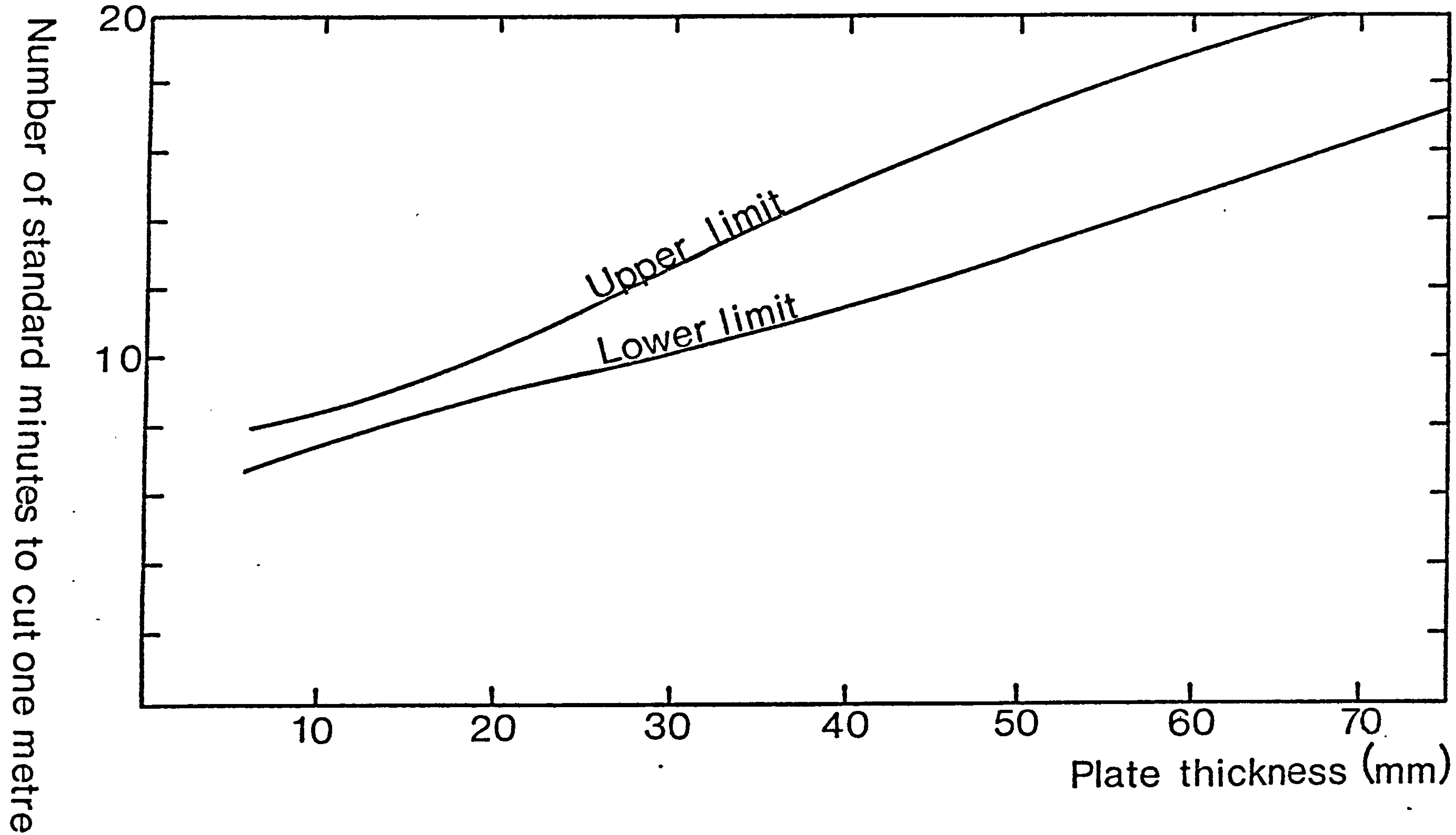


FIG. 5.9 MACHINE FLAME CUTTING DATA (AFTER CORKER)

multiplication factors can also be seen to be very similar for each of the companies.

5.6.7 Cold Sawing

A cold saw is used to cut sections such as universal beams, columns and joists. It is chosen in preference to flame cutting because the cut produced is of a high quality. The component is set up on a bed and the points to be cut are marked. The saw is then placed on the point to be cut and the cut is made. The saw used is a circular saw which may cut at any angle.

The main factors affecting the amount of time taken for sawing are:-

(a) The setting up time.
(b) The geometric dimensions of the section to be cut.
(c) The type of machine used.

The setting up time depends on the type of transport available within the works and the amount of automation of the setting up process. The geometric dimensions which affect the time taken are dependant on the physical dimensions of the cutting blade and the direction in which the cut is made, i.e. across the section or down the section or diagonally. The type of machine in use affects the speed of cutting. The thickness of the section may also affect the cutting speed.

Comparison of labour content is difficult because of the above factors. Four sets of data were collected, three of which were incomplete. The approach chosen to rationalize the data was to try to relate cutting times with a major dimension of the section. These investigations are summarised in figures 5.10, 5.11, 5.12 and 5.13. The standard times collected were plotted against the section breadth, section depth and section weight for each company.

Company "A" provided times for sawing universal beams only,

FIG.5.10 COLD SAWING TIME V SECTION DIMENSIONS FOR COMPANY A

FIG. 5.11 COLD SAWING TIME V SECTION DIMENSIONS FOR COMPANY B

FIG.5.12 COLD SAWING TIME V SECTION DIMENSIONS FOR COMPANY C

FIG. 5.13 COLD SAWING TIME V SECTION DIMENSIONS FOR COMPANY D

these times are shown plotted in figure 5.10. The times can be seen to have a loose relationship to section breadth, no relationship to section depth and a loose relationship to section weight. Company "B" provided times for sawing all types of sections, these times are shown plotted in figure 5.11. The times can be seen to be unrelated to section breadth, loosely related to section depth and unrelated to section weight. Company "C" provided times for sawing all types of sections, these times are shown plotted in figure 5.12. On first examination the times can be seen to be unrelated either to section breadth, section depth or section weight. However closer examination shows that the relationship between time and section breadth and time and section depth consists of two "branches" each of which is substantially linear. One branch consists of times relating to universal beam sections, the other branch consists of times relating to universal column or joist sections. The range between the section with the lowest time and the section with the highest time can be seen to be the narrowest for company "C" indicating the use of a substantially different type of sawing machine to the other companies. Company "D" provided times for sawing all types of sections, these times are shown plotted in figure 5.13. These times can be seen to be loosely related to section breadth, unrelated to section depth and very loosely related to section weight.

It was not found to be possible to correlate the relationships for each fabricator as with other fabrication operations. Therefore a sample was chosen for use in the cost model. The set chosen was that provided by company "D". This set was chosen because firstly the times were a complete set, secondly the times were consistently related to section breadth and finally the times are similar to those for company "A", which was used as "norm" for correlation for the other fabrication processes.

CORKER[7] has examined the cost items in cold sawing. Corker shows that the cost is primarily a function of handling and marking. The value of the cutting speed was not dealt with because it is a variable dependant on the cold sawing machine used. The effects of "mass production", such as cutting more than one section at a time, are shown to be significant because there is a net reduction in handling and setting up times.

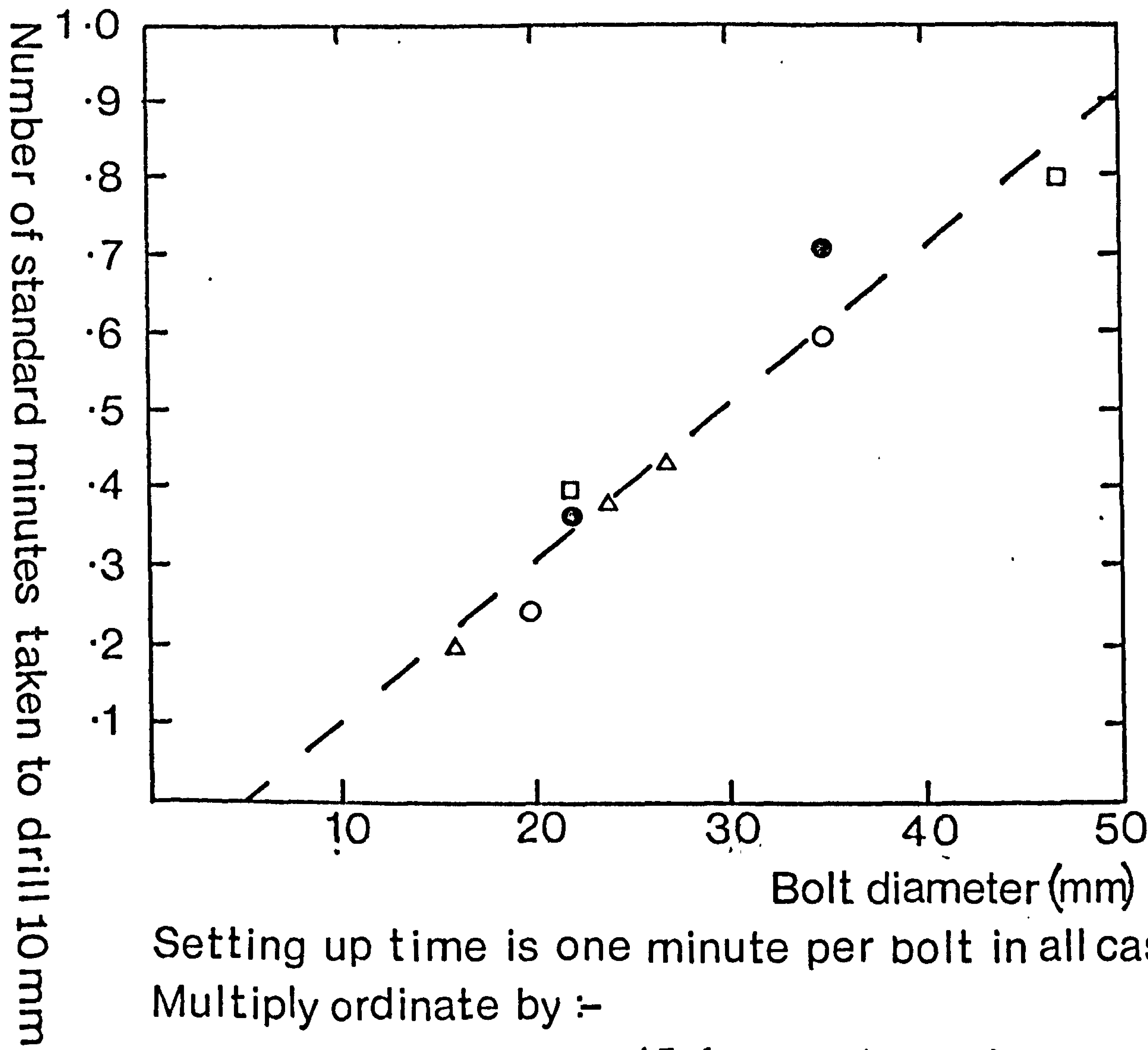
5.6.8 Drilling

Drilling is used to form holes in plates and sections for the insertion of site bolts. The process consists of positioning and clamping the component to be drilled on a bench, and then drilling holes at points which have been previously centre punched, using a variable position drill. Burrs caused by drilling are then removed. The component must be positioned under the drilling machine such that the drill may reach all the hole positions in the bolt group. The operations which may be timed consist of:-

(a) Positioning and clamping the component for each bolt group.
(b) Moving the drill to each hole.
(c) Drilling the hole.
(d) Removing the burrs on each hole.
(e) Sharpening drilling bits.

Clearly the time taken to position and clamp the component depends on the size and type of component to be drilled. The time taken to actually drill the hole is related to the diameter of the hole and the thickness through which the hole has to be drilled.

Four sets of data were collected, none of which was complete. All the data has been rationalized into a setting up time for each hole to be drilled and a drilling time based on the thickness of the plate. The setting up time for company "A" and company "C" included an allowance for positioning and clamping the component,



Setting up time is one minute per bolt in all cases
Multiply ordinate by :-

·45	for company	A	– ○
1·00	" "	B	– □
1·00	" "	C	– △
·35	" "	E	– ●

FIG. 5.14 DRILLING TIME V BOLT DIAMETER

irrespective of the size and type of component. Company "B" and company "E" had a time for positioning and clamping the component, and an individual time for each hole to be drilled. For these two companies the positioning and clamping time has been added to the time for each hole, based on a bolt group of six holes. The drilling time taken for each millimetre of thickness drilled through is dependant on the diameter of the hole, however the setting up time is the same for all hole diameters. The drilling times have been rationalized by multiplying each set of data by a scalar multiplier which made the setting up times for each hole equal to 1 minute. The times per millimetre of thickness drilled through have been plotted against the bolt diameter and the resulting graph is shown in figure 5.14. A linear function fits this data with a reasonable degree of accuracy. Therefore the time taken to drill a hole may be determined from:-

$$T = K\ ((0.02d - 0.10) \times {}^{x}/_{10} + 1.0)$$

where:-

T = number of standard minutes required to drill one hole

d = diameter of the bolt in millimetres

x = thickness to be drilled through in millimetres

K = company multiplication factor given in figure 5.14.

Previous investigators, KNAPTON[18] and BRINKERHOFF[1], have used a cost per bolt only. This approach does not however provide a satisfactory model for drilling costs when the depth of the hole varies considerably such as in column splices.

5.6.9 Handling Within a Works

In order to perform fabrication operations on steelwork it is necessary to move partially fabricated components around the works from process to process. The alternative would be to keep each partially fabricated element in one location and bring the tools to it, however this would prove impractical. Two methods of handling are used commonly. One method involves the use of overhead cranes and the second method involves the use of trolleys.

Movement of components by overhead crane tends to be costly. The process consists of:-

(a) Moving the crane to the component.
(b) Clamping or slinging the component.
(c) Lifting the component.
(d) Moving the crane to the next process.
(e) Setting down the component.

This process normally requires two operatives, one to drive the crane and one to perform the slinging and setting down.

Movement of components by trolley consists of:-

(a) Lifting the component manually onto a trolley.
(b) Moving the trolley to the next process.
(c) Manually lifting the component onto the bench.

Components which weight up to 150 kilogrammes can be moved in this way by two operatives.

One set of data was provided by company "C". Company "A" had very approximate data available, related only to the weight of the component and not the distance travelled. Therefore this was only applicable to their own works and is henceforth omitted. The data collected from company "C" consists of:-

(a) Manual handling (component less than 150kgs).

Lift component from bench to trolley	0.6 minutes/operative
Move trolley 10 metres and return	0.94minutes/operative
Lift component from trolley to bench	0.6 minutes/operative

(b) Overhead crane (component greater than 150kgs)

Clamp, sling and lift component from bench
2.15minutes/operative

Move crane with component 10 metres and return
2.81minutes/operative

Lower component to bench and unclamp 2.15minutes/operative

The application of these times within the cost model is dependant on a number of assumptions:-

(a) The movement is unimpaired, if this is not generally the case a factor must be included to take account of this.
(b) The craneage does not vary and is always available implying a long thin fabrication shop with a surplus of craneage. Again a factor may be included to take account of this.
(c) Errors in the times given will tend to cancel out by using average aggregated times for each component.
(d) The crane or trolley will be nearby whenever it is needed, again a factor can be included to take account of the average calling distance.

A method is used within the cost model which rationalizes the amount of handling data to be used and reduces the data to a total of five figures. As an example of this rationalization, consider the fabrication of a major axis beam with end plates. Firstly it is assumed that any two similar components will follow exactly the same route through the works. The handling processes involved for the end plate up to the time it is subassembled with the beam are as follows:-

(a) Lift full plate from lorry and transport to stockyard.

    time = a minutes

(b) Lift full plate from stockyard and transport to marking out bench.

    time = b minutes

(c) Lift full plate from marking out bench and transport to cutting bench.

    time = c minutes

(d) Lift part plate from cutting bench and transport to drilling bench.

    time = d minutes

(e) Lift part plate from drilling bench and transport to subassembly area.

time = e minutes

If the area of the plate is r square metres and the standard full size of plate used is 20 square metres. The total time taken for handling this endplate is:-

$$\frac{r}{20} \times (a + b + c) + (d + e)$$

Therefore it is only necessary to know the total time taken to move the plate as a whole plate (a + b + c) and the time taken to move the plate as a part plate (d + e). This operation is illustrated in figure 5.15, which shows the processes and times for all the types of components which may be used in multi-storey frames. Sections and whole plates are assumed to be moved by overhead crane and part plates are assumed to be moved manually. For the purposes of this study the distance between processes have been standardized as 15 metres each. This distance represents the conditions existing within a typical fabrication shop. The total times which have been used are shown in figure 5.15.

When handling times for full size plates are divided into the times for individual plates, by scaling down in proportion to the area of the plates, it is found that the resulting handling time per plate is insignificant. Examination of typical details produced by the design programme shows that almost all plate components weigh less than 150kg.

Previous investigators' references have omitted handling costs even though they form a substantial part of the fabrication cost.

5.6.10 Surface Treatment

The processes included within surface treatment are shotblasting, wire brushing and painting. Surface treatment

PROCESSES

1) Unloading

2) Stockholding

3) Marking

4) Flame Cutting

5) Sawing

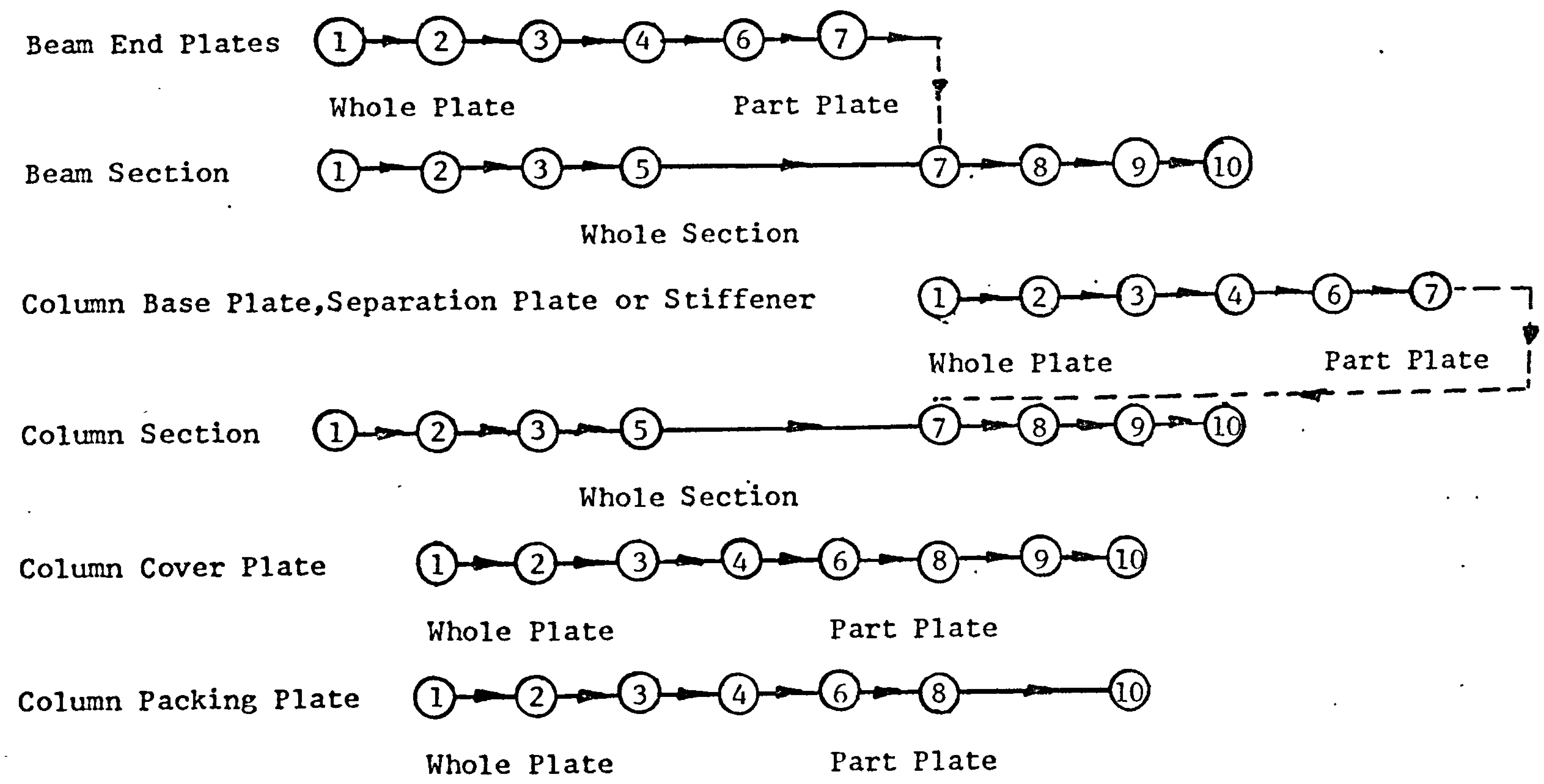
6) Drilling

7) Welding & Clamping

8) Shot Blasting

9) Painting

10) Loading



AGREGATED TIMES USED

1 → 2 → 3 → 4	= 25.92 Minutes as a whole plate
4 → 6 → 7	= 5.22 Minutes as a part plate
1 → 2 → 3 → 5 → 7 → 8 → 9 → 10	= 60.36 Minutes as a whole section
4 → 6 → 8 → 9 → 10	= 10.44 minutes as a pert plate
4 → 6 → 8 → 10	= 7.83 Minutes as a part plate

NOTE:- Whole plates &sections are moved by crane,Part plates are moved manually

FIG. 5.15 HANDLING TIMES FOR VARIOUS COMPONENTS represents a substantial part of the cost of a steel framed building and also they are subject to the widest variation in cost of any fabrication process.

The process of shot blasting may be performed on the raw material as it enters the works or after fabrication and just before painting. The method used is to mount a number of components on a trolley which is placed in a shot blasting tunnel. Lead shot is fired at the components until sufficient mill scale and rust is removed. A less satisfactory alternative is to wire brush the components (usually after fabrication). The components are then ready for fabrication and subsequent painting. The variation between the practice of different fabricators and the fact that subcontractors are often used for this type of work leads to the use of a cost based on surface area of the components. Clearly this approach is not fully representative of the labour cost required but it gives the best available estimate of the cost.

The process of painting consists of brushing the component to clean it and then spray painting with a number of coats, the first coat of which is a priming coat. This process is also estimated on the basis of the surface area to be covered and on the number of coats required. This process may also be undertaken by subcontractors.

Typical costs for three types of surface treatment were collected from company "A". These costs were:-

(a) Wire brush and apply 2 to 3 coats of paint after fabrication = £1.00 to £1.16 per square metre.
(b) Shot blast components prior to fabrication apply 2 to 3 coats of paint after fabrication = £1.80 per square metre.
(c) Shot blast components after fabrication and apply 2 to 3 coats of paint = £3.60 per square metre.

The British Steel Corporation is also prepared, for an extra cost, to shot blast and prime plates on both sides. The cost of shot blasting and applying priming point varies between 1.02£/m^2 and 1.09£/m^2.

Basing the cost of these processes on the surface area of the component to be treated represents the most realistic method available for estimating the cost of surface treatment. This method is realistic if, as in multi-storey frames, all the work to be painted is of similar intricacy. The use of surface area is in accordance with previous work. KNAPTON[18], ZWANZIG[23].

5.7 SIGNIFICANT FABRICATION COSTS OMITTED FROM THE COST MODEL

5.7.1 Subassembly

During the fabrication process it is necessary to clamp the individual components for welding, to manipulate the components during welding and also to check that completed beam and column components will fit together when the are delivered to site. The operations are not suitable in the general case for the application of time study because the times taken and the procedure adopted by the operatives depends to a large extent on the type of fabrication being studied, the method that is used for clamping the components and the practice of the individual fabricator. None of the fabricators contacted had separate times for these operations. Each of the fabricators included an allowance within the fabrication overheads to take account of subassembly. When the fabricator is equipped to produce multi-storey structures these items will form a small part of the fabrication process due to the use of purpose made clamps and manipulators. For other types of construction such as plated structures subassembly costs can form a large proportion of the total cost.

CORKER[14] examined subassembly for plated structures, he concluded that standard times for subassembly are difficult to assess. Corker also showed that for plated structures subassembly costs form a considerable part of the total fabrication cost.

5.7.2 Site Erection

It was found to be very difficult to include site erection within the cost model because no data was available from any of the companies consulted. Company "A" has a bonus scheme for site erection but the data was regarded as "competitive" and was not made available. The problem with assessing the cost of this process is that there is a large number of variables which have to be considered comprising:-

(a) The access to the site and availability of areas for storage.
(b) The availability of craneage on the site.
(c) The size, weight and position of the largest component in the structure.
(d) The height to which the components have to be lifted.
(e) The number and type of bolts to be used in each connection and the complexity of the connections.
(f) The average number of packing pieces in the column splices.
(g) The amount of repetition within the structure.
(h) The number of column splices.
(i) Interference between the erector and other building trades.
(j) The number of members in the frame and the ratio of the number of beams to the number of columns.

The usual method of estimating the cost of site erection is to use experience in deciding a suitable tonnage rate. The resulting cost is empirically related to the variables mentioned above. Clearly this type of cost cannot be included accurately in a cost model in this form as it does not consider the influence of the variables in a logical manner. For this reason this cost has not been included at all within the cost model. Clearly this would have warranted further investigation if cooperation had been given. The whole topic of site erection and planning of site work is a large subject within itself and has, for the sake of rendering the problem tractable, been omitted. 4

If all other considerations are the same, the cost of erection is related to the number of members in the frame because, the more members there are, the more joints there are to make, and the more lifts are required. Clearly an approximate model for erection costs could have been developed on this basis if the relevant information were available.

5.8 FACTORS AFFECTING FABRICATION COSTS

5.8.1 The Use of High Strength Steels

Among the companies approached experience of estimating for large quantities of fabrication in steel of grades 50 and 55 was limited to company "A". The labour times have to be altered to take account of variations in the fabrication procedure. Drilling setting up, handling and marking times remain constant for all grades of steel. Drilling, cold sawing and flame cutting times increase by 10 percent for grade 50 steel and by 25 percent for grade 55 steel. When steel of grade 50 is used for fillet welding the increase is 10 percent. When steel of grade 55 is used for fillet welding the increase is 100 percent, because of the great deal of care required during preheating and cooling to ensure that the steel retains its strength. These figures represent a suitable method for examining the effect on costs of using steels of different grades. Until the use of grade 50 and grade 55 steels becomes more widespread, data for these grades will remain innaccurate.

5.8.2 Performance of Labour and Multiplying Factors

The amount of time taken to perform a fabrication operation is greater than the amount of time during which the equipment is in use. The ratio of these two times is termed the operating factor, which is a measure of the efficiency of the operatives and their equipment. For fillet welding the operating factor varies from about 50 percent down to 15 percent; an average factor is around 33 percent. Other operations have similar factors. Variations in the operating factor are found when standard times are originally derived for use in a bonus incentive scheme which at a later stage is discontinued. The wide variation in the operating factor can be seen to affect the cost of an operation considerably.

Variations in times taken between companies are also due to the policy of the work study team in deciding which operations to include within the standard times for a given operation.

These two factors provide the main reasons for the large variations between the times provided by the companies for the same operation. The effects of these factors are expressed in the scalar multipliers which have been found for some of the fabrication operations.

A related factor is the choice of labour rate for each fabrication operation. A summary of the labour rates provided which are to be applied for each operation are given in Table 5.7. The labour rates given may or may not include a proportion of the overheads and this means that they show variations between companies and fabrication operations.

When the final cost of a frame is being found the differences in overhead factors, labour rates and scalar multipliers between companies cancel each other out in such a way that the final price is similar for all companies.

It is therefore not necessary to consider the role of the scalar multipliers further, if the labour rate and overhead factor are chosen in such a way that the costs produced are realistic.

Operation	Company A	Company B	Company C	Company D	Company E
Marking Out	1.91	N.A.	N.A.	N.A.	N.A.
Welding	1.91	2.00	1.80	1.52	N.A.
Machine Flame Cutting	2.45	2.00	1.80	2.77	2.00
Hand Flame Cutting	1.71	2.00	1.80	1.52	2.00
Cold Sawing	1.44	N.A.	1.80	1.52	2.00
Drilling	1.44	2.00	1.80	N.A.	2.00
Handling	2.45	N.A.	1.80	N.A.	N.A.

Table 5.7 - Labour Rates (£/Hour)

5.8.3 Repetition

Repetition will have the effect of reducing costs when large numbers of the same item have to be produced. Examples of this are:-

(a) When the sections are clipped together for cold sawing.

(b) When plates are clamped together for drilling or cutting.

The cost model makes no attempt to deal with the savings so gained. It is usual when estimating to consider repetition only when very large numbers of the same component are required and in such a case a special study would be made. For a given structure the cost saving due to repetition would be sensibly constant and so should not affect the comparison of two similar designs.

5.9 THE COSTING OF A FRAME

Using the previous cost information, the total frame production cost may be obtained from:-

$$C = \left(\sum_{i=1}^{i=n} L_i \ (t_i) \right) \times O_v + C_s + C_p + C_b + C_x$$

Where:-

L_i = labour rate for process i

t_i = total standard time expended on process i

n = number of processes

O_v = overheads factor

C_s = cost of steel sections

C_p = cost of steel plates

C_b = cost of bolts

C_x = section extra costs for quantity, quality and transport.

The application of this formula to the costing of sections and connections will now be considered.

5.9.1 The Costing of Sections

The application of the cost model to determing the cost of beam and column sections for a frame will now be described. Sections are costed on the basis of the centre to centre distances between joints, this cost is later modified during the costing of connections to take account of the true lengths of the members. The following costs relating to the sections were included.

Material Items

(a) Section of required length and size, cost modified to take extras into account.

Fabrication Items

(a) Shot blasting and painting over the whole length of each member.

Area to be treated = Perimeter length of section x length of section.

5.9.2 Major Axis Beam End Plate

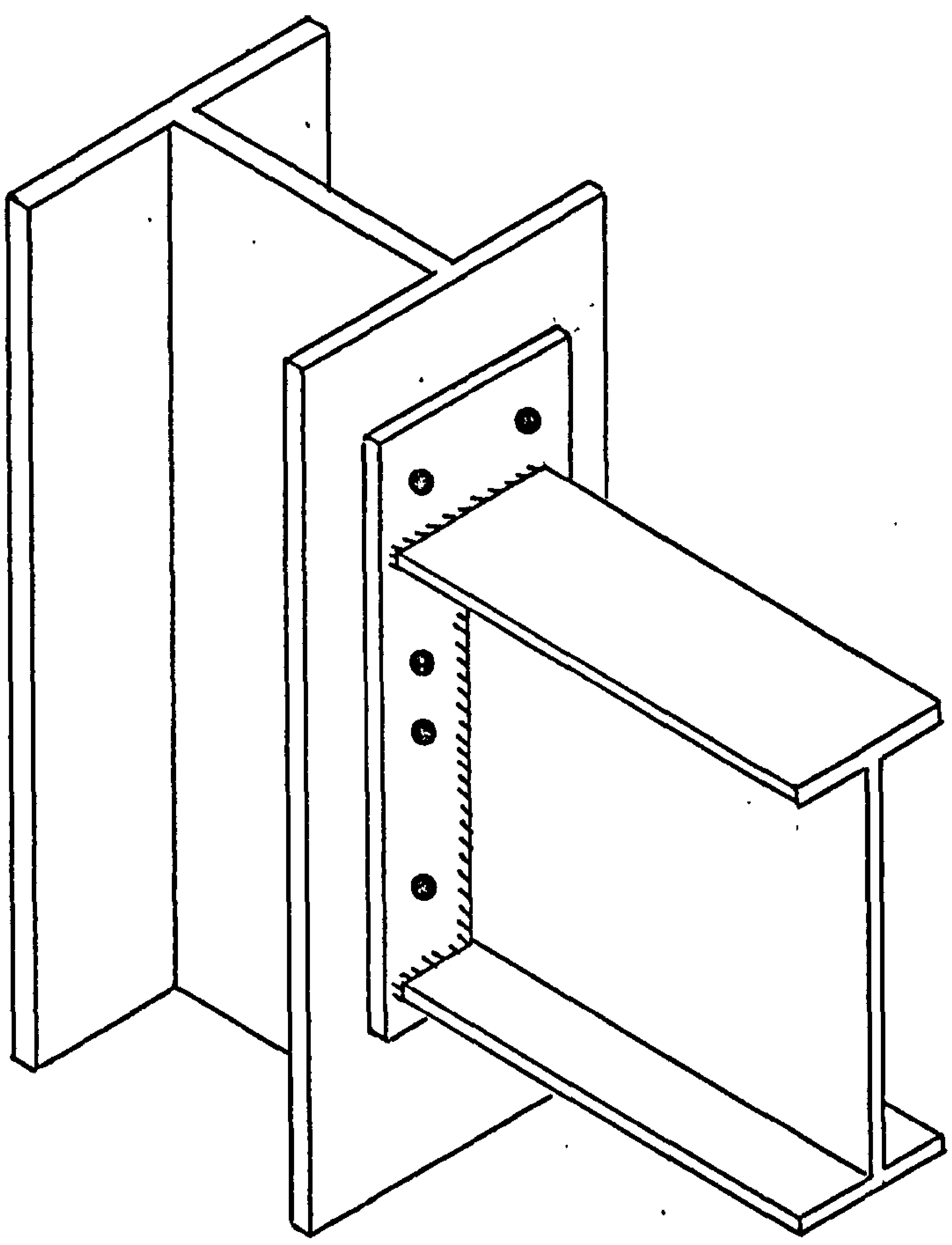


Figure 5.16 - Major Axis Connection

Material Items

(a) Endplate of required thickness, width and depth.

(b) Bolts of required diameter with a minimum grip length equal to the endplate thickness plus the stanchion flange thickness.

(c) Welding rods to construct fillet weld between beam and endplate.

(d) Reduce the cost of the above by the cost of the beam section with a length of half the stanchion depth.

Fabrication Items

(a) Mark out and cut endplate from a large plate
total distance = Depth of plate + Width of plate.

(b) Mark out and drill holes for site bolts on endplate and stanchion flanges.
total number of set ups = number of bolts x 2
total depth to be drilled through = number of bolts x (thickness of endplate + thickness of stanchion flange).

(c) Cold saw end of beam.

(d) Fillet weld beam to endplate.
total length of large welds = 2 x breadth of beam
total length of small welds = 2 x (depth of beam + breadth of beam)

(e) Handle endplate as part of a large plate to cutting process. 3 movements by crane.

(f) Handle endplate separately from cutting to drilling and then to welding. 2 movements by hand.

(g) Handle beam section through all processes.
7 movements by crane
(cost shared between 2 endplates).

(h) Shot blast endplate both sides.
total area = 2 x (depth of plate x width of plate).

(i) Paint endplate one side only.
total area = (depth of plate x width of plate).

(j) Reduce the cost of the above by the cost of painting and shot blasting a length of beam equal to half the stanchion depth.

5.9.3 Minor Axis Beam End Plate Without Continuity

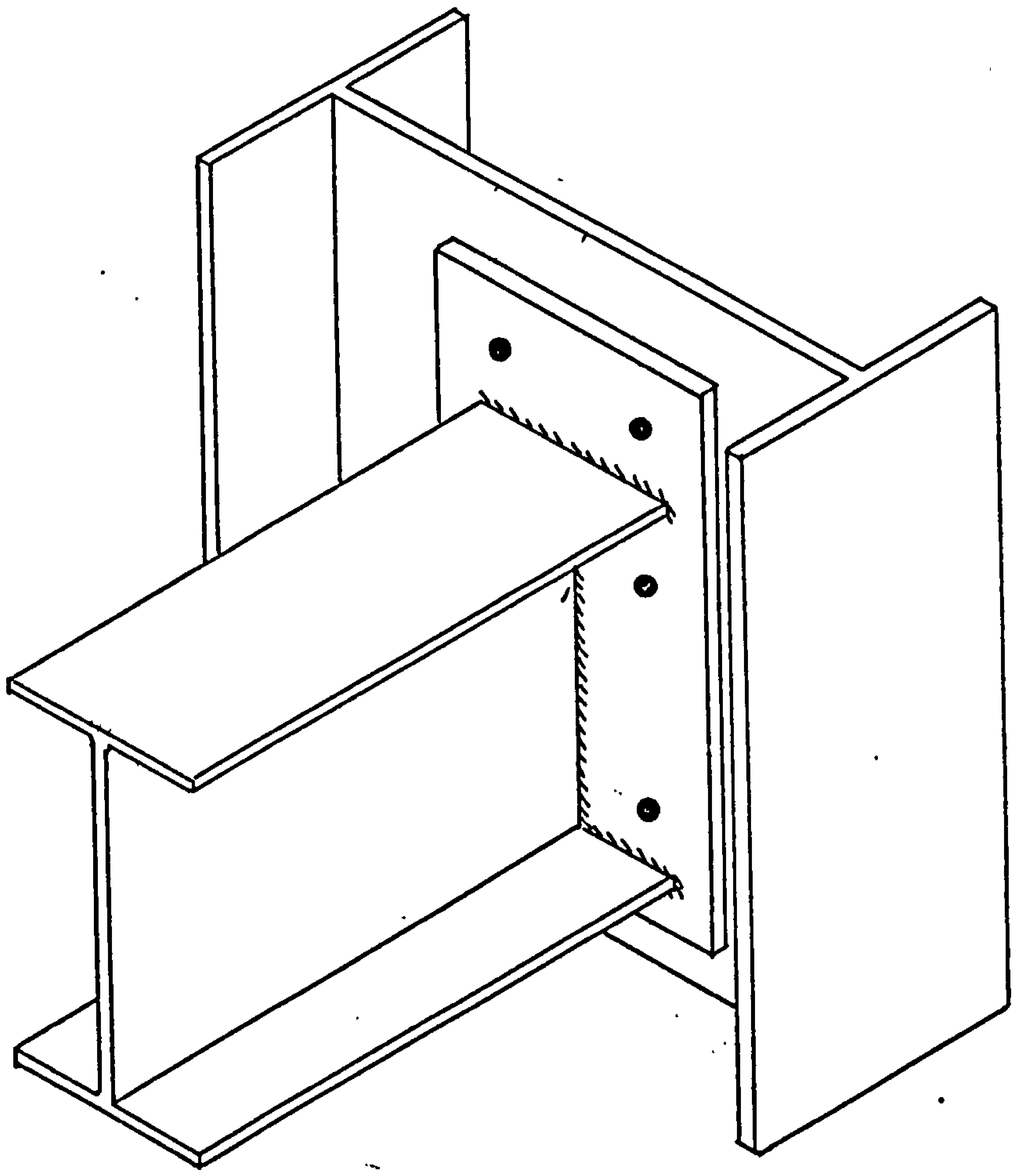


Figure 5.17 - Minor Axis Connection Without Continuity

This connection is similar for costing purposes to the major axis beam endplate connection, requiring the substitution of stanchion web wherever the stanchion flange is mentioned and neglecting the cost reductions for the length of beam. In the case where the endplate is not extended the costing of the connection is also similar.

5.9.4 Minor Axis Beam End Plate with Continuity

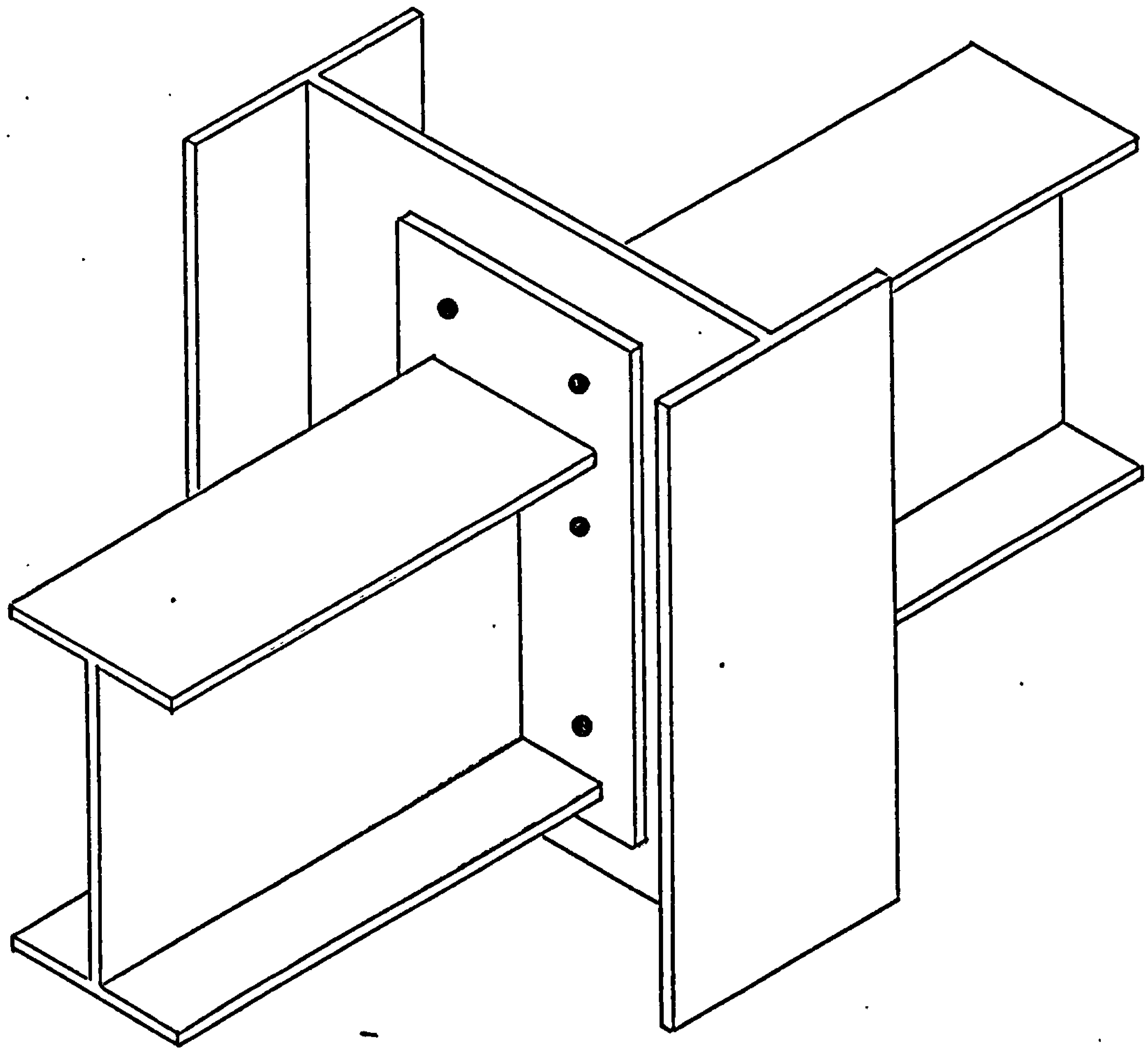


Figure 5.18 - Minor Axis Connection With Continuity

Material Items

(a) Two identical endplates of required thickness, width and depth.

(b) Bolts of required diameter with a minimum grip length equal to twice the endplate thickness plus the thickness of the column web.

(c) Welding rods to construct fillet welds between beams and endplates.

Fabrication Items

(a) Mark out and cut endplates from a large sheet
total distance = 2 x (depth of plate + width of plate)

(b) Mark out and drill holes for site bolts on endplates and stanchion web.

total number of set ups = number of bolts x 3

total depth to be drilled through = number of bolts x 2 x (thickness of endplate + thickness of stanchion web)

(c) Cold saw end of each beam.

(d) Fillet weld beams to endplates.

total length of large welds = 2 x breadth of beam

total length of small welds = 2 x (breadth of beam + depth of beam)

for each of the two beams.

(e) Handle endplates as part of a large plate to cutting process.

6 movements by crane.

(f) Handle endplates separately from cutting to drilling and then to welding.

4 movements by hand.

(g) Handle beam sections through all processes.

7 movements by crane.

(h) Shot blast endplates both sides.

total area = 2 x (depth of plate x width of plate)

5.9.5 Stanchion Stiffeners

Tension Stiffeners

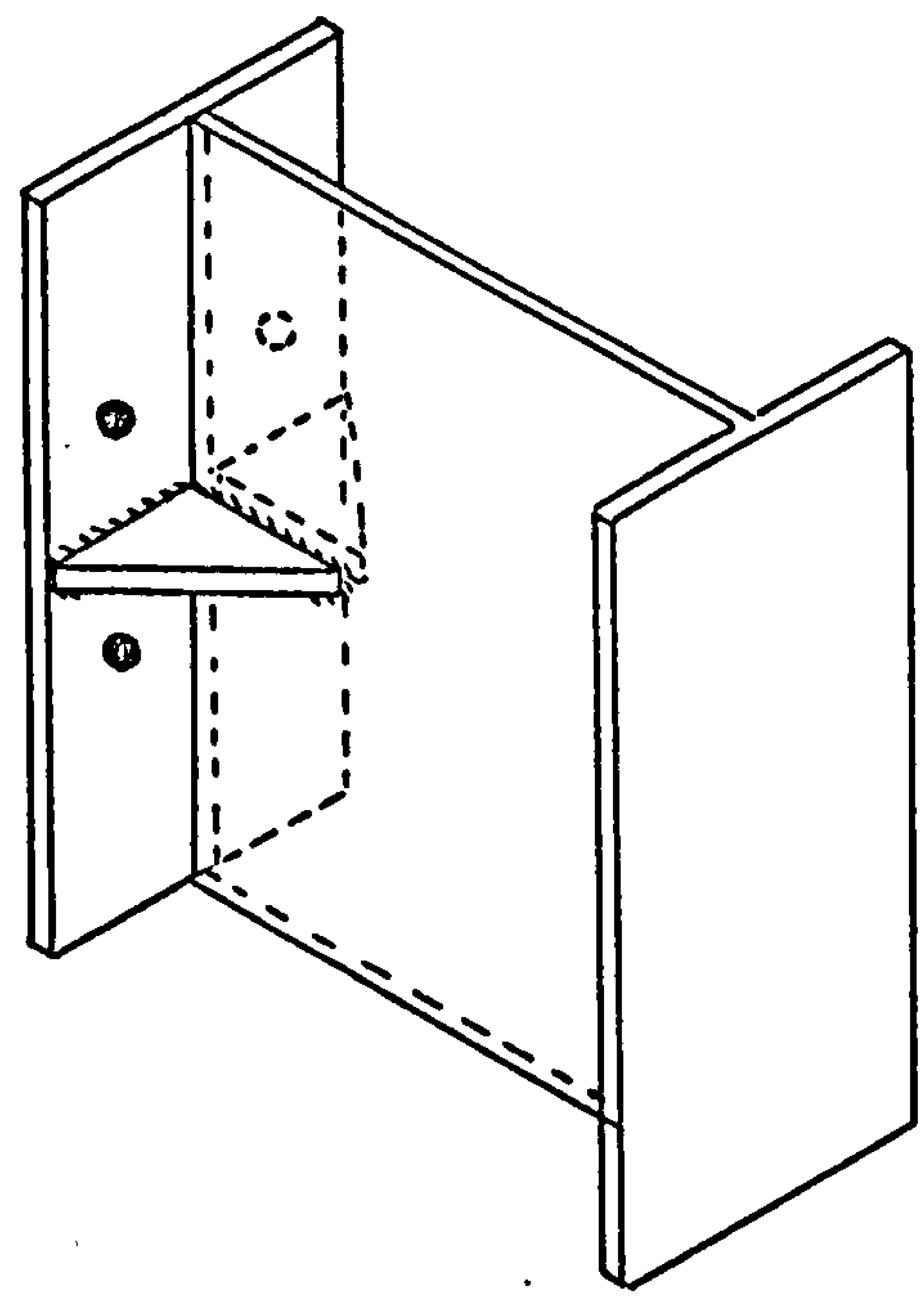


Figure 5.19 - Tension Stiffeners

Material Items

(a) Two identical triangular stiffeners of required thickness.

(b) Welding rods to construct fillet welds between column stanchion and stiffeners.

Fabrication Items

(a) Mark out and cut stiffeners from a large sheet total distance = 3.4 x breadth of stanchion.

(b) Fillet weld stiffeners to stanchio total length of weld = 4 x breadth of stanchion.

(c) Handle stiffeners separately from cutting to welding 2 movements by hand.

(d) Shot blast and paint stiffeners both sides total area = 0.5 x (breadth of stanchion)^2.

Compression Stiffeners

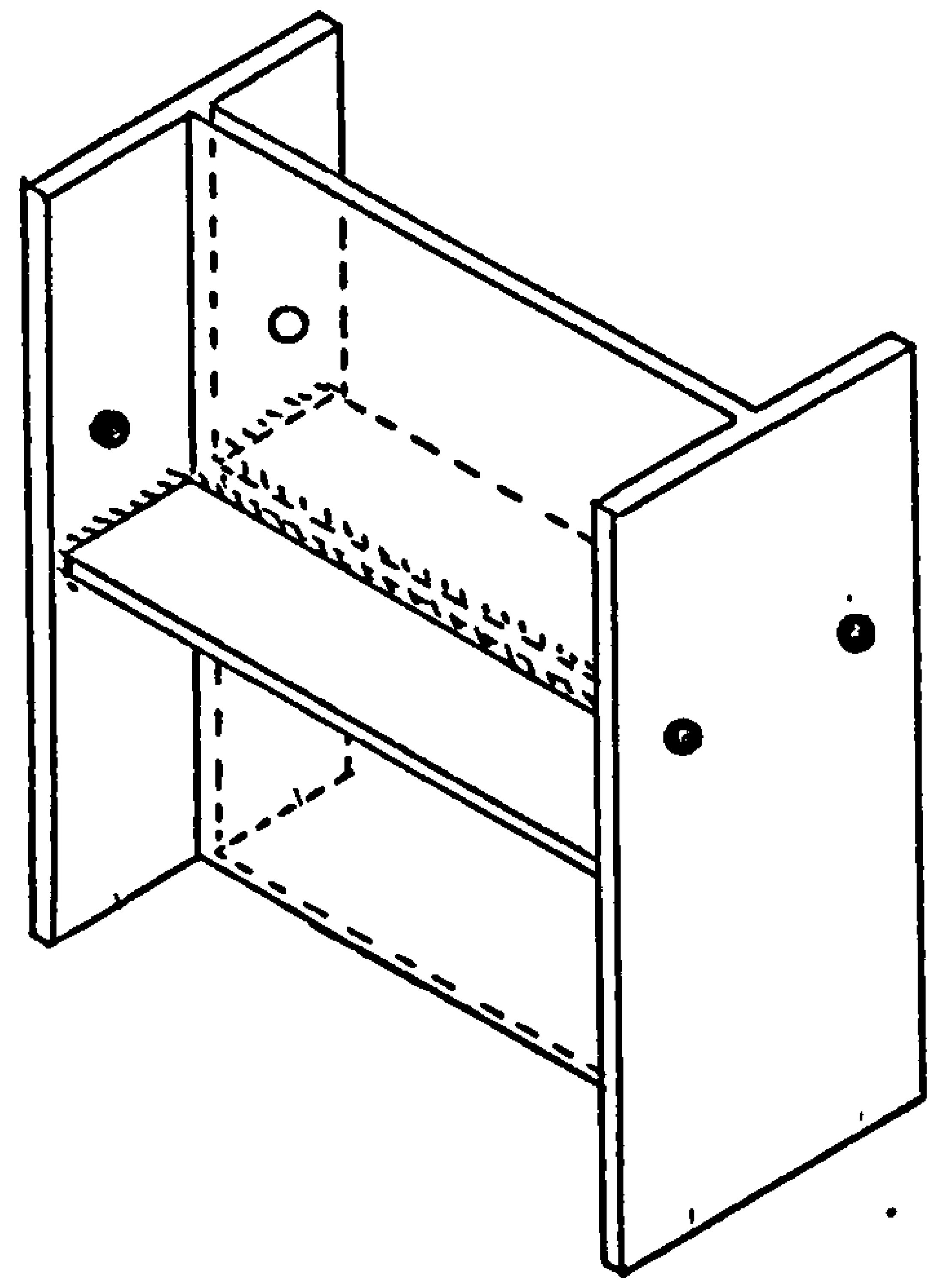


Figure 5.20 - Compression Stiffeners

Material Items

(a) Two identical rectangular stiffeners of required thickness.

(b) Welding rods to construct fillet welds between stanchion and stiffeners.

Fabrication Items

(a) Mark out and cut stiffeners from a large sheet.
total distance = breadth of stanchion + 2 x depth between flanges of the stanchion.

(b) Fillet weld stiffeners to stanchion.
total length of weld = 4 x (breadth of stanchion + depth between flanges of stanchion).

(c) Handle stiffeners separately from cutting to welding.
2 movements by hand.

(d) Shot blast and paint stiffeners both sides.
total area = 2 x (breadth of stanchion x depth between flanges of stanchion).

5.9.6 Stanchion Splice

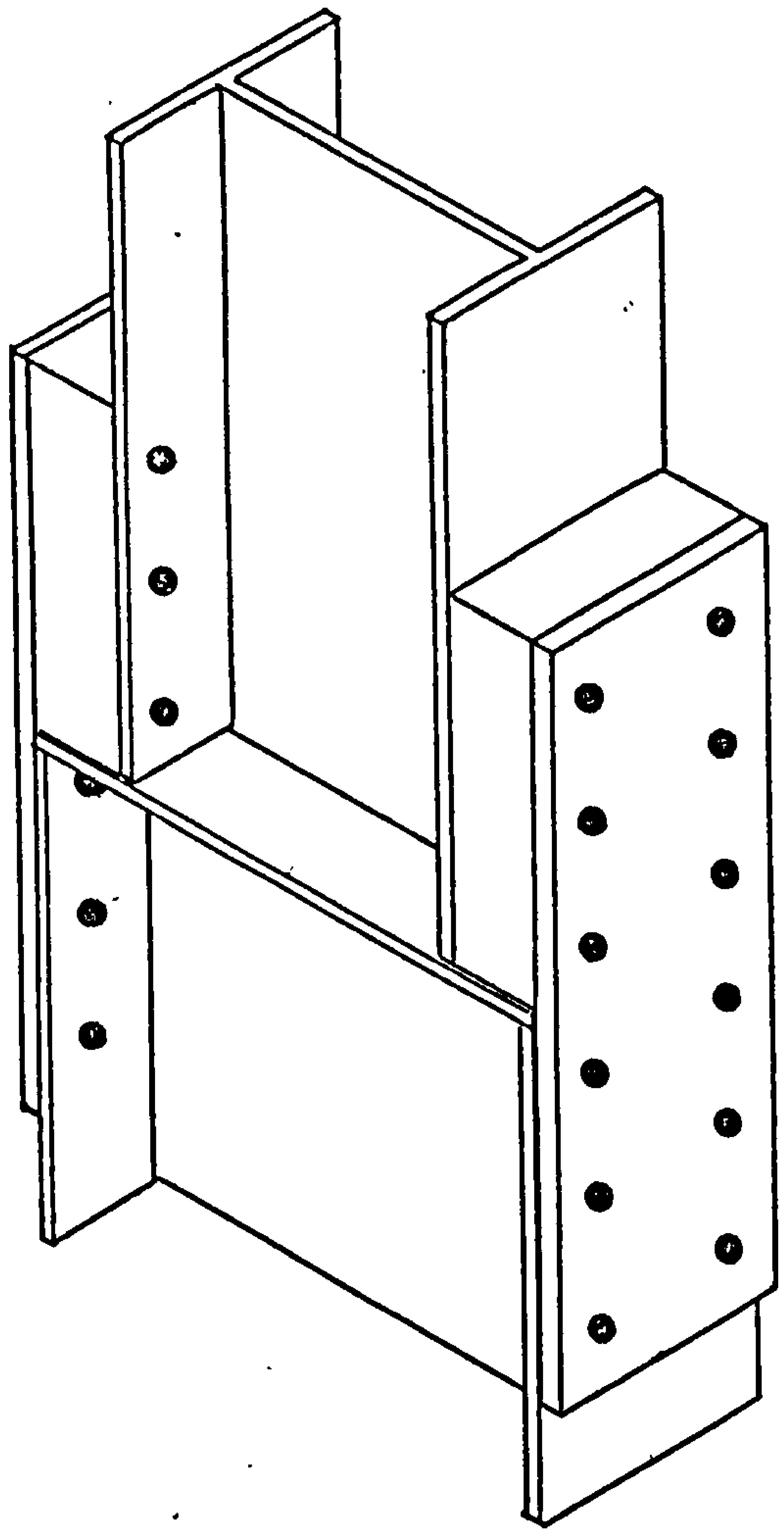


Figure 5.21 - Stanchion Splice

Material Items

(a) Two cover plates of required thickness, width and depth.
(b) Assorted packing pieces of required thickness, width and depth.
(c) One separation plate of required thickness.
(d) Bottom bolts of the required diameter with a minimum grip length equal to the thickness of the cover plate plus the flange thickness of the bottom stanchion.
(e) Top bolts of the required diameter with a minimum grip length equal to the thickness of the cover plate plus the thickness of packing plus the flange thickness of

the upper stanchion.

(f) Welding rods to construct fillet welds between bottom stanchion and separation plate.

(g) Increase the cost of the above by the cost difference between the stanchions above and below, with a length equal to the distance from the top of the beams below to the midpoint of the splice.

Fabrication Items

(a) Mark out and cut cover plates, separation plates, and packing plates from large sheets.
for each plate distance = depth of plate + width of plate.

(b) Mark out and drill holes for site bolts on cover plates and stanchion flanges.
total number of set ups = number of top bolts x 2 (number of packing plates + 2) + number of bottom bolts x 4.
total depth to be drilled through = (number of top bolts x (cover plate thickness + packing plates thickness + flange thickness of top stanchion) + number of bottom bolts x (cover plate thickness + flange thickness of lower stanchion))x 2.

(c) Cold saw both ends of upper stanchion.

(d) Fillet weld separation plate to top of lower stanchion.
total length = 2 x (breadth of lower stanchion + depth of lower stanchion).

(e) Handle plates as part of a large plate up to cutting process.
3 movements by crane per plate.

(f) Handle separation plate from cutting to welding.
1 movement by hand.

(g) Handle packing plates and cover plates from cutting onwards.
4 movements by hand for each cover plate.
3 movements by hand for each packing plate.

(h) Handle one stanchion section through all processes.
7 movements by crane.

(i) Shot blast all plates both sides.
Area for each plate = 2 x (depth of plate x width of plate).

(j) Paint separation plate and one side of each cover plate.

(k) Increase the cost of the above by the cost of painting and shot blasting the lower stanchion for a length from the top of the beam below to the centre of the splice. Reduce the cost by the cost of the same item for the upper stanchion section.

5.9.7 Stanchion Base Plate

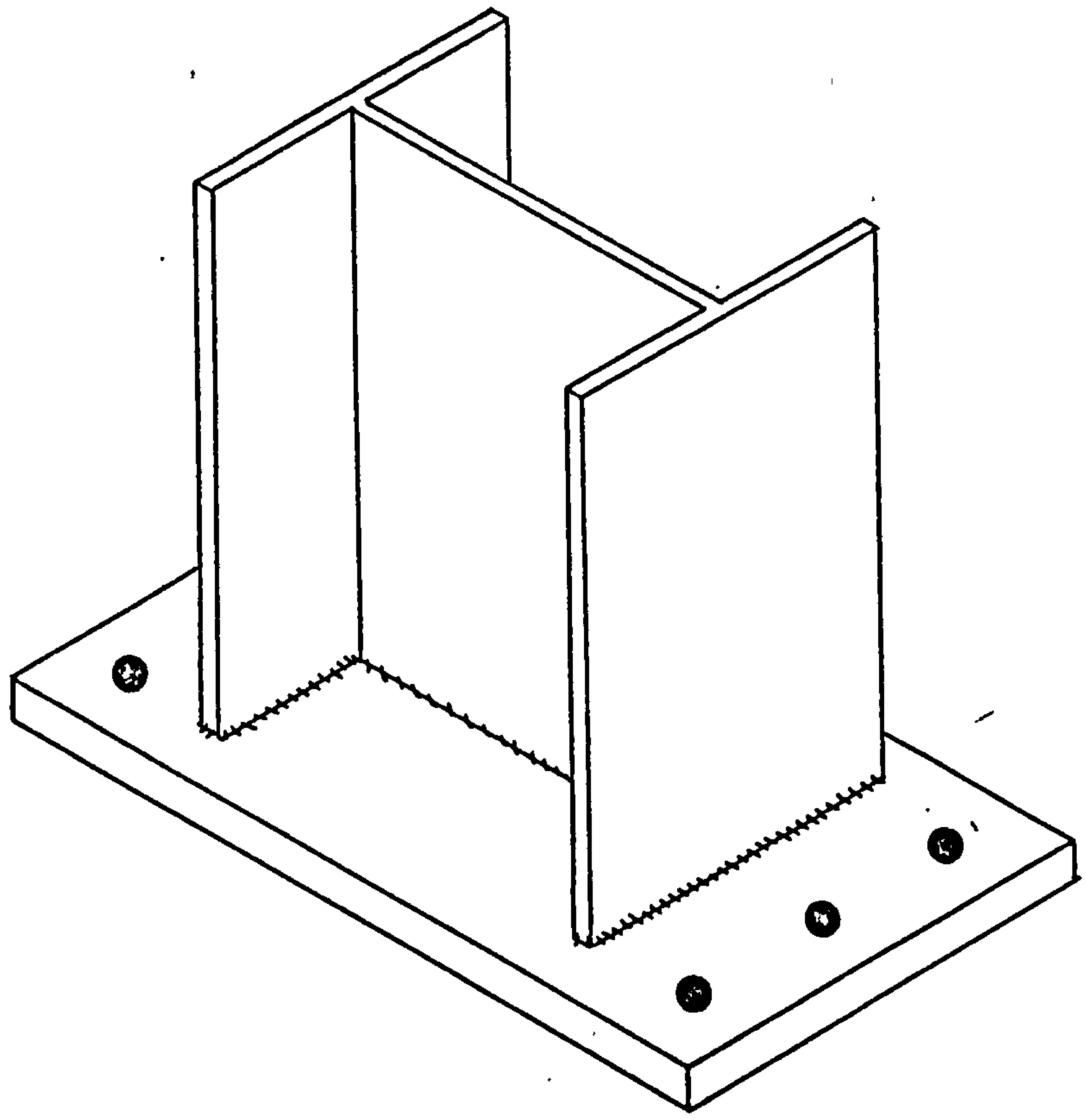


Figure 5.22 - Stanchion Base Plate

Material Items

(a) Base plate of required thickness, width and depth.

(b) Foundation bolts of 25mm diameter, 500mm long.

(c) Welding rods to construct fillet welds between stanchion and base plate.

Fabrication Items

(a) Mark out and cut base plate from a large plate.
total distance = depth of plate + width of plate.

(b) Mark out and drill holes for foundation bolts in base plate.
total number of set ups = number of bolts
total depth to be drilled through = number of bolts x thickness of base plate.

(c) Cold saw both ends of stanchion.

(d) Fillet weld stanchion to endplate.
total length of weld = 2 x depth of stanchion + 4 x breadth of stanchion.

(e) Handle base plate as part of a large plate to cutting process.
3 movements by crane.

(f) Handle base plate separately from cutting to drilling and then to welding.
2 movements by hand.

(g) Handle stanchion section through all processes.
7 movements by crane.

(h) Shot blast and paint both sides of endplate.
total area = 2 x (length of plate x width of plate).

5.10 CONCLUSIONS

The cost model which has been described allows the automatic costing of braced steel frames in accordance with the methods and requirements of the structural steelwork industry. The model represents an attempt to include as many of the production processes involved in the construction of steel frames as possible. Costing data, which has been found in cooperation with industry, has been presented in a unified form, suitable for use by any designer wishing to compare designs on a rational basis. The costs resulting from the use of the cost model have been derived from the most accurate methods of cost estimating available and therefore represents a reasonable basis for the comparison of alternative designs.

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CHAPTER 6

Optimization Techniques

6.1 INTRODUCTION

The optimization of braced frames using the design methods described in Chapter 4 may be stated in a mathematical form as:-

Minimise $Z = \Sigma\ C_i\ (x_i)\ L_i + \Sigma\ C_c\ (\underset{\sim}{x}) + C_x\ (\underset{\sim}{x}, \underset{\sim}{L})$

where $\underset{\sim}{x} = (x_1, x_2\ \ldots\ldots\ x_n)$ = Independant variables (sections chosen for each group of members).

$\underset{\sim}{L} = (L_1, L_2\ \ldots\ldots\ L_n)$ = Total length of members in each group of members.

$C_i\ (x_i)$ = Cost per unit length of section x_i.

$C_c\ (\underset{\sim}{x})$ = Fabrication and material cost of each connection.

$C_x\ (\underset{\sim}{x}, \underset{\sim}{L})$ = Cost of extras for quality, quantity and transport.

n = Number of groups of members.

Subject to constraint functions of the independant variables

$(\underset{\sim}{x})$ and the dependant variables $(\underset{\sim}{\alpha})$

Stress constraints	$f_j\ (\underset{\sim}{x}, \underset{\sim}{\alpha}) \leqslant F_j\ (\underset{\sim}{x})$
Deflection constraints	$d_j\ (\underset{\sim}{x}) \quad \leqslant D_j$
Local buckling constraints	$q_j\ (\underset{\sim}{x}) \quad \leqslant Q_j$
Member buckling constraints	$p_j\ (\underset{\sim}{x}) \quad \leqslant P_j\ (\underset{\sim}{x})$
Geometric constraints	$G_j\ (\underset{\sim}{x}) \quad \leqslant E_k\ (\underset{\sim}{x})$
End moment constraints	$L\ (\underset{\sim}{\alpha}) \quad \leqslant \underset{\sim}{\alpha}_j < U\ (\underset{\sim}{\alpha})$

where k, j = member numbers incremented over all members.
α = end moment factor, the end moments in a beam member when considering moment redistribution.

There may be more than one of each type of constraint for each member. The variables $\underset{\sim}{x}$ are discrete and the variables $\underset{\sim}{\alpha}$ are continuous. The constraints are in general discontinuous. The problem as formulated represents one of the most difficult optimization problems, since:

(a) Some of the variables are discrete.
(b) The objective function is non linear and discontinuous.
(c) The constraints are non linear and cannot always be written in analytical form.
(d) The number of variables may be very large.

One method of rationalising this difficult problem which has been used is that of functionalizing the properties of the sections, which allows a continuous variable formulation of the problem. This approach is not possible for three-dimensional frames because all section properties have to be related to one property (for example the section area). Relationships for two-dimensional frames though inaccurate are possible, however for three-dimensional frames no such relationships are available and even if they were available it would in some cases be impossible to select a section corresponding to the continuous variable solution from a set of available sections. For these reasons this approach has been rejected.

Rather than try to solve the optimization problem by using a single optimization technique, the problem has been solved by firstly obtaining a near optimal solution, using an iterative design algorithm and secondly using this solution as a starting point for a local search algorithm. Two local search algorithms have been used, these are dynamic programming and enumeration. Even after both steps have been carried out, there is no guarantee that the resulting design is optimal.

A description will be given of end moment constraints and geometric constraints and the implementation of these constraints within the design and optimization system. The iterative design algorithm is then described, followed by descriptions of the dynamic programming and enumeration algorithms. Examples are given of the use of the algorithms and finally the two local search algorithms are compared in terms of computational efficiency.

6.2 END MOMENT CONSTRAINTS

The design methods which were considered allow the redistribution of the end moments for beams. This allows the selection of stanchions which are smaller than would be allowed if redistribution was neglected.

Consider a beam framing into an external column. In conventional plastic design without distribution the end and centre moments would be equal. When considering redistribution the beam end moments may be reduced as long as the stanchion moments do not exceed the elastic moment of resistance of the stanchion section. This approach may also be applied to internal stanchions where a large difference in span occurs between the beams on either side of the stanchion, in this case the larger end moment (which can be reduced) must never be less than the smaller end moment (which remains fixed). The approach may be used in order to utilize spare resistance in beams, resulting in a smaller stanchion.

A theoretical investigation of the problem was made in order to examine the characteristics of the problem. The investigation was applied to the B.C.S.A. design method and first order effects were considered. A uniform load was applied to the whole span of a beam and the maximum bending stress was plotted against the value of one beam end moment, the other end moment remaining constant. A typical plot is shown in figure 6.1, it can be seen that the relationship is slightly convex. An investigation was then made of a stanchion. A single load case was considered, analytical expressions were found for the maximum moment at the ends and midheight when only one end moment was varied. The midspan moment would be increased in practice by the second order effects. A plot of end moment against maximum moment is shown in figure 6.2. The relationship can be seen to have a sharply defined minimum and to be made up of linear segments. The plot shows that a reduction in one end moment will not always result in a reduction of maximum moment.

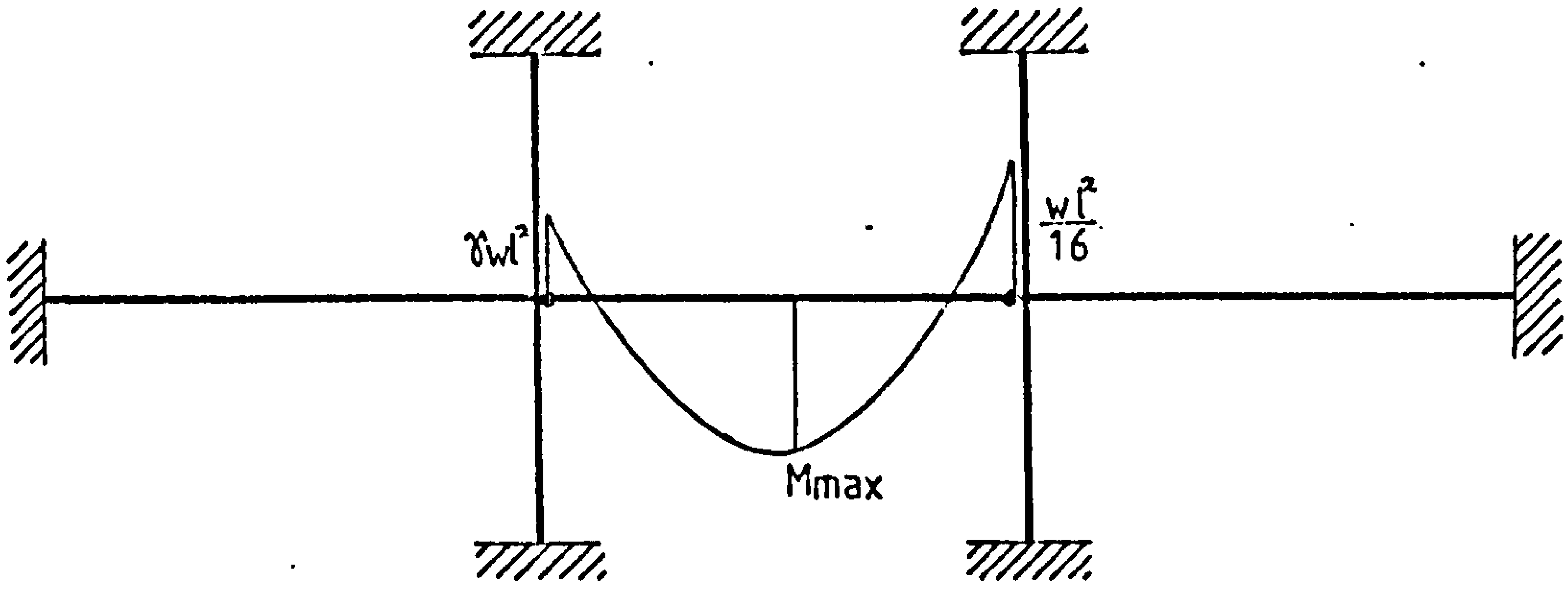
In order to verify the theoretical results, the effect of varying end moments was assessed on two small frames. The investigation was carried out by selecting sections for the beams and columns, the beam end moments were then varied and the maximum stress ratio was plotted. All three design methods were investigated, the results being very similar in each case. The first frame is shown diagramatically in figure 6.3 together with the plot of stress ratios for the B.C.S.A. design method. It can be seen that the design space is non linear and has a sharply defined minimum. The second frame is shown in figure 6.4 together with the plot of stress ratios for the B.C.S.A. design method. The design space is again non linear and has a sharply defined minimum.

From these investigations the principal problems involved in taking advantage of redistribution were seen to be:-

(a) The problem has many variables, each variable end moment is a variable.

(b) The stress ratios are dependant on second order effects which cannot be treated analytically.

(c) The existance of upper and lower bounds on the variables rules out the use of analytical solutions.



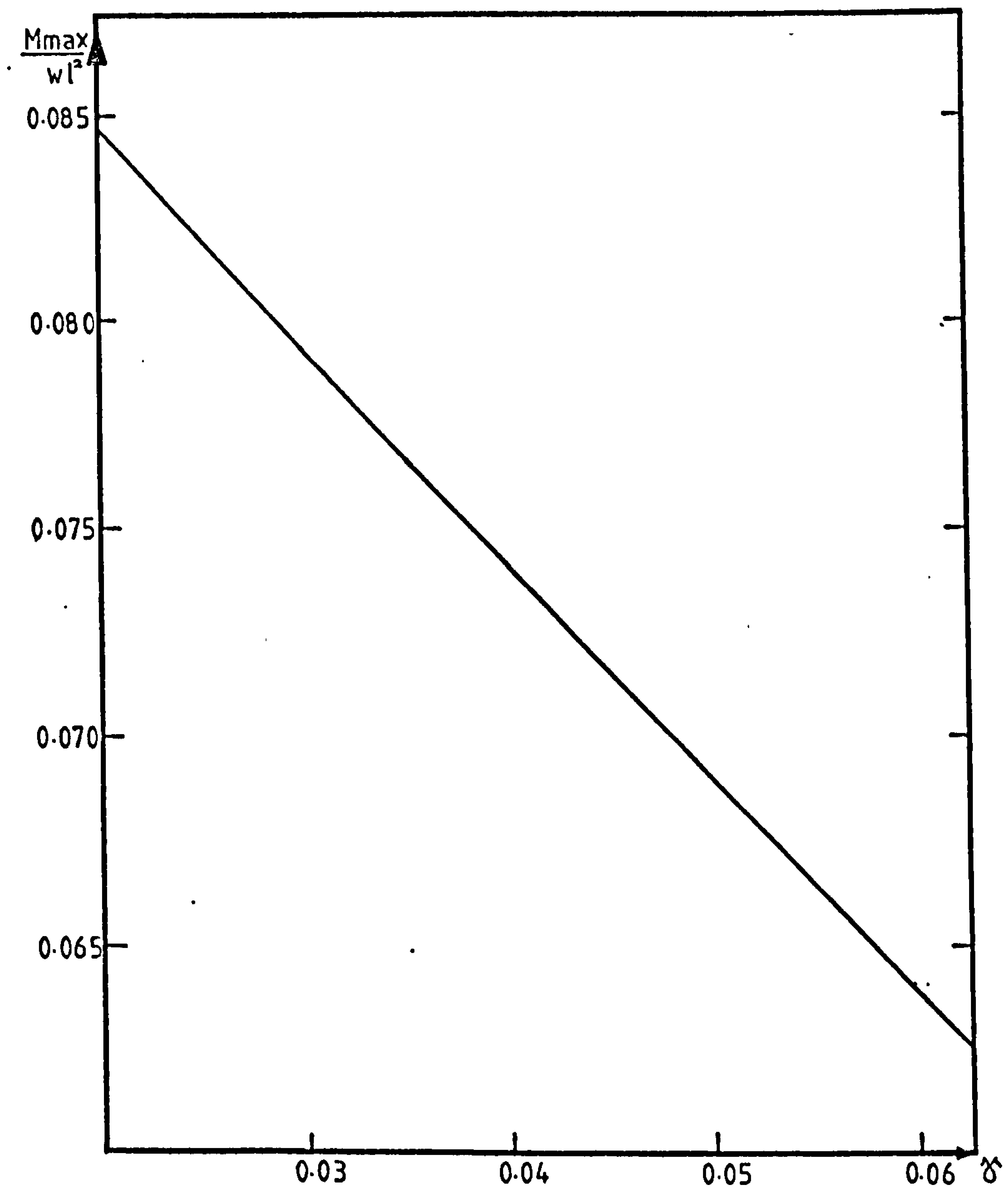
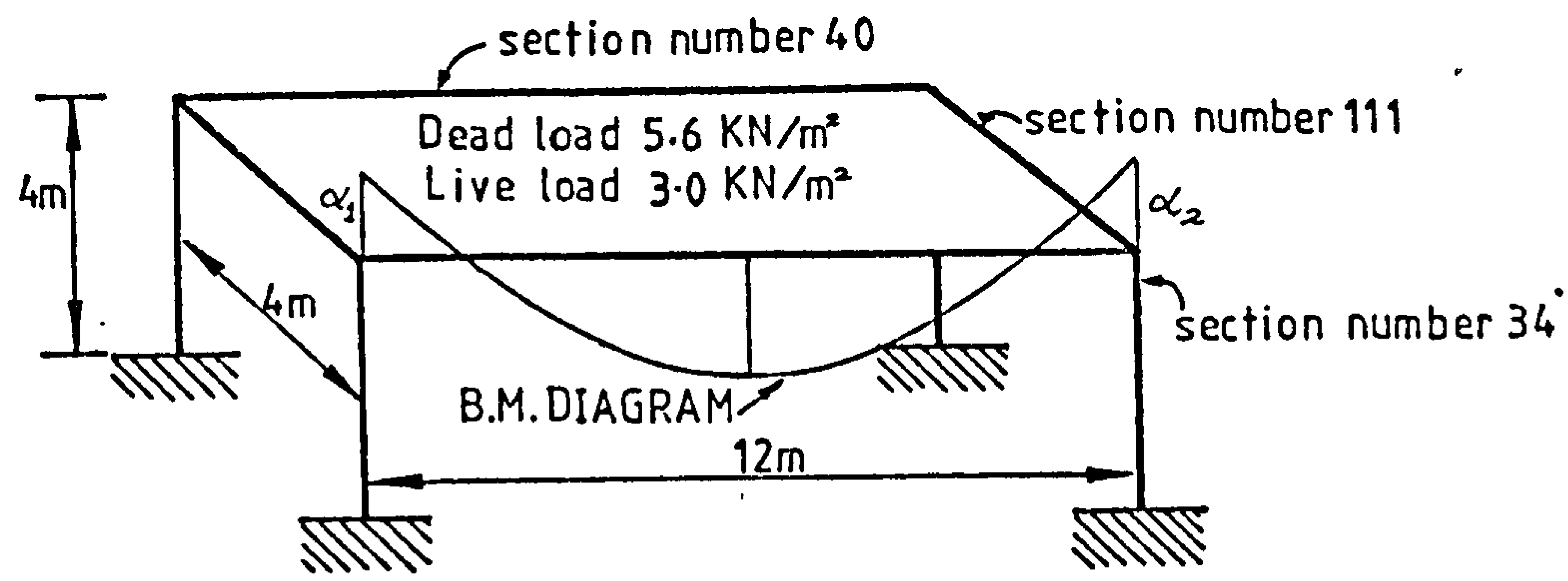


FIG. 6.1 THE RELATIONSHIP OF MAXIMUM MOMENT TO END MOMENT FOR A BEAM

FIG. 6.2 THE RELATIONSHIP OF MOMENTS IN A COLUMN TO THE APPLIED MOMENT AT THE UPPER JOINT



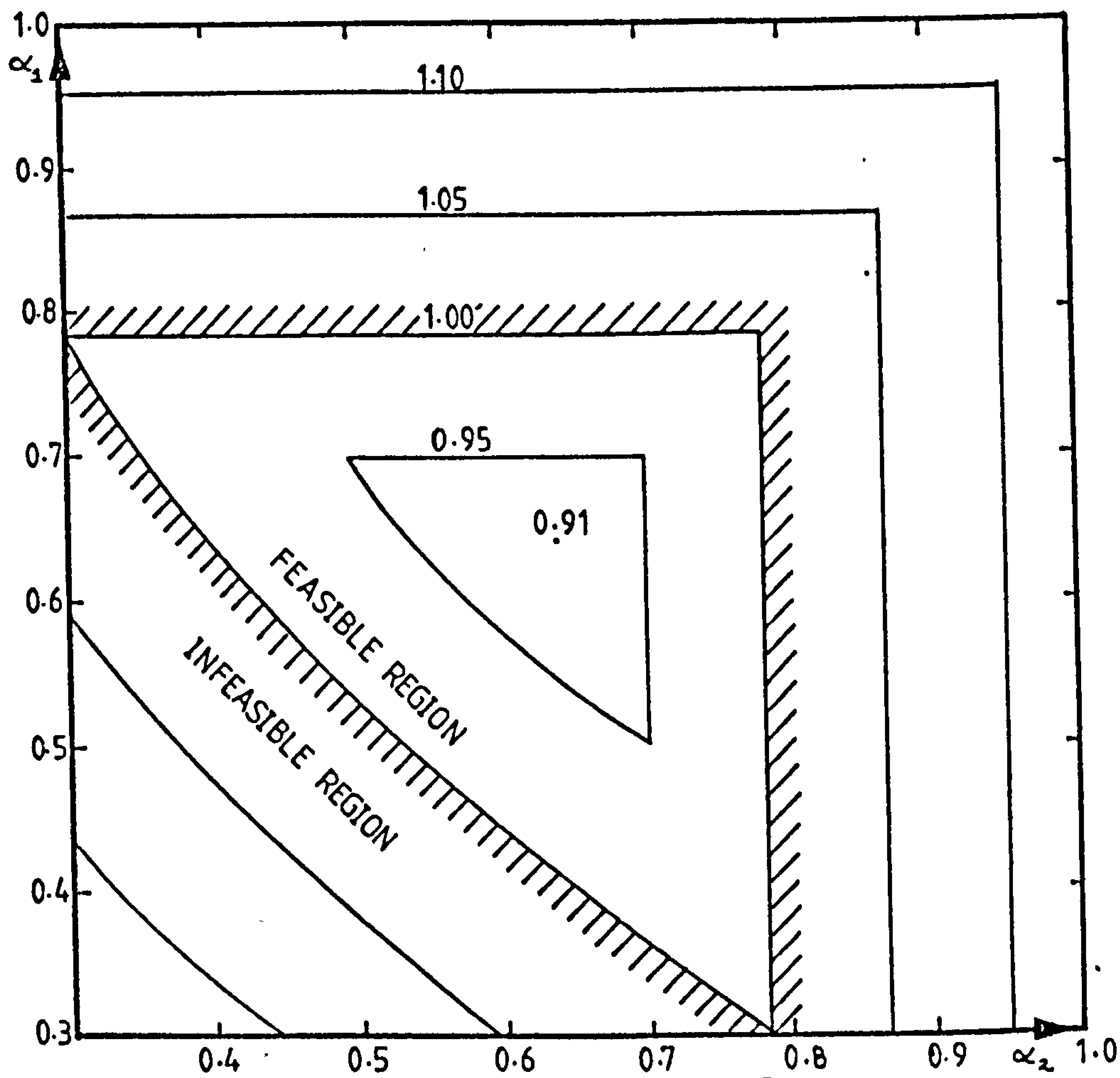


FIG. 6.3 MOMENT REDISTRIBUTION EXAMPLE NUMBER ONE

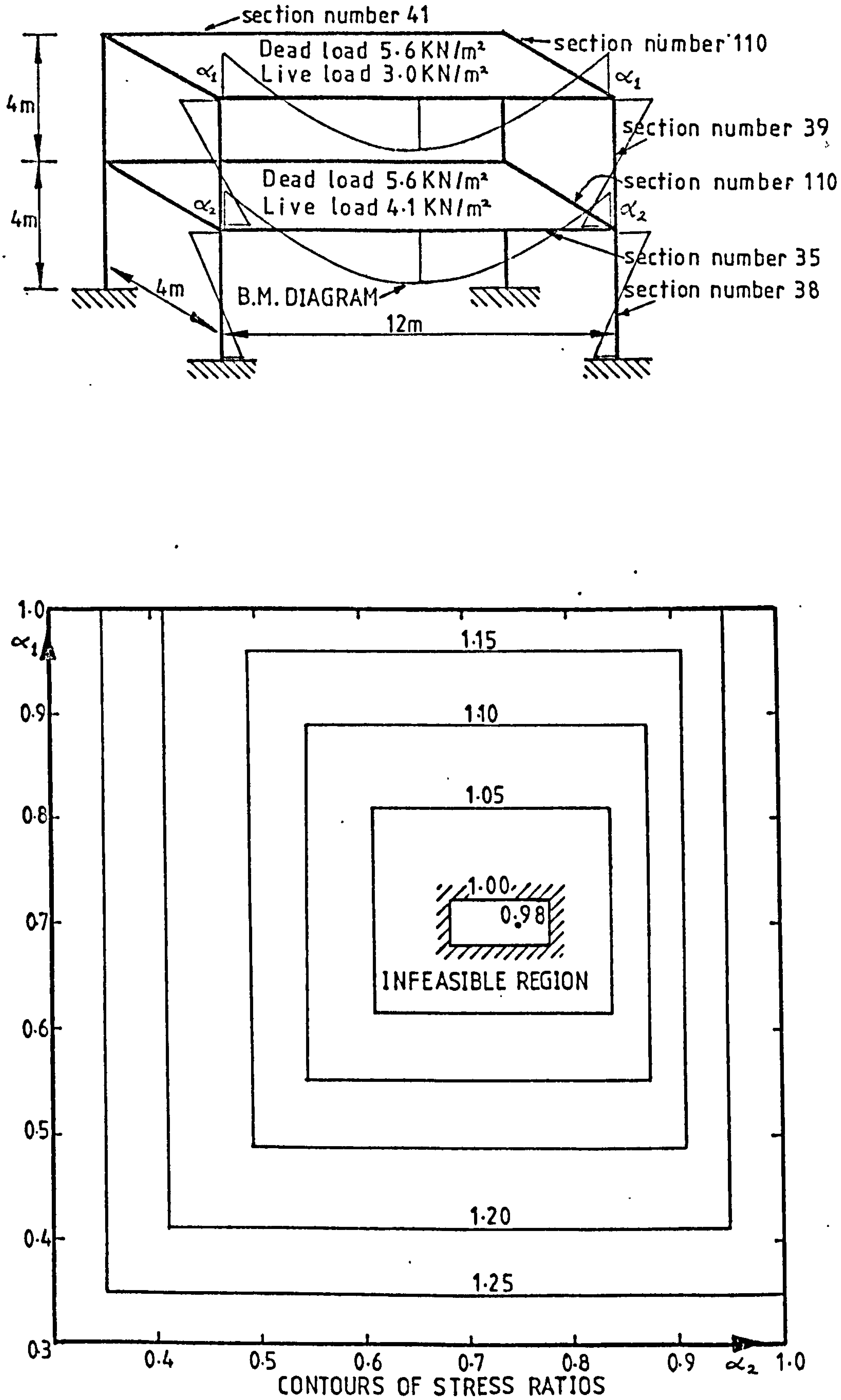


FIG. 6.4 MOMENT REDISTRIBUTION
EXAMPLE NUMBER TWO

A number of properties of the problem could be used to good effect:-

(a) The design space appeared to be unimodal.
(b) The effect of changing one end moment could be easily found by considering only the stress ratios of the attached members.
(c) A solution to the problem is found when all stress ratios are less than one.

An algorithm has been devised to exploit moment redistribution. This algorithm is interposed between the optimization section and the design checking section. The optimization algorithm postulates a set of sections for the frame and the moment redistribution algorithm determines whether these sections are feasible.

The approach of the algorithm consists of starting with an analysis of the frame neglecting redistribution. Each joint is then considered in turn, the moment which may be varied is varied until the point with the smallest maximum stress ratio is found. This procedure is repeated until either the stress ratios are all less than one (a feasible design) or until the maximum stress ratio no longer reduces (an infeasible design). This algorithm, which consists effectively of a series of univariate searches, has a number of advantages:-

(a) The size of the frame does not limit the use of the algorithm.
(b) Even for large frames only three or four full scale iterations are required to bring about convergence.
(c) Designs which fail because certain stress ratios can never be less than one are immediately rejected.

The univariate searches were carried out using "Powell's Algorithm", which makes use of a curve approximation to choose successive trial points. If the design space is convex the algorithm will converge on a minimum. The accuracy of the algorithm cannot be guaranteed and it is not possible to define exactly how many function evaluations are required. In practice convergence of the algorithm is rapid, often requiring only ten function evaluations for each univariate search.

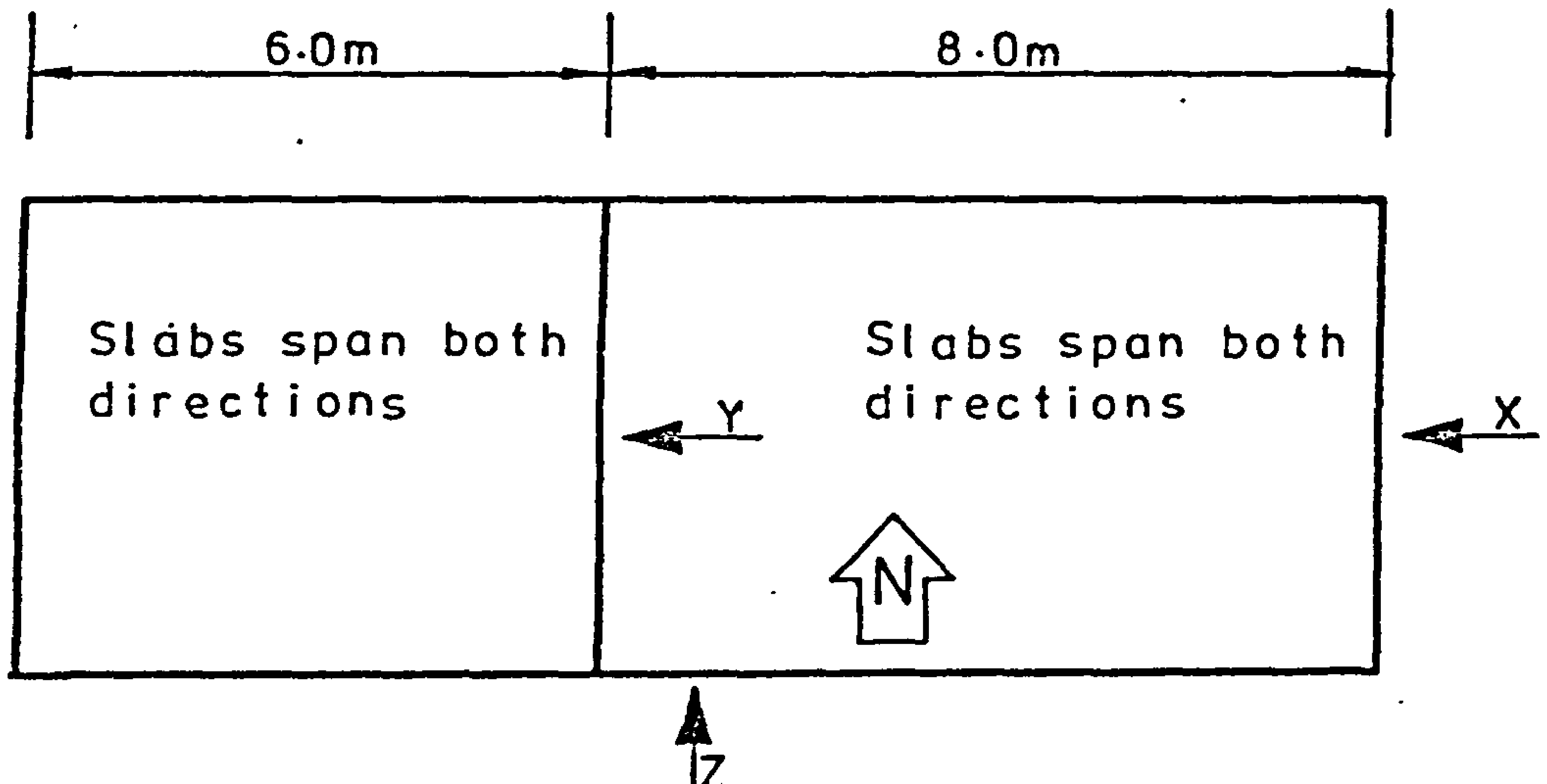
The application of moment redistribution will now be demonstrated using the frame shown diagramatically in figure 6.5. The sections were initially infeasible and the moment redistribution algorithm was used in order to prove the sections feasible. The algorithm converged after two full iterations. The progress of the algorithm can be studied with reference to figure 6.6 which shows the univariate searches at each joint as the algorithm proceeds. It can be seen that in general the relationships between stress ratios and end moments are made up of substantially linear segments. The total number of function evaluations and the value of the objective function for each full iteration are shown in Table 6.1.

Stage of Algorithm	Maximum Stress Ratio	Number of Function Evaluations
Inital Evaluation	1.0695	-
After First Evaluation	1.0196	30
After Second Evaluation	0.99924	34
	Total	64

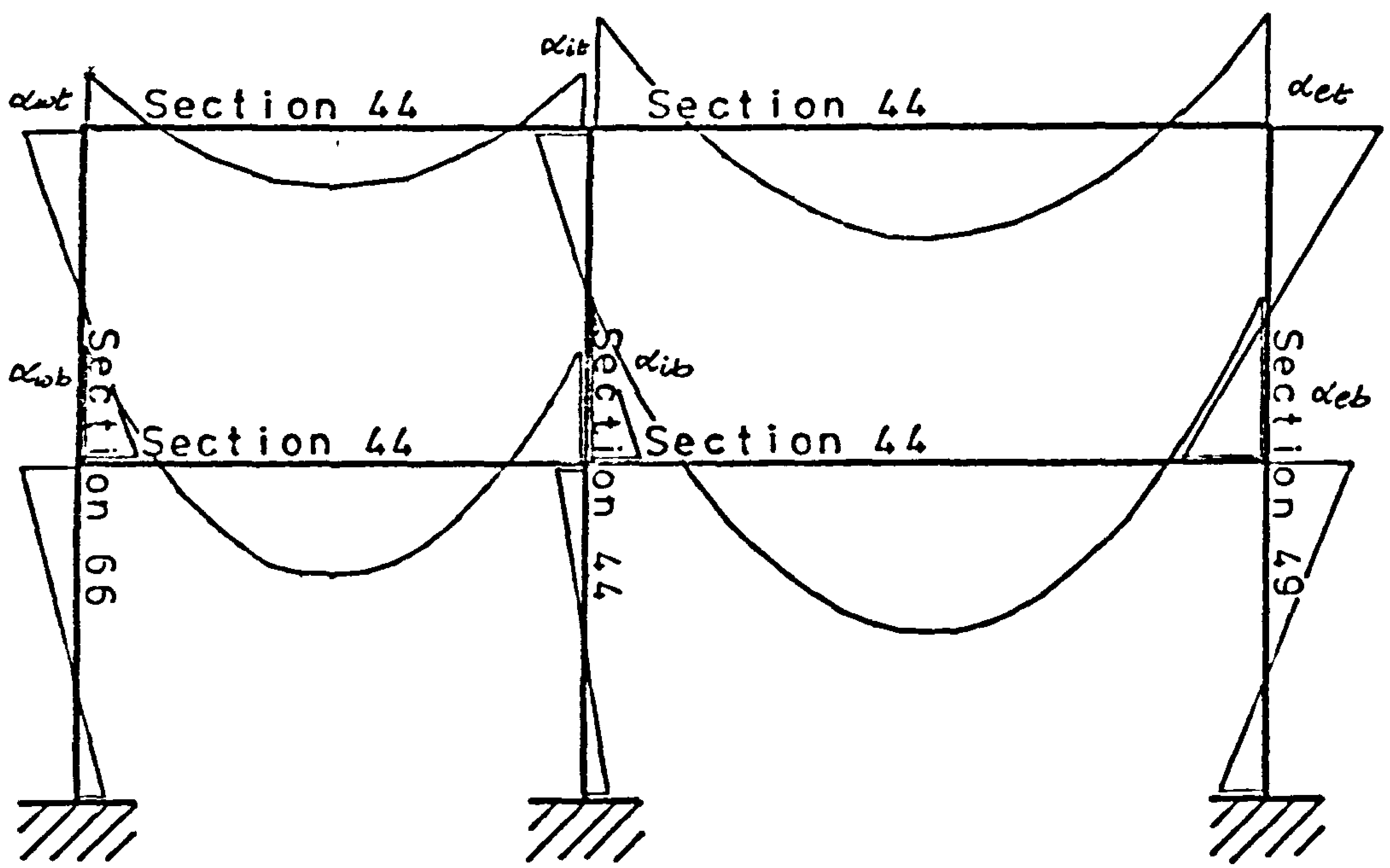
Table 6.1

6.3 GEOMETRIC CONSTRAINTS

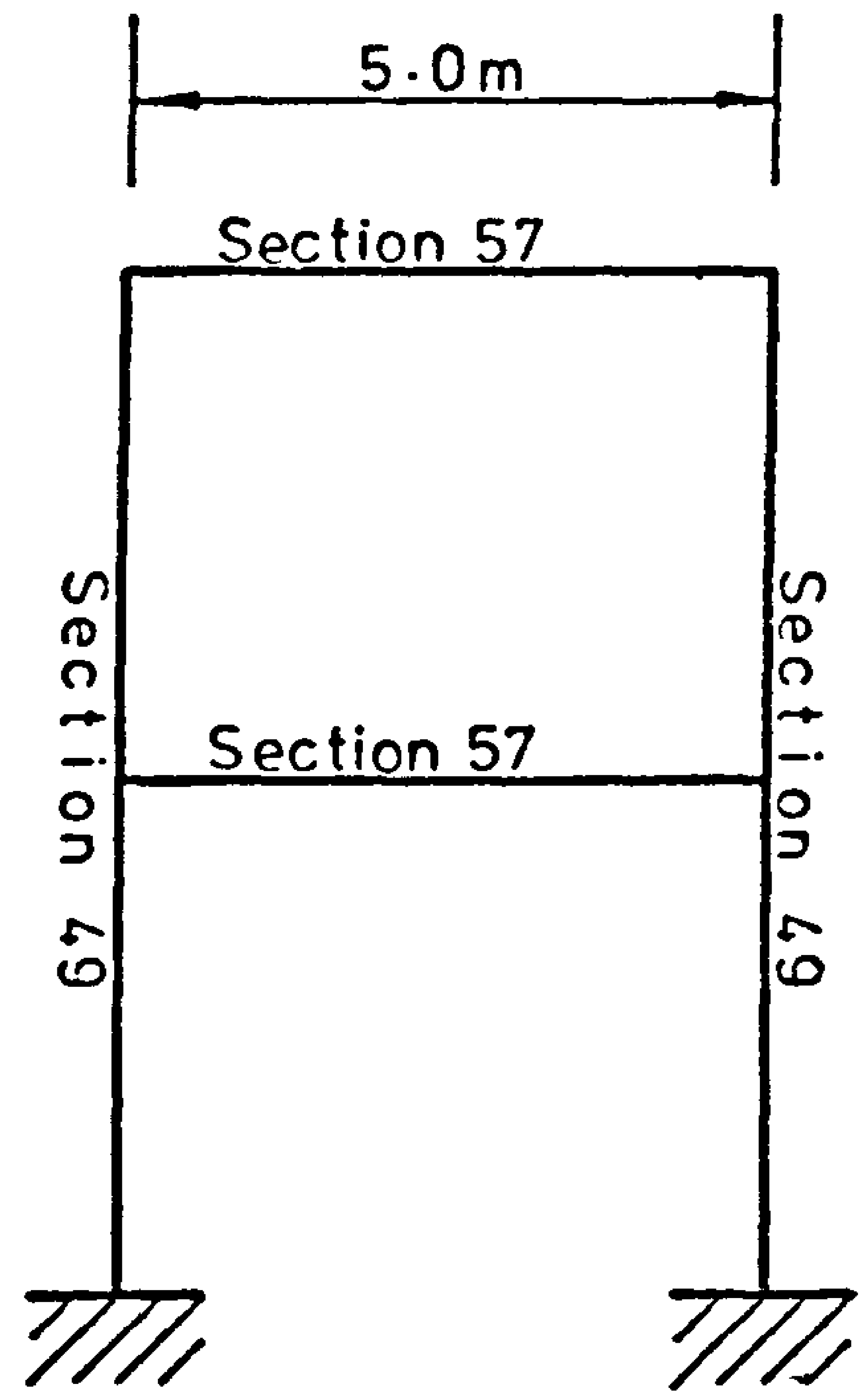
Geometric constraints are used to ensure that the members of the frame will fit together. Specifically, the meeting members must be of such sizes that they may be connected together using those connections



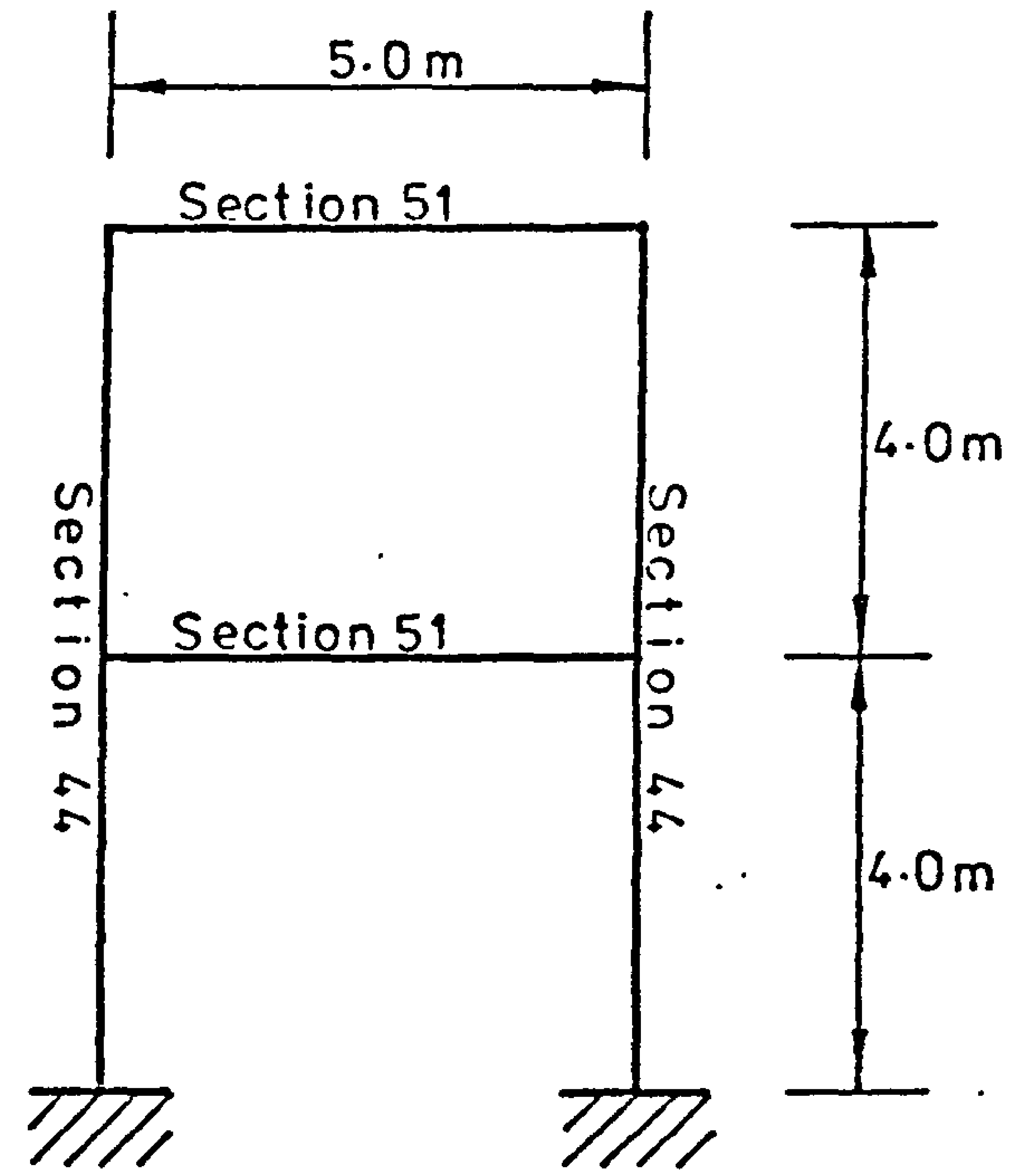
PLAN



ELEVATION Z



ELEVATION X



SECTION Y

α_{wt}, α_{it}, α_{et}, α_{wb}, α_{ib}, α_{eb} are proportions of moments with no redistribution

LOADING

	1st Floor	Roof
Dead	5·8 KN/m²	5·8 KN/m²
Live	6·2 KN/m²	1·6 KN/m²

FIG. 6.5 MOMENT REDISTRIBUTION ALGORITHM EXAMPLE FRAME

FIG. 6.6 MOMENT REDISTRIBUTION ALGORITHM EXAMPLE
RELATIONSHIPS BETWEEN END MOMENTS AND STRESS RATIOS

described previously. For example where stanchions are spliced, the upper section should be shallower than the lower one. These constraints will now be considered with reference to figure 6.7.

a) Stanchion Taper Towards Top of Building

The depth of the column section in the lower group must be larger than the depth of the column section in the upper group.

$$D_j \geqslant D_i \qquad - (1)$$

b) Major Axis Beams to Fit Stanchion Within 25mm

The breadth of the stanchion plus 25mm is greater than the breadth of the beam.

$$\left.\begin{array}{l} B_j + 25 \geqslant B_k \\ B_j + 25 \geqslant B_m \end{array}\right] - (2)$$

c) Minor Axis Beams to Fit Within the Web of the Stanchion

The depth between fillets of the stanchion is greater than the breadth of the beam.

$$\left.\begin{array}{l} D_{BFj} \geqslant B_L \\ D_{BFj} \geqslant B_n \end{array}\right] - (3)$$

d) Minor Axis Beams to Fit Between Column Stiffeners if Required

The minimum size of the minor axis end plate must be less than the depth between fillets of the major axis beams.

$$\left.\begin{array}{l} D_{BF_k} \geqslant 4tf_1 + D_1 \\ D_{BF_k} \geqslant 4tf_n + D_n \\ D_{BF_m} \geqslant 4tf_1 + D_1 \\ D_{BF_m} \geqslant 4tf_n + D_n \end{array}\right] - (4)$$

The last four constraint equations assume that the minor axis beams will always be shallower than the major axis beams. In certain cases, for instance when minor axis beams are more heavily loaded than

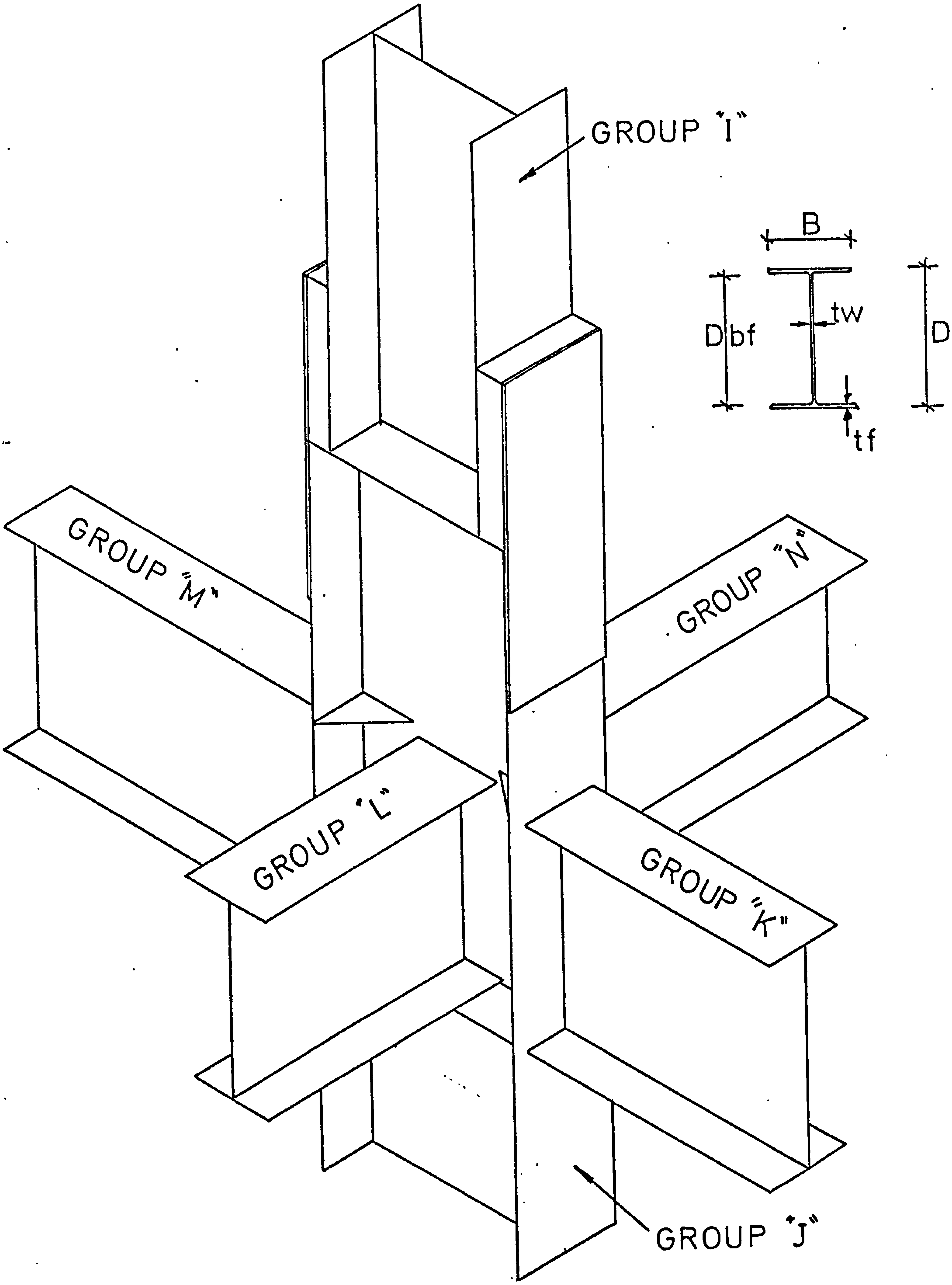


FIG. 6.7 GEOMETRICAL CONSTRAINTS

major axis beams, these constraints may be relaxed by the user. The inclusion of these constraints will reduce considerably the number or feasible combinations in the cases where they are required.

The constraints are formulated automatically at the beginning of the optimization stage. The constraints are stored within constraintmatrices for each type of constraint. Thus if a section is proposed for each of ng groups it is necessary to check the constraints, for example:-

Constraint type 1 if $D_c\,(i,j) = 1$ and $D_i \geqslant D_j$ then the constraint is not violated.

The other constraints are dealt with in a similar manner. This method of dealing with geometric constraints is very efficient because a frame may be checked for the satisfaction of these constraints without the use of the design checking section. This procedure reduces the size of the design space with little effort. This system of dealing with geometric constraints is easily incorporated into the optimization algorithms used and the details are discussed with the description of each algorithm.

6.4 ITERATIVE DESIGN ALGORITHM

The aim of the iterative design algorithm is to select an economic set of sections which will satisfy all constraints. The algorithm uses the results of one design to choose sections for the next design. This type of algorithm was found to be very efficient when discrete sections are used, because rapid improvements in the objective function are usually made in only a few iterations. The resulting design, which may or may not be optimal, can then be used as a starting point for a local search algorithm. Member selection is by the use of iterative formulae and by the satisfaction of geometric constraints.

The constraints which the algorithm considers may be classified as follows:-

(a) Non redistributive constraints:- These constraints are independant of the section sizes chosen for other members and have the same form and value in every iteration.

(b) Redistributive constraints:- These constraints are dependant on the section sizes chosen for other members and vary in form and value from iteration to iteration.

(c) Geometrical constraints:- These constraints have been described previously and they must be satisfied during each iteration.

Each of these types of constraints is considered in a different way by the algorithm in order to ensure a final design which is feasible.

The modus operandi of the algorithm is as follows:-

(a) An initial set of sections for each group of members is chosen by the user.

(b) This set of sections is checked for feasibility with respect to redistributive and non redistributive constraints. Approximate expressions for the redistributive constraints are devised and stored.

(c) For each group of members in turn, all the available sections are tried to see if they are feasible with respect to the approximate constraint functions and with respect to the geometric constraints. The section which satisfies these conditions with the lowest basic section cost is then chosen as the section for that group.

(d) If the algorithm selects the same set of sections for two consecutive iterations convergence has occured and the process is terminated. If convergence has not occured within a preset number of iterations step(e) is executed, otherwise a return is made to step(b).

(e) Additional constraints, which will be described later, are applied to ensure convergence in a finite number of steps, a return is then made to step(b).

The success of the algorithm depends to an extent on the choice of the approximate constraint functions. Properties of these functions which help to bring about convergence and which have been identified include:-

(a) The function must reject sections which are infeasible with respect to the true constraint.

(b) The function must represent a good approximation to the true constraint function over the whole range of sections (i.e. divergence must not occur).

(c) The function must show that a section, which is just feasible with respect to the true constraint function, is feasible with respect to the approximate constraint function. This aids convergence.

It has not been possible to derive approximate functions which satisfy all of the above conditions, for this reason convergence does not always occur. The non redistributive constraints and the redistributive constraints which employ simple iterative equations will not be examined in detail. A simple iterative equation takes the form.

$$x_{new} > x_{old} \times \frac{f_{old}}{f_{max}}$$

where x_{old} = a section property of the current section, such as section modulus.

f_{old} = a stress or deflection produced by the current section, such as bending stress.

f_{max} = the allowable stress or deflection, such as the yield stress.

x_{new} = the same section property of the new proposed section.

6.4.1. Application to the BCSA Design Method

6.4.1.1 Major and Minor Axis Beams

redistributive constraints include bending strength and deflection. The non redistributive constraints include local buckling and shear strength. The application of all these constraints and the use of simple iterative equations is quite straightforward and will not be considered further.

6.4.1.2 Stanchions

The only non redistributive constraint which is considered is that of local buckling. The other constraints consider the stability and strength of the stanchion. The requirement for strength is (as shown in Chapter 2):-

$$\frac{P}{A} + \frac{N_x M_x C_{mx}}{Z_x} + \frac{N_y M_y C_{my}}{Z_y} + f_i < F_y$$

The following constraint function is used to model this constraint:-

$$\frac{C_1}{A} + \frac{C_2}{Z_x} + \frac{C_3}{Z_y} < 1$$

This constraint can also be used for the ends of the stanchion. The factors C_1, C_2 and C_3 can be determined from current design. For each stanchion there are three critical sections and eight load cases, consideration of all such constraints would require much computer storage. The approach used is to store C_1, C_2 and C_3 for the member and load case which produces the greatest stress. Clearly the critical member and load case may change from iteration to iteration, this can result in non-convergence.

It was found necessary to apply further constraints in order to increase the chance of convergence. Consider figure 6.8 which shows two of the stress constraints for a particular stanchion. For simplicity the value of f_i is assumed to be constant. The first constraint is for a load case where M_x is zero. All feasible combinations of

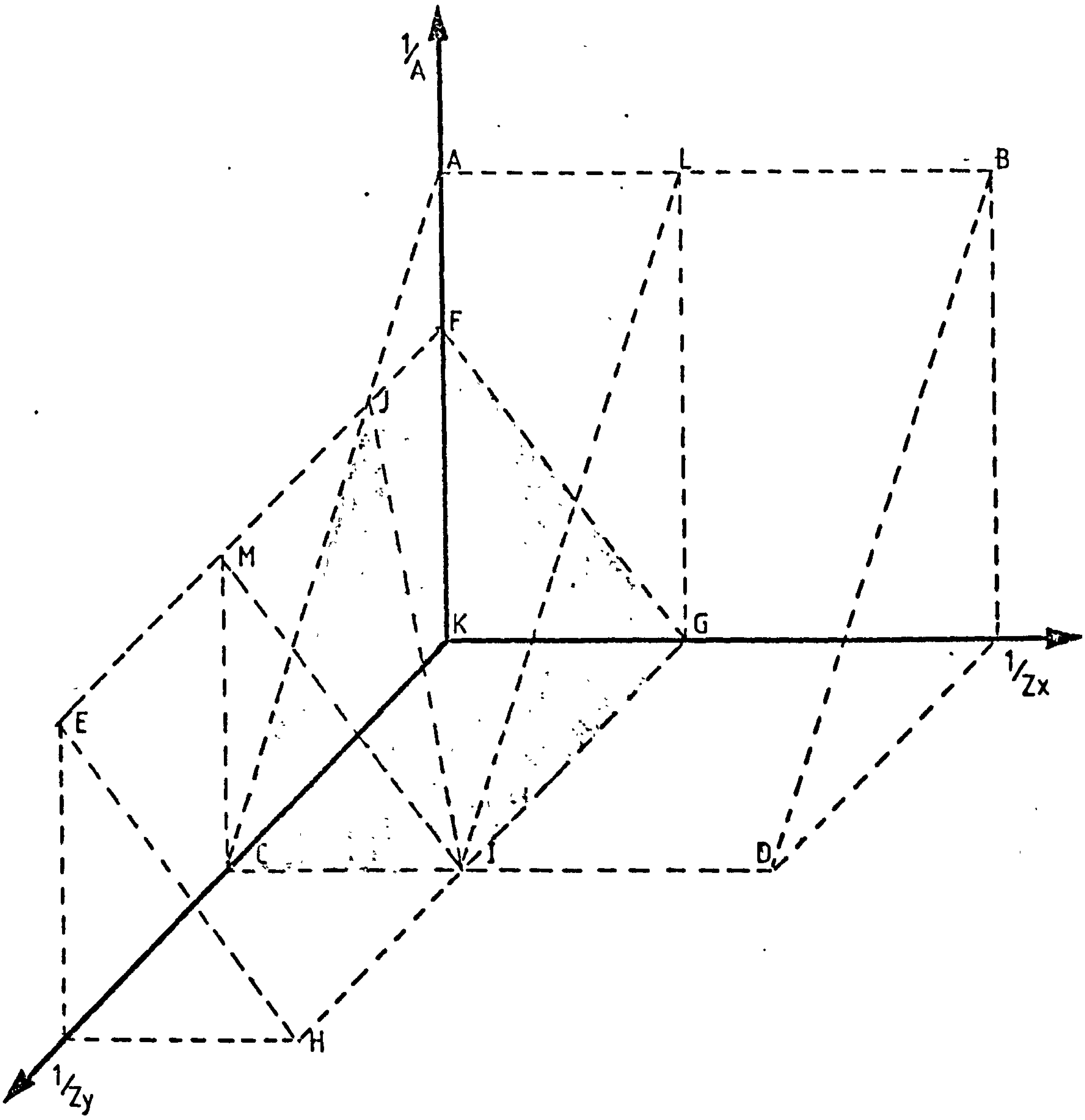


FIG. 6.8 TYPICAL CONSTRAINT SURFACES FOR A STANCHION

Z_x, Z_y and A lie below the plane a, b, d, c. The second constraint is a case where M_y is zero, in this case all feasible combinations of Z_x, Z_y and A lie beneath the plane e, f, g, h. The feasible combinations of Z_x, Z_y and A which satisfy both constraints lie within the five sided space j, f, g, i, c, k. If only one constraint is satisfied at each iteration cycling may occur between the constraints. It is therefore necessary, when considering only one constraint, to take other constraints into account. If it is assumed that A and Z_y may take on any value the minimum value Z_x may take is given by:-

$Z_x \geqslant (C_3)$ max

and similarily:-

$Z_y \geqslant (C_2)$ max

$A \geqslant (C_1)$ max.

In the example given in figure 6.8 the combinations of Z_x, Z_y and A which satisfy these constraints are considerably reduced. For instance, when considering the first constraint and the limit on Z_x the space inside the perimeter a, l, g, i, c is deemed satisfactory. Resulting in a considerable improvement over the case where limits on Z_x, Z_y and A are not considered. The above constraint functions are approximate because the values of C_1, C_2 and C_3 vary with the column section and with the other members in each limited frame. These constraints do not always produce convergence.

Consideration of stability of the stanchion requires that the axial load should not be greater than the Euler buckling load and that the stress due to initial imperfections should never be greater than the yield stress. The stress due to initial imperfections is related to the Euler buckling load and so, the first condition is always met when the second condition is met. The procedure used to ensure meeting the second condition is to derive a minimum value of the minor axis second moment of area. The equations for initial imperfections can be solved to give a value of the minor axis radius of gyration. If the area of the current section is then multiplied by the square of the radius of gyration, a second moment of area can be derived. Iteration using the radius of gyration only was found to produce divergence.

6.4.2 Application to the J.C.R. Design Method

6.4.2.1 Major and Minor Axis Beams

Redistributive constraints include bending strength and deflection. The non redistributive constraints include local buckling and shear strength. The application of all these constraints and the use of simple iterative equations is quite straightforward and will not be considered further.

6.4.2.2 Stanchions

The non redistributive constraints which are considered consist of local buckling and a limit on minor axis slenderness ratio. The other constraints consider the stability and strength of the stanchion. The requirement for strength is (as shown in Chapter 2):-

$$f_a + m\,f_y + f_x + f_{ic} \leqslant F_y$$

The following constraint function is used to model this constraint:-

$$\frac{C_1}{A} + \frac{C_2}{Z_x} + \frac{C_3}{Z_y} \leqslant 1$$

The factors C_1, C_2 and C_3 can be determined from the current design. For each stanchion there are four possible load cases, consideration of all such constraints would require much computer storage. The approach used is to store C_1, C_2 and C_3 for the member and load case which produces the greatest stress. Clearly the critical member and load case may change from iteration to iteration, this can result in non convergence. Further constraints were applied in the same manner as the B.C.S.A. design method, these constraints are:-

$Z_x \geqslant (C_2)$ max

$Z_y \geqslant (C_3)$ max

$A \geqslant (C_1)$ max

Again these constraints do not always produce convergence because the approximate constraints do not fully model the true constraints and the values of C_I, C_2 and C_3 vary as the stanchion and beam sections vary.

Consideration of stability of the stanchion requires that the axial load should not be greater than the elastic critical load. The elastic critical load is dependant on the minor axis second moment of area and the stiffness of the adjoining members. The approach used is to use a simple iterative formula which gives a minimum value of the minor axis second moment of area. The effect of the stiffness of adjoining members is assumed to be small. This approximation was found to be satisfactory.

The derivation of approximate constraint functions which consider lateral torsional buckling proved very difficult because, as previously mentioned, the charts given in the code contain a set of empirically developed, lower bounds for which no analytical solution exists. The problem was further complicated by the fact that there are two dependant variables, the ratio of section depth to flange thickness and the minor axis radius of gyration. If the current section is unsatisfactory and the depth to thickness ratio is reduced for the next section, it is possible that the radius of gyration will also increase. In this case the depth to thickness ratio will have been overestimated with respect to that actually required. The use of depth to thickness ratio alone results in iteration between heavy universal column sections and light universal column sections. The following approximate constraint function was used:-

$$\left({}^{D}/_{T}\right)_{new} \leq \sqrt{\left({}^{D}/_{T}\right)_{req'd} \left({}^{D}/_{T}\right)_{old}}$$

where ${}^{D}/_{T}$ = Stanchion depth to flange thickness ratio

old = The property of the current section

req'd = The required property if the radius of gyration of the current section remains unaltered.

new = The property of the next section.

This iteration equation was derived empirically. The use of the square root function ensures that corrections are made in the right direction but are not as great as would occur if straight iteration was used. The procedure used helps to correct for any changes that occur in the radius of gyration. This procedure has been found satisfactory in most practical cases.

6.4.3 Application to the B.R.S. Design Method

6.4.3.1 Major and Minor Axis Beams

Redistributive constraints include bending strength and deflection. The non redistributive constraints include local buckling and shear strength. The application of all these constraints and the use of simple iterative equations is quite straightforward and will not be considered further.

6.4.3.2 Stanchions

The only non redistributive constraint which is considered is that of local buckling. The other constraints consider the stability of the stanchion. The treatment of stiffness constraints is slightly different to that of the other design methods because the constraint equations are non linear. The approximate constraint functions will now be described. The condition that a plastic hinge must not occur at the end of a stanchion is modelled using a function of the form:-

$$1 \geqslant \frac{C_1}{Z_{ex}} \quad \frac{C_2}{A}$$

where C_1 & C_2 = Constants evaluated during the previous iteration

A = The sectional area of the section

Z_{ex} = The major axis elastic section modulus of the section.

The condition that the stanchion as a whole must not buckle under major or minor axis bending is modelled using the approximate constraint function:-

$$1.0 \geqslant C_3 \frac{C_x}{R_x} \qquad\qquad 1.0 \geqslant C_x C_5$$

where C_3, C_4 & C_5 = Constants evaluated during the previous iteration.

C_x = $f(C_2, I_y, A)$, an approximation to the elastic critical load.

R_x = $f(C_2, C_4, Z_{ex}, A)$, an approximation to the stiffness reduction factor.

I_y = The minor axis second moment of area of the section.

The values of C_1 to C_5 are found for the stanchion and load case which is the most critical. The critical stanchion and load case may not be the same in each iteration, this may result in non convergence. The factors C_1 to C_5 change in each iteration and are dependant on the adjacent members. The factor C_5 is a complex function, which is a lower bound on actual test results, because of this it is not possible to take account of all the relevant variables. The value of C_5 does not vary significantly for sections of similar weight and therefore convergence is not seriously impaired.

The control of lateral torsional buckling is dealt with in the same way as for the J.C.R. design method except that the radius of gyration is multiplied by R_x.

6.4.4 Geometric Constraints

The application of geometric constraints to the iterative design algorithm will now be described.

During the stage of the algorithm in which the next set of trial sections are selected it is necessary to consider the geometric constraints. Consider the selection of the section for group j, the sections have already been selected for all of groups i (i=1, ..., j-1) and the sections for all other groups have not been determined. Therefore only the sections chosen for groups i can be used to help select the section for group j. The section is selected for group j by considering a subset of geometric constraints. For instance the depth of section constraints are:-

$$D_k > D_m$$

for $k = j$ and $m = 1 \ldots\ldots j$

and $m = j$ and $k = 1 \ldots\ldots j$

This treatment has the effect of considering only part of each constraint matrix for each group. When all the groups have been processed the whole matrix will have been considered. This process is shown in figure 6.9, which shows a typical constraint matrix and the constraints considered. The constraints which apply to the sections which are as yet unknown are within the region to the right and below the part of the matrix being considered. The region to the left and above the part of the matrix being considered does not affect the choice of section for the current group.

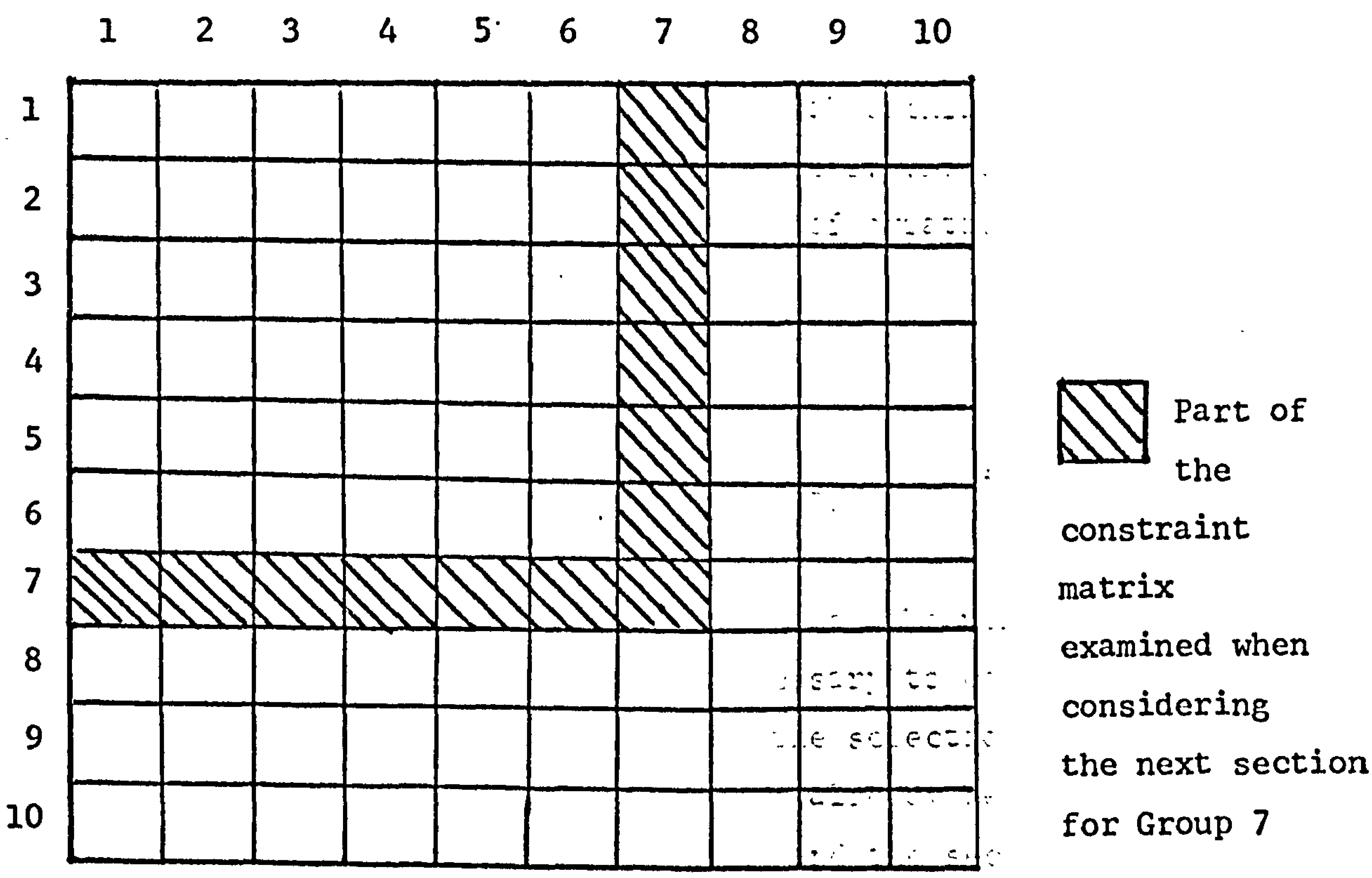


Figure 6.9 Geometric Constraint Checking for the Iterative Design Algorithm

In practice it is necessary to place restrictions on the grouping of members in order that the algorithm can work. The grouping of members must ensure that the selection of a section is always possible. This will not necessarily be the case if the stanchion sections above and below a stanchion have been selected before the stanchion being considered. These restrictions are satisfied if the major axis beams are grouped first, followed by the minor axis beams and then the stanchions, starting at the top of the building and working downwards.

A facility was included within the programme to allow the relaxation of the minor axis beam depth constraints, which will allow the selection of minor axis beam sections deeper than the major axis beam sections. In this case it is necessary to include an approximate constraint function, which ensures that, when a minor axis beam at a joint is deeper than either of the major axis beams, the column section chosen will not require compression stiffeners.

6.4.5 Constraints which bring about Convergence

When convergence has not occured within a specified number of design cycles it becomes necessary to force convergence to occur. The method used to ensure convergence will now be described.

When a design has not converged, the section for a group (or groups) will be cycling between two or more designs of which some will be infeasible. If the section for a particular group is infeasible with respect to stress constraints after a design cycle, any future section is then restricted to being more expensive than the current section. These lower bounds on member cost are updated whenever infeasible sections are used. Therefore after a finite number of iterations the solution will converge on a feasible design. This method of enforcing convergence may not provide the optimum sections, because, when lower limits are set on section cost, a feasible section with a lower basis cost may not have been considered.

6.4.6 Summary of the Iterative Design Algorithm

The iterative design algorithm may be used for any frame and with any starting set of sections. The principle disadvantages of the method are that:-

(a) Section basis costs only are used for comparing designs.

(b) The sections chosen for stanchions at the top of the building will affect the choice of sections lower down, via the geometric constraints, resulting in the final design being non optimal.

(c) End moment constraints cannot be considered because the constraints must not vary significantly from one iteration to the next.

(d) The final sections chosen by the algorithm may be such that suitable connections cannot be designed.

The principle advantages of the algorithm are:-

(a) The formulation of the algorithm is simple and has an intuitive appeal to a designer.

(b) The algorithm will operate with discrete sections and numerous constraints.

(c) The sections do not have to be arranged in any particular way, for instance in order of increasing cost.

The algorithm finds a set of sections for which all the members are feasible for all loading conditions, and which is close to being optimal with respect to the basis cost of the sections. The algorithm provides a solution which is close to optimal and which can be refined using two further algorithms.

6.4.7 Example of the Iterative Design Algorithm

An example of the use of the iterative design algorithm will now be given. The frame to which the algorithm will be applied is shown in figure 6.10. The application of the J.C.R. design method will be given in detail, the B.C.S.A. and B.R.S. design methods will be described briefly.

The heaviest possible section was used as a starting section for all groups. The progress of the iterations are shown in Table 6.2. It can be seen that convergence occurred very rapidly from a starting point which was remote from the

GROUP	ITERATION NUMBER				
	1	2	3	4	5
1	1	42	42	42	42
2	1	27	27	27	27
3	1	63	63	59	59
4	1	52	52	52	52
5	1	46	33	32	32
6	1	42	33	32	32
7	1	31	31	31	31
8	1	41	33	33	33
9	1	40	31	30	30
10	1	32	30	29	29
11	1	26	26	26	26
12	1	25	22	22	22
BASIS COST	33380	4914	5106	5178	5178

TABLE 6.2 - Section Numbers for Each Iteration J.C.R. Design Method

ELEVATION X

ELEVATION Y

STANCHION GROUPS

GROUP	NODES	STOREYS	
5	1 3 7 9	1	2
6	2 8	1	2
7	4 6	1	2
8	5	1	2
9	1 3 7 9	3	4
10	2 8	3	4
11	4 6	3	4
12	5	3	4

BEAM GROUPS

GROUP	NODES	FLOORS
1	1-2-3 7-8-9	ALL
2	4-5-6	ALL
3	1-4-7 3-6-9	ALL
4	2-5-8	ALL

LOADING

	ROOF	ALL OTHER FLOORS
DEAD	5·8 KN/m²	5·8 KN/m²
LIVE	1·6 KN/m	6·2 KN/m²

FIG. 6.10 ITERATIVE DESIGN EXAMPLE FRAME

optimum. The final cost of the frame including fabrication was £9,779.10. The progress of the iterations for each type of member will now be considered.

The major axis beams are represented by groups 1 and 2. It can be seen that the final sections are reached after the first iteration. The two redistributive constraints are bending strength and deflection. The section chosen for group number 1 predicts a minimum plastic section modulus of $1.04 \times 10^6 mm^3$ and a minimum major axis second moment of area of $92.2 \times 10^6 mm^4$ compared with $1.04 \times 10^6 mm^3$ and $185.8 \times 10^6 mm^4$ respectively for the section chosen (42). The corresponding figures for group 2 are a required plastic section modulus of $2.08 \times 10^6 mm^3$ and a required moment of inertia of $209.2 \times 10^6 mm^4$ compared with $2.36 \times 10^6 mm^3$ and $552.5 \times 10^6 mm^4$ respectively for the section chosen (27).

The minor axis beams are represented by groups 3 and 4. For group 3 the final section is reached after three iterations and for group 4 the final section is reached after one iteration. The progress of these iterations are shown in Table 6.3. It can be seen that in all cases the required section property rapidly settles down with little change from iteration to iteration resulting in good convergence properties.

AFTER ITERATION NUMBER	GROUP 3		GROUP 4	
	I required $\times 10^6$	Z_e required $\times 10^6$	I required $\times 10^6$	Z_e required $\times 10^6$
1	17.34	0.298	34.68	0.597
2	13.08	0.288	32.70	0.592
3	12.54	0.283	31.16	0.584
4	12.52	0.281	30.59	0.580

TABLE 6.3

The stanchions represented by groups 5 to 12 can be seen to vary in their convergence properties. Groups 7 to 11 converge after one iteration, groups 8 to 12 converge after two iterations and groups 5,6,9 and 10 converge after three iterations. The iteration history of groups 5 and 9 will be examined more fully. The factors used in the approximate constraint functions for group 5 are shown in Table 6.4. The value of $(C_2)max$ does not vary because the greatest value of C_2 is at the top of the upper stanchion where no elastic redistribution occurs. The values of C_1 and $(C_1)max$ are constant because the lower stanchion is critical in all iterations. All the other factors show slight changes during the iterations showing that the constraints do not vary significantly.

	ITERATION NUMBER			
	1	2	3	4
C_1	1411	1411	1411	1411
$C_2 \times 10^6$	0.564	0.555	0.562	0.563
C_3	19478	56210	63879	65762
$(C_1)max$	1411	1411	1411	1411
$(C_2)max \times 10^6$	0.640	0.640	0.640	0.640
Minor axis moment of inertia required $\times 10^6$	0.827	1.077	1.153	1.169
Maximum Stress Ratio	0.07	1.29	1.06	0.94

TABLE 6.4

The maximum stress ratio shows whether the section used for the iteration is feasible with respect to stress constraints, a ratio less than one shows that the section is feasible. The differences between C_2 and (C_2)max, C_3 and (C_3)max are not very great showing that the use of additional constraints restricts the number of trial designs significantly.

The factors used in the approximate constraint functions for group 9 are shown in Table 6.5. Again the factors can be seen to vary only slightly as the sections are varied resulting in good convergence. It can be seen that in this case C_2 and (C_2)max and C_3 and (C_3)max are equal from the second iteration onwards showing that the maximum moments

	ITERATION NUMBER			
	1	2	3	4
C_1	2833	2169	2169	2169
$C_2 \times 10^6$	0.521	0.660	0.620	0.616
C_3	17240	66872	78218	78524
(C_1)max	2833	2833	2833	2833
(C_2)max $\times 10^6$	0.564	0.660	0.620	0.616
(C_3)max	20005	66872	78218	78524
Minor Axis Moment of Inertia required $\times 10^6$	1.603	1.989	2.110	2.119
Stress Ratio	0.01	1.28	1.02	0.92

TABLE 6.5

in each direction occur concurrently. The use of geometric constraints could tend to require that larger sections are used than are required to satisfy stress constraints, however, in this case, examination of the stress ratios shows that this is not the case. Examination of the factors shows that the constraints are more severe for group 9 than group 5. The behaviour of the other stanchions is similar to groups 5 and 9, however the exact behaviour varies depending on the position of the stanchion within the frame.

The algorithm was also used with the B.C.S.A. design method to design the same frame. The application in this case proved rather different in that the algorithm failed to reach convergence and convergence had to be brought about. The progress of the iterations for the B.C.S.A. design method is shown in Table 6.6. Examination of this iteration history shows that cycling occurs over four iterations and that convergence is brought about rapidly. The final solution is the cheapest feasible solution to be investigated by the algorithm. The differences between successive designs is generally very small, showing that the algorithm rapidly reaches a design close to the optimum. The final cost including fabrication was £9,941.60, slightly greater than the cost of the J.C.R. design.

The B.R.S. design method showed similar behaviour to the J.C.R. design method. The progress of the iterations are shown in Table 6.7

It can be seen that the number of iterations required are again small and that all the designs checked have similar costs. The final cost of the frame including fabrication is £9,33.10, which is substantially less than the cost of the J.C.R. design.

GROUP	ITERATION NUMBER												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	42									Convergence brought about from this Iteration onwards		42
2	1	27											27
3	1	65	104										104
4	1	58											58
5	1	90											90
6	1	90											90
7	1	28	30										30
8	1	33	91	33	91	33	91	33	91	33	91	32	93
9	1	29											29
10	1	90											90
11	1	27	23	25									25
12	1	23	88	22	85	23	88	22	85	23	88	22	85
Basis Cost	-	5165	5226	5218	5231	5202	5249	5218	5231	5202	5249	5226	5230

TABLE 6.6. - SECTION NUMBERS FOR EACH ITERATION FOR B.C.S.A. DESIGN METHOD

GROUP	ITERATION NUMBER				
	1	2	3	4	5
1	1	42	42	42	42
2	1	27	27	27	27
3	1	63	63	63	63
4	1	52	52	52	52
5	1	47	43	48	48
6	1	67	91	61	61
7	1	32	31	31	31
8	1	47	91	40	41
9	1	47	42	42	42
10	1	41	41	41	41
11	1	30	30	30	30
12	1	25	27	27	27
Basis Cost	-	4633	4813	4697	4689

TABLE 6.7 - SECTION NUMBERS FOR EACH ITERATION
B.C.S.A. DESIGN METHOD

GROUP NUMBER	STORIES IN GROUP	SECTION NUMBERS FROM WHICH COMBINATIONS ARE SELECTED				
1	1 & 2	45	36	32	39	35
2	3 & 4	28	34	30	27	29
3	5 & 6	30	27	29	23	26
4	7 & 8	23	26	25	22	24

TABLE 6.8

6.5 THE ENUMERATION ALGORITHM

The enumeration algorithm is an exact method of optimising a small number of variables, when all other variables are held constant. Consider initially the total enumeration of all the combinations of sections for a frame. If there are m groups of members and n available sections for each group, there are n^m possible combinations. Unless m is kept small the amount of computer time required is very large. For this reason it is necessary to allow the user to explore the design space interactively and to use his experience to reduce the computer time required. The enumeration algorithm permits this and will now be described.

The user chooses a set of groups on which to apply the algorithm. A set of available sections is chosen by the user for each group. The user specifies the number of combinations of sections which are to be checked. The algorithm evaluates each combination, checking geometric and stress constraints. Any combination which is infeasible is disregarded. The total frame cost is calculated for each feasible combination. The feasible combination with the lowest cost is the optimum combination. Computational features are used in order to maintain control by the user and to reduce computation, these features will now be described.

6.5.1 Limiting the Number of Combinations

The method used to evaluate the combinations starts with the combination of sections with the lowest basis cost and then works upwards through the available sections. The sequence of enumeration seeks to work upwards from the cheapest combination to the dearest combination. Denote S_{ij} as the sequence of numbers which have a sum of j with i variables. For instance S_{22} consists of the sequence (2,0), (1,1), (0,2). The sequence which is used consists of:-

for m = 0 ..... m maximum
for i = 0 ..... m
Variable n takes the values of i
Variables 1 ..... (n-1) take the values of $S_{(n-1)(m-i)}$

It is relatively simple to generate the next combination from the current terms in the sequence and the sum of the variables (m). The user limits the number of combinations by limiting the maximum value of m.

6.5.2 Limiting the Number of Members to be Checked for Stress Constraints

The limited frame concept allows stress constraints to be checked for members which are in the groups being varied and the members directly attached to those members. This reduces the amount of computation significantly. Failure of any member to satisfy the stress constraints implies failure and therefore rejection of the current combination of sections. This approach is also used when considering end moment constraints.

6.5.3 Checking of Geometric Constraints

The checking of geometric constraints uses less computer time than the checking of stress constraints. Geometric constraints are therefore used to eliminate combinations of sections. In this application all of the constraints are checked for each combination of sections.

6.5.4 Limiting the Basis Cost of the Sections

Consider a feasible frame in which the basis cost of the sections is C_s and the total cost (including fabrication) is C_t. If V_c denotes the fabrication coefficient, defined by:-

$$C_t = C_s \times V_c$$

If the user makes an estimate of the minimum value of V_c, termed (V_c)min. Then any design may be disregarded which satisfies

$$C_s \times (V_c)\text{min} > C_{tf}$$

where C_{tf} = the total cost of the current cheapest feasible design. Values of V_c are provided by the programme for each feasible design. If (V_c)min is taken as 1.0, the equation represents the fact that the basis cost of the sections cannot be greater than the current minimum cost of the frame.

It was also found to be possible to set lower limits on the section basis cost for each group. The lower limits were found by designing each member, ignoring geometric constraints, modifying the limited frames used and modifying some of the constraints. Major axis beams are designed assuming fixed ends. Minor axis beams are designed assuming midspan and end moments are equal and when considering deflections assuming fixed ends. Stanchions are designed assuming all adjoining members are the largest possible section, this has the effect of producing smaller moments and higher collapse loads than is the case in the final design. It is possible to use the resulting lower limiting sections as a guide to help in the choice of sections to be used as possible sections when optimising. This approach cannot be used when end moment constraints are considered.

Another approach, which was tried and rejected, sought to set upper limits on the section basis cost for each group. If the total cost of the frame using the lower bound sections previously chosen is given by:-

$$= \sum_{k=1}^{ng} C_{sk} V_{ck}$$

the section basis cost of sections in group j must satisfy:-

$$C_{sj} < \frac{C_{tf} - \sum\limits_{\substack{k=1 \\ k \neq j}}^{ng} C_{sk} V_{ck}}{V_{cj}}$$

where ng = number of groups

C_{sk} = basis cost of section in group k using the smallest possible section

V_{ck} = estimated fabrication coefficient for group k

C_{sj} = basis cost of section for group j

V_{cj} = estimated fabrication coefficient for group j.

This equation expresses the fact that the cost of the section used in group j must be less than the current cheapest cost of the frame, minus the cheapest cost that all other parts of the frame can take. The fabrication coefficients, estimated by the user, were found to be difficult to estimate. The limiting section cost was found generally to be more costly than the sections chosen by experience. This approach was not therefore followed any further.

The first approach of using a limiting fabrication coefficient was the most useful of the above techniques, however this coefficient was found to be difficult to estimate.

6.5.5 Summary of the Enumeration Algorithm

The enumeration algorithm provides a convenient search method for use under the control of the user, and which uses the experience and observation of the user together with the application of constraints to reduce the number of combinations of sections evaluated. The major advantages of the algorithm are:-

(a) Members may be grouped together in any way

(b) All costs are taken into account

(c) End moment constraints can be applied in conjunction with the algorithm

(d) The algorithm does not involve making approximations of any constraints.

The major disadvantages of the algorithm are:-

(a) The algorithm is very inefficient when compared with dynamic programming

(b) The efficiency of the algorithm relies on the experience of the user, which may not be well developed

(c) The algorithm does not use information found during the design cycle

(d) Only a few groups may be optimised at any one time.

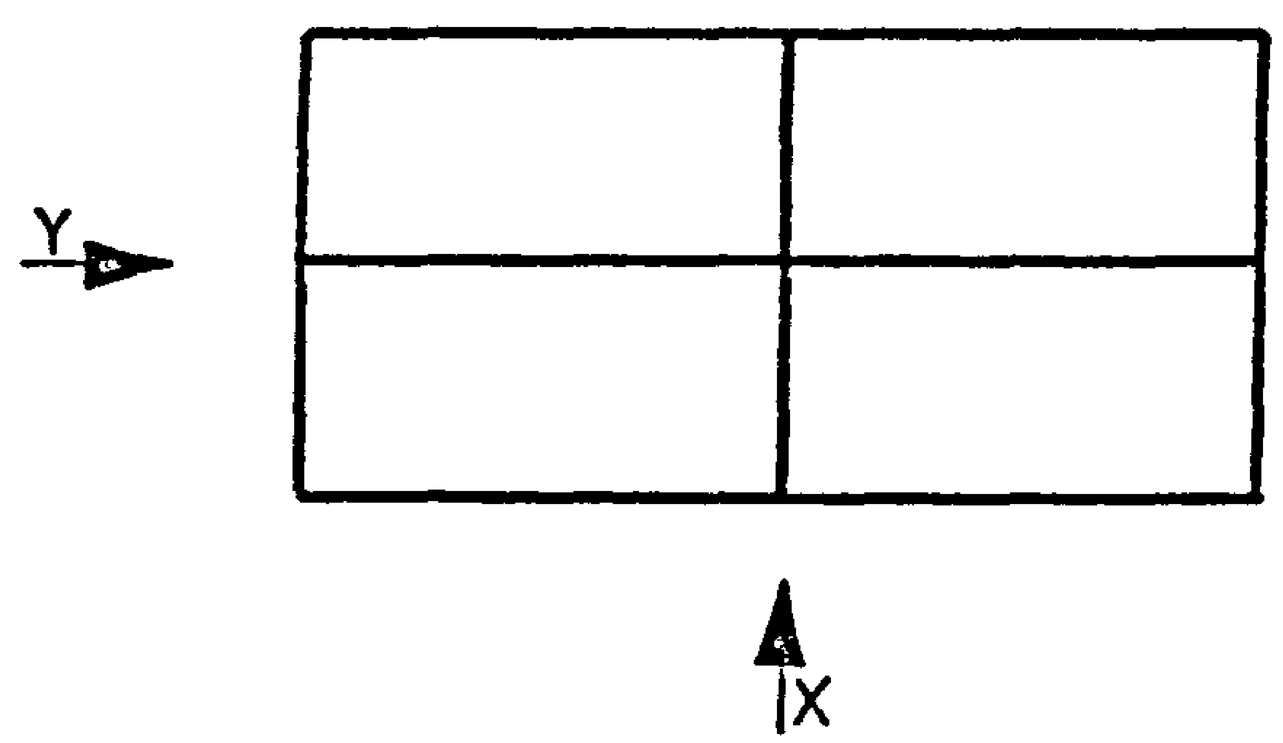
6.5.6 Example of the Use of the Enumeration Algorithm

The enumeration algorithm will be applied to the design of a corner stanchion of the frame shown in figure 6.11. The J.C.R. design method was used and the iterative design algorithm was used to find starting sections for the enumeration algorithm. Five sections were chosen for each stanchion group, two of these sections were cheaper than the starting section. The sections which were used are shown in Table 6.8.

The algorithm was operated considering geometric and stress constraints but neglecting, limiting the number of combinations and limiting the basis cost of the sections. There were 625 possible combinations, of which 225 (36%) failed due to geometric constraint violation, 352 (56%) failed due to stress constraint violation and 48 (8%) were feasible and were costed. The cheapest design was costed as £22,183.31 and consisted of sections 32, 30, 23, 23 for groups 1,2,3 and 4 respectively. The starting sections had a cost of £22,183.57 showing only a small reduction. The use of the resulting sections for all four corner stanchions showed a saving of £39.00 due to the effects of extra costs.

The cost of sections plus fabrication, neglecting extra costs, for the column string is approximately £1,000. The distribution of the designs within certain cost ranges are shown in figure 6.12. It can be seen that the majority of the designs are closer than 6% of the cost of the column string to the optimum.

Figure 6.13 shows a plot of basis section cost versus total frame cost for the feasible designs. The spread of points on this graph shows the difference between designs based on



PLAN ON ROOF

LOADING

	Roof	All Floors
Dead	5·8 KN/m²	5·8 KN/m²
Live	1·6 KN/m²	6·2 KN/m²

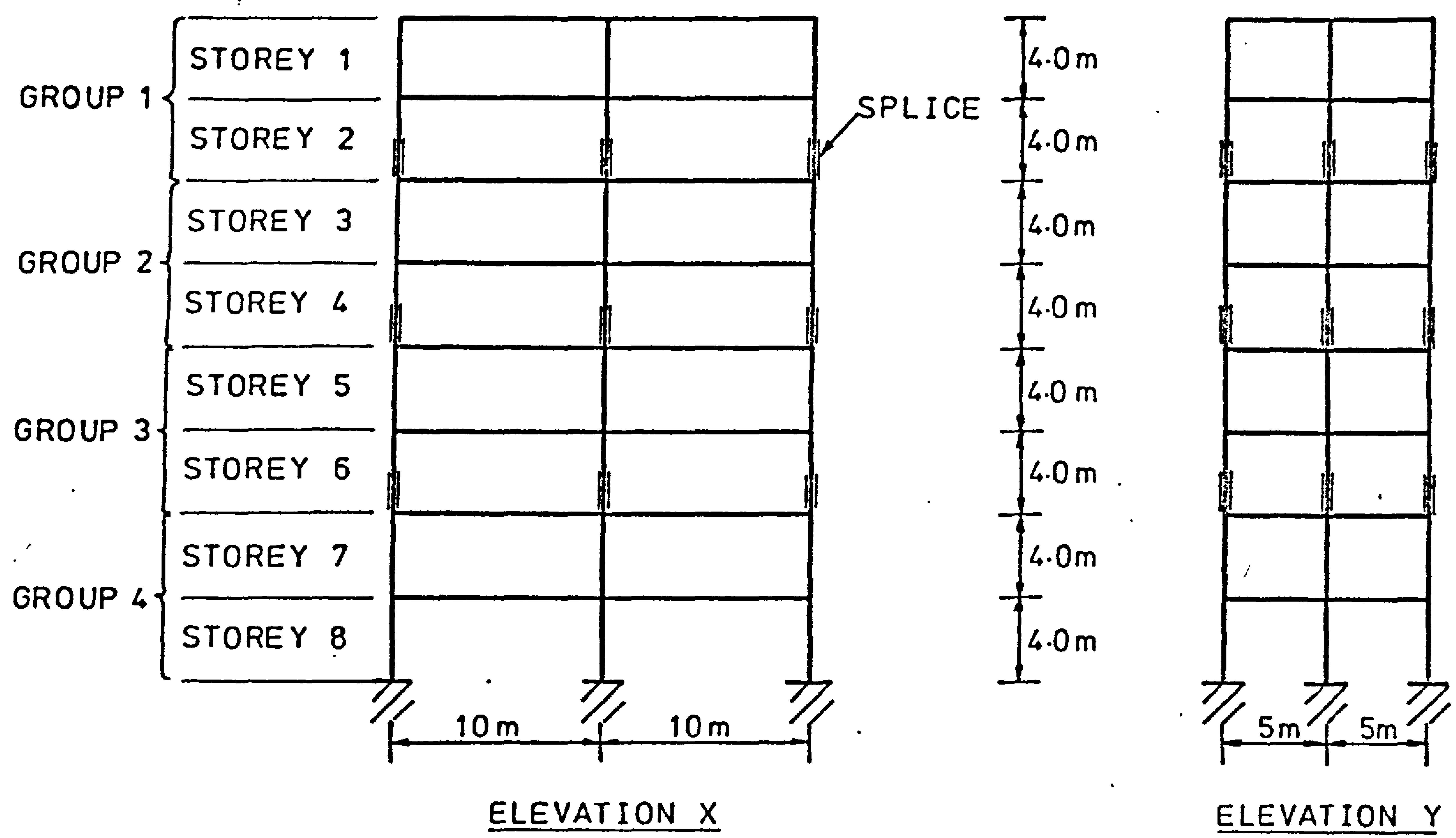


FIG. 6.11 ENUMERATION & DYNAMIC PROGRAMMING EXAMPLE FRAME

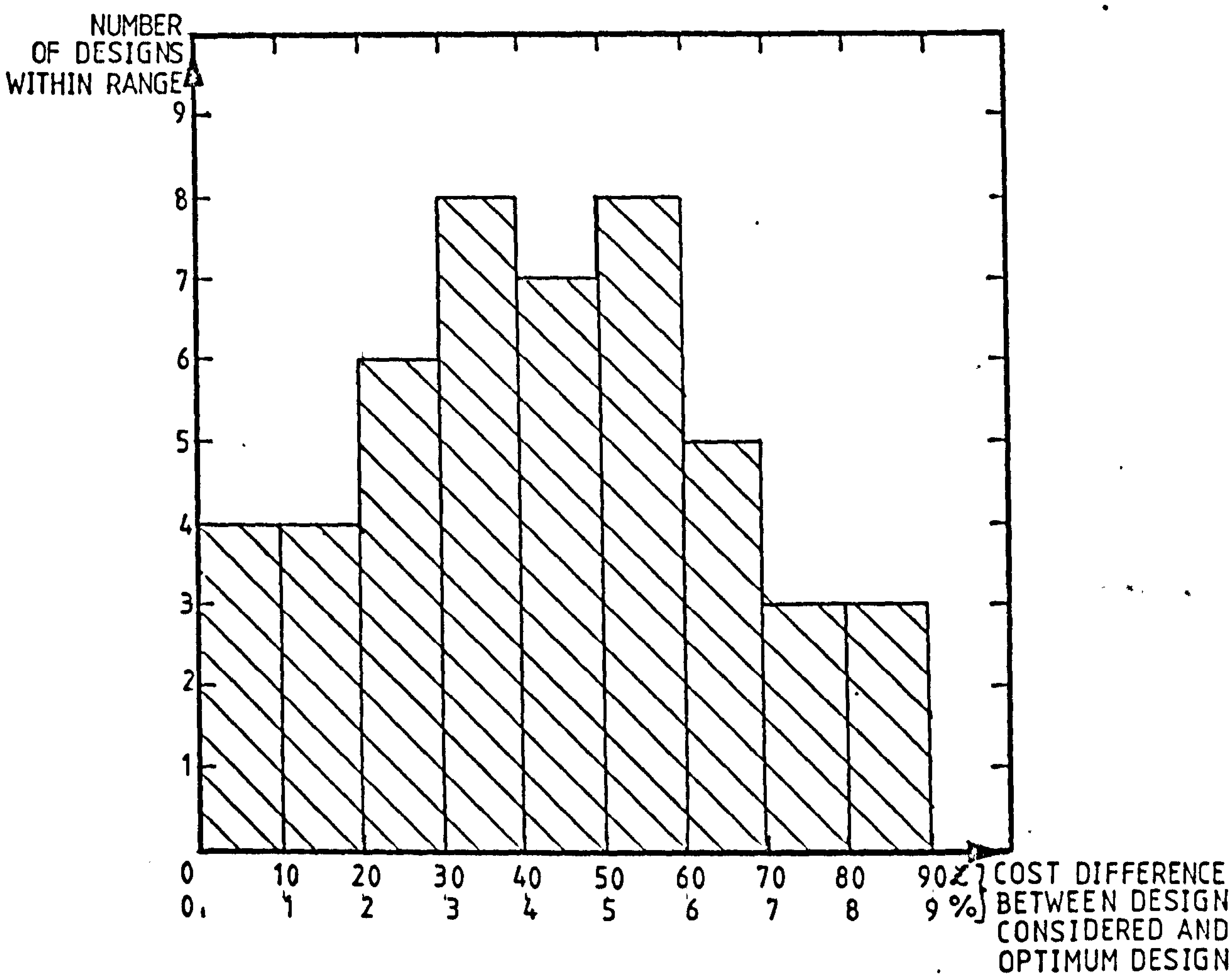


FIG. 6.12 ENUMERATION EXAMPLE
ANALYSIS OF DESIGNS
CONSIDERED BY THE
ALGORITHM

section basis costs and designs based on total costs. This spread is approximately £25.00 which represents 2.5% of the cost of the stanchions.

Figure 6.14 shows a plot of the optimum cost divided by the section basis cost against total frame cost. The vertical axis represents the minimum fabrication coefficient which would cause all designs to be rejected below a horizontal line using the limits on section basis cost previously described. For the example shown, a minimum fabrication coefficient of 1.91 which is within 0.2% of the value for the optimum design will cut out 20 designs and a design which is almost optimal would be cut out if a minimum fabrication coefficient of 1.9114 is used. It can be seen that the estimate of the minimum fabrication coefficient has to be very accurate for it to be of any practical use.

If no constraints were violated the total number of member design checks would be 15,000; the application of geometric constraints reduces this by 5,400; the rejection of a design as soon as one member is found to be infeasible reduces this by a further 3,522. The total number of member stress checks is 6,078, therefore only 40.5% of designs are actually fully checked. For this reason the enumeration algorithm is more attractive than straight enumeration.

6.6 THE DYNAMIC PROGRAMMING ALGORITHM

The dynamic programming algorithm is an approximate method of optimising the choice of sections for column strings. In the method, the design process is turned into a sequence of sequential stages. The method considers each set of stanchions between splices as a stage, as shown in figure 6.15. Each stage is examined in turn starting at the top of the stanchion string. For stage i a section is chosen and each of the combinations of this section with the available sections for stage i-1 is considered. The section in group i-1 which is feasible and results in the minimum cost up to stage i, is termed "the optimal strategy for the section in stage i". This process is repeated for

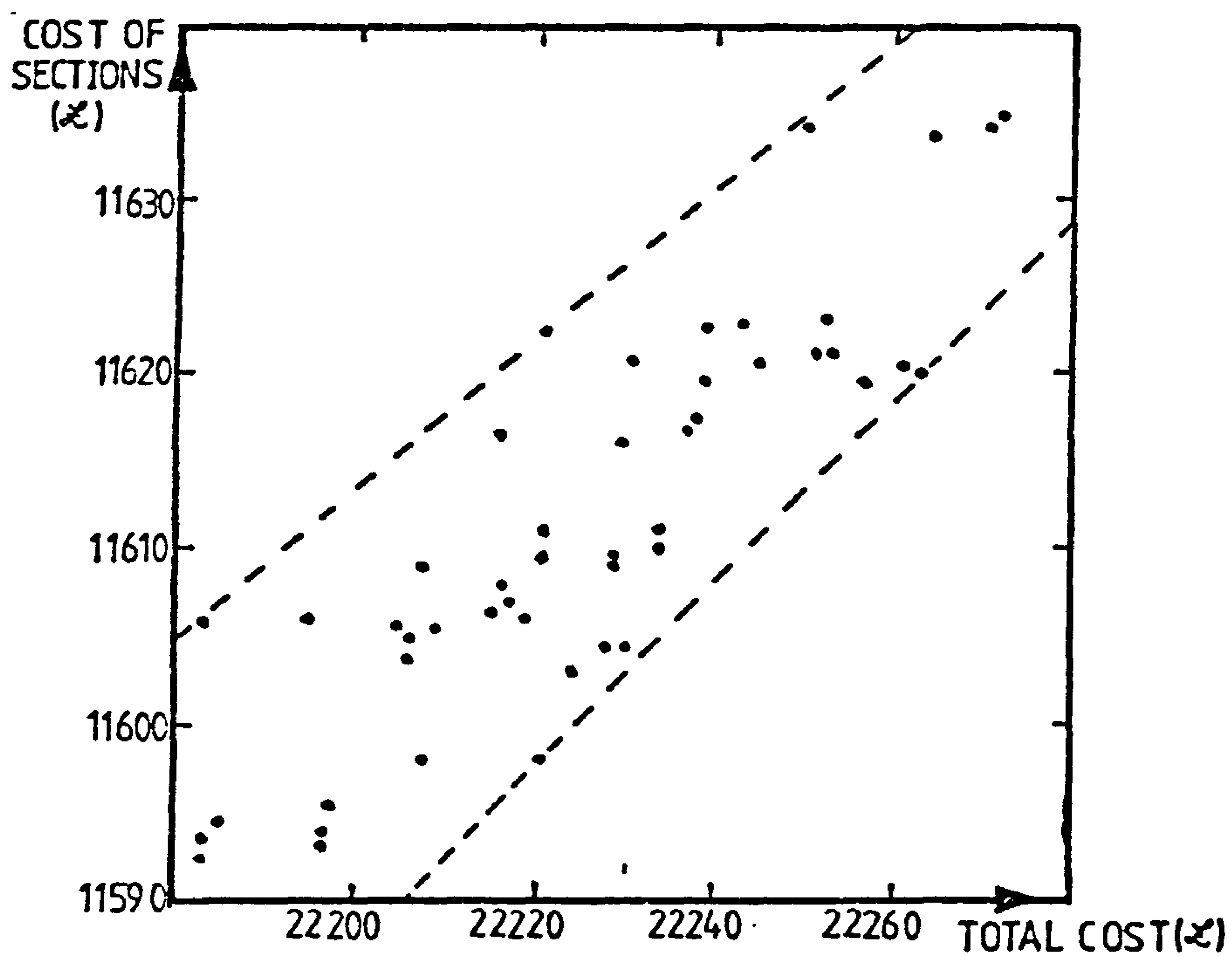


FIG. 6.13 ENUMERATION EXAMPLE
TOTAL COST V COST OF SECTIONS
FOR FEASIBLE DESIGNS CONSIDERED

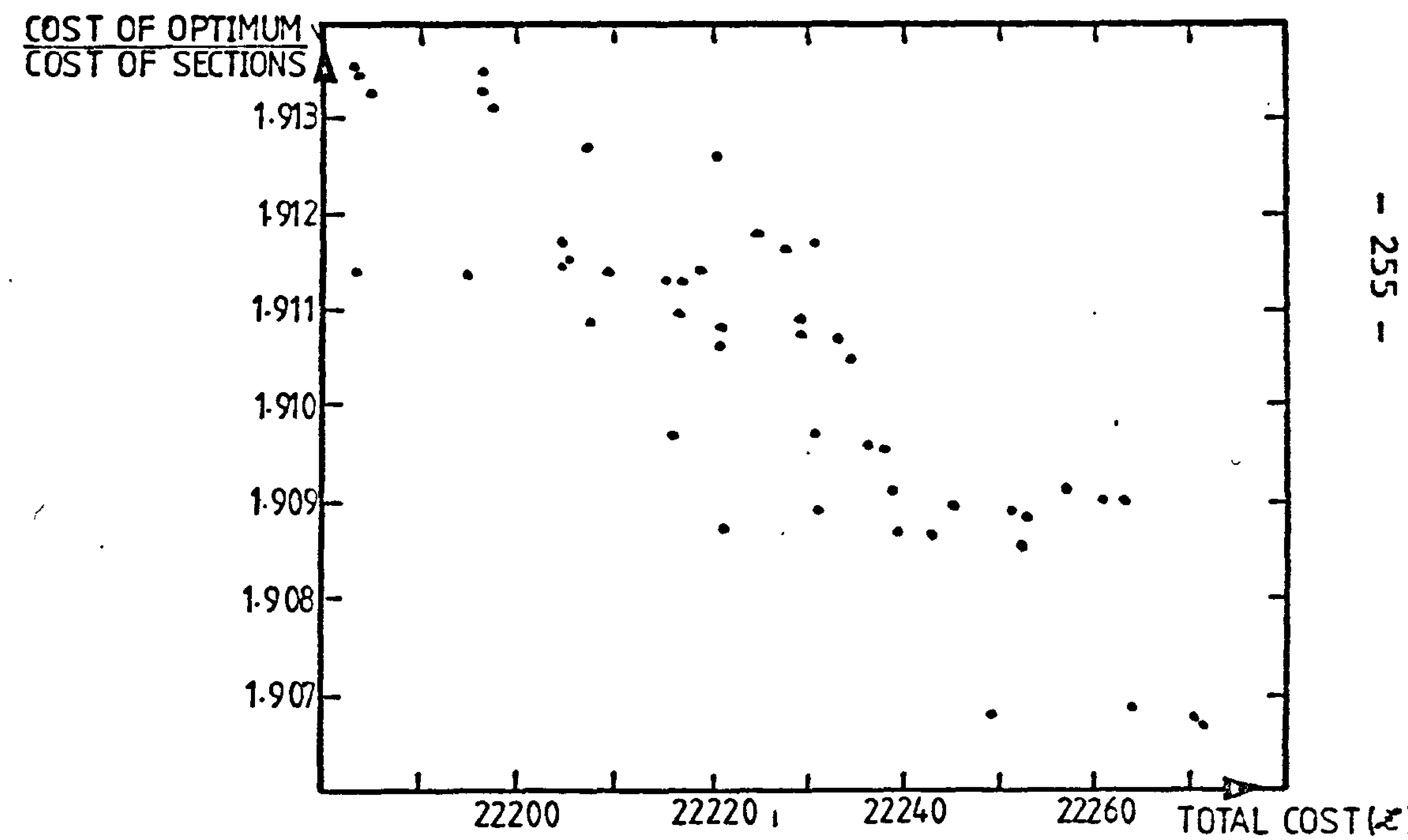


FIG. 6.14 ENUMERATION EXAMPLE
EVALUATION OF THE MINIMUM
FABRICATION COEFFICIENT

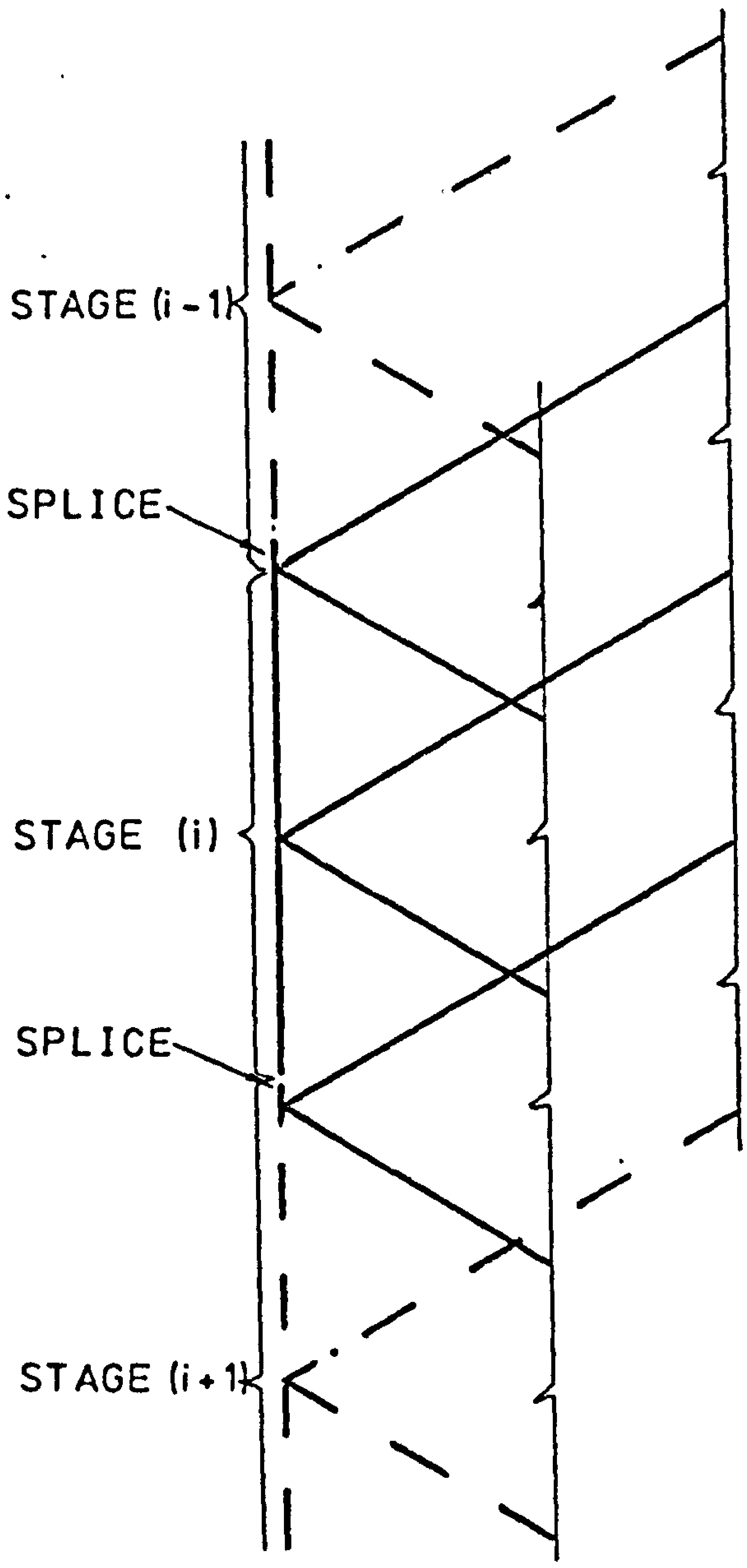


FIG. 6.15 DIVISION OF A CORNER STANCHION INTO STAGES

each of the available sections in stage i. The next stage is then examined in the same way until the final stage is reached. The optimal strategy for the whole string of stanchions can then be found by following the optimal strategy in reverse, resulting in a section for each stage. The method uses a recurrence relationship which can be stated as:-

$$f_i(x_i) = \min_{x_{i-1}} \left\{ C_i\,(x_i,\, x_{i-1}) + g_i(x_i) + f_{i-1}\,(x_{i-1}) \right\}$$

where $f_i(x_i)$ = The minimum cost up to stage i

$C_i(x_i,x_{i-1})$ = The cost of splices between stage i and stage i-1

$g_i(x_i)$ = The cost of beam/column and baseplate connections, plus the section cost for stage i

x_i = The available sections for stage i.

This relationship can be described as; the minimum cost up to stage i is equal to the minimum of, the cost of stage i plus the minimum cost up to stage i-1, where the minimum is found over the available sections for stage i-1.

A more detailed description of the dynamic programming algorithm will now be given:-

(a) For each stage a set of available sections is selected, from which the optimal section for each stage will be chosen.

(b) For stage 1, a check is made for each of the available sections to see if any of the constraints are violated. The cost of using each section is determined, and if any of the constraints are violated the cost of the section is set very high.

(c) Set i = 2.

(d) For each combination of the available sections for stage i and the available sections for stage i-1:-

 (i) Consider all the constraints, and if any are violated set the cost of choosing this section very high.

(ii) Evaluate the cost of beam/column connections, and members in stage i, and the cost of the splice between stages i and i-1.

(iii) Add the cost from (ii) to the minimum cost for stage i-1 to get the cost for stage i.

The combination of sections which has the lowest cost forms the optimal strategy for the sections being considered for stage i.

(e) Increase i by one and return to (d) if more stages remain.

(f) The optimal strategy for the string of stanchions ends with the section which has the cheapest optimal strategy for stage i.

(g) By using a "linking matrix" the sections which result in the optimal strategy can be found.

6.6.1 Computational Points

In order to apply the algorithm, it is necessary to make assumptions about the sectional properties of the section in stage i+1, so that the stress constraints may be checked when considering stage i. It is assumed that the section used in stage i+1 is the same as the section being considered in stage i. In the case where these sections are dissimilar, the lower section will generally be heavier, deeper and therefore stiffer than the upper section. The lower section will therefore attract more of the bending moment at a joint than was assumed. The assumption is therefore conservative with respect to the stress constraints.

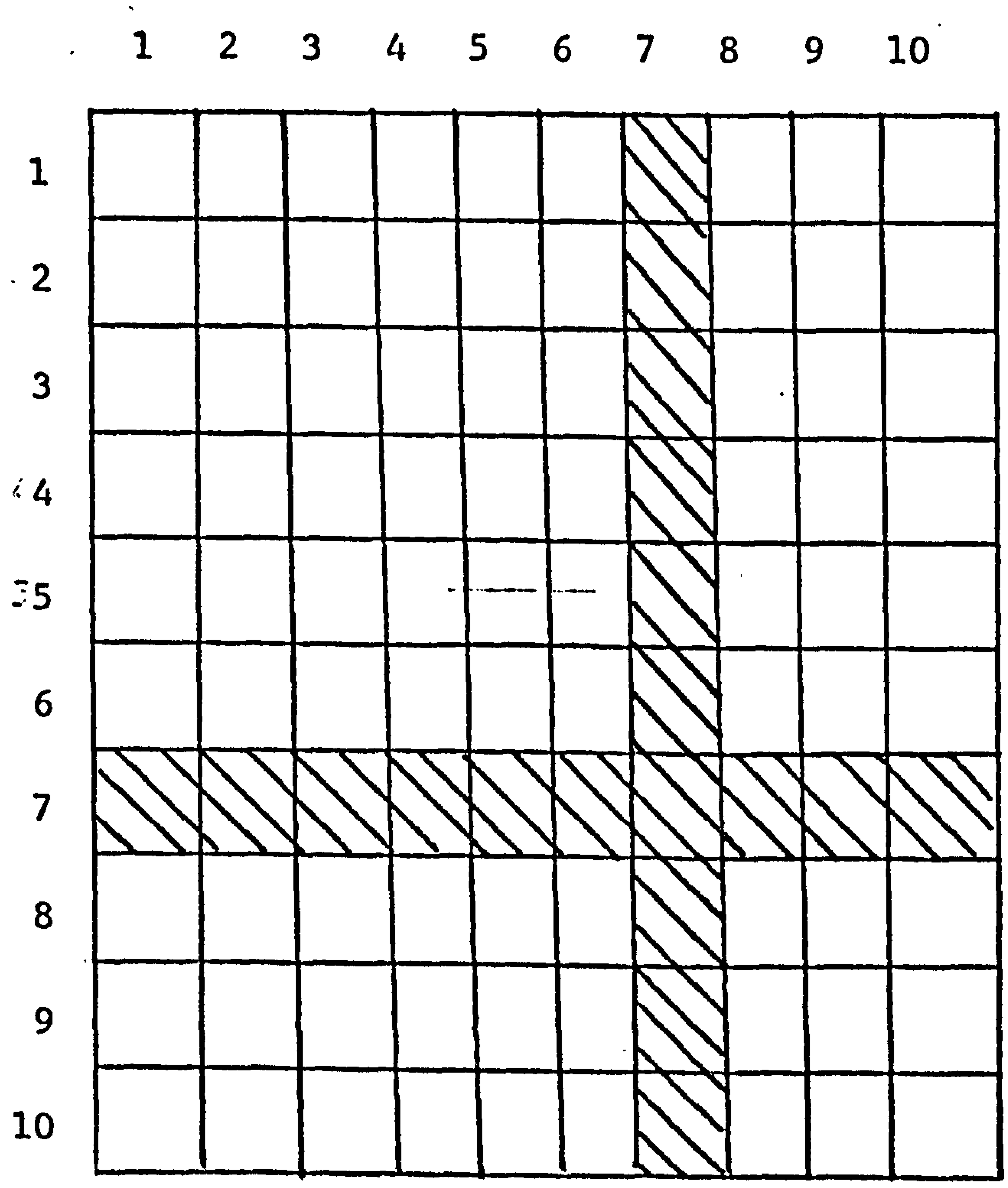
The geometrical constraints can be checked by considering, for each constraint matrix, the constraints which include the stage being considered. All geometric constraints will in this way be satisfied when the final optimal strategy is found. It is only necessary to know the section sizes for adjacent stages in order to check the geometric constraints. For instance the depth constraints which have to be considered for stage k are:-

$$D_i \geqslant D_j \quad \text{for } j = k \text{ and } i = 1 \;\ldots\ldots\; ng$$
$$\text{and } j = 1 \;\ldots\ldots\; ng \text{ and } i = k$$

This represents a row and column in each constraint matrix. The constraints checked at each stage are shown diagramatically in figure 6.16.

When considering a stage the dynamic programming algorithm considers only those members directly attached to the members within the stage, except those members in the next stage. This property limits the number of stress constraints which have to be checked and therefore the algorithm is very efficient.

If the algorithm is to operate correctly it is necessary to observe restrictions on the grouping of members. Firstly, beams and columns must not be in the same group because the "stage wise" nature of the dynamic programming algorithm can only be applied to stanchions. Secondly the stages must be numbered consecutively down the building because of the "stage wise" nature of the algorithm.



Part of constraint matrix examined when considering the stage represented by Group 7.

Figure 6.16

Geometric Constraint checking for the dynamic programming algorithm

6.6.2 Summary of the Dynamic Programming Algorithm

The dynamic programming algorithm provides a convenient search method for designing strings of stanchions, taking into account geometric constraints to reduce computation. The major advantages of this algorithm are:-

(a) The algorithm is very efficient if computation time is considered.

(b) The stages are easily related to groupings of members which form a practical design.

(c) The algorithm can be used with any design method without modification.

The major disadvantages of this algorithm are:-

(a) The algorithm is unable to take section extra costs into account.

(b) The effects of end moment constraints cannot be taken into account.

(c) The stress constraints are treated approximately for the lowest storey in the stage.

(d) The algorithm can be applied only to strings of stanchions.

6.6.3 Example of the Use of the Dynamic Programming Algorithm

The dynamic programming algorithm will be applied to the design of a corner stanchion for the frame previously used (see Section 6.5.6). The J.C.R. design method was used and the iterative design algorithm was used to find suitable starting sections. Nine possible sections were chosen for each stage (a stage being two stories), four sections had a basis cost less than the starting section. The possible sections for each stage are shown in Table 6.9.

The algorithm starts by considering the design of stage one, i.e. the top and next to top stories. Each of the available sections for stage one is checked to see if any

STAGE NUMBER	STORIES IN STAGE	POSSIBLE SECTION NUMBERS								
		1	2	3	4	5	6	7	8	9
1	1 & 2	33	40	45	36	32	39	35	31	28
2	3 & 4	35	31	28	34	30	27	29	23	26
3	5 & 6	28	34	30	27	29	23	26	25	22
4	7 & 8	27	29	23	26	25	22	24	21	16

TABLE 6.9

stress constraints are violated, assuming stage two has the same section as stage one. The results of this stage are shown in Table 6.10. In the following tables S denotes stress constraint failed and G denotes geometric constraint failed.

STAGE 1 SECTION	COST OF STAGE 1
33	S
40	S
45	S
36	S
32	180.57
39	180.31
35	S
31	190.69
28	200.88

TABLE 6.10

STAGE 2 SECTION	STAGE 1 SECTION AND COST OF STAGE 1 AND STAGE 2				OPTIMAL STRATEGY
	32	39	31	28	
35	S	S	S	G	0
31	S	S	S	G	0
28	S	S	S	S	0
34	S	S	S	G	0
30	422.68	424.19	435.03	G	32
27	470.32	465.08	480.16	479.85	39
29	434.73	445.05	444.56	G	32
23	466.16	477.17	476.05	479.07	32
26	457.70	464.52	470.79	469.18	32

TABLE 6.11

STAGE 4 SECTION	OPTIMAL SECTION STAGE 3	OPTIMAL SECTION STAGE 2	OPTIMAL SECTION STAGE 1	COST OF STAGES 1 to 4
27	0	0	0	
29	0	0	0	
23	23	30	32	1029.30
26	26	30	32	1030.90
25	29	29	32	1038.20
22	29	29	32	1057.00
24	29	29	32	1060.30
21	29	29	32	1076.20
16	29	29	32	1087.70

TABLE 6.12

Stage 2 and Stage 1 are considered together. Each of the available sections for Stage 2 is combined with each of the available sections for Stage 1. Five Stage 1 sections failed stress constraints in Stage 1 and therefore do not need to be considered in Stage 2. The combinations which were considered are shown in Table 6.11 together with the cost up to Stage 2. The costs of Stage 1 and Stage 2 for the particular combination are added, the cheapest combination for each Stage 2 section is termed the optimal strategy. It can be seen that four Stage 2 sections can be removed from consideration in Stage 3. When Stage 4 is considered, the optimal strategies for each section are shown in Table 6.12. It can be seen that by using section 23 for Stage 4, and returning along the optimal strategy, the optimal string of stanchions result. This set of sections give exactly the same results as the enumeration algorithm. The total number of member stress checks which were performed during the optimization was 748, which compares favourably with the 2097 member stress checks required if all members and sections were checked for every constraint. Enumeration of all the possible combinations of sections would have resulted in approximately 1×10^9 member stress checks.

6.7 EFFICIENCY OF THE ENUMERATION AND DYNAMIC PROGRAMMING ALGORITHMS

The enumeration and dynamic programming algorithms were compared using a theoretical investigation of the design of an internal stanchion. The stanchion considered consisted of n stages, ns stories per stage and m choices of sections for each stage. Where geometric constraints are considered it has been assumed that the sections are ordered in terms of increasing depth, and that section i in group j will only be compatible with sections i ..... m in group j + 1, where group j is directly above group j + 1.

When the enumeration algorithm is used, ignoring all constraints, the number of member stress checks which are performed is given by:-

$$E = 5ns\ n\ m^n$$

This formula does not consider lower bound constraints, frame cost constraints and geometric constraints. When geometric constraints are included the number of member stress checks which are performed is given by:-

$$E_c = \frac{5ns\ n\ (n + m - 1)!}{n!\,(m - 1)!}$$

If the cost of checking geometric constraints is very small in proportion to the cost of checking member stress constraints, the ratio E_c/E is the reduction in the number of member stress checks brought about by considering geometric constraints. The value of E_c/E is shown in figure 6.17 for various values of n and m. It can be seen that substantial savings can be effected by considering geometric constraints. When beam groups are considered by the enumeration algorithm the reduction in the number of member stress checks will not be as great.

When the dynamic programming algorithm is used, for this example, ignoring geometric constraints the number of member stress checks which are performed is:-

$$D = (5ns + 4)m + m^2((n - 1)(5ns + 5) - 4)$$

When geometric constraints are included the number of member stress checks which are performed is given by:-

$$D_c = (5ns + 4)m + \frac{m}{2}(m + 1)((n - 1)(5ns + 5) - 4)$$

As before the ratio D_c/D is the reduction in the number of member stress checks brought about by considering geometric constraints. The value of D_c/D is shown in figure 6.18 for various values of n & m, the value of ns has a slight effect on these curves. It can be seen that a substantial reduction in computation results from the consideration of geometric constraints. The maximum reduction that can ever be achieved in this case is 50%.

The two algorithms were compared by evaluating the ratio of the number of designs required, by the enumeration algorithm and by the

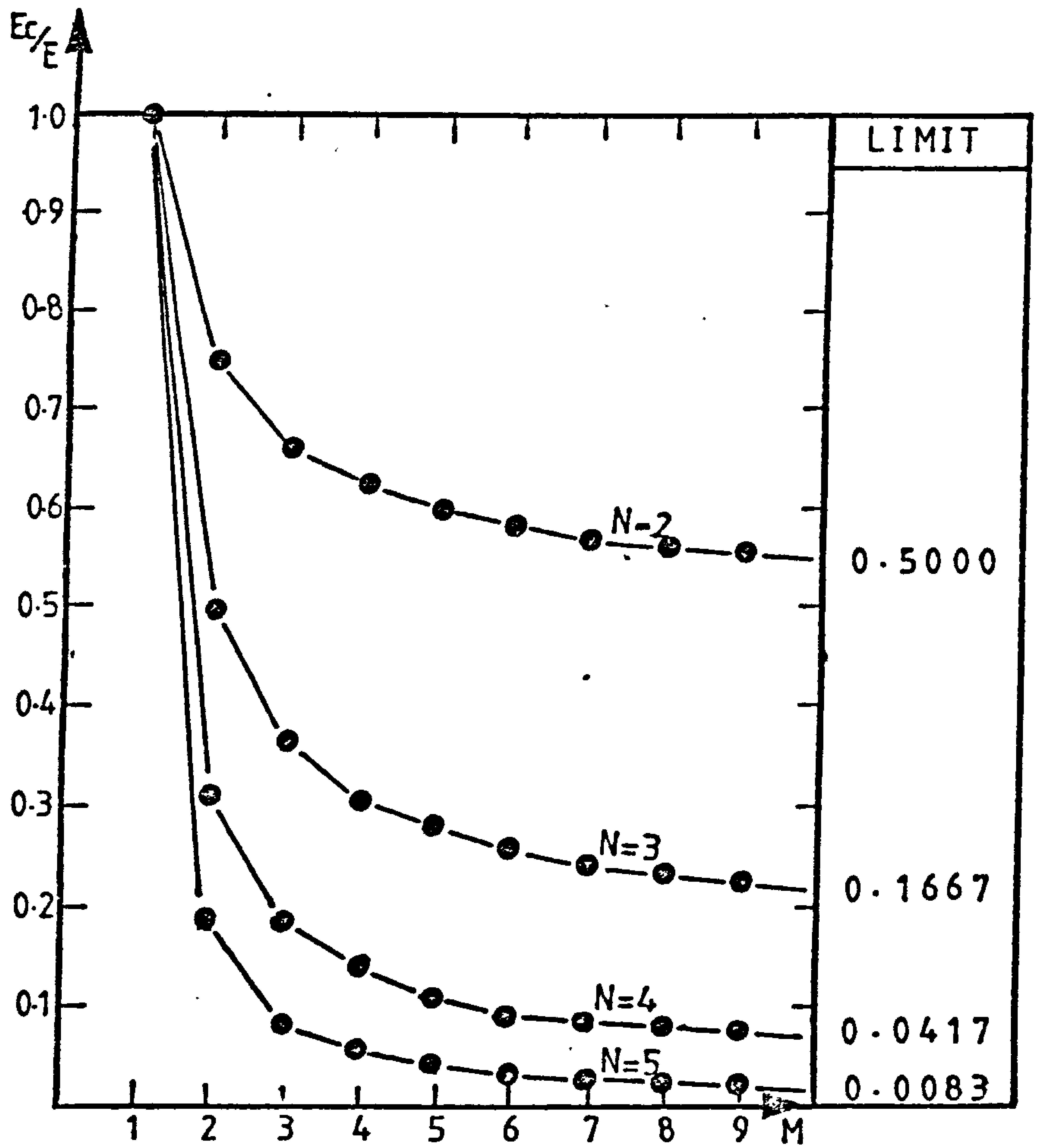


FIG. 6.17 THE EFFECT OF GEOMETRIC CONSTRAINTS ON THE ENUMERATION ALGORITHM

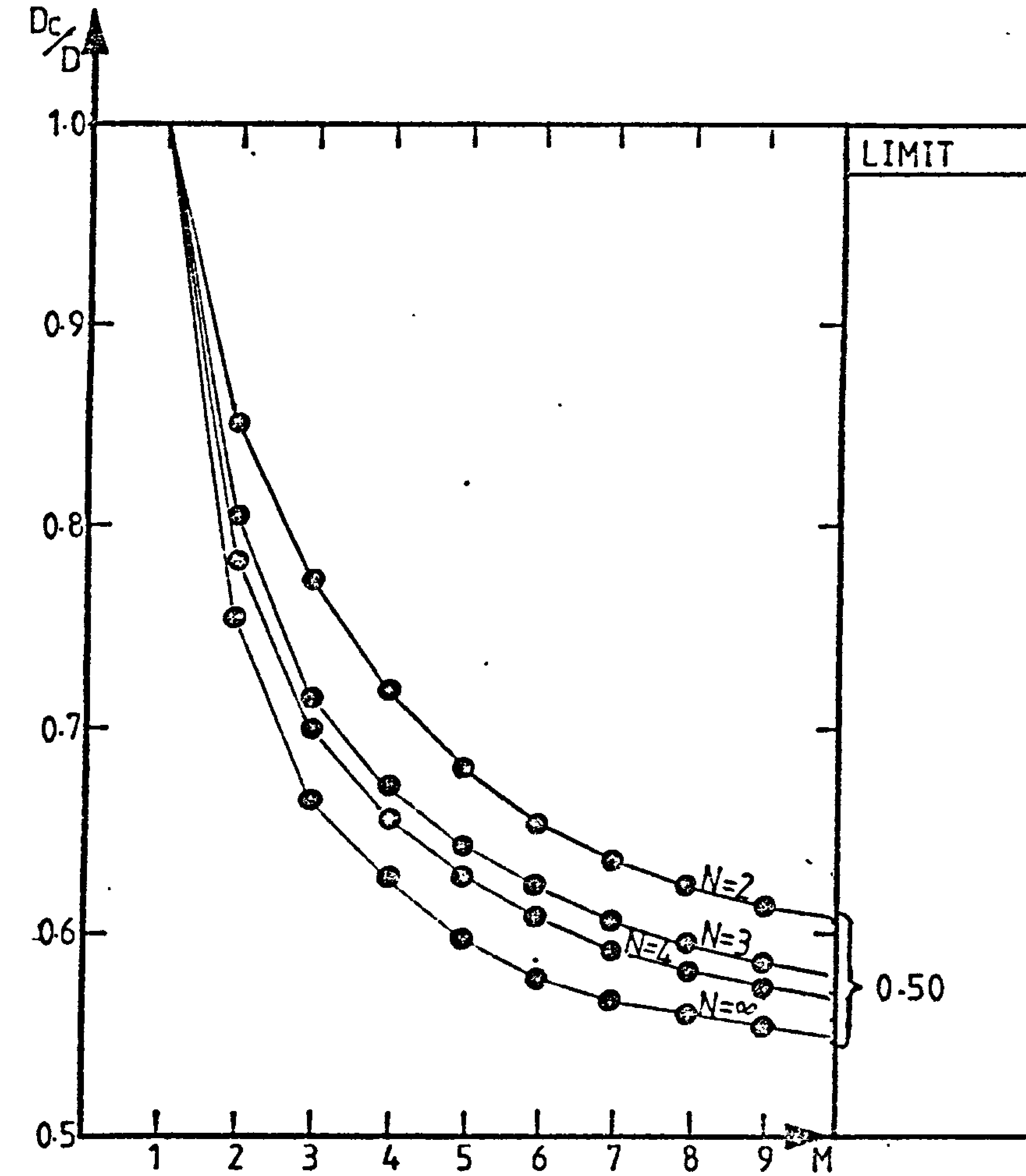


FIG. 6.18 THE EFFECT OF GEOMETRIC CONSTRAINTS ON THE DYNAMIC PROGRAMMING ALGORITHM

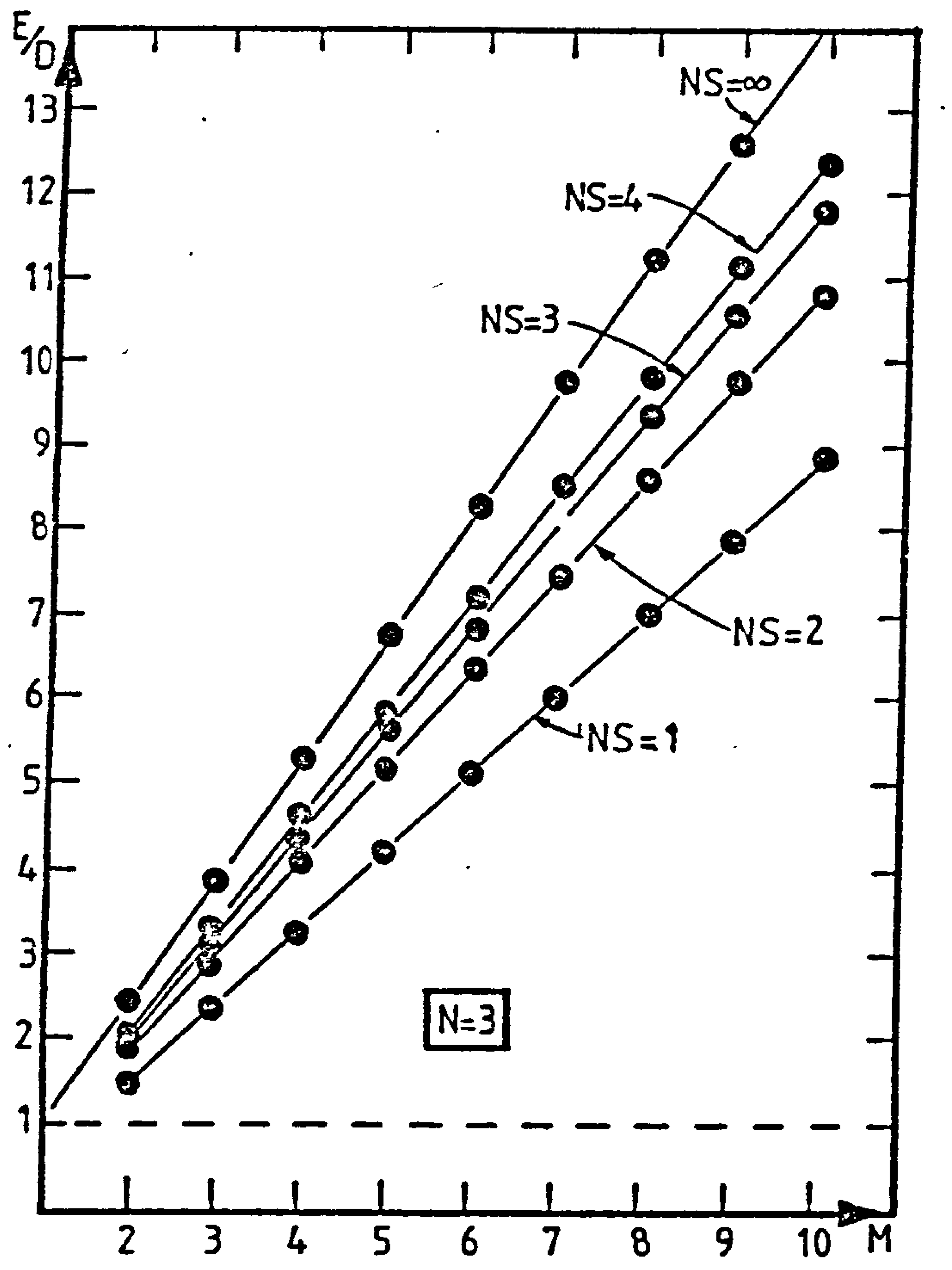


FIG. 6.19 COMPARATIVE EFFICIENCIES OF ALGORITHMS, IGNORING GEOMETRIC CONSTRAINTS

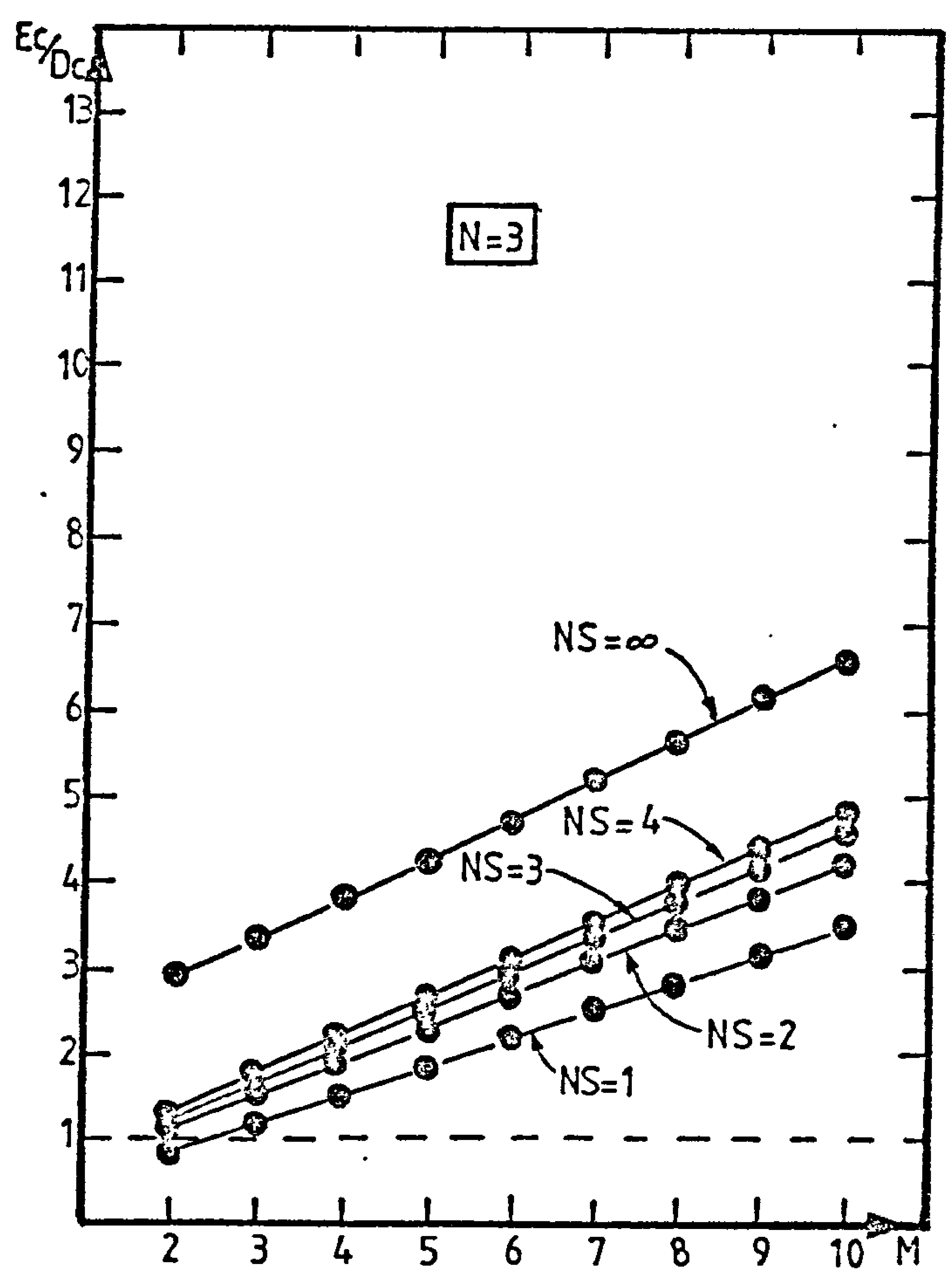


FIG. 6.20 COMPARATIVE EFFICIENCIES OF ALGORITHMS, INCLUDING GEOMETRIC CONSTRAINTS

dynamic programming algorithm. Typical plots of E/D and E_c/D_c for n equal to three are shown in figures 6.19 and 6.20. It can be seen that in almost every case the dynamic programming algorithm is more efficient than the enumeration algorithm. It can also be seen that the inclusion of geometric constraints has a far greater effect on the enumeration algorithm than the dynamic programming algorithm. Further examination of these curves for other values of n show that the enumeration algorithm is the least efficient algorithm in all practical cases.

The example given does not represent a general case, however the relationships which have been derived will be very similar for a practical stanchion. The effect of other computational points on the number of member stress checks required cannot be easily investigated theoretically. These points will increase the efficiency of the algorithms but it is not expected that they will substantially alter the relationships described.

6.8 CONCLUSIONS

Three algorithms, which can be used to aid the optimization of braced rigid frames, have been described. These algorithms have been shown to be efficient when considering certain aspects of the optimization problem. They have all been shown to have disadvantages when applied in certain conditions.

The iterative design algorithm provides an economic design with minimum computation. The enumeration algorithm can be used to optimise the design of small numbers of member groups taking account of all constraints and all costs. The dymanic programming algorithm can be used to optimise the design of strings of stanchions very efficiently neglecting section extra costs.

CHAPTER 7

Results

7.1 INTRODUCTION

In this chapter, investigations of a number of features relating to the design of braced multi-storey steel frames are described. These investigations relate both to the design of complete frames and the design of connections.

The first investigation considers the relationship of cost to the steel sections used for each member. The cost of the frame is plotted against sets of two variables resulting in a "contoured surface". The results of this investigation are used to determine the features of the design space that can be used to facilitate the solution of the optimum design problem. It was also found possible to assess the likely reduction in cost due to the use of the algorithms.

The second investigation consists of deriving a design strategy using the results of the first investigation. The application of the strategy is then evaluated and the most economic procedure is determined.

The third investigation consists of designing a number of frames using the design strategy. The frames utilize various splicing arrangements and the results allow an assessment of the most economic splicing arrangement. The likely reductions in cost which can be achieved by the local search algorithms are also investigated.

A breakdown in the cost items for a typical frame with designs in each grade of steel is then presented, allowing an assessment of the most important fabrication and material cost items.

The effect of varying the load factor, the grade of steel and the design method is then investigated, followed by a limited assessment of the use of minimum weight as a criterion for design.

A limited investigation of the effect of varying the geometry of the frame is then presented. Such an investigation is limited in its scope by costs, which have not been considered, for instance, fire protection, cladding, flooring and foundations.

Finally the procedure used to try to devise standard connections or connection costs for each section is described. Although not implemented in the design programme a number of important features are encountered which increase understanding of the overall problem.

7.2 INVESTIGATION OF A TYPICAL DESIGN SPACE

The purpose of this investigation was to examine the type of surface upon which the algorithms had to operate and to identify any properties of the surface which could be used to simplify the location of the optimum design.

The approach used consisted of designing the frame (described in section 6.4.7 and figure 6.10) using the iterative design algorithm. This resulted in a section for each group of members (see Table 6.2). Two of these groups were selected as variables, all others remaining fixed. A set of possible sections was selected for each of the two variable groups. Each combination of sections for the variable groups was then checked for violations of geometric or stress constraints, feasible combinations were costed. The use of two variables allows the representation of the resulting designs as a "discreet contoured graph". In order to produce these graphs it was necessary to decide on a suitable order to show the possible sections. The sections were ordered in terms of increasing basis cost, this order was chosen, because the basis cost of sections forms a substantial proportion of the total frame cost. It will be seen later that this order is not ideal, however it is probably better than any other. The sections could have been ordered in terms of weight or in terms of a section property, or indeed randomly, clearly the use of any of these alternatives would result in substantial changes in the configuration of the design space. The use of discrete sections also complicates the interpretation of the results because designs only exist at discrete points, and there is no solution to the problem between these points. This means that where a design is feasible the cost is shown as a "spot height" and it is not possible to draw contours.

The investigation has been performed using the J.C.R. design method. The B.C.S.A. and B.R.S. design methods give similar results and have been omitted for the sake of brevity. The main difference between the J.C.R. design method and the other methods is that the degree of minor axis elastic interation is highest for the J.C.R. method. The use of another design method does not affect the application of geometric constraints, it does however show some stress feasible designs as infeasible and vice-versa.

When considering the column groups of members, six sections cheaper and seven section more expensive than that resulting from the iterative design were used. When considering beam groups the sections considered were as expensive or more expensive than the cheapest possible section for the group.

It is worth noting that each of the following investigations examines a small area of a slice through the design space. The true optimum design will probably not be shown on any of these slices, however exhaustive examination of the space would not be practically possible. Each of the investigations will now be described in detail.

7.2.1 Corner Stanchions (Groups 5 and 9)

A plot of the design space for the corner stanchions is shown in figure 7.1. The hundred and ninety six designs which were investigated consisted of seventy one designs that failed to satisfy geometric constraints, one hundred and six designs which failed to satisfy stress constraints and nineteen feasible designs.

The small number of feasible designs are seen to be widely scattered but are concentrated on vertical and horizontal lines. There is an underlying trend towards an increase in frame cost with increase in basis section cost. The minimum point can be seen to be located on the edge of the feasible part of the design space, it is five sections away from the minimum basis cost design and further away from the iterative design. It can be seen that the minimum cost design in this plane has not necessarily been found.

The stress constraints are active all over the design space, some sections (91 and 93) being unsuitable as upper sections irrespective of the lower sections. The area where no designs are stress feasible can be easily identified.

The geometric constraints are active all over the design space. As expected lower stanchion sections that are shallower than upper stanchions are rejected. Designs which fail geometric constraints are generally concentrated on horizontal lines.

The progress of the iterative design algorithm is shown, it can be seen that this algorithm produces a solution, close to the apparent optimum. This solution can be seen to be neither the minimum basis cost design nor the minimum cost design.

7.2.2 North and South Side Stanchions (Groups 6 and 10)

A plot of the design space is shown in figure 7.2. The designs are divided into, eighty seven designs which failed

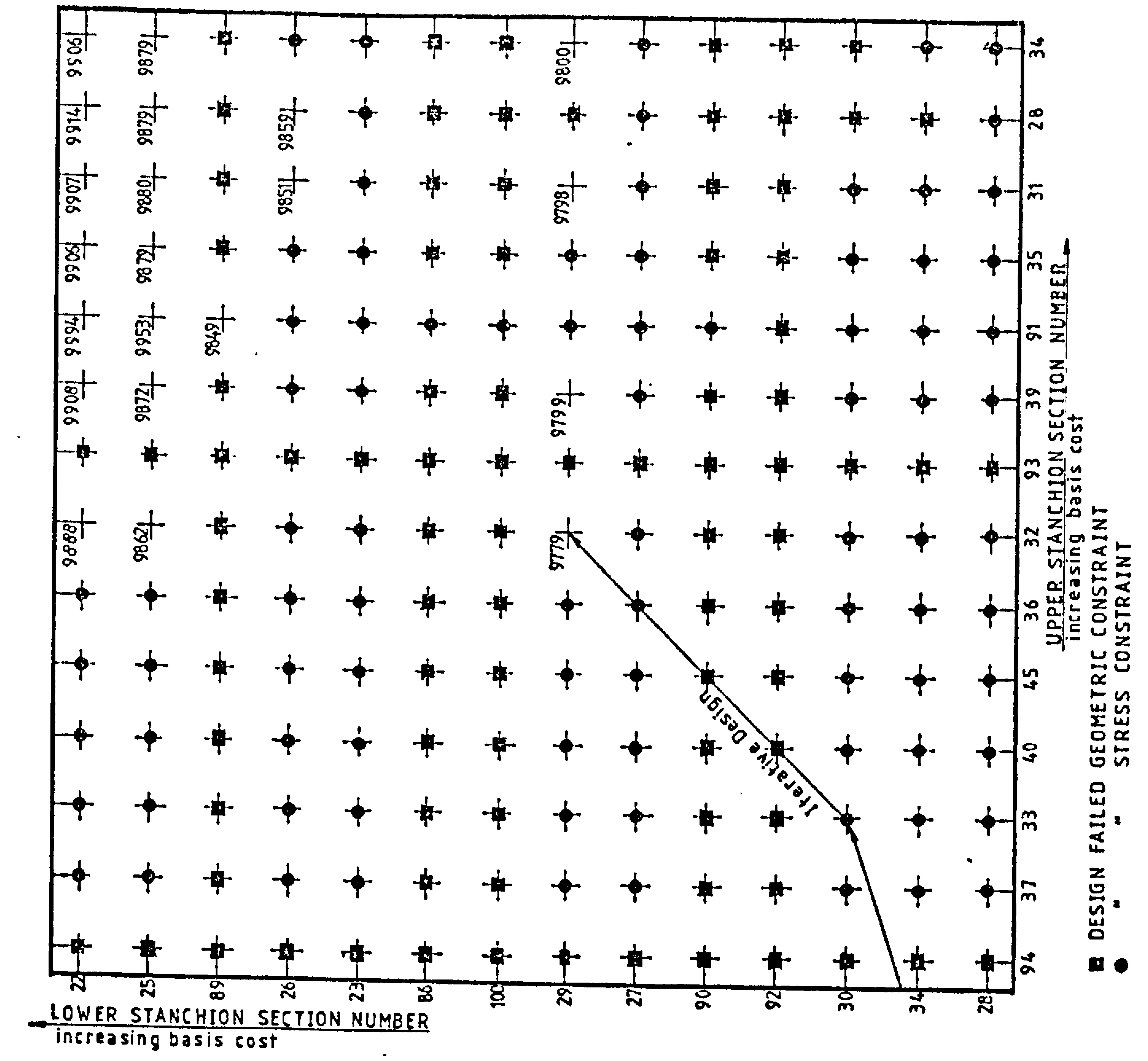


FIG. 7.2 DESIGN SPACE INVESTIGATION
NORTH & SOUTH SIDE STANCHIONS

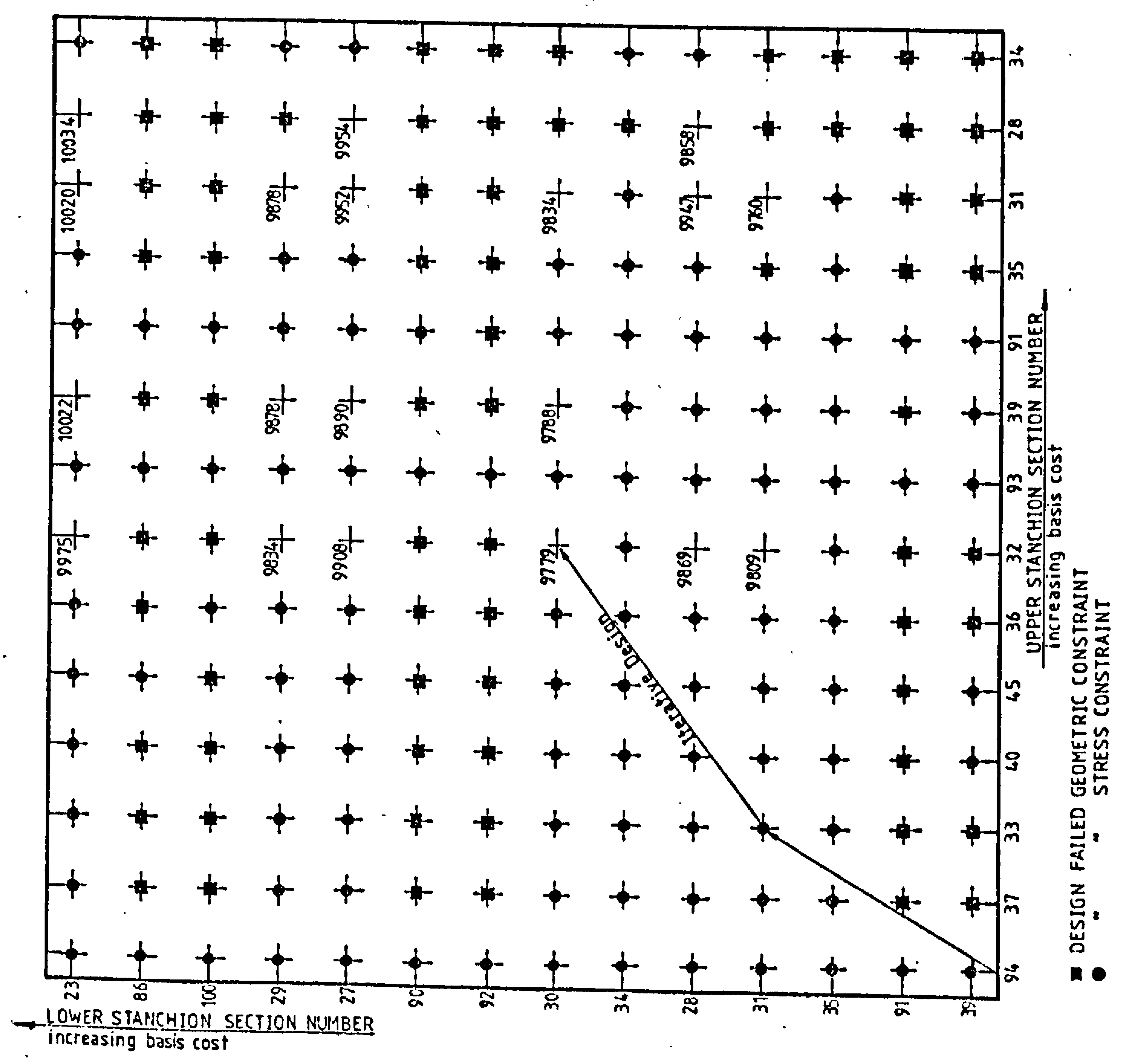


FIG. 7.1 DESIGN SPACE INVESTIGATION
CORNER STANCHIONS

to satisfy geometric constraints, eighty eight designs which failed to satisfy stress constraints and twenty one feasible designs.

The feasible designs again show an underlying trend towards increasing cost with increasing basis cost. The minimum cost design is at the edge of the feasible design space at a vertex. This design corresponds to the minimum section basis cost design. There is a large vertical gap between the optimum design and other feasible designs.

The stress constraints are seen to be active all over the design space. Sections 29 and 26 used as a lower stanchion section are feasible with respect to stress constraints for only certain upper stanchion sections, this shows that elastic interaction occurs between stanchion sections. The area where no designs are stress feasible can again be easily identified.

Geometric constraints which involve beam sections prohibit the use of section 93 as an upper stanchion. Universal column sections such as sections 92, 90 and 86 cannot be used as lower sections sections due to violation of geometric constraints.

The progress of the iterative design algorithm is shown, this algorithm can be seen to produce the optimum design and to approach from the infeasible region.

7.2.3 East and West Side Stanchions (Groups 7 and 11)

A plot of the design space is shown in figure 7.3. The designs are divided into eighty designs which failed to satisfy geometric constraints, seventy one designs which failed to satisfy stress constraints and thirty five feasible designs.

The feasible designs are concentrated on a horizontal and vertical grid. The optimum design does not correspond to the minimum section basis cost design and it is remote from the edge of the feasible design space. Feasible

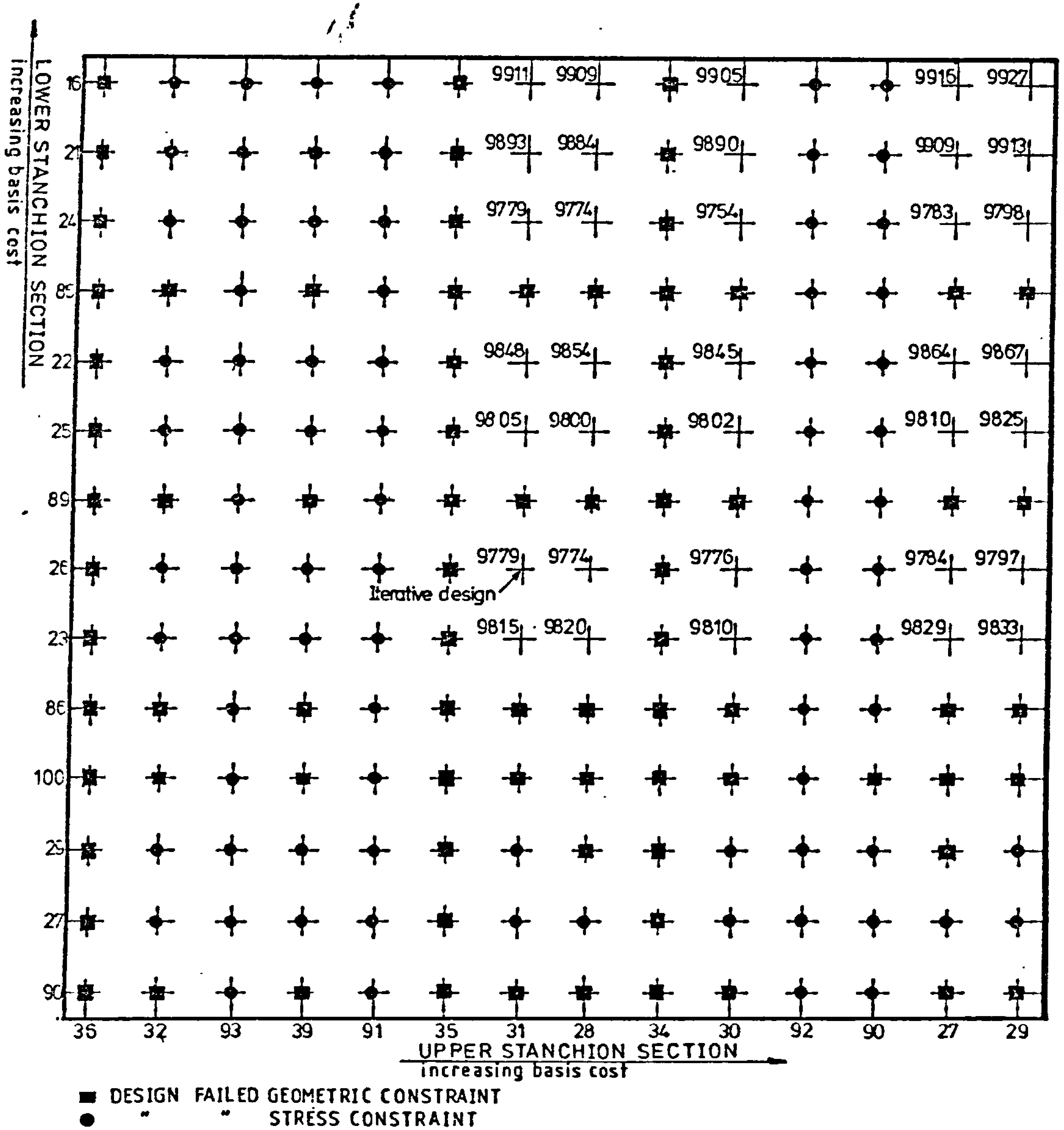


FIG. 7.3 DESIGN SPACE INVESTIGATION
EAST & WEST SIDE STANCHIONS

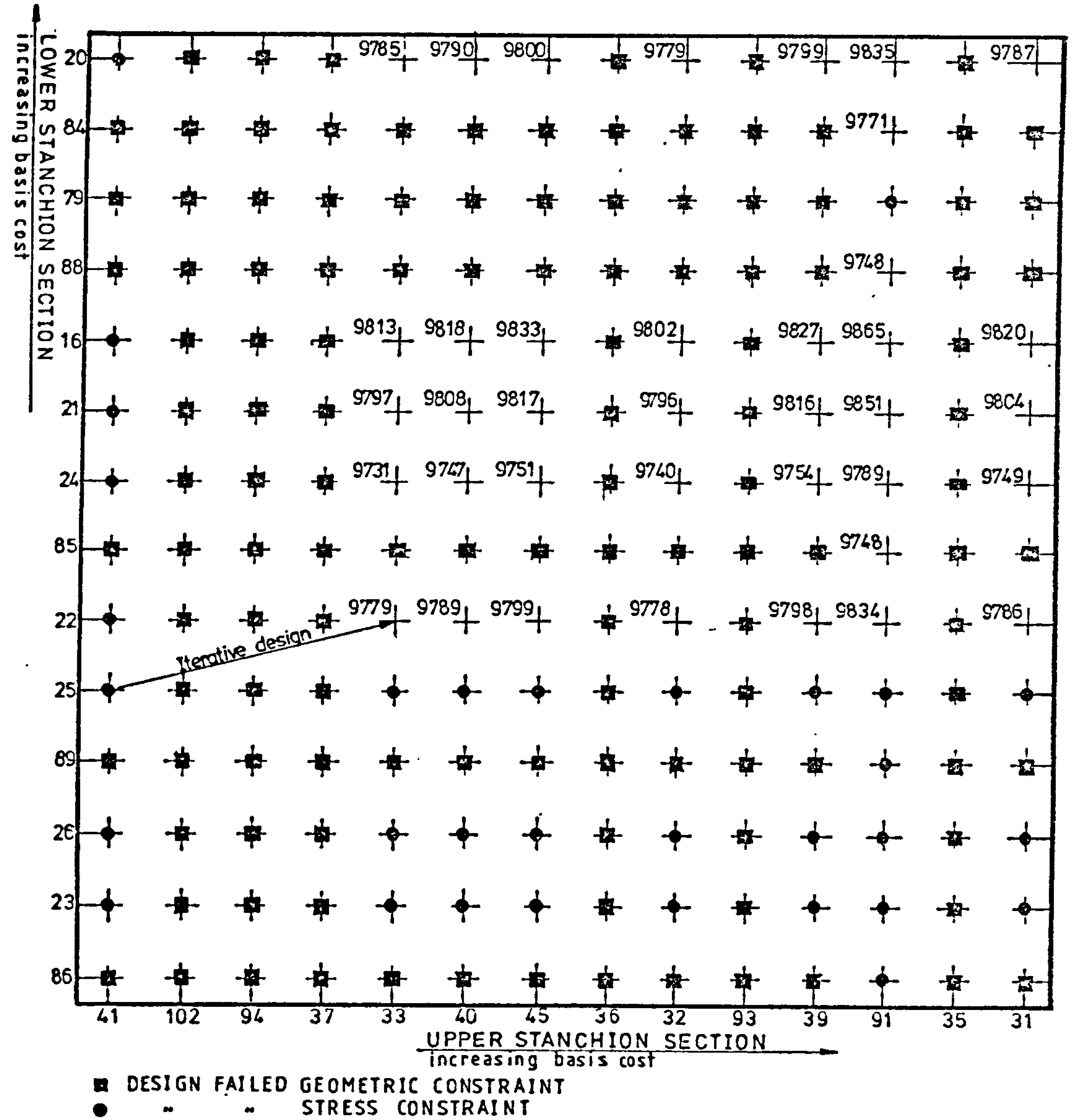


FIG. 7.4 DESIGN SPACE INVESTIGATION
INTERNAL STANCHION

designs with costs very similar to the optimum can be seen to occur all over the area of the design space shown.

The area, where stress constraints are active for all section combinations, can be easily identified. Designs which are infeasible with respect to stress constraints occur within the feasible region. There is no evidence of elastic interation between upper and lower stanchion sections.

Certain sections used as upper stanchions (34, 35 and 36) fail geometric constraints irrespective of the lower stanchion sections due to the influence of the sections used for beams.

The iterative design algorithm chose the design shown in the first iteration, indicating a lack of elastic interaction. The iterative design is close to a local optimum, and remote from the optimum design, direct search algorithms would be unlikely to overcome such an obstacle.

7.2.4 Internal Stanchion (Groups 8 and 12)

A plot of the design space is shown in figure 7.4. The designs are divided into one hundred and twenty six designs which failed geometric constraints, thirty two designs which failed stress constraints and thirty eight feasible designs.

No clearly defined trends can be seen in the cost of feasible designs and feasible designs with costs similar to the optimum exist over all of the portion of the design space shown. The optimum design is on the edge of the stress feasible region and it is not the minimum section basis cost design.

The area where no desings are stress feasible can be easily identified. Within the area where feasible designs exist there is only one design which fails due to active stress constraints.

The geometric constraints dominate the design space. There are two reasons for this, firstly an internal stanchion has to satisfy more geometric constraints than any other type of stanchion and secondly internal stanchion sections are generally smaller than any other type of stanchion and therefore restrict the sections that can be connected to them.

The iterative design algorithm approaches from the infeasible region and produces a design which is not optimal, however this design is close to optimal.

7.2.5 Minor Axis External Beams and Corner Stanchions (Groups 3 and 5)

The plot of this design space is shown in figure 7.5. The designs are divided into forty designs that failed geometric constraints, one hundred and fifty seven designs that failed stress constraints and thirty nine feasible designs.

The feasible designs show a distinct trend towards higher costs with increasing section basis cost. This trend is most noticeable in the direction of increasing beam cost. The optimum design is located on the edge of the stress feasible region. A design with a similar cost to the optimum exists when using a cheaper stanchion and a more expensive beam section. There is evidence of a significant elastic interaction between the beam and column because the stress feasible region is not rectangular.

The edge of the stress feasible area can be easily identified. The stress constraints are active in parts of the stress feasible area. Some sections cannot be used for beams or stanchions irrespective of the other section.

The geometric constraints are only active for certain beam and column sections, because these sections are infeasible throughout this design space it can be deduced that these designs are governed by the sections used in groups which are not being varied.

The progress of the iterative design algorithm is shown, the direction of approach is again from the infeasible region.

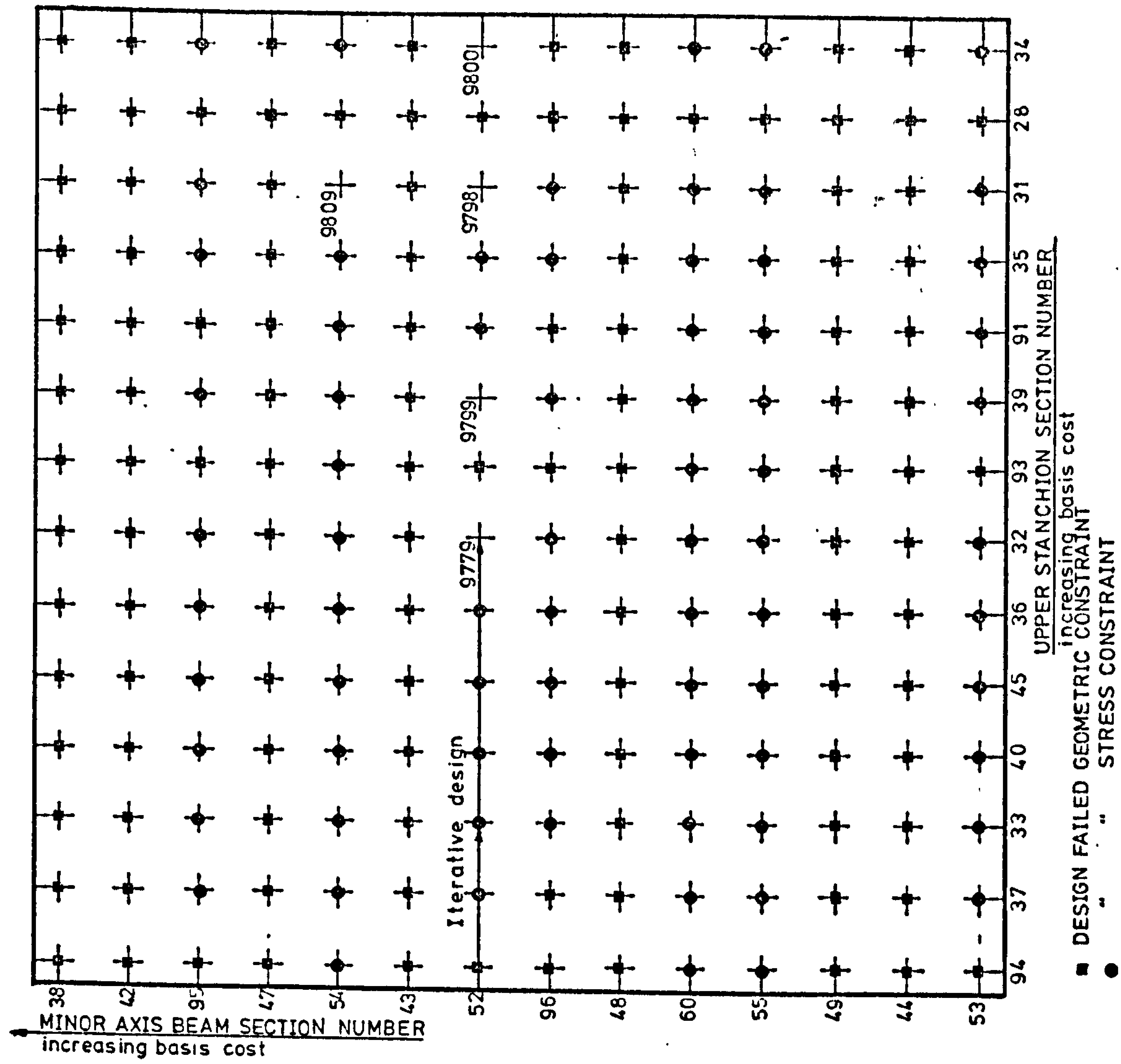


FIG. 7.6 DESIGN SPACE INVESTIGATION
MINOR AXIS INTERNAL BEAMS
AND NORTH & SOUTH SIDE STANCHIONS

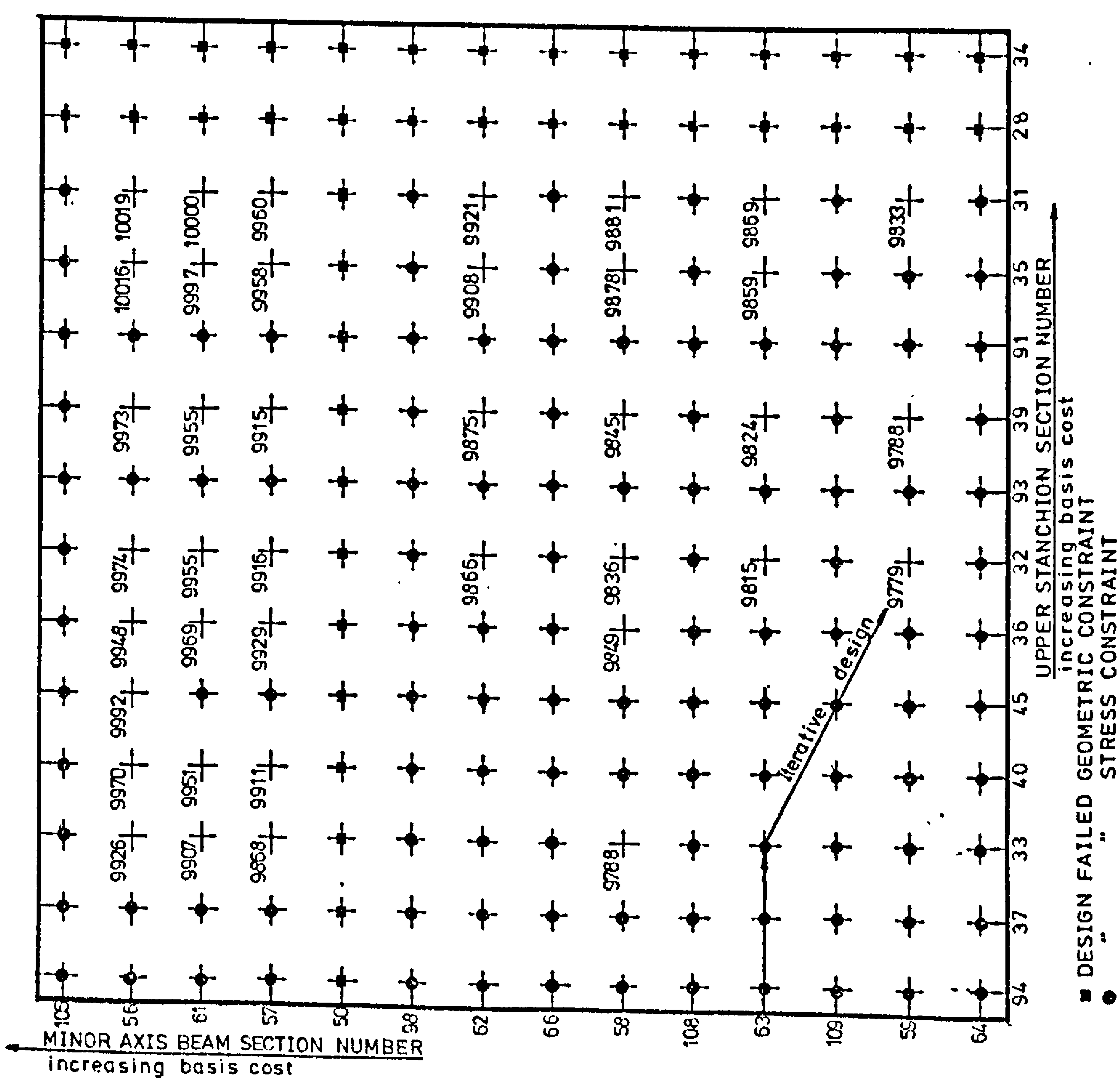


FIG. 7.5 DESIGN SPACE INVESTIGATION
MINOR AXIS BEAM & CORNER STANCHION

7.2.6 Minor Axis Internal Beams and North and South Side Stanchions (Groups 4 and 6)

Two investigations of this design space were performed. The first investigation included the minor axis depth constraints and the second investigation relaxed these constraints.

The design space for the first investigation is shown in figure 7.6. The designs are divided into one hundred and seventeen designs that failed geometric constraints, seventy four designs that failed stress constraints and only five feasible designs.

The small number of feasible designs have very similar costs and are confined to two beam sections. Clearly other sections outside the area considered must be stress feasible.

The boundary of the stress feasible region cannot be located with any degree of accuracy and stress constraints are active in all areas of the design space.

A very large number of designs are infeasible due to violation of geometric constraints. The constraints that are active are likely in many cases to relate to the groups of members that are not being varied, for instance the minor axis depth constraint that relates to the internal stanchion. The iterative design algorithm can be seen to approach from the infeasible region and to reach the apparent optimum design.

The design space of the second investigation is shown in figure 7.7. The designs were divided into only thirty designs which failed geometric constraints, one hundred and fifty five designs which failed stress constraints and eleven feasible desings. The effect of relaxing minor axis depth constraints is quite marked.

The number of feasible designs is increased because section 34 used as a stanchion becomes feasible when combined

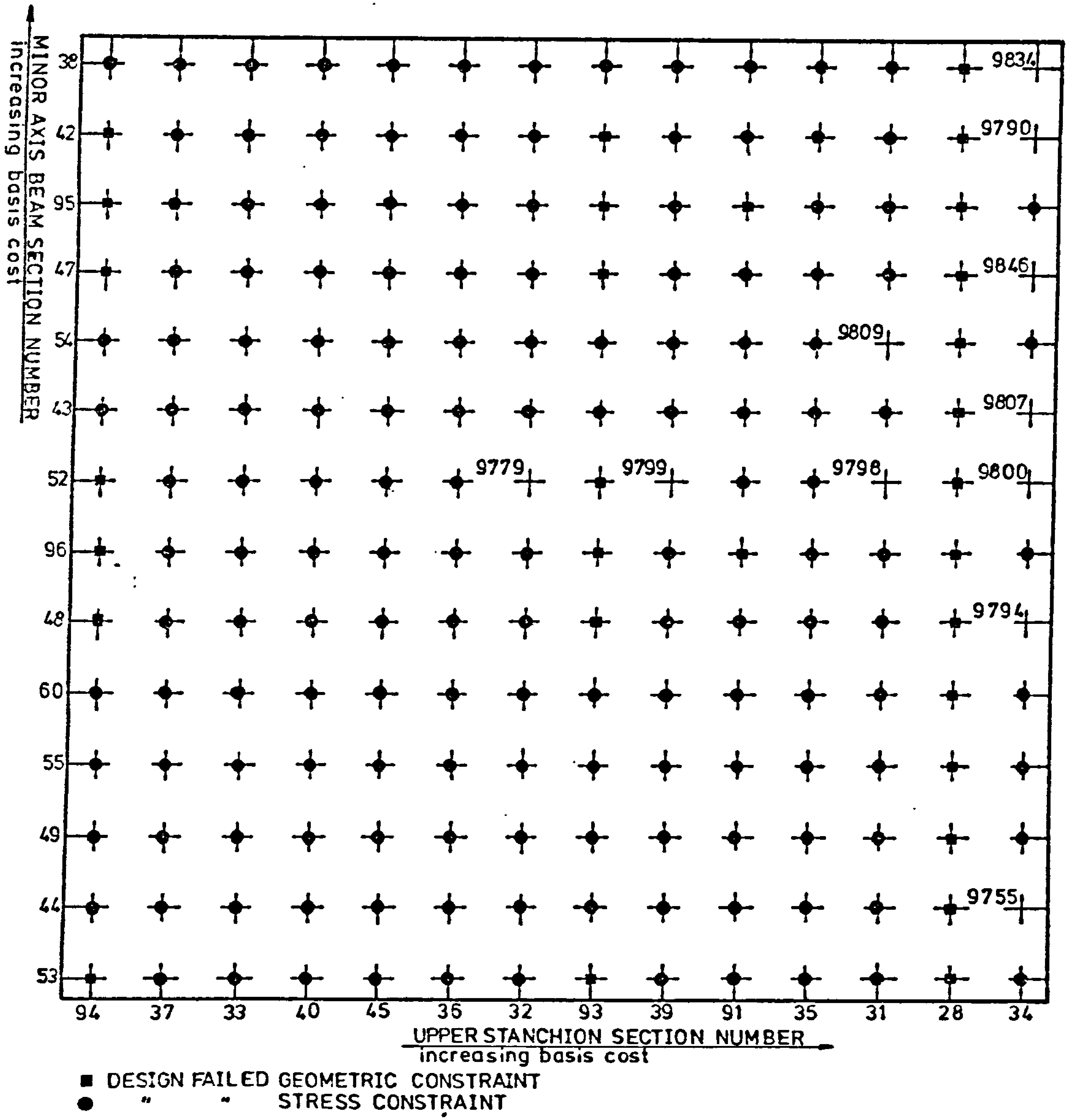


FIG. 7.7 DESIGN SPACE INVESTIGATION
MINOR AXIS INTERNAL BEAMS AND NORTH & SOUTH
SIDE STANCHIONS (CONSTRAINTS RELAXED)

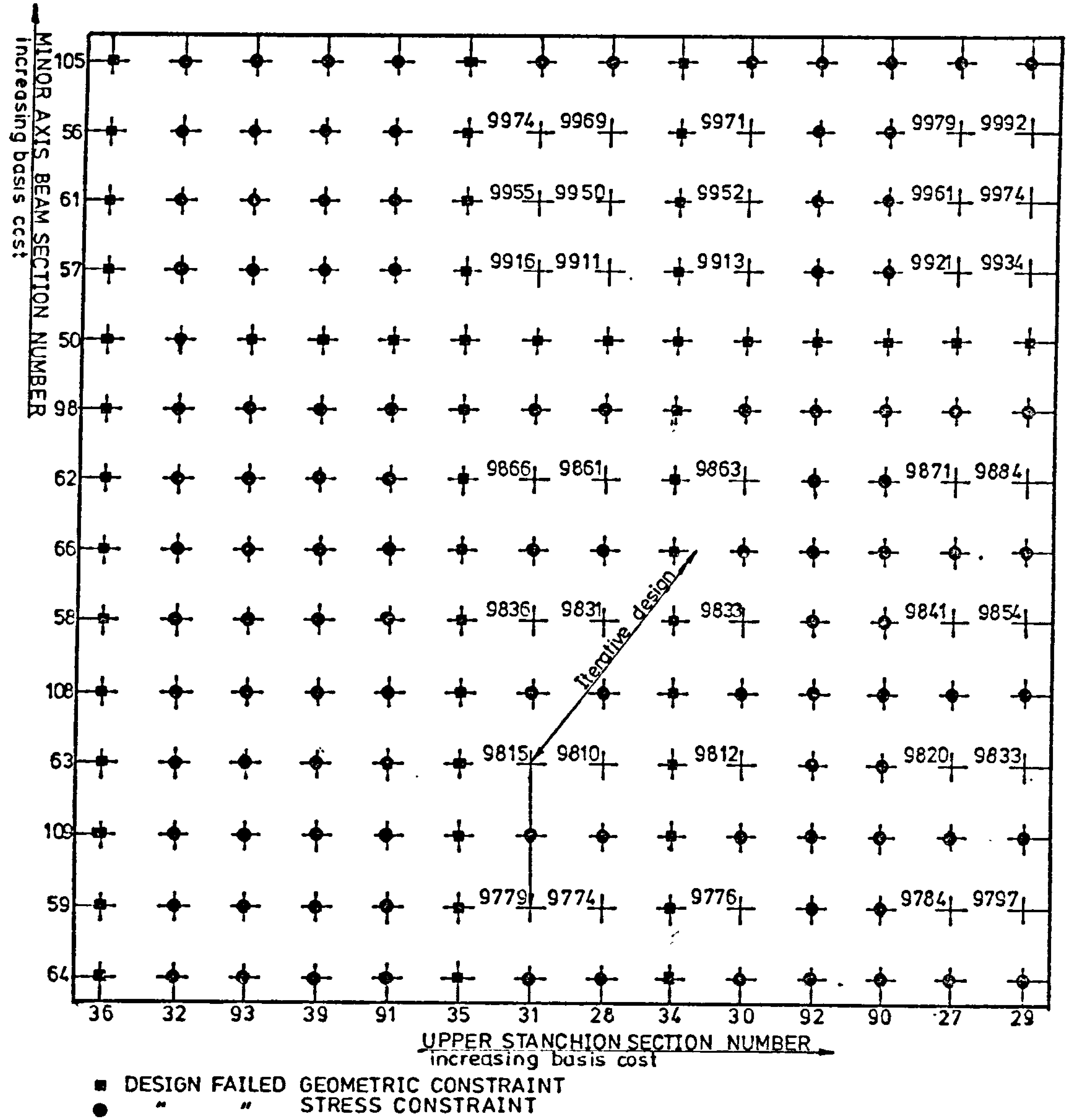


FIG. 7.8 DESIGN SPACE INVESTIGATION
MINOR AXIS EXTERNAL BEAMS
AND EAST & WEST SIDE STANCHION

with seven of the beam sections. Section 34 has a thick flange and web for its size and so does not require stiffeners, thus allowing the use of minor axis beams that are deep in relation to their weight. There is a trend towards increasing cost with increasing basis cost, this is especially so in the direction of increasing beam cost. The optimum design previously found has been superseded by a slightly cheaper design, six beam sections and seven stanchion sections further away.

The stress constrained designs have largely replaced the geometrically constrained designs resulting in a considerable reduction in the number of geometrically constrained designs. The edge of the stress feasible area cannot be easily defined. Many of the stress constrained designs result from the fact that the sections cannot be connected using the standard details.

The geometrically constrained designs have reduced considerably resulting in only a few incompatible combinations. Section 28 used as an upper stanchion is unsuitable throughout because it is deeper than the lower stanchion section, which is fixed. The removal of minor axis depth constraints has resulted in a substantial reduction in the number of geometrically constrained designs, however it has not produced many more feasible designs and has resulted in higher computation costs.

The effect in this case of relaxing minor axis depth constraints was to replace many of the geometrically constrained designs by a few feasible designs and many stress constrained designs (including those for which a connection could not be designed). In this case the effect of relaxing minor axis depth constraints on the optimum design is very small in terms of cost, and large in terms of the location of the optimum design.

7.2.7 Minor Axis External Beams and East and West Side Stanchions (Groups 3 and 7)

The design space is shown in figure 7.8. The designs are divided into fifty three designs which failed geometric constraints, one hundred and eight designs that failed stress constraints and thirty five feasible designs.

The boundary of the stress feasible region can be easily defined and there is no evidence of elastic interation between the beam and column sections. The optimum design is located at the edge of the stress feasible region and is close to the iterative design. The cost increases rapidly with increasing basis beam section cost.

The stress constraints are active within the stress feasible region and some sections are unsuitable either as beam or stanchion sections irrespective of the other section.

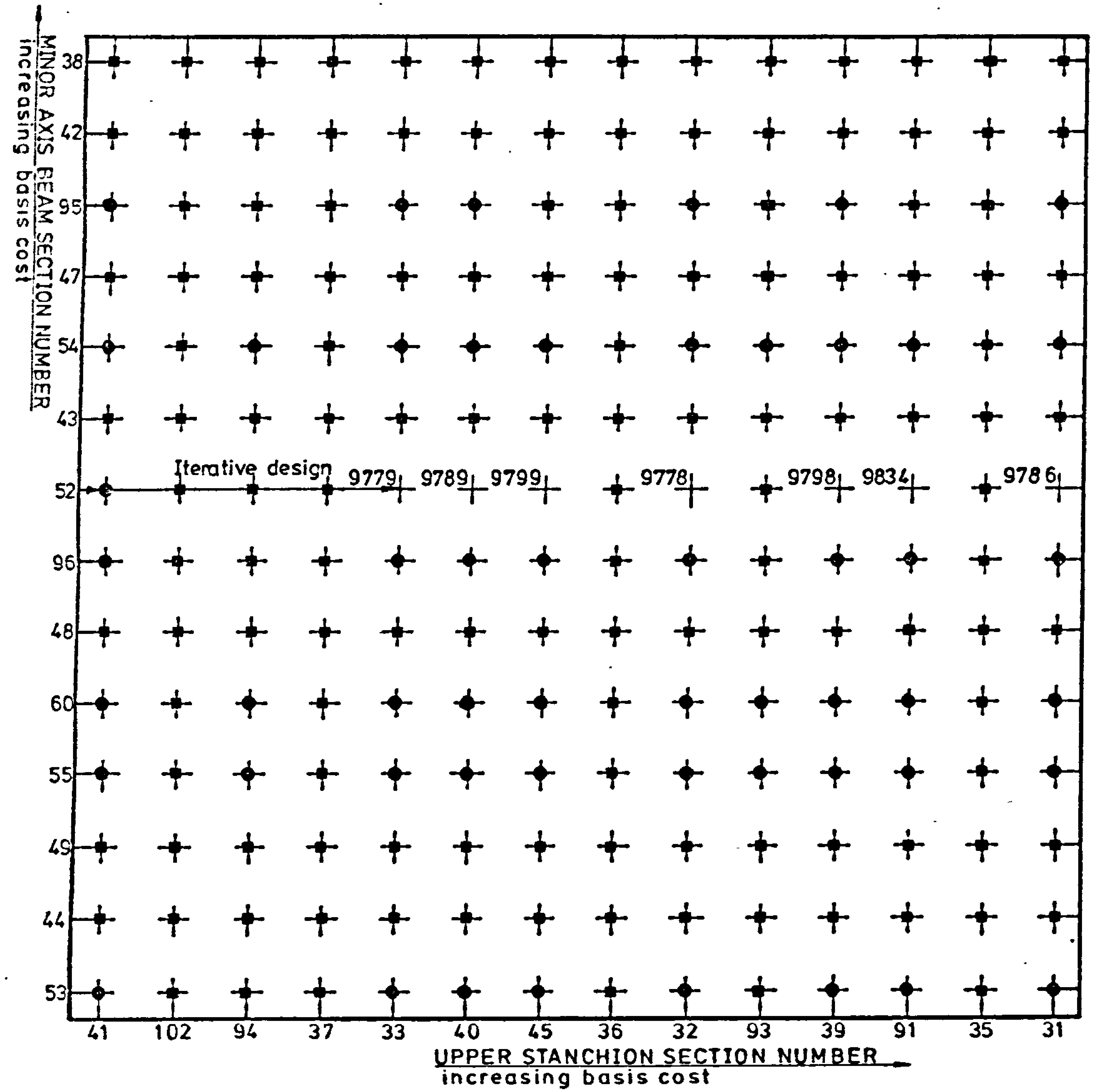
The geometric constaints are active pricipally through the whole length of some horizontal and vertical lines. This feature points to the influence of sections which are not being varied.

The iterative design algorithm approaches through the feasible region and produces a design close to the optimum design.

7.2.8 Minor Axis Internal Beams and Internal Stanchion (Groups 4 and 8)

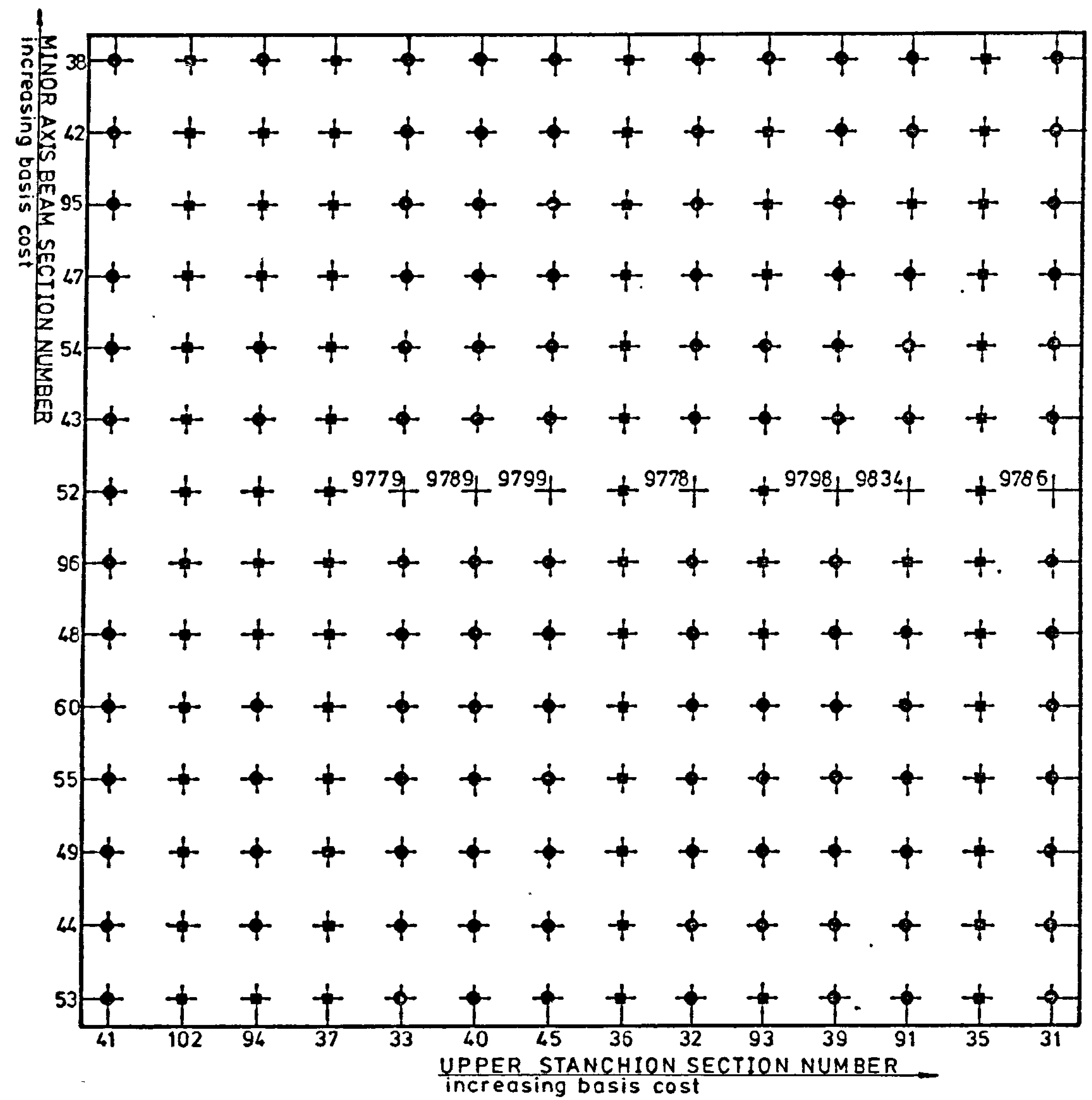
This investigation consisted of three parts. Firstly the design space including minor axis depth constraints was investigated. Secondly the effect of relaxing the minor axis depth constraints was investigated and finally the effect of relaxing the constraints and changing the North and South side stanchion sections.

The first investigation is shown in figure 7.9. The designs are divided into one hundred and thirty seven geometrically constrained designs, fifty two stress constrained designs and seven feasible designs.



■ DESIGN FAILED GEOMETRIC CONSTRAINT
● " " STRESS CONSTRAINT

FIG. 7.9 DESIGN SPACE INVESTIGATION
MINOR AXIS INTERNAL BEAMS.
AND INTERNAL STANCHION



■ DESIGN FAILED GEOMETRIC CONSTRAINT
● " " STRESS CONSTRAINT

FIG. 7.10 DESIGN SPACE INVESTIGATION
MINOR AXIS INTERNAL BEAMS AND INTERNAL
STANCHION (CONSTRAINTS RELAXED)

The feasible designs relate only to one beam section. The boundary of the stress feasible region is not well defined and the shape of the rest of this region cannot be determined. The feasible designs are very similar to each other in cost.

The geometrically constrained designs are concentrated over the whole length of lines of beam or column sections, resulting in some sections being infeasible irrespective of any other variable sections.

The progress of the iterative design algorithm is shown. The algorithm approaches from the infeasible region and reaches a design close to the optimum.

The second investigation is shown in figure 7.10. The designs are divided into seventy two geometrically constrained designs, seventeen stress constrained designs and seven feasible designs. The design space is substantially similar to the first design space, the number of feasible designs remains the same and only twenty geometrically constrained designs are removed. The previous comments are again generally applicable.

The third investigation is shown in figure 7.11. The designs are divided into seventy two geometrically constrained designs, seventy two stress constrained designs and fifty two feasible designs. This design space was produced in order to examine the effect of varying a group of members that was constant, the section for group 6 was changed to section 34. The difference between this design space and the previous design space is considerable.

The feasible designs are concentrated on horizontal lines, these designs replace designs that were geometrically constrained previously. An underlying trend towards increasing cost with increasing beam section cost can be identified. The minimum cost design can be seen to be away from the edge of the stress feasible design space and it can also be seen that the feasible design with the lowest basis cost is not the minimum cost design.

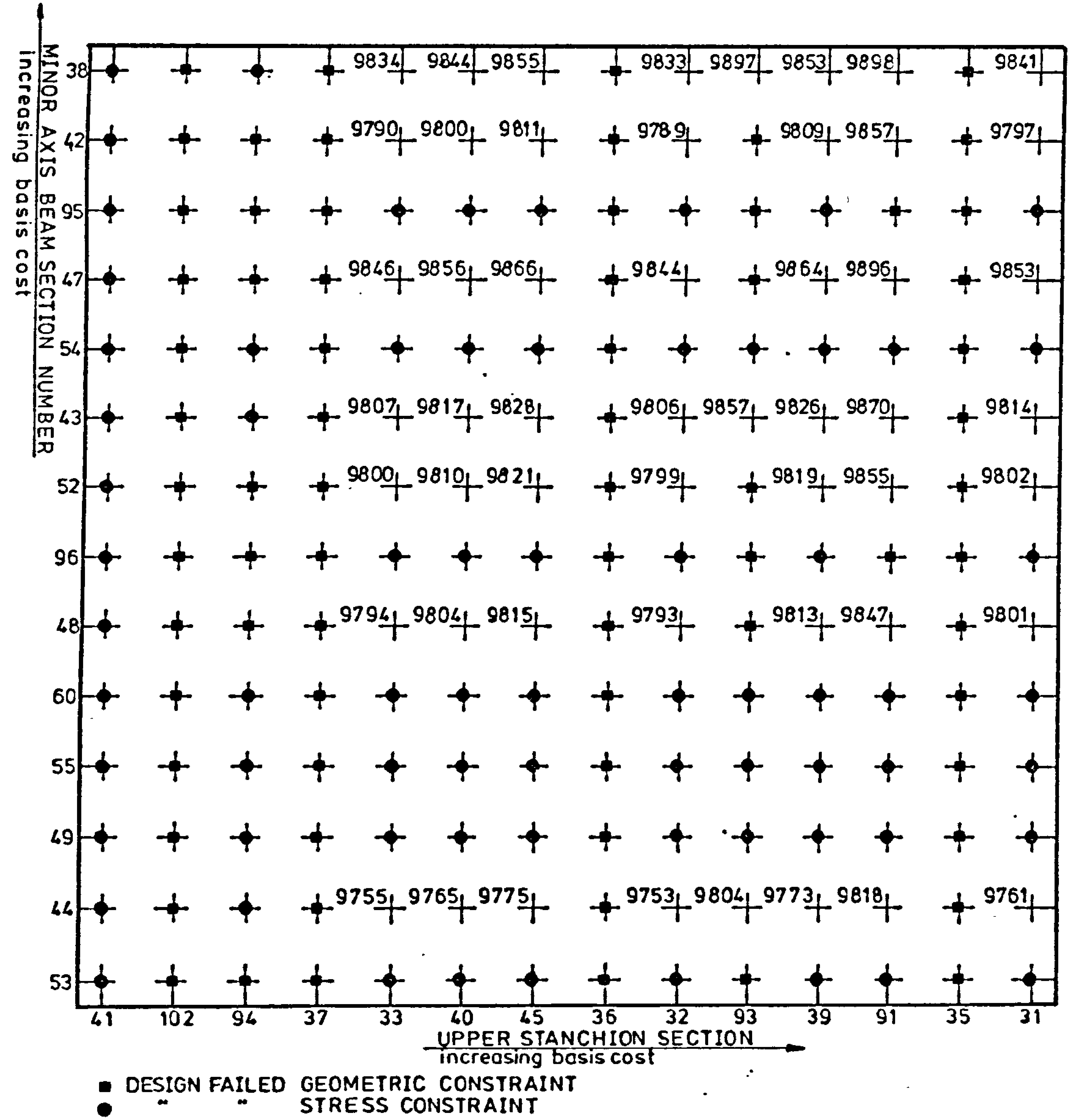


FIG. 7.11 DESIGN SPACE INVESTIGATION
MINOR AXIS INTERNAL BEAMS & INTERNAL STANCHION
(NORTH & SOUTH SIDE STANCHIONS SECTION NUMBER 34)

The area where stress constraints are active for all sections can be readily identified. Some beam sections are stress constrained irrespective of the stanchion section.

The geometric constraints are active principally on vertical lines. Certain stanchion sections cannot be used irrespective of the beam section chosen, because the lower stanchion section constrains the upper section.

These three investigations show that relaxation of minor axis depth constraints does not always increase the number of feasible designs, the geometrically constrained designs often becoming stress constrained. Changing one variable elsewhere in the design space may completely change the configuration of a two variable design space.

7.2.9 External Major Axis Beams and Corner Stanchions

This investigation consisted of two parts. The first part neglected moment redistribution and the second part considered moment redistribution. A slightly larger design space, consisting of eighteen stanchion sections and fourteen beam sections, was used.

The design space for the first investigation is shown in figure 7.12. The designs consisted of one hundred and twenty nine geometrically constrained designs, ninety nine stress constrained designs and twenty four feasible designs.

The feasible designs show a marked trend towards higher cost with increasing beam cost. The edge of the stress feasible region can be readily identified. Each feasible beam section can be seen to be feasible when combined with any of the feasible stanchion sections.

The geometric constraints are active for a number of beam sections irrespective of the stanchion section used, indicating that other members are incompatible with these stanchion sections. Section 102 used as a stanchion is geometrically feasible with some beam sections and infeasible

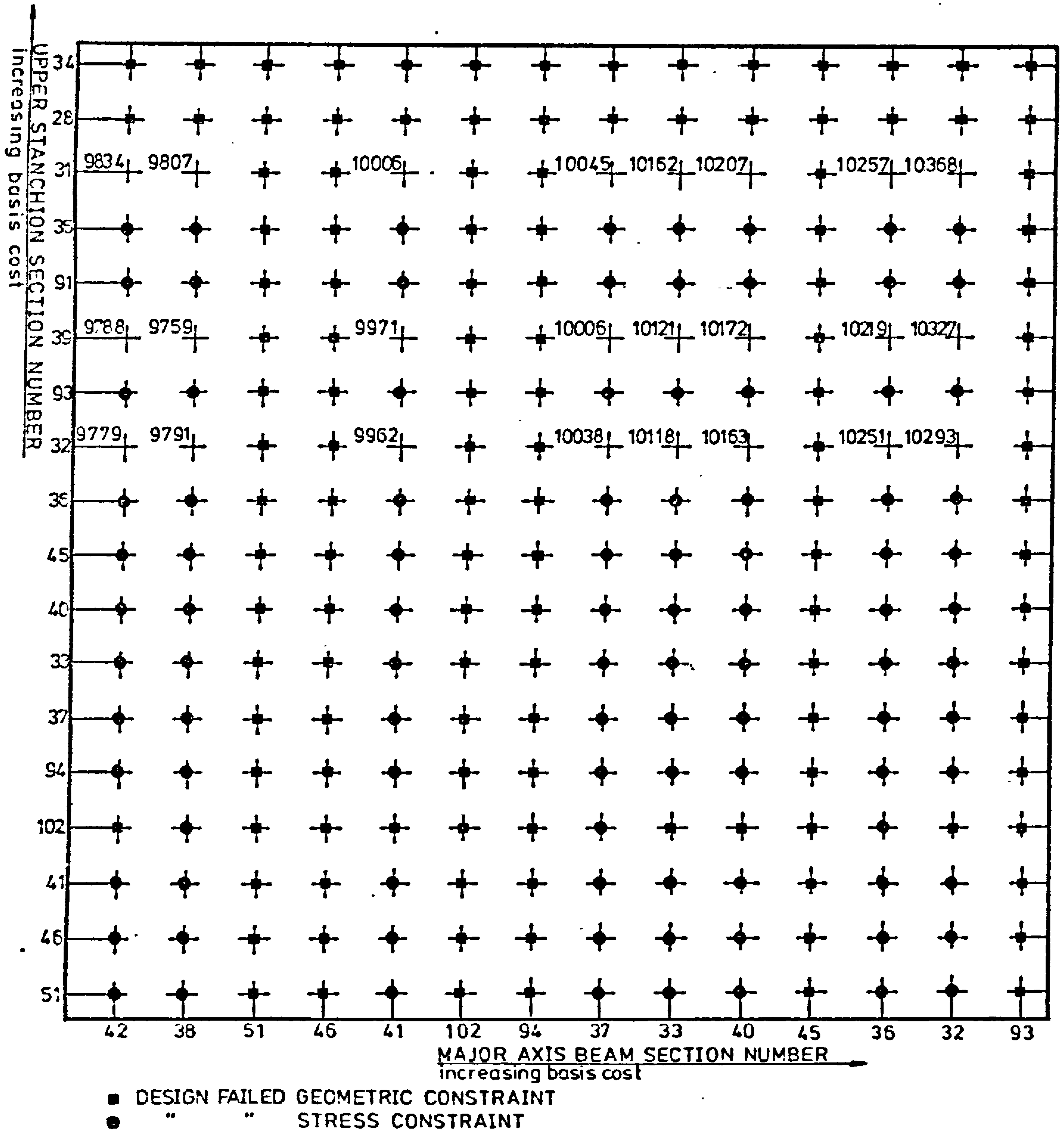


FIG.7.12 DESIGN SPACE INVESTIGATION
EXTERNAL MAJOR AXIS BEAMS & CORNER
STANCHIONS NEGLECTING REDISTRIBUTION

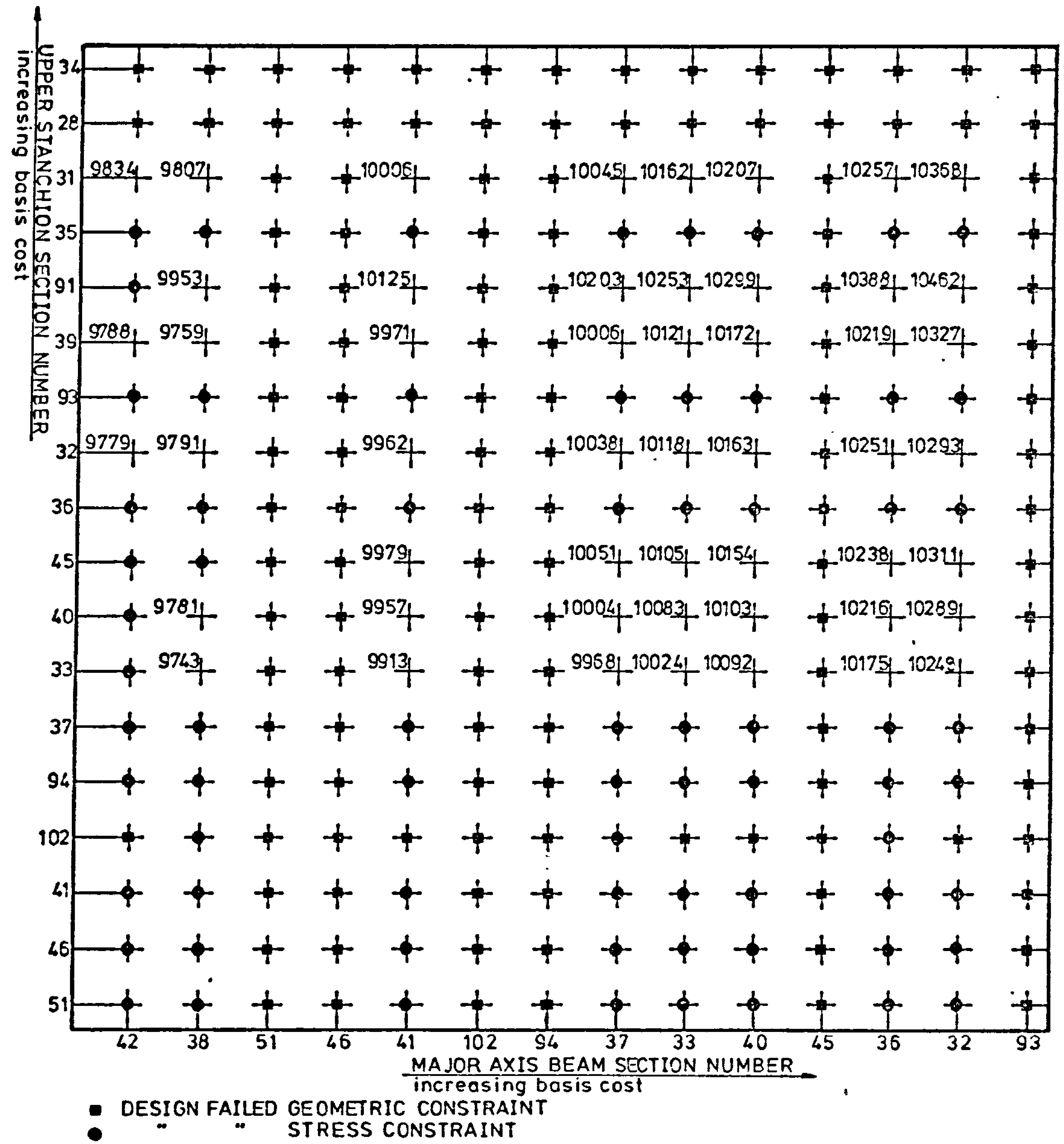


FIG.7.13 DESIGN SPACE INVESTIGATION
EXTERNAL MAJOR AXIS BEAMS & CORNER
STANCHIONS INCLUDING REDISTRIBUTION

with other beam sections, this is due to the influence of the geometric constraint that ensures that the beam flange is not substantially wider than the stanchion flange.

The iterative design corresponds to the minimum section basis cost design and is two stanchion sections and one beam section away from the minimum cost design.

The second investigation, shown in figure 7.13, has many more feasible designs. The designs consist of one hundred and twenty nine geometrically constrained designs, seventy two stress constrained designs and fifty one feasible designs.

The number of feasible designs within the design space shown has almost doubled when considering moment redistribution. The new minimum cost design is only slightly cheaper than the minimum cost design in the first investigation. The designs again show a marked trend towards higher cost with increasing beam section cost. The boundary of the stress feasible region can be readily identified. There is evidence that interaction between beam and stanchion sections occurs,for instance beam section 42 is feasible when combined with some stanchion sections but infeasible with other stanchion sections, which are feasible with other beam sections. Some stanchion sections within the stress feasible region, are infeasible no matter which beam section is used. As would be expected the geometrically constrained designs are exactly the same as the first investigation.

In this case the effect of considering moment redistribution is to considerably increase the number of feasible designs, however the resulting cost reduction is small.

7.2.10 Internal Major Axis Beams and North and South Side Stanchions

This investigation again consisted of two parts, neglecting moment redistribution and considering moment redistribution. An enlarged design space was again considered.

The first investigation, shown in figure 7.14, consists of designs grouped into one hundred and sixty seven geometrically constrained designs, eighty eight stress constrained designs and thirty nine feasible designs.

The boundary of the stress feasible region can be readily identified. The feasible designs show a marked trend towards increasing cost with increasing beam section cost. There is no marked trend with increasing stanchion section cost. The minimum cost design can be seen to be close to the minimum section basis cost design, which is found by the iterative design algorithm. Some stanchion sections are stress infeasible irrespective of the beam section used. The geometric constraints are active for some stanchion sections irrespective of the beam sections and vice-versa.

The second investigation, shown in figure 7.15, is divided into one hundred and sixty seven geometrically constrained designs, seventy one stress constrained designs and fifty six feasible designs.

The number of feasible designs can be seen to have significantly increased due to the consideration of moment redistribution. The boundary of the stress feasible region can be readily identified. The trend towards increasing cost with increasing beam section cost is still evident. A slight trend towards increasing cost with increasing stanchion section cost can also be identified. There is evidence of interaction between beam and stanchion sections. The minimum cost design can be seen to be the minimum section basis cost design. Again the difference in cost between the minimum cost design in each investigation is small and the geometrically constrained designs are exactly the same in each case.

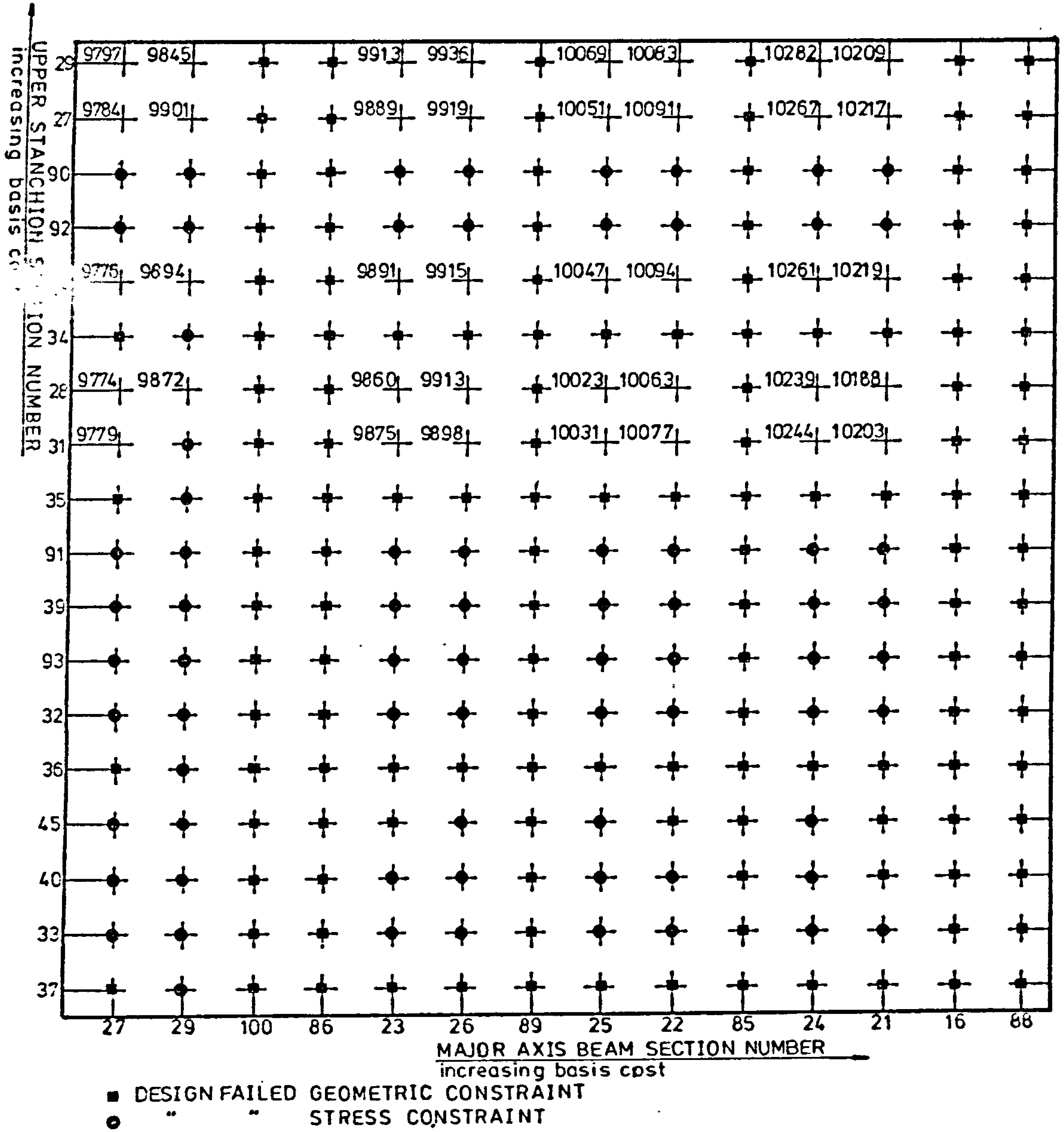


FIG. 7.14 DESIGN SPACE INVESTIGATION
INTERNAL MAJOR AXIS BEAMS & EAST AND WEST
SIDE STANCHIONS NEGLECTING REDISTRIBUTION

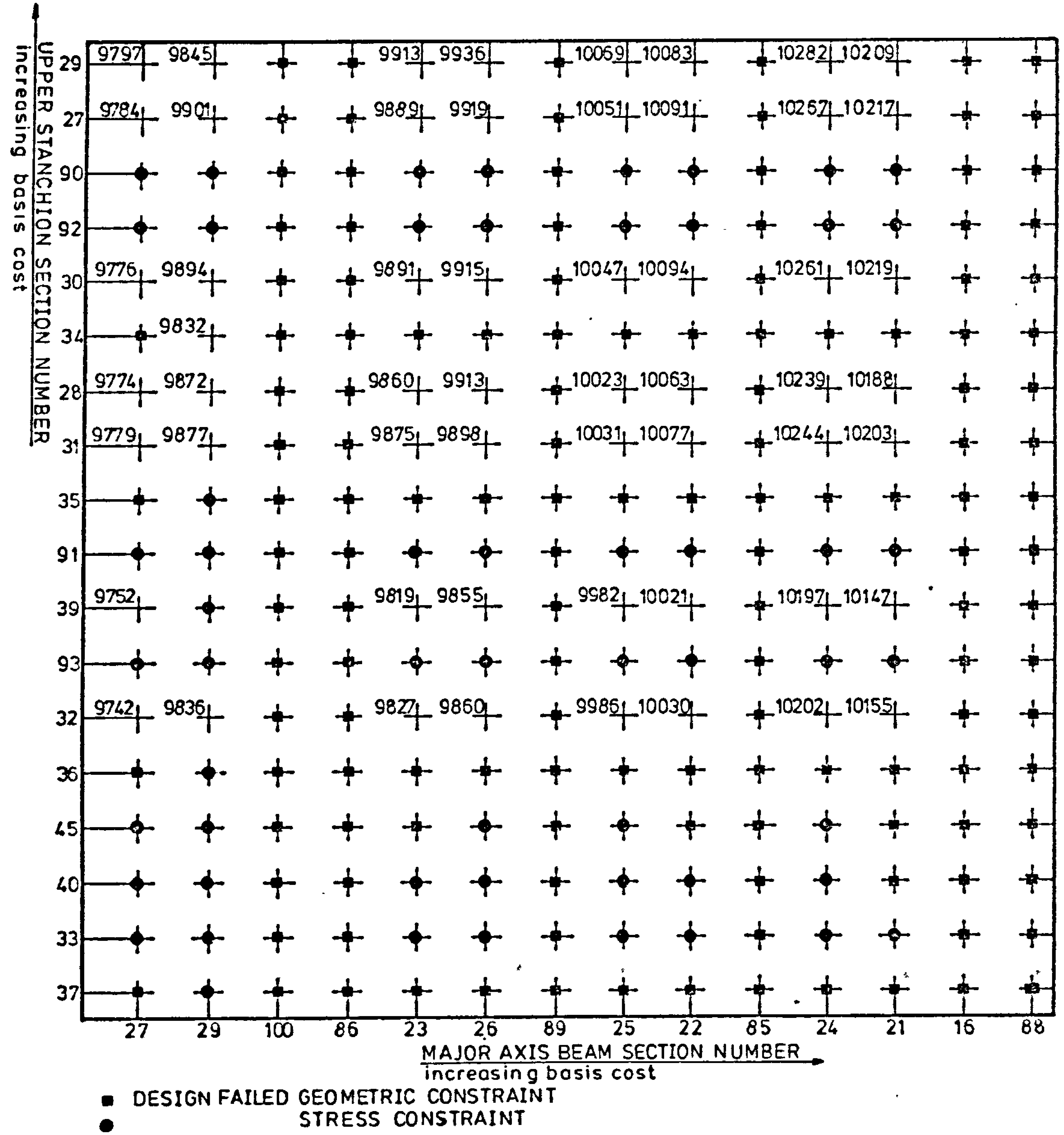


FIG. 7.15 DESIGN SPACE INVESTIGATION
INTERNAL MAJOR AXIS BEAMS & EAST AND WEST
SIDE STANCHION INCLUDING REDISTRIBUTION

7.2.11 Conclusions of the Design Space Investigation

Many optimum design problems consist of a design space which includes a clearly defined feasible region, in which all designs are feasible, and an infeasible region. The feasible region is contoured and the variables are continuously defined, there may be one or more local optima. The design space for the problem considered differs in many respects from the above description. These differences will now be highlighted.

The design space is divided into two regions, the infeasible region and the stress feasible region. The stress feasible region is the area in which all feasible designs are located, some infeasible designs are also located in this region. The boundary between these regions is seldom clearly defined. The distribution of feasible designs may be sparse especially in the region of the optimum design.

The feasible designs are located at discrete points and each one represents a spot height. The disjoint nature of the design space does not allow contours to be drawn. Due to a cost/weight trade-off, increases in the basis cost of stanchion sections do not necessarily result in significant increases in total cost. This may produce a design space in which many designs have a very similar cost. Due to the small amount of fabrication work required for beams, the cost/weight trade-off does not occur to the same extent, resulting in significantly higher costs when the beam section basis cost is increased.

The optimum designs mentioned within the investigation are only optimal with respect to the two variables chosen, all other variables remaining constant, they are therefore not necessarily the optimal design for the whole problem. The "optimum" designs have been found to be on the boundary of the stress feasible region and also away from this boundary. A number of "optimal" designs have been found to be "vertex" optima in that they lie close to the intersection of two or more stress constraints. The "optimum" design may or may not correspond with the minimum section basis cost design and it may be a considerable distance from the design found by the iterative design algorithm. Stress infeasible designs may lie between the "optimum" design and the iterative design. The iterative design algorithm generally locates a design close to the minimum section basis cost design, and with a similar cost to the optimum design. It is worth noting that the true optimum design may have a cost less than or equal to £9,731 (the lowest cost recorded in the investigation), this may be compared with the cost of the iterative design of £9,779.

Geometrically constrained designs occur all over the design space and they are not related to section basis cost, therefore it is not possible to identify a "geometrically feasible region" in the same way as for the stress feasible region. Geometrical constraints complicate the problem because, when varying one member or a group of members, consideration has to be given to the section used for all other connected members. It has been shown that changing the section for one member can completely change the character of the design space which includes variations in other members. These features mitigate against the use of direct search types of algorithms because, a direction of movement which will lead to the optimum design cannot in general be found and such an algorithm may find its progress blocked when it changes one section and changes the character of the surface over which it is moving. The relaxation of minor axis beam geometric constraints has been shown to produce slightly cheaper designs at the expense of additional computation.

The use of moment redistribution results in a significant increase in the number of stress feasible designs and an enlargement of the stress feasible region. In the two cases considered the result of considering moment redistribution was to produce a slightly cheaper design at the expense of a great deal of additional computation.

7.3 DEVELOPMENT OF A DESIGN STRATEGY

During the design space investigation a number of factors have been identified which can be utilised in developing a design strategy. These factors will now be described.

(a) The beam sections produced by the iterative design algorithm were close to optimal due to the fact that the cost of the frame increases consistently with increases in the section basis cost of the beam sections.

(b) Changing any members within a string of stanchion members does not affect the choice of sections for any other stanchion members.

(c) Worthwhile reductions in cost may be achieved by searching locally varying stanchion sections. The extent of this search must be determined with regard to the likely reduction in cost and the additional computation required.

The design strategy developed consists therefore of the following stages:-

(a) Perform an iterative design to select a set of initial sections.

(b) Hold beam sections constant.

(c) Use a local search algorithm on each string of stanchions.

7.3.1 Application of the Design Strategy

It is necessary to establish which of the search algorithms are to be used and the role of moment redistribution and the relaxation of minor axis beam constraints. In order to establish which approach to use, the frame used for the design space investigation, was investigated using various approaches. A summary of the salient features of each investigation is given in Table 7.1. Wherever the dynamic programming and enumeration algorithms were used the choice of sections from each group was taken from the consecutive sections of which five had basis costs less than the current section for the group. The savings shown are based on the reduction due to the algorithm rather than the reduction below £9,779, where necessary the saving based on this figure is mentioned in the text. The cost of computing to a commercial user was approximately £0.33 per second at the time of running. The investigations will now be described.

(a) An iterative design was performed with minor axis beam depth constraints included. The algorithm performed three iterations before converging on a suitable design. Starting the algorithm with the smallest possible sections converged on the same design in four iterations.

(b) Using the iterative design as a starting point, dynamic programming was used on each string of stanchions. It can be seen from Table 7.1 that a worthwhile reduction in cost occurs due to the use of the algorithm. Examination of the design space investigation shows that the reduction achieved is to be expected.

(c) Using the iterative design as a starting point, the enumeration algorithm was used on each string of stanchions. Similar reductions in cost were achieved at the expense of more computation. The number of combinations considered was restricted by limiting the sum of variables to twenty, limiting this sum to ten resulted in a final cost of £9,708 with a cost reduction of £1.43 per second. It was found to be difficult to estimate the number of combinations to be tried because the use of a small number of combinations may result in infeasible designs or very few feasible designs. The dynamic programming algorithm can be seen to be more attractive than enumeration. Comparison of the cost at the end of each stage shows that these algorithms produce different results. The reason for this difference is that the enumeration algorithm considers extra costs whereas the dynamic programming algorithm does not. The design found in the first stage has different sections and a substantially lower section extra cost. However by the end of the final stage the final costs vary by only a few pence, even though the sections used are different.

(d) Using the iterative design as a starting point, the enumeration algorithm was used on each stanchion string, considering moment redistribution. The sum of variables was restricted to ten and any design with a stress ratio greater than 1.1 was neglected. A further reduction in cost was achieved with a substantial amount of computation, this reduction was achieved less efficiently than in the previous investigations. The reduction was due solely to the use of a different section for the lower stanchion on the East and West sides.

(e) The procedure in investigation (d) was repeated but the major axis beams were increased in section modulus (and cost). The results show a substantially similar final cost to the previous investigation even though design started from a higher initial cost. Recomputation of the cost reduction using an initial cost of £9,779 gives a reduction of £0.65 per second, similar to the previous application.

(f) A second iterative design was performed with minor axis beam depth constraints relaxed. This algorithm converged in four iterations from the largest possible sections. The tonnage is less than previously despite the higher total cost, indicating the use of sections with a higher cost per tonne.

(g) Using the second iterative design as a starting point, dynamic programming was used on each stanchion string.

This results in the lowest final cost recorded. If the cost reduction is recalculated as before it reduces to £3.95 per second, still a worthwhile amount. It is worth noting that the feasible designs which result when minor axis beam depth constraints are relaxed, include all the feasible designs which result when these constraints are considered. The result of this is that cheaper designs can often be achieved by relaxing these constraints. Minor axis beam depth constraints were introduced in order to reduce computation, in this case the reduction is from 55 seconds to 44 seconds for a similar number of design combinations. The resulting design would violate minor axis depth constraints if they were applied.

(h) Using the second iterative design as a starting point, the enumeration algorithm was used on each string of stanchions. The result of this investigation was that the dynamic programming algorithm produces a much cheaper design with a far less computation. It is interesting to note that the enumeration algorithm produces a cheaper design at the end of the first stage, this results from the consideration of extra cost, in subsequent stages the enumeration algorithm loses its advantage. Recalculation of the cost reduction, as previously, results in a reduction of £1.37 per second. The reduction in computing cost by considering minor axis depth constraints is from 108 seconds to 84 seconds. Again the resulting design would violate minor axis depth constraints if they were applied.

(i) A selection of sections was made from the design space investigation and this design was evaluated. The resulting design was cheaper than the designs produced using the first iterative design as a starting point, but was not cheaper than those using the second

IDENTIFICATION LETTER AND ALGORITHM	TOTAL COST					COST REDUCTION £/SECOND	TOTAL COST MINUS EXTRAS	FINAL TONNAGE
	INITIAL	STAGE 1	STAGE 2	STAGE 3	STAGE 4			
(a) Iterative Design	-	-	-	-	9779	-	9310	32.18
(b) Dynamic Programme	9779	9779	9779	9755	9702	1.75	9233	32.50
(c) Enumeration	9779	9760	9760	9755	9702	0.92	9284	32.73
(d) Enumeration	9779	9760	9760	9685	9637	0.58	9242	32.23
(e) Enumeration	9887	9751	9747	9677	9638	1.14	9239	32.63
(f) Iterative Design	-	-	-	-	9811	-	9344	31.93
(g) Dynamic Programme	9811	9781	9678	9612	9650	4.53	9710	32.51
(h) Enumeration	9811	9759	9717	9684	9631	1.67	9220	32.54
(i)	-	-	-	-	9682	-	9213	32.38
(j)	-	-	-	-	9651	-	9206	31.91

TABLE 7.1

iterative design as a starting point. This approach, though logical, has not resulted in a significantly cheaper design. It would only be possible to use this type of approach if a full design space was evaluated.

(j) A further selection of sections was made from the design space investigation and this design was evaluated using moment redistribution to ensure a solution. The resulting design was slightly cheaper than the previous one, however it was more expensive than both the previous applications which used moment redistribution. Again this approach could only be used if a full design space investigation was performed.

Examination of Table 7.1 and the foregoing shows that dynamic - 13
programming combined with relaxed minor axis constraints resulted in the cheapest frame together with the highest cost reduction. Where the dynamic programming algorithm was used with active minor axis constraints, the algorithm again showed a high cost reduction and therefore a high efficiency. The enumeration algorithm can be seen to be less efficient than dynamic programming, however the saving does always pay for the computer time used. The use of moment redistribution, though resulting in cheaper designs does tend to be rather expensive in terms of computation. The results show that the maximum reduction that has been achieved is approximately 2.2% of £9,779 showing that the maximum reduction that can be achieved is generally small.

Examination of Table 7.1 shows that the cheapest design is
not the minimum weight design, in fact a design with - 28
almost the lowest weight (of those shown) has the highest cost. The same type of behaviour exists when considering total cost designs and total cost minus extras designs, in this case the cheapest and most expensive designs are the same, however between these designs the order is different. Comparison of designs (b) and (c) shows that

two designs with virtually the same total cost have markedly different tonnages, and markedly different extra costs.

The strategy that has been developed is to perform an 4
iterative design, followed by dynamic programming used on each stanchion string. Minor axis beam depth constraints are neglected throughout. This strategy has the benefit of efficiency combined with modest amounts of computation. In order to evaluate both the likely cost reductions of using the strategy and the economics of various column splicing arrangements, a series of buildings were designed. This investigation will now be described.

7.4 INVESTIGATION OF COLUMN SPLICING AND THE DESIGN STRATEGY

During the design of a multi-storey structure it is necessary to decide on the most economic splicing arrangement for the stanchions. Splicing at every storey reduces the basis cost of sections at the
expense of high splicing costs. Not splicing at all reduces the · 17
fabrication cost but increases the cost of sections, it may also be impractical. Clearly a "trade-off" exists between fabrication and section costs.

A number of small buildings were chosen with various splicing
arrangements, these buildings were then designed using the design - 22
strategy. These buildings had a common storey height of 4m and the same floor loading as the frame shown in figure 6.10. The frames chosen varied over the full range of the capacity of the connection details and sections. The available sections for the dynamic programming algorithm consisted of ten sections of similar cost to the section chosen by the iterative design algorithm.

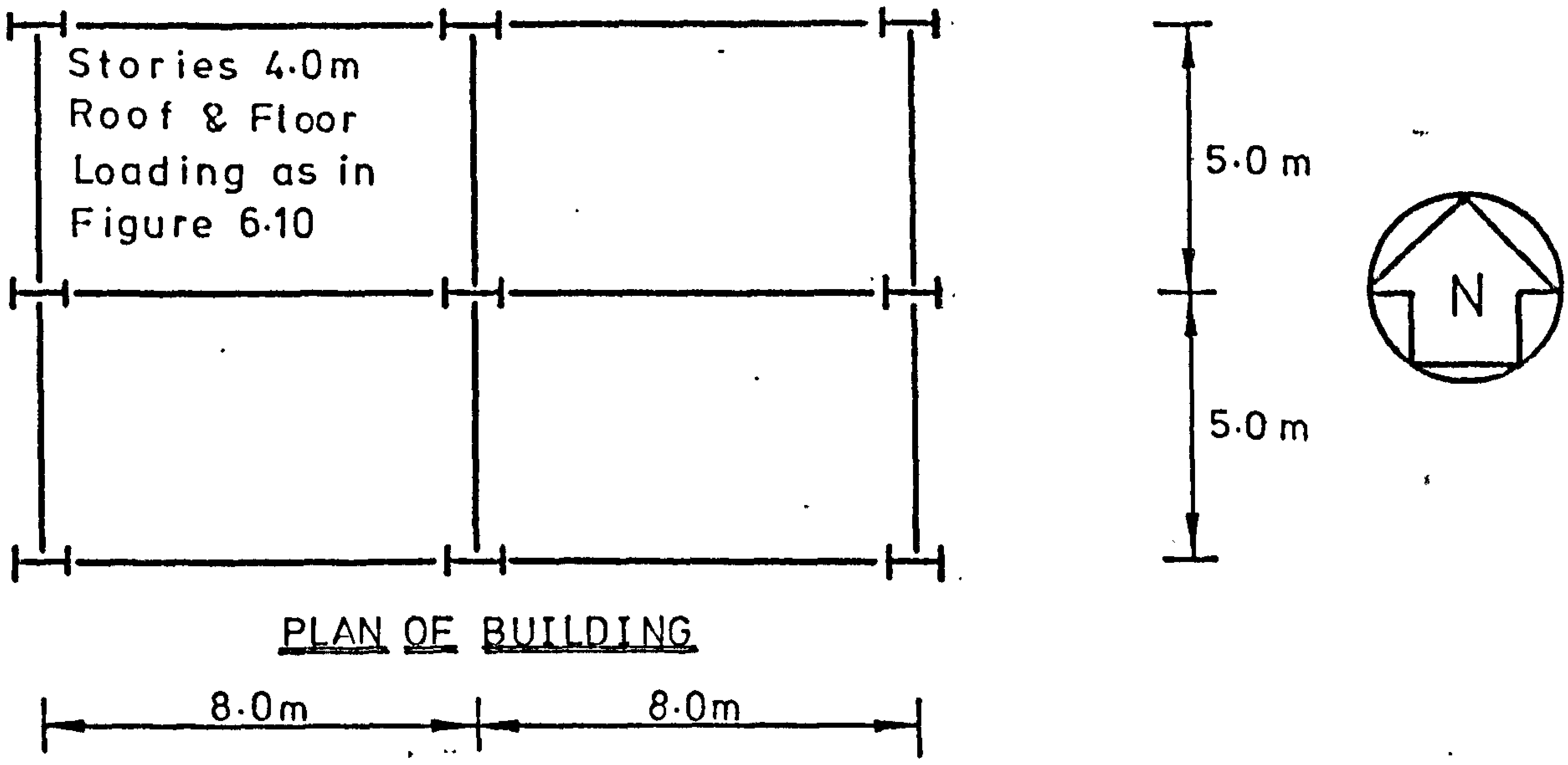
The dynamic programming algorithm costed out all sections and connections attached to the stanchion string, producing a cost for the optimal strategy for each stanchion string. The cost of the optimal strategy does not include section extra or transport costs, because these costs would distort the results. It was found that suitable sections could not always be found for every stanchion string, generally because of the influence of standard connection details.

The results are summarised in figures 7.16, 7.17 and 7.18. The most startling result that can be seen is that in general the most economic way to splice a stanchion is to place the splices as far apart as possible. There are only really three practical column splicing arrangements, splicing at either one, two or three storeys. Examination of the results shows that in every practical case it is cheapest to splice stanchions at every three storeys. The investigation was continued to four, six and twelve storeys between splices, in order to see if the cost of sections ever counteracted the cost of splices. This was seen to occur in each twelve storey building, it being cheapest to splice at every six storeys.

The use of splices at every three storeys can be seen to produce considerable savings over using splices at every storey. For corner stanchions these savings vary between 24 and 33 per cent with an average of 28 per cent, the percentage savings increasing with the height of the building. For the North and South side stanchions the figures are 11%, 43% and 25% respectively. For East and West side stanchions the figures are 26%, 43% and 33% respectively. For the internal stanchions the savings are between 12% and 28% with an average of 20%.

The corresponding savings when comparing splicing at every two storeys with splicing at every three storeys are less. For corner stanchions they vary between 5% and 9%, the average difference being 7%. For North and South side stanchions the corresponding figures are 1%, 14% and 6% respectively. For East and West side stanchions the figures are 6%, 20% and 11% respectively. The internal stanchions show differences of 6% and 7%.

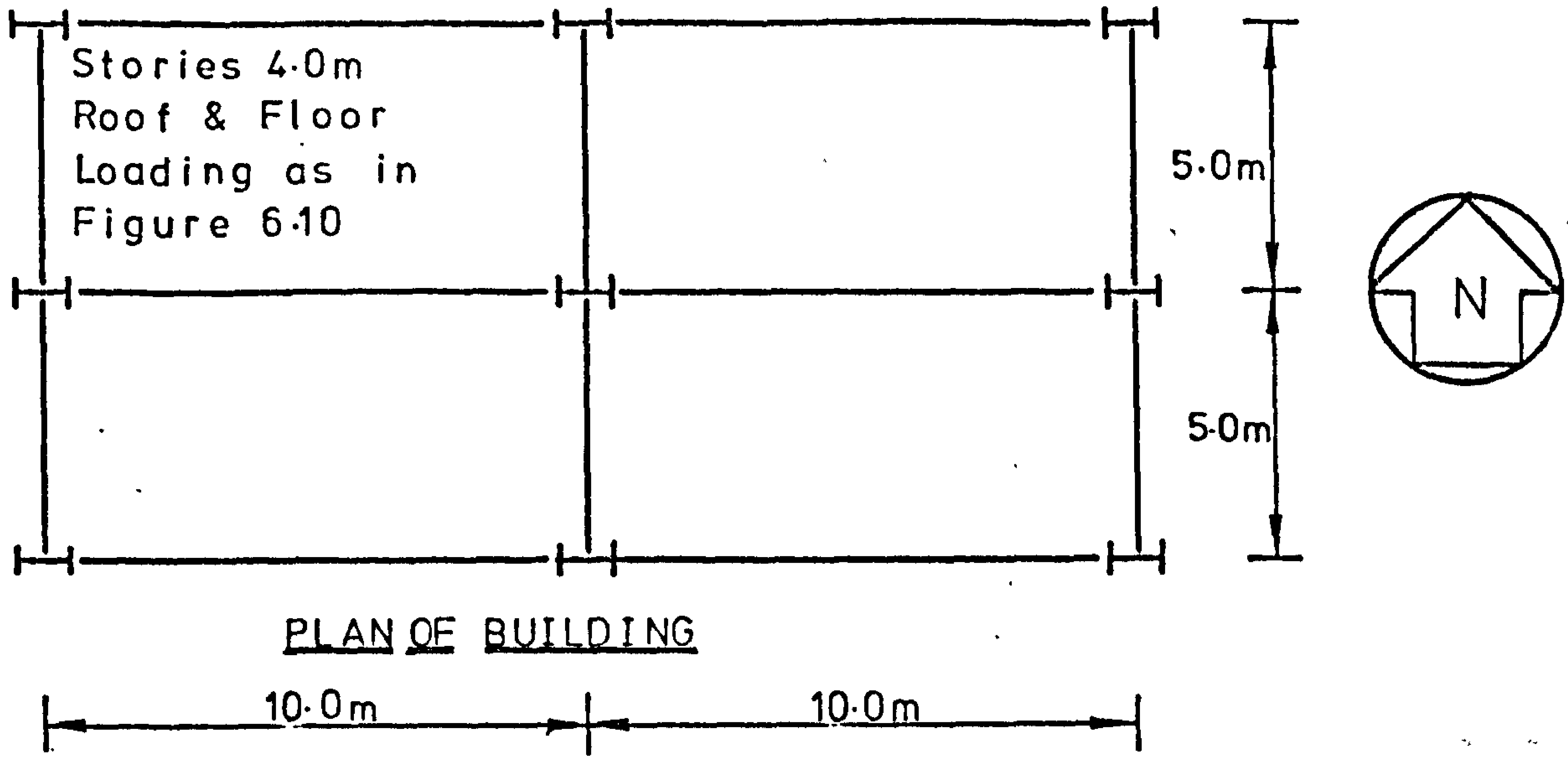
It can be seen that splicing at every storey is unsuitable in all cases. Splicing at every two storeys (which is common in practice) can be seen to involve a small penalty over the optimum of splicing at every three storeys.



COST PER STANCHION PER STOREY (£) NO SECTION EXTRAS

Number of Stories	Total Load at Base (KN)	Number of stories between splices					
		1	2	3	4	6	12
Corner Stanchions							
3	314	98·75	—	79·67	—	—	—
6	674	114·96	96·38	91·92	—	86·33	—
12	1394	144·75	119·10	111·29	106·52	105·35	108·21
North & South Side Stanchions							
3	628	113·83	—	92·50	—	—	—
6	1348	156·50	134·25	128·56	—	127·92	—
12	2788	221·21	183·79	160·75	152·08	149·04	154·63
East & West Side Stanchions							
3	628	129·83	—	102·67	—	—	—
6	1348	158·83	130·67	120·92	—	113·50	—
12	2788	218·08	173·63	157·83	154·58	147·75	154·71
Internal Stanchion							
3	1256	147·33	—	131·00	—	—	—
6	2696	204·17	170·00	160·17	—	161·67	—
12	5576	—	—	225·67	215·25	213·17	232·25

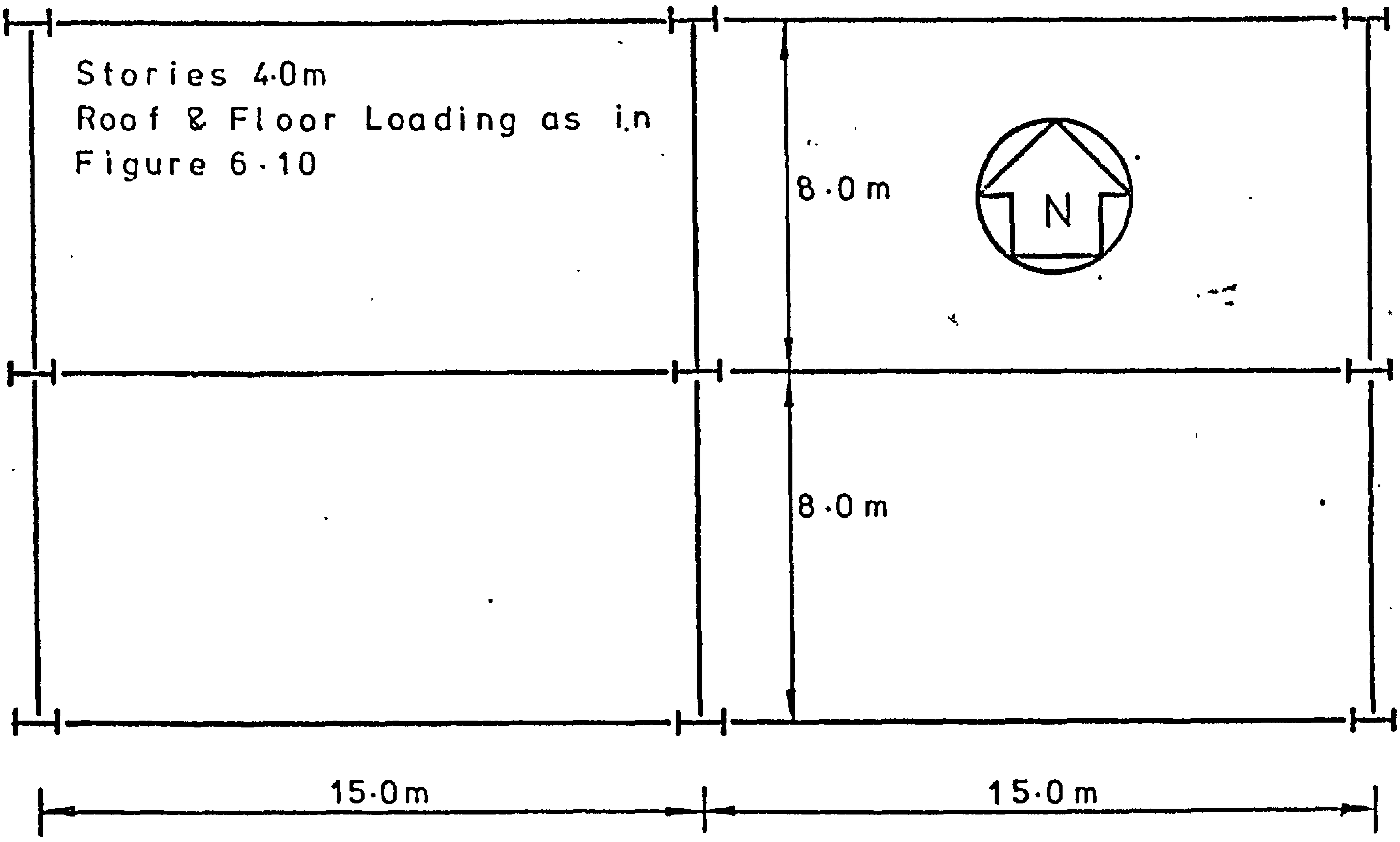
FIG.7.16 STANCHION SPLICE INVESTIGATION
BUILDING TYPE ONE



COST PER STANCHION PER STOREY (£) NO SECTION EXTRAS

Number of Stories	Total Load at Base (KN)	Number of Stories Between Splices					
		1	2	3	4	6	12
Corner Stanchions							
3	392	124·83	—	99·33	—	—	—
6	842	142·83	119·25	112·33	—	105·79	—
12	1742	176·73	146·29	134·02	130·19	125·19	129·92
North & South Side Stanchions							
3	784	140·50	—	126·17	—	—	—
6	1684	178·50	156·58	150·25	—	147·00	—
12	3484	266·79	—	186·63	184·42	177·25	188·38
East & West Side Stanchions							
3	784	173·67	—	135·50	—	—	—
6	1684	204·75	166·67	157·25	—	144·25	—
12	3484	283·58	239·16	199·00	189·42	181·08	185·63
Internal Stanchion							
3	1568	185·67	—	160·00	—	—	—
6	3368	257·84	214·50	201·79	—	200·67	—
12	6968	—	—	—	—	280·00	296·83

FIG.7.17 STANCHION SPLICE INVESTIGATION
BUILDING TYPE TWO



PLAN OF BUILDING

COST PER STANCHION PER STOREY (£) NO SECTION EXTRAS

Number of Stories	Total Load at Base (KN)	Number of stories between splices					
		1	2	3	4	6	12
Corner Stanchions							
3	941	313·08	—	249·17	—	—	—
6	2021	339·92	274·71	255·50	—	233·63	—
North & South Side Stanchions							
3	1882	366·17	—	292·00	—	—	—
6	4042	507·42	423·92	418·42	—	368·67	—
East & West Side Stanchions							
3	1882	413·00	—	312·50	—	—	—
6	4042	—	—	388 50	—	371·67	—
Internal Stanchion							
3	3764	461·33	—	392·33	—	—	—
6	8084	—	—	—	—	—	—

FIG. 7.18 STANCHION SPLICE INVESTIGATION
BUILDING TYPE THREE

As may be expected the cost of a stanchion increases with increasing load. There does not appear to be a simple relationship between total load at the base of the stanchion and the cost of the stanchion, the cost being dependant on the bay sizes as well as the loading.

As previously mentioned the results of this investigation can be used to assess the efficiency of the algorithms and the design strategy. A summary of the costs of the designs is shown in Table 7.2. Three different situations can be seen to occur.

Firstly the iterative design algorithm produces a feasible 10
design in which all the connections can be designed. The dynamic programming algorithm then reduces the cost. The minimum reduction found was 0.09%, the maximum was 5.52% with an average of 1.69%. The reductions can be seen to be small in relation to the cost of the frame, however in most cases the reduction is more than sufficient to pay for the computing cost. Reduction in cost of a particular stanchion string is not guaranteed, the iterative design often producing an optimal design.

Secondly the iterative design algorithm produces a design in 19
which some connections cannot be designed. The dynamic programming algorithm finds a feasible design that can be costed. When this happens the connections for the internal usually cannot be designed due to the high loads involved. The iterative design algorithm considers only the design of sections and therefore failure to design connections is to be expected.

Thirdly the iterative design algorithm may produce a design 26
in which the connections cannot be designed, use of the dynamic programming algorithm does not produce a design that can be costed. The connections on the internal stanchions, near to the bottom, cannot be designed within the range of connections available. The algorithm does however find a set of sections that are suitable with respect to geometric and stress constraints.

BUILDING NUMBER	NUMBER OF FLOORS BETWEEN SPLICES	AT END OF ITERATIVE DESIGN STAGE TOTAL COST	SECTION BASIS COST	SECTION EXTRA COST	DURING THE DYNAMIC PROGRAMMING STAGE COST AFTER CORNER COLUMN	COST AFTER NORTH COLUMN	COST AFTER EAST COLUMN	COST AFTER INTERNAL COLUMN	SECTION EXTRA COST AT END	REDUCTION IN TOTAL COST %
1	1	6012	2632	413	9535	5852	5821	5820	410	3·30
1	3	5167	2668	346	5167	5167	5162	5162	346	0·09
2	1	13654	5763	596	13621	13544	13471	13471	654	1·36
2	2	11893	5846	610	11869	11846	11761	11742	590	1·29
2	3	11436.	5956	518	11481	11481	11380	11325	586	0·98
2	6	11158	6324	492	11139	11139	11016	11016	497	1·29
3	1	—	13752	1182	—	—	—	—	—	—
3	2	—	13935	1124	—	—	—	—	—	—
3	3	26365	13906	1161	26254	26254	26254	25982	1129	1·47
3	4	25478	14022	1031	25458	25458	25458	25261	1045	0·86
3	6	—	14471	823	—	—	—	24617	721	—
3	12	—	16158	605	—	—	—	25168	604	—
4	1	7684	3738	295	7684	7658	7633	7617	372	0·88
4	3	6873	3824	312	6867	6867	6857	6857	338	0·23
5	1	17299	8015	645	17185	17048	16778	16711	571	3·52
5	2	15538	8193	598	15364	15322	15177	15154	516	2·53
5	3	14973	8289	539	14763	14763	14763	14751	608	1·50
5	6	14468	8745	429	14254	14254	14254	14254	465	1·50
6	1	—	18766	1113	—	—	—	—	—	—
6	2	—	18901	1095	—	—	—	—	—	—
6	3	—	18879	993	—	—	—	—	—	—
6	4	—	19096	847	—	—	—	—	—	—
6	6	32060	19549	745	32060	31951	31853	31853	780	0·65
6	12	—	21568	555	—	—	—	32394	565	—
7	1	21634	12110	582	21327	20711	20503	20503	447	5·52
7	3	—	12524	358	—	—	—	18433	441	—
8	1	—	26080	859	—	—	—	—	—	—
8	2	—	25894	863	—	—	—	—	—	—
8	3	—	26196	673	—	—	—	—	—	—
8	6	—	27284	580	—	—	—	—	—	—

TABLE 7·2

It can be seen that if the design strategy is used the result is, at best an economic structure, and at worst a set of reasonably economic sections which cannot always be joined using the available connections.

Further examination of the cost data shows that in most cases the section basis cost increases with increasing distance between splices. There are three cases where this does not occur, indicating that the iterative design has converged on a non optimal design in the case with the smallest distance between splices. The section extra costs generally decrease with increasing distance between splices, this is to be expected due to the smaller number of different types of sections and the larger quantities of these sections contributing to smaller extra cost.

7.5 THE PROPORTIONS OF COST ATTRIBUTABLE TO EACH MATERIAL AND FABRICATION ITEM

The proportions of cost taken by each fabrication and material item was evaluated for one frame, three grades of steel and each design method. This approach allows an assessment of the sensitivity of the final cost to changes in various cost items. The frame chosen, shown in figure 7.19, is around the middle of the span and height range for which the design programme was intended.

The results of the investigation are shown in Table 7.3. 22
The procedure used was to allow the iterative design algorithm to converge, the cost was then subdivided into its component costs. The cost information is arranged so that comparison can be made between grades of steel and between items. The cost breakdowns shown allow the comparison of grades of steel and design methods. The term "section ratio" expresses the difference in section basis cost using the nominal, and true lengths of members, this ratio is always close to one. The section basis cost and tonnage are based on the nominal lengths of members and the total cost includes material, fabrication and section extras. The cost of sections plus extras allows the reduction in cost of sections with increase in grade of steel to be estimated.

	JOINT COMMITEE REPORT DESIGN METHOD (JCR) COSTS AS A % OF THE TOTAL COST FOR GRADE 43			BRITISH CONSTRUCTIONAL STEELWORK ASSOCIATION DESIGN METHOD (BCSA) COSTS AS A % OF THE TOTAL COST FOR GRADE 43			BUILDING RESEARCH STATION DESIGN METHOD (BRS) COSTS AS A % OF THE TOTAL COST FOR GRADE 43		
GRADE	43	50	55	43	50	55	43	50	55
PAINT	9·09	8·00	7·20	8·76	7·74	7·53	9·05	7·98	7·23
CLEAN	4·67	4·12	3·67	4·51	3·98	3·83	4·66	4·10	3·66
WELDING	4·38	4·08	4·20	4·10	3·73	4·43	5·44	4·54	5·02
WELD-RODS	0·63	0·58	0·54	0·60	0·53	0·58	0·81	0·65	0·66
CUTTING	3·37	3·18	2·86	3·48	3·02	3·03	3·78	3·38	2·99
DRILLING	4·18	4·03	4·17	4·37	4·02	4·33	4·37	4·37	4·58
SAWING	1·17	0·90	0·68	1·06	0·85	0·69	1·17	0·90	0·68
HANDLING	4·97	4·98	4·82	4·96	4·91	4·81	5·17	5·03	4·87
PLATE	5·01	4·12	4·18	5·48	4·00	4·35	5·45	4·31	4·12
BOLTS	5·16	5·96	6·48	5·81	6·40	6·21	5·44	5·89	5·96
CLEATS	0·38	0·38	0·38	0·37	0·37	0·37	0·38	0·38	0·38
SECTIONS	54·97	44·00	41·00	54·74	43·82	41·22	52·39	41·44	38·80
SECTION EXTRAS	2·03	5·17	6·31	1·77	5·12	6·03	1·91	4·88	6·08
FABRICATION ITEMS	31·83	29·29	27·60	31·20	28·25	28·65	33·64	30·30	29·03
MATERIAL ITEMS	68·17	60·21	58·88	68·80	60·24	58·77	66·36	57·55	55·99
TOTAL COST (£)	47006	42070	40663	47081	41657	41152	46368	40733	39426
BASIS COST (£)	26515	21000	19536	26329	20929	19831	24929	19592	18222
SECTION TONNAGE	162·01	131·11	117·81	159·98	130·01	120·26	152·16	122·26	108·29
SECTION RATIO	0·974	0·984	0·987	0·979	0·986	0·979	0·975	0·981	0·987
SECTIONS + EXTRAS	57·00	49·17	47·31	56·51	48·94	47·25	54·30	46·32	44·88

TABLE 7·3

BUILDING NUMBER	GRADE OF STEEL	DESIGN METHOD	TOTAL COST (£)	SECTION BASIS COST (£)	FABRICATION COEFFICI-	COST PER TONNE (£)	TONNAGE (TONNE)
1	43	JCR	47029	26515	1·77	290·28	162·01
2	43	JCR	18225	10142	1·80	292·50	62·31
3	43	JCR	19071	10325	1·85	302·65	63·01
1	50	JCR	42105	20999	2·01	321·14	131·11
2	50	JCR	16230	8201	1·98	324·19	50·06
3	50	JCR	17113	8165	2·09	334·74	51·12
1	55	JCR	40663	19536	2·08	345·17	117·81
2	55	JCR	15796	7402	2·13	351·82	44·90
3	55	JCR	17522	8075	2·09	345·18	50·76
1	43	BCSA	47104	26329	1·79	294·43	159·98
2	43	BCSA	18298	10037	1·82	296·99	61·61
3	43	BCSA	19201	10443	1·84	302·25	63·53
1	50	BCSA	41692	20929	1·99	320·70	130·00
2	50	BCSA	16584	8533	1·94	318·38	52·09
3	50	BCSA	17258	8480	2·04	327·68	52·67
1	55	BCSA	41152	19831	2·08	342·18	120·26
2	55	BCSA	16455	8001	2·06	338·60	48·60
3	55	BCSA	16282	7667	2·12	346·23	47·03
1	43	BRS	46391	24929	1·86	304·88	152·16
2	43	BRS	17265	9048	1·91	310·44	55·61
3	43	BRS	18630	9740	1·91	302·25	61·64
1	50	BRS	40768	19591	2·08	333·44	122·26
2	50	BRS	15676	7422	2·11	345·53	45·37
3	50	BRS	16252	7576	2·15	343·65	47·30
1	55	BRS	39426	18221	2·16	364·06	108·30
2	55	BRS	16060	7147	2·25	376·83	42·62
3	55	BRS	16450	7088	2·32	384·65	42·77

TABLE 7·4

By far the largest single cost item in every case is the cost of sections. The basis cost of sections decreases with increases in the strength of steel. The reduction in weight with increase in strength can be determined. The section extra costs increase with the strength of steel due to the extras for quality. Including both the cost of sections and extras shows that grade 50 steel is around 7 to 8% less than grade 43 steel, the difference between grade 50 and grade 55 steels is smaller, showing that by increasing the grade of steel the benefit of higher strength will be offset by the increase in cost of the steel. Painting and cleaning costs decrease with increasing steel strength due to the reduction in the surface area of members. The cost of sawing, handling and cutting generally decreases with increase in the grade of steel, this is due to the use of the same number of smaller components combined with small penalties for the use of higher strength steels. Those items which incur the highest penalties, welding, drilling, welding rods and plates show a marked decrease from grade 43 to grade 50 and usually an increase from grade 50 to grade 55, the difference in costs between grade 43 and grade 55 may be an increase or a decrease. The cost of bolts is the only item that shows a steady increase with increasing grade of steel, this is because the bending moments resisted by the connections are similar in each case, however the member sizes (and so the lever arms) are smaller resulting in higher bolt loads. The cost of all material items decreases considerably between grade 43 and 50, between grade 50 and grade 55 this decrease is far smaller. A similar effect is apparent when considering the total cost of fabrication items.

A comparison will now be made between the B.C.S.A. design method and the J.C.R. design method. The B.C.S.A. design method differs from the J.C.R. design method in two major respects:-

(a) The B.C.S.A. design method produces smaller minor axis beams with highly stressed joints.

(b) The B.C.S.A. design method produces larger column sections, with physically larger splice connections.

Some variations in relative costs should therefore be apparent. The total cost of the frame is very similar with either design method.

The proportions of cost represented by sections and extras are very similar except that the B.C.S.A. designs have lower proportions. Examination of each material and fabrication item shows similar proportions for each design method, however in the case of plates, bolts, cleaning and painting the proportions are higher for the B.C.S.A. design method.

The B.R.S. and J.C.R. design methods will now be examined, the essential differences are that the B.R.S. design method:-

(a) Produces smaller minor axis support moments.

(b) Produces substantially smaller stanchion sections.

(c) Requires the stanchion splices to carry high bending loads.

The total frame costs can be seen to be noticeably less for a B.R.S. design. The proportions of cost represented by sections and extras are similar but in most cases less than those for the B.C.S.A. and J.C.R. design methods. Because the sections form a substantially smaller proportion all other proportions of cost are usually higher than is the case for the other design methods. The fabrication items generally show similar relationships to each other to those already examined. Welding takes a much larger proportion of the total fabrication items than in the other design methods.

This investigation shows that individual cost items are each very small parts of the total cost except for the section cost. This enables us to see that variation of one single cost item will not materially affect the final cost, this property tends to reduce errors in the cost model because any error will form such a minor part of the final cost. The section material costs form approximately half the total cost. Total material costs form approximately two thirds and fabrication costs are approximately one third of the total cost. Painting and cleaning costs form a substantial part of the fabrication costs. The cost of materials reduces with increasing grade of steel. This effect is also apparent with fabrication items but it is less marked.

7.6 THE EFFECT OF VARIOUS FACTORS ON THE COST OF FRAMES

An investigation has been made of various factors on the cost of the three frames shown in figures 7.19a, 7.19b and 7.19c. The iterative design algorithm was used to give the cost of the frames, no further algorithms were used because the iterative design algorithm has been shown to produce designs which are close to the optimum.

7.6.1 The Effect of Local Factor

The frame of figure 7.19a was considered in grade 43 steel and with load factors varying between 1.1 and 2.3. The results are shown in figure 7.20. A linear regression line was fitted to the data. A change from a load factor of one to a load factor of two (an increase in load of 100%) increases the cost of the frame by 45%, this is the case for all three design methods. Similarly the increase in section basis costs is 62%. This shows that with an increase in load the cost of sections becomes a larger proportion of the total cost. This is due to the nominal connections becoming more highly stressed and so taking higher loads for the same cost. Eventually all connections will be stronger than the nominal connections and the proportion of cost represented by the connections will become more constant. The increase in cost can be seen to be approximately linear, however the true shape will be stepped due to the use of discrete sections. In all except two cases the B.R.S. design method gives the lowest cost, likewise the J.C.R. design method is the most expensive in all except two cases.

The relationship between load factor and cost is substantially linear and similar for all design methods. The increase in cost due to increasing load is not the same proportion as the increase in load.

LOADING (KN/m²)

	LIVE	DEAD
ROOF	1.57	5.80
FLOORS	6.20	5.80

FIG.7.19 EXAMPLE FRAMES

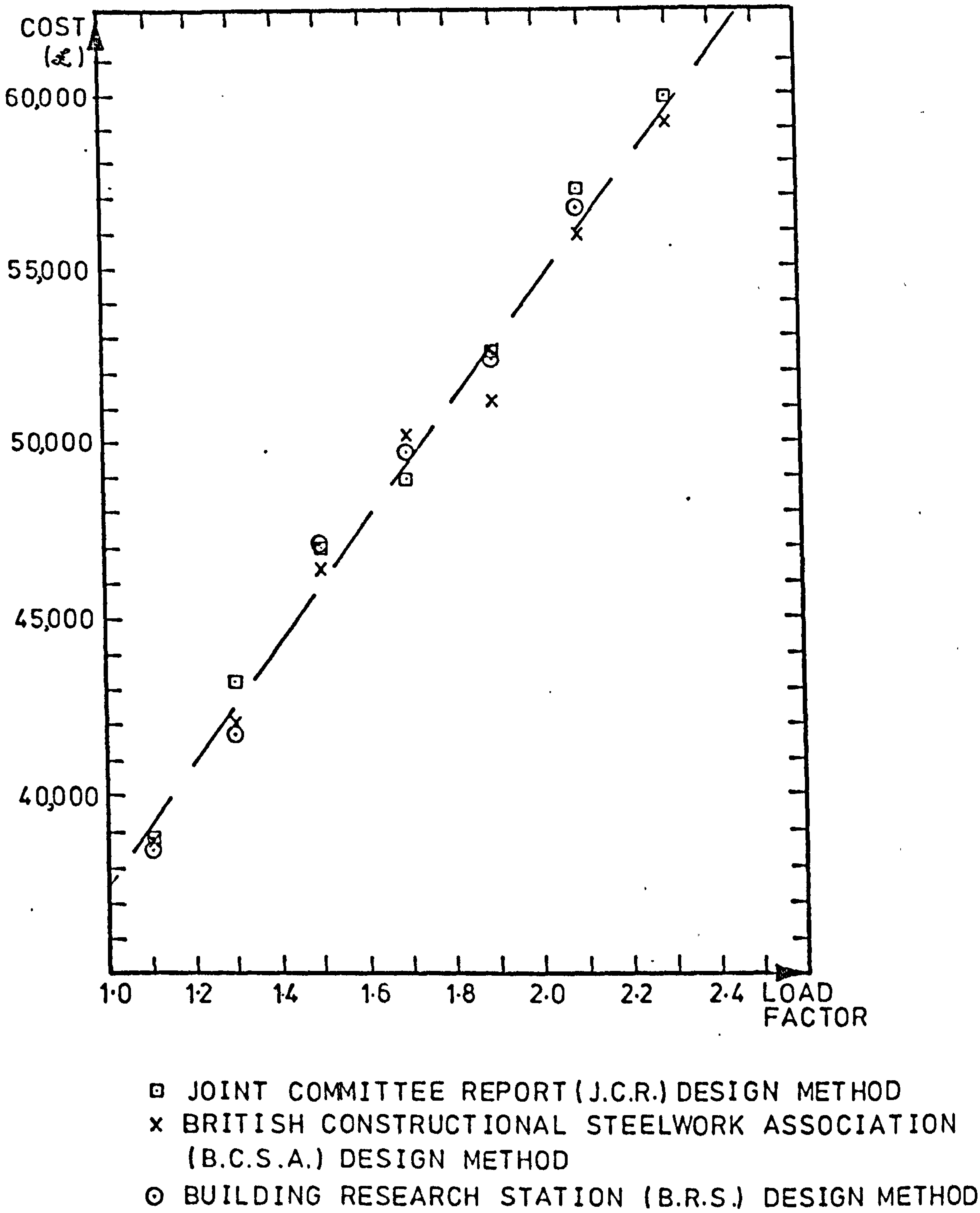


FIG. 7.20 THE RELATIONSHIP BETWEEN COST AND LOAD FACTOR FOR A BUILDING

7.6.2 The Effect of Grade of Steel

This investigation consisted of designing each of the three frames in steel grades 43, 50 and 55 using each design method. The results are shown in Table 7.4.

The total cost of the frame can be seen to reduce as the steel strength increases except for three cases where grade 50 steel gives the cheapest frame. The section basis cost and tonnage decreases in all cases, as the grade of steel increases, the reduction from grade 43 to 50 is greater than that for grade 50 to 55. The cost per tonne and the fabrication coefficient are relatively constant for a given grade of steel and they increase steadily with the higher grade of steel.

The effect of the design method on the various costs can be seen to favour the B.R.S. design method. This method produces both the lowest tonnage, the lowest basis cost and the highest cost per tonne in all cases. The total cost shows seven out of nine cases with the B.R.S. design the cheapest, the reason that it is not the cheapest in all cases is the additional fabrication required. The comparison between the J.C.R. and B.C.S.A. design methods is not as clear cut. Out of nine designs the J.C.R. design method produces the lowest basis cost in five cases and when considering total cost, seven designs prove cheaper, however in nearly every case the difference in total cost is very small.

This investigation shows that grade 43 steel is the most expensive. In most cases grade 55 steel gives the most economic design, however the cost of designs in grade 50 steel is very similar. A small tonnage does not necessarily result in a small final cost. The B.R.S. design method can be seen to produce the cheapest frames irrespective of the larger proportion of fabrication required.

7.6.3 The Effect of a Minimum Weight Objective Function

Investigating the effect of using a minimum weight objective function involved designing each of the frames shown in figure 7.19 using the section weight as the cost of the section and setting all fabrication costs to zero. The designs were evaluated for grade 43 steel using each of the three design methods. The results of the investigation are shown in Table 7.5.

Examination of the results shows that the difference in tonnage may be up to about 4%. The difference in costs is considerably less marked being a maximum of 0.46%. The differences in cost per tonne are an aggregate of the differences in cost and weight and are therefore larger. These small differences in cost, weight and cost per tonne are not unexpected because the section basis cost which forms about 50% of the total cost is related to the weight of the sections.

The use of a minimum weight objective function results in designs with very similar weights and costs to those designs produced using a minimum cost objective function. In the cases considered a slight cost penalty ensues and the designs based on cost and tonnage are never exactly the same. – 12

7.7 THE EFFECT OF BUILDING GEOMETRY

The geometry of a building is likely to have a large effect on the minimum cost of a frame. This phenomenon was investigated using three buildings which are illustrated in figures 7.21, 7.22 and 7.23. The framing arrangement was varied for each building as shown and the cost was found using the iterative design algorithm. The frames chosen were devised in order to minimize the effects of foundation costs, cladding costs and flooring costs. Foundation costs are likely to be approximately proportional to the load carried, cladding costs are constant as long as the area covered is constant and precast floors have the same cost for an identical span.

DESIGN METHOD AND BUILDING NO	Tonnage of Cost Based Design	Tonnage of Weight Based Design	Cost of Cost Based Design	Cost of Weight Based Design	Difference In Weight %	Difference In Cost %
J.C.R. 1	162.01	161.00	47029	47104	0.62	0.16
2	62.31	60.50	18225	18245	2.90	0.11
3	63.01	62.81	19071	19132	0.33	0.32
B.C.S.A. 1	159.98	158.82	47104	47276	0.71	0.37
2	61.61	60.71	18298	18312	1.48	0.08
3	63.53	63.34	19201	19290	0.36	0.46
B.R.S. 1	152.16	151.12	46391	46488	0.76	0.21
2	55.61	54.81	17265	17323	1.46	0.34
3	61.64	59.09	18630	18675	4.12	0.24

TABLE 7.5

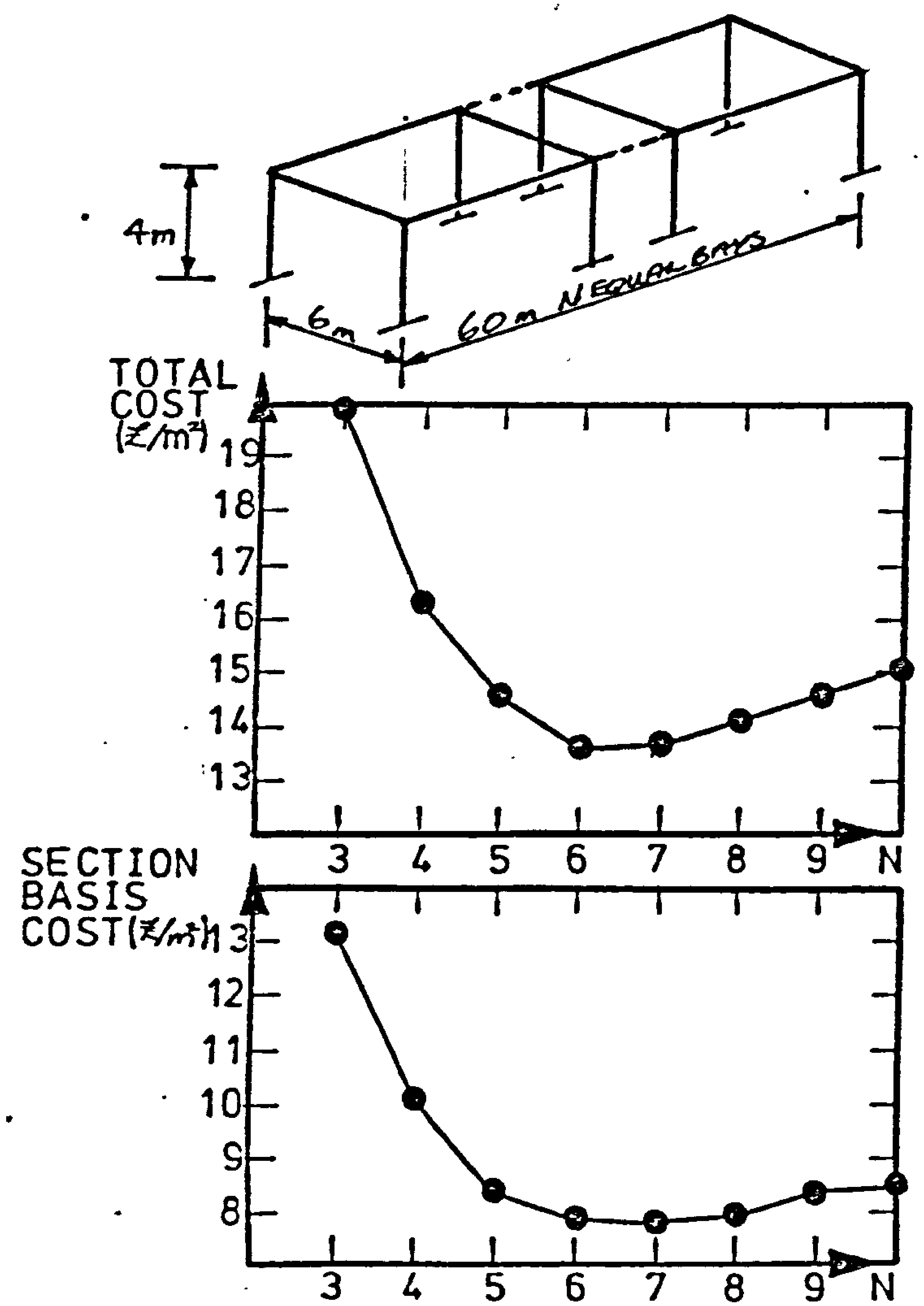


FIG.7.21 THE EFFECT OF BUILDING GEOMETRY 1

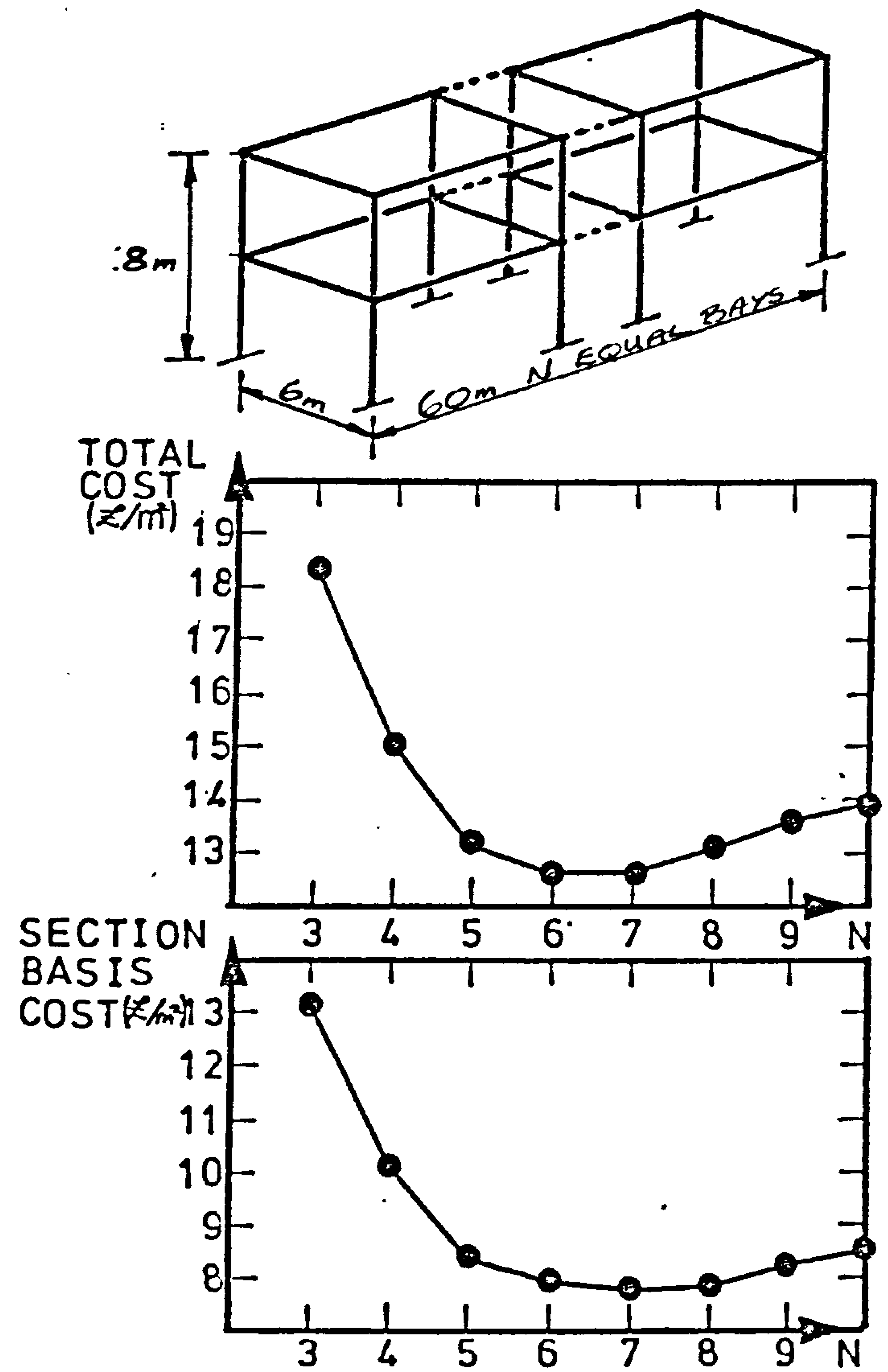


FIG.7.22 THE EFFECT OF BUILDING GEOMETRY 2

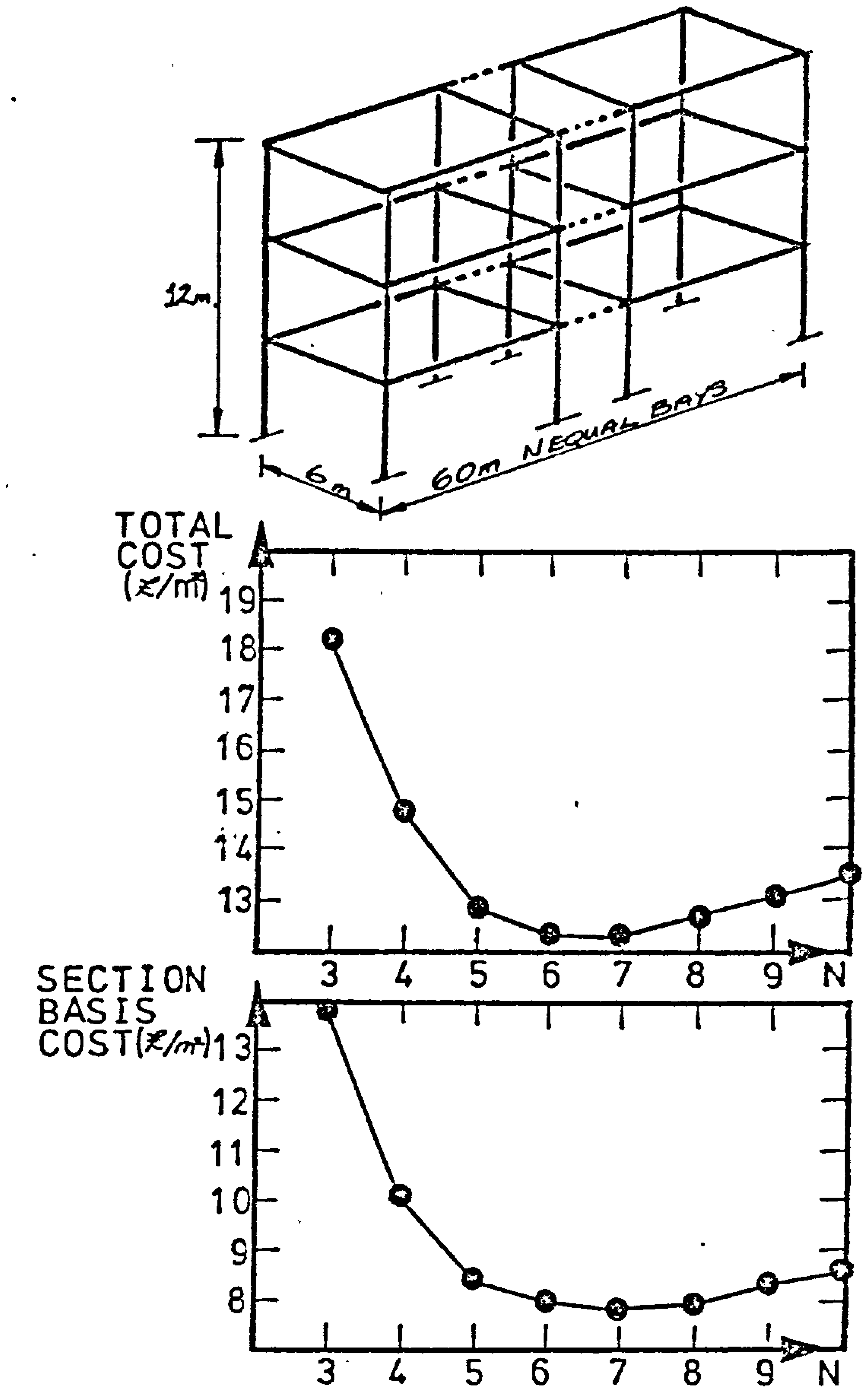


FIG.7.23 THE EFFECT OF BUILDING GEOMETRY 3

The form of the curves can be seen to be approximately similar in each case. The curves consist of a trade-off between two sets of costs.

These sets of costs will now be examined. The types of costs which increase with increasing numbers of bays include items which increase in number, such as the minor axis beams, and their fabrication, and the fabrication to stanchions. The costs which decrease with increasing numbers of bays include items which reduce in cost with reductions in the applied load, such reductions occur in the case of major axis beams which reduce considerably in size as the number of bays increases. It is not possible to tell in certain cases which type of behaviour will occur for certain costs, for instance with increasing numbers of bays, the number of stanchions increase, however the cost of each of those stanchions reduces due to lower loading. Examination of the information shown confirms that the cost of beams reduces considerably with increasing numbers of bays despite the influence of minor axis beams. The behaviour of stanchion costs is towards increasing cost with increasing numbers of bays, however this increase is not a marked as for the beams due to the trade-off between numbers of items and loads on the items. The section basis cost can be seen to have a similar behaviour to the total cost. Examination of the fabrication coefficient shows an increase with increases in the number of bays, however this increase tails off towards the maximum number of bays.

This investigation shows that considerable savings can be —24
achieved by the consideration of the geometry of the frame and that for each frame there is an optimum bay size.

7.8 CONNECTION COST INVESTIGATION

In an effort to simplify the design procedures for connections an investigation of the cost of connections was performed. The object was to find relationships between the connection cost and the load on the connection. The investigation consisted of two parts. Firstly for a variety of sections the relationship between cost and connection load was evaluated. Secondly relationships were derived between the cost of connections and properties of the section to be connected, for given strength levels. These investigations were of necessity very extensive, however they will only be dealt with briefly in order to avoid unnecessary detail.

7.8.1 The Relationship Between Force on a Connection and Cost Of the Connection

The investigation consisted of evaluating the cost of each type of connection, neglecting the cost of welded attachments to the other section being considered. A number of sections were investigated with each type of connection. The effect of all geometric constraints, except those relating to the section considered, have been neglected. The forces used varied up to the capacity of the section being considered.

In each case, contours of cost have been plotted against forces on the connection. This results in a surface which may have an infeasible region, where a connection cannot be designed with the details available, because there are high loads acting simultaneously. There may also be a flat area in regions of low loadings where the nominal connection, i.e. smallest bolts with thinnest plates, is suitable. Where possible, the interaction diagram for the section is shown, the connection details can often carry loads in excess of the loads that the section can carry.

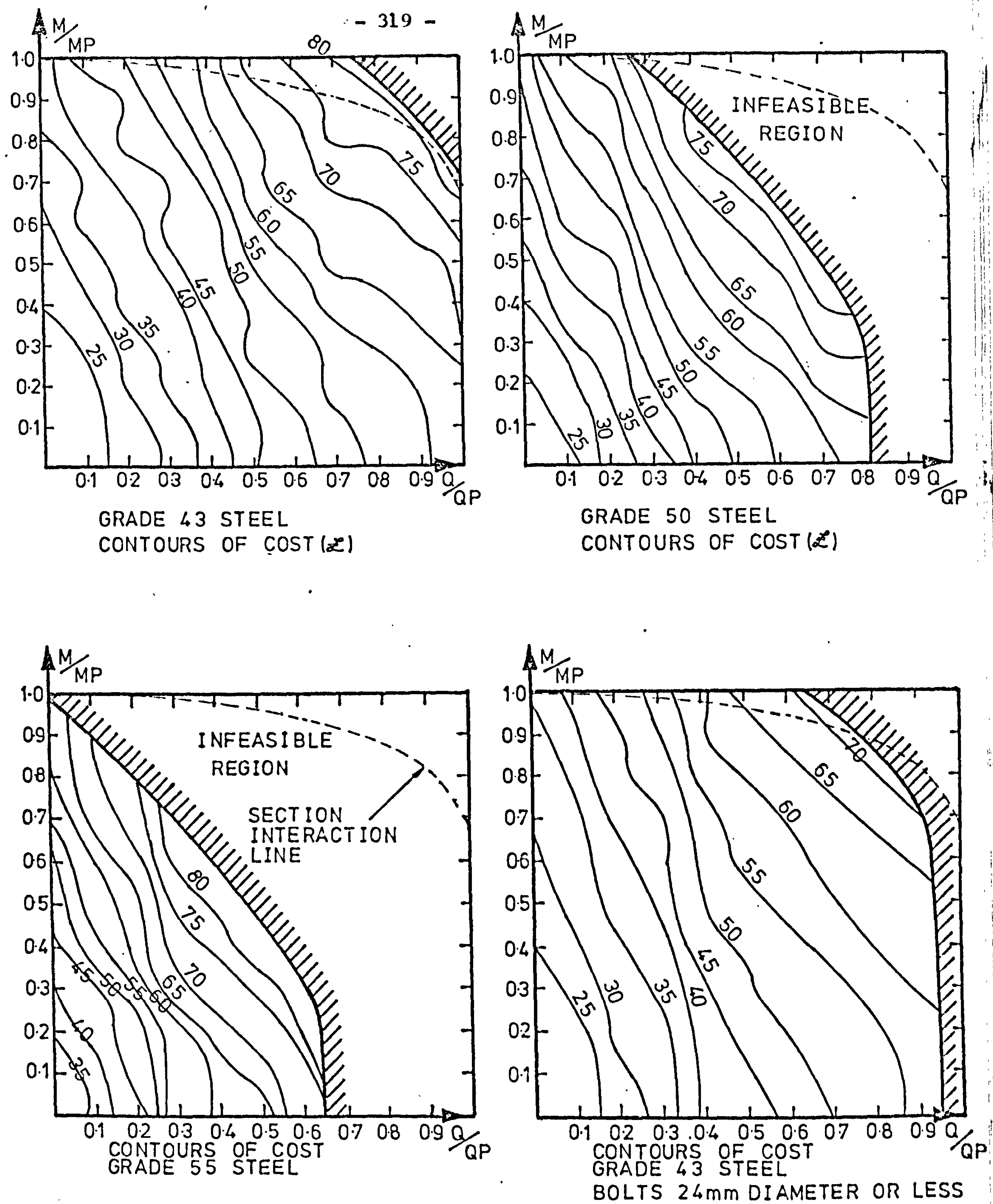
7.8.1.1 Major Axis Beam Connections

The loads considered were bending moments up to the full plastic moment of the section and shear forces up to the shear capacity of the section. The following investigations will be described:-

(a) Section 10 used with steel grades 43, 50 and 55.

(b) Section 10 used with grade 43 steel and limiting the maximum bolt size.

The cost contours are shown in figure 7.24. These contours run diagonally, increasing steadily with



MP=PLASTIC MOMENT OF RESISTANCE OF THE SECTION TO BE CONNECTED
QP=PLASTIC SHEAR CAPACITY OF THE BEAM SECTION TO BE CONNECTED
M =MOMENT APPLIED TO THE CONNECTION
Q = SHEAR APPLIED TO THE CONNECTION

FIG. 7·24 RELATIONSHIP BETWEEN COST & LOAD MAJOR AXIS CONNECTION FOR SECTION NUMBER 10 USED AS A BEAM increasing load. The contours for grade 43 steel show a small infeasible region in the region of high load. Increasing the grade of steel increases both the size of the infeasible region and the cost of the connection. The infeasible region occurs because a sufficient number of bolts cannot be provided within the depth of the section to carry the higher loads. The effect of restricting the size of bolts used, is to increase the size of the infeasible region, this is because a larger number of small bolts is required to carry the load and these will not always fit within the depth of the section. Another effect of restricting the bolt size that can be seen is that the cost is similar where lower loads are involved, but at higher loads the cost is lower. This effect is due to the fact that the cost model indicates that small bolts are cheaper per unit of load carried than larger bolts. The omission of erection costs which are highly dependant on the number of bolts from the cost model is one reason for this discrepancy.

Examination of similar investigations using other sections showed that in some cases a "nominal connection" could carry many combinations of loading resulting in a flat region in the area of low loads. A related feature is that in some cases the surface is composed of plateaus each of which relates to a given arrangement of bolts. In some cases, where a beam section has a thick web, the contours show a steep increase in cost with increasing shear.

7.8.1.2 Minor Axis Beam Connections

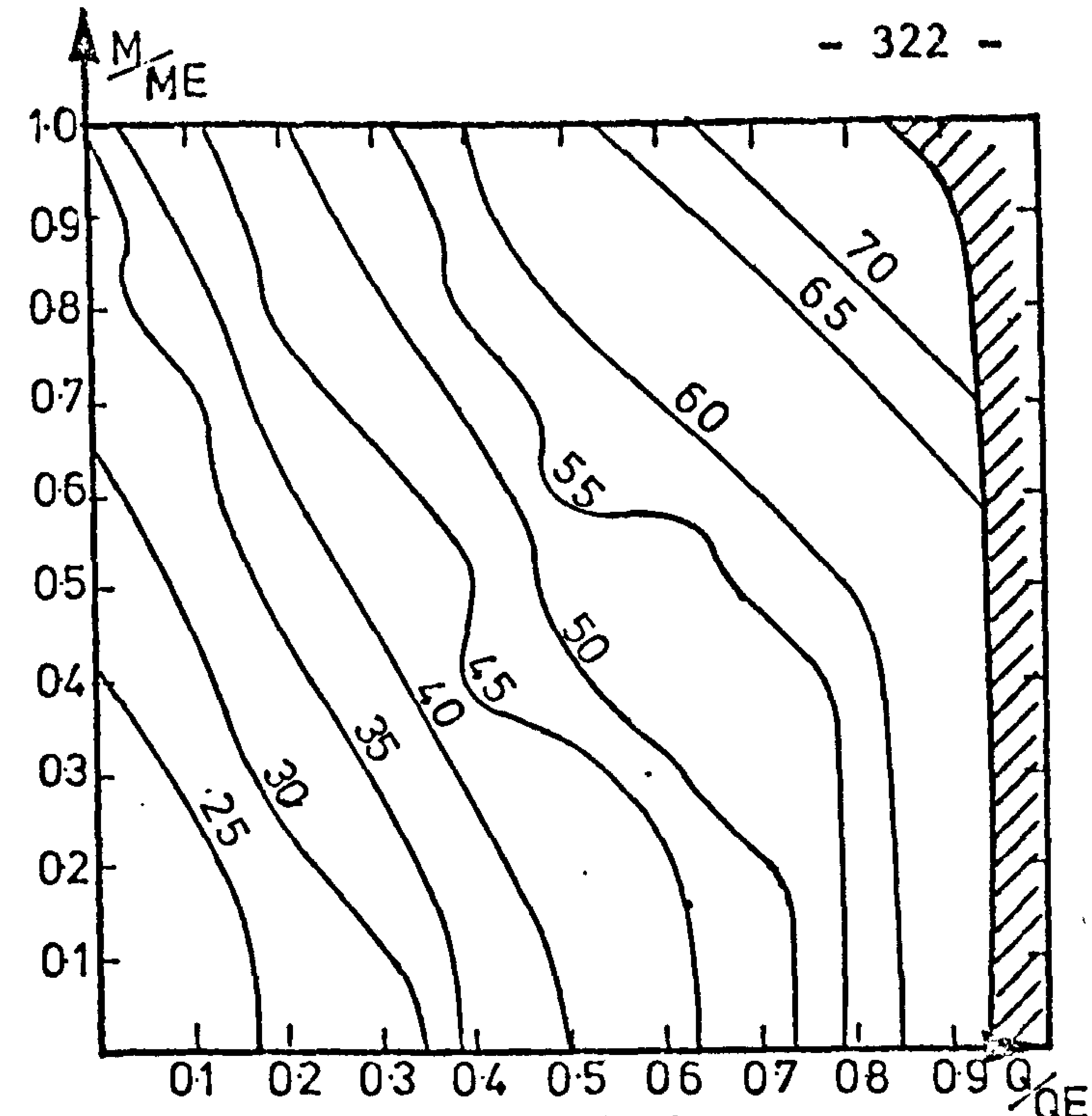
The loads considered in this investigation were bending moments up to the full elastic yield moment and shear forces up to the shear capacity of the section considered. The following investigations will be described:-

(a) Sections 10 and 50 used with grade 43 steel, extended end plate connection without continuity.

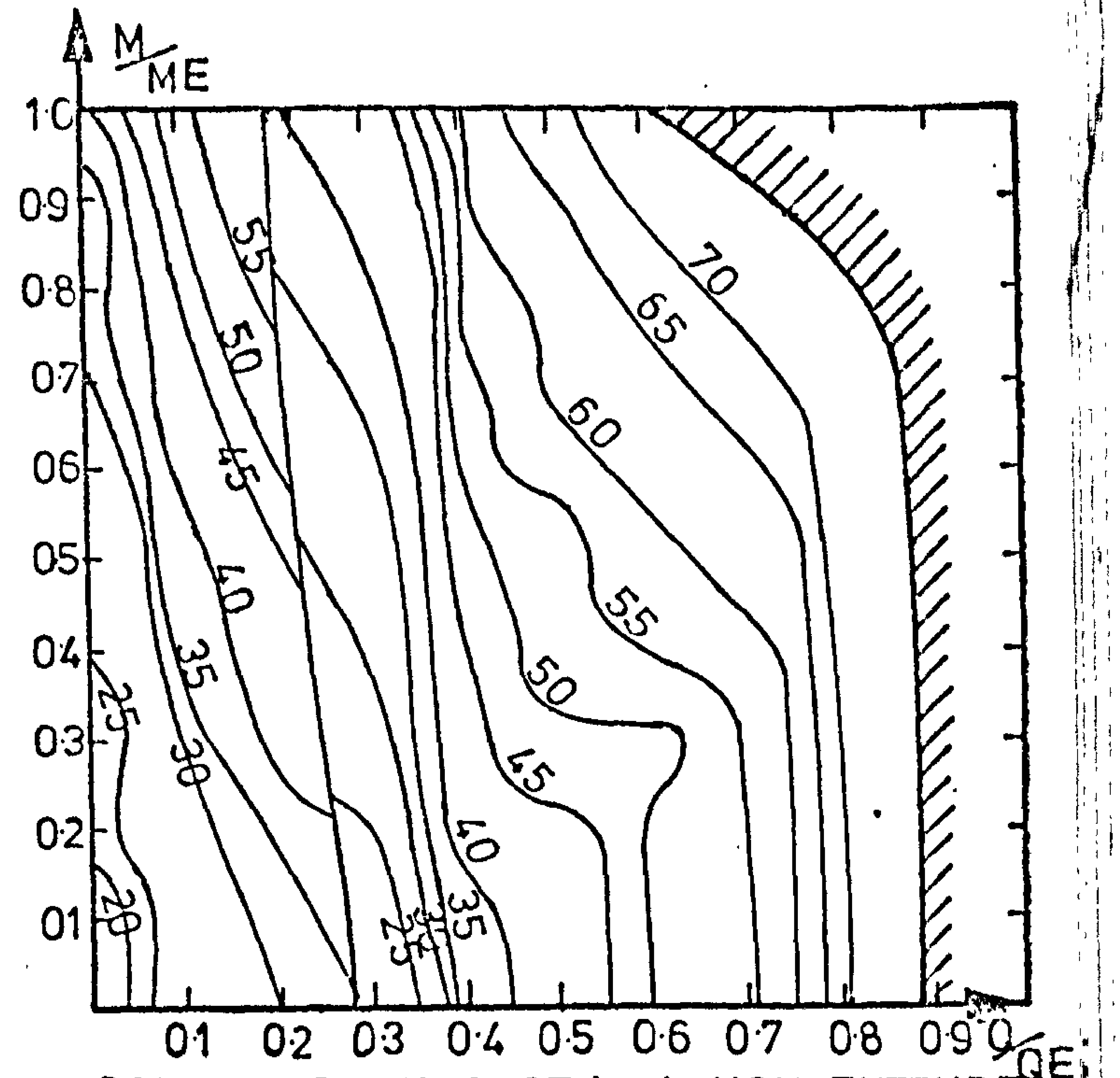
(b) Sections 10 and 50 used with grade 43 steel, non-extended end plate connection without continuity.

The cost contours are shown in figure 7.25. Considering sections 10 and 50 with the extended end plate. Section 10 shows a diagonal trend in the cost contours and the infeasible region has a different shape to the major axis connection. The contours for section 50 are markedly different, showing plateaus, due to the ability of connection arrangements to carry large ranges of load. Each plateau corresponds to a different bolting arrangement. In both cases the costs are similar to those for major axis connections.

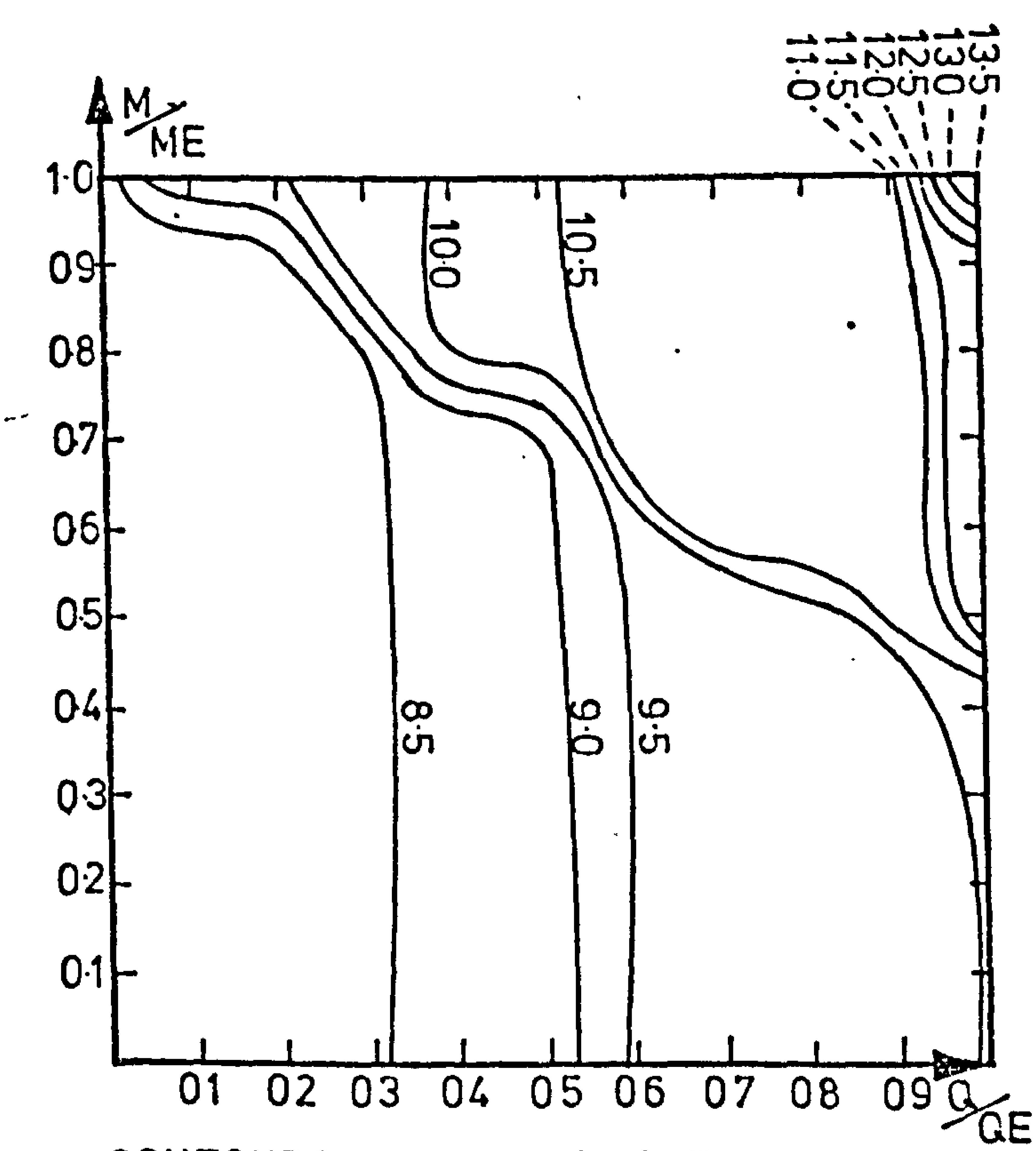
Considering the non-extended end plate connections. The surface evaluated for section 10 is radically different to that for the extended end plates, showing an extensive trough with one steep side in the centre of the surface. This trough exists because increasing the size of bolts may decrease the thickness of the end plate and so, because the plate cost and bolt costs are a substantial part of the total, the cost drops. The infeasible region is seen to be more extensive than is the case with extended end plates due to the reduced bolt lever arm and restricted area available for fitting bolts. The cost surface for section 50 is very similar to that for an extended end plate, there are however more and smaller plateaus indicating that each bolt arrangement is useful



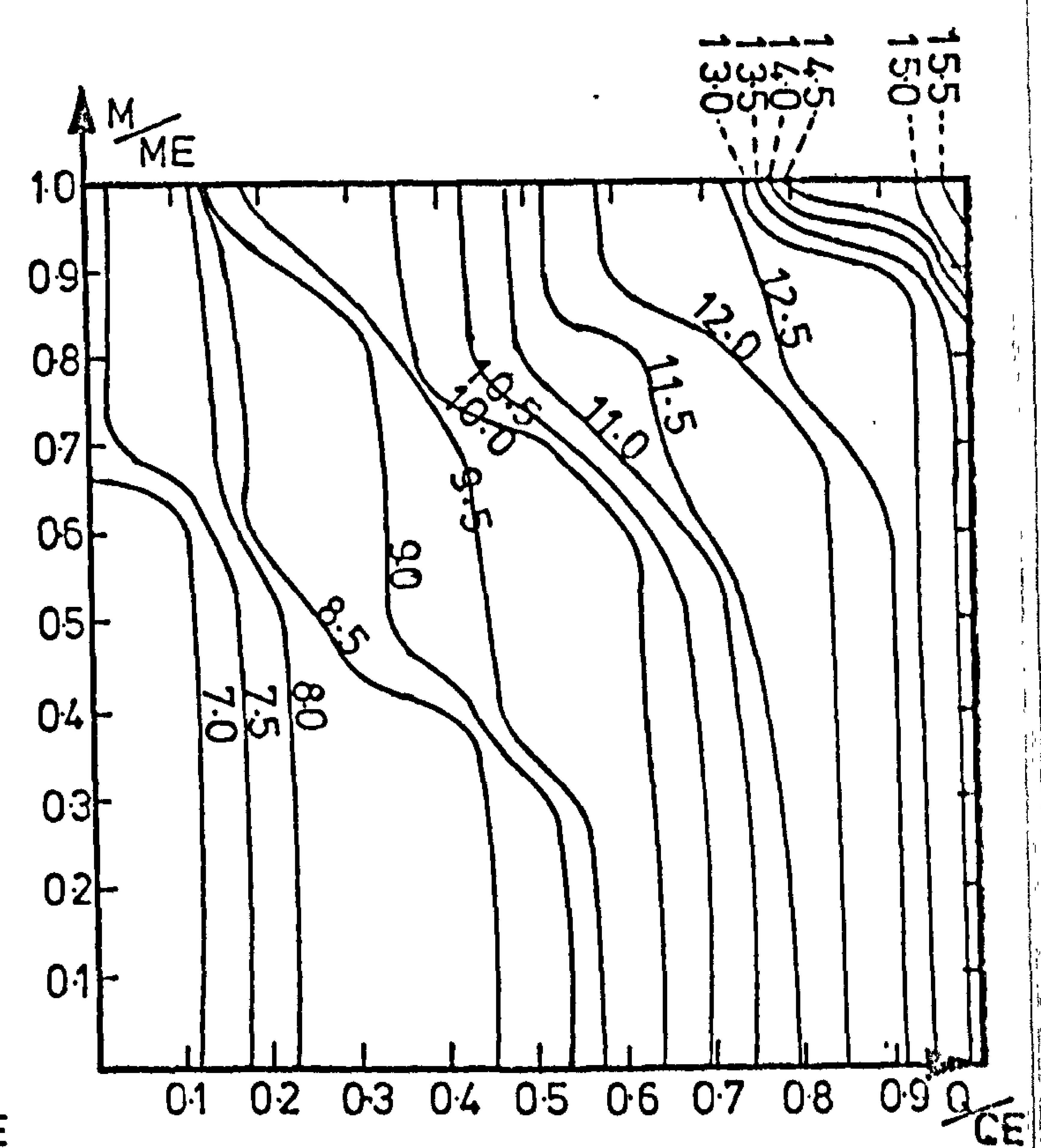
CONTOURS OF COST(£) EXTENDED ENDPLATE CONNECTION FOR SECTION 10 USED AS A MINOR AXIS BEAM



CONTOURS OF COST(£) NON-EXTENDED ENDPLATE CONNECTION FOR SECTION 10 USED AS A MINOR AXIS BEAM



CONTOURS OF COST(£) EXTENDED ENDPLATE CONNECTION FOR SECTION 50 USED AS A MINOR AXIS BEAM



CONTOURS OF COST(£) NON-EXTENDED ENDPLATE CONNECTION FOR SECTION 50 USED AS A MINOR AXIS BEAM

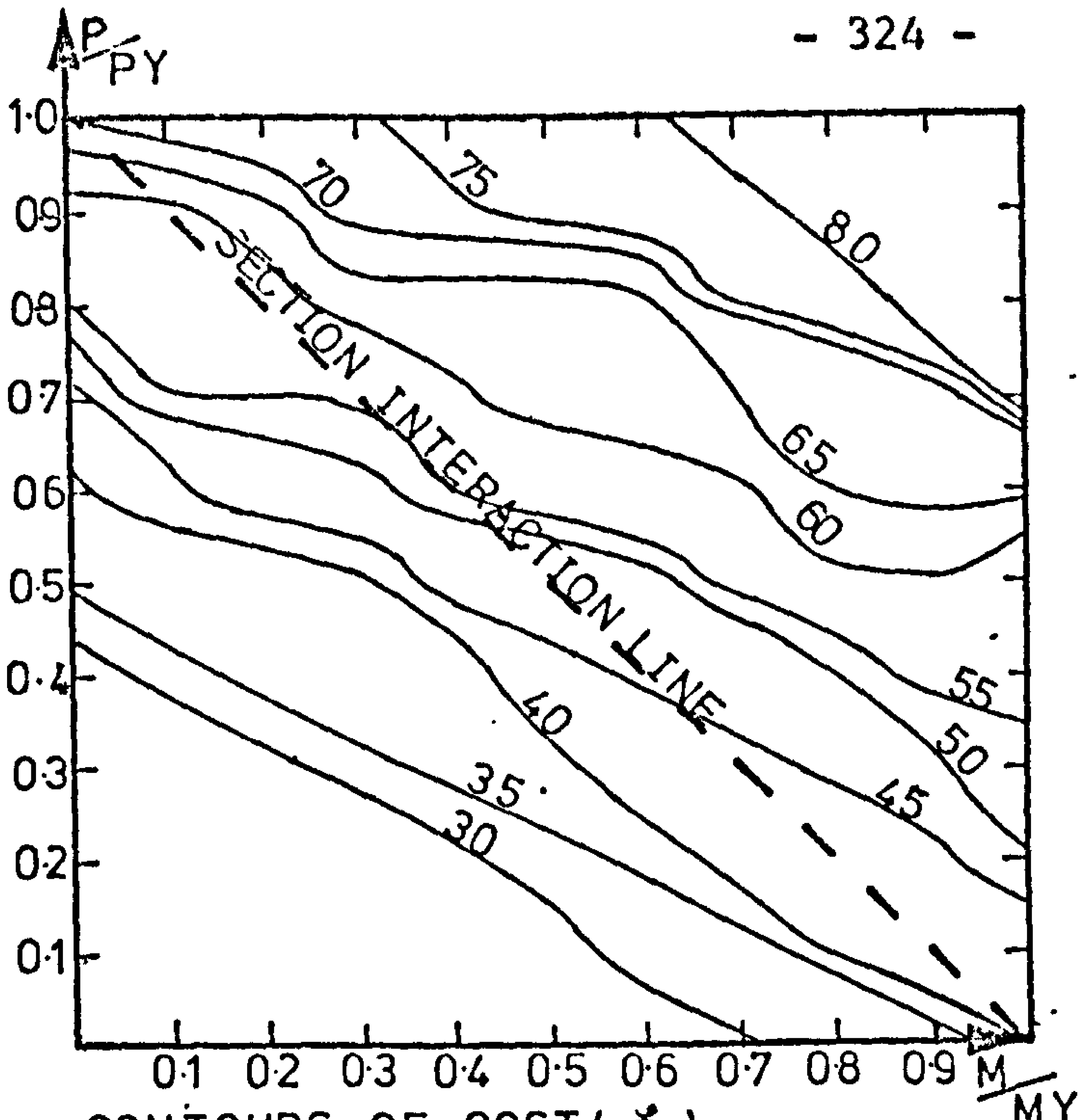
ME=ELASTIC MOMENT OF RESISTANCE OF THE SECTION TO BE CONNECTED
QE=ELASTIC SHEAR CAPACITY OF THE BEAM SECTION TO BE CONNECTED
M =MOMENT APPLIED TO THE CONNECTION
Q= SHEAR APPLIED TO THE CONNECTION

FIG. 7.25 RELATIONSHIP BETWEEN COST & LOAD MINOR AXIS CONNECTION FOR SECTIONS 10 & 50 USED AS BEAMS for fewer loading combinations. The cost is very similar to that for extended end plates, however it is generally greater, where the highest loads are considered.

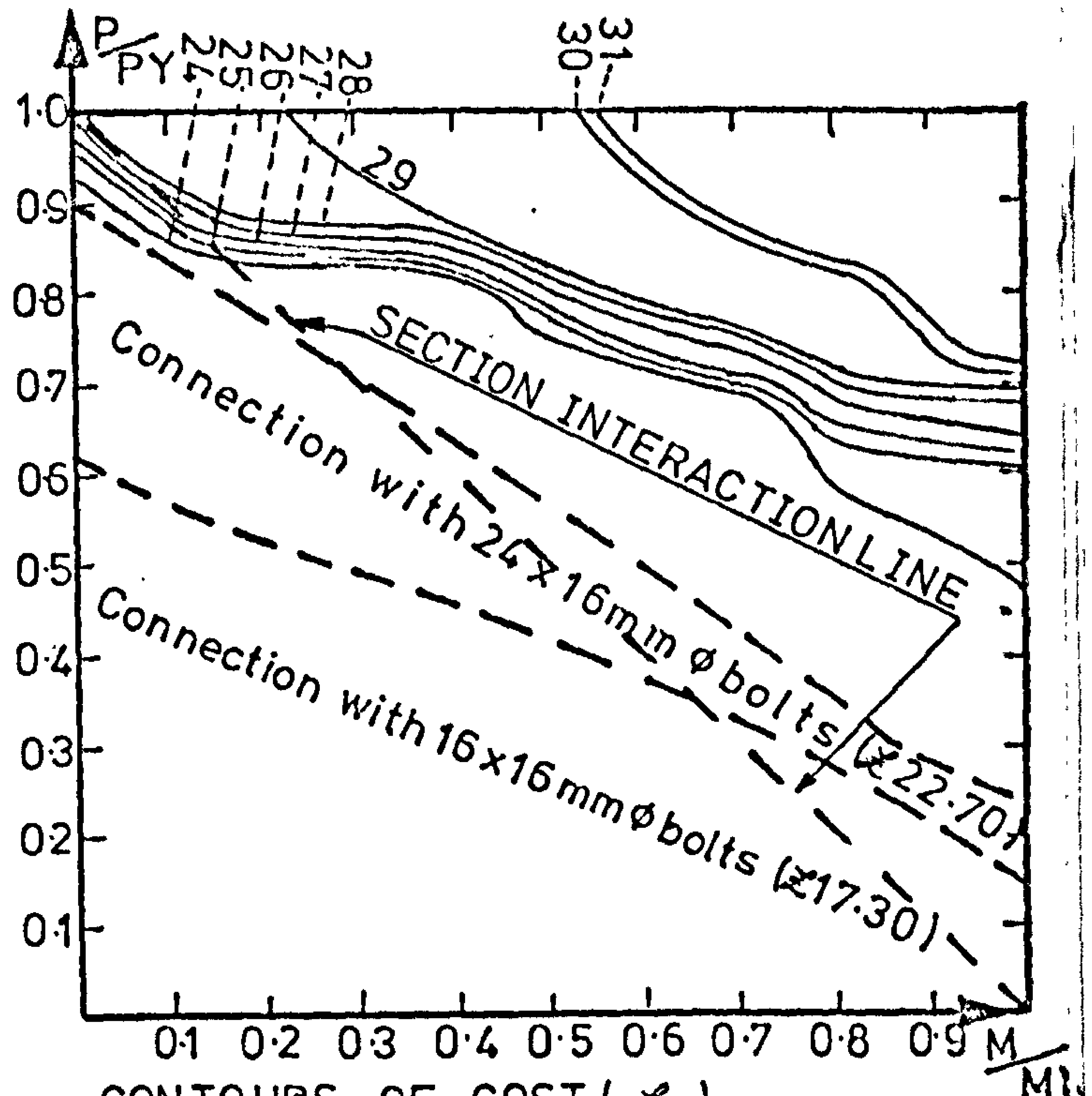
7.8.1.3 Stanchion Splice Connections

In this case the upper stanchion section is used as the reference section because it is generally more highly stressed than the lower stanchion section. It has been assumed that the lower stanchion section has the same breadth, web thickness and flange thickness as the upper section. The loading applied to the splice is separated into two components. Firstly a direct flange force, varying up to the force that causes a stress equal to the yield stress, is applied. This force is an aggregate of both the major axis bending moment and the direct load. Secondly a minor axis bending moment is applied, varying up to the full elastic yield moment of the section. The connections would not be called on to carry both these loads simultaneously. The interaction diagram for the section considered will always be to the left and below the diagonal running from the top left-hand corner to the bottom right-hand corner of any of the cost surfaces. The investigation that will be described consisted of designing a splice to connect sections 30, 50, 70 and 90 with themselves using grade 43 steel throughout.

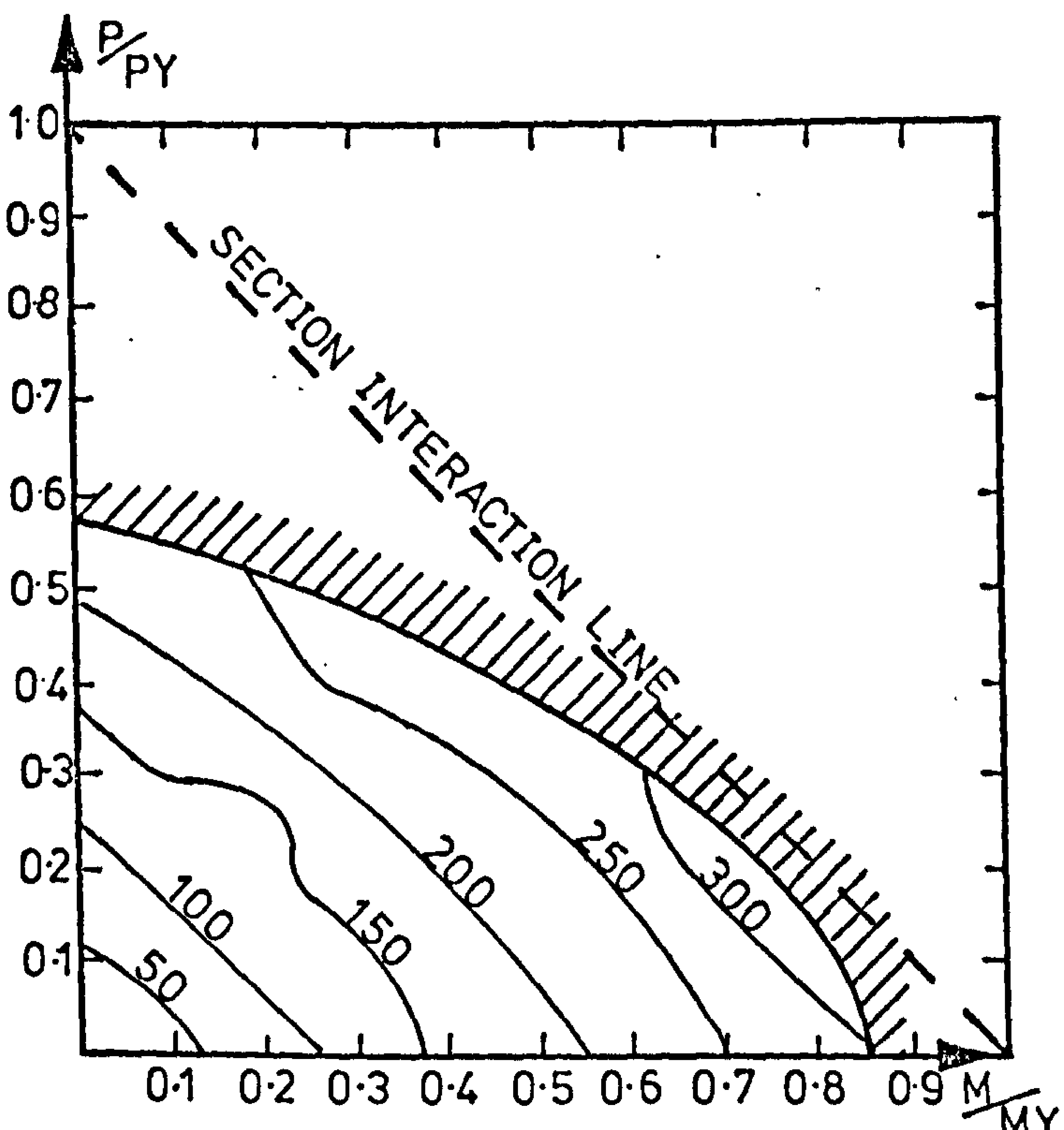
The cost contours are shown in figure 7.26. Examination of the contours shows that for all the sections, the cost contours are orientated diagonally. This indicates that the cost increases steadily with increasing loads, Section 30 shows a flat area in the region of low loads, with a steady increase in cost as the load increases. Section 50



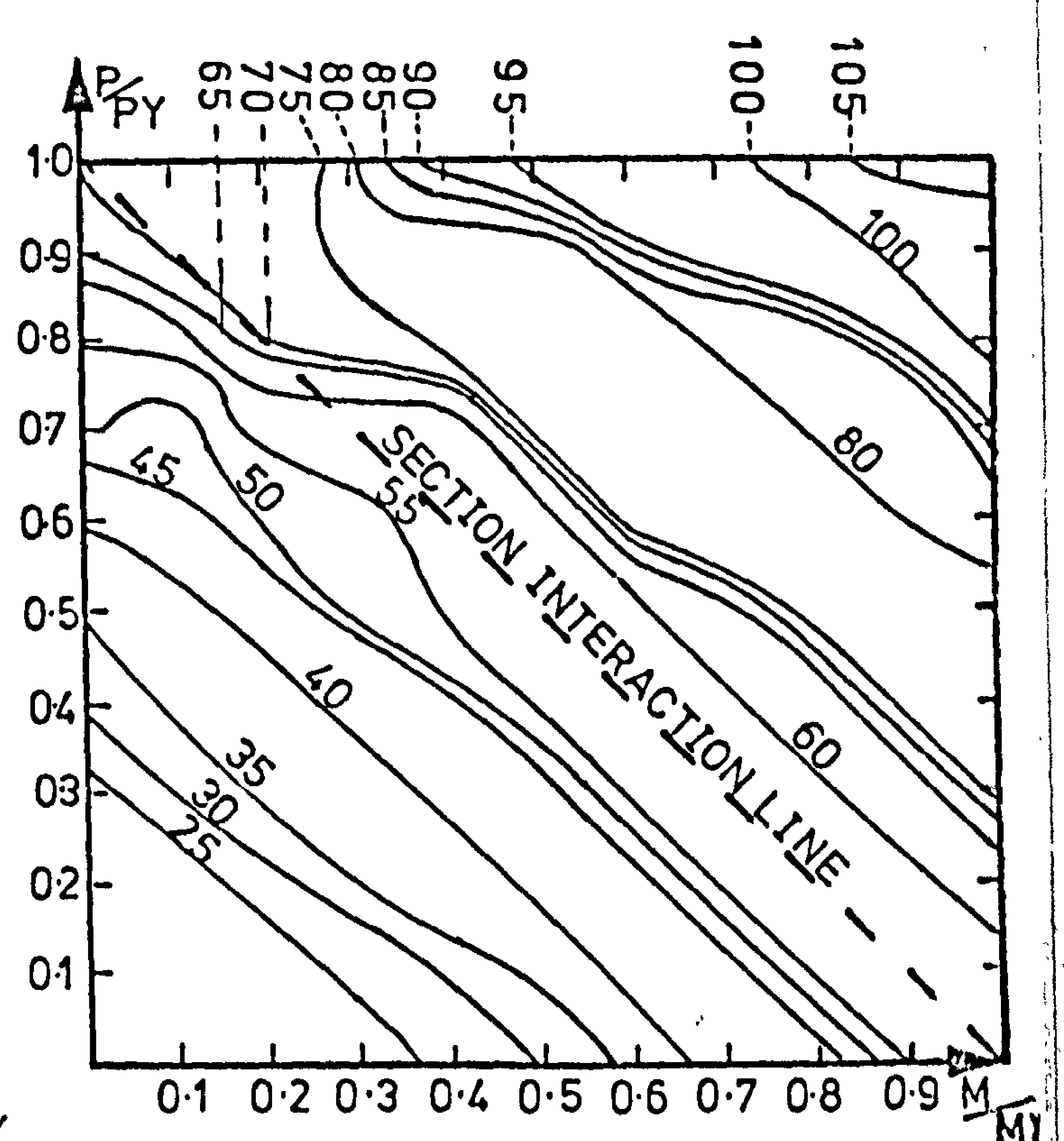
CONTOURS OF COST (£)
SECTION 30 USED AS UPPER STANCHION, LOWER STANCHION SAME DEPTH AND WIDER THAN UPPER



CONTOURS OF COST (£)
SECTION 50 USED AS UPPER STANCHION LOWER STANCHION SAME DEPTH AND WIDER THAN UPPER



CONTOURS OF COST (£)
SECTION 70 USED AS UPPER STANCHION LOWER STANCHION SAME DEPTH AND WIDER THAN UPPER



CONTOURS OF COST (£)
SECTION 90 USED AS UPPER STANCHION LOWER STANCHION SAME DEPTH AND WIDER THAN UPPER

PY = EQUIVALENT FLANGE FORCE FOR SECTION AT YIELD STRESS
MY = MINOR AXIS YIELD MOMENT OF SECTION
P = FLANGE FORCE APPLIED TO CONNECTION
M = MINOR AXIS MOMENT APPLIED TO CONNECTION

FIG. 7.26 RELATIONSHIP BETWEEN COST & LOAD STANCHION SPLICE CONNECTIONS USING GRADE 43 STEEL shows a slightly different pattern in that the nominal connection is suitable for a very large range of forces. A large jump in cost occurs in the area of high forces. The cost surface for section 70 has a very large infeasible region showing that the connection design routines cannot always design suitable connections. The contours for section 90 show three steep cliffs where the numbers of bolts and their sizes change.

Among the other investigations which where undertook was an investigation into the effect of using a deeper lower stanchion section. The lower stanchion was made 1.1, 1.2, 1.3 and 1.5 times deeper than the upper section. The cost contours were similar to those already examined, however as expected the cost increased with the increasing depth. The effect of using higher grade steel is to increase the connection costs, increase the size of the infeasible region and decrease the number of combinations of loads that the nominal connection is suitable for. The cliffs and plateaus that are evident with grade 43 steel are less apparent as the steel grade increases.

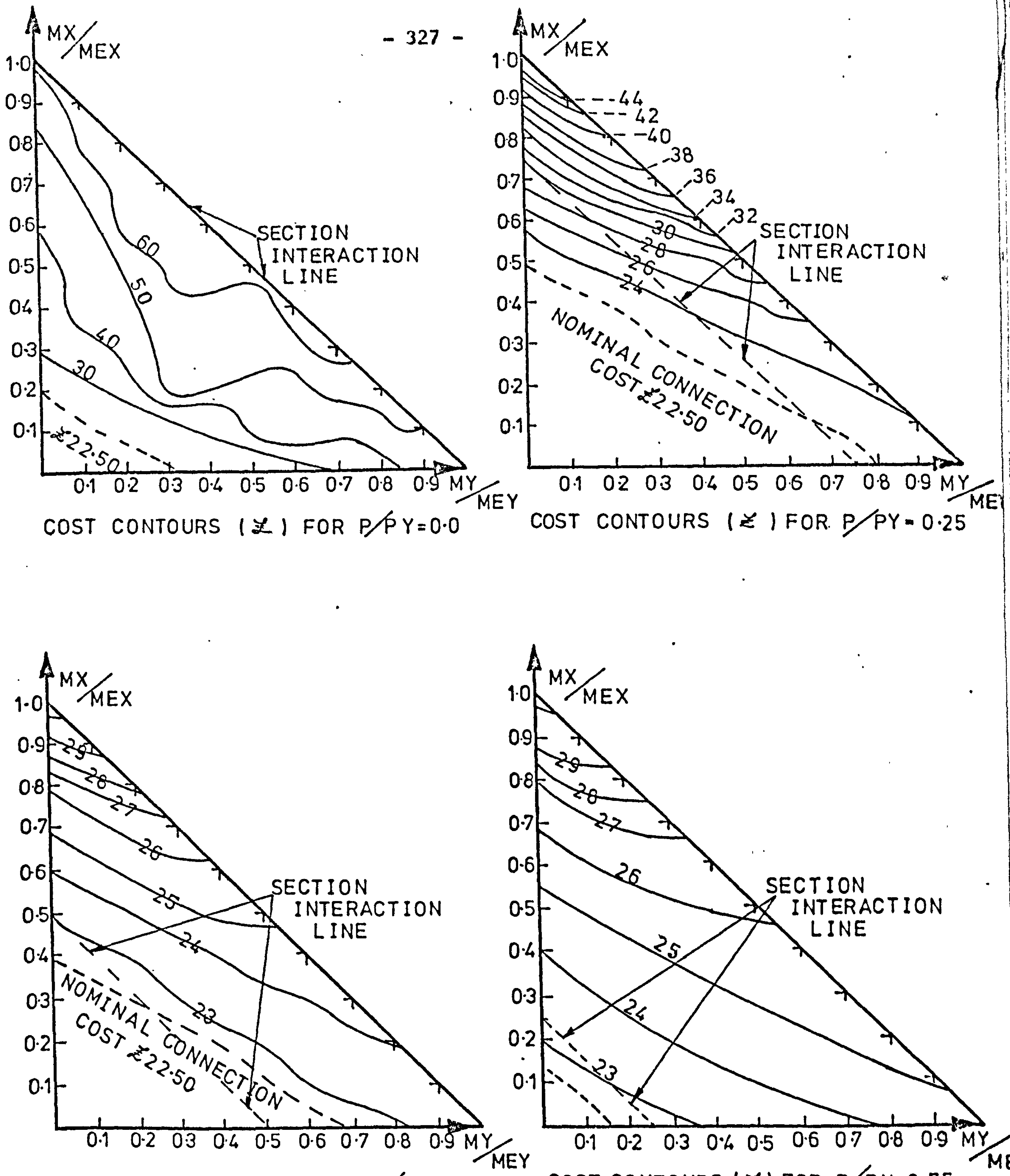
7.8.1.4 Stanchion Base Plates

In this case the loading is more complex than for any of the other connections. The loading considered was the direct load and moments about each axis. The approach used was to consider the base plate with direct loads of 0.0, 0.25 and 0.75 times the direct yield load. The cost of the connection was then evaluated for combinations of the two bending moments. The moments were varied up to the elastic yield moments about each axis. Only the lower left-hand half of the surface was investigated because all practical load case combinations are within the included area. The investigation that will be described is limited to the consideration of section 90 using grade 43 steel. Section 90 is a middle of the range universal column section which one would expect at the bottom of structures up to about six storeys. The cost contours are plotted in figure 7.27. With zero axial load the contours are similar to other types of connections, consisting of an area where the nominal connection can be used and a set of diagonal contours which show a steadily increasing cost with increasing load. As the direct load increases, the size of the area covered by the nominal connection increases and then decreases. This effect is due to the preload caused by the direct load which reduces the bolt loads. The cost of the connections decreases with only a small direct load and the costs are very similar for loads above 0.5 of the yield load. The nominal connection covers a very large proportion of the practical load cases that may occur for direct loads of 0.5 and 0.75 of the yield load.

The effect of using steels of higher grades results in higher costs and smaller areas where the nominal connection is suitable. With no direct load an infeasible area occurs in the region of high moments, this is due to the fact that if the bolts are given a suitable lever arm the base plate becomes thicker than the thickest available plate.

The rationalization of connection cost with respect to the load on the connection can be seen to be very difficult due to a number of factors which will now be summarised:-

(a) The cost contours are non-linear and they include in some cases local minima, maxima, plateaus and cliffs. This does not allow the



COST CONTOURS (£) FOR P/PY=0·0

COST CONTOURS (£) FOR P/PY=0·25

COST CONTOURS (£) FOR P/PY=0·50

COST CONTOURS (£) FOR P/PY=0·75

MEX = MAJOR AXIS MOMENT OF RESISTANCE OF THE SECTION
MEY = MINOR AXIS MOMENT OF RESISTANCE OF THE SECTION
PY = AXIAL YIELD LOAD OF THE SECTION
MX = MAJOR AXIS MOMENT APPLIED TO THE CONNECTION
MY = MINOR AXIS MOMENT APPLIED TO THE CONNECTION
P = AXIAL LOAD APPLIED TO THE CONNECTION

FIG. 7.27 RELATIONSHIP BETWEEN COST & LOAD STANCHION BASE PLATE CONNECTION SECTION 90, GRADE 43 STEEL fitting of simple functions to the resulting data.

(b) The selection of plate thicknesses, sizes of welds, sizes of bolts and numbers of bolts from discrete lists causes discrete jumps and plateaus in the cost surface. The most noticeable effect of this is that a nominal connection may be suitable for many loading combinations.

(c) The effect of geometric constraints which relate to the other section to be joined and the bolts for the connection cannot be easily taken into account.

(d) The infeasible region for each connection cannot be easily defined mathematically.

(e) The cost of a connection which will sustain the worst load case, will not always be sufficient to sustain other load cases, in other words; the connection that suits a set of loading cases may be more expensive than the connections which suit each load case individually.

(f) The design strategy used for the connections causes serious discontinuities in the cost surface. These result from the cost model predicting lower costs for larger numbers of smaller bolts and due to the fact that increasing the number of bolts may result in a decrease in the required end plate thickness. This shows that the cost model is in fact a "macro" cost model, which results in realistic costs for fabricated items, however extreme care should be exercised in using the cost model as an exact or "micro" cost model for individual components.

The definition of functions which model the relationship between cost and load on a connection has not been attempted due to the complexity of the relationship involved. This investigation led to a more satisfactory attempt to functionalise connection costs and this will now be described.

7.8.2 The Relationship Between Connection Costs and Section Properties

In the following, each available section had a connection designed to carry certain proportions of its maximum possible loading. The cost of these connections were then related to a property of the section, which is related in some way to the cost of the connection. Where a connection could not be designed for the loading on a particular section this was ignored. The investigation was limited to the use of grade 43 steel only. For the sake of brevity, only a proportion of the results will be fully described. As before only those items relating directly to the section under consideration were costed.

7.8.2.1 Major Axis Beam Connections

In this case four loading cases were considerd, these were:-

(a) Full plastic moment of resistance and full shear capacity of the section.

(b) Full plastic moment of resistance of the section, no applied shear.

(c) Half the full plastic moment of resistance and full shear capacity of the section.

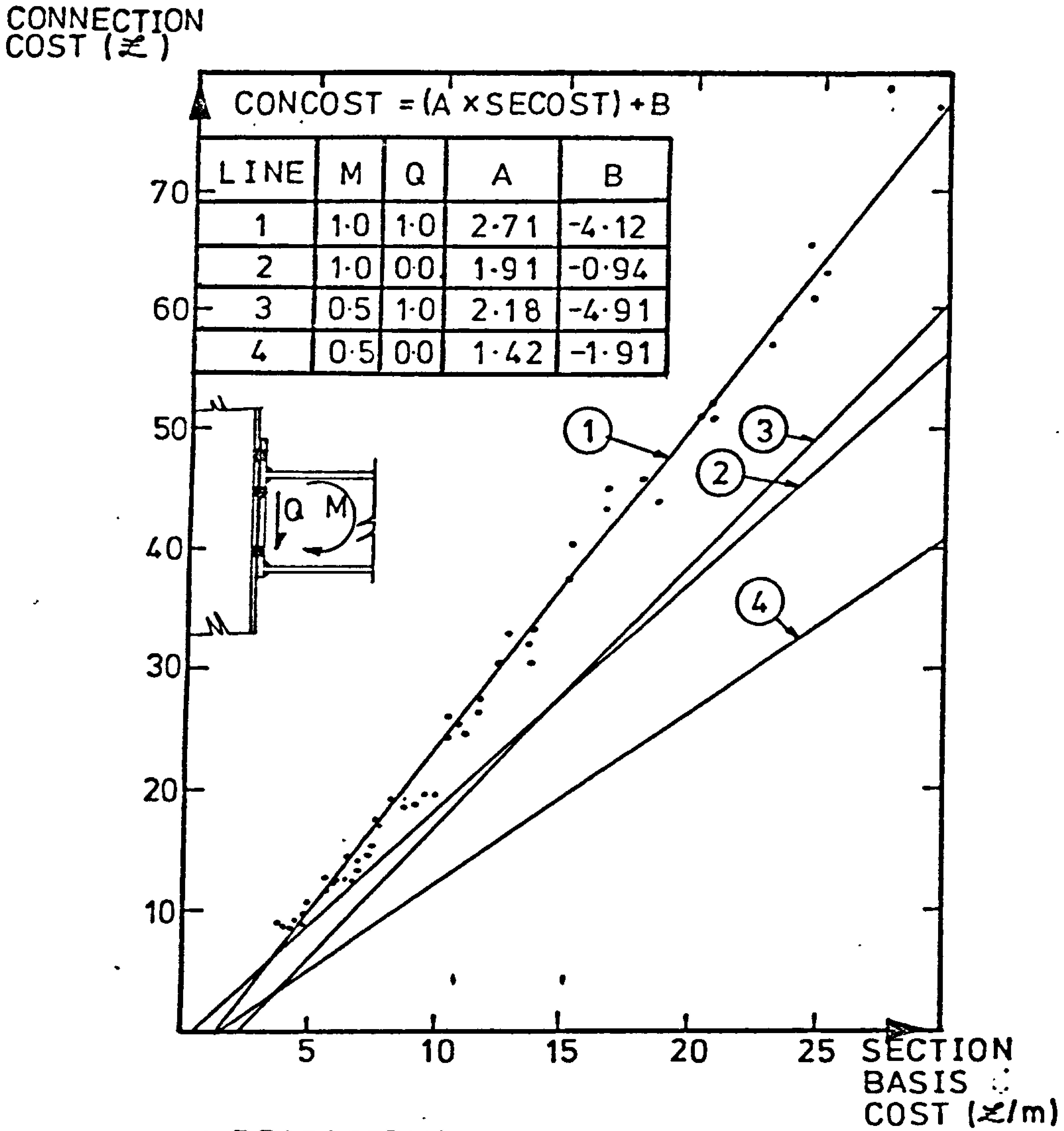
(d) Half the full plastic moment of resistance of the section, no applied shear.

The cost of the connection was first plotted against the basis cost of the section. This property was chosen because it is related to the area of the section and therefore the load that can be carried both in moment and shear. The points for the first load case are shown in figure 7.28, together with regression lines for each load case. The spread about the regression line for the first load case is seen to be quite small.

The cost of the connection was also plotted against the plastic section modulus (not illustrated), this property is proportional to the applied bending moment. In this case there was a greater spread for each load case. The depth times breadth of the section was also used, this property is related to the area of the end plate. A plot of these results (not illustrated) showed an even greater spread about the regression lines.

A summary of the correlation coefficients for each investigation is given in Table 7.6. It can be seen that, in all except one case, the plot against section basis cost had the highest correlation coefficient and so the least spread, it also has the highest mean correlation coefficient. It can therefore be concluded that the straight line relationship to section basis cost is the most accurate of those investigated.

A further relationship that has been investigated was to evaluate the cost of the connection as an extra length of section at the section basis cost. This extra length was evaluated by dividing the connection cost by the section cost in each case. The results of this investigation are shown in bar chart form in figure 7.30. The charts show a wide spread in two cases and a reasonable spread in two other cases. The mean and standard deviation (a measure of spread) have been calculated

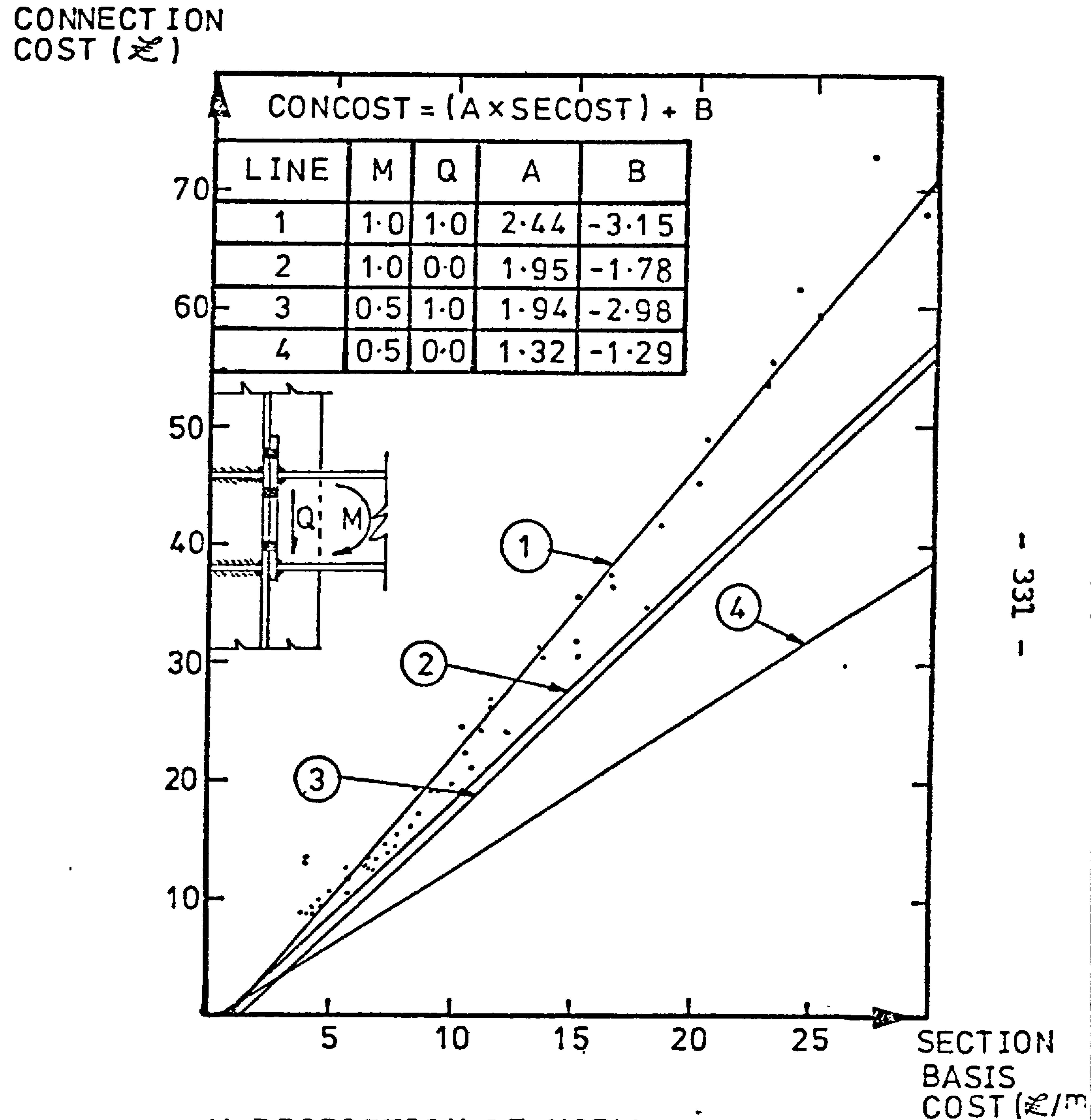


LINE	M	Q	A	B
1	1·0	1·0	2·71	-4·12
2	1·0	0·0	1·91	-0·94
3	0·5	1·0	2·18	-4·91
4	0·5	0·0	1·42	-1·91

M = PROPORTION OF FULL PLASTIC MOMENT OF THE SECTION

Q = PROPORTION OF SHEAR STRENGTH OF THE SECTION

FIG. 7.28 SECTION BASIS COST V CONNECTION COST, MAJOR AXIS CONNECTIONS



LINE	M	Q	A	B
1	1·0	1·0	2·44	-3·15
2	1·0	0·0	1·95	-1·78
3	0·5	1·0	1·94	-2·98
4	0·5	0·0	1·32	-1·29

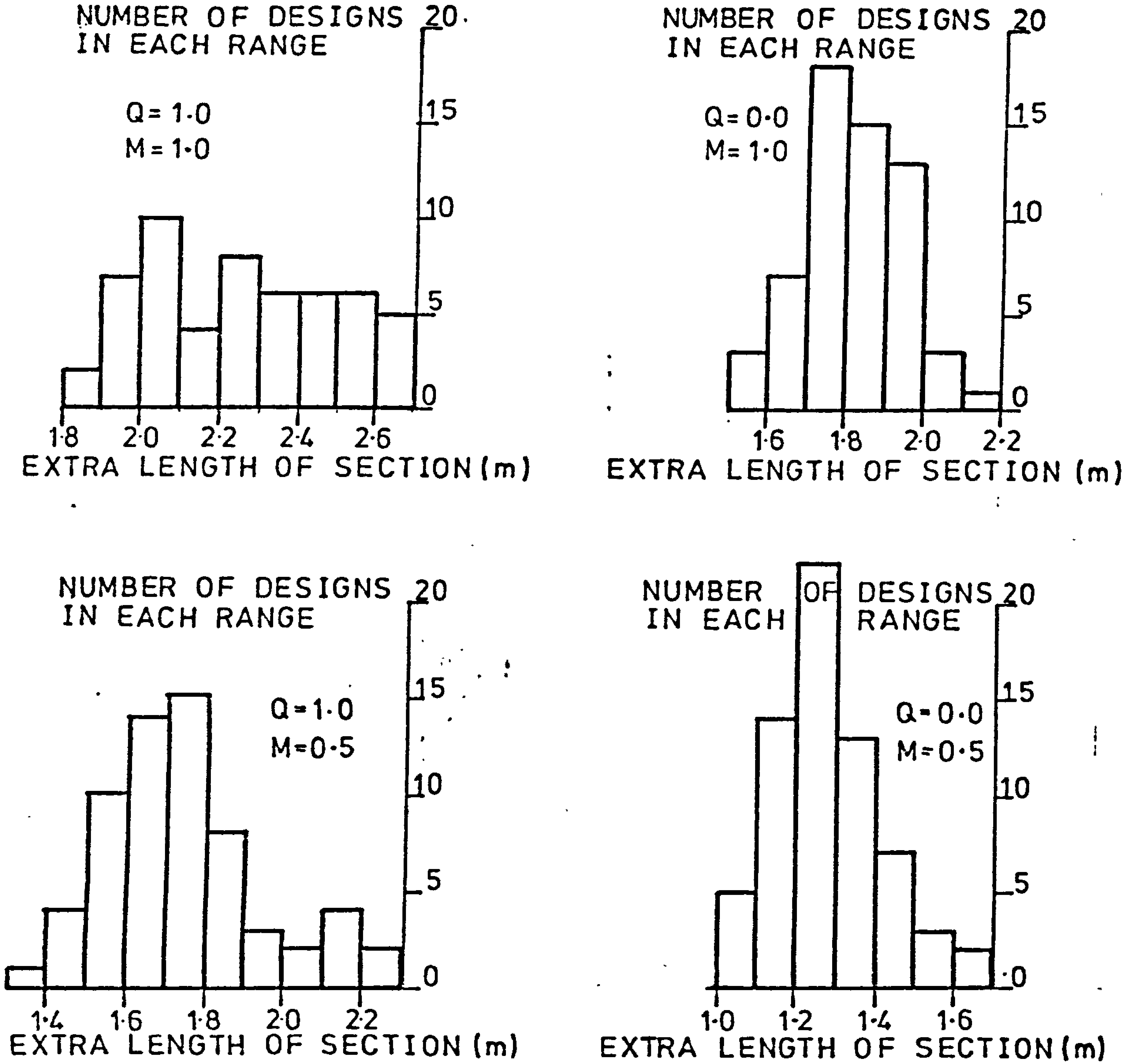
M - PROPORTION OF YIELD MOMENT OF THE SECTION

Q - PROPORTION OF SHEAR STRENGTH OF THE SECTION

FIG. 7.29 SECTION BASIS COST V CONNECTION COST, MINOR AXIS CONNECTIONS

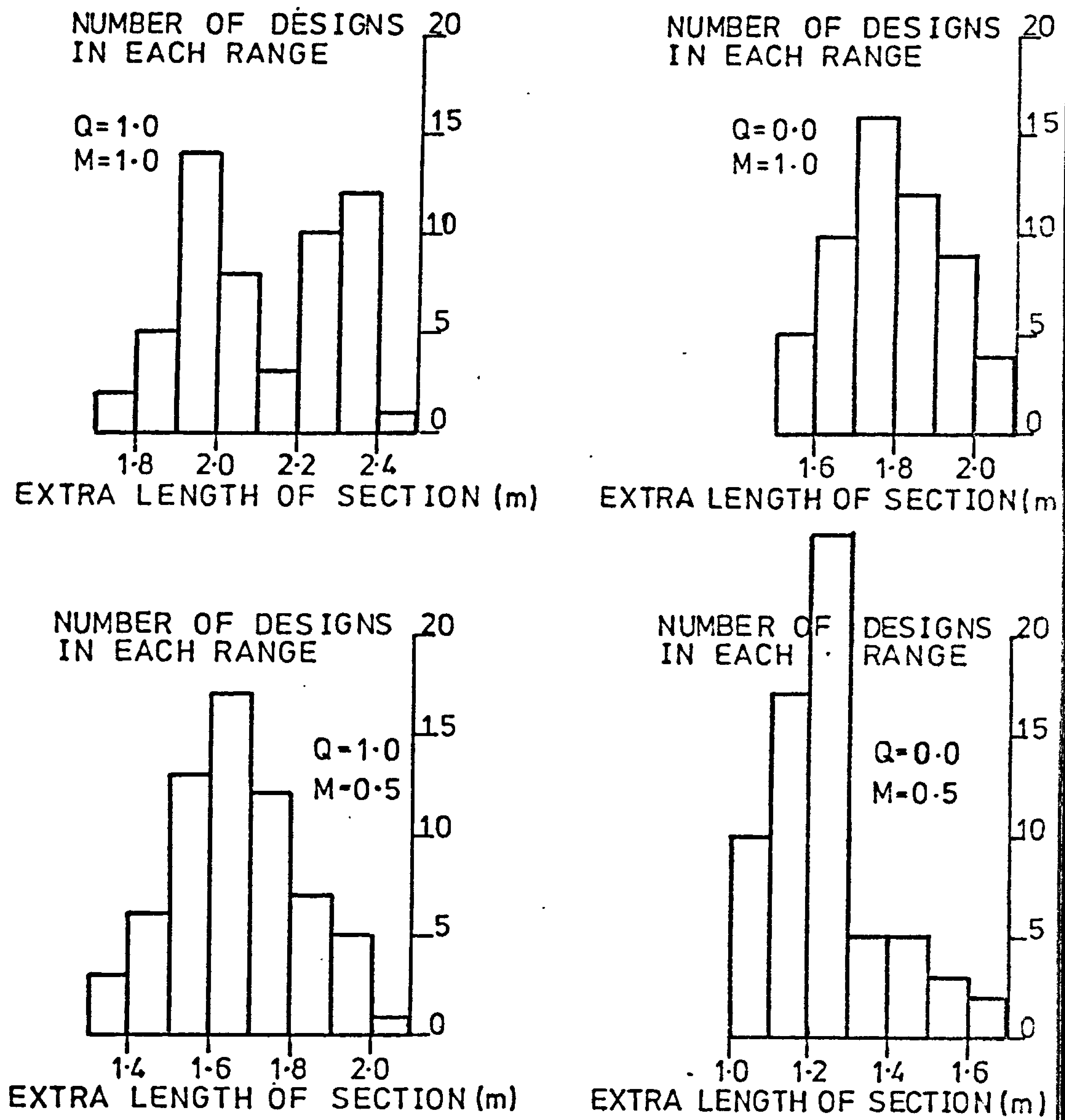
MOMENT IN TERMS OF PLASTIC MOMENT	SHEAR IN TERMS OF SHEAR RESISTANCE	NUMBER OF DESIGNS	CORRELATION COEFFICIENT FOR CONNECTION COST PLOTTED AGAINST:-		
			Section Basis Cost	Plastic Section Modulus	Depth Times Breadth
1.0	1.0	54	0.996	0.987	0.972
1.0	0.0	60	0.996	0.982	0.976
0.5	1.0	63	0.992	0.995	0.957
0.5	0.0	66	0.993	0.991	0.966
	Average		0.995	0.989	0.968

TABLE 7.6



FOR EXPLANATION OF M & Q SEE FIGURE 7·

FIG.7.30 HISTOGRAMS OF CONNECTION COST EXPRESSED AS AN EXTRA LENGTH OF SECTION MAJOR AXIS CONNECTIONS



FOR EXPLANATION OF M Q SEE FIGURE 7·

FIG.7·31 HISTOGRAMS OF CONNECTION COST EXPRESSED AS AN EXTRA LENGTH OF SECTION MINOR AXIS CONNECTIONS and are tabulated in Table 7.7. The standard deviation divided by the mean gives a measure of relative spread and it is also shown. This approach can be seen to provide a simple method of estimating the cost of a connection, however, due to the relatively large spread, it is of little use in structural optimization.

7.8.2.2 Minor Axis Beam Connections

This investigation was similar to that for major axis connections. The connection considered had an extended end plate and had no continuity. Four load cases were again considered using the elastic moment of resistance and the elastic shear capacity instead of the plastic values.

Load Case	Number of Designs	Additional Length of Section		
		Mean	Standard Deviation	S.D./Mean
a	54	2.25	0.234	1.104
b	60	1.82	0.134	0.074
c	63	1.74	0.205	0.118
d	66	1.29	0.139	0.108

TABLE 7.7

The connection cost was plotted against the section basis cost (see figure 7.29), the elastic section modulus (not illustrated) and the depth times breadth of the section (not illustrated). The results were similar to those for the major axis connection. A summary of these results is given in Table 7.8. These results show that the relationship between section basis cost and connection cost produces the smallest spread about the regression line in all but one case and it also produces the best average correllation coefficient.

The evaluation of the connection cost as an extra length of section was repeated in the same manner as for the major axis connections. These results are shown in bar chart form in figure 7.31. The mean and standard deviation were again found and these are shown in Table 7.9. The value of the standard deviation divided by the mean gives an indication of the relative spread. The relative spread is similar to that found with the major axis connections. In all cases the major axis connection was found to be more expensive than the corresponding minor axis connection. The reason for this is that the plastic moment of resistance is greater than the elastic moment of resistance resulting in thicker end plates and greater numbers of bolts. The spread in this case was again found to be too great for use in exact cost optimization.

7.8.2.3 Stanchion Splice Connections

The design of splices is complicated by the fact that the bottom stanchion section may be deeper than the upper section. In the following the lower section is assumed to have the same flange width and thickness as the upper stanchion section. Four load cases were considered for each section:-

(a) Elastic major axis moment of resistance of the section, no other loads.

(b) Axial yield load of the section, no other loads.

MOMENT IN TERMS OF PLASTIC MOMENT	SHEAR IN TERMS OF SHEAR RESISTANCE	NUMBER OF DESIGNS	CORRELATION COEFFICIENT FOR CONNECTION COST PLOTTED AGAINST:-		
			Section Basis Cost	Plastic Section Modulus	Depth Times Breadth
1.0	1.0	55	0.995	0.988	0.976
1.0	0.0	56	0.994	0.985	0.973
0.5	1.0	63	0.993	0.995	0.980
0.5	0.0	66	0.990	0.989	0.960
	Average		0.993	0.989	0.972

TABLE 7.8

(c) Elastic minor axis moment of resistance of the section, no other loads.

(d) One third of the major axis moment of resistance, one third of the minor axis moment of resistance and one third of the axial yield load of the section.

These load cases are all on the edge of the stanchion interaction diagram. The first three load cases are vertices of the interaction diagram. These load cases were combined with four depths of lower stanchion section.

Load Case	Number of Designs	Additional Length of Section		
		Mean	Standard Deviation	S.D./Mean
a	55	2.11	0.189	0.089
b	56	1.78	0.139	0.078
c	63	1.67	0.155	0.093
d	66	1.25	0.145	0.116

TABLE 7.9

The cost of connections was plotted against both the sectional area and the section basis cost for each section. The correlation coefficients are given in Table 7.10. It can be seen that the correlation coefficients are very similar in both cases, with the plot against section basis cost being slightly better than the plot against sectional area. There is no apparent reason that section basis cost is a better variable to use,

MX	MY	P	NUMBER OF FEASIBLE DESIGNS	DIFFERENCE IN SECTION DEPTHS (%)	CORRELATION COEFFICIENT	
					SECTION AREA	SECTION BASIS COST
1	0	0	80	0	0·98831	0·98854
0	0	1	65	0	0·99266	0·99306
0	1	0	89	0	0·95412	0·95439
⅓	⅓	⅓	83	0	0·99078	0·99025
1	0	0	80	10	0·98983	0·99134
0	0	1	65	10	0·98694	0·98905
0	1	0	89	10	0·96915	0·96972
⅓	⅓	⅓	83	10	0·99249	0·99300
1	0	0	80	20	0·98767	0·99067
0	0	1	65	20	0·98457	0·98648
0	1	0	89	20	0·98042	0·97729
⅓	⅓	⅓	83	20	0·98982	0·99181
1	0	0	80	50	0·97404	0·97906
0	0	1	65	50	0·96513	0·96914
0	1	0	89	50	0·98457	0·98748
⅓	⅓	⅓	83	50	0·97627	0·97984
				AVÊRAGE	0·98167	0·98320

MX=PROPORTION OF THE MAJOR AXIS ELASTIC YIELD MOMENT OF THE SECTION
MY=PROPORTION OF THE MINOR AXIS ELASTIC YIELD MOMENT OF THE SECTION
P=PROPORTION OF THE AXIAL YIELD LOAD OF THE SECTION

TABLE 7·10

indeed it is related to the sectional area. The points for the first load case and the regression lines for all load cases plotted against section basis cost are shown in gures 7.32 and 7.33 for four different depths of lower section. It can be seen that in each case there is a wide spread about the regression line. The cost increases steadily as the lower section depth increases, this is to be expected because the cost of packings is the only additional cost item in each case. It may be noted that the correlation coefficients reduce as the lower section depth increases.

The concept of expressing the cost of a connection as an extra length of section was also investigated. The data for the first load case is shown in bar chart form in figure 7.34, bar charts were also plotted for all other cases, however those illustrated are typical. It can be seen that the spread of the data increases with increase in the lower section depth. The mean, standard deviation and the standard deviation divided by the mean for each case is given in Table 7.11. The information in Table 7.11 shows that the relative spread, as defined by the standard deviation divided by the mean, generally increases with the depth of the lower section, except for load case three. As with the beam connections, it can be seen that the spread is generally too great for effective use in an optimization study. The relative spread can be seen to be greater than that for beam connections.

7.8.2.4 <u>Stanchion Base Plates</u>

In this investigation, four loading cases on a stanchion base plate were considered, these are:-

(a) Major axis moment of half the elastic moment of resistance of the section and no other load.

CONCOST = A x SECOST + B (lower stanchion 0% deeper than the upper stanchion)

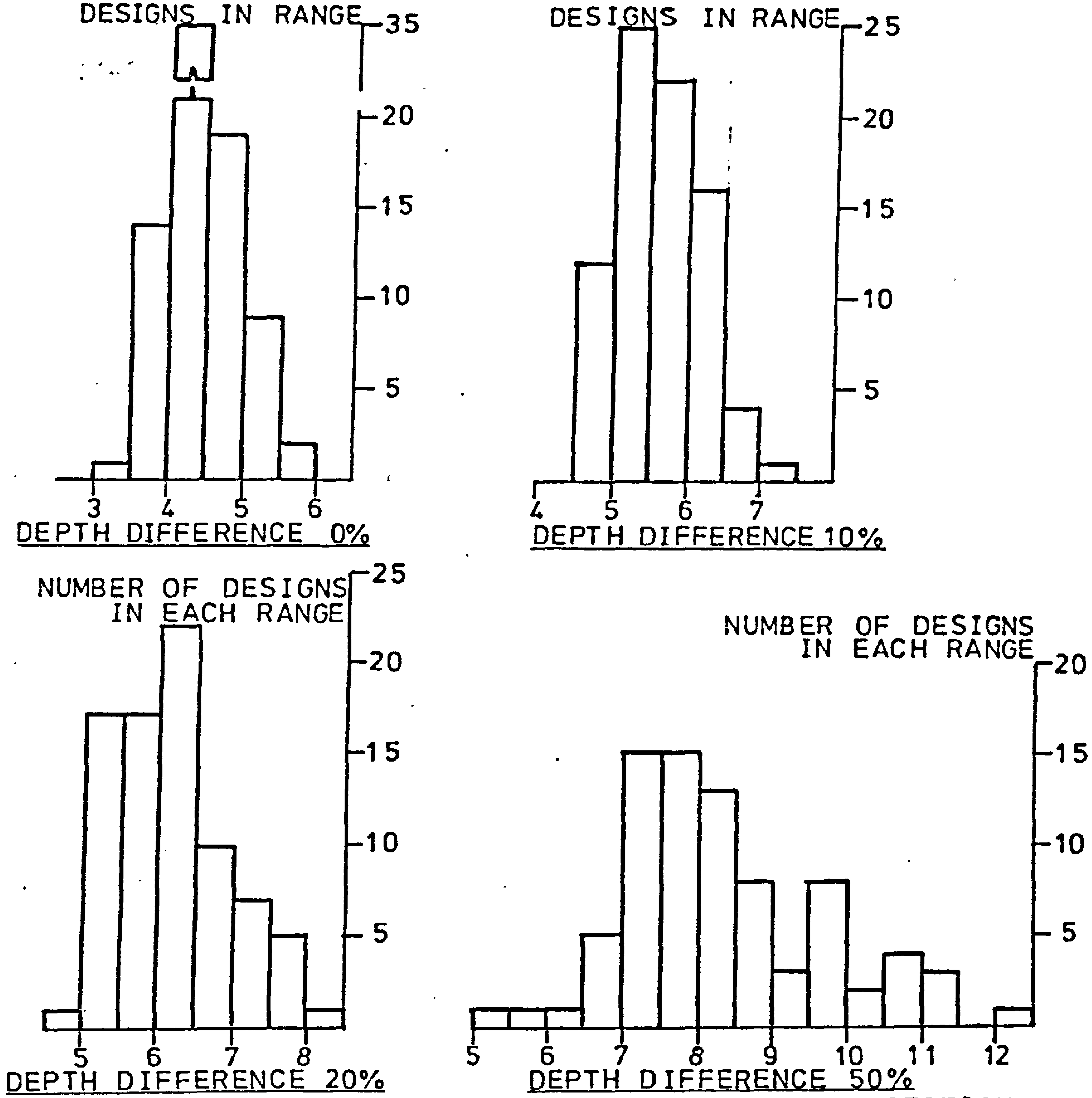
LINE	P	MX	MY	A	B
1	0	1	0	5·18	-9·90
2	1	0	0	8·19	-18·03
3	0	0	1	3 60	-10·10
4	⅓	⅓	⅓	4 75	-8·06

CONCOST = A x SECOST + B (lower stanchion 10% deeper than the upper stanchion)

LINE	P	MX	MY	A	B
1	0	1	0	6·61	-12·66
2	1	0	0	10·05	-21·54
3	0	0	1	4·44	-9·69
4	⅓	⅓	⅓	6 04	-10·08

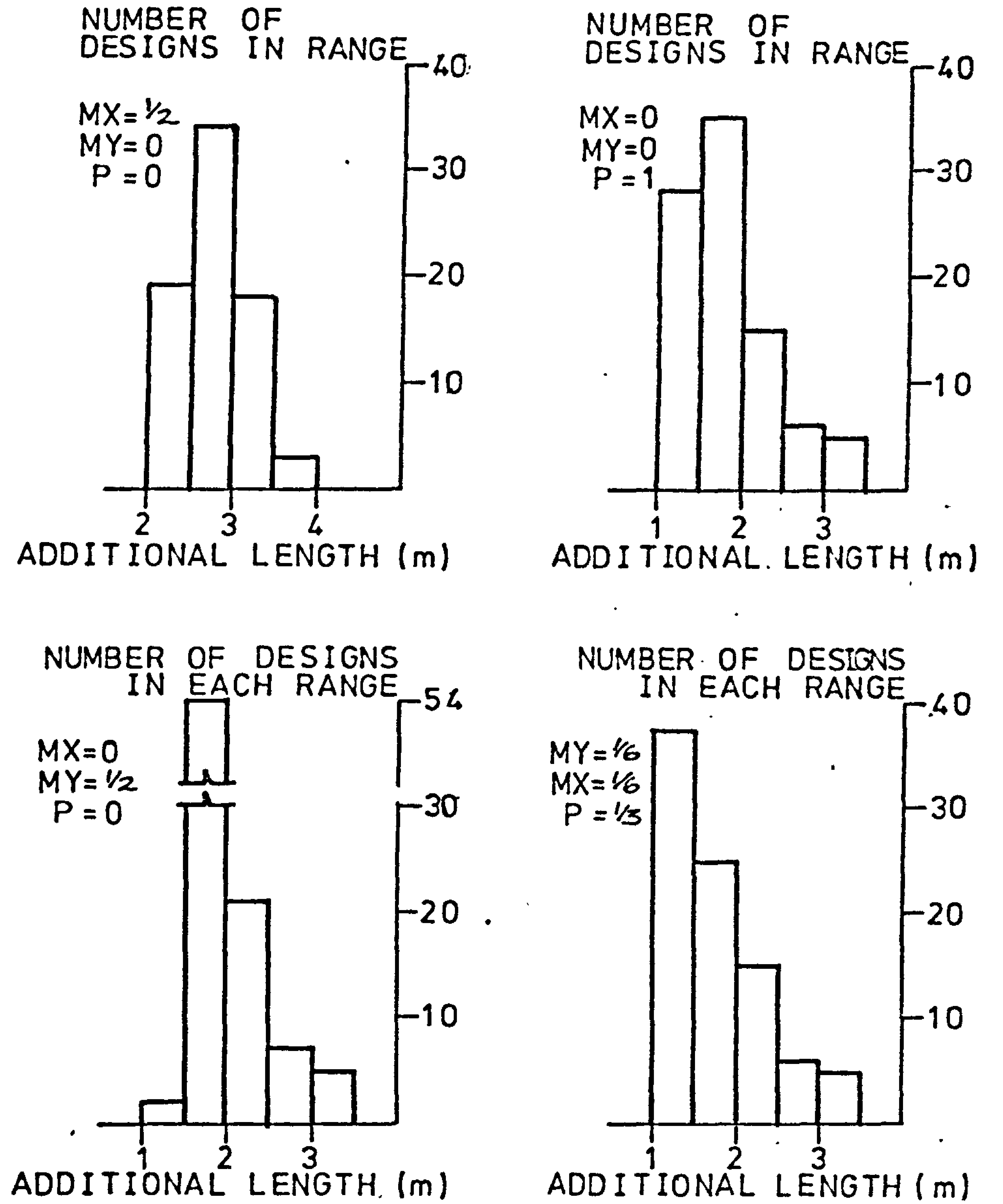
FIG. 7.32 SECTION BASIS COST v CONNECTION COST
STANCHION SPLICE CONNECTIONS

FIG. 7.33 SECTION BASIS COST v CONNECTION COST STANCHION SPLICE CONNECTIONS



LOADING CONSISTS OF MAJOR AXIS YIELD MOMENT OF SECTION

FIG. 7.34 HISTOGRAMS OF EQUIVALENT LENGTH (m) FOR STANCHION SPLICE CONNECTIONS, LOAD CASE, MAJOR AXIS YIELD MOMENT ONLY



FOR EXPLANATION OF LOADCASES (MX, MY & P) SEE FIGURE 7.

FIG. 7.35 HISTOGRAMS OF EQUIVALENT LENGTH STANCHION BASE PLATE CONNECTIONS

Load as a Proportion of the Full Elastic Value for the Section			% Difference in Depths	Number of Designs	Additional Length of Section		
M_x	M_y	P			Mean	Standard Deviation	S.D/Mean
1	0	0	0	80	4.39	0.47	0.11
0	0	1	0	65	6.45	0.81	0.12
0	1	0	0	89	2.93	0.65	0.22
⅓	⅓	⅓	0	83	4.14	0.41	0.10
1	0	0	10	80	5.62	0.57	0.10
0	0	1	10	65	7.98	1.05	0.13
0	1	0	10	89	3.80	0.68	0.18
⅓	⅓	⅓	10	83	5.29	0.52	0.10
1	0	0	20	80	6.16	0.78	0.13
0	0	1	20	65	8.65	1.36	0.16
0	1	0	20	89	4.18	0.73	0.18
⅓	⅓	⅓	20	83	5.80	0.70	0.12
1	0	0	50	80	8.37	1.35	0.16
0	0	1	50	65	11.42	2.06	0.18
0	1	0	50	89	5.68	0.89	0.16
⅓	⅓	⅓	50	83	7.86	1.19	0.15

TABLE 7.11

(b) Direct load of the yield load of the section and no other load.

(c) Minor axis moment of half the elastic moment of resistance of the section and no other load.

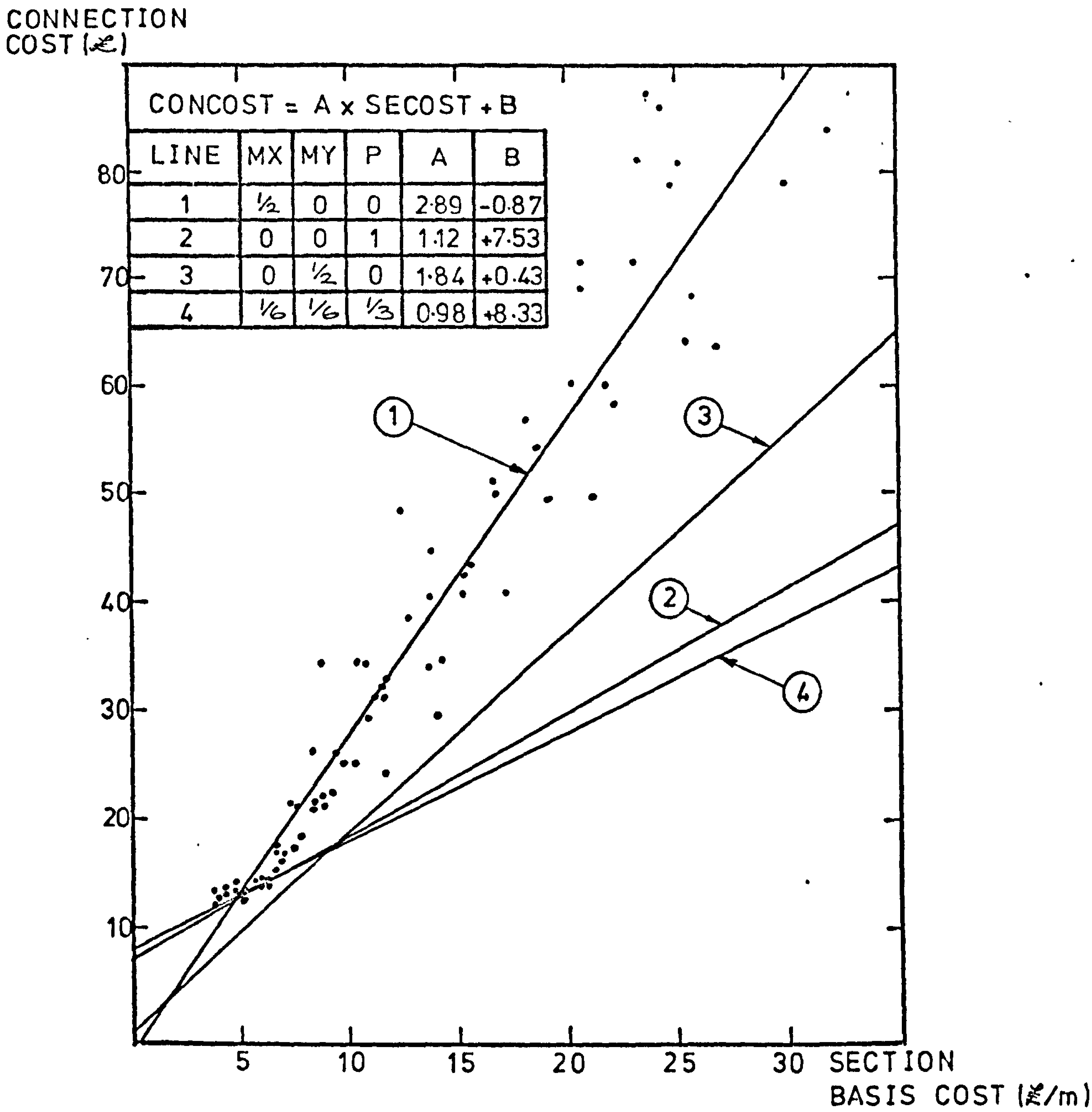
(d) Major axis moment of one sixth of the elastic moment of resistance of the section, minor axis moment of one sixth of the elastic moment of resistance of the section, direct load on one third of the yield load.

Half of the elastic moments are used because, with a fixed base stanchion the moment at the base is half that at the top of the stanchion.

In view of the relative success of previous correlations between connection cost and section basis costs, this investigation is limited to the same correlation. The results are shown in figure 7.36, the points are shown plotted for load case one and regression lines are shown for each load case. The spread of points about the regression line can be seen to be very wide, indicating an unsatisfactory relationship. The different load cases result in very different costs. The correlation coefficients are shown in Table 7.12.

M_x	M_y	P	n	Correlation Coefficient
½	0	0	74	0.96531
0	0	1	89	0.99103
0	½	0	89	0.95831
1/6	1/6	⅓	88	0.99191

TABLE 7.12



LINE	MX	MY	P	A	B
1	½	0	0	2·89	-0·87
2	0	0	1	1·12	+7·53
3	0	½	0	1·84	+0·43
4	⅙	⅙	⅓	0·98	+8·33

MX = PROPORTION OF MAJOR AXIS YIELD MOMENT OF THE SECTION
MY = PROPORTION OF MINOR AXIS YIELD MOMENT OF THE SECTION
P = PROPORTION OF AXIAL YIELD LOAD OF THE SECTION

FIG. 7.36 SECTION BASIS COST v CONNECTION COST, STANCHION BASE PLATES

It can be seen that load cases two and four give good correlation, whereas the other load cases have a worse correlation coefficient.

The cost of a connection in terms of an extra length of section at the section basis cost is shown in bar chart form in figure 7.35. The values of the mean, standard deviation and the standard deviation divided by the mean are shown in Table 7.13. The spread in this case is larger than for any other type of connection. Again, this approach could be used for estimating, however the spread is so large that satisfactory estimates may not be obtained.

The cost of connections for various load cases can be related to the section basis cost. This approach is difficult to use practically because only a small number of load cases can be treated practically, other load cases can only be treated by analogy, which is a difficult procedure to programme. The additional length of section concept could be used for estimation and preliminary design but it is not a practical tool for accurate optimization.

Load as a Proportion of the Yield Load						
M_x	M_y	P	Number of Designs	Mean	Standard Deviation	$^{S.D}/_{Mean}$
⅓	0	0	74	2.8009	0.3926	0.1402
0	0	1	89	1.8432	0.5356	0.2906
0	½	0	89	1.9946	0.4666	0.234
$^1/_6$	$^1/_6$	⅓	88	1.7758	0.5782	0.326

TABLE 7.13

7.9 CONCLUSIONS

In this chapter a number of investigations of the design of multi-storey braced steel frames have been described. The general conclusion that can be drawn is that the relationship between the cost of the frame and the primary variables (the sections chosen) are very complex. Examination of these relationships by means of examining the design space for a particular frame, shows that the design problem as formulated is subject to constraints on isolated designs and that there is seldom a definite trend between the variables and the cost. Properties of the relationship between the primary variables and the cost have been identified, and these have been utilised in order to develop a design strategy. The design strategy developed makes use of two of the algorithms that have been developed, it neglects the consideration of moment redistribution because it is uneconomic to apply. It also neglects the use of minor axis depth constraints because these are shown to reject economic designs. The strategy has been shown to produce merely an economic design which has been searched for sequentially and which will not necessarily be optimal. The design strategy has been applied to the problem of optimal splicing arrangements for stanchions. The strategy shows modest improvements in the cost of a frame between the initial design and the final design. The optimal splicing arrangement was seen to be when splices were placed as far apart as is practically possible, this was seen to be in accordance with current industrial practice.

The sensitivity of the final cost to each material and fabrication cost item was examined and it was shown that the fabrication items each represented only a small proportion of the total cost, however in total they represent about half the total cost. The section basis cost was shown to represent by far the largest single cost item. Errors in individual fabrication cost items could be seen to have a very small effect on the total cost.

The effect of various secondary variables (load factor, grade of steel and geometry) were then considered. Increases in the load factor were shown to produce increases in cost which were not in proportion but were substantially linear. The effect of using various grades of steel was examined and it was shown that grade 43 steel was generally uneconomic, grade 55 steel was shown to be the most economic, with grade 50 steel a close second. The limited availability of grade 55 steel and the specialist skills required when welding would seen to point to the use of grade 50 steel as the ideal material. The use of a minimum weight objective function was also examined, and it was concluded that small differences exist between cost and weight based designs. The effect of varying the geometry of a building was to produce a trade-off between cost items which are dependant on loading and cost items dependant on numbers of items. If this trade-off was evaluated it was possible to find an optimal geometry for a frame.

Finally accounts of investigations into the cost of connections have been presented. These investigations show that the relationship between the variables that define connections (loads and sections) and the cost of connections are also very complex and that in order to rationalise the costing of connections serious approximations have to be made which if used may affect the validity of the optimal design produced.

In conclusion, it can be stated that the relationship between primary variables and the cost of either the frame or individual connections is very complex and cannot be adequately predicted in order to find the true optimum design, however an economic design strategy has been developed which finds designs that are an improvement over the designs produced by iterative design. Relationships between costs and the secondary variables are seen to be more predictable and these relationships can be used as a basis for deciding such questions as which grade of steel to use.

CHAPTER 8

Conclusions

8.1 INTRODUCTION

In this chapter conclusions are drawn about the formulation and the solution of the optimum structural design problem. The design of braced multi-storey structures is also examined. Finally the relationship of this research to the wider field of structural optimization is summarised, together with an examination of ways in which it could be extended and improved.

8.2 PROBLEM FORMULATION

The problem stated in non mathematical terms was to design a rectangular three-dimensional framework to carry the loads applied by the walls and floors of a multi-storey structure.

The problem formulation consisted of identifying the three major components of an optimization problem so that the problem could be stated mathematically. These components will now be examined.

8.2.1 Variables

The independant variables consist of the steel sections chosen for each group of members in the frame. These variables were of necessity of discrete form due to the finite range of sizes and shapes of sections that are available. The dependant variables consist of the connections and the internal forces in the members. The connections are evaluated by the use of a design strategy based on current practice. Evaluation of the internal forces is based on a codified rigid frame design method.

8.2.2 Objective Function or Cost Model

Given a set of variables, a cost may be evaluated using the cost model. This model was derived by examining all the major cost items and then relating these costs to the variables. The model seeks to be as comprehensive as possible, however certain items were omitted for which data was unavailable. The cost model produces realistic prices which can be used with confidence for the costing of complete structures.

8.2.3 Constraints

The stress constraints consist of limits on stresses and deflections. These constraints apply to the sections and the connections. Once the design method has been selected, these constraints can be tested for feasibility.

In addition to the stress constraints, a further set of constraints was identified. These geometric constraints express the practical problems associated with the fitting together of sections to form a framework. Rapid methods of checking these constraints had to be devised in conjunction with the optimization algorithms.

To sum up the problem was identified as having truly discrete variables, a discontinuous objective function, and a disjoint feasible region.

8.3 PROBLEM SOLUTION

8.3.1 Available Algorithms

An examination and classification of available optimization algorithms showed that many algorithms exist for the solution of continuous and discretized variable problems. Few algorithms were found to exist for the solution of truly discrete variable problems. Problem orientated algorithms showed some promise.

8.3.2 The Algorithms Developed

The initial requirement was for an algorithm which would show rapid improvement and find a solution close to optimal. This resulted in the development of the iterative design algorithm which is a problem orientated algorithm developed from fully stressed iterative design.

The second requirement was for an algorithm which would improve on the solution produced by the iterative design algorithm. In order to keep computation to a minimum the algorithm was required to optimize a few variables at a time. The enumeration algorithm evaluates designs represented by combinations of variables and exploits features of the problem that save computation. It was also recognised that the construction of a stanchion allowed the use of a stage-wise formulation, resulting in the dynamic programming algorithm.

8.3.3 The Design Strategy Developed

An investigation was made of a typical design space, in order to identify the features of the problem which could be used to develop a design strategy, which uses the algorithms efficiently. A strategy was developed which consisted of using the iterative design algorithm to generate a design, followed by the optimization of stanchion strings using the dynamic programming algorithm. The strategy can be said to generate a cheap design and then systematically improve this design.

8.3.4 Comments on the Algorithms and the Strategy

The exploitation of the properties of the problem made solution with an economic amount of computation possible. In particular the identification of geometric constraints and their inclusion reduced computation considerably.

The iterative design algorithm, though considering only section basis cost in its objective function, showed rapid progress towards designs which are close in total cost to the cheapest designs found.

Local searches using the enumeration algorithm operating on strings of stanchions showed disappointing returns despite the inclusion of numerous cost saving features. The dynamic programming algorithm however showed an economic return on computation costs.

The use of the design strategy does not necessarily 17
guarantee the production of a feasible design, however in most cases feasible designs resulted. Where the iterative design algorithm produced a design that could be costed, application of local searches resulted in small, but significant, cost reductions.

The design strategy as presented takes the form of a sequence of operations which could be automated. This was necessary in order to give a common basis for comparison of algorithms and the resulting designs. The ability of an experienced designer to use his intuition and experience to select sections would compliment the ability of the local 28
search algorithms to investigate combinations of variables efficiently. It can be seen that both automated and interactive design have a part to play in the solution of optimum structural design problems.

8.4 MULTI-STOREY BRACED RIGID STEEL FRAMES

8.4.1 Design Methods

The design methods that are available for the design of braced multi-storey frames were reviewed and three that represent current British practice were selected for consideration. It was found that the requirements of each method were very similar and that they could all be included within the same data structure. In each case the design space had similar characteristics, differing only in the position of the stress feasible region and in the cost of feasible designs. In most cases it was found that 11
the Building Research Station design method produced the cheapest designs, however it must be emphasised that any such conclusions are dependant on the load factors employed.

8.4.2 The Optimum Design

Limited investigations of frames designed using the algorithms were performed. The main conclusions of these investigations will now be described. Firstly a large proportion of the total fabrication is included within the stanchion splices, resulting in optimal splicing arrangements that use as few splices as possible.

Secondly minimum weight designs use different steel sections to minimum cost designs. The difference in the price and weight of such designs is only small vindicating the use of 24
minimum weight when fabrication costs are unknown. It was also found that the cost per tonne (often used as an aid 26
to estimating) varied considerably from one building to the next.

Taking advantage of moment redistribution resulted in a reduction in frame cost, however the operation was not cost effective. This may again be a place where the designer could play a part in reducing computation costs.

Variation of building geometry was seen to have a marked effect, however the cost of the framework represents only a small proportion of the total structure cost and therefore other factors will generally dictate the form of the framework.

8.4.3 Connections

An extensive investigation of connection costs was carried out. This produced useful relationships that could be used for preliminary estimating. It was apparent however that until erection and extra costs can be determined and rationalised it will not be possible to truly optimize an individual connection.

8.5 OPTIMUM STRUCTURAL DESIGN

This thesis describes an attempt to design practical structures in accordance with codified design methods, using the criterion of minimum cost. The few investigators that have considered truly discrete problems, taking into account fabrication costs, have done so with small problems and few variables. The optimization of designs using comprehensive interactive design programmes with sub-optimization routines for the design of details is believed to form a logical development. In future research it is recommended that emphasis should be placed on the satisfaction of practical constraints and the development of comprehensive cost models. It is recommended that the approach used within this investigation should be carefully studied in order that some of the many pitfalls may be avoided.

To sum up, this thesis shows that the coupling of a practical design programme with suitable optimization algorithms ~~can~~ provide economic designs cost effectively.

APPENDIX

AVAILABLE STRUCTURAL STEEL SECTIONS

NOTATION

Serial Size = Depth X Width (mm)
UB = Universal Beam Section
UC = Universal Column Section

Nº	SERIAL SIZE	TYPE	WEIGHT (Kg/m)	BASIS COST (£/m)
1	914 X 419	UB	388	66·23
2			343	58·55
3	914 X 305	UB	289	48·99
4			253	42·88
5			224	37·97
6			201	34·07
7	838 X 292	UB	226	37·79
8			194	32·44
9			176	29·43
10	762 X 267	UB	197	32·70
11			173	28·72
12			147	24·40
13	686 X 254	UB	170	28·22
14			152	25·23
15			140	23·24
16			125	20·75
17	610 X 305	UB	238	39·51
18			179	29·71
19			149	24·73
20	610 X 229	UB	140	23·07
21			125	20·60
22			113	18·62
23			101	16·64
24	533 X 210	UB	122	20·25
25			109	18·09
26			101	16·77
27			92	15·27
28			82	13·61
29	457 X 191	UB	98	15·31
30			89	13·90
31			82	12·81
32			74	11·56
33			67	10·47
34	457 X 152	UB	82	13·76
35			74	12·42
36			67	11·24
37			60	10·07
38			52	8·73
39	406 X 178	UB	74	11·64
40			67	10·54
41			60	9·44
42			54	8·49
43	406 X 140	UB	46	7·76
44			39	6·58
45	356 X 171	UB	67	10·92
46			57	9·29
47			51	8·31
48			45	7·34
49	356 X 127	UB	39	6·69
50			33	5·66
51	305 X 165	UB	54	8·80
52			46	7·50
53			40	6·52
54	305 X 127	UB	48	7·91
55			42	6·92
56			37	6·10
57	305 X 102	UB	33	5·72
58			28	4·86
59			25	4·34

Nº	SERIAL SIZE	TYPE	WEIGHT (Kg/m)	BASIS COST (£/m)
60	254 X 146	UB	43	6·96
61			37	5·99
62			31	5·02
63	254 X 102	UB	28	4·82
64			25	4·31
65			22	3·79
66	203 X 133	UB	30	4·93
67			25	4·11
68	356 X 406	UC	634	108·22
69			551	94·06
70			467	79·72
71			393	67·09
72			340	58·04
73			287	48·09
74			235	40·11
75	COLUMN CORE	UC	477	81·42
76	356 X 368	UC	202	34·24
77			177	30·00
78			153	25·93
79			129	21·87
80	305 X 305	UC	283	45·65
81			240	38·71
82			198	31·94
83			158	25·49
84			137	22·10
85			118	19·03
86			97	15·65
87	254 X 254	UC	167	26·94
88			132	21·29
89			107	17·26
90			89	14·36
91			73	11·77
92	203 X 203	UC	86	14·02
93			71	11·57
94			60	9·78
95			52	8·48
96			46	7·50
97	152 X 152	UC	37	6·20
98			30	5·03
99			23	3·85
100	254 X 203	JOIST	82	15·55
101	254 X 114	"	37	6·30
102	203 X 152	JOIST	52	9·66
103	203 X 102	"	25	4·14
104	178 X 102	JOIST	22	3·62
105	152 X 127	JOIST	37	6·17
106	152 X 89	"	17	2·93
107	152 X 76	"	18	2·88
108	127 X 114	JOIST	30	4·84
109	127 X 114	"	27	4·35
110	127 X 76	"	16	2·77
111	127 X 76	"	13	2·26
112	114 X 114	JOIST	27	4·29
113	102 X 102	JOIST	23	3·72
114	102 X 64	"	10	1·67
115	102 X 44	"	7	1·26
116	89 X 89	JOIST	19	3·14
117	76 X 76	JOIST	15	2·67
118	76 X 76	"	13	2·30