Container Shipping Contract Design under Financial Constraints and Default Risks

By

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Abstract

Due to the unexpectedly rapid outbreak of the epidemic, the global economy is experiencing a sluggish market situation. In the context of the global shipping network, liner shipping has been serving as a critical link to connect the global economy. The liner companies are currently struggling with solvency problems resulting from the utilisation of the leasing tactics, while the shipping market has not yet experienced a comprehensive recovery. This thesis is a compilation of three essays that deal with the intersection of business operations and financial management in container shipping chains that consist of liner companies, non-vessel operating common carriers (NVOCCs), financial institutions, and customers. Each of these essays focuses on a different aspect of this intersection.

The first essay mainly focuses on the contract design of liner companies collaborating with NVOCCs under the financial constraint of which contract is more conducive to repayment. To achieve the above objectives, the default cost is involved to compare the default loss associated with each contract. Besides, three types of slot purchase contracts are compared in detail: buyback contracts, revenue sharing contracts, and quantity discount contracts. The research finds that buyback contracts and revenue sharing contracts are identical for the whole container shipping chains. Regarding expected profit, the profitability of buyback contracts and revenue sharing contracts are more conducive to the repayment of the financial lease than quantity discount contracts. In terms of profit under deterministic demand, the quantity discount contracts show better solvency than others in the face of low-rent financial lease contracts. In other cases, buyback contracts and revenue sharing contracts still outperform quantity discount contracts.

The second essay investigates the interface between the optimal contract parameters and financial lease size under each contract type selection. Different from the first essay, the contract in this essay isolates the direct selling profit of the liner companies from the contract negotiation and achieves equilibrium conditions by invoking liner companies to assist NVOCCs in operational cost sharing. Still, the profitability of buyback contracts and revenue sharing contracts are identical. However, the different preferences of the liner companies and the NVOCC in buyback contracts and revenue sharing contracts will result in these two contracts being executed differently. Meanwhile, this will also lead to the buyback contract being more favourable to the repayment of liner companies.

The final essay presents the liner companies' equilibrium condition of operating and financial leasing from a long-term perspective. We apply a prospect theory-based credit rating model to present the long-term willingness of financial institutions to provide financial leases to liner companies. Besides, we provided the optimal contract parameters for the slot purchase contract between NVOCCs and liner companies with insufficient shipping capacity. By comparing with the liner companies with sufficient capacity in the previous essay, our results indicate that the reduction of finance leases cannot significantly improve the balance between profit and repayment. Instead, a moderately excessive financial lease will help liner companies balance operations and financing. Additionally, it also assists liner companies in establishing their creditworthiness and competitiveness to improve future revenues.

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In loving memory of my grandmother

I hope I am making you pound up there in heaven

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Chapter 1. Introduction

The history of maritime shipping may date back thousands of years. From ancient civilisations using boats to transport goods along rivers and coastlines to the development of long-distance shipping in the 19th century, the industry has been playing a key role in facilitating trade and connecting individuals all over the world (Lambert, 2013). With the advent of steam-powered ships to the development of containerisation and automation, the shipping industry has seen significant transformations over time. According to the 'Review of Maritime Transport' from the United Nations Conference on Trade and Development (UNCTAD) covering the recent five years from 2018 to 2022, liner shipping has been one of the essential pillars of the globalised economy, with ships of all sizes transporting a variety of products, including raw materials, consumer goods, and energy resources, across over great distances. Especially in the epidemic, with countries under lockdown due to the spread of the pandemic, liner shipping transport preserves the physical connection to international value chain deployment and contributes particularly to global interconnectedness. However, the maritime shipping market has also been subject to several challenges and advancements, driven by factors such as global trade patterns, the economic climate and technological progress.

First, the shipping industry has gone through a great amount of consolidation over the past ten years. According to Monacelli (2018), the market share of the top three shippers was around 24% in the early 2000s, increasing to 39.9% in 2016 and 42.8% in 2017. In addition, the figure below from 'Maritime Transport Review 2022' can also

Figure 1.1 Market shares of top four, top ten, and top twenty carriers, 2011-2022 (%)

(Source: UNCTAD based on data provided by Alphaliner.)

prove this trend of consolidation (UNCTAD, 2022). Particularly, there have been a considerable number of mergers and acquisitions involving large container shipping companies such as APM-Maersk, Hapag-Lloyd, and CMA CGM Group (UNCTAD, 2022). Besides, Tran (2022) also found that some major shipping lines also collaborate with each other in the form of strategic alliances to expand economies of scale and scope. According to data provided by the UNCTAD in Review of Maritime Transport 2022, the top three deep-sea container shipping alliances currently control more than 80% of the market. This suggests a transformation in the global trade patterns of the shipping industry, characterized by a growing trend of consolidating market shares through mergers or collaboration. Since the huge network of routes and connections is an important factor in achieving extensive coverage of the shipping network, these could help the container shipping companies achieve economies of scale, reduce costs and remain competitive. In addition, this could also enable them to mitigate risk and increase vessel utilisation by sharing vessels (Hu et al., 2019). However, it could make it difficult for other shipping companies to match major alliances in terms of pricing and service capabilities, potentially reducing their market share and profitability (Cariou and Guillotreau, 2021). Besides, these shipping alliances heavily relies on effective operational integration and coordination among member companies. Second, despite the continued expansion of the above two patterns, the current shipping market is facing the challenge of overcapacity because the growth in trade volume has not kept up with this expansion. Since many vessels are left idle or stranded, resulting in port congestion, shipping companies also have to face the challenges of delays and increased costs (Okur and Tuna, 2022). UNCTAD (2020) stated that the shipping market experienced a period of recovery between 2016 and 2019, during which freight rates rose and demand for vessels increased. However, the outbreak of the pandemic in 2020 had a significant impact on the shipping market, causing disruptions to supply chains and a decrease in global trade. During this period, freight rates have experienced a period of sharp increases due to the increase in operating costs. With the smooth recovery of the economic climate, the condition of overcapacity might be improved gradually (UNCTAD, 2022). However, Due to changes in some regional markets, there may be an upsurge in demand for shipping services along one trade route, which could result in a shortage of services and higher freight costs (Song, 2021). On the contrary, other routes with lower demand might accumulate aggravating excess capacity and experience lower freight rates. Peng and Bai (2022) proposed that this imbalance of supply and demand could also lead to periods of large fluctuations in freight rates for shippers and tighter profit margins for shipping companies. Therefore, the prediction under uncertain demand over time is critical to the operations of shipping companies.

Other advancements faced by the shipping market are mainly from digitalisation. According to UNCTAD (2022), digitalisation has helped developed countries react faster to emergencies than least developed countries and facilitated the smooth flow of goods during the pandemic. The use of digital technology also involves the use of blockchain technology for secure and efficient cargo tracking, the development of autonomous ships, and the use of data analytics to optimise vessel routes and improve operational efficiency (Yang, 2019). Therefore, digitalization can also be used to analyse the situation faced by shipping companies in a timely manner and help the company make timely decisions.

Based on the above analysis regarding the current state of the shipping industry, it is evident that the container shipping sector is confronted with considerable challenges and opportunities in maintaining efficiency and productivity, given the complex economic, social, and environmental influences. Therefore, this thesis aims to assist the liner companies in matching the other stakeholders and addressing the above challenges from the scale of vessels, with a specific focus on efficiently identifying changes in market supply and demand as well as adapting to the continuous expansion of company vessel sizes through flexible adjustments. Continuous expansion of the scale of vessels can enrich the shipping network and productivity but also bring enormous liabilities

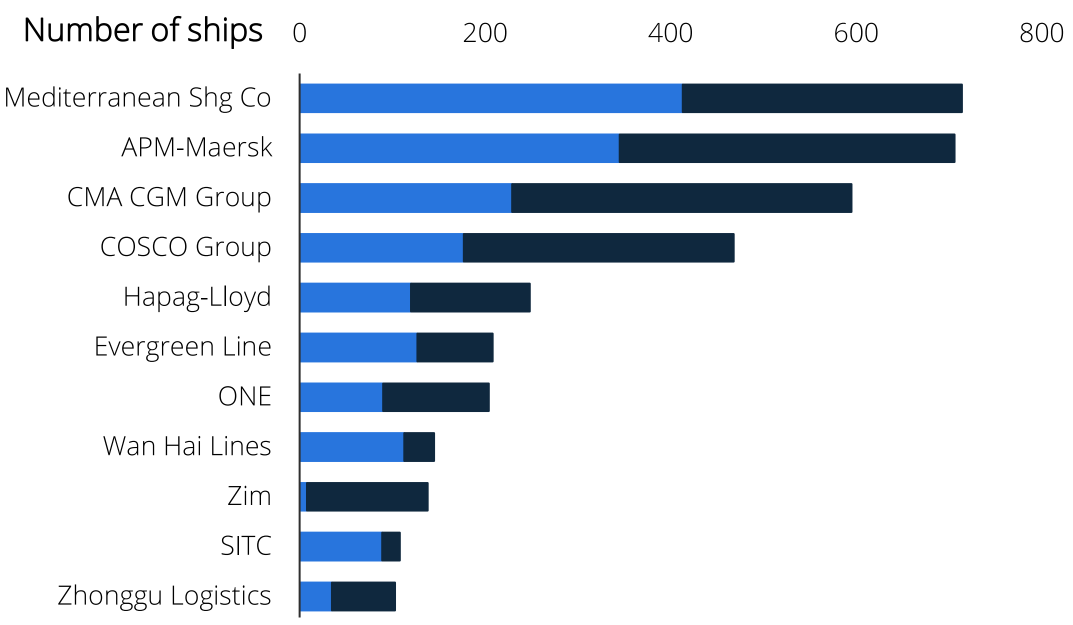

Figure 1.2 Leading container ship operators - owned and chartered ships.

(Source: Statista based on data provided by Alphaliner)

(Tran, 2022), making it difficult for liner companies to manage their supply chain efficiently. Especially in the depressed freight market, liner companies are constrained by the lack of capital liquidity and might generate excessive net losses (Li and Dong, 2021). Besides, the lessons from Hanjin's insolvency case (Shin et al., 2019) and the bankruptcy of an increasing number of cross-border liner companies (Li and Dong, 2021) are all reminders of the importance of efficient liability management in the shipping industry. Based on the state of the shipping industry shown in Figure 1.2 above from Statista, the chartered vessels account for a large portion of the vessels operated by these leading shipping companies. Without the heavy financial stress of purchasing or owning vessels, chartering vessels provides companies with operational flexibility and the ability to respond quickly to fluctuations in market demand (Shin et al., 2019). Consequently, this strategy enables liner companies to increase their market share and generate a steady income stream, as fleet size and composition can be adjusted in response to fluctuating market conditions. As many leading companies in the shipping industry have gradually shown resilience and adaptability, it is worthwhile to analyse the function of chartered vessels in the operation of liner companies.

Motivated by this, we construct a container shipping chain comprising liner companies, non-vessel operating common carriers (NVOCCs), financial institutions, and customers to investigate the equilibrium conditions for liner companies' operating and financial activities. The collaboration between liner companies and NVOCC is through the slot purchase contract in the form of a supply chain contract like revenue-sharing, buyback, and quantity discount contracts. On this basis, this thesis explores the equilibrium conditions for liner companies under financial constraints and default risks from the following three aspects:

1. How might slot purchase contracts be structured to be more to be more effective in cooperating with NVOCC and conducive to liner company's repayment under the constraints of financial leasing and default costs?
2. How can liner companies and financial institutions effectively balance the operational risk associated with slot purchase contracts and the financial risk associated with financial lease contracts while market circumstances continue to fluctuate?
3. In the case of continuous market fluctuations, how should liner companies deal with the collaboration with NVOCCs if it does not have sufficient shipping

capacity and whether it can be more conducive to the balance of operation and financial leasing than liner companies with sufficient shipping capacity?

In order to answer the questions above, a theoretical model is developed to quantify the interaction of financial lease contracts and slot purchase contracts of various contract types under different scenarios.

For the first question of this thesis, we initially investigate the optimal contract parameters of slot purchase contracts under different contract models. By evaluating the disparities in profits and default costs under different contract models, the contract type with the highest profit and the lowest default cost is the contract that is most conducive to repayment. In the first essay, it should be noted that the operating cost of the liner company and NVOCC are generated according to their holding shipping capacity, and the goodwill loss is solely attributable to their respective shortfalls. Since they make decisions based on their own intentions, the contract cannot achieve optimal equilibrium unless the liner company shares the direct selling profit with the NVOCC.

Subsequently, in the second essay, the liner company's direct selling profits can be extracted from the slot purchase contract design by involving the liner company in sharing some of NVOCC's operating costs and goodwill losses. Then, the liner company could preserve any additional income from direct sales. To answer this thesis's second question, we initially researched the ideal size of the finance lease and then obtained the financial lease rent at this time. After making a comparison to the profit generated from the improved contract that was previously mentioned, the conditions for the liner company to achieve a balance between the operating and financing are accomplished. In light of the above calculation results, the slot purchase contract and the financial lease contract are both amenable to flexible modification by inspecting the developing pattern of the equilibrium condition in response to fluctuating market parameters.

To answer the third question of this thesis, we initially designed the optimal contract parameters of slot purchase contracts for liner companies with insufficient shipping capacity. By comparison, it is evident that liner companies with insufficient capacity cannot achieve the same level of profits as those with sufficient capacity because they must pay an extra cost to acquire the capacity to fulfil the order volume of the slot purchase contract. However, due to the small scale of financial leasing, liner companies with insufficient capacity will also face less financial lease rent. To assess the financial and operational stability between these two types of liner companies, a prospect theory-based credit rating was established in the third essay.

Our investigation resulted that for liner companies to achieve the most effective operating and financial leasing methods, they should primarily compare the profitability based on the size of the financial lease rent that is payable, even if different situations may apply. Furthermore, profit under stochastic demand and profit under deterministic demand differ significantly among different scenarios. Therefore, the forecast results under stochastic demand may deviate from results when deterministic demand is actually met. Therefore, in many situations, we take into account the worst-case scenario for the liner company, even though there is quite a low chance of it happening. In most instances, the actual findings do not depart from the projected outcomes by an excessive amount.

Chapter 2. Literature Review

2.1 Theoretical Foundations for Supply Chain Contract Design

The effectiveness of supply chain management relies on efficient coordination between different stakeholders, each following their individual intentions while jointly aiming for optimal performance. Therefore, supply chain contracts play a crucial role in adjusting strategies, managing risks, and enhancing collaboration among partners. This chapter lays the groundwork by introducing the fundamental concepts of supply chain contracts and coordination. We delve into the principles of game theory and the Newsvendor model, which provide valuable insights into decision-making under uncertainty and risk. Additionally, we explore the taxonomy of supply chain contracts, showcasing various contract types and their implications on supply chain performance. By establishing these theoretical foundations, we aim to equip readers with a comprehensive understanding of the key elements that support effective supply chain contract design.

2.1.1 Game theory and Newsvendor model

This section will start with an overview of game theory and its key concepts, including Nash equilibrium and Pareto efficiency. Next, the newsvendor model, a classic model that employs game theory, will be discussed. Since the newsvendor model is often used in supply chain management to determine optimal inventory levels, it is highly applicable for liner companies to forecast and manage the scale of vessels.

Since Von Neumann proposed the basic principle of game theory in 1928 and extended the two-player game to the n-player game structure in another work with Oskar Morgenstern in 1953, the mechanism of game theory has been disciplined and widely appreciated in the field, from the economic field to all fields of social science. From an economics and business perspective, game theory is a powerful tool to analyse strategic decision-making among several stakeholders. It is based on the terms that at least two players join the game and choose from a set of strategies. The outcome of each choice is the payoff to each player, and there is no randomness when making choices (Neumann and Morgenstern, 2007). When there is complete information, which means there is no information asymmetry in the decision system, the game theory can be typically divided into two types, namely the static game and the dynamic game (Gibbons, 1992). Under the static game, every player makes their decision simultaneously in a round of the game. The Prisoner's Dilemma, framed by Merrill and

Melvin in 1950 at an experiment, is a classic example of a normal-form game with complete information. In this gamble, the two prisoners choose the confession strategy to benefit themselves instead of not confessing (Mérő, 1998). John Forbes Nash Jr. used the fixed-point theorem to prove the existence of equilibrium points and presented the concept of Nash equilibrium and the equilibrium existence theorem from 1950 to 1951. The equilibrium point in this prisoner's dilemma is the Nash equilibrium, which is the outcome that none of the players could improve the payoff by changing their choice unilaterally. The outcome called Pareto's efficiency is another outcome where resources are efficiently allocated and not allocated to improve the outcome of one player by making the other players worse off. Therefore, the equilibrium point in this prisoner's dilemma is not Pareto's efficiency since the players benefit themselves by betraying others. When the selection sequence is in a dynamic game, the later player can make a choice based on the previous player's decision, which is different from the normal form of the game. The representative dynamic game, which was proposed by Von Stackelberg in 1934 and is called the Stackelberg Game, shows the strategies for each party in the game that a follower company make the decision after a leader company. This game of strategic interaction emphasises that the players must consider the decisions of their opponents when making their own (Von Stackelberg, 2010). This dynamic gamble is often applied to the trading process, such as the newsvendor model, where companies must make decisions based on the choices of other companies.

Under the structure of the Stackelberg Game, the newsvendor model provides a mathematical framework for supply chain management to evaluate the decision-making process of a single-period supply chain. Through the concept of decision-making under uncertainty, all parties involved must make their decision before knowing how many newspapers will be sold the next day. This uncertainty creates a risk of stock-outs, which can result in lost sales and potential reputation damage. At the same time, ordering too many products leads to carrying costs, which can be expensive. Therefore, this model could capture the trade-offs between inventory costs and stock-out costs, making it a valuable tool for understanding supply chain risk management. There will be a simple agreement between the supplier and the retailer to connect stakeholders, which is the supply chain contract. Through regulation, the outcome of this contract gamble could reach the Nash equilibrium point, which is also the Pareto efficiency point for the whole supply chain (Lariviere, 1999). This contract is not to force the companies to choose the Pareto efficiency point but to restructure the payoff outcome for each strategy to make the Nash equilibrium also a Pareto efficiency point (Snyder et al., 2019). This is the main idea for supply chain contract design in the newsvendor model, which will be introduced in detail in the next section.

2.1.2 Supply Chain Contracts and Coordination

The starting point for analysing these supply chain contracts is the basic one-period newsvendor setting, in which there is a Stackelberg Game between supplier and retailer. The early literature predominantly focused on contract design for the contracted game-theoretic model, which mainly used contracts to facilitate cooperation in the supply chain (Höhn, 2010). Through proper information and incentive provisions, the contract could optimise supply chain performance, which is also known as coordination between stakeholders. Since the mid-1990s, a great number of studies have researched the mechanisms of coordination inside the whole supply chain. In the past works of literature, the main measurement of the coordination of contract is the whether the profit in the decentralised supply chain could be maximised under the same order quantity as in the centralised chain (Höhn, 2010). In the decentralised model, stakeholders execute independent decisions separately but are bound by a set-up contract. The centralised model illustrates an ideal form of collaboration among stakeholders when the supplier and retailer are treated as a virtual entity. Since numerous studies have conducted literature reviews about contract negotiation and supply chain coordination with contracts in the newsvendor model (Cachon, 2003; Höhn, 2010; Liu et al., 2015; Guo et al., 2017; Bart et al., 2020), supply chain contracts, as a critical part of modern business management, could provide a mechanism for coordinating the actions of different companies within the newsvendor model.

In literature, the taxonomy of supply chain contracts could be divided into wholesale price contracts (Bresnahan and Reiss, 1985), buyback contracts (Pasternack, 1985), revenue-sharing contracts (Cachon and Lariviere, 2005), quantity-flexibility contracts (Tsay, 1999), sales-rebate contracts (Taylor, 2002) and quantity-discount contracts (Moorthy, 1987). According to Guo et al. (2017), buyback contracts, revenue-sharing contracts, and quantity-discount contracts are the most widely used contracts in the study of single supply chain contracts. Our paper focuses on the research behind these three contract types. Basically, the buyback contract refers to the retailer acquiring the items at wholesale prices but receiving a partial payback on unsold stock (Pasternack, 1985). Cachon and Lariviere's revenue sharing contract (2005) requires retailers to split a portion of their sales income with the supplier to coordinate the entire chain. The quantity discount contract organises the chain by selling all goods at a reduced wholesale price to the retailer (Moorthy, 1987).

The primary objective of these contracts is to determine the parameters that enable each party to make optimal decisions in order to enhance the performance of the supply chain (Xiong et al., 2011). However, when applying these contracts to different situations, these contracts might be expressed in diverse ways (Höhn, 2010). With the development of globalisation and outsourcing, some decentralised supply chains, like outsourcing of production, are widely spread, forcing the companies to decentralise decision power to coordinate the whole supply chain, which is a significant assessment to measure the performance of supply chains.

2.2 Contract Design and Comparison under Constraints and Preference

Within the dynamic and evolving global economic environment, the design of contracts stands a crucial role in influencing collaboration, risk mitigation, and performance optimization. When stakeholders negotiate in complex environments with limitations and distinct preferences, such as the shipping industry in this thesis, the contract design becomes more intricate and essential. This chapter focuses on the relationship between contract design and the interplay of constraints and preference considerations. We explore the challenges posed by financial constraints and the strategies that can be employed to incorporate financial considerations into contract design. Furthermore, we conduct a comparative analysis of different contract types, evaluating their performance under various financial constraints. We also examine prospect theory and behavioural operations management to enrich the understanding of behavioural biases that influence contract design decisions. This chapter aims to provide valuable insights into how financial constraints and preference can be navigated while designing effective supply chain contracts.

2.2.1 Contract Design under Financial Constraints

Many studies have illustrated the coordination condition of supply chain contracts for the supply chain system with limited funds. Recent research in this segment mainly addressed the problem of coordinating the financial and operational decisions in the newsvendor model. Yan et al. (2014) found the optimal strategy for the supply chain system with a manufacturer, a retailer, and a commercial bank where the retailer and the manufacturer cooperate via a wholesale price contract, and both are capital constrained. By considering the risk faced by financing participants, Huang et al. (2020) set up a supply chain finance framework based on a general supply chain contract. The effect of capital constraint on the decision of contract parameters has garnered a great deal of attention in academia. Xiao et al. (2017) demonstrated that the all-unit discount contract fails to coordinate the financially constrained newsvendor model, while the revenue-sharing and buyback contracts can facilitate coordination only when the supply chain has adequate total working capital. Nocke and Thanassoulis (2014) examined the effects of credit constraints and market risk on optimal contracts. Considering the difference between the credit ratings of suppliers and retailers, Kouvelis and Zhao (2018) examined the interaction of the financial decision and operational decision via the early payment discount contract in a newsvendor model.

Many other investigations have also considered contract setting, including the financial return of the contract, economic conditions, financial circumstances, and the specified industry or commercial products. Zhou and Groenevelt (2008) measured a three-parties supply chain constrained by asset financing, considering that suppliers could help the retailers repay the loan interest without an early payment discount in trade credit. Kouvelis and Zhao (2008) researched the trading coordination contract for a supply chain, including a retailer and a supplier under financing. In their model, the supplier plays a positive role in providing the trade credit contract to help the retailer. They also researched the optimal order quantity under a wholesale price contract based on the assumption that additional costs are incurred due to the default on the loan contract (Kouvelis and Zhao, 2011). Jiang and Liu (2018) analysed a supply chain that includes an overconfident supplier, a retailer and a bank. They found that this overconfidence pushes these three parties into making irrational decisions; however, the bank should keep a positive attitude toward this overconfidence.

Some other studies also investigated the financial strategies of suppliers or retailers (Yan and Sun, 2013; He, 2017; Yang et al., 2018; Liu and Wang, 2019; Lai et al., 2021). Yang et al. (2018) intend to present a deeper understanding of how trade credit improves supply chain efficiency by encouraging the retailer to partially share the supplier's demand risk. Lai et al. (2021) investigated two financing methods (credit finance and supplier green investment) for a supplier-manufacturer green supply chain with cost-sharing and quantity discount contracts. The impact of the business environment on supply chain finance has also received substantial interest (Holzhacker et al., 2015; Basu et al., 2019; Lin and Xiao, 2018; Li et al., 2021). Basu et al. (2019) study the issue of hedging demand uncertainty in a supply chain consisting of a risk-neutral supplier and a risk-averse retailer under a buyback contract. Lin and Xiao (2018) study the credit guarantee scheme used in a supply chain finance system, including a manufacturer with capital constraint, a retailer, and a bank in the competitive credit market.

Based on these leading studies, the effect of financial constraints on the decision within a supply chain contract has garnered a great deal of attention in academia. However, few of them compare different contracts under such conditions, which will be introduced in next subsection.

2.2.2 Contract Comparison under Financial Constraints

Based on above leading studies, the effect of financial constraints on the decision within a supply chain contract has garnered a great deal of attention in academia. However, few of them compare different contract types under such conditions, which will be introduced in this section. Kouvelis and Zhao (2008) researched the financing problem of the supply chain of a supplier trading with a retailer via the supply chain contract. In their model, they compared how bank financing, trade credits and quantity discount contracts affected the profit of both the retailer and the supplier. Kouvelis and Zhao (2011) found unique equilibrium solutions for the newsvendor model to adopt wholesale price contracts when the retailer is under bankruptcy risk. They also structured a contract in which the supplier offers a discount on the wholesale price if the retailer pays early but charges interest if the retailer pays late (Kouvelis and Zhao, 2012). Based on all these works, Kouvelis and Zhao (2016) compared several contract types between a supplier and a retailer, both of whom faces financial constraints from short-term borrowing. They also pointed out that most previous works only consider contract coordination but do not consider the impact of debt issues. Our work is closely related to this Kouvelis and Zhao's work in 2016. Xiao et al. (2017) demonstrated that the all-unit discount contract fails to coordinate the financially constrained newsvendor model, while the revenue-sharing and buyback contracts can facilitate coordination only when the supply chain has adequate total working capital. However, regarding the comparison of contracts in the shipping industry, only Wang and Liu (2019) compared a revenue-sharing contract and a wholesale contract for two parallel competing shipping service chains. Our work will fill this gap.

2.2.3 Prospect Theory and Behavioural Operations Management

Prospect theory, initially presented by Kahneman and Tversky in 1979, shows investors value losses more than gains in times of uncertainty. They think investors tend to change their decisions when the expression of choice changes (isolation effect) and are risk-averse when facing gains (certainty effect), but risk-seeking when facing losses (reflection effects). Since their experiment only considered one outcome at a time and the data were not selected randomly, it has two issues. One is that it does not satisfy stochastic dominance and the other is that it cannot be extended to multiple outcomes. In this case, Quiggin (1982) and Schmeidler (1989) thought expected utility would solve the first problem, while Tversky and Kahneman (1992) also set up cumulative prospect theory to solve both problems. Since then, a significant amount of more recent research has examined how irrational decision-makers enforce contracts differently. Schweitzer and Cachon (2000) proposed models based on loss aversion and prospect theory to describe decision bias in the experiment and found that the pattern of order bias does not fit the certainty effect or the reflection effect proposed by prospect theory. Considering this, Ren and Croson (2013) provided two experiments to support the result of a theoretical model that demonstrates underestimating demand variance causes actual orders to differ from the predictable optimal outcome. Surti et al. (2020) also conducted two experiments to highlight the significance of the reference point in defining newsvendor behaviour using the prospect theory model.

Supported by the experimental works presented above, the majority of behavioural research involving prospect theory in operations management is to investigate the impact of irrational preferences on the contract parameter decisions of suppliers and retailers in response to different contract structures in the newsvendor problem. Zhang et al. (2016) indicated that loss-averse suppliers prefer the buyback contract in the chain with low-critical-ratio products and the revenue sharing contract with high-critical-ratio products. Here, the critical ratio indicates the underlying newsvendor model at the channel level. Wu et al. (2018) investigated a loss-averse competitive newsvendor model and discovered that competition causes newsvendors to place more orders but may not result in a loss of profits. Lond and Nasiry (2015) suggested that zero payoffs should not be employed as a reference point and proposed a reasonable reference point for the newsvendor model to match the previous experimental results. Uppari and Hasija (2019) examined the results of several prospect theory-based newsvendor models under the expected profit maximization assumption. Based on the stochastic reference point, Vipin and Amit (2021) showed the non-linear order behaviour in the newsvendor model. They found that the wholesale price contract might outperform than buyback contract when involving a behavioural retailer in the traditional newsvendor model. In addition, under these conditions, the wholesale price contract could coordinate the chain while the buyback contract fails to.

Although prospect theory is widely applied in the newsvendor model to demonstrate decision bias, very little research has used it to assess financial risk following biased decisions. Accordingly, we attempt to fill this gap by simulating a financial institution’s willingness to offer finance leases through prospect theory. Because the original model by Tversky and Kahneman (1992) utilised the log-concave cumulative distribution functions and the power utility functions, which make the function curves of decision-makers’ willingness nonlinear and complex. All the above papers simplify the expected utility function into a two-piece piecewise linear utility function and use the concept of loss aversion based on the heart of prospect theory. In this setting, we form a four-piece piecewise linear credit rating function to present the function curves of the financial institution's willingness toward a financial lease.

In the context of overconfidence, Moore and Healy (2008) reviewed over 350 works and analysed different examples of overconfidence behaviour, including overestimation, over-placement and over-precision. Following Moore and Healy (2008), we specify the overconfidence behaviours of liner companies in our paper as over-placement. Specifically, these liner companies are overconfident that expanding the scale of shipping can capture more market shares and increase competitiveness. In contrast, we define the liner company with under-placement behaviour as an over-conservative liner company.

2.3 Application of Contract Design in the Container Shipping Industry

As containerized trade continues to grow, the significance of well-designed contracts becomes increasingly evident in addressing the unique challenges and opportunities faced by the container shipping sector. In this chapter, we turn our attention to the application of contract design in the container shipping industry. We examine the diverse contract types commonly used by carriers, shippers, freight forwarders, and NVOCCs to structure their engagements. Additionally, we explore how limited inventory and its associated risks impact contract design decisions within the shipping industry. Through these explorations, we attempt to highlight the critical role that effective contract design plays in optimising container shipping operations in a dynamic and competitive environment.

2.3.1 Container Shipping Contracts

Within the domain of container shipping contract design, numerous studies have integrated supply chain contract mechanisms to construct container shipping agreements that guarantee fair resource distribution and efficiently coordinate container supply chains. Song (2021) outlined that most of the literature is mainly about calculating the precise contract parameters for a given contract type.

In terms of buyback contracts, Xie et al. (2017) developed a bilateral buyback agreement for rail and liner companies to exchange empty containers. Kong et al. (2017) employed the buyback contract to optimise the slot amount supplied by liner firms and the quantity booked by shipping agencies. Regarding revenue sharing contracts, Wang and Liu (2019) compared two competing shipping service chains, each with a single carrier and a single port. They identified that when both chains choose different contracts, the winner is the party that chooses the revenue-sharing contract. However, a lose-lose situation will emerge if both chains select revenue-sharing agreements. Tan et al. (2018) compared the competition between an ocean carrier and an inland shipping company with their cooperation under a revenue-sharing contract. Zhang et al. (2019) developed a revenue-sharing contract for logistics service providers to promote horizontal logistics collaboration in a decentralised model and coordinate the system. Li et al. (2013) constructed a bidirectional revenue sharing contract to solve the preventative lateral transport problem between two locations. Liu et al. (2013) proposed the fairest revenue-sharing contract strategy for the chain consisting of logistics service integrators and functional logistics service providers. Wang et al. (2017) considered the effect of the canvassing strategy on the shipping service supply chain formed by an ocean shipping (OS) company and an inland shipping (IS) company and the revenue-sharing rate between them.

Regarding quantity discount contracts, Song et al. (2019) built the model with quantity discount contracts between a liner company and a forwarder and illustrated how the profit of each party and order quantity change with a canvassing strategy. Cai et al. (2013) identified the optimal decision for a fresh product supply chain that consists of a producer, a distributor, and a third-party logistics operator. In this supply chain, a wholesale-market contract was applied between the producer and the distributor, whereas another quantity discount contract was adopted for the producer and the third-party logistics provider. Wang et al. (2021) compared the coordination conditions of the quantity discount contract with the two-part tariff contract in the carrier-shipper chain. Song et al. (2017) proposed a modified quantity discount contract for an online retailer and a delivery operator. Yin and Kim (2012) measured the optimal quantity discount price of the container shipping company when trading with several freight forwarders. Qiu and Lee (2019) built a model wherein shippers export their cargo worldwide through seaports. To reduce transportation costs, dry port systems are used to connect shippers and seaports. They designed a quantity-discount contract for dry ports to trade with multiple shippers.

Apart from these three specific contracts, several other researchers have investigated the design and application of other types of supply chain contracts. For example, Xu et al. (2015) designed a subsidy contract for the sea cargo service chain to reposition empty containers between one carrier and two freight forwarders across two ports. Song et al. (2017) compared two canvassing tactics for carriers, which involved freight forwarders or NVOCCs. In their model, general supply chain contracts are used between NVOCCs and carriers. The existing literature on container shipping contract design has primarily formulated contracts based on contract coordination theory, with little consideration given to the impact of funds and inventories (for example, vessels and containers) constraints on contract decisions.

2.3.2 Limited Inventory and Contract Design

In the shipping sector, the literature on limited inventories can be divided into two segments: inventory financing and inventory management. The first section demonstrates the effect of financial decisions regarding fleet development (including leasing schemes, shipping funds, shipbuilding credit, and so forth) on the operation of shipping companies. The literature on this area is vast. But most of them are empirical research. Examples include Gómez-Padilla et al. (2021) and Cariou and Wolff (2013). Our paper falls into the second category, focusing on matching the inventory level with uncertain demand. Liu et al. (2015) studied a two-stage batch ordering strategy for the logistics service integrator to satisfy the updated demand. Feng et al. (2015) proposed a tying mechanism to allocate air cargo capacity reasonably. Song et al. (2017) compared two canvassing strategies to enable a carrier to resist variability in market demand. Xie et al. (2017) developed the empty container inventory-sharing strategy for an intermodal transport system to meet the demand for empty containers. None of the above papers linked the issue of financial risk and inventory risk with contract coordination, and we explicitly explore this issue in addition to contract comparison.

2.4 Research gaps and Opportunities

Through the above review, this chapter create a comprehensive depiction of supply chain contract design covering theoretical foundations and a comprehensive account of practical implementation, focusing on aligning conflicting objectives, risk mitigation, and performance optimization. The integration of theoretical frameworks such as game theory, newspaper vendor model, and behavioural preferences seamlessly combined with various practical situations in the shipping industry can effectively provide effective solutions to previous research questions.

First and foremost, the research revolves around the development of supply chain contract design, which plays a crucial role in enabling collaboration and managing operational risks. This is also the first barrier that the shipping sector will need to overcome before expanding more in-depth. Although the theoretical model was established by the structuring of economic models, such as game theory and the Newsvendor model, it was the strategic selection and application of these methods that enabled stakeholders in the shipping industry to gain significant insight into the decision-making process and the factors that influence it. Our further investigation of the container shipping contract indicates that the previous research has shifted from theoretical exploration to practical application. Besides, numerous research efforts in the field have provided support for the investigation of limited inventory studies and their intricate interplay with contract design, which suggests that the current research trends encompass both the practical implementation of the contract and methodological innovation. Therefore, our research into the preference in operations management when designing contracts could offer practical insights for irrational stakeholders in a dynamic and competitive environment.

As we look through the above publications, none of them has compared different contract types in the field of container shipping supply chain and taken into account default behaviour due to financial constraints. Regarding the comparison of contracts in the shipping industry, only Wang and Liu (2019) compared a revenue-sharing contract and a wholesale contract for two parallel competing shipping service chains. Further, most of them researched capital-constrained small and medium-sized businesses in need of a loan or trade credit for their operations. These companies are characterised by having poor access to financial leases, whilst it is one of the main financing sources for liner companies to get operating control of vessels. In addition, these companies use capital-based financing methods, whereas financial leasing is a financing method based on high-quality credit ratings. Considering the particularity of liner companies, our research also highlights that shipping capacity has no residual value after limited life, which is different from the specified products in these previous works. Furthermore, traditional supply chain contract modelling is mainly aimed at maximizing profits but lacks a demonstration of how biased orders in the face of increasing market competition affect the relative performance of these contracts from a financial point of view. In the Chapter 5, we also weaken the contract decision and focus more on the influence that the decision of liner companies (as a supplier) and financial institutions will have on future operations and financing.

Chapter 3. The Impacts of Financial Lease Constraints on the Design of Container Shipping Contracts

3.1 Introduction

Since the introduction of containerisation to maritime transport and the rapid development of economic globalisation, the liner shipping industry has experienced fast and continuous growth (Song, 2021). Nowadays, approximately 50% of the total value of global trade is shipped by liner vessels (Brooks and Faust, 2018). Liner companies operate vessels and containers between fixed geographical locations with regularly scheduled services. The main goal for liner companies is to achieve higher volumes of customer orders and, therefore, higher profits. To achieve the target, liner companies must put more effort into canvassing orders and managing vessels effectively.

According to Wang et al., (2017), the canvassing strategy that liner companies adopt in the dynamic and competitive shipping market involves collaborating with non-vessel-operating common carriers (hereafter NVOCCs) (Wang et al., 2017). By leveraging the expertise of NVOCCs, liner companies can assess a range of benefits, including expanded market coverage, enhanced operational efficiency, and improved cargo allocation. Therefore, the implementation of NVOCCs not only can enhance financial benefits but also contribute to the overall growth and effectiveness of liner company operations. The employed NVOCCs serve as wholesalers but do not fully operate vessels (Song et al., 2017). They reserve blocks of container slots from liner companies through slot purchase contracts and sell them to shippers at the market price. Although liner companies and NVOCCs make their own decisions separately, their decisions are constrained by slot purchase contracts, in which the liner companies specify the total amount of container capacity (measured in the number of TEUs) to be sold and the corresponding payment parameters. NVOCCs decide the order quantity and other contract parameters based on the market conditions. Their contracts usually take one of the following three forms: revenue-sharing contract, buyback contract or quantity discount contract (Snyder and Shen, 2019). The design of the slot purchase contract plays a significant role in the canvassing tactics and the solvency of liner companies (Song et al., 2017).

Concerning the acquisition of vessel capacity, more and more liner companies rely on financial entities (i.e., banks) to finance the lease of vessels rather than buying them. Cariou and Wolff (2013) have outlined the data from the top 100 liner companies in 2011 and found that around 50% of the container ships used in the shipping industry were chartered. However, the use of a finance lease still carries a risk of default, which may cause default costs when the operating income of liner companies is lower than the required rental fee in the lease. At each accounting period, liner companies must make a series of decisions about vessel leasing. The main factor influencing the decision-making is the repayment of finance lease, which is subject to liner companies' operating incomes. Further, the number and the size of vessels leased from financial institutions greatly affect the parameters in the slot purchase contract. Therefore, there is a high dependency on the decisions around financial leasing contracts (between liner companies and financial institutions) and slot purchasing contracts (between liner companies and NVOCCs).

In the literature, similar topics have been addressed in a few studies (Zhou and Groenevelt, 2008; Kouvelis and Zhao, 2011; Kouvelis and Zhao, 2016; Jiang and Liu, 2018). However, most of them neglect the financial institution's effect on the debtor's decision and do not account for peculiar elements in the container shipping supply chain (e.g., no salvage value for the shipping capacity). Only Jiang and Liu (2018) use the loan-to-value ratio as a metric for banks to decide the amount of loan they would like to offer to suppliers. However, their loan-to-value ratio is measured after the supply chain contract. Besides, the study does not compare multiple contracts from the view of their ability to guarantee repayment. Though Kouvelis and Zhao (2016) compared the impact of capital constraints on three contracts from the perspective of default costs, this work did not provide the optimal contract settings and neglected goodwill losses and residual values for products in order to simplify the model. Finally, they all concentrate on borrowing money or cash, which differs from the financial leasing studied in this paper. Financial leasing is a more prevalent financing form in container shipping chains, with the financial institution providing funds to support liner companies to obtain operational control of vessels. Unlike loans, there is no cash transfer from financial institutions to liner companies in financial leases, and assets are not required as collateral. The key to applying for financing leases is maintaining an excellent credit rating. Most importantly, the liner companies can only have the ownership of vessels at the end of the finance lease term. Therefore, we underline in this research that the design of slot purchase contract needs to meet the important conditions of guaranteed repayment to emphasise the binding force of financial leasing on the slot purchase contract.

In this study, we develop a theoretical model to identify the best contractual setting which is beneficial to the repayment to financial institutions and the effective cooperation between liner companies and NVOCCs through slot purchasing contracts. In designing the slot purchase contracts, we will aim to coordinate the container shipping chain to ensure participants act in a way that maximises the profit of the whole chain. Although the three researched contract types use similar adjustment strategies on wholesale prices according to market demand to achieve coordination (Höhn, 2010), their profitability and repayment capacities vary even when confronted with the identical financial leasing contract. While providing the optimal contract parameters for each contract type, we found that the guiding effect of profitability on repayment ability was invalid under certain circumstances.

This paper is structured as follows. In Section 2, we will review the related literature. Section 3 describes the model and the associated assumptions. Section 4 will briefly present the profit functions of liner companies and NVOCCs and examine the case of coordination under each contract type before the repayment. Secondly, we will compare the profitability of liner companies and NVOCCs under different contract types. In Section 5, we will compute the income of financial institutions, default cost and optimal loan-to-value ratio and the effect of financial lease on the choice of contract types. In Section 6, numerical examples demonstrate how the profits of the liner company, the NVOCC and the financial institution change according to contract types and related parameters. Finally, Section 7 closes the paper with a summary of the results and some future research directions.

3.2 Literature Review

This section will provide an overview of extant studies on contract design and supply chain coordination. We will highlight previous results on the contract comparison and design under different constraints in different industries.

3.2.1 Contract Design and Application

Several contract types regulating supply chains have been defined in the past. These contracts mainly focus on the relationship between suppliers and retailers. Cachon and Netessine (2004) examined the application of game theory to supply chain contract negotiation processes and proved the existence of a unique equilibrium point in the newsvendor model, where suppliers trade with retailers under a contract. Pasternack (1985) designed a buyback contract based on the idea that it might be better to relate the profit of a retailer with the profit of a supplier in the contract. Under this contract, a retailer purchases the products at wholesale price but obtains a partial refund of the unsold inventories. Similarly, the revenue-sharing contract, proposed by Cachon and Lariviere (2005), requires a retailer to pay a supplier a wholesale price for purchased units and share a portion of the revenue. Finally, a quantity discount contract generally coordinates a supply chain by trading all products to retailers at a discounted wholesale price (Moorthy, 1987). Höhn (2010) reviewed the literature on contract negotiation and supply chain coordination with contracts, concluding that wholesale price contracts (Bresnahan and Reiss, 1985) fail to achieve coordination in the newsvendor model, while buyback contracts (Pasternack, 1985), revenue sharing contracts (Cachon and Lariviere, 2005) and quantity discount contracts (Moorthy, 1987) can. From a profit perspective, Cachon and Lariviere (2005) stated that the revenue-sharing contract is equivalent to the buyback contract in the newsvendor model.

In the domain of container shipping contract design, several types of contracts have been proposed to achieve collaborations. Firstly, regarding buyback contracts, Xu et al. (2015) examined sea cargo service chains between a carrier and two freight forwarders at two ports. They considered a type of buyback contract and analysed how this contract influences empty equipment decisions. Song et al. (2017) studied carriers' canvassing tactics involving freight forwarders or NVOCCs. A commission fee-based contract and a buyback contract were used to compare the advantages and disadvantages of hiring freight forwarders over NVOCCs. Xie et al. (2017) designed a buyback contract for rail and liner firms to share empty containers. Kong et al. (2017) used a buyback contract to optimally coordinate slot purchasing quantity between liner companies supply and shipping agencies. Second, regarding the applications of quantity discount contracts, Yin and Kim (2012) measured the optimal quantity discount price of container shipping companies when they trade with several freight forwarders. Cai et al. (2013) identified the optimal decision for a fresh product supply chain that consists of a producer, a distributor, and a third-party logistics operator. In this supply chain, a wholesale-market contract was applied between the producer and the distributor, whereas another quantity discount contract was adopted for the producer and the third-party logistics provider. Yin et al. (2019) presented a genetic algorithm to show quantity discount schemes in multi-leg network services. Song et al. (2019) examined how a quantity-discount contract is affected by a canvassing strategy of a shipping supply chain involving a liner company and a forwarder at two ports. Qiu and Lee (2019) studied a model wherein shippers export their cargo worldwide through seaports. To reduce transportation costs, dry port systems are used to connect shippers and seaports. They designed a quantity-discount contract for dry ports to trade with multiple shippers. Lastly, regarding the applications of revenue-sharing contracts in transportation, Li et al. (2013) analysed how preventive lateral transhipments affect order quantities in bidirectional contracts. Liu et al. (2013) proposed a revenue-sharing contract to coordinate a supply chain which consists of a logistics service integrator and a functional logistics service provider. Tan et al. (2018) investigated the competition between an ocean carrier and an inland shipping company when cooperating under an excess revenue-sharing contract. Wang and Liu (2019) compared two parallel competing shipping service chains, each involving a carrier and a port. They found that the "winner" that can obtain the highest profit was due to using a revenue-sharing contract. But a lose-lose situation appears when both chains adopt a revenue-sharing contract. However, most of the literature in these three contract areas mentioned above considers only a single type of contract when examining the collaboration problem in container shipping chains. They focus more on the design and implementation of a single type of contract and do not investigate the possible relative implications of applying multiple types of contracts.

3.2.2 Contract Comparison under Financial Constraints

When comparing several types of contracts, some other works have considered the impact of financing issues on contract design. Zhou and Groenevelt (2008) measured a three-parties supply chain constrained by asset financing, considering that suppliers could help the retailers repay the loan interest without an early payment discount in trade credit. Jiang and Liu (2018) analysed a supply chain that includes an overconfident supplier, a retailer and a bank. They found that this overconfidence pushes these three parties into making irrational decisions; however, the bank should keep a positive attitude toward this overconfidence. Kouvelis and Zhao (2008) researched the trading coordination contract for a supply chain, including a retailer and a supplier under financing. In their model, the supplier plays a positive role in providing the trade credit contract to help the retailer. They also researched the optimal order quantity under a wholesale price contract based on the assumption that additional costs are incurred due to the default on the loan contract (Kouvelis and Zhao, 2011). Kouvelis and Zhao (2016) compared several contract types between a supplier and a retailer, both of whom faces financial constraints from short-term borrowing. They also pointed out that most previous works only consider contract coordination but do not consider the impact of debt issues. Regarding the comparison of contracts in the shipping industry, only Wang and Liu (2019) compared a revenue-sharing contract and a wholesale contract for two parallel competing shipping service chains.

To the best of our knowledge, no studies have compared different contract types in the field of container shipping supply chain and taken into account default behaviour due to financial constraints. Further, the existing studies involving financing issues and the coordination of generic supply chains mainly focus on loans. Essentially, our paper extends the work of Kouvelis and Zhao (2016) by evaluating the performance of slot-purchasing contracts in container shipping supply chains, considering an interest rate greater than 0 and an industry-specific financing method and product. Secondly, the financial institutions in this study use a book-to-market ratio, also called a loan-to-value ratio in the financing industry, as an indicator to determine whether to invest in financial leasing contracts, whereas Jiang and Liu (2018) used a similar setting to measure short-term borrowing from bank to retailer. In this paper, however, it is the liner company (as suppliers) that needs financial leasing support from financial institutions. Lastly, the collaboration between liner companies and NVOCCs will include the direct sales channels of liner companies, which have not been taken into account in previous studies.

3.3 The Model

In this section, we first describe our problem and main assumptions. Next, we formulate the objective functions of the stakeholders involved. Notations to be used for the model are summarised in Table 3.1. To better illustrate, we will use the subscripts ‘c’, ‘n’ and ‘l’ to denote the centralised company, NVOCC and the liner company, respectively. The subscript and superscript ‘d’ both stand for the decentralised model.

Parameters

d	The market demand quantity
R	Financial lease interest rate
B	Unit default cost incurred when liner company could not repay the loan $B = \alpha\xi + \beta$ (ξ is the profit before repayment, see variable for details)
α	Variable default cost rate to the profit before repayment
β	Fixed default cost
L	The cost for the financial institution to support the finance lease for each accounting period
Q	The capacity that liner company get from financially leased vessels
q_0	The initial capacity that liner company has from owned vessels
ω	Loan-to-value ratio
P_r	The freight rate paid by shippers
c	Unit operating cost
g	Unit goodwill loss
μ	= E(d), the expected value of demands
Variables	
$x(x_c, x_n^d, x_l^d)$	The order quantity of NVOCC, x ∈ [0, $Q + q_0$].
P_s	Contract parameter, freight fee paid from the NVOCC to the liner company
$\theta(\theta_R^*, \theta_B^*, \theta_Q^*)$	Contract parameter, the NVOCC's revenue share from the sales
T	Contract parameter, fixed money transferred from the liner company to the NVOCC if T ＞0
π_N, π_L, π_c	The profit function of NVOCC, liner company and centralised company
ξ_L, ξ_c	Profit before repayment of liner company in decentralised model and centralised company
Δ_d, Δ_c	The repayment from the liner company to the financial institution in decentralised model and centralised company
δ_d, δ_c	The income of financial institution in decentralised model and centralised company ($\delta \leq \Delta$)

Table 3.1 Notations of Chapter 3

3.3.1 Problem description

We study a stylised container shipping supply chain consisting of a financial institution, a liner company, an NVOCC and a group of shippers. The sequence of the events occurring in the model is depicted in Figure 3.1. First, at the beginning of each accounting period, the liner company consider whether or not to renew a successive vessel finance lease contract with the financial institution. Second, the liner company signs a slot purchase contract with the NVOCC. Third, from the effective date of the slot purchasing contract, the NVOCC canvasses orders. After the canvassing is completed, the liner company provides services to shippers and completes all transportation tasks. Fourth, the liner company and the NVOCC clear the payment as specified in the preceding slot purchase contract. Finally, the liner company repays the rental obligations to the financial institution. The default cost may be incurred when the liner company cannot pay the full rental at the end of each accounting period.

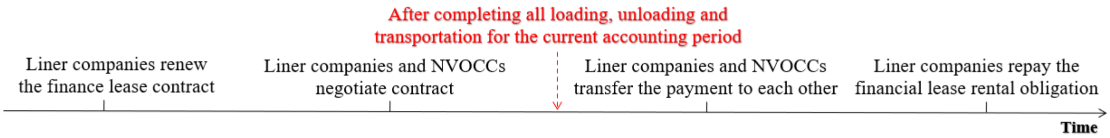

Figure 3.1 The Model Timeline

We denote the quantity of future demand from shippers as *d*, which is a stochastic variable measured with TEUs (The Twenty-foot Equivalent Unit) with a Probability Density Function (*PDF*) *f(d)* and a Cumulative Distribution Function (*CDF*) *F(d)*. The shippers are charged the freight rate of P_r for each TEU of capacity sold. The cost *c* is applied to each TEU of sold capacity. The goodwill loss *g*, which is also the lost sales cost, is applied for each TEU of unmet market demand due to insufficient capacity. We assume that the liner company operates the vessels under a stable setting (e.g., routes, network and other realistic factors) and that the amount of capacity, *Q*, that the liner company obtains from the financial lease is known at the beginning of the accounting period. The overall cost borne by the financial institution to support the finance lease contract is the total fund used to acquire these vessels. We amortise this cost in equal amounts, *L*, throughout the finance lease contract. According to Elliott and Dawson (2015), the regular rental payment of a finance lease has a variable relationship with the asset value, the interest rate and the period of the finance lease contract. Based on this and in line with Jiang and Liu (2018), we use the loan-to-value ratio to link the financial lease rental and asset value. The loan-to-value ratio ω is calculated by dividing the loan amount by the asset's appraised value, denoted as $\omega = \frac{L}{P_r \mathrm{Q}}$. Therefore, the financial lease cost on the financial institution's account at each repayment period can be denoted as $L = P_r \mathrm{Q} \omega$, and the rental that the liner company needs to pay is $\Delta = L(1 + R) = P_r \mathrm{Q} \omega (1 + R)$, which is in line with Elliott and Dawson (2015). The liner company's profit before the repayment is denoted as ξ, and the actual payment that the financial institution receives is δ. The default costs occur when the operating income of the liner company is lower than the rental fee as required in the finance lease, which means $\xi < L(1 + R)$. Here, the default costs include fixed and variable management fees such as $B = \alpha \xi + \beta$, which is similar to the setting of Kouvelis and Zhao (2011). It should be noted that the financial institution does not receive or bear the cost of default, but rather it is received by other institutions that charge court costs, reasonable attorney's fees, and various additional charges and expenses related to default proceedings. Therefore, when the liner company default on the rental fee, the rent paid by the liner company will lose a portion in the form of default costs. The actual payment that the financial institution receives, which is $\delta = \xi - B$, will also incur this deduction.

In addition to the capacity acquired via successive vessels finance lease contracts, we assume that the liner company has the initial capacity, q_0 , from owned vessels. Hence, the liner company has a total shipping capacity of $Q + q_0$ to meet the customer demands at each accounting period. When the liner company hires the NVOCC to canvass orders to sell its capacity, the liner company needs to sign a specific format of slot purchase contracts, e.g., a revenue-sharing contract, a buyback contract, or a quantity discount contract (Snyder and Shen, 2019). Here, we use one set of contract parameters (x, P_s, θ, T) to represent these three types of contracts following the earlier study by Kouvelis and Zhao (2016). x represents the order quantity of NVOCC; P_s the wholesale price that the liner company set up to provide vessel capacity to the NVOCC; θ the NVOCC's revenue sharing parameter; T the transfer payment between the NVOCC and the liner company. $T \geq 0$ represents the amount of money that the liner company transfers to the NVOCC and $T < 0$ means the fixed amount of money transferred from the NVOCC to the liner company. The market demand d will be first fulfilled by the NVOCC using their ordered slots x. Afterwards, if there is still unmet demand such as $d > x$ and remaining slots such as $Q + q_0 - x > 0$, the liner company can fulfil customer orders directly.

To find the optimal design of slot purchase contracts for the cooperation between the liner company and the NVOCC, we consider a decentralised as well as a centralised model. In the decentralised model, each stakeholder makes decision independently and separately but is bound by the pre-determined slot purchase contract. In our study, the liner company first designs the contract parameters before the start of the accounting period. Next, the NVOCC chooses how much capacity to order based on the market conditions and the selected contract. Two sales channels are available in the system: one is the sales of NVOCCs to the customers and the other is the sales of liner companies directly to the customers. The centralised model depicts the perfect collaboration between the liner company and the NVOCC, where the liner company and the NVOCC are considered as a virtual entity that makes decisions centrally to optimise the resource of the entire supply chain. It should be noted here that financial institutions provide identical financial leases for both models and do not participate in the collaboration between liner companies and NVOCCs. According to Snyder and Shen (2019), we use the centralised model as the benchmark to find the optimal contract for the decentralised model because the latter can never be more efficient than the former. See Figure 3.2 for a graphical representation of the two models.

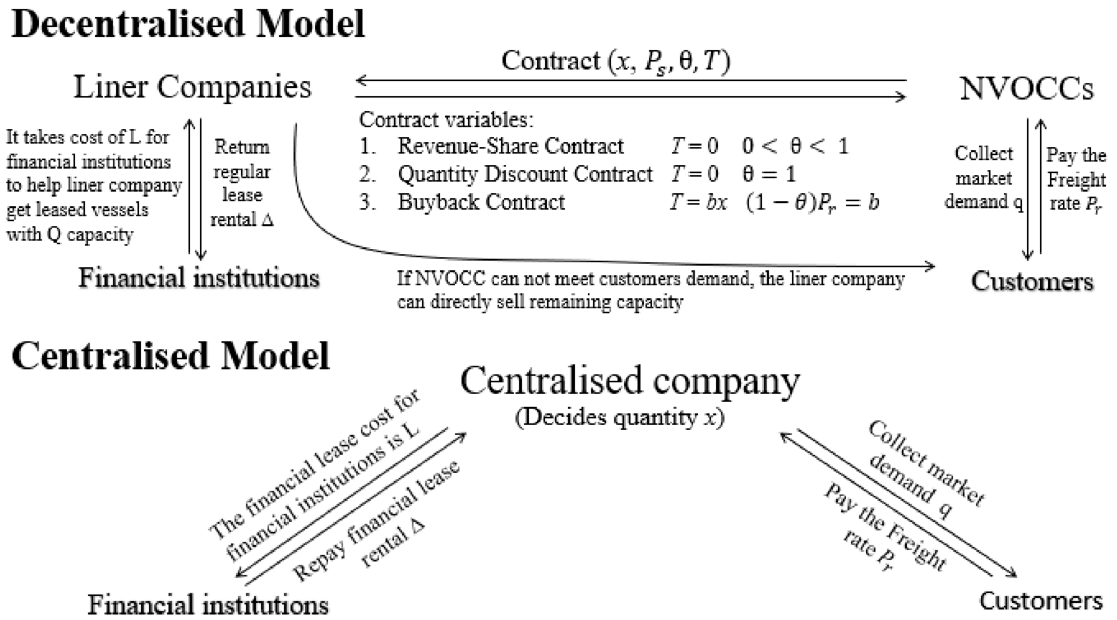

Figure 3.2 Decentralised and Centralised Models

The assumptions to be adopted in the decentralised and the centralised models are as follows:

1) The expected income of the financial institution and the default cost are the key criteria by which to evaluate the solvency of the liner company.

2) The planning horizon considered is one accounting period extending from the renewal of the finance lease contract to the repayment of the rental obligation.

3) The stakeholders are risk-neutral and have full information.

4) The liner company has no intention to break the financial lease contract and strives to fulfil its repayment obligations.

5) The NVOCC first sells the shipping capacity to the market. If the NVOCC cannot meet shippers' demands, the liner company can then directly sell the remaining capacity to the shippers.

It should be noted that assumptions (3) and (4) are similar to those of Kouvelis and Zhao (2016). Besides, let d^* denote the minimum amount of demand that enables the liner company to repay the full rental, then the NVOCC's initial order quantity *x* should be more than d^* since the liner company's profit initially comes from the slot purchase contract, i.e., $d^* \leq x$.

3.3.2 The Expected Profit Functions

Let $S_N(x)$ denote the expected amount of capacity sold by the NVOCC:

$$S_N(x) = E[min(x, d)] = x - \int_0^x F(d)dd$$

Let $I_N(x)$ denote the unsatisfied demand of the NVOCC:

$$I_N(x) = E[(d - x)^+] = \mu - x + \int_0^x F(d)dd$$

The amount of capacity of the liner company directly sold to customers is:

$$S_L(x) = E[min\,(Q + q_0 - x, (d - x)^+)] = \ (Q + q_0 - x) - \int_x^{Q+q_0} F(d)dd$$

The capacity stockout of the liner company is:

$$I_L(x) = E[(d - \mathrm{Q} - \mathrm{q}_0)^+] = \ \mu - (\mathrm{Q} + \mathrm{q}_0) + \int_0^{Q+q_0} F(d)dd$$

In the decentralised model, the NVOCC's expected profit function is:

$$\pi_N(x, P_s, \theta, \mathrm{T}) = \theta P_r S_N(x) + T - P_s x - cx - I_N(x)\mathrm{g} \quad (1)$$

In equation (1), the profit of NVOCC is the fraction of the revenue from selling the capacity to customers plus the transfer payment to the liner company minus the capacity purchasing cost, the operating cost and the goodwill penalty.

Similarly, the liner company's expected profit function is:

$$\pi_L(x, P_s, \theta, T) = P_s\, x + (1 - \theta)P_r S_N(x) - T + (P_r - c)S_L(x) - I_L(x)\mathrm{g} - E\Delta_d \quad (2)$$

The expected profit of the liner company in equation (2) consists of the payment from the NVOCC for providing the capacity, the fraction of NVOCC's shared revenue, the transfer payment T to the NVOCC, the revenue from direct selling, the goodwill penalty and the repayment of rental. Besides, $\pi_L = \xi_L - E\Delta_d$.

In the centralised model, the sold capacity of the centralised virtual entity is:

$$S_c(x) = S_N(x) + S_L(x) = E[min(Q + q_0, d)] = (Q + q_0) - \int_0^{Q+q_0} F(d)dd$$

$I_c(x)$ is set as the unsatisfied demand of the centralised virtual entity:

$$I_c(x) = I_L(x) + I_N(x)$$

The expected profit of the centralised virtual entity is, therefore:

$$\pi_c = P_r S_c(x) - [S_L(x) + x]c - [I_L(x) + I_N(x)]\mathrm{g} - \mathrm{E}\Delta_d \qquad (3)$$

Equation (3) indicates the total profit of the centralised virtual entity is the sum of the NVOCC's profit and the liner company's profit. And $\pi_c = \xi_c - E\Delta_d$.

After assessing the profit function of each party, it is necessary to clarify and define the contract parameters and their boundaries for each contract type. First, in the revenue-sharing contract, the $(1-\theta)P_r S_N(x)$ element is the money that the NVOCC transfers to the liner company. Therefore, $T = 0$ and $0 < \theta < 1$. Second, in the quantity discount contract, $\theta = 1$, which leads to $(1-\theta)\mathrm{P_r} S_N(x) = 0$. In this case, there is no revenue-sharing part in the calculation and T= 0. The only special part of this contract is that the liner company offers a lower price to the NVOCC than other contracts when other contract parameters are fixed. Third, in the buyback contract, we set the buyback price provided by the liner company to the NVOCC as $b = (1-\theta)P_r$ and$T = bx$. Therefore,$(1-\theta)P_r S_N(x) - T = bS_N(x) - bx = -b[x - S_N(x)]$which is the cost for the unsold capacity that the liner company buy back from the NVOCC. To better illustrate, we will use the subscripts 'R', 'B' and 'Q' to denote the revenue-sharing contract, the buyback contract and the quantity discount contract, respectively. Then, slot purchase contracts are set to be the revenue-sharing contract $(x, P_s, \theta_R, 0)$, buyback contract $(x, P_s, \theta_B, T(\theta_B))$and quantity discount contract $(x, P_s, 1{,}0)$, respectively.

3.4 Container Shipping Supply Chain Coordination Before the Financial Institution's Involvement

In this section, we analyse how to choose the contract parameters that enable the slot purchase contract between the liner company and the NVOCC to coordinate the supply chain before the liner company repays the rental. We also compare the profits of the liner company and the NVOCC under both deterministic and stochastic demands. We first restrict our discussion to the simple coordination between the liner company and the NVOCC without the financial institution's involvement. Proofs of all the following theoretical results are included in Appendix A.

3.4.1 Conditions Required for the Coordination

According to Cachon (2003), a container supply chain is coordinated when the optimal order quantities that maximise the profit for each party in the decentralised model are identical to those in the centralised model and that each party can earn a positive profit; meanwhile, a coordinated supply chain needs to ensure that the profit of the decentralised model is equal to that of the centralised model under perfect collaboration. These conditions guarantee that the relevant companies' profitability under the decentralised model can reach the same level as the centralised model when it reaches coordination.

Let x_c denote the order quantity that maximises the total profit of the centralised model and x_n^d and x_l^d indicate the order quantity which maximises the profit of the NVOCC and the liner company in the decentralised model. In light of the above discussion, a coordinated container supply chain satisfies the following condition: **Condition 1**: $x_c = x_n^d = x_l^d$.

Corollary 1. π_c is strictly concave in x. The optimal order quantity x_c for the centralised model satisfies:

$$F(x_c) = \frac{g}{g + c} \tag{4}$$

Since the first order derivative of the profit function π_c in the centralised model is continuous and strictly decreasing, there exists a unique optimal order quantity x that the NVOCC places for any (P_s, θ, T) to the liner company in the centralised model. In addition, the second order derivative of the profit function of the centralised model also implies that there exists a maximum value. Based on **Corollary 1**, we derive the following propositions that characterise the optimal strategy for the decentralised model.

Proposition 1. Under the revenue-sharing contract and the quantity discount contract**,** the optimal wholesale price should be set as follow:

$$P_s(\theta) = \frac{\theta P_r c - c^2}{g + c} \tag{5}$$

Under the buyback contract, the optimal wholesale price should be set as follow:

$$P_s(\theta) = \frac{(1-\theta)P_r g + P_r c - c^2}{g + c} \tag{6}$$

If the relation between contract parameters P_s and θ is set appropriately according to **Proposition 1**, the optimal order capacity of NVOCC in both the decentralised and centralised models will be identical such as $x_c = x_n^d = x_l^d$. It is worth mentioning that the settings of these contract parameters in **Corollary 1** and **Proposition 1** are very similar to those set by Cachon and Lariviere (2005), Moorthy (1987) and Pasternack (1985) for revenue-sharing contracts, quantity discount contracts and buyback contracts, respectively.

In addition, we also compare the total profit of the decentralised model with that of the centralised model. Under the revenue-sharing contract and the quantity discount contract, the relationship between the expected profit function of the NVOCC, the liner company and the virtual central entity is:

$$\begin{aligned}
&\pi_N(x, P_s, \theta, T) + \pi_L(x, P_s, \theta, T) \\
&= \left[\theta P_r S_N(x_n^d) - \frac{\theta P_r c - c^2}{g + c} x_n^d - c x_n^d - I_N(x_n^d) g\right] \\
&+ \left[\frac{\theta P_r c - c^2}{g + c}\, x_l^d + (1-\theta) P_r S_N(x_l^d) + (P_r - c) S_L(x_l^d) - I_L(x_l^d) g\right] \\
&= P_r S_c(x_c) - [S_L(x_c) + x_c] c - [I_L(x_c) + I_N(x_c)] g = \pi_c
\end{aligned}$$

Under the buyback contract, the relationship between the expected profit function of the NVOCC, the liner company and the centralised virtual entity is:

$$\begin{aligned}
&\pi_N(x, P_s, \theta, T) + \pi_L(x, P_s, \theta, T) \\
&= \left[\theta P_r S_N(x_n^d) + T - \frac{(1-\theta) P_r g + P_r c - c^2}{g + c} x_n^d - c x_n^d - I_N(x_n^d) g\right] \\
&+ \left[\frac{(1-\theta) P_r g + P_r c - c^2}{g + c}\, x_l^d + (1-\theta) P_r S_N(x_l^d) - T + (P_r - c) S_L(x_l^d) - I_L(x_l^d) g\right] \\
&= P_r S_c(x_c) - [S_L(x_c) + x_c] c - [I_L(x_c) + I_N(x_c)] g = \pi_c
\end{aligned}$$

The above calculation indicates that the sum of NVOCC's and the liner company's maximum profits in the decentralised model equals the maximum profits obtained in the centralised model before the repayment. As a result, it turns out that these three contracts can coordinate the container supply chain if the contract parameters are set appropriately.

According to the previous work of Kouvelis and Zhao (2016) and Jiang and Liu (2018), the traditional contracts could coordinate the supply chain with a single sale channel from one supplier to one retailer under financial constraints. However, in our study, there are dual sales channels in the container shipping supply chain as discussed above.

Proposition 2. The establishment of direct sales channel from liner company to customers will contribute to the liner company and NVOCC reaching a reasonable slot purchase contract that maximises the profits of both parties.

Due to the peculiar element of direct sale channel in the container shipping chain, improvements as **Proposition 2** are made to facilitate the coordination of the container supply chain. Although direct sales of liner companies can allow liner companies to obtain higher returns, unknown market demand will bring huge risk of lost sales. Collaboration with NVOCC can help liner companies share this risk of market demand. Based on **Proposition 2,** the slot purchase contract after joining the direct selling can prompt the liner company to obtain a higher profit from the direct sales and ensure a stable collaboration between the liner company and the NVOCC.

3.4.2 Profit split between the liner company and the NVOCC before Repayment

Palepu et al. (2016) stated that profitability and solvency analysis are two main criteria in evaluating the overall performance of any project. Based on this, the profits under the three types of contracts are the key factor for the liner company and the financial institution in making their decisions. In this sub-section, we examine the functional relationship between contract parameters associated with each contract and the corresponding profit function and compare the profits of the liner company and the NVOCC under different contracts. According to equation (4), the goodwill loss and the operating cost are the main elements influencing the order quantity x. Therefore, we define $x = G(\mathrm{g}, c)$ (as shown in **Corollary 1**: $F(x_c) = \frac{\mathrm{g}}{\mathrm{g}+c}$).

According to equation (1), (2) and (3), the income that the liner company and the NVOCC have on hand before the repayment is made to the financial institution is:

$$\xi_L = P_s x + (1-\theta)P_r S_N(x) - \mathrm{T} + (P_r - c)S_L(x) - I_L(x)\mathrm{g}$$

$$\xi_c = P_r[S_N(x) + S_L(x)] - [S_L(x) + x]c - [I_L(x) + I_N(x)]\mathrm{g}$$

$$= [\theta P_r S_N(x) + T - P_s x - cx - I_N(x)\mathrm{g}] + [P_s x + (1-\theta)P_r S_N(x) - \mathrm{T} + (P_r - c)S_L(x) - I_L(x)\mathrm{g}]$$

$$= \pi_N + \xi_L$$

It is clear that P_s, θ and T have no relationship with the operating revenue of the centralised virtual entity. Therefore, this operating revenue of the centralised virtual entity is fixed after the NVOCC has decided on the order quantity. Here, we first consider the conditions under stochastic market demand.

Corollary 2: There exists an optimal θ^* for the liner company and the NVOCC to divide the total operating revenue equally. Besides, let

$$\theta_1 = \frac{P_s x + cx + I_N(x)\mathrm{g}}{P_r S_N(x)}$$

$$\theta_2 = 1 + \frac{P_s x + (P_r - c)S_L(x) - I_L(x)\mathrm{g}}{P_r S_N(x)}$$

Then,

1. If $\theta < \theta_1$, then the NVOCC earns negative profit, and the liner company earns more than ξ_c.

2. If $\theta = \theta_1$, then the liner company earns the entire ξ_c.

3. If $\theta_1 < \theta < \theta_2$, then the liner company and NVOCC sharing the ξ_c.

4. If $\theta = \theta_2$, then the NVOCC earns the entire ξ_c.

5. If $\theta > \theta_2$, then the liner company earns negative profit and the NVOCC earns more than ξ_c.

To promote the collaboration with the NVOCC, the liner company needs to guarantee that the NVOCC can obtain an appropriate income when deciding other contract parameters (P_s, θ, T). From **Proposition 1**, it is evident that the liner company will first choose θ and then determine the suitable contract price P_s and T. From the proof of **Corollary 2** in appendix A, we see that the correlation between the operating revenue of the liner company and θ is opposite to the correlation between the income of the NVOCC and θ. As the sum of ξ_L and π_N is fixed, the following equilibrium condition must hold at θ^*:

$$\underbrace{P_s(\theta^*)\,x + (1-\theta^*)P_r S_N(x) - \mathrm{T}(\theta^*) + (P_r - c)S_L(x) - I_L(x)\mathrm{g}}_{\xi_L} - \underbrace{\theta^* P_r S_N(x) - T(\theta^*) + P_s(\theta^*)x + cx + I_N(x)\mathrm{g}}_{\pi_N} = 0 \quad (7)$$

Because $S_L(x)$ and $I_L(x)$ will change with $\mathrm{Q} + \mathrm{q}_0$, from equation (7), we find that the optimal θ^* changes with the original capacity q_0 (which the liner company could provide), the capacity Q (which is from leased vessels), and the market parameters (g, c). We fix (Q, q_0) and transform the left side of equation (7) into function $H(\mathrm{g}, c|\,\theta)$ to examine the connection between θ and the market parameters (g, c).

$$H(\mathrm{g}, c|\,\theta) = 2P_s(\theta)x + (1-2\theta)P_r S_N(x) - 2T + (P_r - c)S_L(x) + cx + [I_N(x) - I_L(x)]\mathrm{g} \quad (8)$$

If $H(\mathrm{g}, c|\,\theta)$ is greater than 0, then liner company could earn more profit than the NVOCC. If $H(\mathrm{g}, c|\,\theta)$ is smaller than 0, NVOCC will earn more than half of the centralised virtual entity's profit.

Proposition 3: For any market parameters (g, c) satisfying equation (9), the quantity discount contract will be a better choice for both the liner company and the NVOCC. Then, the liner company will obtain more than half of the centralised virtual entity's profit.

$$\mathrm{H}\,(\mathrm{g}, c|\theta = 1) = 2G(\mathrm{g}, c)P_s(\mathrm{g}, c) + G(\mathrm{g}, c)\mathrm{g} + P_r\int_0^{G(\mathrm{g},c)} F(d)dd + \left(\mathrm{Q} + \mathrm{q}_0 - 2x - \int_{G(\mathrm{g},c)}^{\mathrm{Q}+\mathrm{q}_0} F(d)dd\right)(P_r + \mathrm{g} - \mathrm{c}) \geq 0 \quad (9)$$

Note that θ should be no more than 1 in the buyback contract, revenue-sharing contract or quantity discount contract. With reference to **Corollary 2**, we know that $\theta_2 > 1$, which leads to cases 4 and 5 in **Corollary 2** will not happen. Therefore, the three kinds of contracts can prevent the NVOCC from earning the entire ξ_c, which also ensures that the liner company can obtain a positive profit. When the optimal θ^* in **Corollary 2** more than 1, $\theta = 1$ (the quantity discount contract) will be the superior practical option for the liner company and the NVOCC. First, $P_s\,x + (P_r - c)S_L(x) - I_L(x)\mathrm{g}$ in θ_2 (see **Corollary 2**) happens to be ξ_L (the profit of the liner company before repayment) under the quantity discount contract. Therefore, when $\xi_L > 0$, $\theta_2 > 1$.

Since the quantity discount contract holds $\theta = 1$, which is closer to θ_2 compared to the θ that of buyback contract and revenue-sharing contract, the quantity discount contract could ensure the NVOCCs could get a higher profit than other contracts. Second, the NVOCCs will choose $\theta = 1$ under the revenue sharing contract to ensure that it can obtain a greater profit. Finally, under the buyback contract, the liner company will choose $\theta_1 < \theta \leq 1$ only in the condition of $S_N(x) \geq \frac{P_s x + cx + ug}{P_r + \mathrm{g}}$, otherwise there is a high chance that NVOCC will get a negative profit and hinder cooperation. Therefore, from the perspective of NVOCC, the liner company's selection of $\theta = 1$ in the buyback contract can decrease the probability of NVOCC's refusal to collaborate. It is worth noting that the choice of $\theta = 1$ will make the revenue-sharing contract and buyback contract enforced as a quantity discount contract. Based on these, we could get the result of **Proposition 3**.

Apart from the condition in **Proposition 3,** that is $H\ (\mathrm{g}, c|\theta = 1) < 0$, then the optimal θ^* in **Corollary 2** will be in the range 0 to 1. For better expression, we introduce θ_B^* as a specific form of θ^* in the buyback contract, and thus we can formulate the liner company's optimal buyback price $b^* = (1 - \theta_B^*)P_r$ in buyback contract; θ_R^* as the NVOCC's optimal revenue share under the revenue-sharing contract and $\theta_Q^* = 1$ in the quantity discount contract. From equations (5) and (6), the optimal wholesale price that the NVOCC pays to the liner company under each contract could be summarised as:

$$P_s(\theta) = \begin{cases} \dfrac{(1 - \theta_B^*)P_r \mathrm{g} + P_r c - c^2}{\mathrm{g} + c}, & \text{buyback contract} \\ \dfrac{\theta_R^*\ P_r c - c^2}{\mathrm{g} + c}, & \text{revenue sharing contract} \\ \dfrac{P_r c - c^2}{\mathrm{g} + c}, & \text{quantity discount contract} \end{cases} \tag{10}$$

Corollary 3: The buyback contract and the revenue-sharing contract are identical to generate profits for both the liner company and the NVOCC.

From **Corollary 3**, we can see that $(x, P_s(\theta_R^*), \theta_R^*, 0)$ and $(x, P_s(\theta_B^*), \theta_B^*, T)$ are identical for both the liner company and the NVOCC. This result is consistent with the previously mentioned work of Cachon and Lariviere (2005). Therefore, the liner company could achieve the same profit under the buyback and revenue sharing contracts. Then, we have

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T)$$

This result also holds under the determined market demand.

Proposition 4: When the market demand is stochastic and $H\ (\mathrm{g}, c|\theta = 1) < 0$, the following results hold:

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T) > \xi_L\big(x, P_s\big(\theta_Q^*\big), 1{,}0\big)$$

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_N(x, P_s(\theta_B^*), \theta_B^*, T) < \pi_N\big(x, P_s\big(\theta_Q^*\big), 1{,}0\big)$$

In **Proposition 4**, we compare the liner company's profit before the rental is paid under the revenue-sharing contract, the buyback contract and the quantity discount contract under the stochastic market demand. It implies that the revenue sharing contract $(x, P_s(\theta_R^*), \theta_R^*, 0)$ and the buyback contract $(x, P_s(\theta_B^*), \theta_B^*, T)$ could generate more expected profit for the liner company than the quantity discount contract $\big(x, P_s\big(\theta_Q^*\big), 1{,}0\big)$. Therefore, the liner company is better off under the revenue-sharing and the buyback contracts than the quantity discount contract. Since the profit of total centralised operating revenue is the same under these three contracts, the profit for the NVOCC is lower under the revenue-sharing contract and the buyback contract than it is under the quantity discount contract.

However, the profit of each party under deterministic demand will show a different result when that under stochastic demand. Since the expected profit under the stochastic demand model recognises that demand is uncertain and subject to random volatility, this model reflects the predicted profit associated with the probabilities. Whereas the expected profit under deterministic demand is a single fixed estimate and does not take into account the probabilities associated with profit predictions. It could provide a baseline estimate of expected profit that can be used to identify the minimal profit generated so as to justify the launch of the financial lease. Therefore, considering the predicted profit under deterministic demand provides a more comprehensive and realistic view of the potential repayment outcomes and helps liner companies plan for a range of potential default scenarios. Under the quantity discount contract and the assumption that $d^* \leq x$, when facing any deterministic market demand less than x, the realised profit for the liner company before repayment is,

$$\xi_L\big(x, P_s(\theta_Q^*), 1{,}0\big) = \frac{P_r c - c^2}{\mathrm{g} + c} x \quad (11)$$

From **Corollary 3**, when under the revenue-sharing contract and the buyback contract, the realised profit of the liner company before the rental payment is,

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T) = \frac{\theta^* P_r c - c^2}{\mathrm{g} + c} x + (1 - \theta^*) P_r d^* \quad (12)$$

From equations (11) and (12), we see that there are two cases for the profit of the liner company and the NVOCC, which lead to **proposition 5** as follows.

Proposition 5: When market demand is deterministic,

i. If the realised market demand is $d \le \frac{cx}{\mathrm{g}+c}$, the profit of the liner company, the NVOCC, and the centralised model under each contract could be summarised as follows:

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T) \le \xi_L\big(x, P_s(\theta_Q^*), 1{,}0\big)$$

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_N(x, P_s(\theta_B^*), \theta_B^*, T) \ge \pi_N\big(x, P_s(\theta_Q^*), 1{,}0\big)$$

$$\pi_c(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_c(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_c\big(x, P_s(\theta_Q^*), 1{,}0\big)$$

Then, the liner company is better off under the quantity discount contract. However, the NVOCC might suffer a more significant loss under the quantity discount contract.

ii. If realised market demand is $d > \frac{cx}{\mathrm{g}+c}$, the following equations show the profit of the liner company, the NVOCC, and the centralised model under different contracts, respectively.

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T) > \xi_L\big(x, P_s(\theta_Q^*), 1{,}0\big)$$

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_N(x, P_s(\theta_B^*), \theta_B^*, T) < \pi_N\big(x, P_s(\theta_Q^*), 1{,}0\big)$$

$$\pi_c(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_c(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_c\big(x, P_s(\theta_Q^*), 1{,}0\big)$$

Then, the liner company is better off under revenue-sharing and buyback contracts. The profit of the NVOCC is lower under both the revenue-sharing contract and the buyback contract than the quantity discount contract but is still positive.

From **Proposition 5**, when the realised market demand is less than $\frac{cx}{\mathrm{g}+c}$, the liner company can earn a fixed higher positive operating revenue before the rental is paid to the financial institution under the quantity discount contract than that under the revenue-sharing contract and the buyback contract. Therefore, the liner company is better off under the quantity discount contract than the revenue-sharing contract and the buyback contract when the realised demand is lower than $\frac{cx}{\mathrm{g}+c}$. However, the NVOCC with the quantity discount contract might earn a higher negative profit as demands drop. Suppose the market demand is better than $\frac{cx}{\mathrm{g}+c}$. In that case, the revenue-sharing contract and the buyback contract can generate an identical higher operating revenue for the liner company before the rental payment is made to the financial institution than the quantity discount contract. Therefore, the liner company is better off under the revenue-sharing contract and the buyback contract when the market demand is higher than $\frac{cx}{\mathrm{g}+c}$. Although the NVOCC's profit is lower under both the revenue-sharing contract and the buyback contract than the quantity discount contract, it reduces the risk for the NVOCC to earn a higher negative profit.

From the above analysis in this section, it is worth noting that the expected profit of liner companies before repayment under the revenue-sharing contract and the buyback contract is still higher than that under the quantity discount contract when $H\ (\mathrm{g}, c|\theta = 1) \geq 0$. However, in this case, the execution of the revenue-sharing contract and the buyback contract will have certain preferences according to **Corollary 2** due to the selfishness of the liner company and NVOCC, which eventually leads to the execution of the revenue-sharing contract and the buyback contract approaching the quantity discount contract. Thus, the **Proposition 3** presents the condition for them to choose a quantity discount contract under stochastic market demand. Then, we looked into the deterministic market demand and found that in a certain smaller demand range, the quantity discount contract is better for liner companies than the revenue-sharing contract and the buyback contract. The reason why the profitability of the liner company in this instance differs from the conclusion that under stochastic market demand is that calculations under deterministic market demand eliminate the probability associated with each demand.

3.5 Performance of Contracts with the Presence of Financial Institution

In this section, we will first discuss the performance of the contracts in both the decentralised and the centralised model with the presence of the financial institution.

To understand which contract is favourable to the financial institution, we will also compare the performance of each contract from the perspective of solvency.

3.5.1 The Financial Institution's Expected income, Default Loss and LTV ratio

In the decentralised model, the repayment from the liner company to the financial institution is expressed as:

$$\Delta_d(x, P_s, \theta, T) = \begin{cases} P_r Q\omega_d(1+R) & if\ \xi_L \geq P_r Q\omega_d(1+R) \\ \xi_L & if\ \xi_L < P_r Q\omega_d(1+R) \end{cases} \tag{13}$$

From equation (13), the liner company just pays the full rental fee if they do not have to default on the financial lease contract. However, if defaults occur due to insufficient market demands, the liner company can only pay the partial rental fee up to its total profit. The profit that the liner company has on hand before the repayment is made to the financial institution can be expressed as:

$$\xi_L(x, P_s, \theta, T) = P_s\, x + (1-\theta)P_r S_N(x) - T + (P_r - c)S_L(x) - I_L(x)g \tag{14}$$

Similar to Kouvelis and Zhao (2011), the default cost is set as $B_d(\xi_L) = \alpha\xi_L + \beta$. The income that the financial institution will receive is:

$$\delta_d = \begin{cases} P_r Q\omega_d(1+R) & \xi_L \geq P_r Q\omega_d(1+R) \\ \xi_L - B_d(\xi_L) & \xi_L < P_r Q\omega_d(1+R) \end{cases} \tag{15}$$

From equation (15), the financial institution will receive the full rental fee if the liner company does not have to default on the financial lease contract. Since some portion of the liner company's profit will be used to cover the default costs, the rental that the financial institution receives will be less than this amount if a default occurs.

In the centralised model, the expression of the repayment from the centralised virtual entity to the financial institution is:

$$\Delta_c(x, P_s, \theta, T) = \begin{cases} P_r Q\omega_c(1+R) & \xi_c \geq P_r Q\omega_c(1+R) \\ \xi_c & \xi_c < P_r Q\omega_c(1+R) \end{cases} \tag{16}$$

Equation (16) implies that the virtual central entity just pays the entire rental if it does not default on the financial lease contract. The virtual central entity will pay the total profit before repayment if it defaults on the finance lease contract. Thus, ξ_c can be expressed as:

$$\xi_c(x, P_s, \theta, T) = P_r[S_N(x) + S_L(x)] - [S_L(x) + x]c - [I_L(x) + I_N(x)]g \tag{17}$$

Since the calculation of default cost is: $B_c(\xi_c) = \alpha\xi_c + \beta$, the income that the financial institution will receive in the decentralised model is:

$$\delta_c = \begin{cases} P_r Q\omega_d(1+R) & \xi_c \geq P_r Q\omega_c(1+R) \\ \xi_c - B_c(\xi_c) & \xi_c < P_r Q\omega_c(1+R) \end{cases} \tag{18}$$

To compare the repayment of the liner company to the financial institution and the income of financial institution under decentralised model and centralised model, equation (17) will be rewritten as the following equation:

$$\xi_c = P_r[S_N(x) + S_L(x)] - [S_L(x) + x]c - [I_L(x) + I_N(x)]g$$

$$= \underbrace{\theta P_r S_N(x) + T - P_s x - cx - I_N(x)g}_{\pi_N} + \underbrace{P_s\, x + (1-\theta)P_r S_N(x) - T + (P_r - c)S_L(x) - I_L(x)g}_{\xi_L}$$

Therefore, $\xi_c = \pi_N + \xi_L$ and $B_c \geq B_d$.

Considering the default costs, the income of the financial institution δ is not always the same as the actual repayment Δ_d of the liner company. Thus, the income of financial institution in decentralised model and centralised model might be different. If the liner company defaults on the lease contract, a default loss has to be included in the actual payment Δ_d that the liner company can make. Because $\xi_c \geq \xi_L$, the centralised model behaves better than the decentralised model for the collaboration between the liner company and the NVOCC. Since the financial institution is excluded from the collaboration, we investigate the attitude of financial institution toward the centralised model and the decentralised model when $B_c \geq B_d$ happens.

Proposition 6. With the financial institution involved in the container supply chain, there are four possible cases in relation to the financial institution's income and default costs:

i. if the liner company does not default on the finance lease contract, the financial institution's income and default costs under the decentralised model are equivalent to those under the centralised model.

ii. if there is only a fixed default cost, i.e., $B_d = \beta = B_c$, the default costs under the decentralised model are equivalent to that under the centralised model. However, the financial institution's income under the centralised model is higher than that under the decentralised model.

iii. if there is only the variable default cost, i.e., $B_d = \alpha\xi_L < \alpha\xi_c = B_c$, the sum of default costs under the centralised model is higher than that under the decentralised model. However, the income of the financial institution in the centralised model is higher than that under the decentralised model.

iv. if both the variable and fixed default costs exist, i.e., $B_d = \alpha\xi_L + \beta < \alpha\xi_c + \beta = B_c$, the sum of default costs under the centralised model is higher than that of the decentralised model. However, the income of the financial institution under centralised model is still higher than that under the decentralised model.

According to the above four cases, the centralised management of the container supply chain is more favourable to the financial institution than the decentralised management model.

Note that the decentralised and centralised models are equivalent only in case i. The other cases in **Proposition 6** indicate that the financial institution in the centralised model will have more income from the financial lease contract than the financial institution in the decentralised model. In general, it will be easier for the centralised virtual entity under the centralised model to repay the rental obligation than the financial institution in the decentralised model. The reason for this conclusion is that the centralised model engages the NVOCC in debt sharing. Therefore, the centralised virtual entity will be more favourable to the financial institution than the liner company in the decentralised one.

However, the three contracts in the decentralised model can reduce default loss, the risk of decreased market demand and guarantee the profit of the NVOCC. In addition, the best loan-to-value ratio to attract a financial institution's investment in the decentralised model should be in case i of **Proposition 6,** where the liner company does not default on the finance lease contract. Therefore, the decentralised model might reach the efficiency of the centralised model in case i of **Proposition 6.**

We then use ω^* to denote the optimal loan-to-value ratio for the financial institution to sign a finance lease contract with the liner company in the decentralised model. Since the liner company will not default on the finance lease contract if there is enough market demand or customer orders, there is a certain threshold of market demand below which a default may occur. We set this threshold of market demand as d^* which satisfies the following equation:

$$P_r Q\omega(1+R) = P_s\, x + (1-\theta)P_r S_N(x) - \mathrm{T} + (P_r - c)S_L(x) - I_L(x)\mathrm{g} \qquad (19)$$

In equation (19), the goodwill cost element should be zero because the liner company is in a situation with enough market demand to support earnings at the threshold demand. In other words, this threshold demand point d^* should be less than the maximum capacity the liner company can provide. Because the profit of liner company will fall after this range due to insufficient shipping capacity. Besides, if the liner company could not repay within this range, there would be no change for the liner company to use operating profit to repay the financial lease rental. In addition, there are two sources of income for the liner company: one is from the NVOCC for providing vessel slots via the purchase contract, and the other is from direct sales to the market. Since the profit of the liner company comes initially from the purchase contract, the point where the liner company simply repays the rental with the profit should be within this range. Therefore, this supposition leads to $d^* \leq x$. Then, equation (19) could be simplified as:

$$P_r Q\omega(1+R) = P_s x + (1-\theta)P_r d^* - \mathrm{T} \tag{20}$$

Then we obtain δ_d as follows:

$$\begin{aligned}\delta_d &= \int_0^{d^*} \big((1-\theta)P_r d^* - \mathrm{T} + P_s x\big) f(d)dd + \int_{d^*}^{\infty} P_r Q\omega(1+R) f(d)dd \\ &= \big((1-\theta)P_r d^* - \mathrm{T} + P_s x\big) - \big[\alpha\big((1-\theta)P_r d^* - \mathrm{T} + P_s x\big) + \beta\big]F(d^*) \\ &\qquad + (\alpha-1)(1-\theta)P_r \int_0^{d^*} F(d)dd \end{aligned} \tag{21}$$

Corollary 4: In the decentralised model, the income of the financial institution is concave in ω. In addition, the loan-to-value ratio ω has a positive correlation with threshold demand d^* when $\theta < 1$.

Here, we first consider the condition that $\theta < 1$ and move to the other situations. By taking the first order derivative of the threshold demand with respect to the loan-to-value ratio, we can see that the loan-to-value ratio ω has a positive correlation with threshold demand d^*. When under the quantity discount contract ($\theta = 1$) and the market demand is low, the profit of the liner company before repayment will be $P_s x$, which has no relation with market demand d. This proves that the threshold demand $d^* \leq x$ may not exist for the case under the quantity discount contract. This will be demonstrated in detail in the latter part of this study.

Proposition 7: The optimal ω^* for the financial institution must satisfy the equation:

$$(1-\theta)P_r - (1-\theta)P_r F(d^*) - \{\alpha[(1-\theta)P_r d^* - \mathrm{T} + P_s x] + \beta\} f(d^*) = 0 \tag{22}$$

It should be noted here that the relationship of equation (20) is maintained between the threshold demand d^* in equation (22) and the optimal loan-to-value ratio ω^*. Since in **Corollary 4** we came to the conclusion that there exists a loan-to-value ratio ω^* that maximises the income of financial institutions, we then get the above equation (22) for the financial institution to make informed decisions.

The above analysis measures the solvency and profitability of the liner company under a general contract and briefly compares the centralised and the decentralised models. As with the traditional literature, the income of the financial institution in the centralised model serves as an upper boundary to that in the decentralised model. In our subsequent analysis, we will compare the various indicators under each specific contract.

3.5.2 The Impact of Financial Institutions on Contracts Design

In this section, we will first present the default cost under each contract type and examine how this affects the financial institution's income. According to **Corollary 3**, it is clear that the threshold market demand d^* for the liner company to repay the rental obligation is the same under the revenue-sharing contract and the buyback contract. That is,

$$d^*(x, P_s(\theta_R^*), \theta_R^*, 0) = d^*(x, P_s(\theta_B^*), \theta_B^*, T)$$

Based on this, the repayments under the revenue-sharing contract and the buyback contract are,

$$\Delta_R(x, P_s(\theta_R^*), \theta_R^*, 0) = \Delta_B(x, P_s(\theta_B^*), \theta_B^*, T) = \begin{cases} \dfrac{\theta^* P_r c - c^2}{g + c} x + (1 - \theta^*) P_r d, & d < d^* \\ P_s Q \omega (1 + R), & d \geq d^* \end{cases} \quad (23)$$

The expression of default cost and the financial institution's income under the revenue-sharing contract and the buyback contract are:

$$\begin{aligned} & B_R(x, P_s(\theta_R^*), \theta_R^*, 0) = B_B(x, P_s(\theta_B^*), \theta_B^*, T) \\ &= \int_0^{d^*} \left\{ \alpha \left[\frac{\theta^* P_r c - c^2}{g + c} x + (1 - \theta^*) P_r d \right] + \beta \right\} f(d) dd \\ &= \left(\frac{\theta^* P_r c - c^2}{g + c} \alpha x + \beta \right) F(d^*) + \alpha (1 - \theta^*) P_r \int_0^{d^*} d f(d) dd \qquad (24) \end{aligned}$$

$$\delta_R(x, P_s(\theta_R^*), \theta_R^*, 0) = \delta_B(x, P_s(\theta_B^*), \theta_B^*, T) = \Delta - B$$
$$= \int_0^{d^*} \left\{\frac{\theta^* P_r c - c^2}{g + c} x + (1 - \theta^*) P_r d\right\} f(d) dd + P_s Q \omega (1 + R)[1 - F(d^*)]$$
$$- \int_0^{d^*} \left\{\alpha \left[\frac{\theta^* P_r c - c^2}{g + c} x + (1 - \theta^*) P_r d\right] + \beta\right\} f(d) dd$$
$$= \left[\frac{\theta^* P_r c - c^2}{g + c}(1 - \alpha) x - \beta\right] F(d^*) + (1 - \alpha)(1 - \theta^*) P_r \left[d^* F(d^*) - \int_0^{d^*} F(d) dd\right]$$
$$+ P_s Q \omega (1 + R)[1 - F(d^*)] \tag{25}$$

Proposition 8: The financial leasing cost of the liner company is the same under each contract which is $P_r Q\omega$. From a financial perspective, the revenue-sharing contract is still equivalent to the buyback contract. Besides, we have

i. When $P_s Q \omega (1 + R) < \frac{\theta^* P_r c - c^2}{g + c} x$, the default cost is zero under these three contracts. The expected income of the financial institution is also the same under these three contracts.

ii. When $\frac{\theta^* P_r c - c^2}{g + c} x \leq P_s Q \omega (1 + R) \leq \frac{P_r c - c^2}{g + c} x$, the default cost under quantity discount contract is zero, which is lower than the revenue-sharing contract and the buyback contract. The expected income for the financial institution is higher under the quantity discount contract.

iii. When $P_r Q \omega (1 + R) > \frac{P_r c - c^2}{g + c} x$, the default cost under the quantity discount contract is higher than it is under revenue-sharing contract and the buyback contract. The income for the financial institution under the revenue-sharing contract and the buyback contract is more than the financial institution's income under the quantity discount contract.

We first consider the condition $\frac{\theta^* P_r c - c^2}{g + c} x \leq P_s Q \omega (1 + R) \leq \frac{P_r c - c^2}{g + c} x$. Under this condition, the liner company's default threshold under the revenue-sharing contract and the buyback contract is $0 \leq d_R^* = d_B^* \leq \frac{c}{g + c} x$. But the liner company could repay the rentals fully under the quantity discount contract no matter how market demands change. Therefore, the default cost for the liner company under the quantity discount contract is zero, meaning the default cost under the revenue-sharing contract and the buyback contract is higher. The financial institution's income under the quantity discount contract is fixed as $P_r Q\omega(1+R)$. When combining **Proposition 8** and **Proposition 4**, we notice that even when the quantity discount contract makes a less expected profit for the liner company when facing stochastic market demand, it could still bring more income to the financial institution. The main reason for this is that the profit of the liner company before repayment under the revenue-sharing contract and the buyback contract is lower than the rental payment $P_r Q\omega(1+R)$ when facing a deterministic market demand lower than $\frac{cx}{g+c}$. In addition, this will lead to the default of the liner company which not only leads to a reduction in the repayment amount but also impairs the income of financial institution. Therefore, the financial institution would prefer the liner company to opt for the quantity discount contract when trading with the NVOCC. This contract could guarantee that no impairment losses occur to the income of the financial institution. In this case, profitability cannot be used as a measure of solvency.

Next, we consider the second condition that $P_r Q\,\omega\,(1+R) > \frac{P_r c - c^2}{g+c}x$. Under this condition, the liner company's default threshold demand under the revenue-sharing contract and the buyback contract is $\frac{c}{g+c}x < d_R^* = d_B^* < x$. The default threshold for the liner company under the quantity discount contract is $d_Q^* = \frac{P_r Q\omega(1+R)}{P_r - c} + \frac{g}{g+c}x > x$. Then, it costs more for the liner company to default on the finance lease contract under the quantity discount contract than the revenue sharing contract and the buyback contract. Therefore, the income of the financial institution under the revenue-sharing contract and the buyback contract will be more than the financial institution's income under the quantity discount contract. In this case, profitability can be used as a measure of solvency.

3.6 Numerical Example

In this section, by means of a numerical example, we will analyse how the three contracts promote the cooperation between liner companies and the NVOCCs under financial constraints. The parameters are set as follows: $Q = 100, q_0 = 50, P_r = 400, c = 21, g = 50$. The demand of market customers follows a normal distribution $d \sim N(100, 30)$. Based on Equation (4), the NVOCC should order $x \approx 116$.

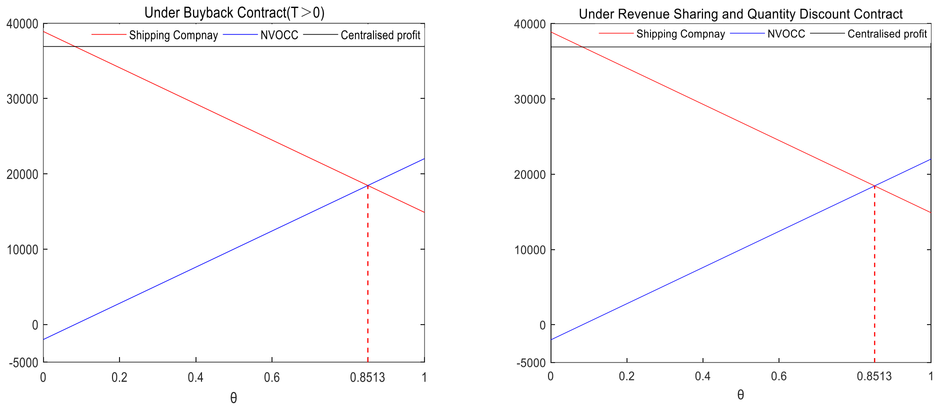

Figure 3.3 Profit under Contracts $(\mathrm{x}, \mathrm{P_s}, \theta, \mathrm{T})$

In Figure 3.3, the vertical axis represents the profit of each entity, and the horizontal axis is the contract parameter θ, which is decided by the liner company. From this diagram, it is clear that the changes in the profit under the buyback and revenue-sharing contracts are identical with respect to the contract parameter θ. This confirms **Corollary 3**. When the liner company chooses $\theta = 0.8513$ (which sets the buyback price as $b = 59.48$), it will divide the total profit equally with the NVOCC. Consequently, the optimal contract price that the liner company provides is $P_r = 94.51$ under the revenue-sharing contract, $P_r = 153.99$ under the buyback contract, $P_r = 112.10$ under the quantity discount contract.

With equations (1), (2) and (3), we compute the profits of the decentralised model and the centralised model before repayment, respectively. The sum of π_L and π_N is 36910, confirming that the supply chain is coordinated under three contracts. In addition, the profits of the liner company under the buyback contract and the revenue-sharing contract are always higher than the profit under the quantity discount contract, which confirms the results of **Proposition 4**.

$$\pi_L(116, 94.51, 0.8513, 0) = \pi_L(116, 153.99, 0.8513, 6905) = 18455$$
$$> \pi_L(116, 112.10, 0, 0) = 14882$$

$$\pi_N(116, 94.51, 0.8513, 0) = \pi_N(116, 153.99, 0.8513, 6905) = 18455$$
$$< \pi_N(116, 112.10, 0, 0) = 22028$$
$$\pi_C(116, 94.51, 0.8513, 0) = \pi_C(116, 153.99, 0.8513, 6905)$$
$$= \pi_C(116, 112.10, 0, 0) = 36910$$

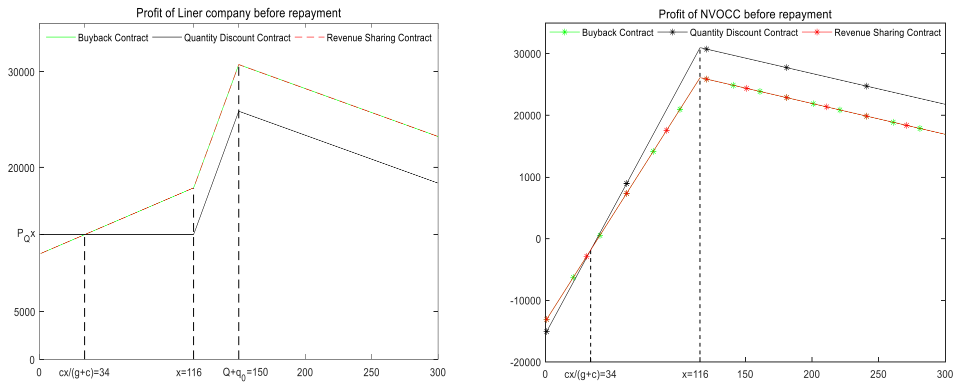

Figure 3.4 The Profits under three contracts with deterministic demands

Figure 3.4 shows the profit of the liner company and the NVOCC with deterministic demands. The horizontal axis of this Figure is the market demand d. From this diagram, we can see that when $d < 34$, the profit of the liner company under the buyback contract and the revenue sharing contract is less than the profit under the quantity discount contract. Whereas the profit of the NVOCC under the buyback contract and the revenue sharing contract is more than the profit under the quantity discount contract, indicating that both the buyback contract and revenue-sharing contract reduce the risk of the NVOCC suffering a bigger loss. This is in line with the outcomes of **Proposition 5**.

Based on the profit of the liner company in Figure 3.4, there are two cases for the repayment to the financial institution under deterministic demands. The horizontal axis of Figure 3.5 and Figure 3.6 represent the market demand. If the loan-to-value ratio is $\omega =$ 0.3 and the interest rate is $R = 8\%$ (Figure 3.5), the income of the financial institution and default cost is shown in Figure 3.5. When market demand is $d < 34$, the profits of the liner company under the buyback contract and the revenue-sharing contract are lower than the required repayment, whereas the liner company's profits under the quantity discount contract are larger. However, when market demand is $d \geq 34$, these three contracts do not affect the financial institution's income and default cost.

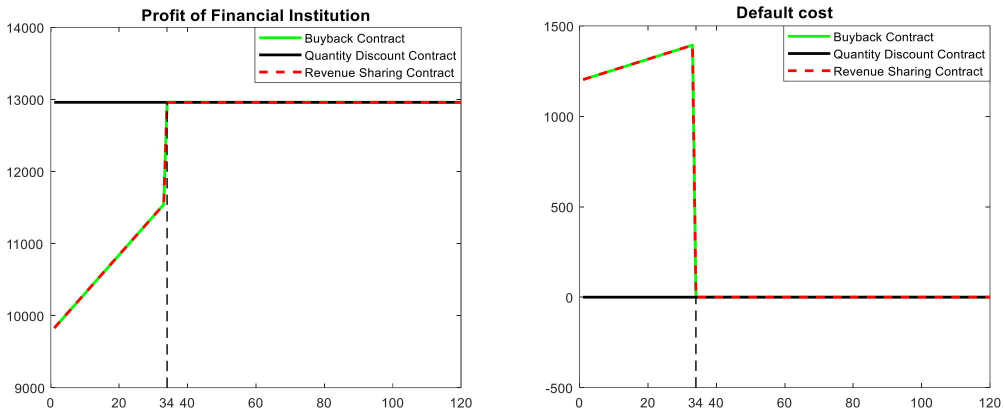

Figure 3.5 Financial institution's income and Default Cost (Case 1: ω = 0.3 R = 8%)

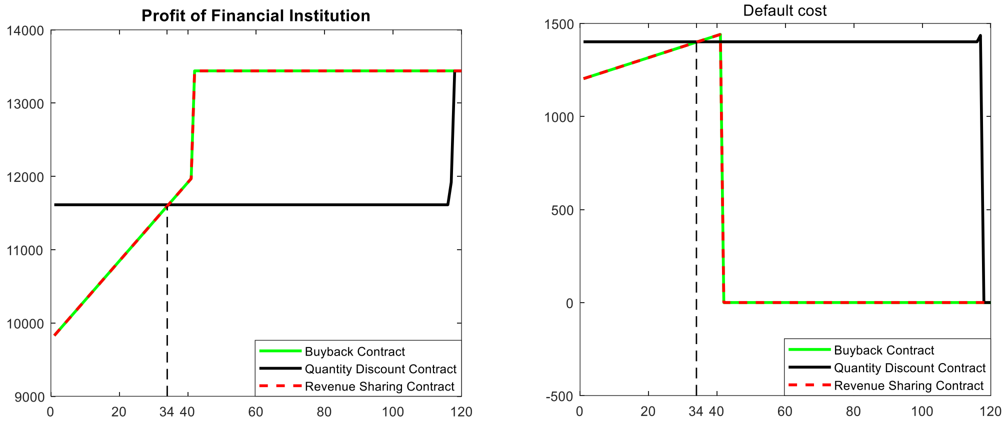

Figure 3.6 Financial institution's income and Default Cost (Case 1: ω = 0.3 R = 12%)

In the case that the loan-to-value ratio is $\omega = 0.3$ and the interest rate is $R = 12\%$ (Figure 3.6), the financial institution's income and default cost are shown in Figure 3.6. Similarly, if the market demands $d < 34$, the profit of the liner company under the buyback and revenue-sharing contracts is lower than the liner company's profit under the quantity discount contract. Therefore, the default costs under the buyback contract and the revenue-sharing contract are initially lower than that under the quantity discount contract. However, eventually, the default cost gradually increases and is slightly higher than the default cost under the quantity discount contract. When market demand is greater than or equal to 34, the default cost under the buyback contract and the revenue-sharing contract remains lower than that under the quantity discount contract. This leads to the financial institution's income under the buyback contract and the revenue-sharing contract being higher than its profits under the quantity discount contract.

The profit of the liner company in the above two cases is shown in Figure 3.7. In the above two cases, the profit of the liner company is generally higher with the buyback contract and the revenue-sharing contract. However, in the case of $\omega = 0.3, R = 8\%$, the profit of the liner company after repayment is slightly lower when choosing the buyback contract and the revenue sharing contract when the market demand is $d < 34$.

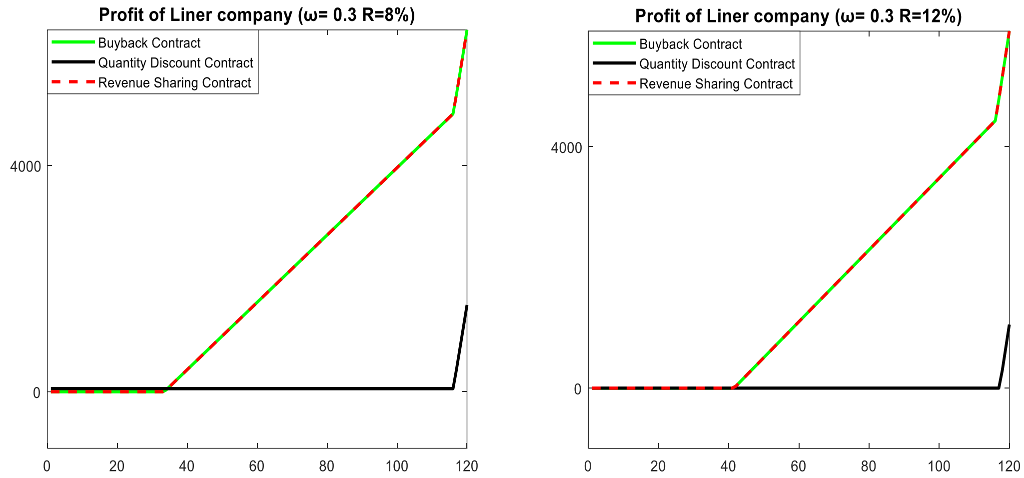

Figure 3.7 The Profit of the liner company

When facing uncertain demand, the first case is the loan to value ratio $\omega = 0.3$ and interest rate $R = 8\%$, which leads to case ii of **Proposition 8**. Under the buyback contract and the revenue-sharing contract, the financial institution's income is 12934.27, and the default cost is 17.67, which makes the profit of the liner company after repayment 5503.20. Under the quantity discount contract, the income for the financial institution is 12960, and the default cost is 0, which makes the profit of the liner company after repayment 1922.64. The second case is the loan to value ratio $\omega = 0.3$ and interest rate $R = 12\%$, which leads to case iii of **Proposition 8**. Under the buyback contract and the revenue-sharing contract, the income for the financial institution is 13387.68, and the default cost is 35.22, which makes the profit for the liner company after repayment of 5032.24. Under the quantity discount contract, the income for the financial institution is 12134.26, and the default cost is 1003.3, which makes the profit for the liner company after repayment 1745.07. From this, it is clear that if the liner company could control the repayment amount within an acceptable range in the slot purchase contract, a win-win situation can achieve for financial institutions and liner companies.

3.7 Conclusion and Future Research

In this paper, we have considered the case of liner companies that lease ships from financial institutions and are constrained to repay a series of rentals or instalments. The repayment depends on the net income of the liner companies when cooperating with the NVOCC, who are appointed to canvass orders for their owned and rented vessels. We have assumed that cooperation between liner companies and NVOCCs is through purchase contracts such as revenue-sharing contracts, buyback contracts, or quantity discount contracts. Because the failure to repay leads to variable and fixed default costs, we have investigated and compared the performance of these three contracts in the leveraged supply chains.

First, we sought the optimal contract parameters for a liner company and an NVOCC to address the issue of supply chain coordination before the rental payment and without default costs. In particular, we benchmarked this performance against a centralised model depicting the perfect collaboration between the liner company and the NVOCC. Then, we carried out an analysis of the profit under three types of contracts before repayment. We found that quantity discount contracts outperformed buyback contracts and revenue-sharing contracts in some cases. First of all, if it focuses on the decision-making power of contract parameters, a quantity discount contract can make both parties more likely to obtain a suitable non-negative profit before repayment under the condition of equation (9). Although a buyback contract can allow liner companies to increase profits by regulating the buyback price ($b = (1-\theta)P_r$) under the condition of $S_N(x) \geq \frac{P_s x + cx + ug}{P_r + g}$, it also increases the chance of NVOCC obtaining negative profits. In this way, the revenue-sharing contract at the same time will approach the quantity discount contract because the choice of θ belongs to NVOCC. Secondly, considering the profit before repayment in the face of deterministic market demand, the quantity discount contract is better than the revenue-sharing contract and buyback contract when $d \leq \frac{cx}{g+c}$.

Secondly, we analysed three types of contracts from the perspective of financial institutions. After substituting the functional relationship between several contract parameters into the profit function under stochastic market demand, the expected profits of the liner company entering into a revenue-sharing or buyback contract are identical and greater than the profit obtained through a quantity discount contract. However, after comparing the several cases of required repayment, we found that if the minimum profit of contract before repayment is higher than the rental, it is more conducive to the repayment. When the required repayment is low, there is zero default cost for the liner company when choosing the quantity discount contract, which guarantees that the financial institution receives the full rental and interest. Thus, although the expected profit for the liner company before repayment under both the buyback contract and the revenue-sharing contract is higher, the quantity discount contract is still more favourable to the financial institution. In this case, it is ineffective for financial institutions to use the profitability of liner companies to measure their solvency.

In summary, our results indicate that a favourable outcome for the financial institution might not correspond to the expected profit. The focus of the financial institution is less on profit and more on ensuring that the liner company is solvent enough to fully repay the financial leasing payment. Therefore, it implies the importance of controlling the deposit and interest within an acceptable range in financing activities and the necessity of adjusting the operating decision to support solvency.

Since our research is based on information symmetry, future research could look into the impact of stakeholders' biased decisions on the balance of operational decisions in slot purchase contracts and financing decisions in financial leasing. The law of supply and demand between market demand and market price might also impact the profit and cost of the liner company, the NVOCC and the financial institution. Another research direction is to consider the customers on the opposite port. Since the customer’s need on the opposite port is unknown, the liner company needs to manage the amount of the containers well, which could lead to a series of costs. After that, the research could focus on analysing the cost of each entity.

3.8 List of Symbols

d	The market demand quantity
d^*	The market demand when the liner company repays the finance lease
μ	$= E(d)$, the mean value of demand
$f(d)$	Probability density function of market demand
$\mathrm{F}(d)$	Cumulative distribution function of market demand
P_r	The freight rate paid by shippers
x	The order quantity of NVOCC
x_c	The order quantity that maximises the profit of centralised virtual entity
x_n^d	The order quantity that maximises the profit of NVOCC
x_l^d	The order quantity that maximises the profit of liner company
P_s	Contract parameter, freight fee paid from the NVOCC to the liner company
θ	Contract parameter, a percentage of the revenue that NVOCC keeps
θ_R^*	The percentage of the revenue that NVOCC keeps in revenue sharing contract
θ_B^*	The percentage of the revenue that NVOCC keeps in buyback contract
θ_Q^*	The percentage of the revenue that NVOCC keeps in quantity discount contract
T	Contract parameter, fixed money transferred from the liner company to the NVOCC if $T > 0$
b	The buyback price of unsold products
R	Financial lease interest rate
L	The cost for the financial institution to support the finance lease for each accounting period
Q	The capacity that liner company get from financially leased vessels
q_0	The initial capacity that liner company has from owned vessels
ω	Loan-to-value ratio
c	Unit operating cost
g	Unit goodwill loss
$S_N(x)$	The expected sales of the NVOCC
$I_N(x)$	The unsatisfied demand of the NVOCC
$S_L(x)$	The expected sales of the liner company

$I_L(x)$	The unsatisfied demand of the liner company
$S_c(x)$	The expected sales of the centralised virtual entity
$I_c(x)$	The unsatisfied demand of the centralised virtual entity
π_N	The expected profit of the NVOCC
π_L	The expected profit of the liner company
π_c	The expected profit of centralised virtual entity
ξ_L	Profit before repayment of liner company in decentralised model
ξ_c	Profit before repayment of centralised virtual entity in centralised model
Δ_d	The repayment from the liner company to the financial institution in decentralised model
Δ_c	The repayment from centralised virtual entity to the financial institution in centralised model
B	Unit default cost incurred when liner company could not repay the loan
B_d	The default cost of liner company in decentralised model
B_c	Unit default cost of centralised virtual entity in centralised model
δ_d	The income of financial institution in decentralised model
δ_c	The income of financial institution in centralised model

Chapter 4. The Interface between Financial lease contract and Container Shipping Contract

4.1 Introduction

The COVID-19 pandemic has highlighted the prominent role of container shipping as being a key sector for sustained global supply delivery and resumption of normalcy. According to the annual review of maritime transport in 2020 by the United Nations Conference on Trade and Development, roughly 80% of the commodities are transported by sea, and more than 13% of the world fleet by volume is carried by container ships, which reinforces the importance of the container shipping sector. The past decade has seen the rapid development of the container shipping network of suppliers and consumers (Yang et al., 2015). However, the 2020 annual review report also indicates that the shipping industry is facing severe challenges amid the significant decrease in world trade, demand contractions and global economic uncertainty induced by the epidemic. To address these issues, liner companies have employed several strategies to reduce costs and enhance economies of scale, including leasing vessels and cooperating with NVOCC (Li, 2006; Song et al., 2017).

In response to the unfavourable shipping market conditions and to keep pace with competitors, most liner companies have to devise charter/ownership plans and take their fleet management and financial risk into consideration. According to Cariou and Wolff (2013), around 50% of the containerships were chartered, and an increasing number of liner shippers lean on financial institutions for financing the lease of vessels rather than purchasing them. The financial lease contract for vessels is characterised by the fact that the financial institution (for example, a bank) offers funds to liner companies in exchange for operating control of the target vessels but receives the principal and interest back as a regular rental without extra collateral. However, underestimating chartering risks and shipping market circumstances might increase the insolvency risk of liner companies. Shin et al. (2019) indicated that inadequate chartering inventory management and weak market circumstances in the global maritime industry ultimately led to Hanjin's bankruptcy. In addition, though no collateral is required in applying for a finance lease contract, only companies with a favourable track record and creditworthiness can apply for the finance lease (Alexopoulos and Stratis, 2016; Li, 2006). Braglia et al. (2019) also outlined that inventory management cannot be ignored because the characteristics of major properties deteriorate with time. For vessels, it is the net profit brought by shipping capacity. In the container shipping field, the previous research on leasing issues mainly focused on the management of inventory (Gómez-Padilla et al., 2021; Cariou and Wolff, 2013) and limited inventory (Song et al., 2017; Li, 2006). None of them explores the ideal financial leased vessel quantity for the liner companies and its potential impact on profitability and solvency, which is the subject under investigation in this paper. As the loan-to-value ratio is commonly used in the finance industry, we will utilise this ratio to indicate the likelihood of liner companies defaulting on the financial lease contract, which is similar to the setting in Jiang and Liu (2018).

When cooperating with a liner company, the principal role of NVOCCs is to significantly assist liner companies in selling their shipment capacity to shippers in the container shipping industry. According to Song et al. (2017), NVOCCs function as wholesalers but do not operate vessels directly. Through purchasing blocks of container capacity through some formats of the slot purchase contract from liner companies, NVOCCs can sell this capacity block to shippers at market pricing. All capacities and demands are measured by TEU. The slot purchase contract proposed by the liner company may typically be in three forms: revenue-sharing contract, buyback contract or quantity discount contract (Snyder and Shen, 2019). If they wish to achieve the coordination of the container supply chain, the profit of each entity should reach the maximum value under the same optimal order quantity in the slot purchase contract. Furthermore, NVOCCs offer freight movement services for shippers and liner companies. The relationship between liner companies and NVOCCs can be characterised as a decentralised supply chain model where each entity operates independently as a supplier and retailer. In addition, a centralised model will be employed for comparison, where liner companies and NVOCCs can be regarded as a total centre to maintain pivotal roles in the long-term operational and financial aspects of liner companies. This centralised model enables the decision-making between these entities to be more consistent, leading to potentially more efficient and effective supply chain operations. However, it also requires greater cooperation and collaboration between liner companies and NVOCCs to ensure their respective interests are mutually beneficial.

During each accounting period, liner companies need to make decisions about the quantity and size of leased vessels, which would impact liner companies' shipping capacity (measured in TEUs). Therefore, this leased amount is significantly connected to NVOCC's selection of order amount in the purchase contract with liner companies.

Furthermore, the major influencing element on the payable of financial lease rent is the operational revenue of liner companies, which is the functional outcome of the contract between liner companies and NVOCCs. As a result, the contract selection will influence this revenue, directly influencing the risk of whether the lease contract is defaulted on due to the liner company being unable to pay the required rental. It is worth noting that defaulting on the financial lease contract may negatively influence the future operating and reputation (Li, 2006), which is tied to the financial strength to support future financing. The financial lease and slot purchase contract decisions will interact in this case.

An increasing number of studies investigate the design and application of specific forms of contracts in container shipping (Li et al., 2013; Wang et al., 2021; Xie et al., 2017; Zhang et al., 2019 and so on). However, they did not evaluate contract types in terms of a company's capacity to maintain creditworthiness under financial limitations. To fill this gap, we model a contract between an NVOCC and a liner company that has signed successive finance leasing contracts with a financial institution. The primary objective is to develop an optimum trade-off that ensures profit for the financial institution while also facilitating commerce between the liner company and the NVOCC. In this paper, shortages can be considered before the supply chain contract.

This paper is organised as follows: In Section 2, the corresponding literature will be reviewed. The models and related assumptions are introduced in Section 3. In Section 4, we will first briefly present the profit models of the liner company and NVOCC under the slot purchase contract and examine the case of coordination under each contract before repaying the financial lease rent. Secondly, we will compare the profit of the liner company and NVOCC under these three contract types and evaluate the impact of the market parameters, such as unit operating cost and unit goodwill loss on the contract variables. In Section 5, we will examine the relationship between the leasing amount and loan-to-value ratio. Section 6 provides numerical examples illustrating how the profit margins of liner companies, NVOCCs, and financial institutions vary according to the contract type. Section 7 closes the paper by summarising the findings and suggesting future areas for investigation.

4.2 Literature Review

In this section, a thorough and systematic analysis of the pertinent literature will be presented with the aim of identifying gaps in the current research and showing our contributions to the literature. Our review will include a wide variety of empirical and theoretical works, covering both foundational and cutting-edge research in the newsvendor game and coordination of supply chain contracts, with the objective of providing a thorough and detailed understanding of the research landscape in this study.

4.2.1 Contract Design and Comparison

Supply chain contracts have been explored in the literature from various points of view and for numerous purposes. The early literature predominantly focused on contract design for the contracted game-theoretic model, which mainly used contracts to facilitate cooperation in the supply chain (Höhn, 2010). Through proper information and incentive provisions, the contract could optimise supply chain performance. According to Cachon (2003) and Höhn (2010), the supply chain's contractual negotiation begins with the newsvendor model, where suppliers negotiate a wholesale pricing contract with retailers. Numerous studies have conducted literature reviews about contract negotiation and supply chain coordination with contracts in the newsvendor model (Cachon, 2003; Höhn, 2010; Liu et al., 2015; Guo et al., 2017; Bart et al., 2020). In literature, the contract types could be divided into wholesale price contracts (Bresnahan and Reiss, 1985), buyback contracts (Pasternack, 1985), revenue-sharing contracts (Cachon and Lariviere, 2005), quantity-flexibility contracts (Tsay, 1999), sales-rebate contracts (Taylor, 2002) and quantity-discount contracts (Moorthy, 1987). These contracts aim to choose the parameters that allow each party to make the best decision to optimise the supply chain's performance (Xiong et al., 2011).

According to Guo et al. (2017), buyback contracts, revenue-sharing contracts, and quantity-discount contracts are the most widely used contracts in the study of single supply chain contracts. Our paper focuses on the research behind these three contract types. Basically, the buyback contract refers to the retailer acquiring the items at wholesale prices but receiving a partial payback on unsold stock (Pasternack, 1985). Cachon and Lariviere's revenue sharing contract (2005) requires retailers to split a portion of their sales income with the supplier to coordinate the entire chain. The quantity discount contract organises the chain by selling all goods at a reduced wholesale price to the retailer (Moorthy, 1987).

In addition to the above research on single contracts, many recent papers focus on comparing different contract types. Guo et al. (2017) noted that a substantial fraction of recent publications covered more than one contract type. Within them, the prevalent comparisons are the buyback contract and revenue-sharing contract (Bart et al., 2020). Several studies indicate that buyback contracts are equivalent to revenue-sharing contracts when facing the same market price (Cachon and Lariviere, 2005; Cachon, 2003). By comparing the retailer's order quantity, Cachon (2003) found that the buyback contracts cannot coordinate the newsvendor with price-dependent demand, while revenue sharing contracts could coordinate if there is no goodwill loss, and the quantity discount contracts could coordinate if there is no goodwill loss for suppliers. From laboratory research, Katok and Wu (2009) found that the order quantity outcomes between the buyback and revenue-sharing contracts differ because of loss aversion. Zhang et al. (2016) proved that revenue-sharing contracts are more beneficial for the supplier than buyback contracts in a high critical ratio scenario. Kalkanci et al. (2011) examined the behaviour of suppliers under a quantity discount contract and discovered that the performance of this contract is worse than a basic wholesale pricing contract. Like these previous studies, our research will focus on comparing and designing contracts for rational suppliers and retailers in a specific industry, which can lead to differences in contract parameters due to the characteristics of the products, such as no residual value.

4.2.2 Contract Design under Financial Constraints

Many studies have illustrated the coordination condition of supply chain contracts for the supply chain system with limited funds. Recent research in this segment mainly addressed the problem of coordinating the financial and operational decisions in the newsvendor model. Yan et al. (2014) found the optimal strategy for the supply chain system with a manufacturer, a retailer, and a commercial bank where the retailer and the manufacturer cooperate via a wholesale price contract, and both are capital constrained. Jiang and Liu (2018) illustrated how the buyback contract between one overconfident supplier and one retailer changes with the loan that the retailer obtains from the bank.

Kouvelis and Zhao (2008) researched the financing problem of the supply chain of a supplier trading with a retailer via the supply chain contract. In their model, they compared how bank financing, trade credits and quantity discount contracts affected the profit of both the retailer and the supplier. Kouvelis and Zhao (2011) found unique equilibrium solutions for the newsvendor model to adopt wholesale price contracts when the retailer is under bankruptcy risk. They also structured a contract in which the supplier offers a discount on the wholesale price if the retailer pays early but charges interest if the retailer pays late (Kouvelis and Zhao, 2012). Considering the difference between the credit ratings of suppliers and retailers, Kouvelis and Zhao (2018) examined the interaction of the financial decision and operational decision via the early payment discount contract in a newsvendor model. Our paper is closely related to Kouvelis and Zhao's work in 2016. In their work, they compared the coordination condition of several types of contracts for the chain with one supplier and one retailer when both are facing bankruptcy risk (Kouvelis and Zhao, 2016). However, they assume that the interest rate for the loan is zero if the retailer and supplier are able to repay the debt obligation in full. Compared to these relevant publications, our paper's interest rate for the financial lease changes with the loan-to-value ratio, reflecting the current industrial practice.

4.2.3 Container Shipping Contract

Many studies have applied the mechanism of supply chain contracts to construct container shipping contracts that coordinate container supply chains and distribute resources fairly. Song (2021) outlined that most of the literature is mainly about calculating the precise contract parameters for a given contract type. In terms of buyback contracts, Xie et al. (2017) developed a bilateral buyback agreement for rail and liner companies to exchange empty containers. Kong et al. (2017) employed the buyback contract to optimise the slot amount supplied by liner firms and the quantity booked by shipping agencies. Regarding revenue sharing contracts, Wang and Liu (2019) compared two competing shipping service chains, each with a single carrier and a single port. They identified that when both chains choose different contracts, the winner is the party that chooses the revenue-sharing contract. However, a lose-lose situation will emerge if both chains select revenue-sharing agreements. Tan et al. (2018) compared the competition between an ocean carrier and an inland shipping company with their cooperation under a revenue-sharing contract. Zhang et al. (2019) developed a revenue-sharing contract for logistics service providers to promote horizontal logistics collaboration in a decentralised model and coordinate the system. Li et al. (2013) constructed a bidirectional revenue sharing contract to solve the preventative lateral transport problem between two locations. Liu et al. (2013) proposed the fairest revenue-sharing contract strategy for the chain consisting of logistics service integrators and functional logistics service providers. Wang et al. (2017) considered the effect of the canvassing strategy on the shipping service supply chain formed by an ocean shipping (OS) company and an inland shipping (IS) company and the revenue-sharing rate between them.

Regarding quantity discount contracts, Song et al. (2019) built the model with quantity discount contracts between a liner company and a forwarder and illustrated how the profit of each party and order quantity change with a canvassing strategy. Wang et al. (2021) compared the coordination conditions of the quantity discount contract with the two-part tariff contract in the carrier-shipper chain. Song et al. (2017) proposed a modified quantity discount contract for an online retailer and a delivery operator. Yin and Kim (2012) measured the optimal quantity discount price of the container shipping company when trading with several freight forwarders. Qiu and Lee (2019) built a model wherein shippers export their cargo worldwide through seaports. To reduce transportation costs, dry port systems are used to connect shippers and seaports. They designed a quantity-discount contract for dry ports to trade with multiple shippers.

Apart from these three specific contracts, several other researchers have investigated the design and application of other types of supply chain contracts. For example, Xu et al. (2015) designed a subsidy contract for the sea cargo service chain to reposition empty containers between one carrier and two freight forwarders across two ports. Song et al. (2017) compared two canvassing tactics for carriers, which involved freight forwarders or NVOCCs. In their model, general supply chain contracts are used between NVOCCs and carriers. The existing literature on container shipping contract design has primarily formulated contracts based on contract coordination theory, with little consideration given to the impact of funds and inventories (for example, vessels and containers) constraints on contract decisions.

4.2.4 Limited inventory

In the liner shipping sector, the literature on limited inventories can be divided into two segments: inventory financing and inventory management. The first section demonstrates the effect of financial decisions regarding fleet development (including leasing schemes, shipping funds, shipbuilding credit, and so forth) on the operation of shipping companies. The literature on this area is vast. But most of them are empirical research. Examples include Gómez-Padilla et al. (2021) and Cariou and Wolff (2013). Our paper falls into the second category, focusing on matching the inventory level with uncertain demand. Liu et al. (2015) studied a two-stage batch ordering strategy for the logistics service integrator to satisfy the updated demand. Feng et al. (2015) proposed a tying mechanism to allocate air cargo capacity reasonably. Song et al. (2017) compared two canvassing strategies to enable a carrier to resist variability in market demand. Xie et al. (2017) developed the empty container inventory-sharing strategy for an intermodal transport system to meet the demand for empty containers. None of the above papers linked the issue of financial risk and inventory risk with contract coordination, and we explicitly explore this issue in addition to contract comparison.

4.2.5 Research gaps and opportunities

Overall, this study could contribute to a more comprehensive understanding of how slot purchase contracts perform in a complex and constantly evolving market environment and provides valuable insights for liner companies and financial institutions looking to improve their financial leasing strategies. The findings of this research are expected to add to the existing literature on the coordination of supply chain contracts and contribute to the broader discourse on shipping finance. Specifically, this analysis aims to provide insight into which slot purchase contracts can fulfil the requirements of financial institutions and enable liner companies to repay their financial lease obligations in the face of constantly fluctuating market conditions. It is worth noting that prior research has overlooked the complexities of dynamic market conditions and their implications for contractual design and implementation in the context of liner shipping. By addressing this research gap, the results of this study could offer a practical financial lease guideline for liner companies and financial institutions as they seek to make informed decisions about their leasing and financing strategies in the shipping industry.

4.3 Model Setup and Preliminaries

The problem considered in this article is whether the liner company's finance leasing and operation are mutually beneficial and constrained by one another. To study the liner company's optimal financing lease strategy, we constructed a stylised container shipping system involving a financial institution, a liner company (supplier), NVOCC (retailer) and shippers (consumers). One of the major objectives of our research is to find an equilibrium which will achieve channel coordination and the full repayment of financial lease rent. The second part of our research objective is to compare the preferences among buyback contracts, revenue-sharing contracts, and quantity discount contracts from the perspective of a liner company's profitability and solvency. Table 4.1 lists the major notations mentioned in our article. To better illustrate, we will use the subscripts ‘c’, ‘n’ and ‘l’ to denote the centralised company, NVOCC and the liner company, respectively. The superscript ‘d’ both stand for the decentralised model.

Parameters

d	The market demand quantity
$R(\omega)$	Financial lease interest rate, $R(\omega) = \rho\omega + \varepsilon$
Q	The capacity that the liner company gets from financially leased vessels
q_0	The initial capacity that the liner company has from owned vessels
P_r	The freight rate paid by shippers for each TEU capacity sold
c, c_1, c_2	Operating cost per unit of TEU capacity sold, $c = c_1 + c_2$
$\mathrm{g}, \mathrm{g}_1, \mathrm{g}_2$	Goodwill loss per unit of unmet TEU capacity, $\mathrm{g} = \mathrm{g}_1 + \mathrm{g}_2$
μ	= E(d), the expected value of demands

Decision Variables

$x(x_c, x_n^d, x_l^d)$	The order quantity of NVOCC, x ∈ [0, $Q + q_0$]
$\theta(\theta_R^*, \theta_B^*, \theta_Q^*)$	Contract parameter, the NVOCC's revenue share from the sales
P_s	Wholesale price, freight fee paid from the NVOCC to the liner company
T	Fixed money transferred from the liner company to the NVOCC if T > 0

ω	Loan-to-value ratio
L	The cost for the financial institution to support the finance lease for each accounting period

Other Variables

π_N, π_L, π_c	The profit function of stakeholders
ξ_L^1, ξ_L^2, ξ_L	The profit of the liner company from slot purchase contract before the repayment, the profit of the liner company from direct selling before the repayment and the sum of them, respectively
Δ	The rent payable from the liner company to the financial institution
δ_F	The income of financial institutions ($\delta_F \leq \Delta$)
B	Unit default cost incurred when the liner company could not repay the loan $B = \alpha\xi + \beta$

Table 4.1 Notations of Chapter 4

4.3.1 Sequence of Events

The sequence of decision events is depicted in Figure 4.1. During the contract negotiation period, two contracts are to be signed, namely the financial lease contract (between the liner company and financial institution) and the slot purchase contract (between the liner company and NVOCC). At the start of each accounting period, the liner company will first renew the successive vessels' financial leasing contracts $\left(Q,\ R(\omega)\right)$ to remain afloat (event 1). Under these contracts, the financial institution will provide the financial lease cost L and the interest rate R based on the financial lease amount Q and loan-to-value ratio ω. After these, event 2 happened, that is, the liner company and NVOCC engaged in negotiation for the slot purchase contract $(x, P_s, \theta,\ T)$. Behind event 2, the NVOCC is committed to canvassing orders from the market, and the liner company is devoted to completing all relevant transportation, loading and sailing. During the payment transfer period, the liner company will first clear the payment involved in the slot purchase contract and then repay the rental to the financial institution (to be specific, event 3 occurred prior to event 4). After event 4, one operating season ends, and the next cycle begins.

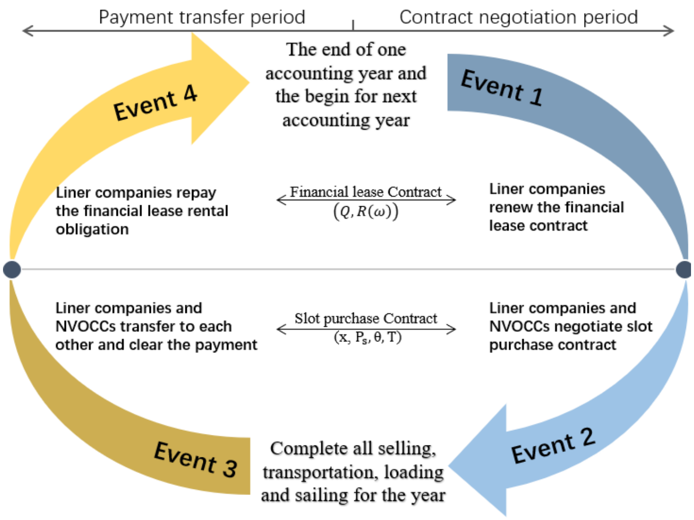

Figure 4.1 The Sequence of Decision Events

During events 2 to 3, we consider two models to normalise the contract: decentralised and centralised. In the decentralised model, each party makes their decision separately. Firstly, NVOCC decides the order quantity x according to market conditions. Then, the liner company designs the contract parameters (P_s, θ, T), which includes the wholesale price P_s, the NVOCC's revenue share parameter θ and correlative payment T. T ≥ 0 represents the amount of money that the liner company transfers to the NVOCC and T < 0 means the fixed amount of money transferred from the NVOCC to the liner company. There are two-echelon selling processes in the decentralised model. First, the NVOCC sells the purchased capacity to the shipper, which is selling process 1. Thereafter, the liner company sells the surplus capacity directly to the shipper to fully satisfy market demand, which is set to be selling process 2. Since this amount of profit has no relationship with NVOCC and the slot purchase contract, the profit that the liner company makes from this direct sale will not be included in the decentralised model. In the centralised model, the liner company and the NVOCC will be considered as a single entity. Specifically, they will be treated as a single entity to facilitate decision-making and resource optimisation. Within this model, the entire entity directly sells all the slot capacity x to meet the market demand. It should be noted here that part of the capacity over x is not considered in the centralised model because the direct sales channel of the liner company is not covered in the decentralised model. Since the decentralised model can never be more efficient than the centralised model, we will utilise the centralised model as a normative benchmark for the decentralised model. The two models are depicted in Figure 4.2.

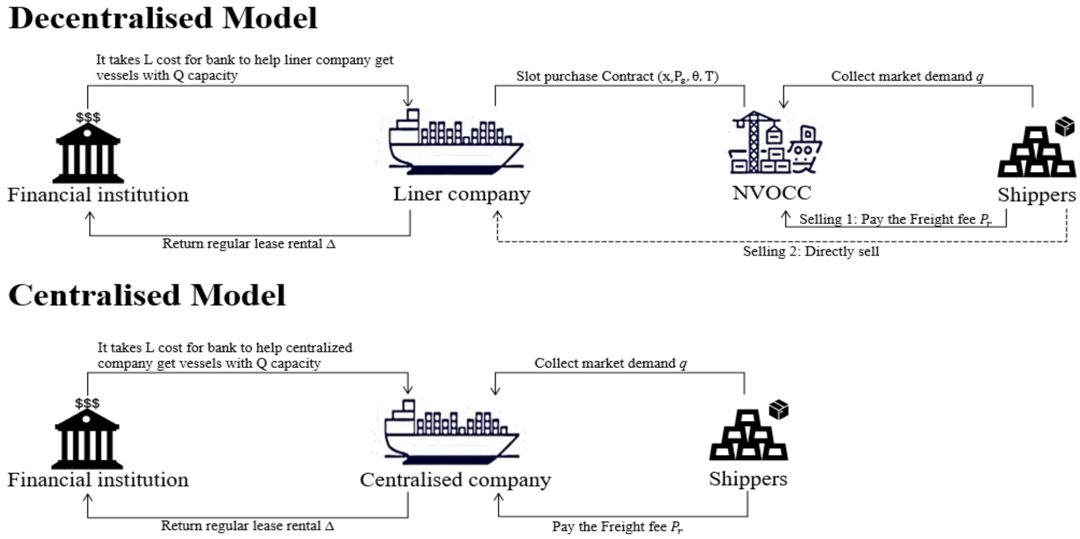

Figure 4.2 Relationships in the Container Shipping Supply Chain

4.3.2 Symbolic Description and Assumptions

Let d denote the demand from shippers over the selling period, which is a stochastic variable with a probability density function (*PDF*) *f(d)* and a cumulative distribution function (*CDF*) *F(d)*. The freight fee paid by shippers is P_r per unit of TEU capacity sold. Similar to the market notation set by Snyder and Shen (2019), the NVOCC incurs an operating cost c_2 for each unit of TEU capacity, and the liner company has a corresponding operating cost c_1 for each TEU capacity. For each TEU unsatisfied demand, the liner company will incur a goodwill loss g_1 and NVOCC will incur a goodwill loss g_2. Here, we set $c = c_1 + c_2$ as the unit cost for each TEU capacity sold in the centralised model. $g = g_1 + g_2$ is the goodwill loss for each unmet TEU capacity in the centralised model, which occurs when the demand of the market exceeds the capacity that the centralised company can provide. Since the collaboration between the liner company and NVOCC is in the form of a supplier-retailer model, this container shipping chain contract is reverted to a slot purchase contract to enable coordination between them. We use a set of contract parameters (x, P_s, θ, T) provided by Kouvelis and Zhao (2016) to represent buyback contracts, revenue sharing contracts and quantity discount contracts at the same time.

The closest work to our problem is the study carried out by Kouvelis and Zhao (2016), which contrasted these three forms of contracts when both suppliers and retailers face the risk of bankruptcy but still do not provide the company with the optimal contract parameters or financing strategies to avoid bankruptcy. Besides, the product in our work has the characteristic of no salvage value. Therefore, the second part of our problem is finding the appropriate financial lease amount Q and financial lease cost L for the liner company and establishing the corresponding loan-to-value ratio ω for the financial institution to rate the financial lease contract. The difficulty of our work is that the supply chain contact interacts with the financial lease contract. We will solve this by assuming that no financial institution is engaged in the design of slot purchase contracts to isolate operations decisions from financing decisions. In addition, this financial lease is highly related to the maximum capacity that a liner company could provide. Since the financial lease is equivalent to a loan in terms of functionality (Risk, 2014), we set the loan-to-value ratio $\omega = \frac{L}{P_r \mathrm{Q}}$ as Jiang and Liu (2018) to build the relationship between the loan cost in each period and the market price of the product. It is evident that the top side of the fraction is the financial lease cost L, and the bottom part is the market value of the financially leased vessels. However, unlike the fixed interest rate set by Jiang and Liu (2018) and Kouvelis and Zhao (2016), we set a positive correlation between the interest rate and the loan-to-value ratio as $R(\omega) = \rho\omega + \varepsilon$, $\rho > 0, 1 > \varepsilon > 0$. The primary rationale for this setup is that financial institutions use the loan-to-value ratio to determine the degree of financial risk they are incurring with their investment selections (Jiang and Liu, 2018). Typically, the higher ratio means that the lender will need to provide additional funding to support the financial lease contract or that the asset's value is less than its market value. Then, the borrower is deemed to have a higher default risk. As a result, the financial institutions would offer a higher interest rate to offset the financial risk.

Based on the above improvements, the financial lease cost to each accounting period on the financial institution's account is $L = P_r \mathrm{Q}\omega$ and the rent payable on the liner company's account is $\Delta = L \times [1 + R(\omega)] = P_r \mathrm{Q}\omega[\rho\omega + \varepsilon + 1]$. After clear transfer in event 3, the profit that liner company has is ξ_L and the payment that financial institution receives in event 4 is δ_F. If $\xi_L \geq \Delta$, the liner company could repay the full financial lease rental. Otherwise, the liner company does not have enough profit to repay the financial lease contract. Then, default costs will occur on the profit that liner company has ξ_L after event 3 and the payment δ_F that financial institution receive will be less than $\Delta = P_r Q\omega[\rho\omega + \varepsilon + 1]$. The default cost is the impairment loss of the recoverable amount. It is not received by the financial institution but by other institutions that charge the court costs, reasonable attorney's fees, and other additional charges and expenses that are associated with the default activity. According to Kouvelis and Zhao (2016), we set the default costs B comprising a fixed administrative fee β and a variable fee which has a proportional relationship α with the profit that liner company has ξ_L after event 3. Therefore, $B = \alpha\xi_L + \beta$.

Assumptions about the model are as follows:

(1) We assume that the total capacity held by the liner company exceeds the quantity that NVOCC might order, $Q + q_0 \geq x$. Considering this is a two-stage contract, the financial leasing is negotiated prior to the slot purchase contract. However, the slot purchase contract payment is clear prior to the payment in the financial lease contract. All the participants in these two contracts are risk neutral.

(2) We assume that the liner company operates the vessels according to fixed round-trip and other realistic factors are also kept fixed. Then, the capacity from the leased vessels is equal for each fixed accounting period.

(3) The total financial lease cost for the financial institution is the price of leased vessels. We amortise this cost into equal amounts L over the term of the financing lease contract.

(4) The liner company sells its remaining capacity directly to shippers only when the NVOCC does not have enough capacity to meet market demand. Since the profit from direct selling process 2 will be considered a separate profit, it will be excluded from the liner company's profit from the slot purchase contract.

(5) There is no moral hazard in the supply chain, which implies everyone will fulfil their obligations to comply with the contract and will not breach it. The information system is symmetrical, meaning all participants have the same information.

4.3.3 Mathematical model

We will formulate the expected profit of all participants under each event stage. To facilitate the explanation of our formulations, we use subscripts N, L, F, d and c to denote the NVOCC, liner company, financial institution, the decentralised model and the centralised model, respectively. We set $S_N(x)$ denote the satisfied demand of NVOCC at the end of the selling process 1:

$$S_N(x) = E[min(x,d)] = x - \int_0^x F(d)dd$$

The unsatisfied demand parts of NVOCC at the same time is:

$$I_N(x) = E[(d-x)^+] = \mu - x + \int_0^x F(d)dd$$

After the selling process 1, the expected sales of the liner company in the direct selling process is:

$$S_L(x) = E[(\mathrm{Q}+\mathrm{q}_0 - x - d)^+] = (\mathrm{Q}+\mathrm{q}_0 - x) - \int_x^{\mathrm{Q}+\mathrm{q}_0} F(d)dd$$

Then, the stockout capacity of liner company during the selling process 2 is:

$$I_L(x) = E[(d - \mathrm{Q} - \mathrm{q}_0)^+] = \mu - (\mathrm{Q}+\mathrm{q}_0) + \int_0^{Q+q_0} F(d)dd$$

Within event 2 to event 3, the NVOCC's expected profit function is:

$$\pi_N(x, P_s, \theta, \mathrm{T}) = \theta P_r S_N(x) + T - P_s x - c_1 x - I_N(x)\mathrm{g}_1 \tag{1}$$

We use superscripts 1 and 2 to denote the liner company's expected cash flow from selling process 1 and selling process 2, respectively. Then, the liner company's expected profit function during selling process 1 is:

$$\xi_L^1(x, P_s, \theta, \mathrm{T}) = P_s\, x + (1-\theta)P_r S_N(x) - \mathrm{T} - c_2 x - I_N(x)\mathrm{g}_2 \tag{2}$$

Whereas the liner company's expected profit from selling process 2 is:

$$\xi_L^2(x, P_s, \theta, \mathrm{T}) = P_r S_L(x) - (Q + q_0 - x)c - I_L(x)\mathrm{g} \tag{3}$$

Then, the total expected profit function of liner company after event 3 is:

$$\xi_L(x, P_s, \theta, T) = \xi_L^1 + \xi_L^2$$

The expected profit of centralised company is, therefore:

$$\begin{aligned} \pi_c &= \pi_N(x, P_s, \theta, T) + \xi_L^1(x, P_s, \theta, T) \\ &= P_r S_N(x) - (c_1 + c_2)x - (g_1 + g_2) I_N(x) \end{aligned} \tag{4}$$

After event 3, the liner company needs to face the rent payable, which is $L[1 + R(\omega)]$. If the total expected profit of the liner company after event 3 is more than the rent payable, the financial institution could receive the full rent repayment. If this profit is lower than the rent payable, the liner company can only use the sales revenue to cover rental obligation and the financial institution can only receive a portion of the rent. Therefore, the payment that the financial institution receives in event 4 is:

$$\delta_F = \begin{cases} P_r Q\omega[1 + R(\omega)] & if\ no\ default \\ \xi_L - B(\xi_L) & if\ default\ occurs \end{cases} \tag{5}$$

Here, $B(\xi_L)$ are the default costs, which can be rewritten as $B(\xi_L) = \alpha\xi_L + \beta$.

Our main research aim is to investigate how the decision around the financial lease contract varies with the contract parameters of the slot purchase contract. The decentralised model may generate the same maximum profit as the centralised model does by using a particular set of contract parameters. The fundamental criterion for measuring this condition is that the order quantity that maximises profit in both models is the same. This profit is tied to the financial lease rental, which is the major indicator for the financial institution and the liner company when making a decision.

4.4 Slot Purchase Contracts with NVOCCs: Operational Analysis

In this section, we first discuss the optimal contract parameters in greater detail for the liner company and NVOCC to collaborate in a decentralised model while using the centralised model as a normative benchmark. Additionally, we compare the profitability of the NVOCC and the liner company under both deterministic and stochastic demands.

4.4.1 Optimal Slot Purchase Contract Parameters

After formulating each participant's profit function in the previous section, it is vital to define the contract parameters (x, P_s, θ, T) and their boundaries for each contract type. We take equation (2) as an example and start with the revenue sharing contract. $(1-\theta)\mathrm{P_r}S_N(x)$ could refer to the funds transferred by the NVOCC to the liner company in the revenue sharing contract. As a result, T= 0 and $0 < \theta < 1$. Second, in a quantity discount contract, $\theta = 1$, resulting in $(1-\theta)\mathrm{P_r}S_N(x) = 0$. In this situation, no revenue-sharing component is included in the profit function of liner company and $\mathrm{T} = 0$. The unique feature of this contract is that the liner company gives a lower price for the NVOCC than it does in other contracts. Third, we fix the buyback price provided by the liner company to the NVOCC in the buyback contract as $\mathrm{b} = (1-\theta)\mathrm{P_r}$ and $\mathrm{T} = \mathrm{b}x$. As a result, $(1-\theta)\mathrm{P_r}S_N(x) - \mathrm{T} = \mathrm{b}S_N(x) - \mathrm{b}x = -\mathrm{b}[x - S_N(x)]$, which is the buyback portion paid by the liner company to the NVOCC. Proofs of all theoretical results are included in the Appendix B.

Corollary 1. π_c is strictly concave in *x*.

Corollary 1 indicates that there exists an optimal order quantity *x* that maximises the profit of centralised model under given market parameters. Then, we use superscript * to denote the corresponding cases when the stakeholders choose the optimal contract parameters. In the centralised model, the optimal amount of shipping capacity is x_c^*, with the objective of maximising the expected profit of centralised company π_c^*. Under this, the following conditions must hold:

$$\underset{x}{Maximize} \qquad \pi_c^* = P_r S_N(x) - (c_1 + c_2)x - (g_1 + g_2) I_N(x)$$

$$s.t. \qquad x \geq 0$$

$$\frac{\partial \pi_c}{\partial x} = 0$$

From these, we could solve the relationship between the optimal order quantity of the centralised model and the market parameters as:

$$F(x_c^*) = \frac{P_r - c + g}{P_r + g} \tag{6}$$

Because Snyder and Shen (2019) assert that the contract could coordinate the container shipping chain when the order quantities that optimise each participant's profit function are identical. Therefore, $x_c^* = x_N^* = x_L^*$. Since F (·) is a monotone increasing function, $F(x_c^*) = F(x_N^*) = F(x_L^*)$.

Proposition 1. In the decentralised model, when the optimal order quantity in the slot purchase contract satisfies the above equation (6), the optimal wholesale price satisfies the following three equations according to different contract types:

$$P_s(\theta) = \begin{cases} (1 - \theta_B^*)P_r + \dfrac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}, & \text{buyback contract} \\ \dfrac{\theta_R^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}, & \text{revenue sharing contract} \\ \dfrac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}, & \text{quantity discount contract} \end{cases}$$

From **Corollary 1** and equation (6), we characterise the relationship between contract decisions as **Proposition 1** when the objective is to maximise the expected profit of each entity under the same order quantity decision. From equation (4), the profit of the centralised company is the summary of the NVOCC's profit and the liner company's profit. Therefore, both the NVOCC and the liner company can allocate the profit of the centralised model arbitrarily under **Proposition 1**.

Corollary 2. If the NVOCC and the liner company want to divide the total profit equally, the following equilibrium condition must hold:

$$\theta^* = \frac{1}{2} + \frac{I_N(x)P_r + I_N(x)g + cx}{S_N(x)P_r + S_N(x)g - cx} * \frac{2g_1 - g}{2P_r} \tag{7}$$

It is evident that the division of goodwill loss g will highly influence the contract decision θ (revenue share rate), which affects the profit of both the NVOCC and the liner company. After this selling process 1, the liner company has $Q + q_0 - x$ shipping capacity reserved for the direct sales process. Therefore, the profit of the liner company in selling process 2 will also be affected by the contract decision x. To find the relationship between the order quantity x and the expected profit from selling process 2, we provide the following equation and constraint:

$$\underset{x}{Maximize} \qquad \xi_L^2(x, P_s, \theta, \mathrm{T}) = P_r S_L(x) - (Q + q_0 - x)c - I_L(x)\mathrm{g}$$

$$s.t. \qquad \frac{\partial \xi_L^2(x, P_s, \theta, \mathrm{T})}{\partial x} = 0$$

Because $\frac{\partial^2 \xi_L^2(x,P_s,\theta,\mathrm{T})}{\partial x^2} = P_r f(x) > 0$, we find that when the following equation (8) is satisfied, the liner company minimises expected profit from selling process 2.

$$F(x') = \frac{P_r - c}{P_r} \tag{8}$$

When comparing it with the optimal order quantity in equation (6), we find that:

$$F(x_L^*) = F(x_c^*) = \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}} > \frac{P_r - c}{P_r} = F(x')$$

It indicates that the change from x' to x_L^* will increase the liner company's expected profit from the selling process 2. Then, the liner company is delighted with the NVOCC's decision of $x_N^* = x_L^*$. Besides, it is essential to highlight that the selection of x_L^* will also help the entire shipping chain to generate higher profits, as compared to x'. This result highlights the potential benefits of coordinated supply chain management, in which individual entity optimisation is aligned with the optimisation of the whole shipping chain.

4.4.2 Profit allocation

Based on the prior findings, we will preliminarily examine the preference of the liner company and NVOCC between these three contract types where the contract terms are ideal from a profit maximisation perspective in this sub-section.

From **Corollary 1** and **Proposition 1**, we can understand the decision sequence to get the optimal contract parameters to maximise the profit function of the liner company and the NVOCC under each contract type. First, the NVOCC will choose the same x that satisfies equation (6) as the optimal option to maximise the expected profit in each contract. For better expression, we define this optimal order quantity $x^* = G(\mathrm{g}, c)$. Second, θ will be determined by NVOCC under the revenue sharing contract and by the liner company under the buyback contract as an indicator to fix the buyback price. Then, for θ under each contract type, the liner company will select the corresponding $P_s(\theta)$ in **Proposition 1**. To illustrate the effect of contract modifications on each participant's profit during each process, we substitute the functional connection between the contract parameters under each contract into each profit function. Therefore, the expected profit for the liner company and the NVOCC will satisfy the following equation under a buyback contract:

$$\xi_{L_B}^{1} = \left[(1-\theta_B^*)P_r + \frac{\theta_B^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}}\right] x^* + (1-\theta_B^*)P_r S_N(x^*) - T - c_2 x^* - I_N(x^*)\mathrm{g}_2$$

$$\xi_{L_B}^{2} = P_r S_L(x^*) - (Q + q_0 - x^*)c - I_L(x)\mathrm{g}$$

$$\pi_{N_B} = \theta_B^* P_r S_N(x^*) + \mathrm{T} - \left[(1-\theta_B^*)P_r + \frac{\theta_B^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}}\right] x^* - c_1 x^* - I_N(x)\mathrm{g}_1$$

Similarly, the expected profit for the liner company and the NVOCC under a revenue sharing contract will satisfy:

$$\xi_{L_R}^{1} = \left[\frac{\theta_R^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}}\right] x + (1-\theta_R^*)P_r S_N(x) - c_2 x^* - I_N(x^*)\mathrm{g}_2$$

$$\xi_{L_R}^{2} = P_r S_L(x^*) - (Q + q_0 - x^*)c - I_L(x^*)\mathrm{g}$$

$$\pi_{N_R} = \theta_R^* P_r S_N(x^*) - \frac{\theta_R^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}} x^* - c_1 x^* - I_N(x^*)\mathrm{g}_1$$

While under a quantity discount contract, the expected profit of the liner company and the NVOCC are:

$$\xi_{LQ}^{1} = [\frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}]\, x^* - c_2 x^* - I_N(x^*) g_2$$

$$\xi_{LQ}^{2} = P_r S_L(x^*) - (Q + q_0 - x^*)c - I_L(x^*) g$$

$$\pi_{N_Q} = P_r S_N(x^*) - [\frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}] x^* - c_1 x^* - I_N(x^*) g_1$$

As a result of the above equations, we have evidence that altering contract types will have no effect on the liner company's profit in the selling process 2.

Corollary 3. When the market is stable, the buyback and revenue-sharing contracts are identical since they both generate equivalent profit for the liner company and the NVOCC. By simply adjusting θ, the liner company and NVOCC can determine their distribution of centralised profit under buyback and revenue-sharing contracts. Under the quantity discount contract, the liner company will earn a negative expected profit in the selling process 1.

Based on **Corollary 3** and **Corollary 2**, we can conclude that there exists the same θ as equation (7) for the liner company and the NVOCC to split the centralised profit equally under the buyback contract and the revenue sharing contract. However, if **Proposition 1** holds for the relationship between P_s and θ, the three contracts can coordinate the entire supply chain and:

1. If $\theta < \frac{\mu g_1 P_r - \pi_c^*(x) g_1}{\pi_c^*(x) P_r + \mu g P_r}$, then the NVOCC earns negative profit, and the liner company earns more than π_c^*.

2. If $\theta = \frac{\mu g_1 P_r - \pi_c^*(x) g_1}{\pi_c^*(x) P_r + \mu g P_r}$, then the liner company earns the entire π_c^*.

3. If $\frac{\mu g_1 P_r - \pi_c^*(x) g_1}{\pi_c^*(x) P_r + \mu g P_r} < \theta < \frac{\pi_c^*(x) P_r + \pi_c^*(x) g_2 + \mu g_1 P_r}{\pi_c^*(x) P_r + \mu g P_r}$, then the liner company and NVOCC share the π_c^*.

4. If $\theta = \frac{\pi_c^*(x) P_r + \pi_c^*(x) g_2 + \mu g_1 P_r}{\pi_c^*(x) P_r + \mu g P_r}$, then the NVOCC earns the entire π_c^*.

5. If $\theta > \frac{\pi_c^*(x) P_r + \pi_c^*(x) g_2 + \mu g_1 P_r}{\pi_c^*(x) P_r + \mu g P_r}$, then the liner company earns a negative profit, and the NVOCC earns more than π_c^*.

Since the optimal x^* remains constant as equation (6), we can observe the effect of other contract parameters on the profit allocation of the liner company and the NVOCC from this point. Here, π_c^* is the maximum centralised profit that can be earned when an optimal order quantity of $x^* = G(\mathrm{g}, c)$ is placed by NVOCC. In addition, the stability of the market in **Corollary 3** means c_1, c_2, g_1 and g_2 maintain a consistent condition. However, the market demand is unpredictable, resulting in two profit comparison options: one is based on stochastic market demand and the other is based on deterministic market demand.

Proposition 2. When market demand is stochastic, the profit of the liner company and NVOCC before the repayment stage have the following relationships:

$$\xi_{L_B}(x, P_s(\theta_B^*), \theta_B^*, T) = \xi_{L_R}(x, P_s(\theta_R^*), \theta_R^*, 0) \geq \xi_{L_Q}\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

$$\pi_{N_B}(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_{N_R}(x, P_s(\theta_R^*), \theta_R^*, 0) \leq \pi_{N_Q}\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

Therefore, liner companies are more profitable under buyback contract and revenue sharing contracts than they are under quantity discount contracts. However, the NVOCC earns less profit under revenue-sharing and buyback contracts than it does under quantity discount contracts. Additionally, under the quantity discount contract, the liner company will incur a loss in the first selling process. This point can also be proved through the analysis of the previous profit distribution. In the previous section, we clarified that θ should be equal to 1 when the quantity discount contract is selected. Because of $\frac{\pi_c^*(x)P_r + \pi_c^*(x)\mathrm{g}_2 + \mu \mathrm{g}_1 P_r}{\pi_c^*(x)P_r + \mu \mathrm{g} P_r} = 1 - \frac{[\mu P_r - \pi_c^*(x)]\mathrm{g}_2}{\pi_c^*(x)P_r + \mu \mathrm{g} P_r} < 1$, the situation of liner company under quantity falls to the above condition 5.

However, when market demand is deterministic, there are two cases for the profit of each stakeholder. When market demand is low because $d < x$, the realised profit of the liner company under the buyback contract, revenue sharing contract and quantity discount contract are, respectively,

$$\xi_{L_B} = \xi_{L_R} = \frac{\theta_B^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}} x + (1 - \theta_B^*)P_r d - c_2 x - (Q + q_0 - x)c$$

$$\xi_{L_Q} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_2 x - (Q + q_0 - x)c$$

The realised profit of the NVOCC under the buyback contract, the revenue sharing contract and the quantity discount contract are, respectively,

$$\pi_{N_B} = \pi_{N_R} = \theta_B^* P_r d - \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x$$

$$\pi_{N_Q} = P_r d - \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x$$

From the equation, we find that, although in the previous **Proposition 2**, the profit of the liner company under the buyback contract and revenue sharing contract is greater than that of the quantity discount contract when the market demand is stochastic. However, when the market demand is deterministic, there is a deficient market demand $d = \frac{cx}{P_r + g}$, which leads to equal realised profits under these three contracts.

Proposition 3. When it is possible to forecast the market demand situation,

i. If the realised market demand $d \leq \frac{cx}{P_r + g}$,

$$\xi_{L_B}(x, P_s(\theta_B^*), \theta_B^*, T) = \xi_{L_R}(x, P_s(\theta_R^*), \theta_R^*, 0) \leq \xi_{L_Q}(x, P_s(\theta_Q^*), 1{,}0)$$

$$\pi_{N_B}(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_{N_R}(x, P_s(\theta_R^*), \theta_R^*, 0) \geq \pi_{N_Q}(x, P_s(\theta_Q^*), 1{,}0)$$

ii. If realised market demand $d > \frac{cx}{P_r + g}$,

$$\xi_{L_B}(x, P_s(\theta_B^*), \theta_B^*, T) = \xi_{L_R}(x, P_s(\theta_R^*), \theta_R^*, 0) > \xi_{L_Q}(x, P_s(\theta_Q^*), 1{,}0)$$

$$\pi_{N_B}(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_{N_R}(x, P_s(\theta_R^*), \theta_R^*, 0) < \pi_{N_Q}(x, P_s(\theta_Q^*), 1{,}0)$$

From **Proposition 3**, we suggest that when the predicted market outlook is unfavourable, the liner company can suffer less loss when choosing the quantity discount contract. Otherwise, the buyback and revenue sharing contracts can be the best options for the operation of the liner company.

4.4.3 Impact of market climate

According to the previous findings, when the market parameters are fixed, the buyback contract is equivalent to revenue sharing contract. However, the decision of θ

is made by the liner company in the buyback contract (because we set the buyback price $b = (1-\theta)P_r$) and by the NVOCC in the revenue sharing contract. Therefore, when market parameters exhibit a specific pattern of change, the decisions of the liner company and the NVOCC might have a certain preference in these two contracts. In this subsection, we will analyse the broader influence of changes in market parameters on contract selection in greater detail. First, from the perspective of the NVOCC's order quantity x, according to equation (6), this contract parameter is determined by the market freight fee P_r, unit operating cost c and unit goodwill loss g.

Corollary 4. When keeping other contract parameters do not change in lockstep with market trends, x will increase with the uptrend of g or the downtrend of c.

Since there is no special market parameter $(c_1, c_2, \mathrm{g}_1, \mathrm{g}_2)$ in the expression of x, the allocation of c and g will not change the order quantity of NVOCC or the contract preference of the liner company. Additionally, **Corollary 4** states that, regardless of how operating costs c_1 and c_2 are assigned to the liner company and NVOCC, the increase in unit total operating cost $c = c_1 + c_2$ will result in a decrease in NVOCC's order quantity. In addition, because unmet market demand will generate a goodwill loss, the NVOCC will order more shipping capacity to offset this loss if the goodwill loss increases.

Second, as previously stated, the change in θ will influence the profit distribution of the liner company and the NVOCC to the profit of the centralised model. Since θ is determined by different stakeholders under different contract types, the stakeholders will be biased towards their own interests. Therefore, we here choose the specific θ^* in **Corollary 2** as an example to analyse.

Corollary 5. When keeping other contract parameters constant in relation to market parameters,

i. If g remains constant and $\mathrm{g}_1 \neq \mathrm{g}_2$:

case a. When $S_N(x)(P_r + \mathrm{g}) > cx$ and c remains constant, θ^* has an upward trend with g_1 increasing or g_2 decreasing. When $S_N(x)(P_r + \mathrm{g}) < cx$ and c remains constant, θ^* have an upward trend with g_1 decreasing or g_2 increasing.

case b. The proportion of c_1 and c_2 to c will not affect the choice of θ^*. However, θ^* will have an upward tendency with the increase of c if g_1 accounts for more than 50% of g and a downward tendency if g_1 accounts for more than 50% of g.

ii. If g has a proclivity towards change:

case c. When $\mathrm{g}_1 = \mathrm{g}_2$, the change of c, the proportion of c_1 and c_2 to c and the proportion of g_1 and g_2 to g will not affect the choice of θ^*.

case d. When $S_N(x)(P_r + \mathrm{g}) > cx$: If $\mathrm{g}_1 > \frac{\mathrm{g}}{2}$, θ^* has an upward trend when g decreases. If $\mathrm{g}_1 < \frac{\mathrm{g}}{2}$, θ^* has an upward trend when g increases.

case e. When $S_N(x)(P_r + \mathrm{g}) < cx$: If $\mathrm{g}_1 > \frac{\mathrm{g}}{2}$, θ^* will have an upward trend when g increases. If $\mathrm{g}_1 < \frac{\mathrm{g}}{2}$, θ^* will have an upward trend when g decreases.

Corollary 5 indicates that the trend and allocation of unit operating cost c and unit goodwill loss g between the liner company and the NVOCC have a significant impact on the choice of the slot purchase contract. From the calculation in **Proof of Corollary 3**, the revenue sharing rate θ has a positive correlation with the profit of NVOCC and a negative correlation with the profit of liner company from selling process 1. Therefore, the increase in θ will lead to an increase in the profit of NVOCC and a decrease in the profit of liner company during the selling process 1.

Finally, we consider the movement from the perspective of the freight fee P_s, which is decided by the liner company. From **Proposition 1** and **Corollary 3**, it is obvious that P_s has a positive correlation with the revenue sharing rate θ. Therefore, the trend of market parameters will influence P_s in the same way with θ. Since equation (7) is the result of substituting **Proposition 1** into the profit function of the NVOCC and the liner company, P_s will react to the trend of market parameters similarly to θ^* when keeping θ unchanged but only changing P_s.

Proposition 4. With regard to the NVOCC and the liner company's response to market climate change trends, we have the following three scenarios:

1. When the NVOCC and the liner company have negotiated their respective unit goodwill loss and this satisfies $g_1 = g_2$, there will be no deviation between the decision of the NVOCC in the revenue sharing contract and the decision of the liner company in the buyback contract.

2. When $g_1 \neq g_2$, the choices of NVOCC (θ_N) and liner company (θ_L) for revenue sharing rate θ are shown in the following table:

	g remains constant		g tends to change	
	$S_N(x)(P_r + g) > cx$	$S_N(x)(P_r + g) < cx$	$S_N(x)(P_r + g) > cx$	$S_N(x)(P_r + g) < cx$
$2g_1 > g$	$\theta_N \uparrow \theta_L \downarrow$	$\theta_N \downarrow \theta_L \uparrow$	$\theta_N \downarrow \theta_L \uparrow$	$\theta_N \uparrow \theta_L \downarrow$
$2g_2 > g$	$\theta_N \downarrow \theta_L \uparrow$	$\theta_N \uparrow \theta_L \downarrow$	$\theta_N \uparrow \theta_L \downarrow$	$\theta_N \downarrow \theta_L \uparrow$

Table 4.2 Trend of θ with market

Based on the above analysis, **Corollary 4** reveals the influence of market parameters (c, g) on contract parameter x will not lead to deviations in the choice of contract type between NVOCC and liner company. However, the influence of market parameters (c_1, c_2, g_1, g_2) on contract parameter θ and $P_s(\theta)$ will result in similar differences in the contract type selection between NVOCC and liner company. If the market environment results in an upward trend for the contract parameter θ, the NVOCC's profit will increase. At this time, the NVOCC will also be willing to choose this larger θ at this point. However, the liner company's profit will fall accordingly. There is no doubt that the liner company will choose a minor deviation at this point to maintain the expected profit. When NVOCC chooses a slightly higher θ in the revenue sharing contract, the liner company could respond to the NVOCC with a corresponding $P_s(\theta)$ to maintain future profits. When the liner company chooses a slightly smaller θ to maintain future profits in the buyback contract, a corresponding $P_s(\theta)$ will also be given simultaneously to ensure that it is consistent with the collaboration of the NVOCC. These will not lead to a change in the profit of each stakeholder but will have an impact on the choice of contract type.

4.5 Financial Lease Contracts with Financial Institutions: Financial Impact

The previous chapter conducted an operational analysis of the supply chain contracts between Liner Companies and NVOCCs. In this section, the focus will shift to the financial perspective of the relationship within the supply chain. Specifically, we will study the liner company's optimal financial lease policy which aims to fully repay the financial lease rental and maintain credibility. By analysing the solvency of the liner company, we characterise the financial institution's response loan-to-value curve and the reasonable loan amount for both the financial institution and the liner company. The potential risks and opportunities associated with these financing lease contracts will also be discussed.

4.5.1 The Financial Lease Contract: Expected Default Costs and Profit

To clearly show the cash flows associated with the financial lease contract, here we will show the inflows and outflows of the financial institution and the liner company in detail. First, the financial institution generates a cash outflow of L in event 1. Based on the previous definition of loan-to-value ratio ω, this loan amount L could be presented by $P_r Q\omega$. From the collaboration with NVOCC from event 2 to event 3, the liner company's surplus profit is ξ_L. In the subsequent event 4, the outflow that the liner company is permitted to repay will vary depending on the following circumstances:

$$\Delta_c(Q) = \begin{cases} P_r Q\omega[1+R(\omega)] & if\ P_r Q\omega[1+R(\omega)] \geq \xi_L(Q) \\ \xi_L(Q) & if\ \xi_L(Q) < P_r Q\omega[1+R(\omega)] \end{cases} \tag{9}$$

Therefore, the cash inflow that the financial institution receives will be:

$$\delta_B = \begin{cases} P_r Q\omega[1+R(\omega)] & if\ P_r Q\omega[1+R(\omega)] \geq \xi_L(Q) \\ (1-\alpha)\xi_L(Q) - \beta & if\ \xi_L(Q) < P_r Q\omega[1+R(\omega)] \end{cases} \tag{10}$$

Here, $R(\omega) = \rho\omega + \varepsilon, \rho > 0, 1 > \varepsilon > 0$. When the loan-to-value ratio ω is low, the project is worth investing in, and the interest rate is relevantly low. We keep the parameters of the slot purchase contract (x, P_s, θ, T) fixed (follow section 4) but Q will change, then

$$\begin{aligned} \xi_L(Q) &= \xi_L^1 + \xi_L^2 \\ &= P_s\, x + (1-\theta)P_r S_N(x) - \mathrm{T} - c_2 x - I_N(x)\mathrm{g}_2 + P_r S_L(x) \\ &\quad - (Q + q_0 - x)c - I_L(x)\mathrm{g} \end{aligned} \tag{11}$$

From the preceding equation (9), it is clear that ξ_L^1 has no relationship with Q. And ξ_L^2 is the main section of the formula that relates to Q.

Thereafter, we want to investigate the relation between Q and ω. Typically, they should not have a particular relationship other than $\omega = \frac{L}{P_r \mathrm{Q}}$. Under assumption 2, when Q increases, L will increase. When we double the financial lease size Q, the financial lease cost might double to 2L, and the market value of financial leased vessels will be $2P_r$Q. Therefore, ω should be fixed. However, solvency should be the main indicator that determines the loan-to-value ratio. This ratio will adjust according to the liner company's ability to repay the financial lease rent. In addition, the financial lease cost L within our model might not have a linear relationship with Q. From this, while assessing the relationship between Q and ω, we can also measure the relationship between L and Q. Here, Q' is specified as the minimum size of a financial lease at which the liner company's profit is just sufficient to cover the financial lease rent. In the face of stochastic market demand, this threshold Q' will lead the $\xi_L(\mathrm{Q}')$ equal to the rental. Therefore, we keep:

$$V_1 = P_r \mathrm{Q}'\omega(\rho\omega + \varepsilon + 1) - \xi_L(\mathrm{Q}') = 0$$

In the face of deterministic market demand, the default threshold satisfies the following:

$$V_2 = P_r \mathrm{Q}\omega(\rho\omega + \varepsilon + 1) - \xi_L(Q|d = d') = 0$$

Here, d' is the threshold market demand point. It is reasonable that the market demand is insufficient to enable the liner company to earn enough profit to repay the rental. Then d' is below the maximum capacity offered by the NVOCC in the selling process 1, which results in the profit of liner company in event 3 as:

$$\xi_L(Q|d = d_1') = P_s\, x + (1 - \theta)P_r d_1' - \mathrm{T} - c_2 x - (Q + q_0 - x)c \qquad (12)$$

However, there may also be situations where the liner company can only afford debt when the market is slightly larger. In this situation ($x \le d' < Q + q_0$), the profit of liner company in event 3 will be:

$$\xi_L(Q|d = d_2') = P_s\, x + (1 - \theta)P_r x - \mathrm{T} - c_2 x - (Q + q_0 - x)c + (d' - x)(P_r - \mathrm{g}_2) \quad (13)$$

The worst case should be when the market needs to match the maximum capacity that the liner company has, which is $d' = Q + q_0$ but no more than that amount. From the following liner company's profit function, it is obvious that the goodwill cost part will partially offset the profit when market demand exceeds $Q + q_0$.

$$\xi_L(Q) = P_s\, x + (1-\theta)P_r S_N(x) - \mathrm{T} - c_2 x - (d' - x)\mathrm{g}_2 + P_r(Q + q_0 - x) - (Q + q_0 - x)c - (d' - Q - q_0)\mathrm{g}$$

Therefore, if the previous market demand cannot help the liner company meet the repayment, the demand in this range will not help the liner company satisfy the condition of repayment.

According to V_2, the threshold market in the first case that $d' \le x$ will be:

$$d_1' = \frac{P_r \mathrm{Q}\omega(\rho\omega + \varepsilon + 1) - P_s\, x + \mathrm{T} + c_2 x + (Q + q_0 - x)c}{P_r - \theta P_r}$$

The market threshold under the situation of $x \le d' < Q + q_0$ will be:

$$d_2' = \frac{P_r \mathrm{Q}\omega(\rho\omega + \varepsilon + 1) - P_s\, x + \mathrm{T} + c_2 x + (Q + q_0 - x)c + (\theta P_r - \mathrm{g}_2)x}{P_r - \mathrm{g}_2}$$

According to the observation of the above formula, when $\theta P_r = \mathrm{g}_2$, $d_1' = d_2' = x$. Then, the profit of the financial institution could be rewritten as:

$$\delta_B = \int_0^{d'} [\xi_L(Q|d = d') - B(\xi_L)]\, f(d)dd + \int_{d'}^{\infty} [P_r \mathrm{Q}\omega(\rho\omega + \varepsilon + 1)] f(d)dd$$

And the expected default cost will be:

$$B(\xi_L) = \begin{cases} \int_0^{d_1'} \{\alpha\xi_L(Q|d = d_1') + \beta\} f(d)dd\,, & d' = d_1' \\ \int_0^{x} \{\alpha\xi_L(Q|d \le x) + \beta\} f(d)dd + \int_x^{d_2'} \{\alpha\xi_L(Q|d = d_2') + \beta\} f(d)dd\,, & d' = d_2' \end{cases}$$

Therefore, the expected default cost under $d' = d_2'$ is higher than that under $d' = d_1'$. However, from equation (12) and equation (13), it is clear that the deterministic profit of the liner company under d_2' is higher than under d_1'. On this basis, we conclude that although the liner company's deterministic profit under d_2' is greater, the final repayment is the same due to the higher default cost under d_2'. This also demonstrates that the liner company's profitability cannot be used as the main criterion for financial institutions to measure repayment.

4.5.2 Optimal financial lease policy

From the liner company's perspective, the amount of leased capacity Q is the crucial decision in the financial leasing contract. Not only because it is tied to whether the default cost will damage the liner company's existing profit available for repayment, but it is also related to the liner company's final profit after repayment.

From the profit function of the financial institution, the liner company could get the full rental if the liner company does not default on the financial lease contract. The default behaviour of the liner company depends on whether its expected profit could cover the total rent. Based on this and from the analysis of the slot purchase contract in 4.4.1, we can see that none of the contract parameters is related to the previous decision of the financial lease amount Q in event 1. Therefore, the decision of Q will not influence the decision around contract type. In other words, equation (11) will not be affected by the changes in the contract parameters (x, P_s, θ, T). To determine the optimal financial leasing strategy, we must first figure out the conditions under which the liner company will have a higher chance to repay. Namely, no matter how the financial lease contract changes, the total maximum profit of liner company should be greater than or equal to the rental repayment amount to ensure that there exists a possibility of not defaulting. Then, we use Q^* to denote the optimal capacity from financial lease contract with the objective of maximising the expected profit of liner company before repayment.

Corollary 6 Since $\xi_L(Q)$ is strictly concave in Q, the optimal financial lease amount Q^* must satisfy the following equation:

$$\begin{aligned} \underset{Q^*}{Maximize} \quad & \xi_L(Q^*) = P_s\, x + (1-\theta)P_r S_N(x) - \mathrm{T} - c_2 x - I_N(x)\mathrm{g}_2 \\ & \qquad + P_r S_L(x) - (Q^* + q_0 - x)c - I_L(x)\mathrm{g} \\ s.t. \quad & Q^* \geq 0 \\ & \frac{\partial \xi_L(Q^*)}{\partial Q^*} = 0 \end{aligned}$$

The optimal financial lease amount Q^* satisfies $F(Q^* + q_0) = \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}}$.

When combined with **Corollary 1**, it is not difficult to find that the optimal financial leased capacity Q^* is equal to the optimal order quantity x_c^* of NVOCC minus the initial capacity q_0 that the liner company has from owned vessels. That is to say, the sales of the liner company in the first selling process will be the main source for increases in the company's profit. Combined with the results of equation (8), an increase in Q^* does not increase profit in the second selling process, while an increase in x_c^* does. However, the order quantity x of NVOCC could not exceed the quantity of $Q + q_0$, which also shows that the financial lease contract has certain constraints on the decision of the slot purchase contract. Nevertheless, as a result of **Corollary 6**, the slot purchase contract has no impact on the selection of Q.

Based on the assumption of $Q \geq x - q_0$ and **Corollary 6**, the profit function $\xi_L(Q)$ is monotonically decreasing over the range of Q. However, it can be seen from the observation that rent $P_r Q\omega(\rho\omega + \varepsilon + 1)$ is monotonically increasing over the range of Q. To guarantee the repayment, the liner company could only choose the leased capacity Q that makes the maximum profit of liner company greater than or equal to the rental.

Proposition 5 The financial institutions are less likely to reject the application of the liner company for the financial lease when the loan-to-value ratio is in the range of $\left(0, \frac{-(\varepsilon+1)+\sqrt{(\varepsilon+1)^2+4\rho M}}{2\rho}\right]$.

Here, $M = \frac{(P_s+c_1+g_2)\,x-\theta P_r-T-\mu(2g_2+g_1)+(\theta P_r-g_2)\int_0^x F(d)dd+(P_r+g)\int_0^{Q^*+q_0} df(d)dd}{P_r Q^*}$

When the NVOCC and the liner company pick the optimal slot purchase contract in **Chapter 4.1** as the contract parameters, the liner company could only use the profit from slot purchase contract as the main source for repayment because $Q + q_0 = x_c^*$ and $\xi_L^2 = -I_L(x)g$. Then, the expected profit of liner company is:

$$\xi_L = \xi_L^1 = P_s\, x_c^* + (1-\theta)P_r S_N(x_c^*) - T - c_2 x_c^* - I_N(x_c^*)(g_2 + g)$$

Based on **Proposition 5**, the liner company should then control the loan amount from financial institution below $\frac{P_r Q^*}{2\rho}[-(\varepsilon+1)+\sqrt{(\varepsilon+1)^2+\frac{4\rho\xi_L}{P_r Q^*}}]$. But this **Proposition 5** can only help the liner company to reduce the rejection risk of the financial leasing contract.

Proposition 6 In the face of stochastic market demand, the threshold financial lease amount Q' has a negative correlation with loan-to-value ratio ω. In addition, the relationship between L, ω and Q' can be expressed as the following equations:

$$\begin{cases} \omega = \dfrac{-(\varepsilon+1)+\sqrt{(\varepsilon+1)^2+4\rho\dfrac{\xi_L(\mathrm{Q}')}{P_r\mathrm{Q}'}}}{2\rho} \\ L = \dfrac{P_r Q^*}{2\rho}\left[-(\varepsilon+1)+\sqrt{(\varepsilon+1)^2+\dfrac{4\rho\xi_L(\mathrm{Q}')}{P_r\mathrm{Q}'}}\right] \end{cases} \tag{14}$$

Proposition 6 explores the condition for the liner company to guarantee repayment. Although we know from **Corollary 6** that the profit of the liner company decreases as Q increases, in **Proposition 6** we find that when the value of Q' increases ω will decrease. Since a smaller loan-to-value ratio suggests that the liner company is more capable of repaying, **Proposition 6** demonstrates that a larger Q' is preferable for the liner company but with a smaller expected profit. Therefore, if the liner company want to facilitate the financial leased contract with financial institution, company should prioritise demonstrating improved solvency over profit margins. Furthermore, **Proposition 6** could also provide guidance for the financial lease policy of the liner company. Namely, for any interest rate formula $R(\omega)=\rho\omega+\varepsilon$ proposed by financial institutions, if the liner company chooses Q' as the financial lease amount, and if L and ω are lower than the values in the equation (14), there is no need to be concerned about the reject risk in event 1.

However, the approval of the financial lease contract by the financial institution only means that the liner company has the possibility of repayment. Since the market demand is unknown, the above results only guarantee that the liner company can repay the rental at the expected profit. If the deterministic market demand cannot support the liner company in generating a sufficient profit, both the liner company and the financial institution will face the risk of default.

4.5.3 Risk allocation

As shown in the previous chapters, the contract type of slot purchase contract will not influence the decision around the financial lease contract in event 1. However, the decision around contract parameters (x, P_s, θ, T) in event 3 will affect subsequent repayment in event 4, which relates to the allocation of financial risk.

Based on equation (9) and equation (10), if the profit of the liner company can reach the financial lease rental, there will be no default cost and there will be a win-win situation for both liner company and financial institution. However, if the profit is lower than the repayment amount, the default cost will lead to a lose-lose situation. Additionally, combined with the formula of default cost, we can see that when the repayment gap is smaller, even though the profit of the financial institution is larger, the impairment loss of payment is greater, thus worsening the liner company's situation.

As demonstrated in **Chapter 4.4.2**, the sum of the profits earned by the NVOCC and the liner company under the condition of equation (6) and **Proposition 1** is constant as π_c^*. However, a change in θ can result in a change in their segmentation of π_c^*. From **Corollary 3** and following the conclusion that the θ interval affects the profit allocation, we can see that the liner company can adjust θ to increase its profit share in π_c^* when the liner company is about to repay the rental. In this case, the liner company could use its decision-making power over the parameters of the slot purchase contract to infringe on the profit of the NVOCC to fill the repayment shortfall. Since the liner company can only decide θ under the buyback contracts, NVOCC is forced to assist the liner company in assuming a portion of the financial risk. Under revenue-sharing contracts and quantity discount contracts, the liner company will bear the financial risk alone. Nevertheless, in an ideal state, buyback contracts and revenue-sharing contracts can bring more profits to the liner company. Therefore, buyback contracts are more beneficial to the liner company and to the financial institution.

When the liner company has no other way to make up the shortfall in repayment, it can be seen from equation (9) that the liner company alone can cover all the profits and the profit of the financial institutions will suffer further losses. At this point, the financial institution's financial risk is actually the liner company's operational risk associated with the mismatch between supply and demand. However, the liner company is also exposed to the risk of lower credibility, which is in turn related to rising financial costs in the future. Therefore, this is a no-win situation.

4.6 Numerical examples

In this section, numerical examples will be utilised to demonstrate how those three contracts foster collaboration between the liner company and the NVOCC when the liner company is financially constrained. We here select the standard distribution function $d \sim N(100, 30)$ as market demand. All other parameters are listed as follows: $q_0 = 50, P_r = 400, c_1 = 20, c_2 = 30, \mathrm{g}_1 = 50, \mathrm{g}_2 = 40$. From the previous equation (6), the optimal quantity that the NVOCC should order should be $x \approx 138$. Based on **Corollary 2**, $\theta^* = 0.5023$ could help the NVOCC, and the liner company divide the total profit equally. To better demonstrate the gap between the different contracts, we first choose $\theta = 0.66$ and $Q_1 = 100$. Then, according to **Proposition 1**, the prices offered by liner companies according to different contract types are 148 in the buyback contract (BB) (buyback price as b=136), 12 in the revenue sharing contract (RS) and 26 in the quantity discount contract (QD). To better present the optimal decision in the slot purchase contract, Figure 4.3 shows how the profit of the liner company (in selling process 1 only) and the profit of the NVOCC changes with the order quantity x under each contract when facing stochastic demand. The vertical axis of Figure 4.3 represents the profit of each entity. The horizontal axis is the order quantity of the NVOCC.

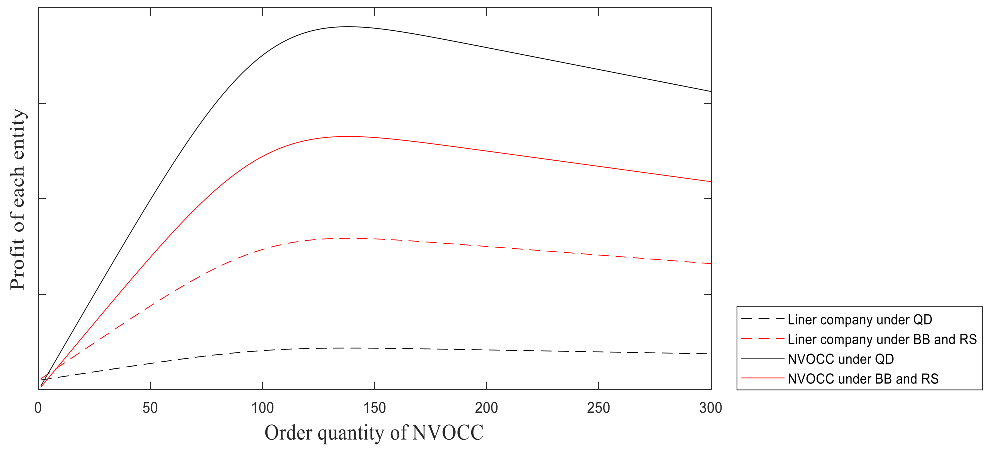

Figure 4.3 Profit of each entity under Contracts (x, P_s, θ, T)

As illustrated in the above figure, because $\theta = 0.66$ is greater than the point $\theta^* = 0.5023$ that divides the centralised profit π_c^* equally, the NVOCC receives a larger portion of the centralised profit. Therefore, to make more profits, liner companies need to strive for control of θ. Additionally, it is evident that buyback contracts and revenue-sharing contracts bring more profit to the liner company in the selling process 1 than in the quantity discount contracts.

Then, the expected profit of NVOCC and the total expected profit of the liner company in event 3 (which is the sum profit of sales process 1 and sales process 2) under the optimal order quantity x and $Q_1 = 100$ before repayment are as follows:

$$\pi_L(138, 12, 0.66, 0) = \pi_L(138, 148, 0.66, 18781) = 14120 > \pi_L(138, 26, 0, 0) = 2634$$

$$\pi_N(138, 12, 0.66, 0) = \pi_N(138, 148, 0.66, 18781) = 21519 < \pi_N(138, 26, 0, 0) = 33005$$

According to the different choices of the liner company for Q, Figure 4.4 shows how the profit of the liner company changes with Q when facing stochastic demand.

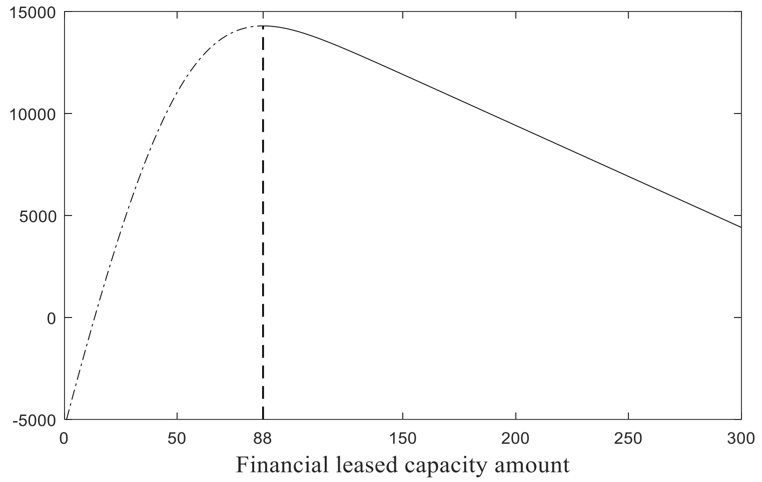

Figure 4.4 Total expected profit of Liner Company

Therefore, the optimal financial leased capacity for the liner company will be $Q_2 = 88$. In this case, the expected profit of the liner company before repayment is as follows:

$$\pi_L(138, 12, 0.66, 0) = \pi_L(138, 148, 0.66, 18781) = 14293 > \pi_L(138, 26, 0, 0) = 2807$$

$$\pi_N(138, 12, 0.66, 0) = \pi_N(138, 148, 0.66, 18781) = 21519 < \pi_N(138, 26, 0, 0) = 33005$$

When compared with the previous condition $Q_1 = 100$, it is found that neither the parameters of the slot purchase contract nor the profit of the NVOCC has changed, but the profit of the liner company has indeed increased.

Figure 4.5 shows the profit of the liner company facing a deterministic demand in event 3 when the liner company chooses the optimal financial leasing capacity $Q_2 = 88$ and higher $Q_1 = 100$.

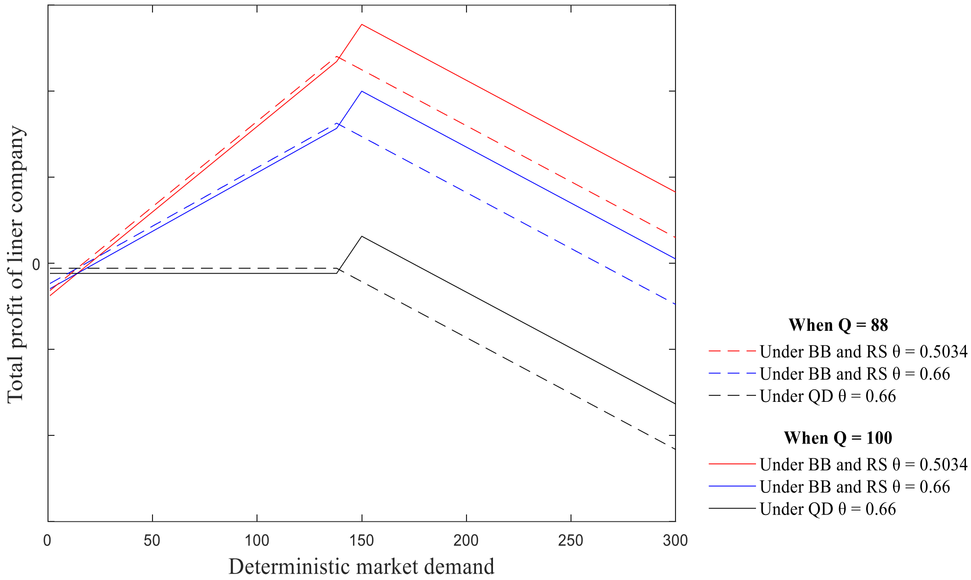

Figure 4.5 Profit of liner company under several conditions.

The most obvious result from this figure is the enormous increase in profit caused by the decrease in θ. Secondly, the optimal financial leased policy brings higher profits in the early stage of market demand. This is due to the increase in leased capacity, which results in more holding costs. However, because the subsequent market demand is not met, the liner company's profits do not continue to rise as $Q_1 = 100$, and immediately show a downward trend. In addition, due to the insufficient quantity of capacity, more goodwill loss is paid. However, this is not to imply that expanding leased volumes would be beneficial to the liner company. After all, the operation of vessels is limited by the financial expenses of financial lease.

If $\theta = 0.66$ and the financial institution proposes the interest rate as $R(\omega) = 0.8\omega + \frac{1}{5}$, the loan-to value-ratio for the liner company will be $\omega = 1.75$ when the liner company chooses $Q_1 = 100$ and $\omega = 1.78$ when the liner company choose $Q_2 = 88$. To guarantee repayment, the liner company needs to control the borrowing amount below $L = 70075$ when the liner company chooses $Q_1 = 100$ and $L = 62812$ when the company chooses $Q_2 = 88$. If $\theta = 0.5023$, the loan-to value-ratio for the liner company will be $\omega = 1.83$ when the company chooses $Q_1 = 100$ and $\omega = 1.87$ when the company chooses $Q_2 = 88$. To guarantee repayment, the liner company needs to control the borrowing amount below $L = 73271$ when the liner company chooses $Q_1 = 100$ and $L = 65900$ when the company chooses $Q_2 = 88$. From the above calculations, we can find that although the liner company's expected profit in $Q_2 = 88$ is more, but the solvency shown from loan-to value-ratio is weaker than $Q_1 = 100$. This is because the range of deterministic market demand that could help the liner company cover the repayment is more comprehensive when choosing $Q_1 = 100$ than $Q_2 = 88$. It can be seen from this that it is not bad for the liner company to bear more debt. Instead, it benefits the liner company's operational and financial activity more. The conservative decisions of risk-averse companies based solely on expected parameters may be less effective than the innovative decisions of less risk-averse companies.

4.7 Conclusions and Future Research

In this article, we discuss the financing and operating issue of liner companies that finance leasing vessels from financial institutions and collaborate with NVOCCs to sell shipping capacity. In terms of financing, the liner companies are required to repay the financial lease via a series of rents or instalments contingent upon the liner company's net profitability in the following operating activity. In terms of operating, the prepared capacity relies heavily on the financial leased amount in the previous financing activity. We expect that the collaboration will take place via various types of slot purchase contracts, such as revenue-sharing, buyback, or quantity discount contracts. From the perspective of profit allocation and risk allocation, we analyse and evaluate the performance of these three contracts in this leveraged container shipping chain.

From an operational aspect, we present optimal solutions for each contract type to coordinate the collaboration between the liner company and the NVOCC. We also show that the buyback contract is equivalent to the revenue sharing contract when the market is stable. Additionally, in most cases, the buyback and revenue sharing contracts can bring more profit for the liner company to repay the financial lease than the quantity discount contract. Only when the market is sluggish will the performance of quantity discount contracts be better than the buyback and revenue sharing contracts. It is also worth noting that both the liner company and the NVOCC will have a specific deviation in the choice between the buyback contract and revenue sharing contract when the market is unstable. However, the final profit they generate is identical. The deviation has only reflected the selection of contract parameters.

From a financial aspect, we propose the optimal financial lease policy for a liner company and some appropriate response strategies for the financial institution regarding their decision-making process. During the financial analysis, we found that using only profitability as an indicator to measure a liner company's ability to repay the rental has a distorting effect. In other words, as long as the liner company can control repayments within an affordable range, its profitability is irrelevant to financial institutions. In this regard, we proposed threshold formulas for liner companies and financial institutions to control and judge the solvency of a liner company. In addition, combined with the previous conclusions on profit allocation under different types of slot purchase contract, we find that the buyback contracts are more favourable to the repayment of liner company, followed by the revenue-sharing contracts, and finally the quantity discount contracts.

When combining the operating and financial activities of the liner company, we found that participants within the financial contract should first frame the repayment amount as a reference before demonstrating more profitability. In other words, profitability is only useful if the profit can be higher than the repayment amount. Otherwise, no matter how profitable the liner company is, its solvency cannot support the liner company's access to financing. From the calculation in **Proposition 6**, we also found that apart from reducing the loan amount and financing costs, liner companies can also control other partners to share financial risks by exercising control over operating activities. Therefore, when cooperating in operating activities, companies should be alert to the financial situation of other partners so as not to be affected by their financial risks.

In this paper, we build our model based on a fixed accounting period. However, the operating and financing activities are ongoing. In addition, apart from the financial leases, operating leases are also widely used and they are potentially a faster and more efficient way for a liner company to acquire vessels within the short term. When these factors are considered in the model, the outcome might be different. Regarding the price proposed by the liner company, in addition to the price mechanism that may in turn affect this price, the cost of financing activities may also impact it. In this case, more complex applications can be added to the model to discuss the impact of leverage on operating choices.

4.8 List of Symbols

d	The market demand quantity
d', d_1', d_2'	The threshold market demand when the liner company repays the finance lease
μ	$= E(d)$, the mean value of demand
$f(d)$	Probability density function of market demand
$\mathrm{F}(d)$	Cumulative distribution function of market demand
P_r	The freight rate paid by shippers for each TEU capacity sold
$R(\omega)$	Financial lease interest rate
L	The cost for the financial institution to support the finance lease for each accounting period
Q	The capacity that liner company get from financially leased vessels
Q^*	The optimal financial leased capacity
q_0	The initial capacity that liner company has from owned vessels
ω	Loan-to-value ratio
c	Operating cost per unit of TEU capacity sold, $c = c_1 + c_2$
c_1	Operating cost of NVOCC per TEU capacity sold
c_2	Operating cost of liner company per TEU capacity sold
g	Goodwill loss per unit of unmet TEU capacity, $\mathrm{g} = \mathrm{g}_1 + \mathrm{g}_2$
g_1	Goodwill loss of NVOCC per unmet TEU capacity
g_2	Goodwill loss of liner company per unmet TEU capacity
P_s	Wholesale price, freight fee paid from the NVOCC to the liner company
x	The order quantity of NVOCC
x^*	The optimal order quantity
x_c	The order quantity that maximises the profit of centralised company
x_n^d	The order quantity that maximises the profit of NVOCC
x_l^d	The order quantity that maximises the profit of liner company
θ	Contract parameter, a percentage of the revenue that NVOCC keeps

θ_R^*	The percentage of the revenue that NVOCC keeps in revenue sharing contract
θ_B^*	The percentage of the revenue that NVOCC keeps in buyback contract
θ_Q^*	The percentage of the revenue that NVOCC keeps in quantity discount contract
θ_N	The revenue sharing rate θ decided by NVOCC
θ_L	The revenue sharing rate θ decided by liner company
T	Contract parameter, fixed money transferred from the liner company to the NVOCC if $T > 0$
b	The buyback price of unsold products
$S_N(x)$	The expected sales of the NVOCC
$I_N(x)$	The unsatisfied demand of the NVOCC
$S_L(x)$	The quantity of unsold products
$I_L(x)$	The quantity of unsold products
π_N	The expected profit of the NVOCC
π_L	The expected profit of the liner company
π_c	The expected profit of centralised company
ξ_L^1	The profit of the liner company from slot purchase contract before the repayment
ξ_L^2	The profit of the liner company from direct selling before the repayment
ξ_L	The total profit of the liner company before the repayment
Δ_c	The repayment from centralised virtual entity to the financial institution in centralised model
B	The default cost incurred when liner company could not repay the loan
δ_F	The income of financial institution

Chapter 5. The Financial Strategy for a Liner Company Operating under Multiple Constraints: Application of the Prospect Theory in Credit Rating

5.1 Introduction

In recent years, with the severe impact of the COVID-19 pandemic, maritime shipping has played a prominent role in sustainable global supply delivery and in the resumption of normalcy. Along with rail and road freight, the maritime shipping industry's importance is bolstered by the fact that it enables effective transportation over long distances with little influence from government policies geared toward addressing the pandemic (United Nations Conference on Trade and Development, 2021). As disclosed in UNCTAD’s 2021 Annual Review of Maritime Transport, around 80% of the global commodities are carried by maritime.

Although the pandemic has caused less disruption to maritime transport than was initially feared, limited vessels are a common constraint for liner companies. Lack of funding for business expansion can be a challenge that impedes growth, so numerous liner companies may approach financial institutions for financial support. Financial leasing, as one of the most effective methods of accessing vessels, has been commonly adopted in the maritime shipping industry (Tran, 2022). The Mediterranean Shipping Company (MSC), as the largest container ship operator with a 17.2% market share, discloses that more than 50% of operational vessels were leased. However, the fluctuation in the control measures for the outbreak has resulted in severe market volatility and frequent occurrences of unexpected situations, which may expose the maritime shipping industry to high financial risks (UNCTAD, 2021). Aside from financial risk, a more common risk for liner companies associated with their operations is the mismatch between a vessel’s shipping capacity and market demand. In real-world industrial practice, NVOCCs are often involved in the container shipping chain to help liner companies interact and trade with shippers in more efficient ways. Like the newsvendor model, their collaboration is constrained by slot purchase contracts which can be in the form of a revenue-sharing contract, a buyback contract or a quantity discount contract (Snyder and Shen, 2019). With different contract forms, liner companies will be able to show different levels of profit and – consequently – different attitudes toward financial risk.

In determining the appropriate amount of finance lease vessels, liner companies will not only be constrained by an uncertain market reaction but also by whether they can achieve the expected operating results. Specifically, financial leasing can support liner companies in expanding their operating scale to capture more market share (Shin et al., 2019). In turn, the operating profit of the liner company must be sufficient to support its obligation to pay the cost of the finance lease. Otherwise, the credit rating of liner companies will suffer, which will impair their future operations and financing (Li, 2006). Motivated by this, it is worth exploring the interplay between operational and financial risk. According to cumulative prospect theory, decision-makers react more negatively to losses than they do positively to gains (Tversky and Kahneman, 1992). In practice, if a company is overly cautious about the financial risks it takes, this may not necessarily guarantee better operation, which may cause the company to be eliminated from the market due to its lack of innovation and competitiveness. Joo and Parhizgari (2021) also present that credit downgrades have a positive impact on the adjustment speed for overleveraged firms. Therefore, it is worth seeking a balance and reasonable rating level in these conflicting boundary conditions.

From the available literature, a rising number of works have observed and investigated the specific form and application of container shipping contracts of liner companies by a variety of limitations (Xie et al., 2017; Wang et al. 2017; and Song et al., 2019). In this paper, we concentrate on the issue of whether the liner company should adopt over-placement strategies or over-conservative strategies in financial leases rather than restricting the design of container shipping contracts through financial leasing. From the perspective of decision preference, numerous studies around supply chains focus on the intersection of prospect theory and the newsvendor model (Zhang et al., 2016; Uppari and Hasija, 2019; and Vipin and Amit, 2021). Most of the studies focus on the concept and function of reference points in using prospect theory to measure the behaviour in the newsvendor model. However, they use prospect theory to describe how suppliers and retailers respond to revenue and expenditures rather than as a reference model for describing investment willingness. To fill this gap, we established a prospect theory-based credit rate model linking financial institutions and liner companies to investigate the financial strategies that liner companies should adopt in financial leasing.

To examine the above problem, we consider a container shipping chain consisting of a financial institution, a liner company, an NVOCC and shippers. In the model, the liner company extends its capacity to a wider market through the financial leasing contract and enthusiastically canvasses orders from shippers through NVOCCs. As the size of the financial lease will limit the capacity that the liner company can provide to the NVOCC, we assume that there are two types of liner companies. First are the over-placement liner companies that lease large-scale vessels, allowing them to provide sufficient shipping capacity to meet the NVOCC's order quantity. The second group consists of over-conservative liner companies that lease small-scale vessels, which enables them to reduce rental costs but may result in a lower order quantity from NVOCC. Furthermore, we consider there are two stages operating to show their differences, which will be introduced in detail in the Methodology. Our research contributes to the existing literature in two ways. First, we will propose the contract tactics for a liner company to collaborate with an NVOCC when the liner company cannot provide enough capacity. Second, based on prospect theory, we will compare the attitudes of financial institutions against over-conservative liner companies and over-placement liner companies. In addition, the analytical results propose the conditions in which the over-placement liner companies outperform the over-conservative liner companies.

The remainder of the paper is arranged as follows: In Section 2, we review the related literature. Our work focuses on studies around the supply chain contract, prospect theory and their application. Section 3 details the model setup and preliminaries. We will particularly consider the prospect theory-based credit rate model and related assumptions in this session. In Section 4, we will first analyse the contracts under the case of a liner company with sufficient capacity and then we will compare this with the case of a liner company with insufficient capacity. In Section 5, we will examine the financial lease rental and credit rating of the above two liner companies to compare their solvency. Numerical experiments are performed in Section 6 to compare the over-placement liner companies and over-conservative liner companies under stochastic demand and deterministic demand. Section 7 summarises the results and provides future research directions.

5.2 Literature review

This study sits at the intersection of supply chain management and behavioural finance which has been widely researched. In this section, we will highlight gaps in the previous literature and demonstrate our contribution.

5.2.1 Supply chain contract theory and financial risk

Most of the research on supply chain contract theory examines the optimal decisions of risk-neutral decision-makers according to the assumption of expected profit maximization. Shen et al. (2019) and (Höhn, 2010) presented a broad taxonomy for understanding several supply chain contract theories in the newsvendor model, where suppliers and retailers negotiate a wholesale pricing contract (Bresnahan and Reiss, 1985). The model includes the cases of buyback contracts (Pasternack, 1985), wherein suppliers offer retailers a partial payback strategy on unsold stock and revenue-sharing contracts (Cachon and Lariviere, 2005), where retailers share a portion of their sales revenue with the retailers. Guo et al. (2017) reviewed the recent state literature (2006–2016) on supply chain contracts with a focus on logistics systems. They found that the quantity-discount contract (Moorthy, 1987) was the most studied contract, along with the wholesale price contract and the two contracts listed above.

Based on these leading studies, many other investigations have also considered contract setting, including the financial return of the contract, economic conditions, financial circumstances, and the specified industry or commercial products. From a financial perspective, the effect of capital constraint on the decision of contract parameters has garnered a great deal of attention in academia (Nocke and Thanassoulis, 2014; Kouvelis and Zhao, 2016; Xiao et al., 2017; Huang et al., 2020). Kouvelis and Zhao (2016) examined the impact of financial constraints on the operational and financial decisions of a supply chain in which supplier and retailer interact through revenue-sharing, buyback contracts and quantity discount contracts. Xiao et al. (2017) demonstrated that the all-unit discount contract fails to coordinate the financially constrained newsvendor model, while the revenue-sharing and buyback contracts can facilitate coordination only when the supply chain has adequate total working capital. By considering the risk faced by financing participants, Huang et al. (2020) set up a supply chain finance framework based on a general supply chain contract. Many studies also investigated the financial strategies of suppliers or retailers (Yan and Sun, 2013; He, 2017; Yang et al., 2018; Liu and Wang, 2019; Lai et al., 2021). Yang et al. (2018) intend to present a deeper understanding of how trade credit improves supply chain efficiency by encouraging the retailer to partially share the supplier's demand risk. Lai et al. (2021) investigated two financing methods (credit finance and supplier green investment) for a supplier-manufacturer green supply chain with cost-sharing and quantity discount contracts. The impact of the business environment on supply chain finance has also received substantial interest (Holzhacker et al., 2015; Basu et al., 2019; Lin and Xiao, 2018; Li et al., 2021). Basu et al. (2019) study the issue of hedging demand uncertainty in a supply chain consisting of a risk-neutral supplier and a risk-averse retailer under a buyback contract. Lin and Xiao (2018) study the credit guarantee scheme used in a supply chain finance system, including a manufacturer with capital constraint, a retailer, and a bank in the competitive credit market.

Most of them researched capital-constrained small and medium-sized businesses in need of a loan or trade credit for their operations. These companies are characterised by having poor access to financial leases, whilst it is one of the main financing sources for liner companies to get operating control of vessels. In addition, these companies use capital-based financing methods, whereas financial leasing is a financing method based on high-quality credit ratings. Considering the particularity of liner companies, our research also highlights that shipping capacity has no residual value after limited life, which is different from the specified products in these previous works. Furthermore, traditional supply chain contract modelling is mainly aimed at maximizing profits but lacks a demonstration of how biased orders in the face of increasing market competition affect the relative performance of these contracts from a financial point of view. In this study, we also weaken the contract decision and focus more on the influence that the decision of liner companies (as a supplier) and financial institutions will have on future operations and financing.

5.2.2 Behavioural operations management

Prospect theory, initially presented by Kahneman and Tversky in 1979, shows investors value losses more than gains in times of uncertainty. They think investors tend to change their decisions when the expression of choice changes (isolation effect) and are risk-averse when facing gains (certainty effect), but risk-seeking when facing losses (reflection effects). Since their experiment only considered one outcome at a time and the data were not selected randomly, it has two issues. One is that it does not satisfy stochastic dominance and the other is that it cannot be extended to multiple outcomes. In this case, Quiggin (1982) and Schmeidler (1989) thought expected utility would solve the first problem, while Tversky and Kahneman (1992) also set up cumulative prospect theory to solve both problems. Since then, a significant amount of more recent research has examined how irrational decision-makers enforce contracts differently. Schweitzer and Cachon (2000) proposed models based on loss aversion and prospect theory to describe decision bias in the experiment and found that the pattern of order bias does not fit the certainty effect or the reflection effect proposed by prospect theory. Considering this, Ren and Croson (2013) provided two experiments to support the result of a theoretical model that demonstrates underestimating demand variance causes actual orders to differ from the predictable optimal outcome. Surti et al. (2020) also conducted two experiments to highlight the significance of the reference point in defining newsvendor behaviour using the prospect theory model.

Supported by the experimental works presented above, the majority of behavioural research involving prospect theory in operations management is to investigate the impact of irrational preferences on the contract parameter decisions of suppliers and retailers in response to different contract structures in the newsvendor problem. Zhang et al. (2016) indicated that loss-averse suppliers prefer the buyback contract in the chain with low-critical-ratio products and the revenue sharing contract with high-critical-ratio products. Here, the critical ratio indicates the underlying newsvendor model at the channel level. Wu et al. (2018) investigated a loss-averse competitive newsvendor model and discovered that competition causes newsvendors to place more orders but may not result in a loss of profits. Lond and Nasiry (2015) suggested that zero payoffs should not be employed as a reference point and proposed a reasonable reference point for the newsvendor model to match the previous experimental results. Uppari and Hasija (2019) examined the results of several prospect theory-based newsvendor models under the expected profit maximization assumption. Based on the stochastic reference point, Vipin and Amit (2021) showed the non-linear order behaviour in the newsvendor model. They found that the wholesale price contract might outperform than buyback contract when involving a behavioural retailer in the traditional newsvendor model. In addition, under these conditions, the wholesale price contract could coordinate the chain while the buyback contract fails to.

Although prospect theory is widely applied in the newsvendor model to demonstrate decision bias, very little research has used it to assess financial risk following biased decisions. Accordingly, we attempt to fill this gap by simulating a financial institution’s willingness to offer finance leases through prospect theory. Because the original model by Tversky and Kahneman (1992) utilised the log-concave cumulative distribution functions and the power utility functions, which make the function curves of decision-makers’ willingness nonlinear and complex. All the above papers simplify the expected utility function into a two-piece piecewise linear utility function and use the concept of loss aversion based on the heart of prospect theory. In this setting, we form a four-piece piecewise linear credit rating function to present the function curves of the financial institution's willingness toward a financial lease.

In the context of overconfidence, Moore and Healy (2008) reviewed over 350 works and analysed different examples of overconfidence behaviour, including overestimation, over-placement and over-precision. Following Moore and Healy (2008), we specify the overconfidence behaviours of liner companies in our paper as over-placement. Specifically, these liner companies are overconfident that expanding the scale of shipping can capture more market shares and increase competitiveness. In contrast, we define the liner company with under-placement behaviour as an over-conservative liner company.

5.3 Methodology

We set up a stylized container shipping chain for a liner company with an upstream financial institution that invests in financial leases and a downstream NVOCC that sells slots to shippers. The timeline of decisions for each lease term is shown in Figure 5.1. At the beginning of each lease term, the liner companies with a qualified credit rating could renew the financial lease contract with financial institutions. Based on their market demand forecast, the liner companies could update the number of vessels and corresponding shipping capacity in the financial lease contract. Then, the financial institutions will give corresponding quotations based on the leased volume and credit rating. After this, liner companies and NVOCCs could negotiate a slot purchase contract to sell the shipping capacity to the market. After completing all the operations for the year, the liner companies and NVOCCs will transfer the payment negotiated in the slot purchase contract. At the end of the event cycle, the financial institutions will renew the credit ratings of the liner companies according to their ability to repay the financial lease obligations. In this section, we will profile notations, assumptions and models related to operations and financing over the lease term. The assumptions to be adopted in the models are as follows:

1) There is no salvage value for the unsold shipping capacity.
2) Shippers first order shipping capacity from NVOCC and then from the liner company.
3) To achieve the balance of negotiated contract with the NVOCC, the liner company with insufficient capacity can purchase a small portion of shipping capacity from its peers (or the market).
4) We assume that liner companies operate vessels on fixed routes and regular schedules, and that all other aspects of reality remain unchanged. This is done to

reduce the impact of other unnecessary additional factors on the fixed number of vessels so that the capacity of the financially leased vessels remains constant throughout each lease term.

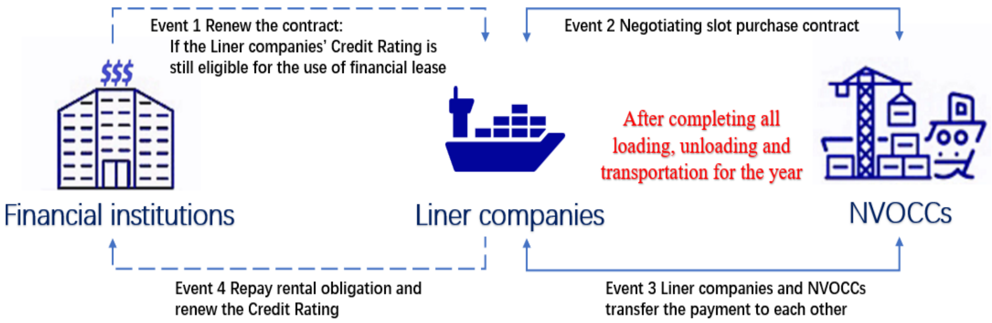

Figure 5.1 Sequence of Decision events

5.3.1 Notation and Assumptions

In the slot purchase contracts, the NVOCC will order the shipping capacity x from the liner company with the wholesale price as $\mathrm{P_s}$. The contract types of slot purchase contracts will be represented by $(x, \mathrm{P_s}, \theta)$ for the revenue sharing contract, $(x, \mathrm{P_s}, \mathrm{b})$ for the buyback contract and $(x, \mathrm{P_s}(x))$ for the quantity discount contract, respectively. θ is the NVOCC's revenue sharing percentage and b is the buyback price. The wholesale price $\mathrm{P_s}(x)$ in the quantity discount contract has a linear regression with NVOCC's order quantity x. The relationship between each stakeholder is as Figure 5.2. Here, the solid line represents the flow of goods or services, and the dashed line represents the flow of cash. For the process of selling shipping capacity, the NVOCC will sell the shipping capacity to the market first. After this, the liner company could sell the surplus shipping capacity directly to the shipper market. During this two-selling process, the freight fee P_r that shippers pay to the NVOCC is the same as the freight fee that shippers pay to the liner company. Here, we define $c = c_1 + c_2$ as the unit operating cost for each capacity and $\mathrm{g} = \mathrm{g}_1 + \mathrm{g}_2$ as the unit goodwill loss, which occurs when market demand exceeds the centralised company's supply capacity. The shipping capacity obtained by the liner company through the financial leasing contract is Q and the initial capacity from its owned vessels is q_0. We also assume the market demand d following the stochastic distribution with the probability density function (PDF) of $f(\cdot)$ and cumulative distribution function (CDF) of $F(\cdot)$.

Figure 5.2 Production/services and cash flows of the liner company

After the market demand is realised, the liner companies should repay the financial lease contracts with the rental of $L(r) * \frac{i+1}{1+e^r}$. According to Gilroy et al. (2007), the sigmoid function (having a typical S-shaped curve) is used to control the $interest\, rate\ + 1 = \frac{i+1}{1+e^r}$ within the rage of $(0, i+1)$. Here, i is a non-variance constant base interest rate and r is liner company's credit rating. $L(r)$ presents the financial institution's acceptance of financial lease cost (or allowance to support the liner company's financial lease). Liner companies with higher credit scores can get a higher quota for finance leases and lower interest rates. Therefore, let $L(r) = \rho r + \varepsilon$. For allowance $L(r)$, it could support the liner company to lease the vessels with a shipping capacity of $Q = \frac{L(r)}{P_r \omega}$. Here, ω is used to assess the asset value for each shipping capacity with its market value P_r, which is also known as the loan-to-value ratio (LTV). To facilitate the exposition, we normalize the LTV ratio $\omega = 1$, $\rho = 1$ and $\varepsilon = 0$. Then, $L(r) = P_r Q = r$. Note that r represents the company's real-time credit rating, whereas a company could choose a lower financial leased size $L(r_0) = P_r Q_0 = r_0$ under the lower rate $r_0 \leq r$ with lower shipping capacity $Q_0 \leq Q$. The revenue from the slot purchase contracts and direct sales will be used to pay back the financial lease rental. To facilitate an exploration of the constraint effect of financial leasing on various contract types, we compare liner companies with sufficient capacity $Q + q_0$ to those with insufficient capacity $Q_0 + q_0$. The list of other notations used in this study is given in Table 5.1.

Parameters

d	The market demand quantity
μ	The expected value of market demand, $\mu = E(d)$
i	Fixed financial lease interest rate
q_0	The initial capacity that the liner company has from owned vessels
P_r	The freight rate paid by shippers for each TEU capacity sold
c, c_1, c_2	Operating cost per unit of TEU capacity sold, $c = c_1 + c_2$
$\mathrm{g}, \mathrm{g}_1, \mathrm{g}_2$	Goodwill loss per unit of unmet TEU capacity, $\mathrm{g} = \mathrm{g}_1 + \mathrm{g}_2$
k_1, k_2	The level of pessimism among financial institutions towards the repayment of financial lease
$\delta k_1, \delta k_2$	The level of optimism among financial institutions towards the repayment of financial lease
M^-, M^+	The point at which pessimism and optimism become milder

Decision Variables

Q	The capacity that over-placement liner company get from financially leased vessels
Q_0	The capacity that over conservative liner company get from financially leased vessels
$x(x^*, \tilde{x})$	The order quantity of NVOCC, x ∈ [0, $Q + q_0$]
$\theta(\theta^*, \tilde{\theta})$	The NVOCC's revenue share from the sales
$\mathrm{b}(b^*, \tilde{b})$	The buyback price from the liner company
P_s	Wholesale price, freight fee paid from the NVOCC to the liner company

Other Variables

Π_N, Π_L, Π_c	The profit function of stakeholders under sufficient capacity case in stage 1
π_N, π_L, π_c	The profit function of stakeholders under insufficient capacity case in stage 1

Π_L', π_L'	The profit function of over-placement liner company and over conservative liner company in stage 2
m, m_0	The surplus profit of over-placement liner company and the surplus profit of over conservative liner company, respectively
r	The updated liner company's credit rating
$r(m)$	The amount of change in the credit rating of the liner company
$L(r)$	Allowances to support finance leases for the liner company, $L(r) = \rho r + \varepsilon$
Δ, Δ_0	The financial lease rental over-placement liner company and over conservative liner company in stage 1
Δ', Δ_0'	The financial lease rental over-placement liner company and over conservative liner company in stage 2

Table 5.1 Notations of Chapter 5

5.3.2 Symbolic Model Settings

According to Moore and Healy (2008), we treat the over-conservative liner company (risk-neutral) as the benchmark to investigate the credit rating of over-placement liner company (risk-seeking). We will first examine the equilibrium contract conditions with sufficient shipping capacity and then compare them with the equilibrium contract conditions with insufficient capacity. The profitability of these two cases also plays an essential role in our investigation. We will also compare the solvency of the two scenarios from a credit rating perspective. We assume the financial institution is risk-averse when facing the financial lease application from liner companies. Based on the credit rating model of Moody's Investor Service, Standard & Poor's (S&P) and the Fitch Group, we simply divide the investment grades into AA, A, B and BB (Dimitrov et al., 2015; Toscano, 2020). We build our credit rating model following these credit rating definitions (see Table 8.1 in Appendix C.1 for definition details) and combine it with prospect theory (Kahneman and Tversky, 1979) to construct a theoretical model.

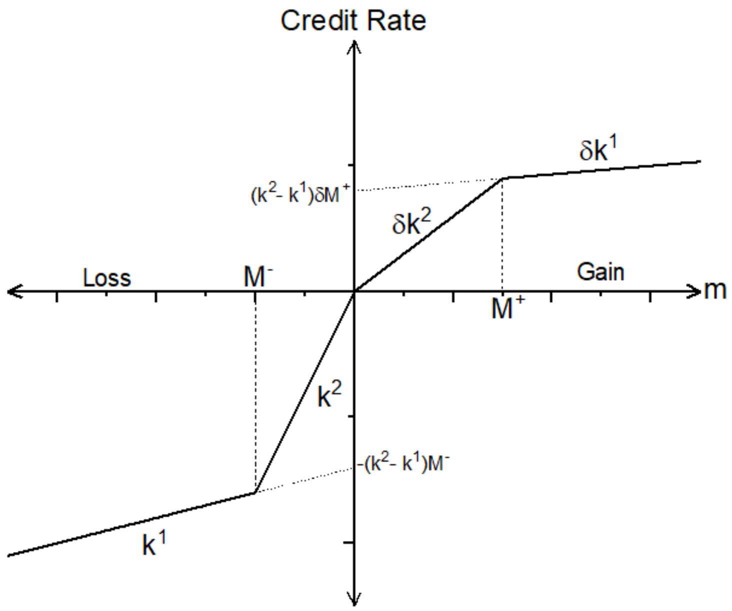

Figure 5.3 Prospect-dependent Credit Rate Model

Specifically, we set the credit rating of company as BB when the overdraft of liner company is less than $|M^-|$, and as B when the overdraft of liner company is greater than $|M^-|$. If the liner company's surplus profit after payback is greater than M^+, we will assign it the rating of AA, and if it is less than M^+, we will assign it a rating of A. According to these four grades, we have established the following theoretical model with reference to the linear utility function (see Long and Nasiry, 2015; Zhang et al., 2016; Wu et al., 2018 for similar models). Here, we denote the liner company's profit minus rent by m (see Appendix C.1 for proof of $r(m)$). $r'$is the updated credit rating and r is the credit rating before repayment.

$$r(m) = r' - r = \begin{cases} k_1 m + (k_2 - k_1)M^-, & B\ : m < M^- \\ k_2 m, & BB: M^- \leq m < 0 \\ \delta k_2 m, & A\ : 0 \leq m < M^+ \\ \delta k_1 m + (k_2 - k_1)\delta M^+, & AA: M^+ \leq m \end{cases} \quad (1)$$

Where $0 < k_1 < k_2$ and $\delta M^+ < |M^-| < M^+$. k_1 and k_2 present the pessimism level of financial institutions towards a liner company's shortfalls in repayment. $0 < \delta < 1$ controls the financial institution's more negative response to losses than its positive response to gains. Figure 5.3 illustrates the above relationship between increases and decreases in credit ratings with the difference between profits and rental.

To make the comparison clearer we characterise the financial lease term in the supply chain as a two-stage operating model for the over-conservative liner company and the over-placement liner company to show their differences.

Stage 1: Competition stage. At this stage, liner companies finance and lease more vessels to gain more shipping capacity to grab a larger market share.

Stage 2: Restoration Rating Phase. The liner company which suffers a loss needs to restore its credit rating score. The liner company enjoying surplus profit could gain an acceptable amount of losses.

Given that the liner companies have two financial leasing terms, they need to address the question of which negotiation should be conducted first. The following Figure 5.4 shows the timeline of their decision on finance leases in two stages. We assume that initial losses during stage 1 are not always detrimental but can lead to an increase in competitiveness that cannot be measured by the short-term profit from operations. We need to consider scale economies, service, environmental impact, amongst others (Ha and Seo, 2017). Because the actual market demand is unknown, according to Jin et al. (2022), the liner companies will have certain deviations according to market conditions in determining the amount of financial lease in stage 1. If the liner companies think that the market prospect is better, not only will the liner company completely book the order quantity of NVOCC in the slot purchase contract, but also will lease additional shipping capacity to obtain more profit from direct sales. The main role of stage 2 is to predict the benefits that the operational results of the stage 1 can provide and future considerations that need to be made.

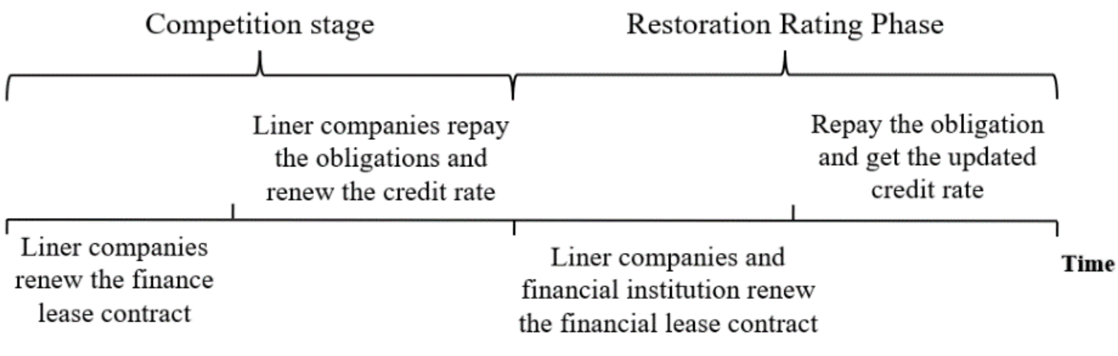

Figure 5.4 Timing of two financial leasing terms

5.4 The restrictive impact of financial leasing on slot purchase contracts

5.4.1 Equilibrium Analysis with Sufficient Capacity

In this session, we will first define the optimal contract solution to these three contract types and then investigate the profit allocation of liner company and NVOCC in terms of their total expected profit under each contract type. First, we set $S_N(x)$ as the expected sales of NVOCC:

$$S_N(x) = E[min(x, d)] = x - \int_0^x F(d)dd$$

Let $I_N(x)$ be the unsatisfied demand of NVOCC:

$$I_N(x) = E[(d - x)^+] = \mu - x + \int_0^x F(d)dd$$

When the NVOCC completes the sale, the expected sales of the liner company in subsequent direct sales is:

$$S_L(x) = E[(\mathrm{Q} + \mathrm{q}_0 - x - d)^+] = (\mathrm{Q} + \mathrm{q}_0 - x) - \int_x^{\mathrm{Q}+\mathrm{q}_0} F(d)dd$$

Let $I_L(x)$ be the stockout capacity of the liner company in direct sales:

$$I_L(\mathrm{Q} + \mathrm{q}_0) = E[(d - \mathrm{Q} - \mathrm{q}_0)^+] = \mu - (\mathrm{Q} + \mathrm{q}_0) + \int_0^{Q+q_0} F(d)dd$$

To facilitate visualisation, we use superscripts 1 and 2 to denote the liner company’s expected cash flow from slot purchase contracts and direct sales, respectively. Regardless of the contract type, the liner company's expected profit from direct sales will always be:

$$\Pi\,_L^2 = P_r S_L(x) - (Q + q_0 - x)c - I_L(\mathrm{Q} + \mathrm{q}_0)\mathrm{g} \tag{1}$$

Here, we use subscripts BB, RS and QD to denote the buyback contract, the revenue-sharing contract and the quantity discount contract, respectively. Under the buyback contract, the liner company's expected profit function from the slot purchase contract is:

$$\Pi\,_{L_{BB}}^1(x, P_s, \mathrm{b}) = P_s\, x - \mathrm{b}[x - S_N(x)] - c_2 x - I_N(x)\mathrm{g}_2 \tag{2}$$

Then, the NVOCC's expected profit function is:

$$\Pi_{N_{BB}}(x, P_s, \mathrm{b}) = P_r S_N(x) + \mathrm{b}[x - S_N(x)] - P_s x - c_1 x - I_N(x)\mathrm{g}_1 \tag{3}$$

Under a revenue-sharing contract, the liner company's expected profit function from the slot purchase contract is:

$$\Pi_{L_{RS}}^{1}(x, P_s, \theta) = P_s\, x + (1-\theta)P_r S_N(x) - c_2 x - I_N(x)\mathrm{g}_2 \tag{4}$$

Then, the NVOCC's expected profit function is:

$$\Pi_{N_{RS}}(x, P_s, \theta) = \theta P_r S_N(x) - P_s x - c_1 x - I_N(x)\mathrm{g}_1 \tag{5}$$

Under a quantity discount contract, the liner company's expected profit function from slot purchase contract is:

$$\Pi_{L_{QD}}^{1}(x, \mathrm{P_s}(x)) = \mathrm{P_s}(x)\, x - c_2 x - I_N(x)\mathrm{g}_2 \tag{6}$$

Then, the NVOCC's expected profit function is:

$$\Pi_{N_{QD}}(x, \mathrm{P_s}(x)) = P_r S_N(x) - \mathrm{P_s}(x) x - c_1 x - I_N(x)\mathrm{g}_1 \tag{7}$$

Then in the slot purchase contract, the total profit of the NVOCC and the liner company under these contract types is the same as:

$$\begin{aligned}\Pi_c &= \Pi_N + \Pi_L^{1} \\ &= P_r S_N(x) - (c_1 + c_2)x - (\mathrm{g}_1 + \mathrm{g}_2) I_N(x)\end{aligned} \tag{8}$$

The proof of all theoretical results can be found in Appendix C.2. Here, the superscript $*$ represents the optimal choice of the corresponding parameter.

Proposition 1. The profit of the liner company and the NVOCC are maximised by setting

$$F(x^*) = \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}}$$

$$P_s = \begin{cases} c_2 + b^* - (b^* + \mathrm{g}_2)\dfrac{c}{P_r + \mathrm{g}}, & \text{buyback contract,} \\ c_2 - (P_r - \theta^* P_r + \mathrm{g}_2)\dfrac{\mathrm{c}}{(P_r + \mathrm{g})}, & \text{revenue sharing contract,} \\ c_2 - \mathrm{g}_2 \dfrac{c}{(P_r + \mathrm{g})}, & \text{quantity discount contract} \end{cases}$$

First, **Proposition 1** demonstrates the relationships between contract parameters that maintain the effectiveness of the three contracts in terms of the liner company's and the NVOCC's maximised profits. In this container shipping chain, it should be emphasised that only by setting the contract wholesale price to be fixed can the quantity discount contract maximise the profit of each stakeholder and the overall profit at the same time. Secondly, the optimal choice of the contract parameter x^* is independent of both the other contract parameters and Q. Therefore, the change in contract types will not affect the optimal order quantity of the NVOCC. However, it is not difficult to find from the profit function of each stakeholder that Q only limits the maximum value of the order quantity x. Besides, the expected profit made by the liner company from direct sales were not included into the contract negotiation in **Proposition 1**.

Corollary 1 If the direct selling profit is considered in the slot purchase contract, total profit function is monotonically increasing over the range of x and none of these contracts can maximise the profit for each stakeholder as well as the total profit function at the same order quantity. However, when $F(\mathrm{Q}^* + \mathrm{q}_0) = \frac{P_r+\mathrm{g}-\mathrm{c}}{P_r+\mathrm{g}}$, the total profit function can take the maximum value and the change of contract type will not change this result.

Corollary 1 indicates that the joining of direct selling profit will disrupt the balance of the optimal contract decision-making between the liner company and NVOCC. *Therefore, the direct sale of liner companies should not be included in the lot purchase contract*. When combining **Proposition 1** and **Corollary 1**, the profit for each stakeholder and the total profit function can be maximised simultaneously at $\mathrm{Q}^* = x^* - \mathrm{q}_0$. It is worth noting that the contract type will not affect this conclusion. However, this conclusion is acceptable only if the liner company holds capacity greater than the potential order quantity that NVOCC might make.

Corollary 2. Here, similar to Zhang et al. (2016), we let

$$\lambda = \begin{cases} \dfrac{P_r + g_1 - b}{P_r + g}, & \text{buyback contract,} \\ \dfrac{\theta P_r + g_1}{P_r + g}, & \text{revenue sharing contract,} \\ \dfrac{P_r + g_1}{P_r + g}, & \text{quantity discount contract} \end{cases}$$

When $\lambda_1 = \frac{\mu g_1}{\Pi_c^* + \mu g}$ and $\lambda_2 = \frac{\Pi_c^* + \mu g_1}{\Pi_c^* + \mu g}$ (where Π_c^* is the maximised profit for the sum of Π_N and Π_L^1), this leads to the following results:

1. If $0 \leq \lambda < \lambda_1$, the NVOCC earns negative profit, and the liner company holds more than Π_c^*
2. The liner company extract the entire Π_c^* if $\lambda = \lambda_1$
3. The liner company and NVOCC sharing the Π_c^* if $\lambda_1 < \lambda < \lambda_2$
4. The NVOCC extract the entire Π_c^* if $\lambda = \lambda_2$
5. If $\lambda_2 < \lambda \leq 1$, the liner company earns negative profit and the NVOCC holds more than Π_c^*

If the optimal wholesale price is set accounting to **Proposition 1**, then combined with **Corollary 2** we get $P_s(\mathrm{b}) = -c_1 + b + \lambda c$, $P_s(\theta) = -c_1 + \lambda c$ and $P_s(x) = -c_1 + \lambda c$. Here, λ is set like the channel profit allocation index in Zhang et al. (2016). It is used to more clearly and simply point out the profit allocation of the liner company and NVOCC to the overall contract profit under different contract types at the same time. Through λ, we get the optimal wholesale price under the revenue sharing contract and the buyback contract that can be regulated by the following equations:

$$P_s(\mathrm{b}) - b = P_s(\theta) \tag{9}$$
$$b = (1 - \theta)P_r$$

Therefore, there is a one-to-one correspondence between the parameters under the revenue sharing contract and the buyback contract. Note that the contract types will also not change the way that the total profit Π_c^* is split between the liner company and NVOCC according to λ in **Corollary 2**.

5.4.2 The Standard Setting with Insufficient Capacity

To explore the constraint effect of financial leasing on various contract types, we first investigate its limitations on the capacity that the liner company can provide. It is well known in the literature that available shipping capacity and market coverage are essential for the competitiveness of liner companies (Ha and Seo, 2017; Jin et al., 2022). However, only long-term good reputation and a good credit record will allow the liner company to obtain sufficient capital to continuously expand and improve (Alexopoulos and Stratis, 2016), which is not what most companies can achieve. To consider this characteristic of the maritime industry, we will discuss further the situation where the liner company cannot provide sufficient capacity x^* given by **Proposition 1.** If the liner company does not have enough capacity to support the optimal reservation of the NVOCC, the NVOCC might not accept the contract or will perhaps order the total capacity that the liner company holds to obtain as much profit as possible. This will result in the equilibrium of the above optimal contracts being disturbed and the negotiation unable to reach an agreement.

If the capacity held by the liner company is set to be $Q_0 + q_0 = \frac{1+\sin\alpha}{2}x, \alpha \in \mathbb{R}$, the liner company's capacity will be less than the order quantity x. Then the quantity that NVOCC order will be $x = \frac{2(Q_0+q_0)}{1+\sin\alpha}$. To meet the NVOCC's order requirements, the liner company needs to order additional capacity as $\frac{1-\sin\alpha}{2}x$ from the market at higher market price P_r. Then, we restructure the profit of both liner company and NVOCC under each contract type to find the new equilibrium conditions.

Since the liner company can still provide the shipping capacity that the NVOCC wants to book through additional purchases from market, the profit function of the NVOCC will not be affected. Under the buyback contract, the profit of liner company will be as follows:

$$\pi^1_{L_{BB}}(x, P_s, \mathrm{b}) = P_s\, x - \mathrm{b}[x - S_N(x)] - c_2 x - I_N(x)\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r \qquad (10)$$

Under the revenue sharing contract, the profit function of liner company will be:

$$\pi^1_{L_{RS}}(x, P_s, \theta) = P_s\, x + (1-\theta)P_r S_N(x) - c_2 x - I_N(x)\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r \qquad (11)$$

Under the quantity discount contract, the profit function liner company will be:

$$\pi_{LQD}^{1}\big(x, P_s(x)\big) = \mathrm{P_s}(x)\,x - c_2 x - I_N(x)\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r \tag{12}$$

At the same time, the profit of NVOCC is $\pi_N = \Pi_N$. The total profit of the NVOCC and liner company under these contract types will change to:

$$\begin{aligned}\pi_c &= \pi_N + \pi_L^1 = \Pi_c - \frac{1-\sin\alpha}{2}xP_r \\ &= P_r S_N(x) - \frac{1-\sin\alpha}{2}xP_r - (c_1 + c_2)x - (\mathrm{g}_1 + \mathrm{g}_2)I_N(x)\end{aligned} \tag{13}$$

Because the NVOCC has ordered more than $Q_0 + q_0$, the liner company will not have any capacity left for direct sales, so there will be no directselling $(Q + q_0 - x)$ for this kind of liner company. Therefore, $\pi_L^2 = -I_L(Q_0 - \mathrm{q}_0)\mathrm{g} = -E[(d - Q_0 - \mathrm{q}_0)^+]\mathrm{g}$. *Here, the superscript ~ denotes the new optimal choice of each corresponding parameter.*

Proposition 2 Under the condition of $Q_0 + q_0 = \frac{1+\sin\alpha}{2}x \le x$**,** the NVOCC will accept the contract when the optimal choice of other contract parameters becomes:

$$F(\tilde{x}) = \frac{P_r + \mathrm{g} - c + \frac{\sin\alpha - 1}{2}P_r}{P_r + \mathrm{g}}$$

$$P_s = \begin{cases} -c_1 + b + \dfrac{(P_r - \tilde{b} + \mathrm{g}_1)(c - \frac{\sin\alpha - 1}{2}P_r)}{P_r + \mathrm{g}}, & \text{buyback contract,} \\ -c_1 + \dfrac{(\tilde{\theta}P_r + \mathrm{g}_1)(c - \frac{\sin\alpha - 1}{2}P_r)}{P_r + \mathrm{g}}, & \text{revenue sharing contract,} \\ -c_1 + \dfrac{(P_r + \mathrm{g}_1)(c - \frac{\sin\alpha - 1}{2}P_r)}{P_r + \mathrm{g}}, & \text{quantity discount contract} \end{cases}$$

When comparing **Proposition 2** with **Proposition 1**, we find that as the profit of the liner company decreases, the NVOCC also decreases the predetermined amount accordingly. But to stabilise the balance of the contract, the liner company will reduce the some portion of the optimal wholesale price to compensate the NVOCC for the component it intended to order but was unable to.

Corollary 3 According to the profit function under stochastic demand, let $\mathrm{H}(x) = \Pi_{LRS}^1(x, P_s, \theta) - \Pi_{LQD}^1\big(x, P_s(x)\big)$ and $h(x) = \pi_{LRS}^1(x, P_s, \theta) - \pi_{LQD}^1\big(x, P_s(x)\big)$. Through the calculation in Appendix C.2, we get $0 \leq h(x) \leq \mathrm{H}(x)$. Combine with **Corollary 2,** this implies the following properties:

(a) $\Pi_{LQD}^1\big(x, P_s(x)\big) \leq \Pi_{LRS}^1(x, P_s, \theta) = \Pi_{LBB}^1(x, P_s, \mathrm{b})$

(b) $\pi_{LQD}^1\big(x, P_s(x)\big) \leq \pi_{LRS}^1(x, P_s, \theta) = \pi_{LBB}^1(x, P_s, \mathrm{b})$

(c) $\Pi_{LQD}^1\big(x, \mathrm{P_s}(x)\big) \leq \pi_{LRS}^1(x, P_s, \theta)$

Considering equations (4), (6), (11) and (12), it is not difficult to find that the profit of the liner company under the revenue sharing contract has $(1-\theta)P_r S_N(x)$ $(0 < \theta < 1)$ part more than that under the quantity discount contract. However, the wholesale price under the revenue sharing contract is lower than that of the quantity discount contract. **Corollary 3** implies that the profit of the liner company is higher under the revenue sharing contract (equivalent to the buyback contract) than under a quantity discount contract. Since the total profit of liner company and NVOCC is the same under different contract types, we can easily prove that the profit of the NVOCC is higher under a quantity discount contract than under the other two contract types. Note that this result is the same under the liner company's sufficient capacity case or insufficient capacity case. However, the difference between them is lower in the sufficient capacity case than in the insufficient capacity case.

From equations (8) and (13), it is obvious that the total profit of both NVOCC and liner company is lower under the insufficient capacity case than sufficient capacity case. However, the liner company only needs a small number of leased vessels to support shipping capacity allowing it to reduce a portion of its debt financing costs. Therefore, there are two issues here: the first is that even if the financial lease amount is lower, the profit is also less. The second issue is that, regardless of whether the lease is cheaper, the revenue will be greater than the expenses. Cachon and Lariviere (2005) asserted that there is a significant distinction between revenue derived from deterministic demand and revenue generated by a stochastic demand function. Therefore, whether the financial lease rental can be repaid by the revenue under the deterministic demand setting is another issue.

5.5 The impact of over-conservative and over-placement on credit rating

5.5.1 Optimal decisions related to financial lease

In the previous chapter, we determined the impact of limited financial leasing capacity on the contract parameters and on the stakeholder's profit in the equilibrium situation. However, both company operators and investors are attracted by long-term excess profits, which is based on a large economy of scale (Ha and Seo, 2017). As a result, a long-term business strategy that balances debt levels and profitability is critical for liner companies. To investigate the conditions for shipping companies to achieve long-term competitive growth, we set the liner company under insufficient capacity $Q_0 + q_0$ as an over-conservative company and the liner company under sufficient capacity $\mathrm{Q} + \mathrm{q}_0$ as an over-placement company and investigate their performance in the face of risk-averse financial institutions. The over-conservative liner company only holds the capacity that is less than the order quantity that the NVOCC intends to place, whereas the over-placement liner company holds the shipping capacity that is greater than the order quantity that the NVOCC may place. Therefore, $Q_0 + q_0 < x < \mathrm{Q} + \mathrm{q}_0$.

We assume these two companies start their lease term with the same credit rating r_1. Then, both could have the allowance $L(r_1) = P_r Q = r_1$ for financial lease. The over-placement liner company, who choose to use the total allowance, should repay the financial lease rental as:

$$\Delta = r_1 \frac{i+1}{1+e^{r_1}} = P_r \mathrm{Q} \frac{i+1}{1+e^{r_1}} \tag{14}$$

The over-conservative liner company will only use part of the allowance as $L(r_0) = P_r Q_0 = r_0 < r_1 = L(r_1)$. Here, the credit rating is $r_0 < r_1$. The financial lease rental that the over-conservative liner company should repay is given by the formula:

$$\Delta_0 = r_0 \frac{i+1}{1+e^{r_1}} = P_r Q_0 \frac{i+1}{1+e^{r_1}} \tag{15}$$

If the liner company cannot take out the full rental amount, the credit rating score of the company will be partially deducted based on the missing amount. Meanwhile, if the liner company has profit left over after the full repayment is made, the credit score of the liner company will receive bonus points. Let m be the surplus profit that is the profit of the over-placement liner company minus the repayment:

$$m = \Pi_L - \Delta \tag{16}$$

Let m_0 be the surplus profit of the over-conservative liner company:

$$m_0 = \pi_L - \Delta_0 \tag{17}$$

Updates to credit ratings are calculated based on the prospect-dependent credit rate model in equation (1). In the competition stage, the liner company could maintain the same credit rating when the following formula is satisfied:

$$\begin{aligned}\Pi_L &= \Pi_L^1 + \Pi_L^2 \\ &= P_s x - \mathrm{b}[x - S_N(x)] - c_2 x - I_N(x)\mathrm{g}_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(Q + q_0)\mathrm{g} = \Delta \end{aligned} \tag{18}$$

$$\pi_L = \pi_L^1 + \pi_L^2 = P_s x - \mathrm{b}[x - S_N(x)] - c_2 x - I_N(x)\mathrm{g}_2 - \frac{1 - \sin\alpha}{2} x P_r - I_L(Q_0 - \mathrm{q}_0)\mathrm{g} = \Delta_0 \tag{19}$$

Note that there is no profit from the direct sales for the over-conservative company. This is because the capacity Q_0 that the liner company will control is less than the order quantity that NVOCC might make.

Proposition 3 We set **Condition 1** as the liner company which made profits m^+ in stage 1 and can tolerate a certain amount of overdraft in stage 2 and **Condition 2** as the liner company which made losses m^- in stage 1 and could earn profits in stage 2 to restore its rating. Then, there are the following results:

(1) There are three cases for each condition:

condition 1: The maximum loss amount for liner company is

$$\mathrm{Loss}(m^+) = \begin{cases} \text{case 1: } -\delta M^+ < -\delta m^+ \le 0, & 0 \le m^+ < M^+ \\ \text{case 2: } M^- < -\dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2} \le -\delta M^+, & M^+ \le m^+ < M^+ - \dfrac{k_2(M^- + \delta M^+)}{\delta k_1} \\ \text{case 3: } -\dfrac{\delta k_1 m^+ + (k_2 - k_1)(\delta M^+ + M^-)}{k_1} \le M^-, & M^+ - \dfrac{k_2(M^- + \delta M^+)}{\delta k_1} \le m^+ \end{cases}$$

condition 2: The minimum profit that a liner company should earn is

$$\mathrm{Profit}(m^-) = \begin{cases} \text{case 4: } 0 < -\dfrac{m^-}{\delta} \le M^+, & -\delta M^+ \le m^- < 0 \\ \text{case 5: } M^+ < \dfrac{-k_2 m^- - (k_2 - k_1)\delta M^+}{\delta k_1} \le M^+ - \dfrac{k_2(M^- + \delta M^+)}{\delta k_1}, & M^- \le m^- < -\delta M^+ \\ \text{case 6: } M^+ - \dfrac{k_2(M^- + \delta M^+)}{\delta k_1} \le -\dfrac{k_1 m^- + (k_2 - k_1)(\delta M^+ + M^-)}{\delta k_1}, & m^- \le M^- \end{cases}$$

(2) No matter how the credit rating of the liner company changes within stages 1 and 2, the net profit of liner company is positive. The specific net profit in **Condition 1** and **Condition 2** is:

$$\text{Condition 1:} \quad \begin{cases} m^+(1-\delta) > 0, & case\ 1 \\ m^+ - \dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2} > 0, & case\ 2 \\ m^+ - \dfrac{\delta k_1 m^+ + (k_2 - k_1)(\delta M^+ + M^-)}{k_1} > 0, & case\ 3 \end{cases}$$

$$\text{Condition 2:} \quad \begin{cases} m^-(1-\dfrac{1}{\delta}) > 0, & \text{case 4} \\ m^- - \dfrac{k_2 m^- + (k_2 - k_1)\delta M^+}{\delta k_1} > 0, & \text{case 5} \\ m^- - \dfrac{k_1 m^- + (k_2 - k_1)(\delta M^+ + M^-)}{\delta k_1} > 0, & \text{case 6} \end{cases}$$

(3) Since the loss and gain in conditions 1 and 2 share the same interval, the following equations could be applied to these two conditions.

$$\begin{cases} m^- = -\delta m^+, & Case\ 1\ and\ Case\ 4 \\ m^- = -\dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2}, & Case\ 2\ and\ Case\ 5 \\ m^- = -\delta m^+ - \dfrac{(k_2 - k_1)(\delta M^+ + M^-)}{k_1}, & Case\ 3\ and\ Case\ 6 \end{cases}$$

Note that $M^- + \delta M^+ < 0$. The above results outline the minimal requirements for a liner company to preserve its rating in subsequent operations when it gains profits or suffers losses under its credit rating.

When comparing result (1) and result (2) in **Proposition 3**, it is obvious that when the liner company makes a profit in the first stage, even if it suffers loses in the subsequent operations, it can still avoid relegation. In addition, the more surplus profits that a liner company could obtain, the stronger the ability that the company will have to bear losses in subsequent operations. In contrast, when the liner company made a loss in the first stage, it only needs to achieve a certain amount of profit to return to the original rating in subsequent operations.

Through the first derivative of the loss function with respect to the profit in Condition 1, there is a decrease in the amount that the liner company can lose in the future for each previous increase in profit. Based on the result (3) in **Proposition 3,** by taking the first derivative of the profit with respect to the loss in Condition 2, the result in case 2 and case 5 is higher than in the other two cases. Therefore, the liner company needs to earn a higher profit to offset the previous loss. Thus, liner companies should try to avoid making profits at AA rating while suffering losses at BB rating. Based on this, we excluded case 2 in Condition 1 and the case 5 in Condition 2. Like the result (3) of **Proposition 3**, we found the following relationship between profits and losses, regardless of the order of profits and losses in the first or second stages and the credit rating intervals:

$$m^{-} = -\delta m^{+} + \epsilon(\epsilon > 0) \tag{20}$$

Corollary 4 When facing the stochastic market demand, **Corollary 2** and equation (9) still holds for the profit function of the over-conservative liner company and the NVOCC.

Based on **Corollary 4**, the over-conservative liner company could also control the distribution of centralised profit through the same λ as the over-placement liner company in **Corollary 2**. The only difference between the above result and **Corollary 2** is the relationship between λ and the decision of contract parameters (x, P_s, θ), (x, P_s, b) and $\left(x, P_s(x)\right)$.

Proposition 4 For over-placement liner companies who fully use the allowance under their credit rating, there is a certain maximum value as $(r^{*} - 1)(1 + i)$ for their financial lease rental when $\frac{1}{1+e^{r^{*}}} = \frac{r^{*}-1}{r^{*}} < 1$.

With the increase in credit rating, the financial lease rental will first increase until $\frac{r}{1+e^{r}} - r + 1 = 0$. After this, $\frac{r}{1+e^{r}} - r + 1 < 0$ and the financial lease rental will decrease with the increase in credit rating. Therefore, for over-placement liner companies, using more allowance does not pose a persistent risk to solvency. Instead, it can not only reduce the debt to be serviced, but it can also bring in more capacity to support competitiveness. Additionally, if the expected profitability of the liner company cannot support the maximum financial lease rental, the company can reduce the financial lease rental like over-conservative liner companies, which is reducing the allowance proportionally to $\frac{r_0}{r^*}\Delta$. We assume that the over-placement liner companies need to pay the maximum financial lease rental as $(r_1 - 1)(1 + i)$.

5.5.2 Strategies to Guarantee Credit Ratings

Through the predictions, we can compare the liner company's expected profits and the financial lease repayments before financing and operating activities happen in order to make optimal decisions. Therefore, our analysis below will focus on the equilibrium condition for liner companies. Here, let the superscript ' denote the correlated variable in the second stage. Since the financial lease rental will not exceed $(r^* - 1)(1 + i)$, we assume that the over-placement liner companies need to pay the maximum financial lease rental as $(r_1 - 1)(1 + i)$. Here, $r_1 = r^* > r_0$. If the over-placement liner companies could repay this amount of financial lease rental, there is no need to worry about solvency and credit rating.

Based on the previous assumption, the surplus profit function of the over-placement liner company when facing the stochastic market demand is:

$$m(Q,\lambda) = (1-\lambda)\Pi_c^* - \mu(\lambda \mathrm{g} - \mathrm{g}_1) + P_r S_L(x) - (Q + q_0 - x)c - I_L(Q + q_0)\mathrm{g} - (r_1 - 1)(1 + i)$$

If the liner company cannot repay the rental, the updated credit rating will be:

$$r_1 + r(m) = \begin{cases} r_1 + k_2 m, & BB \\ r_1 + k_1 m + (k_2 - k_1)M^-, & B \end{cases}$$

The largest financial lease size will reduce:

$$L(r) = P_r Q' = r_1 + r(m) \rightarrow \; Q' = Q + \frac{r(m)}{P_r}$$

According to equation (20), if the liner company wants to restore the original rating, the minimum surplus profit that needs to be obtained in the second stage is: $(0 < \delta < 1)$

$$-\frac{m(Q,\lambda) - \epsilon}{\delta} = \Pi_L'(Q,\lambda) - \Delta' = \Pi_L' - [r_1 + r(m)]\frac{i+1}{1 + e^{r_1 + r(m)}}$$

Therefore,

$$\Pi_L'(Q,\lambda) = \frac{\Delta}{\delta} - \frac{\Pi_L(Q,\lambda)}{\delta} + \Delta' + \frac{\epsilon}{\delta}$$

If the liner company could repay the rental, the updated credit rating will be:

$$r_1 + r(m) = \begin{cases} r_1 + \delta k_2 m, & A \\ r_1 + \delta k_1 m + (k_2 - k_1)\delta M^+, & AA \end{cases}$$

The largest financial lease size will increase to:

$$L(r) = P_r Q' = r_1 + r(m) \rightarrow \ Q' = Q + \frac{r(m)}{P_r}$$

Assume the liner company keeps using the largest allowance, the financial lease repayment will decrease to:

$$\Delta_0' = [r_1 + r(m)]\frac{i+1}{1 + e^{r_1 + r(m)}} < (r_1 - 1)(1 + i)$$

To avoid being downgraded, the maximum amount that a liner company can lose in the second stage is:

$$-\delta m_0(\Pi_L) + \epsilon = \Pi_L'(Q, \lambda) - \Delta' = -\delta[\Pi_L(Q_0, \lambda) - \Delta] + \epsilon$$

Therefore,

$$\Pi_L'(Q, \lambda) = \Delta' - \delta[\Pi_L(Q, \lambda) - \Delta] + \epsilon$$

For the over-conservative liner company, the surplus profit function is:

$$m_0(Q_0, \lambda) = (1 - \lambda)\pi_c - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - I_L(Q_0 - \mathrm{q}_0)\mathrm{g} - \frac{r_0}{r_1}(r_1 - 1)(1 + i)$$

According to **Proposition 2**, $\pi_c = \Pi_c - \frac{1 - \sin\alpha}{2} x P_r$. If the liner company can repay the rental, the updated credit rating will be:

$$r_1 + r(m) = \begin{cases} r_1 + \delta k_2 m, & A \\ r_1 + \delta k_1 m + (k_2 - k_1)\delta M^+, & AA \end{cases}$$

The largest financial lease size will increase to:

$$L(r) = P_r Q_0' = r_1 + r(m) \rightarrow \ Q_0' = Q_0 + \frac{r(m)}{P_r}$$

Assume the liner company keeps using the same allowance, the financial lease repayment will decrease to:

$$\Delta_0' = r_0 \frac{i+1}{1 + e^{r_1 + r(m)}}$$

To avoid being downgraded, the maximum amount that a liner company can lose in the second stage is:

$$-\delta m_0(\pi_L) + \epsilon = \pi_L'(Q_0, \lambda) - \Delta_0' = -\delta[\pi_L(Q_0, \lambda) - \Delta_0] + \epsilon$$

Therefore,

$$\pi_L'(Q_0, \lambda) = \Delta_0' - \delta[\pi_L(Q_0, \lambda) - \Delta_0] + \epsilon$$

Proposition 5 When facing stochastic market demand, let $\Lambda(m)$ represent the surplus profit of the over-placement liner company minus the surplus profit of the over-conservative liner company.

$$\Lambda(m) = (1-\lambda)(\Pi_c^* - \pi_c) + \Pi_L^2 + I_L(Q_0 - q_0)g + \frac{r_0 - r_1}{r_1}(r_1 - 1)(1+i) \quad (21)$$

i. If the over-conservative liner companies can repay their financial lease rental $\frac{r_0}{r_1}(r_1 - 1)(1+i)$, the over-placement liner company can also repay larger financial lease rental $(r_1 - 1)(1+i)$ when $\Lambda(m) = 0$.

ii. When $\Lambda(m) \geq 0$ and both liner companies can repay the rental, $m(Q,\lambda) > m_0(Q_0,\lambda) > 0$. The over-placement liner companies can accumulate more credit scores at each stage and more limits to loss at stage 2 than the over-conservative liner companies when $\Lambda(m) \geq 0$. Based on **Proposition 3**, it would be better for liner companies to keep their surplus profits within the range of $M^+ \geq m(Q,\lambda) > m_0(Q_0,\lambda)$.

iii. When $\Lambda(m) \geq 0$ and both liner companies cannot repay the rental, $0 > m(Q,\lambda) > m_0(Q_0,\lambda)$. The surplus profit that over-placement liner companies need to earn in the second stage is less than the over-conservative liner companies. In addition, the interest of the over-placement liner company will be lower than that of the over-conservative liner companies. Because of the market share foundation laid in the first stage, there will be greater future profits for over-conservative liner companies.

iv. When $\Lambda(m) < 0$, if the over-conservative liner companies happen to repay the rental, the over-placement liner companies cannot repay the rental. Although the interest of the over-conservative liner companies' financial lease decreases, the over-placement liner companies will also lower their rental due to the lowering of the credit rating.

v. When $\Lambda(m) < 0$, if both liner companies cannot repay the rental, the surplus profit that the over-conservative liners need to earn in the second stage is less than the over-placement liner companies. Under this case, the financial lease rental of both companies will decrease. However, once both liner companies return to their original ratings in the second stage and maintain good profitability and solvency, then the difference between the two liner companies will go back to case ii.

Note that the result in **Proposition 5** is based on **Corollary 4** and **Proposition 4**. From the profit function of the over-conservative liner company and the over-placement liner company, when the liner company negotiates the same λ with the NVOCC, the profit that the over-placement liner company can get is more than the profit that the over-conservative liner company can get. However, because the over-conservative liner company leases fewer vessels and the interest rate is the same for both liner companies, the financial lease rental of the over-conservative liner company is less than that of over-placement liner company. **Proposition 5** compared the profit and financial lease rental of the over-conservative liner company and the over-placement liner company. If the over-conservative liner company can repay the total rental, the result shows that the over-placement liner company can also repay the largest amount of financial lease rent when the sum of Π_L^2 (the over-placement liner company's expected profit from directly selling), the goodwill loss of the over-conservative liner company and $(1-\lambda)(x-Q_0+q_0)P_r$ could cover the difference between their financial lease rentals. Here, $(1-\lambda)(x-Q_0+q_0)P_r$ is a portion of the cost that over-conservative liner company used to fulfil the NVOCC's order. The portion rate $1-\lambda$ depends on the decision of the liner company in the slot purchase contracts, which is also related to how the liner company and the NVOCC distribute the centralised profit.

From **Proposition 5**, it is obvious that the over-conservative liner company and the over-placement liner company share the same solvency under the case i. However, the over-placement liner company has stronger profitability than the over-conservative liner company. Additionally, the larger financial lease size will increase liner companies' market share and awareness, which will lead to additional future income. Therefore, the over-placement liner company performs better than the over-conservative liner company under these conditions. In addition, the over-conservative liner companies may not be able to avoid heavy debt by leasing fewer vessels. However, the deterministic market demand may result in a dramatically varied realization of expected profits.

Proposition 6 When facing the deterministic market demand, the profit of the over-placement liner companies will reach maximum value when the market demand $d = Q + q_0$ and the profit of over-conservative liner companies will reach maximum value when the market demand $d = x$. Combined with **Proposition 3**, the maximum overdrafts that over-placement liner companies and over-conservative liner companies can take to balance operations and debt and the minimum surplus profits for both liner companies to restore their ratings are shown in Appendix C.2. In addition, we have the following conclusions:

i. The over-conservative liner companies can only fully outperform the over-placement liner companies when,

$$(\lambda P_r - c)(Q + q_0 - x) + (Q - Q_0)\mathrm{g} < \Delta - \Delta_0 = \frac{r_1 - r_0}{r_1}(r_1 - 1)(1 + i)$$

ii. The over-placement liner companies can deliver positive profits in a broader market demand than over-conservative liner companies.

iii. When the market demand is small, if the gap between the over-conservative liner companies and the over-placement liner companies' financial lease rent cannot exceed the gap between their profit, the over-conservative liner companies need greater control of overdrafts than the over-placement liner companies. In this instance, the following formula holds, and as market demand increases, over-placement liner companies will be more conducive to rent payable than over-conservative liner companies.

$$(x - Q_0 - q_0)P_r(1 - \lambda) - (Q + q_0 - x)c < \frac{r_1 - r_0}{r_1}(r_1 - 1)(1 + i)$$

iv. When $P_r\lambda + \mathrm{g}\lambda - 2\mathrm{g} - c \geq 0$, the over-placement liner companies show better profitability than the over-conservative liner companies in all cases of market demand. When both liner companies suffer loss in the same range in stage 1, it will be easier for the over-placement liner companies to repay the financial lease rental in stage 2.

When **Proposition 5** and **Proposition 6** are combined, the advantages of over-placement liner companies are even more obvious. In response to a deterministic market demand, over-placement liner companies have a greater chance to obtain more surplus profit than over-conservative liner companies. In addition, once the gap between them exceeds the difference between the two financial leases, the advantages over-conservative liner companies gain by reducing the financial leases no longer exist. Once the difference between their profit surpasses the difference between their financial lease rental, the benefits over-conservative liner companies gain by reducing the financial leases are no longer applicable. From the perspective of short-term operations, it is evident that the over-conservative liner companies might be more dominant than over-placement liner companies in the depressed freight market. However, the liner shipping industry is characterized by a denser shipping network which can lead to greater market share and competitiveness. Consequently, during the same operations period, the market demand that can be achieved by over-placement liner companies will exceed the market demand that over-conservative liner companies can achieve. In addition, **Proposition 6** demonstrates once more that the over-conservative liner companies may not be able to avoid their debt by lowering their leased vessels. However, over-placement liner companies can repay by regulating the cooperation with NVOCC to obtain better profits.

5.6 Numerical examples

In this section, numerical examples are provided to explain the research subject we examined and to offer insights into credit rating. For operating problems, we first derive a pair of optimal conditions. To obtain insights into the effects of financial leasing on the operating outcomes between liner company and NVOCC, we consider the over-placement liner companies with fully financial leased size $Q = 100$ and over-conservative liner companies with less financial leased size $Q_0 < Q$, respectively. Then, we perform a numerical study on the equilibrium financial lease rental and corresponding profits.

We suppose the market demand reflects a standard normal distribution with parameters $d \sim N(100, 30)$. Accordingly, the parameters used are as follows: $q_0 = 50, P_r = 400, c_1 = 20, c_2 = 30, \mathrm{g}_1 = 50, \mathrm{g}_2 = 40$. Correspondingly, $x^* \approx 138$ is the ideal quantity that NVOCC should order when collaborating with over-placement liner companies. No matter how Q changes, the range of critical point λ which is used to adjust profit distribution between the over-placement liner company and the NVOCC is fixed at $[0.1208, 0.9033]$. However, the critical point λ interval that balances over-conservative liner companies and NVOCC profits is $[0.1539, 0.8769]$ when Q_0 is 50, $[0.1335, 0.8932]$ when Q_0 is 60, and $[0.1427, 0.8858]$ when Q_0 is 70. Here we mainly focused on the result when $Q_0 = 60$. Under this case, the optimal order quantity that the NVOCC will order is $\tilde{x} \approx 125$.

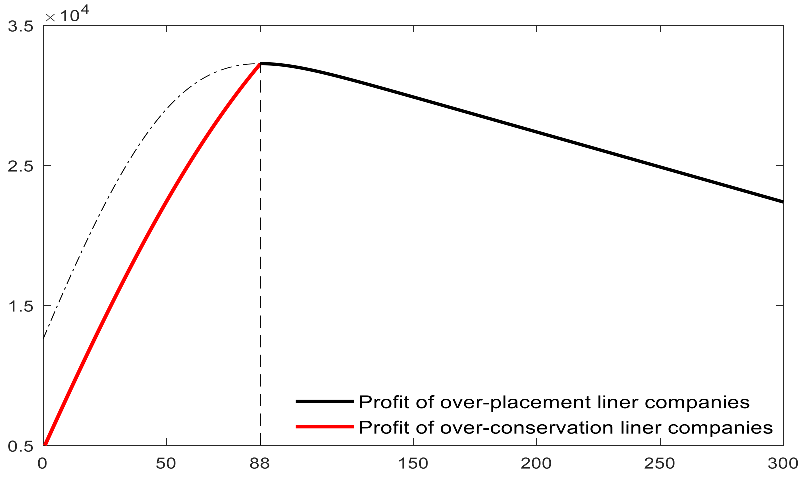

Figure 5.5 Profit of both liner companies under stochastic demand.

To better compare over-placement liner companies and over-conservative liner companies, Figure 5.5 presents how the profits of two liner companies vary with the size of the finance lease volume. The vertical axis of this Figure represents the profit of each liner company, and the horizontal axis is the quantity of financial lease volume. It is obvious that the profit of over-conservative liner companies shows a large increase trend. In particular, when the leasing volume of over-conservative liner companies is infinitely close to over-placement liner companies, over-conservative liner companies' profits are also close to over-placement liner companies' profit. But in contrast, the profit of over-placement liner companies shows a small decrease trend with the increase of financial lease volume.

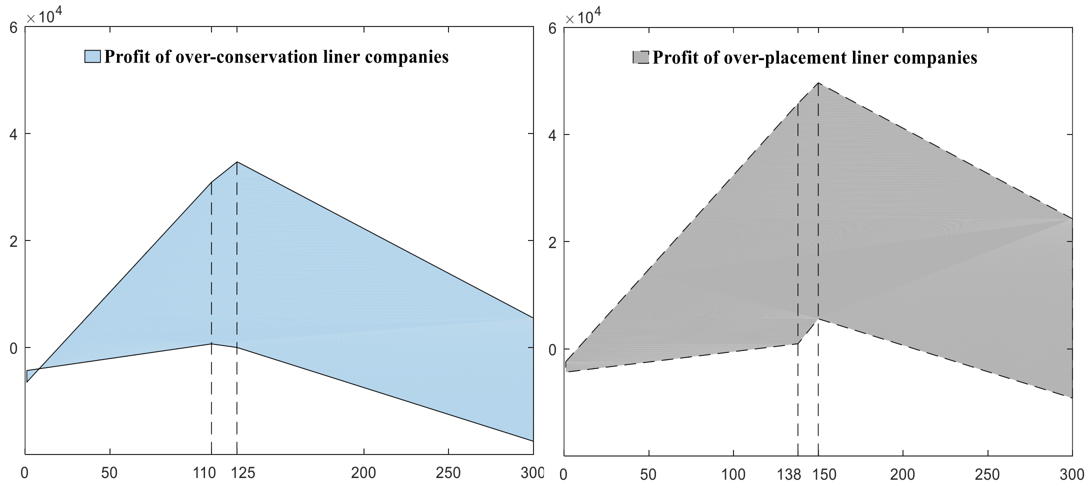

Figure 5.6 Profit of both liner companies under deterministic demand

Although the profits of the two liner companies converge under certain conditions, the above Figure 5.6 better shows the profits of two liner companies under the deterministic market demand. The vertical axis of Figure 5.6 represents the profit of each liner company, and the horizontal axis is the market demand volume. As illustrated in Figure 5.6, the profit of over-placement liner companies can achieve a higher profit than over-conservative liner companies. In addition, it is evident that the negative profit area of over-placement liner companies is less than that of over-conservative liner companies. To better illustrate this, both liner companies choose the critical point λ that maximise their profits when collaborating with NVOCC, namely $\lambda = 0.1208$ for over-placement liner companies and $\lambda = 0.1335$ for over-conservative liner companies.

Then, the optimal wholesale price selected by the over-placement liner companies based on the previous parameters is $P_s(\mathrm{b}) = 376.84$ under the buyback contract, $P_s(\theta) = -13.96$ under the revenue-sharing contract, and $P_s(x) = 25.92$ under the quantity discount contract. In addition, the NVOCC will choose $\theta = 0.023$ under the revenue-sharing contract and the over-conservative liner companies will also choose $b = 395.63$ under the buyback contract. By contrast, the optimal wholesale price selected by the over-conservative liner companies based on the previous parameters is $P_s(\mathrm{b}) = 374.11$ under the buyback contract, $P_s(\theta) = -5.95$ under the revenue-sharing contract, and $P_s(x) = 70.40$ under the quantity discount contract. In addition, the NVOCC will choose $\theta = 0.0498$ under the revenue-sharing contract and the over-conservative liner companies will also choose $b = 380.06$ under the buyback contract. In this instance, the primary reason behind over-conservative liner companies' profitability is worse than for over-placement liner companies' because over-conservative liner companies need to purchase additional shipping capacity owing to the reduction in financial leasing.

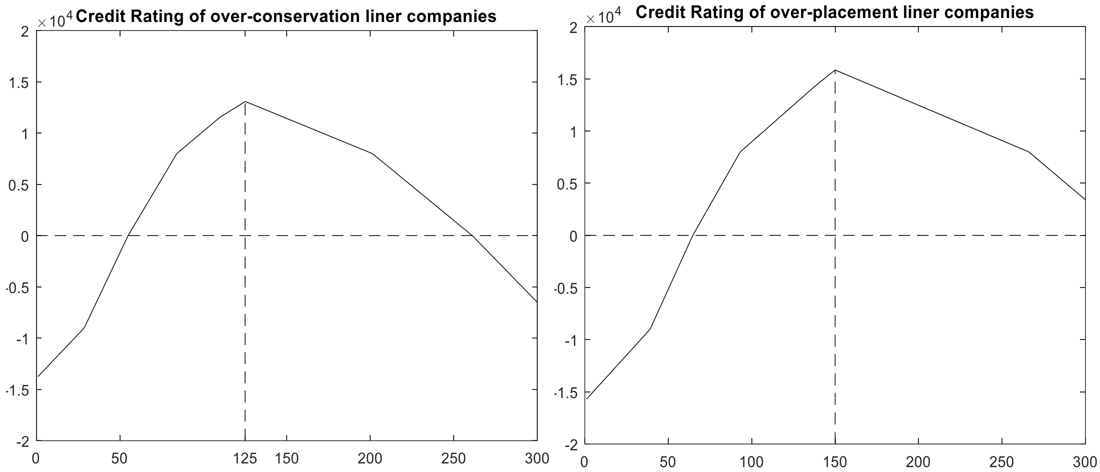

Figure 5.7 Changes in the Credit Rate of two liner companies

Based on the Prospect-dependent Credit Rate Model, Figure 5.7 depicts how the credit rates of over-placement and over-conservative liner companies fluctuate in response to market demand. Then, in terms of the equilibrium between the actual operation and financial lease payback in the first stage, over-placement liner companies seem to perform better than over-conservative liner companies. Due to the large downgrade of the credit rating, there is a high potential for both companies to reduce a large amount of finance leasing.

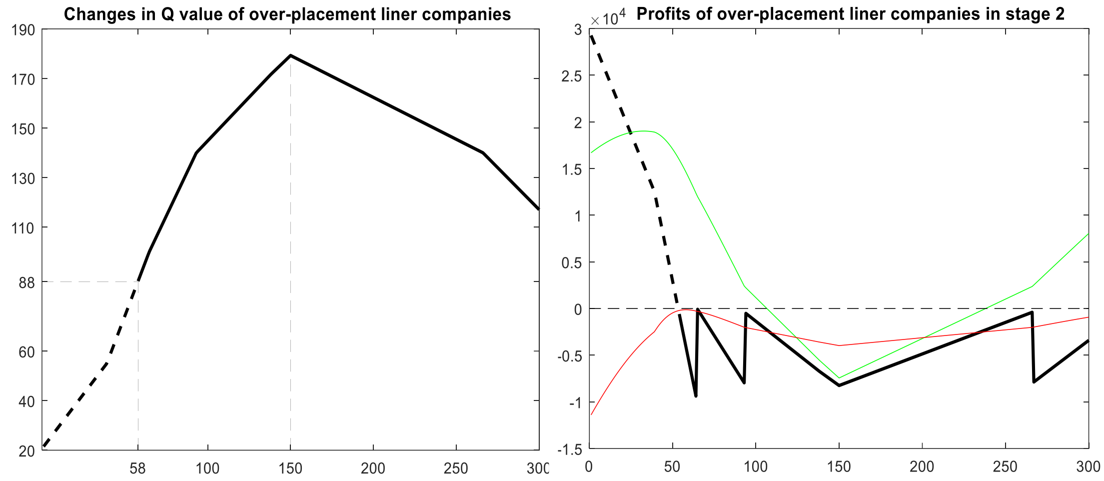

Figure 5.8 Changes of over-placement liner companies

Due to the impossibility of calculating the impact of a change in the amount of financial leasing on all the key parameters of the over-conservative liner companies, we focus primarily on those of the over-placement liner companies. The horizontal axis of Figure 5.8 represents the market demand volume. The vertical axis of the left figure is the maximum amount that the over-placement liner companies can lease in stage 2. The dotted line indicates the situation in which the over-placement liner companies may become an over conservative company. This is because the overdraft induced by market demand during this period has led the company's ratings to plummet. Under this case, the credit rate of a company cannot support the financial lease of vessel to provide enough shipping capacity. Fortunately, this part only dominates a small fraction of all real situations when facing deterministic market demands. According to the surplus profit in the first stage, the black line on the right figure represents the minimum profit (dotted line) that over-placement liner companies need to achieve in the second stage and the maximum overdraft (solid line) that can be lost. Here, the red and green lines represent the profits that over-placement liner companies can achieve in the second stage at maximum λ and minimum λ, respectively. Therefore, the area between the two lines is all the possible profits that the over-placement liner companies can obtain by regulating critical point λ. When comparing them with the black line, we find that when the over-placement liner companies face a large loss in the first stage, the company can control the profit to recover ratings by regulating the slot purchase contract with the NVOCC in most cases. Additionally, over-placement liner companies' minimum profit in most cases will not be lower than the maximum overdraft that can be lost.

5.7 Conclusions and Future research

This paper studies the restrictions of financial leasing on the capacity of liner companies to balance operating profit and financial lease rental and adopts the prospect theory to present the credit rating of liner companies, which is also the willingness financial institutions show toward financial leases. We calculate the optimal slot purchase contact for liner companies and the NVOCC in over-conservative and over-placement situations respectively and then prove the over-placement liner companies seem to perform better than the over-conservative liner companies in many cases.

Our analysis on the profitability of over-conservative liner companies and over-placement liner companies shows that overly conservative behaviour could reduce the rent payable to a certain extent but can also lead to lower profits. In contrast, due to the improvement of credit rating, the financial institutions hold a positive attitude towards the financial leases of over-placement liner companies which leads to the existence of a certain maximum value for the finance lease rental of over-placement liner companies. However, the profitability of over-placement liner companies is higher than that of over-conservative liner companies under stochastic demand. Therefore, the advantage over-conservative liner companies obtain from reducing financial leases is offset by the absolute profitability of over-placement liner companies.

Moreover, through the Prospect-dependent Credit Rate Model, we also find that the risk-averse behaviour of financial institutions will not have a huge negative impact on the risk preference behaviour of liner companies in this paper. The main reason is that the risk preference behaviour of over-placement liner companies is used to expand their operating scale to capture more market share and increase competitiveness. In addition, based on the analysis of the profit of over-placement liner companies and over-conservative liner companies under deterministic demand, we show that both types of liner company could reach similar profitability when the market demand is low, but over-placement liner companies' profitability remains high when the over-conservative liner companies' profit is reduced due to goodwill loss caused by high market demand. Consequently, when both liner companies face the same demand, the over-conservative liner companies outperform over-placement liner companies in the depressed freight market. While in a booming freight market, the over-placement liner companies outperform the over-conservative liner companies. However, the fact that the denser shipping network of over-placement liner companies can bring more market demand should also be considered. In fact, the numerical study also proves that over-placement liner companies can adjust their profits more flexibly to face financial lease rents. This adjustment on the critical point of the contract could also ensure the future collaboration with the NVOCC. From the perspective of operating risk, the NVOCC could help the liner companies keep a certain profitability to deal with financial risk. Additionally, our study confirms that when liner companies make profits at the AA rating while suffering losses at the BB rating, they will receive lower future losable overdrafts when holding surpluses, or they will require higher surpluses to restore their ratings when they experience losses. Therefore, the over-placement liner companies' flexible adjustment of critical points makes the company more competitive than the over-conservative liner companies.

Finally, it is not possible for this paper to cover all the issues connected to the utilisation of the financial lease of the liner company for shipping capacity. However, based on the numerical study, the over-placement liner companies are forced to adopt conservative strategies due to excessive losses. Under this condition, it is worthwhile investigating the circumstances under which over-placement liner companies are compelled to change their risk preference strategies, as well as the circumstances under which over-conservative liner companies could benefit from adopting an over-placement strategy to boost their competitiveness. Additionally, **Proposition 3** also shows that if the liner company is overly conservative, retaining enormous residual profits is of little value. It should invest this portion of the profits in financial leasing so that the profits can be maintained within the A range.

According to this idea and combined with the slot purchase contract, it is also worth investigating how to set the contract parameters to adjust the profit within an acceptable range and the optimal strategy for balancing financial lease and operation.

5.8 List of Symbols

d	The market demand quantity
μ	The expected value of market demand, $\mu = E(d)$
$f(d)$	Probability density function of market demand
$\mathrm{F}(d)$	Cumulative distribution function of market demand
i	Fixed financial lease interest rate
q_0	The initial capacity that the liner company has from owned vessels
P_r	The freight rate paid by shippers for each TEU capacity sold
c	Operating cost per unit of TEU capacity sold, $c = c_1 + c_2$
c_1	Operating cost of NVOCC per TEU capacity sold
c_2	Operating cost of liner company per TEU capacity sold
g	Goodwill loss per unit of unmet TEU capacity, $\mathrm{g} = \mathrm{g}_1 + \mathrm{g}_2$
g_1	Goodwill loss of NVOCC per unmet TEU capacity
g_2	Goodwill loss of liner company per unmet TEU capacity
k_1, k_2	The level of pessimism among financial institutions towards the repayment of financial lease
$\delta k_1, \delta k_2$	The level of optimism among financial institutions towards the repayment of financial lease
M^-	The point at which pessimism becomes milder
M^+	The point at which optimism becomes milder
Q	The capacity that over-placement liner company get from financially leased vessels
Q_0	The capacity that over conservative liner company get from financially leased vessels
Q^*	The optimal financial leased capacity
x	The order quantity of NVOCC
x^*	The optimal order quantity of NVOCC with over-placement liner company
$\tilde{x}$	The optimal order quantity of NVOCC with over conservative liner

	company
θ	The percentage of the revenue that NVOCC keeps
θ^*	The NVOCC's revenue sharing rate with over-placement liner company
$\tilde{\theta}$	The NVOCC's revenue sharing rate with over-conservative liner company
b	The buyback price from liner company
b^*	The buyback price from over-placement liner company
$\tilde{b}$	The buyback price from over-conservative liner company
P_s	Wholesale price, freight fee paid from the NVOCC to the liner company
Π_N	The profit function of NVOCC under sufficient capacity case in stage 1
Π_L	The profit function of liner company under sufficient capacity case in stage 1
Π_c	The profit function of the centralised company under sufficient capacity case in stage 1
π_N	The profit function of NVOCC under insufficient capacity case in stage 1
π_L	The profit function of liner company under insufficient capacity case in stage 1
π_c	The profit function of centralised company under insufficient capacity case in stage 1
Π_L'	The profit function of over-placement liner company in stage 2
π_L'	The profit function of over-conservative liner company in stage 2
m, m_0	The surplus profit of over-placement liner company
m, m_0	The surplus profit of over conservative liner company
r	The updated liner company's credit rating
r_1	The original credit rating of over-placement and over conservative liner company
r_0	The original credit where the rental allowance is chosen by over conservative liner company

$r(m)$	The amount of change in the credit rating of the liner company
$L(r)$	Allowances to support finance leases for the liner company, $L(r) = \rho r + \varepsilon$
Δ	The financial lease rental of over-placement liner company in stage 1
Δ_0	The financial lease rental of over conservative liner company in stage 1
Δ'	The financial lease rental of over-placement liner company in stage 2
Δ'_0	The financial lease rental of over conservative liner company in stage 2
$S_N(x)$	The expected sales of the NVOCC
$I_N(x)$	The unsatisfied demand of the NVOCC
$S_L(x)$	The number of unsold products
$I_L(x)$	The number of unsold products
BB	The buyback contract
RS	The revenue-sharing contract
QD	The quantity discount contract

Chapter 6. Conclusion and Contribution

Over the last decade, an increasing number of scholars have focused on the financial risk in supply chain operations and begun to examine the link between supply chain operations and finance. Most of them emphasised using financial tools to decrease operational risk and improve operational efficiency. Few of them conduct empirical research to identify the negative impact of excessive use of financial tools on operational risks. Regarding the liner shipping sector, we devote our research effort to similar areas and develop theoretical models to propose methods to reduce the impact of operational risk on financial risk and improve operational competitiveness by controlling financing strategies. The outcome of this thesis also aims to implement a novel method for liner companies to diagnose their liquidity.

In 1st theoretical chapter (Chapter 3), we investigate the ideal contract parameters of the slot purchase contract between liner companies and NVOCC and its impact on the defaults on finance leases. After assessing the profits before repayment and default costs of liner companies under the buyback contract, the revenue sharing contract and the quantity discount contract, we reveal the importance of contrasting operating profits and finance lease rents when measuring contracts under financial constraints. We contribute to the existing literature on supply chain management by researching the detrimental effects of excessively using financial tools to support operations. By comparing the default risk associated with the excess use of financial tools, the objective of contract design is to mitigate the possibility that market demand risk would jeopardise solvency and thus threaten future financing capacity. Our result indicates that quantity discount contracts are more conducive to financial lease repayment than buyback contracts and revenue sharing contracts when the finance lease is small, despite the fact that buyback contracts and revenue sharing contracts are more likely to generate greater profits for liner companies. This is due to the fact that quantity discount contracts have a greater likelihood of ensuring repayment under a low scale of financial lease. When it comes to paying back a financial lease on a large scale, buyback contracts and revenue sharing contracts are still more favourable.

In 2[nd] theoretical chapter (Chapter 4), the focus of the theoretical model is mainly on the equilibrium conditions of operation and financing. Although we also investigated the ideal contract parameters of the slot purchase contract between liner companies and NVOCC, some adjustments were made to the content of the contracts. Compared to the contract model in the 1st theoretical chapter, the liner companies in the 2nd theoretical chapter share some of NVOCC's operating costs and goodwill loss in exchange for the profit from the direct selling process. According to Appendix D, the optimal order quantity from NVOCC is higher under the strategy for slot purchase without direct selling profit than with direct selling profit. When adding the slot purchase contract in Chapter 3 to the optimal contract parameters in Chapter 4, only the buyback contracts will be accepted by NVOCC. Because under other contract types, the wholesale prices are all negative. In addition, according to the results of Chapter 4, because the liner company has greater control over the parameters of the buyback contract, it will be more flexible for the liner company to regulate the contract to address the repayment of the financial lease when the market parameters change. Therefore, in accordance with the conditions for the liner company to balance operations and financing provided in Chapter 4, the buyback contract is more conducive to the repayment of the liner company.

In 3[rd] theoretical chapter (Chapter 5), the emphasis of the theoretical model has shifted from short-term operations to long-term operations and financing of the liner company. Based on this, financial institutions relax their strict demands for repayment but maintain a risk-averse attitude toward overdue. Therefore, a credit ratings model based on prospect theory is employed to depict the attitudes of the financial institutions towards the financial leasing conducted by liner companies. Besides, we also investigate the ideal contract parameters of the slot purchase contract between NVOCC and liner companies without insufficient shipping capacity. In contrast to the slot purchase contract for a liner company with sufficient shipping capacity, we conclude that if the liner company cannot enhance its solvency in a depressed freight market by drastically decreasing the finance lease scale, then it is desirable for the liner company to maintain the finance lease scale at the order volume level of the NVOCC in the slot purchase contract. In a market with strong demand, keeping insufficient capacity poses a significant risk to the company's ability to remain competitive over the long term. Besides, if the operation is carried out over a lengthy period, a certain amount of excess profits can compensate for the impairment to the financing ability caused by the prior overdraft. Consequently, from a long-term commercial point of view, it is desirable for the development of companies in the liner shipping sector to maintain a particular over-placement strategy.

This thesis contributes to the literature in three stages. The first step is to offer contract design and comparison within a short-term commercial cycle, and the second step is to give balance strategies for dynamic situations. The last step is to develop long-term strategies for the liner company to keep a stable balance between operating and financial lease. Since the judgment of liquidity, profitability and solvency from the accounting point of view is based on complete financial statements of the company, the analysis of this thesis based on one phase of operating activities is slightly limited to the judgment of the liner company's ability from all aspects. Moreover, the actual market environment will be more complicated than the scenario assumed in this thesis. Therefore, the establishment of simulation experiments based on this paper can make up for the above limitations and verify the conclusions of this paper. Furthermore, it would be worthwhile to conduct additional research into the connections between the variables that were not considered in this thesis, such as the connection between the scale of financial leasing and market demand.

Chapter 7. References

Alexopoulos, I. and Stratis, N. (2016) 'Structured finance in shipping', *The International Handbook of Shipping Finance: Theory and Practice*, pp. 191-211.

Anupindi, R. and Bassok, Y. (1999) 'Supply contracts with quantity commitments and stochastic demand', *Quantitative models for supply chain management*, pp. 197-232.

Bart, N., Chernonog, T. and Avinadav, T. (2021) 'Revenue-sharing contracts in supply chains: a comprehensive literature review', *International Journal of Production Research,* 59(21), pp. 6633-6658.

Basu, P., Liu, Q. and Stallaert, J. (2019) 'Supply chain management using put option contracts with information asymmetry', *International Journal of Production Research,* 57(6), pp. 1772-1796.

Braglia, M., Castellano, D., Marrazzini, L. and Song, D. (2019) 'A continuous review,(Q, r) inventory model for a deteriorating item with random demand and positive lead time', *Computers & Operations Research,* 109, pp. 102-121.

Bresnahan, T. F. and Reiss, P. C. (1985) 'Dealer and manufacturer margins', *The RAND Journal of Economics*, pp. 253-268.

Brooks, M. R. and Faust, P. (2018) *50 years of review of maritime transport, 1968-2018: Reflecting on the past, exploring the future*.

Cachon, G. P. (2003) 'Supply chain coordination with contracts', *Handbooks in operations research and management science,* 11, pp. 227-339.

Cachon, G. P. and Lariviere, M. A. (2005) 'Supply chain coordination with revenue-sharing contracts: strengths and limitations', *Management science,* 51(1), pp. 30-44.

Cachon, G. P. and Netessine, S. (2006) 'Game theory in supply chain analysis', *Models, methods, and applications for innovative decision making*, pp. 200-233.

Cai, X., Chen, J., Xiao, Y., Xu, X. and Yu, G. (2013) 'Fresh-product supply chain management with logistics outsourcing', *Omega,* 41(4), pp. 752-765.

Cariou, P. and Wolff, F.-C. (2013) 'Chartering practices in liner shipping', *Maritime Policy & Management,* 40(4), pp. 323-338.

Cariou, P. and Guillotreau, P. (2021) 'Capacity management by global shipping alliances: findings from a game experiment', *Maritime Economics & Logistics,* pp. 1-26.

Dimitrov, V., Palia, D. and Tang, L. (2015) 'Impact of the Dodd-Frank act on credit ratings', *Journal of Financial Economics,* 115(3), pp. 505-520.

Elliott, B. and Elliott, J. (2007) *Financial accounting and reporting.* Pearson Education.

Feng, B., Li, Y. and Shen, H. (2015) 'Tying mechanism for airlines’ air cargo capacity allocation', *European Journal of Operational Research,* 244(1), pp. 322-330.

Gibbons, R. S. (1992) *Game theory for applied economists.* Princeton University Press.

Gilroy, S., Cavazza, M., Chaignon, R., Mäkelä, S.-M., Niiranen, M., André, E., Vogt, T., Billinghurst, M., Seichter, H. and Benayoun, M. 'An emotionally responsive AR art installation'. *International Symposium of Mixed and Augmented Reality 2007*.

Gómez-Padilla, A., González-Ramírez, R. G., Alarcón, F. and Voß, S. (2021) 'An option contract model for leasing containers in the shipping industry', *Maritime Economics & Logistics,* 23, pp. 328-347.

Guo, S., Shen, B., Choi, T.-M. and Jung, S. (2017) 'A review on supply chain contracts in reverse logistics: Supply chain structures and channel leaderships', *Journal of Cleaner Production,* 144, pp. 387-402.

Ha, Y. S. and Seo, J. S. (2017) 'An analysis of the competitiveness of major liner shipping companies', *The Asian Journal of Shipping and Logistics,* 33(2), pp. 53-60.

He, Y. (2017) 'Supply risk sharing in a closed-loop supply chain', *International Journal of Production Economics,* 183, pp. 39-52.

Höhn, M. I. and Höhn, M. I. (2010) 'Literature review on supply chain contracts', *Relational Supply Contracts: Optimal Concessions in Return Policies for Continuous Quality Improvements,* pp. 19-34.

Holzhacker, M., Krishnan, R. and Mahlendorf, M. D. (2015) 'Unraveling the black box of cost behavior: An empirical investigation of risk drivers, managerial resource procurement, and cost elasticity', *The Accounting Review*, 90(6), pp. 2305-2335.

Hu, J., Huang, Y. and Zhao, X. (2019) 'Research on the slot allocation of container liner under E-commerce environment', *Computers & Industrial Engineering,* 129, pp. 556-562.

Huang, J., Yang, W. and Tu, Y. (2020) 'Financing mode decision in a supply chain with financial constraint', *International Journal of Production Economics,* 220, pp. 107441.

Jiang, W. and Liu, J. (2018) 'Inventory financing with overconfident supplier based on supply chain contract', *Mathematical Problems in Engineering,* 2018.

Jin, L., Chen, J., Chen, Z., Sun, X. and Yu, B. (2022) 'Impact of COVID-19 on China's international liner shipping network based on AIS data', *Transport Policy,* 121, pp. 90-99.

Joo, M. H. and Parhizgari, A. (2021) 'A behavioral explanation of credit ratings and leverage adjustments', *Journal of Behavioral and Experimental Finance,* 29, pp. 100435.

Kai-Ineman, D. and Tversky, A. (1979) 'Prospect theory: An analysis of decision under risk', *Econometrica,* 47(2), pp. 363-391.

Kalkanci, B., Chen, K.-Y. and Erhun, F. (2011) 'Contract complexity and performance under asymmetric demand information: An experimental evaluation', *Management science,* 57(4), pp. 689-704.

Katok, E. and Wu, D. Y. (2009) 'Contracting in supply chains: A laboratory investigation', *Management Science,* 55(12), pp. 1953-1968.

Kong, F., Qu, L. and Xu, L. (2017) 'Decision and Coordination Strategy for Liner Slots Booking Based on Demand Updating', *Journal of University of Shanghai for Science and Technology,* 39(3), pp. 255-261.

Kouvelis, P. and Zhao, W. (2008) 'Financing the newsvendor: Supplier vs. bank, optimal rates, and alternative schemes', *Olin Business School-WashingtonUniversity-2008*.

Kouvelis, P. and Zhao, W. (2011) 'The newsvendor problem and price-only contract when bankruptcy costs exist', *Production and Operations Management*, 20(6), pp. 921-936.

Kouvelis, P. and Zhao, W. (2012) 'Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts', *Operations research*, 60(3), pp. 566-580.

Kouvelis, P. and Zhao, W. (2016) 'Supply chain contract design under financial constraints and bankruptcy costs', *Management Science*, 62(8), pp. 2341-2357.

Kouvelis, P. and Zhao, W. (2018) 'Who should finance the supply chain? Impact of credit ratings on supply chain decisions', *Manufacturing & Service Operations Management*, 20(1), pp. 19-35.

Lai, Z., Lou, G., Zhang, T. and Fan, T. (2021) 'Financing and coordination strategies for a manufacturer with limited operating and green innovation capital: Bank credit financing versus supplier green investment', *Annals of Operations Research*, pp. 1-35.

Lambert, A. (2013) 'The World's Key Industry: History and Economics of International Shipping', *The Journal of Transport History*, 34(2), pp. 217.

Lariviere, M. A. (1999) 'Supply chain contracting and coordination with stochastic demand', *Quantitative models for supply chain management*, pp. 233-268.

Li, G., Wu, H., Sethi, S. P. and Zhang, X. (2021) 'Contracting green product supply chains considering marketing efforts in the circular economy era', *International Journal of Production Economics*, 234, pp. 108041.

Li, X., Sun, L. and Gao, J. (2013) 'Coordinating preventive lateral transshipment between two locations', *Computers & Industrial Engineering*, 66(4), pp. 933-943.

Li, Y. (2006) 'The pros and cons of leasing in ship financing: —Theoretical perceptions versus practitioner’s views', *WMU Journal of Maritime Affairs*, 5, pp. 61-74.

Li, Z. and Dong, B. (2021) 'A game theory analysis of China’s maritime cross-border insolvency policy: from the perspective of Hanjin shipping’s bankruptcy case', *Maritime Policy & Management*, 48(3), pp. 419-431.

Lin, Q. and Xiao, Y. (2018) 'Retailer credit guarantee in a supply chain with capital constraint under push & pull contract', *Computers & Industrial Engineering*, 125, pp. 245-257.

Liu, W.-h., Xu, X.-c. and Kouhpaenejad, A. (2013) 'Deterministic approach to the fairest revenue-sharing coefficient in logistics service supply chain under the stochastic demand condition', *Computers & Industrial Engineering*, 66(1), pp. 41-52.

Liu, W., Liu, X. and Li, X. (2015) 'The two-stage batch ordering strategy of logistics service capacity with demand update', *Transportation Research Part E: Logistics and Transportation Review*, 83, pp. 65-89.

Liu, X., Xu, Q. and Xu, L.-X. (2015) 'A literature review on supply chain contracts selection and coordination under competing multi manufacturers', *International Journal of Business and Management*, 10(7), pp. 196.

Liu, Z. and Wang, J. (2019) 'Supply chain network equilibrium with strategic financial hedging using futures', *European Journal of Operational Research*, 272(3), pp. 962-978.

Long, X. and Nasiry, J. (2015) 'Prospect theory explains newsvendor behavior: The role of reference points', *Management Science*, 61(12), pp. 3009-3012.

Méró, L. (1998) *Moral calculations: Game theory, logic, and human frailty*. Springer Science & Business Media.

Monacelli, N. (2018) 'Improving maritime transportation security in response to industry consolidation', *Homeland Security Affairs*, 14.

Moore, D. A. and Healy, P. J. (2008) 'The trouble with overconfidence', *Psychological review*, 115(2), pp. 502.

Moorthy, K. S. (1987) 'Comment—managing channel profits: Comment', *Marketing science*, 6(4), pp. 375-379.

Nocke, V. and Thanassoulis, J. (2014) 'Vertical relations under credit constraints', *Journal of the European Economic Association*, 12(2), pp. 337-367.

Okur, İ. G. and Tuna, O. (2022) 'Schedule Reliability in Liner Shipping: A Study on Global Shipping Lines', *Pomorstvo,* 36(2), pp. 389-400.

Palepu, K. (2016) *Business Analysis and Valuation.* Cengage Textbooks.

Pan, K., Lai, K. K., Leung, S. C. and Xiao, D. (2010) 'Revenue-sharing versus wholesale price mechanisms under different channel power structures', *European Journal of Operational Research,* 203(2), pp. 532-538.

Pasternack, B. A. (1985) 'Optimal pricing and return policies for perishable commodities', *Marketing science,* 4(2), pp. 166-176.

Peng, W. and Bai, X. (2022) 'Prospects for improving shipping companies’ profit margins by quantifying operational strategies and market focus approach through AIS data', *Transport Policy,* 128, pp. 138-152.

Qiu, X. and Lee, C.-Y. (2019) 'Quantity discount pricing for rail transport in a dry port system', *Transportation Research Part E: Logistics and Transportation Review,* 122, pp. 563-580.

Quiggin, J. (1996) ''A Theory of Anticipated Utility', Journal of Economic Behavior and Organization, 3 (2-3), June/September, 323-43', *INTERNATIONAL LIBRARY OF CRITICAL WRITINGS IN ECONOMICS,* 73, pp. 97-117.

Ren, Y. and Croson, R. (2013) 'Overconfidence in newsvendor orders: An experimental study', *Management Science,* 59(11), pp. 2502-2517.

Risk, M. (2014) 'Lease Financing'.

Schmeidler, D. (1989) 'Subjective probability and expected utility without additivity', *Econometrica: Journal of the Econometric Society,* pp. 571-587.

Schweitzer, M. E. and Cachon, G. P. (2000) 'Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence', *Management science,* 46(3), pp. 404-420.

Shen, B., Choi, T.-M. and Minner, S. (2019) 'A review on supply chain contracting with information considerations: information updating and information asymmetry', *International Journal of Production Research,* 57(15-16), pp. 4898-4936.

Shin, S.-H., Lee, P. T.-W. and Lee, S.-W. (2019) 'Lessons from bankruptcy of Hanjin Shipping Company in chartering', *Maritime Policy & Management,* 46(2), pp. 136-155.

Snyder, L. V. and Shen, Z.-J. M. (2019) *Fundamentals of supply chain theory.* John Wiley & Sons.

Song, D. (2021) 'A literature review, container shipping supply chain: Planning problems and research opportunities', *Logistics,* 5(2), pp. 41.

Song, J., Yin, Y. and Huang, Y. (2017) 'A coordination mechanism for optimizing the contingent-free shipping threshold in online retailing', *Electronic Commerce Research and Applications,* 26, pp. 73-80.

Song, Z., Tang, W. and Zhao, R. (2017) 'Ocean carrier canvassing strategies with uncertain demand and limited capacity', *Transportation Research Part E: Logistics and Transportation Review,* 104, pp. 189-210.

Song, Z., Tang, W. and Zhao, R. (2019) 'Encroachment and canvassing strategy in a sea-cargo service chain with empty container repositioning', *European Journal of Operational Research,* 276(1), pp. 175-186.

Surti, C., Celani, A. and Gajpal, Y. (2020) 'The newsvendor problem: The role of prospect theory and feedback', *European Journal of Operational Research,* 287(1), pp. 251-261.

Tan, Z., Meng, Q., Wang, F. and Kuang, H.-b. (2018) 'Strategic integration of the inland port and shipping service for the ocean carrier', *Transportation Research Part E: Logistics and Transportation Review,* 110, pp. 90-109.

Taylor, T. A. (2002) 'Supply chain coordination under channel rebates with sales effort effects', *Management science,* 48(8), pp. 992-1007.

Toscano, F. (2020) 'Does the Dodd-Frank Act reduce the conflict of interests of credit rating agencies?', *Journal of Corporate Finance,* 62, pp. 101595.

Tran, N. K. (2022) 'Market structure and horizontal growth strategies–A case study of the container shipping industry', *German Economic Review,* 23(3), pp. 423-461.

Tsay, A. A. (1999) 'The quantity flexibility contract and supplier-customer incentives', *Management science,* 45(10), pp. 1339-1358.

Tversky, A. and Kahneman, D. (1992) 'Advances in prospect theory: Cumulative representation of uncertainty', *Journal of Risk and uncertainty,* 5, pp. 297-323.

Unctad (2021) *Review of maritime transport 2021.* UN.

Uppari, B. S. and Hasija, S. (2019) 'Modeling newsvendor behavior: A prospect theory approach', *Manufacturing & Service Operations Management*, 21(3), pp. 481-500.

Vipin, B. and Amit, R. (2021) 'Wholesale price versus buyback: A comparison of contracts in a supply chain with a behavioral retailer', *Computers & Industrial Engineering,* 162, pp. 107689.

Von Neumann, J. and Morgenstern, O. (2007) 'Theory of games and economic behavior', *Theory of games and economic behavior*: Princeton university press.

Von Stackelberg, H. (2010) *Market structure and equilibrium.* Springer Science & Business Media.

Wang, F., Zhuo, X., Niu, B. and He, J. (2017) 'Who canvasses for cargos? Incentive analysis and channel structure in a shipping supply chain', *Transportation Research Part B: Methodological,* 97, pp. 78-101.

Wang, J. and Liu, J. (2019) 'Vertical contract selection under chain-to-chain service competition in shipping supply chain', *Transport Policy,* 81, pp. 184-196.

Wang, K. Y., Wen, Y., Yip, T. L. and Fan, Z. (2021) 'Carrier-shipper risk management and coordination in the presence of spot freight market', *Transportation Research Part E: Logistics and Transportation Review,* 149, pp. 102287.

Wu, M., Bai, T. and Zhu, S. X. (2018) 'A loss averse competitive newsvendor problem with anchoring', *Omega,* 81, pp. 99-111.

Xiao, S., Sethi, S. P., Liu, M. and Ma, S. (2017) 'Coordinating contracts for a financially constrained supply chain', *Omega*, 72, pp. 71-86.

Xie, Y., Liang, X., Ma, L. and Yan, H. (2017) 'Empty container management and coordination in intermodal transport', *European Journal of Operational Research,* 257(1), pp. 223-232.

Xiong, H., Chen, B. and Xie, J. (2011) 'A composite contract based on buy back and quantity flexibility contracts', *European Journal of Operational Research,* 210(3), pp. 559-567.

Xu, L., Govindan, K., Bu, X. and Yin, Y. (2015) 'Pricing and balancing of the sea–cargo service chain with empty equipment repositioning', *Computers & Operations Research,* 54, pp. 286-294.

Yan, N., Dai, H. and Sun, B. (2014) 'Optimal bi-level Stackelberg strategies for supply chain financing with both capital-constrained buyers and sellers', *Applied Stochastic Models in Business and Industry,* 30(6), pp. 783-796.

Yan, N. and Sun, B. (2013) 'Coordinating loan strategies for supply chain financing with limited credit', *Or Spectrum,* 35, pp. 1039-1058.

Yang, C.-S. (2019) 'Maritime shipping digitalization: Blockchain-based technology applications, future improvements, and intention to use', *Transportation Research Part E: Logistics and Transportation Review,* 131, pp. 108-117.

Yang, S. A. and Birge, J. R. (2018) 'Trade credit, risk sharing, and inventory financing portfolios', *Management Science,* 64(8), pp. 3667-3689.

Yin, M. and Kim, K. H. (2012) 'Quantity discount pricing for container transportation services by shipping lines', *Computers & Industrial Engineering,* 63(1), pp. 313-322.

Yin, M., Wan, Z., Kim, K. H. and Zheng, S. Y. (2019) 'An optimal variable pricing model for container line revenue management systems', *Maritime Economics & Logistics,* 21, pp. 173-191.

Zhang, M., Fu, Y., Zhao, Z., Pratap, S. and Huang, G. Q. (2019) 'Game theoretic analysis of horizontal carrier coordination with revenue sharing in E-commerce logistics', *International Journal of Production Research,* 57(5), pp. 1524-1551.

Zhang, Y., Donohue, K. and Cui, T. H. (2016) 'Contract preferences and performance for the loss-averse supplier: Buyback vs. revenue sharing', *Management Science*, 62(6), pp. 1734-1754.

Zhou, J. and Groenevelt, H. (2008) 'Impacts of financial collaboration in a three-party supply chain', *University of Rochester*.

Chapter 8. Appendices

Appendix A To Chapter 3

Here, we provide the proofs for corollaries and propositions. We use the subscripts $'$ ($''$) to denote the relevant functions' first (second) order derivative. The container shipping chain is coordinated when the optimal order quantities are identical for each party, and all parties can earn a maximum positive profit. Further, it is also essential to ensure that the sum of each party's maximum profits in the decentralised model equals the centralised company's maximum profit in the centralised model.

Proof of Corollary 1. From equation (3), we have:

$$\pi_c'(x) = -F(x)(\mathrm{g} + c) + \mathrm{g}$$

Since $F(x)$ is continuous and strictly increasing in the range of $[0, 1]$, $\pi_c'(x)$ is continuous and strictly decreasing from $-c$ to g. Therefore, for all $x \in I$ and $x_c \in I$, $\pi_c(x) - \pi_c(x_c) \leq \pi_c'(x)(x - x_c)$.

$$\pi_c''(x) = -f(x)(\mathrm{g} + c) \leq 0$$

This implies that the profit function of the centralised model is strictly concave in x. Therefore, the profit function of the centralised company could get a maximum value. Let x_c be the order quantity that maximises the centralised model's total profit. Therefore,

$$\frac{\partial \pi_c(x_c)}{\partial x_c} = -F(x_c)c + \mathrm{g} - \mathrm{F}(x_c)\mathrm{g} = 0$$

Then, we can determine the x_c using the following equation:

$$F(x_c) = \frac{\mathrm{g}}{\mathrm{g} + c} \tag{4}$$

Proof of Proposition 1. Let's set x_n^d and x_l^d as the order quantities that maximise the profit of the NVOCC and liner company in the decentralised model, respectively. Note that $F(\cdot)$ is a monotone increasing function, and the supply chain is coordinated when $x_n^d = x_l^d$, $F(x_n^d) = F\left(x_l^d\right)$. By taking the first order derivative for the profit function of the NVOCC and the liner company before repayment, we have:

$$\frac{\partial \pi_N(x_n^d)}{\partial x_n^d} = 0 = \theta P_r[1 - F(x_n^d)] + \frac{dT}{dx_n^d} - P_s - c - [F(x_n^d) - 1]\mathrm{g} \qquad (1a)$$

$$\frac{\partial \pi_L(x_l^d)}{\partial x_l^d} = 0 = P_s + (1-\theta)P_r\left[1 - F(x_l^d)\right] - \frac{dT}{dx_l^d} + (P_r - c)\left[F(x_l^d) - 1\right] \qquad (2a)$$

Because the condition of T is different under different contract types, we analyse the optimal order quantity according to the contract type. First, the contracts with zero transfer payment T are the revenue-sharing and the quantity discount contracts. Therefore, equation (1a) is simplified as follows:

$$\frac{d\pi_N(x_n^d)}{x_n^d} = \theta P_r[1 - F(x_n^d)] - P_s - c - [F(x_n^d) - 1]\mathrm{g} = 0$$

$$\Rightarrow F(x_n^d) = \frac{\theta P_r - P_s - c + g}{\theta P_r + g} \qquad (3a)$$

For the profit of the liner company before the repayment, it could be simplified as follows:

$$\frac{d\pi_L(x_l^d)}{dx_l^d} = P_s + (1-\theta)P_r\left[1 - F(x_l^d)\right] + (P_r - c)\left[F(x_l^d) - 1\right] = 0$$

$$\Rightarrow F(x_l^d) = \frac{\theta P_r - P_s - c}{\theta P_r - c} \qquad (4a)$$

The supply chain is coordinated if $x_n^d = x_l^d$. Based on (3a) and (4a), it turns out that:

$$P_s(\theta) = \frac{\theta P_r c - c^2}{\mathrm{g} + c} \qquad (5)$$

Second, the contract with positive transfer T will be the buyback contract. Then, equation (1a) is simplified as:

$$\frac{d\pi_N(x_n^d)}{x_n^d} = \theta P_r[1 - F(x_n^d)] + (1-\theta)P_r - P_s - c - [F(x_n^d) - 1]\mathrm{g} = 0$$

$$\Rightarrow F(x_n^d) = \frac{P_r - P_s - c + g}{\theta P_r + g} \qquad (5a)$$

Equation (2a) is simplified into:

$$\frac{d\pi_L(x_l^d)}{dx_l^d} = P_s + (1-\theta)P_r\left[1 - F(x_l^d)\right] - (1-\theta)P_r + (P_r - c)\left[F(x_l^d) - 1\right] = 0$$

$$\Rightarrow F(x_l^d) = \frac{P_r - P_s - \mathrm{c}}{\theta P_r - c} \qquad (6a)$$

Since the supply chain is coordinated when $x_n^d = x_l^d$, it turns out that:

$$P_s(\theta) = \frac{(1-\theta)P_r g + P_r c - c^2}{g + c} \tag{6}$$

The supply chain is coordinated when the optimal order quantities $x_n^d = x_l^d = x_c$ are the same for each party. Because $F(\cdot)$ is a monotone increasing function, $F(x_c) = F(x_n^d) = F\left(x_l^d\right)$. Note that in the above calculation**,** we only obtained equation (5) and equation (6) based on the setting of $F(x_n^d) = F\left(x_l^d\right)$. To further verify the coordination of these three contracts, we need to examine whether we can obtain the same result as equation (5) and equation (6) based on the setting of $F(x_c) = F(x_n^d)$ and $F(x_c) = F\left(x_l^d\right)$.

First, we consider the revenue-sharing contract and the quantity discount contract. Because equation (4) is equal to equation (3a),

$$F(x_c) = \frac{g}{g + c} = \frac{\theta P_r - P_s - c + g}{\theta P_r + g} = F\left(x_l^d\right)$$

we can get the same result as equation (5):

$$P_s(\theta) = \frac{\theta P_r c - c^2}{g + c}$$

When equation (4) is equal to equation (4a),

$$F(x_c) = \frac{g}{g + c} = \frac{\theta P_r - P_s - c}{\theta P_r - c} = F\left(x_l^d\right)$$

we can also get the same result as (5):

$$P_s(\theta) = \frac{\theta P_r c - c^2}{g + c}$$

Second, we consider the buyback contract. Because equation (4) is equal to equation (5a),

$$F(x_c) = \frac{g}{g + c} = \frac{P_r - P_s - c + g}{\theta P_r + g} = F(x_n^d)$$

we can get the same result as (6):

$$P_s(\theta) = \frac{(1-\theta)P_r g + P_r c - c^2}{g + c}$$

When equation (4) is equal to equation (6a),

$$F(x_c) = \frac{\mathrm{g}}{\mathrm{g}+c} = \frac{P_r - P_s - \mathrm{c}}{\theta P_r - c} = F\left(x_l^d\right)$$

we can also get the same result as (6):

$$P_s(\theta) = \frac{(1-\theta)P_r\mathrm{g} + P_r c - c^2}{\mathrm{g}+c}$$

Under **Proposition 1,** $F(x_c) = F(x_n^d) = F\left(x_l^d\right) = \frac{\mathrm{g}}{\mathrm{g}+c}$. Thus, when the contract parameters are set according to equation (4), equation (5) and equation (6), the profit of the liner company, NVOCC and centralised company could achieve the maximum value at the same optimal order quantities. This means these three contracts can coordinate and allocate the profits before repayment arbitrarily in the chain.

Proof of Proposition 2. To better illustrate, we split the profit of liner company before repayment into two parts as $\xi_L = \xi_L^1 + \xi_L^2$. Here, ξ_L^2 represents the gross profit of liner company from direct selling part. In order to study the profit composition of the liner company, we will take the derivative with respect to x for each segment's gross profit separately. Let x_l^1 and x_l^2 be the order quantity that maximise the gross profit ξ_L^1 and ξ_L^2 separately.

Firstly, the gross profit of liner company from direct selling part could be expressed as:

$$\xi_L^2 = (P_r - c)S_L(x) - I_L(x)\mathrm{g}$$

Then,

$$\frac{d\xi_L^2}{dx} = (P_r - c)[F(x) - 1] = 0 \implies F(x_l^2) = 1$$

$$\frac{\partial^2 \pi_c}{\partial x^2} = (P_r - c)f(x) > 0$$

The above results show that the direct selling profit could get a minimum value when $F(x) = 1$. However, $F(\cdot)$ is a cumulative distribution function within the range of [0,1]. Therefore, the direct selling profit is minimised when the order quantity of NVOCC x approaches infinity. Besides, because $F(\cdot)$ is a monotone increasing function, within the range of x, the gross profit of liner company from direct sales will decrease as x increases.

When under the revenue-sharing and the quantity discount contracts, the gross profit of liner company excluding direct sales is:

$$\xi_L^1 = P_s\, x + (1-\theta)P_r S_N(x)$$

Then,

$$\frac{d\xi_L^1}{dx} = P_s\ + (1-\theta)P_r[1-F(x)] = 0 \ \Longrightarrow\ F(x_l^1) = \frac{P_s\ + P_r - \theta P_r}{P_r - \theta P_r}$$

$$\frac{\partial^2 \xi_L^1}{\partial x^2} = -(1-\theta)P_r f(x) < 0$$

Because $P_s\ + P_r - \theta P_r > P_r - \theta P_r$, $F(\xi_L^1) > 1$.

When under the buyback contract, the gross profit of liner company excluding direct sales is:

$$\xi_L^1 = P_s\, x + (1-\theta)P_r S_N(x) - T = P_s\, x - b[x - S_N(x)]$$

Then,

$$\frac{d\xi_L^1}{dx} = P_s\ + (1-\theta)P_r\left[1 - 1 - F\left(x_l^d\right)\right] = 0 \ \Longrightarrow\ F(x_l^1) = \frac{P_s}{P_r - \theta P_r}$$

$$\frac{\partial^2 \xi_L^1}{\partial x^2} = -(1-\theta)P_r f(x) < 0$$

Because $b =\ (1-\theta)P_r < P_s$, $F(x_l^1) > 1$. Based on the above results about the gross profit of liner company excluding direct sales, this portion of the gross profit can only reach its maximum value when $F(\cdot)$ is greater than 1, however, which is outside the range of $F(\cdot)$.

When combining the above results with **Corollary 1** we find that the optimal order quantity should satisfy $F(x_c) = \frac{g}{g+c} < 1$. Since $F(\cdot)$ is a monotone increasing function, $x_c < x_l^2 < x_l^1$. Therefore, the change from x_l^2 and x_l^1 to x_c allows the gross profit from direct sales to obtain a higher value and also allows the order quantity in the slot purchase contract to settle in a reasonable range, thus promoting the cooperation between NVOCC and the liner company.

Proof of Corollary 2. When operating under the revenue-sharing contract and the quantity discount contract, the transfer is T = 0. Then,

$$\frac{d\xi_L}{d\theta}=\frac{P_r c}{\mathrm{g}+c}x-P_rS_N(x)=[1-F(x)]xP_r-\left(\int_0^x df(d)dd+[1-F(x)]x\right)P_r$$
$$=-P_r\int_0^x df(d)dd<0$$
$$\frac{d\pi_N}{d\theta}=P_rS_N(x)-\frac{P_r c}{\mathrm{g}+c}x>0$$

When under the buyback contract, the fixed transfer payment T＞0. Then,

$$\frac{d\xi_L}{d\theta}=-\frac{P_r\mathrm{g}}{\mathrm{g}+c}x-P_rS_N(x)+P_rx=\frac{P_r\mathrm{c}}{\mathrm{g}+c}x-P_rS_N(x)<0$$
$$\frac{d\pi_N}{d\theta}=P_rS_N(x)+\frac{P_r\mathrm{g}}{\mathrm{g}+c}x-P_rx=P_rS_N(x)-\frac{P_r\mathrm{c}}{\mathrm{g}+c}x>0$$

Therefore, $\frac{d\xi_L}{d\theta}=-\frac{d\pi_N}{d\theta}$. Note that the results under both $T=0$ and $T>0$ are the same. Since ξ_L decreases with the increase of θ and π_N increases with the increase of θ and the sum of $\xi_L$and π_N is independent of θ, there exists an θ^* for ξ_L and π_N to divide the π_c equally.

Based on ξ_c and ξ_L, when the liner company could earn the same profit as the total profit of the centralised virtual entity:

$$P_rS_c(x)-[S_L(x)+x]c-[I_L(x)+I_N(x)]\mathrm{g}$$
$$=P_s\,x+(1-\theta)P_rS_N(x)-\mathrm{T}+(P_r-c)S_L(x)-I_L(x)\mathrm{g}$$
$$\theta=\frac{P_sx+cx+I_N(x)\mathrm{g}}{P_rS_N(x)}$$

Based on ξ_c and π_N, when the NVOCC could earn the same profit as the total profit of the centralised virtual entity:

$$P_rS_c(x)-[S_L(x)+x]c-[I_L(x)+I_N(x)]\mathrm{g}=\theta P_rS_N(x)+T-P_sx-cx-I_N(x)\mathrm{g}$$
$$\theta=1+\frac{P_s\,x+(P_r-c)S_L(x)-I_L(x)\mathrm{g}}{P_rS_N(x)}$$

Proof of Proposition 3. From **Corollary 2,** the optimal value of θ^* for the liner company and the NVOCC to divide the total operating revenue equally is greater than 0. However, when turning the general contract into a specific contract type, the range of θ is $0<\theta<1$ under the buyback contract, $0<\theta<1$ under the revenue-share contract and $\theta = 1$ for the quantity discount contract. However, there exists market data which divides the total operating revenue equality for the liner company and the NVOCC but with the optimal $\theta^* > 1$. In this case, the liner company can only choose the quantity discount contract (where $\theta = 1$) to ensure that the NVOCC can earn a fair profit. Otherwise, the liner company can earn more than half of the total decentralised profit.

Proof of Corollary 3. Since equation (10) indicates the optimal wholesale price for both the liner company and the NVOCC, equations (7) and (10) must hold at the same time under the buyback contract and the revenue-sharing contract. In this case, equation (8) can be rewritten as follows:

$$H(\mathrm{g}, c|\ \theta_B^*) = 2x\frac{(1-\theta_B^*)P_r\mathrm{g} + P_r c - c^2}{\mathrm{g}+c} + (1-2\theta_B^*)P_r S_N(x) - 2x(1-\theta_B^*)P_r + (P_r - c)S_L(x) + cx + (I_N - I_L)(x)\mathrm{g}$$

$$= 2x\frac{\theta_B^*\ P_r c - c^2}{\mathrm{g}+c} + (1-2\theta_B^*)P_r S_N(x) + (P_r - c)S_L(x) + cx + (I_N - I_L)(x)\mathrm{g} = 0$$

$$H(\mathrm{g}, c|\ \theta_R^*) = 2x\frac{\theta_R^*\ P_r c - c^2}{\mathrm{g}+c} + (1-2\theta_R^*)P_r S_N(x) + (P_r - c)S_L(x) + cx + (I_N - I_L)(x)\mathrm{g} = 0$$

When the liner company chooses both contracts to deal with the same market condition, it is clear that θ_B^* and θ_R^* is the same under $H(\mathrm{g}, c|\ \theta_B^*) = 0$ and $H(\mathrm{g}, c|\ \theta_R^*) = 0$ which implies $\theta_B^* = \theta_R^*$.

Since **Corollary 1** indicates that the order quantities for the buyback and revenue-sharing contracts are related to goodwill loss and operating costs, the optimal order quantities for both contracts are the same. To prove that the buyback contract is equivalent to the revenue-sharing contract, we rewrite the profit function of the liner company and the NVOCC based on equation (16). Under the buyback contract, the profit function of the liner company and the NVOCC are:

$$\pi_L(x, P_s(\theta_B^*), \theta_B^*, T)$$

$$= \frac{(1-\theta_B^*)P_r\mathrm{g} + P_rc - c^2}{\mathrm{g}+c}\ x + (1-\theta_B^*)P_rS_N(x) - (1-\theta_B^*)P_rx + (P_r - c)S_L(x) - I_L(x)\mathrm{g} - \mathrm{E}\Delta_d$$

$$= \frac{\theta_B^*\ P_rc - c^2}{\mathrm{g}+c}x + (1-\theta_B^*)P_rS_N(x) + (P_r - c)S_L(x) - I_L(x)\mathrm{g} - \mathrm{E}\Delta_d$$

$$\pi_N(x, P_s(\theta_B^*), \theta_B^*, T)$$

$$= \theta_B^*P_rS_N(x) + (1-\theta_B^*)P_rx - \frac{(1-\theta_B^*)P_r\mathrm{g} + P_rc - c^2}{\mathrm{g}+c}x - cx - I_N(x)\mathrm{g} = \theta_B^*P_rS_N(x) - \frac{\theta_B^*\ P_rc - c^2}{\mathrm{g}+c}x - cx - I_N(x)\mathrm{g}$$

Under the revenue-sharing contract, the profit function of the liner company and the NVOCC are:

$$\pi_L(x, P_s(\theta_R^*), \theta_R^*, 0)$$

$$= \frac{\theta_R^*\ P_rc - c^2}{\mathrm{g}+c}x + (1-\theta_R^*)P_rS_N(x) + (P_r - c)S_L(x) - I_L(x)\mathrm{g} - \mathrm{E}\Delta_d$$

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \theta_R^*P_rS_N(x) - \frac{\theta_R^*\ P_rc - c^2}{\mathrm{g}+c}x - cx - I_N(x)\mathrm{g}$$

Since $\theta_B^* = \theta_R^*$, it is apparent that the profit function of the liner company and NVOCC under the buyback and revenue-sharing contracts is the same.

Proof of Proposition 4. When facing stochastic market demand, the profit the liner company before repayment under the revenue-sharing contract, the buyback contract and the quantity discount contract are:

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T)$$

$$= \frac{\theta^*P_rc - c^2}{\mathrm{g}+c}x + (1-\theta^*)P_rS_N(x) + (P_r - c)S_L(x) - I_L(x)\mathrm{g}$$

$$> \frac{P_rc - c^2}{\mathrm{g}+c}x + (P_r - c)S_L(x) - I_L(x)\mathrm{g} = \xi_L\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

which implies $(x, P_s(\theta_R^*), \theta_R^*, 0)$ and $(x, P_s(\theta_B^*), \theta_B^*, T)$ could bring more expected profit for the liner company than $\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$. While the profit of the NVOCC under these three contracts is:

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_N(x, P_s(\theta_B^*), \theta_B^*, T)$$

$$= \theta^* P_r S_N(x) - \frac{\theta^* P_r c - c^2}{\mathrm{g} + c} x - cx - I_N(x)\mathrm{g}$$

$$< P_r S_N(x) - \frac{P_r c - c^2}{\mathrm{g} + c} x - cx - I_N(x)\mathrm{g} = \pi_N\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

which indicates $\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$ could bring more expected profit for the NVOCC when facing stochastic market demand. The total profit of the centralised company is the same under these three contracts.

$$\pi_c(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_c(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_c\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

Proof of Proposition 5. According to equations (11) and (12), we compared the profit of the liner company before repayment, the profit of NVOCC and the total profit of the centralised company under each contract type. When under the condition of the deterministic market demand $d \le \frac{cx}{\mathrm{g}+c}$,

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T) = P_s(\theta_R^*)x + (1 - \theta)P_r d$$

$$\le P_s\left(\theta_Q^*\right)x = \xi_L\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_N(x, P_s(\theta_B^*), \theta_B^*, T) = \theta P_r d - P_s(\theta_R^*)x - cx$$

$$\ge P_r d - P_s\left(\theta_Q^*\right)x - cx = \pi_N\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

$$\pi_c(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_c(x, P_s(\theta_B^*), \theta_B^*, T) = P_r d - cx = \pi_c\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

which indicates that the liner company could earn a higher profit before rental payment under the quantity discount contract than it could under the revenue-sharing and the buyback contracts. Therefore, the liner company is better off under $\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$ than it is under $(x, P_s(\theta_R^*), \theta_R^*, 0)$ and $(x, P_s(\theta_B^*), \theta_B^*, T)$. But the

NVOCC might earn a negative profit under $\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$. When the deterministic market demand $d > \frac{cx}{\mathrm{g}+c}$,

$$\xi_L(x, P_s(\theta_R^*), \theta_R^*, 0) = \xi_L(x, P_s(\theta_B^*), \theta_B^*, T) > \xi_L\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

$$\pi_N(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_N(x, P_s(\theta_B^*), \theta_B^*, T) < \pi_N\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

$$\pi_c(x, P_s(\theta_R^*), \theta_R^*, 0) = \pi_c(x, P_s(\theta_B^*), \theta_B^*, T) = \pi_c\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$

which indicates that the revenue-sharing contract and the buyback contract could bring an identical higher profit before the repayment for the liner company than the quantity discount contract. Therefore, the liner company is better off under $(x, P_s(\theta_R^*), \theta_R^*, 0)$ and $(x, P_s(\theta_B^*), \theta_B^*, T)$ than $\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$. Even though the NVOCC makes less profit with the revenue-sharing contract and the buyback contract than with the quantity discount contract, the risk of the NVOCC suffering a negative profit is lower than the NVOCC under quantity discount when deterministic market demand $d \leq \frac{cx}{\mathrm{g}+c}$.

Proof of Proposition 6. To measure the liner company's solvency and profitability under the centralised and the decentralised models, we compare the repayment of the liner company, the income of the financial institution and the default costs under the following 4 cases.

We first consider **Case 1**, where the liner company does not default on the financial lease contract. Because the leased vessels are the same in both models, the lease cost should be the same in both the centralised and the decentralised models. Therefore, the repayment from the liner company in the decentralised model should be the same as in the centralised model, which means $\Delta_c = P_r \mathrm{Q} \omega_c (1 + R) = \Delta_d = P_r \mathrm{Q} \omega_d (1 + R)$. Therefore, ω is the same in both models, which means the centralised model is equivalent to the decentralised model for the financial institution when engaging with the financial lease contract. In this instance, the financial institution will receive the same income in both models with no default cost.

However, if the company defaults on the financial lease contract, the liner company needs to use the total profit on hand (i.e., ξ_L) to pay the rental. Then, the total profit of the decentralised model and the profit of the centralised model are $\pi_d = \pi_N + \xi_L - \xi_L > \pi_c = \xi_c - \xi_c$. The payment that the financial institution receives should be:

$$\delta_d = \xi_L - \alpha\xi_L - \beta_d = (1-\alpha)\xi_L - \beta_d$$

$$\delta_c = \xi_c - \alpha\xi_c - \beta_c = (1-\alpha)\xi_c - \beta_c$$

After that, we show **Case 2**, where with only fixed default cost $\beta_d = \beta_c$. Similarly, the payment that the financial institution receives is $\delta_d = \xi_L - \beta_d < \delta_c = \xi_c - \beta_c$. If the company defaults on the financial lease contract, the decentralised model could leave a portion of the total profit after repayment (π_N) rather than the centralised model. However, the repayment the financial institution could receive in the centralised model is larger. This means that the centralised model demonstrates a greater degree of solvency. Therefore, the centralised model is more favourable to the financial institution when the whole container shipping chain is under financial constraints.

In **Case 3**, with only variable default cost $B_d = \alpha\xi_L < \alpha\xi_c = B_c$, the centralised model will have more profit on hand to repay the financial lease rental than the liner company with the decentralised model, $\delta_d = (1-\alpha)\xi_L < \delta_c = (1-\alpha)\xi_c$. Hence, the centralised model would be more favourable to this container shipping chain.

For **Case 4** with both variable and fixed default costs, $B_d = \alpha\xi_L + \beta_d < \alpha\xi_c + \beta_c = B_c$. Then, $\delta_d = (1-\alpha)\xi_L - \beta_d < \delta_c = (1-\alpha)\xi_c - \beta_c$. This means the centralised virtual entity in the centralised model could pay more income to the financial institution than the liner company in the decentralised model. It would be easier for the company in the centralised model to repay the rental obligation than it would be in the decentralised model. Therefore, the centralised model would be more favourable to the financial institution.

Based on the above analyses, we can surmise that the financial institutions favour the centralised model more than the decentralised model.

Proof of Corollary 4. Given case i in **Proposition 6,** the loan-to-value ratios for the decentralised and the centralised model are the same when the liner company does not default on the finance lease contract. From equation (15), we obtain δ_d as follows:

$$\delta_d = \int_0^{d^*} ((1-\theta)P_r d - \mathrm{T} + P_s x - B) f(d)dd$$
$$+ \int_{d^*}^{\infty} ((1-\theta)P_r d^* - \mathrm{T} + P_s x) f(d)dd$$

To investigate the best ω leading to the maximum income for the financial institution, we need to analyse the relationship between several variables and ω (or $d^*(\omega)$), especially $(1-\theta)P_r d - \mathrm{T} + P_s x - B$. First, there is no variable in $(1-\theta)P_r$ that has any relationship with ω. Second, as shown in the previous section, $T = 0$ in the revenue-sharing contract and the quantity discount contract but $T = bx$ in the buyback contract. Therefore, T has no relationship with ω. Third, $B = \alpha[(1-\theta)P_r d - T + P_s x] + \beta$. After replacing these three parts into δ_d , we will get:

$$\delta_d = \int_0^{d^*} \left((1-\theta)P_r d^* - \mathrm{T} + P_s x\right) f(d)dd + \int_{d^*}^{\infty} P_r Q \omega (1+R) f(d)dd$$
$$= \left((1-\theta)P_r d^* - \mathrm{T} + P_s x\right) - \left[\alpha\left((1-\theta)P_r d^* - \mathrm{T} + P_s x\right) + \beta\right] F(d^*)$$
$$+(\alpha - 1)(1-\theta)P_r \int_0^{d^*} F(d)dd$$

Form equation (20),

$$d^* = \frac{P_r Q \omega (1+R) + \mathrm{T} - P_s x}{P_r - \theta P_r}$$

By taking the first order derivative of threshold demand with respect to the loan-to-value ratio, we have

$$\frac{dd^*}{d\omega} = \frac{P_r Q(1+R)}{(1-\theta)P_r} > 0$$

Therefore, it is clear that the loan-to-value ratio ω has a positive correlation with threshold demand d^*. Here, we set $A = (1-\theta)P_r > 0$ and $K = \mathrm{T} - P_s x$ for simple expression in the following calculation. From equation (21), in order to get the optimal loan-to-value ratio for the financial institution, let

$$V^1 = \frac{d\delta_d}{d\omega} = \frac{d\delta_d}{dd^*} \times \frac{dd^*}{d\omega} = 0$$

$$V^1 = \frac{d\delta_d}{dd^*} \times \frac{dd^*}{d\omega} = \{\mathrm{A} - \mathrm{A}F(d^*) - [\alpha(\mathrm{A}d^* - K_1) + \beta] f(d^*)\} \times \frac{P_r Q(1+R)}{\mathrm{A}} = 0$$

From these calculations, it is evident that there is an extreme value point for δ_d. However, it is not clear whether the financial institution's income is the maximum value or the minimum value under this equation. Hence, it is necessary to make a further calculation:

$$V^2 = \frac{dV^1}{d\omega} = \frac{dV^1}{dd^*} \times \frac{dd^*}{d\omega}$$

$$= (\frac{P_r Q(1+R)}{\mathrm{A}})^2 \times \{-(1+\alpha)\mathrm{A}f(d^*) - [\alpha(\mathrm{A}d^* - K) + \beta]f^{-1}(d^*)\} < 0$$

Proof of Proposition 7. $V^2 < 0$. Thus, the expected income for the financial institution is concave in ω and could reach the maximum value when $\frac{d\Delta}{dq^*} = 0$. Therefore, the optimal $\omega^*$for the financial institution must satisfy the equation:

$$A - \mathrm{A}F(d^*) - [\alpha(\mathrm{A}d^* - K) + \beta]f(d^*) = 0 \tag{22}$$

Proof of Proposition 8. For the condition i in **Proposition 6** that the liner company does not default, the range of the required repayment is $P_s Q\omega(1+R) < \frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x$. Based on the previous assumption that $d^* \leq x$, the range of $P_s Q\omega(1+R)$ is from $\frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x$ to $\frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x + (1 - \theta) P_r x$. However, there are two cases for the repayment of the liner company under quantity discount contracts: $\frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x \leq P_r Q\omega(1+R) \leq \frac{P_r c - c^2}{\mathrm{g}+c} x$ and $P_r Q\omega(1+R) > \frac{P_r c - c^2}{\mathrm{g}+c} x$.

When $P_s Q\omega(1+R) < \frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x$, the liner company could repay in full the rental payment under each contract type. Therefore, the default cost under the quantity discount contract is zero (under the proposed three contracts), and the financial institution will get $P_s Q\omega(1+R)$ under these three contracts. When $\frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x \leq P_s Q\omega(1+R)$, there are another two cases for the repayment of the liner company under the quantity discount contract (see details in Subsection 5.1). When under the condition of $\frac{\theta^* P_r c - c^2}{\mathrm{g}+c} x \leq P_r Q\omega(1+R) \leq \frac{P_r c - c^2}{\mathrm{g}+c} x$, the realised profit for the liner company before the rental payment is as shown equation (11) which is higher than the full rental.

Then, the liner company could repay the full rental no matter the changes in market demand. The expression of repayment of the liner company under the quantity discount contract is:

$$\Delta_Q\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right) = P_r Q \omega (1+R) \tag{7a}$$

The financial institution's income under the quantity discount contract is:

$$\delta_Q\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right) = P_r Q \omega (1+R) \tag{8a}$$

While the liner company's default threshold under the revenue-sharing contract and the buyback contract is $0 \leq d_R^* = d_B^* \leq \frac{c}{g+c}x$. Based on the above equations $(7a)$ and (8a), it is evident that $\delta_R(x, P_s(\theta_R^*), \theta_R^*, 0) = \delta_B(x, P_s(\theta_B^*), \theta_B^*, T) < \delta_Q\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$. Therefore, the quantity discount contract could reduce the risk of the liner company defaulting on the financial lease contract. When the rental is lower than $\frac{P_r c - c^2}{g+c}x$, the financial institution would prefer the liner company to choose the quantity discount contract when trading with the NVOCC.

We next consider the second condition that $P_r Q \omega (1+R) > \frac{P_r c - c^2}{g+c}x$. Under this condition, the repayment of liner company could be divided into three parts:

$$\Delta_Q\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right) = \begin{cases} \dfrac{P_r c - c^2}{g+c}x, & d < x \\ \dfrac{P_r c - c^2}{g+c}x + (P_r - c)(d - x), & x < d < d_Q^* \\ P_r Q \omega (1+R), & d_Q^* \leq d \end{cases} \tag{9a}$$

$$= \frac{P_r c - c^2}{g+c} x F\left(d_Q^*\right) + (P_r - c)\left[(d-x)F\left(d_Q^*\right) - \int_x^{d_Q^*} F(d)dd\right] + P_s Q \omega (1+R)\left[1 - F\left(d_Q^*\right)\right]$$

Here, the $d_Q^* = \frac{P_r Q \omega (1+R)}{P_r - c} + \frac{g}{g+c}x > x$. Therefore, the expression of the default cost and financial institution's income under the quantity discount contract is:

$$B_Q\left(x, P_s\left(\theta_Q^*\right), 1{,}0\right)$$
$$= \int_0^x \left(\alpha \frac{P_r c - c^2}{g+c}x + \beta\right) f(d)dd + \int_x^{d_Q^*} \left[\alpha \frac{P_r c - c^2}{g+c}x + \alpha(P_r - c)(d - x) + \beta\right] f(d)dd$$
$$= \left(\frac{P_r c - c^2}{g+c}\alpha x + \beta\right) F\left(d_Q^*\right) + \alpha(P_r - c)\left[\left(d_Q^* - x\right)F\left(d_Q^*\right) - \int_x^{d_Q^*} F(d)dd\right] \tag{10a}$$

$$\delta_Q\big(x, P_s(\theta_Q^*), 1{,}0\big)$$
$$= [\frac{P_r c - c^2}{\mathrm{g} + c}(1-\alpha)x - \beta]F(d_Q^*) + (1-\alpha)(P_r - c)[(d-x)F(d_Q^*)$$
$$- \int_x^{d_Q^*} F(d)dd] + P_s Q\omega(1+R)[1 - F(d_Q^*)] \qquad (11a)$$

When comparing equation (24) with (10a), it is evident that the default cost under the quantity discount contract is more than it is under the revenue-sharing and buyback contracts. Moreover, equations (25) and (11a) indicate that the financial institution's income under the revenue-sharing and the buyback contracts is greater than that under the quantity discount contract. In this case, the financial institution prefers the liner company to trade with the NVOCC using either the revenue-sharing or the buyback contract.

Appendix B To Chapter 4

Proof of Corollary 1. x_C^* denote the optimal order quantities that maximise the profit of centralised model π_c. Then, it satisfies

$$\frac{\partial \pi_c}{\partial x} = P_r[1 - F(x)] - (c_1 + c_2) - (g_1 + g_2)[F(x) - 1] = 0$$

Form this first order derivative for the function, we can get

$$F(x_C^*) = \frac{P_r - c + g}{P_r + g}$$

Max or min needs second order derivative

$$\frac{\partial^2 \pi_c}{\partial x^2} = -(P_r + g_1 + g_2) f(x) < 0$$

This denotes that the centralised company will achieve the maximised profit when order quantity satisfies the previous equation.

Proof of Proposition 1. The container supply chain is coordinated when the optimal order quantities are coincident for each participant and all participants could earn a positive profit. Additionally, the coordination of slot purchase contracts also needs to ensure the sum of maximum profits that each party obtains in the decentralised supply chain equal to the maximum system profits in centralised supply chain. From **Corollary 1**, we get the π_c is concave in x and the function of optimal order quantity. Let x_N^* and x_L^* denote the optimal order quantities that maximise $\pi_N(x, P_s, \theta, \mathrm{T})$ and $\xi_L^1(x, P_s, \theta, \mathrm{T})$, respectively. It is clear that there are two conditions for the transfer T. Then, we can divide the optimal order quantity into the following two cases. When $T = 0$, x_N^* satisfies

$$\frac{\partial \pi_N(x, P_s, \theta, \mathrm{T})}{\partial x} = \theta P_r[1 - F(x)] - P_s - c_1 - g_1[F(x) - 1] = 0$$

$$F(x) = \frac{\theta P_r - P_s - c_1 + g_1}{\theta P_r + g_1}$$

$$\frac{\partial^2 \pi_N(x, P_s, \theta, \mathrm{T})}{\partial x^2} = -(\theta P_r + g_1) f(x) < 0$$

and x_L^* satisfies

$$\frac{\partial \xi_L^1(x, P_s, \theta, \mathrm{T})}{\partial x} = P_s + (1 - \theta) P_r[1 - F(x)] - c_2 - g_2[F(x) - 1] = 0$$

$$F(x) = \frac{P_s + P_r - \theta P_r - c_2 + g_2}{P_r - \theta P_r + g_2}$$

$$\frac{\partial^2 \xi_L^1(x, P_s, \theta, T)}{\partial x^2} = -(P_r - \theta P_r + g_2) f(x) < 0$$

When $F(x_N^*) = F(x_L^*)$,

$$P_s = \frac{\theta P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}$$

When $T > 0$, x_N^* satisfies

$$\frac{\partial \pi_N(x, P_s, \theta, T)}{\partial x} = \theta P_r[1 - F(x)] + (1 - \theta) P_r - P_s - c_1 - g_1[F(x) - 1] = 0$$

$$F(x) = \frac{P_r - P_s - c_1 + g_1}{P_r + g_1}$$

$$\frac{\partial^2 \pi_N(x, P_s, \theta, T)}{\partial x^2} = -(\theta P_r + g_1) f(x) < 0$$

and x_L^* satisfies

$$\frac{\partial \xi_L^1(x, P_s, \theta, T)}{\partial x} = P_s + (1 - \theta) P_r[1 - F(x)] - (1 - \theta) P_r - c_2 - g_2[F(x) - 1] = 0$$

$$F(x) = \frac{P_s - c_2 + g_2}{P_r - \theta P_r + g_2}$$

$$\frac{\partial^2 \xi_L^1(x, P_s, \theta, T)}{\partial x^2} = -(P_r - \theta P_r + g_2) f(x) < 0$$

When $F(x_N^*) = F(x_L^*)$,

$$P_s = (1 - \theta) P_r + \frac{\theta P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g}$$

Then, we the optimal contract decisions when the objective is to maximise each participants' expected profit.

Proof of Corollary 2. To divide the profit in the centralised model equally, the following equilibrium condition must hold at θ^*:

$$\theta P_r S_N(x) + T - P_s x - c_1 x - I_N(x) g_1 = P_s x + (1 - \theta) P_r S_N(x) - T - c_2 x - I_N(x) g_2$$

When $T = 0$,

$$2\theta P_r S_N(x) = 2P_s x + P_r S_N(x) + c_1 x - c_2 x + I_N(x) g_1 - I_N(x) g_2$$

When $T = (1 - \theta) P_r x > 0$,

$$\begin{aligned} 2\theta P_r S_N(x) &= 2P_s x + P_r S_N(x) - 2T + c_1 x - c_2 x + I_N(x) g_1 - I_N(x) g_2 \\ &= 2P_s x + P_r S_N(x) - 2P_r x(1 - \theta) + c_1 x - c_2 x + I_N(x) g_1 - I_N(x) g_2 \end{aligned}$$

After simplification, we get:

$$\theta^* = \frac{1}{2} + \frac{I_N(x)P_r + I_N(x)\mathrm{g} + cx}{S_N(x)P_r + S_N(x)\mathrm{g} - cx} * \frac{2\mathrm{g}_1 - \mathrm{g}}{2}$$

Proof of Corollary 3. When the cooperation between the liner company and NVOCC is through the buyback contract, the profit of each participant's profit can be simplified as:

$$\xi_{L_B}^1(x, P_s, \theta, \mathrm{T}) = \frac{\theta_B^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}} x^* + (1 - \theta_B^*) P_r S_N(x^*) - c_2 x^* - I_N(x^*)\mathrm{g}_2$$

$$\pi_{N_B}(x, P_s, \theta, \mathrm{T}) = \theta_B^* P_r S_N(x^*) - \frac{\theta_B^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}} x^* - c_1 x^* - I_N(x)\mathrm{g}_1$$

When the cooperation between the liner company and NVOCC is through the revenue sharing contract, the profit of each participant's profit can be simplified as:

$$\xi_{L_R}^1(x, P_s, \theta, \mathrm{T}) = \frac{\theta_R^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}} x^* + (1 - \theta_B^*) P_r S_N(x^*) - c_2 x^* - I_N(x^*)\mathrm{g}_2$$

$$\pi_{N_R}(x, P_s, \theta, \mathrm{T}) = \theta_R^* P_r S_N(x^*) - \frac{\theta_R^* P_r c - P_r c_1 + \mathrm{g}_1 c_2 - \mathrm{g}_2 c_1}{P_r + \mathrm{g}} x^* - c_1 x^* - I_N(x)\mathrm{g}_1$$

Clearly, the only difference between the two profit functions is the value of θ, which is decided by NVOCC in revenue sharing contract and by liner company in buyback contract. To find the relation between each participant's profit and θ, we will take the first order derivative of the profit functions with respect to θ. When under the buyback contract,

$$\frac{\partial \xi_{L_B}^1}{\partial \theta} = \frac{P_r c}{P_r + \mathrm{g}} x^* - P_r S_N(x^*) < 0$$

$$\frac{\partial \pi_{N_B}}{\partial \theta} = P_r S_N(x) - \frac{P_r c}{\mathrm{g} + c} x > 0$$

When under the revenue sharing contract,

$$\frac{d\xi_{L_R}^1}{d\theta} = \frac{P_r c}{P_r + \mathrm{g}} x^* - P_r S_N(x^*) < 0$$

$$\frac{d\pi_{N_R}}{d\theta} = P_r S_N(x) - \frac{P_r c}{\mathrm{g} + c} x > 0$$

From the above calculation, the results under both conditions are the same. Due to the fact that $\xi_{L_B}^1$ and $\xi_{L_R}^1$ have the same relationship to θ and the optimal order quantity

x^* is the same under the other two contracts, it is apparent that the buyback and revenue sharing contracts are identical to liner company. Similarly, this also can be proved for NVOCC.

Proof of Proposition 2. Here, we will present two proofs for this proposition. The first is to prove from the perspective of profit function under three contracts, and the second is to prove based on **Corollary 3**.

Proof 1: When dealing with stochastic market demand, the profits obtained by the shipping company under buyback contract, revenue sharing contract and quantity discount contract are, respectively,

$$\xi_{L_B} = \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_B^*) P_r S_N(x) - c_2 x - I_N(x) g_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(x) g$$

$$\xi_{L_R} = \frac{\theta_R^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_R^*) P_r S_N(x) - c_2 x - I_N(x) g_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(x) g$$

$$\xi_{L_Q} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_2 x - I_N(x) g_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(x) g$$

Based on the **Corollary 3** the θ_B^* is equal to θ_R^*. Since the optimal order quantity x^* is the same under these three contracts, the profit gap between ξ_{L_B}, ξ_{L_R} and ξ_{L_Q} is the part that NVOCC transferred to liner company.

$$\xi_{L_B} = \xi_{L_R} = \xi_{L_Q} - (1 - \theta_B^*) P_r [S_N(x) - \frac{cx}{P_r + g}]$$

Here, we set $H(x) = S_N(x) - \frac{cx}{P_r + g} = x - \int_0^x F(d)dd - \frac{cx}{P_r + g} = F(x)x - \int_0^x F(d)dd$.

Since $h(x) = \frac{dH(x)}{dx} = F(x) + f(x)x - F(x) = f(x)x > 0$, $H(x)$ is a monotone increasing function. When returning to the definition of x, it is the order quantity of NVOCC, which should be within the value range of greater than or equal to 0. Therefore, $H(x) \geq H(0) = 0$. In this case, $(1 - \theta_B^*) P_r [S_N(x) - \frac{cx}{P_r + g}] \geq 0$.

As for the profits obtained by the NVOCC under buyback contract, revenue sharing contract and quantity discount contract are, respectively,

$$\pi_{N_B} = \theta_B^* P_r S_N(x) - \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x - I_N(x) g_1$$
$$= \theta_R^* P_r S_N(x) - \frac{\theta_R^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x - I_N(x) g_1 = \pi_{N_R}$$
$$\pi_{N_Q} = P_r S_N(x) - \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x - I_N(x) g_1$$

Similar to the proof of liner company's profit, the result can be proved for NVOCC.

$$\pi_{N_B} = \pi_{N_R} = \pi_{N_Q} + (1 - \theta_B^*) P_r [S_N(x) - \frac{cx}{P_r + g}]$$

Since $(1 - \theta_B^*) P_r [S_N(x) - \frac{cx}{P_r + g}] \geq 0$, $\pi_{N_B} = \pi_{N_R} \leq \pi_{N_Q}$.

Proof 2: When under the quantity discount contract, for the liner company's profit from selling process 1, it could be simplified as:

$$\xi_{LQ}^{1} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x^* - c_2 x^* - I_N(x^*) g_2$$
$$= \frac{P_r c x^* - P_r c_1 x^* + g_1 c_2 x^* - g_2 c_1 x^* - P_r c_2 x^* - g c_2 x^*}{P_r + g} - I_N(x^*) g_2$$
$$= -\frac{g_2 c x^*}{P_r + g} - I_N(x^*) g_2 \leq 0$$

From this, it seems that the liner company could only count on the profit from the selling process 2. Due to the fact that the liner company get the same profit from the selling process 2, the profit of liner company under quantity discount contract will be less than that under the buyback contract and revenue sharing contract.

Proof of Proposition 3. From the profit function of liner company and NVOCC under quantity discount contract, the market demand d that cause the equal realised profits under these three contracts must satisfy the following equation:

$$\xi_{L_B} = \xi_{L_R} = \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_B^*) P_r d - c_2 x - (Q + q_0 - x) c$$
$$= \xi_{L_Q} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_2 x - (Q + q_0 - x) c$$
$$\pi_{N_B} = \pi_{N_R} = \theta_B^* P_r d - \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x = \pi_{N_Q}$$
$$= P_r d - \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_1 x$$

Therefore,

$$\frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_B^*) P_r d = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x$$

$$\theta_B^* P_r d - \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x = P_r d - \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x$$

Then,

$$(1 - \theta_B^*)\left[P_r d - \frac{P_r c x}{P_r + g}\right] = 0 \rightarrow d = \frac{cx}{P_r + g}$$

Therefore, the change of θ will not affect this threshold market demand. When the market demand is lower than this threshold point, it is evident that the profit of liner company under buyback contract and revenue sharing contract is lower than that under quantity discount contract because of θ. Since the centralised profit under the three contracts is the same, the profit of NVOCC under buyback contract and revenue sharing contract is higher than that under quantity discount contract.

When $\frac{cx}{P_r+g} < d < x$,

$$\xi_{L_B} = \xi_{L_R} = \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_B^*) P_r d - c_2 x - (Q + q_0 - x)c$$

$$\xi_{L_Q} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_2 x - (Q + q_0 - x)c$$

Therefore, $\xi_{L_B} = \xi_{L_R} = \xi_{L_Q} + (1 - \theta_B^*) P_r \left[d - \frac{cx}{P_r+g}\right]$

Since $d > \frac{cx}{P_r+g}$, $(1 - \theta_B^*) P_r \left[d - \frac{cx}{P_r+g}\right] > 0$. Then, $\xi_{L_B} = \xi_{L_R} > \xi_{L_Q}$.

When $x < d \le Q + q_0$,

$$\xi_{L_B} = \xi_{L_R} = \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_B^*) P_r x - c_2 x - (d - x) g_2 + P_r (d - x) - (Q + q_0 - x)c$$

$$\xi_{L_Q} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_2 x - (d - x) g_2 + P_r (d - x) - (Q + q_0 - x)c$$

Therefore, $\xi_{L_B} = \xi_{L_R} = \xi_{L_Q} + (1 - \theta_B^*) P_r x \left[1 - \frac{c}{P_r+g}\right]$

Then, $\xi_{L_B} = \xi_{L_R} > \xi_{L_Q}$.

When $Q + q_0 < d$,

$$\xi_{L_B} = \xi_{L_R} = \frac{\theta_B^* P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x + (1 - \theta_B^*) P_r x - c_2 x - (d - x) g_2 + P_r (Q + q_0 - x) - (Q + q_0 - x)c - (d - Q - q_0) g$$

$$\xi_{L_Q} = \frac{P_r c - P_r c_1 + g_1 c_2 - g_2 c_1}{P_r + g} x - c_2 x - (d - x) g_2 + P_r (Q + q_0 - x) - (Q + q_0 - x)c - (d - Q - q_0) g$$

Therefore, $\xi_{L_B} = \xi_{L_R} = \xi_{L_Q} + (1-\theta_B^*)P_r x\left[1-\frac{c}{P_r+\mathrm{g}}\right]$

Then, $\xi_{L_B} = \xi_{L_R} > \xi_{L_Q}$.

In summary, we could get the result that $\xi_{L_B} = \xi_{L_R} > \xi_{L_Q}$ when $d > \frac{cx}{P_r+\mathrm{g}}$. Since the centralised profit under three contracts is the same, $\pi_{N_B} = \pi_{N_R} > \pi_{N_Q}$.

Proof of Corollary 4. From equation (6), it is obvious that the optimal order quantity x^* has a negative correlation with unit operating cost c. In order to know the relationship between x^* and g, we first find the relation between $F(x^*)$ with g. Here, we take the first order derivative of equation (6):

$$\frac{\partial F(x^*)}{\partial \mathrm{g}} = \frac{c}{(P_r+\mathrm{g})^2} > 0$$

Since $F(\cdot)$ is a monotone increasing function, if g has an upward trend, x^* will likewise increase. This is consistent with case c. To further analyse the effect of c and g on θ^*, we need to calculate how c and g influence x^* in detail.

$$\frac{\partial F(x^*)}{\partial \mathrm{g}} = \frac{\partial F(x^*)}{\partial x^*} * \frac{\partial x^*}{\partial \mathrm{g}} = f(x^*)\frac{\partial x^*}{\partial \mathrm{g}}$$

Therefore, $\frac{\partial x^*}{\partial \mathrm{g}} = \frac{c}{(P_r+\mathrm{g})^2} * \frac{1}{f(x^*)}$.

Similarly,

$$\frac{\partial F(x^*)}{\partial c} = \frac{\partial F(x^*)}{\partial x^*} * \frac{\partial x^*}{\partial c} = f(x^*)\frac{\partial x^*}{\partial c} = -\frac{1}{P_r+\mathrm{g}}$$

Therefore, $\frac{\partial x^*}{\partial c} = -\frac{1}{(P_r+\mathrm{g})f(x^*)}$

Proof of Corollary 5. From observations of equation (7), we can easily draw conclusions that θ^* will stay fixed at 0.5 when $\mathrm{g}_1 = \mathrm{g}_2 = \frac{\mathrm{g}}{2}$. Then, we begin to consider the following two scenarios:

i. If g remains constant:

For case a, we take the first order derivative of the equation (6) to find the relation between θ^* with g_1.

$$\frac{\partial\theta^*}{\partial g_1}=\frac{I_N(x)P_r+I_N(x)g+cx}{S_N(x)P_r+S_N(x)g-cx}*\frac{1}{P_r}$$

Since $g_2=g-g_2$, $\frac{\partial\theta^*}{\partial g_2}=-\frac{I_N(x)P_r+I_N(x)g+cx}{S_N(x)P_r+S_N(x)g-cx}*\frac{1}{P_r}$.

For case b, it is obvious that only the sum of c_1 and c_2 will influence θ^*. To find the relation between θ^* with c, we did the following calculations:

$$\frac{\partial\theta^*}{\partial c}=\frac{(2g_1-g)(P_r+g)\mu x}{2P_r}$$

From this, we could conclude that θ^* will have an upward tendency if $g_1>\frac{g}{2}$ and a downward tendency if $g_2>\frac{g}{2}$.

ii. If g has a proclivity towards change, we then first explore the influence of g on θ^*. We set g_1 accounts for $0\leq\rho\leq 1$ of g.

$$\frac{\partial\theta^*}{\partial g}=\frac{1-2\rho}{2}*\frac{[I_N(x)P_r+I_N(x)g+cx][S_N(x)P_r+S_N(x)g-cx]+cxg\mu}{[S_N(x)P_r+S_N(x)g-cx]^2}$$

Therefore, when $S_N(x)(P_r+g)>cx$ (For case d), $\frac{\partial\theta^*}{\partial g}<0$ if $\rho>0.5$ and $\frac{\partial\theta^*}{\partial g}>0$ if $\rho<0.5$. When $S_N(x)(P_r+g)<cx$ (For case e), $\frac{\partial\theta^*}{\partial g}>0$ if $\rho>0.5$ and $\frac{\partial\theta^*}{\partial g}<0$ if $\rho<0.5$.

Proposition 4 is the summary of **Corollary 4** and **Corollary 5**.

Proof of Corollary 6 The optimal leased quantity that maximises the profit of liner company in event 3 satisfies

$$\frac{\partial\xi_L(Q)}{\partial Q}=P_r[1-F(Q+q_0)]-c-[-1+F(Q+q_0)]g=0$$

From this first order derivative for the function, we can get

$$F(Q+q_0)=\frac{P_r-c+g}{P_r+g}$$

However, in order to check whether it is the maximum or minimum value at this point, the second derivative is required.

$$\frac{\partial^2\xi_L(Q)}{\partial Q^2}=P_r[-f(Q+q_0)]-f(Q+q_0)g=-(P_r+g)f(Q+q_0)<0$$

Then, $\xi_L(Q)$ is strictly concave in Q. Therefore, any leased quantity that satisfies the previous equation will help the liner company achieve the maximised profit in event 3.

Proof of Proposition 5 We then need to plug the optimal financial lease amount Q^* into the profit function of the liner company to show the liner company's maximum profit, which should be greater or equal to the rental.

$$P_s\, x + (1-\theta)P_r\left[x - \int_0^x F(d)dd\right] - \mathrm{T} - c_2 x - \left[\mu - x + \int_0^x F(d)dd\right]\mathrm{g}_2$$

$$+ P_r\left[(\mathrm{Q} + \mathrm{q}_0 - x) - \int_x^{\mathrm{Q}+\mathrm{q}_0} F(d)dd\right] - (Q + q_0 - x)c$$

$$- \left[\mu - (\mathrm{Q} + \mathrm{q}_0) + \int_0^{Q+q_0} F(d)dd\right]\mathrm{g} \geq P_r Q\omega(\rho\omega + \varepsilon + 1) \geq 0$$

From $P_r Q\omega(\rho\omega + \varepsilon + 1) \geq 0$, we could get that when the liner company choose to use financial lease to expand capacity, the loan-to-value ratio should satisfy:

$$\omega(\rho\omega + \varepsilon + 1) > 0$$

Therefore, $\omega < -\frac{\varepsilon+1}{\rho}$ or $\omega > 0$.

From the left inequation,

$$\frac{\begin{array}{c} P_s\, x + (1-\theta)P_r\left[x - \int_0^x F(d)dd\right] - \mathrm{T} - c_2 x - \left[\mu - x + \int_0^x F(d)dd\right]\mathrm{g}_2 \\ +P_r\left[(\mathrm{Q} + \mathrm{q}_0 - x) - \int_x^{\mathrm{Q}+\mathrm{q}_0} F(d)dd\right] - (Q + q_0 - x)c - \left[\mu - (\mathrm{Q} + \mathrm{q}_0) + \int_0^{Q+q_0} F(d)dd\right]\mathrm{g} \end{array}}{P_r Q}$$

$\geq \omega(\rho\omega + \varepsilon + 1)$

We then use M to simplify the left side of inequality as

$$M \geq \omega(\rho\omega + \varepsilon + 1)$$

Therefore, $\omega(\rho\omega + \varepsilon + 1) - M \leq 0$. By using the solution for the roots of the quadratic equation, we could get the loan-to-value ratio to be within the range as follows:

$$\frac{-(\varepsilon+1) - \sqrt{(\varepsilon+1)^2 + 4\rho M}}{2\rho} \leq \omega \leq \frac{-(\varepsilon+1) + \sqrt{(\varepsilon+1)^2 + 4\rho M}}{2\rho}$$

Next, we simplify M to:

$$P_rQM = P_s\,x + (1-\theta)P_r\left[x - \int_0^x F(d)dd\right] - \mathrm{T} - c_2x - \left[\mu - x + \int_0^x F(d)dd\right]\mathrm{g}_2$$

$$+ P_r\left[(\mathrm{Q}+\mathrm{q}_0 - x) - \int_x^{\mathrm{Q}+\mathrm{q}_0} F(d)dd\right] - (Q+q_0-x)c$$

$$- \left[\mu - (\mathrm{Q}+\mathrm{q}_0) + \int_0^{Q+q_0} F(d)dd\right]\mathrm{g}$$

$$= P_s\,x - \theta P_r - T - c_2x - \mu\mathrm{g}_2 + x\mathrm{g}_2 + xc - \mu\mathrm{g}$$

$$+ (\theta P_r - \mathrm{g}_2)\int_0^x F(d)dd + (P_r - c + \mathrm{g})(Q^* + q_0)$$

$$- (P_r + \mathrm{g})\int_0^{Q^*+q_0} F(d)dd$$

Since $(P_r + \mathrm{g})F(Q^* + q_0) = \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}}$,

$$P_rQM = (P_s + c_1 + \mathrm{g}_2)\,x - \theta P_r - T - \mu(2\mathrm{g}_2 + \mathrm{g}_1) + (\theta P_r - \mathrm{g}_2)\int_0^x F(d)dd$$

$$+ (P_r + \mathrm{g})\int_0^{Q^*+q_0} df(d)dd$$

Proof of Proposition 6 Since $V_1 = P_r\mathrm{Q}'\omega(\rho\omega + \varepsilon + 1) - \xi_L(\mathrm{Q}') = 0$

$$\frac{\partial V_1}{\partial Q} = P_r\omega(\rho\omega + \varepsilon + 1) + P_r\mathrm{Q}'\frac{\partial \omega}{\partial \mathrm{Q}'}(\rho\omega + \varepsilon + 1) + P_r\mathrm{Q}'\omega\rho\frac{\partial \omega}{\partial \mathrm{Q}'}$$

$$- [P_r - c + \mathrm{g} - (P_r + \mathrm{g})F(\mathrm{Q}' + q_0)] = 0$$

Therefore,

$$\frac{\partial \omega}{\partial \mathrm{Q}'} = \frac{P_r - c + \mathrm{g} - (P_r + \mathrm{g})F(\mathrm{Q}' + q_0) - P_r\omega(\rho\omega + \varepsilon + 1)}{P_r\mathrm{Q}'(\varepsilon + 1) + 2P_r\mathrm{Q}'\omega\rho} < 0$$

In order to guarantee the existence of solutions for w and L, the following inequality relations must hold:

$$\xi_L(\mathrm{Q}') > \frac{-P_r\mathrm{Q}'(\varepsilon + 1)^2}{4\rho}$$

Appendix C To Chapter 5

C.1 Model Setup and Preliminaries

According to Dimitrov et al. (2015) and Toscano (2020), the following table summarises the meaning of each rating model assigned by Moody's Investor Service, Standard & Poor's (S&P) and Fitch Group, respectively.

Model	Credit Rating	Meaning
Moody's	Aa1/ Aa2/Aa3	Having a high-quality rating and a minimal credit risk rating.
	A1/ A2/A3	Having an upper-middle grade and a low credit risk rating.
	Baa1/Baa2/Baa3	Having a medium grade with certain speculative characteristics and moderate credit risk.
	Ba1/Ba2/Ba3	Considered to contain speculative characteristics and a high credit risk.
S&P	AA+/AA/AA-	The obligor has a strong capacity to fulfil its financial obligations.
	A+/A/A-	The obligor has enough capacity to fulfil its financial obligations but are considerably impressionable to the negative impacts of economic conditions and circumstances.
	BBB+/BBB/BBB-	The obligor has enough capacity to fulfil its financial obligations. However, unfavourable economic conditions or shifting circumstances are more likely to impair the debtor's ability to fulfil its financial obligations.
	BB+/BB/BB-	The obligor is less sensitive to unfavourable corporate, financial and economic conditions in the short-term, but faces significant long-term risks.
Fitch	AA	AA' ratings indicate a very low default risk. They suggest a very powerful capacity for paying financial obligations. This capacity is not particularly susceptible to unexpected situations.
	A	A' ratings indicate modest default risk assumptions. The capacity to meet financial obligations is regarded as powerful. However, this capability may be more sensitive to unfavourable business or economic conditions than capacities with higher ratings.
	BBB	Ratings of BBB imply that the default risk is low. The capacity to fulfil financial obligations may be sufficient, but poor operating or economic situations are more likely to weaken it.
	BB	Ratings of BB imply that an increased sensitivity to default risk, particularly in the event of severe changes in operating or economic situations over time.

Table 8.1 Rating Definitions in Dimitrov et al. (2015) and Toscano (2020)

Based on these three models, we divide the solvency of shipping companies into four areas: B, BB, A, AA. Here, A and AA are used to show the liner companies which have the capacity to repay the financial lease obligations. Companies rated AA have more excess profits than companies rated A. BB and B are used to show the liner companies which do not have enough capacity to repay the financial lease obligations. Similarly, companies rated B have more deficits than companies rated BB.

The prospect theory can be expressed as the following function and the redline in the following Figure 8.1.

$$Pr(x) = \begin{cases} \log_a \dfrac{1}{1-x}, & x < 0 \\ \log_b x + 1\,, & x \geq 0 \end{cases} \quad (where\ 1 < a < b)$$

Therefore, the value function used is generally concave for incomes greater than the target repayment amount and convex for incomes with lower target repayments. For the purposes of demonstrating the diverse responses of financial institutions to increased surplus profits and deficits our study simplifies the model into piecewise linear with four pieces, which represent the grades of B, BB, A, AA, respectively.

$$\boldsymbol{r(m)} = \begin{cases} \boldsymbol{k_1 m + (k_2 - k_1)M^-}, & \boldsymbol{m < M^-} \\ \boldsymbol{k_2 m}, & \boldsymbol{M^- \leq m < 0} \\ \boldsymbol{k_3 m}, & \boldsymbol{0 \leq m < M^+} \\ \boldsymbol{k_4 m + (k_3 - k_4)M^+}, & \boldsymbol{M^+ \leq m} \end{cases}$$

Where $k_3 = \delta k_2$; $k_4 = \delta k_1$; $0 < k_1 < k_2$ and $|M^-| < |M^+|$. The model is shown as a black line in Figure 8.1 below.

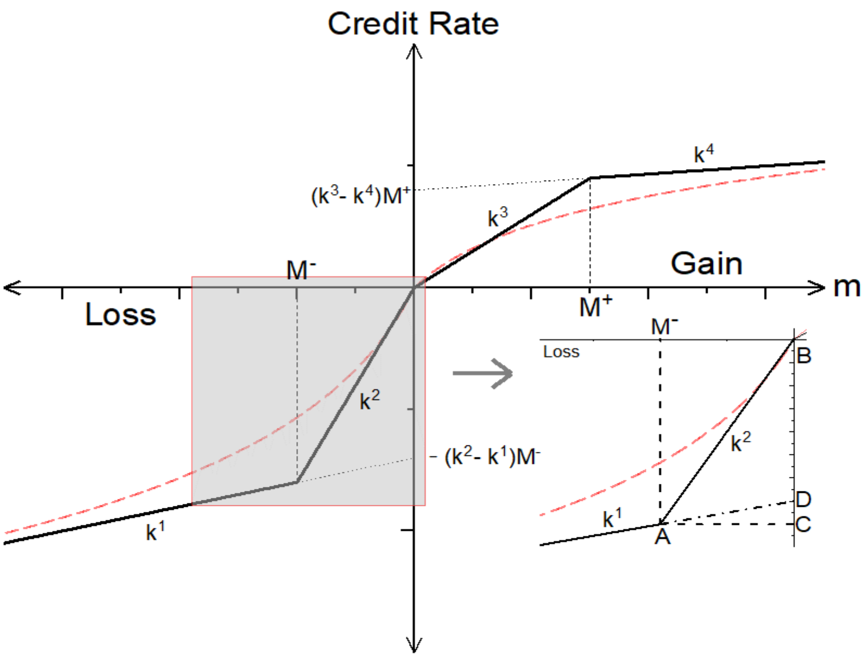

Figure 8.1 Prospect Theory based Credit Rate Model

Here, $k_1(k_2)$ and $k_3(k_4)$ respectively show the financial institutions' perception of the company's surplus profits and deficits. The role of $0 < \delta < 1$ is to control the financial institution's perceived level of gains to be less than that of losses. The y-intercept of the first and fourth segments of the function will be proved by the trigonometric function at the geometric level. According to the enlarged view in Figure 8.1, it can be seen that the y-intercept of the first segment of the function is the length of BD. From the slopes of the first and second segment functions, we see that the following formulas must hold in triangle ΔABC ABC and triangle ΔADC.

$$\tan \angle BAC = \frac{\text{the length of BC}}{\text{the length of AC}} = k_2$$
$$\tan \angle DAC = \frac{\text{the length of DC}}{\text{the length of AC}} = k_1$$

Therefore,

$$\text{the length of BD} = \text{BC} - \text{DC} = (k_2 - k_1) * \text{the length of AC} = -(k_2 - k_1)M^-$$

In the same way, the intercept of the fourth segment function is:

$$(k_3 - k_4)M^+ = \delta(k_2 - k_1)M^+$$

Therefore, the change of credit rate function can be rewritten as:

$$r(m) = \begin{cases} k_1 m + (k_2 - k_1)M^-, & m < M^- \\ k_2 m, & M^- \le m < 0 \\ \delta k_2 m, & 0 \le m < M^+ \\ \delta k_1 m + (k_2 - k_1)\delta M^+, & M^+ \le m \end{cases}$$

C.2 Proofs

Proof of Proposition 1. According to the sum profit of the NVOCC and the liner company, the following conditions must hold to maximize the total profit.

$$\underset{x}{Maximise} \quad \Pi_c^*(x) = P_r S_N(x) - (c_1 + c_2)x - (\mathrm{g}_1 + \mathrm{g}_2) I_N(x)$$

$$s.t. \quad x \geq 0$$

$$\frac{\partial \Pi_c}{\partial x} = P_r[1 - F(x)] - c - \mathrm{g}[F(x) - 1] = 0$$

From these, we can solve the relationship between optimal order quantity and the market parameters as:

$$F(x^*) = \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}}$$

Under the buyback contract, the optimal order quantity of the NVOCC must satisfy:

$$\underset{P_s, b}{Maximise} \quad \Pi_{N_{BB}}(x, P_s, \mathrm{b})$$

$$s.t. \quad \frac{\partial \Pi_{L_{BB}}^{1}(x, P_s, \mathrm{b})}{\partial x} = P_s - c_2 + \mathrm{g}_2 - (\mathrm{b} + \mathrm{g}_2)F(x) = 0$$

$$\frac{\partial \Pi_{N_{BB}}(x, P_s, \mathrm{b})}{\partial x} = P_r - P_s - c_1 + \mathrm{g}_1 - (P_r - \mathrm{b} + \mathrm{g}_1)F(x) = 0$$

$$x \geq 0$$

Therefore,

$$F(x^*) = \frac{P_s - c_2 + \mathrm{g}_2}{\mathrm{b} + \mathrm{g}_2} = \frac{P_r - P_s - c_1 + \mathrm{g}_1}{P_r - \mathrm{b} + \mathrm{g}_1}$$

Then, $P_s(\mathrm{b}^*) = \mathrm{b} + c_2 - (b + \mathrm{g}_2)\frac{c}{P_r + \mathrm{g}}$

Under revenue sharing contract, the optimal order quantity of the NVOCC must satisfy:

$$\underset{P_s, \theta}{Maximise} \quad \Pi_{N_{RS}}(x, P_s, \theta)$$

$$s.t. \quad \frac{\partial \Pi_{L_{RS}}^{1}(x, P_s, \theta)}{\partial x} = P_s + (1 - \theta)P_r - c_2 + \mathrm{g}_2 - ((1 - \theta)P_r + \mathrm{g}_2)F(x) = 0$$

$$\frac{\partial \Pi_{N_{RS}}(x, P_s, \theta)}{\partial x} = \theta P_r - P_s - c_1 + \mathrm{g}_1 - (\theta P_r + \mathrm{g}_1)F(x) = 0$$

$$x \geq 0$$

Therefore,

$$F(x^*) = \frac{P_s + (1 - \theta)P_r - c_2 + \mathrm{g}_2}{(1 - \theta)P_r + \mathrm{g}_2} = \frac{\theta P_r - P_s - c_1 + \mathrm{g}_1}{\theta P_r + \mathrm{g}_1}$$

Then, $P_s(\theta^*) = c_2 - (P_r - \theta^* P_r + g_2)\frac{c}{(P_r+g)}$

Under quantity discount contract, the optimal order quantity of the NVOCC must satisfy:

$$\underset{P_s(x)}{Maximise} \quad \Pi_{N_{QD}}(x, P_s(x))$$

$$s.t. \qquad \frac{\partial \Pi_{L_{QD}}^{1}(x, P_s(x))}{\partial x} = \frac{\partial P_s(x)}{\partial x}x + P_s(x) - c_2 + g_2 - g_2 F(x) = 0$$

$$\frac{\partial \Pi_{N_{QD}}(x, P_s(x))}{\partial x} = P_r - \frac{\partial P_s(x)}{\partial x}x - P_s(x) - c_1 + g_1 - (P_r + g_1)F(x) = 0$$

$$x \geq 0$$

Therefore,

$$F(x^*) = \frac{\frac{\partial P_s(x)}{\partial x}x^* + P_s(x^*) - c_2 + g_2}{g_2} = \frac{P_r - \frac{\partial P_s(x)}{\partial x}x^* - P_s(x^*) - c_1 + g_1}{P_r + g_1}$$

Here we get $P_s(x^*) = -\frac{\partial P_s(x)}{\partial x}x^* + c_2 - \frac{g_2 c}{(P_r+g)}$. Because there is a linear functional relationship between $P_s(x)$ and x,

$$\frac{\partial P_s(x)}{\partial x} = -\frac{\partial P_s(x)}{\partial x} = 0$$

Then, $P_s(x^*) = c_2 - \frac{g_2 c}{(P_r+g)}$

Proof of Corollary 1. From the equation (1) and (8), the total profit of the NVOCC and the liner company under this condition will be:

$$\Pi_c = \Pi_N + \Pi_L^1 = P_r[S_N(x) + S_L(x)] - (Q + q_0)c - g[I_N(x) + I_L(x)]$$

To study the relationship between Π_c and x and Q:

$$\frac{\partial \Pi_c}{\partial x} = g[1 - F(x)] > 0$$

$$\frac{\partial^2 \Pi_c}{\partial x^2} = -gf(x)g < 0$$

Then, Π_c is strictly concave in x. The first order derivative for the function denotes that the total profit function is monotonically increasing over the range $x \in \mathbb{N}$.

$$\frac{\partial \Pi_c}{\partial Q} = P_r + g - c - (P_r + g)F(Q + q_0) = 0 \qquad \Rightarrow F(Q + q_0) = \frac{P_r + g - c}{P_r + g}$$

$$\frac{\partial^2 \Pi_c}{\partial Q^2} = -(P_r + \mathrm{g})f(\mathrm{Q} + \mathrm{q_0}) < 0$$

Therefore, the total profit function Π_c is strictly concave in Q and will take the maximum value when $F(\mathrm{Q} + \mathrm{q_0}) = \frac{P_r + \mathrm{g} - \mathrm{c}}{P_r + \mathrm{g}}$. From the equation (1), (2), (4) and (6) the total profit of the liner company under a buyback contract will be:

$$\Sigma_{LBB}(x, P_s, \mathrm{b}) = \Pi\,{}^1_{LBB}(x, P_s, \mathrm{b}) + \Pi\,{}^2_L$$

$$= P_s\, x - \mathrm{b}[x - S_N(x)] - c_2 x - I_N(x)\mathrm{g}_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(x)\mathrm{g}$$

$$\frac{\partial \Sigma_{LBB}(x, P_s, \mathrm{b})}{\partial x} = P_s - \mathrm{b}F(x) - c_2 - [F(x) - 1]\mathrm{g}_2 + P_r[F(x) - 1] + c$$

Combined with the calculation in **Proposition 1**, $(\mathrm{g}_1 + \mathrm{g}_2)(\mathrm{b} - P_s - c_1) = 0$. Because the goodwill loss will bigger than 0, $P_s(\mathrm{b}) = \mathrm{b} - c_1$.

The total profit of the liner company under a revenue-sharing contract is:

$$\Sigma_{LRS}(x, P_s, \theta) = \Pi\,{}^1_{LRS}(x, P_s, \theta) + \Pi\,{}^2_L$$

$$= P_s\, x + (1 - \theta)P_r S_N(x) - c_2 x - I_N(x)\mathrm{g}_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(x)\mathrm{g}$$

$$\frac{\partial \Sigma_{LRS}(x, P_s, \theta)}{\partial x} = P_s + (1 - \theta)P_r[1 - F(x)] - c_2 - [F(x) - 1]\mathrm{g}_2 + P_r[F(x) - 1] + c$$

Combined with the calculation in **Proposition 1**, $(\mathrm{g}_1 + \mathrm{g}_2)(P_s + c_1) = 0$. Because the goodwill loss will bigger than 0, $P_s(\mathrm{b}) = -c_1$.

The total profit of the liner company under a quantity discount contract is:

$$\Sigma_{LQD}(x, \mathrm{P_s}(x)) = \Pi\,{}^1_{LQD}(x, \mathrm{P_s}(x)) + \Pi\,{}^2_L$$

$$= \mathrm{P_s}(x)\, x - c_2 x - I_N(x)\mathrm{g}_2 + P_r S_L(x) - (Q + q_0 - x)c - I_L(x)\mathrm{g}$$

$$\frac{\partial \Sigma_{LQD}(x, \mathrm{P_s}(x))}{\partial x} = \frac{\partial \mathrm{P_s}(x)}{\partial x} x + \mathrm{P_s}(x) - c_2 - [F(x) - 1]\mathrm{g}_2 + P_r[F(x) - 1] + c$$

Combined with the calculation in **Proposition 1**, $(\mathrm{g}_1 + \mathrm{g}_2)\left(-\frac{\partial \mathrm{P_s}(x)}{\partial x} x - \mathrm{P_s}(x) - c_1\right) = 0$. Because the goodwill loss will bigger than 0, $P_s(\mathrm{b}) = -c_1$.

Based on the above calculation, there is no way to structure a contract that maximises the profit for each stakeholder as well as the overall profit. Therefore, we exclude the direct selling of liner companies from the lot purchase contract. From the following first and second derivation of the direct selling profit function, we see that this part of profit is the minimum when $F(x) = \frac{P_r - \mathrm{c}}{P_r}$.

$$\frac{\partial \Pi_{L_{RS}}^{2}}{\partial x} = P_r[F(x) - 1] + c = 0 \qquad \Rightarrow F(x) = \frac{P_r - \mathrm{c}}{P_r} < \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}}$$

$$\frac{\partial^2 \Pi_{L_{RS}}^{2}}{\partial x^2} = P_r f(x) > 0$$

If $P_s + \mathrm{g}_2 + c_1 > P_r > \mathrm{b} + \mathrm{g}_2$ and $P_s + \mathrm{g}_2 + c_1 > \theta P_r > \mathrm{g}_2$, the first derivative of the total profit of the liner company with respect to x is:

$$\frac{\partial \Sigma_{L_{BB}}(x, P_s, \mathrm{b})}{\partial x} = P_s + \mathrm{g}_2 + c_1 - P_r + (P_r - \mathrm{b} - \mathrm{g}_2)F(x) > 0$$

$$\frac{\partial \Sigma_{L_{RS}}(x, P_s, \theta)}{\partial x} = P_s + \mathrm{g}_2 + c_1 - \theta P_r + (\theta P_r - \mathrm{g}_2)F(x) > 0$$

$$\frac{\partial \Sigma_{L_{QD}}(x, \mathrm{P_s}(x))}{\partial x} = \mathrm{P_s}(x) + \mathrm{g}_2 + c_1 - P_r + (P_r - \mathrm{g}_2)F(x) > 0$$

Therefore, the total profit of the liner company is monotonically increasing over the range of x.

$$\frac{\partial \Sigma_{L_{BB}}(x, P_s, \mathrm{b})}{\partial Q} = \frac{\partial \Sigma_{L_{RS}}(x, P_s, \theta)}{\partial Q} = \frac{\partial \Sigma_{L_{QD}}(x, \mathrm{P_s}(x))}{\partial Q}$$

$$= P_r + \mathrm{g} - \mathrm{c} - (P_r + \mathrm{g})F(\mathrm{Q} + \mathrm{q}_0)$$

Proof of Corollary 2. When excluding direct selling profits, from equation (8) the total profit of the liner company and the NVOCC could be simplified as:

$$\Pi_c = P_r S_N(x) - (c_1 + c_2)x - (\mathrm{g}_1 + \mathrm{g}_2)[\mu - S_N(x)]$$

$$= (P_r + \mathrm{g})S_N(x) - cx - \mu \mathrm{g}$$

Under the buyback contract, the profits of the liner company and the NVOCC could be simplified as:

$$\Pi_{L_{BB}}^{1}(x, P_s, \mathrm{b}) = (\mathrm{b} + \mathrm{g}_2)S_N(x) + (P_s - \mathrm{b} - c_2)x - \mu \mathrm{g}_2$$

$$\Pi_{N_{BB}}(x, P_s, \mathrm{b}) = (P_r - \mathrm{b} + \mathrm{g}_1)S_N(x) - (P_s - b + c_1)x - \mu \mathrm{g}_1$$

Let

$$\lambda = \frac{P_r + \mathrm{g}_1 - b}{P_r + \mathrm{g}} \leq 1$$

Since $P_s(b) = -c_1 + b + \frac{(P_r + \mathrm{g}_1 - \mathrm{b})c}{P_r + \mathrm{g}}$, $P_s(\mathrm{b}) = -c_1 + b + \lambda c$. Then,

$$\lambda = \frac{P_s(\theta) - b + c_1}{c} \geq 0$$

Therefore,

$$\Pi_{N_{BB}}(x, P_s, \mathrm{b}) = \lambda(P_r + \mathrm{g})S_N(x) - \lambda cx - \mu \mathrm{g}_1 = \lambda \Pi_c + \mu(\lambda \mathrm{g} - \mathrm{g}_1)$$

$$\Pi\,{}^{1}_{LBB}(x, P_s, \mathrm{b}) = \Pi_c^* - \Pi_{N_{BB}} = (1 - \lambda)\Pi_c^* - \mu(\lambda \mathrm{g} - \mathrm{g}_1)$$

In particular, the liner company will earn the entire Π_c^* when

$$\Pi\,{}^{1}_{LBB}(x, P_s, \mathrm{b}) = (1 - \lambda_1)\Pi_c^* - \mu(\lambda_1 \mathrm{g} - \mathrm{g}_1) = \pi_c^*$$

$$\lambda_1 = \frac{\mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

The NVOCC will earn the entire Π_c^* when

$$\Pi_{N_{BB}}(x, P_s, \mathrm{b}) = \lambda_1 \Pi_c + \mu(\lambda_1 \mathrm{g} - \mathrm{g}_1) = \pi_c$$

$$\lambda_2 = \frac{\Pi_c^* + \mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

Under the revenue sharing contract, the profits of the liner company and the NVOCC can be simplified as:

$$\Pi\,{}^{1}_{LRS}(x, P_s, \theta) = \big((1 - \theta)P_r + \mathrm{g}_2\big)S_N(x) + (P_s - c_2)x - \mu \mathrm{g}_2$$

$$\Pi_{N_{RS}}(x, P_s, \theta) = (\theta P_r + \mathrm{g}_1)S_N(x) - (P_s + c_1)x - \mu \mathrm{g}_1$$

Let

$$\lambda = \frac{\theta P_r + \mathrm{g}_1}{P_r + \mathrm{g}} \leq 1$$

Because $P_s(\theta) = -c_1 + \frac{(\theta P_r + \mathrm{g}_1)c}{P_r + \mathrm{g}}$, $P_s(\theta) = -c_1 + \lambda c$. Therefore,

$$\lambda = \frac{P_s(\theta) + c_1}{c} \geq 0$$

Then,

$$\begin{cases} \Pi_{N_{RS}}(x, P_s, \theta) = \lambda \Pi_c^* + \mu(\lambda \mathrm{g} - \mathrm{g}_1) \\ \Pi\,{}^{2}_{LRS}(x, P_s, \theta) = \Pi_c^* - \Pi_{N_{RS}} = (1 - \lambda)\Pi_c^* - \mu(\lambda \mathrm{g} - \mathrm{g}_1) \end{cases}$$

Like the buyback contract, the liner company will earn the entire π_c^* when

$$\xi\,{}^{1}_{LRS}(x, P_s, \theta) = (1 - \lambda_1)\Pi_c^* - \mu(\lambda_1 \mathrm{g} - \mathrm{g}_1) = \Pi_c^*$$

$$\lambda_1 = \frac{\mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

The NVOCC will earns the entire Π_c^* when

$$\Pi_{N_{RS}}(x, P_s, \theta) = \lambda_1 \Pi_c + \mu(\lambda_1 \mathrm{g} - \mathrm{g}_1) = \Pi_c$$

$$\lambda_2 = \frac{\Pi_c^* + \mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

When under the quantity discount contract, the profits of the liner company and the NVOCC could be simplified as:

$$\Pi\,{}^{1}_{L_{RS}}(x, P_s, \theta) = \mathrm{g}_2 S_N(x) + (P_s - c_2)x - \mu \mathrm{g}_2$$

$$\Pi_{N_{RS}}(x, P_s, \theta) = (P_r + \mathrm{g}_1)S_N(x) - (P_s + c_1)x - \mu \mathrm{g}_1$$

Let

$$\lambda = \frac{P_r + \mathrm{g}_1}{P_r + \mathrm{g}} \leq 1$$

Because $P_s(x) = -c_1 + \frac{(P_r+\mathrm{g}_1)c}{P_r+\mathrm{g}}$, $P_s(x) = -c_1 + \lambda c$. Therefore,

$$\lambda = \frac{P_s(x) + c_1}{c} \geq 0$$

Then,

$$\begin{cases} \Pi_{N_{QD}}(x, \mathrm{P_s}(x)) = \lambda \Pi_c^* + \mu(\lambda \mathrm{g} - \mathrm{g}_1) \\ \Pi^1_{L_{QD}}(x, \mathrm{P_s}(x)) = \Pi_c^* - \Pi_{N_{QD}} = (1-\lambda)\Pi_c^* - \mu(\lambda \mathrm{g} - \mathrm{g}_1) \end{cases}$$

Like the buyback contract, the liner company will earn the entire Π_c^* when

$$\Pi\,{}^{1}_{L_{RS}}(x, P_s, \theta) = (1 - \lambda_1)\Pi_c^* - \mu(\lambda_1 \mathrm{g} - \mathrm{g}_1) = \Pi_c^*$$

$$\lambda_1 = \frac{\mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

The NVOCC will earn the entire Π_c^* when

$$\Pi_{N_{RS}}(x, P_s, \theta) = \lambda_1 \Pi_c + \mu(\lambda_1 \mathrm{g} - \mathrm{g}_1) = \Pi_c$$

$$\lambda_2 = \frac{\Pi_c^* + \mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

From this, the formula for the fraction λ that governs the profit distribution is the same under these contract types and can be summarized as follows:

$$\lambda_1 = \frac{\mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}} \, and \; \lambda_2 = \frac{\Pi_c^* + \mu \mathrm{g}_1}{\Pi_c^* + \mu \mathrm{g}}$$

Here, the profit of the liner company is decreasing in λ and the profit of the NVOCC is increasing in λ. Taking the buyback contract as an example, λ is decreasing b because the sum profit of liner company and NVOCC is fixed, with the increase of λ, the liner company will earn a greater proportion of Π_c^* and $\lambda_1 < \lambda_2$.

Proof of Proposition 2. When the liner company does not have enough capacity to support the previous optimal reservation of the NVOCC, the new profit function of the liner company will be:

$$\begin{cases} \pi_{L_{BB}}^1(x, P_s, \mathrm{b}) = \Pi_{L_{BB}}^1(x, P_s, \mathrm{b}) - \dfrac{1-\sin\alpha}{2} x P_r, & \text{buyback contract,} \\ \pi_{L_{RS}}^1(x, P_s, \theta) = \Pi_{L_{BB}}^1(x, P_s, \theta) - \dfrac{1-\sin\alpha}{2} x P_r, & \text{revenue sharing contract,} \\ \pi_{L_{QD}}^1(x, P_s(x)) = \Pi_{L_{BB}}^1(x, P_s(x)) - \dfrac{1-\sin\alpha}{2} x P_r, & \text{quantity discount contract} \end{cases}$$

Given that the profit function of the NVOCC $\pi_N = \Pi_N$ remains unchanged, the total profit of the NVOCC and the liner company under these contract types will change to:

$$\begin{aligned} \pi_c = \pi_N + \pi_L^1 &= \Pi_c - \frac{1-\sin\alpha}{2} x P_r \\ &= P_r S_N(x) - \frac{1-\sin\alpha}{2} x P_r - (c_1 + c_2)x - (\mathrm{g}_1 + \mathrm{g}_2) I_N(x) \end{aligned}$$

From the first order derivative for the profit function:

$$\frac{\partial \Pi_c}{\partial x} = P_r[1 - F(x)] - \frac{1-\sin\alpha}{2} P_r - c - \mathrm{g}[F(x) - 1] = 0$$

we can solve the relationship between optimal order quantity and the market parameters as:

$$F(\tilde{x}) = \frac{\frac{1+\sin\alpha}{2} P_r - c + \mathrm{g}}{P_r + \mathrm{g}} \leq F(x^*)$$

When under the buyback contract, the optimal contract setting to maximize profit of both the liner company and the NVOCC must satisfy:

$$\begin{aligned} &\underset{P_s, b}{Maximise} \quad \pi_{N_{BB}}(x, P_s, \mathrm{b}) \\ &s.t. \quad \frac{\partial \pi_{L_{BB}}^{1}(x, P_s, \mathrm{b})}{\partial x} = P_s - c_2 + \mathrm{g}_2 - (\mathrm{b} + \mathrm{g}_2)F(x) - \frac{1-\sin\alpha}{2} P_r = 0 \\ &\qquad \frac{\partial \pi_{N_{BB}}(x, P_s, \mathrm{b})}{\partial x} = P_r - P_s - c_1 + \mathrm{g}_1 - (P_r - \mathrm{b} + \mathrm{g}_1)F(x) = 0 \\ &\qquad x \geq 0 \end{aligned}$$

Therefore,

$$F(\tilde{x}) = \frac{P_s - c_2 + \mathrm{g}_2 - \frac{1-\sin\alpha}{2} P_r}{\mathrm{b} + \mathrm{g}_2} = \frac{P_r - P_s - c_1 + \mathrm{g}_1}{P_r - \mathrm{b} + \mathrm{g}_1}$$

Then, $P_s(\widetilde{b}) = c_2 + \widetilde{b} - \frac{(\widetilde{b}+g_2)c + \frac{\sin\alpha - 1}{2} P_r (P_r - \widetilde{b} + g_1)}{P_r + g}$

Then under the revenue sharing contract, the optimal order quantity of the NVOCC must satisfy:

$$\underset{P_s,\theta}{Maximise} \quad \pi_{N_{RS}}(x, P_s, \theta)$$

$$s.t. \qquad \frac{\partial \pi^{1}_{L_{RS}}(x, P_s, \theta)}{\partial x} = P_s + (1-\theta)P_r - c_2 + g_2 - ((1-\theta)P_r + g_2)F(x) - \frac{1-\sin\alpha}{2}P_r = 0$$

$$\frac{\partial \pi_{N_{RS}}(x, P_s, \theta)}{\partial x} = \theta P_r - P_s - c_1 + g_1 - (\theta P_r + g_1)F(x) = 0$$

$$x \geq 0$$

Therefore,

$$F(\tilde{x}) = \frac{P_s + (1-\theta)P_r - c_2 + g_2 - \frac{1-\sin\alpha}{2}P_r}{(1-\theta)P_r + g_2} = \frac{\theta P_r - P_s - c_1 + g_1}{\theta P_r + g_1}$$

Then, $P_s(\tilde{\theta}) = c_2 - \frac{(P_r - \theta P_r + g_2)c + \frac{\sin\alpha - 1}{2} P_r(\theta P_r + g_1)}{(P_r + g)}$

Then under quantity discount contract, the optimal order quantity of the NVOCC must satisfy:

$$\underset{P_s(x)}{Maximise} \quad \pi_{N_{QD}}(x, P_s(x))$$

$$s.t. \qquad \frac{\partial \pi^{1}_{L_{QD}}(x, P_s(x))}{\partial x} = \frac{\partial P_s(x)}{\partial x}x + P_s(x) - c_2 + g_2 - g_2 F(x) - \frac{1-\sin\alpha}{2}P_r = 0$$

$$\frac{\partial \pi_{N_{QD}}(x, P_s(x))}{\partial x} = P_r - \frac{\partial P_s(x)}{\partial x}x - P_s(x) - c_1 + g_1 - (P_r + g_1)F(x) = 0$$

$$x \geq 0$$

Therefore,

$$F(\widetilde{x}) = \frac{\frac{\partial P_s(x)}{\partial x}x + P_s(x) - c_2 + g_2 - \frac{1-\sin\alpha}{2}P_r}{g_2} = \frac{P_r - \frac{\partial P_s(x)}{\partial x}x - P_s(x) - c_1 + g_1}{P_r + g_1}$$

Here we get $P_s(\widetilde{x}) = -\frac{\partial P_s(x)}{\partial x}\widetilde{x} + c_2 - \frac{g_2 c + \frac{\sin\alpha - 1}{2}P_r(P_r + g_1)}{(P_r + g)}$. Because there is a linear functional relationship between $P_s(x)$ and x,

$$\frac{\partial P_s(x)}{\partial x} = -\frac{\partial P_s(x)}{\partial x} = 0$$

Then, $P_s(\widetilde{x}) = c_2 - \frac{g_2 c + \frac{\sin\alpha - 1}{2}P_r(P_r + g_1)}{(P_r + g)}$

Proof of Corollary 3. By substituting the optimal wholesale price in **Proposition 1** into equation (4) and (6), we can get:

$$\Pi_{LQD}^{1}\big(x,\mathrm{P_s}(x)\big)=\left\{c_2-\mathrm{g}_2\frac{\mathrm{c}}{(P_r+\mathrm{g})}\right\}x-c_2x-I_N(x)\mathrm{g}_2$$

$$\Pi\,_{LRS}^{1}(x,P_s,\theta)=\left\{c_2-(P_r-\theta^*P_r+\mathrm{g}_2)\frac{\mathrm{c}}{(P_r+\mathrm{g})}\right\}x+(1-\theta)P_rS_N(x)-c_2x-I_N(x)\mathrm{g}_2$$

$$=\left\{c_2-\mathrm{g}_2\frac{\mathrm{c}}{(P_r+\mathrm{g})}\right\}x-c_2x-I_N(x)\mathrm{g}_2-\frac{(1-\theta)P_r}{P_r+\mathrm{g}}\mathrm{c}x+(1-\theta)P_rS_N(x)$$

$$=\Pi_{LQD}^{1}\big(x,\mathrm{P_s}(x)\big)-\frac{(1-\theta)P_r}{P_r+\mathrm{g}}\mathrm{c}x+(1-\theta)P_rS_N(x)$$

$$=\Pi_{LQD}^{1}\big(x,\mathrm{P_s}(x)\big)+(1-\theta)P_r\left\{S_N(x)-\frac{\mathrm{c}x}{P_r+\mathrm{g}}\right\}$$

From **Proposition 1**, we get $F(x^*)=\frac{P_r-c+\mathrm{g}}{P_r+\mathrm{g}}$. Let $\mathrm{H}(x)=S_N(x)-\frac{\mathrm{c}x}{P_r+\mathrm{g}}=x-\int_0^x F(d)dd-\frac{\mathrm{c}x}{P_r+\mathrm{g}}=F(x^*)x-\int_0^x F(d)dd.$

$$\frac{\partial \mathrm{H}(x)}{\partial x}=F(x)+f(x)x-F(x)=f(x^*)x>0$$

Therefore, $\mathrm{H}(x)$ is a monotone increasing function. $H(x)\geq H(0)=0$

$$\Pi_{LQD}^{1}\big(x,P_{\mathrm{s}}(x)\big)\leq\Pi_{LRS}^{1}(x,P_s,\theta)=\Pi_{LBB}^{1}(x,P_s,\mathrm{b})$$

And substituting the optimal wholesale price **Proposition 2** into equation (11) and (12), we can get:

$$\pi_{LQD}^{1}\big(x,P_{\mathrm{s}}(x)\big)=\left\{-c_1+\frac{(P_r+\mathrm{g}_1)\left(c-\frac{\sin\alpha-1}{2}P_r\right)}{P_r+\mathrm{g}}\right\}x-c_2x-I_N(x)\mathrm{g}_2-\frac{1-\sin\alpha}{2}xP_r$$

$$\pi_{LRS}^{1}(x,P_s,\theta)=\left\{-c_1+\frac{(\tilde{\theta}P_r+\mathrm{g}_1)\left(c-\frac{\sin\alpha-1}{2}P_r\right)}{P_r+\mathrm{g}}\right\}x+(1-\theta)P_rS_N(x)-c_2x-I_N(x)\mathrm{g}_2$$

$$=\left\{-c_1+\frac{(P_r+\mathrm{g}_1)\left(c-\frac{\sin\alpha-1}{2}P_r\right)}{P_r+\mathrm{g}}\right\}x-c_2x-I_N(x)\mathrm{g}_2-\frac{1-\sin\alpha}{2}xP_r$$

$$-\frac{(1-\theta)P_r}{P_r+\mathrm{g}}\left(c-\frac{\sin\alpha-1}{2}P_r\right)x+(1-\theta)P_rS_N(x)$$

$$=\Pi_{LQD}^{1}\big(x,\mathrm{P_s}(x)\big)-\frac{(1-\tilde{\theta})P_r\left(c-\frac{\sin\alpha-1}{2}P_r\right)}{P_r+\mathrm{g}}x+(1-\theta)P_rS_N(x)$$

$$=\Pi_{LQD}^{1}\big(x,\mathrm{P_s}(x)\big)+(1-\theta)P_r\left\{S_N(x)-\frac{\left(c-\frac{\sin\alpha-1}{2}P_r\right)x}{P_r+\mathrm{g}}\right\}$$

From **Proposition 2**, we get $F(\tilde{x})=\frac{P_r-c+\mathrm{g}-\frac{1-\sin\alpha}{2}P_r}{P_r+\mathrm{g}}$. Let $h(x)=S_N(x)-\frac{\left(c-\frac{\sin\alpha-1}{2}P_r\right)x}{P_r+\mathrm{g}}=$ $x-\int_0^x F(d)dd-\frac{\left(c-\frac{\sin\alpha-1}{2}P_r\right)x}{P_r+\mathrm{g}}=F(\tilde{x})x-\int_0^x F(d)dd.$

$$\frac{\partial \mathrm{h}(x)}{\partial x}=F(x)+f(\tilde{x})x-F(x)=f(\tilde{x})x>0$$

Therefore, $h(x)$ is a monotone increasing function. $h(x)\geq h(0)=0$.

$$\pi^1_{LQD}\big(x,P_\mathrm{s}(x)\big)\leq \pi^1_{LRS}(x,P_s,\theta)=\pi^1_{LBB}(x,P_s,\mathrm{b})$$

For the quantity discount contract under the sufficient capacity and the revenue sharing contract under the insufficient capacity case, the profit of the liner company satisfies the following equation:

$$\pi^1_{LRS}(x,P_s,\theta)=\left\{-c_1+\frac{\left(\tilde{\theta}P_r+\mathrm{g}_1\right)\left(c-\frac{\sin\alpha-1}{2}P_r\right)}{P_r+\mathrm{g}}\right\}x+(1-\theta)P_rS_N(x)-c_2x-I_N(x)\mathrm{g}_2$$

$$=\left\{-c_1+\frac{(P_r+\mathrm{g}_1)c}{P_r+\mathrm{g}}\right\}x-c_2x-I_N(x)\mathrm{g}_2-\frac{1-\sin\alpha}{2}xP_r-\frac{(P_r+\mathrm{g}_1)\frac{\sin\alpha-1}{2}P_r}{P_r+\mathrm{g}}x$$

$$=\Pi^1_{LQD}(x,\mathrm{P_s}(x))+\frac{(P_r+\mathrm{g}_1)}{P_r+\mathrm{g}}*\frac{1-\sin\alpha}{2}xP_r$$

$$\Pi^1_{LQD}\big(x,\mathrm{P_s}(x)\big)\leq \pi^1_{LRS}(x,P_s,\theta)$$

Next, we compare H(x) and h(x)

$$\mathrm{H}(x)=F(x^*)x-\int_0^x F(d)dd$$

$$h(x)=F(\tilde{x})x-\int_0^x F(d)dd$$

Since $F(x)$ is a monotone increasing function,

$$F(x^*)\geq F(\tilde{x})$$

$$\mathrm{H}(x)\geq h(x)$$

Proof of Proposition 3. For ease of explanation, we divide the function from left to right into four regions: B, BB, A and AA. Condition 1: for any company that benefits from AA and A, there are four situations listed in Table 8.2 to limit the loss in B or BB. Condition 2: For any company that suffers losses in the BB and B areas, there are also four situations shown in Table 8.3 to restore the rating in the A or AA area.

Under Condition 1: Companies with a surplus profit of $0\leq m^+<M^+$ should be rated

A at the end of the competition stage. Then, the company could earn a reward of $\lambda k_2 m^+$ on the credit rating score. Companies with a surplus profit of $M^+ \leq m^+$ should be rated A at the end of the competition stage. Then, the company could earn a reward of $\lambda k_1 m^+ + (k_2 - k_1)\delta M^+$ on the credit rating score.

	A $(0 \leq m^+ < M^+)$	AA $(M^+ \leq m^+)$
BB	$-\delta m^+$	$-\dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2}$
B	$\dfrac{-\delta k_2 m^+ - (k_2 - k_1)M^-}{k_1}$	$-\dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+ + (k_2 - k_1)M^-}{k_1}$

Table 8.2 Amount for companies that made profits m^+ in the stage 1 could lose in the stage 2

Note that all the numbers in the above table are negative. Additionally, because $k_2 M^- < -\delta k_2 M^+ < -\delta k_2 m^+ \leq 0$, there is no chance for a liner company with rate A to get a loss at rate B in stage 2. To keep the liner company with rate AA at stage 1 and to get a loss amount within the rate BB at stage 2, $M^- \leq -\frac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2} < 0$ must hold. Similarly, $-\frac{\delta k_1 m + (k_2 - k_1)\delta M^+ + (k_2 - k_1)M^-}{k_1} < M^-$ must hold to keep the liner company with rate AA at stage 1 to get a loss amount at the rate B in stage 2. We can summarise and conclude that the amount of possible loss in stage 2 must satisfy:

$$\text{Loss}(m^+) = \begin{cases} \text{case 1: } -\delta M^+ < -\delta m^+ \leq 0, & 0 \leq m^+ < M^+ \\ \text{case 2: } M^- < -\dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2} \leq -\delta M^+, & M^+ \leq m^+ < M^+ - \dfrac{k_2(M^- + \delta M^+)}{\delta k_1} \\ \text{case 3: } -\dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+ + (k_2 - k_1)M^-}{k_1} \leq M^-, & M^+ - \dfrac{k_2(M^- + \delta M^+)}{\delta k_1} \leq m^+ \end{cases}$$

Here, $M^- + \delta M^+ < 0$. When the liner company suffers the losses according to the above equation, the net profit through stage 1 and 2 will be:

$$\begin{cases} m^+(1 - \delta) > 0, & case\ 1 \\ m^+ - \dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2} > 0, & case\ 2 \\ m^+ - \dfrac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+ + (k_2 - k_1)M^-}{k_1} > 0, & case\ 3 \end{cases}$$

Here,

$$\begin{aligned} m^+ - \frac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2} &= \frac{k_2 m^+ - \delta k_1 m^+ - (k_2 - k_1)\delta M^+}{k_2} \\ &= \frac{k_2 m^+(1 - \delta) + (k_2 - k_1)\delta m^+ - (k_2 - k_1)\delta M^+}{k_2} \\ &= \frac{k_2 m^+(1 - \delta) + \delta(k_2 - k_1)(m^+ - M^+)}{k_2} > 0 \end{aligned}$$

$$m^+ - \frac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+ + (k_2 - k_1)M^-}{k_1}$$
$$= \frac{k_1 m^+ - \delta k_1 m^+ - (k_2 - k_1)\delta M^+ - (k_2 - k_1)M^-}{k_1}$$
$$= \frac{k_1 m^+(1-\delta) - (k_2 - k_1)(M^- + \delta M^+)}{k_1} > 0$$

Therefore, no matter how the credit rate of the liner company changes, the net profit of the company under the above three cases will be positive.

Under Condition 2: Companies with a difference of $M^- \le m^- < 0$ who have not repaid should be rated BB at the end of the competition stage. Thereafter, companies suffer a deduction of $k_2 m^-$ on the credit rating score. Companies with a difference of $m^- < M^-$ who have not repaid should be rated B at the end of the competition stage. Then, the company suffers a deduction of $k_1 m^- - (k_2 - k_1)M^-$ on the credit rating score.

	BB ($M^- \le m^- < 0$)	B ($m^- < M^-$)
A	$-\frac{m^-}{\delta}$	$\frac{-k_1 m^- - (k_2 - k_1)M^-}{\delta k_2}$
AA	$\frac{-k_2 m^- - (k_2 - k_1)\delta M^+}{\delta k_1}$	$-\frac{k_1 m^- + (k_2 - k_1)M^- + (k_2 - k_1)\delta M^+}{\delta k_1}$

Table 8.3 Amount for companies that made losses m^- in the stage 1 should earn in the stage 2

Note that $m^- < 0$. Given that the credit rating of the liner company with rate B at stage 1 will drop $k_1 m + (k_2 - k_1)M^- < k_2 M^-$, there is no chance for the company to recover the credit rating with the profit at rate A area in stage 2. To keep the liner company with rate BB at stage 1 could recover the rating within the rate A at stage 2, $0 < -k_2 m^- \le \delta k_2 M^+$ must hold. As for the liner company who was rated BB and wants to recover the rating within rate AA at stage 2, $\delta k_2 M^+ < -k_2 m^- \le -k_2 M^-$ must hold. $k_1 m^- + (k_2 - k_1)M^- \le k_2 M^-$ is necessary for the company rated at B to recover the rating within rate AA at stage 2. Then, we can summarise and conclude that the profit range for the liner company to restore the credit rating in stage 2 must satisfy:

$$\text{Profit}(m^-) = \begin{cases} \text{case 4: } 0 < -\frac{m^-}{\delta} \le M^+, & -\delta M^+ \le m^- < 0 \\ \text{case 5: } M^+ < \frac{-k_2 m^- - (k_2 - k_1)\delta M^+}{\delta k_1} \le M^+ - \frac{k_2(M^- + \delta M^+)}{\delta k_1}, & M^- \le m^- < -\delta M^+ \\ \text{case 6: } M^+ - \frac{k_2(M^- + \delta M^+)}{\delta k_1} \le -\frac{k_1 m^- + (k_2 - k_1)M^- + (k_2 - k_1)\delta M^+}{\delta k_1}, & m^- \le M^- \end{cases}$$

Note that $M^- + \delta M^+ < 0$. When the liner company can earn the profits according to the above equation in stage 2, the net profit through stage 1 and 2 will be:

$$\begin{cases} m^-(1-\frac{1}{\delta}) > 0, & \text{case 4} \\ m^- - \frac{k_2 m^- + (k_2 - k_1)\delta M^+}{\delta k_1}, & \text{case 5} \\ m^- - \frac{k_1 m^- + (k_2 - k_1)M^- + (k_2 - k_1)\delta M^+}{\delta k_1}, & \text{case 6} \end{cases}$$

Here,

$$\begin{aligned} m^- - \frac{k_2 m^- + (k_2 - k_1)\delta M^+}{\delta k_1} &= \frac{\delta k_1 m^- - k_2 m^- - (k_2 - k_1)\delta M^+}{\delta k_1} \\ &= \frac{-k_2 m^- + \delta k_2 m^- + \delta k_1 m^- - \delta k_2 m^- - (k_2 - k_1)\delta M^+}{\delta k_1} \\ &= \frac{(\delta - 1)k_2 m^- + \delta m^-(k_1 - k_2) + (k_1 - k_2)\delta M^+}{\delta k_1} \\ &= \frac{(\delta - 1)k_1 m^- + (k_1 - k_2)(m^- + \delta M^+)}{\delta k_1} > 0 \end{aligned}$$

$$\begin{aligned} m^- - \frac{k_1 m^- + (k_2 - k_1)M^- + (k_2 - k_1)\delta M^+}{\delta k_1} \\ &= \frac{\delta k_1 m^- - k_1 m^- - (k_2 - k_1)M^- - (k_2 - k_1)\delta M^+}{\delta k_1} \\ &= \frac{(\delta - 1)k_1 m^- + (k_1 - k_2)(M^- + \delta M^+)}{\delta k_1} > 0 \end{aligned}$$

Therefore, no matter how the credit rating of a liner company changes, the net profit of the company under the above three cases will be positive.

When comparing the range of profits and losses in condition 1 with condition 2, it is obvious that the profits in Stage 1 of Condition 1 share the same range as the profits in Stage 2 of Condition 2, the same as the losses in Stage 2 of Condition 1 and the losses in Stage 1 of Condition 2. Based on this, we presuppose that m^+ and m^- have the following relationship to simplify the model:

$$\begin{cases} m^- = -\delta m^+, & Case\ 1\ and\ Case\ 4 \\ m^- = -\frac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+}{k_2}, & Case\ 2\ and\ Case\ 5 \\ m^- = -\frac{\delta k_1 m^+ + (k_2 - k_1)\delta M^+ + (k_2 - k_1)M^-}{k_1}, & Case\ 3\ and\ Case\ 6 \end{cases}$$

Here, the profit in stage 1 of case 1 and the profit in stage 2 of case 4 is:

$$-\frac{m^-}{\delta} = m^+ \rightarrow m^- = -\delta m^+$$

Similarly, the profit in stage 1 of case 2 and the profit in stage 2 of case 5 is:

$$m^{+}=\frac{-k_{2}m^{-}-(k_{2}-k_{1})\delta M^{+}}{\delta k_{1}}$$

$$\rightarrow\ m^{-}=-\frac{\delta k_{1}m^{+}+(k_{2}-k_{1})\delta M^{+}}{k_{2}}$$

And the profit in stage 1 of case 3 and the profit in stage 2 of case 6 is:

$$m^{+}=-\frac{k_{1}m^{-}+(k_{2}-k_{1})M^{-}+(k_{2}-k_{1})\delta M^{+}}{\delta k_{1}}$$

$$\rightarrow\ m^{-}=-\frac{\delta k_{1}m^{+}+(k_{2}-k_{1})\delta M^{+}+(k_{2}-k_{1})M^{-}}{k_{1}}$$

Based on these, the profits and losses in conditions 1 and 2 could be exchanged through the above equations. Then, by taking the first derivative of the loss in condition 1 with respect to the profit and taking the first derivative of the profit in condition 2 with respect to the loss, we can obtain the following formula:

$$\frac{\partial \text{Loss}(m^{+})}{\partial m^{+}}=\begin{cases}-\delta, & case\ 1\\ -\frac{\delta k_{1}}{k_{2}}, & case\ 2\\ -\delta, & case\ 3\end{cases}$$

$$\frac{\partial \text{Profit}(m^{-})}{\partial m^{-}}=\begin{cases}-\frac{1}{\delta}, & case\ 4\\ \frac{-k_{2}}{\delta k_{1}}, & case\ 5\\ -\frac{1}{\delta}, & case\ 6\end{cases}$$

Proof of Corollary 4. Combined with **Corollary 2**, equation (13) can be rewritten as:

$$\begin{aligned}\pi_{c}&=P_{r}S_{N}(x)-\frac{1-\sin\alpha}{2}xP_{r}-(c_{1}+c_{2})x-(\mathrm{g}_{1}+\mathrm{g}_{2})I_{N}(x)\\&=P_{r}S_{N}(x)-\frac{1-\sin\alpha}{2}xP_{r}-cx-g[\mu-S_{N}(x)]\\&=(P_{r}+\mathrm{g})S_{N}(x)-cx-\mu\mathrm{g}-\frac{1-\sin\alpha}{2}xP_{r}\end{aligned}$$

Under the buyback contract, the profit of the liner company will become as follows:

$$\begin{aligned}\pi_{LBB}^{1}(x,P_{s},\mathrm{b})&=P_{s}\,x-\mathrm{b}[x-S_{N}(x)]-c_{2}x-I_{N}(x)\mathrm{g}_{2}-\frac{1-\sin\alpha}{2}xP_{r}\\&=P_{s}\,x-\mathrm{b}[x-S_{N}(x)]-c_{2}x-[\mu-S_{N}(x)]\mathrm{g}_{2}-\frac{1-\sin\alpha}{2}xP_{r}\\&=(\mathrm{b}+\mathrm{g}_{2})S_{N}(x)+(P_{s}-\mathrm{b}-c_{2})x-\mu\mathrm{g}_{2}-\frac{1-\sin\alpha}{2}xP_{r}\end{aligned}$$

Let $\lambda = \frac{P_r + \mathrm{g}_1 - b}{P_r + \mathrm{g}}$. Because

$$P_s(b) = -c_1 + b + \frac{(P_r + \mathrm{g}_1 - \mathrm{b})\left(c - \frac{\sin\alpha - 1}{2}P_r\right)}{P_r + \mathrm{g}}$$
$$= -c_1 + b + \lambda\left(c - \frac{\sin\alpha - 1}{2}P_r\right)$$

Then,

$$\pi_L^1(x, P_s, \mathrm{b}) = (1-\lambda)\pi_c - \mu(\lambda \mathrm{g} - \mathrm{g}_1).$$

Under the revenue sharing contract, the profit function of the liner company will become:

$$\pi_{LRS}^1(x, P_s, \theta) = P_s\, x + (1-\theta)P_r S_N(x) - c_2 x - I_N(x)\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r$$
$$= P_s\, x + (1-\theta)P_r S_N(x) - c_2 x - [\mu - S_N(x)]\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r$$
$$= [(1-\theta)P_r + \mathrm{g}_2]S_N(x) + (P_s - c_2)x - \mu \mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r$$

Let $\lambda = \frac{\theta P_r + \mathrm{g}_1}{P_r + \mathrm{g}}$. Because

$$P_s(\theta) = -c_1 + \frac{(\tilde{\theta}P_r + \mathrm{g}_1)\left(c - \frac{\sin\alpha - 1}{2}P_r\right)}{P_r + \mathrm{g}}$$
$$= -c_1 + \lambda\left(c - \frac{\sin\alpha - 1}{2}P_r\right)$$

Then,

$$\pi_L^1(x, P_s, \theta) = (1-\lambda)\pi_c - \mu(\lambda \mathrm{g} - \mathrm{g}_1).$$

Under the quantity discount contract, the profit function liner company will be:

$$\pi_{LQD}^1\big(x, P_s(x)\big) = \mathrm{P_s}(x)\, x - c_2 x - I_N(x)\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r$$
$$= \mathrm{P_s}(x)\, x - c_2 x - [\mu - S_N(x)]\mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r$$
$$= \mathrm{g}_2 S_N(x) + (P_s - c_2)x - \mu \mathrm{g}_2 - \frac{1-\sin\alpha}{2}xP_r$$

Let $\lambda = \frac{P_r + \mathrm{g}_1}{P_r + \mathrm{g}}$. Because

$$P_s(x) = -c_1 + \frac{(P_r + \mathrm{g}_1)\left(c - \frac{\sin\alpha - 1}{2}P_r\right)}{P_r + \mathrm{g}}$$
$$= -c_1 + \lambda\left(c - \frac{\sin\alpha - 1}{2}P_r\right)$$

Then,

$$\pi_L^1\big(x, P_s(x)\big) = (1-\lambda)\pi_c - \mu(\lambda g - g_1).$$

We can summarise and conclude that the profit function of over-conservative for the liner company could be simplified as:

$$\pi_L^1 = (1-\lambda)\pi_c - \mu(\lambda g - g_1)$$

Proof of Proposition 4. The financial lease rental could be expressed as:

$$\Delta = P_r Q \frac{i+1}{1+e^r} = r\frac{i+1}{1+e^r}$$

To help with subsequent calculations we first calculate

$$\frac{d\frac{i+1}{1+e^r}}{dr} = -\frac{(i+1)e^r}{(1+e^r)^2} = -(i+1)\left[\frac{1}{1+e^r}\times\frac{e^r}{1+e^r}\right]$$
$$= -(i+1)\left[\frac{1}{1+e^r}\times\frac{1+e^r-1}{1+e^r}\right]$$
$$= -(i+1)\left\{\frac{1}{1+e^r}\times\left[1-\frac{1}{1+e^r}\right]\right\}$$

Then, the first derivative of financial lease rental with respect to the credit rating is:

$$\frac{d\Delta}{dr} = \frac{i+1}{1+e^r} - r(i+1)\left\{\frac{1}{1+e^r} - \left[\frac{1}{1+e^r}\right]^2\right\}$$
$$= \frac{i+1}{1+e^r} - \frac{r(i+1)}{1+e^r} + r(i+1)\left[\frac{1}{1+e^r}\right]^2$$
$$= (i+1)\left\{\frac{1}{1+e^r} - \frac{r}{1+e^r} + r\left[\frac{1}{1+e^r}\right]^2\right\}$$
$$= \frac{i+1}{1+e^r}\left\{1 - r + \frac{r}{1+e^r}\right\} = 0$$

Since $\frac{i+1}{1+e^r} \neq 0$, when r satisfies $1 - r + \frac{r}{1+e^r} = 0$ the financial lease rental will reach extreme value. The second derivative of financial lease rental with respect to the credit rating as:

$$\frac{d^2\Delta}{dr^2} = -\frac{i+1}{1+e^r}\left[1-\frac{1}{1+e^r}\right]\left\{1-r+\frac{r}{1+e^r}\right\} + \frac{i+1}{1+e^r}\left\{-1+\frac{1}{1+e^r} - \frac{r}{1+e^r}\times\left[1-\frac{1}{1+e^r}\right]\right\}$$
$$= -\frac{i+1}{1+e^r}\times\left[1-\frac{1}{1+e^r}\right]\left\{1-r+\frac{r}{1+e^r}\right\} - \frac{i+1}{1+e^r}\times\left[1-\frac{1}{1+e^r}\right]\left\{1+\frac{r}{1+e^r}\right\}$$
$$= -\frac{i+1}{1+e^r}\times\left[1-\frac{1}{1+e^r}\right]\left\{2-r+\frac{2r}{1+e^r}\right\}$$

Then, we can get that $\frac{d^2\Delta}{dr^2} < 0$ when $1 - r + \frac{r}{1+e^r} = 0$. Therefore, the above extreme value is the maximum value. Based on this, the maximum financial lease payment is:

$$\Delta = r\frac{i+1}{1+e^r} = r(1+i)\frac{r-1}{r}$$
$$= (r-1)(1+i)$$

Proof of Proposition 5. Based on **Corollary 4**, we compare the difference between over-conservative liner companies and over-placement liner companies from profit function and financial lease rental perspective:

$$\begin{aligned}
\Lambda(m) &= m(Q,\lambda) - m_0(Q_0,\lambda) \\
&= (1-\lambda)\Pi_c^* - \mu(\lambda \mathrm{g} - \mathrm{g}_1) + P_r S_L(x) - (Q + q_0 - x)c - I_L(Q+q_0)\mathrm{g} \\
&\qquad - (r_1 - 1)(1+i) - (1-\lambda)\pi_c + \mu(\lambda \mathrm{g} - \mathrm{g}_1) + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} \\
&\qquad + \frac{r_0}{r_1}(r_1 - 1)(1+i) \\
&= (1-\lambda)(\Pi_c^* - \pi_c) + P_r S_L(x) - (Q + q_0 - x)c - I_L(Q+q_0)g \\
&\qquad + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} + \frac{r_0 - r_1}{r_1}(r_1 - 1)(1+i) \\
&= (1-\lambda)(\Pi_c^* - \pi_c) + \Pi_L^2 + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} + \frac{r_0 - r_1}{r_1}(r_1 - 1)(1+i)
\end{aligned}$$

Based on **Proposition 4**, the following equation must hold for the profit function of over-conservative liner companies who can repay the total financial lease rental:

$$\pi_L = (1-\lambda)\pi_c - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - I_L(Q_0 - \mathrm{q}_0)\mathrm{g} = r_0\frac{i+1}{1+e^{r_1}} = r_0\frac{r_1 - 1}{r_1}(1+i)$$

Here, $P_r Q_0 = r_0 < P_r Q = r_1$. Then, the over-placement liner companies can be simplified as:

$$\begin{aligned}
\Pi_L &= (1-\lambda)\Pi_c^* - \mu(\lambda \mathrm{g} - \mathrm{g}_1) + \Pi_L^2 \\
&= (1-\lambda)\left\{\pi_c + \frac{1-\sin\alpha}{2}xP_r\right\} - \mu(\lambda \mathrm{g} - \mathrm{g}_1) + \Pi_L^2 \\
&= (1-\lambda)\pi_c - \mu(\lambda \mathrm{g} - \mathrm{g}_1) + (1-\lambda)\frac{1-\sin\alpha}{2}xP_r + \Pi_L^2 \\
&= r_0\frac{r_1 - 1}{r_1}(1+i) + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} + (1-\lambda)\frac{1-\sin\alpha}{2}xP_r + \Pi_L^2
\end{aligned}$$

When the the over-placement liner companies can repay the financial lease rental, the follow equation must hold:

$$\Pi_L = r_0 \frac{r_1 - 1}{r_1}(1+i) + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} + (1-\lambda)\frac{1-\sin\alpha}{2} x P_r + \Pi_L^2$$
$$= (r_1 - 1)(1+i)$$

Therefore,

$$\frac{r_1 - r_0}{r_1}(r_1 - 1)(1+i) = (1-\lambda)\frac{1-\sin\alpha}{2} x P_r + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} + \Pi_L^2$$
$$= (1-\lambda)(x - Q_0 - q_0)P_r + I_L(Q_0 - \mathrm{q}_0)\mathrm{g} + \Pi_L^2$$

Apart from the above result, the over-placement liner companies could accumulate more credit ratings at each stage, and more limits to loss at stage 2 than over-conservative liner companies when $\Lambda(m) \geq 0$. Additionally, when $0 > m(Q,\lambda) > m_0(Q_0,\lambda)$ and both liner companies could not repay the rental, $r_1 + r(m) > r_1 + r(m_0)$.

When $\Lambda(m) < 0$ and $m(Q,\lambda) < m_0(Q_0,\lambda) < 0, r_1 + r(m) < r_1 + r(m_0)$. The profit that both liner companies need to earn are:

$$\Pi_L'(Q,\lambda) = -\frac{m(Q,\lambda)}{\delta} + \Delta' + \frac{\epsilon}{\delta}$$
$$\pi_L'(Q_0,\lambda) = -\frac{m_0(Q_0,\lambda)}{\delta} + \Delta_0' + \frac{\epsilon}{\delta}$$

Then,

$$m'(Q,\lambda) = \Pi_L'(Q,\lambda) - \Delta' = -\frac{m(Q,\lambda)}{\delta} + \frac{\epsilon}{\delta}$$
$$m_0'(Q_0,\lambda) = \pi_L'(Q_0,\lambda) - \Delta_0' = -\frac{m_0(Q_0,\lambda)}{\delta} + \frac{\epsilon}{\delta}$$

Therefore,

$$m'(Q,\lambda) - m_0'(Q_0,\lambda) = -\frac{m(Q,\lambda)}{\delta} + \frac{m_0(Q_0,\lambda)}{\delta} = -\frac{\Lambda(m)}{\delta} > 0$$

Proof of Proposition 6. When facing the deterministic market demand, there are four cases for the over-placement liner companies and over-conservative liner companies. The profit function of over-placement liner companies will be:

$$\Pi_L(d,Q,\lambda)=\begin{cases}(1-\lambda)\{P_r d-cx\}-\mu(\lambda g-g_1)-(Q+q_0-x)c, & d<Q_0+q_0\\ (1-\lambda)\{P_r d-cx\}-\mu(\lambda g-g_1)-(Q+q_0-x)c, & Q_0+q_0<d<x\\ (1-\lambda)\{P_r x-cx-g(d-x)\}-\mu(\lambda g-g_1)+P_r(d-x)-(Q+q_0-x)c, & x<d<Q+q_0\\ (1-\lambda)\{P_r x-cx-g(d-x)\}-\mu(\lambda g-g_1)+P_r(Q+q_0-x)-(Q+q_0-x)c-(d-Q-q_0)g, & Q+q_0<d\end{cases}$$

The profit function of over-conservative liner companies will be:

$$\pi_L(d,Q_0,\lambda)=\begin{cases}(1-\lambda)\{P_r d-cx-(x-Q_0-q_0)P_r\}-\mu(\lambda g-g_1), & d<Q_0+q_0\\ (1-\lambda)\{P_r d-cx-(x-Q_0-q_0)P_r\}-\mu(\lambda g-g_1)-(d-Q_0-q_0)g, & Q_0+q_0<d<x\\ (1-\lambda)\{P_r x-cx-g(d-x)-(x-Q_0-q_0)P_r\}-\mu(\lambda g-g_1)-(d-Q_0-q_0)g, & x<d<Q+q_0\\ (1-\lambda)\{P_r x-cx-g(d-x)-(x-Q_0-q_0)P_r\}-\mu(\lambda g-g_1)-(d-Q_0-q_0)g, & Q+q_0<d\end{cases}$$

By taking the first derivative of over-placement liner companies' profit function and over-conservative liner companies' profit function with respect to the market demand, we can obtain the following formula:

$$\frac{\partial\Pi_L(d,Q,\lambda)}{\partial d}==\begin{cases}(1-\lambda)P_r, & d<Q_0+q_0\\ (1-\lambda)P_r, & Q_0+q_0<d<x\\ P_r-(1-\lambda)g, & x<d<Q+q_0\\ -(1-\lambda)g-g, & Q+q_0<d\end{cases}$$

$$\frac{\partial\pi_L(d,Q_0,\lambda)}{\partial d}=\begin{cases}(1-\lambda)P_r, & d<Q_0+q_0\\ (1-\lambda)P_r-g, & Q_0+q_0<d<x\\ -(1-\lambda)g-g, & x<d<Q+q_0\\ -(1-\lambda)g-g, & Q+q_0<d\end{cases}$$

Because $(1-\lambda)P_r - \mathrm{g} > 0$ and $-(1-\lambda)\mathrm{g} - \mathrm{g} < 0$, the profit of over-placement liner companies will reach maximum value when the market demand is $d = Q + q_0$ and the profit of over-conservative liner companies will reach maximum value when the market demand is $d = x < Q + q_0$.

The initial minimum value of the profit function for both liner companies happens at $d = 0$, where

$$\Pi_L(d, Q, \lambda) = -(1-\lambda)cx - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - (Q + q_0 - x)c$$
$$\pi_L(d, Q_0, \lambda) = -(1-\lambda)cx - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - (x - Q_0 - q_0)P_r(1-\lambda)$$

If $\Pi_L(d, Q, \lambda) = \pi_L(d, Q_0, \lambda)$, then

$$\frac{c}{P_r(1-\lambda)} = \frac{x - Q_0 - q_0}{Q + q_0 - x}$$

In this case, the profit of the over-conservative liner companies approaches the profit of over-placement liner companies. However, the rental that over-placement liner companies need to pay is larger than that of the over-conservative liner companies. Therefore, the larger rental causes the over-placement liner companies to be inferior to the over-conservative liner companies in terms of solvency. It should be noted that the number of orders x for which the NVOCC collaborates with the over-conservative liner companies is close to but slightly lower than the number of orders chosen when collaborating with the over-placement liner companies. Therefore, when the following formula is satisfied, the solvency of the over-placement liner companies is better than the solvency of the over-conservative liner companies.

$$\Pi_L(d, Q, \lambda) - \pi_L(d, Q_0, \lambda) = (x - Q_0 - q_0)P_r(1-\lambda) - (Q + q_0 - x)c$$
$$\geq \Delta - \Delta_0 = \frac{r_1 - r_0}{r_1}(r_1 - 1)(1 + i)$$

When facing the same market demand $d < Q_0 + q_0 < x < Q + q_0$, the profit of over-placement liner companies and the profit of over-conservative liner companies share the same growth trend. When facing the market demand $Q_0 + q_0 < d < x$, for each unit of order demand, over-placement liner companies earn g more than over-conservative liner companies. In this case, the advantage of over-conservative liner companies due to lower financial lease rental is slowly diminishing. When the following formula is satisfied, the surplus profit of over-placement liner companies approaches the surplus profit of over-conservative liner companies.

$$\Pi_L(d, Q, \lambda) - \pi_L(d, Q_0, \lambda) = (d - Q_0 - q_0)\mathrm{g} + (1-\lambda)(x - Q_0 - q_0)P_r - (Q + q_0 - x)c$$
$$\geq \Delta - \Delta_0 = \frac{r_1 - r_0}{r_1}(r_1 - 1)(1 + i)$$

When market demand is greater than x, over-conservative liner companies' profit has started a long-term downward trend and over-placement liner companies' profit will continue to have an upward trend until the market demand is greater than $Q+q_0$. The over-conservative liner companies can fully outperform the over-placement liner companies when the difference in their profit cannot cover the difference in their rent when demand is $Q+q_0$. Therefore,

$$(\lambda P_r - c)(Q+q_0-x)+(Q-Q_0)\mathrm{g} < \Delta-\Delta_0 = \frac{r_1-r_0}{r_1}(r_1-1)(1+i)$$

Otherwise, with the further reduction of their profit gap, over-placement liner companies may perform better than over-conservative liner companies within a certain market demand range. Since both liner companies have the same downward trend when $Q+q_0<d$, the profit of over-placement liner companies will fall to a level where demand is 0 under the following condition:

$$(1-\lambda)\{P_r x - cx - \mathrm{g}(d-x)\} - \mu(\lambda\mathrm{g}-\mathrm{g}_1) + P_r(Q+q_0-x) - (Q+q_0-x)c - (d-Q-q_0)\mathrm{g} = -(1-\lambda)cx - \mu(\lambda\mathrm{g}-\mathrm{g}_1) - (Q+q_0-x)c$$

Therefore,

$$d_{OP} = \frac{(P_r+\mathrm{g})(Q+q_0-\lambda x)+\mathrm{g}x}{(2-\lambda)\mathrm{g}}$$

The profits of over-conservative liner companies will fall to a level where demand is 0 under the following condition:

$$(1-\lambda)\{P_r x - cx - \mathrm{g}(d-x) - (x-Q_0-q_0)P_r\} - \mu(\lambda\mathrm{g}-\mathrm{g}_1) - (d-Q_0-q_0)\mathrm{g} = -(1-\lambda)cx - \mu(\lambda\mathrm{g}-\mathrm{g}_1) - (x-Q_0-q_0)P_r(1-\lambda)$$

Therefore,

$$d_{OC} = \frac{(P_r+\mathrm{g})(1-\lambda)x+(Q_0+q_0)\mathrm{g}}{(2-\lambda)\mathrm{g}}$$

Since,

$$(1-\lambda)P_r x = P_r x - \lambda P_r x < (Q+q_0)P_r - \lambda P_r x = (Q+q_0-\lambda x)P_r$$

Therefore, $d_{OP} > d_{OC}$. When the market demand is particularly large, the loss of goodwill will cause the liner companies to generate negative profits. Through the above calculation, we found the condition for both companies reduced to their initial minimum levels. It doesn't make much sense to consider a situation where market demand grows indefinitely. After all, the profit of over-conservative liner companies starts to decrease first. Therefore, the maximum overdraft is computed based on the profits of the two liner companies when $d=0$. Additionally, by comparison, it is obvious that over-placement liner companies could bear a wider range of market demand than over-conservative liner companies.

The maximum surplus profits of over-placement liner companies and over-conservative liner companies are:

$$m^{+}(Q,\lambda)=(P_r+\lambda\mathrm{g}-\mathrm{g}-c)(Q+q_0)-(P_r\lambda-\mathrm{g}+\mathrm{g}\lambda-c\lambda)x-\mu(\lambda\mathrm{g}-\mathrm{g}_1)-\Delta$$
$$=(P_r-\lambda P_r+\mathrm{g})(Q+q_0)-(c-c\lambda+\mathrm{g})x-\mu(\lambda\mathrm{g}-\mathrm{g}_1)-\Delta$$
$$+(\lambda P_r+\lambda\mathrm{g}-2\mathrm{g}-c)(Q+q_0-x)$$

$$m^{+}(Q_0,\lambda)=(P_r-\lambda P_r+\mathrm{g})(Q_0+q_0)-(c-c\lambda+\mathrm{g})x-\mu(\lambda\mathrm{g}-\mathrm{g}_1)-\Delta_0$$

In addition, the maximum overdrafts of the over-placement liner companies and the over-conservative liner companies are:

$$m^{-}(Q,\lambda)=cx\lambda-\mu(\lambda\mathrm{g}-\mathrm{g}_1)-(Q+q_0)c-\Delta$$

$$m^{-}(Q_0,\lambda)=(Q_0+q_0)P_r(1-\lambda)-(1-\lambda)(c+P_r)cx-\mu(\lambda\mathrm{g}-\mathrm{g}_1)-\Delta_0$$

To ensure that both liner companies can repay the payment, the following conditions must be hold:

$$m^{+}(Q,\lambda)\geq 0 \text{ and } m^{+}(Q_0,\lambda)\geq 0$$

Therefore,

$$Q=\frac{r_1}{P_r}\geq\frac{(P_r+\mathrm{g}\lambda-\mathrm{g}-c)(x-q_0)+(P_r-c)(\lambda-1)x+\mu(\lambda\mathrm{g}-\mathrm{g}_1)-(1+i)}{\lambda\mathrm{g}-\mathrm{g}-c-P_r i}$$
$$=(x-q_0)+\frac{(1+i)(P_r x-P_r q_0-1)+(P_r x-cx)(\lambda-1)+\mu(\lambda\mathrm{g}-\mathrm{g}_1)}{\lambda\mathrm{g}-\mathrm{g}-c-P_r i}$$

$$Q_0=\frac{r_0}{P_r}\geq\frac{\mathrm{g}(x-q_0)+(P_r q_0-cx)(\lambda-1)+\mu(\lambda\mathrm{g}-\mathrm{g}_1)}{i+1+(\mathrm{g}-\lambda P_r-P_r i)\mathrm{Q}}\mathrm{Q}$$

When the surplus profits of the over-placement liner companies in stage 1 is in the A range, their largest financial lease size will increase to:

$$Q'=Q+\frac{\delta k_2 m}{P_r}$$
$$=Q+\frac{\delta k_2}{P_r}\{(P_r+\lambda\mathrm{g}-\mathrm{g}-c)(Q+q_0)-(P_r\lambda-\mathrm{g}+\mathrm{g}\lambda-c\lambda)x-\mu(\lambda\mathrm{g}-\mathrm{g}_1)$$
$$-\Delta\}$$

Here, $m^{+}\leq\boldsymbol{M}^{+}$.

Then, range of the maximum loss overdraft of over-placement liner companies is:

$$m^{-\,\prime}(Q',\lambda)=cx\lambda-\mu(\lambda\mathrm{g}-\mathrm{g}_1)-\left(Q+q_0+\frac{\delta k_2 m^{+}}{P_r}\right)c-\Delta'$$
$$\geq m^{-}(Q,\lambda)-\frac{k_2 c}{P_r}\delta\boldsymbol{M}^{+}>m^{-}(Q,\lambda)+\frac{k_2 c}{P_r}M^{-}$$

To keep the credit rating, the over-placement liner companies should control $m^{-}(Q,\lambda)$

in stage 2 to avoid exceeding the maximum loss amount that the company can bear. Then,

$$m^{-\,\prime}(Q',\lambda)=cx\lambda-\mu(\lambda \mathrm{g}-\mathrm{g}_1)-\left(\frac{\delta k_2 m^+}{P_r}+Q+q_0\right)c-\Delta'\geq -\delta m^+ +\epsilon$$

Therefore, the over-placement liner companies need to control the surplus profits in stage 1 must meet the following condition:

$$m^+\geq \frac{\mu(\lambda \mathrm{g}-\mathrm{g}_1)-cx\lambda+(Q+q_0)c+\Delta'+\epsilon}{\delta\left(1-\frac{k_2 c}{P_r}\right)}$$

When the surplus profits of the over-placement liner companies in stage 1 is in the AA range, their largest financial lease size will increase to:

$$Q'=Q+\frac{\delta k_1 m^+ +(k_2-k_1)\delta M^+}{P_r}$$

$$=\frac{\delta k_1}{P_r}\{(P_r+\lambda \mathrm{g}-\mathrm{g}-c)(Q+q_0)-(P_r\lambda-\mathrm{g}+\mathrm{g}\lambda-c\lambda)x-\mu(\lambda \mathrm{g}-\mathrm{g}_1)-\Delta\}$$

$$+\frac{(k_2-k_1)\delta M^+}{P_r}+Q$$

Here, $m^+\geq \boldsymbol{M}^+$.

Then, range of the maximum loss overdraft of over-placement liner companies is:

$$m^{-\,\prime}(Q',\lambda)=cx\lambda-\mu(\lambda \mathrm{g}-\mathrm{g}_1)-\left[Q+q_0+\frac{\delta k_1}{P_r}m^+ +\frac{(k_2-k_1)\delta M^+}{P_r}\right]c-\Delta'$$

$$\leq m^-(Q,\lambda)-\frac{k_2 c}{P_r}\delta\boldsymbol{M}^+ < m^-(Q,\lambda)+\frac{k_2 c}{P_r}M^-$$

To keep the credit rating, the over-placement liner companies should control $m^-(Q,\lambda)$ in stage 2 so as not to exceed the maximum loss amount that the company can bear. Then,

$$m^{-\,\prime}(Q',\lambda)=cx\lambda-\mu(\lambda \mathrm{g}-\mathrm{g}_1)-\left[Q+q_0+\frac{\delta k_1}{P_r}m^+ +\frac{(k_2-k_1)\delta M^+}{P_r}\right]c-\Delta'$$

$$\geq -\delta m^+ +\epsilon$$

Therefore, the over-placement liner companies need to control the surplus profits in stage 1 must meet the following condition:

$$m^+\geq \frac{\mu(\lambda \mathrm{g}-\mathrm{g}_1)-cx\lambda+(Q+q_0)c+\Delta'+\epsilon+\frac{(k_2-k_1)c\delta M^+}{P_r}}{\delta\left(1-\frac{k_2 c}{P_r}\right)}$$

Based on **Proposition 3**, the over-placement liner companies and the over-conservative liner companies should try to avoid making profits at the AA rating while suffering losses at the BB rating.

When the maximum overdraft of the over-placement liner companies in stage 2 is at the BB range, their largest financial lease size will decrease to:

$$Q' = Q - \frac{k_2 m}{P_r}$$

Here, $m^- \geq \boldsymbol{M}^-$. To restore the original rating, the maximum profit that the over-placement liner companies can earn is:

$$m^{+\prime}(Q', \lambda)$$
$$= (P_r + \lambda g - g - c)\left(Q - \frac{k_2 m}{P_r} + q_0\right) - (P_r\lambda - g + g\lambda - c\lambda)x - \mu(\lambda g - g_1) - \Delta'$$
$$\leq m^+(Q, \lambda) - (P_r + \lambda g - g - c)\frac{k_2}{P_r}\boldsymbol{M}^- < m^+(Q, \lambda) + (P_r + \lambda g - g - c)\frac{k_2}{P_r}\boldsymbol{M}^+$$

To keep the credit rating, the over-placement liner companies should control $m^{+\prime}(Q', \lambda)$ in stage 2 to exceed the minimum profit that the over-placement liner companies need to earn. Then,

$$m^{+\prime}(Q', \lambda) = (P_r + \lambda g - g - c)\left(Q + q_0 - \frac{k_2 m^-}{P_r}\right) - (P_r\lambda - g + g\lambda - c\lambda)x$$
$$- \mu(\lambda g - g_1) - \Delta' \geq \frac{\epsilon - m^-}{\delta}$$

Therefore, the over-placement liner companies need to control the surplus profits in stage 1 must meet the following condition:

$$m^-$$
$$\geq \frac{\epsilon + \delta(P_r\lambda - g + g\lambda - c\lambda)x + \delta\mu(\lambda g - g_1) + \delta\Delta' - \delta(Q + q_0)(P_r + \lambda g - g - c)}{1 - (P_r + \lambda g - g - c)\frac{\delta k_2}{P_r}}$$
$$= \frac{\epsilon + \delta x(\lambda A + B) + \delta\mu(\lambda g - g_1) + \delta\Delta' - \delta(Q + q_0)(A + B)}{1 - (A + B)\frac{\delta k_2}{P_r}}$$

Here, $A = P_r - c \;\; B = \lambda g - g$. The over-placement liner companies should try to avoid making profits at the AA rating while suffering losses at the BB rating. Under the condition that the repayment can be guaranteed, the over-placement liner companies should increase λ so that the maximum surplus profits do not exceed the A range.

When the maximum overdraft of the over-placement liner companies in stage 2 is at the B range, their largest financial lease size will decrease to:

$$Q' = Q - \frac{k_1 m + (k_2 - k_1)M^-}{P_r}$$

Here, $m^- \leq M^-$. To restore the original rating, the maximum profit that the over-placement liner companies can earn is:

$$m^+{}'(Q',\lambda) = (P_r + \lambda \mathrm{g} - \mathrm{g} - c)\left(q_0 - \frac{k_1 m + (k_2 - k_1)M^-}{P_r}\right)$$
$$- (P_r\lambda - \mathrm{g} + \mathrm{g}\lambda - c\lambda)x - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - \Delta'$$
$$\geq m^+(Q,\lambda) - (P_r + \lambda \mathrm{g} - \mathrm{g} - c)\frac{k_2}{P_r}\boldsymbol{M}^- > m^+(Q,\lambda) + (P_r + \lambda \mathrm{g} - \mathrm{g} - c)\frac{k_2}{P_r}\boldsymbol{M}^+$$

To keep the credit rating, the over-placement liner companies should control $m^+{}'(Q',\lambda)$ in stage 2 to exceed the minimum profit that the over-placement liner companies need to earn. Then,

$$m^+{}'(Q',\lambda) = (P_r + \lambda \mathrm{g} - \mathrm{g} - c)\left(q_0 - \frac{k_1 m^- + (k_2 - k_1)M^-}{P_r}\right)$$
$$- (P_r\lambda - \mathrm{g} + \mathrm{g}\lambda - c\lambda)x - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - \Delta' \geq \frac{\epsilon - m^-}{\delta}$$

Therefore, over-placement liner companies need to control the surplus profits in stage 1 must meet the following condition:

$$m^-$$
$$\geq \frac{\epsilon + \delta(P_r\lambda - \mathrm{g} - c\lambda + \mathrm{g}\lambda)x + \delta\mu(\lambda \mathrm{g} - \mathrm{g}_1) + \delta\Delta' - \delta(P_r - \mathrm{g} - c + \lambda \mathrm{g})\left[Q + q_0 - \frac{(k_2 - k_1)M^-}{P_r}\right]}{1 - \delta(P_r - \mathrm{g} - c + \lambda \mathrm{g})\frac{k_1}{P_r}}$$
$$= \frac{\epsilon + \delta x(\lambda A + B) + \delta\mu(\lambda \mathrm{g} - \mathrm{g}_1) + \delta\Delta' - \delta(A + B)\left[Q + q_0 - \frac{(k_2 - k_1)M^-}{P_r}\right]}{1 - (A + B)\frac{\delta k_1}{P_r}}$$

Here, $A = P_r - c\ \ B = \lambda \mathrm{g} - \mathrm{g}$. Under the condition that the repayment can be guaranteed, the over-placement liner companies should increase λ so that the maximum surplus profits do not exceed the range of A rating.

Since the main purpose of the over-conservative liner companies is to control the outstanding financial lease rental, it is no need to control the company's surplus profit. However, the over-conservative liner companies still need to control the overdraft. When the maximum overdraft of the over-conservative liner companies in stage 2 is at the BB range, their largest financial lease size will decrease to:

$$Q_0' = -\frac{k_2 m^-}{P_r} = -\frac{k_2}{P_r}\{(Q_0 + q_0)P_r(1 - \lambda) - (1 - \lambda)(c + P_r)cx - \mu(\lambda \mathrm{g} - \mathrm{g}_1)$$
$$- \Delta_0\}$$

Here, $m^- \geq \boldsymbol{M}^-$. To restore the original rating, the maximum profit that the over-conservative liner companies could earn is:

$$m^+{}'(Q_0', \lambda) = (P_r - \lambda P_r + \mathrm{g})\left(Q_0 - \frac{k_2 m^-}{P_r} + q_0\right) - (c - c\lambda + \mathrm{g})x - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - \Delta_0'$$

$$\leq m^+(Q_0, \lambda) - (P_r - \lambda P_r + \mathrm{g})\frac{k_2}{P_r}\boldsymbol{M}^-$$

$$< m^+(Q_0, \lambda) + (P_r - \lambda P_r + \mathrm{g})\frac{\delta k_2}{P_r}\boldsymbol{M}^+$$

To keep the credit rating, the over-conservative liner companies should control $m^+{}'(Q_0', \lambda)$ in stage 2 to exceed the minimum profit that the over-conservative liner companies need to earn. Then,

$$m^+{}'(Q_0', \lambda) = (P_r - \lambda P_r + \mathrm{g})\left(-\frac{k_2 m^-}{P_r} + q_0\right) - (c - c\lambda + \mathrm{g})x - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - \Delta_0'$$

$$\geq \frac{\epsilon - m^-}{\delta}$$

Therefore, the over-conservative liner companies need to control the surplus profits in stage 1 must meet the following condition:

$$m^- \geq \frac{\epsilon + (c - c\lambda + \mathrm{g})\delta x + \delta\mu(\lambda \mathrm{g} - \mathrm{g}_1) + \delta\Delta_0' - \delta(Q_0 + q_0)(P_r - \lambda P_r + \mathrm{g})}{1 - (P_r - \lambda P_r + \mathrm{g})\frac{\delta k_2}{P_r}}$$

When the maximum overdraft of the over-conservative liner companies in stage 2 is at the B range, their largest financial lease size will decrease to:

$$Q_0' = Q_0 - \frac{k_1 m + (k_2 - k_1)M^-}{P_r}$$

Here, $m^- \leq M^-$. To restore the original rating, the maximum profit that the over-conservative liner companies can earn is:

$$m^+{}'(Q_0', \lambda) = (P_r - \lambda P_r + \mathrm{g})\left(Q_0 + q_0 - \frac{k_1 m + (k_2 - k_1)M^-}{P_r}\right) - (c - c\lambda + \mathrm{g})x - \mu(\lambda \mathrm{g} - \mathrm{g}_1) - \Delta_0'$$

$$\geq m^+(Q_0, \lambda) - (P_r - \lambda P_r + \mathrm{g})\frac{k_2 \boldsymbol{M}^-}{P_r}$$

$$> m^+(Q_0, \lambda) + (P_r - \lambda P_r + \mathrm{g})\frac{k_2 \boldsymbol{M}^+}{P_r}$$

To maintain the credit rating, the over-conservative liner companies should control $m^+{}'(Q_0', \lambda)$ in stage 2 to exceed the minimum profit that the over-conservative liner companies need to earn. Then,

$$m^{+\prime}(Q_0', \lambda) = (P_r - \lambda P_r + \mathrm{g})\left(Q_0 + q_0 - \frac{k_1 m + (k_2 - k_1)M^-}{P_r}\right) - (c - c\lambda + \mathrm{g})x$$

$$- \mu(\lambda \mathrm{g} - \mathrm{g}_1) - \Delta_0' \geq \frac{\epsilon - m^-}{\delta}$$

Therefore, the over-conservative liner companies need to control the surplus profits in stage 1 must meet the following condition:

$$m^- \geq \frac{\epsilon + \delta(c - c\lambda + \mathrm{g})x + \delta\mu(\lambda \mathrm{g} - \mathrm{g}_1) + \delta\Delta_0' - \delta(P_r - \lambda P_r + \mathrm{g})\left[Q_0 + q_0 - \frac{(k_2 - k_1)M^-}{P_r}\right]}{1 - \delta(P_r - \lambda P_r + \mathrm{g})\frac{k_1}{P_r}}$$

Under the condition that the repayment can be guaranteed, the over-conservative liner companies should increase λ so that the maximum surplus profits do not exceed the rang of A rating.

When $P_r\lambda + \mathrm{g}\lambda - 2\mathrm{g} - c \geq 0$ and both liner companies face a loss in the same range at stage 1, the surplus profit that over-placement liner companies can achieve in stage 2 is higher than that of over-conservative liner companies, and the requirement that over-placement liner companies need to restore their rating is easier to reach than over-conservative liner companies. Therefore, over-placement liner companies show better solvency than over-conservative liner companies. By comparing the above results of over-placement liner companies and over-conservative liner companies, because the surplus profits and overdrafts of the over-conservative liner companies is less good than that of the over-placement liner companies, the over-conservative liner companies need greater control over overdrafts than over-placement liner companies.

Appendix D

This proof section is dedicated to comparing the improvements made by the second essay (Chapter 4) over the first essay (Chapter 3). First, we compare the optimal contract parameter in both essays. To compare the optimal order quantity from NVOVV:

$$
\begin{aligned}
F(x_c^*)_{\text{ in Chapter 4}} - F(x_c^*)_{\text{ in Chapter 3}} &= \frac{P_r - c + \mathrm{g}}{P_r + \mathrm{g}} - \frac{\mathrm{g}}{\mathrm{g} + c} \\
&= \frac{(P_r - c + \mathrm{g})(\mathrm{g} + c) - (P_r + \mathrm{g})\mathrm{g}}{(P_r + \mathrm{g})(\mathrm{g} + c)} \\
&= \frac{P_r\mathrm{g} - c\mathrm{g} + \mathrm{gg} + P_r c - cc + \mathrm{g}c - P_r\mathrm{g} - \mathrm{gg}}{(P_r + \mathrm{g})(\mathrm{g} + c)} \\
&= \frac{(P_r - c)c}{(P_r + \mathrm{g})(\mathrm{g} + c)} > 0
\end{aligned}
$$

Therefore, the optimal order quantity from NVOCC is higher in Chapter 4 than in Chapter 3. Therefore, When the direct sales profit of the liner company is not included in the collaboration, NVOCC will reserve more capacity. Since liner company and NVOCC are only responsible for their respective operating costs and loss of goodwill in Chapter 3, this can be analogous to $c_1 = c$, $c_2 = 0$ $\mathrm{g}_1 = \mathrm{g}$ and $\mathrm{g}_2 = 0$ in Chapter 4. Therefore, the optimal wholesale price in Chapter 4 can be rewritten into the following:

$$
P_s(\theta) = \begin{cases} (1 - \theta_B^*)P_r + \dfrac{\theta_B^* P_r c - P_r c}{P_r + \mathrm{g}}, & \text{buyback contract} \\ \dfrac{\theta_R^* P_r c - P_r c}{P_r + \mathrm{g}}, & \text{revenue sharing contract} \\ \dfrac{P_r c - P_r c}{P_r + \mathrm{g}}, & \text{quantity discount contract} \end{cases}
$$

Here,

$$
\begin{aligned}
P_s(\theta) &= (1 - \theta_B^*)P_r + \frac{\theta_B^* P_r c - P_r c}{P_r + \mathrm{g}} = \frac{P_r(P_r + \mathrm{g}) + \theta_B^* P_r c - \theta_B^* P_r(P_r + \mathrm{g}) - P_r c}{P_r + \mathrm{g}} \\
&= \frac{P_r(P_r + \mathrm{g} + \theta_B^* c - \theta_B^* P_r - \theta_B^* \mathrm{g} - c)}{P_r + \mathrm{g}} = \frac{P_r(1 - \theta_B^*)(P_r + \mathrm{g} - c)}{P_r + \mathrm{g}} > 0
\end{aligned}
$$

Then, it is obvious that the optimal wholesale price is only positive under a buyback contract. To better compare the profitability and solvency of the liner company under both contract strategies, we assume that the scale of financial leasing under the two contract strategies is the same and that the liner companies adopt the strategy of splitting the centralised profit equally with the NVOCC. It should be noted that the operating cost of the liner company from direct sales in Chapter 3 depends on the sell volume, while in Chapter 3, it depends on the holding shipping capacity. Therefore, to ensure the convergence of the two contract strategies, this part of the cost will be added to the additional expenditure of the liner company in Chapter 3. Therefore, the profit of the liner company before repayment in Chapter 3 will be as follows:

$$\pi_L(x, P_s, \theta, T) = \frac{\pi_c}{2} - [Q + q_0 - S_L(x)]c$$
$$= P_r \frac{S_N(x) + S_L(x)}{2} - \frac{S_L(x) + x}{2} c - \frac{I_L(x) + I_N(x)}{2} \mathrm{g}$$
$$- [Q + q_0 - S_L(x)]c$$
$$= P_r \frac{S_N(x) + S_L(x)}{2} - \frac{2(Q + q_0) + x - S_L(x)}{2} c - [Q + q_0 - S_L(x)]c$$
$$- \frac{I_L(x) + I_N(x)}{2} \mathrm{g}$$

And the profit of the liner company before repayment in Chapter 4 will be:

$$\pi_L(x, P_s, \theta, T) = \frac{\pi_c}{2} + \xi_L^2 = P_r \left[\frac{S_N(x)}{2} + S_L(x)\right] - (Q + q_0)c - \left[\frac{I_N(x)}{2} + I_L(x)\right] \mathrm{g}$$

Therefore, the contract strategy in Chapter 3 could be used by liner companies to involve NVOCC in sharing the operational risk of direct sales. However, the contract strategy in Chapter 4 could improve the profitability of the liner company. Since the order quantity of NVOCC is higher under the contract strategy in Chapter 4 and the financial lease rental of the two contract strategies is the same, the solvency of the liner company under the contract strategy in Chapter 4 will be better than the contract strategy in Chapter 3.