Through Weber to Cage:
the Theory of Rationality and the *Solo for Piano*

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Abstract

The theory of rationality is a dynamic concept that evolves in relation to music. In the context of indeterminate music, where compositions are no longer grounded in harmonic structures or emotional expression, rationality emerges in the composer's intentions and the relationships between notations and their constituent elements. Conversely, irrationality is reflected in the use of unconventional materials, multiple interpretative choices within notations, and the unpredictable nature of individual realisations. These developments suggest that Cagean indeterminacy offers a valuable framework for reinterpreting Weber's ideas. By focusing on Cage's expansive *Solo for Piano*, I wish to explore both sets of ideas and interpret them through the use of a computer visualisation programme, Gephi.

Three key features are examined in relation to the theory of rationality: (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive, all of which are recognisable in the *Solo for Piano*. Cage transformed random imperfections in paper into semi-determinate compositional materials by assigning function to these materials and granting performers interpretive freedoms and varied approaches to the notations. This composition thus creates a score that invites performers to actively 'interact' with the notations, resulting in significantly different realisations.

Through a comprehensive analysis of the notations, this research employs computational methods using Gephi to uncover individual systems and the internal structure of the work, emphasising that the analyses are inherently shaped by the analyst's understanding and the chosen metalanguage. This research investigates the complexities between rationality and irrationality in both European art music and indeterminate music and proposes a computational method for studying graphic notations, offering new insights into their interpretation and analysis.

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Chapter 1. Introduction

What is the uniqueness of European art music? And what can be done when we recognise it? Max Weber (1864–1920), a German sociologist, was investigated in these questions and recorded his findings in an unfinished book, *Die rationalen und soziologischen Grundlagen der Musik* (1921, *The Rational and Social Foundations of Music*, 1958). This unfinished book is Weber's major contribution in the field of music, as he is the first to have identified rationalisation as a crucial concept for researching European art music,[1] and ground a foundation of modern musicologists' philosophies of music.[2] Specifically, he explained the specialities of European art music through the relationships between music and societal factors, such as climate conditions, the rise of bourgeoisie, and market demands, and he emphasised the rational actions and rational features involving in musical elements and the development of music.

However, his idea of ultimate rationalisation would restrict musical expression is questionable. Valerie Ann Malhotra points out that this prophecy is unsubstantiated because composers have turned their expression from emotion to philosophical and technological decisions.[3] Her explanation points out a prospective direction to develop Weber's ideas of rationality. As James Wierzbicki asks, what would Weber respond to music featuring mathematically structured or ultra-rationalised organisation if he lived long enough to witness the development of avant-garde music?[4] Following their opinions, I aim to investigate applicability of the theory of rationality; does the theory of rationality remain the same since 1921, or might it have transformed since then? This thesis aims to apply it to John Cage's (1912–1992) indeterminate music work, the *Solo for Piano* (1958), to discover three rational features – (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive – and present the transformation of the theory of rationality.

[1] Adorno, *Sound Figures*; Konoval, 'Max Weber and the Sociology of Music', 468.
[2] Feher, 'Weber and the Rationalization of Music', 147.
[3] Malhotra, 'Weber's Concept of Rationalization and the Electronic Revolution in Western Classical Music', 114–16.
[4] Wierzbicki, 'Max Weber and Musicology', 282.

1 Research Design

1.1 The Scope of the Study

Reviewing previous research on Weber's rationality, I notice that many of them concentrate on meanings and analyses of the ideas of rationality,[5] and some focus on the root of Weberian rationality.[6] Based on these studies, it is noticeable that the ideas of rationality is open to interpretations. As Roth concludes, the scholarly value of Weber's text lies in its openness,[7] which is developed through Weber's examination of the interactions between individuals and religion and his analysis of how religion shaped individuals' value systems, how they responded through their actions, and how societies evolved as a result.

To put it another way, he recognised historically and culturally significant phenomena and explored their importance as a unique process of rationalisation in Western societies. When he further developed rationalisation in European art music, he once again emphasised the significance of cultural and historical factors, alongside economic, technical, and climatic factors as prerequisites for the process of rationalisation.[8] Although Weber's observations are criticised for its Eurocentrism,[9] his ideas of applying historically and culturally significant phenomena are not limited to narrative history, but expand to causal explanations of historical

[5] Swidler, 'The Concept of Rationality in the Work of Max Weber'; Mueller, 'The Notion of Rationality in the Work of Max Weber'; Kalberg, 'Max Weber's Types of Rationality: Cornerstones for the Analysis of Rationalization Processes in History'; Barker, 'Kant as a Problem for Weber'; Gronow, 'The Element of Irrationality'; Stauth, 'Nietzsche, Weber, and the Affirmative Sociology of Culture'; Brubaker, *The Limits of Rationality*; Schluchter, 'Weber's Sociology of Rationalism and Typology of Religious Rejections of the World'.

[6] Levine, 'Rationality and Freedom'; Rutgers and Schreurs, 'The Morality of Value- and Purpose-Rationality'; Roth, 'Rationalization in Max Weber's Developmental History'; Derman, 'Max Weber and the Idea of the Occident'. To maintain the focus on the aspect of musicology, this section intends to avoid excessive discussions about these studies. The following scholars who have published influential works that research Weber's concepts in the field of sociology: Jürgen Habermas, Stephen Kalberg, Alan Sica, and Sam Whimster. Regarding Weber's biography, I recommend Joachim Radkau and Harry Zohn. The two scholars have thoroughly and meticulously discussed Weber's lifetime through their unique interpretations. Of course, many more scholars have contributed to Weber's research. Additional research can be found in *Max Weber Studies* (ISSN: 1470-8078), a journal dedicated to applying and disseminating his concepts. It includes topics, such as, the moral dilemmas of modern life, the impact of rationalisation processes, the disenchantment and resurgence of magic, the analysis of power stratification, and the viability of an interpretative science of social reality. With such a broad scope, this journal collects a wide variety of research about Weber's concepts from around the world. Available at: https://maxweberstudies.org (Accessed: 17 July 2023).

[7] Roth, 'Rationalization in Max Weber's Developmental History', 90.

[8] Käsler, *Max Weber*; Segady, 'Consequences of the Increasing Rationality of Music', 459; Roth, 'Rationalization in Max Weber's Developmental History', 93.

[9] Derman, 'Max Weber and the Idea of the Occident', 521.

and cultural developments and results.[10] The causality supplies a sense of openness allowing readers to interpret Weber's ideas freely. Building upon openness of his ideas, this thesis endeavours to extend the theory of rationality, encompassing rationality, irrationality, rationalisation, and irrationalisation, to indeterminate music.

This thesis examines Cage's *Solo for Piano* to observe Weberian rationality within it. The examination encompasses aspects of Cage's intentions, his conceptual framework, and the composing techniques and compositional elements he employed to compose this musical work. Focusing on this work, inspirations from Zen Buddhism are essential. Zen emerged as a crucial influence in Cage's life, inspired him to incorporate chance-derived materials into his compositions and prompted him to redefine the roles of the composer and performers. Following the ideas of Zen, he applied source from surroundings and unforeseen actions in his composition and emphasised the experience in performances. The philosophical pursuits reflect in his music of chance operation, such as the seminal work *4'33"* (1952), which subverted conventional notions of music and offered a novel definition of what it could be.

Examining *4'33"* through Weber's rationality and irrationality, this musical work presented a new conceptual framework – an uncontrolled silence that embodied irrationality, yet demonstrated free realisations that were influenced by Zen, thereby showing rationality to his audience. This new framework implies the possibility of development of the theory of rationality, and therefore, the research aim of this thesis is to uncover the intricate relationship between rationality and irrationality and to identify transformation of the ideas of Weberian rationality. To do so, I propose three research questions: (i) Does the theory of rationality undergo transformation when a composer applied his own notational system rather than the conventional notational system? (ii) How can we understand the deeper relationship between rationality and irrationality resulting from the composer's actions? (iii) When a musical work is based on free realisations and multiple choices, is this freedom truly liberating, or is it another form of control? To investigate the three questions, I shall start with the meaning of rational features, then apply these features to music to discuss the similarity and differences, and finally present visualising results to illustrate rational features.

[10] Weber, Roth, and Wittich, *Economy and Society*, 20; Mommsen, 'Personal Conduct and Societal Change', 36.

1.2 Research Objectives

To investigate the three research questions, research objectives include: (i) examination of the theory of rationality in Weber's two publications, (ii) comparison between J. S. Bach's Prelude in C major BWV.846 and Cage's *Solo for Piano* regarding Weberian rationality, and (iii) illustration of rational features in the *Solo for Piano*.

The examination of the theory of rationality focuses on rational features, which I plan to analyse from Weber's *Die protestantische Ethik und der Geist des Kapitalismus* (1904–05, *The Protestant Ethic and the Spirit of Capitalism*, 1930) and *The Rational and Social Foundations of Music*. To analyse the central notion of Weber's ideas of rationality, I shall concentrate on the Protestants' responses to the doctrine of predestination, including the mental influences, living styles, and pursuits of calling to clarify Weber's ideas of rational actions and process of rationalisation and obtain the rational features in the domain of sociology. The examination will move on to *The Rational and Social Foundations of Music* to investigate how Weber extended the rational features to music. In summary, I propose that the three rational features are: (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive.

After the examination, three features will be applied to J. S. Bach's Prelude in C major BWV.846 and Cage's *Solo for Piano* to examine the applicability of the theory of rationality in two types of music. The comparison will clarify nuanced differences between participation from the composer and the performers through corresponding types of acts in two musics, and hence clarify the transformations of the theory of rationality. The investigations of the composer and performers' participation will employ David Clarke's revision of Nattiez's tripartite model to elucidate the formation of Cage's indeterminacy. The tripartite model outlines the processes through which a producer constructs a message, and a receiver interprets and understands it. By examining these interconnected processes, the model emphasises that the comprehension of a message is inherently influenced by individual perspectives. David Clarke proposes a revision in which the tripartite model highlights the differences between each performer's realisations;[11] hence, the revised tripartite model places a strong emphasis on the participation of both parties. Consequently, any performance of the *Solo for Piano* results from the bilateral participation of both the composer and the performers.

[11] Clarke, 'Musical Indeterminacy and Its Implications for Music Analysis'.

The final objective will begin with the deconstruction of compositional elements in Cage's composition, including the uses of compositional materials, compositional materials' functions, types of freedom, and performing approaches. This means that I will analyse eighty-four types of graphic notations and the corresponding performing instructions to delve into the relationships between compositional elements and notations. The analytical results will be used as qualitative data, which will be processed using the visualisation software, Gephi. Gephi employs the force-directed layout algorithm, ForceAtlas2, to illustrate the proximity or separation between compositional elements and notations. This method shall establish individual systems and reveal the inner structure to uncover the rational features of the musical work.

In summary, this research will be carried out in attempts to extract rational features from *The Protestant Ethic and the Spirit of Capitalism* and *The Rational and Social Foundations of Music*, compare the theory of rationality in J. S. Bach's Prelude in C major BWV.846 and Cage's *Solo for Piano*, and present rational features in the *Solo for Piano* through visualisation.

1.3 Findings and Contributions

This research extends the theory of rationality by focusing on music itself and suggests its developments. Reviewing the research of rationality in music, they mostly concentrate on the ideas of social foundations. For instance, constructing overarching framework of how the music and society jigsaw fit together;[12] considering potential musical matters as one's engagement of cultural actions;[13] or arguing for the global status of black music through sense of liberation and modern social rationalisation.[14] Meanwhile, as previously mentioned, some research focus on meanings, analyses, and the root of Weber's ideas of rationality.[15] It appears that the research highlighting rationality, irrationality, rationalisation, and irrationalisation in music does not have the attention that it deserves. For this reason, I choose to focus on music and the theory of rationality. The findings highlight the evolution of the concept of rationality and uncover

[12] Prior, 'Bourdieu and Beyond', 352.
[13] Martin, *Sounds and Society*.
[14] Radano, 'MUSIC, RACE, AND THE FIELDS OF PUBLIC CULTURE', 310.
[15] Swidler, 'The Concept of Rationality in the Work of Max Weber'; Mueller, 'The Notion of Rationality in the Work of Max Weber'; Kalberg, 'Max Weber's Types of Rationality: Cornerstones for the Analysis of Rationalization Processes in History'; Barker, 'Kant as a Problem for Weber'; Gronow, 'The Element of Irrationality'; Stauth, 'Nietzsche, Weber, and the Affirmative Sociology of Culture'; Brubaker, *The Limits of Rationality*; Schluchter, 'Weber's Sociology of Rationalism and Typology of Religious Rejections of the World'. Levine, 'Rationality and Freedom'; Rutgers and Schreurs, 'The Morality of Value- and Purpose-Rationality'; Roth, 'Rationalization in Max Weber's Developmental History'; Derman, 'Max Weber and the Idea of the Occident'.

unexpected rational connections between compositional elements and notations. These findings contribute to the theoretical aspect and the research method.

1.3.1 The Theoretical Contributions

For the theoretical contributions, this research extends the current knowledge about Weberian rationality by way of three aspects: Cage's interpretation and practice of Zen Buddhism, his use of the graphic compositional system, and the bilateral participation from himself and the performers.

Firstly, Cage's interpretation and practice of Zen indicate the first stage of rationalisation, referring to his process of receiving, interpreting, and producing. This process is homologous with Protestants' rationalisation; they received the doctrine of predestination, built the value concepts upon this, and decided to pursue their calling to ease the inner loneliness, anxiety, and fear. When Weber applied this idea of rationalisation to European art music, it was about the development of musical instruments, music genres, and the notational system, which were distinctively influenced by market demands, aesthetic preferences, techniques, and other societal factors. Reasonably, as a sociologist, his focus was on the influences upon the societies. However, he did not fully discuss individuals' rationalisation as he did for Protestants' rationalisation. Thus, a discussion about Cage's process of receiving the ideas of Zen, interpreting the ideas with his understanding, and producing a musical work based on the interpretation provides the absent part in Weber's theory of rationality.

Secondly, his graphic compositional system demonstrates his intentions, purposes, and value concepts that were influenced by Zen and implies a methodical construction of the *Solo for Piano*. Through these elements, this thesis will delve into a deeper layer of the relationship between rationality and his indeterminate music. Taken from the theory of rationality, two types of rationality are crucial – value rationality and purposive rationality. As the name implies, value rationality refers to actions based on value concepts, while purposive rationality indicates actions driven for specific purposes. The findings highlight that Cage's value-rational actions are rooted in his acceptance and interpretations of Zen, while his purposive-rational actions are based upon chance operation music, the application of the graphic compositional system, and the emphasis of experiences. This means that the composition of the *Solo for Piano* demonstrates a means-end approach that Cage took it to practise his comprehension of Zen. This approach contributes to the development of the theory of rationality.

In the notational system of European art music, the process of conventionalisation is essential. This system conveys intrinsic relationships between musical symbols, allowing composers to translate their creativity into these symbols, performers to bring the music to life, and listeners to appreciate the performers' interpretations. Simply put, all participants communicate in the same language, enabling performers to focus on musical expression. However, conventionalisation does not fully apply to Cage's *Solo for Piano*. With the graphic compositional system, this work incorporates intentional ambiguity, which leads to diverse interpretations by performers. As a result, the performers' understanding becomes critical to realising the musical work. By exploring the differences between these two types of notational systems, we can uncover the development of the theory of rationality.

Finally, the bilateral participation of Cage and the performers demonstrates the two-stage rationalisation of the *Solo for Piano*. Cage's participation of applying graphic compositional system rationalised chance-derived paper imperfections to semi-determinate compositional materials, while the performers' participation revealed the second stage of rationalisation: turning semi-determinate compositional materials into rational realisations. The contribution of the second stage of rationalisation clarifies the formation of Cagean indeterminacy. Based on this finding, a complex intersubjectivity between Cage and the performers can be discuss as a case study, while performance research can deepen the performers' consideration and reasons of why they realise the notation in such way and discuss the unexpected events on the stage.

In essence, as Weber's rationality demonstrates his observations of causality drawing from the certain societal circumstances, subjective meaning, and value-derived actions,[16] I suggest that rationality can also be found in an indeterminate music work, from its structure to the participation of its composer and performers.

1.3.2 The Contributions in the Computational Method

The contribution of research method lies in the visualisation of methodical features of indeterminate music through Gephi and discovery of unexpected connections by computational method. Gephi's modularity class applies the formula: $Q = \frac{1}{2m} \sum_{i,j} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta\left(c_i, c_j \right)$ to calculate how many communities in the *Solo for Piano*. The computation analysis reveals that this musical work comprises ten distinct communities, each interconnected with one another through shared compositional elements. A detailed discussion of these findings will be

[16] Kalberg, 'Max Weber', 139.

presented in Chapter 6 Section 2.1 Visualisation of the *Solo for Piano*: Clusters and Modularity Class.

Furthermore, research in visualisation tends to focus on visualising information in harmonic music. For instance, 'A Survey on Visualizations for Musical Data' (2020) by Khulusi et al. investigates current techniques of visualisation, such as Arc Diagrams, Isochords, and ImproViz, that had been applied to analyse the musical structure.[17] Other researches focus on information visualisation techniques, for instance, Hiraga et al. visualise expressive musical performance through computer graphics;[18] Chan et al. illustrate macro and micro relationships of music works to explain the semantic structure;[19] Malandrino et al. mapped similar tonoalities to similar colours to illustrate structural features of musical compositions.[20] These innovative techniques undoubtedly provide new visualising methods to show static representations and animations progressing of musical works, as well as provide approachable methods for untrained music enthusiasts to comprehend affinity of tones and consonance through colours. However, their research objectives predominantly focus on specific types of music, leaving a research gap in the visualisation of indeterminate music.

Additionally, I notice that Gephi's main uses have been to visualise social network analyses in management studies, bibliometric studies, and biomedicine,[21] specifically focusing on networks of individuals, groups of people, web resources, archives, transportation, citations, and higher education. Although Gephi has been utilised in musicological studies, the

[17] Khulusi et al., 'A Survey on Visualizations for Musical Data'. Wattenberg uses Arc Diagrams to visualise the repeated musical themes and phrases as translucent arcs in his article 'Arc diagrams: visualizing structure in strings' (2002); Bergstrom, Karahalios, and Hart invented Isochords, and explained in the article 'Isochords: visualizing structure in music' (2007) that Isochords effectively highlights the consonant intervals between notes and chords and conveys information about interval quality, chord quality, and the chord progression; ImproViz is a visualising technique highlighting the signature patterns of a jazz musicians' improvisational style, in order to assist learners a new way to study jazz theory, as Snydal and Hearst introduced in their article 'ImproViz: visual explorations of jazz improvisations' (2005).

[18] Hiraga, Mizaki, and Fujishiro, 'Performance Visualization'.

[19] Chan, Huamin Qu, and Wai-Ho Mak, 'Visualizing the Semantic Structure in Classical Music Works'.

[20] Malandrino et al., 'A Color-Based Visualization Approach to Understand Harmonic Structures of Musical Compositions'.

[21] A library search of relevant research with Gephi as the keyword, with the creation date from 2011 to 2023 shows eighty-five articles, nine conference papers, nine book chapters, and two dissertations. The disciplines of these research extend across networks of social media, biomedicine, student engagement, tourism, bibliometric analysis, business studies (such as marketing, sustainable supplier selection, and national resources distribution), and digital archive analysis.

applications are rather limited to visualising melody and rhythm,[22] the network of the music-print trade,[23] and an analysis of musicians and musical genre. The limited research suggests the musicological applications of Gephi are not fully developed, and therefore, applying Gephi in graphic score analysis can widen the practical use of this tool.

Through Gephi, I uncover the methodical feature of the *Solo for Piano* and suggest the structure of this musical work is open to interpretation, meaning that its structure is dependent on the analysts' points of views. This finding contributes to reassessment of free realisations and reconceptualisation the structure of the indeterminate music work. The free realisations do not go along with the binary idea of either under control or absolute freedom, but to jump out of the box and stress on the performers' experiences, which indicate corresponding rationalisation.

The emphasis on performers' experience leads to my new interpretation of the *Solo for Piano*'s structure. The *Solo for Piano* requires the analysts' perspective on understanding and realising the notations. As each analyst has different ways to interpret the notations and performing instructions, the structure of this musical work will present differently. Meanwhile, each performer experiences unique rationalisation, and therefore, the structure can be different. To put it simply, a realisation presents the intersubjectivity between Cage and the analyst/the performer.

1.4 Outline of the Thesis

In Chapter 1, I shall provide a general introduction of the thesis, the literature review, and the methodology. The Chapter 2 will focus on Weber's rationality in *The Rational and Social Foundations of Music* and rational and irrational features of European art music, taking J. S. Bach's Prelude BWV. 846 as the example. The Chapter 3 will reassess the theory of rationality, and then suggest differentiating affectual function and affectual results. The Chapter 4 will start with Cage's life, Zen's influence, and his interpretation on it. The Chapter 5 focuses on analysing Cage's *Solo for Piano.* The analyses of eighty-four types of graphic notations will be provided to reveal the three rational features. The analyses will be used for visualising the individual systems and the inner structure in the Chapter 6. Specifically, I discuss the

[22] The conference proceeding, 'Bridging People and Sound', in *Melody and Rhythm Through Network Visualization Techniques,* Blot, Saurel, and Rousseaux propose a rhythm representation by a force-directed layout of Gephi.

[23] This is a research course at Duke University, which helps students to visualise important information regarding connections in the music print trade, then visualise the networks of prominent figures in the business. Available at: https://bigdata.duke.edu/projects/visualizing-restoration-englands-music-print-trade-with-gephi/ (Accessed: 19 July 2023).

interrelations between compositional elements and notations, as well as the conjunctions between groups of compositional elements and notations. With these visualising results, the feature of systems and structures can be presented clearly. The final chapter summarises the finding of this thesis, reinforces my points of view in terms of the application of theory of rationality to indeterminate music, and proposes the potential research direction that this research has not explored.

2 Literature Review

2.1 Weber and the Theory of Rationality

As a sociologist, Weber's investigation started with religion, history, and economics, but did not stop at these disciplines; rather, he expanded to music. He kept beliefs, desires, and choices as the foundation of his studies, and then delved into the relationships between societal factors, groups, and individuals. His socio-musicological study, *The Rational and Social Foundations of Music*, includes historical, cultural and comparative discussions, based on music theory and ethnomusicological insights.[24] Weber observed the differences between European, Eastern, and primitive music and suggested that a harmonic and melodic systems are uniqueness of European art music.[25] This is because the harmonic and melodic systems were produced by two approaches to rationalise the asymmetrically arithmetical structures: (i) bypassing or covering irrationality and (ii) taking irrationality as a type of source.[26] Based on these systems, Weber proposed basic rational components of these systems is symmetrically arithmetical structures of intervals and chords. In composition, these musical elements produce a new form of irrationality involving the composer, performers, and listeners' emotions. Through his investigation, an intertwined relationships between rationality, irrationality, rationalisation, and irrationalisation can be discovered.

This publication provides fresh connections among Western Roman Catholic monasticism, feudal structures in the Middle Ages and uses of language with developments of musical notations, musical instruments, and compositions.[27] This type of novel relationships firmed a foundation of sociology of music, and therefore, Weber's contributions are recognised by

[24] Darmon, 'Weber on Music', 21.
[25] Martin, *Sounds and Society*, 58.
[26] Darmon, 'Weber on Music', 24.
[27] Käsler, *Max Weber*, 169.

scholars.[28] Nevertheless, sources he used, writing styles, and disproportionate contents were major doubts that scholars questioned.

First of all, Weber's writing surrounded with the significant scholars in the study, such as Hermann von Helmholtz (1821–1894), Carl Stumpf (1848–1936), and Erich von Hornbostel (1877–1935). Although Weber built his elaborations upon their research, the contents were too hard to follow for readers.[29] As a result, *The Rational and Social Foundations of Music* became an unengaging publication. This issue relates to the next criticism: disproportionate technical contents. The disproportionate technical contents of this driest publication were one of reasons why this study was not as popular as it may have deserved in spheres of musicology and ethnomusicology;[30] without a certain understanding of music, readers hardly understood the contents. Finally, the sources in Weber's research were mixed primary and secondary musicological research and the fieldwork of others, these resources seemed to be reliable, but Weber did not examine the sources.[31]

Despite the criticisms that motioned above, one key fact needs to be borne in mind is that this study was a collection of notes for a future study of rationality in all of the arts.[32] This means that when Weber wrote down his observations of rationalisation in European, Eastern, and primitive music, he was in a preparation for investigating rationality in the field of the arts. He did not aim for publishing it as a book; not as what he intended to do in *The Protestant Ethic and the Spirit of Capitalism*, and therefore, it may be unreasonable to put this type of criteria upon this study.

In terms of the first and the second criticisms, Christoph Braun explained, Pythagorean-based contents are discussions addressing social conditions of evolution of European art music, and Weber did not concern all types of music, but specifically focused on modern European culture.[33] With this focus, it is reasonable that Weber especially centred on the well-known

[28] Turley, 'Max Weber and the Sociology of Music', 637; Fend, 'Witnessing a "Process of Rationalisation"? A Review-Essay of Max Weber's Study on Music', 105; Konoval, 'Max Weber and the Sociology of Music', 465.
[29] Fend, 'Witnessing a "Process of Rationalisation"? A Review-Essay of Max Weber's Study on Music', 103.
[30] Ibid.
[31] Ibid., 113.
[32] Wierzbicki, 'Max Weber and Musicology', 280.
[33] Braun, *Max Webers Musiksoziologie*; as cited in Wierzbicki, 'Max Weber and Musicology', 281.

scholars and the elements building the foundations of modern European culture.[34] Nevertheless, these technical contents certainly create difficulties for its readers. Moving to the resources in this study. Braun clarifies Weber's empirical resources were from phonographic records, undertaken by the Berlin Phonographische Institut under Carl Stumpf and Erich Moritz von Hornbostel, in order to investigate the differences between the European tonal system and non-European system.[35] Considering this clarification along with the first criticism, I suggest that Weber's study was similar to a case study, rather than a full application of developed theory.

Moreover, these criticisms may be caused by a rushed process of publishing, Kroyer focused on making the manuscript legible only, even he was sceptical of some controversial pronouncements.[36] During such a process, Kroyer and Marianne improperly emphasised the 'rational foundation of music'; instead, 'social conditions' seems to be the more appropriate lens through which to view Weber's ideas regarding rationality in European art music.[37] This false emphasis led readers to look for the meaning of rationality and an absolute definition of rationality, yet these were not the main purposes of this study; *The Rational and Social Foundations of Music* was a study extending rationality to the arts, as Weber was attracted by alleged irrationality of this area.[38]

Although these criticisms made their points in some ways, the importance of Weber was not erased from sociology of music. The inspiration of his research towards scholars and studies in sociology of music is undeniable. His ideas consider causal explanations of historical and cultural developments and results, musical life, and musical logics[39] showed scholars the path

[34] In the discussions of the foundations of modern European culture, Heinrich Schenker (1868–1935) was recognised as one of influential and original music theorists of the twentieth century, even though he was not listed in Weber's observations. Examining Schenker's background, it is noticeable that his theoretical analysis was deeply associated with his political views, conservatism, and resistance to modernity, concluded by Leon Botstein. He and Weber shared the German culture arrogance, they both saw rationalisation would end individuality and human spirit. William Drabkin concluded that Schenker's understanding of music surrounds with a concept of hierarchy suggesting the harmonic functions are subordinate to the tonic chord. This idea may point out a new angle to interpret Weber's rational features in harmonic music; I shall provide further explanations in Chapter 2.
[35] Braun, 'The "Science of Reality" of Music History: On the Historical Background to Max Weber's Study of Music', 181, 190.
[36] Mueller, 'Reviewed Work(s): The Rational and Social Foundations of Music by Max Weber, Don Martindale, Johannes Riedel and Gertrude Neuwirth', 149.
[37] Wierzbicki, 'Max Weber and Musicology'.
[38] Käsler, *Max Weber*, 169.
[39] Weber, Roth, and Wittich, *Economy and Society*, 20; Mommsen, 'Personal Conduct and Societal Change', 36; Braun, 'The "Science of Reality" of Music History: On the Historical Background to Max Weber's Study of Music', 185.

to researching the relationships between music and society.[40] More rather, he attracted scholars' curiosities about: What is the uniqueness of European art music? What is the central concept of his research in music? Most importantly, what can be done when someone becomes aware of rationality in music? Reflecting on these questions, the answer surrounds with conventionalisation and standardisation of musical development, including musical instruments and musical notations. Turley suggests that modern musical instruments and modern musical notation are two essential factors stimulating rationalisation in music, as the modern musical instruments and modern musical notation are rational results of an organised society.[41] To be specific, modern musical instruments are produced by standard processes of manufactures, and musical notations show a systematic way of recording and transmitting music. On the other hand, the standardisation of manufacturing musical instruments and annotating musical notations serve as a precondition of rationalisation in music.

Following the idea of conventionalisation, we may trace back to Weber's widely known origin of his approach to rationalisation, *The Protestant Ethic and the Spirit of Capitalism*. In this publication, he discussed his ideas of rationality in the cultural domain of action and engagement. Specifically, Weber showed how the Protestant living style, and the doctrine of predestination were deeply related with one another and explained that the religion motivated the process of rationalisation by divesting its magic and systematically unifying the relationship between God and believers' ethics,[42] and therefore, while the believers' actions were shaped by the religion, they were also value-rational and purposive-rational actions. Following his ideas, the actions do not simply refer to any actions, but especially highlight the ones producing a certain pattern of logics and resonating with societal backgrounds. For instance, for Protestants, the pursuit of a methodical living style did not come out of nowhere but was a solution dealing with irrational feelings (i.e. anxiety, uncertainty, and fear) that corresponding to their religion. This means that Protestants perceived the doctrine and under influences of irrational feelings; in order to cope with this effect, they worked out a solution in terms of obtaining success in their calling. To put it simply, the doctrine of predestination was conventionalised among believers, and they took actions after internalising the doctrine, and therefore their actions are the logic of practice. Nevertheless, I am not suggesting the methodical way of living is the 'destined' result in this case. Before they took actions, nothing was certain. This then brings out

[40] Paddison, *Adorno's Aesthetics of Music*, 135.
[41] Turley, 'Max Weber and the Sociology of Music', 638.
[42] Weber, *The Religion of China: Confucianism and Taoism*, 226; Käsler, *Max Weber*, 108.

other essential elements of Weber's rationalisation: unpredictability and freedom. These elements resonate with the various dimensions of reality via the logic of actions.[43]

Reflecting this to developments of European art music, Weber recognised the causal relationships between developments in European music and societies; for instance, stable markets show musical guilds and performer's freedom, and technical development of the organ demonstrates ecclesiastical use of this musical instrument and chordal harmony.[44] These developments are the result of coalescence between reasons, beliefs, desires, choices, action, unpredictability, and freedom. Between 1910 and 1912, Weber started his investigations about rationality in the arts through these elements. In his investigation, he used European music together with Eastern music, primitive music, and societies as starting points to obtain the finding advising that rationality in music is about the formation of musical elements and interactions between music, societies, and citizens. Of interest here is that out of all his examples, he did not offer the definition or criteria of rationality and irrationality in music, nor did he directly draw any conclusions in his thinking. Weber had positioned himself more as an observer, who examined music through his unique sociological lens. With such a wide scope of research objectives and the distinct backgrounds across religion, history, economics, and music, Weber's research provided a solid basis for the sociology of music.

As Weber found, rationality and irrationality are inherent features in mathematical structures of European art music; the approaches solving irrationality construct rationalisation and produce another form of rationality within in the theorical basis of European art music. Yet, this theoretical basis is not the end of rationality. Rather, it produces another form of irrationality; the theoretical basis came from our intelligence and reckoning, when composers created music based on it, the needs of aesthetics took over, and then constructed an externally asymmetrical form, namely, irrationality.[45] This type of irrationality involves tensions motivating the movement of music. In Weber's ideas, this irrationality is based on unconsciously emotional expression, which carries the composer and the performers' value, and contagious emotions among people. However, his ideas seem to mingle the affectual function with the affectual result.

[43] Gafijczuk, 'Max Weber's Science of Composition', 109.
[44] Braun, 'The "Science of Reality" of Music History: On the Historical Background to Max Weber's Study of Music', 180.
[45] Gafijczuk, *Identity, Aesthetics, and Sound in the Fin de Siècle*, 100.

When discussing emotional expression, one detailed difference between rational actions and emotional expressions needs to be carefully handled. According to Weber, social action can be classified as four types of actions: (i) by purpose, (ii) by value, (iii) by affect, and (iv) by tradition.[46] The first indicates how do people employ means to achieve ends emerging from external expectations; the second explains the action is derived by unconditional and intrinsic value, which can be ethical, aesthetic, or religious; the third refers to the actions seek for instant pleasure or stratification based on current emotions; the final is through ingrained habituation. It is true that when we relate the rational actions to music, systematic discussions regarding value, emotional-expressive, or traditional ends can be developed;[47] however, this does not mean that emotions – e.g. enthusiasm, rapture, and fervour – are rational. Rather, there is an ontological difference between rational action and expressive action;[48] rational actions consider purposes, values, intrinsic rewards, and costs, while expressive actions consider emotion itself. The expressive action of music is a type of tension deriving from inherent irrationality of music's asymmetrical structure, and this means that this expressive action serves the emotions, instead of serving other purposes. For this reason, I suggest differentiating rational actions and expressive actions by dividing affectual function with affectual results of European art music. A reassessment of the theory of rationality will be discussed in Chapter 3.

With the awareness of the nuanced difference between affectual function and affectual result, I am curious about the applicability of the theory of rationality in the music, which does not prioritise emotion-expressive actions. Indeterminate music does not consider the same needs of emotional-derived aesthetics as the European art music, and it is not constructed by harmonic and melodic systems. In this case, where is rationality and irrationality, and what is the developments of them? At the beginning of this section, I address rationality with reasons, consisting of beliefs, desires, and choices, and then suggest causality is the key to view *The Rational and Social Foundations of Music*. Reason results from our beliefs, desires, and choices, and hence, it considers individuality as an essential element. For this reason, reason can never be simplified as rule, procedure, or calculus,[49] so as one's actions. A sound reason is the foundation of one's actions; [50] keeping this in mind, I investigate the beliefs, desires, and

[46] Weber, *Economy and Society*, 101.
[47] Malhotra, 'Weber's Concept of Rationalization and the Electronic Revolution in Western Classical Music', 103.
[48] Goldthorpe, 'Rational Action Theory for Sociology', 188.
[49] Wood, *Kantian Ethics*, 17.
[50] Guyer, *Kant on the Rationality of Morality*, 6.

choices of the composer, along with causality on both a micro and macro scale, in order to research rationality and irrationality in indeterminate music.

2.2 Cage and His Music

The first question of my investigation is which composer should I focus on, and which representative piece should I choose? Tracing back to Weber's initial intention of researching rationality, I had my answer. Weber suggested that rationalisation of prioritising efficiency, profits, and interests through division of labour, calculations, or other means-end approaches is a distinct phenomenon of Western societies. On the contrary, Chinese Buddhism adherents abstain from relying on external approaches and instead strive for emptiness and mindfulness to obtain inner peace, and therefore, they posit that virtue ought to emanate from a commitment to themselves.[51] In essence, two ways of thinking from different culture demonstrate corresponding types of rationalisation; Protestants tended to seek for external solutions, such as, obtain successes in their callings, while Buddhism believers look for internal solutions, such as accepting the fact of life and embracing emptiness.

Despite both Protestantism and Buddhism presenting the process of rationalisation, the different intentions led to the corresponding developments. When the ideas of Protestantism reinforced profits, interests, and means-end approaches, they became the foundation for the emergence of capitalism and resulted in the unique Western rationalisation. On the other hand, the ideas of Buddhism did not emphasise such purposive-rational thinking, so the rationalisation aligned with Western one did not emerge. Yet, when Cage applied Eastern philosophical thinking to music, he also demonstrated a purposive-derived thinking – how should we think of this? Following this question, I chose to focus on John Cage's music as the research object.

The forms of his creations include acoustics, graphic scores, and visual arts. Through these forms, he successfully practised his interpretations of Eastern philosophies. One of the well-known ones is Zen Buddhism, which defies concept-making and looks for pure facts and experience of being.[52] Based on this core idea, Cage interpreted Zen as freedom,[53] unimpededness, and interpenetration,[54] and applied different compositional materials, techniques, and notational systems to produce chance music and indeterminate music. His philosophical decisions and composing intentions can provide in-depth discussions in terms of

[51] Weber, *The Religion of India*, 265–67.
[52] Suzuki, *An Introduction to Zen Buddhism*, 42,44,51.
[53] Silverman, *Begin Again*, 121.
[54] Revill, *The Roaring Silence*, 113.

rationality and irrationality. Following this direction, I shall stress the fact that rationality and irrationality are not two separate ideas, but an idea presenting both rational and irrational aspects. Rationality demonstrates individuals' actions appropriate to the beliefs and ends,[55] while irrationality demonstrates the contradictions between impulsive desires and logics. Thus, I shall focus on clarifying the intertwined relationships between rational and irrational aspects of indeterminate music, rather than assertively classifying what is rationality and what is not rationality.

As previously explained, harmonic and melodic systems are two results of rationalisation; meanwhile, standardisation of musical instruments and conventionalisation of musical notation are the precondition of rationalisation. These types of rationalisation reveal a fact that relationships between notes, intervals, chords, and melodies are interrelated. Yet, this fixed interrelationship is not applicable to indeterminate music. Taking Cage's *Solo for Piano* as example, it was composed by the graphic compositional system. This system was based on Cage's arrangements to construct the relationships between compositional elements. In addition to this, indeterminate music involves possibility, actions of individuals, and the process of making sounds.[56] When music is not only about tonality, this can be difficult to discover systems and structures of a conventional meaning. Nyman emphasises the difference between European art music and music in the avant-garde style is that the former considers emotional expressions and personal interpretations from the same musical score, while the latter focuses on substantial uniqueness, including, but not limited to, different speaking speeds, choices of instruments, selections of performing movements, or any other natural individual differences.[57] Comparing this type of uniqueness with European art music, it seems to be obvious that the two genres demonstrate different features. However, it does not mean that indeterminate music cannot be rational. Nyman's emphasis points out the process of irrationalisation in indeterminate music and shows us a possible way to construct rationality and the process of rationalisation in indeterminate music – by bringing compositional materials and performing instructions into consideration.

One benefit of analysing compositional materials and performing instructions is that the analyses can effectively deal with the difficulties of 'criticising a random act'. As Pritchett identified, the way out of the dilemma of criticising and analysing a randomly made

[55] Benn and Mortimore, 'Can Ends Be Rational? The Methodological Implications', 282.
[56] Nyman, *Experimental Music: Cage and Beyond*, 4.
[57] ibid.

composition is to delve into the ideas behind the music.[58] However, when going in this direction it seems to face an unavoidable argument regarding Cage's identity as a composer or as a philosopher, and the true value of his music. Charles Wuorinen (1938–2020) once criticised Cage, saying that he left some composing questions behind, and these unsolved questions then led to a series of issues of devalued composing, his 'offers' to his audiences, and the meaning of his compositions.[59] Meanwhile, Eric Salzman (1933–2017) pointed out that the lack of identity is the essence of Cage's chance works, and this deficiency seemed to bring simplification, showing no differences between different chance works.[60]

However, these criticisms seem to overlook the importance of innovations of music after World War II and misunderstand Cage's intentions. First of all, in the post-war period, the federal, state, and local patronage of arts councils provided supports to experimentalists, meaning that the experimental movement benefitted from institutional support at the national level, as the government of the United States was economically sufficient for developing the arts, and the government became concerned about the international image in this area.[61] In this circumstance, the experimentalists had their resources and supports to explore new sounds, techniques, and ideas. Their compositions were released from the chains of harmony and turned to emphasise their composing intentions and creativity. These elements relate to the second point of my rebuttals. As Wuorinen argued, the composition is devalued when the composing is all about randomness, and there are no meanings for the audience to receive.[62] However, Hamilton clarifies, Cage's works contain essential incompleteness, which requires the audience to complete in a different sense.[63] This means that the audience needs to actively participate in the completion, rather than passively take the information in music from the composer's one-sided arrangement. To be specific, the completion of this sense of music is not limited to transmissions of sounds but expands to every element that the audience can and cannot notice, such as the performers' talking speed, actions, behaviours, or auxiliary instruments in the realisations. Certainly, this elaboration leads back the debate about Cage's identity as a composer or as a philosopher.

[58] Pritchett, *The Music of John Cage*, 2.
[59] Wuorinen, 'The Outlook for Young Composers', 60.
[60] Salzman, *Twentieth-Century Music*, 163.
[61] Cameron, 'Fighting with Words', 431–32.
[62] Wuorinen, 'The Outlook for Young Composers', 60.
[63] Hamilton, 'The Aesthetics of Imperfection: The Finished Work, and Process versus Product', 16.

Pritchett addressed Cage in two terms: Cage-the-composer and Cage-the-philosopher and explained that Cage's composition and philosophy are connected with one another.[64] This can be a starting point to research Cage's value of his composing. In Cameron's article, titled 'Fighting with Words: American Composers' Commentary on Their Work', she clearly explains the core intensions of Cage and other composers who seem to have the same purposes.[65] For instance, they intend to replace traditional forms with new American music, and get rid of the inherited baggage of the past; they intend to strive for originality, and seek some new idea; they propose that music needs to be expanded conceptually to include a larger universe of sound: street sounds, animal calls, jet airplanes, silence, and electronic sounds; they suggest that the highest purpose in composition is to have no purpose, while non-Western musical cultures are the most important source of sound and ideas, and other good sources are Thoreau, Zen, Classical Greek music, and the visual arts; they believe music should reflect life, and music is more interesting as process; and finally, they propose to liberate sounds from hierarchical forms by stressing timbre and rhythm over melody, and replacing equal temperament with a natural acoustical basis for scales.[66] With these such clear intentions, these composers bring out their own value by their unique composition, instead of building upon tonality or musical theory.

Finally, these clear intentions oppose the issue of simplification which was caused by a lack of identity. Once again, when composing techniques are inspired by a certain life experience and carry specific intentions, the search of identity may not concern the acoustic results, but the whole composing process – what inspired the composer, how did the element inspired the composer, and how did the composer transform the inspiration into compositions. Consequently, I suggest that the identity is not about how music sounds but should be how is music composed. When critics examined his musical works in a conventional fashion and looked for something they were familiar with, how could they get into the core of his musical works? When analysing a musical work which intends to stay away from the genius of European art music, this work should not be confined by pre-established conventions.[67] If the analysis focuses the silent work *4'33''* with a conventional analytical approach, it is certainly hard to develop further discussions, as the musical sheet is literally nothing but staffs with a musical term, Tacet. Moreover, as I previously suggested, Nyman's emphasis presents the substantial difference between European art music and music in the avant-garde style. Through his point of view, a

[64] Pritchett, *The Music of John Cage*.
[65] Cameron, 'Fighting with Words'.
[66] Cameron, 'Fighting with Words', 452–55.
[67] Song, 'Music Analysis and the Avant-Garde Compositions of Post-World War II', 185.

new approach of analysing musical works is presented. Reflecting on this about Cage's *Solo for Piano,* the compositional materials and performing instructions are the crucial elements to delve into Cage's purposes, value, intentions, and approach of forming the musical work, and so construct rationality and the process of rationalisation in indeterminate music.

In terms of the construction of rationality and rationalisation, Philip Thomas's realisations can be an entry for us to understand a new way for the musical analysis. In his article, 'Understanding Indeterminate Music through Performance: Cage's *Solo for Piano*' (2013), he categorises graphic notations in the *Solo for Piano* into three categories: (i) most determined, (ii) partly determined, and (iii) radically indeterminate, and he selected Notations C, E, I, and S for the first category; Notations B, D, M, Q, T, X, Y, Z, and AG for the second category; Notations U, AR, AV, BB, BJ, BT, BV, BW, and BX for the third category. The different levels of indeterminacy lead the performers to consider various aspects of aesthetics in music, including density and texture, continuity and pacing, noise and pitch, but also demonstrate degrees of conformity within notations.[68] The distinction between three categorises is according to Cage's descriptions of performing instructions, which include choices showing large- and small-scale influences on the realisations. Interestingly, such radical indeterminate music work as the *Solo for Piano* still demonstrates a sense of consistency itself.[69] When realising the notations, it is noticeable that one compositional material may be used in various ways representing different functions or performing approaches, or one function can be presented by various types of compositional materials. Through the compositional material, the function, or the performing approach, relationships across the whole musical work can be built. Engaging this sense of consistency with Weber's theory of rationality, I suggest using 'individual systems' and 'the inner structure' to represent the sense of consistency.

What are individual systems and the inner structure, and how should I construct them? In the case of the *Solo for Piano*, Cage's composing technique directs us to investigate a further commonality between notational types[70] that are determined by a composer's rules.[71] This means that how Cage formulated each notation and how he designed the realising approach will guide us to discover the relationships between compositional elements and notations. In the performing instructions, Cage referred one notation in the other notation's performing

[68] Thomas, 'Understanding Indeterminate Music through Performance', 110–11.
[69] Ibid.
[70] Thomas, 'Understanding Indeterminate Music through Performance', 111.
[71] Lochhead, 'Performance Practice in the Indeterminate Works of John Cage', 234; as cited in Miller, 'The Shapes of Indeterminacy', 18.

instruction. For instance, Notations O and Q both mention Notation M[72] and Notation BB is mentioned in Notations BJ and BV.[73] The direct referrals are similar to spines, which demonstrate clear relationships between notations and the corresponding elements. These spines are the individual systems, which can be used to discover the inner structure.

After analysing all notations in the *Solo for Piano*, I notice that some notations share more compositional elements, while some notations share less communal elements. Inputting this finding in Gephi, it is clear to observe the dense connections between notations in the notations sharing more communal elements, while relatively loose connections can be found in notations sharing less communality. Putting it differently, notations with dense connections will be close to each other and construct a cluster. By discovering more clusters, the inner structure can be presented. With individual systems and the inner structure, rationality of this musical work can be revealed. The further discussion will be provided in Chapter 6 Section 2 Application of the *Solo for Piano.*

In summary, features of rationality and irrationality and processes of rationalisation and enchantment are two main elements in Weber's ideas of the theory of rationality. The investigations of features will focus on analyses of *Solo for Piano*, and the examination of rationalisation will concentrate on the process of composing and realising. Researching *Solo for Piano* through the theory of rationality not only carries and extends the research direction as Philip suggested: discovering further commonalities between notational types, but also presents the transformation of the theory of rationality.

[72] Performing instruction of Notation M: 'Begin at left, end at right, changing direction at the intersections if desired. May be expressed as one voice, a 'counter point', or as 3 or 4 voices. Pedals only in areas indicated, not obligatory'; performing instruction of Notation O: 'Audible (as chords, lines, arpeggiations as in D but free, etc.) only between horizontal lines. Pedals as in M'; performing instruction of Notation Q: 'Like M, but in time (any units); going back incurs need for increased speed'.
[73] Performing instruction of Notation BB is 'Notes are single sounds. Lines are duration (D), frequency (F), overtone structure (S), amplitude (A), and occurrence (succession) (O). Proximity to these measured by dropping perpendiculars from notes to lines gives respectively, longest, lowest, simplest, loudest, and earliest'; performing instruction of Notation BJ is: 'A single sound. Boundaries are frequency, duration, amplitude, and overtone structure. Proximity as in BB'; performing instruction of Notation BV is: 'Three large (4 or more sounds), six less large (3 sounds), 10 small (two sounds), 4 very small points (single sounds). The 5 lines and the 4 boundaries to be used as in BB and BJ. When obtaining measurements for 3 frequencies use 3 different lines and likewise for other measurements'.

3 Methodology

3.1 The Key Concepts

After examining Weber's *The Protestant Ethic and the Spirit of Capitalism* and *The Rational and Social Foundations of Music*, I propose the three rational features: (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive.[74] Following these features, I conclude that the first rational feature shows in the individual systems and the inner structure of the *Solo for Piano*, as previously explained. The first rational feature is connected to the second rational feature, which focuses on the function of the compositional materials. Cage applied chance-derived elements in his composition and rationalised them by arranging functions for them. He used a great variety of drawings, performing approaches, and different types of freedoms to build these functions into his arrangements, which means that the meanings of these compositional materials are not as instinctive as in conventional notations – i.e. clef signs and staff showing pitch of notes, and note value differentiates duration of notes accordingly, etc. Based on this difference, the second rational feature in Cage's graphic notations reinforces participation from the performers. The stress leads to the third rational feature, interactivity/being interactive. This feature shows the interactions between the composer and the performers. Regarding this interaction, the revised tripartite model can be applied to provide deeper considerations.

Cage's intentions, value standards, purposes, and graphic notations are his practices of Zen. He carried the essence of Zen throughout the whole production of the *Solo for Piano,* in that he composed it upon a concept called 'unimpededness', which sees each thing and each human in all of space as a centre.[75] Hence, the centre of this work is not limited to the composer, but can be shifted to performers, listeners, and the compositional materials. With so many different centres, Cage's composition allows for fundamentally free realisations from each performer.[76] The shifting centre is one of the major reasons why I apply the revised tripartite model along with the theory of rationality in this research. Umberto Eco (1932–2016) explains, one message can be interpreted in various ways, depending on the receiver's angle, which is formed by their education, value concepts, and backgrounds, meaning that apart from the producer's original

[74] The connection between these rational features and Weber's rationality will be discussed in Chapter 2. The methodology mainly discusses how three rational features will be applied in this thesis.
[75] Revill, *The Roaring Silence*, 113.
[76] The discussion of the 'shifting centre' will be limited to the performer's realisation in order to keep the focus on the theory of rationality in indeterminate music.

denotation, the receiver's interpretation may produce different ones.[77] Based on this idea, Molino designed a schema of communication, in which he identified the message as a material reality, and took the process of producing and receiving into consideration (Figure 1).[78] Thus, the schema reinforces the position of the receiver and the open interpretation of the message.

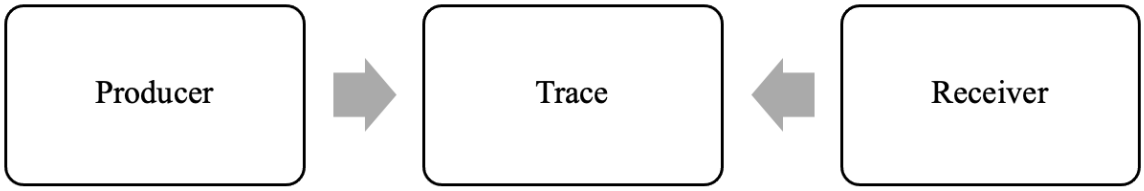

Figure 1 A revised schema of communication by Molino.[79]

Applying the receivers' open interpretations of the message to the doctrine of predestination, the schema reveals a part of rationalisation; the believers were under indoctrination of Protestantism, they accepted the doctrines, built their value concepts upon it and then took actions accordingly. Applying the idea of open interpretations to Cage's shifting centre, it emphasises the performers' understanding, choices, and decisions of the realisations. In essence, the open interpretation of the schema, the acceptance of doctrine from the Protestants, and Cage's shift centre all involve aspects of individuals' comprehension and interpretation. Addressing the aspects of producer and receiver, 'participation' can be a term to include both.

The participation of different subjectivities is the essence of Nattiez's tripartite model, he designed this model based on the symbolic phenomenon: the poietic dimension, the esthesic dimension, and the trace.[80] The poietic level, the esthesic level, and the neutral level are the three levels involved in compositional creation and perspective approach, from which physical scores emerged.[81] He uses these levels to demonstrate subjectivities involving in the creation, and thus, when applied to music, the poietic level indicates the composer's choice during the process of creation, the esthesic level represents the performer's perception of the work, and the neutral level refers to the physical scores of the work. However, this tripartite model does not fully consider the importance of performers' participation. As Philip Thomas points out that performances of indeterminate music are distinct from European art music, as the performances of former cannot be conducted upon melodic, harmonic, and motivic relationships; instead,

[77] Eco, *A Theory of Semiotics*, 114; as cited in Nattiez and Abbate, *Music and Discourse*, 21.
[78] Ibid., 15.
[79] Molino, Underwood, and Ayrey, 'Musical Fact and the Semiology of Music', 105–6.
[80] Nattiez and Abbate, *Music and Discourse*, 11–2.
[81] Ibid., 46.

these performances are based on performers' intentions, selections, and choices.[82] This fact emphasises the transformation of performers' role, from an executor to a decision maker.

This essential difference between European art music and indeterminate music is acknowledged by David Clarke, who advocates that this fact should be remained whilst investigating indeterminacy[83] and proposes a revised tripartite model based on it. The revised tripartite model precisely presents the cooperation of each participation throughout the three levels, as it embodies a successive process that produces a new poietic level by the performers, a new esthesic level by listeners, and a new neutral level by the performer's performance (Figure 2). This means that in Clarke's revision, he considers participation from the composer and performers; while the positions of scores, realisations, and performances are as messages that are comprehended and interpreted by the performers and listeners in this process. Through the revised tripartite model, it is noticeable that participation refers to corresponding types of actions that are either based on the composer's compositional creation, performers' realisations, and listeners' appreciation. Following these actions, I suggest combining the theory of rationality with the revised tripartite model to delve into the two-stage rationalisation at the poietic and esthesic levels and the transformation of compositional materials' status at the neutral level.

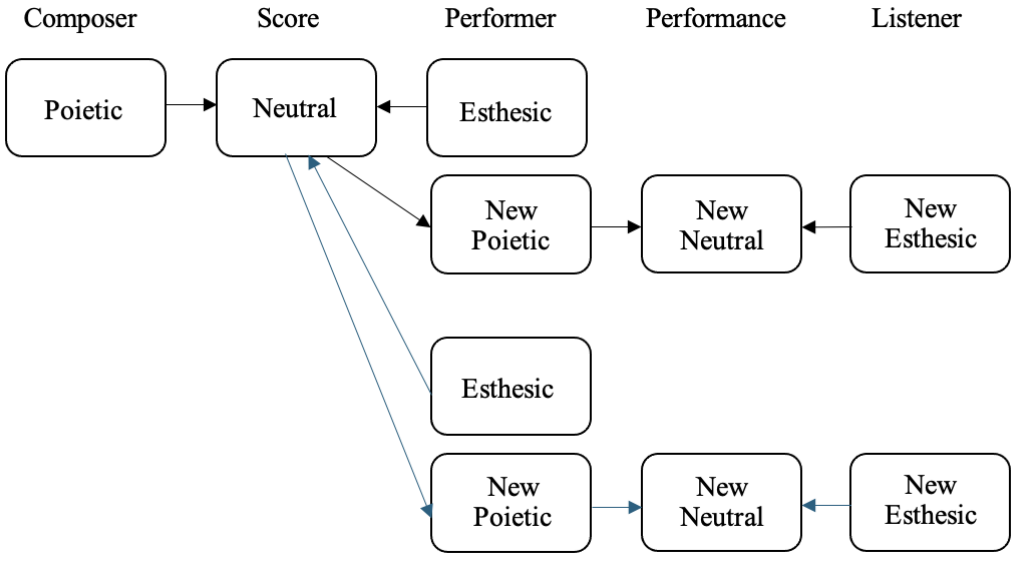

Figure 2 This figure shows the successive process involving the original three levels and the new three levels. [84]

[82] Thomas, 'Understanding Indeterminate Music through Performance', 92.
[83] Clarke, 'Musical Indeterminacy and Its Implications for Music Analysis', 178.
[84] Ibid., 179.

3.1.1 The Combination of the Theory of Rationality and the Revised Tripartite Model

The aim of combining the theory of rationality with the model is to clarify the two-stage rationalisation, consists of Cage's practices of indeterminacy and the performers' decisions. To do so, it is necessary to recognise two types of compositional materials and changeable status of compositional materials.

The compositional materials include determinate and indeterminate statuses. The determinate compositional materials refer to the materials provide clear information, for instance, numbers in Notation J (Figure 3) clearly show the numbers of notes should be played, or the clef signs in Notation BX (Figure 4) show the pitch of the notes. The indeterminate compositional materials refer to the materials requiring the performers' decisions, for example, the long straight lines in Notation J show the pitch limit, the performers ought to choose certain numbers of notes from the limit, or the curved lines in Notation BX that Cage did not state the performing approach in detail (i.e. performing by which hands, starting with which direction, or realising in what loudness, etc), but rather he guided the performer to realise this notation 'all at once like a moment of a plant'.[85] This arrangement leaves interpreting freedom to the performer, the realisation can be with *fff* as a flourishing plant, or with *ppp* as a dying plant. With this performing instruction, the realisations can be diverse.

Nevertheless, even though determinate compositional materials offer clear information, it does not mean that the performers can understand the meaning without referring to Cage's performing instructions. Graphic notations are similar to a jigsaw puzzle; even though you can clearly recognise the pattern on the piece of puzzle, you still need to refer to the whole picture to obtain a better understanding of where this piece should go. For instance, the compositional material, number, is used to represent time in Notations F, Q, R, and Y, to show loudness in Notations T and Z, and to show the number of performing notes in Notations A, B, D, G, J, L, and U. Hence, the performers cannot judge the meaning or the use of the determinate compositional materials without the performing instructions.

[85] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

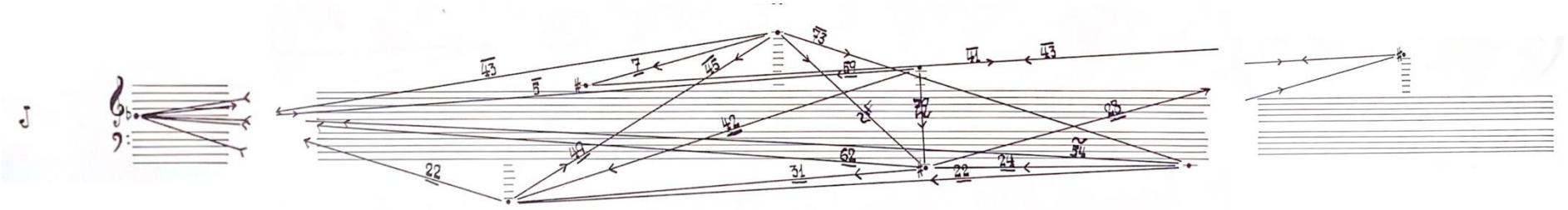

Figure 3 Notation J on pages 5–7.

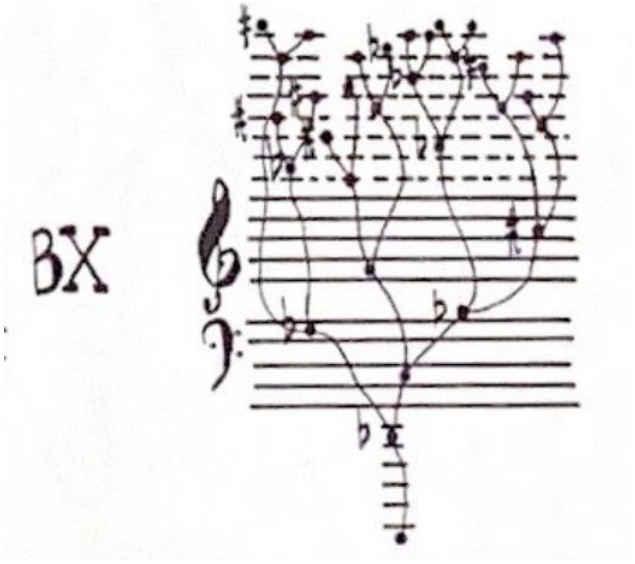

Figure 4 Notation BX on page 56.

With the intentional vagueness and the individuals' rationalisation, it is hard to predict or duplicate realisations, not even the performers themselves. This unique Cagen indeterminacy reveals: (i) Cage's participation of turning chance-derived paper imperfection into determinate realisations, and (ii) the performers' participation of transforming the purposive vagueness into determinate realisations.

The first point reflects the first stage of rationalisation in Cage's composing process. In the process, he transformed the nature of paper imperfections into determinate and semi-determinate – different compositional materials show determinate and indeterminate nature, accordingly. As Holzaepfel explained, Cage placed a solid notehead on the paper imperfections, then overlaid other musical symbols to turn noteheads into notes.[86] This gradual process illustrates how Cage assigned meaning to these paper imperfections, and, more importantly, how he imbued value into the composition by incorporating musical symbols and designing performing approaches and types of freedom. Applying the rationalisation along with poietic and esthesic levels, the esthesic level points out Cage's process of receiving and interpreting the ideas of Zen, while the poietic level refers to his process of producing the *Solo for Piano*.

The other focuses on the second stage of rationalisation in performers' participation. By following the performing instructions to decode meanings of compositional materials and produce realisations, the performers rationalise these compositional materials, and therefore, they transform the statuses of indeterminate graphic notations to determinateness. The realisation process includes choosing notes, deciding on timing and loudness, and other approaches that Cage arranged in graphic notations. Schematically, intentional vagueness of graphic notations represents both rationality in Cage's design and irrationality in the compositional materials simultaneously. Applying the rationalisation along with poietic and esthesic levels, the esthesic level points to the decoding process and the poietic level refers to the process of making decisions.

In summary, Cage's composition subtly manipulates indeterminacy and determinateness. He took intentional ambiguity as an essential source of graphic notations and required performers to directly participate in the process of transforming the status of materials. Consequently, when more compositional elements are analysed, their participation at poietic and esthesic

[86] Holzaepfel, 'Cage and Tudor', 176.

levels in the two-staged rationalisation is getting distinct. Hence, I consider applying the revised tripartite model and the theory of rationality to research the *Solo for Piano*.

3.2 Data Collection and Analysis Method

3.2.1 Data Collection: Analyses of Solo for Piano

The following section will explain the rationale of the data collection. In order to show the rational features in this musical work, I deconstruct the compositional elements in eighty-four graphic notations to collect qualitative data, including compositional materials, functions of compositional materials, the performing approaches, and types of freedom. As previously explained, the realisations require both Cage and performers' participation; Cage built the framework and offered the performing instructions, while the performers decide the details for the realisations. Thus, my analyses of the *Solo for Piano* examine performing instruction to investigate what functions are used in the notations, which performing approaches are assigned to the notations, and what types of freedom are allowed in the notation. I choose Notations A and AS to demonstrate the process of collection because the instruction of Notation AS simply describes 'A single note',[87] while Notation A's instruction is comparatively clear. With the different styles of instructions, I expect to elaborate how I obtain the qualitative data from the notations.

Notation A's performing instruction describes 'Following the perimeter, from any note on it, play in opposite directions in the proportion given. Here and elsewhere, the absence of indications of any kind means freedom for the performer in that regard'.[88] To obtain the performing approaches, I especially focus on the description with actions, such as 'following' the perimeter and 'play' in the opposite direction in the proportion given. For the types of freedom, I concentrate on words relating to performers' decisions, details ought to be decided, and any compulsory information for the realisations but Cage did not specifically point out.[89] Consequently, the known performing approaches are: (i) following the perimeter, (ii) playing in opposite directions, and (iii) playing the given proportion; the types of freedom include: (i)

[87] Cage, *Concert for Piano and Orchestra: Solo for Piano*.
[88] Ibid.
[89] The words relating to performers' decisions, such as 'free', 'freedom', 'absence', 'disregard', 'ambiguity', 'if desired', 'not obligatory'. The details to be decided, for instance, 'in opposite directions' means the performers should decide which hand play in which directions. The missing compulsory information, such as quality of sounds.

free to choose notes as the start, (ii) free to determine the performing direction, and (iii) free to decide the quality of sounds.

For the compositional materials and their functions, I will go through the process of realisation to examine what functions they carry and what compositional materials are used in notations. The reason for going through the realising process is because the compositional materials are not limited to the visible materials but also include the materials involving performers' performing approaches or freedom. Referring to Notation A itself (Figure 5), I recognise the used compositional materials include numbers, clef signs, accidentals, staff, the perimeter for grouping, and paper imperfections, while the applied compositional materials in the performing instruction include the perimeter, notes, direction, and the given proportion. With these materials, the performers will take numbers as the given proportion of notes and use the perimeter as the performing direction and the performing orders of notes, and hence, the function of number is representing performing numbers of notes, and the function of perimeter is to indicate the performing directions and orders. On the contrary, Notation AS and its instruction are simple and direct. The instruction describes 'A single note' and the notation consists of a note, an accidental, a clef sign, and a staff (Figure 6). When I realise this notation, the performing approach I recognise is performing the given note, and the type of freedom is free to decide the quality of notes, including dynamics and duration. In short, the compositional materials can be recognised in both the notation and its performing instruction; the function of materials can be obtained by going through the realising process.

Comparing the two notations and their instructions, I would like to emphasise that one notation can be realised diversely. Even though the realisation is based on a straightforward notation as Notation AS or a clear one as Notation A, different esthesic and poietic processes will result in different rationalisation and enchantment, and hence, my realisations do not represent all possibilities of the musical work. Further analyses of the *Solo for Piano* will be discussed in Chapter 5 Section 2 Analyses of Compositional Materials: Notation A to Notation CF.

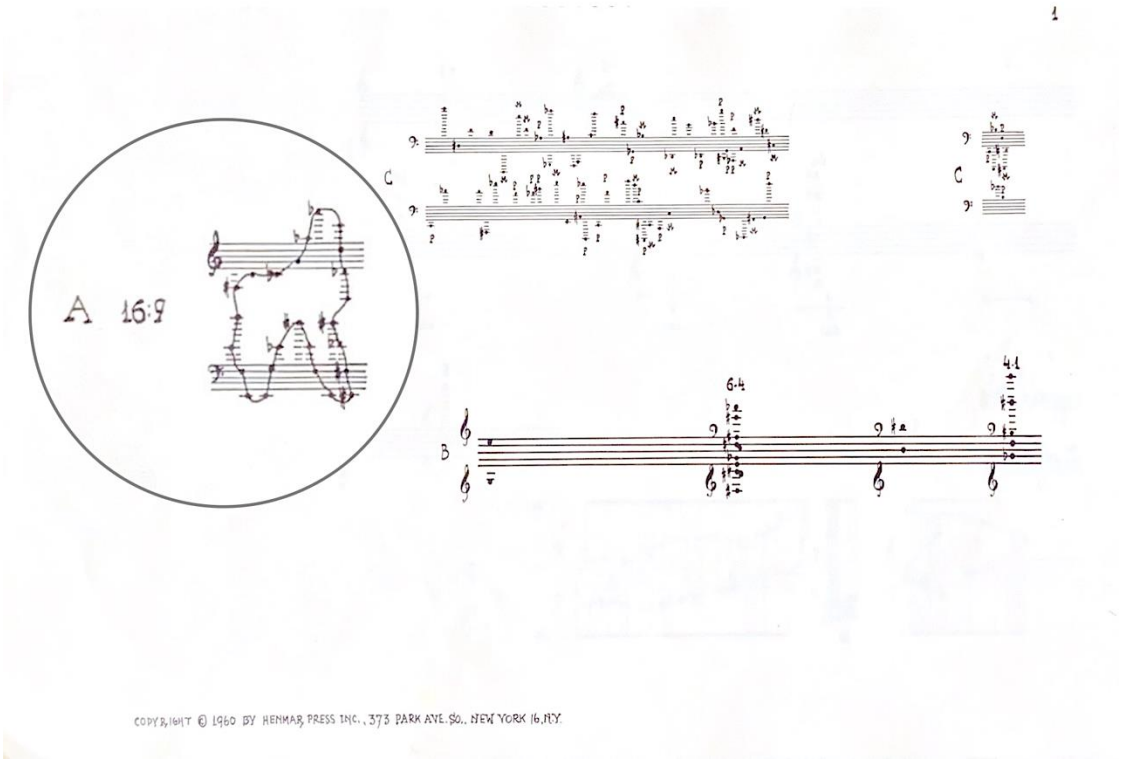

Figure 5 Notation A on page 1 in the *Solo for Piano*.

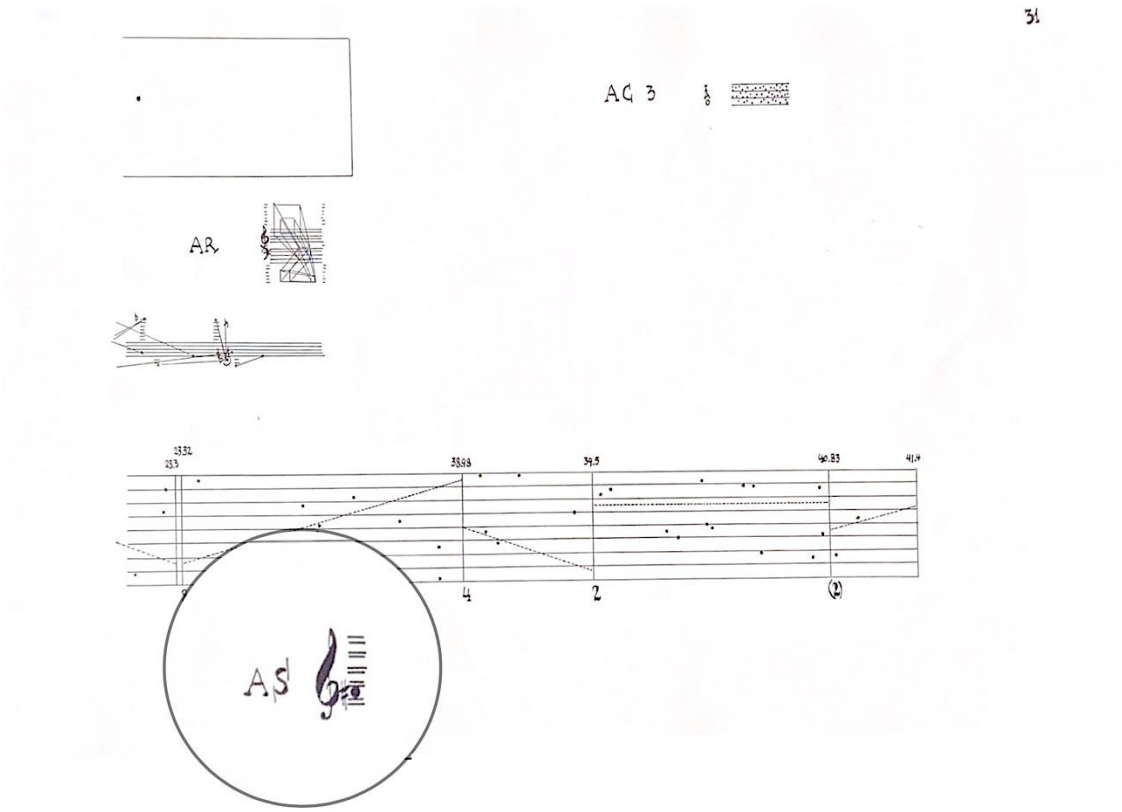

Figure 6 Notation AS on page 31 in the *Solo for Piano*.

3.2.2 *Analysis Method: Gephi*

Gephi, a software visualises, spatialises, and transforms networks into maps, has been widely applied in social network analysis, biology, and genomics, etc.[90] One of the advantage of Gephi is its accessibility, as it does not require advanced programming experience; hence, it is highly accessible for users who do not have any program training.[91] In this research, I shall consider three elements of Gephi: component, cluster, and centrality, in order to demonstrate the relationships between notations and their compositional elements. The first element, component, includes nodes and edges, which are the basic elements of composing the network.[92] An edge represents a type of link, connection, or relationship that bridges two or more nodes (Figure 7). For instance, a node can represent anything, such as a message, a person, or a train station, while the edges can represent the how the connections between the objectives are built; it can be the message history between two people or the routes between two train stations, depending on its context.

Edges can be directed and undirected. The directed edges represent relationships towards another node, and the undirected edges present non-directional relationships. For instance, a mother writes a letter to her daughter; the mother and the daughter are nodes, and the letter is the directed edge that connects two nodes together. The mother actively contacts with the daughter, so they are directedly connected with each other by the letter.[93] Putting it differently, the subject takes an action, and the object receives the action. On the other hand, undirected edges represent the existing relationships with no positions of the subject or the object. Applying nodes and edges to the *Solo for Piano*, the compositional materials, the materials' functions, the performing approaches, and the types of freedom are presented as nodes, while edges present the relationships between these compositional elements. Meanwhile, the compositional elements and their corresponding notations do not interact with one another, meaning that the relationships between them are not directional, and therefore, the edges are non-directional.

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[90] Jacomy et al., 'ForceAtlas2, a Continuous Graph Layout Algorithm for Handy Network Visualization Designed for the Gephi Software'.
[91] Bastian, Heymann, and Jacomy, 'Gephi : An Open Source Software for Exploring and Manipulating Networks'.
[92] Grandjean, 'GEPHI – Introduction to Network Analysis and Visualization'.
[93] The judgment of nodes depends on researchers and research focus. When the research focuses on what communication methods are popular between family members, the letter can be included as a node, however, if the research focuses on the frequency of contact between the family members, the letter would be represented as a directional edge.

The second element is cluster, which consists of a set of two nodes and an edge at least. The size of a cluster can be varied, depending on the number of nodes and the density of edges. The presentation of clusters relates to the final element, centrality, which indicates a focal of a cluster within a network. Centrality includes four primary centralities: closeness centrality, betweenness centrality, eigenvector centrality, and degree centrality. Ken Cherven explained that closeness centrality quantifies how close a selected node is to all the other nodes within a graph, meaning that a node with stronger closeness centrality normally shows less edges to all other nodes within the network; betweenness centrality means a node bridging two or more clusters together, and by bridging nodes from different clusters, the node shows high betweenness centrality; eigenvector centrality is defined by the connectedness of its closest nodes, meaning that if a node is connected to other highly connected nodes, this node shows a higher level of influence; degree centrality indicates a node act as a centre for information flow, it shows high connectivity with other nodes.[94]

Considering Cage's composing technique of the *Solo for Piano* – he started with paper imperfections and then applied other symbols, shapes, or musical symbols to offer meanings – I choose applying closeness centrality to node size to reflect the uses of compositional materials in this musical work. Since the stronger closeness centrality means fewer less edges to all other nodes, the size of nodes shall reveal which materials are commonly used in notations and which materials are less used in notations. As a result, bigger nodes equal more uses, and smaller nodes equal less uses.[95] For instance, the Node E shows one edge connecting with the Node B, the Node C, the Node D, the Node F, and the Node G. Even for the farthest two nodes, the Node A and the Node H, the Node E only traverses two edges to reach the two nodes, and hence, the Node E shows the stronger closeness centrality (Figure 8).

[94] Cherven, *Mastering Gephi Network Visualization*, 14–17.

[95] For other types of centralities, betweenness centrality emphasises the node connecting clusters together rather than the commonly used nodes. This does not fit for my purpose of using Gephi. Eigenvector centrality and degree centrality are less applicable in an object without interactions, as they emphasise level of influence and information flow.

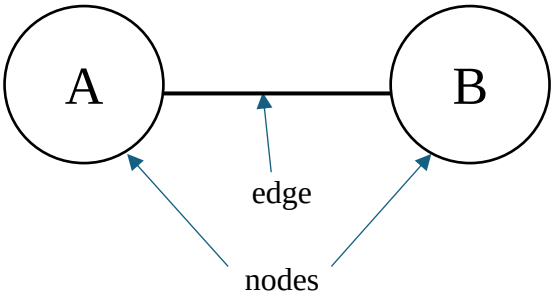


Figure 7 Node A and Node B is connected by an edge.

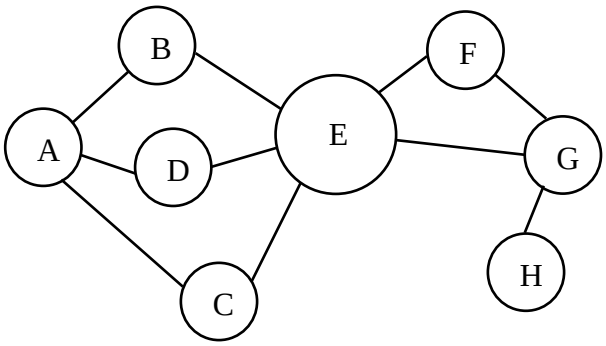


Figure 8 Node E shows stronger closeness centrality than other nodes.

Moving on to Gephi's algorithm. Andreas Noack explains that ForceAtlas2 is a force-directed layout that locates each node depending on their connections, and therefore, it optimises the sense of communities in the visualisation, meaning that the nodes from the same group form an individual community.[96] With this feature, the visualising results illustrate centres of each cluster. As Figure 9 illustrates, each notation connects to the relevant compositional elements, so when the compositional elements are from the same notation, the positions of nodes are closed to the corresponding notations.

Looking into Notation S of this example, six sets of nodes construct this notation (Figure 10). For instance, nodes from Notation S itself (marked as colour blue), from Notation C (marked as colour dark pink), from Notation H (marked as colour grey), from Notation AC (marked as colour blue-green), from Notation BK (marked as colour green), and other commonly used compositional elements (marked as colour purple) (Figure 10). As Figure 10 shows, nodes from both Notation S and the other notation are located in the middle of two notations. The compositional elements (noise, interior piano construction, and outer piano construction) are

[96] Noack, 'Modularity Clustering Is Force-Directed Layout', 3; Jacomy et al., 'ForceAtlas2, a Continuous Graph Layout Algorithm for Handy Network Visualization Designed for the Gephi Software', 2.

used in both Notation S and Notation BK, so the position of these elements lie between them. The commonly used compositional materials (e.g., G-clef, F-clef, staff, accidentals, notes, and paper imperfections) are located in the middle of this graph since they are shared by the notations.

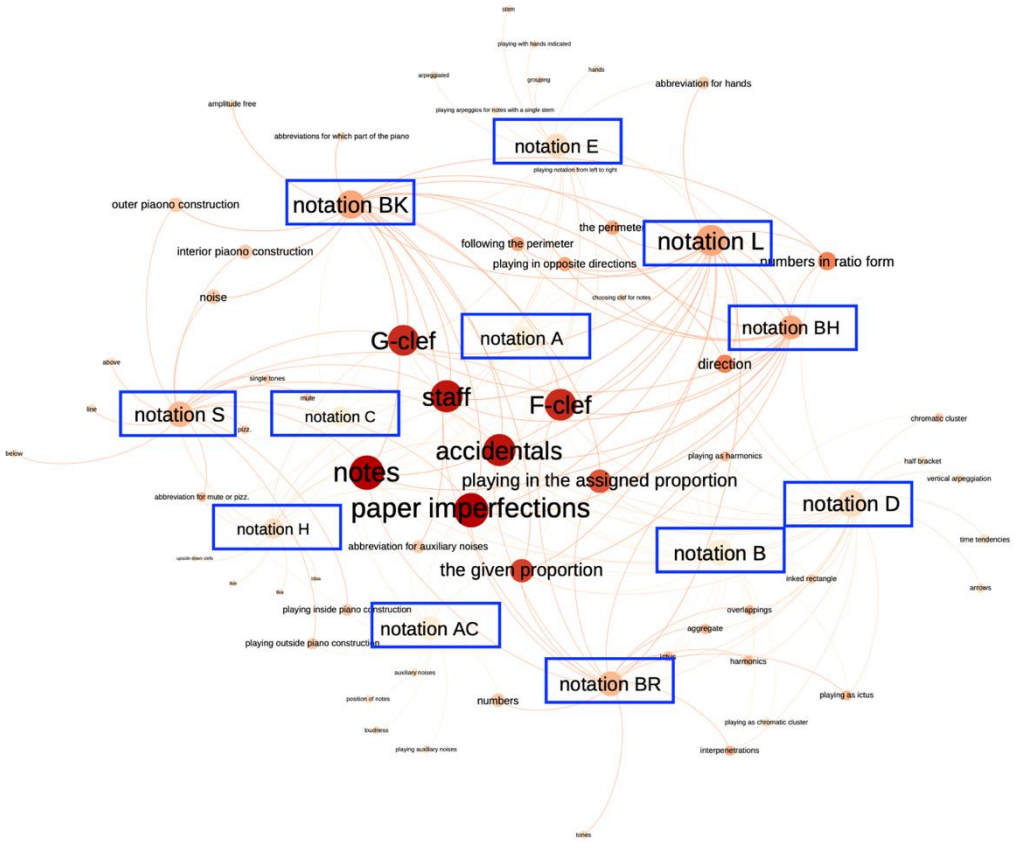


Figure 9 This visualising result is based on twelve graphic notations in the *Solo for Piano*. It shows how ForceAtlas2 can locate the position of nodes based on their connections.

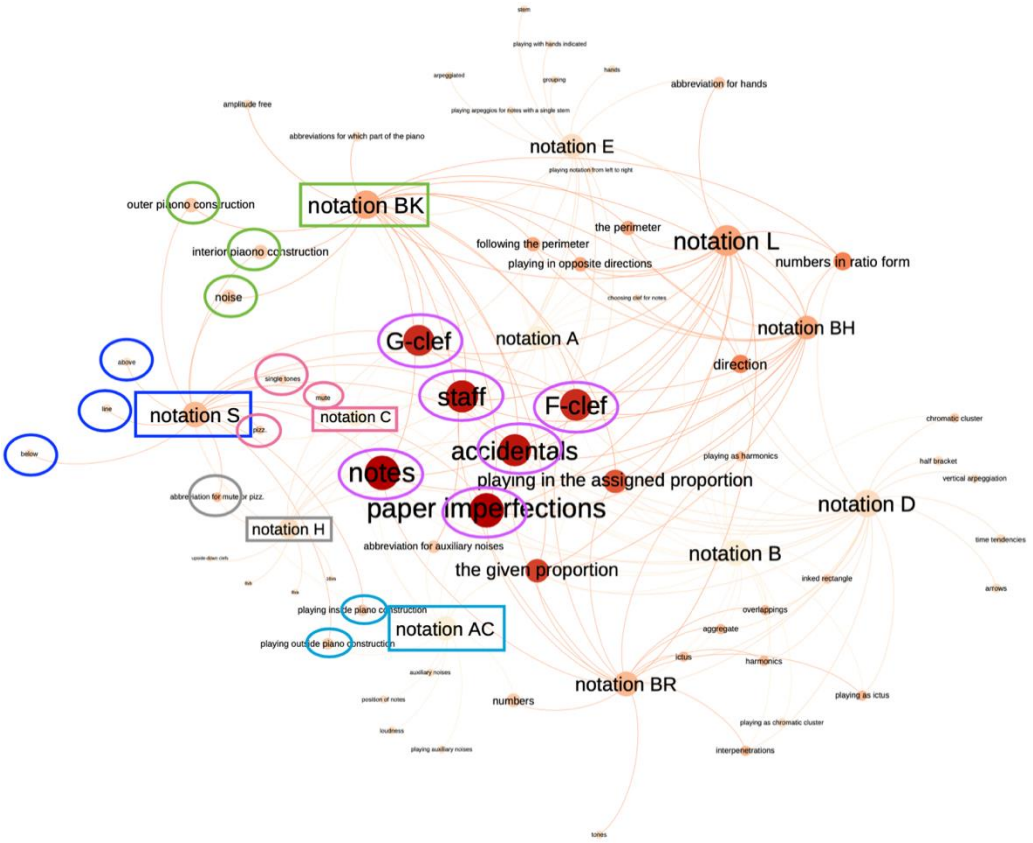


Figure 10 This visualising result demonstrates how ForceAtlas2 locates the position of nodes.

Gephi user interface includes three parts: Overview, Data Laboratory, and Preview. Overview includes two settings, Appearance and Layouts, and the user can choose node colours and node size based on different node attributes (Figure 11; Figure 12; Figure 13). In this research, I chose modularity class for the node colour and applied closeness centrality for the node size. Modularity class is a measure of a particular division of networks, interactions, or structural constructions;[97] it applies colours to the nodes based on their communities in Gephi, if the nodes belong to the same community, they will have the same colour, and vice versa. Closeness centrality shows the proximity of a selected node to all other nodes.[98] This setting shows how compositional elements connect with one another and reveals which compositional elements are most commonly used in the *Solo for Piano*; applying this centrality to node size, the nodes of commonly used elements are bigger than others.

[97] Blondel et al., 'Fast Unfolding of Communities in Large Networks', 2.
[98] Cherven, *Mastering Gephi Network Visualization*, 14, 187.

Layout in Gephi means to select an algorithm and set parameters, including scaling, gravity, and behaviour alternatives. As mentioned above, I applied the force-directed layout, ForceAtlas2, to illustrate the relationships between notations and the relevant compositional elements (Figure 13). The second Gephi user interface is Data Laboratory, which shows nodes and edges attributes in a table (Figure 14). The final user interface is Preview, where the users can choose different ways to present nodes and edges, such as node label font and node label colours, edge label font, edge colours, edge thickness, and edge curved/straight line, and background colour (Figure 15). Based on the explanations above, a basic understanding of Gephi may be acquired. The official website of Gephi (https://gephi.org) provides further details.

In summary, visualisation is an effective approach for discovering features hidden within massive data.[99] By applying Gephi to the *Solo for Piano*, the relationships between notations and the compositional elements can be revealed, and so illustrate the rational features of this work. Three main settings that I applied in Gephi: ForceAtlas2, closeness centrality, and modularity class. The force-directed layout, ForceAtlas2, forms the clusters based on the uses of compositional elements; closeness centrality shows which elements are commonly shared among notations; modularity class offers the same colours to the elements from the same communities. With Gephi's verification of the rational features of indeterminate music and the graphic score, it opens opportunities to illustrate consistency within chance operation music and graphic notations.

[99] Bastian, Heymann, and Jacomy, 'Gephi : An Open Source Software for Exploring and Manipulating Networks'.

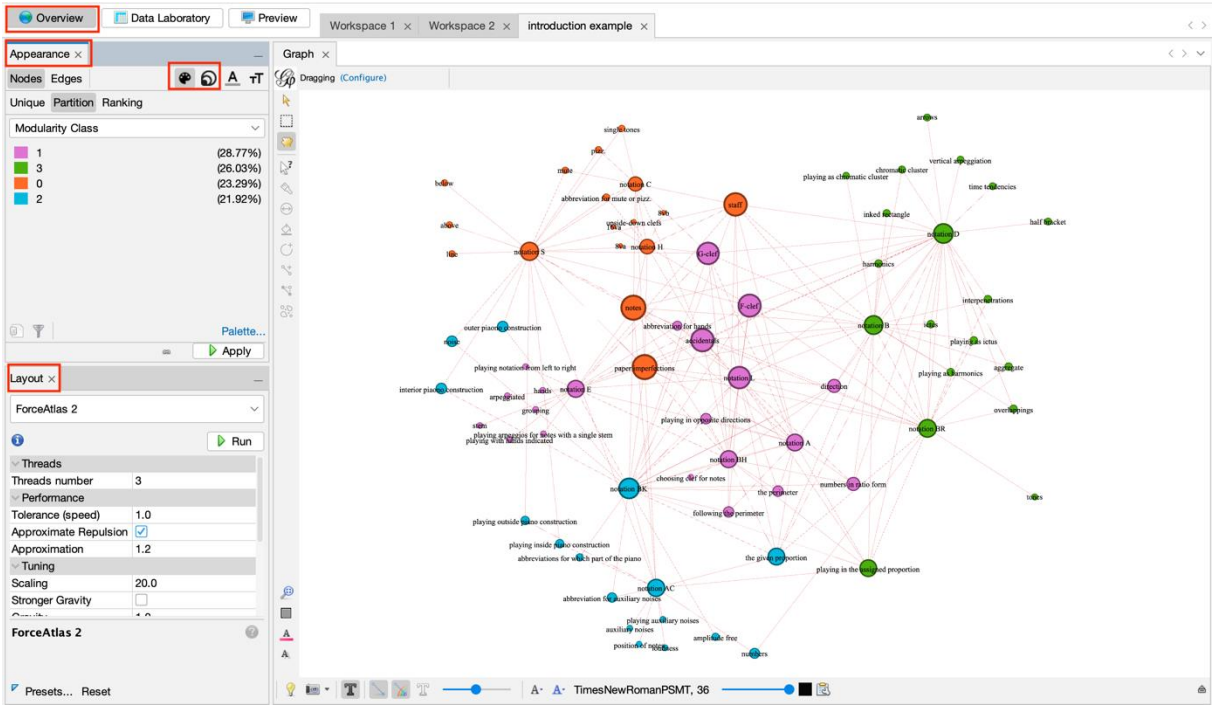


Figure 11 Gephi user interface, Overview. The user can choose different attributes of nodes, as the red squares marked (close to the top of this figure, on the left-hand side).

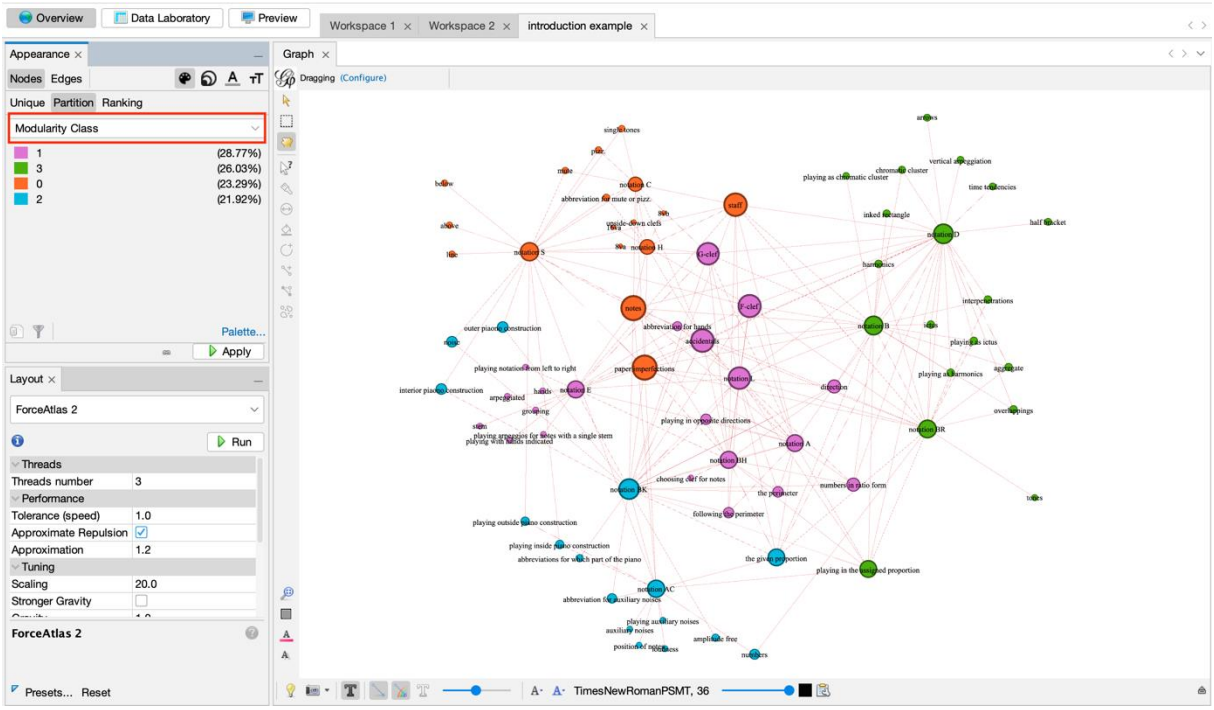


Figure 12 Node colour setting at Gephi user interface (Overview → Appearance → Nodes colour →Partition).

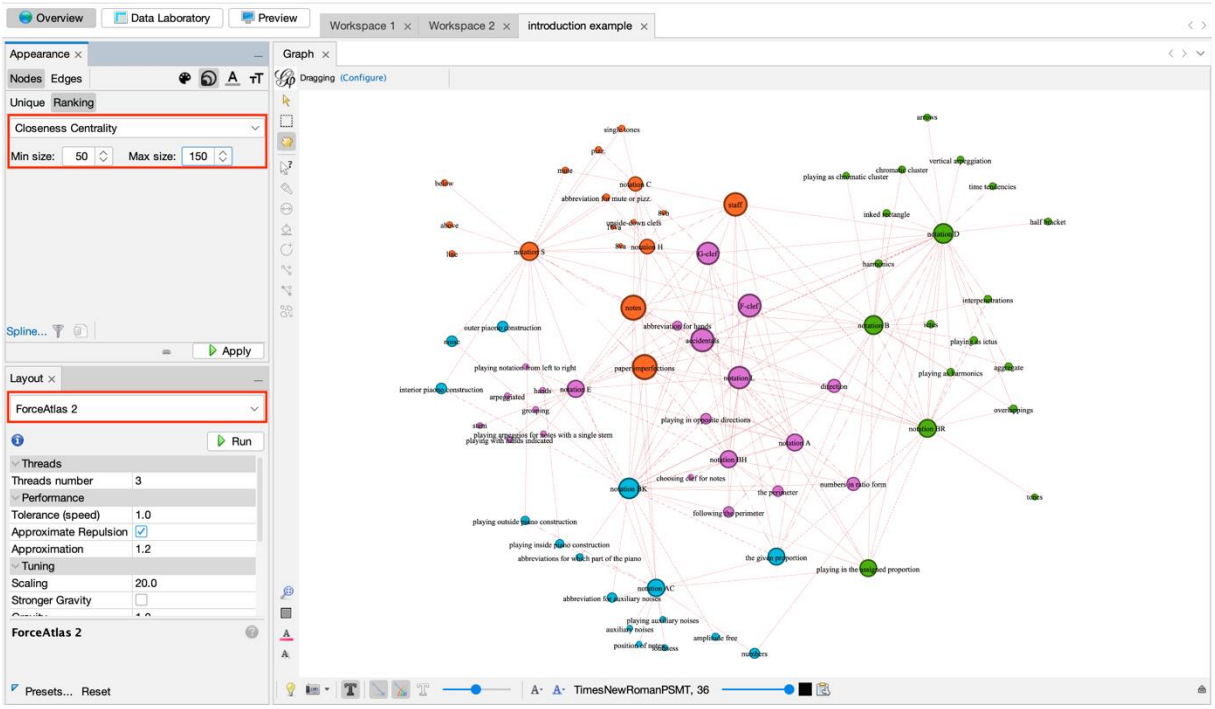
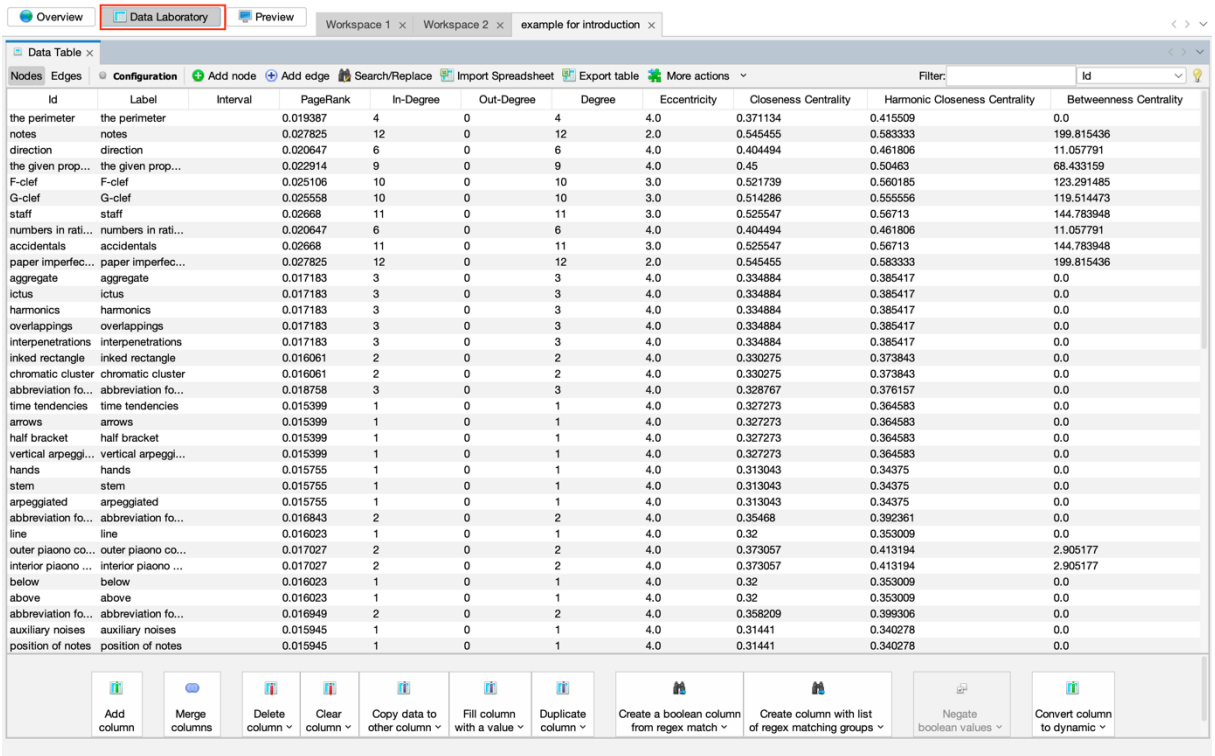


Figure 13 Node size and layout at Gephi user interface (Overview → Appearance → Nodes size → Ranking). Layout provides different choices of presenting the visualising results (Overview → Appearance → Layout).



Id	Label	Interval	PageRank	In-Degree	Out-Degree	Degree	Eccentricity	Closeness Centrality	Harmonic Closeness Centrality	Betweenness Centrality
the perimeter	the perimeter		0.019387	4	0	4	4.0	0.371134	0.415509	0.0
notes	notes		0.027825	12	0	12	2.0	0.545455	0.583333	199.815436
direction	direction		0.020647	6	0	6	4.0	0.404494	0.461806	11.057791
the given prop...	the given prop...		0.022914	9	0	9	4.0	0.45	0.50463	68.433159
F-clef	F-clef		0.025106	10	0	10	3.0	0.521739	0.560185	123.291485
G-clef	G-clef		0.025558	10	0	10	3.0	0.514286	0.555556	119.514473
staff	staff		0.02668	11	0	11	3.0	0.525547	0.56713	144.783948
numbers in rati...	numbers in rati...		0.020647	6	0	6	4.0	0.404494	0.461806	11.057791
accidentals	accidentals		0.02668	11	0	11	3.0	0.525547	0.56713	144.783948
paper imperfec...	paper imperfec...		0.027825	12	0	12	2.0	0.545455	0.583333	199.815436
aggregate	aggregate		0.017183	3	0	3	4.0	0.334884	0.385417	0.0
ictus	ictus		0.017183	3	0	3	4.0	0.334884	0.385417	0.0
harmonics	harmonics		0.017183	3	0	3	4.0	0.334884	0.385417	0.0
overlappings	overlappings		0.017183	3	0	3	4.0	0.334884	0.385417	0.0
interpenetrations	interpenetrations		0.017183	3	0	3	4.0	0.334884	0.385417	0.0
inked rectangle	inked rectangle		0.016061	2	0	2	4.0	0.330275	0.373843	0.0
chromatic cluster	chromatic cluster		0.016061	2	0	2	4.0	0.330275	0.373843	0.0
abbreviation fo...	abbreviation fo...		0.018758	3	0	3	4.0	0.328767	0.376157	0.0
time tendencies	time tendencies		0.015399	1	0	1	4.0	0.327273	0.364583	0.0
arrows	arrows		0.015399	1	0	1	4.0	0.327273	0.364583	0.0
half bracket	half bracket		0.015399	1	0	1	4.0	0.327273	0.364583	0.0
vertical arpeggi...	vertical arpeggi...		0.015399	1	0	1	4.0	0.327273	0.364583	0.0
hands	hands		0.015755	1	0	1	4.0	0.313043	0.34375	0.0
stem	stem		0.015755	1	0	1	4.0	0.313043	0.34375	0.0
arpeggiated	arpeggiated		0.015755	1	0	1	4.0	0.313043	0.34375	0.0
abbreviation fo...	abbreviation fo...		0.016843	2	0	2	4.0	0.35468	0.392361	0.0
line	line		0.016023	1	0	1	4.0	0.32	0.353009	0.0
outer piaono co...	outer piaono co...		0.017027	2	0	2	4.0	0.373057	0.413194	2.905177
interior piaono ...	interior piaono ...		0.017027	2	0	2	4.0	0.373057	0.413194	2.905177
below	below		0.016023	1	0	1	4.0	0.32	0.353009	0.0
above	above		0.016023	1	0	1	4.0	0.32	0.353009	0.0
abbreviation fo...	abbreviation fo...		0.016949	2	0	2	4.0	0.358209	0.399306	0.0
auxiliary noises	auxiliary noises		0.015945	1	0	1	4.0	0.31441	0.340278	0.0
position of notes	position of notes		0.015945	1	0	1	4.0	0.31441	0.340278	0.0

Figure 14 Gephi user interface, Data Laboratory.

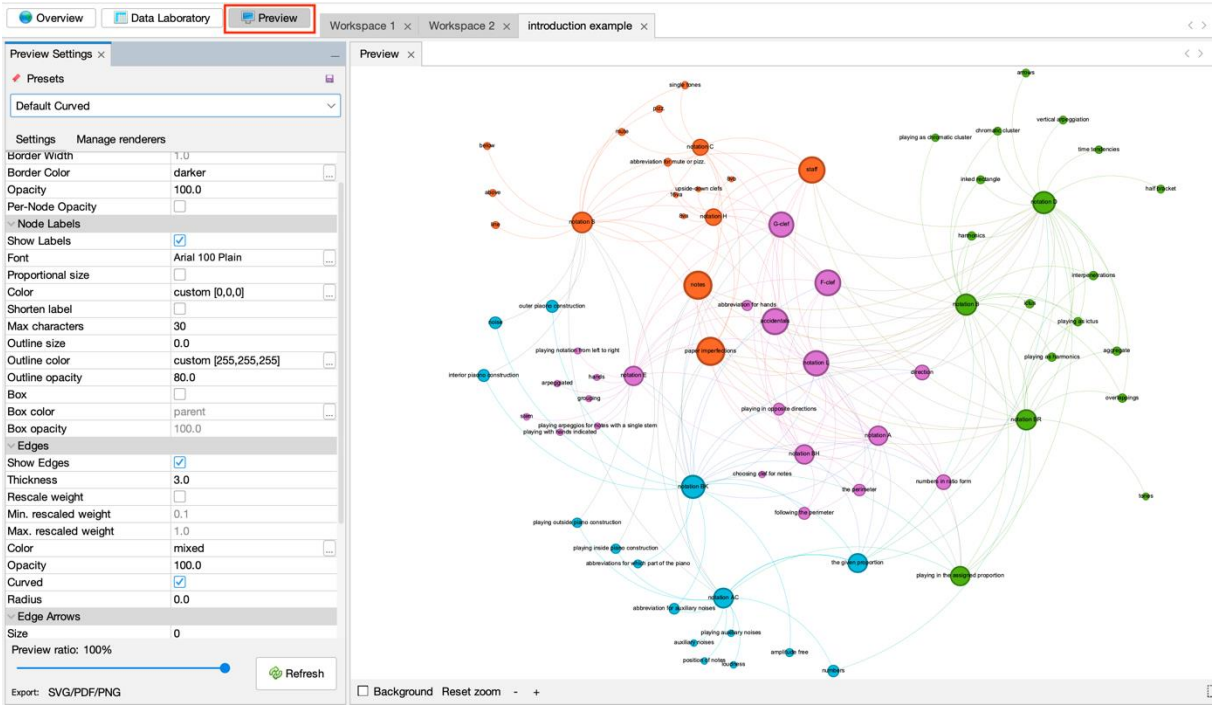


Figure 15 Gephi user interface, Preview.

Chapter 2. Weber, Rationality and European Art Music

1 Introduction

Weber extended his interests to politics, economics, religion, and the arts, and his many contributions also included music. However, his contributions to music have been controversial and criticised by many academics, who are still debating his opinions, and his legacy, until today. When scholars criticise the writing style, unclear statements, and scratchy handwriting, and question the validity of his ideas of *The Rational and Social Foundations of Music*, we should keep in mind that he did not intend to publish the study but took it as a preparation of sociology of art; also, causality is the key factor to understand his idea of this study. For these reasons, his ideas should be revised according to social changes, and they should be applied in terms of current thinking. Thus, in spite of such overwhelming criticisms, Weber's contributions still full of potential. Looking into his thinking and concepts, many of which are quite rationally founded, his complex and beautiful ideas ran through his life. Consequently, he extended them as far as he was able. After the first edition of *The Protestant Ethic and Spirit of Capitalism* was published, Weber started his investigations on the sociology of art, and among all the artistic formulations, he chose music as his first attempt since he had acquired a musical education in his childhood.[100]

Between 1910 and 1912, Weber developed the idea of rationality in music, which he observed through its various aspects, such as musical theories, notational systems, listening effects, and social needs. He recorded his observations in an unfinished draft named *The Rational and Social Foundations of Music*, which was published by his wife, Marianne Weber (1870–1954), and a musicologist, Theodor Kroyer (1873–1945).[101] Weber was the first to identify rationalisation in the development of music;[102] Weber listed various factors to prove his statements; for instance, he used Pythagoras' studies to show how mathematics constructs music in order to explain the functions of ancient music, and he illustrated religious relationships with musical instruments by showing their development in relationships to

[100] Baumgarten, *Max Weber Werk Und Person.*, 482; Blaukopf, *Musical Life in a Changing Society*, 118.
[101] Weber, *The Rational and Social Foundations of Music*.
[102] Adorno, *Sound Figures*; Konoval, 'Max Weber and the Sociology of Music', 468.

weather and social class.[103] Moreover, he forged novel connections between musical language, technique, and notation, as well as the social division of labour between composers, performers, and listeners.[104] Among these examples it is noticeable that he attempted to extend the idea of rationality to music by explaining the process of rationalisation and postulating parallels between the rationality of Protestantism and European art music.

With the ideas of rationalisation in European art music, Weber is considered as one of influential scholars of the sociology of music.[105] His research has been carried on and diversely expanded to various aspects of the sociology of music. For instance, constructing overarching framework of how the music and society jigsaw fit together;[106] considering potential musical matters as one's engagement of cultural actions;[107] or arguing for the global status of black

[103] The relationship between weather, social class, and musical development will be explained in 3.1.3. The final rational feature: interactivity/being interactive. In brief, the weather influenced the people's activities, and so collateral impact on the development of musical instruments.
[104] Hennion, 'MUSIC AND MEDIATION: Toward a New Sociology of Music', 254.
[105] Along with Weber, other scholars, such as, Herbert Spencer (1820–1903), Georg Simmel (1858–1918), Charles Seeger (1886–1979), John H. Mueller (1895–1965), Alfred Schutz (1899–1959), Theodor Adorno (1903–1969) are also recognised as significant scholars. In 'The Origin and Function of Music' (1961[1854]) Spencer explained that music facilities the development of emotion and relates to our feeling of happiness and sympathy, and therefore, music is a means of communication for human emotions. For Simmel, his research on psychological and ethnological foundations of music investigated the social meaning in music and the position and function of music in a society; specially, he suggested that the artists and audiences' socialisations associate with representative items of artistic productions, as K. Peter Etzkorn summarised in his article, 'Georg Simmel and the Sociology of Music'. In terms of Seeger's contributions, as Ellie M. Hisama emphasised in her review article 'Understanding Charles Seeger, Pioneer in American Musicology. Edited by Bell Yung and Helen Rees. Urbana: University of Illinois Press, 1999', Seeger's importances lie in his advocacy of researching music in contexts, the inclusion of world music, folk music, and orally transmitted music in repertoire studies, and his use of melograph in the 1950s. Mueller's *The American Symphony Orchestra: A Social History of Musical Taste* (1951) examines the relationships between music and its audiences to demonstrate that the musical taste can be considered as an aesthetic phenomenon and an element of sociological account, and therefore, the rise and decline of the forms and styles can be investigated through the performed music. Schutz's 'Making Music Together: A Study in Social Relationship' (1951) proposes when performers produce music together they experience identical flux of inner time, while the performers and the listeners experience the unified outer time and community of space based on the 'we-relationship'. Thus, for people involving in the music performance, music produces 'an interplay of recollections, retentions, protentions, and anticipations which interrelate the successive elements' (Schutz, *Collected Papers II: Studies in Social Theory*, 170). Adorno's *Introduction to the Sociology of Music* (1976) broadly includes popular music, opera, chamber music, public opinion and critics, and mediation as the research objects. His research provides a critical framework for comprehending the interrelations between culture, society, and individual consciousness. From the above research, it is noticeable that these significant scholars contributed to the sociology of music through their insightful opinions.
[106] Prior, 'Bourdieu and Beyond', 352.
[107] Martin, *Sounds and Society*.

music through sense of liberation and modern social rationalisation.[108] The focus of these research seems to concentrate on the relationships between music and societies; the one-sided focus seems to result in a lack of further investigations into the theory of rationality. Thus, this chapter will firstly introduce the main ideas in Weber's *The Rational and Social Foundations of Music* to show how he extended the theory of rationality from Protestantism to European art music and will then take J. S. Bach's BWV 846 Prelude in C major as an example to explain his stance whereby European art music shows its rational features in harmonic progressions. This chapter will also introduce Schenkerian analysis, which presents another form of the structure within European art music.

2 Weber's Lifetime

Maximilian Karl Emil Weber was born in Erfurt, Prussia, on 21st April 1864. Five years later, when his father, Max Weber senior (1836–1897), was appointed as a councillor, the Weber family moved to Berlin. Weber senior invisibly influenced young Weber at his early age, as at that time, Weber senior pursued a dual career – one was as a national liberal representative in the Reichstag and the Prussian parliament, and the other was the head of the Berlin building department. As a result academics and political nobles used to meet in the Weber home, and therefore, young Weber had his chance to have frequent contact with men such as Wilhelm Dilthey (1833–1911), Theodor Mommsen (1817–1903), Julian Schmidt (1818–1886), and Friedrich Kapp (1824–1884).[109] Young Weber showed talent when he was just a thirteen-year-old when he wrote two historical essays, entitled 'Concerning the Course of German History, with Special Regard to the Positions of Kaiser and Pope' and 'About the Roman Imperial period from Constantine to the migration of nations' as Christmas presents, and then two years later, he wrote another essay, entitled 'Observations on the Ethnic Character, Development, and History of the Indo-European Nations', in which, Weber demonstrated his intelligence on philosophy of history clarifying the laws governing their development.[110] In 1882, Weber finished high school and moved to Heidelberg for studying law. In the following year, he undertook military training in Strasburg, then backed to Berlin and switched his major to economics in 1884.

[108] Radano, 'MUSIC, RACE, AND THE FIELDS OF PUBLIC CULTURE', 310.
[109] Dilthey was a German philosopher, Mommsen's expertise was Roman history, Schmidt was a German journalist and historian of literature, and Kapp was a German American lawyer, writer, and politician. The frequent contact with them seemed to inspire young Weber.
[110] Weber, *Max Weber*, 46.

In 1889, Weber gained his doctoral degree with a thesis on the history of trading companies in the Middle Ages. In 1891, during which he passed his second law examination, he wrote another essay about the history of agrarian institutions, in which he analysed the sociological, economic, and cultural aspects of ancient societies.[111] In the same year, he worked as a lecturer teaching Roman agrarian history.[112] In 1893, he married his cousin, Marianne Schnitger (1870–1954), who later took is surname. The following year, he began to actively participate in teaching politics and religions, and he accepted professionalship in political economics at the University of Freiburg,[113] and took over the chair of economics and finance studies at Heidelberg from Karl Knies in 1896.[114] He then started teaching at Heidelberg and Munich in the same field.

In the summer of 1898, he suffered a mental illness and was unable to work; following a few weeks' treatment in a mental institution, he appeared to get better. In 1902, he continued his work at Heidelberg, and started both his research into art history and to examine problems of methods in the social sciences; however, to do this he had to relinquish his professorship, because he could not undertake too many tasks.[115] The following year, his colleague Werner Sombart (1863–1941) invited him to join the professional journal *Archiv fur Sozialwissenschaft und Sozialpolitik* (1904–1933, *Archive for Social Science and Social Policy*) as an editor, which offered him opportunities to be in contact with scholars and politicians again. This lucky chance enlivened him to produce the first part of his influential publication, *The Protestant Ethic and the Spirit of Capitalism*, which was published in 1904. This editorship was not the only stimulation, for in the same year of joining *Archives for Social Science and Social Policy*, he travelled to America, where he saw the problems of labourers. With this awareness, he obtained primary resources by interviewing and talking to people, and then returned to Germany to draft his findings and finish the second part of *The Protestant Ethic and the Spirit of Capitalism* before Christmas.

In August 1912, Weber attended concerts of Wagner with the pianist Mina Tobler (1880–1967); soon he started to reflect his ideas to music and made a rough draft of his work on the sociology

[111] Ibid., 114.
[112] Radkau, *Max Weber*, 5.
[113] Ibid., 137.
[114] Ibid., 251.
[115] Weber, Gerth, and Mills, *From Max Weber*, 13–14.

of music.[116] Between 1904 and 1914, Weber and his brother, Alfred Weber (1868–1958), maintained academic connections with other scholars, such as the sociologists Ferdinand Tönnies (1855–1936), Georg Simmel (1858–1918), Werner Sombart, and Robert Michels (1876–1936), the philosopher Paul Hensel (1860–1930), the psychologist Hugo Münsterberg (1863–1916), and the linguist Karl Vossler (1872–1949), during which several generations of scholars sought stimulation from them.[117] In August 1914, Weber officially retired; in midsummer in 1919 he became seriously ill, and succumbed to pneumonia on 14th June 1920.

Weber was interested in various disciplines, broadly gaining knowledge in history, politics, economics, religion, philosophy, and culture. This broad interest may have been inherited from both his father and mother (Helene Fallenstein, 1844 –1919) – Weber senior was an intelligent lawyer, employed by the Berlin municipal government; he got the position of a Liberal minister when he was twenty-four, and Weber's mother was a devout believer who had received higher education.[118] Apart from the inheritance of bloodline, Weber's education also benefited from his father who brought him into a duelling fraternity, from which eminent circle he accessed social resources. With this internal and external assistance, he cultivated his intuitions and insights from an early age. It is noticeable that he travelled to America in 1904, when he visited cities, such as New York, Chicago, Brooklyn, New Orleans, Tuskegee, Virginia, Washington, Baltimore, and Boston. This is where he noticed social phenomena and where he became particularly interested in labour problems, immigrant issues, and the problems of political management. His observations there inspired him to draft the first part of *The Protestant Ethic and the Spirit of Capitalism.*

His work on such linear developments may have been influenced by Immanuel Kant (1724 – 1804). His continuous contact with the philosopher may have started in his youth, when Weber was exploring philosophy on his own, from Baruch Spinoza (1632–1677) and Arthur Schopenhauer (1788–1860) and finally to Kant.[119] His contact with Kant initially came when he entered university, from his professor, Karl Fischer, and also from one of his colleagues, Heinrich Rickert (1863–1936).[120] Weber's thoughts were cultivated under the influence of

[116] Max Weber Studies, 'Chronology of Max Weber's Life'.
[117] Weber, Gerth, and Mills, *From Max Weber*, 21.
[118] Weber, *Max Weber*, 26–30.
[119] Ibid., 45.
[120] Rutgers and Schreurs, 'Weber's Neo-Kantian Roots', 104–5.

Kant. By studying his biographical facts, the Kantian influences are noticeable, particularly in the forms he used in his investigations into action and methodology.[121] This may be the reason why his work on rationality was reinforced by both a Kantian view of ethical law,[122] and by the Kantian philosophical paradigm of idealism.[123] At this point in his career, the influence of Kant seems undeniable.[124] Hence his emphasis on formations of rationality and rationalisation are not purely either sociological or philosophical, but combining paradigms of both. This may echo with my stance about Weber's rationality should be comprehended through causal relationships of the societal factors, which can be found in Chapter 1. When he delved into societal problems in America and recorded his findings about Protestantism, he did not base these on positivism; instead, he searched for their causes in terms of humanity in order to interpret reasons for actions of individuals. Such an integration can also be found in Cage's composing intentions, which I will discuss in Chapter 4.

When Weber proposed rationality and irrationality from his findings on Protestant living styles, actions and beliefs, he brought the formation of capitalism into the discussions. His inclusion of capitalism is fundamental to his findings – not just because *The Protestant Ethic and the Spirit of Capitalism* is a sociological study, but also because it presents a flexibly interpretable potentiality of rationality. That is, Weber intended to show his readers that rationality is a concept rather than a fixed notion. In terms of this, he applied rationality to European art music and observed the rational and irrational features in musical elements, he recorded his observations in a manuscript, which was published and entitled '*The Rational and Social Foundations of Music*'. When this publication was released, it attracted scholars' attentions describing rationality as akin to reasonableness and logic, while rationality in European art music refers to tone-systems, scales, chords, systems of notation, and the development of instruments.[125] However, rationality is more than just reasonableness and logic. As explained in the Chapter 1 Introduction, the value of Weber's ideas is its openness;[126] the concept of

[121] Mackinnon, 'Max Weber's Disenchantment', 330, 334.
[122] Karl, *Leonardo, Descartes, Max Weber (Routledge Revivals)*, 462.
[123] Lenhardt, 'Max Weber and the Legacy of Critical Idealism', 22.
[124] For more studies and researches about Kant's influences on Weber, refer to Martin Barker's 'Kant as a Problem for Weber' (1980), Bryan S. Turner's *For Weber: essays on the sociology of fate* (1996), Malcolm H. Mackinnon's 'Max Weber's Disenchantment: Lineages of Kant and Channing' (2001), and Mark R. Rutgers and Petra Schreurs's 'The Morality of Value- and Purpose-Rationality: The Kantian Roots of Weber's Foundational Distinction' (2006).
[125] Lundin, 'Reviewed Work(s): The Rational and Social Foundations of Music by Max Weber, Don Martindale, Johannes Riedel and Gertrude Neuwirth', 116.
[126] Roth, 'Rationalization in Max Weber's Developmental History', 90.

rationality can be applied in various disciplines. In medical science, it can refer to medical technique with a biochemical foundation, employed through statistics and clinical data. In mathematics, it indicates Greek foundations and calculations.[127] In politics, it means a society ruled by an equitably written constitution and trained officials.[128] In bureaucracy, it implies the clear division of departments, the separation of powers and systematic rules in organisations, reasonable employment, efficiency, and fairness. With his passion for music, he extended rationality to the field of music and documented his observations in *The Rational and Social Foundations of Music* to discusses religious affiliations, individual choices, the development of society, and the influences of these factors on music. The discussions highlight both rational and irrational elements within music – how did people rationalise irrationality, and how did the rationalisation produce irrationality in terms of emotional-expressive actions.

3 *The Rational and Social Foundations of Music* (1921)

This posthumous publication contains Weber's findings regarding connections between mathematics and European art music, between monastic and European art music, and between Western societies and European art music. It was by examining these connections that Weber revealed the unique centrality of Western culture, which he interpreted as rationality, including the rationalisation of societies, culture and civilisation. By referring to *The Protestant Ethic and the Spirit of Capitalism* and *The Rational and Social Foundations of Music*, I extract the three main rational features: (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive, to illustrate Weberian rationality.

3.1 The Rational Features in European Art Music

3.1.1 The First Rational Feature: Method/Methodical

The first feature is method/methodical. In *The Protestant Ethic and the Spirit of Capitalism*, irrationality, a purpose emerging from irrationality, and the process of rationalisation can be recognised. The irrationality is irrational emotions that derived from predestination, the purpose refers to glorification, while the process of rationalisation refers to the process of value formation and dealing with the irrationality.

[127] Weber, *Protestant Ethic and the Spirit of Capitalism*, xxix.
[128] Ibid.

As Weber described, Calvinists believed in predestination, they could not be certain whether they were the chosen ones, and therefore, they were shrouded in uncertainty, anxiety, and fear; to ease these emotions, they decided to conduct their calling through a methodical way of living.[129] From this example, I notice that the value formation appeared in the belief and the irrational emotions, which motivated the believers to live in an assiduous lifestyle and so initiated the process of rationalisation. This means that this decision is not a spontaneous result, but an action to deal with irrational emotions. The aspect of emotions reveals the link between expectation and predictability. As mentioned above, Calvinist chose to live methodical lives because they were influenced by predestination; based on this belief, their methodical actions brought hope to them and eased their anxiety, uncertainty, and fear, which meant that their belief sparked their actions. It also made them believe in a better afterlife. Thus, being methodical is a continuous process that shows believers were struggle with irrational emotions, but most importantly, it also shows the process of how the believers took actions based on their considerations, thinking, and purposes.

Reflecting this feature to European art music, it starts with the methodical structure of musical elements and the approach of dealing with irrational elements. In *The Rational and Social Foundations of Music*, Weber introduced the basic facts of the harmonic chord system, the irrational properties of the dominant seventh chord, the melodic determination of chord progressions, and the rationalisation in terms of the numbers. He dissected Pythagoras' studies and presented a formula $\frac{n}{(n+1)}$ resting upon a vibration ratio, which he used to present a subdivision of intervals to demonstrate the arithmetical existence of music, which he demonstrated with an octave ($\frac{1}{2}$) that harmonically divided into a fourth ($\frac{3}{4}$) and a fifth ($\frac{2}{3}$); the fourth could be subdivided into a major third ($\frac{4}{5}$) and a major halftone ($\frac{15}{16}$), and the fifth that comprised a major third ($\frac{4}{5}$) and a minor third ($\frac{5}{6}$). Those constructions of intervals, presented in arithmetical formation, are the fourth $\frac{4}{5} \times \frac{15}{16} = \frac{3}{4}$, the fifth $\frac{4}{5} \times \frac{5}{6} = \frac{2}{3}$, and the octave $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$.[130]

[129] Weber, *Protestant Ethic and the Spirit of Capitalism*, xiii, 60.
[130] Weber, *The Rational and Social Foundations of Music*, 4.

Weber explained that when an octave is the basis of the tonal system, the fifth and the fourth play their roles of being the theoretical basis of tuning and melodic fundamental interval.[131] These positions consist of the foundation of modern tonality and went on to build the rationalisation of the tone system and temperament. Along with this, Weber extended the idea of the formation of chords, whereby he listed the irrational property of chords as the minor seventh in the minor scale and pointed out that numbers 2 and 3 were the foundations of rationalisation.[132] For the approach of dealing with irrational elements, e.g. asymmetrical structure of intervals, by harmonic and melodic systems. These systems then produce expectation and predictability of acoustic results. For instance, when a harmonic progression stops at the dominant seventh chord, it is expected that the tonic chord will follow with a musical phrase, raising an expectation that the harmonic progression will lead to a tonic chord. Hence, expectation and predictability signal an inherent methodical feature. It appears that Weber slightly altered his original meaning in order to make it fit to music, but the central concept remained the same. His short description also conveys other aspects that are worthy of attention, such as the functions of religion and societal interactions.

3.1.2 The Second Rational Feature: Function/Functional

The 'functional' feature gives rise to a wide-ranging discussion. Considering Weber's theory of rationality was started from his observations of Protestantism, this section focuses on asking, what is the religion's position of the developments of a society? How did religion influence believers? How did people act, including by innovations and improvements, while under the influence of religion?

In Weber's description in *The Protestant Ethic and the Spirit of Capitalism*, the religious influences can be seen at a psychological level. As previously explained, Calvinists' belief of predestination trapped them in the irrational fears, however, by following the doctrine they also devised a method to ease these irrational emotions. This means that the religion created loneliness, anxiety, uncertainty, and fear, but it also provided solutions to the mental predicament, and therefore, these emotions, which Weber saw as deriving from the idea of predestination, became motivations that stimulated believers into achieving success. Thus, religion acted as a catalyst that spurred believers to live worldly, methodical lives, convincing

[131] Ibid., 54.
[132] Ibid., 6.

them that success in a particular calling was a sign of being a chosen one.[133] From the description, the process of rationalisation can be recognised; believers received the idea of predestination, they interpreted the idea, and then they took actions accordingly. When applying this idea to the development of European art music, the importance of monastery is distinctive.

In *The Rational and Social Foundations of Music*, he dissected the development of organs and presented the importance of church and monastery to demonstrate their functions. He emphasised that 'only ecclesiastical use offered a solid basis for the development of this instrument', and that 'the monastery organisation was the only possible base on which it could prosper'.[134] The monastery organisations were considered to be ideal fields for the musical instrument organ, because of the multiple identities of monks. The custom was that organists and organ builders had to be affiliated to monasteries, either as monks or technicians, hence they were responsible for upkeep of the organs, which led to the development of polyvocality.[135] Because organists and organ builders were either monks or technicians, they were in advantageous positions to adjust their organs in order to further develop polyvocality, which was an embryo of harmony.

In summary, when Weber postulated the rationality of music, he emphasised the function of religion, showing how he altered the original idea of Protestantism to fit with music without changing its central concept. From examples of Protestantism, it can be seen how religion motivated believers to pursue their calling and achieve success; these suggest values of purposive rationality. For the development of music, the function of religion refers to an invisible field that offered an ideal condition for organs.

3.1.3 The Final Rational Feature: Interactivity/Being Interactive

The final feature is general interactions at the societal level, regarding which Weber covered a huge range of periods; he directly pointed out that the popularity of the piano was deeply connected to market needs, the rise of the middle class, and local weather. Weber emphasised the advantages of the piano, such as its ability to sustain its tone, increase or decrease the

[133] Mitzman, *The Iron Cage*, 190.
[134] Weber, *The Rational and Social Foundations of Music*, 114.
[135] Ibid., 115.

volume, and the beautiful perfection of its chords.[136] The 'accidental' appearance of Mozart, Liszt, and other virtuosos drove people's appreciation, which increased the needs of music publishers and concert managers, finally leading to the success of the piano. From a long-term perspective, this success was also supported by the rise of the middle class. The piano was particularly popular in Nordic countries because cold weather made people stay at home; therefore, the piano, which required only a moderate space, became popular.[137]

When examining the ideas of the theory of rationality, I notice that Weber's focus was the interactions at the societal level, as his emphasis in relation to Protestantism related to how Protestants were affected by their beliefs, how their decisions shaped societies, and how their actions generated the spirit of capitalism. Following this idea, people's decisions and preferences are clearly visible in his descriptions. The interaction at the societal level can be unintentional and intentional. This is because the formation of capitalism is not an intentional result that the Calvinists aimed for when they contributed themselves to pursue callings. Rather, they purposively decided to live in an assiduous living style by underwent a serial process of rationalisation in terms of how they took actions based on their considerations, thinking, and purposes. This means that when Calvinists worked hard, looked for profits, and lived methodical lives, their efforts accidentally generated the spirit of capitalism. The formation of capitalism was an unpredictable development. The surprising development can also be found in influences of virtuoso; Mozart increased the needs of music publishers and of concert managers to satisfy the large music consumption of the mass market, Chopin entirely contributed his techniques to piano, and Liszt enlarged expressive possibilities of piano,[138] all the developments contributed to the success of modern piano.

To put it simply, such unpredictable influences stimulate more intentional interactions and thereby form the development of societies. Capitalism concerns individual people making profits, calculating interests, and developing business plans; their calculations and decisions indicate different forms of rationality, and together, these rationalities construct the society. Analogous conditions appear in music. For instance, different musical periods produced corresponding musical styles that represented integral musical tastes involving an individual's

[136] Ibid., 121.
[137] Ibid., 125.
[138] Ibid., 121–2.

preferences, aesthetics, and perceptions. The interactions at the societal level can be variable, the central idea focuses on formation.

In short, from the discussion above, internal and external conditions that drove rationality and rationalisation are presented. In *The Protestant Ethic and the Spirit of Capitalism*, individuals' emotions and desires act as internal conditions, while religious influences act as external conditions. When Weber applied these ideas to music, internal conditions meant tonal materials, from attunement, compositional techniques, and aesthetics, while the external conditions referred to the development of bourgeois musical culture, technologies, and productions.[139] Examining Weberian rationality in Protestantism and European art music, I propose the rational features are: (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive. These features will be applied to the *Solo for Piano* to demonstrate the applicability of the theory of rationality.

3.2 The Irrational Features in European Art Music

When spontaneous expression cannot be comprehended by common sense, or it does not follow rules or obligations, it tends to be 'irrational'. Following this condition, the irrationalities in European art music can be recognised as: (i) unsymmetrical structure, (ii) melodiousness, and (iii) intuitiveness.[140]

Firstly, beginning with unsymmetrical structure, as previously explained, European art music is rational because of its methodical, systematic, and structural construction. Yet, it does not mean that every instance of arithmetic leads to rationality. Weber used the diminished triad of a dominant seventh chord as an example of irrationality, illustrating that the diminished triad

[139] Konoval, 'Max Weber and the Sociology of Music', 479.

[140] Weber declared 'primitive' music also to be an example of irrationality. However, this research focuses on European art music, so primitive music is not included in the discussion. Other examples of irrationality, for instance, Thomas Segady (1993) suggested the irrationalities may include: harmonic-structural irrationality, interpretational irrationality, and interactional irrationality in his article 'Consequences of the increasing rationality of music: A reassessment of Weberian rationalization'(1993). He dissected Weber's irrationality through the mathematical basis of European art music, musicians' unprecise interpretations, and hierarchies in symphonic organisations. Looking into his assessment, it shows another interpretation of irrationality, and surrounds with mathematical components, the influence of musical instruments' developments, and monopolisation within symphonic organisations. His interpretations are certainly different from my suggestion of irrationality in European art music; however, as Roth (2007, p.90) concludes, Weber's text has shown scholarly value through its very openness. Thus, I contemplate three irrational features: (i) unsymmetrical structure, (ii) melodiousness, and (iii) intuitiveness, to show a reassessment that is closed to the origin of the theory of rationality.

acts as a remainder, which comes from an unsymmetrical structure in the chord.[141] The first unsymmetrical structure in Weber's finding was the difference between apotome and limma, called the 'Pythagorean comma'; this difference was produced by different calculations. Weber extended this idea to the dominant seventh chord and found that the diminished triad was the remainder from an unsymmetrical structure of chords. Following this idea, he proposed the unsymmetrical structure to be the irrationality in European art music. The intertwined relationship of rationality and irrationality emerged in an unsymmetrical structure, which both produced the remainder, diminished triad, and methodically formed chords. The dissonance of the diminished triad plays an important role in European art music; it provides tension and source of emotional expression to music. Without this dissonance, music would become a series of movements of parallel intervals. In this way, the unsymmetrical structure takes essential part of produces melodies.[142]

By following this, I propose the second rational feature: 'melodiousness', which is produced from the first irrational feature. This feature reveals spontaneous expression that elicits performers' emotions and intuition. When speaking of emotions, a clear connection between emotions and irrationality can be found in *The Protestant Ethic and the Spirit of Capitalism*; Weber judged that superstition, self-interest, impulsiveness, anxiety, uncertainty, and fear are primitive sensations that do not contain purpose, value standards, or a sense of motivation, and therefore, he classified them as irrational.[143] Applying this idea to European art music, this irrationality is in the form of spontaneous expression. The spontaneous expression relates to the performers' expressive actions, which prioritise emotions[144] and aim to build the affective connections between the performers and the listeners.[145] Putting it differently, when the performers execute their interpretations, their expressive actions concretise and transmit the embedded emotions to the listeners. The spontaneous expression leads to the last feature of irrationality: intuitiveness. For instance, a common harmonic progression in C major is I—IV—V—V^7—I, in the absence of any expression marks, performers will emphasise the dominant seventh chord, then proceed gently to the final cadence. From a listeners' perspective, on hearing the dominant seventh chord, they expect a soft cadence acoustic effect. This intuitive action and expectation carved into performers and listeners' mind; we will feel odd if

[141] Weber, *The Rational and Social Foundations of Music*, 7.
[142] Ibid., 10.
[143] Weber, *Protestant Ethic and the Spirit of Capitalism*, 18, 38, 112.
[144] Goldthorpe, 'Rational Action Theory for Sociology', 188.
[145] Loeffler, '"The Only Real Magic"', 13.

the harmonic progression stops at the dominant seventh chord. An interesting question here is, since intuitive reaction is such reasonable, why should it not be rational? To answer this, I suggest, the intuitive reaction is both rational and irrational; it can be rational because it was derived from harmonic functions; it should be irrational, as it arises from impulsive and intuitive expressions. To present my point, I will provide further discussions in Chapter 2 Section 4.2 and Chapter 3 Section 2.

In summary, in Protestantism, irrationality as being spontaneous expression – an emotion that is either unreasonable, self-interested, intuitive, or impulsive. When Weber attempted to stretch this idea to music, the three irrational features in European art music are evident in the construction and expression of music. The former refers to the unsymmetrical structure, which produced the diminished triad and resulted in melodiousness. The feature of melodiousness derives from the tensions between dissonance and consonance, while the tensions relate to the performers' expressive and intuitive actions.

4 Rationality and Irrationality in J. S. Bach's Prelude BWV. 846

In this section, I will apply the three rational features: (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive, to demonstrate how rationality works in J. S. Bach's Prelude BWV. 846 to show how rationality brings out irrationality. Additionally, at the end of this section I will present another angle of interpreting rational features and take Schenkerian analysis as the example.

4.1 The Methodical Feature and Emotional Result

As the title suggests, methodical feature refers to arithmetical construction, such as the construction of the fifth as $\frac{4}{5} \times \frac{5}{6} = \frac{2}{3}$ and other intervals, as described in the previous section. When this type of construction is investigated in respect to European art music, it can also be found in harmonic progression. The intention here is to show the feasibility of there being multiple interpretations of a methodical feature, and to bridge the gap that Weber left in his application of the theory of rationality.

The methodical feature contains three factors: (i) arithmetical, (ii) structural, and (iii) theoretical (Figure 16), and each factor connects with the others, forming this rational feature. Taking Prelude BWV. 846 as an example, the measures 1 to 11 show the harmonic progression

to be $\mathrm{I}-\mathrm{ii}^{7}-\mathrm{V}_{5}^{6}-\mathrm{I}-\mathrm{vi}^{6}-\mathrm{V}_{2}^{4}-\mathrm{I}^{6}-\mathrm{IV}_{2}^{4}-\mathrm{ii}^{7}-\mathrm{V}^{7}-\mathrm{I}$ (Figure 17). This can clearly be regarded as a structural factor, especially as the progression shows the structural construction vertically. For instance, notes in the first measure are C, E, G, C, E, G, C, and E, which is a broken chord of a triad in C major. When mixing these notes together, chordal verticality is restored. Along with being a vertically structural construction, the horizontally structural construction is also evident. For instance, in measures 1 to 2, it can be seen that the horizontally structural construction is formed of a series of semiquavers ($\mathrm{C}-\mathrm{E}-\mathrm{G}-\mathrm{C}-\mathrm{E}-\mathrm{G}-\mathrm{C}-\mathrm{E}$), which maintains the same rhythmic structure with different notes in the following measures (Figure 18, noted by arrows). Meanwhile, another horizontally structural construction is shown in the first note of each measure (Figure 18, noted in boxes).

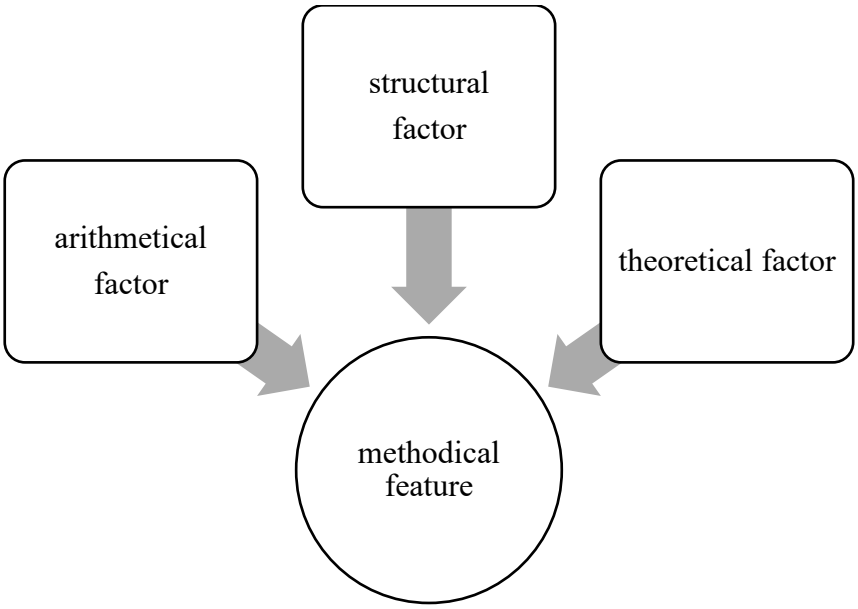


Figure 16 Three rational factors of a methodical feature.

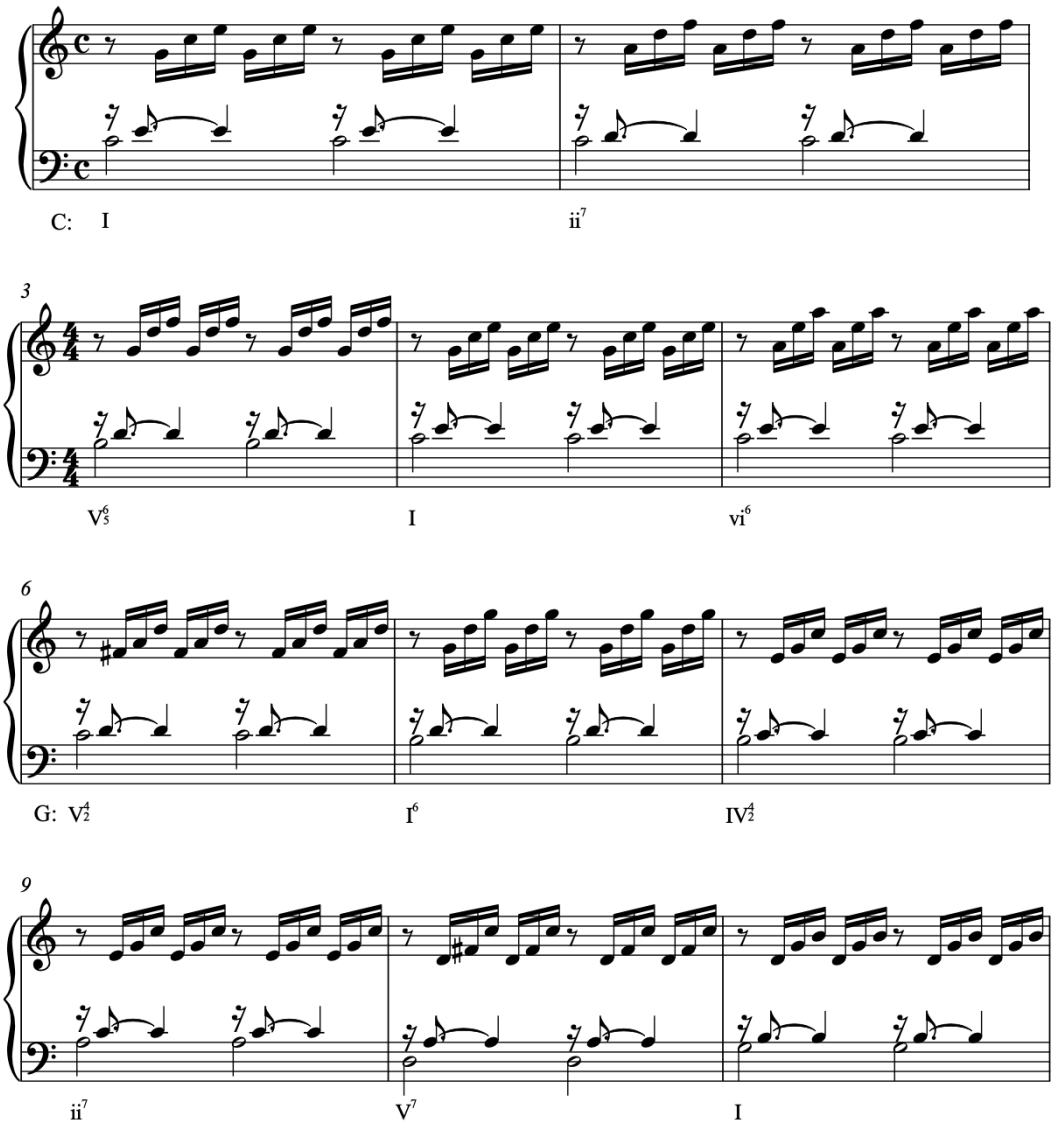


Figure 17 Harmonic progression of Measures 1 to 11 in Prelude BWV. 846.

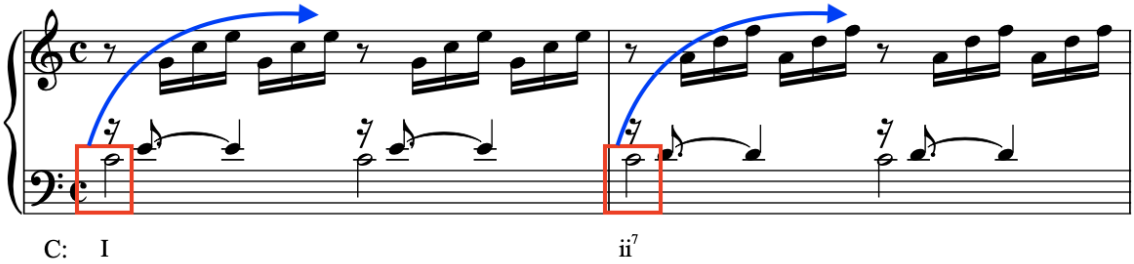


Figure 18 The horizontally structural construction in Prelude BWV. 846.

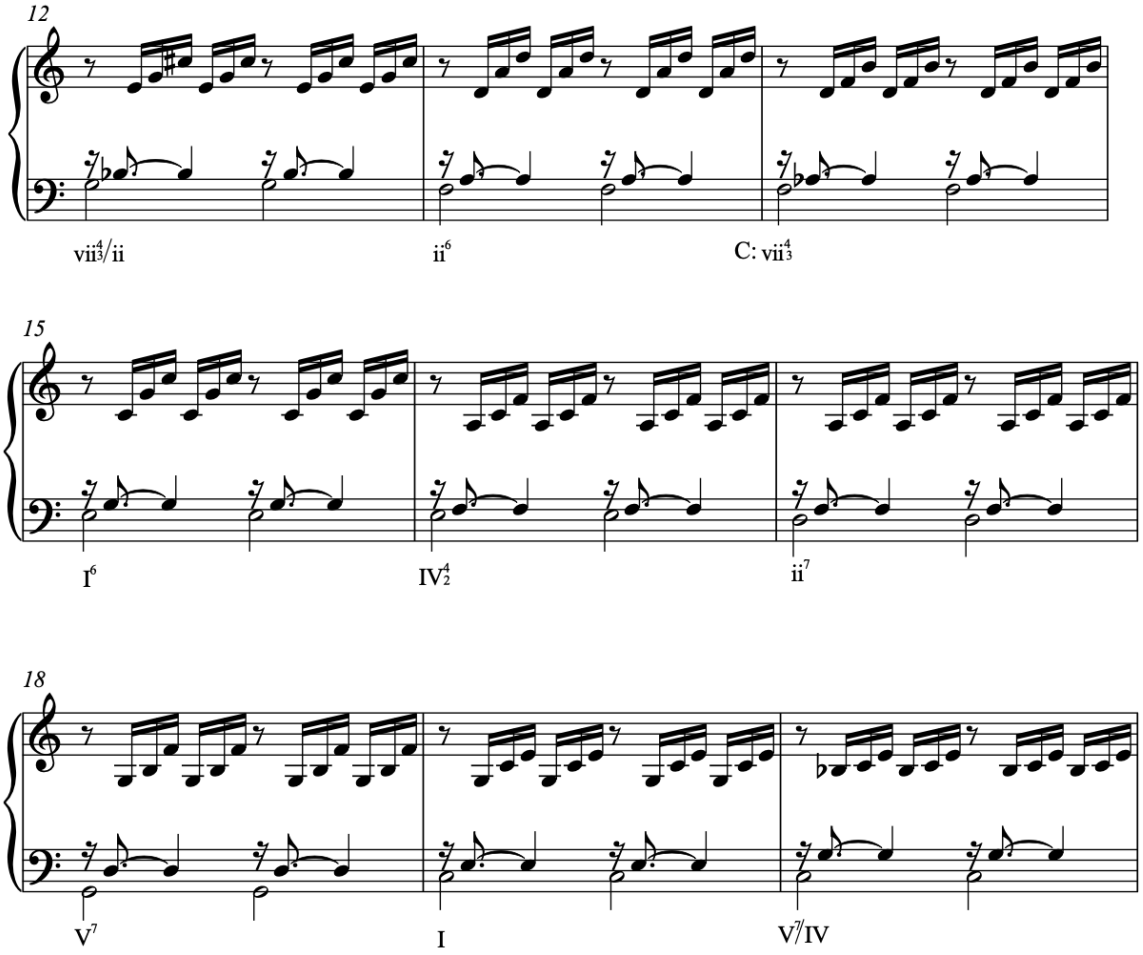


Figure 19 Harmonic progression in Measures 12 to 20 in Prelude BWV. 846.

Looking at the following musical phrase, the harmonic progression is vii$^{4}_{3}$/ii - ii^{6} - vii$^{4}_{3}$ - I^{6} - IV$^{4}_{2}$ - ii^{7} - V^{7} - I - V^{7}/IV (Figure 19). The harmonic progression of measures 21 to 35 shows as IV^{7} - vii^{7}/V - IV$^{4}_{2}$ - V^{7} - I$^{6}_{4}$ - V$^{7}_{4}$ - V^{7} - vii^{7}/V - I$^{6}_{4}$ - V$^{7}_{4}$ - V^{7} - V^{7}/IV - IV$^{6}_{4}$ - V^{7} - I (Figure 20). Along with the harmonic progression, repeated rhythmic structures are interlaced with different broken chords.

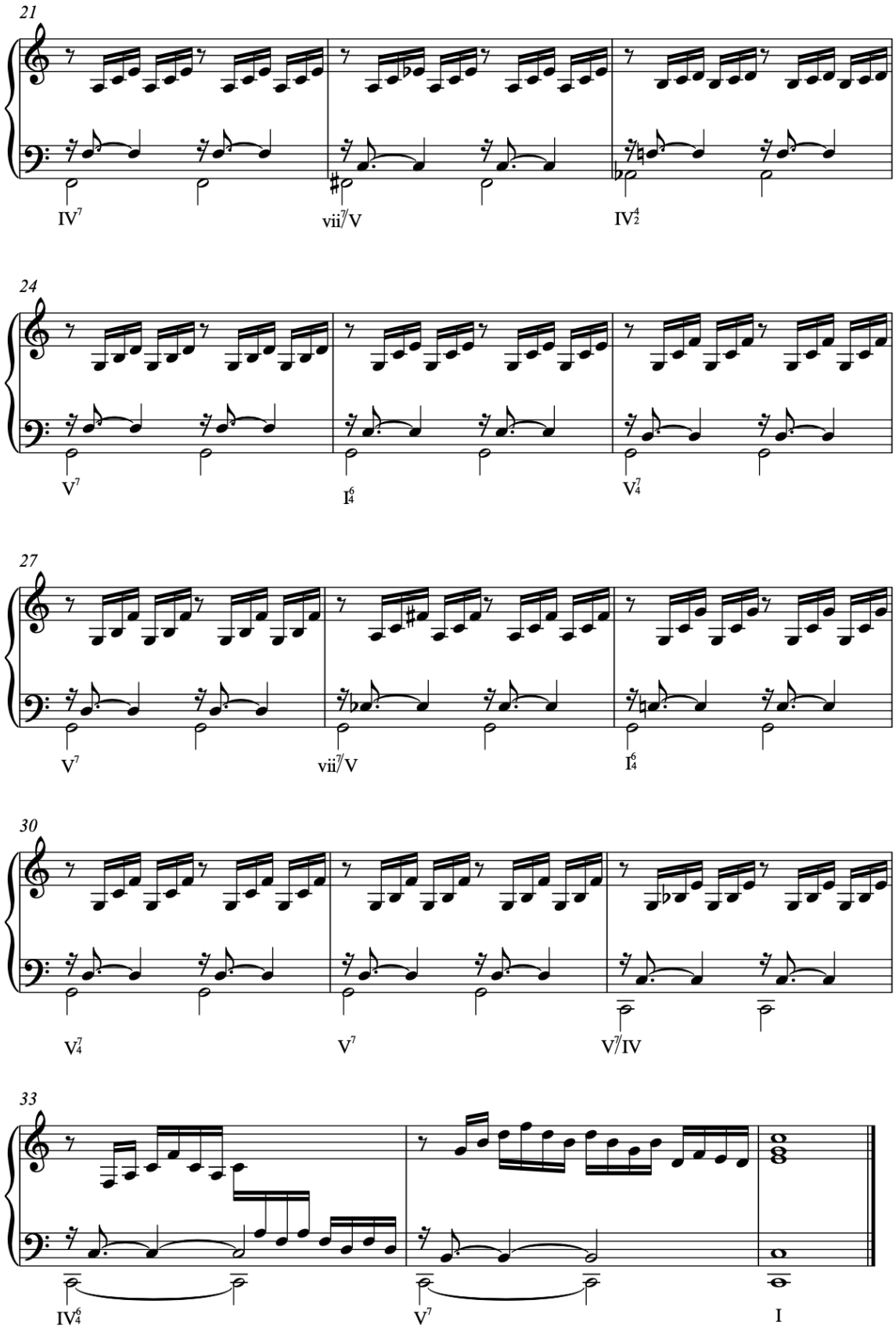


Figure 20 Harmonic progression in Measures 21 to 35 in Prelude BWV. 846.

When Weber interpreted European art music and discovered structural and systematic characters, he took mathematics, chords, and harmonic progression as examples to show different layers of structure, where musical elements function as introducers or connectors to structure music. In this way, he also presented functions of musical elements. While focusing on functions, another type of analytical approach may be worthy of attention – Schenkerian analysis – which provides another layer of structure. Schenkerian analysis is a dominant approach to analysing tonal music, by using terms such as 'prolongation', 'diminution', and 'transformation' to explain recursive musical events and to show the hierarchical structure of music.[146] By using this approach, another layer of structural and theoretical factors can be added to a work. The previous discussion showed the harmonic progression in Measures 1 to 14 is $\mathrm{I}-\mathrm{ii}^7-\mathrm{V}_5^6-\mathrm{I}-\mathrm{vi}^6-\mathrm{V}_2^4-\mathrm{I}^6-\mathrm{IV}_2^4-\mathrm{ii}^7-\mathrm{V}^7-\mathrm{I}-\mathrm{vii}_3^4/\mathrm{ii}$ - ii^6 - vii_3^4. This progression can be interpreted as the process of C major—G major—C major (Figure 21, the red and blue boxes).

The idea of prolongation of Schenkerian analysis indicates how a melody extends itself with a clear, or unclear, sense of direction, while the melody aims for its ultimate goal, which is the full cadence at the end of the work. During the process, the melody may experience modulation, superposition, or voice exchange, no matter the journey it undertakes, it is always on the track of harmony. Regarding the example above, the C major modulates to G major, and then returns to C major in Measure 14 (Figure 21). These three measures reveal three small sections: C major (Measures 1 to 5), G major (Measures 6 to 13), and C major (Measures 14 to 19), or in Schenkerian terms, a prolongation of the I of C major. Cook described these types of motions within music as showing a goal-directed implication, meaning that the motion is towards a harmonic goal.[147] Inserting this idea to this piece, Measures 1 to 19 includes a process of stasis–motion–stasis.

[146] Temperley, 'Composition, Perception, and Schenkerian Theory', 146.
[147] Cook, *A Guide to Music Analysis*, 29.

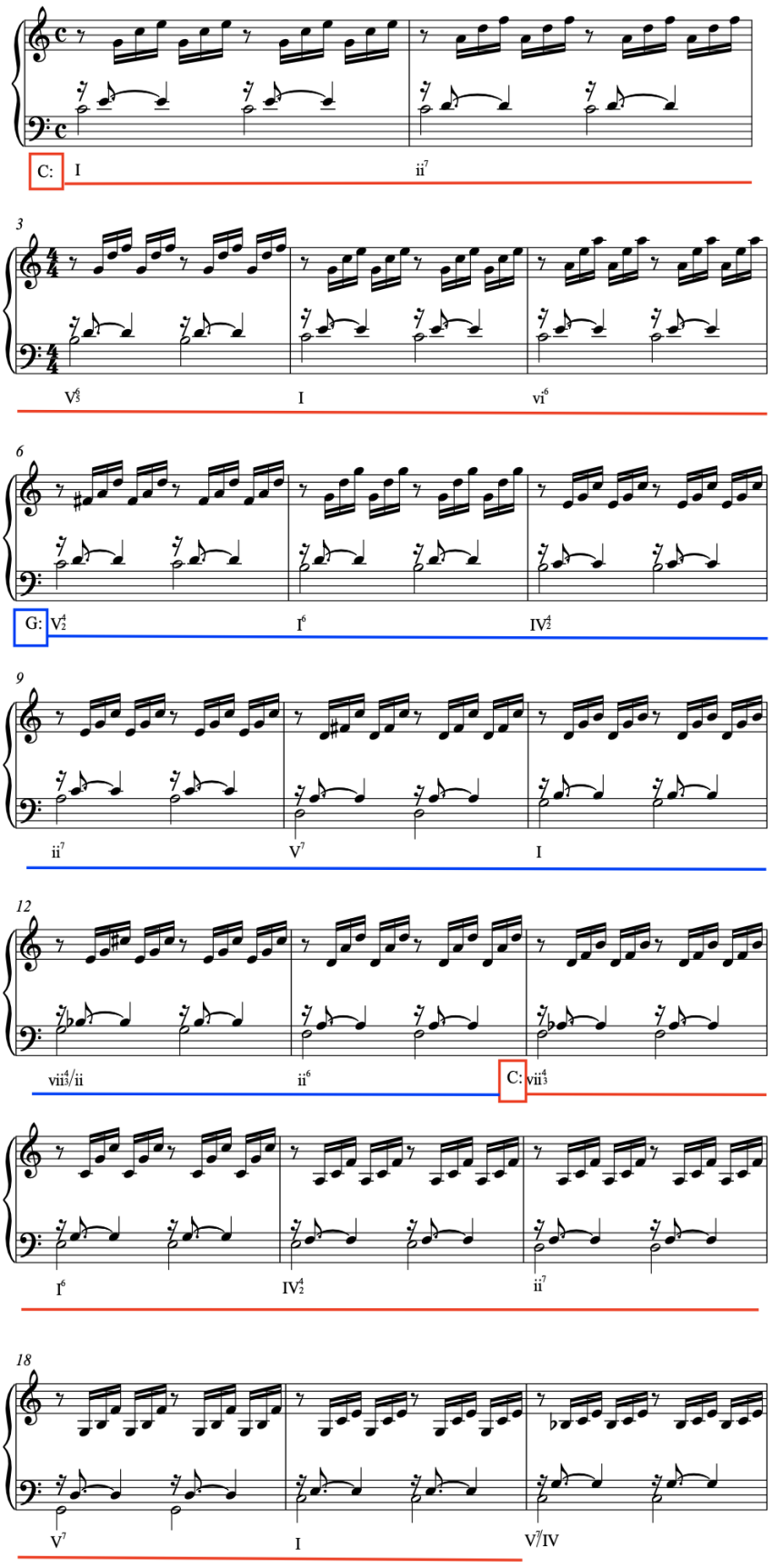


Figure 21 Sections in C and G major in Prelude BWV. 846.

By applying the process of stasis–motion–stasis to the rest of the measures, it may be seen that a larger process of stasis and motion occurs; first, the beginning and the end need to be clarified. As explained above, Measures 1 to 19 show the prolongation of the I of C Major, at this point this prolongation represents the beginning (stasis). However, where is the next and the last stasis – at the end? Based on Schenker's analyses, the cadence starts in the Measure 32 (Figure 22). Nevertheless, it does not mean that the end starts in the same measure. Considering the expectation of the arrival of the final cadence, the V in the Measure 24 must be brought into account, which means Measures 24 to 32 represent the end – stasis. Returning to Measures 22 and 23, two special notes, F♯ and A♭, create a sense of uncertainty, while the Measure 21 includes notes F and E, dissonance emanating from them bring out tension. Meanwhile, the Measure 20 (V^7/IV) acts as a preparation for Measure 21 (Figure 23). Taking account of the sense of uncertainty, dissonance, tension, and preparation, Measures 20 to 23 belong together, in that senses both motivate the piece and offer a sense of direction; therefore, they constitute that part of motion that appears between its beginning and its end. In short, this Prelude shows a process of stasis (Measures 1 to 19) to motion (Measures 20 to 23), and motion to stasis (Measures 24 to 35).

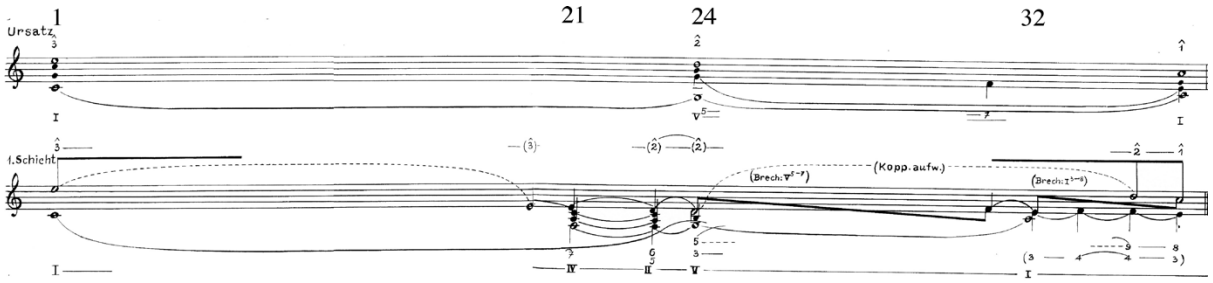


Figure 22 Schenker's middleground and background analysis of J. S. Bach's Prelude in C major.[148]

[148] Temperley, 'Composition, Perception, and Schenkerian Theory', 161.

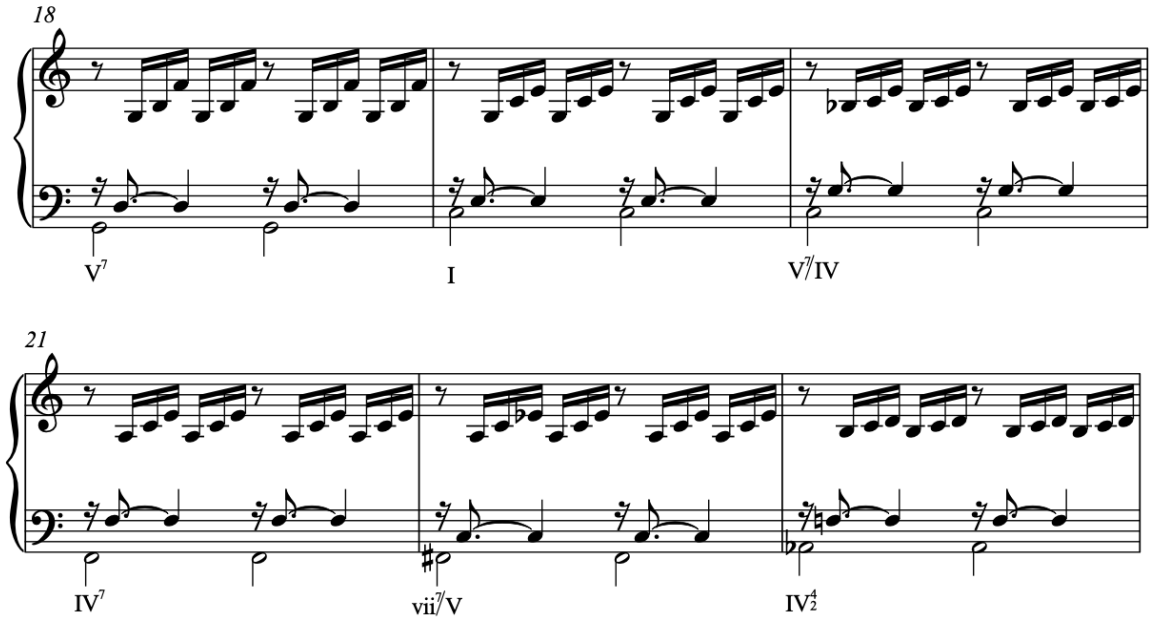


Figure 23 Harmonic progression in Measures 18 to 23 in Prelude BWV. 846.

Regarding this aspect, another layer of structural and theoretical factors widens their effect so that harmonic progression can be generalised by way of a progression of keys and harmonic functions. The idea of Schenkerian analysis is to interpret rationality differently, suggesting that the rational feature of methodicalness in this section is not limited in its harmony. Schenkerian analysis was disseminated by way of publications by Adele Katz's *Challenge to musical tradition: a new concept of tonality*[149] ([1945] 2007) and Felix Salzer's *Structural hearing: tonal coherence in music*[150] ([1952] 1982).[151] Therefore, it can be assumed that Weber would not have known about this approach, since he passed away in 1920. This development provides us to rethink and revisit the theory of rationality, and therefore, I will keep open-minded when applying the idea of rationality to Cage's *Solo for Piano*.

In brief, the methodical feature consists of arithmetical, structural, and theoretical factors with each factor showing at different angles through similar musical elements. The arithmetical factor indicates the arithmetical construction of an inner level, such as intervals and chords. This factor also involves integral structures; in this sense, the structural factor is another appropriate description, as it includes harmonic progression and vertical and horizontal

[149] Katz, *Challenge to Musical Tradition*.
[150] Salzer, *Structural Hearing*.
[151] Bent, 'Analysis', 58.

constructions. These constructions can be found in different combinations of notes, such as broken chords, and in a series of rhythmical structures. Such combinations refer to a theoretical factor, such as harmony. Along with these, methodical features may, according to Schenkerian analysis, be extended, which reveals a different aspect of both structure and theory. Under these comprehensive arrangements, European art music seems to be rational and full of arithmetical, structural, and theoretical factors. However, when performers interpret music, they mostly involve their emotions; hence, listeners who appreciate music are affected by the expressions of its performers, which produces irrational results; hence, such irrationality explain both emotions and affective expressions. The contradiction between rational constructions and irrational interpretations may appear clearer when viewed through two other concepts – disenchantment and enchantment.

Disenchantment may be explained as 'dis-godded', a synonym Weber borrowed from Schiller; alternatively, it could be seen as a rejection of mysterious powers, such as miracles and magic.[152] When this term is applied to European art music, it refers to method, structure, and theory. Interestingly, when written music is transferred to sound, it enters a process of enchantment – from rigorous structure to free interpretation. Consequently, music shows its domain of affectivity through which people share affective connections with others.[153] Take Glenn Gould's interpretations as an example. In his album entitled, *Bach: The Well-Tempered Clavier, Books I & II, BWV 846-893* (catalogue number: 88725412692)',[154] he performs measures one to seven gently and calmly with his interpretations that feature clear leaps at the end of each motif (C–E–G–C–E–G–C–E, and subsequent broken chords). When the first musical phase is played, listeners expect streamlined melodies; even though his interpretation is calm, with no clear difference between loud and soft, they remain attracted by the compositional arrangement, although they expected a peak to the phrase.

Another example is Franz Schubert's *Gretchen am Spinnrade* Op.2 D.118 (1814). When the accompanist pressingly plays a series of semiquavers, to listeners it seems as if they can 'see' the wheels of the carriage spinning, and they can 'feel' the disappointment and broken heart of Gretchen. In this piece, not only is the D minor key an important position, but also the rhythmical structure plays an essential role in implying musical meaning, thereby informing

[152] Yelle, *The Language of Disenchantment*, 7.
[153] Loeffler, '"The Only Real Magic"', 13.
[154] Gould, *Bach: The Well-Tempered Clavier Books I & II, BWV 846-893*.

the scene, ambience, and mood. With such scope of emotions and feelings, by its disenchanted structure, it is noticeable how the music bespeaks of 'affective inheritance'. Returning to the theme of the irrational features of European art music, in this case, its melodic and intuitive features, the affective inheritances that are carried out by music was why Weber considered it to be irrational. However, Weber may have missed the function of musical elements. As described in the examples above, motive, musical phrase, key and rhythmic structure convey meanings and are responsible for building scenes and for transmitting messages to listeners; these functions become clearer when comparing European art music with indeterminate music. Therefore, I propose that the theory of rationality should be revised. This will be discussed later in the following chapter.

4.2 The Functions of Neglected Musical Elements

When Weber used ratios of the Common Practice scales to illustrate the rational features of European art music, the example showed his intentions and the inner construction of music clearly. Yet, this is hard to build connections with readers, and it may lose the central idea of music. When I read it, my head felt as if it was experiencing explosions; too many questions were posed, but no answers. I wondered where exactly the music was. This doubt continued throughout the remaining parts. When Weber wrote about 'the second rational feature', which is function/functional, it was mainly about how religion played a role in society – as I mentioned previously, religion built an invisible field, which, for him, became the essential condition for developments of musical organs. Surely, these contents should be understood as cultural and historical, without which, no musicological research can be completed.

However, when he referred to unsymmetrical structure of arithmetic and emotions as being 'irrational', he may overlook the functions of musical elements. Putting this in another way, musical elements present their functions on motivating musical movements, which produce emotions, but Weber seemed to conclude rational movements and irrational emotions as the same finding. By following this, I intend to carry on discussing the functions of musical elements, by suggesting that the harmonic function can be discussed diversely and potentially developed overtimes. From the theoretical aspects, I expect to provide a brief discussion suggesting that melodiousness is built upon harmonic functions, which are distinctly carry rational thinking of individuals.

As suggested earlier, Schenkerian analysis leads to the process of stasis ─ motion ─ stasis in the whole Prelude. Regarding 'motion', I considered its basic condition its sense of direction, which is formed by uncertainty, dissonance, tension, and preparation; yet, these senses are produced by harmonic functions, which, is a contestable topic, about which scholars have expressed different views. Taking a IV chord as examples. Allen Forte advocated that a IV chord should be regarded as a predominant chord,[155] which acts as a preparation for dominant chords, however, this stance was partially disagreed with by Deborah Stein, who suggested that a IV chord includes two functions: (i) as a preparation for the dominant chord, and (ii) as an extension of a tonic chord,[156] Kostka & Payne then clarified, a IV chord contains three harmonic functions: predominant, subdominant, or pre-predominant,[157] while Swinden proposed, the function of chords should depend on harmonic contexts – when a IV chord approaches an I chord it can only be a subdominant chord, and when the IV chord should be dominant preparation.[158]

Among the researchers mentioned above, one distinct fact emerges – that chords can act as a connection between what went before and what comes after, because they contain harmonic functions regarding relations between functions and emotions; again, it was on these grounds that Weber claimed that European art music contained irrationality because of its emotional expressions. However, as I explained in the previous section, music underwent a process of enchantment, from rigorous structure to affective interpretation; the main factor affecting this transformation was harmonic function, which led to affective functioning. Based on Schenkerian analysis, the functions are evident in the process of motion, which derived from dissonance. By analysing Weber's description, I notice that in his observations he listed the function of dissonance by stating that 'a dissonant chord demands resolution into a new chord to relax its inherent tension, so it is the basic fact that motivates harmonic progression'.[159]

This statement may differentiate the functions – i.e. the inherent tensions that motivate the music, resulting in harmonic progression, which contains stasis and motion, which, in turn, motivates performers and listeners through their emotional experiences. By applying emotional

[155] Forte, *Tonal Harmony in Concept and Practice*.
[156] Stein, 'The Expansion of the Subdominant in the Late Nineteenth Century', 153.
[157] Kostka and Payne, *Tonal Harmony, with an Introduction to Twentieth-Century Music*, 114.
[158] Swinden, 'When Functions Collide', 253; as cited in White and Quinn, 'Chord Context and Harmonic Function in Tonal Music', 314–15.
[159] Weber, *The Rational and Social Foundations of Music*, 6.

expression to conventional music, it therefore becomes melodic and lyrical and is rendered singable. The formation of these features is based on rational musical elements – i.e., structural, systematic, and functional. Consequently, harmonic functions produce harmonic anticipations, assisting performers to prepare mentally.[160] Therefore harmonic functions not only show their importance for carrying out stasis and motion, but they also reveal aspects of anticipations and mental preparation; together, therefore, that is their affective function. Here, then, is the feature Weber ignored, since he turned his attention to the transformation of the process of music production, by focusing on the relationships between dissonance, harmonic function, tension, and musical rationalisation. Consequently, his discussions were concerned with calculation, musical instruments, and musical rules.[161] Therefore, the irrational facts that Weber found – for instance, dissonance from unsymmetrical structures, together with the features of melodies, which derive from tension – carry harmonic functions that offer a sense of direction to music, providing possibilities to value of musical rationalisation; however, in back-and-forth discussions, Weber neglected the functions that hide behind irrationality.

Regarding affective function, it may be rephrased as interpretations, how performers set their minds before, during, and after performances, and how they arrange their interpretations in advance. Hence, the third feature of rationality – interactivity – comes into account; therefore, the section covering the final rational feature, will discuss the original interactivity, which focuses on shaping societies by Protestants' decisions, preferences, and actions. When Weber extended this feature to European art music, he referred to the influences resulting from the development of musical instruments, by social conditions and people's preferences. Yet, as I expound in the final section of my thesis, the functions of harmonic chords will build up anticipations and mental preparation. Put another way, performers' interpretations reflect the interaction between the composer, the performers, and the musical work, all which shape both performers and performance in many ways. Based on the aspect of interactions, Jean-Jacques Nattiez and Molino's semiotic schema, as described below, may lead to intelligent discussions.

[160] Ford, 'The Harmonic Anticipation of Charlie Parker', 55.
[161] Weber, *The Rational and Social Foundations of Music*, li.

4.3 The Feature of Interactivity within Interpretations

Jean-Jacques Nattiez and Molino's semiotic schema explains both the relationship and statuses between the composer, the performers, the music work, and the audience. Nattiez explained the schema as communications, message delivery, and interpretational information. The classical schema of communication starts with producers, who deliver messages regarding their ways and intentions; this message will be received by receivers, who understand the message through their interpretations, knowledge, and backgrounds (Figure 24). However, according to Molino, the classic schema of communication is not precise, since it does not fully consider the possibility of 'interpretation'.

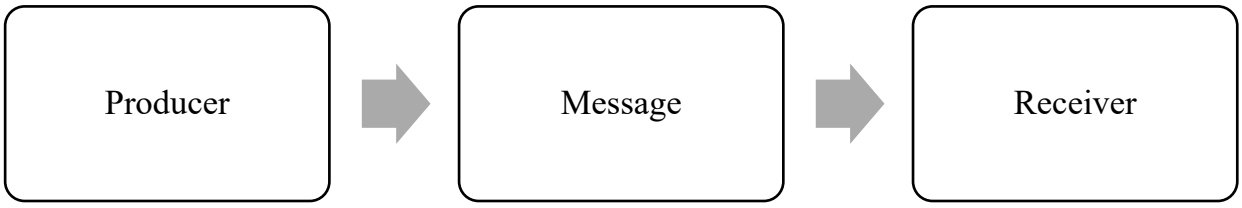


Figure 24 A classic schema for communication.[162]

According to Molino, the interpretation is not only about understanding messages, it also involves reconstructing messages from the receiver's angle. Since receivers reconstruct the message, the message should not be an intermediary existence in the process, rather, it should be neutral in that it cannot compel receivers to understand messages by following a producer's logic. Thus, words and a producer's action when delivering a message does not relate to a receiver's message comprehension. Consequently, Molino designed another schema of communication (Figure 25) by bringing both poietic and esthesic processes into consideration and identifying the message as material reality and immanent configuration of analyses.[163]

[162] Nattiez and Abbate, *Music and Discourse*, 16.
[163] Ibid., 15.

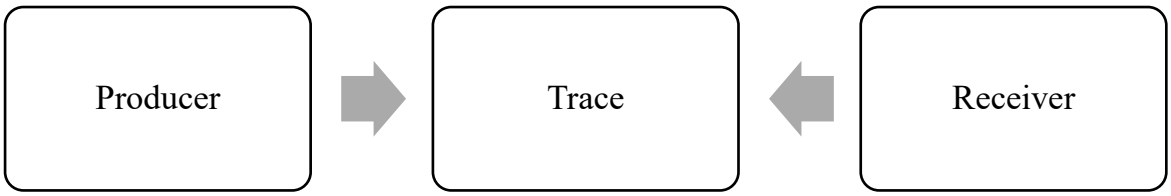


Figure 25 A revised schema of communication.[164]

Applying this schema to music, the position of producers is equal to composers, trace is equal to music work, and receiver is equal to performers. When composers compose a music work, they produce a material trace, which will be interpreted by performers. Putting this schema to Bach's Prelude, it is equal to Bach composing Prelude in C major, the work is then interpreted by Glenn Gould. Thus, interactivity is formed by both Bach and Gould; Bach created the musical framework, Gould practised the musical framework based on his understanding, interpretation, and, of course, his emotional expression. The main concept here is the two-way creation of trace – the first is from Bach, while the second is from Gould – they formed the Prelude in C major mutually. This schema will be applied to Cage's Solo for Piano, which is described in Chapter 5.

[164] Molino, Underwood, and Ayrey, 'Musical Fact and the Semiology of Music', 17.

Chapter 3. Revisiting Rationality

1 Introduction

Along with the criticisms that have been discussed in Literature Review, Turley and other critics point out flaws in Weber's rationality, specifically focusing on the rational result of musical organisations. Weber suggested that bureaucracy is an effectual means of forming rationally ordered societal actions, because it maintains authority that coordinates people to achieve tasks efficiently.[165] Nevertheless, this social instrument may not be an ideal measure in terms of the autonomy of musicians. As Stephen R. Couch argues, authoritarian work structures do not allow musicians to get sufficiently involved in an orchestra, no matter how professional they are, and a factory-like environment reduces the autonomy of individual musicians.[166] Judith R. Blau suggests that the level of bureaucracy is influenced by the urban environment, but a large music group is non-bureaucratic.[167] These arguments against Weber's prediction about how rationalisation will lead musical development to become highly formalised, standardised, and bureaucratised structures. This is because when Weber formulated his theory of rationality, he took an economically oriented view that was especially focused on market needs and the production of musical instruments and works; therefore, his rationality lacked a consideration of musical communities, he did not consider the role of musicians or different classes of musicians, and he was overly focused on the elite structures of medieval music.[168]

Therefore, with his research biases, Weber's prediction was problematic. These criticisms may reveal another flaw in his ideas about rationality, as he centralised relationships between people and other societal factors based on an economic-oriented method. Moreover, when reviewing other research regarding Weber's rationality in music, I noticed that some of them examined rationality through societal relationships rather than musical works. Is this because rationality in music too obvious to carry on? Or have we become caged in a consensus about Weber's undeniable contribution to the scope of the sociology of music? By acknowledging the above

[165] Weber, Gerth, and Mills, *From Max Weber*, 228; as cited in Blau, *The Shape of Culture*, 80.
[166] Couch,
'The Orchestra as Factory: Interrelationships of Occupational Change, Social Structure and Musical S tyle', 295.
[167] Blau, *The Shape of Culture*, 88,179.
[168] Turley, 'Max Weber and the Sociology of Music', 640, 647, 648.

criticisms and holding these questions in mind, this thesis intends to revisit Weber's rationality, but with a concrete core focused on music itself to demonstrate his central concept of rationality and rationalisation of music, which is still full of potential. Thus, this chapter will aim to revisit the theory of rationality through Cage's *Solo for Piano* to see how rationality can be different in terms of indeterminate music and graphic scores.

2 Rethinking Weber's Rationality in Music

As mentioned above, this chapter's focus will be on musical works – one is from European art music, Bach's Prelude BWV. 846, and the other is indeterminate music, Cage's *Solo for Piano* – to examine the development of rationality. Before moving into this examination, the central concepts of rationality need to be recognised. In Chapter 2 Section 3 *The Rational and Social Foundations of Music* (1921), I extracted three main rational features – (i) method/methodical, (ii) function/functional, and (iii) interactivity/being interactive – and applied them to Bach's Prelude BWV. 846 to demonstrate these features. When focusing on this musical work, a simple summary can be drawn, rationality of European art music is about theoretical foundation, methodical structure, and systematic notation. This summary does not oppose my statement arguing that the theory of rationality concerns reasons, beliefs, desires, choices, actions, unpredictabilities, and freedom. Rather, I suggest that all of these features are the core elements of constructing the theory of rationality.

When examining the elements of theoretical foundation, methodical structure, and systematic notation of European art music, it is not difficult to find traces of people. The two ways of dealing with inherent irrationality in music are evident examples revealing such traces. People took the irrationalities as a type of source for musical creation by bypassing or covering the irrationalities, and hence, the harmonic and the melodic systems were created from both approaches.[169] Thus, within the two systems, a new type of irrationality was formed, and this type of irrationality motivated the movement of music to create musical tensions. Putting this schematically, people devised approaches to deal with irrational elements. For instance, established the positions of musical instruments' holes by calculation; correcting pitch by hearing, detached music from religion, practised autonomy of art; and created market demands based on preferences, whether for aesthetic, practical, or religious purposes.

[169] Darmon, 'Weber on Music', 24.

Just as interactions in societal and personal levels were recognised, Weber tried to explain the popularity of the modern piano by it having been influenced by the rise of the middle class, market demand, geographical location, and weather – hence, when the middle class rose, pianos became their chosen instruments, since the organ required giant indoor space; in Nordic countries, the piano was more popular than other instruments, because severe weather conditions caused people to spend much more time at home.[170] In a bigger picture, these developments revealed the rational elements, forming the rationalisation of societies. Based on this, he emphasised the actions initiated by reasons, purposes, value, and intentions as rational actions, and took affect and tradition as the source of irrational actions. However, this sort of classification resulted in one essential rationality in European art music that seemed to be overlooked in Weber's ideas of theory of rationality in music.

My intention is not to overturn Weber's contributions or suggest a way to reclassify rationality in European art music; rather, I aim to suggest how his ideas can be interpreted after differentiating the affectual function and affectual result of music, as well as push the theory of rationality further along by emphasising subjectivity and bringing the focus back to music. Keeping this in mind and taking the openness of Weber's rationality into account, I propose that the intertwined relationship between rationality and irrationality may need to be reconstructed and suggest that the function of musical elements is a part of rationality, while the result of interpretations and expressions remains a part of irrationality.

2.1 Rationality or Irrationality: Affectual Function and Affectual Result

Firstly, turning to the basics of harmonic music. As previously explained, rationality is based on arithmetical, structural, and theoretical factors, while irrationality is based on asymmetrical

[170] Weber, *The Rational and Social Foundations of Music*, 124.

structure, melodiousness, and intuitiveness from them.[171] The interesting contradiction of Weber's statement is that the musical elements, such as seventh chords, are derived from melodic needs and show irrationality,[172] while this chord plays an important role in musical movement as it creates tensions and acts as a connection between different tonalities. With the crucial function, I am curious as to why he excluded the seventh chord from the spectrum of rationality? Was it because of its unequal divisions? My answer is because of the affectual function that are carried by seventh chords.

According to Weber's ideas, irrationality is not fixed or intrinsically irrational, but it shows contradiction among impulsive desires and logics.[173] Therefore, it seems reasonable for Weber to have divided elements involving affectuality, emotions, and expressions into irrationality, as they show the features of impulsiveness. However, as I discuss 'affective inheritance' in Schubert's *Gretchen am Spinnrade* as an example in Chapter 2 Section 4.1 The Methodical Feature and Emotional Result, affectuality, emotion, and expressions play their own roles in serving the purposes of informing a scene, ambience, and mood. These functions along with the expressive action that I discussed in the Literature Review, in which elements containing affectuality, emotions, and expressions may be carried out by purposive actions.

This means that the musical elements that Weber considered to be irrational are more than irrationality, they also reveal a deeper level of rationality. I do not intend to contradict Weber's statement about asymmetric structures showing irrationality, rather, I shall focus on his

[171] When discussing rationality and irrationality in music, we shall keep in mind that no system is complete or fully rationalised; rational and irrational elements interplay with one another. Weber pointed it out that even the harmonic tone system is unsymmetrical. However, it does not mean these elements are all irrational. The example for the asymmetrical structure is the diminished triad of a dominant seventh chord, which consists of major triad, diminished triad, perfect fifth, and minor seventh. The speciality of the diminished triad is that it acts as the remainder, which produces the tension, and it should resolve downward by step, so that the unbalanced acoustic can resolve to consonance. In other words, the resolution is for melodic needs, and therefore, the diminished triad should be accounted for through the needs. Putting this simply, to classify a musical element as irrational, the decisive factors include the asymmetrical structure, melodiousness, and intuitiveness. When focusing on the melodic needs of the diminished triad, this chord demonstrates irrational features. Nevertheless, one may argue that the diminished triad is the symmetrical triad as it contains two minor chords. This is true if we only consider the construction of the triad; however, the judgement should consider all features before classifying the diminished triad as rational or irrational. Considering the tension and melodiousness from the diminished triad, I suggest this element should be considered in irrationality.
[172] Weber, *The Rational and Social Foundations of Music*, 10.
[173] Kalberg, 'Max Weber's Types of Rationality: Cornerstones for the Analysis of Rationalization Processes in History', 1156.

statement saying irrationality is inherent in the process of rationalisation,[174] and suggest dividing his irrationality of musical elements into affectual function and result. This division may possibly extend his theory of rationality further, which can be helpful to compare rationality and irrationality in both European art music and indeterminate music. As I discussed in Methodology in Chapter 1, Nattiez's tripartite model can be an effective way to dissect participation from both the composer and the performer, as it considers the poietic level of the composer and the esthesic level of the performer. Here, based on this feature, I suggest using this model to discuss the affectual function and the affectual result to discuss the composer's arrangement and the performer's emotional expression.

For the affectual function, as the name suggests, this is an emotional-expressive function inherent in musical elements. In J. S. Bach's Prelude BWV. 846, which is based on C major, from Measures 1 to 7, harmonic progressions are I–ii^7–V_5^6–I–vi^6–V_2^4–I^6 (Figure 26). In this musical work, the composer used a tonic chord in the Measure 1, and then used a supertonic seventh chord in the Measure 2 and composed a dominant seventh chord in the Measure 3, and then return to the tonic chord in the Measure 4. In the Measure 5, the composer used vi^6, followed by V_2^4 (secondary dominant chord) in the Measure 6. The secondary dominant chord carries important information implying the coming of modulation. in the Measure 7, the key modulates to G major with I^6.

Taking Measures 1 to 4 (I–ii^7–V_5^6–I) as the first short musical phrase, interesting arrangements is found in Measures 3 and 6. In Measure 3, the composer chose V_5^6 (B–D–F–G, the dominant seventh chord of C major) which includes a leading note (note B) attracting the listeners' attention; in Measure 6, V_2^4 (C–D–F#–A, secondary dominant chord of G Major) occurs, this chord implies the coming of modulation and changes the colour of the phrase. In the listeners' act of listening, this musical phrase is not static but move towards a destination; in the performers' acts of interpreting, this compositional arrangement offers a chance to demonstrate their imaginations of this phrase; in the composer's act of composing, the whole musical work is logically designed. Consequently, when the performers perform this musical work, they enchant the music with their emotional expressions. Nevertheless, this enchantment is carried out by the affectual function, which is produced by the composer's purposive-rational action in the act of composing.

[174] Gronow, 'The Element of Irrationality', 330.

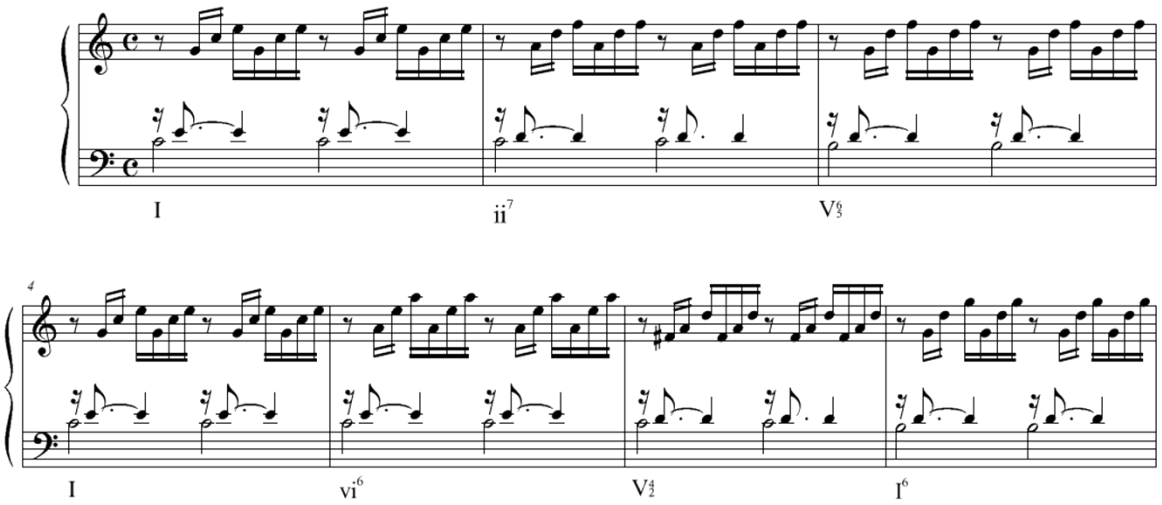


Figure 26 J. S. Bach's Prelude BWV. 846, from Measures 1 to 7.

The enchantment is the affectual result producing by the performers' acts of interpreting. Taking a Canadian pianist Glenn Gould's interpretation as an example in his album, '*Bach: The Well-Tempered Clavier, Books I & II, BWV 846-893 (2012)*',[175] he performs Measures 1 to 7 gently and calmly, his interpretations with clear leaps at the end of each motif (C–E–G–C–E–G–C–E, and the following broken chords). Even his interpretation is calm, no clear differences between loud or soft, listeners are still attracted by the compositional arrangement; the listeners notice the hint that the composer inserted in the musical phrase and expect the peak during the performance. The other interpretation from an American pianist, Keith Jarrett (1945–), whose interpretation may be a good example to show how different interpretations can be. In his album, '*J. S. Bach: Das Wohltemperierte Klavier Buch I*' (ECM 1362/63) (1988),[176] he chose to interpret this piece in a faster tempo, and with no staccato on the note E and note F, which were interpreted as staccato in Gould's interpretation. In Measures 6 and 7, their interpretations are evidently different. Gould seemed to prefer dolce; he performs these notes with a gentle and slightly soft touch. On the other hand, Jarrett's interpretation shows a relatively fast tempo.

The beauty of the acts of interpreting is that there is no right or wrong in their interpretations. The performers analyse and appreciate the musical work at the esthesic level and concretise their perceptions of the musical work in performances at the poietic level. This means that one

[175] Gould, *Bach: The Well-Tempered Clavier Books I & II, BWV 846-893*.
[176] Jarrett, *J. S. Bach: Das Wohltemperierte Klavier Buch I*.

musical work can be interpreted diversely, even interpretations from the same performer may exist nuanced differences.[177] This diverse interpretation refers to Weber's ideas of impulsive emotional expression, which he classified as irrational. The important fact here is that the performers' expressions are stimulated by the compositional arrangements. Following this idea, I suggest locating affectual function in rationality and labelling affectual result as irrationality.

Reflecting this to the relation of affectual expressions and degrees of rationality, as P. S. Cohen (1928–1999) asked, does the greater the expression demonstrate the lower the degree of rationality?[178] I answer, in the sphere of European art music, a valid discussion may consist of both aspects of the composer and the performers. The composer's arrangement applying the affectual function of musical elements may not be considered as irrational actions, as the actions is based on the composer's purposes, the actions are conscious, while the affectual result is conducted by the performers, and may not be considered as rational actions, as their actions contain uncontrollable emotional expressions. As Goldthorpe suggests, an ontological difference between rational and expressive action is that the former considers purposes, while the latter prioritises emotions.[179] In the case of European art music, the affectual function relates to the composer's rational action, whilst an affectual result is an emotionally expressive action of the performers.

As previously mentioned, the division of affectual function and affectual result can be useful to compare rationality and irrationality in both European art music and indeterminate music. This is because the expression in indeterminate music was longer limited in emotional expressions but expand to substantial uniqueness of each realisation, such as, the choices of instruments or auxiliary tools, selection of graphic notations or movements, speaking speeds, or other elements in the realisation.[180] The shift of the focus led to the development of enchantment; enchantment was about affectual results of performers' interpretations, but it transformed to unpredictable realising results in performers' realisations.

The graphic notations in the *Solo for Piano* can be realised diversely because Cage offered the performers different types of freedom and intentional ambiguity in the performing instructions.

[177] Payne, 'Creativity beyond Innovation', 341.
[178] Cohen, 'Rational Conduct and Social Life', 142.
[179] Goldthorpe, 'Rational Action Theory for Sociology', 188.
[180] Nyman, *Experimental Music: Cage and Beyond*, 4.

Taking Notation T as the example. The performing instruction of which is: 'Influences in pitch and time notated as shapes with centre points, to be audible as clusters, a single one changing in its course. Numbers refer to loudness (1-64) (soft to loud or loud to soft)'.[181]

From the performing instruction, I noticed that Cage indicated pitch and time through shapes with centre notes and showed loudness by numbers, as shown in Notation T on page 12 (Figure 79). With the given information, the performers need to compare the positions of shapes to obtain ranges of pitch and performing timing and decide the relationship of numbers. The substantial difference of performers' realisations may show up in dynamics, the timing of performing clusters and single notes, and duration of this notation. The performer A may decide the number 1 represent the loudest and the number 64 is the softest, while the performer B may choose the other way round; the performer A may decide the duration is 2 minutes, while the performer B may plan the realisation as a 5-minute performance; or the performer A may decide to perform Notation T solely, but the performer B choose to present Notation T with Notation Z, since Cage specially pointed out 'Dynamics as in T ' in the performing instruction of Notation Z.[182] In essence, performers' realisations are constructed by their act of decoding, which consists of their considerations, and therefore, this act is a rational action. Although the realisations are carried out by their rational actions, their decisions are varied so the acoustic results are enchanted and unpredictable.

To sum up, when Weber divided rationality and irrationality, he considered the lyrical and melodic features as irrationality, as they enchant music, and make it indeterminate and unpredictable. Indeed, I cannot deny that such features are the main factors of why interpretations of European art music are so different. Nevertheless, when considering the composer's participation – how he or she arranged the harmonic progression, or how he or she practised the imagination through the musical elements – it is hard to claim that those actions are irrational simply because of the application of irrational musical elements, it is also hard to avoid discussing the composer's intentions, which represent rationality. Consequently, I suggest locating the affectual function of musical elements as an element of rationality, whilst locating the affectual result as an element of irrationality.

[181] Cage, *Concert for Piano and Orchestra: Solo for Piano*.
[182] Ibid.

Despite the affectual function of musical elements and the affectual result of performers' interpretations are not applicable in indeterminate music, the composer's rational actions and the musical work's enchantment can be discovered in the *Solo for Piano*. The former refers to Cage's practise of philosophical ideas and composing arrangements, while the latter points to substantially different and unpredictable realisations from the performers. Through the example discussed above, I argue that the ideas of theory of rationality developed along with the development of music, and therefore, they are applicable in two musics. The following section will continue the comparison of rationality and irrationality in European art music and indeterminate music by focusing on the notational system in BWV. 846 and the graphic compositional system in the *Solo for Piano*.

2.2 Rationalising and Conventionalising

This section will briefly discuss the differences between rationality and irrationality in European art music and indeterminate music. When I researched Weber and Cage, one common thing they had between them is that they both identified themselves as observers looking for meaning, purpose, intention, and value of societies, individuals, and philosophies, and being producers of knowledge based on their comprehension and findings. Following their paths, I also identify myself as an observer and ask, what is happening here?

When analyses are built upon theoretical dissection, they can be too abstract and intricate to understand. To cope with this, I need a medium to concretise the analyses. Truax's investigations in electroacoustic music may be worth looking at.[183] He studied electroacoustic music at physical, social, and psychological levels, step by step, then investigated the relationship between music itself and the outside world. In his investigation, he considered the recordings, electroacoustic techniques, soundscapes, and perceptions of listeners; accordingly, he discussed space, time, acoustics, and the environment at a physical level; he also analysed the social level, which comprises individual, institutions, and cultural heritages; then he used emotions, archetypes, imagery, metaphors, myths, and symbols to reflect the psychological level. Though these categories bring out effective discussions, after considering the abstractions of indeterminate music, I found that one concrete focus at the physical level may be a better way to approach an abstract idea. For this reason, this section will examine the

[183] Truax, *Acoustic Communication*.

notational system, and the compositional materials required to deliver valid analyses and comparisons.

2.2.1 The Notational System and Compositional Materials

This section and Chapters 5 and 6 will consistently refer to compositional materials, which are slightly different from musical symbols of the notational system in conventional notations. The intention to differentiate the compositional materials from the notational system is to emphasise two types of relationships; one is between musical symbols in a notational system, and the other is between compositional materials in the graphic compositional system in the *Solo for Piano*.

Examining the notational system of European art music, it is hard to find a division between symbols within the system and symbols outside it. A good question to ask is, what does 'outside or inside the system' mean? In the notational system, each symbol plays its role, whether it carries musical meaning, function, or expressive interpretation. Music learners gain the ability of reading all conventional scores, though they need clef signs to read notes and time signatures to determine note value. This means that the musical symbols in the notational system are connected with one another and have been conventionalised; the system is like a toolbox filled with various types of tools, allowing composers to choose which and how to utilise them to compose their music. To put it differently, the relationships between musical symbols in the notational system are separate from the composers; they do not build the relationships in their compositions but apply the existing relationships to produce their music. However, this is not the case in the *Solo for Piano*.

Solo for Piano is presented as a graphic notation, which offers information regarding the construction of sound through graphic elements; the composer experiences a process of thinking and decision-making during the act of composing.[184] This musical work is composed by the graphic compositional system, in which Cage firstly marked on chance-derived paper imperfections, and then turned them into meaningful and performable compositional materials by numbers, shapes, lines, arrows, areas, drawings, and performing instructions for each notation. With the performing instructions for each notation, the performers will conduct a

[184] Mongini, 'Sign and Information: On Anestis Logothetis' Graphical Notations', 227; Fay, 'Towards a Rattling of the Everyday', 140.

serial action of decoding to determine the details for their realisations.[185] In this case, the relationships are formed in a two-stage rationalisation by participation from Cage and the performers; the first one is Cage's act of composing, and the second one is the performers' act of decoding. This means that the relationships between compositional materials are changeable, different performers will form different relationships in their realisations, and hence, musical symbols in the notational system and compositional materials in the graphic compositional system are substantially distinct.

The distinction of the two types of relationships demonstrates particular processes of meaning construction which relate to corresponding rationalisation. Based on Weber's proposition in *The Rational and Social Foundations of Music*, one of the rational features of European art music is the relationships in the notational system. The worthwhile highlight hiding within the system is the conventionalisation of notation, which demonstrates a gradational process of how notations were developed, spread, and commonly used. Before carrying on the conventionalisation of notations, I shall refer to Weber's observations of Protestants' living style to present the development of his ideas of rationalisation.

In *The Protestant Ethic and the Spirit of Capitalism*, Weber proposed that the process of rationalisation originated from a methodical way of life influenced by values of individuals and societies.[186] He noticed how religion deeply affected people's thinking, motivations, and values, and how they took actions and made decisions in order to pursue salvation and ease the anxieties that emerged from their religious doctrines. Out of these actions and methodical living styles emerged the spirit of capitalism.[187] Weber recognised that the origin of the spirit of

[185] In discussions of performances of indeterminate music, the term 'realisation' seems to be broadly used. Alternatively, the term 'interpretation' refers to 'expressive' interpretations. Considering the radical difference between European art music and indeterminate music, I applied the use of 'realisation' regarding indeterminate music.

[186] Kalberg, 'Max Weber's Types of Rationality: Cornerstones for the Analysis of Rationalization Processes in History', 1149.

[187] Another crucial foundation of modern civilisation is an ability to calculate, which scholars agree is a distinct power that replaces the mysterious (Evans and Evans, 'Religion and Science', 91; Grosby, 'Max Weber, Religion, and the Disenchantment of the World', 301; Han, 'Disenchantment Revisited', 79). Thus, when that is gained, advanced scientific ability follows, stimulating the process of forming modern civilisation. In Weber's notion of formal rationality, purposeful calculation is the basis of modern civilisation, by bringing out the most efficient means to achieve targets (Cockerham, 'Max Weber: Bureaucracy, Formal Rationality and the Modern Hospital', 124). This does not mean that Weber only considered calculation to be rational; if individuals' actions are appropriate to their beliefs and ends, their actions can be rational, regardless of the rationality, or irrationality, of their ends (Benn and Mortimore, 'Can Ends Be Rational? The Methodological Implications', 282).

capitalism came from the irrational fears of uncertainty. This irrationality motivated people to live a worldly life, while their religion acted as a catalyst spurring them to achieve the best performance in their calling.[188] This process demonstrates a solution dealing with irrational fears; people took their actions to achieve the ultimate target, and the solution spread and was accepted by a certain group of people. Thus, the idea to 'work hard in this life in order to gain a better afterlife' was conventionalised. Delving into the process of conventionalisation, the process of rationalisation can be discovered.

A similar process also appeared in the gradational progression of the conventionalisation of notations. Before modern staff was proposed, in the 9th century, people used neumes, which could not specify notes, but only roughly stated melodies; then Guido of Arezzo proposed the four-line staff with C and F clef, or with coloured lines. This means that people could state notes and melodies much more clearly;[189] the five-line staff became popular in the 13th century,[190] and then the notational system spread all over Europe. Using this process, monks firstly regulated the notational system, including note value, time signature, and staff. This fully functional system became familiar to musicians and this group of professionals then helped with the conventionalisation process. Comparing rationalisation and conventionalisation of the cases of Protestantism and the notational system, the process of value unification and spread were two fundamental conditions. However, these are applicable to Cage's indeterminate music.

As previously explained, the relationships between compositional materials in the *Solo for Piano* were constructed by both Cage and the performers. When the performers realise the graphic notations in their own ways, they also form the distinct relationships between materials in their realisations. This shows that the processes of value unification and spread are not in Cage's composition and performers' realisations. Putting it another way, rationalisation of graphic notations transformed into new a form, which emphasises the composer's philosophical and technical decisions and the performers' considerations for the realisation. Meanwhile, conventionalisation of notation appears inapplicable. I shall take musical elements

[188] Mitzman, *The Iron Cage*, 190.
[189] Palisca, 'Guido of Arezzo'.
[190] Hiley, 'Staff'.

regarding time, such as length, duration, tempo, and rhythm, to illustrate my point in the following section.

Speaking of length, duration, tempo, and rhythm, the evident examples in European art music are note value and the time signature. When people learn music, they are taught that a semibreve equals two minims; it also equals four crotchets and eight quavers. The equivalent of notes values between one semibreve, two minims, four crotchets, and eight quavers equals results from both conventionalisation notations and rationalisations. This equivalence is closely connected with the time signature, which is presented by numbers. People were taught that the number on top means how many beats there are in one measure, while the other number indicates which note value is equivalent to a beat. At this point, two types of relationships can be found; the first one is the relationship between different types of notes, and the second one the relationships between notes value and the time signature. Based on those, the first rational feature (method/methodical) in European art music is presented, it shows the feature of system and structure in notation and composition.

Looking at the Prelude in C Major BWV 846, Bach's composition was following strict musical theory by using a time signature of common time (C or 4/4) to offer information about rhythm to performers. Based on this time signature, and the fixed relationship of notes value between different notes, the performers will be able to understand this work by skimming the musical score. Putting it differently, conventionalisation of notations, rationalisation, and the rational feature make scores much more accessible since the composers and the performers speak the same language by using notes and musical symbols. On the other hand, as I argue in this section, rationalisation and conventionalisation of notations of indeterminate music were transformed into new forms, because Cage's composition was not limited in composing music but expanded to decide the compositional materials, build the general relationships between the materials, and suggest the realising approach of the graphic notations. When he finished his composition, this musical work remained undone until the performer started the serial action of decoding. In essence, conventionalisation and rationalisation in the *Solo for Piano* cannot be generally applied to other musical works or other realisations, they remain between Cage and the realisation from the performer.

Taking Notations F, Q, R, and Y in the *Solo for Piano* as examples, the four notations show two ways of offering information about time. As previously emphasised, the performers need

to decode the graphic notations based on the corresponding performing instructions. The instruction for Notation F is 'Numbers are seconds or other time units'; for Notation Q is 'Like M, but in time (any units); going back incurs need for increased speed'; for Notation R, he annotated, 'Rigorously in time. Notes above staff: treble; below: bass; on: ambiguous. Lines were part of composing means, the intersections producing tones other than the extreme piano keys'; and for Notation Y, he annotated, '8 pitch areas, chromatically adjustment, and having number of chromatic tones given (large numbers below staff). When notes are on a line, they belong to either the upper or the lower area. Time in space seconds as indicated. Loudness given by position of note with respect to area vertically. High is *fff*. Low is *ppp*'.[191]

Focusing on the information about time, in Notations F and Q, Cage indicated numbers showing the basic timing frame, while he guided the performers to choose any time units they liked to realise the notations with the allocating numbers. In Notations R and Y, he also stated the timing with numbers, and therefore the performers need to realise sound based on the allocated seconds. From the given guidance, it is noticeable that Cage rationalised Notations F and Q partially and required the performers to determine the further details. Nevertheless, in Notations R and Y, Cage did not offer the same types of freedom to the performers; instead, the realisations of two notations are required to be done in given seconds. Consequently, rationalisation of the graphic notations is dependent on Cage and the performers, the aspect of subjectivity has been emphasised. This development leads to disappearance of conventionalisation of notation, meaning that the compositional materials in the *Solo for Piano* cannot be generally applied to other music works and the performers cannot anticipate the acoustic results before they conduct the realisations. This alteration brings out different rationality and irrationality in the notational system of European art music and the graphic compositional system in the *Solo for Piano*, as well as the development of the theory of rationality.

[191] Cage, *Concert for Piano and Orchestra: Solo for Piano*. The detailed analyses of these notations, refer to Chapter 5 Section 2.1 Compositional Materials in Notation A to Notation Z.

Chapter 4. Zen Buddhism and Rationality in Cage's *Solo for Piano*

1 Introduction

In the Introduction and Literature Review in Chapter 1, I suggested that one of the effective ways to unpack Weber's rationality is through causality, based on scholarly research. As Braun argued, Weber's rationality in music is not limited to a consensus about rationalism; readers need to comprehend this type of rationality through practice, life, and logic.[192] Mommsen suggests that Weber's ideas about rationality delve into causal explanations of historical and cultural developments and results,[193] while Kalberg described Weber's observations about societal circumstances as using subjective meanings and value-oriented actions on which to build his rationality, hence causality is a key factor.[194]

Accordingly, I plan to apply causality in order to gain a better understanding of what led Cage to his unique composing techniques and what were his intentions, how he came to make the choices he did and what inspired him. What was his belief and his purpose, and what actions did he take to achieve that purpose? An effective way for these questions to be answered is to look into his other musical works and into his life as a whole.

In retracing his life, one can see that his novel musical creations did not mean he had no ability to create conventional music, but he seemed intent to show his audiences how determined he was to break the invisible boundaries between rigor and immutability. Indeed, following his lead, the New York School inaugurated a new era of unusual music. In this type of music there are barely any conventional structure, systems, or familiar notational symbols. Instead, his music is full of novelty and his arrangements were individually organised and structured. Does this mean that his music shows by its features that it is opposite to Weber's rationality? My answer is that, considering his composing techniques, intentions, and purposes, it still demonstrates rational features, which can also be the key to pushing rationality further.

[192] Braun, 'The "Science of Reality" of Music History: On the Historical Background to Max Weber's Study of Music'.
[193] Mommsen, 'Personal Conduct and Societal Change'.
[194] Kalberg, 'Max Weber'.

The questions now are: What influenced him to create music in that way? Therefore, the next section will look into how his composing techniques matured, followed by a focus on influences from oriental philosophy, then moving on to his interpretation of Zen, and finally I will examine rationality, irrationality, rationalisation, and irrationalisation within his compositions.

2 Cage's Lifetime

Reflecting Cage's life, his experimental spirit was inherited by his father, John Milton Cage senior, who was an inventor who made an airplane engine in 1918,[195] and a crystal radio set in the mid-1920s.[196] The intense aspiration in Cage junior seemed like an obstacle when he studied composition with Arnold Schoenberg (1874–1951) in 1934,[197] Cage once said he would spend his life against the wall that constructed by melodic music.[198] The first surprise of Cage was using twenty-five notes of two successive chromatic octaves, with no repeated notes allowed until all notes had been used; this compositional technique seems to resonate with Schoenberg's twelve-tone technique, as it used an organisational method way to compose music. Most importantly, with this application, Cage started his investigations with radical ideas. In the interview with Will Duckworth (1943–2012, composer, performer, author, and a friend of Cage), Cage said, 'I do not have an ear for music, and I do not hear music in my mind before I write it. I have other qualities which are, I would say, more radical than those things which most musicians have'.[199] In addition to a rebellious spirit from his father, Cage's teacher, friends, colleagues, and partner are crucially important.

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[195] Revill, *The Roaring Silence*, 7.
[196] Silverman, *Begin Again*, 6.
[197] Although Hans Keller (1980, p.20) attempted to clarify the relationship between Schoenberg and Cage, he said that 'Schoenberg would have denied them [Cage's studies with Schoenberg]. Whatever he did with Schoenberg, he never came to "study" with him', and hence the relationship between Schoenberg and Cage was a fantasy of Cage's (as cited in Revill, 2014, p.48). However, multiple resources evidently suggest their relationship are genuine – whether as a teacher and a student, an inspirer and a receiver, or any other possible ways to address this. Aaron Copland (1900 –1990) (1948, p.54) concludes Cage's music derived not only from Balinese and Hindu music, but also from Schoenberg; in Reynolds's (1934–) interview with Cage, Cage said 'In fact, that I learned from him [Schoenberg] what it was that led me to use chance operations' (Reynolds and Cage, 'John Cage and Roger Reynolds: A Conversation', 593) ; Richard Kostelanetz (1940–) sees the interactions between Schoenberg and Cage as motivation to rethink the need and purpose of self-expression (Kostelanetz and Cage, 'The Aesthetics of John Cage: A Composite Interview', 109) ; and Michael Hick's (1956–) 'John Cage's Studies with Schoenberg' (1990) outlines the process of how Cage learned composition under Schoenberg.
[198] Rich, *American Pioneers*, 145.
[199] Duckworth, *Talking Music*, 7.

Henry Cowell (1897–1965) was an essential presence in Cage's new music explorations since 1933. Cowell, as a composition teacher for Cage, offered opportunities to learn contemporary and non-Western music, along with a scholarship from Charles Ives (1874–1954), and an introduction to his former student/colleague Lou Harrison (1917–2003), who directed a percussion concert with Cage in 1940 in San Francisco. Most importantly, Cowell was the one who aroused Cage's interest in music,[200] supported him in learning composition, created networks and financial support, and inspired Cage's second major composing technique – prepared piano.[201] The idea of prepared piano was from Cowell's *The Banshee* (1925), in which he produced glissando of harmonics using a darning egg, and stummed, struck, and plucked strings that inside the piano.[202] This concept would then be expanded by Cage. He enhanced the prepared piano by inserting objects into the strings that are inside the piano. *Bacchanale* (1940) is a well-known prepared piano work where Cage used a small bolt, a screw with nuts, and some fibrous weather-stripping to produce two difference sounds.[203] An interesting finding emerging here is that notations were no longer annotations offering performers imaginative interpretation, but a way of producing specific sounds;[204] this change enhanced Cage's graphic scores and indeterminate music.

In 1936, Oskar Fischinger (1900–1967, German American abstract animator, filmmaker, and painter) invited Cage to compose for his projects. During their collaborations, one of Fischinger's idea inspired Cage's appreciation of different dimensions of music. Cage examined everything he saw in relation to audible existence,[205] and concretised this inspiration through vibration and timbre, such as the percussion work *Trio* (1936). This work is for three percussionists performing on pieces of woods of gradated lengths, tom toms, bass drum, and bamboo sticks. Later Cage applied the sixteen repetitions of a rhythmic pattern of 4+3+2+3+4 bars in the *First Construction (in Metal)* (1939).[206] In 1938, a lifelong inspiration showed up in Cage's life – Merce Cunningham (1919–2009); he was an American choreographer, but most importantly, he was an inspiration, a business partner, and a life partner for Cage. In 1943, Cage, Xenia Kashevaroff (1913–1995), and Cunningham held their first percussion concert at

[200] Revill, *The Roaring Silence*, 37.
[201] Rich, *American Pioneers*, 149.
[202] Revill, *The Roaring Silence*, 65.
[203] Ibid., 66.
[204] Cage and Charles, *For the Birds*, 160.
[205] Revill, *The Roaring Silence*.
[206] Rich, *American Pioneers*; Silverman, *Begin Again*; Pritchett, Kuhn, and Garrett, 'Cage, John (Milton, Jr.)'; Revill, *The Roaring Silence*.

the Museum of Modern Art; after this concert Cage and Cunningham's collaboration was closer than before. On 5th April 1944, Cage and Cunningham presented their first concert together; it contained Cunningham's dance solo and Cage's prepared-piano music. Although they collaborated in the concert, the form was free – dance and music were together at the beginning, independent in the middle of the work, then came together again at the end.[207] From their collaborations, Cunningham and Cage were not just close to each other in their personal lives, but also as artists.

In 1945, Cage met a philosophical scholar who inspired his third major application: the graphic compositional system. Daisetz T. Suzuki (1870–1966), a philosophical scholar of Zen Buddhism, taught Eastern philosophy and religion from 1945 to 1957 at Columbia University; the attendees were diverse, including philosophy students, painters, sculptors, scientists and psychoanalysts, while Cage was the only musician who regularly participated in lectures.[208] From these lectures and three books (*I Ching*, *Chuang-tze*, and *Huang Po*), Cage concluded that one of main ideas of Zen Buddhism is freedom,[209] which refers to giving up seeking anything. Applying this understanding to music, Cage tried to apply a new composition that could deconstruct the musical element that we call structure, produce unpredictable performing results, and allow the performers a greater freedom when processing their performances. Reflecting on his experimental innovation through his life experience, we can see that his experimental applications were not just from himself, but from his surroundings, his family, teachers, colleagues, friends, and partner, from birth to death. These influences expanded Cage's mind, and he embraced the diversity of music.

3 Influences from Oriental Philosophy: Chance and Indeterminacy

3.1 Chance and Indeterminacy

The two major elements Cage used in his musical works were chance and indeterminacy. The distinguishing features between those two elements seems to be relatively blurred; they are not like the usual elements of European art music, which come with inherent meaning, and systematic and structural features. Nevertheless, referring to his own explanations, a clear distinction can be obtained. In *Silence: lectures and writings* (2011 [1961]), Cage listed J. S.

[207] Silverman, *Begin Again*.
[208] Revill, *The Roaring Silence*, 125.
[209] Silverman, *Begin Again*, 121.

Bach, Karlheinz Stockhausen (1928–2007), Morton Feldman (1926–1987), and Earle Brown's (1926–2002) works which he interpreted in terms of which were determinate or which indeterminate, which he suggested was where he came to his interpretation of indeterminacy.[210] In his analysis, he listed the structures, methods, forms, and materials of their works, using specific terms and metaphorical descriptions to illustrate his perspectives. For instance, he stated that, 'a performer [in *The Art of Fugue*] acts as a colourist who can fill in colours with or without consciousness and in an organised or unorganised way'.[211] This unique writing style suggests a tripartite relationship between the composer, the performer, and the musical work, it also reflects Cage's interpretation of indeterminacy.

As Cage explained in *The Art of Fugue*, its method, form, frequency and duration are determined, but it lacks instrumentation, timbre, dynamics, amplitude and articulation. Additionally, the performer can decide which fugue to perform in this piece. Free choices and the absence of elements lead to the possibility of interpretations, so the performer is free to interpret within the performance. In Cage's terms, this situation is called the 'performer centre', and the performer's function is to be a colourist. In Stockhausen's *Klavierstück XI*, the materials, method and structure are determined, while the form is indeterminate; therefore, each performance consists of different expressive contents. Thus, the function of the performer is to provide the continuity of the expressed content.[212] In Feldman's *Intersection 3*, the determinate parts are timbre, frequency and form, while the part of indeterminacy is its method and structure could be both.

Therefore, because structure could be both indeterminate and determinate, the performer occupies an important position, as does a photographer.[213] Anyone in this position decides whether the work is analysable or unanalysable because the position allows either for a mechanical construction or an arbitrary arrangement, just as a photographer either intentionally takes photos, or casually captures street views. In Brown's *4 Systems*, the structure and the form consist of indeterminacy; method, timbre, amplitude, and frequency construct the

[210] Cage, *Silence*.
[211] Ibid., 35.
[212] Ibid., 35–6.
[213] Ibid., 36–7

determinacy of this work, while its duration could be both. Thus, the performer is a provider who presents continuity from the form and the structure.[214]

Wolff's Duo II for Pianists is unlike any of the pieces listed above. The elements are all indeterminate. Each performance determines only the beginning and the ending. *Thus, this piece* is not limited in its time frame. The performer's function, therefore, is as a traveller who needs to catch the train, where the conductor is about to announce its departure time.[215] In addition to these works, Brown's *Indices* is a crucial example that illustrates the opposite of indeterminate music. As Cage explained, this work is not the case for indeterminate music because 'it does not permit the conductor or the players any such identification'. [216] Consequently, the conductor must conduct from the centre of the work, while the performers ought to follow the instruction from the conductor; none of them 'owns their stance'. From these examples, one may see that indeterminate music is unpredictable and manipulatable, as it allows the performers to realise the musical work from their perspectives, and therefore, the form of realisations, the use of materials, the timbre, the amplitude, the frequency, and the duration can be different each time it is performed. Putting this more schematically, indeterminacy offers the performers freedom to realise musical works; it emphasises the centre of different composers, and the different results of realisations from different performers' centre.

Conversely, however, chance music deeply involves randomness and probability. The performers may throw a dice or a coin to make decisions or to decide the details for the realisations; the performers, on the other hand, may do nothing, and let the environment 'create' the realisation. In short, the apparent difference between indeterminacy and chance is a centre – indeterminate music is performed by a centre, while chance music is constructed by aleatory results.[217]

[214] Ibid., 37–8.
[215] Ibid., 38–9.
[216] Ibid., 37.
[217] Pritchett explained that chance refers to the use of randomness in the composition; indeterminacy, on the other hand, refers to substantially different realisations of one piece of music. This means that chance music focuses on the randomness, while indeterminate music focuses on how this musical work can be realised differently. From his explanations, the performers' approach to realisation are the key to differentiate chance and indeterminate music.

3.1.1 Music of Chance Operation: 4'33''

John Cage was an influential experimental composer, but he certainly did not investigate chance music and indeterminate music alone. He worked along with his fellows of The New York School, including Earle Brown, Morton Feldman, and Christian Wolff, and the pianist David Tudor (1926 –1996), he also worked with a painter and a dancer. For this reason, this section starts with relevant inspirations and cooperation that influenced him, both directly and indirectly, and which pushed him to the next level, and to successes. In his investigations, music was not the only medium he paid attention to. Although his visual arts were created in the latter stages of his investigations, the influence of the visual arts, which he gained from other artists, is undeniable. For example, the American abstract expressionist Robert Rauschenberg's (1925–2008) artwork *White Painting* (1951) was the first materialised silence, emptiness, whiteness, and the void,[218] and it inspired Cage to compose his silent work *4'33''* (1952).[219]

Cage emphasised the importance of this art work in his publication *Silence*, he explained: 'the *White Painting* came first; my silent piece came later'.[220] The philosophical idea that inspired Cage was Rauschenberg's idea that three white panels collaborated with the environment, so anything from its surrounding could be a part of this artwork.[221] The artist created an artwork which was logically pure, he did not intend to pursue ontological purity. This means that white remains white, when shadow, dust, or any other uncontrolled objects cover the white, ontological white no longer exists, but the logical white is always on the three panels. Therefore, once the artwork is created, it could never be the same as it had been at the very beginning. Every existence of any substance can change the status of an artwork. Thus, based on this idea, the artwork is no longer about painting technique, rather, it is about the experience of the audience and the creation of the painter.[222] Rauschenberg's *White Painting* led to Cage rethinking the different experiences between a logical outline and ontological facts; hence, thereafter, he included the environment in his compositions.

When such radical changes take place, what happens with Weber's rationality and irrationality? Apparently, no performers, audiences, or even Cage himself, could foresee any of the acoustic

[218] Larson, *Where the Heart Beats*, 227.
[219] Iddon, *John Cage and David Tudor: Correspondence on Interpretation and Performance*, 23.
[220] Cage, *Silence*, 98.
[221] Fielding, 'An Aesthetics of the Ordinary', 167.
[222] Cage, *Silence*, 101.

results of *'4'33'*. The unpredictability and randomness of this work appears to reinforce irrational features, which also seem to eliminate conventional rationality from it. Insistence of discovering conventional rational features, such as a notational system and a harmonic structure, which could lead into a dark tunnel; hence, I suggest turning the focus on the first and the third rational of the three rational features that I proposed in Chapter 2, which are method/methodical and interactivity/being interactive. Before connecting those two rational features to *4'33''*, however, one fact should be kept in mind, when music is detached from harmonic, melodic, and lyrical features, its value no longer concentrates on conventional and fashionable aesthetics. Rather, it extends to the idea behind the compositions. This means that the approach of examining rationality in music was not only about music itself, but also about the relationships between the composer and the musical work. By wrapping up Weber's rationality, *4'33''* demonstrates John Cage's value rationality and purposive rationality.

4'33'', as I propose, this silent work reflects Cage's value and his purpose. In terms of value rationality, it is a musical result that he intended to blur the boundaries between art and life,[223] and certainly, his intensions deeply connected with Zen. Around 1945, Cage experienced sexual reorientation and divorce;[224] Cage intended to look for inner peace, and Zen provided the path for Cage. As mentioned before, the core idea of Zen is defying all concept-making, looking for pure facts of life. Specifically, Zen includes seven features: aloneness, simplification, poverty, disregard of form, beauty of imperfection, asymmetry, and imbalance.[225] All of these features refer to central ideas, called 'emptiness' and 'nothingness', which were the central ideas that Cage attempted to achieve. He intended to lead the audiences into a mindful practice and suggested the audiences to notice their mind and body through different senses.[226] To achieve this target, he applied Rauschenberg's approach to design logical silence and offered an ontologically different experience. His action of designing logical

[223] Woodward, 'John Cage and the Aesthetic Pedagogy of Chance & Silence', 2.
[224] Taruskin, 'The Scary Purity of John Cage', 266.
[225] Shigeta, 'Suzuki Daisetsu's and Hisamatsu Shin'ichi's Discussions on East Asian Arts, Dramatic Art and "Zen"', 171. To be specific, Zen include seven main points: (i) concentrating on spirits, instead of forms; (ii) detecting in form of any description the presence of the spirit; (iii) embracing the imperfection of beauty, which makes the spirit is more expressive; (iv) rejecting formalism, conventionalism, and ritualism, so it tends to make the spirits stand in all its nakedness, aloneness and solitariness; (v) representing a transcendental aloofness; (vi) showing no attachments to nonessentials; (vii) depositing itself in all beings of nature.
[226] Pong, 'True or False as It Is: Zen Enlightenment in John Cage's Music 真謬之間如如觀：凱吉音樂中的禪意解讀', 224.

silence shows purposive rationality. However, Cage's practices of Zen Buddhism's emptiness and nothingness are not flawless.

As Low criticised, Cage's *Silence* reveals his misunderstanding of emptiness and nothingness.[227] Cage stated, 'each moment presents what happens. I derived the method I use for writing music by tossing coins from the method used in the Book of Changes. It may be objected that from this point of view anything goes. Actually, anything does go but only when nothing is taken as the basis. In an utter emptiness anything can take place'; however, emptiness refers to awareness of 'the reality is empty',[228] rather than something that people can look for in the world.[229] This means that when Cage intended to pursue emptiness and nothingness by designing a composition with nothing, he failed this task at the moment when he took action. For this reason, in *4'33''*, emptiness and nothingness moved far away from their original meaning, and the realisations of this musical work became a space with nothing.[230] Despite the flaws in Cage's understanding, a misinterpreted result (*4'33''*) does not subvert his intention, purpose, and actions. Scholars may criticise his understanding as not authentic and his approach as not effective, but they cannot ignore the influence of Zen and its connection with *4'33''*. Rather, to a certain degree, this result shows that chance became a tool in Cage's compositions.[231] This reveals a potential connection between Cage's use of chance and Weber's rationality, meaning that the silent work still validly demonstrates Cage's value rationality and purposive rationality.

Let us go back to the rational features – method/methodical and interactivity/being interactive – of *4'33''*. The feature of method/methodical refers to Cage's design of using silence as a main compositional material. The rational feature in this case does not refer to its actual musical structure or to any of its performances, but it relates to Cage's method of constructing it. The feature of interactivity/being interactive shows in the composer's purposive arrangement of this work and the unaware participation of the audience; because the compositional materials are the sounds from the audience, an invisible interactivity exists. In other words, the composer and the audience both participate in the realising process to formulate the performance. Cage

[227] Low, 'Religion and the Invention(s) of John Cage'.
[228] Cage, *Silence*, 175.
[229] Low, 'Religion and the Invention(s) of John Cage' 170.
[230] Ibid.
[231] Pong, 'True or False as It Is: Zen Enlightenment in John Cage's Music 真謬之間如如觀：凱吉音樂中的禪意解讀', 225.

practised his value concepts in this work, he took action to achieve his intention, and all of the value concept, purpose, and intention came from his life experience, meaning that there was a causal relationship between his life experience and the musical creations. Consequently, his actions should be considered as rational, while the unpredictable acoustic results and random noises from the audiences demonstrate irrationality.

In summary, when music is not composed of musical symbols, its uniqueness brings out a new form of rationality and irrationality; therefore, rationality contains the composer's philosophical pursuit, intentions, values, and actions, while irrationality remains unpredictability and randomness; however, it does not carry emotional expression.

3.1.2 Music of Indeterminacy: Solo for Piano

The influence of Zen Buddhism not only inspired Cage in terms of chance music, but also regarding the application of indeterminacy. As mentioned in Section 2, Cage's journey into Zen started in 1945 when he attended Suzuki's lectures for the first time. It took him several days to understand their contents. As I explained in the Literature Review, his interpretation includes three elements: 'freedom, unimpededness, and interpenetration'. Each of these directed him to decrease the composer's authority and seemed to him to offer countless possibilities for constructing realisations. Cage described it as:

> 'Now this unimpededness is seeing that in all of space each thing and each human being is at the centre and furthermore that each one being at the centre is the most honoured one of all. Interpenetration means that each one of these most honoured ones of all is moving out in all directions penetrating and being penetrated by every other one no matter what the time or what the space'.[232]

However, his unimpededness and interpenetration are not fully constructed as in Zen. The original ideas of unimpededness and interpenetration focus on how individuals acknowledge, perceive, and interact with one another, but Cage emphasised the equal importance of every individual instead.[233] The misinterpretation also shows in his intentional persistence on nothing, as the previous criticism explains; Cage practised the ideas of Zen, but not authentic enough. By saying this, I do not mean to fundamentally subvert his own interpretation of Zen, but to

[232] Cage, *Silence*, 46.
[233] Pong, 'True or False as It Is: Zen Enlightenment in John Cage's Music 真謬之間如如觀：凱吉音樂中的禪意解讀', 231.

keep his central idea, yet with adjustments; therefore, I choose to return to his analysis of indeterminacy – for instance, he analysed the structures, methods, forms, and materials in J. S. Bach, Karlheinz Stockhausen, Morton Feldman, and Earle Brown's works and suggested which parts of them were determinate, and which indeterminate. By following this, I suggest that the term 'the shifting centre' to include freedom, unimpededness, and interpenetration. As he explained, he took both the composer's and conductor's functions, elements, centres, and indications as primary components in order to analyse those musical works and illustrate what indeterminacy is. Therefore, I conclude that indeterminate music is the idea that everyone can be the centre of being, regardless of time and space. In the following discussion, I will look into how Cage practised freedom and how he saw the shifting centre in his famous indeterminate musical work, the *Solo for Piano*.

The *Solo for Piano* is the piano part of *Concert for Piano and Orchestra*; it includes sixty-three pages and eighty-four notations. Some notations are originally from his other musical works, such as *Music for Piano* (1953–56) and *Winter Music* (1957).[234] The notations in the *Solo for Piano* are categorised by alphabet, A to Z, AA to AZ, BA to BZ, and CA to CF; some are used repetitively, while others are only used once. For instance, Notations A and B are used six times, G, I, M, and O four or five times, while those after AE are only used one to three times.[235] The composing technique is based on a two-step graphic compositional system consisting of a drawing process and a means of translation.[236] During composition, Cage drew points, shapes or lines and used musical symbols to give them meaning. For instance, he marked Notation A on uneven surfaces and added staffs, clefs, accidentals, numbers and groupings to give musical meaning and performing guidance. In this notation, performers must choose sixteen notes for the left or right hand, then another nine for the right or left hand; he did not state the rhythm, volume, or length, therefore, they are free to decide such details. Thus, Cage offered the performers a huge range of freedom.

Here, the distinct difference between graphic scores and conventional scores are noticeable. The generation of such a graphic score requires the composer to answer numerous questions and to make many decisions, from the use of compositional materials to the degree of free

[234] Campana, 'As Time Passes', 127; Thomas, 'Understanding Indeterminate Music through Performance', 93.
[235] Thomas, 'Understanding Indeterminate Music through Performance', 93.
[236] Pritchett, *The Music of John Cage*, 110.

interpretation, while for the conventional score, the composer concentrates more on the musical content.[237] Here, Cage used chance-oriented elements, therefore the performer's centre focus is crucial. Such a shifting centre emphasises the core of his indeterminacy, with its emphasis on the character of process, rather than on using notations as fixed objects.[238] However, because he designed ambiguous notations to emphasise individual centres, he also offered a huge range of freedom, which may have caused the performer confusion. In Peter Dickinson's (1934–2023) interview, Earle Brown worried that Cage's indeterminate music generally might not be treated seriously, since he did not give enough information to the performers; hence they might not be confident enough to know what to do.[239] Carolyn Brown (1927–) agreed, stating that, although Cage tried to let sounds be sounds, his idea might be misunderstood and lead performers to think they could do whatever they liked.[240] This seems to me to be true, for when reading such an unconventional score at the very first time, an overly free interpretation might resemble no rules, no structures, and no system at all.

Such misunderstandings highlight the structure and systems of the stereotype that has developed as a result of European art music's dominance over several centuries, where rules, structures, and systems can only be governed by a standard notational, harmonic and melodic system. Graphic scores are not constructed for conventional music; instead, they demonstrate a deeper level of composers' arrangements. Non-determinate notations can be seen as catalysts, which indicate new ways to examine the relationships between notations, the performers, and their performances.[241] In this thesis, by way of the *Solo for Piano*, I propose to deconstruct Cage's graphic notations to examine his use of compositional materials and their functions, the types of freedom, and performing approaches to reveal any hidden systems and structures.

Other elements that I shall examine will be the source of performers' confusion and the important position of Cage as a composer. It was he who used the graphic compositional system in order to achieve his specific purpose; he consciously manipulated indeterminacy and chance for his compositions, and he even saw through the true value of sounds to eliminate external meaning and let them 'be themselves'; however, without his clear guidance none of them could

[237] Fay, 'Towards a Rattling of the Everyday', 140.
[238] Pritchett, Kuhn, and Garrett, 'Cage, John (Milton, Jr.)'.
[239] Brown and Dickinson, 'Earle Brown', 142–43; as cited in Iddon, *John Cage and David Tudor: Correspondence on Interpretation and Performance*, 82.
[240] Brown, *Chance and Circumstance*, 198–99.
[241] Lewis, 'Improvisation and the Orchestra', 431.

be understood. Hence, in my view, the confusion that may have resulted from their use, can offer a good dividing point to separate Cage's poietic level and the score's neutral level. This I will discuss in Section 4, in which I will present three potential connections: (i) ambiguity/confusion and irrationality, (ii) purpose/intention and rationality, and (iii) value formation and the process of rationalisation.

4 Visiting Rationality through the *Solo for Piano*

In the previous section, I stated that the performer's confusion can act as a useful divider to separate the composer's poietic level and the score's neutral level. The poietic and neutral levels here refer to Nattiez's tripartite model, which is derived from a semiotic schema whereby its levels consist of corresponding objects and actions, while each has a linear relationship over levels. For instance, composers, working at the poietic level, exert their creativity, arrange the materials, design the musical formulation, establish the score on paper and decode it; performers understand the score and realise it at the esthesic level, while the musical work acts at the neutral level.

Regarding the *Solo for Piano,* John Cage was clearly aware of his intentions and actions, thus these at the poietic level, can be considered to have been rational, in that he fulfilled his sense of values and purpose and solidified his philosophy during its composition. However, by intentionally offering his performers a great degree of freedom, they seemed to become lost in such novel circumstances. They did not understand his intentions or why he composed it in such way, which means that, as a result of his rational actions, the work was transformed into an irrational one from the performers' perspective, at an esthesic level. On the other hand, Cage applied chance as a tool in order to avoid personal taste and memory, an application that obstructs the conventional transmission of a musical message.[242] This indicates that any such graphic scores are unlike conventional ones, and that all such differences involve changes to the theory of rationality. In this circumstance, rationality, irrationality, rationalisation, and irrationalisation in the *Solo for Piano* are different from conventional ones in European art music.

[242] Lewallen, 'Cage and the Structure of Music', 236.

4.1 Jean-Jacques Nattiez's Tripartite Model

4.1.1 Tripartite Model and its Revision

One of Cage's intentions is to compose musical works that allows multiple centres, in order to decrease the control of the composer and increase the freedom of the performers. This idea of multiple centres resonates with the core of the tripartite model. From the aspect of multiple centres, Nattiez's tripartite model can investigate the difference between potential centres. This model consists of three levels – a poietic level, a neutral level, and an esthesic level, through three levels, the process of composing, perception, and interpretation can be discovered. When the composer composes music, the essence of the music is messages that he or she tends to transmit and intentions behind the composition, while notes, chords, or notations are alike tools using for concretising the messages and intentions. Schematically, composition and communication are both approaches of sending messages. In this circumstance, the origin of the tripartite model – the schema of communications – may be helpful to clarify the transmission of the message.

The classical schema of communication starts with producers, who deliver a message in their ways and intentions; this message will then be received by receivers, who understand the message with their interpretations, knowledge, and backgrounds (Figure 27). However, Jean Molino opposes this, as this classic schema does not fully consider different interpretations from the aspect of receivers, so it is not precise. Eco proposes a similar idea consistent with Molino's; he points out the same message can be understood in many points of view, and therefore, along with the original denotation from the producer, the receiver can have different interpretations of the message.[243] When receivers build their interpretation to the message, they also reconstruct the message from their angle; in the process of reconstruction, they apply their knowledge, perception, value concept, or other elements of themselves to process the reconstruction. This means that the interpretation is not just about understanding the message, but also involves reconstruction of the message from the angle of the receiver (Figure 28).

[243] Eco, *A Theory of Semiotics*, 114; as cited in Nattiez and Abbate, *Music and Discourse*, 21.

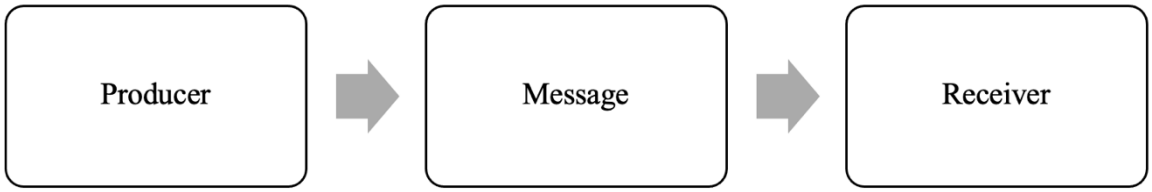


Figure 27 A classic schema for communication.[244]

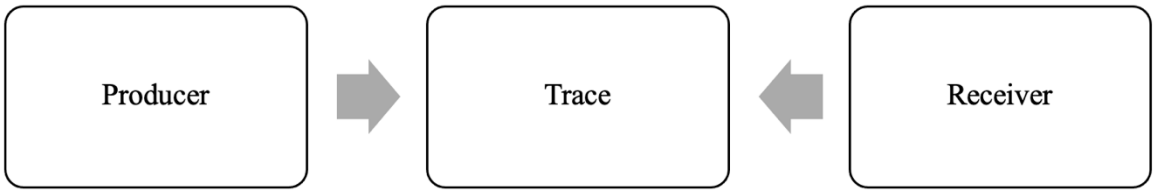


Figure 28 A revised schema of communication by Molino.[245]

Nattiez then carries this revision and develops this model for musical analyses. In his version, a poietic level refers to the composer's intentions, inspirations, and purposes, or other elements that may be relevant to the composer's creative process, while an esthesic level links with the performer's perceive process, for instance, how does the performer understand the musical work, and how does the performer interpret the musical work, etc.[246] The neutral level refers to the material trace of composer's poietic level, the musical work, in this case. As this material trace is open to interpretation, Nattiez did not categorise it as either the poietic level or the esthesic level; instead, he chose to keep its neutrality. With these levels we can recognise conceptions of the work from paper to performance.

Combining these levels with the communication schema, the tripartite model is completed. The producer shows in the poietic level, the trace is at the neutral level, and the receiver represents the esthesic level. Applying this model to music, the position of producers equals to composers, trace equals to music work, and receivers equals to performers. When composers compose a music work, they produce a material trace, which will be interpreted by performers. Applying this model to Bach's prelude and Glenn Gould's performance as examples, the model shows that Bach when composed the Prelude in C major, the Prelude is the trace, which is interpreted

[244] Nattiez and Abbate, *Music and Discourse*, 16.
[245] Molino, Underwood, and Ayrey, 'Musical Fact and the Semiology of Music', 105–6.
[246] Nattiez and Abbate, *Music and Discourse*, 92.

by Glenn Gould. The other layer of these levels also shows up at the poietic level of Gould; he (the producer) interpreted the work with his understanding at the poietic level, his interpretation/performance (the trace) entered the neutral level, the work will be appreciated by listeners (the receiver) at the esthesic level. From this example, bilateral participation across the composer, the performer, and listeners can be found (Figure 29). The first tripartite mode consists of the first direction of J.S. Bach's participation of composing the Prelude at the poietic level and the second direction of Gould's participation regarding his perception of the musical work at the esthesic level. In Figure 29, the directions are annotated as numbers 1 and 2. The second tripartite model consists of the first direction of Gould's participation at the poietic level, in terms of his interpretation; the second direction of participation considers the listeners' participation at the esthesic level.

Applying Weber's rationality to this example, as I explained in Chapter 3, the affectual functions and affectual results are different, and therefore, I suggest locating the affectual function of musical elements as a part of rationality, whilst locating the affectual result as a part of irrationality. Through this revision and the tripartite model, the differences between rationality and irrationality in European art music can be observed. The composer in European art music applies the harmonic and melodic systems to create musical works and applies a notational system to record them; the performers interpret the musical works based on their interpretations. In this process, the rational elements include the structure of composition and notations, and musical elements' functions, while the irrational elements refer to the performers' emotional expressive. Following this finding, the difference between Prelude BWV. 846 and the *Solo for Piano* can also be recognised.

In composing the *Solo for Piano*, as explained earlier in this chapter, the composing technique gradually forms the graphic notations, from chance-derived paper imperfections to semi-determinate compositional materials; meanwhile, the formation of graphic notations requires the composer's decisions and arrangements,[247] from choosing compositional materials, building the relationships between materials, to the degree of free realisation. Putting it differently, the *Solo for Piano* and Prelude BWV. 846 are distinct from each other in the aspects of acts of composing and interpreting. As explained in Chapter 3, the act of composing in BWV. 846 refers to the composer's purposive-rational action of applying existing musical elements

[247] Fay, 'Towards a Rattling of the Everyday', 140.

and the inherent relationships within the notational system to concretise the musical creativity. On the other hand, the act of composing in the *Solo for Piano* is about Cage's rationalisation, including receiving the ideas of Zen, interpreting the ideas as shifting centres, and producing musical works based on interpretations. The different acts of composing lead to the different acts of interpreting.

The interpretations in BWV. 846 focus on how performers carry out the affectual results. Despite the interpretations being different from one another, listeners can recognise the acoustic features when paying attention to the melodies. However, in the *Solo for Piano* the act of interpreting equals the act of decoding, meaning that what the performers must do is determine the details to finalise their realisations. To realise a graphic notation, the performers have to follow the Cage's guidance to decode the meaning of every symbol, and therefore, they experience the second stage of rationalisation, transforming semi-determinate compositional materials into determinate realisations.

In Figure 30, the first arrow shows the composer rationalising the imagination to produce the musical work; this is then determined by the performer in a process that they logically understand by having considered all the performing details, including tempo, rhythm, length and style. The interpretation irrationalises the work, since it involves affectual results, which means that, even though the listeners may have a good sense about how the work will sound, they cannot anticipate the performer's interpretation; hence, at the end of the second tripartite model, listeners appreciate the irrationalisation. On the other hand, in the case of the *Solo for Piano*, Cage rationalised the paper imperfections step by step by applying them to his graphic compositional system. Thus, the *Solo for Piano* contains semi-determinate compositional materials that require the performers' decisions. Such a process creates the second stage of rationalisation as it involves the performers' choices, actions, and thinking. I annotate this by using the second arrow of the first tripartite model in Figure 30. The first arrow in the second tripartite model shows the performers irrationalising their realisation. However, this irrationalisation indicates a different irrationality from European art music, which involves affectual results, whereas the same element in the *Solo for Piano* focuses on unpredictable acoustic results.

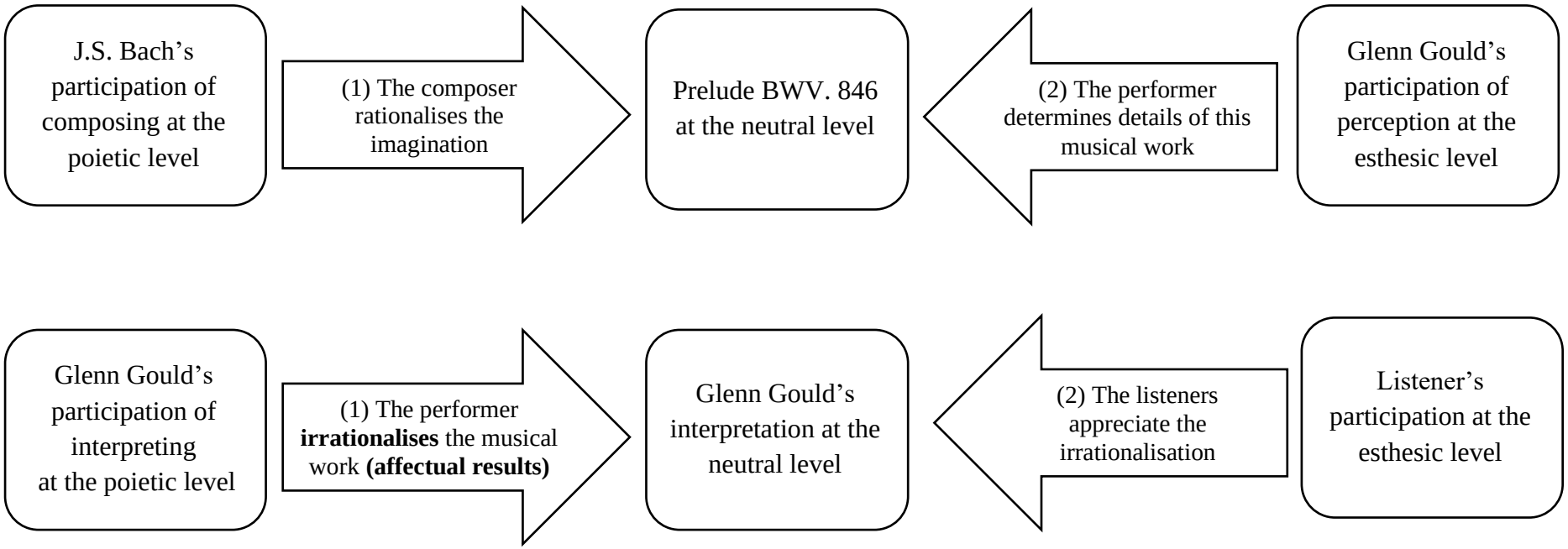


Figure 29 A bilateral participation across the composer, the performer, and the listeners in Prelude BWV. 846.

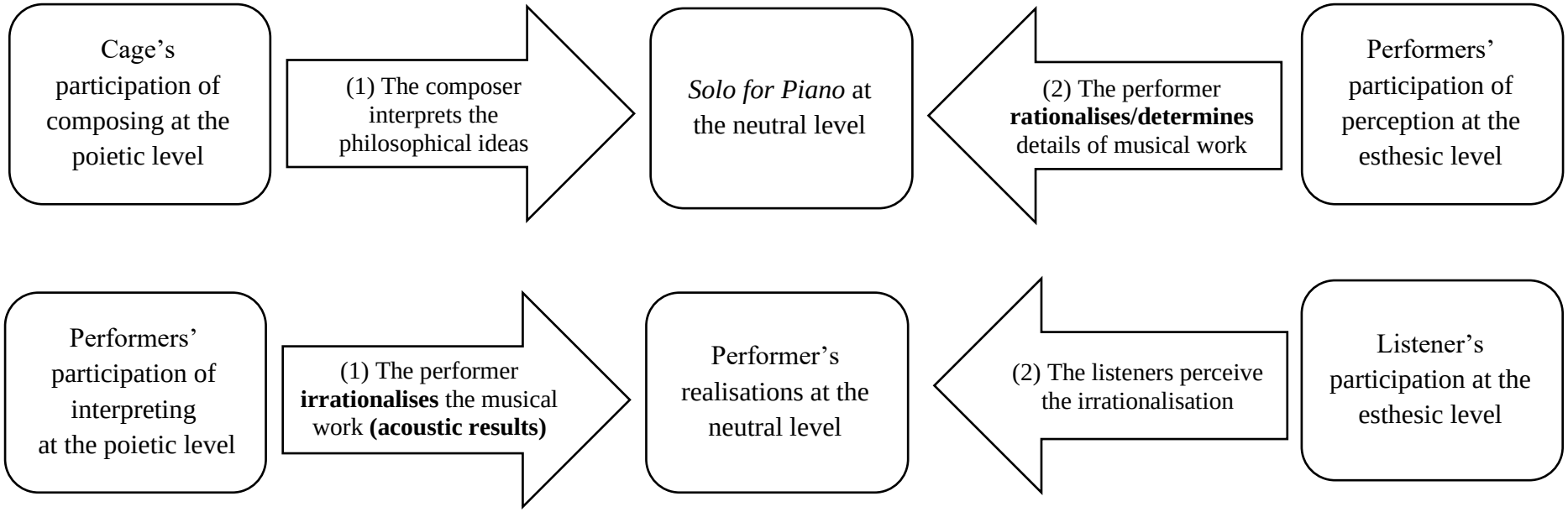


Figure 30 A bilateral participation across the composer, the performer, and the listeners in the *Solo for Piano*.

Certainly, when discussing the open interpretation of a musical work, indeterminate music can be a tricky example. As mentioned in early this chapter, one of Cage's intentions is to create a shifting centre of a musical work. From this aspect of multiple centres, this tripartite model may be able to investigate the difference between centres. As the tripartite model reveals three levels of musical creation, the poietic level, the neutral level, and the esthesic level, in order to explain how music is composed, perceived, and interpreted, this model looks into the two forms of participation, one from the composer, the other from the performer. Nevertheless, the original tripartite model does not consider a unique feature of indeterminate music: an unduplicated feature of each realisation, and therefore, it does not fully consider the potentially different realisations of indeterminate music works. Clarke suggests a revised tripartite model, for which he took Cage's *Solo for Piano* as an example to correct the situation of the marginalisation of the performer.[248] In the correction, he points out the composer produces a material trace of the musical work, while the performers are responsible for transforming this material trace to sounds; with this type of collaboration, a completed musical work is produced. Based on this point, he advises that with the ambiguity of the material trace, indeterminate music may emphasise the position of the performers, as they take a crucial position in turning material trace to sound. Hence, he modifies the tripartite model by adding a new poietic level, a new neutral level, and a new esthesic level in the revision.

In the revision, the composer produces a piece of music at the poietic level; the musical score becomes a material trace at the neutral level and requires a decoding process from the performers, who then interpret the musical score at the esthesic level; the process of realisation of the musical score creates a new poietic level, while the realising result acts as a neutral material trace (Figure 31).[249] From Clarke's revision, the position of the performers has been emphasised; meanwhile, neutral material traces are no longer limited in the musical score itself, but also refers to the realising result of the performers. From this modification, I suggest neutral status not just emphasises the performers' position, but also provides analysts a chance to discover rational features of indeterminate music. Based on this, in Chapter 5 I will focus on the neutral level, the graphic score of the *Solo for Piano*, to see where the rational features are.

[248] Clarke, 'Musical Indeterminacy and Its Implications for Music Analysis'.
[249] Ibid.

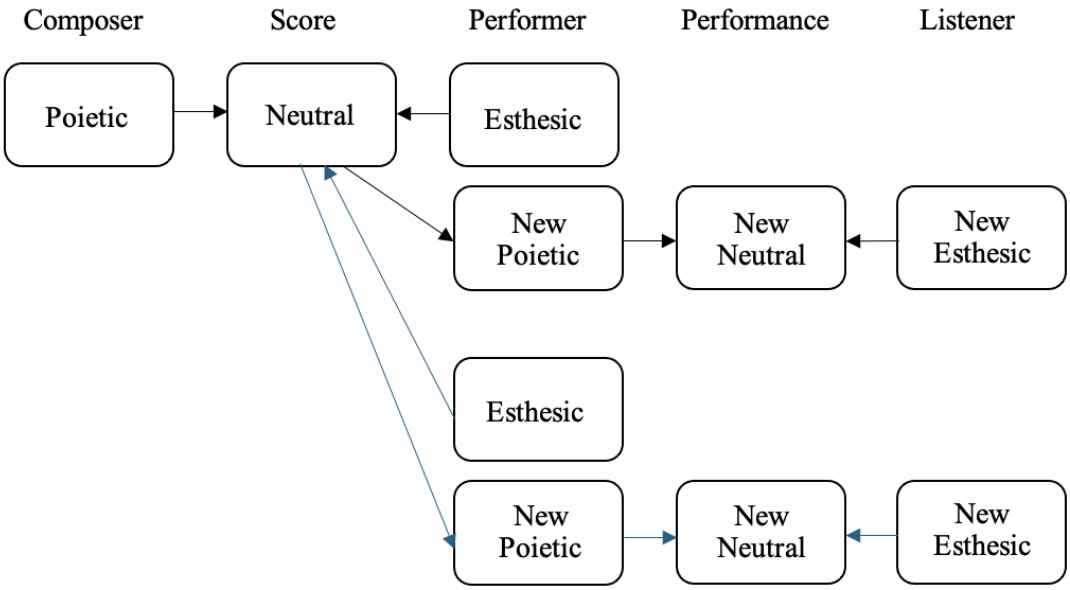


Figure 31 This figure is from David Clarke's revision.[250] P means poietic level; N means neutral level; E means esthesic level.

4.2 Value Formation in the Graphic Compositional System

In addition to the rational features, the revised tripartite model can also be used to delve into rationalisation, as the gradational process makes compositional materials' changeable status apparent. In his graphic compositional system, Cage applies paper imperfections to practise his ideas of the shifting centre. Pritchett describes this as involving two steps: a drawing process and a means of translation.[251] This means that the whole process involves: (i) chance-derived materials, (ii) semi-determinate compositional materials, and (iii) rational-oriented realising results. To be specific, Cage marks the paper imperfections, so these imperfections undergo a determining process that transforms them into performable compositional materials. However, at this stage the compositional materials are only semi-determinate; given their uncertain status, they require performers to determine the final details through the performers' realisations. For this reason, I render the performers' realisation as rational results. This graphic compositional system demonstrates the rationalisation process of these compositional materials. As Figure 32 shows, applying Clarke's revision of tripartite model to Cage, the *Solo for Piano*, and the performer, two-stage of rationalisation can be discovered.[252]

[250] Clarke, 'Musical Indeterminacy and Its Implications for Music Analysis'.
[251] Pritchett, *The Music of John Cage*, 110.
[252] The number 1 refers to the actions at the poietic level, and the number 2 indicates the actions at the esthesic level.

To discover the two-stage rationalisation, deconstruction of the notation is the first step, then referring to Cage's performing instruction of this notation. Taking Notation A on page 1 as the example (Figure 33). From the notation, the compositional materials include the paper imperfections, notes, staff, clef signs, accidentals, numbers in ratio form (16:9), and the perimeter. These are the visible materials, but the elements constructing this graphic notation are more than these visible materials, Cage's performing instructions are equally important since they direct performers regarding how to read and understand the graphic notations. For instance, his performing instruction for Notation A states, 'following the perimeter, from any note on it, play in opposite directions in the proportion given. Here and elsewhere, the absence of indications of any kind means freedom for the performer in that regard'.[253] The instruction suggests the performers to follow the perimeter, play in the opposite directions, and play the assigned proportions; meanwhile, the instruction suggests the performers to choose starting notes freely, and decide the quality of sounds freely.

The first rationalisation focuses on Cage marking the twenty-five paper imperfections that he showed as twenty-five dots, which remained meaningless until in the second step of the graphic compositional system, he overlapped them with other compositional materials, such as, staff, clef signs, and accidentals. After the second step, he turned the twenty-five dots into twenty-five notes, which retained a neutral status as semi-determinate compositional materials in Notation A. For the second rationalisation, as Figure 32 shows, the process depends on the performers. Therefore, Performer 1 and Performer 2 will produce different realisations, use different starting notes, perform directions differently, and produce different quality of sounds. Alongside these differences, at the esthesic level, both performers will decide which notes to start with and which performing direction they will use both hands for; they will also determine all the other details for their particular realisations. Despite them being consciously careful when making their realisation decisions, when they produce them at the new poietic level, they irrationalise them regarding their acoustic results, since Cage has inserted indeterminacy into his notation, therefore with a great number of decisions to be made by each performer, listeners cannot predict the results.

Referring back to the three potential connections that I mentioned in the last section: (i) ambiguity/confusion and irrationality, (ii) purpose/intention and rationality, and (iii) value

[253] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

formation and the process of rationalisation, one crucial variable within these is noticeable – that is the actor who takes the actions. For Cage, the *Solo for Piano* was the practice based on his value and purposive rationality, whereby he applied chance as his primary tool to create its foundation, after which, by offering diverse choices to the performers, indeterminacy became his second tool. From his composition, a gradational transformation of paper imperfections can also be recognised; he created value for his compositional materials, which offered his musical work the status of rationality.

However, as Earle Brown and Carolyn Brown both pointed out, such a degree of freedom caused the performers confusion, since they were not confident enough to know what to do with it, or to take the work seriously.[254] Therefore, even though Cage utilised these compositional elements with clear purposes and intentions, the performers did not share the same values, and so the result of his rationalisation (the *Solo for Piano*) turned into ambiguity, confusion, and irrationality to the performers. Yet, if the performers jumped out of the Western conventional stereotypical structure and system and followed Cage's performing instructions, – choosing notes, directions, and tools as Cage requested, conducting required actions as he suggested, and used different parts of piano constructions as he assigned – the ambiguity and confusion representing irrationality can be solved, and the performers will be able to recognise the rational parts of the *Solo for Piano.*

In short, during this process, they finalise the value formation step by step and transform ambiguity and confusion (indeterminacy) into certainty (determinacy), and so turn irrationality into rationality. However, once again, the process of rationalisation is only open to the performers who participate in the process, which means that the listeners do not recognise this rationality, hence, from their perspective, it reverts to irrationality.

[254] Brown and Dickinson, 'Earle Brown', 142–43; as cited in Iddon, *John Cage and David Tudor: Correspondence on Interpretation and Performance*, 82; Brown, *Chance and Circumstance*, 198–99.

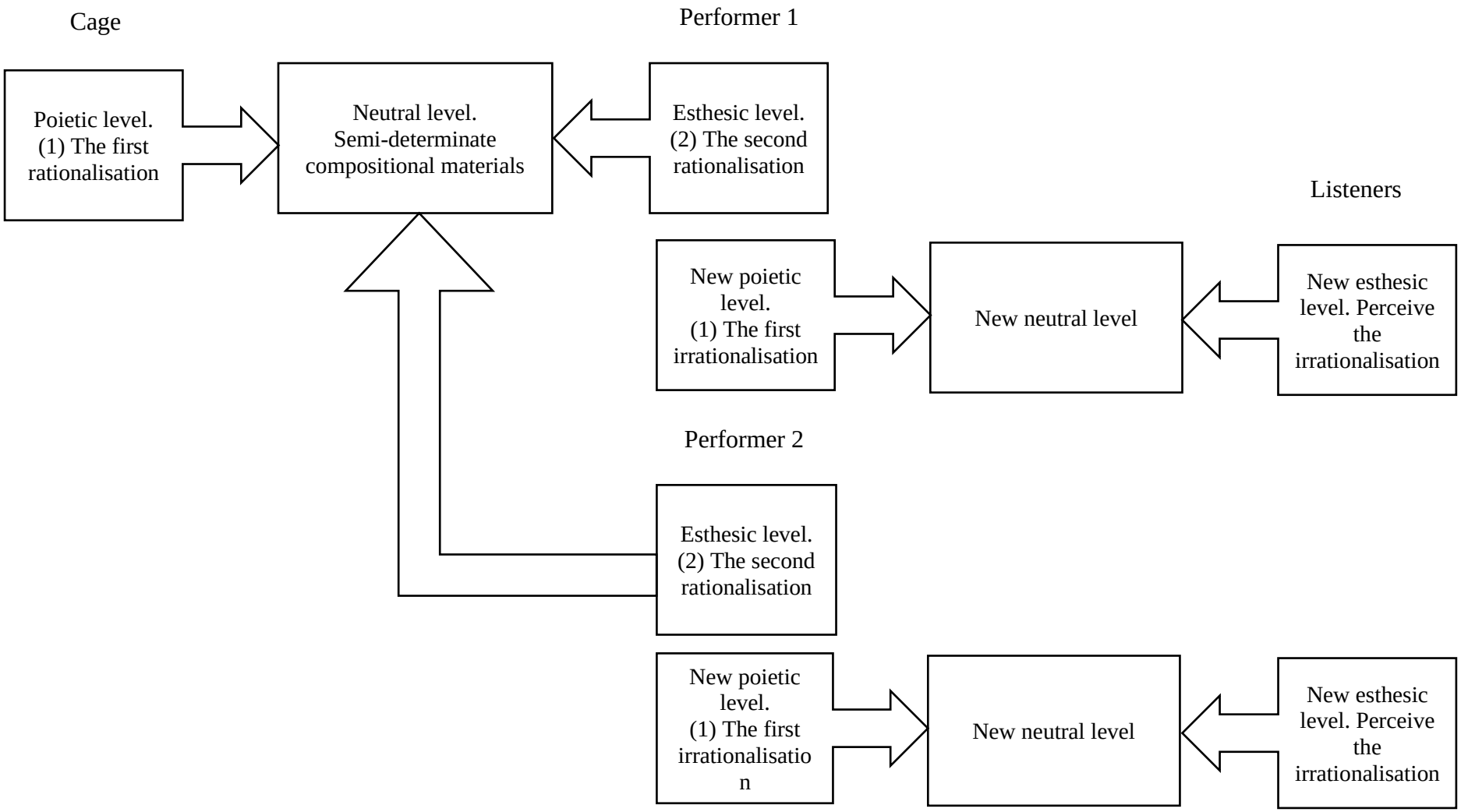


Figure 32 The figure shows two-stage rationalisation and irrationalisation at three levels.

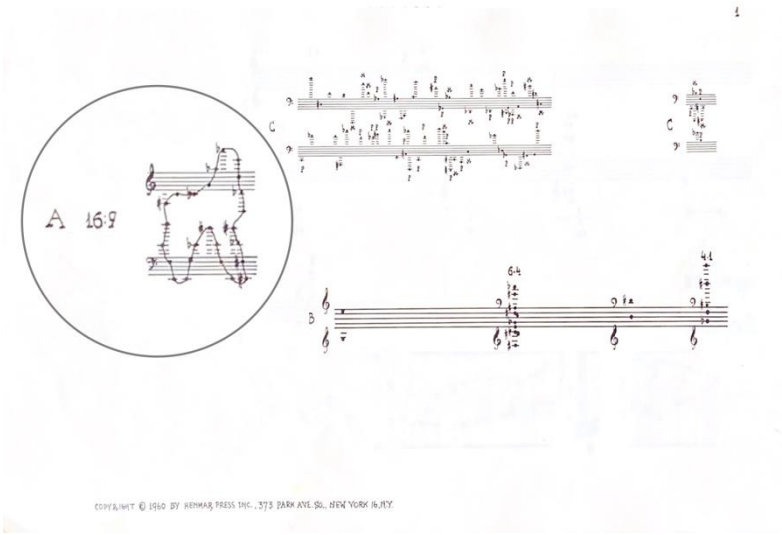


Figure 33 Notation A on page 1 of *Solo for Piano*.

In conclusion, in Weber's rationality, value rationality refers to the individual's expression of preferences, value concepts, and beliefs. This rationality connects with Cage's interpretations of Zen, and his compositions emerged under its influence. This awareness led to an idea that a composer should not be the sole authority of the music he composes; consequently, in order to achieve this ultimate purpose, he produced his interpretations by way of the graphic compositional system. This application fulfilled the conditions of his value-rational action, which he imposed on himself, based on his philosophical enthusiasm. The action of applying and composing are the elements of his purposive rationality as well – a process that required calculation, planning and deep consideration. However, since rationality concerns individuals' intentions, purposes, values, and corresponding actions, Cage's rational composition cannot retain its rationality throughout the poietic, neutral, and esthesic level; rather, the composition turns into irrational at the performer's perspectives. Nevertheless, this irrational status can be gradationally rationalised by following Cage's performing instructions. The changeable status of compositional materials in the *Solo for Piano* presents a great opportunity to revisit the theory of rationality. In the next chapter, I shall focus on the uses of compositional materials and their functions, the types of freedom, and the performing approaches that construct the rational features, and the systems and the inner structures that are hidden within the *Solo for Piano*.

Chapter 5. Analyses of the *Solo for Piano*

1 Introduction

As we already recognise Cage's indeterminate music is based on chance operation, is this possible to discover the structure, if so, how? Delving into Cage's ideas of the composition to observe how the various components are integrated and produce musical sense may offer analysts a good understanding of structure.[255] The analysis of this chapter refers to Nattiez's proposition of differentiating between variables that are extrinsic to the musical traces and intrinsic variables, to present the structure of the musical trace.[256] He explains that the analysis can reflect the analyst's perspective, methods, and background, meaning that the way analysts perceive and describe music is influenced by their own understanding, which he refers to as 'the metalanguage of the analyst'; the metalanguage is shaped by the structure of analysis and the influence of analytical discourse on music.[257] Applying this idea to this research, I will build my analysis upon compositional materials, materials' functions, performing approach, and types of freedom to present the structure of this musical work.

Solo for Piano is based on participation from the composer and the performers; the composer creates the musical framework, while the performers transform the framework into sounds. In the transforming process, the performing instructions play an important role. Nevertheless, Cage did not offer any instruction regarding the rate, or number of, notations and neither did he provide a concrete description to guide the performers about how to realise the sounds.[258] Such a purposive ambiguity authorises the performers to adopt a deep level of autonomy, thereby implementing Cage's philosophical practices through this piece; this purposive ambiguity – whether by way of performing instructions, or in graphic notations – is not only considered as irrationality, but it can also be rationality carrying the composer's value concept, intentions, and purposes. Moreover, as Cage inserted ambiguity differently in graphic notations, the similar type of ambiguity may lead to similar performing approach and types of freedom, and vice versa. This means that ambiguity may allude to a co-dependent relationship between rationality and irrationality and reveal unnoticed systems and structures.

[255] Pritchett, 'The Development of Chance Techniques in the Music of John Cage, 1950-1956', 5.
[256] Nattiez and Abbate, *Music and Discourse*, 143, 178.
[257] Ibid., 134–35.
[258] Thomas, 'Understanding Indeterminate Music through Performance', 93.

When analysing the performing instructions, I notice that the descriptions of instructions are varied; Cage sometimes only mentioned one specific notation, sometime pointed out which compositional materials should be free, and sometimes stated abbreviations only. This means that he did not use a fixed format but was much more flexible with his instructions; thus, one instruction might contain compositional materials, one or more functions, one or more performing approach, and/or one or more types of freedom. In this way, he guided the performers towards a certain way of realisation, while remained purposive ambiguity in each notation.

In order to investigate the new co-dependent relationship between rationality and irrationality, and rational features in the *Solo for Piano*, it is necessary to examine both performing instructions and notations themselves, since Cage did not always mention every used compositional material in the instructions. For instance, in Notation A, where he used numbers to indicate proportions of notes, he did not point to this material (numbers) in the performing instruction. This is also the case in Notation BX, where he applied lines to draw a plant, denoting the sound effect of his imagination. However, in the performing instruction for Notation BX, he only stated 'All at once like a moment of a plant'.[259] With such a brief and simple performing instruction, the performers may find it difficult to recognise the meaning without a graphic notation. For this reason, if the analysis is only based on the performing instructions, some compositional materials may be omitted; therefore, in this section, I shall also deconstruct compositional materials from each notation.[260]

By analysing words and notations, this chapter aims to build five types of relationships within the *Solo for Piano*. As explained above, the first relationship is based on compositional materials, including those based on instructions and notations. The second relationship is based on the functions of compositional materials, the reason for examining functions is to link

[259] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

[260] When examining graphic notations, I notice Cage's composition was built upon paper imperfections, which he took as fundamental materials that offered meanings through other compositional materials, such as drawings and musical symbols. This shows that compositional materials can be a potential foundation to construct the systems and the inner structure of *Solo for Piano*. To put it differently, Cage's composition may include two stages of rationalisation: (i) turning paper imperfections into compositional materials with different level of indeterminacy, and (ii) turning these compositional materials into realisations. The major difference of two rationalisation is paper imperfections and notes/dots. In the first stage of stage, Cage applied paper imperfections to form compositional materials, and therefore, these paper imperfections are under Cage's arrangements – acting as notes or dots – in the second stage of rationalisation. The two stages of rationalisation reinforce the participation of both the composer and the performers. Consequently, this chapter discusses paper imperfections and notes/dots separately and deconstructs the performing instructions and compositional materials.

compositional materials with the same function together to present the inner structure as an aspect of function. The third relationship is based on the types of freedom that Cage offered to the performer, and the fourth relationship is the connection based on performing approaches.

Along with insertions of different ambiguity, as mentioned previously, the types of freedom and the performing approach are the main means by which the performers can participate in the formulation of the *Solo for Piano*. As Philip Thomas emphasised, when he studied the David Tudor interview,[261] one fundamental difference between conventional music and experimental music is that the first involves feeling, whilst the second requires actions. The importance of creating actions experimental music is the equivalent of the performers' expressive interpretations in conventional music. Thus, actions are not only elements of a composer's poietic and esthesic practices, but they are also an essential condition in transforming indeterminate notations into the status of determinacy, thereby presenting the performers with an understanding of the shape of the music itself.[262] Hence, types of freedom and performing approach will be the third and fourth relationships when constructing the systems and the inner structure of the *Solo for Piano*.

The final, and fifth, relationship is based on the appearance of the notations, which is a possible extension of Thomas, who suggests the composer can be recognised based on the acoustic features, even if the composition is based on chance or indeterminacy.[263] Applying this idea to the appearance of graphic scoring, the graphic scores demonstrate a visually recognisable style of composers. For instance, Feldman's *Projection 1* (Figure 34), it is noticeable that some signatures relate to compositional materials: i.e. lines, dotted lines, squares, and rectangles, whilst it can be seen that the similar signature in *Intersection 1* (Figure 35). On the other hand, in Earle Brown's *Folio and 4 Systems* (Figure 36; Figure 37) vertical lines and horizontal lines refer mainly to compositional materials, with various widths and lengths. Thus, a sense of similarity of the composers may be observed based on their graphic style. Certainly, I cannot claim that all the music of one composer is presented in one style only; however, I suggest that the appearance can imply a connection by comparing one notation with another. Hence, I have built the fifth relationship upon the appearance of graphic notations.

[261] Thomas, 'A Prescription for Action', 77. The interview is recorded in the CD *David Tudor – Music for piano*, edition RZ 1018-19, published in 2007.
[262] Ibid., 77–78.
[263] Ibid., 79.

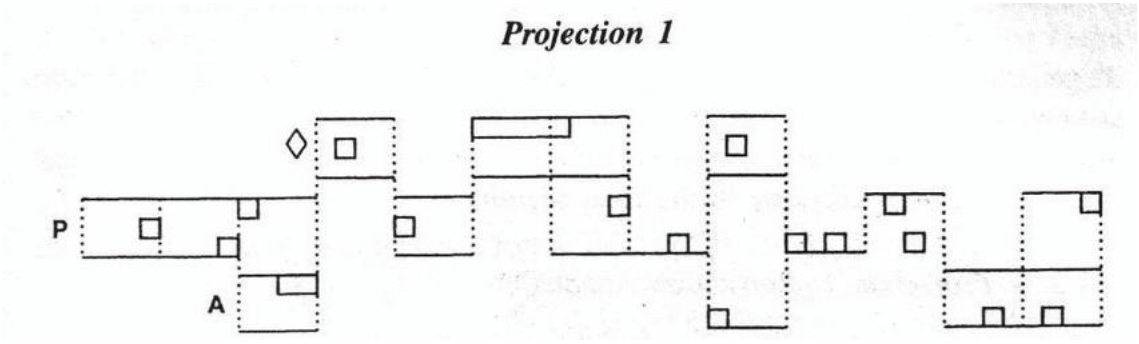


Figure 34 *Projection 1* (1950) by Morton Feldman.

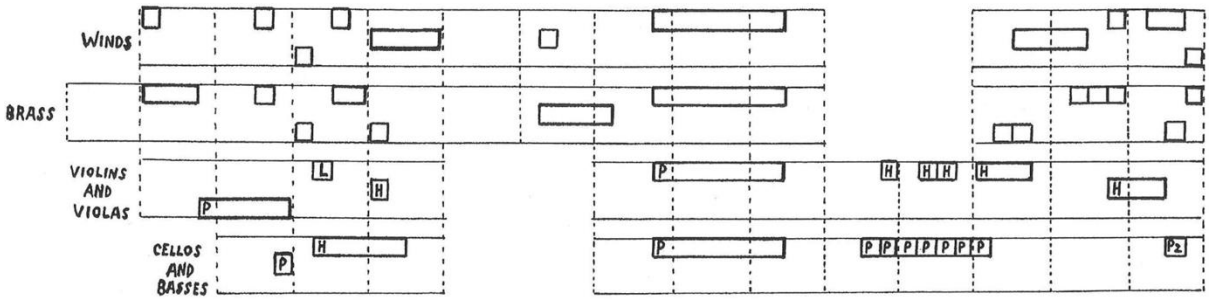


Figure 35 *Intersection 1* (1951) by Morton Feldman.

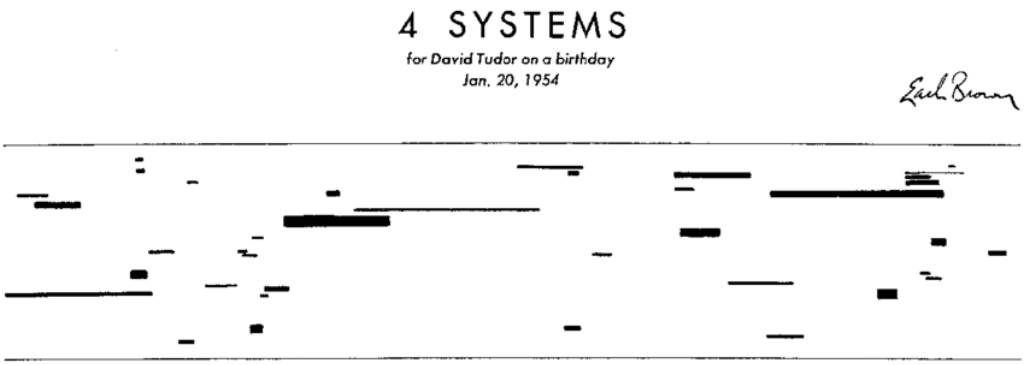


Figure 36 *4 systems* (1954) by Earle Brown.

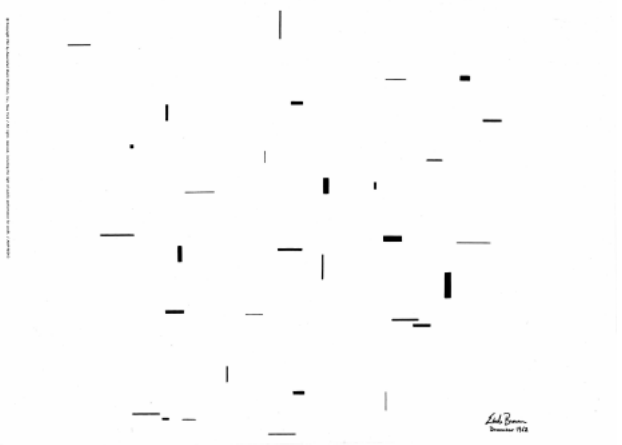


Figure 37 *December 1952* (1952) by Earle Brown.

2 Analyses of Compositional Materials: Notation A to Notation CF

2.1 Compositional Materials in Notation A to Notation Z

As described above, the analyses start with categorising compositional materials from the performing instructions and corresponding notations, followed by discussing freedom and the possible performing approach. The performing instruction in Notation A is: 'Following the perimeter, from any note on it, play in opposite directions in the proportion given. Here and elsewhere, the absence of indications of any kind means freedom for the performer in that regard'.[264] From this instruction, the compositional materials can be found as follows: the perimeter, notes, directions, and the given proportion; the performing approach includes two instructions: (i) following the perimeter and (ii) playing in opposite directions in the assigned proportion; while the type of freedom is free to decide starting point and performing direction of this notation, together with an absence of indications. The compositional materials in Notation A, which was not mentioned in the performing instruction, are numbers in ratio format, clef signs, accidentals, paper imperfections, and notes (Appendix Table 48).

Based on types of freedom (free to choose the note as the start; free to determine the performing direction; free to decide the quality of the sounds), the performers need to decide which note to start with, the direction for either the right or left hand, together with other potential aspects of sound production, such as dynamics and duration. This means that the performers are responsible for decisions related to note selection, direction, dynamics, and duration. The performing approach involves following the perimeter, playing in opposite directions, and performing the given proportion. This implies that the performers should adhere to the perimeter, with both hands should move in different directions, and perform the given proportions of notes. Taking Notation A on page 1 (Figure 38) as the first example; the instructions regarding types of freedom suggest that the performer can begin with any note on the perimeter and should play sixteen notes with the right hand while playing nine notes with the left hand, or vice versa. Thus, the performer may choose a note C as the starting point for the right hand and a note C♭ as the first note for the left hand.

[264] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

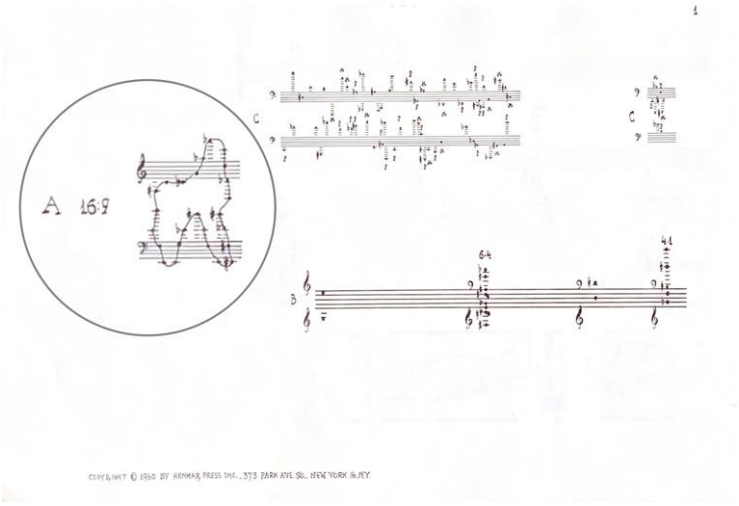


Figure 38 Notation A on page 1 of the *Solo for Piano*.

The interesting part lies in the instructions for the performing approach. After examining Notation A several times, the same question always come to mind: can the performers choose both hands to start the same note with? Or can they select notes in an alternating pattern, such as even and odd numbers – e.g., choosing the first, third, and fifth notes for the right hand, and the second, fourth, and sixth notes for the left hand? Or can the performers skip notes? These questions are addressed in Cage's instructions for the performing approach. Firstly, I wondered whether both hands could start with the same note. The instructions emphasise playing according to the assigned proportion, making it impossible to use the same note as a starting point for both hands. The performers need to play twenty-five notes in total, with sixteen notes for the right/left hand and nine notes for the left/right hand, this performing approach limits the performers in starting with the different notes. The next issue concerns the order of note selection. The instruction specifies that the performing order should follow the perimeter, indicating that the selection order should be based on the perimeter. This can be demonstrated by using a website application, named 'Solo for Piano App'.[265] This app provides clear realisations from Notation A to Notation AZ. Therefore, I have utilised it to demonstrate two realisations of Notation A on page 1, in order to answer the previous questions.

To begin with, I numbered the twenty-five notes in Notation A (Figure 39), starting with note C and following the perimeter until reaching the last note, F♯. The numbered notes are listed in

[265] This app stems from the research output of the AHRC-funded project led by Philip Thomas, Martin Iddon, Emily Payne, and Christopher Melen. It enables users to create their own interpretations from Notations A to AZ by setting the notations and their corresponding pages. Website address: https://solo.cageconcert.org/app/ (Accessed: 19 September 2023).

Table 1. I then used these numbered notes for comparison with the realisations from the Solo for Piano App. As shown in Figure 40, the first note of the G clef is G, followed by A♯, G♭, G, E, E, F, G, C, F♯ (number 25), C (number 1), C♭, G, A♭, C♭, and E. In contrast, the first note of the F clef is F, followed by E, E♯, F♯, A♭, A♭, B, D♭, and D. Comparing this with Table 1, it is observed that the order of notes in the G-clef is from numbers 16 to 25, followed by numbers 1 to 6. Meanwhile, in the F-clef, the order is from numbers 15 to 7. This realisation demonstrates that the order of notes, indeed, follows the perimeter.

The second realisation also reaffirms that the order must adhere to the perimeter. When examining the notes in a counterclockwise direction starting from the second-to-last note C (number 1) of the F-clef, it presents a sequence from numbers 1 to 8, with number 25 as the final note in the F-clef. In the G-clef, the sequence starts with number 9 and ends with number 24 (Figure 41). Based on these two realisations, it may be deduced that Cage informed the performers about the order of note selection. This also answers the third question regarding skipping notes: the performer cannot skip any notes; otherwise, the performance would deviate from Cage's instructions to follow the perimeter and play the given proportion.

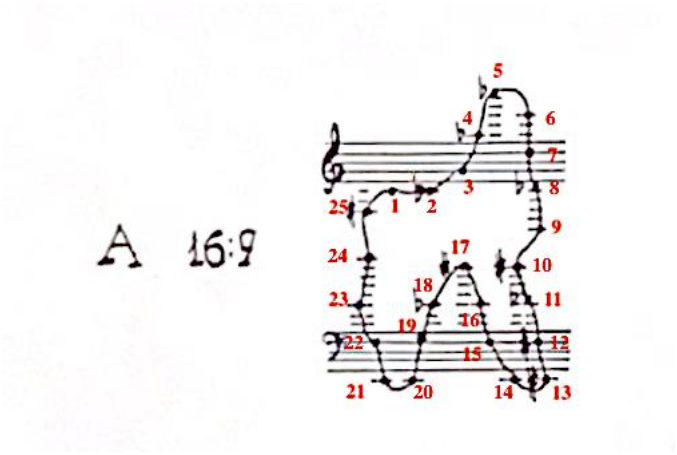


Figure 39 Numbered Notation A on page 1.

Number	1	2	3	4	5	6	7	8	9
Note	C	C ♭	G	A♭	C♭	E	D	D♭	B
Number	10	11	12	13	14	15	16	17	18
Note	A♯	A♭	F♯	E♯	E	F	G	A♯	G♭
Number	19	20	21	22	23	24	25		
Note	G	E	E	F	G	C	F♯		

Table 1 This table shows 25 numbered notes in Notation A on page 1.

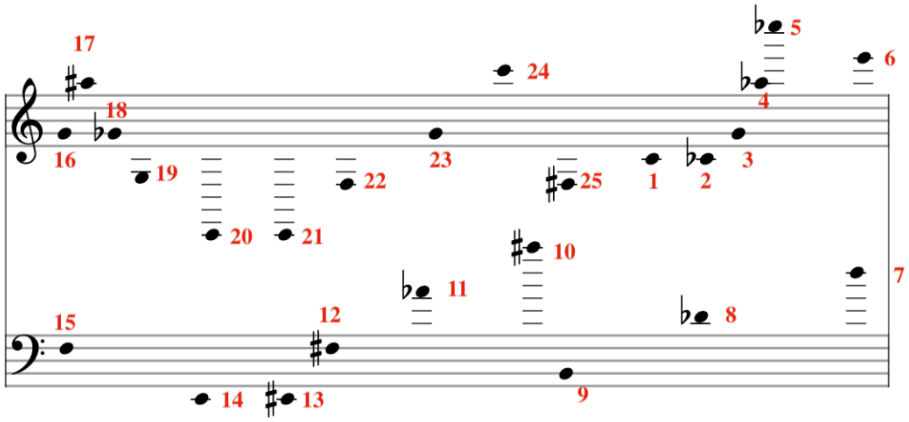


Figure 40 The first realisation of Notation A on page 1 from the Solo for Piano app. The numbers here show the order of notes in the perimeter.

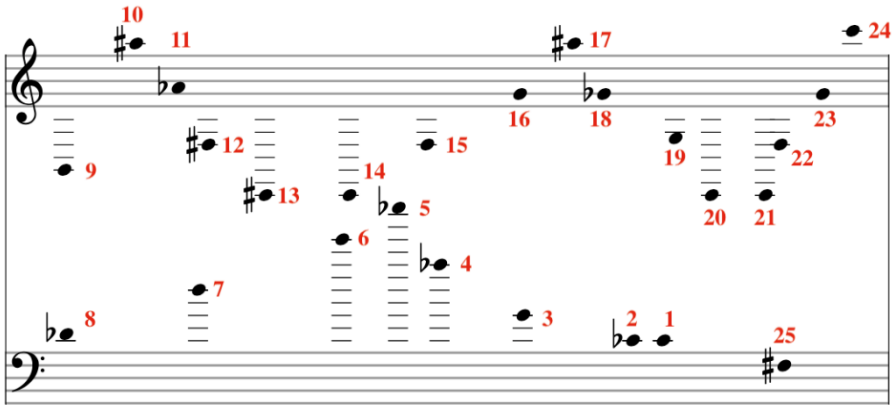


Figure 41 The second realisation of Notation A on page 1 from the Solo for Piano app. Similarly with Figure 38, the numbers here show the order of notes in the perimeter.

Before analysing Notation A, it may appear that the instructions for freedom and the performing approach are seemingly identical. However, on closer, I discovered that the instruction for the performing approach aligns much more closely with 'limitations' – i.e. what does the performer have to do? Which conditions must be fulfilled? How should the performer go about realising the notation? However, types of freedom in interpretation arises, not only from Cage's performing instructions, but also from what he did 'not' specify in them, which shows how performers decode the notations based on their understanding. This distinction is the primary reason why I intend to discuss types of freedom and the performing approach separately. Hence, this section discusses eighty-four notations by dissecting compositional materials, types of freedom, and performing approach.

The performing instruction of Notation B (Figure 42; Figure 43) is 'An aggregate must be played as a single ictus, where this is impossible, the unplayable notes shall be taken as

harmonics prepared in advance. Harmonics may also be produced where they are not so required. Resonances, both of aggregates and individual notes of them, may be free in length. Overlappings, interpenetrations, are also free. The single staff is provided with 2 clef signs, where these differ, ambiguity obtains in the proportion indicated by the 2 numbers above the aggregate, the first of these applying to the clef sign above the staff. An inked in rectangle above a pair of notes indicates a chromatic cluster' (Appendix A Table 49).[266] From these performing instructions, it may be observed that Cage used descriptions of acoustics to guide the performer regarding how to execute realisations, including overlappings, harmonics, interpenetrations, ictus,[267] and chromatic clusters.

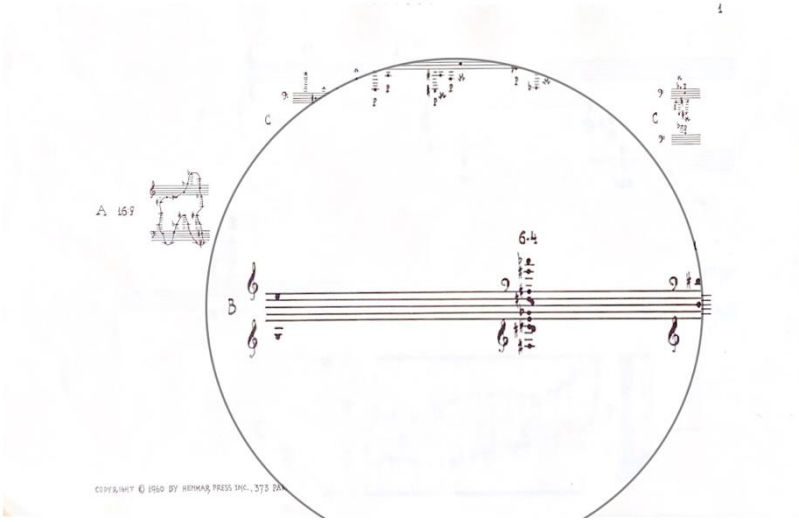


Figure 42 Notation B on page 1.

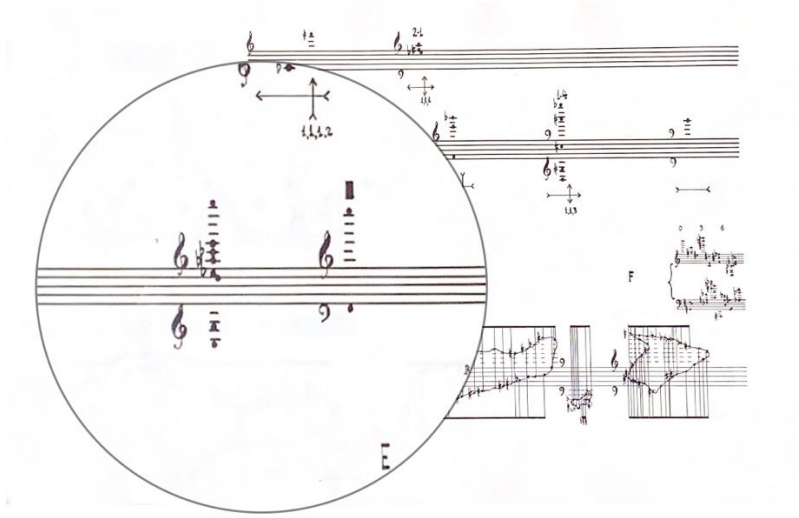


Figure 43 Notation B on page 2.

[266] Ibid.

[267] According to Robert Donington (2001), when ictus is used in a music context it refers to a comparable stress on a certain beat of a bar or in a certain metre. Applying this explanation to Cage's performing instruction of Notation B, the performers shall choose notes from one aggregate and stress them in the realisation, while the unperformable notes shall be realised before the aggregate.

As mentioned in Notation A, the instruction for the performing approach resembles more of a limitation. In Notation B, Cage treats the aggregate as a single ictus, and unplayable notes must be performed as harmonics. Additionally, the clef signs indicate the specific number of notes to be played. This limitation is similar to that in Notation A, where choosing notes is free, but skipping or repeating notes is not allowed.

A closer examination of Notation B on page 1 clarifies this limitation further. In Notation B on page 1, there are two aggregates, which I have numbered as 1 and 2. In aggregate 1, there are a total of ten notes, with two ambiguous clef signs. The F-clef is positioned above the staff, requiring the performer to choose six notes for it, while the G-clef, positioned below the staff, should have four notes selected (Figure 44). To organise all potential notes, they are numbered from top to bottom, ranging from 1 to 10. It is evident that the performer has twenty choices in this aggregate (Table 2). The performers select six notes from the first row of Table 2 and four notes from the second row, or vice versa. The first realisation from the Solo for Piano App illustrates one possible execution: F♯ (number 10), D♯ (number 9), G (number 7), E (number 5), F♯ (number 4), and A♯ (number 3) for the F-clef, and C♯ (number 8), G♭ (number 6), E♯ (number 2), and A♭ (number 1) for the G-clef (Figure 45). In Notation B, aggregate number 2 comprises a total of five notes, but with ten potential notes – five for the F-clef and five for the G-clef. Numbering them in the same way as Table 2, and listing them in Table 3, which displays all potential notes. In the same realisation, aggregate number 2 is presented as B♭ (number 5), F (number 4), B♯ (number 3), and D♯ (number 2) for the F-clef, and C (number 1) for the G-clef (Figure 45). The preliminary analysis of Notation A and Notation B reveals a commonality: both notations share a performing approach based on playing in assigned proportions. This connection between the two notations will become evident when the results of visualisation are discussed.

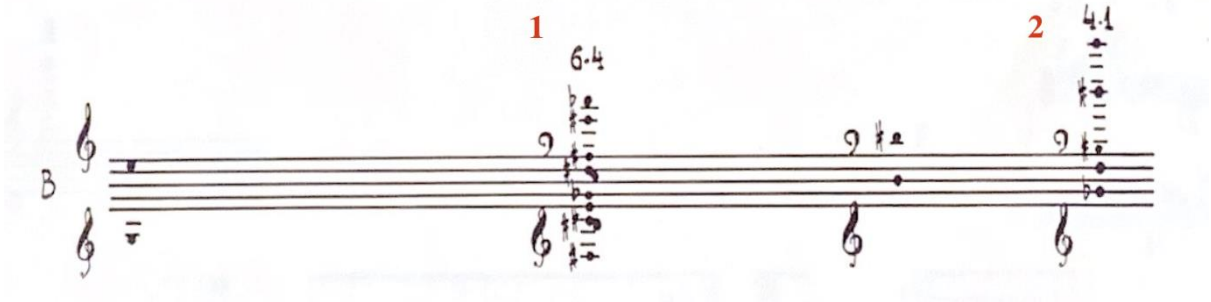


Figure 44 Numbered Notation B on page 1.

Number	1	2	3	4	5	6	7	8	9	10
Notes for F-clef	C ♭	G♯	A♯	F♯	E	D♭	G	E♯	D♯	F♯
Number	1	2	3	4	5	6	7	8	9	10
Notes for G-clef	A♭	E♯	F♯	D♯	C	G♭	E	C♯	B♯	D♯

Table 2 20 numbered notes in Notation B on page 1.

Number	1	2	3	4	5
Notes for F-clef	E	D♯	B♯	F	B♭
Number	1	2	3	4	5
Notes for G-clef	C	B♯	G♯	D	G♭

Table 3 10 numbered notes in Notation B on page 1.

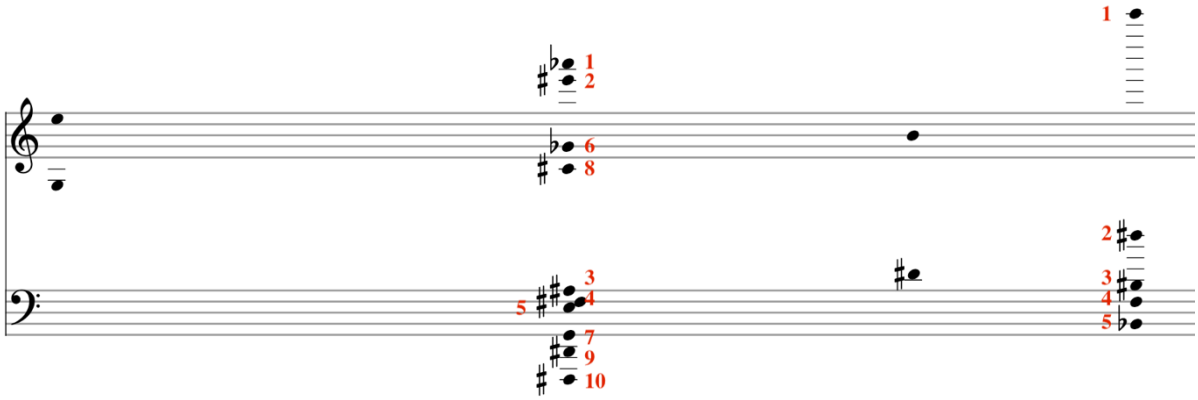


Figure 45 The first realisation of Notation B on page 1 from the Solo for Piano app.

As mentioned above, freedom in Notation B encompasses aspects, such as harmonics, overlapping, interpenetrations, and lengths. This type of freedom allows performers to determine freely how they produce harmonic sounds, overlapping notes, and to decide the duration, dynamics, and amplitude of the notes. In essence, these freedoms can be categorised as choices related to playing acoustics, and sound quality. Additionally, performers' freedom to choose notes for the aggregate is also evident in Notation B.

The performing instruction of Notation C is relatively simple and direct: 'M = mute. P = pizz. All single tones'. [268] This performing instruction shows its performing approach as abbreviations; the other compositional materials found here include the G-clef, accidentals, staff, paper imperfections, and notes (Appendix A Table 50). In Notation C, performers are not required to choose specific notes, or to decide on a direction; instead, they must consider how

[268] Ibid.

to produce acoustics in accordance with Cage's annotations and make decisions regarding the duration and dynamics of the notes; the symbols showing levels of dynamics can be found above the notes (Figure 46). Thus, the types of freedom present here revolve around the choice of duration and the choice of dynamics. Cage's annotations specifying which notes should be played as mute and pizzicato.

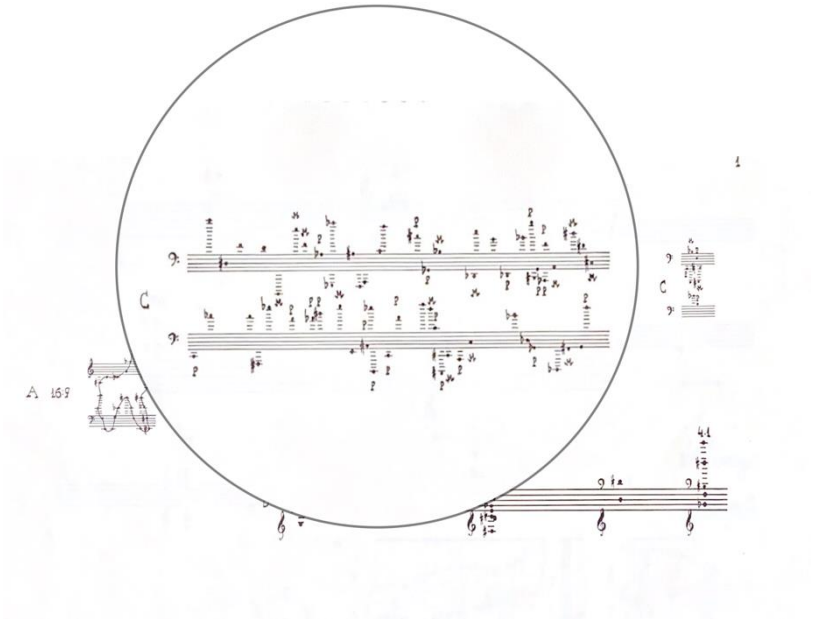


Figure 46 Notation C on page 1.

Notation D is the first notation that Cage connected to a previous one. Here, he provided a performing instruction, stating, 'Like B, but with vertical arpeggiation and time tendencies making use of harmonics unnecessary. The arpeggiation is indicated by double arrows accompanied by a series of numbers. A sign with 1, 1, 1 means 1 note in the middle followed by 1 note higher or lower followed by 1 note lower if the 2nd was higher, higher if the second is lower. A sign with 2,1 means two notes of which at least 1 is the highest of the three followed by a third lower note. The horizontal arrows refer to time and the tendency of the tones to sound sooner, later, or at the point of notation'.[269] From this performing instruction, I noticed that Cage especially described this notation as being based on Notation B, with vertical arpeggiation and time tendencies. When examining the compositional materials, I found that double arrows indicating relationships between notes had been added. One of the performing instructions of Notation D that he provided was, 'The sign [arrows with numbers] 1, 1, 1 means 1 note in the middle followed by 1 note higher or lower followed by 1 note lower if the 2nd was higher, higher if the second is lower' (Figure 47).[270]

[269] Ibid.
[270] Ibid.

Along with this performing approach, Cage also mentioned that the horizontal arrows indicate time and the tendency of the tones, which can be either sooner, later, or at the point of notation. The horizontal arrows can point in different directions – left, right, inside, and outside. Based on this performing instruction, the performing approach of this notation includes choosing notes under the instructions of the sign (where the performing notes should be higher or lower depending on other notes), and different horizontal arrows indicate the timing and tendency of the sounds. The types of freedom in this notation include optional harmonics, a free choice of the length of individual notes, and free overlapping and interpretation, which are inherited from Notation B. Additionally, Notation D provides the type of freedom related to the choice of notes and the choice of timing and tendency (Appendix A Table 51).

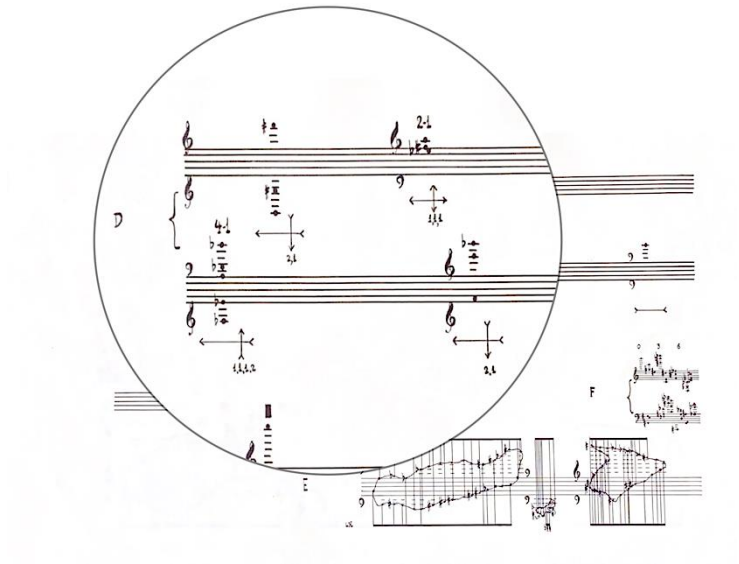


Figure 47 Notation D on page 2.

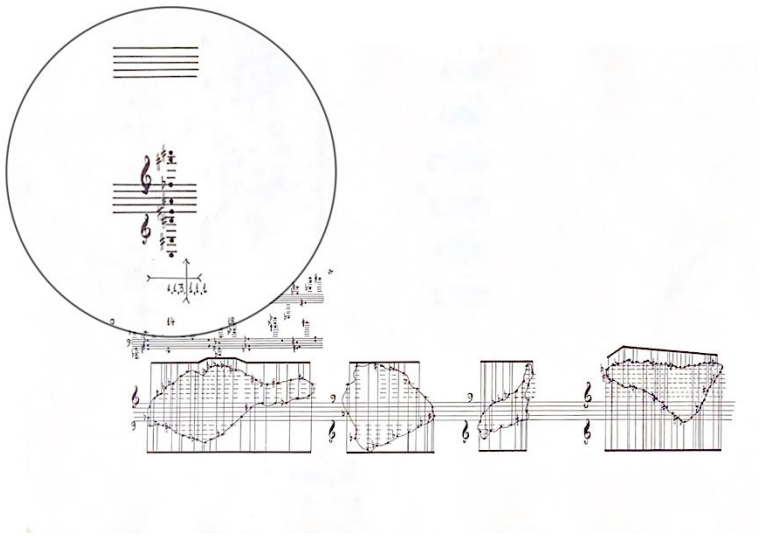


Figure 48 Notation D on page 3.

The next notation, Notation E, exhibits a similar appearance to Notation A, as Cage used grouping to connect every note in both notations (Figure 49; Figure 50; Figure 51). Furthermore, the presentation of clef signs and staff resembles that of Notation B, featuring a single staff with one clef sign above it and one below it. These shared visual elements suggest two potential relationships between Notations A and E, as well as Notations B and E. These relationships are based on the consistent use of the same compositional materials and the similar presentation style across them. This connection also relates to the performing approach that Cage designed for Notation E.

In Notation B, Cage instructed performers to choose specific numbers of notes, based on the assigned clef signs. Notation E follows a similar realising method, guiding performers to play notes with either the clef signs above or below the staff. Although he did not explicitly explain which clef signs should be assigned to specific notes, as he did in Notation B – where he indicated that notes should be performed with either the clef signs above or below the staff using two numbers, with the first one applying to the above sign – he used the abbreviation of hands (R.H. and L.H.) and the directions of stems (either up or down) to provide guidance. This allows for the realisations of notes with upward stems to correspond to the clef signs above the staff, and vice versa.

The performing instruction of Notation E – 'Play with hands indicated, where clefs differ, a note is either bass or treble. The notes having a single stem are to be arpeggiated (up or down). Play notations from left to right'.[271] This encompasses various compositional materials, including hands, clefs, notes, bass or treble, stems, arpeggiation, direction (left to right), and the notation itself, which also presents additional compositional elements, such as accidentals, groupings, staff, paper imperfections, and abbreviations for hands. From the performing instruction, it becomes evident that Cage offered relatively limited types of freedom for this notation. The performing hands are indicated, the order of notes is determined by the stems, and the clef signs are provided. One type of freedom found in this notation, however, pertains to the quality of sounds, allowing performers the freedom to decide on aspects like duration, dynamics, and timbre, among others. Also, since freedom is relatively constrained in Notation E, the performing approach is quite comprehensive. In the performing instruction, Cage specified hands, the order of notes, and the performing direction, so performers may not need to make further decisions, except regarding the quality of sounds (Appendix A Table 52).

[271] Ibid.

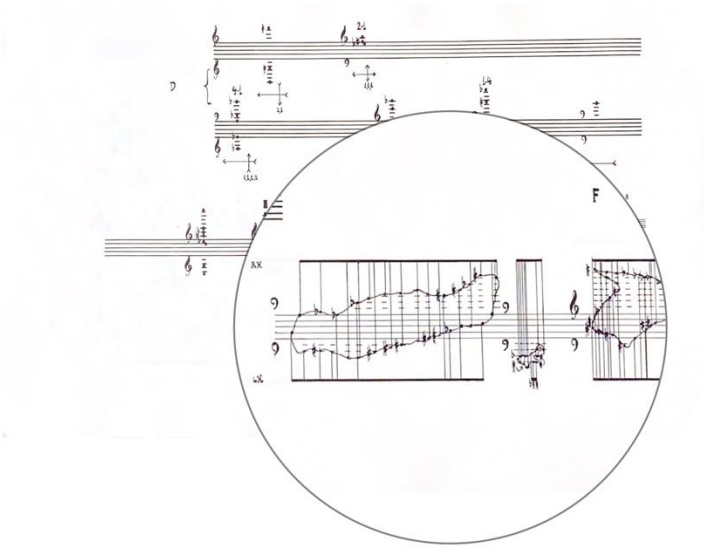


Figure 49 Notation E on page 2.

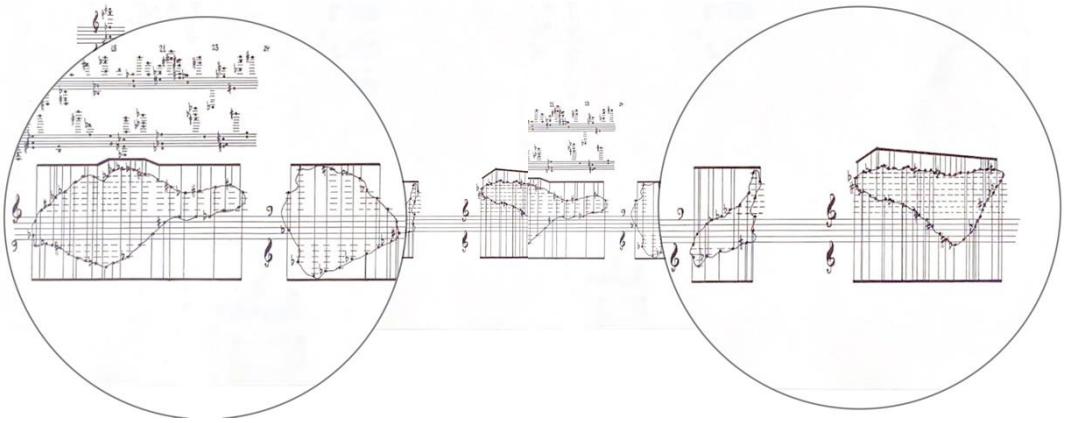


Figure 50 Notation E on page 3.

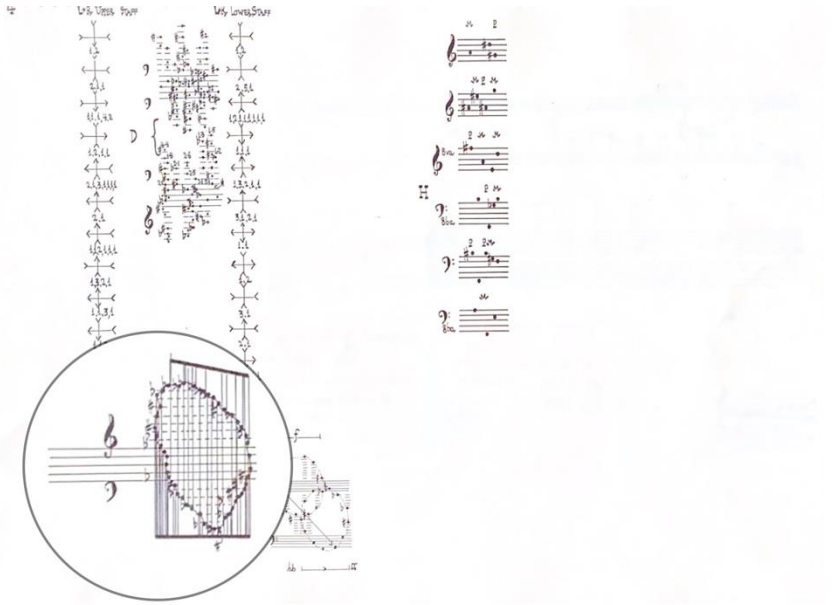


Figure 51 Notation E on page 4.

The performing instruction of Notation F is 'Numbers are seconds or other time units'.[272] This notation represents the first instance where Cage employs numbers to indicate time units and where he restricts the duration of notes by these numbers (Appendix A Table 53).[273] In essence, the performer is expected to play the notes between 0 and 3 within three seconds (Figure 52) – assuming the chosen time unit is seconds, so the number 3 means the third second. This limitation constitutes the sole restriction within this notation.

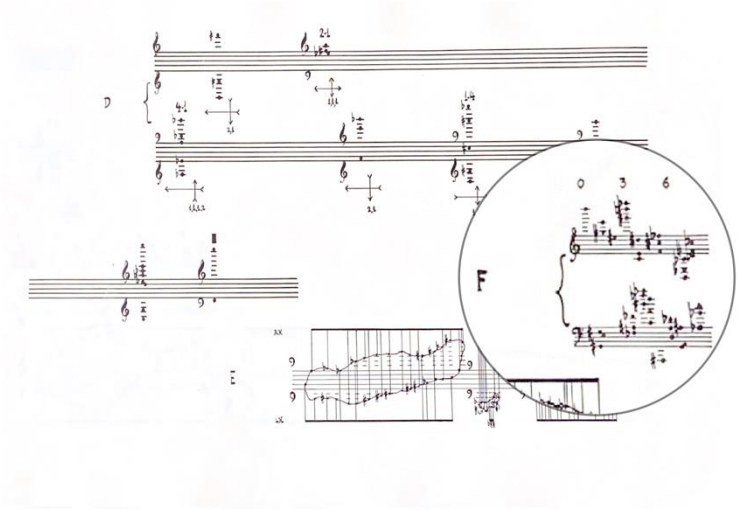


Figure 52 Notation F on page 2.

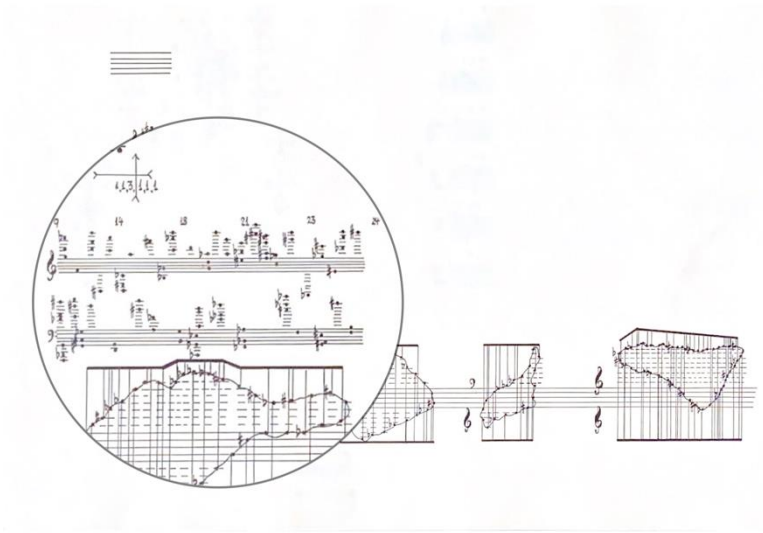


Figure 53 Notation F on page 3.

[272] Ibid.

[273] By describing Notation F as the first instance here, I do not mean to clarify this notation is the first one in the performing order, but the first in the alphabetical order (Notations A to CF), as my analyses of notations are based on the sequence of performing instructions so the order of my analyses will follow the performing instructions.

Notation G comprises compositional materials, such as numbers, keys, harp, arrows, dynamic indications, circles, circumferences, G-clefs, F-clefs, accidentals, groupings, notes, and paper imperfections. The performing instruction for this notation is as follows: 'Of notes written play number given in any manner (keys, harp) beginning and ending as indicated by arrow. Dynamic indications accompany each circle on the circumference of which the notes are placed. (scale ppp-fff)'.[274] From the performing instruction, it is understood that the performers need to play the specified number of notes, with the first and last indicated by the arrows. Instructions for dynamics are provided through lines above and below each circumference (Appendix A Table 54).

Taking Notation G on page 4 as an example (Figure 54), it features two circles, each with pairs of straight lines and arrows indicating the first and last notes of the circle. In the first circle, the straight line starts with D♭ (number 1) and ends with C♯ (number 9), while the second circle starts with C♯ (number 9) and ends with G (number 19). The performer must follow the order of D♭, C♯, and G, regardless of the number of notes in each circle. Following the performing instruction, the performer selects nineteen out of twenty-four notes for realisation, with three notes assigned by Cage: note number 1 at the beginning, note number 9 in the middle, and note number 19 at the end. When choosing notes from the left circle (from notes number 1 to 12) they should be played as *f*, whereas notes from the right circle (from numbers 13 to 24) require dynamic variations from *pp* to *ff* (Figure 55). Thus, the performing approach of Notation G involves playing with keys or harp, starting with a given note and ending with another given note, while following the dynamic indications. The types of freedom in this notation include the freedom to choose the performing method and the freedom to choose a certain number of notes.

[274] Ibid.

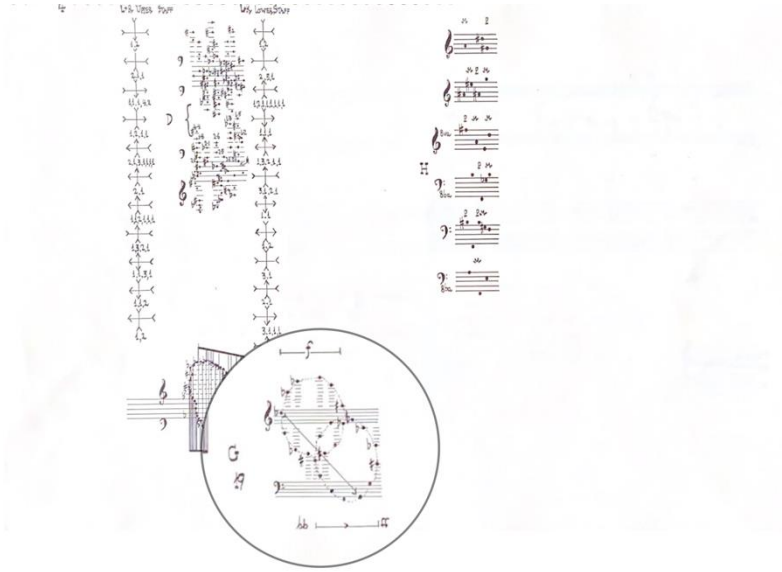


Figure 54 Notation G on page 4.

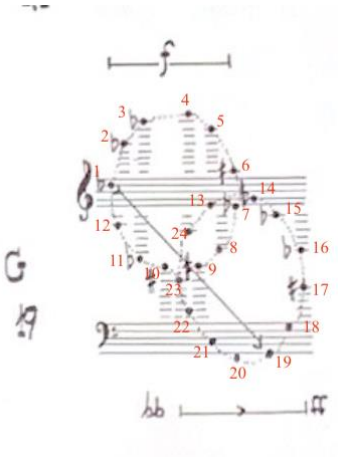


Figure 55 Numbered Notation G on page 4.

Next is Notation H (Figure 56), and its performing instruction is: 'Any one of these [staffs], as in C, or any number (including all) [staffs] in sequence, maintaining, in the latter case, the clef sign of the one first played'.[275] Once again, from the performing instruction, Cage specifically mentions 'as in C',[276] establishing a relationship between Notations C and H (Appendix A Table 55). As indicated in the performing instruction, the performer needs to choose which staff or staffs will be included for the realisation. The number of performing staffs can range from one to them all. When all staffs have been selected, the clef sign of the first one played is retained for the realisation.

[275] Ibid.

[276] Notation C provides the performing instruction, 'M = mute. P = pizz. All single tones' (ibid.); the performing approach refers to performing mute or pizzicato for annotated notes.

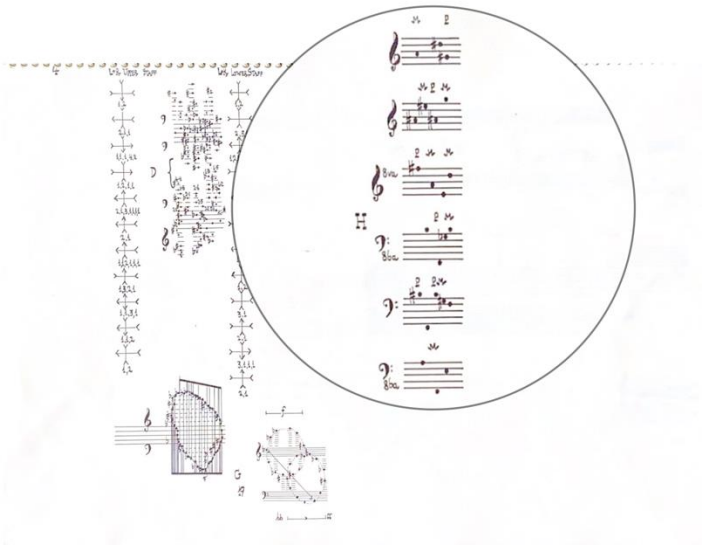


Figure 56 Notation H on page 4.

Notation I, which Cage stated, 'Pizz. where indicated. A single tone, interval, or a 3 note aggregate. Reappearances of tones to be played as originally'.[277] Noticeably, pizzicato and mute constitute key elements of the performing approach for this notation, which also encompasses single tones, intervals, and three-note aggregates. However, Cage does not give an instruction for the mute technique (Figure 57). Another important aspect is the 'reappearances of tones to be played as originally', meaning that the performers need to adhere to the performing approach that was established when the notes first appeared. When examining the performing instruction and the notation, I noticed that Cage does not explicitly specify the types of freedom present in Notation I. Nevertheless, based on the patterns I have seen in previous notations, it can also be inferred that performers have the freedom to shape the quality of sounds, including decisions regarding duration, dynamics, and timbre, in their realisations (Appendix A Table 56).

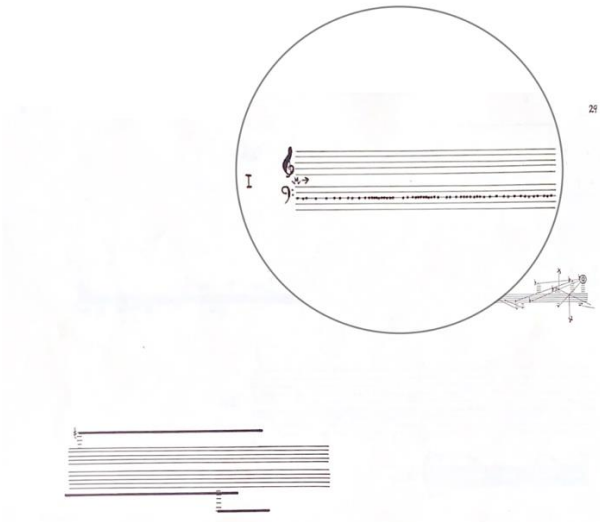


Figure 57 Notation I on page 29.

[277] Ibid.

From Notation J (Figure 58; Figure 59; Figure 60), there are several straight lines with directional arrows and numbers with short lines above or below. These compositional materials serve important purposes in this notation, during which performers play the given number of notes within the range of straight lines, and that the direction can be backwards, or forwards, depending on the directions of the arrows. The short lines above and below the numbers indicate that the notes can be an ascending gamut or descending gamut, and ∾ in the performing instruction mean that notes to be played as ascending and descending gamut between limits connected by lines. Arrows indicate direction in space-time backwards and forwards. Based on this performing approach, the performers are seemingly free to decide performing ascending or descending gamut. However, when reflect this performing approach to the notation, I notice some arrow's directions limit the performers in certain performing direction. For instance, numbers 22, 7, and 31 are with moving backwards arrows, and number 73 is with a moving-forwards arrows in Notation J on pages 5–7 (Figure 61). By following the arrows, the performers are limited in performing ascending gamut for the number 22, and performing descending gamut for the numbers 7, 31, and 73. I conclude the performers' approach will be to perform the according number of notes, taking their direction from the arrows, and performing ascending gamut or descending gamut, based on the short lines, the symbol ∾. The types of freedom of Notation J are deciding notes within a range of notes and deciding quality of sounds (Appendix A Table 57).

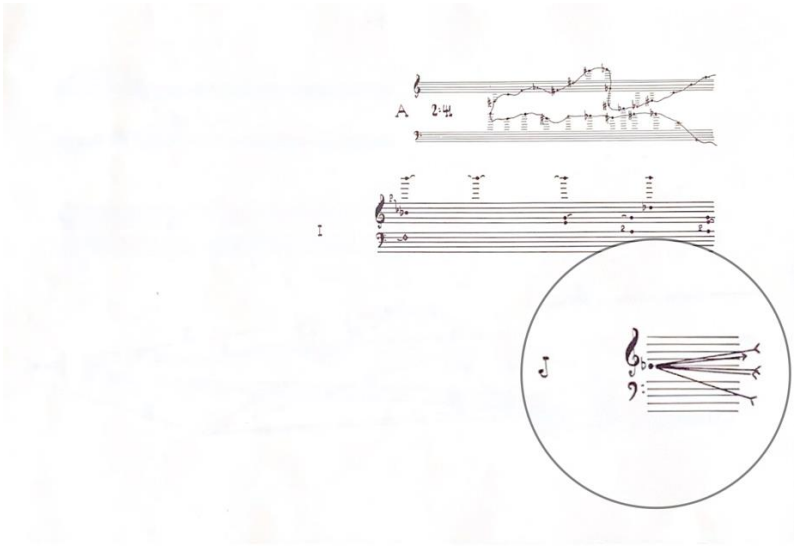


Figure 58 Notation J on page 5.

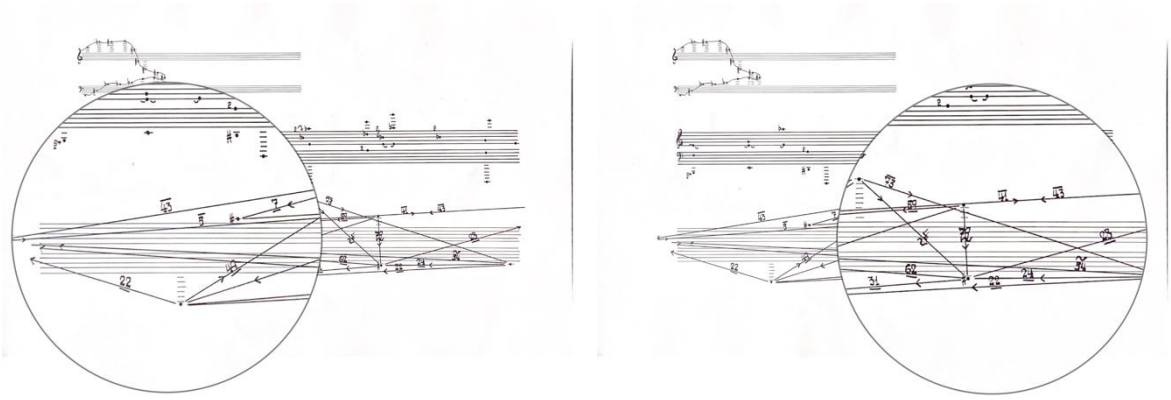


Figure 59 Notation J on page 6.

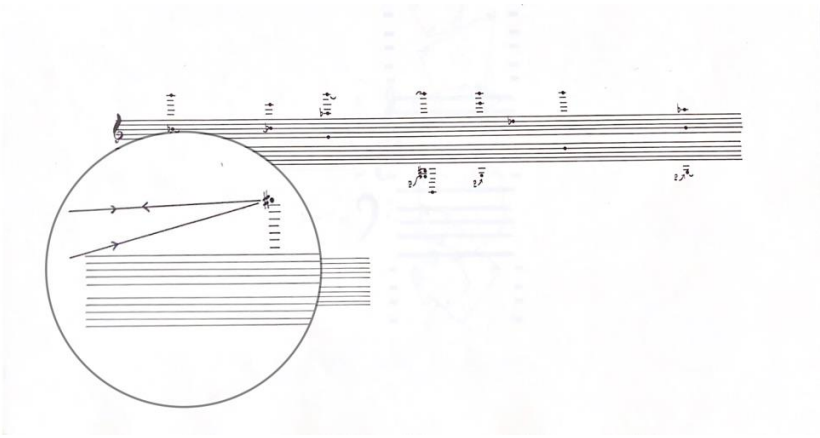


Figure 60 Notation J on page 7.

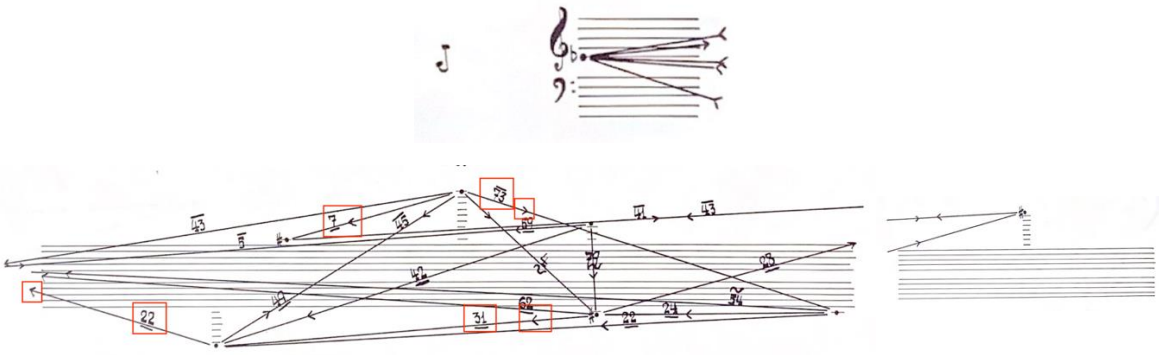


Figure 61 Notation J on pages 5–7.

Notation K presents intriguing graphical elements, including triangles, squares, pentagons, and hexagons, each marked with note names (Figure 62). These shapes serve as graces or punctuations, which means that Cage both limits the performers' options to these notes and offers a form of freedom whereby the performers can choose which notes to include in the realisations. Another aspect of freedom in this notation is that it is not bound by time. The performer is free to decide the duration of notes and the overall realisation, as indicated in the

instruction: 'Disregard time. Play only odd or even number of tones in a performance, using others of a given 3, 4, 5 or 6 sided figure as graces or punctuations' (Appendix A Table 58).[278]

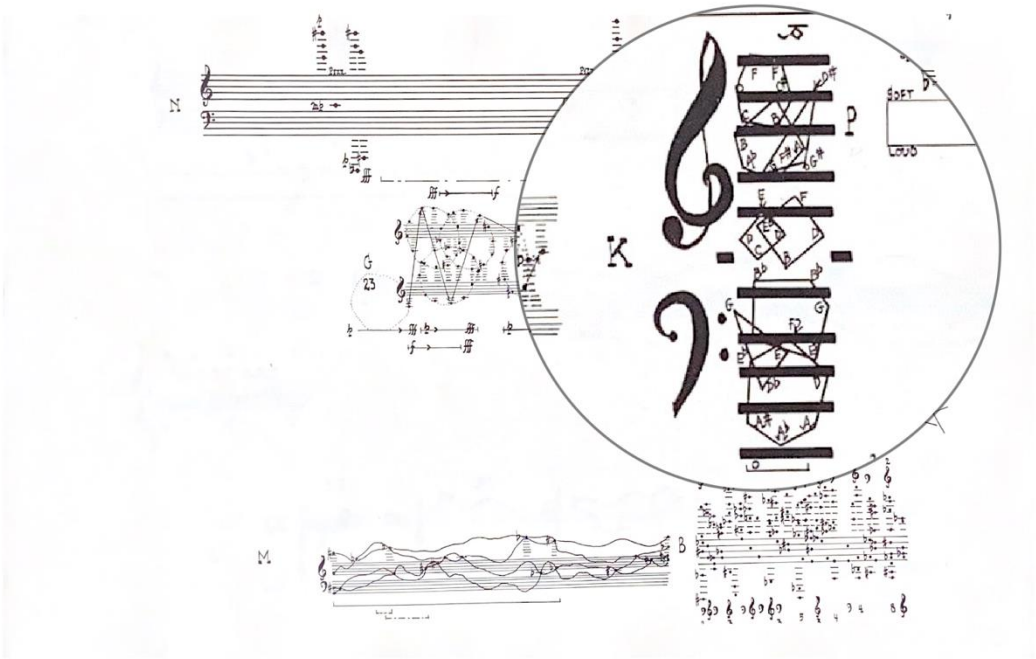


Figure 62 Notation K on page 9.

Notation L demonstrates a relationship between Notations A and B. In the performing instruction, Cage specifies 'Play from left to right with hands indicated. Clef ambiguity as in B. Perimeters where composing means and do not here affect time, as they do in A'.[279] Drawing from the performing approach of both Notation A (starting with any note, playing with the opposite direction, and following the perimeter) and Notation B (clef signs assigned as in Notation B, with the first number applying to the clef sign above the staff), this notation serves as a bridge between the two. Consequently, this notation incorporates performing approaches from three notations, including its own left-to-right sequence. The types of freedom present here encompass the selection of notes, the choice of the first note for both hands, and the determination of sound quality. Taking Notation L on page 10 as an example (Figure 63) the performer follows the performing instruction in terms of ambiguous clefs – as Notation B states 'The single staff is provided with 2 clef signs, where these differ, ambiguity obtains in the proportion indicated by the 2 numbers above the aggregate, the first of these applying to the clef sign above the staff'[280] – meaning that performers are free to choose three notes for the F-clef, while they select five notes for the G-clef. Even with the ambiguous clefs, performers are

[278] Ibid.
[279] Ibid.
[280] Ibid.

limited to a certain performing order, which is indicated by the perimeters, as in Notation A, Appendix A Table 59).

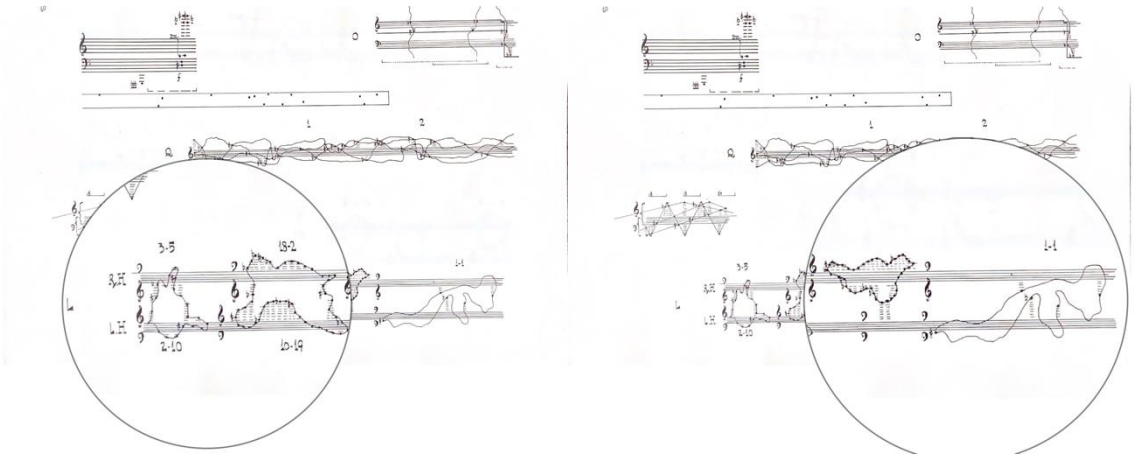


Figure 63 Notation L on page 10.

In Notation M, Cage offers performers several types of freedom. Firstly, when encountering intersections, the performers can choose to change the direction or maintain the original direction; secondly, they have the freedom to decide how many voices to incorporate into the realisation, and finally, they can choose whether to use pedals, as indicated, or not to use them. Therefore, the types of freedom determine performing directions, selecting the number of voices in the realisation, and deciding whether to use pedals. As indicated in the instruction, the performing approach can be summarised as beginning at the left and ending at the right, with the option to express it as a single voice, a counterpoint, or as three or four voices. Notably, the performing approach aligns closely with the types of freedom Cage offered, as evidenced by his use of phrases such as 'if desired', 'may be', and 'not obligatory'. This is evident in the performing instruction: 'Begin at left, end at right, changing direction at the intersections if desired. May be expressed as one voice, a 'counter point', or as 3 or 4 voices. Pedals only in areas indicated, not obligatory' (Appendix A Table 60). [281] Examining the notation itself, it is noticeable that a significant compositional element that he did not explicitly mention in the instruction is 'curved lines' (Figure 64), which play a crucial role in indicating the direction of the realisation and the placement of notes. In essence, they function similarly to the straight lines in Notation J, allowing performers the freedom to decide which notes to include within a specified range.

[281] Ibid.

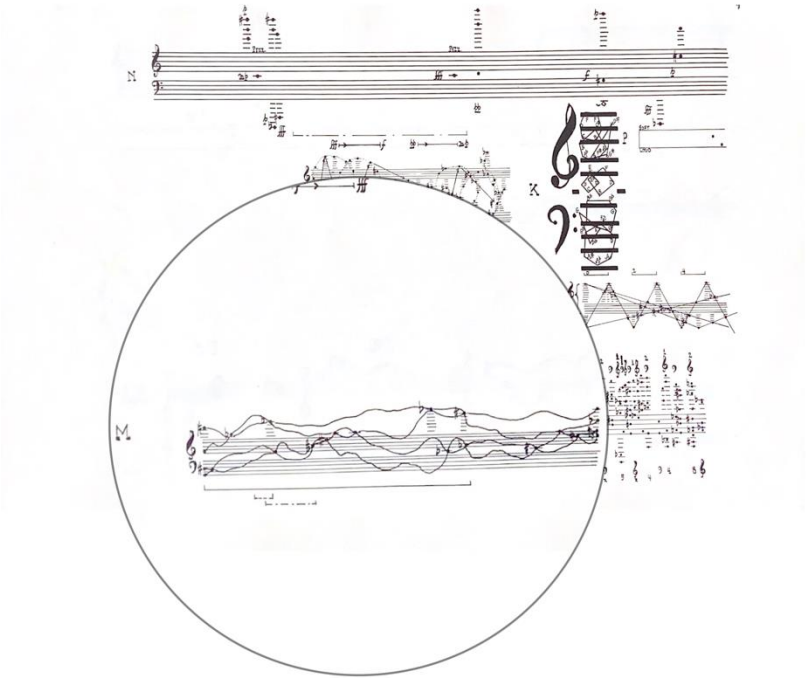


Figure 64 Notation M on page 9.

The performing instruction in Notation N is, 'Like I, but with varying dynamics. Sostenuto pedal given'.[282] Based on this, Notation I may act as fundamental compositional material, therefore, their types of freedom may be similar: i.e. free to decide on the duration of notes; meanwhile, the performing approach includes either mute and pizzicato, performing as single tones, intervals, or three-note aggregates, and repeating the same performing approach for the repeated tones, as in Notation I (Appendix A Table 56). The differences between the two notations are the given dynamics and sostenuto pedals; these elements are only used in Notation N (Appendix A Table 61) (Figure 65; Figure 66).

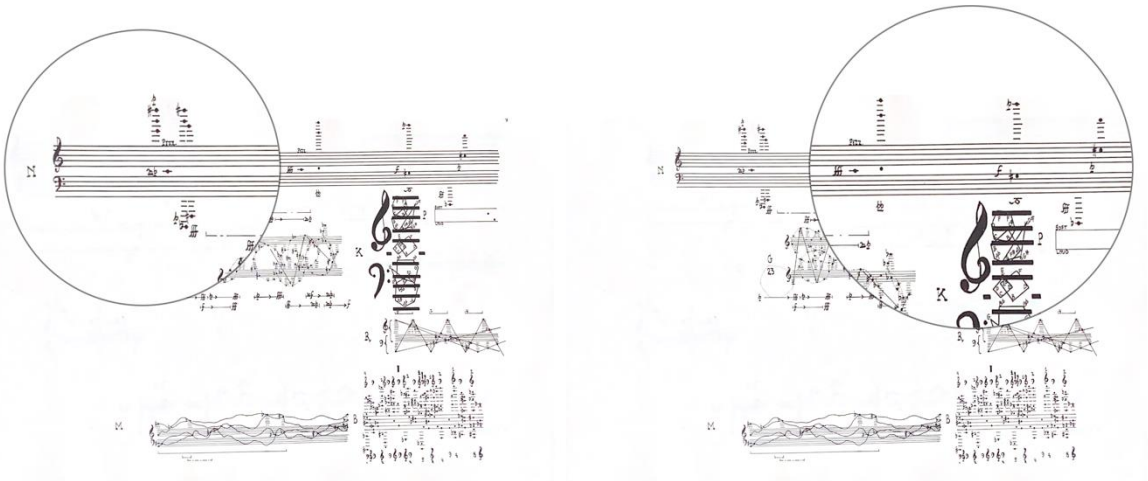


Figure 65 Notation N on page 9.

[282] Ibid.

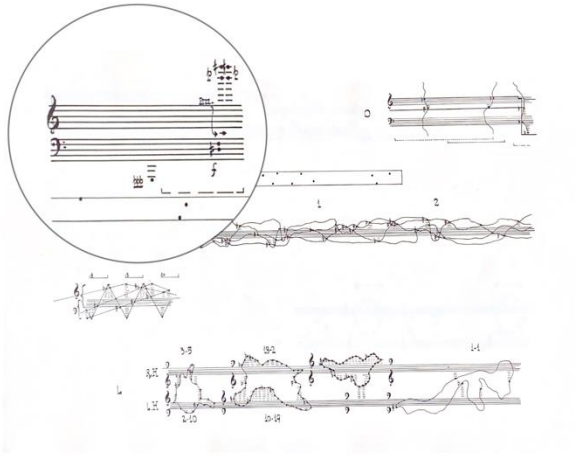


Figure 66 Notation N on page 10.

Notation O (Figure 67; Figure 68; Figure 69) is a notation that based on both Notations D and M. It shares similarities with Notation D in terms of chords and arpeggiations, but it grants performers the freedom to choose their preferred performing methods. Additionally, it borrows elements from Notation M regarding the use of pedals, although their use is not mandatory (Appendix A Table 62).[283] The notation itself prominently features two key compositional materials: curved lines connecting notes, and pitch spaces constructed by straight lines. With these elements at their disposal, performers have the flexibility to determine how to execute the clusters of notes, be it as chords or arpeggiations.

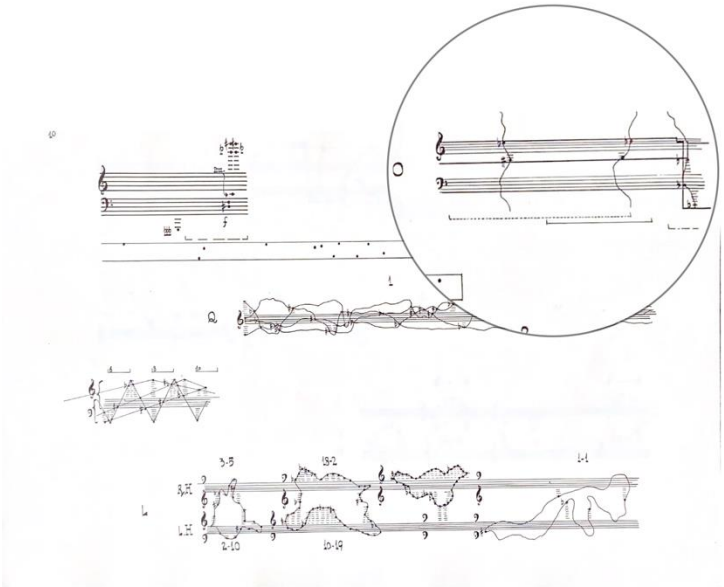


Figure 67 Notation O on page 10.

[283] The performing instruction of Notation O is 'Audible (as chords, lines, arpeggiations as in D but free, etc.) only between horizontal lines. Pedals as in M' (ibid.).

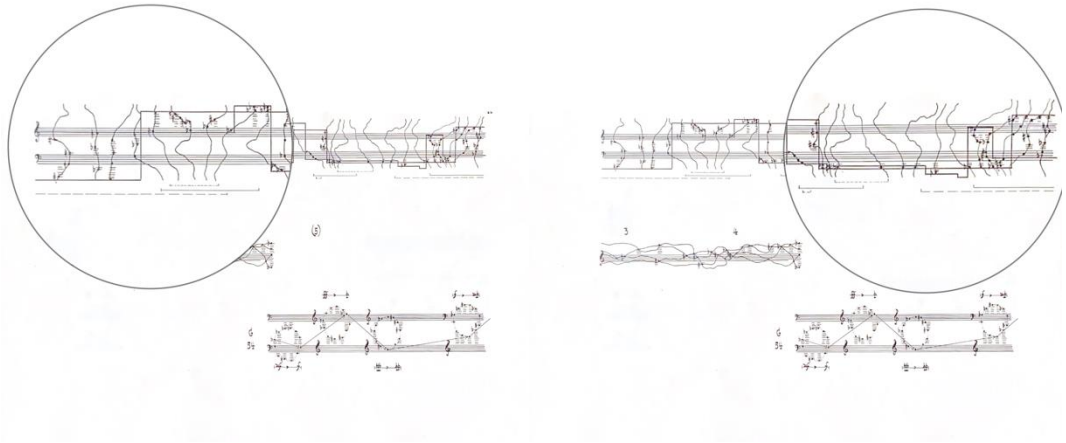


Figure 68 Notation O on page 11.

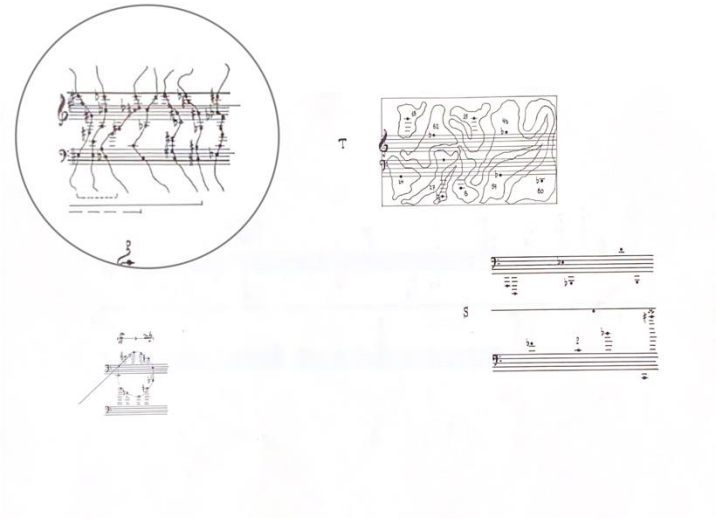


Figure 69 Notation O on page 12.

Notation P is where Cage applied noise and offered performers the freedom to choose auxiliary tools, while the dynamics are assigned; the performing instruction states: 'Any noises (including auxiliary), dynamics of which are notated'.[284] It can be seen, when comparing this design to the notation, that an area constructed by straight lines that have the instructions of dynamics, soft and loud, on the top and at the bottom of the area (Figure 70; Figure 71). Consequently, performers are free to decide duration of sounds, while they need to follow the instructions regarding dynamics (Appendix A Table 63). Another novelty of Notation P is that Cage did not use staff here, rather he chose a much more free and vague way (the area) to function as the staff.

[284] Ibid.

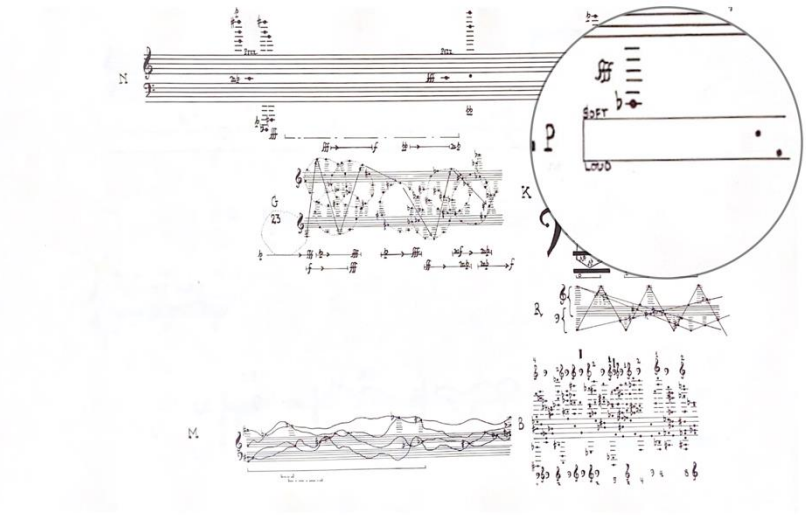


Figure 70 Notation P on page 9.

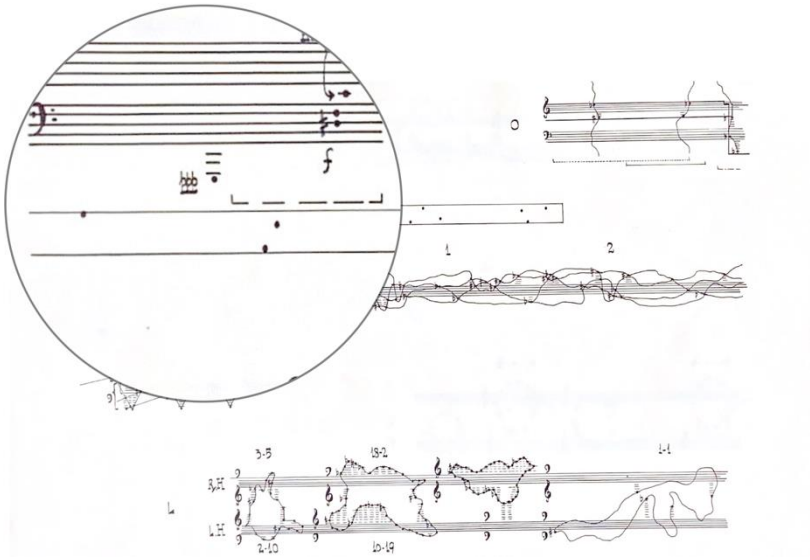


Figure 71 Notation P on page 10.

Notation Q (Figure 72; Figure 73), the performing instruction of which is, 'Like M, but in time (any units); going back incurs need for increased speed'. As the performing instruction suggests, this is the same performing approach as Notation M. Thus, Notation Q also begins on the left and ends on the right, and the change of direction at the intersections, as the performers prefer; it can also be expressed as one, or in multiple voices, and the pedals are optional. The only difference is that Notation Q needs to consider time, Cage stated numbers in this notation, and in the performing instruction he explained that performers are free to decide time units. Hence, the performing approach of Notation Q is as Notation M, where speed increases with the change of direction. The types of freedom are whether to change direction at the intersection or not,

free to decide once or more voices, free to decide time units, and also to decide the range of increased speed (Appendix A Table 64).

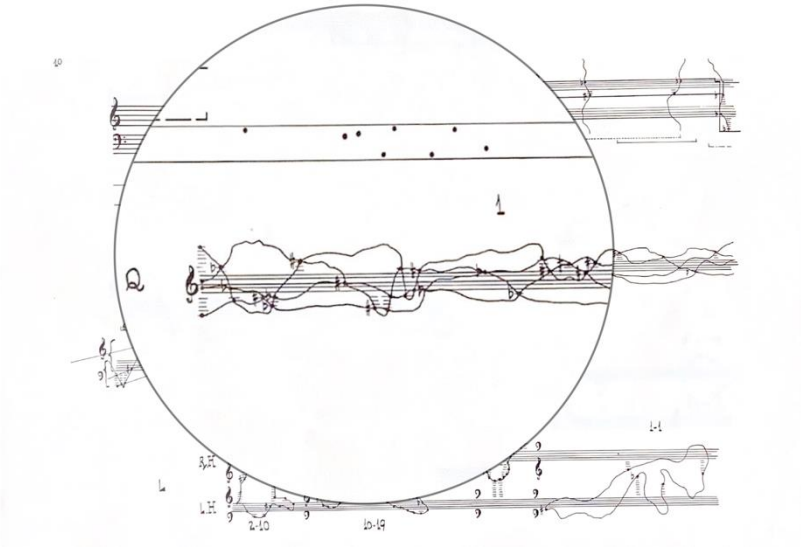


Figure 72 Notation Q on page 10.

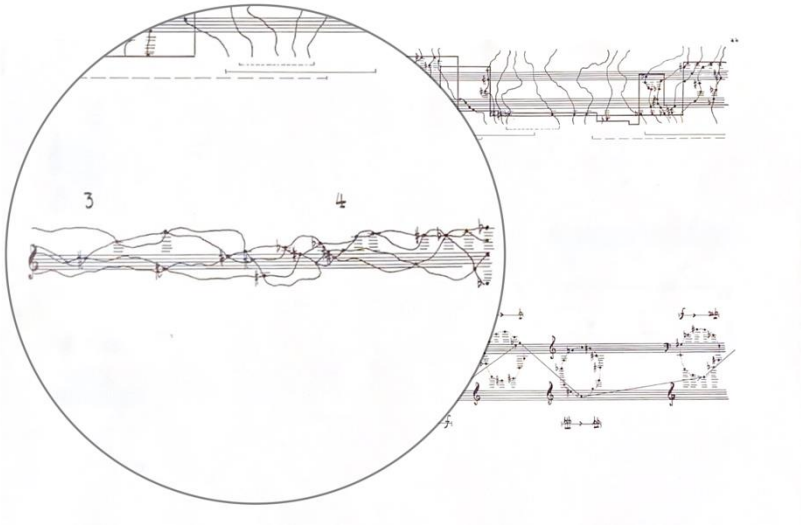


Figure 73 Notation Q on page 11.

The performing instruction of Notation R (Figure 74; Figure 75) is 'Rigorously in time. Notes above staff: treble; below: bass; on: ambiguous. Lines were part of composing means, the intersections producing tones other than the extreme piano keys'.[285] Based on the performing instruction, I notice that time is an important compositional material, which Cage used numbers and square brackets to represent. Here, he used clef signs on the staff that performers have the freedom to choose. The performing approach here is that performers may play the notes above the staff with treble, while they play those below the staff with bass; the freedoms here are

[285] Ibid.

deciding which clef signs should be applied to the notes on the staff, and also deciding the quality of the sounds (Appendix A Table 65).

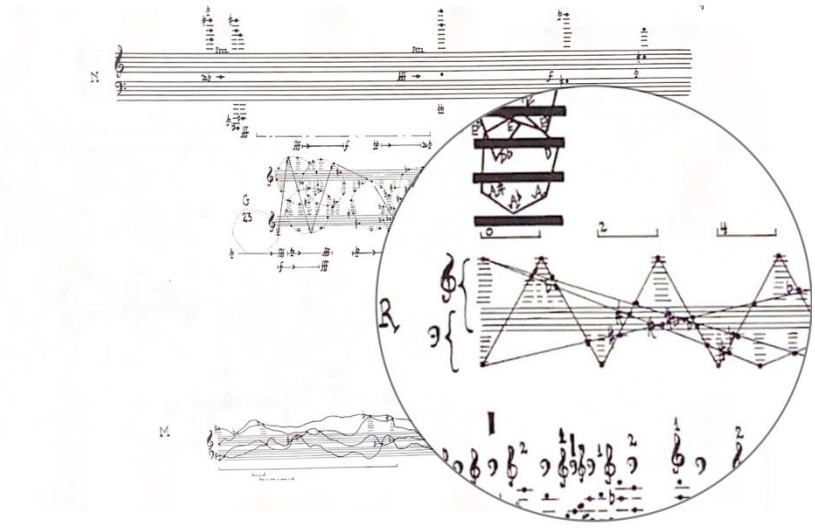


Figure 74 Notation R on page 9.

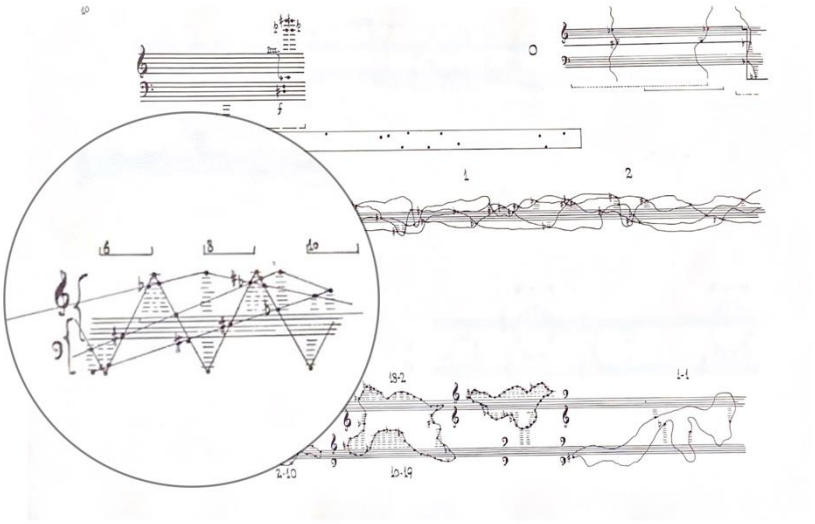


Figure 75 Notation R on page 10.

In Notation S, Cage explained that it is, 'Like C, but with noises; above line: inside piano constructions; below: outside piano constructions'.[286] The notation shows a straight line between two staffs (Figure 76; Figure 77; Figure 78). He used this straight line to indicate where the performers can produce the noises; above the line being inside piano construction, and below the line, outside it. Alongside this, he stated mute and pizzicato. As Notation S was built on Notation C, the performing approaches are to perform mute and pizzicato where the indications suggest, all notes are single tones, and notes above the lines should be played inside

[286] Ibid.

piano construction, while notes below the lines should be played outside it. The types of freedom here are for performers to decide the duration of notes and to decide the dynamics of notes (Appendix A Table 66).

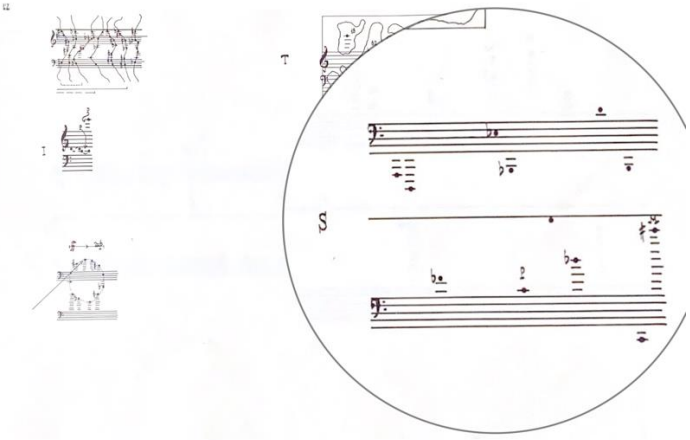


Figure 76 Notation S on page 12.

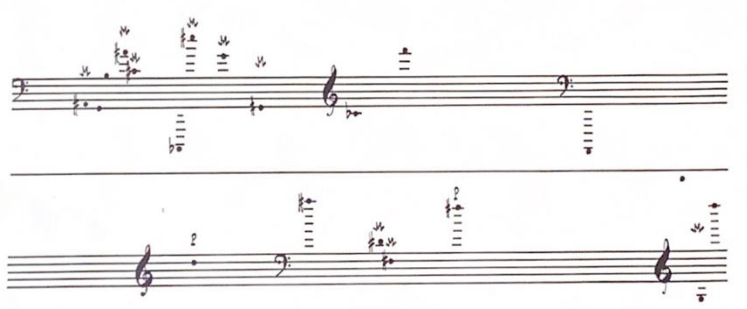


Figure 77 Notation S on page 13.

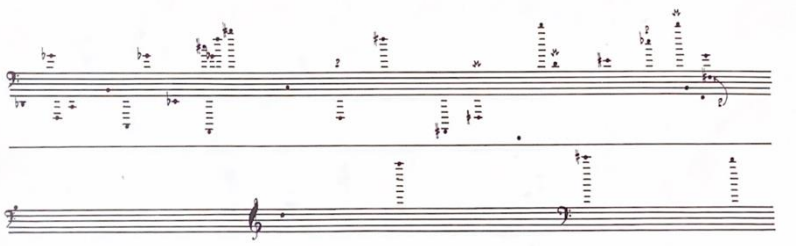


Figure 78 Notation S on page 14.

The performing approach of Notation T is 'Influences in pitch and time notated as shapes with centre points, to be audible as clusters, a single one changing in its course. Numbers refer to loudness (1-64) (soft to loud or loud to soft)'.[287] From the performing instruction it is noticeable that Cage indicated pitch and time through shapes with centre notes, as shown in Notation T on page 12 (Figure 79). Since the shapes indicate time and pitch, and the numbers represent loudness, performers need to realise this notation through a certain direction, and to compare

[287] Ibid.

the positions of shapes in order to realise the timing of the notes, while also comparing their positions to realise the pitch ranges. Thus, the performing approach of Notation T is to follow the positions of the shapes and notes in order to decide timing and pitch, and to follow the numbers indicating the degree of dynamics. The types of freedom here are being free to decide relationship between numbers and dynamics (either loud to soft or soft to loud), and to choose performing notes with a centre note (Appendix A Table 67).

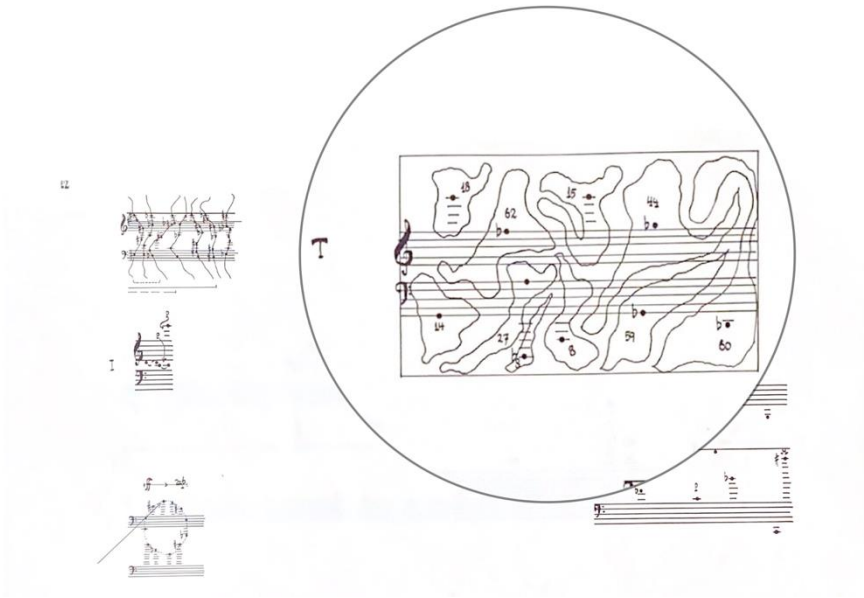


Figure 79 Notation T on page 12.

The performing instruction of Notation U, 'Choose one of 3 mutually exclusive areas, bounded by straight lines. Proceed L. to R. using number of tones given within dotted parts'.[288] This means that performers choose the three areas divided by dotted lines, to perform the given proportion of notes, from left to right. Taking Notation U on page 16 as the example (Figure 80), the performer decides three out of nine areas, and then must decide performing notes, based on the pitch areas that are constructed by dotted lines and numbers. In short, the performing approach is to perform from left to right and perform the given proportions, choosing notes from the areas; while the performers are free to choose performing notes within the given proportions, they are also free to decide the quality of sounds (Appendix A Table 68).

[288] Ibid.

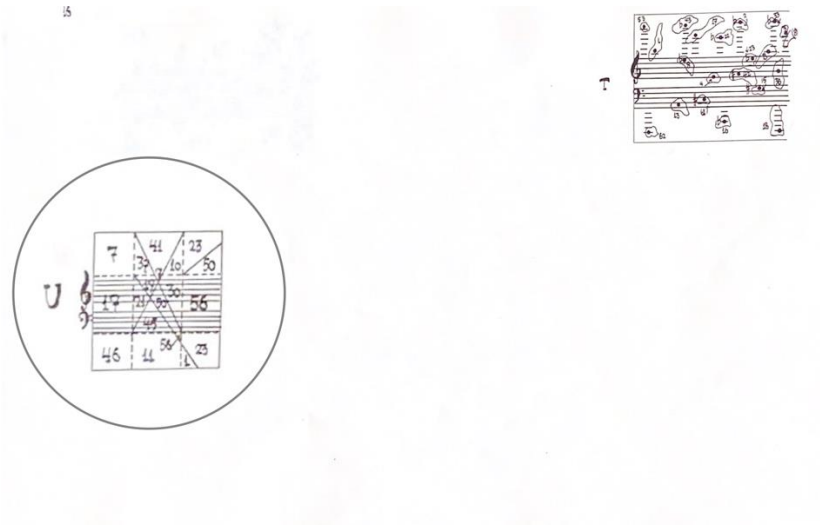


Figure 80 Notation U on page 16.

Notation V's (Figure 81) performing instruction states, 'Performance indications above (reading down) are degree of force most-least; vertical distance of attack, far-close; speed of attack, slow-fast. Any noises (below line between staves)'.[289] This shows two areas, including performance indications and two staffs with a straight line between them. The performance indications consist of the degree of force, vertical distance attack, and speed of attack, meaning that performers ought to follow the indication in order to realise the notes on the staff. The dots below the straight line indicate any noises. Hence, the performing approach is to perform the notes based on the indications, while making noises as the dots indicate. The type of freedom is to decide the performing approach regarding the noises (Appendix A Table 69).

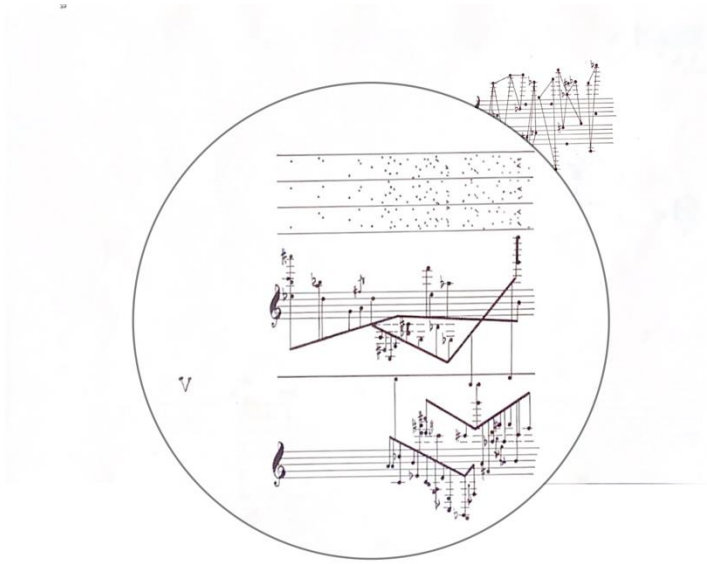


Figure 81 Notation V on page 18.

[289] Ibid.

Notation W (Figure 82) is constructed by shapes (triangles) and single notes. As the performing instruction suggests, the triangles indicate the need for it to be performed as legato, and the single notes need to be performed as staccato – 'Legato (triangles) and staccato (isolated notes)'.[290] Notation W on page 18, shows that there are nine triangles and seven single notes. The performing approach is to follow the nine triangles to perform legato, to perform staccato for isolated notes, and to perform the notes based on their order in the notation. For instance, the performers may realise notes G1, F1, and F4♭ as legato, and realise a note G6 as staccato (Figure 83). The types of freedom are the freedom to decide the duration of legato, and the freedom to decide the dynamics of single notes (Appendix A Table 70).

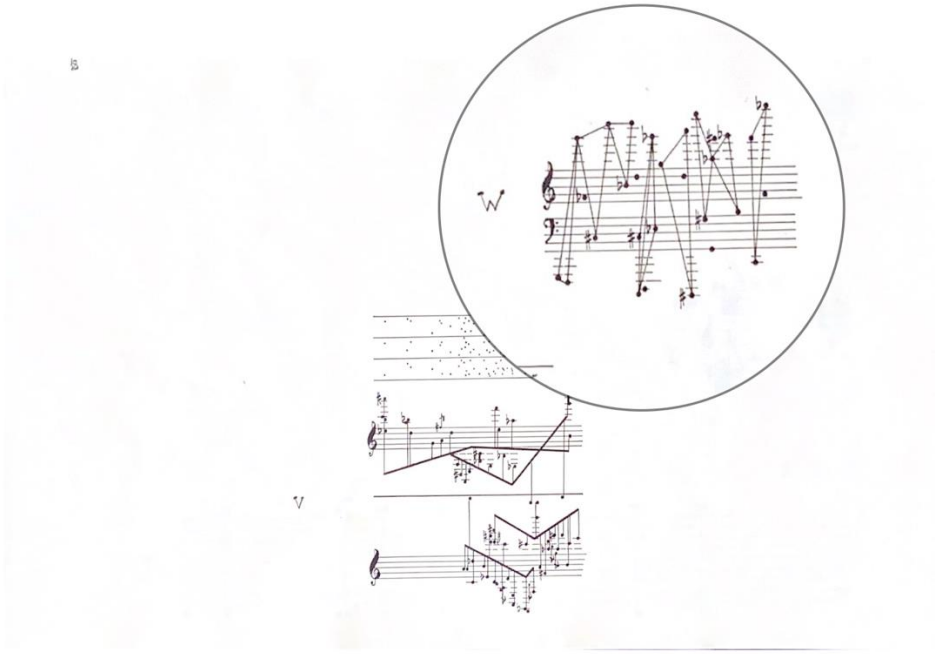


Figure 82 Notation W on page 18.

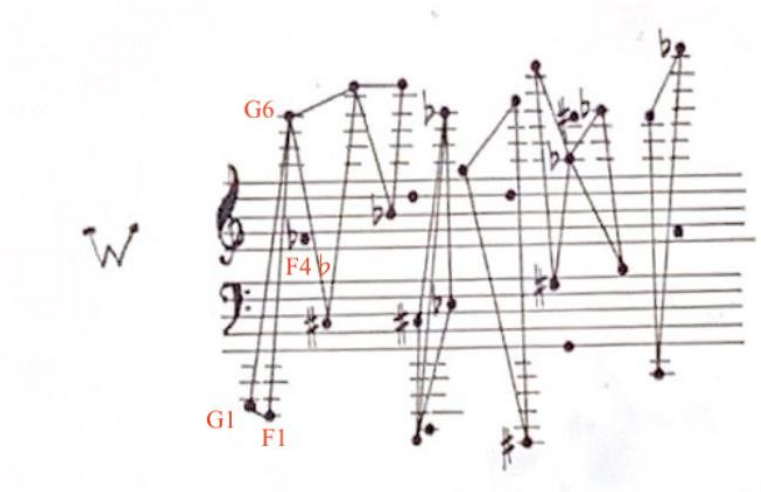


Figure 83 Realisation of Notation W on page 18.

[290] Ibid.

The performing instruction for Notation X is, '𝅘𝅥𝅮s are punctuations (before at, during, or end of internal they accompany). 𝄋 means repeat something (or all, or any amt.). Played before, but changing amplitude. Pedals are optional'.[291] Notation X uses symbols: 𝅘𝅥𝅮 and 𝄋, 𝅘𝅥𝅮 is for punctuations, can appear be before, during, or at the end of, the collection of notes. Here, he not only assigned the performing approach, but also offered a type of freedom to allow performers to conduct the realisation. When he uses the symbol 𝄋, the performers are free to choose what to repeat but need to change amplitude. The pedals are optional in this notation (Figure 84; Figure 85; Figure 86; Figure 87) (Appendix A Table 71).

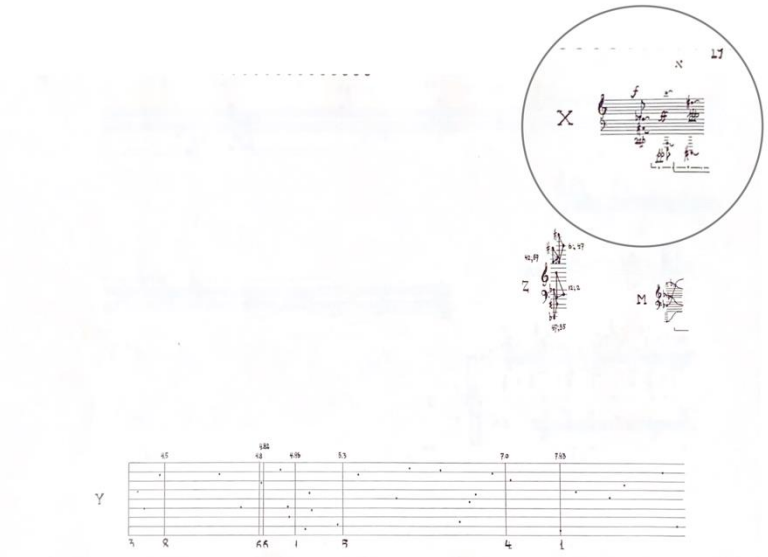


Figure 84 Notation X on page 19.

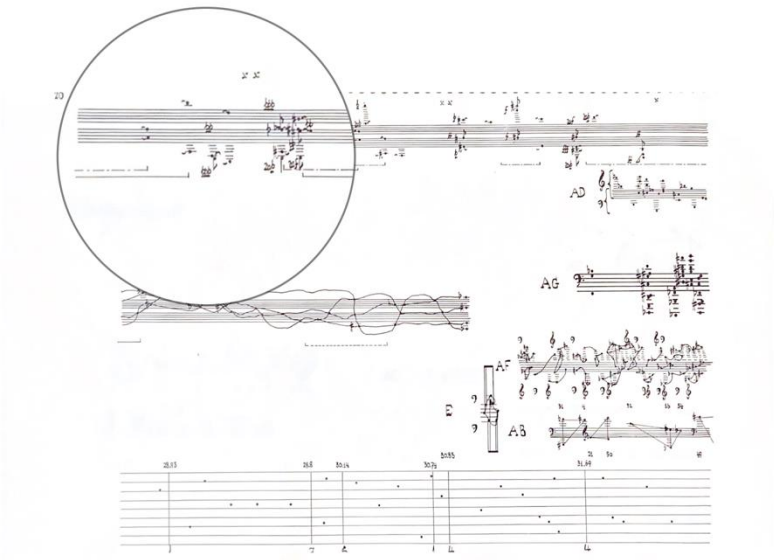


Figure 85 Notation X on page 20.

[291] Ibid.

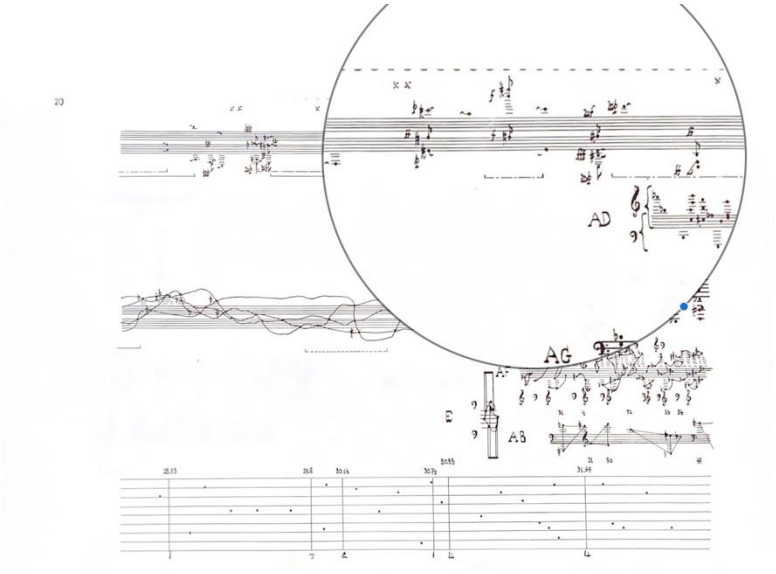


Figure 86 Notation X on page 20.

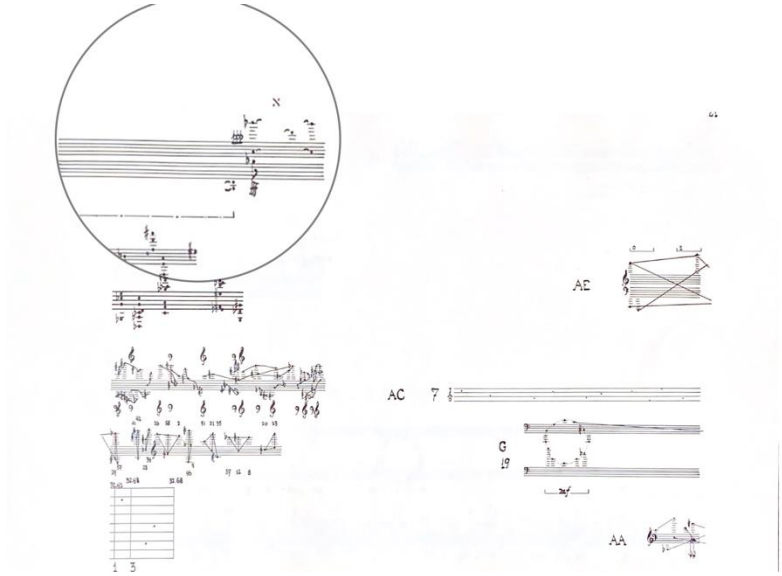


Figure 87 Notation X on page 21.

The performing instruction of Notation Y states, '8 pitch areas chromatically adjustment, and having number of chromatic tones given (large numbers below staff). When notes are on a line, they belong to either the upper or the lower area. Time in space seconds as indicated. Loudness given by position of note with respect to area vertically. High is *fff*. Low is *ppp*'.[292] This demonstrates notes in eight pitch areas, using numbers, with seconds as the time unit, and the position of notes, where high represents loudness and low represents softness, as guides to help performers' realisations (Figure 88; Figure 89; Figure 90; Figure 91). The performing approach involves playing the specified number of notes and maintaining the relationships between pitch areas; playing notes at the indicated times in seconds, and adjusting loudness based on the

[292] Ibid.

position of the notes. This allows freedom in choosing specific notes, while still adhering to the overall relationships between them (Appendix A Table 72).

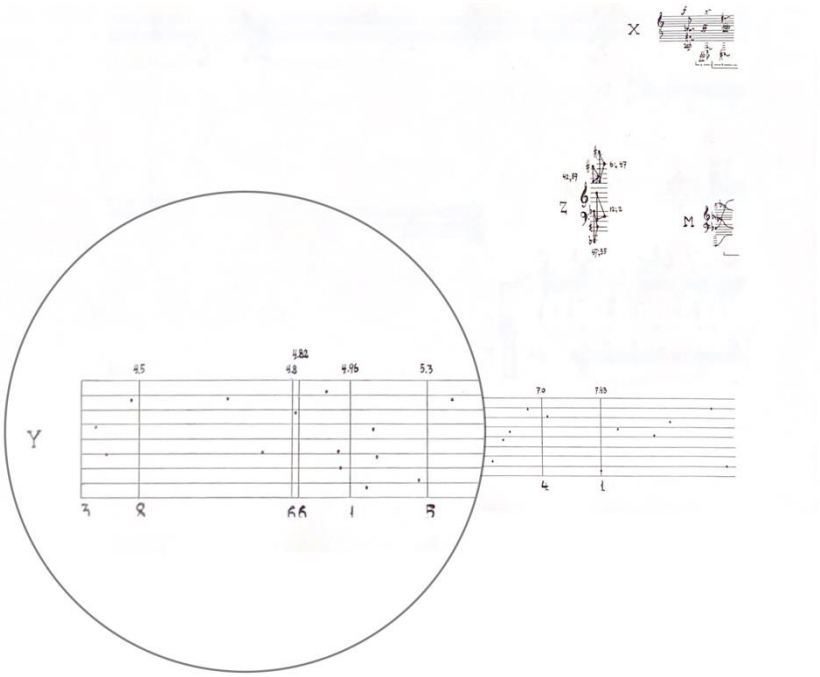


Figure 88 Notation Y on page 19.

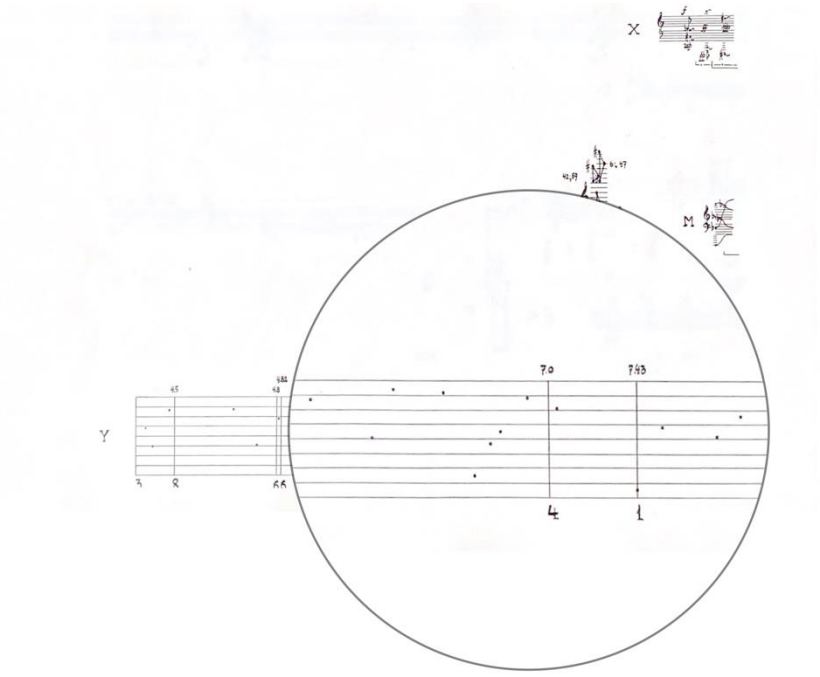


Figure 89 Notation Y on page 19.

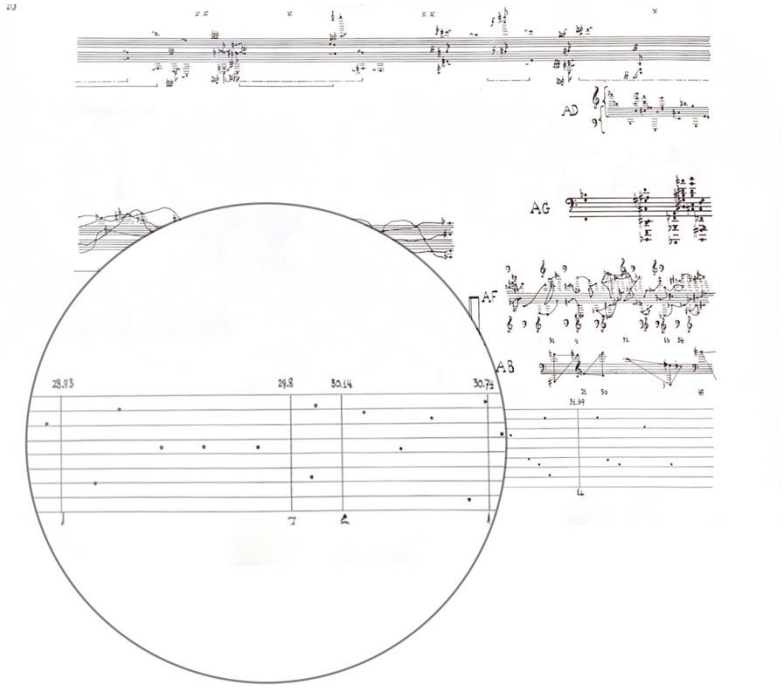


Figure 90 Notation Y on page 20.

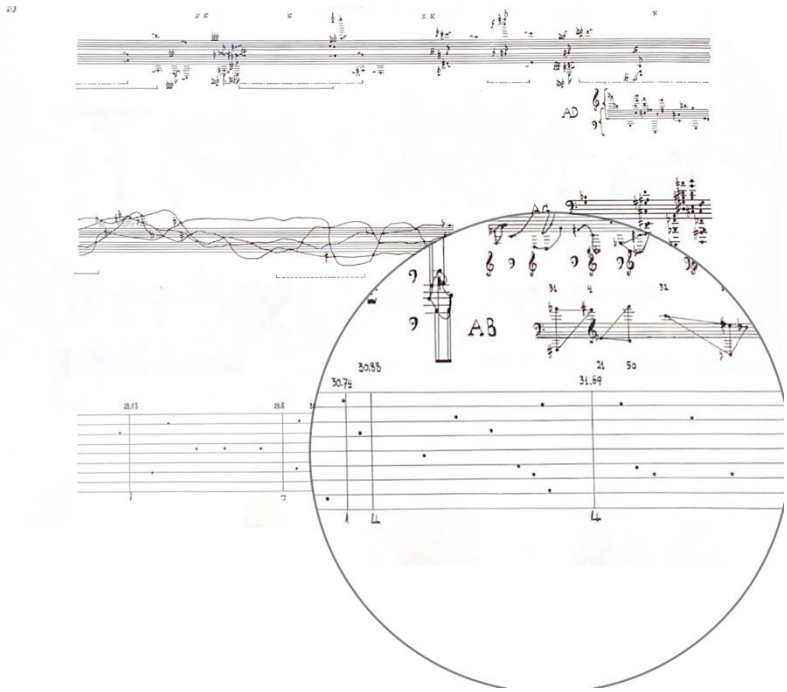


Figure 91 Notation Y on page 20.

Notation Z, which Cage stated as. 'Clusters ending as single tones. Dynamics as in T '.[293] This means that, by using numbers 1–64 to indicate dynamics, from loud to soft or from soft to loud, this notation shares the same performing approach of dynamics as Notation T. The other performing approach is the clusters should end as single notes, meaning that the left sides of triangles should be realised together, and the right side of the triangles should be performed as

[293] Ibid.

a single note. For instance, Notation Z on page 19 (Figure 92), the performers may realise notes E3 and B4 as a cluster, a note F3 as a single note, while the numbers 12 and 2 may represent to the cluster is louder than the single note, or the other way round (Figure 93). In short, the performing approaches are following the numbers of dynamics, performing notes together as clusters ending at single notes. The types of freedoms allow performers to decide the relationship between numbers and dynamics – either loud to soft or soft to loud – and to remain free to choose the duration of notes (Appendix A Table 73).

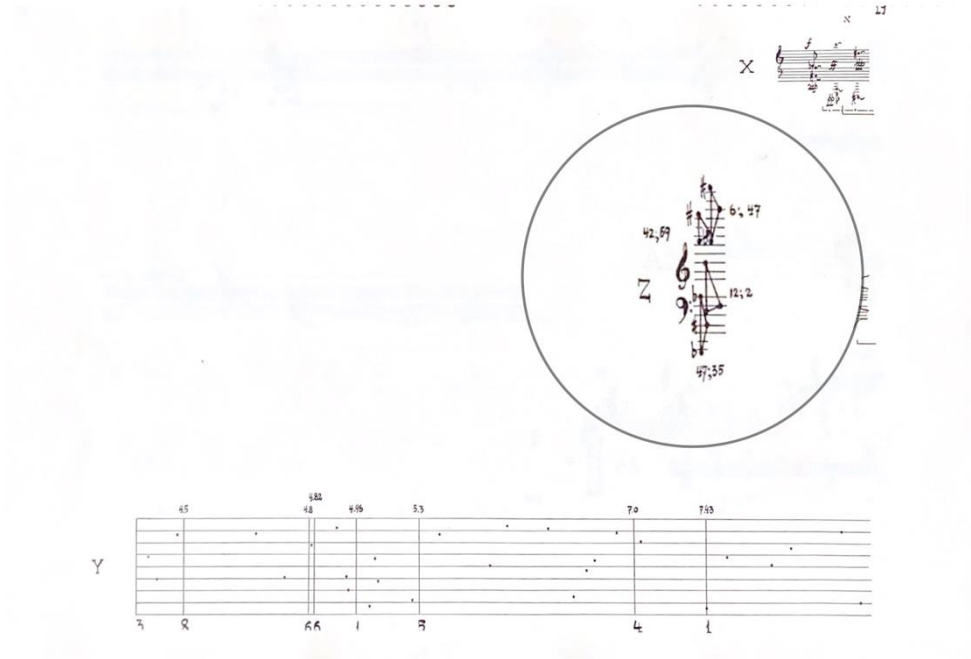


Figure 92 Notation Z on page 19.

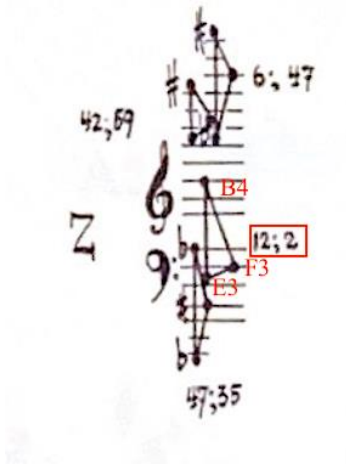


Figure 93 Realisation of Notation Z on page 19.

2.2 Compositional Materials in Notation AA to Notation AZ

This section deconstructs the compositional elements of Notation AA to Notation AZ, similar to the previous deconstruction; the deconstructions look into the performing instructions of corresponding notations and the notation before categorising the compositional elements into a table.

Notation AA (Figure 94; Figure 95; Figure 96; Figure 97), Cage stated, 'Clef at all times ambiguous. "Sticks" to be played freely (single tones, intervals, aggregates, events, etc. within pitch limits and relative time units indicated by their extremities (notes), using graces as "assistance". Give emphasis by dynamics, harmonics, repetition or other means to circled tones and more extremely to those circled twice or 3 times'.[294] From this it is noticeable that performers are free to decide which notes to play with which clef sign; also that he applied 'sticks' on the paper imperfections to inform performers that these notes can be played as single notes, intervals, aggregates, or any other way performers prefers. However, along with this type of freedom (how to perform the sticks) he limited them to the given notes and time; hence, in this notation, time is relative, as he did not clearly state what time units, or any other compositional material, represented time. Hence, time in this notation, is relatively free, or, rather, it depends on the notes that come before and after. The performing instruction that clarifies the circled tones, can be emphasised by dynamics, harmonics, repetition, or any other means; more circles mean louder, or more repetitions, etc. Based on the above, the performing approach is ambiguous, such as clef sign, performing the sticks with given notes and relative time, and differentiating degrees of emphasising the notes by circles; while the types of freedom mean that performers are free to decide clef signs for notes, the performing ways of sticks, and to decide the quality of sounds (i.e. the means of emphasising notes) (Appendix A Table 74).

[294] Ibid.

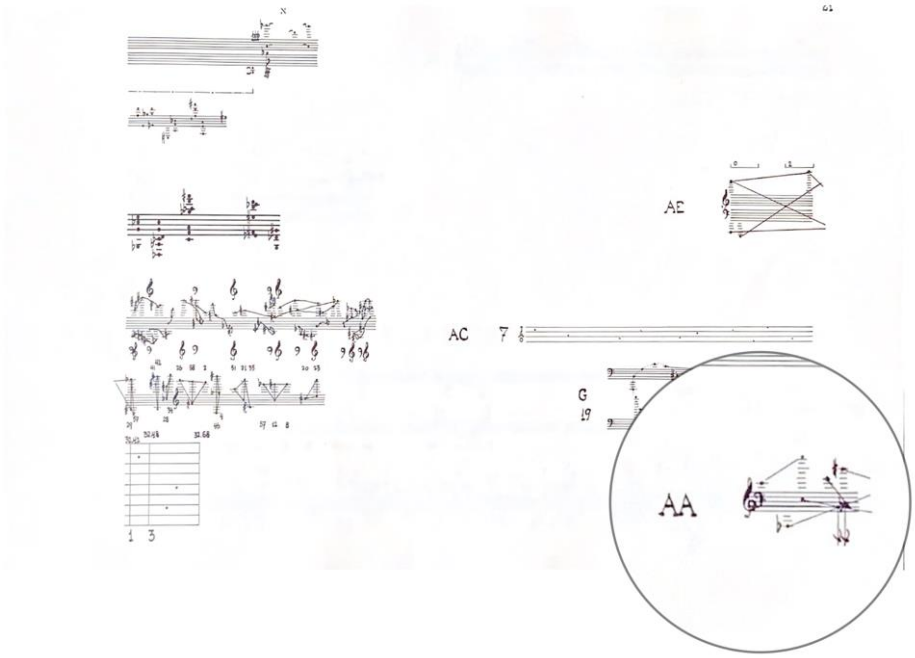


Figure 94 Notation AA on page 21.

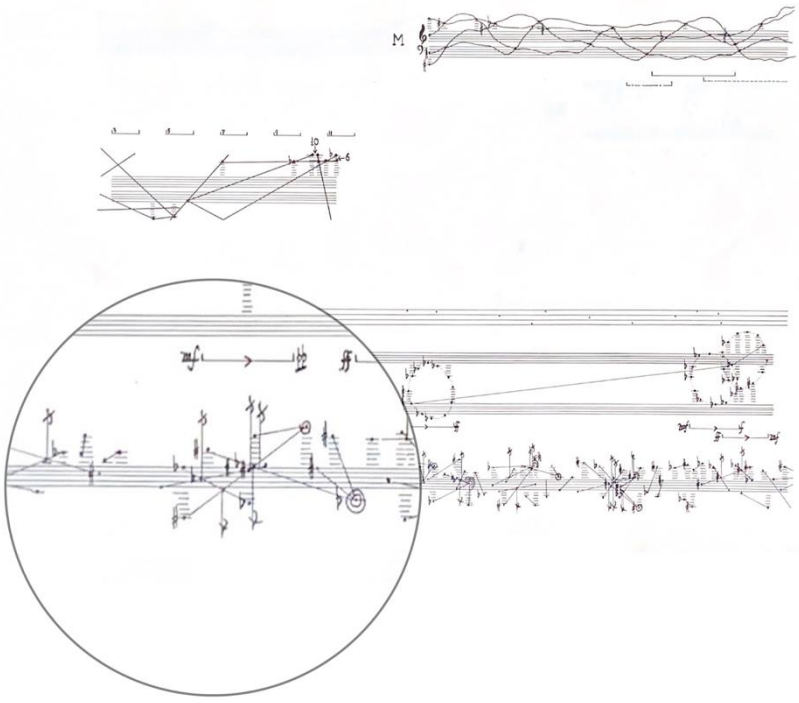


Figure 95 Notation AA on page 22.

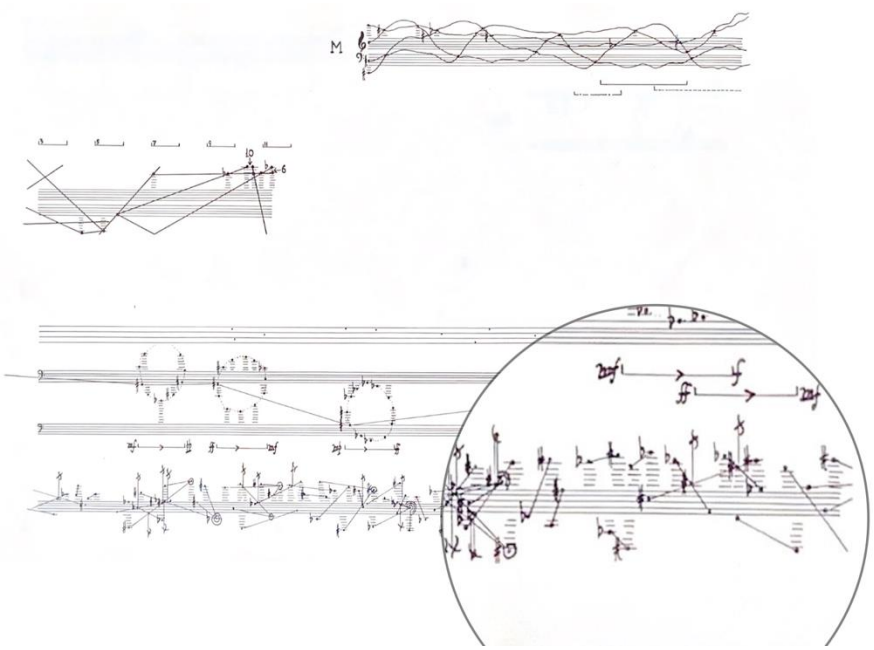


Figure 96 Notation AA on page 22.

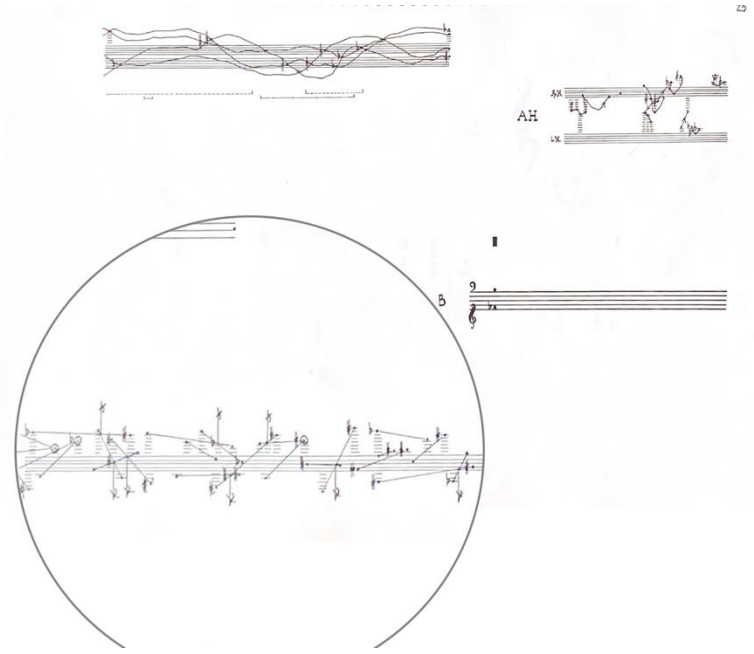


Figure 97 Notation AA on page 23.

Notation AB's (Figure 98; Figure 99) performing instruction states, 'Clusters as in Z. Sometimes beginning as single tones (numbers are dynamics)'.[295] Cage specifically pointed out that the clusters in Notation AB should be performed as in Notation Z, which means that they should end as single tones. Here, he again applied numbers to indicate the level of dynamics. This indication can be found in Notations T and Z, as they use numbers 1–64 to indicate dynamics; thus, performers can choose number 1 to be the loudest or the softest, and number

[295] Ibid.

64 to be the softest or the loudest. From this performing instruction, it is noticeable that Cage has applied two identical performance approaches – from Notations Z and T – in order to construct Notation AB, suggesting that a single relationship regarding a performing approach can be built from three notations. The type of freedom here is also from Notation T, where performers may decide the relationship between numbers and dynamics (loud to soft or soft to loud); and, from Notation Z, to decide the duration of notes (Appendix A Table 75).

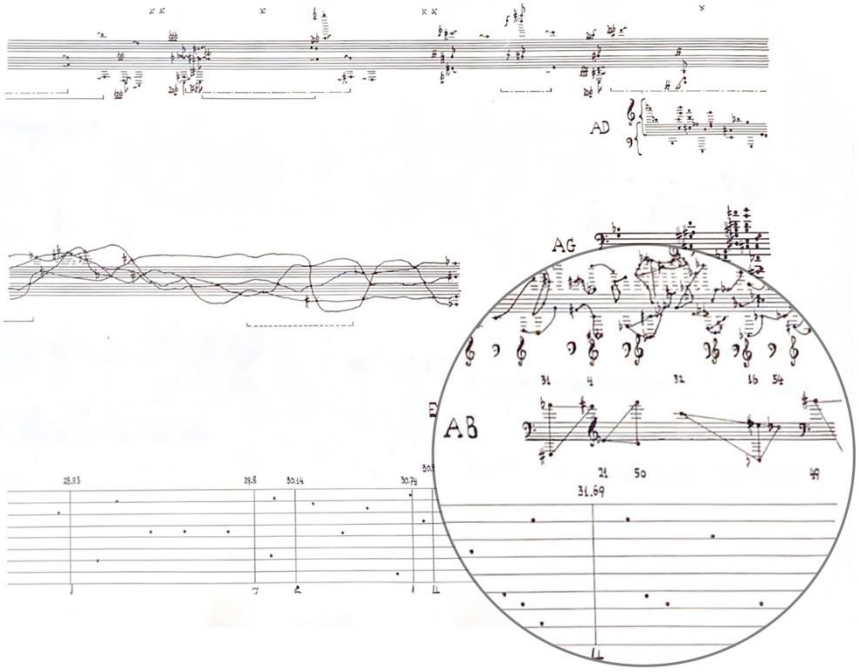


Figure 98 Notation AB on page 21.

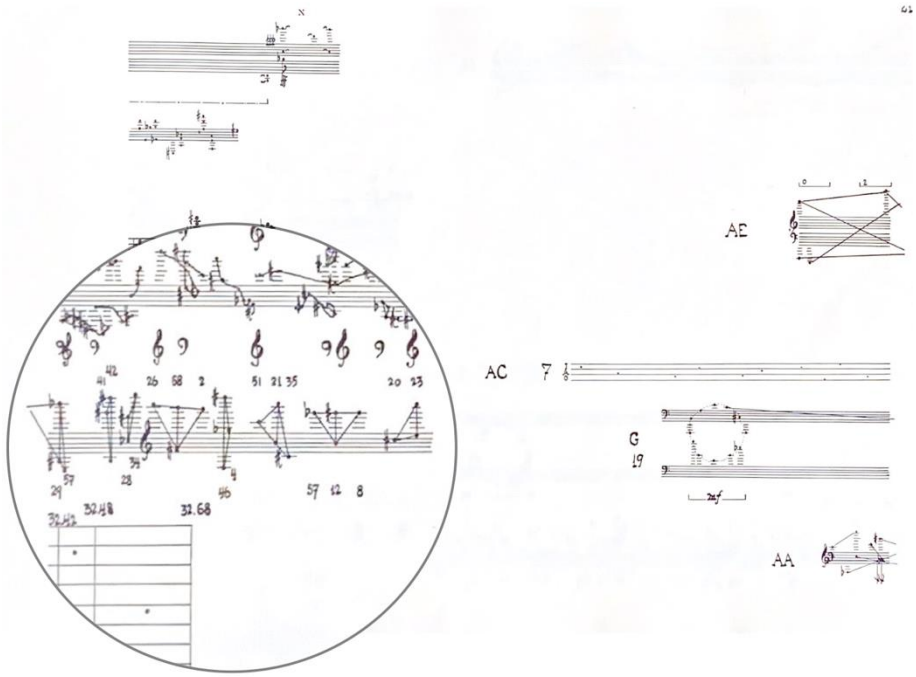


Figure 99 Notation AB on page 22.

The performing instruction for Notation AC is, 'Noises. Of those notated play only that number given. I = interior piano construction. A = auxiliary noises. O = outer piano construction. The position of the notes vertically gives its loudness (High: *fff*) (low is *ppp*)' .[296] From this performing instruction, it is noticeable that Notation AC is based on noises that can be produced by interior piano construction, outer piano construction, or auxiliary tools. For instance, Notation AC on pages 21–23 (Figure 100; Figure 101; Figure 102; Figure 103), in which Cage offered freedom to the performer to choose seven noises; the performers are free to decide any of seven noises based on the notation. The position of the noises represents their loudness, with the higher position referring to louder, while the lower position refers to softer. A similar performing approach can be found in Notation P (Appendix A Table 63), which is also based on noises, and which allows the performer to choose auxiliary tools. Another similarity in the two notations is the position of the notes; in both notations, Cage used higher and lower positions to indicate the loudness level, the only difference Notation AC is in higher position is louder, but in Notation P the higher position indicates a softer sound. Regardless of the detailed difference in the position, he applied the same compositional materials (position of noises) to demonstrate the performing approach (level of loudness) (Appendix A Table 76). Consequently, he built two potential relationships between two notations on the compositional materials and performing approach.

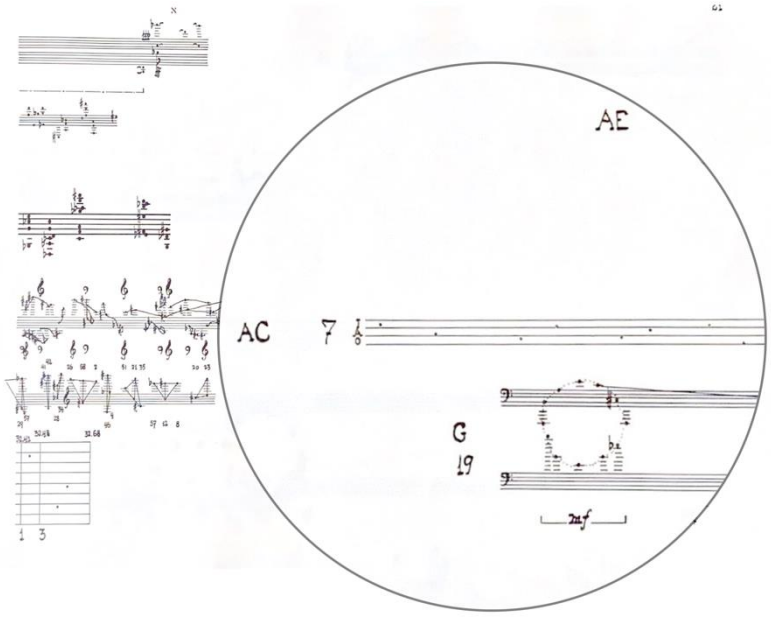


Figure 100 Notation AC on page 21.

[296] Ibid.

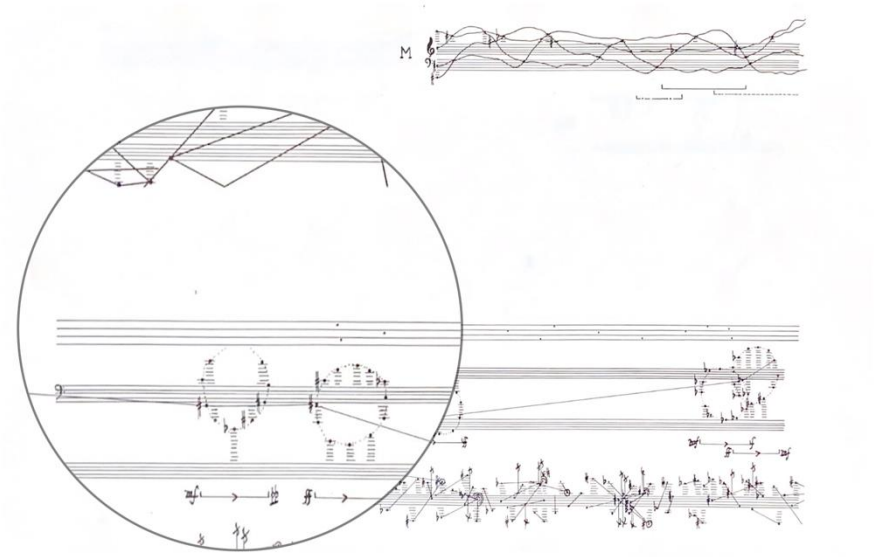


Figure 101 Notation AC on page 22.

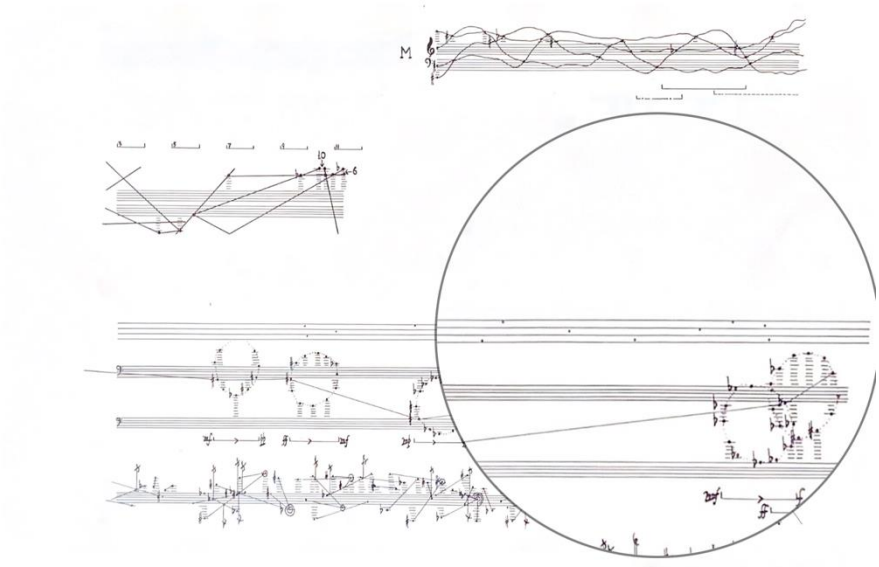


Figure 102 Notation AC on page 22.

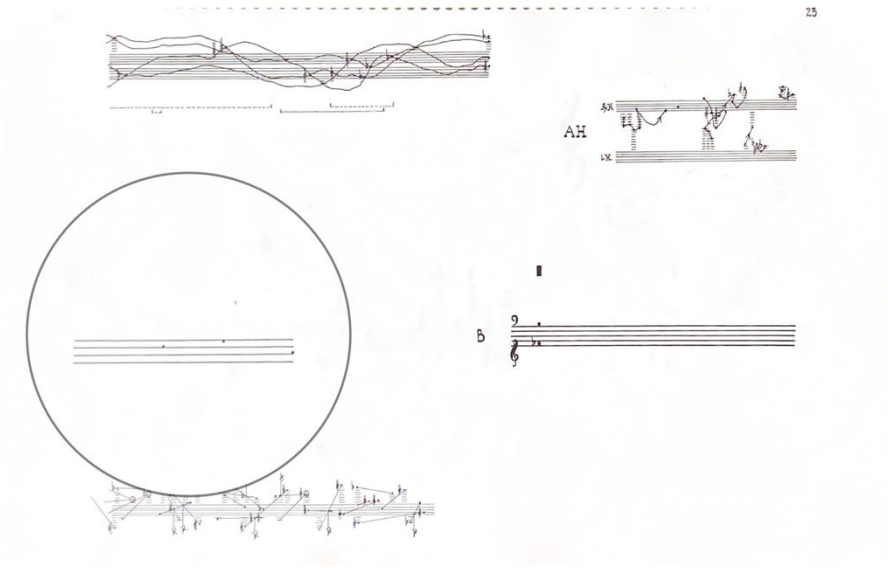


Figure 103 Notation AC on page 23.

In Notation AD (Figure 104; Figure 105), Cage's performing instruction states, 'Single tones, intervals and three note aggregates with ambiguous staff, ledger lines above, treble, below, bass'.[297] Cage divided this notation into three parts: above the staff, on the staff, and below the staff, with each part corresponding to a treble, either the treble or a bass, and the bass, respectively. Hence, performers are free to decide which clef signs should apply to the notes on the staff, and also free to decide the quality of sounds. The performing approaches here are, notes on ledger lines above the staff are with the treble, and notes on ledger lines below the staff are with the bass, notes should be performed as single tones, intervals, and three-note aggregations (Appendix A Table 77).

[297] One interesting observation regarding Notation AD is that Cage phrased the performing instruction with 'ambiguous staff' instead of 'ambiguous clef signs'. The meaning of different description is unclear.

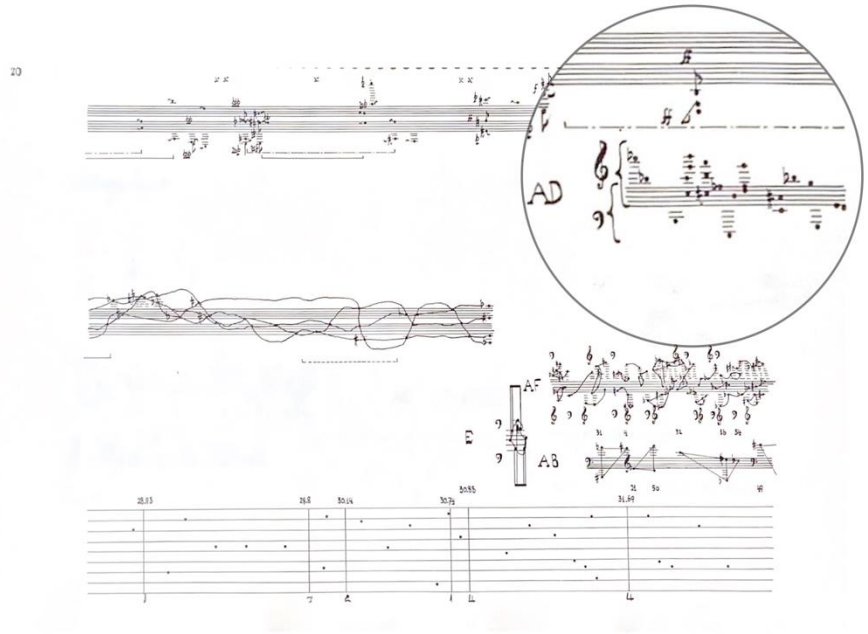


Figure 104 Notation AD on page 20.

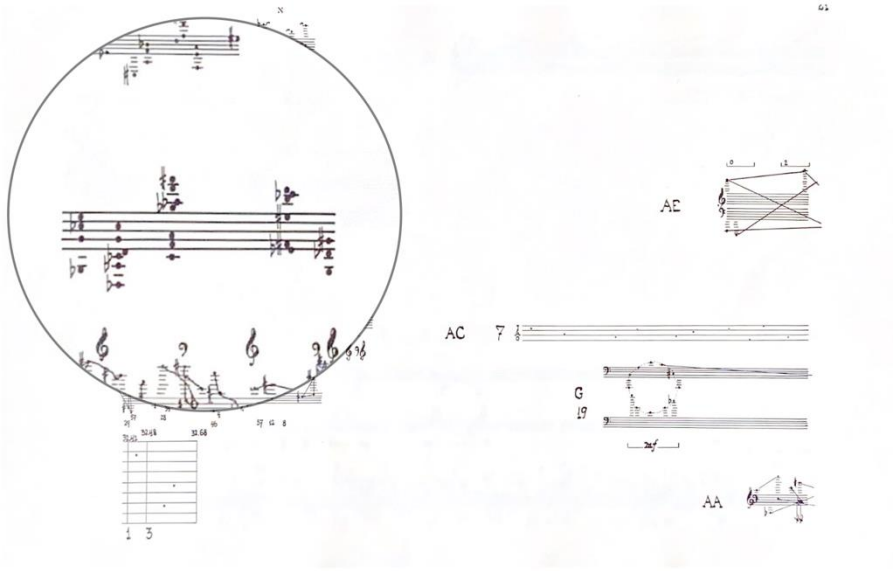


Figure 105 Notation AD on page 21.

Notation AE's (Figure 106; Figure 107; Figure 108) performing instruction is, 'Pitch-time areas silent unless accompanied by numbers, meaning number of tones (any) to be played'.[298] Here, it appears that Cage applied straight lines to delimit the pitch area, used numbers with an upside-down bracket above the staff to indicate time, and used numbers to guide performers as to how many notes should be performed under certain conditions. For instance, in Notation AE on pages 56–57, around the fourteenth to the seventeenth second, there is a triangle-like pitch-time

[298] Ibid.

area constructed by notes C4♯, E4, and F3, with a given number 11 indicating the performing number of notes. Here, performers are free to choose eleven notes from the pitch-time area and are free to decide the quality of sounds. Nevertheless, the realisations of this pitch-time are limited to allocated time – the fourteenth and the sixteenth second – meaning that the performers' realisations will be influenced based on allocated time; the pitch range of the fourteenth second is from B3 to C4♯, while the pitch range of the sixteenth second is from C3 to E4 (Figure 109) (Appendix A Table 78).

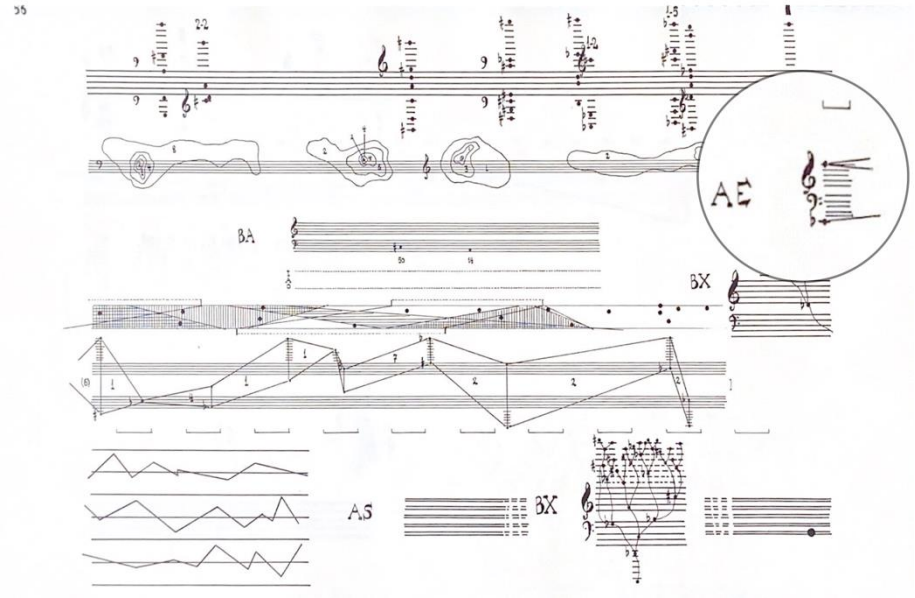


Figure 106 Notation AE on page 56.

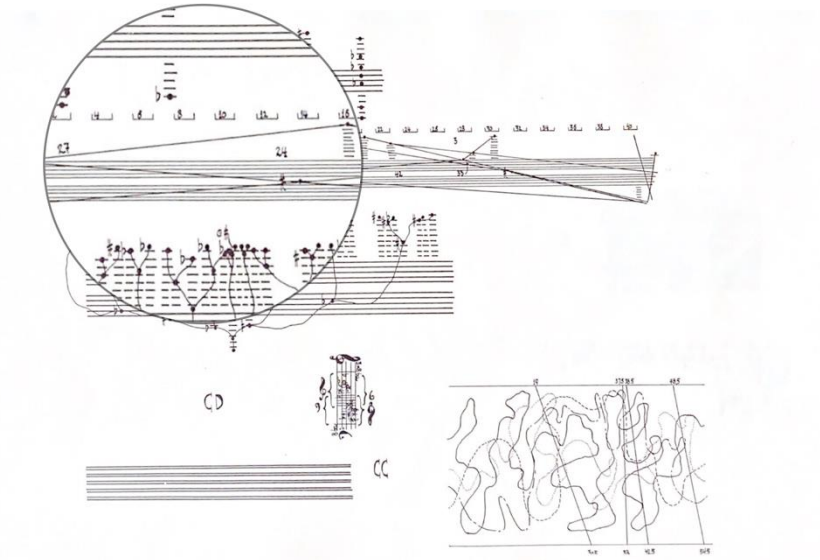


Figure 107 Notation AE on page 57.

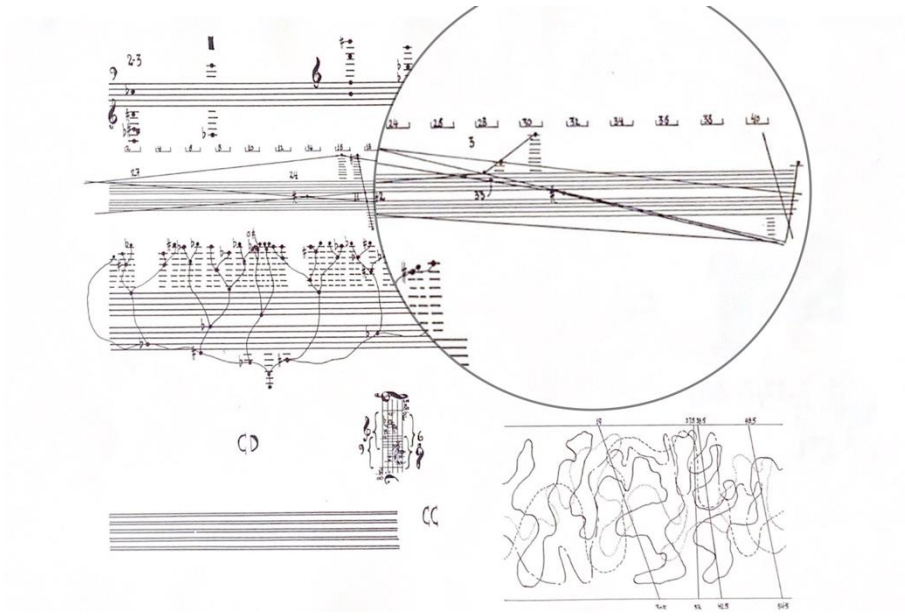


Figure 108 Notation AE on page 57.

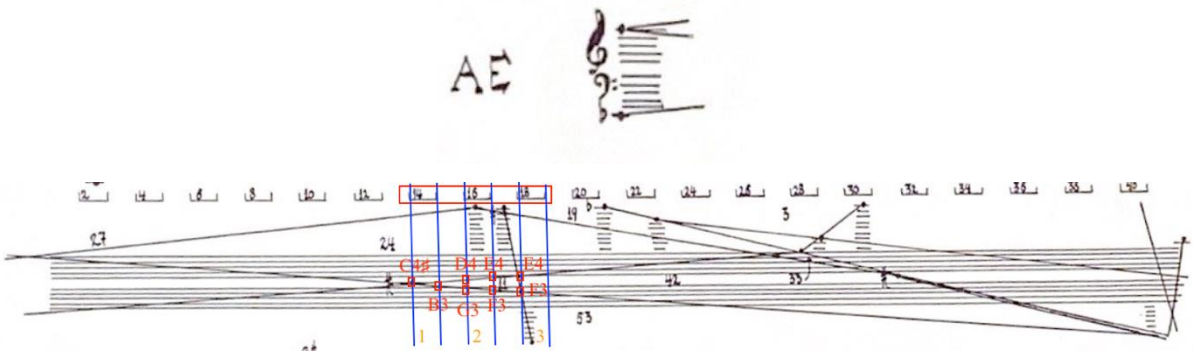


Figure 109 Realisation of Notation AE on pages 56–57.

Notation AF's performing instruction states, 'Each event (line-connected notes) is to be played by one hand only. The clef signs above are for the right hand, below for the left'.[299] Here, it is noticeable that Cage intended to limit performers' realisations to a certain hand, by clef signs; however, he did not strictly assign which event should be realised by which clef signs, meaning that performers are free to decide which event is performed by which clef signs (Appendix A Table 79). For example, in Notation AF on pages 20–21(Figure 110; Figure 111), the first event above the staff can be F♭, C♯, B♯, A, B♯, B when it is performed with the bass.

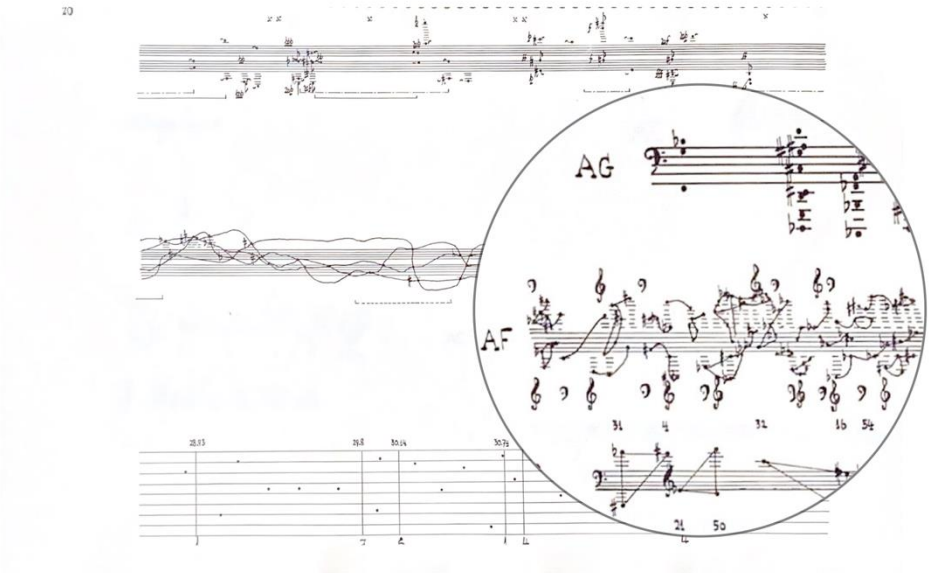


Figure 110 Notation AF on page 20.

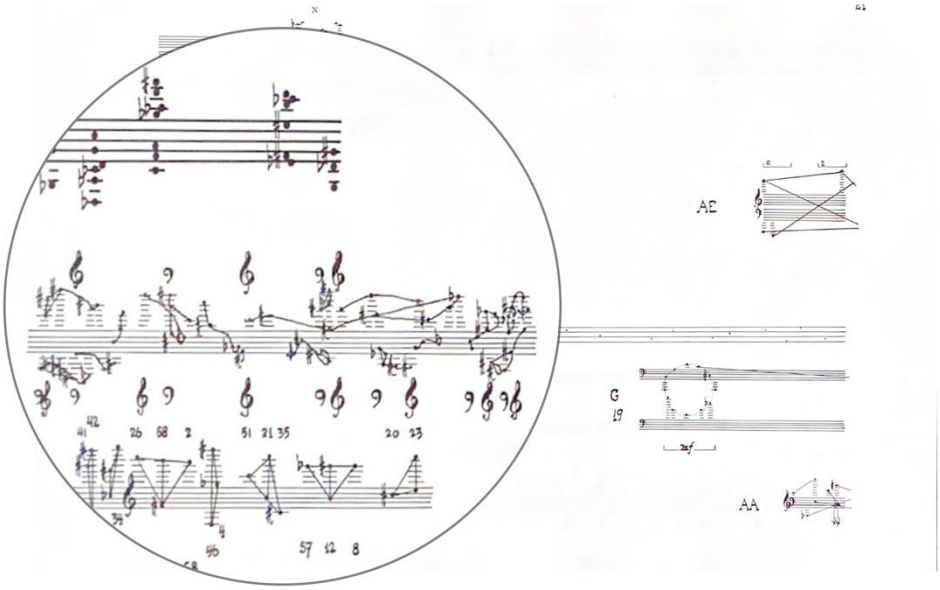


Figure 111 Notation AF on page 21.

[299] Ibid.

In Notation AG, Cage did not tell performers how to realise it, instead, the performing instruction states, 'Omit any 2 notes of each aggregate'.[300] Therefore, a freedom here is a choice of notes; for instance, performers can choose not to play any two notes from notes F, G and B♭ of the first aggregation in Notation AG on pages 19–20 (Figure 112; Figure 113). Performers are free to omit two notes, but this freedom is also built upon a performing approach; as performers 'need to' omit two notes, although they are free to decide which two. Along with this type of freedom, performers are also free to decide the quality of sounds (Appendix A Table 80).

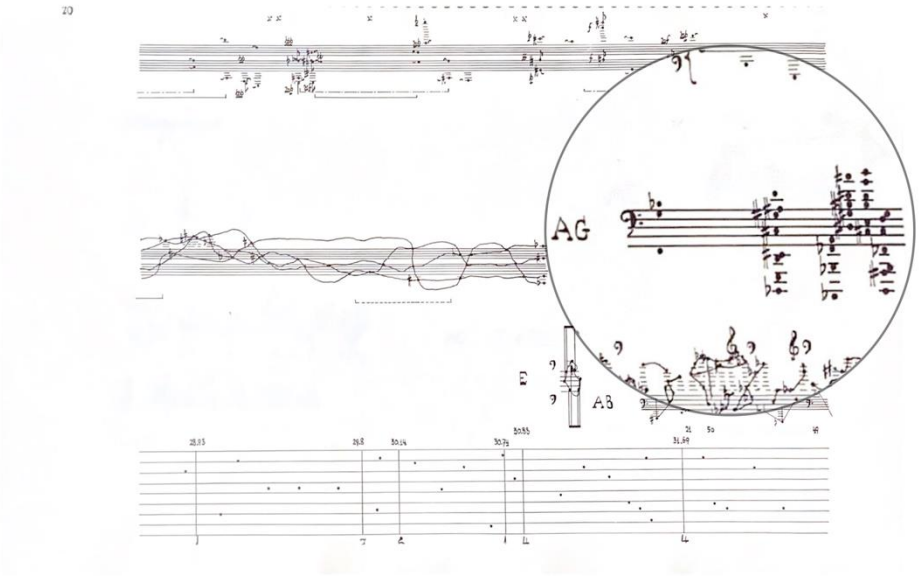


Figure 112 Notation AG on page 20.

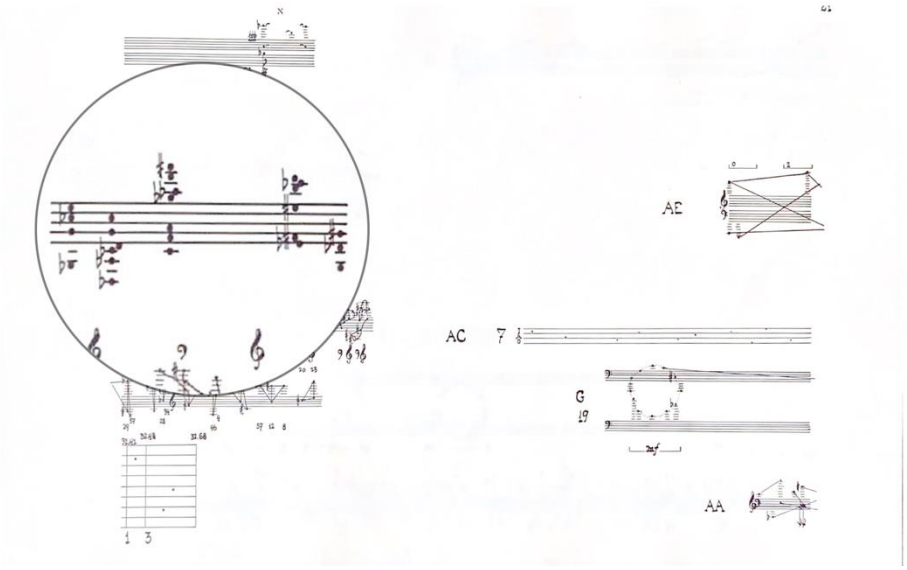


Figure 113 Notation AG on page 21.

[300] Ibid.

For Notation AH, Cage guided the performer thus, 'Clefs free (treble or bass) hands given. Follow lines in directions given by arrows'.[301] From the performing instruction, the known compositional materials are hands, lines, directions, and arrows. In this notation, he did not assign clef signs, instead he indicated that the upper staff should be performed with the right hand, and the lower staff with the left hand (Appendix A Table 81). Along with this arrangement, the other important element of this notation is direction. In Notation AH (Figure 114), he applied arrows to lines to let performers follow the performing directions that he expected. This compositional material is used in Notations J, M, and Q, expecting performers to follow the directions, but are free to decide the performing direction when encountering intersections (Appendix A Table 57; Table 60; Table 64). The similar arrangement of this compositional material alludes to a potential relationship between these notations.

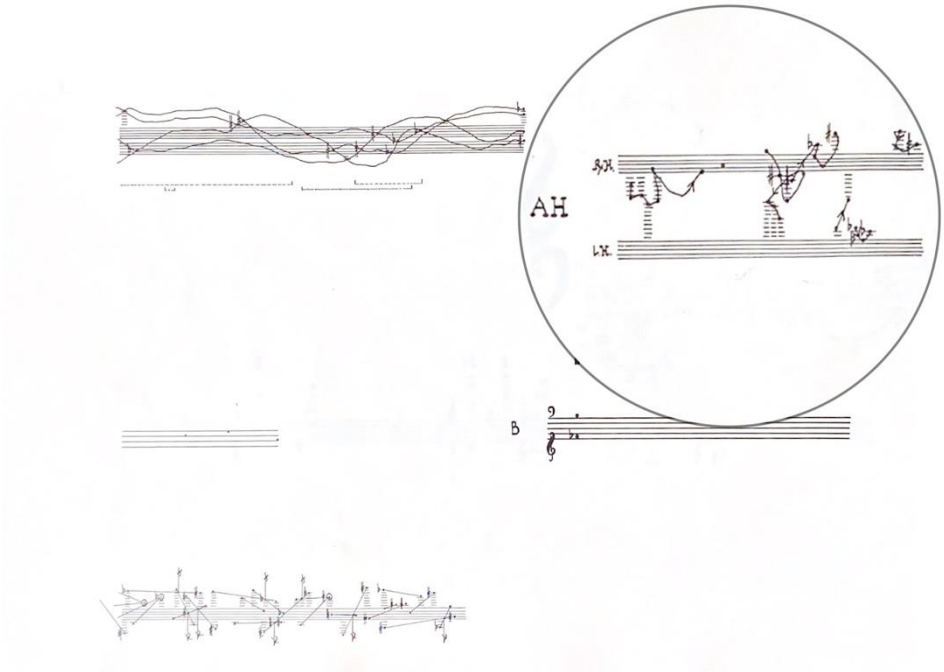


Figure 114 Notation AH on page 23.

In Notation AI (Figure 115; Figure 116), Cage stated, 'Play 'wheel' or 'axle' using one(s) not played as harmonic(s). Play from left to right'.[302] From the performing instruction, the compositional materials include wheel and axle, the two graphs that Cage used to represent notes. The notation includes eight wheels, each containing two to five axles. Therefore, performers are free to choose one or more wheels (axles) in their realisations. The interesting freedom here is that he did not limit the order of notes, but guided them into the performing direction, which is from left to right. This limits the notes to a fixed order; hence performers

[301] Ibid.
[302] Ibid.

should be aware of the distance between each note (Appendix A Table 82). For instance, in Notation AI on pages 24–25, the first wheel (located on the bottom of the notation) and the second wheel (located on the top of the first wheel), the notes from the two wheels are staggered. Following this, the note E from the second wheel and the note C♯ from the first wheel are performed at the same time, followed by the notes A and B of the first wheel, then is the note C from the second wheel, and so on (Figure 117). As Cage specifically mentioned, it should not be played as harmonic(s), the performers need to follow this limitation, while they are free to decide the quality of notes and duration.

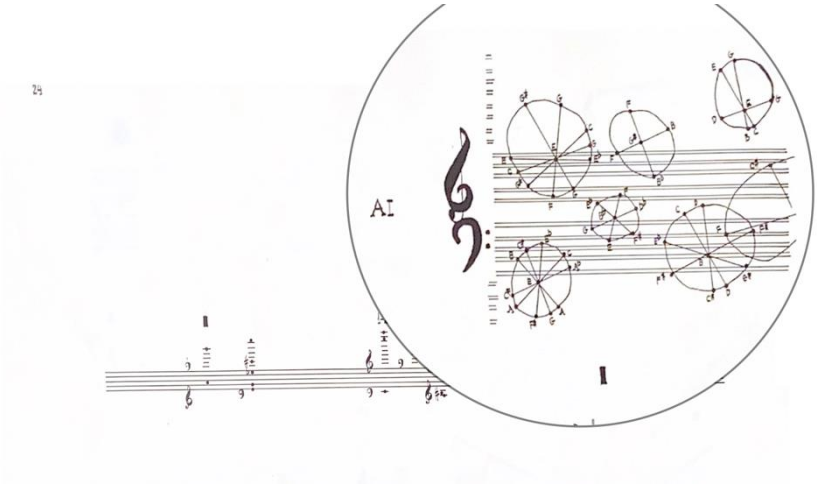


Figure 115 Notation AI on page 24.

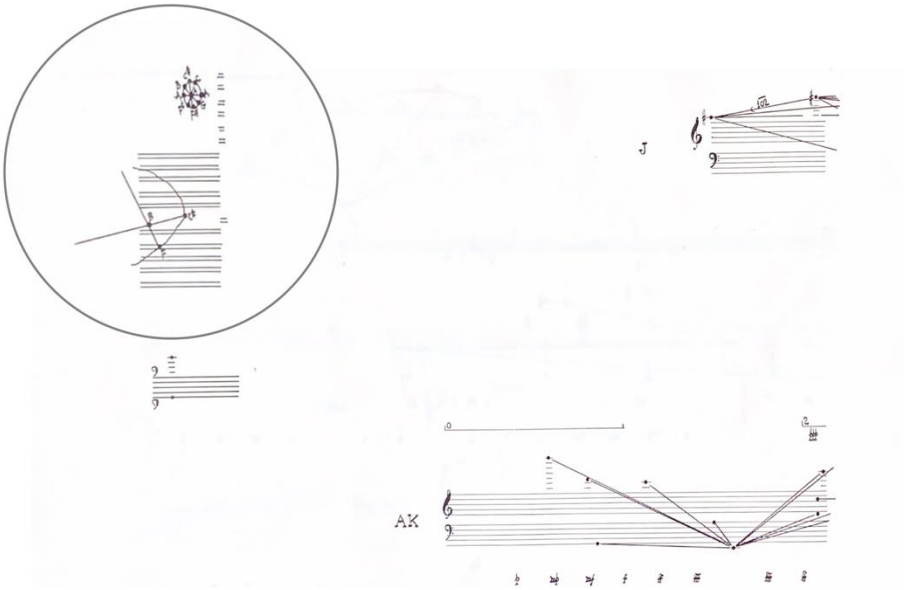


Figure 116 Notation AI on page 25.

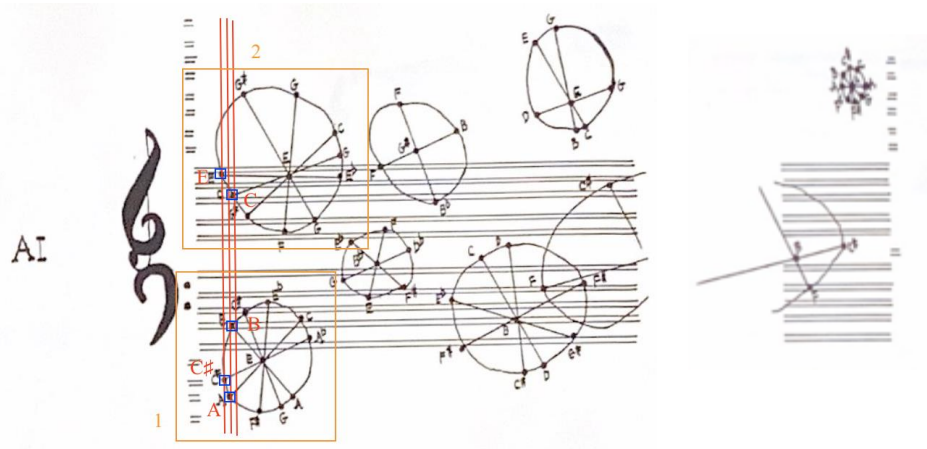


Figure 117 Realisation of Notation AI on pages 24–25.

The performing instruction for Notation AJ is, 'Numbers are proportional of notes to be played in directions given by arrows changing clef at points where signs appear'.[303] Cage applied straight lines, arrows, and direction in this notation, requiring performers to follow the direction and change to the assigned clef sign (Appendix A Table 83). Here, the straight lines indicate the pitch limit, arrows indicate the direction, and numbers represent the number of performing notes. For instance, the straight line in the middle, shows a pitch limit, from G to A of F-clef, with number 6 at the start of the straight line, so indicating that the performer should choose six notes from G to A (Figure 118). This notation can be realised in many segments, as Cage applied clef signs to divide the straight lines into sections. Thus, the realisation may contain either one or multiple tracks.

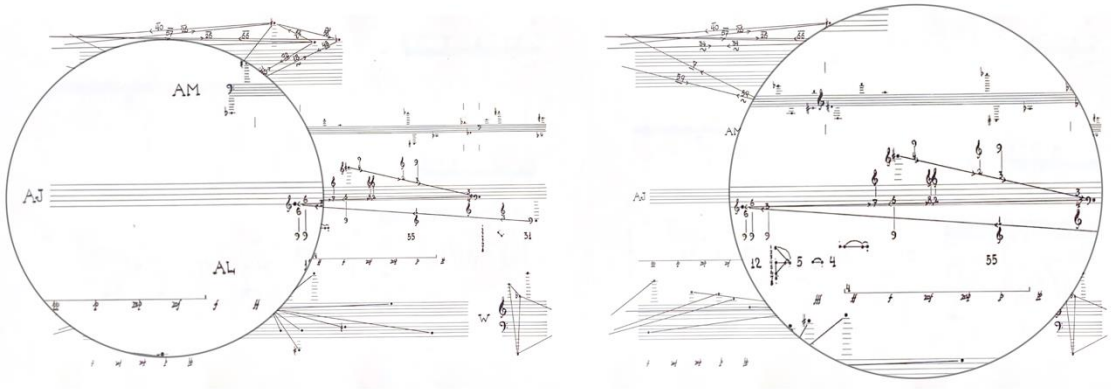


Figure 118 Notation AJ on page 26.

[303] Ibid.

In Notation AK, the performer should follow, 'Play any 1 note in each 'universe' according to time and amplitude given'.[304] 'Universe' refers to pitch limits, which are presented as straight lines, with two notes at both ends. In this notation, performers need to choose one note from each pitch limit and follow the time and amplitude that he indicated above and below the staff (Appendix A Table 84). When combine the indications of time and amplitude, it is noticeable that he subtly offered relationships between notes, time, and amplitude; consequently, each universe includes a wide range of notes, while straight lines act as indicators of pitch range. With lines, time, and amplitude indicators, the performers have the information they need for the realisations. For example, in Notation AK on pages 25–26 (Figure 119; Figure 120), the first straight line shows the pitch limit, from F of G-clef to E of F-clef. Based on the indication of time (from the zeroth second to the first second), this pitch limit can be divided into two sections: F of G-clef to D of G-clef, and D of G-clef to E of F-clef. Meanwhile, the indication of amplitude divides this pitch limit into five sections: *mp, mf, f, ff, fff*. When choosing the performing note, performers need to consider these sections. If the chosen note is D6 of F-clef, this note should be played before the first section, and with *mf.* From this type of arrangement, it is noticeable that Cage offered a type of freedom, allowing performers to choose a note; however, he also limited them to a certain time and degree of amplitude.

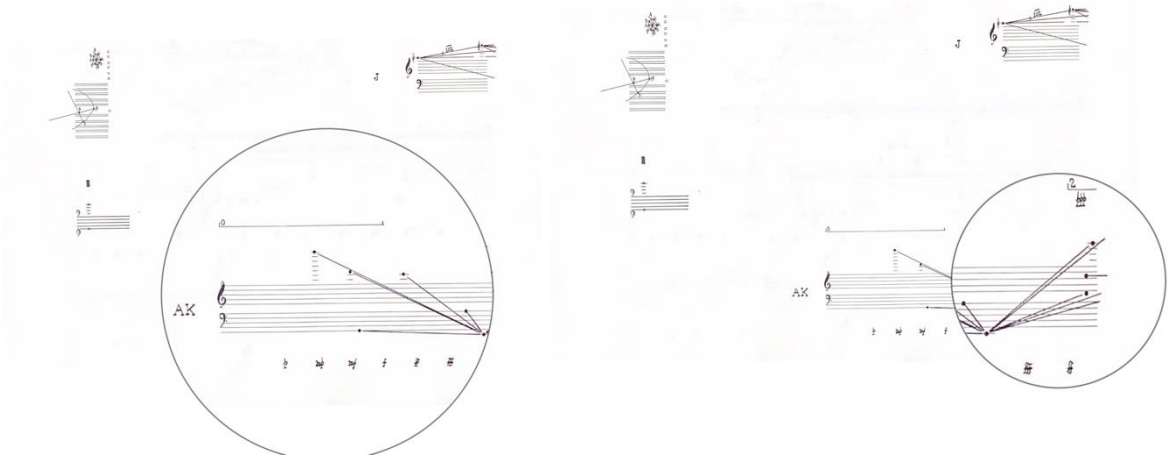


Figure 119 Notation AK on page 25.

[304] Ibid.

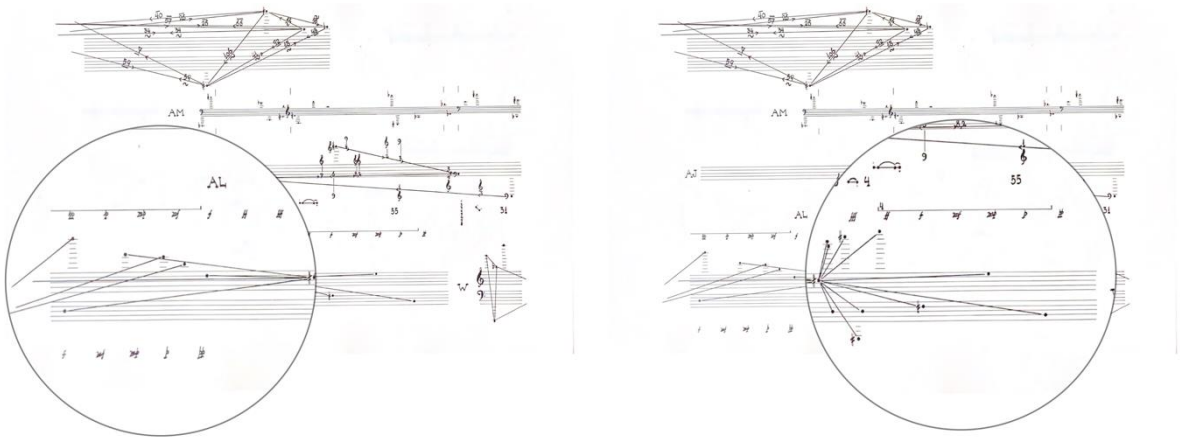


Figure 120 Notation AK on page 26.

The performing instruction for Notation AL is, 'Any 8 tone gamut. Large numbers are proportional time between sound events. ⌒ = legato · = staccato'.[305] In this instruction, Cage used numbers to indicate proportional time and range of tones. Here he stated the proportions of each section, but not which time unit that should be used; neither did he state the range of tones. Hence, performers are free to choose any time unit, as long as the proportions remain the same (i.e., either one second, or ten seconds); they are also free to decide the range of tones (Appendix A Table 85). Alongside the larger numbers, there are small numbers that showing how many sound events there should be. Based on these performing instructions, he used two drawings, ⌒ and ·, to indicate legato and staccato. For instance, in Notation AL on pages 26–27 (Figure 121; Figure 122), the antepenultimate larger number is 11; however, before 11, there is a small bracketed number, (7) above which there is a dot with a drawing indicating staccato locates in the seventh tone gamut. This indicates that the dot with a drawing may realise as seven sound events, performed as staccato, and based on the seventh tone gamut.

[305] Ibid.

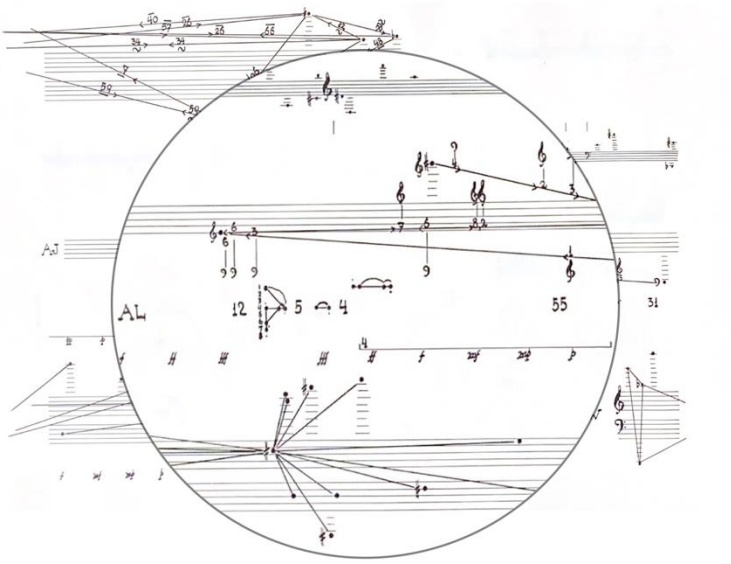


Figure 121 Notation AL on page 26.

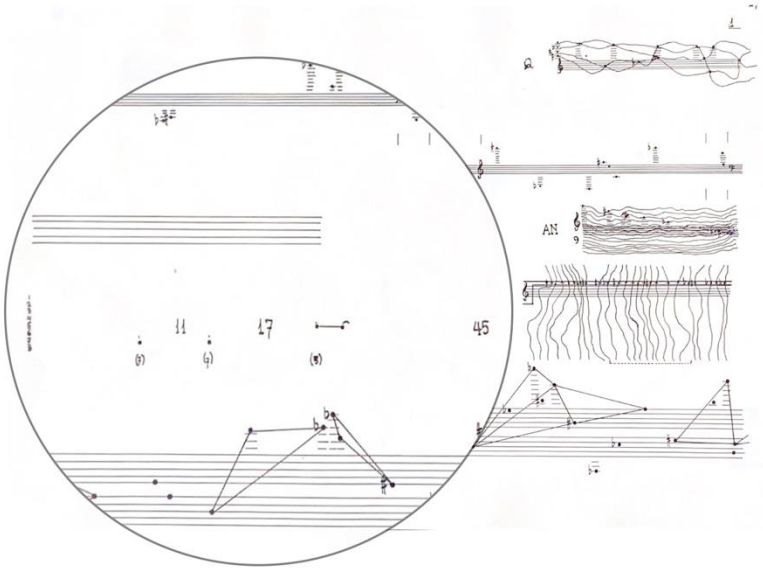


Figure 122 Notation AL on page 27.

Notation AM's (Figure 123; Figure 124; Figure 125) performing instruction is, 'Single tones and intervals equal lengths of time between marks above and below staff'.[306] Therefore, the compositional materials are single tones, intervals, length of time, marks, and staff. Cage applied these compositional materials to offer a fundamental performing approach – single notes and intervals should be performed to the same duration; however, performers need to perform either single notes and intervals, although they are free to decide the quality of notes

[306] Ibid.

and intervals between marks, they are limited to their assigned starting points (Appendix A Table 86).

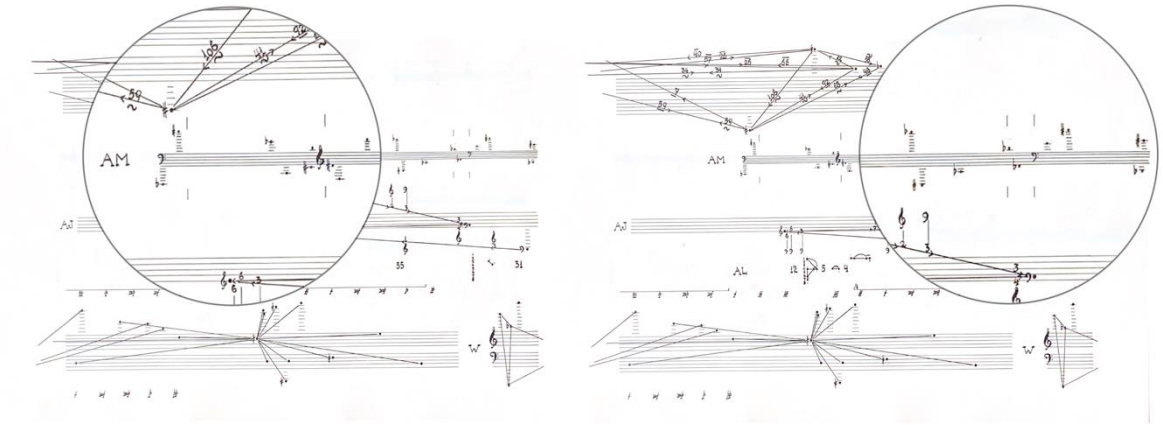


Figure 123 Notation AM on page 26.

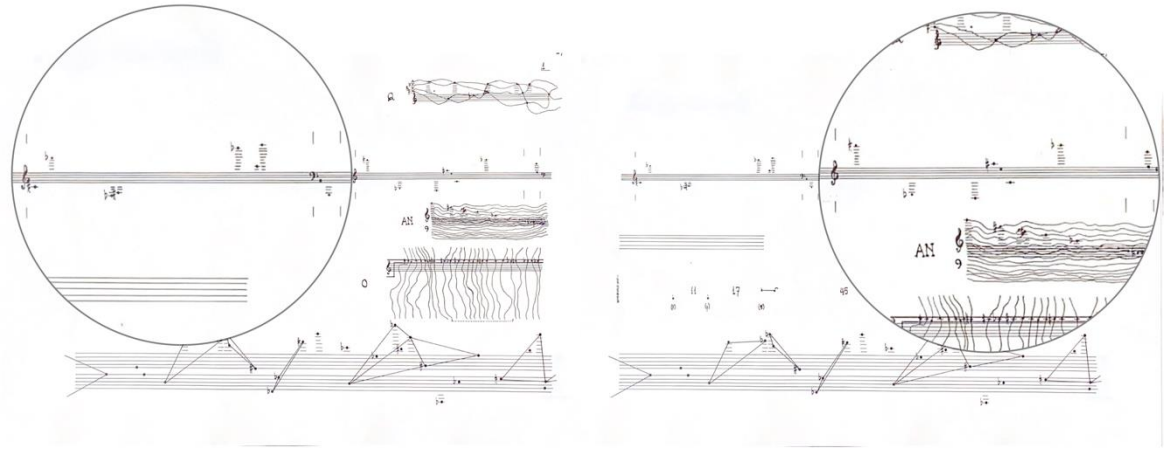


Figure 124 Notation AM on page 27.

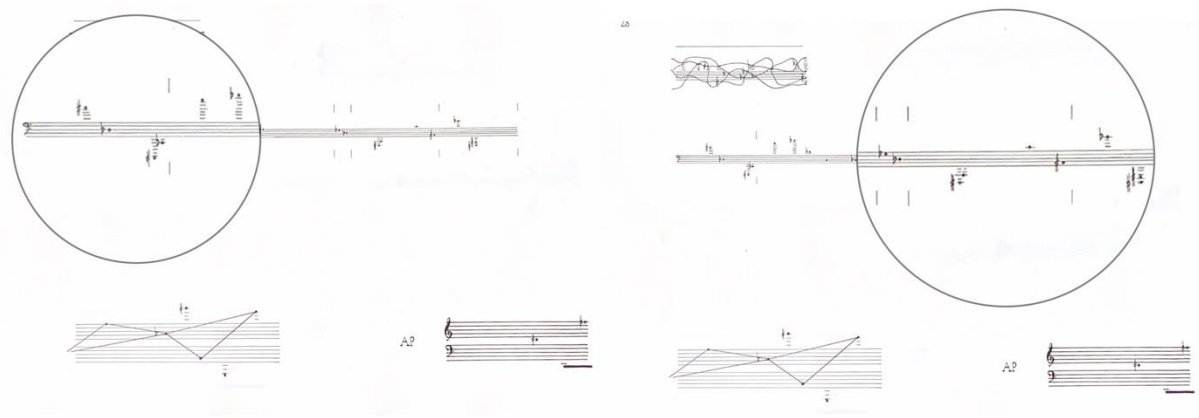


Figure 125 Notation AM on page 28.

Notation AN's performing instruction is, 'Clefs refer to hands. Following the line from left to right play any number of notes with one hand followed by a different number with left (etc., if numbers chosen do not use all the notes)'.[307] The compositional materials here are clef signs, hands, numbers of notes, and lines. Hence, performers can decide both the number of performing notes, and which to choose, as long as they use different hands for different clef signs (Appendix A Table 87). For Notation AN on pages 47–48 (Figure 126; Figure 127), there are two lines, one curved and one straight, with five notes, B♭, G, A G, D♯, and C♯, for G-clef, or D♭, B, B, F♯, and E♯ for the F-clef, locating on intersections. Performers may realise the straight line with the right hand and the G-clef and apply the left hand to the curved line and the F-clef. They may choose any number of notes, based on the tendency of lines, while two hands may perform different numbers of notes.

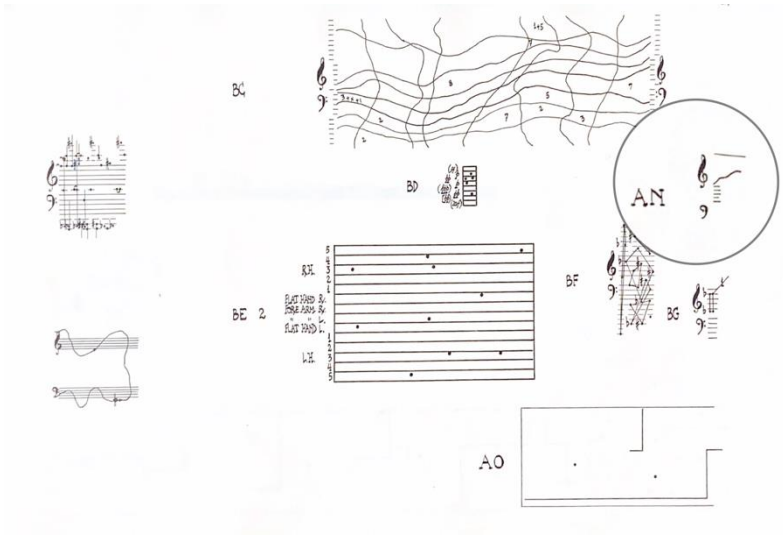


Figure 126 Notation AN on page 47.

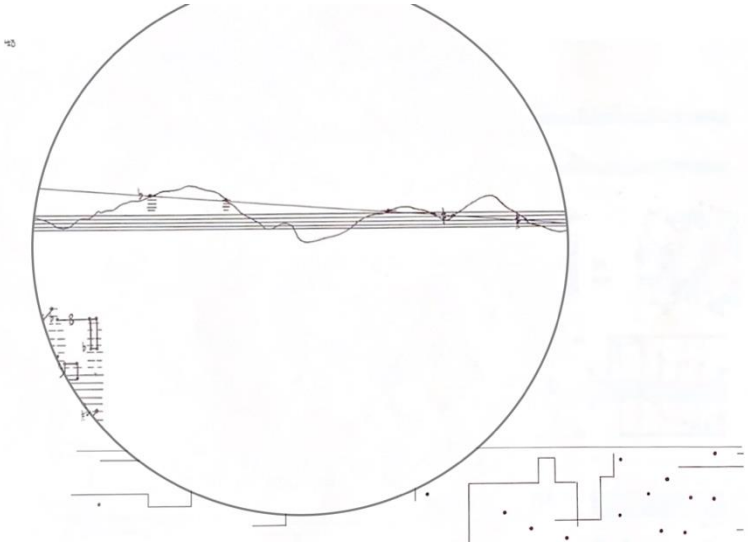


Figure 127 Notation AN on page 48.

[307] Ibid.

In Notation AO on pages 30–31 (Figure 128), Cage did not apply staff and clef signs, instead he used areas, dots, and lines. His performing instruction is, 'Any pitch area having at least 20 chromatic tones. Space vertically = frequency. Horizontally = time. Horizontal lines = duration of single tones. Vertical lines = clusters or legato. Points = short single tones'.[308] Here, performers can choose freely any pitch area containing at least twenty chromatic tones; however, they need to realise this based on vertical and horizontal lines and spaces (Appendix A Table 88). Hence, the performer may choose a pitch area based on C major, from C4 to G5, since Cage annotated vertical spaces to mean frequency, therefore, the pitch area, which should be applied as C4, is at the bottom of the notation, while G5 is at the top, meaning that the first point can be C4, and the fourth point, G5. Alongside these points, he drew vertical and horizontal lines. Because vertical lines signify either clusters or legato, performers can choose to take the horizontal lines to mean single notes. For instance, for the first vertical and horizontal lines the performers can realise these compositional materials as a single note C4, with C4♯, D4, and D4♯ as the cluster.

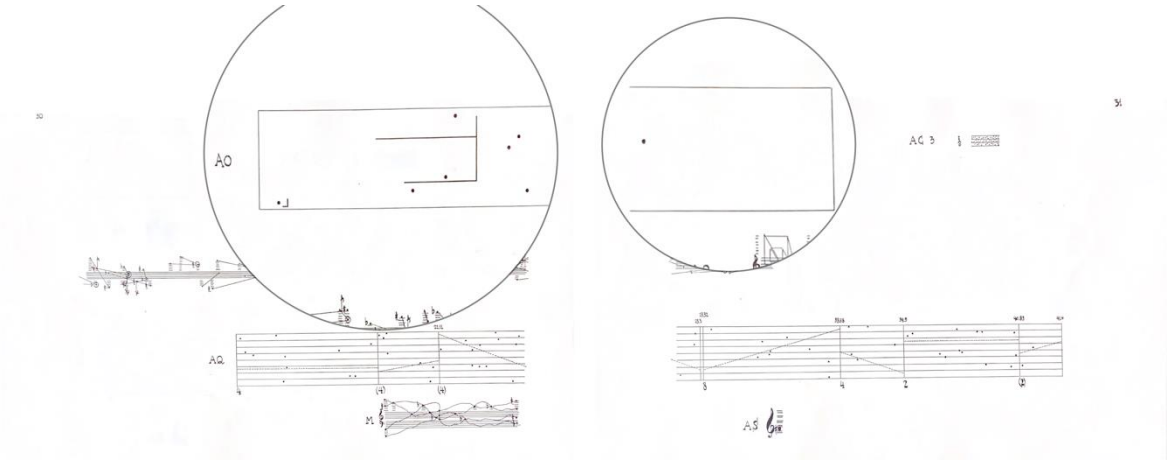


Figure 128 Notation AO on pages 30–31.

In Notation AP, Cage's performing instruction is, 'Lines give duration. Notes equal staccato'.[309] Here, the performers play the given notes, which can be short and quick , or long notes, depending on whether they appear with or without lines, which signify their duration (Appendix A Table 89). For instance, in Notation AP on pages 28–29 (Figure 129), the first note D♯ of the G-clef should be staccato, while the first note D of F-clef should remain until the final note B of F-clef is performed.

[308] Ibid.
[309] Ibid.

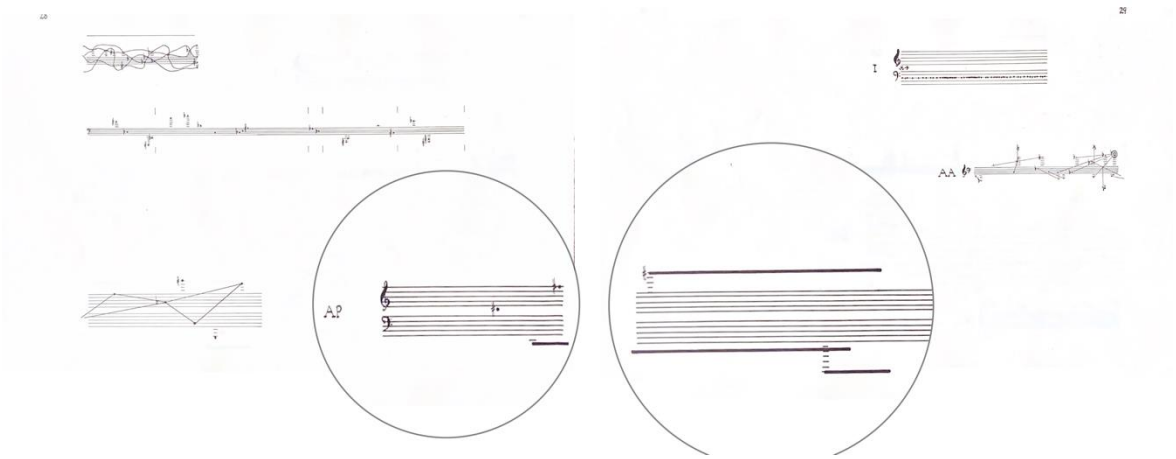


Figure 129 Notation AP on pages 28–29.

For Notation AQ (Figure 130; Figure 131), Cage stated that it should be 'Like Y, but use in each area only notations above or below dotted line'.[310] Because the performing instruction describes this notation to be like Notation Y, the performing approach and types of freedom are based on that. This means that performers may choose eight pitch areas and the given chromatic tones. However, they must perform the notes within the indicated time, and follow the location of the notes as higher, *fff*, and lower as, *ppp*. Their performing approach allows for a type of freedom, such as to choose the realising areas to be either below, or above, the dotted lines (Appendix A Table 90). For instance, they may choose the area below the dotted line for the first section, and the area above the dotted line for the second.

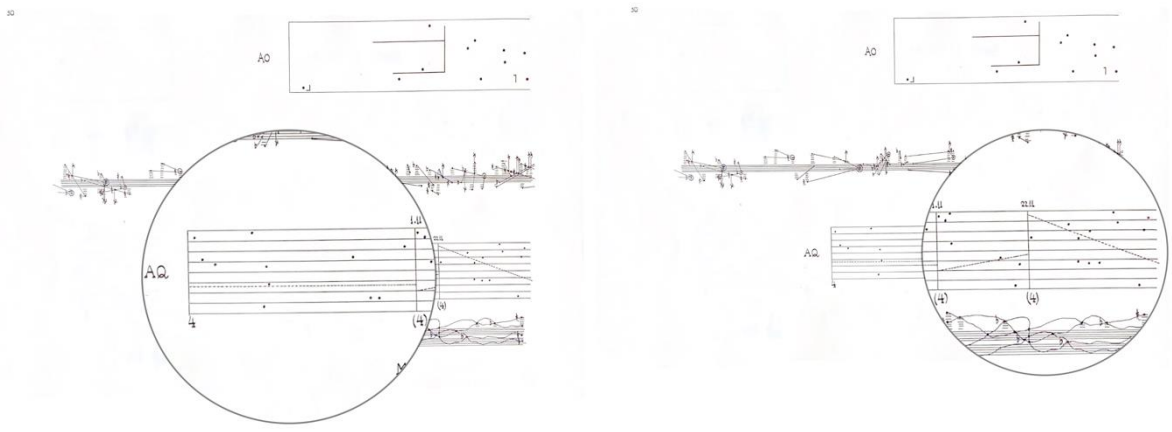


Figure 130 Notation AQ on page 30.

[310] Ibid.

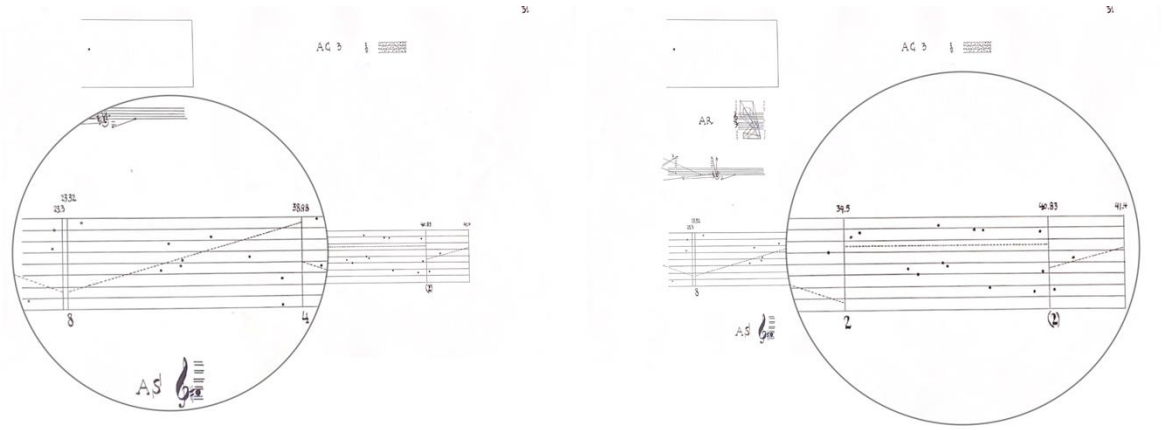


Figure 131 Notation AQ on page 31.

The performing instruction of Notation AR is, 'Play in any way that is suggested by the drawing'.[311] As Cage did not provide more detailed instructions, performers are allowed a significant degree of freedom. The compositional materials consist of a drawing that includes squares and rectangles, as well as lines, staff, clef signs, ledger lines, and paper imperfections (Appendix A Table 91). Taking Notation AR on page 31 as an example (Figure 132), from this drawing, performers may realise the vertical line as an interval (C5 and F7) and choose to follow an oblique line to perform a series of intervals until they reach another interval (C3 and G3) within a small square. However, this represents just one possible realisation. The interpretation of the edges of the stereoscopic squares can vary, depending on the performers' perspectives. Therefore, the path from C5 and F7 to C3 and G3 (following the oblique line) may not be the only performing direction. Alternatively, performers may choose a parallel line within the square (from C5 and F7 to C5 and F7) and perform these notes as staccato, or as two separate intervals. In essence, the stereoscopic drawing here offers numerous realisation possibilities.

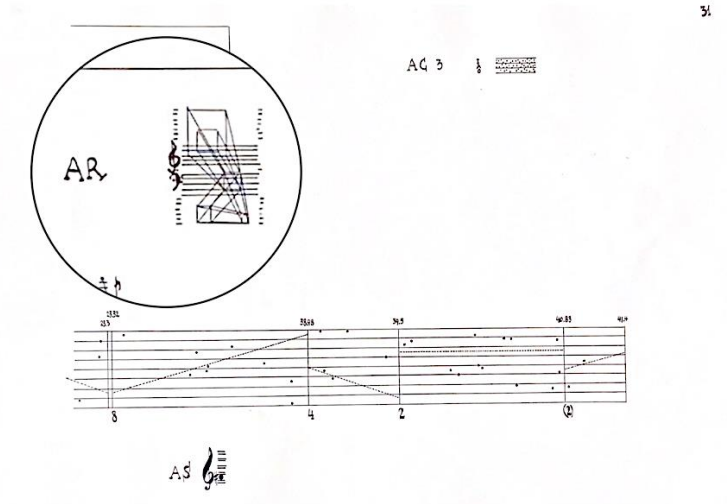


Figure 132 Notation AR on page 31.

[311] Ibid.

Compared to Notation AR, Notation AS (Figure 133) is quite simple. The performer must play a single note, since Gage's instruction simply states, 'A single note'.[312] Apart from the given note, the performer is free to decide its dynamics and duration (Appendix A Table 92).

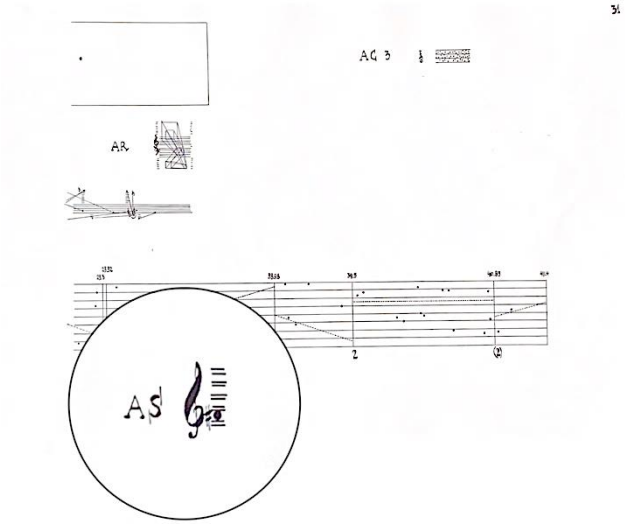


Figure 133 Notation AS on page 31.

In Notation AT (Figure 134), Cage's instruction is that it should be performed as Notation AE – 'Perform as in AE',[313] in which he uses pitch-time areas and numbers as the main compositional materials for certain numbers of notes to be played in an assigned time. Based on Notation AE, performers of AT follow the same performing approach and types of freedom (Appendix A Table 93).

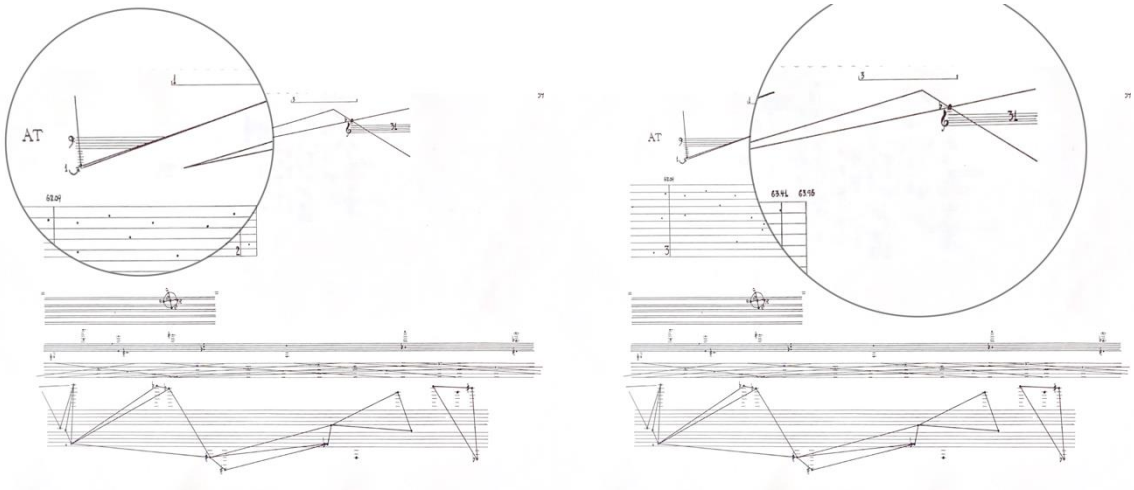


Figure 134 Notation AT on page 39.

[312] Ibid.
[313] The performing instruction of Notation AE is: 'Pitch-time areas silent unless accompanied by numbers, meaning number of tones (any) to be played' (ibid.).

Cage designed Notation AU (Figure 135) based on two previously described notations – Q and M. In its performing instruction, he stated, 'As in Q, but each line having its own clef sign brings about pitch ambiguity of some of the intersection notes'.[314] However, the performing instruction for Notation Q are built on Notation M, as its performing instruction states: 'Like M, but in time (any units); going back incurs need for increased speed'. It is noticeable that the difference between Q and M is the allocated time.[315] The minor difference between two notations can also be found in Notations Q and AU; the performing approach and type of freedom of Notation AU, consist of the major guidance for Notation Q, but each line should be realised with its own clef sign. In short, Cage composed Notation AU based on Notation Q, and Notation Q is built upon Notation M. This sort of compositional technique suggests a potential relationship between three notations, based on similar, but not identical, performing approaches and types of freedom (Appendix A Table 94).

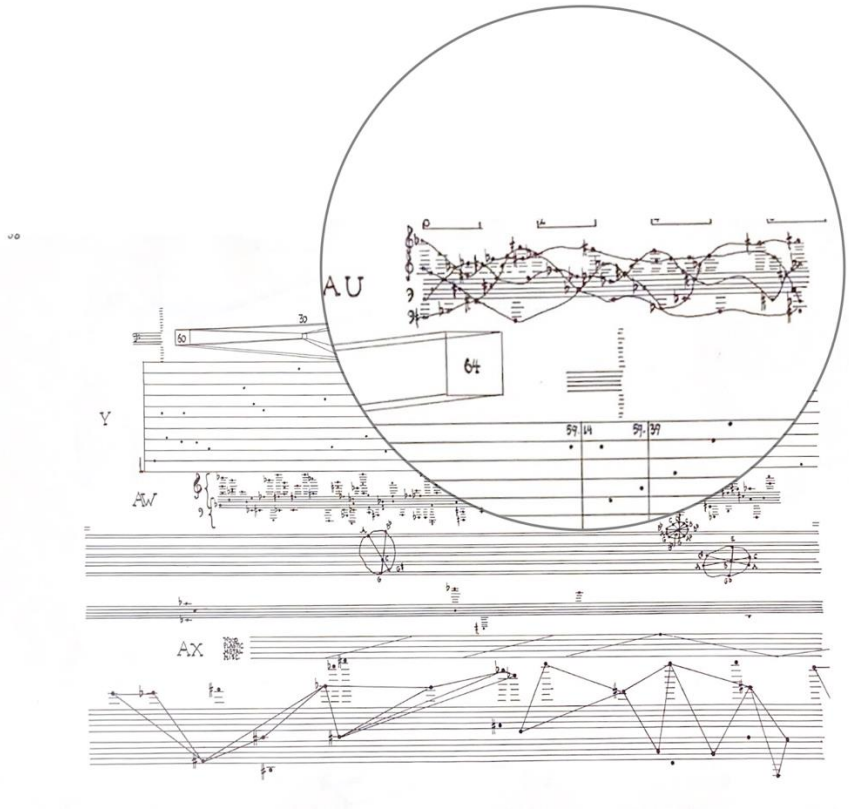


Figure 135 Notation AU on page 38.

[314] Ibid.

[315] The performing instruction for Notation M is 'Begin at left, end at right, changing direction at the intersections if desired. May be expressed as one voice, a 'counter point', or as 3 or 4 voices. Pedals only in areas indicated, not obligatory'. The analyses of Notation M, refers to the Section 2.1 Compositional Materials in Notations A to Z.

The performing instruction for Notation AV (Figure 136; Figure 137) is 'AR with the addition of numbers giving amplitude (1-64: *fff-ppp* or *ppp-fff*)' and the performing instruction of Notation AR is, 'Play in any way that is suggested by the drawing'.[316] This indicates that it should be performed as the drawing suggests, with the numbers indicating amplitude. Performers can decide if numbers 1 to 64 should be *fff* to *ppp*, or the other way round. Based on this instruction, the similarity and difference between the two notations can be seen; the first allows the freedom to decide amplitude, while the other has been assigned amplitude (Appendix A Table 95).

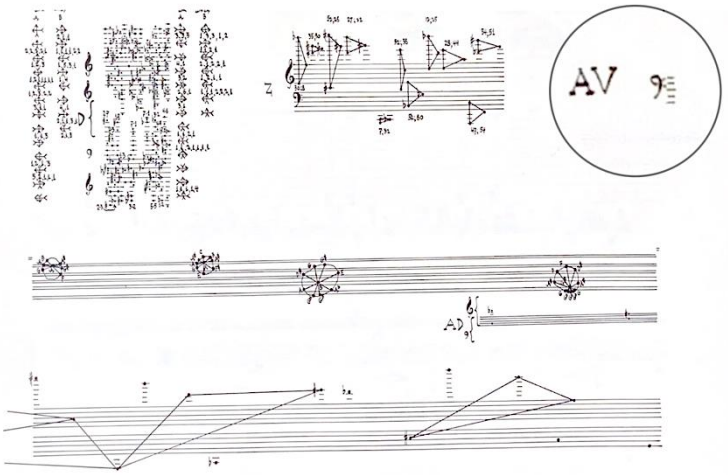


Figure 136 Notation AV on page 37.

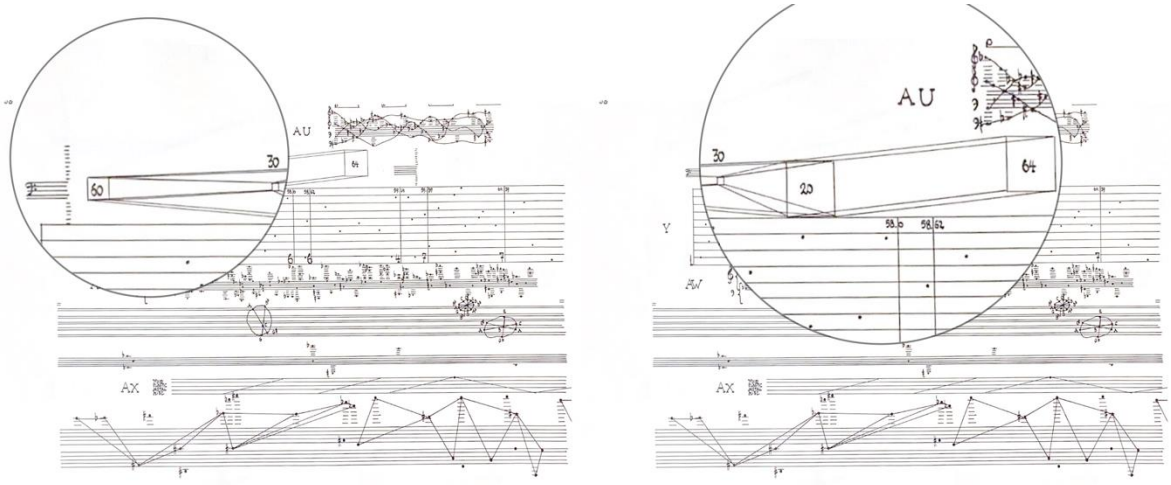


Figure 137 Notation AV on page 38.

[316] Ibid.

For Notation AW (Figure 138), Cage stated 'See AD'.[317] The main compositional material of Notation AD is the ambiguous staff, the notes on the staff may belong to either F-clef or G-clef, performers are free to judge this. Identically, Notation AW applies the ambiguous staff to allow performers to decide the relationships between notes on the staff and the clef signs (Appendix A Table 96).

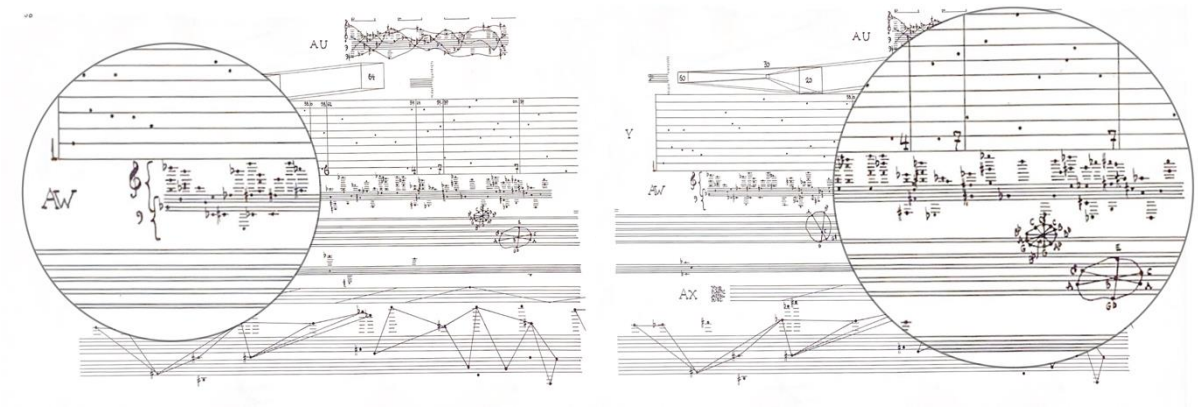


Figure 138 Notation AW on page 38.

In Notation AX (Figure 139; Figure 140; Figure 141), Cage's performing instruction are, 'Noises of any amplitude. Beaters given. On lines between areas = 2 beaters'.[318] The compositional materials are noise, amplitude, beaters, lines, and areas. The notation indicates some oblique lines and areas signifying four types of materials – wood, plastic, metal, and miscellaneous. Hence, performers need to produce sounds with certain materials when indicated by the notation; the freedom is in sound materials and amplitude (Appendix A Table 97).

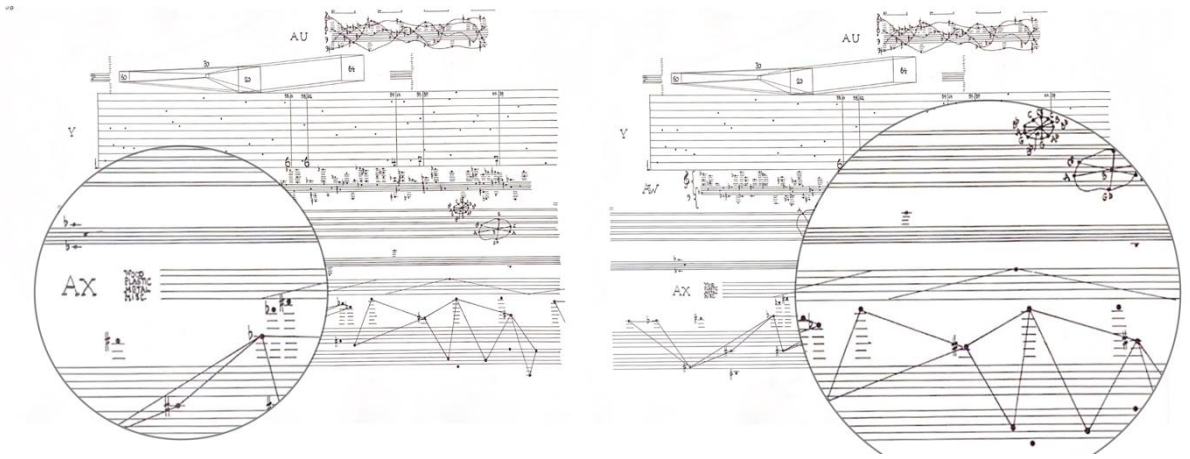


Figure 139 Notation AX on page 38.

[317] The performing instruction for Notation AD is, 'Single tones, intervals and three note aggregates with ambiguous staff, ledger lines above, treble, below, bass'.
[318] Ibid.

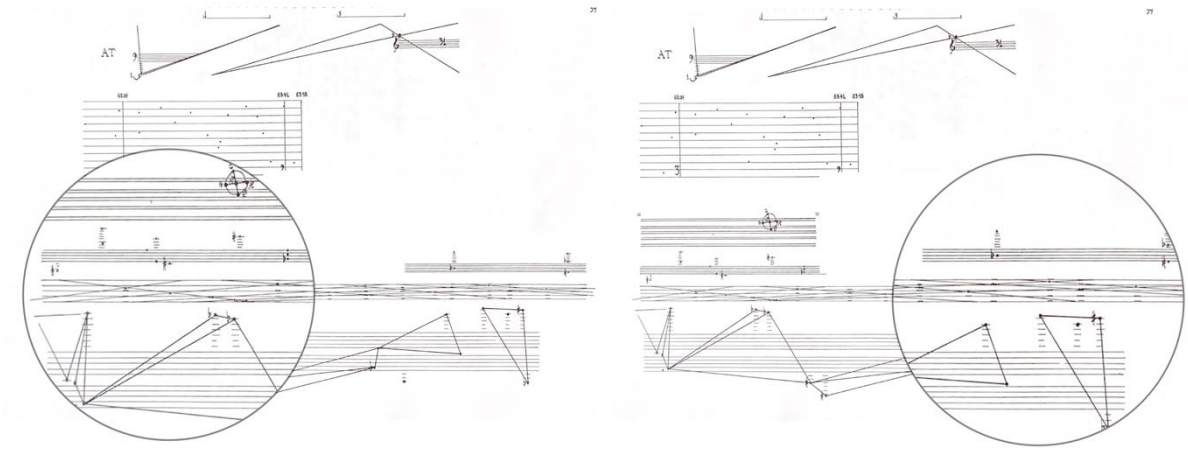


Figure 140 Notation AX on page 39.

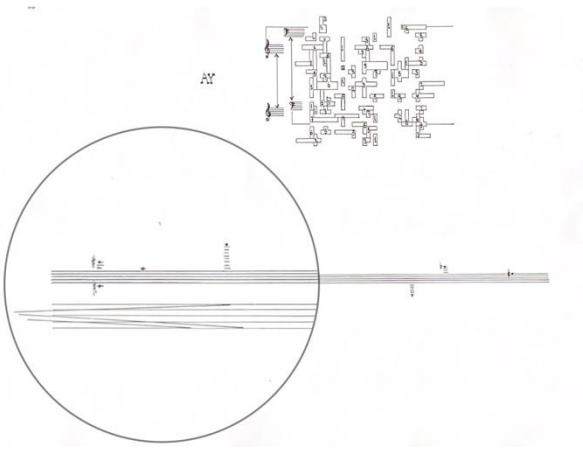


Figure 141 Notation AX on page 40.

Notation AY (Figure 142), for which Cage's performing instruction is, 'Graph music. 1/10 inch squared = time unit. Numbers within are of tones that may complete their appearance within any amount of time area given them by graph. Vertical graph is frequency, the treble and bass areas mobile as indicated'.[319] Notation AY applies inch squared, numbers, and graphs to guide performers regarding time, amounts and frequency of notes. Cage drew an 'exchanging' symbol for performers to decide whether to apply the G-clef or the F-clef, which suggests they may realise this notation based on these compositional materials, the freedom being the choice of graphs (Appendix A Table 98). Here, the performers need to measure the horizontal graphs to obtain the time unit, then choose which graph to use for the realisation, then determine the clef signs for the graphs, and, finally, determine the performing notes based on the given number of notes, the chosen clef signs, and the vertical graphs.

[319] Ibid.

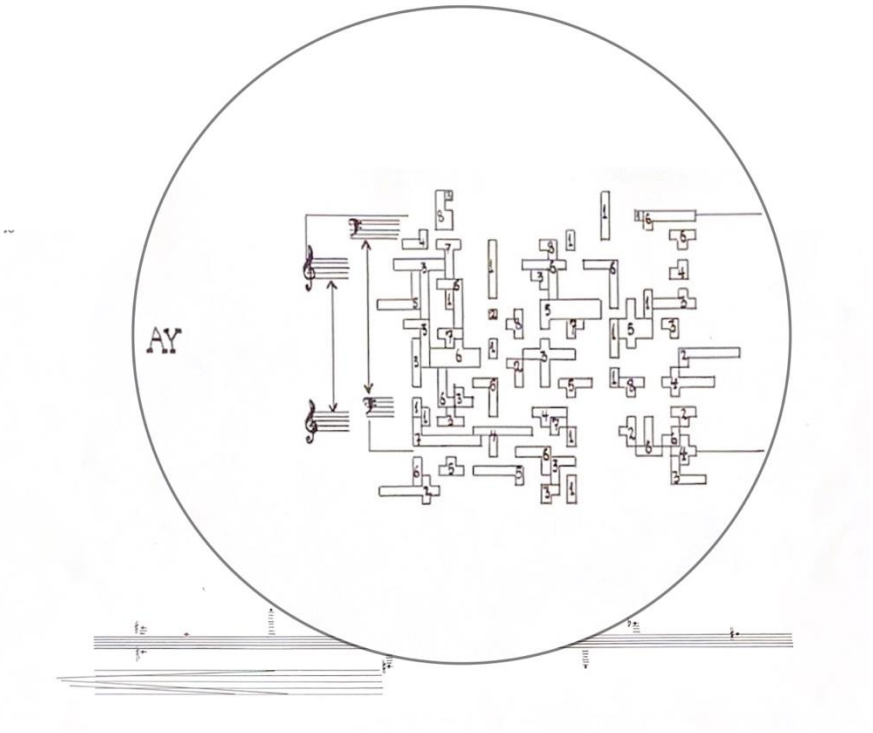


Figure 142 Notation AY on page 40.

Notation AZ's performing instruction (Figure 143; Figure 144) state, 'Numbers indicate time (any units). Notes connected by lines, vertical, are clusters'.[320] Here, Cage asked performers to follow the numbers that indicate time, with a freedom to decide any time units (Appendix A Table 99). Alongside the performing approach of following numbers, performers also need to follow the vertical lines and realise the lines as clusters.

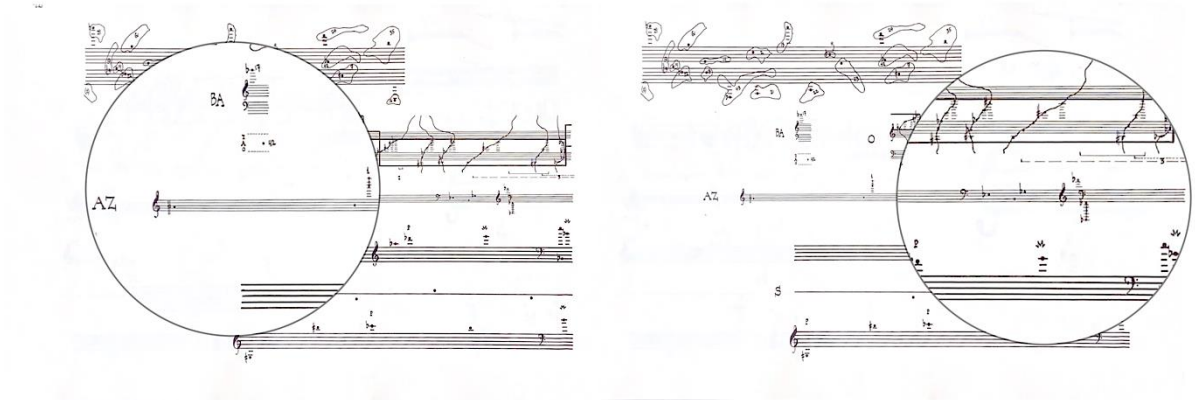


Figure 143 Notation AZ on page 42.

[320] Ibid.

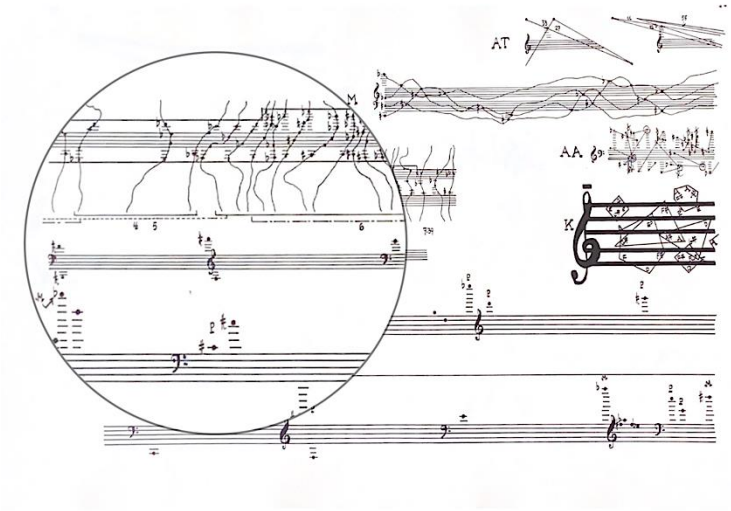


Figure 144 Notation AZ on page 43.

2.3 Compositional Materials in Notation BA to Notation BZ

In Notation BA, the performing instruction states, 'Numbers as in T. Sources of noise as in AC'.[321] Similarly, from this description, Notation T and Notation AC play fundamental roles, since Cage stated that the function of the numbers is identical to Notation T – referring to loudness; here, performers are free to decide whether number 1 to number 64 indicates either from soft to loud, or from loud to soft. The sources of noise are as in Notation AC; Cage used the abbreviations, I, A, and O to indicate the interior piano construction, auxiliary noises, and the outer piano construction. The instance here is Notation BA, on page 42 (Figure 145), which includes two parts, (i) staff with clef signs, a note, and the number 17, and (ii) an area with three abbreviations and the number 42. Because the function of numbers refers to loudness, performers need to obey this performing instruction and decide whether number 17 is louder than number 42, or number 42 is louder than number 17. This notation indicates a type of freedom similar to that in Notation T (Appendix A Table 100).

[321] The performing instruction for Notation T is, 'Influences in pitch and time notated as shapes with centre points, to be audible as clusters, a single one changing in its course. Numbers refer to loudness (1–64) (soft to loud or loud to soft)'. The performing instruction for Notation AC is 'Noises. Of those notated play only that number given. I = interior piano construction. A = auxiliary noises. O = outer piano construction. The position of the notes vertically gives its loudness (High: *fff*) (low is *ppp*)'.

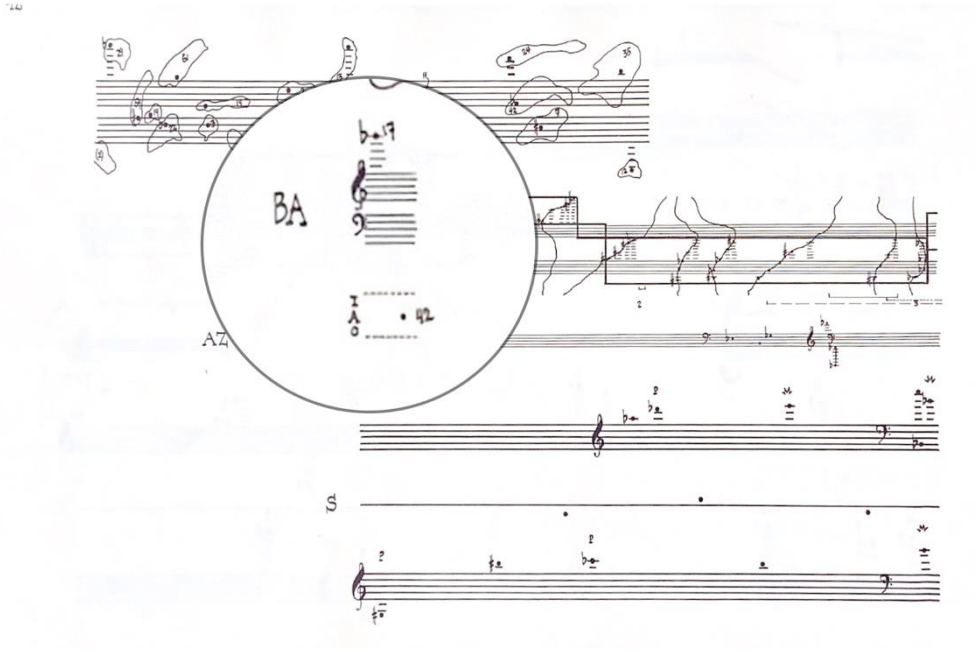


Figure 145 Notation BA on page 42.

In Notation BB, the performing instruction states, 'Notes are single sounds. Lines are duration (D), frequency (F), overtone structure (S), amplitude (A), and occurrence (succession) (O). Proximity to these measured by dropping perpendiculars from notes to lines gives respectively, longest, lowest, simplest, loudest, and earliest'.[322] For Notation BB (Figure 146), Cage drew one straight line representing duration, and four different types of dotted lines representing frequency, overtone structure, and amplitude, within an area. Performers need to measure distances between lines and each note to obtain these note elements. Although Cage guided performers to draw perpendiculars for measuring distances, he did not mention which result the composer should take when the perpendiculars show two contacts with the lines; neither did he guide the performer what to do if there is no contact between the perpendiculars and the lines. For this reason, the decision of how to use the results of measurements appear to be types of freedoms (Appendix A Table 101).

[322] Ibid.

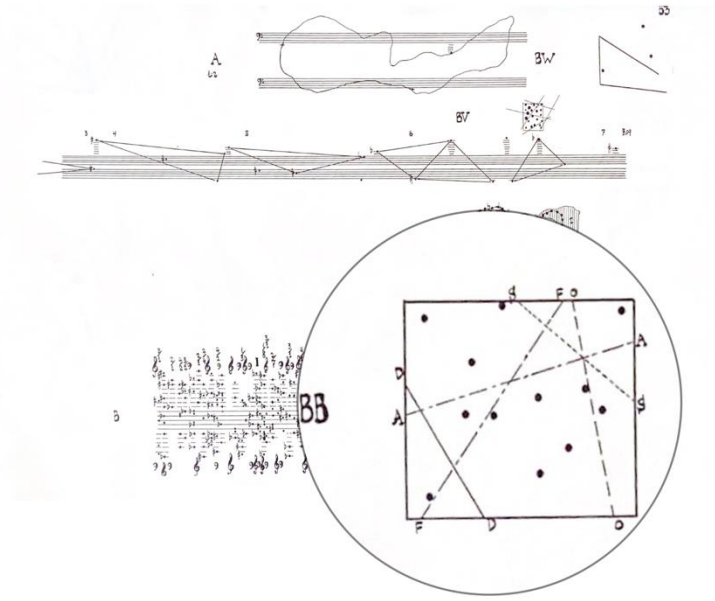


Figure 146 Notation BB on page 53.

Notation BC (Figure 147) shows several curved lines and numbers, from its performing instruction: 'Play number of tones in pitch-areas given. X = any number',[323] the known compositional materials are number of tones, pitch-areas, and numbers; as Cage annotated X is equal to any numbers, meaning that choosing numbers is the type of freedom that Cage offered in this notation. The other type of freedom is choosing notes from certain pitch-areas, this is also a performing approach, as the composer can only choose notes from the pitch-areas with numbers (Appendix A Table 102).

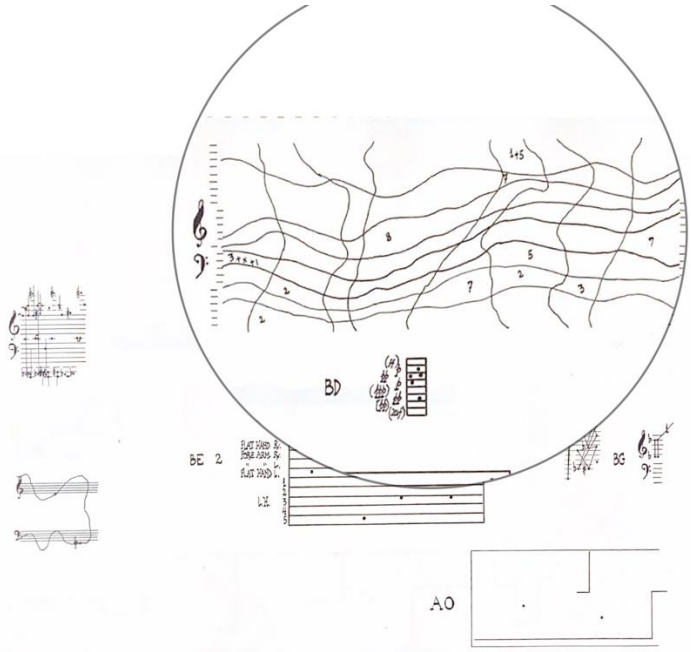


Figure 147 Notation BC on page 47.

[323] Ibid.

The performing instruction of Notation BD (Figure 148) is, 'Notes with amplitude given. Adjacent areas may be used to affect attack'.[324] The compositional materials include notes, given amplitude, and areas; the graphic notation includes eight areas presenting different amplitudes, from top to bottom is *ff*, *p*, *pp*, *p*, *ppp*, *pp*, *pp*, and *mf*. However, Cage did not set this performing approach as compulsory, so performers may decide on the amplitude. Added to this freedom, performers may decide which note can be just a single note throughout, or more than one, and performers are also free to decide their duration (Appendix A Table 103).

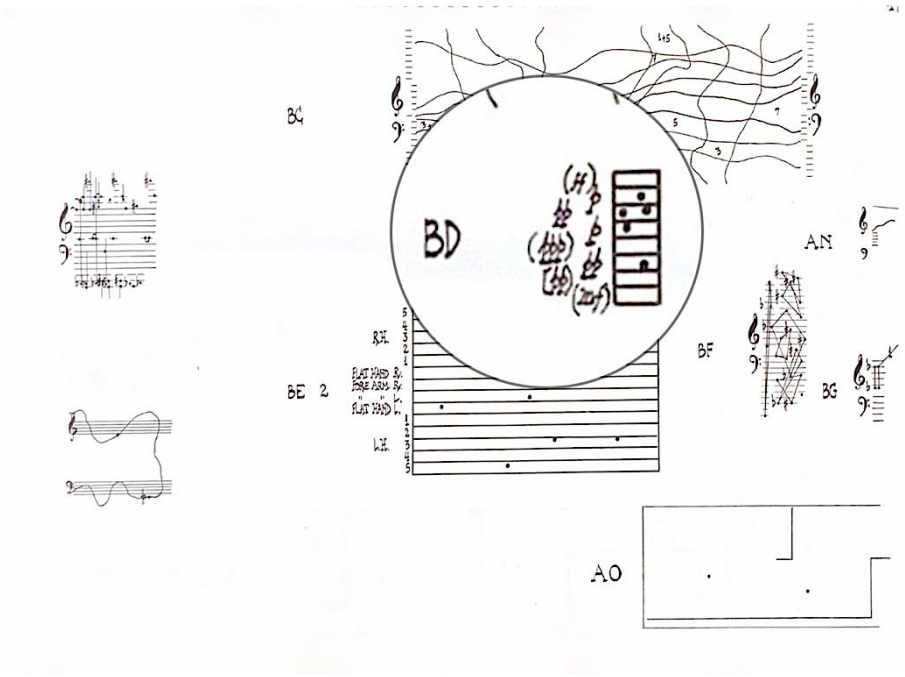


Figure 148 Notation BD on page 47.

The performing instruction of Notation BE states, 'Number = events to be expressed. Notes refer to fingers, hands, forearms to be used in playing'.[325] In Notation BE on page 47 (Figure 149), performers are free to choose two out of ten events and perform the two events with corresponding part of body. For instance, they may choose the first two events, and decide which notes for the chosen events, and finally realise the events by the third finger of the right hand and palm of the left hand (Appendix A Table 104).

[324] Ibid.
[325] Ibid.

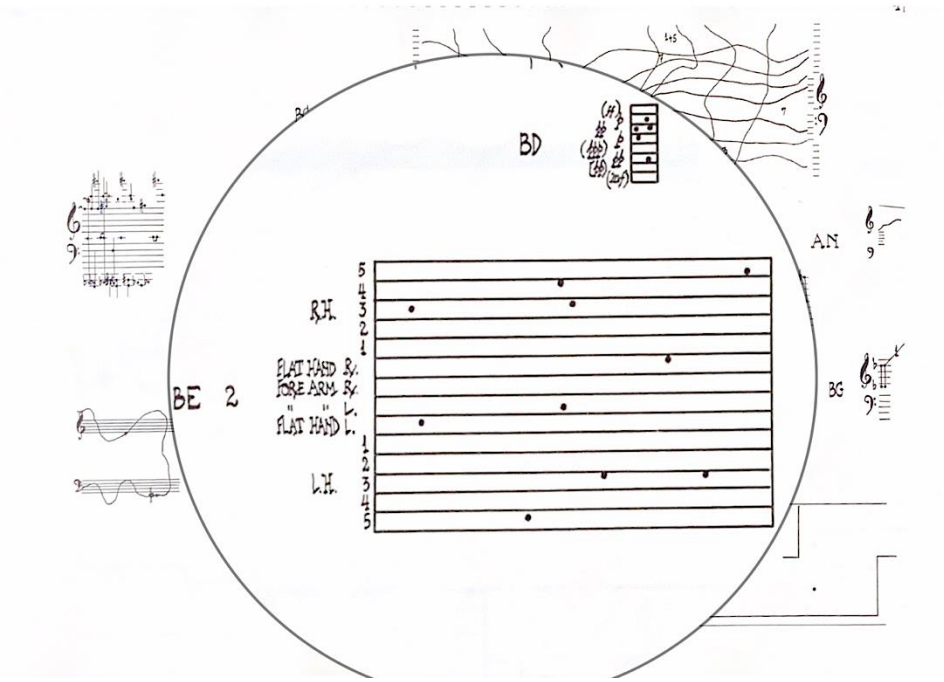


Figure 149 Notation BE on page 47.

In Notation BF (Figure 150), the performing instruction states, 'Notes connected by lines to be played legato. Single note = Staccato. Make super impositions as suggested by notation'.[326] Therefore, performers need to follow the notes in the order of notation, and to follow lines to realise legato for connected notes and staccato for single note (Appendix A Table 105). Based on this performing approach, I notice that Notation W is based on a similar performing approach, which guides the performer to realise triangles as legato and perform isolated notes as staccato (Appendix A Table 70).[327] The only difference are the shapes in the two notations, Notation W only uses triangles, while BF uses triangles and quadrilaterals to connect legato notes, and separate staccato notes. Regardless of the difference, a potential relationship between the two notations is shown in the performing approach and the compositional materials.

[326] Ibid.
[327] The performing instruction for Notation W is, 'Legato (triangles) and staccato (isolated notes)'.

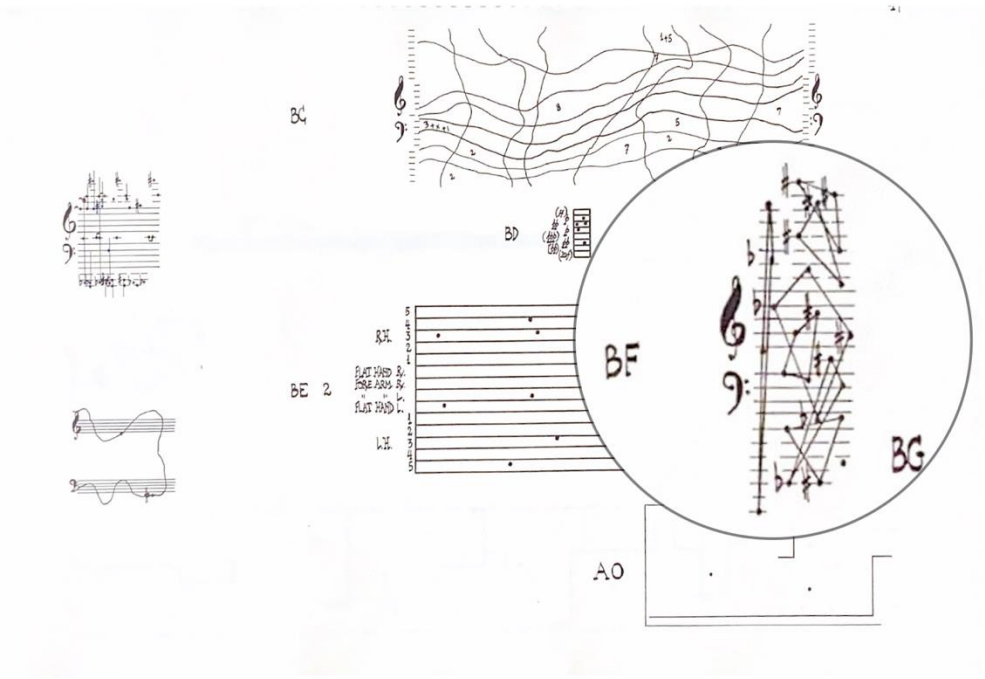


Figure 150 Notation BF on page 47.

The performing instruction for Notation BG is 'Intervals with free approaches, departures and simultaneities. Numbers indicate number of tones to be played within range notated'.[328] This shows the need to perform the intervals at the same time, but they are free to decide the duration and quality of sounds of the intervals (Figure 151) (Appendix A Table 106). Thus, the numbers represent the number of notes within the given pitch range, meaning that performers need to follow the number to choose the given proportion of notes for the realisation. It is interesting to note that Cage applied numbers in horizontal, vertical, and inclined lines; although the numbers indicate the same function, they have different purposes depending on the different types of lines.

In this Notation BG for instance (Figure 152), the first inclined line is connected by notes ♭D and B, the number in this pitch range is, 1; hence performers should choose one note from ♭D and B, and perform the chosen note (blue line and blue squares in Figure 152). Similarly, for the vertical lines, performers need to choose to play a certain number of notes; however, in the horizontal lines they may be unable to exercise a choice, since both ends consist of the same note. Consequently, the numbers on the horizontal lines demonstrate how many times they need to perform the note. For example, the horizontal line connects a note ♭D and the same note ♭D, with a number 4 on the line (red line and red squares in Figure 152). Thus, performers should

[328] Ibid.

perform note ♭D four times. As the performing instruction suggests, the intervals can be performed with any approach and departure timing, these are two types of freedom that Cage offered here.

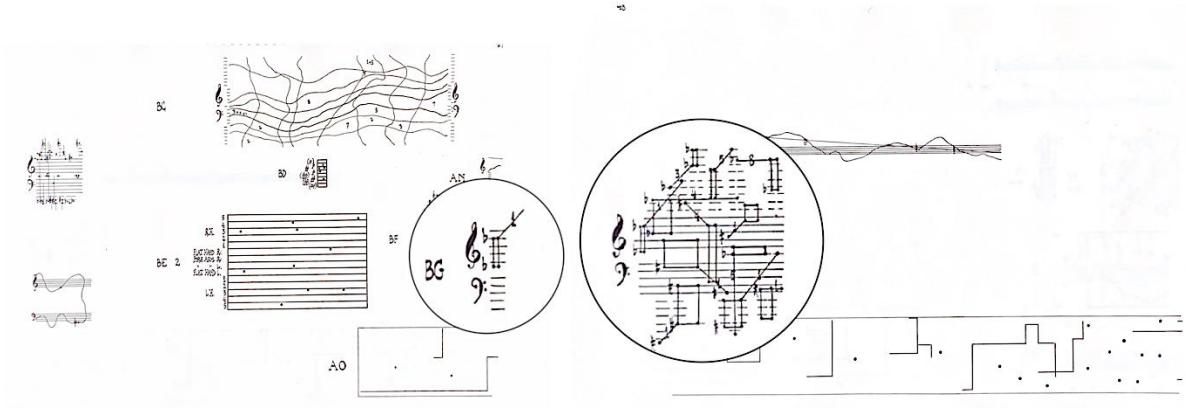


Figure 151 Notation BG on pages 47–48.

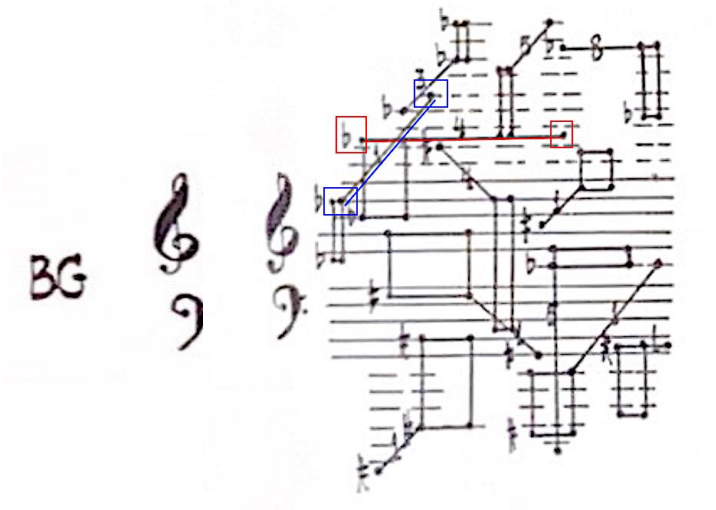


Figure 152 Realisation of Notation BG on pages 47–48.

Notation BH shows a potential relationship with Notation A, as Cage stated, 'Like A, but with ambiguous clef'.[329] In the performing instruction, and with the irregular shape in Notation BH (Figure 153). Like A, means performers are free to decide, (i) which note to start, (ii) the notes for each proportion, (iii) the performing direction of hands, and (iv) the quality of sounds, while the performers need to be aware of ambiguous clef signs. The performing approach noticeably relates to Notation A; therefore, the performers should follow the perimeter and play in the opposite direction (Appendix A Table 107).

[329] Ibid.

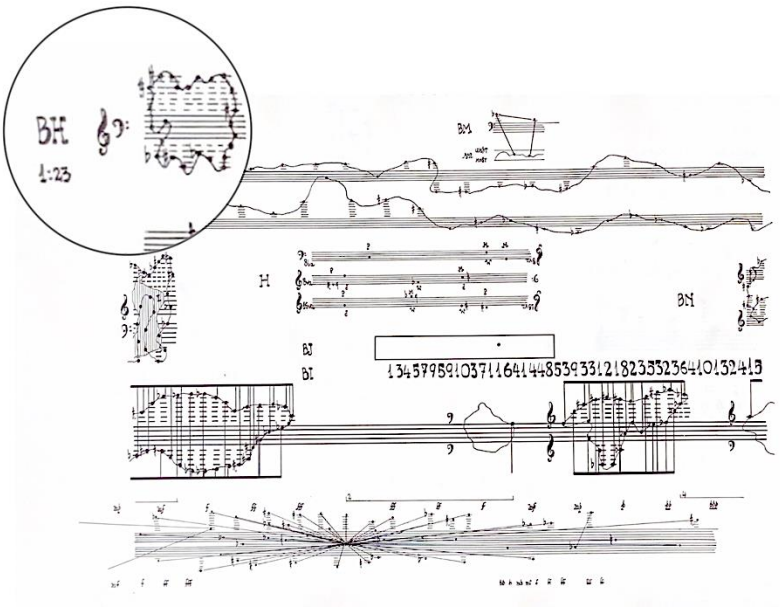


Figure 153 Notation BH on page 50.

In Notation BI, the performing instruction states, 'Use 1 or 2 numbers followed by 1 or 2 numbers, the first = frequencies, the second, time units, continue or not'.[330] Cage composed this notation based on a series of numbers. Here, he allowed performers to choose two sets of numbers; the first set for frequency of notes, the second for time, thus performers are free to decide time units, and whether, or not, to continue the circle (Appendix A Table 108). These choices include four possibilities: (i) one number for both sets, (ii) one number for the first set, and two for the second set, (iii) two numbers for the first set, and one for the second set, or (iv) two numbers for both sets.

Taking the first four numbers, 1, 3, 4, and 5 from Notation BI on pages 50–51, for example (Figure 154; Figure 155) the first possibility is to realise the first set and the second set, which include numbers 1 and 3, respectively. This means playing a combination of one note for three seconds (performers may choose another time unit for this, and for all the following possibilities). The second possibility is to play the first set, including number 1, and the second set, including numbers 3 and 4; here, one note must be played within a timeframe of thirty-four seconds. The third possibility may include numbers 1 and 3 in the first set, and number 4 in the second set. Alternatively, performers may choose thirteen notes and perform them within four second. The final possibility is for performers to choose thirteen notes, finishing them within forty-five seconds.

[330] Ibid.

The above possibilities show that the series of numbers does not mean performers are absolutely free to decide how to use them; rather, Cage's design needs to be followed for both sets. However, the performers can divide the numbers into sets and decide how many sets to play. Nevertheless, this type of freedom is undertaken within Cage's framework, which shows his constructed indeterminacy of both control and freedom.

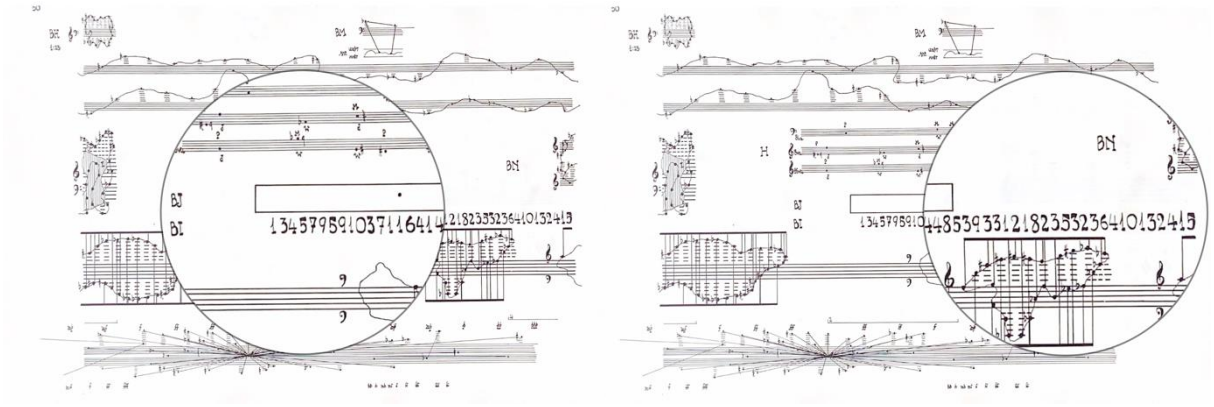


Figure 154 Notation BI on page 50.

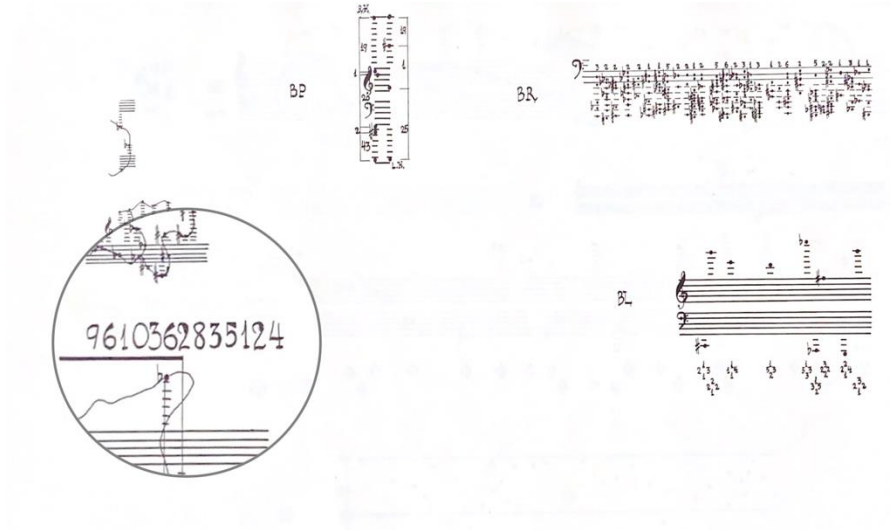


Figure 155 Notation BI on page 51.

The performing instruction for Notation BJ is, 'A single sound. Boundaries are frequency, duration, amplitude, and overtone structure. Proximity as in BB'.[331] The performing instruction shows compositional materials are a single sound, boundaries, frequency, duration, amplitude, overtone structure, and Notation BB; thus, similar with Notation BB, the performers need to measure the distances from the note to the boundaries, and the performer is free to decide which

[331] The performing instruction of Notation BB is, 'Notes are single sounds. Lines are duration (D), frequency (F), overtone structure (S), amplitude (A), and occurrence (succession) (O). Proximity to these measured by dropping perpendiculars from notes to lines gives respectively, longest, lowest, simplest, loudest, and earliest'.

side is frequency, duration, amplitude, and overtone structure (Appendix A Table 109). For instance, Notation BJ (Figure 156), performers must firstly decide the meaning of four sides, and then measure the distance between the notes and the sides.

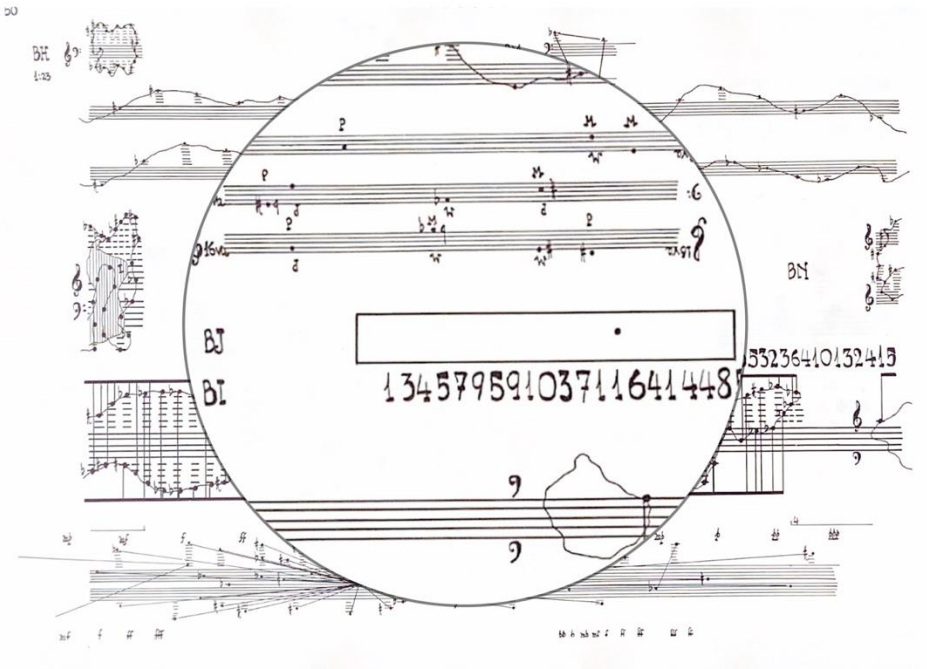


Figure 156 Notation BJ on page 50.

The performing instruction for Notation BK is, 'Like A, but with noises. A, I and O as in AC (amplitude free)'.[332] This shows potential relationships between Notations A and BK; also, between Notations AC and BK; as Cage pointed out, BK is like A, while its abbreviations are like AC. This means that performers need to follow the perimeter, choose a first note, play in the opposite direction, and play the given proportion of notes with either the right or the left hand. As with Notation AC, BK includes noises, while these positions are based on the performing instruction of the abbreviation Notation AC. The abbreviation I, stands for interior piano construction, the abbreviation, A means auxiliary noises, and the abbreviation, O, means outer piano construction (Appendix A Table 110).[333] Taking Notation BK on pages 49–50 as the example (Figure 157), it shows coloured areas with abbreviations I, A, and O, a perimeter with notes, and the given proportion. As a first step, performers may choose a note E on the G-

[332] Ibid.

[333] The performing instruction for Notation A is, 'Following the perimeter, from any note on it, play in opposite directions in the proportion given. Here and elsewhere, the absence of indications of any kind means freedom for the performer in that regard'. The performing instruction of Notation AC is, 'Noises. Of those notated play only that number given. I = interior piano construction. A = auxiliary noises. O = outer piano construction. The position of the notes vertically gives its loudness (High: *fff*) (low is *ppp*)'.

clef as the first note. As a second step, they may use their right hand to performer the following eleven notes, C, G, E♭, C, E♭, A♭, E, B, G, F, and E, whilst the remainder of the thirty-four notes are performed with the left hand in the opposite direction. As the performing instruction states, they need to make noises when they encounter coloured areas, therefore the final step is to delimit the timing of noises, based on the coloured areas. For instance, for the coloured area on the top, the delimitation is from notes C to G; between these, noises may be produced with any auxiliary tools they choose.

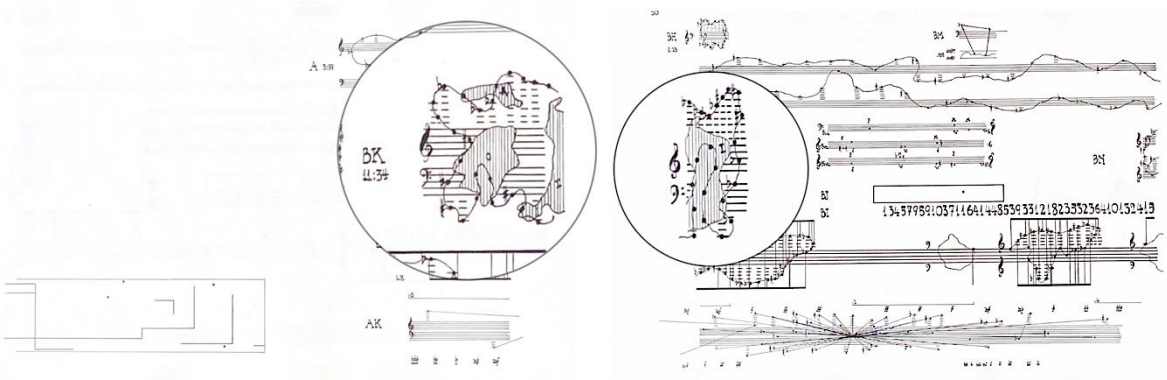


Figure 157 Notation BK on pages 49–50.

The performing instruction for Notation BL is, 'Single notes accompanied by numbers giving number of tones to appear above, below, before and after the one notated'.[334] In this performing instruction, Cage applied the compositional material as the same as in BK. However, the numbers in Notation BL refer to notes above, below, before, and after central notes; therefore, the performers are both free to choose notes based on the central note and to decide the quality of sounds, while they realise the given number of notes based on the central note (Appendix A Table 111). Of interest here is the notes both above and below the central notes refer to the same group of notes; similarly, the notes below and after the central notes offer the same range of notes. From the performing instruction, the variety of compositional materials is worth noting, in that he applied different dimension of the notes, even though the result may refer to the same grouping. One exemplar from pages 51–52 (Figure 158; Figure 159), is that the fourth set of numbers is 5, 1, 3, and 2, and the central note is D, therefore, performers may choose five notes and two notes before and below note D, and one note and three notes above and after note D.

[334] Ibid.

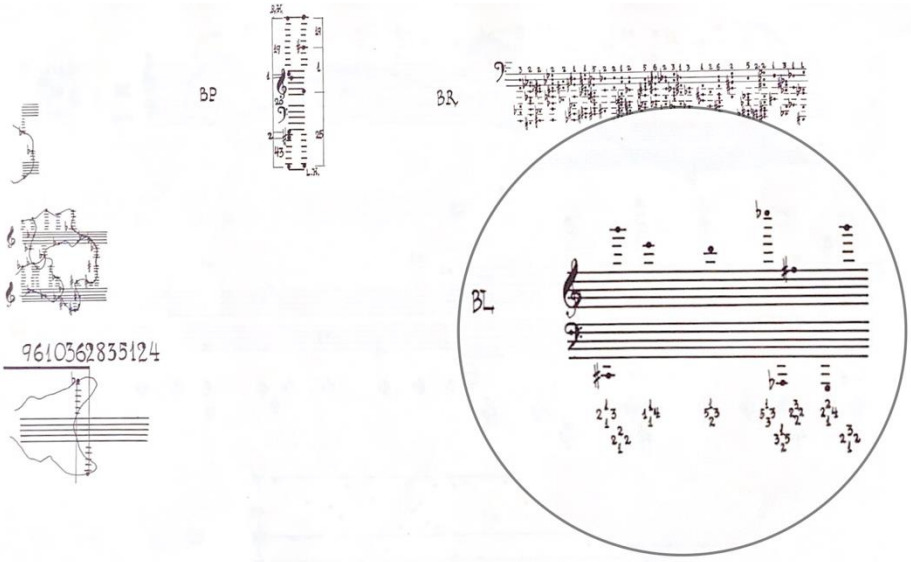


Figure 158 Notation BL on page 51.

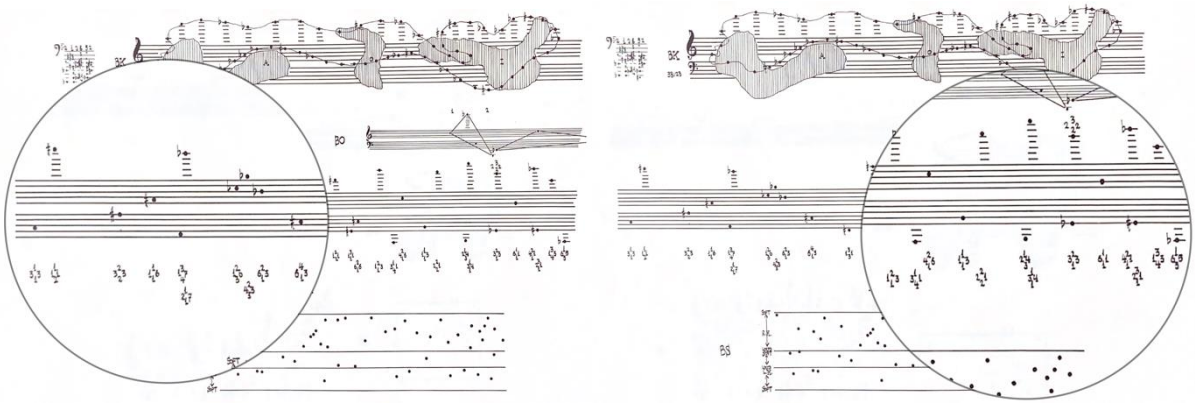


Figure 159 Notation BL on page 52.

The performing instruction for Notation BM (Figure 160) is, 'Pitches with amplitude graphically given. The horizontal difference between a pitch and its amplitude gives time available for tone'.[335] In this notation, Cage used lines to indicate amplitude and time; the curved line in the area annotating 'AMP', 'least', and 'most' represent the given amplitude, while the straight line between two notes is used for showing the performing time. Thus, the performers ought to decide the loudness range for B4♭ and D4 and following the instruction that B4♭ should be softer than D4. Looking into the curved line representing amplitude of the given notes, I notice that it is difficult to perform the given amplitude by two attacks; thus, I deduce that the performers need to perform the given notes more than two times in order to follow the given amplitude. The known compositional materials of this notation are: pitch, amplitude, time, lines, areas, clef sign, staff, accidental, and paper imperfections; the type of

[335] Ibid.

freedom is free to decide the frequency of performing two given notes; and performing approach is following dynamic indications, performing based on time arrangement, and performing the given notes (Appendix A Table 112).

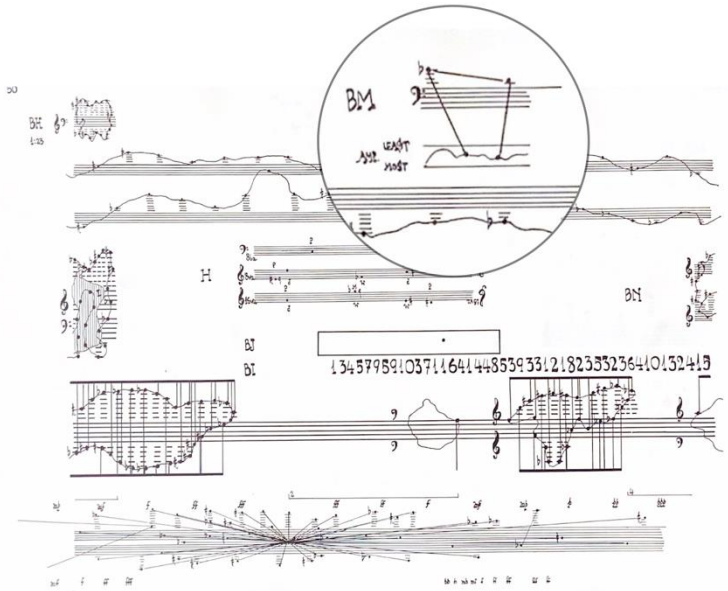


Figure 160 Notation BM on page 50.

The performing instruction for Notation BN (Figure 161) is, ‘2 hands starting at two different points on perimeter arrive eventually at centre together by any paths’. Thus, the known compositional materials are hands, points, the perimeter, the centre, and paths. Performers should choose two notes as two points to start the realisation, and then follow the perimeter to perform the given notes, and finally two hands will end at the same point at the centre (Appendix A Table 113).

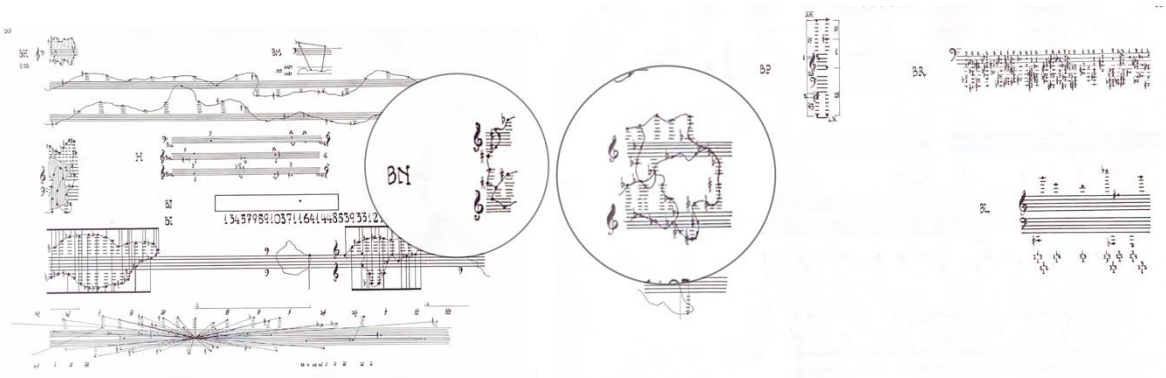


Figure 161 Notation BN on pages 50–51.

The performing instruction for Notation BO shows its relationship with Notation W, as it states, 'Like W with time units given'; meaning that the difference between the two notations is the application of time units. Because of this difference, numbers 1, 2, 3, 4, 5, 6, 7, and 8.09 in Notation BO on pages 52–53 indicate the 1st second, the 2nd second, the 3rd second, the fourth second, and so on (Figure 162; Figure 163). One notable limit of the numbers is the distances between each number; although the numbers are from 1 to 8.09, but the distances between them are unequal, meaning that Cage used numbers and their different distances to govern the duration and speed of the performers' realisations. Hence, performers need to follow the given time units in terms of the distances between them, while they are free to decide the dynamic of the notes, as in Notation W (Appendix A Table 114). Regarding the performing instruction of Notation W, it guides performers to realise triangles as legato and isolated notes as staccato,[336] as such, Notation BO follows this performing approach.

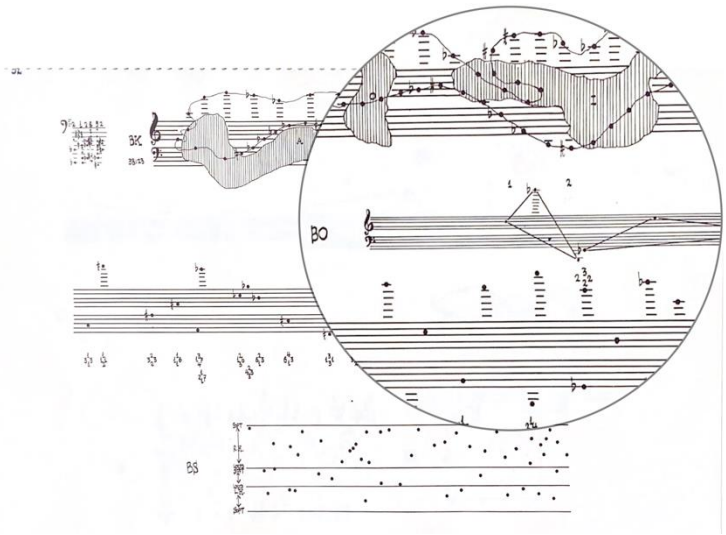


Figure 162 Notation BO on page 52.

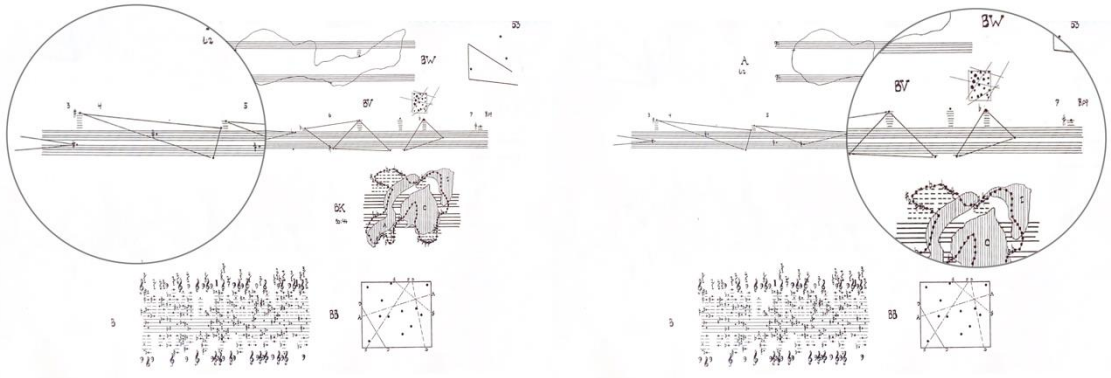


Figure 163 Notation BO on page 53.

[336] The performing instruction for Notation W states, 'Legato (triangles) and staccato (isolated notes)' .

For Notation BP (Figure 164), Cage stated that, 'Numbers of tones within ranges given for each hand'.[337] Therefore, the known compositional materials are numbers, given pitch ranges, and hands. Performers need to decide the performing notes based on the range and proportion that Cage indicated, meaning that the freedoms here are choosing the performing notes and deciding quality of sound, while the performing approaches are performing the given numbers of notes and choosing notes from the pitch ranges (Appendix A Table 115). For instance, Notation BP on page 51, the performers may choose one note from the given pitch range (D5 to E5), meaning that the choices of notes include D5, D5♯, and E5 (Figure 165).

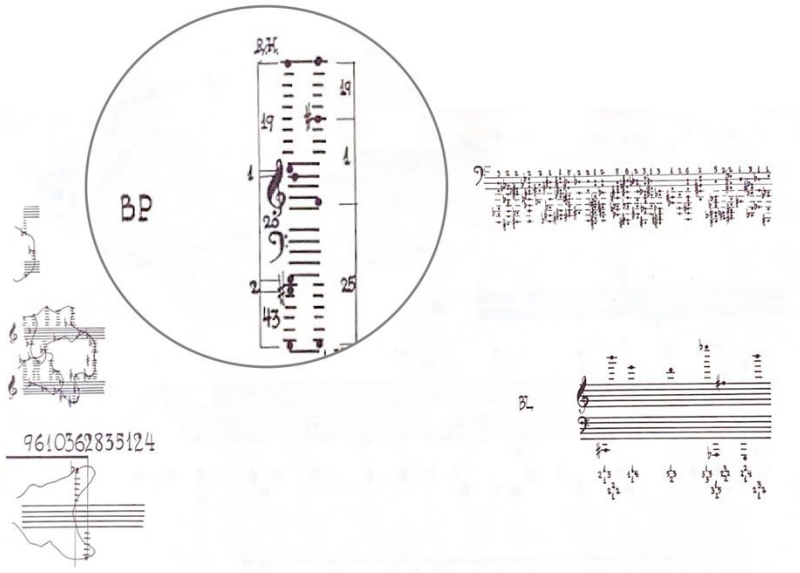


Figure 164 Notation BP on page 51.

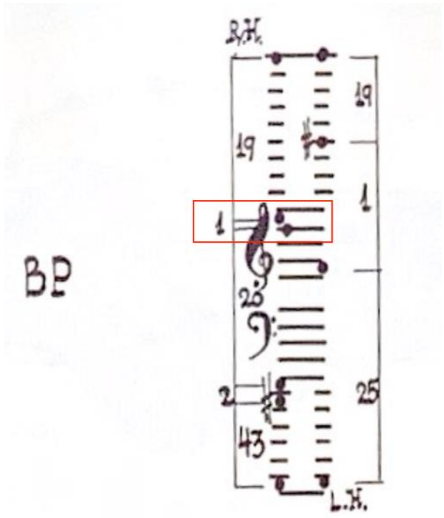


Figure 165 Notation BP on page 51.

[337] Ibid.

The Notation BQ (Figure 166) states 'Single tones at any point (i.e. pitch, duration) within triangles. Hypotenuse gives dynamics available'.[338] From the performing instruction, Cage used triangles as pitch areas to demonstrate pitch, duration, and dynamics. The performers decide one note for each triangle; the pitch, duration, and dynamics follow the clef signs and staff on both sides, numbers below the triangles, and the dynamic symbols on hypotenuses (Appendix A Table 116). Figure 167 marks three notes in three triangles; the blue arrows present the pitch of the note, the purple arrows indicate the duration of the note, and the green arrows show the dynamics of the note. Based on Cage's instruction, the performer may perform three notes, a note A for five seconds with *p*, a note D for ten seconds with *mp*, and a note E for seven seconds with *mp*.

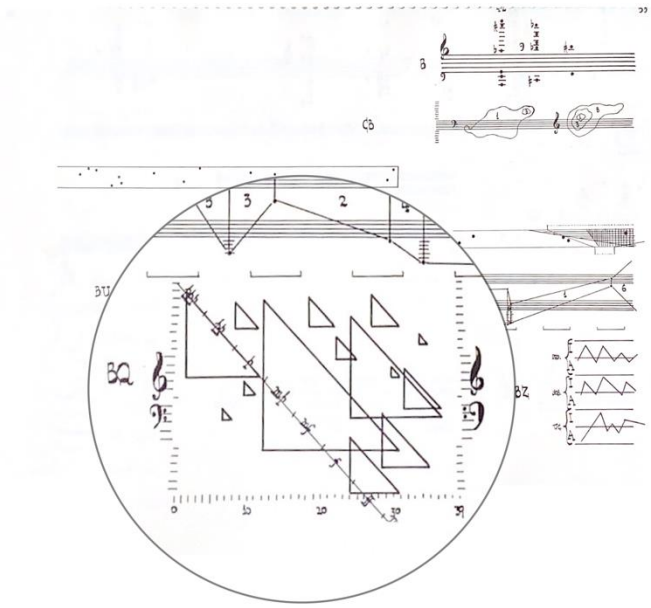


Figure 166 Notation BQ on page 55.

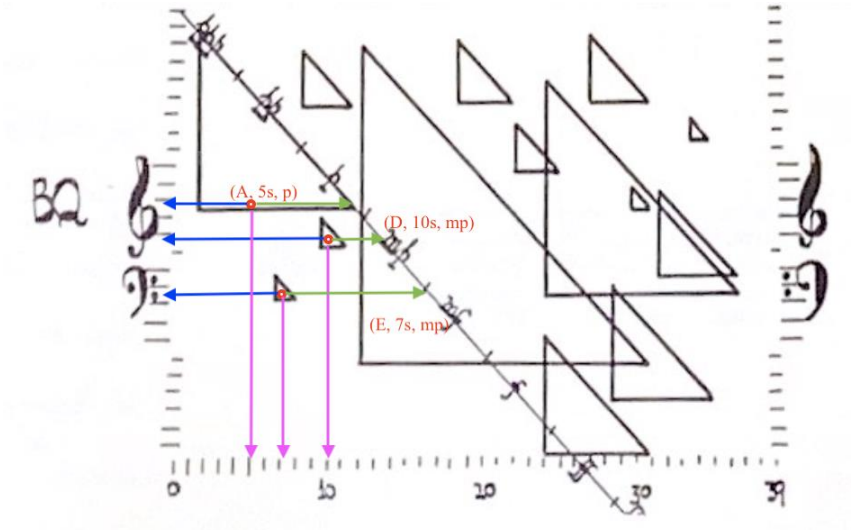


Figure 167 Notation BQ on page 55.

[338] Ibid.

The performing instruction for Notation BR (Figure 168) is: 'Number of tones that may be taken in advance for production of harmonics given above each aggregate. Play as in B'.[339] From this instruction, we know that this notation is a variation based on Notation B. The compositional materials in performing instruction of Notation BR includes tones, harmonics, and aggregate; combining this instruction with the first performing instruction of Notation B (an aggregate must be played as a single ictus, where this is impossible, the unplayable notes shall be taken as harmonics prepared in advance), it shows that the performers need to play the given amount of harmonics with notes, while these notes are the notes that cannot be played as single ictus.

Along with single ictus, Notation B also includes overlappings and interpenetrations as compositional materials, which are the part of freedom from Cage. Putting this in Notation BR, it shows that beat of each aggregate can be closed, separated, or overlapped. From this performing approach and freedom, the compositional materials in Notation BR are more than it shows in the performing instruction and the notation; it also includes single ictus, overlappings, and interpenetrations from Notation B. This means that when Cage annotated 'play as in B' for Notation BR, a certain level of closeness between two notations are formed, whether at aspects of compositional materials, performing approach, or freedom (Appendix A Table 117).

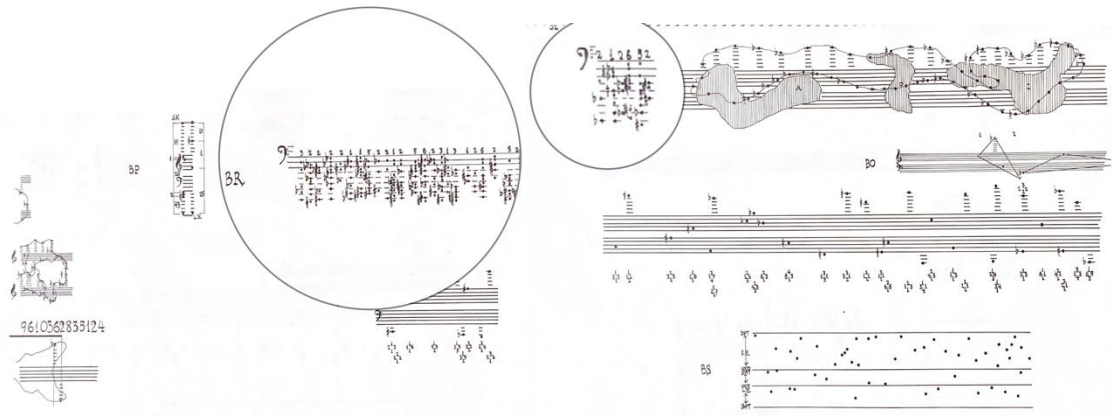


Figure 168 Notation BR on pages 51–52.

[339] Ibid.

The performing instruction of Notation BS (Figure 169) is 'Dynamics as notated for both hands'.[340] This notation requires the performer to perform notes with the given hands and dynamics. Cage did not state the instruction of time for this notation, so the duration of notes is free to decide, with the relative distance as the condition (Appendix A Table 118).

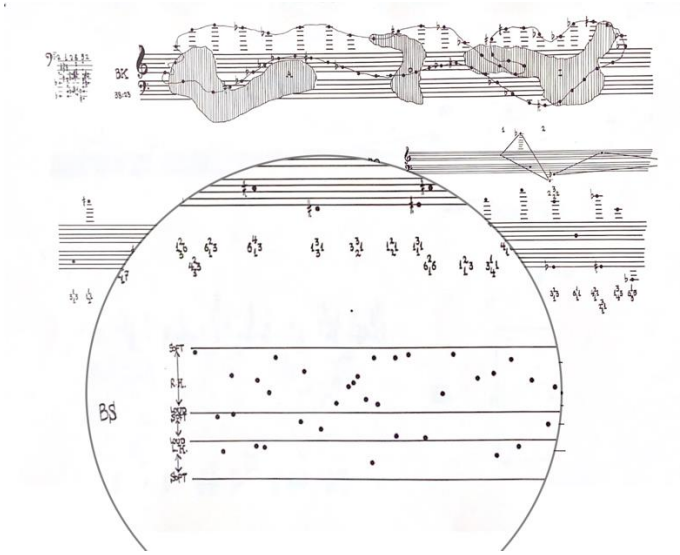


Figure 169 Notation BS on page 52.

For Notation BT (Figure 170), Cage stated 'Notes given place of performance with respect to piano'. [341] This notation requires the performers to perform the notes on the assigned constructions of two pianos. For this notation, Cage drew the shape of pianos by dotted lines, and the part of piano was drawn by solid lines. The performers follow the position of dots to make sounds. Based on this performing instruction, the types of freedom are free to decide the approach, free to decide the duration of this notation, and free to decide the quality of sounds. The performing approach is following the given position (Appendix A Table 119).

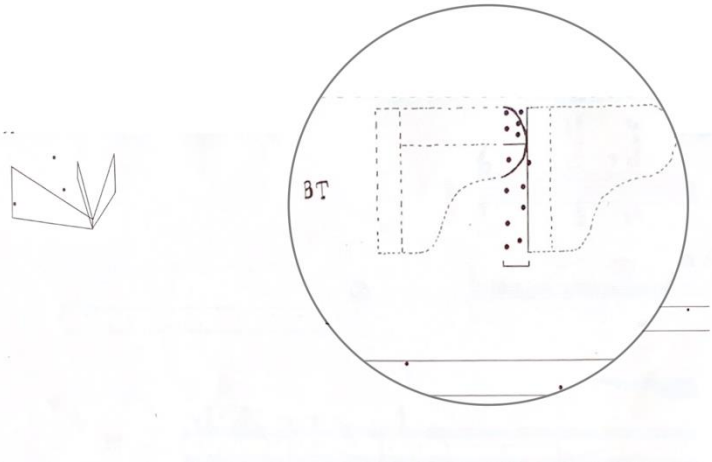


Figure 170 Notation BT on page 54.

[340] Ibid.
[341] Ibid.

The performing instruction of Notation BU shows 'Play sounds given plus number of sounds within areas. ⌜ between staves = noise area. ⌞ below = time units'.[342] In Notation BU, Cage applied areas of tones, areas of noises, numbers, and time units to guide the performers how to realise this notation (Figure 171). For instance, during the zeroth and the first second, the performers may choose six sounds from a pitch area constructing by E♭, F, C, and A, and from the noise area; during the 1st and the 2nd second, the performers may choose two sounds from both a pitch area and the noise area, and so on (Figure 172). From this realisation, the performers are free to choose notes from both pitch areas and noise areas by following given amounts of sounds and the allocated time, and so the types of freedom are free to decide performing notes and noises, free to decide duration of notes and noises, and free to decide the quality of sounds (Appendix A Table 120). Meanwhile, the performing approach of this notation are performing the given proportion of sounds, performing sounds in the allocated time, and choosing notes or noises from the given areas.

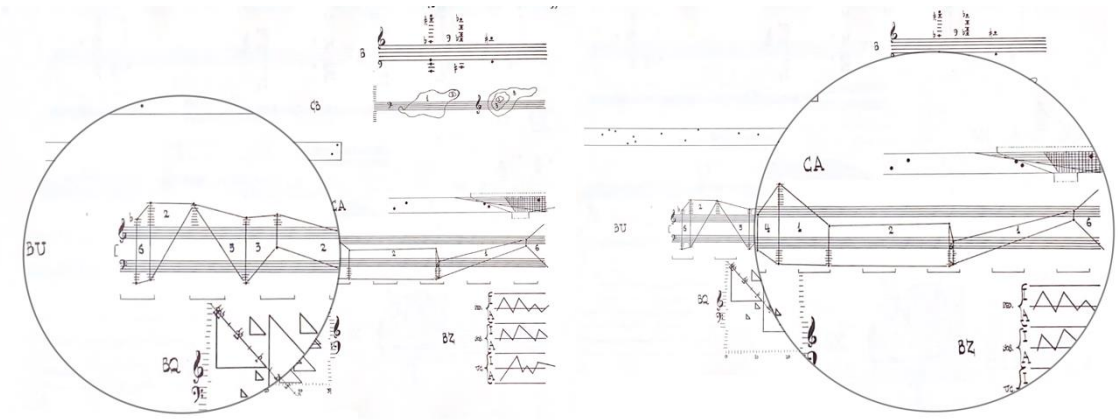


Figure 171 Notation BU on page 55.

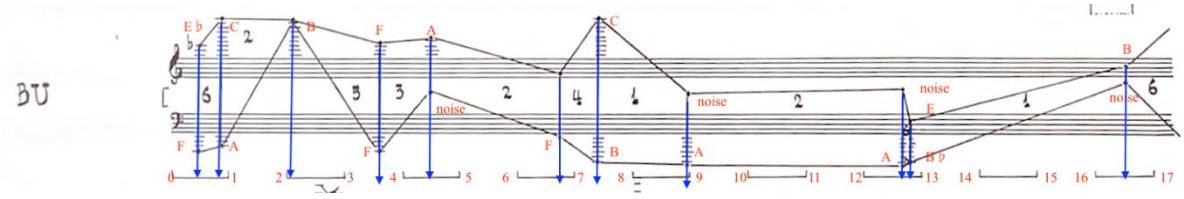


Figure 172 Realisation of Notation BU on page 55.

[342] Ibid.

In Notation BV (Figure 173), Cage offered the performing instruction describing 'Three large (4 or more sounds), six less large (3 sounds), 10 small (two sounds), 4 very small points (single sounds). The 5 lines and the 4 boundaries to be used as in BB and BJ. When obtaining measurements for 3 frequencies use 3 different lines and likewise for other measurements'.[343] Cage built this notation upon Notation BB and Notation BJ, as the functions of lines and boundaries are identical to these two notations.[344] Although the function of lines and boundaries are as the same as Notation BB and Notation BR, the assignment of lines to parameter is not fixed.[345] The four size points represent corresponding sound events, with this arrangement, the performer may produce much indeterminate result. The performers are free to choose to apply either lines or boundaries for the sound events, so the type of freedom is free to choose parameter for sound events; meanwhile, the performing approaches are performing different amounts of sound events by corresponding point size, measuring the distances between the note and the boundaries, and following the measurement results to determine the quality of sound events (Appendix A Table 121).

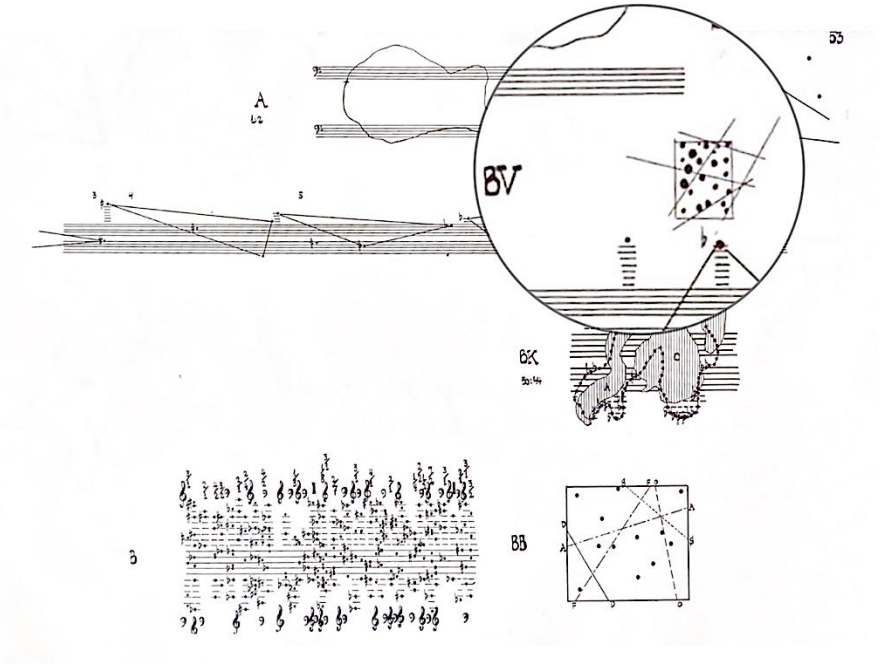


Figure 173 Notation BV on page 53.

[343] Ibid.
[344] The performing instructions for Notation BB is, 'Notes are single sounds. Lines are duration (D), frequency (F), overtone structure (S), amplitude (A), and occurrence (succession) (O). Proximity to these measured by dropping perpendiculars from notes to lines gives respectively, longest, lowest, simplest, loudest, and earliest'; the performing instruction for Notation BJ is, 'A single sound. Boundaries are frequency, duration, amplitude, and overtone structure. Proximity as in BB'.
[345] Pritchett, *The Music of John Cage*, 135.

The Notation BW's performing instruction is, '4 sided figures give frequency, amplitude, duration and overtone structure. The illusion of perspective gives occurrence, closest to the observer = earliest in time' (Figure 174).[346] The compositional materials include shapes, frequency, amplitude, duration, and overtone structure, time, dot, and paper imperfections. In this notation, Cage required the performers to observe these shapes as three-dimensional. By following the performing instruction, the performing approaches are observing the order of three shapes, and following the observing result to decide the performing order, and the types of freedom are free to decide the representatives of the sides (i.e., frequency, amplitude, duration, and overtone structure), and free to decide which dot corresponds with which shape (Appendix A Table 122).

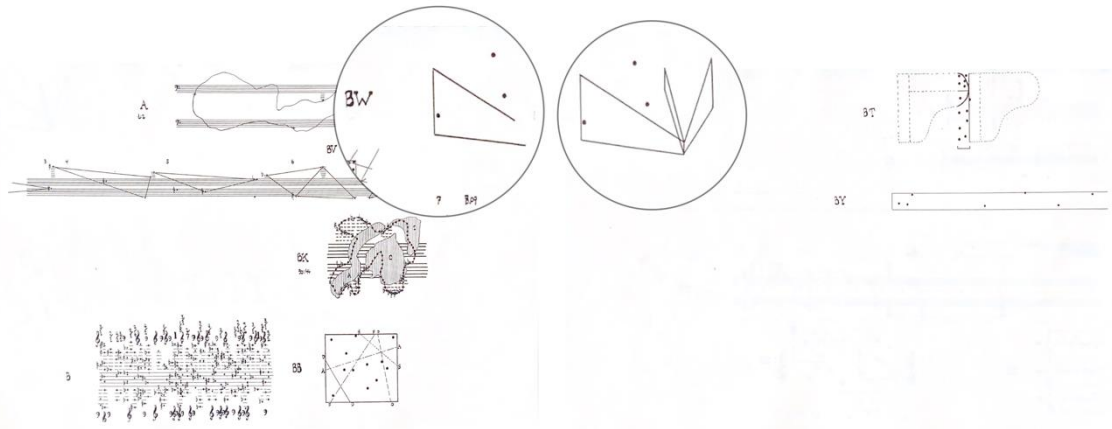


Figure 174 Notation BW on pages 53–54.

[346] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

The Notation BX's performing instruction is, 'All at once like a moment of a plant'.[347] This notation shows an image of a plant (Figure 175). For this, performers need to perform the image of a plant when playing the given notes. As Cage did not offer instruction about dynamics, performers are free to decide the quality of sound. Regarding the performing instruction, the performing approach follows the shape of the graph when performing the given note (Appendix A Table 123).

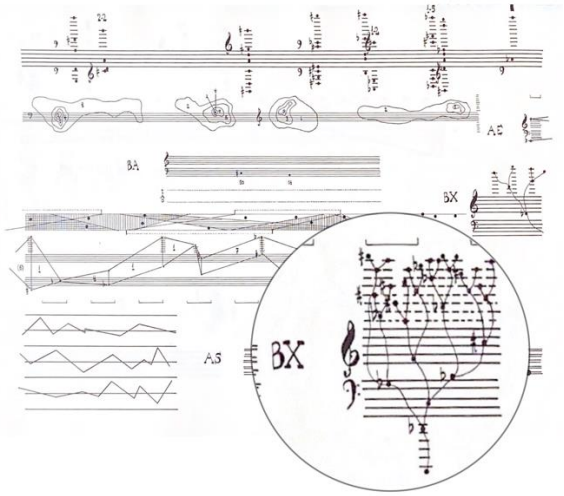


Figure 175 Notation BX on page 56.

For Notation BY (Figure 176), Cage stated, 'Any noises. Their relative pitch given graphically (Up = high, down = low)'.[348] Therefore, performers need to determine which parts of the piano produce different noise pitches, then match them with the position of points. Thus, Cage's instruction means that higher pitch and lower pitch refers to different parts of the piano. He did not offer instructions about noise dynamics, leaving performers free to decide which part of the piano will produce the noises and sound quality they want, while their performing approach is to follow the relative position of points (Appendix A Table 124).

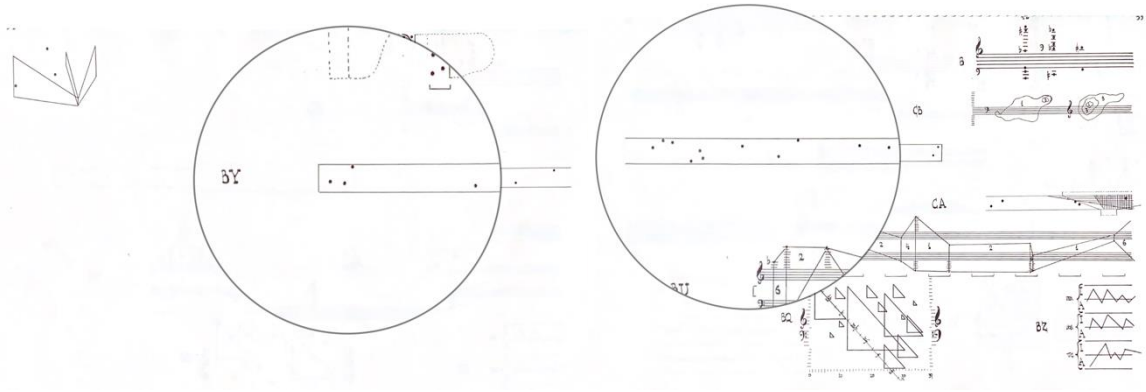


Figure 176 Notation BY on pages 54–55.

[347] Ibid.
[348] Ibid.

The performing instruction for Notation BZ (Figure 177) shows 'The 3 pedals with I = inactivity, and A = activity. Any or no keyboard, harp or noise sounds'.[349] The three pedals refer to the piano's sustaining pedal, the sostenuto pedal, and the una corda pedal (soft pedal). Performers apply pedals in their realisations by following Cage's indicator showing inactivity and activity. This may be realised with any combination of keyboard, harp, or noises. Hence, the several freedoms in this notation are to perform tones using both keyboard and harp, or one of them, noises, and the quality of sounds. The performing approach is to follow the movements of lines and to apply three pedals (Appendix A Table 125).

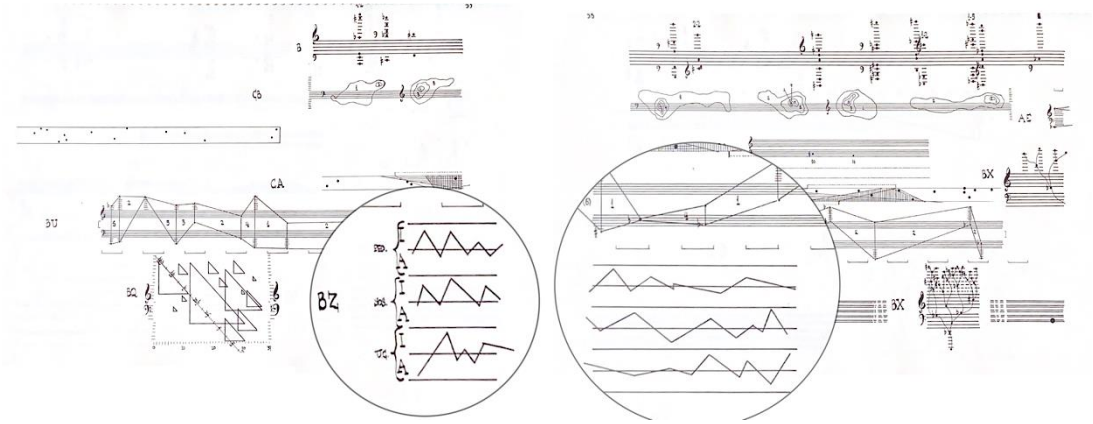


Figure 177 Notation BZ on pages 55–56.

2.4 Compositional Materials in Notation CA to Notation CF

For Notation CA (Figure 178; Figure 179), Cage offered this performing instruction, 'Keyboard (white), mute (vertical lines), pizz. (bracketed by dotted lines), and friction (horizontal lines) areas given. Notes of any pitch. When areas overlap, either both or more timbres may be produced'.[350] He applied keyboard, mute, pizzicato, and friction areas to produce timbre diversity. He did not state the range of tones, therefore performers are free to decide the range; then they can follow the position of points for their pitch. Thus, four areas may overlap. Figure 182 shows a green arrow pointing to the overlaps of friction, mute, and pizzicato, a blue arrow pointing out the overlaps of pizzicato and mute, a purple arrow pointing out the overlaps of friction and mute, and a dark yellow arrow pointing out the overlaps of pizzicato and keyboard. Therefore, performers are free to decide which areas to present. The performing approach are, follow both the drawing areas and the position of points (Appendix A Table 126).

[349] Ibid.
[350] Ibid.

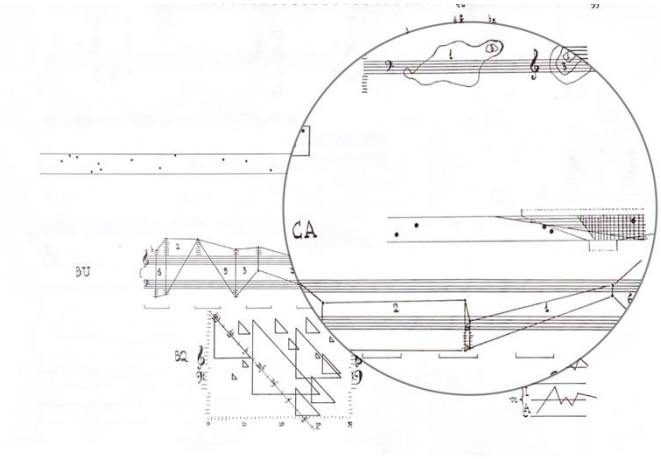


Figure 178 Notation CA on page 55.

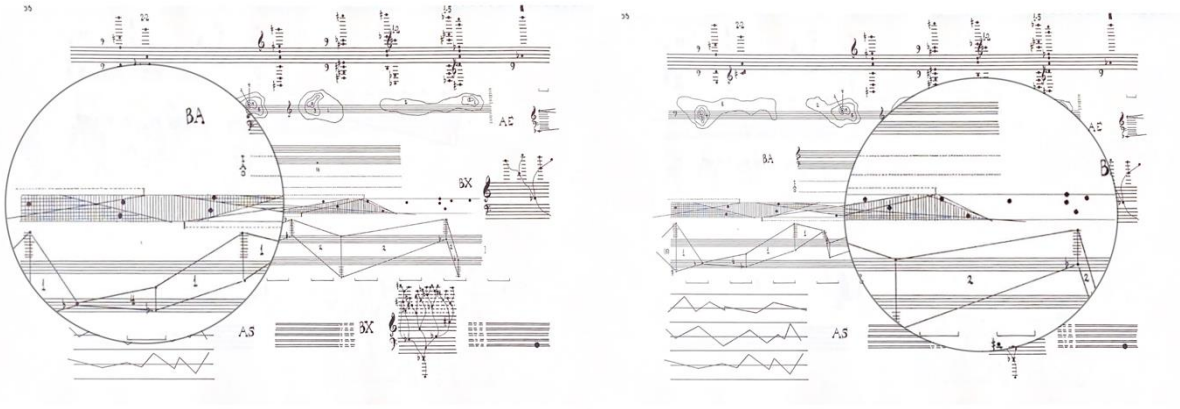


Figure 179 Notation CA on page 56.

For Notation CB (Figure 180; Figure 181), the performing instruction states, 'Numbers of tones in pitch areas given'.[351] Cage circled pitch areas and offered numbers stating the amounts of performing notes. For instance, as the Figure 183 shows, the red dotted lines act as parameters to clarify the pitch areas of each irregular shape, and the blue dots show the corresponding notes of their edges. The first irregular shape (the green number 1) shows the pitch area from E1 to A5, the second is from G4 to F5 (the green number 2), the third is from E3 to G7 (the green number 3), the fourth consists of the range from C4 to C7 (the green number 4), and the final irregular shape include the notes from E5 to E6 (the green number 5). Reading this notation with the numbers showing the amounts of playing notes, the performers may choose one note from E1 to A5 for the first irregular shape, choose three notes from G4 to F5 for the second irregular shape, select eight notes from E3 to G7 for the third pitch area, select three notes from C4 to C7 for the fourth pitch area, and choose one note from E5 to E6 for the fifth pitch area. Thus, the types of freedom are: (i) to choose performing notes from the given pitch areas, and (ii) to decide the duration and dynamics of notes, while the performing approaches are: (i) to

[351] Ibid.

follow the given pitch areas to determine the performing notes, (ii) follow the numbers to determine the amount of performing notes, and (iii) to follow the clef signs to read the pitch areas (Appendix A Table 127).

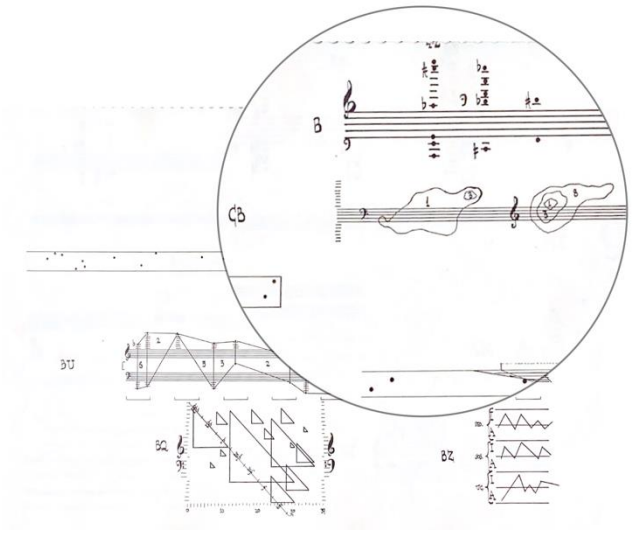


Figure 180 Notation CB on page 55.

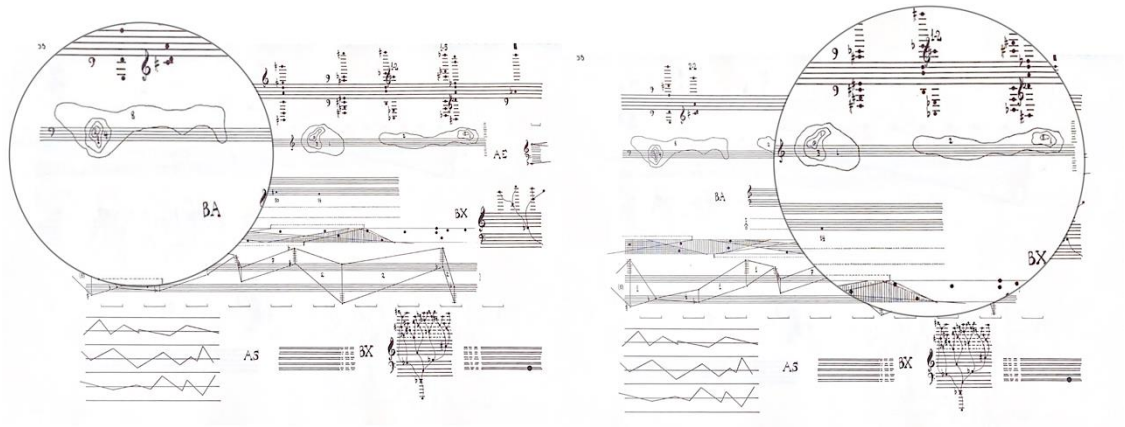


Figure 181 Notation CB on page 56.

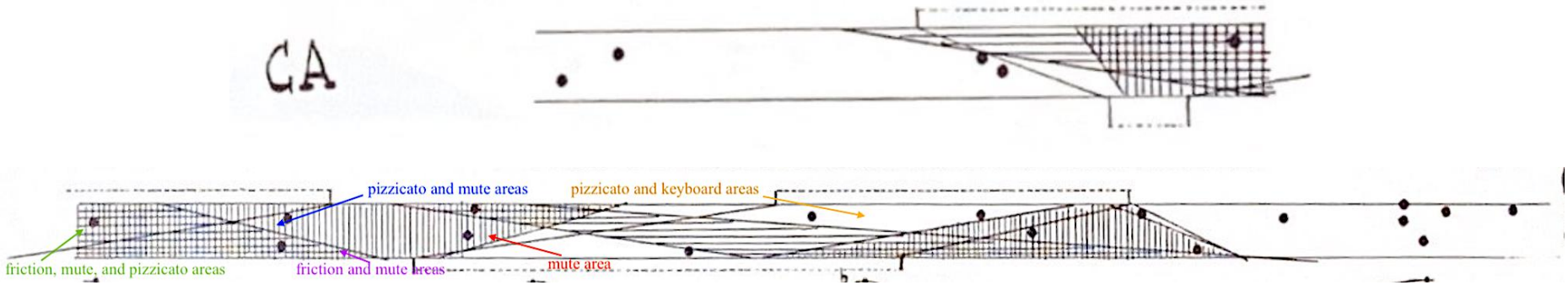


Figure 182 Realisation of Notation CA on pages 55–56.

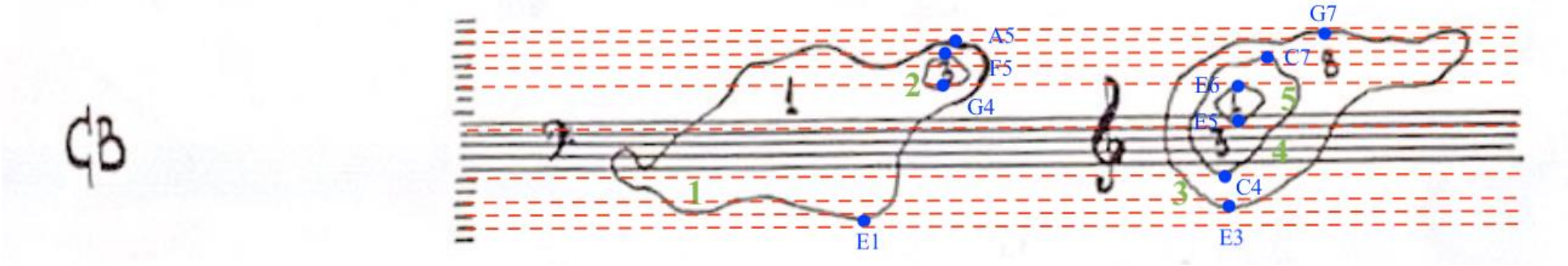


Figure 183 Realisation of Notation CB on page 55.

The performing instruction for Notation CC (Figure 184) sates, 'The four differently drawn lines = frequency, duration, amplitude, overtone structure, in any correspondence measurements defining these are to be perpendicularly from straight lines above or below to their points of intersection with slanting lines. Numbers at ends of these given by their difference time available for sounds'.[352] For example, Notation CC on page 57; I set the solid line as frequency in order to proceed my realisation of this line. Firstly, start with the two straight lines, one horizontal, and the other vertical, to obtain intersections of slanting and solid lines; these will represent the frequency of notes. Secondly, read these by the numbers above the graph; the result suggests five sounds between the 19th second and the 30.5th second, six sounds between the 37.5th second and the 38th second, six sounds between the 38.5th second and 42.5th second, and one sound around the 48.5th second (Figure 185). Thirdly, as the annotation in Figure 185 suggests, the intersections (the red points with red squares) of horizonal lines (green dotted lines) and vertical lines (blue dotted lines) show the frequency of notes; with the red arrows suggesting the corresponding time. Therefore, based on this interpretation of the realisation, the performer is free to decide what the drawn lines represent. The performing approach for this notation is: follow the intersections of the drawn lines and slanting lines, and follow the time allocated, as marked above and below the graph (Appendix A Table 128).

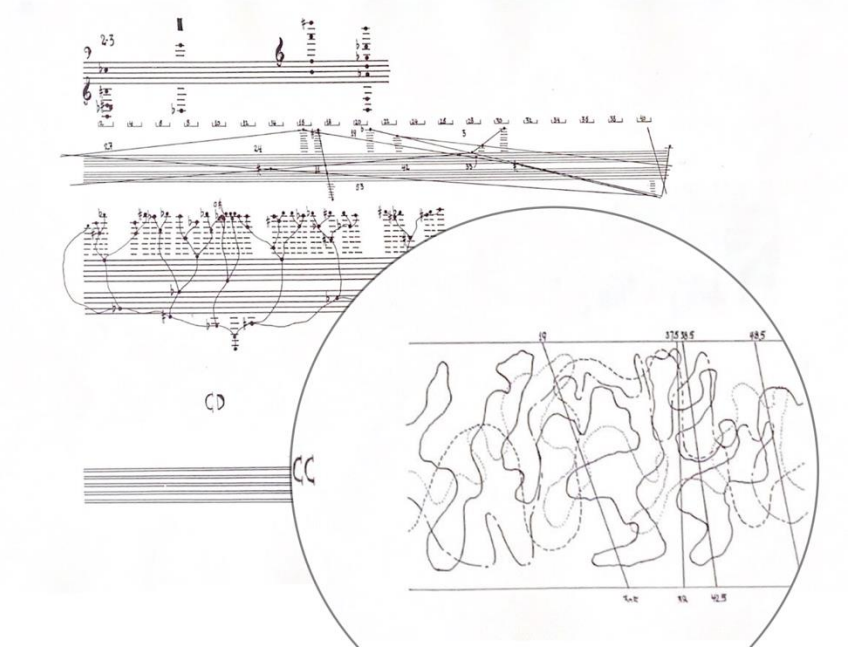


Figure 184 Notation CC on page 57.

[352] Ibid.

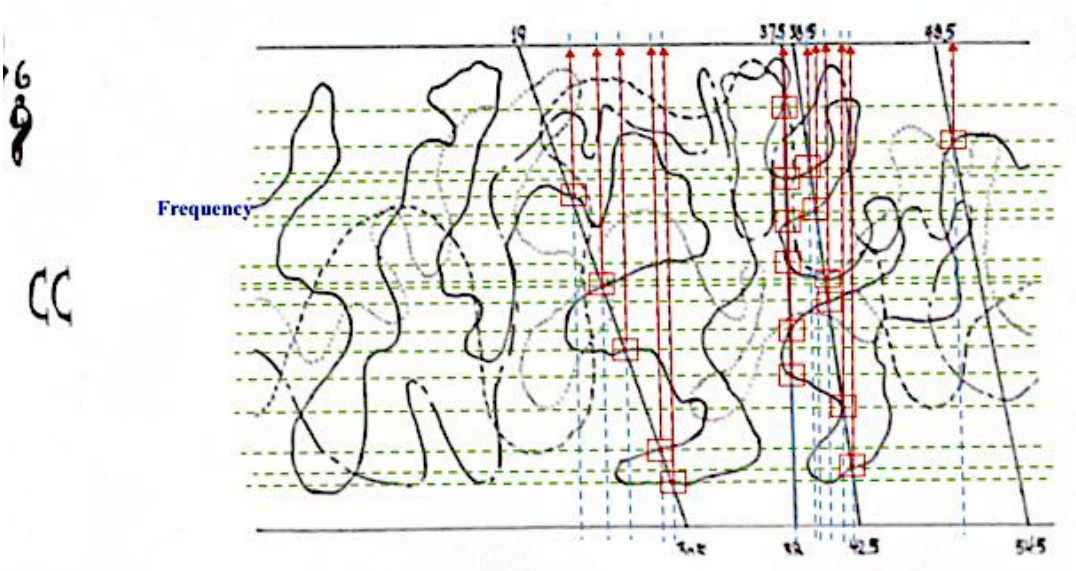


Figure 185 Realisation of Notation CC on page 57.

The performing instruction for Notation CD (Figure 186) sates, 'For · use 1 of 4 readings, for ◦ use 2 of 4 readings, for X use 3 of 4 readings. Horizontal readings = keyboard, vertical reading = harp'.[353] Readings, refer to four directions to read clef signs, from top, left, right, and the bottom of the notation. As the performing instruction describes, three symbols: · and ◦, and X should be realised by one, two, and three readings, respectively. By following this, the performers are free to decide which direction to determine clef signs. Meanwhile, Cage also set the performing approaches based on the direction of readings; horizontal readings are for keyboard, while the vertical readings are for harp.

For instance, Figure 187 circled one of X in Notation CD on page 57. Based on the performing instruction, x needs to be realized by three readings, and therefore, I choose: (i) the G-clef on the left-hand side, (ii) the F-clef on the bottom, and (iii) the G-clef on the top; the realising results are C6 by keyboard (the red arrow in Figure 187), G3 by harp (the orange arrow Figure 187), and F4 by harp (the blue arrow in Figure 187). The performing approaches of this notations are performing by keyboard or harp and choosing given numbers of clef signs; the types of freedom are free to choose clef signs, and free to decide quality of sounds (Appendix A Table 129).

[353] Ibid.

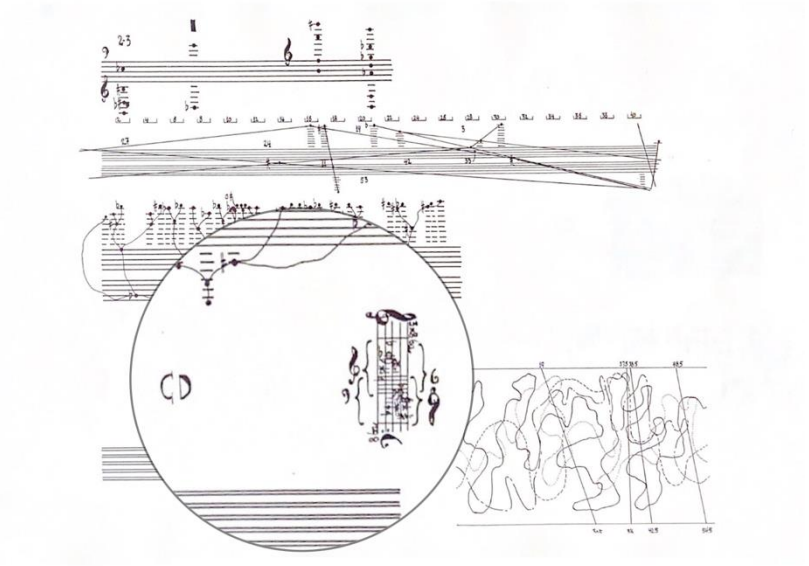


Figure 186 Notation CD on page 57.

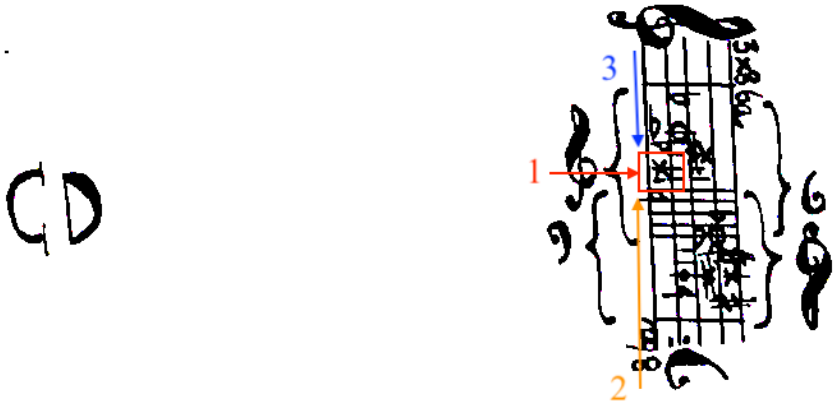


Figure 187 Realisation of Notation CD on page 57.

The performing instruction for Notation CE (Figure 188) sates ‘Clefs ambiguous, ledger lines above 𝄢 = 15, below 𝄞 = 13. Make intervals and aggregates where suggested by notation’.[354] Compositional materials of Notation CE include ambiguous clef signs, ledger lines, numbers (given proportion of notes), intervals, aggregates, notes, accidentals, staff, and paper imperfections. From the performing instruction, it is noticeable that the performers have to make decisions regarding clef signs, the numbers of performing notes, and the quality of sounds.

[354] Ibid.

They also need to follow the performing approach of performing certain format of sounds, e.g. intervals and aggregates (Appendix A Table 130).

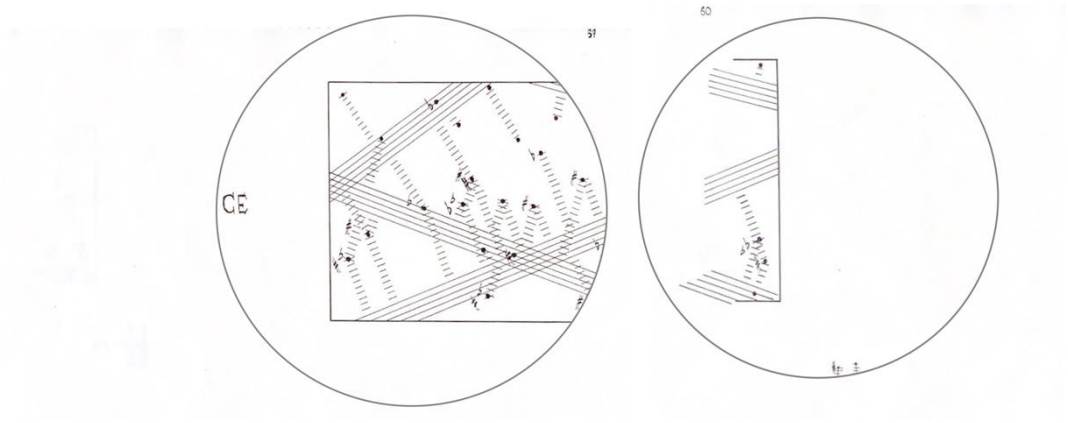


Figure 188 Notation CE on pages 59–60.

The performing approach for Notation CF states it should be realised as Notation BZ, without offering other guidance. The performing instruction for Notation BZ states, 'Three pedals with I = inactivity, and A = activity. Any or no keyboard, harp or noise sounds'.[355] Consequently, the types of freedom of this notation are: (i) free to decide performing manners (by keyboard, harp, or noises), and (ii) free to decide the quality of sounds (notes and noises), while the performing approach is to: (i) perform with pedals, and (ii) follow movements of lines to apply three pedals (Figure 189) (Appendix A Table 131).

Figure 189 Notation CF on page 62.

[355] Ibid.

3 Five Types of Relationships: Compositional Materials, Functions of Compositional Materials, Types of Freedom, Performing Approaches, and Similarity of Appearance

This section demonstrates individual systems and their inner structure by establishing five types of relationships that based on: (i) compositional materials, (ii) functions of compositional materials, (iii) types of freedom, (iv) performing approaches, and (v) appearances. In the previous section, I deconstructed notations into compositional materials, the performing approach, and the types of freedom. Through this deconstructive process, a sense of relevance becomes evident as repetitions of certain compositional materials, performing approaches, and types of freedom regularly appear. One example is Cage's annotation stating, 'as in Notation XXX' or 'like Notation XXX'. For instance, Notation H states that the staff should be understood as in Notation C; Notation L points out that the function of ambiguous clefs is similar to Notation B, and the function of the perimeters is like that in Notation A. These simple performance instructions reveal potential relationships between two or more notations, similar to the relationships between Notations C and H and between Notations A, B, and L, which suggest that Cage composed them in reference to each other. Another example is Cage's repetitive uses of numbers, lines, shapes, arrows, area(s), and dots. Based on these compositions, I discovered that each notation is interconnected with others through compositional materials, types of freedom, and performing approaches. Based on this finding, this section demonstrates the five types of relationships in the *Solo for Piano*.

3.1 The First and Second Types of Relationships: Compositional Materials and their Functions

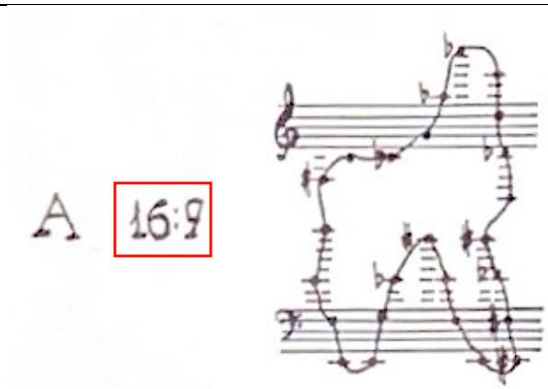
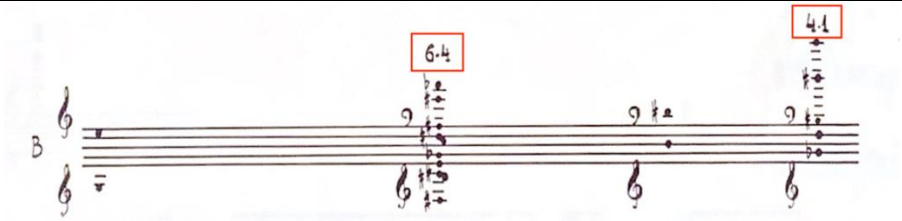
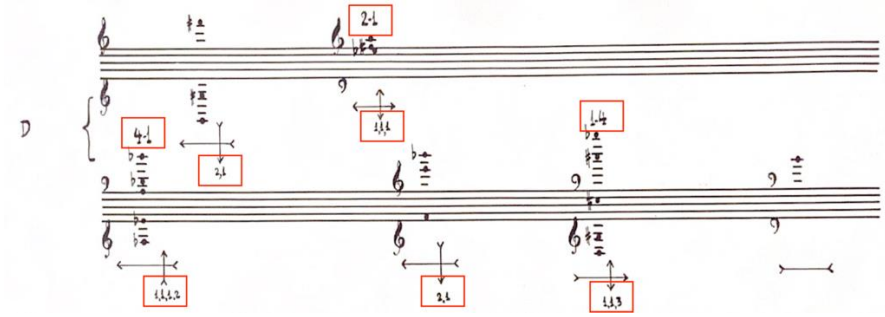
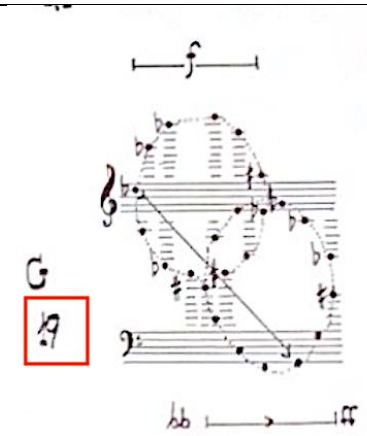
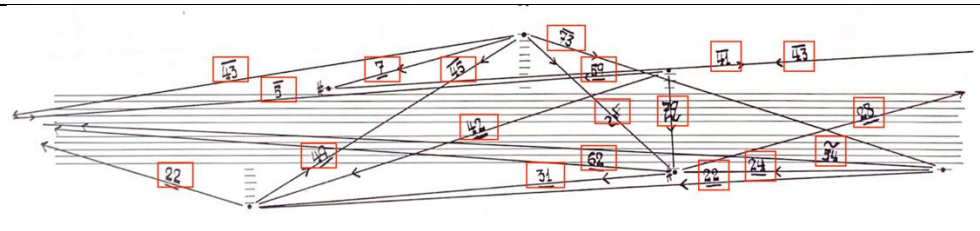
In this composition, Cage repeatedly used compositional materials, such as, numbers, lines, shapes, arrows, areas, and dots. As this musical work includes eighty-four types of notations, discussions can be difficult when dealing with such large numbers of notations; therefore, I shall divide them into four notation sections: (i) Notations A to Z, (ii) Notations AA to AZ, (iii) Notations BA to BZ, and (iv) Notations CA to CF in order to discuss common compositional materials in each notation section and present the relationships.

3.1.1 Compositional Materials in the Notation Section One

The first compositional material is numbers, which Cage broadly applied this compositional material in the *Solo for Piano*. Following the decomposition of Notations A to Z, it is clear that

the compositional material was used in Notations A, B, D, F, G, J, L, Q, R, T, U, Y, and Z. When delving deeper, I noticed that the numbers indicated different information. In Notation A, the numbers refer to the proportions of playing notes for the right and left hands; Notation B shows how many notes need to be played with the clef sign above, or below, the staff; Notation D demonstrates the proportions of playing notes; in Notation F the numbers indicate performing time, for instance, a number can mean any time unit; in Notation G the number shows the numbers of playing notes; Notation J shows the number of playing notes, with one note as the beginning and one note as the end; Notation L functions in the same way as Notation B; Notation Q represents time with any type of time units that the performer prefers; Notation R is shown as time; Notation T clarifies the degree of loudness; Notation U shows the numbers of playing notes; Notation Y indicates seconds as numbers; in Notation Z, the numbers are used to indicate degrees of loudness, as does Notation T. This brief summary describes how the numbers were used in these notations, including three ways, (i) the number of playing notes, (ii) time, and (iii) degrees of loudness.

Cage used the function of showing the numbers of playing notes in the following seven notations, A, B, D, G, J, L, and U. Although these numbers give the same information, the way they are presented varies, for instance, the numbers in Notation A uses a ratio format to demonstrate the number of playing notes; similarly, Notation B uses a ratio format; however, the symbol between two numbers is a dot, instead of a colon. Notation D shows two presenting ways, the first is the same as Notation B, in that it shows the number of playing notes with clef signs above, or below, the staff, while the other one is shown together with arrows. The numbers with arrows show the numbers of playing notes, but these also indicate the pitch relationship between notes. Notation G is relatively simple, just one number below the alphabet G. The other combinations with numbers are long straight lines, short straight line, or a sign as in Notation J. In this notation, Cage used long straight lines to indicate the pitch limit, used a short straight line above or below a number to indicate to the performers that the playing notes should be an ascending gamut or a descending gamut. The number with the sign can be either an ascending or a descending gamut. The following one, in Notation L presents numbers in the same way as Notation B. The final one is Notation U, in which Cage applied numbers along with pitch areas (Table 4).

Notation	Presentation of numbers
Notation A	
Notation B	
Notation D	
Notation G	
Notation J	

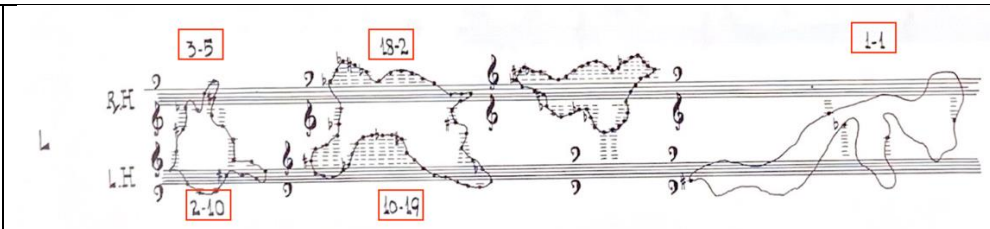
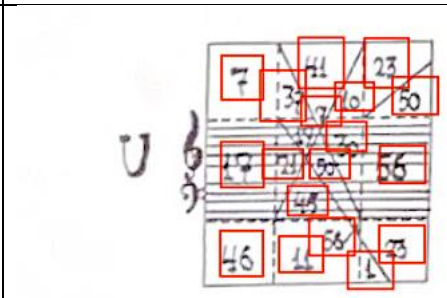
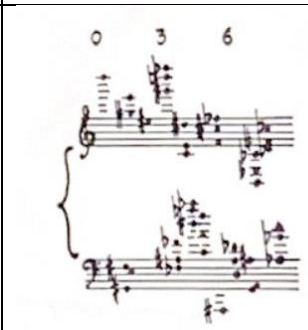
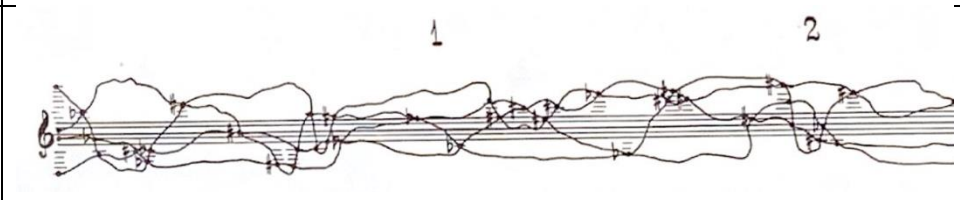
Notation L	
Notation U	

Table 4 Different ways of showing numbers of playing notes in notations.

The second function of numbers is to show time, which was used in Notations F, Q, R, and Y. There seems to be no big difference in the way time between notations is presented. The only difference here is that the performers are either free to decide a time unit or have to follow the exact scheduled time. For instance, in Notation F, Cage said numbers are either in seconds or in other time units; in Notation Q, the performing instruction is in time with any units. Hence, 0, 3, and 6 in Notation F can be from the 0th second to the 3rd second, then the 6th second, or change a second to a minute, the 0th minute to the 3rd minute, then the 6th minute, etc. The same use can be applied to Notation Q, also. However, Notations R and Y do not allow performers to decide the time unit, they need to use seconds to time their realisations (Table 5).

Notation	Presentation of numbers
Notation F	
Notation Q	

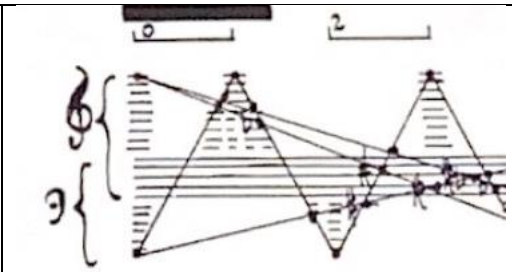
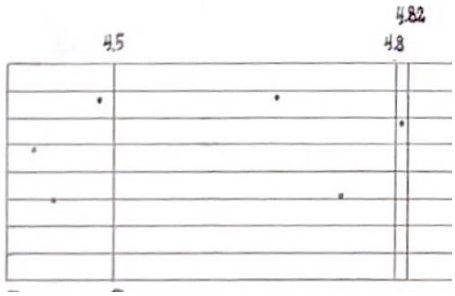
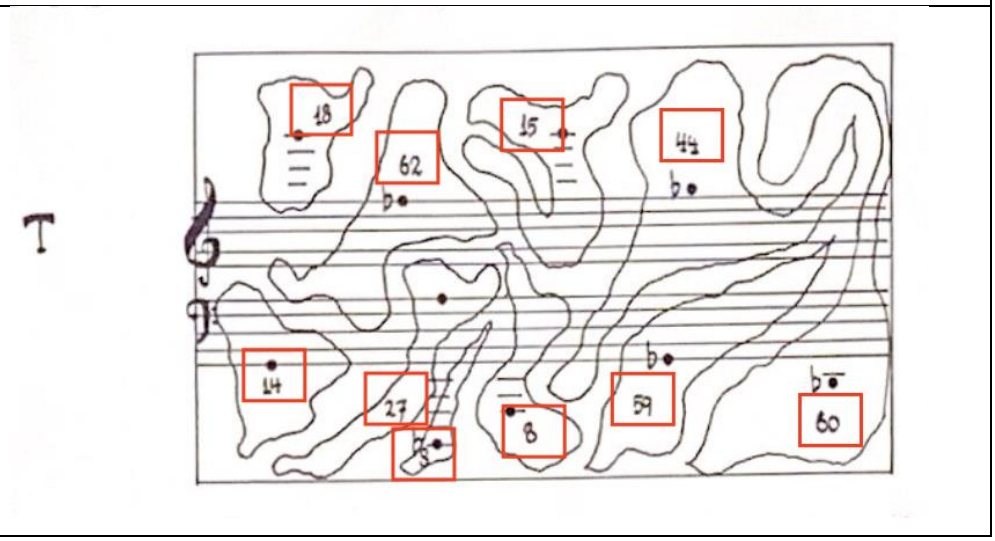
Notation R	
Notation Y	

Table 5 Different ways of showing time in notations.

The final function of numbers is to show degree of loudness, which can be found in Notation T and Notation Z. In Notation T, the range of numbers is from 1 to 64, the performer can choose either from soft to loud or vice versa. The numbers in this notation should be read alongside the irregular shapes, which are used to show pitch areas, meaning that the performer can play any notes in the corresponding pitch area, with the given degree of loudness. The same meaning of number also shows in Notation Z, where numbers present as a pair, divided by a semi-colon (Table 6). The former number indicates the degree of loudness of the earlier notes in the triangles, while the latter number refers to the later notes. As with Notation T, the performer can decide whether the number is from loud to soft, or from soft to loud.

Notation	Presentation of numbers
Notation T	

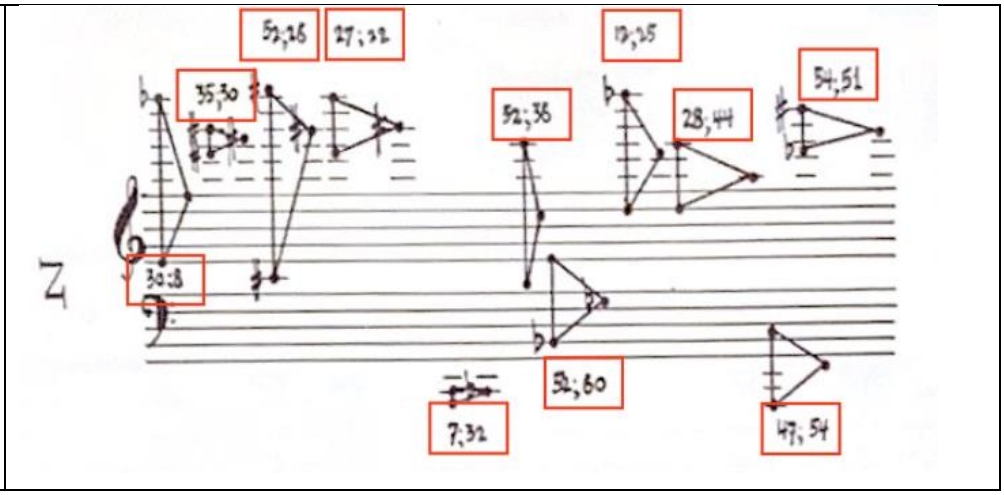
Notation Z	

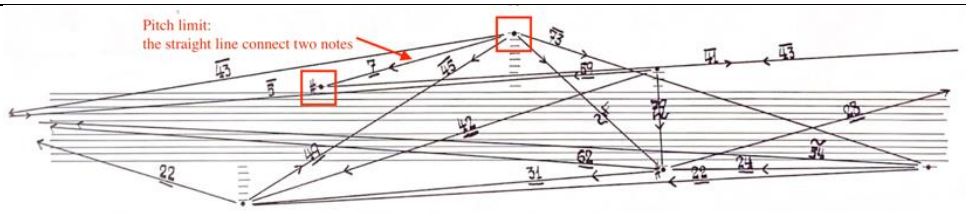
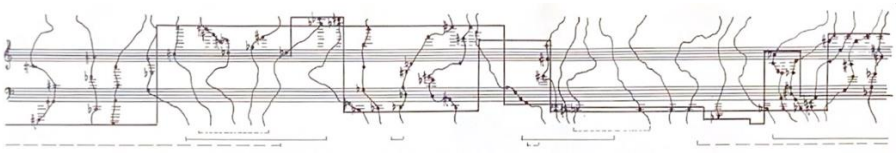
Table 6 Different presentations of degrees of loudness.

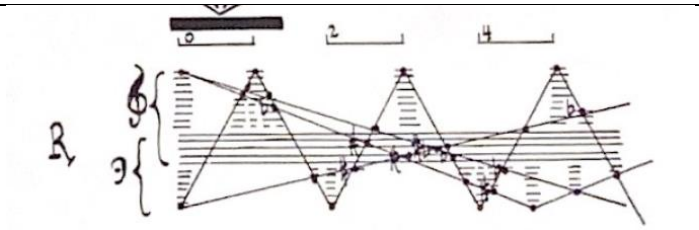
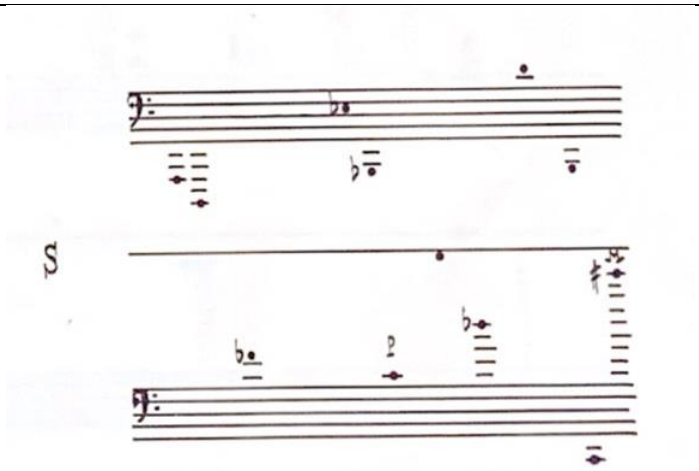
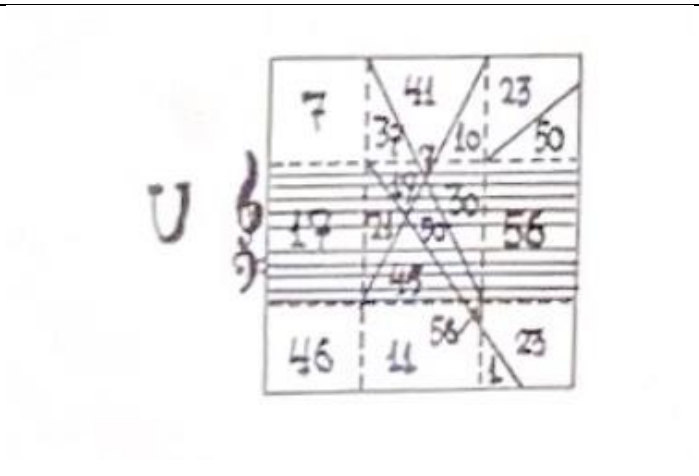
Cage used numbers in thirteen notations – Notations A, B, D, F, G, J, L, Q, R, T, U, Y, and Z – in the A to Z section in order to demonstrate the number of playing notes, their timing, and degree of loudness. The different functions of numbers, I suggest, enhance the relationship between notations; should the notations use numbers with the same functions build the first and the second type of relationships among them, while the notations use numbers with different functions build the first type of relationships only. This means that thirteen notations build first type of relationships together; while the second type of relationships build among three groups of notations: (i) Notations A, B, D, G, J, L, and U, (ii) Notations F, Q, R, and Y, and (iii) Notations T and Z.

The next compositional material is lines, which includes straight, dotted, and curved lines. Straight lines can be found in Notations J, O, R, S, U, V, W, and Z. Again, Cage used them for different purposes, including showing pitch limits/areas, grouping notes together, dividing areas, and indicating noises. The first function, showing pitch limits/areas, is shown in Notations J, O, and U. In Notation J, Cage used two notes at the beginning and the end of straight lines, which he called 'pitch limits', such as in Notation J in Table 7, which shows a straight-line linking notes A♯ and A. A similar function occurs in Notation O, with a slightly different presentation; here, Cage did not always link two notes with a straight line in order to limit the range of playing notes; instead, he would take two straight lines, sometimes crossed with one another, and sometimes parallel, to form pitch areas. With these, performers need to play notes as chords, or arpeggiations, within the pitch areas. These areas can also be found in Notation U, where he used both straight lines and dotted lines to form the areas.

The second function of straight lines is grouping notes together, such as in Notations R, W, and Z; here, both short and long lines link the notes by representing their relationships. In Notation R, the straight lines show a relatively fixed relationship between notes, meaning that performers cannot change notes above or below the staff, but they can decide which notes on the staff are played with which clef. Thus, Cage applied straight lines to group all these notes. In Notation W, linked notes should be played as legato, while isolated notes should be played as staccato (Notation W in Table 7). In Notation Z, he used the same approach to group three notes together as a triangle; thus, two notes grouped by a vertical line should be played together, and the other note should end as a single note. In Notation S, the third and the final function of straight lines is to indicate dividing areas and indicating noises, whereby a straight line divides the inside piano construction (above the line) or outside piano construction (below the line), as well as guides the performers when to realise the noises. The final function of line can also be found in Notation V, where Cage used lines to separate two staffs and state the notes below and above the staffs need to be performed as noises.

From the above description, it is clear that Notations J, O, R, S, U, V, W, and Z form the first type of relationships through the application of straight lines, while the three functions of straight lines form the second type of relationships among notations – the third function, dividing areas, is only applied in Notation S. For instance, Notations J, O, and U build the second type of relationships as they use straight lines to show pitch limits/areas; Notations R, W, and Z share this relationship as they use straight lines to group notes together; while Notations S and V both indicate noises by straight lines, and therefore, they form the second type of relationships among them.

Notation	Presentation of straight lines
Notation J	
Notation O	

Notation R	
Notation S	
Notation U	

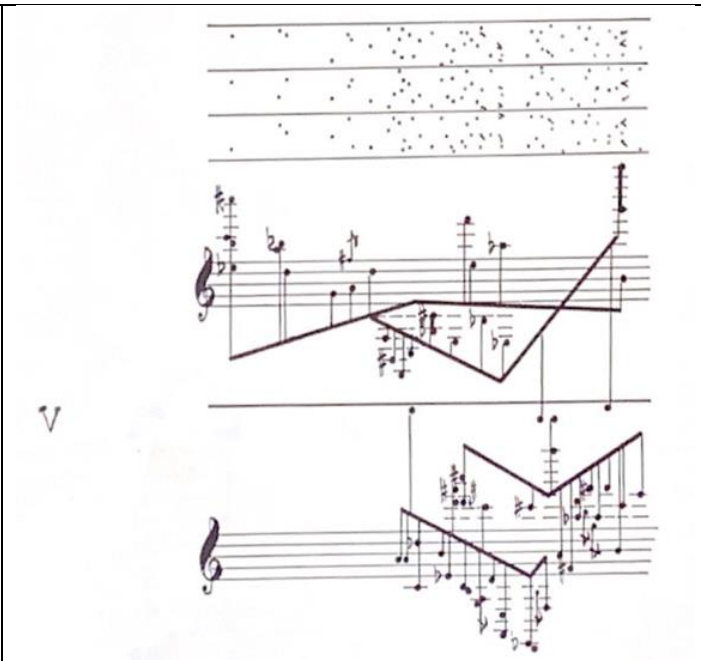
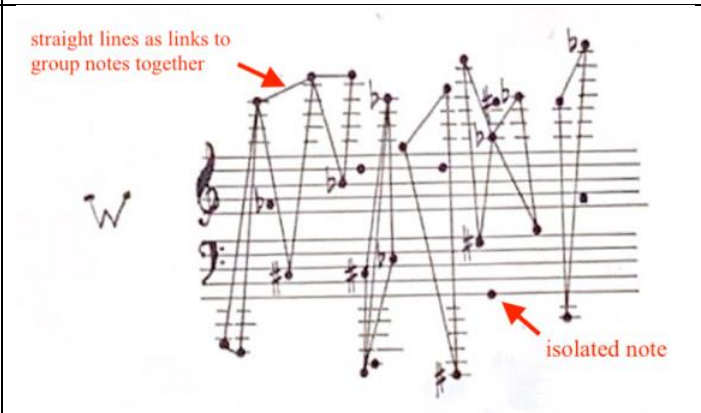
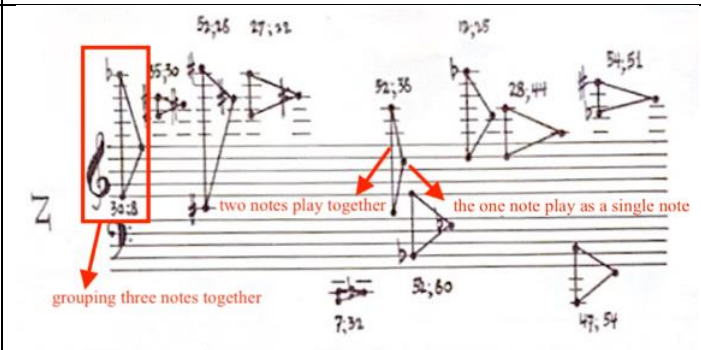
Notation V	
Notation W	
Notation Z	

Table 7 Different straight lines in notations.

Curved lines can be found in Notations M, O, and Q (Table 8). This compositional material has two functions: to show directions and grouping notes. In Notations M and Q, the curved lines are horizontal, with notes located only at intersection. Comparing this design to Cage's performing instructions – change direction at the intersections if desired – it shows routes with notes, which performers may choose. Thus, Notations M and Q share two relationships, one is

compositional material curved lines, the other is curved lines' function, showing directions. Notation O demonstrates the other function, grouping notes, as its curved lines are vertical, and its performing instruction guides the performers to realise the notes either as chords or arpeggiations, by placing the notes on the same curved lines. Consequently, Notation O does not share the relationship of the function with Notations M and Q; however, the three notations share the relationship of curved lines.

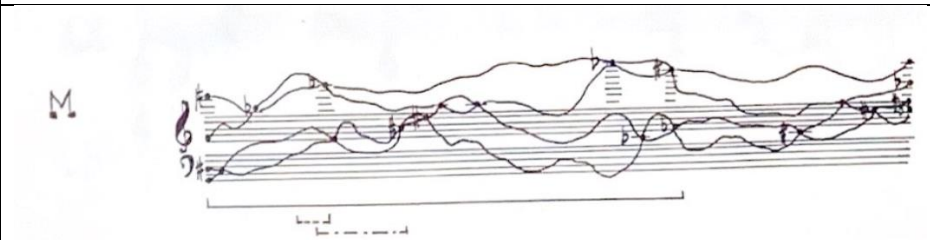
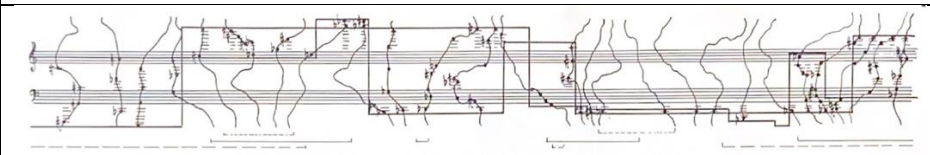
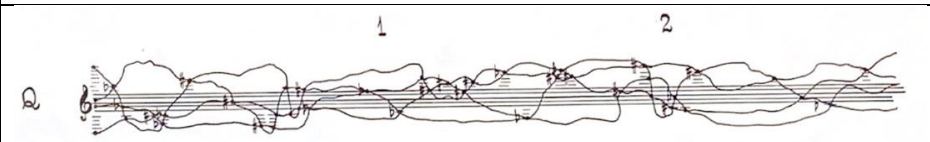
Notation	Presentation of curved lines
Notation M	
Notation O	
Notation Q	

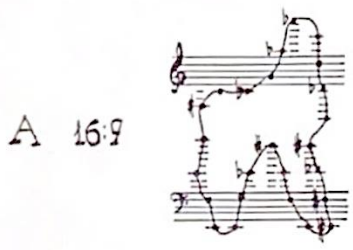
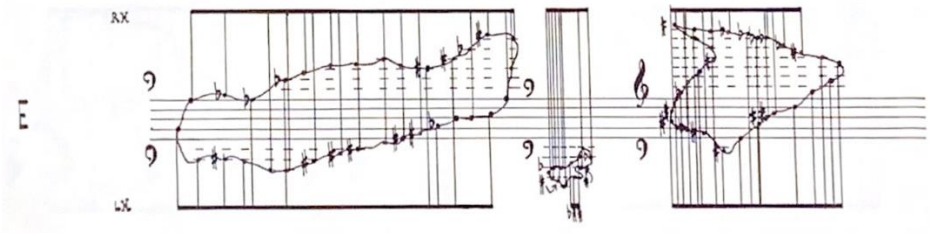
Table 8 Different curved lines in notations.

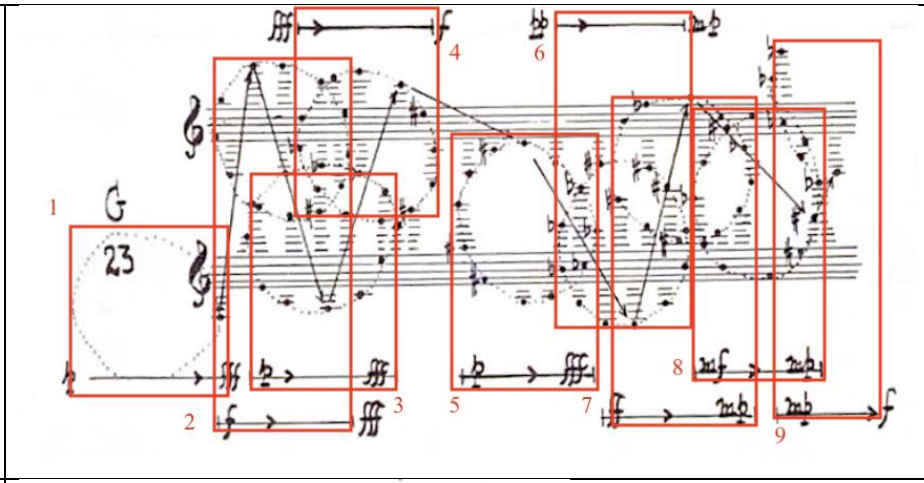
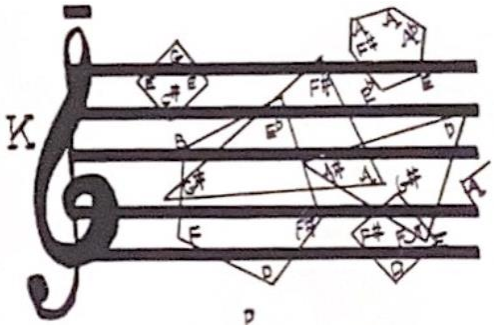
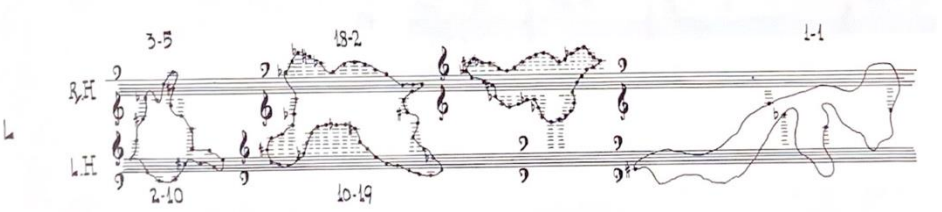
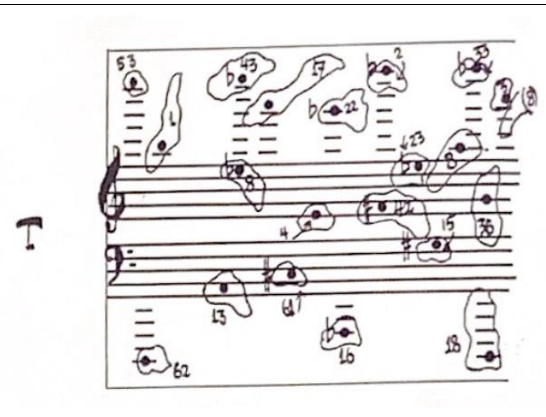
The next compositional material is shapes, that Cage broadly used in notations with the following types: irregular, as in Notations A, E, L, and T; circles, as in Notation G, triangles, as in Notations W and Z; triangles, squares, pentagons, and hexagons, as in Notation K. Analogous to numbers and straight and curved lines, different shapes serve five functions in order to show, (i) directions, (ii) sequences, (iii) groups of notes, (iv) pitch, and (v) time. Directions and sequences can be found in Notations A and L, where Cage gives the number of notes and directs performers to choose a given number of notes for both hands; however, he limited the performers in following the perimeter of the irregular shape. This means that the performers can only follow the irregular shape to practise realisations, therefore, the shapes show directions and sequence.

The third function, grouping notes, was used in Notations A, E, G, L, W, and Z. Notations A and L used irregular shapes to show the groups of notes, and so the performers may follow the

group to realise the performing directions and sequences. In Notations E and L, the notes were grouped together to show which pair of clef signs should be played with; however, under this grouping, performers are still offered freedom to choose which clef sign they want to apply to the notes. The same function also shows in Notation G, although the difference in the grouping is not for clef signs, but for dynamics. As Notation G in Table 9 shows, Cage used nine circles and nine sets of dynamics; one circle matches with one set of dynamics, the notes in the circle need to follow the corresponding set of dynamics. In Notations W and Z, as I explained in the section of lines' functions, the two notations show the function of grouping notes together to differentiate performing approaches (legato and staccato in Notation W, and cluster and single notes in Notation Z); considering the appearances of lines are presented as triangles, I include the two notations in this discussion.

The next function shows notes as in Notation K; here, Cage used triangles, squares, pentagons, and hexagons to guide performers regarding which notes should be included in their realisations. Therefore, performers need to play either odd or even numbers of notes from the shapes, the remaining notes can be graces. The final function of shapes shows time and pitch, meaning that the shapes indicate a relative relationship of duration for each note, it also shows possible pitch areas with a centre note, as Notation T presents. Again, based on this compositional material, these notations share the first type of relationships, while the notations share the same functions construct the second type of relationships.

Notation	Presentation of shapes
Notation A	
Notation E	

Notation G	
Notation K	
Notation L	
Notation T	

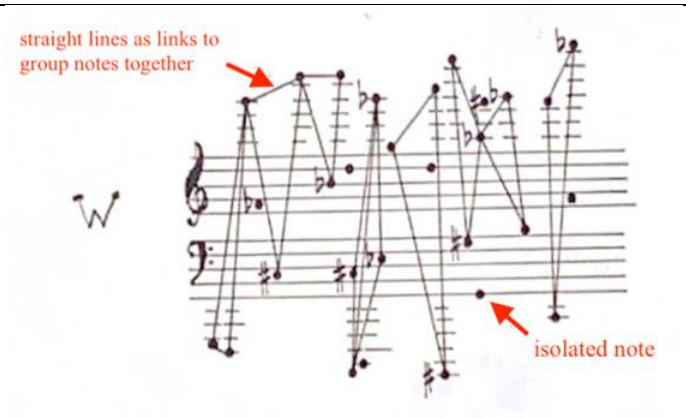
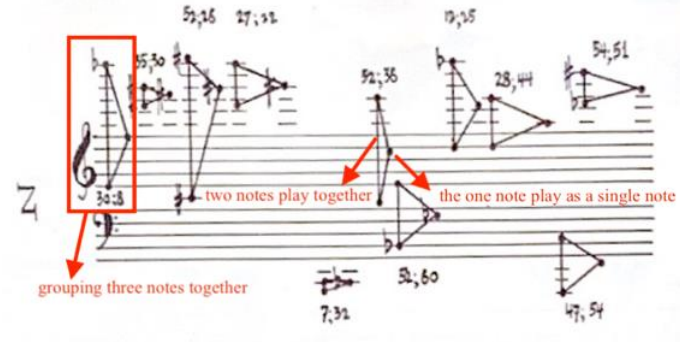
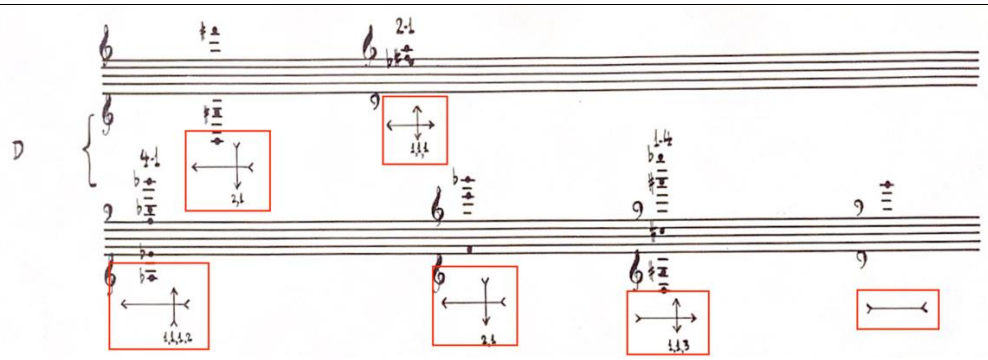
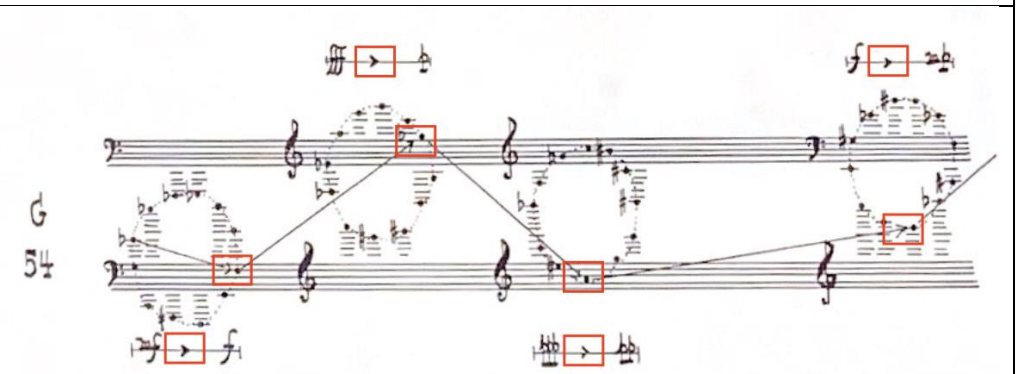
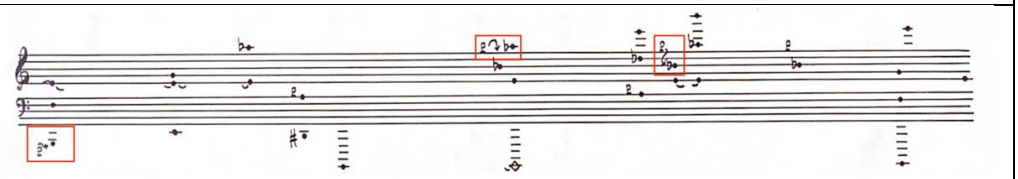
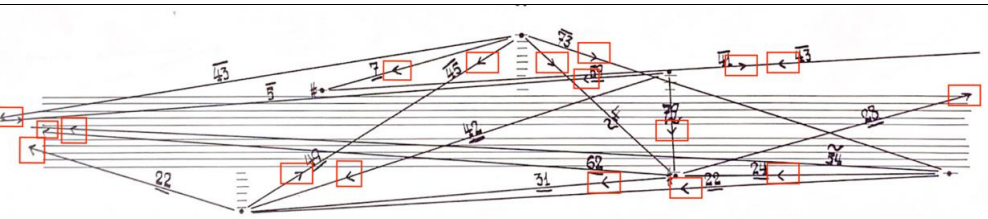
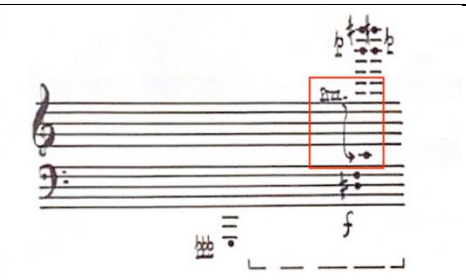
Notation W	
Notation Z	

Table 9 Different shapes in notations.

The next compositional material is arrow, which Cage used in Notations D, G, I, J, N, S, and T (Table 10). He used this compositional material for directing pitch and time, performing directions, and indications. For instance, Notation D includes horizontal and vertical arrows. A horizontal arrow shows the pitch relationship between notes, and the horizontal arrow shows the relationship of time. The notes can be played sooner, later, or at the point of notation. In Notations G and J, the arrows point in the same function, indicating directions. In Notation G, the arrows indicate starting and ending notes. Thus, the arrows point in a general direction, from a note in one circle to another note in the same circle, or to another note in another circle. In addition to the directions of notes, the arrows also show directions of dynamics; from *mf* to *f*, from *fff* to *p*, from *ppp* to *p*, or from *f* to *mp*. In Notation J, the arrows point out relatively specific performing directions. Unlike the performing direction in Notation G is from one circle to the other, the performing directions in Notation J is detailed to the directions between notes. In Notations I, N, S, and T, he used arrows as indications to point out both the performing approach and the degree of loudness. Thus, notations that have the same function build a relationship based on that function, while those with different functions are not included in the relationship.

Notation	Presentation of arrows
Notation D	
Notation G	
Notation I	
Notation J	
Notation N	

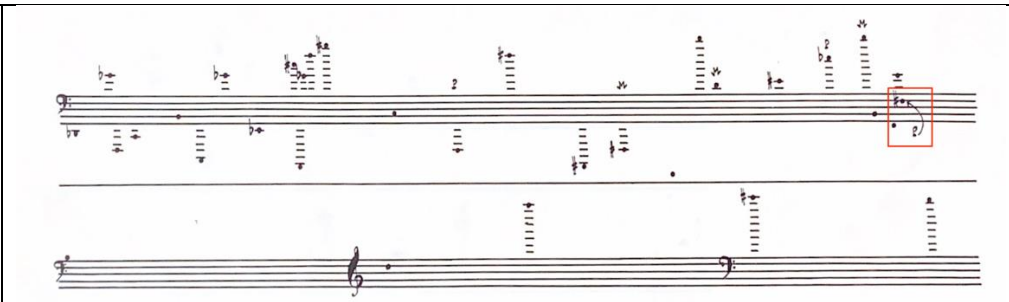
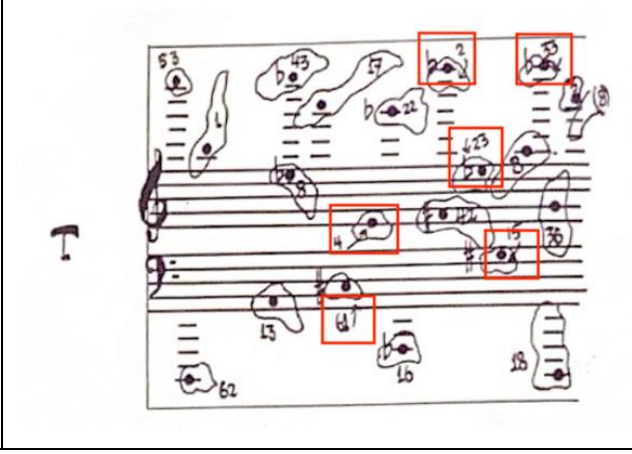
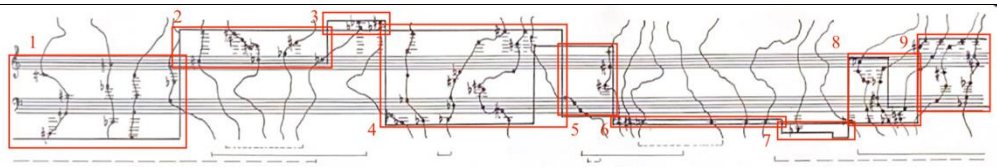
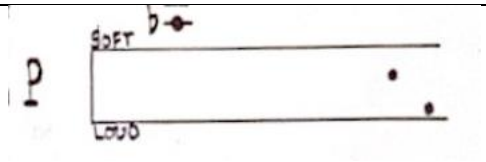
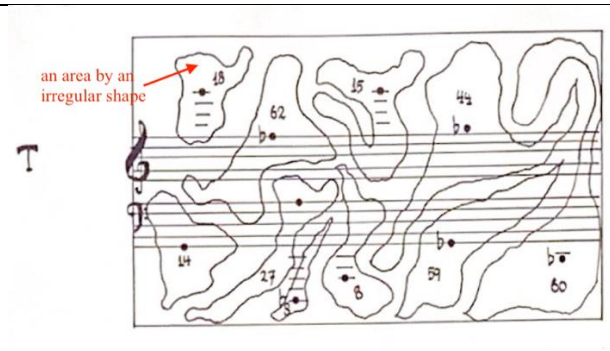
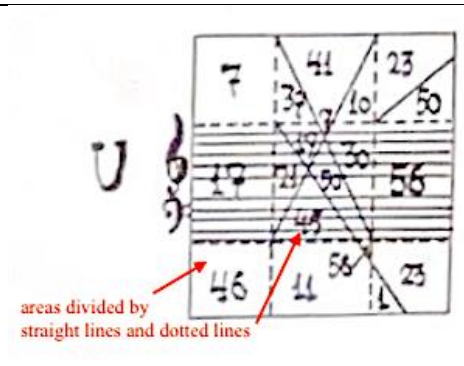
Notation S	
Notation T	

Table 10 Different types of arrows in notations.

The next compositional material is areas in Notations O, P, T, U, V, and Y. This compositional material indicates the performing approach, the degree of loudness, speed, and the pitch. The performing approach function appears in Notation O; here, Cage stated that the notes between two horizontal lines can be realised as chords and arpeggiations. The loudness function can be found in Notations P, V, and Y. As Table 11 shows, in Notation P, Cage drew an area and indicated soft at its top and loud at its bottom. Performers practise the two dots in the area with the indications of loudness – the higher note should be softer, while the lower note should be louder. Notation V shows the function of degree of loudness and speed; it includes three areas, from top to bottom, the first shows the degree of loudness, dots in higher positions show louder sounds, and the dots in lower positions show softer sounds. The second indicates the vertical distance of attack, higher dots represent further, while lower dots represent closer; and the third, the speed of attack, showing the faster and slower speed of attack by higher and lower positions of dots. Here, performers refer to the three areas to practise their realisations. Similarly, in Notation Y, the areas indicate the degree of loudness, based on the position of notes, high is *fff* and low is *ppp* for horizontal direction.

The final function, showing pitch, is shown in Notations T and U. As analysed in the section on lines and shapes, Notation U shows its areas by lines, and Notation T uses the irregular

shapes to delimit the areas, by considering the overlapping between areas and lines in Notation U and between areas and shapes in Notation T, I suggest that one function may be formulated by two compositional materials, which may mean that the relationships between compositional materials may be much intertwined than it looks. Back to the relationships of this compositional material and its functions, Notations O, P, T, U, V, and Y construct the first type of relationships, which was based on the areas. Meanwhile, Notations P, V, and Y built the second type of relationships, since their areas performed the same function, and Notations T and U as well as built the second type of relationships between them.

Notation	Presentation of area(s)
Notation O	
Notation P	
Notation T	
Notation U	

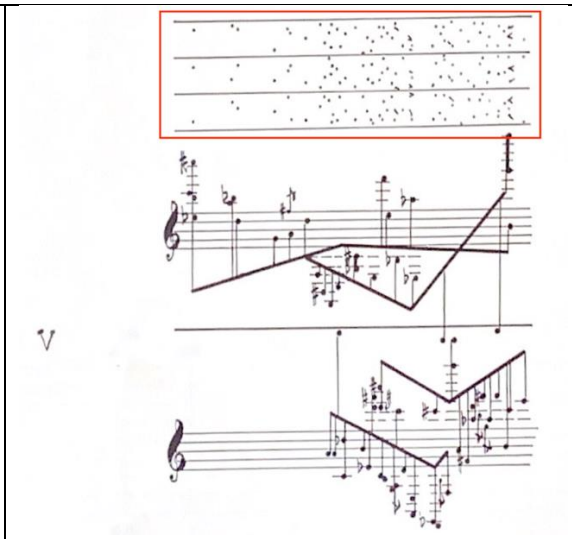
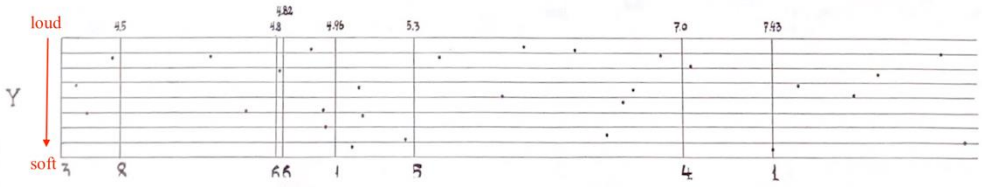
Notation V	
Notation Y	

Table 11 Different types of areas in notations.

The final compositional material is dots, Cage used in Notations P, S, V, and Y to show performers how he arranged his notations. His dots do not carry meaning in themselves, this material has to be comprehended alongside other materials. For instance, in Notation P, the area showing different dynamics – i.e. top to bottom means soft to loud – with positions of two dots, the performers are able to recognise the corresponding level of dynamics. The same situation can be found in Notations S, V, and Y; these use dots to indicate the timing of noise, levels of loudness, speed together with other compositional materials. As Notation S shows, he arranged his dots under straight lines in order to divide areas; above the straight line means inside piano construction, below it means outside piano construction. Performers need to follow the relative positions of the dots in order to practise the noises. A similar reading appears in Notation V, which also includes a straight line and, in three areas, many other dots, which performers need to comprehend to realise the corresponding levels of loudness and speed. In Notation Y, the dots read vertically and are arranged in different pitch areas. Horizontally positioned dots need to be understood in terms of time, as Cage stated 4.5, 4.8 and 4.82 seconds in the notation (Table 12). Regarding the use of this compositional material, I suggest, Notations P, S, V, and Y increase their closeness between one another.

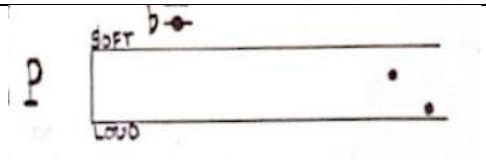
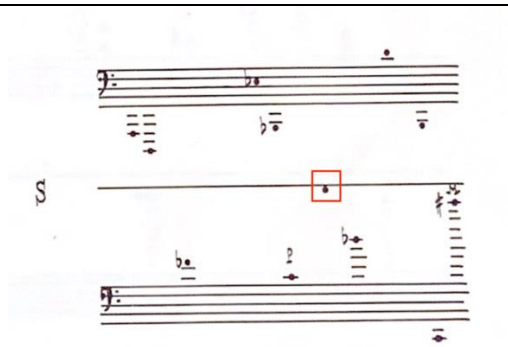
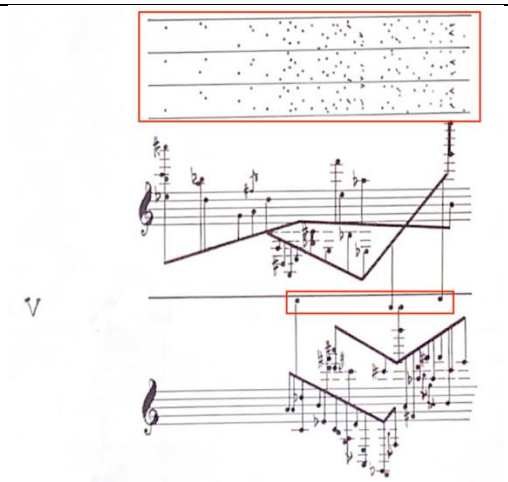
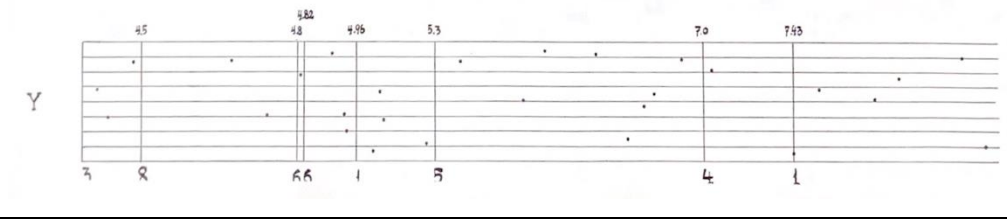
Notation	Presentation of dots
Notation P	
Notation S	
Notation V	
Notation Y	

Table 12 Different ways of presenting dots in notations.

In summary, one compositional material may be assigned a variety of functions. As previously explained, numbers indicate the numbers of playing notes, time, and degree of loudness; lines include both straight and curved lines, the straight ones indicating pitch limits/areas and noises, grouping notes together, and dividing areas; the curved lines indicate directions and group notes together; shapes show direction/performing sequences, notes, pitch, and group notes; arrows act as indicators of pitch, time, direction, and conventional indications, and areas show the degree of loudness, pitch, and speed. I have discovered two types of relationships

throughout the notations, the first type is based on the uses of compositional materials, which means that notations share the same compositional material to build the first type of relationships, whilst the second type can only be found in those notations that share the same functions.

3.1.2 Compositional Materials in Notation Section Two

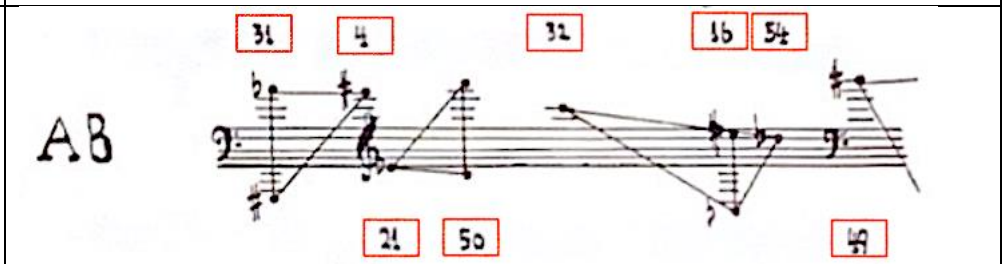
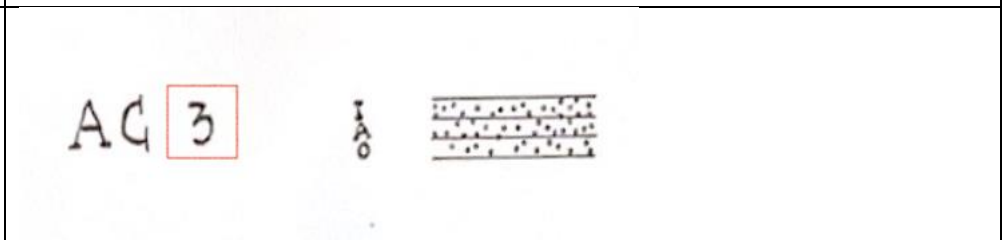
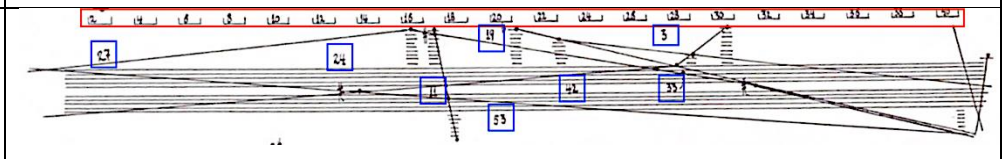
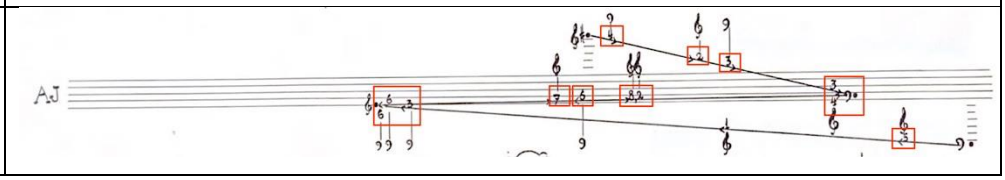
As discussed above, numbers, lines, shapes, arrows, areas, and dots are the common compositional materials in Cage's composition. In this section, I will discuss these compositional materials in Notation Section Two, from Notations AA to AZ. The first compositional material is numbers, Cage applied this in Notations AB, AC, AE, AJ, AK, AL, AT, AU, AV, and AZ, with the functions: (i) indicating different degrees of dynamics, (ii) the numbers of playing notes, (iii) showing time, and (iv) chosen notes.

The first function can be found in Notation AB, which uses numbers to indicates different degrees of dynamics, as in Notation T. AB applies numbers either above, or below, the staff to indicate the dynamics of the clusters and the single notes (Table 13). The second function, showing the numbers of playing noises, can be discovered in Notation AC. In the example below, a number 3 is next to the areas of I, A, and O; the number informs performers to choose three noises from the given dots and areas (Table 13).

In Notation AE, the numbers show two different uses; as the example demonstrates, the red square shows the third function of numbers, showing time, while blue squares indicate the number of notes to be played. Notation AJ, again, uses numbers as the given proportion of notes. Notation AK shows the third function of numbers as in Notation AE. In Notation AL, the numbers indicate a similar function as Notations AE and AK; however, there is a difference in numbers, where the large ones, which are surrounded by red squares, represent the proportional of time between sound events; consequently, those numbers have a relative relationship with time. However, that proportional time does not refer to the actual timing when sounds are produced, therefore, performers need to measure the distance between the large numbers to obtain the timing of sounds. Alongside this function, the small numbers, which are surrounded by blue squares, indicate the final function, representing the chosen notes. In Notation AT, numbers have the second and the third function, showing the number of notes played and showing time; the second function is similar to Notations AC, AE, and AJ; while the third function is closed to Notations AE, AK, and AL. As the example in Table 13, the red

squares indicate the given number of notes, and the blue square indicates the given time. Notation AU suggests the performing time by numbers above the notation, as in Notations AE, AK, AL, and AT. Notation AV uses numbers 1 to 64 to indicate either *fff* to *ppp*, or *ppp* to *fff*, the function of numbers is similar to Notations T, Z, and AB, indicating different degrees of dynamics. Notation AZ uses numbers to indicate performing time, just as in Notations AE, AK, AL, AT, and AU.

In short, in this section Cage used numbers in ten notations. The function of this compositional material indicates degrees of dynamics, representing the numbers of playing sounds, showing time, and chosen notes. Notations AB, AC, AE, AJ, AK, AL, AT, AU, AV, and AZ build the first type of relationship, while those that share the same function also build the second type of relationship.

Notation	Presentation of numbers
Notation AB	
Notation AC	
Notation AE	
Notation AJ	

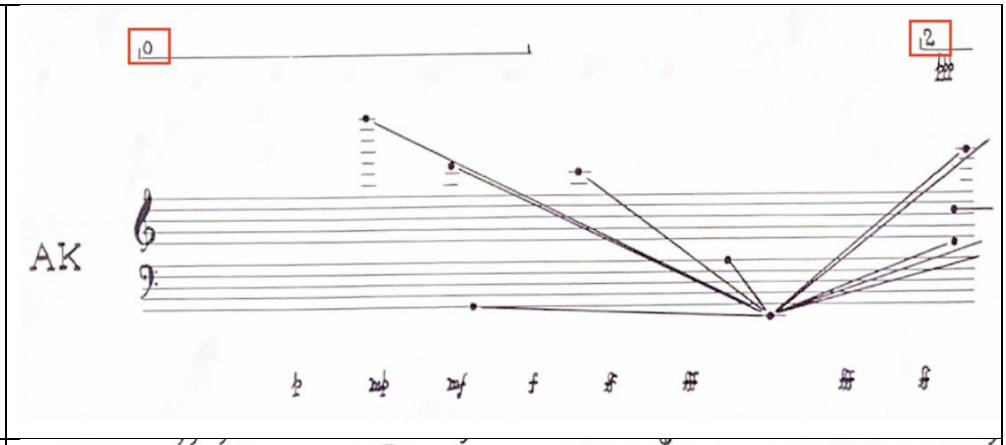
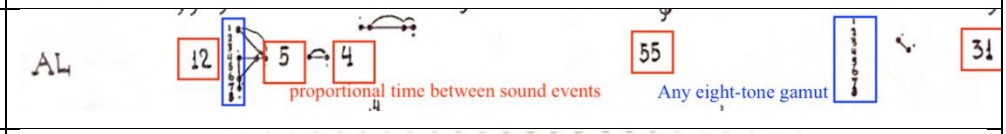
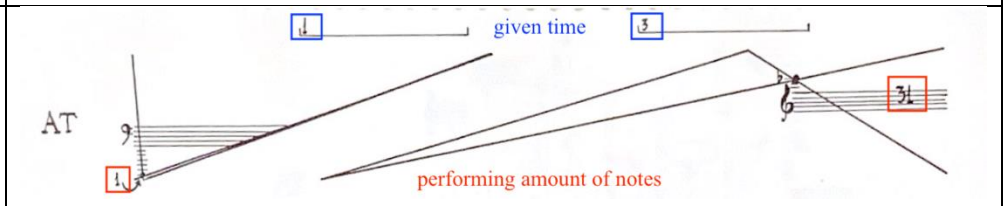
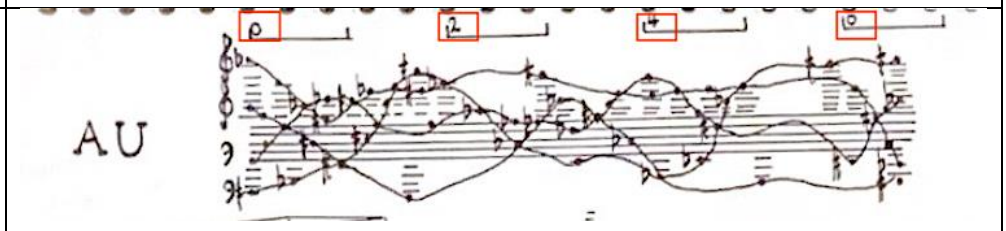
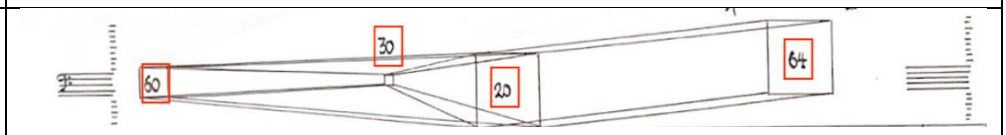
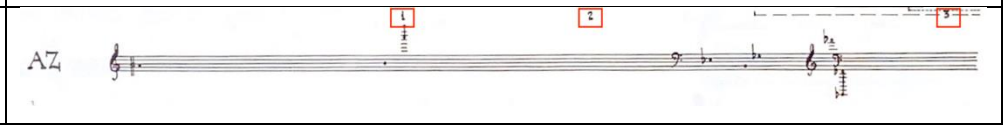
Notation AK	
Notation AL	
Notation AT	
Notation AU	
Notation AV	
Notation AZ	

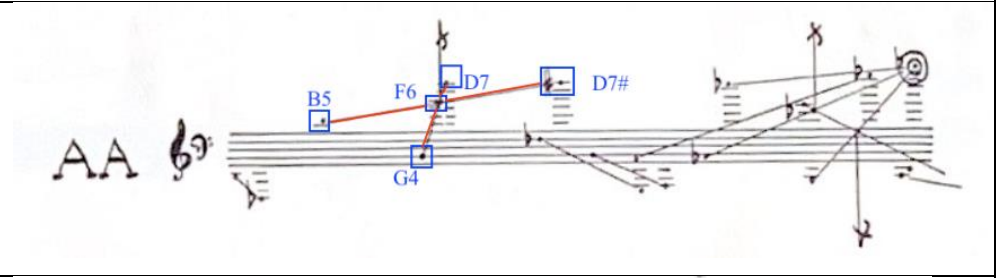
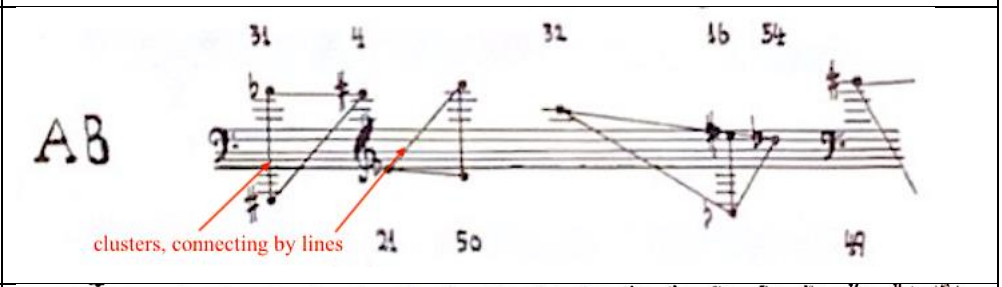
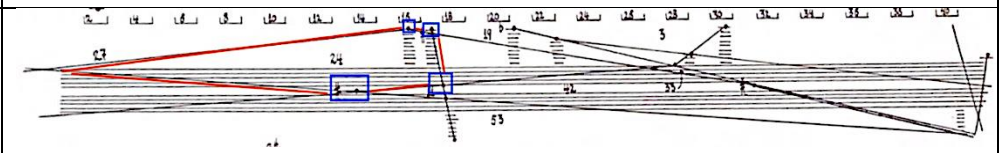
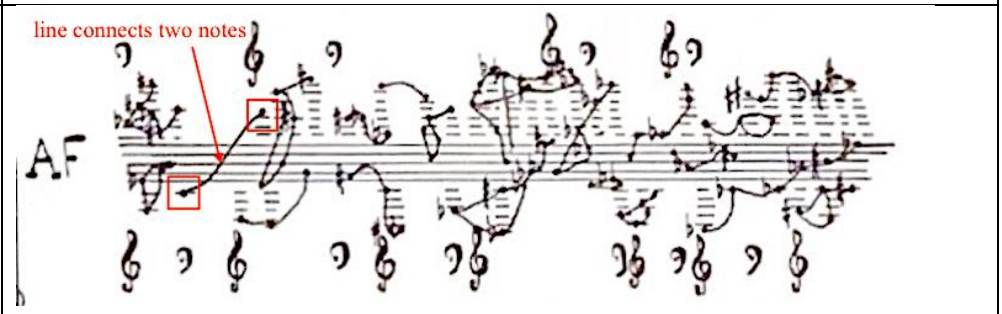
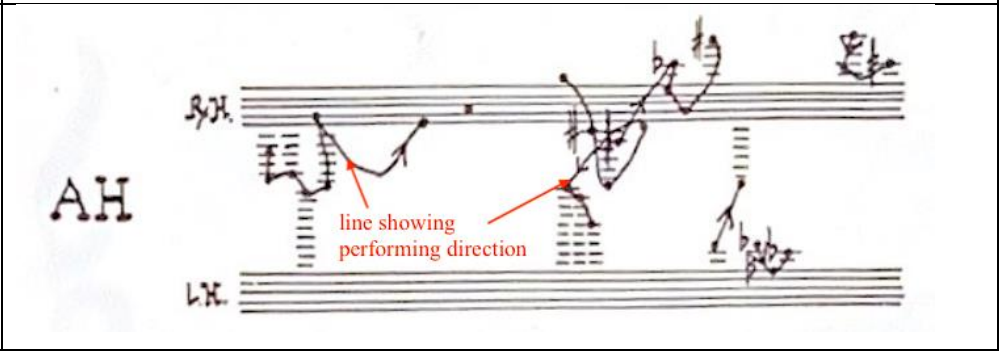
Table 13 Different types of numbers in notations.

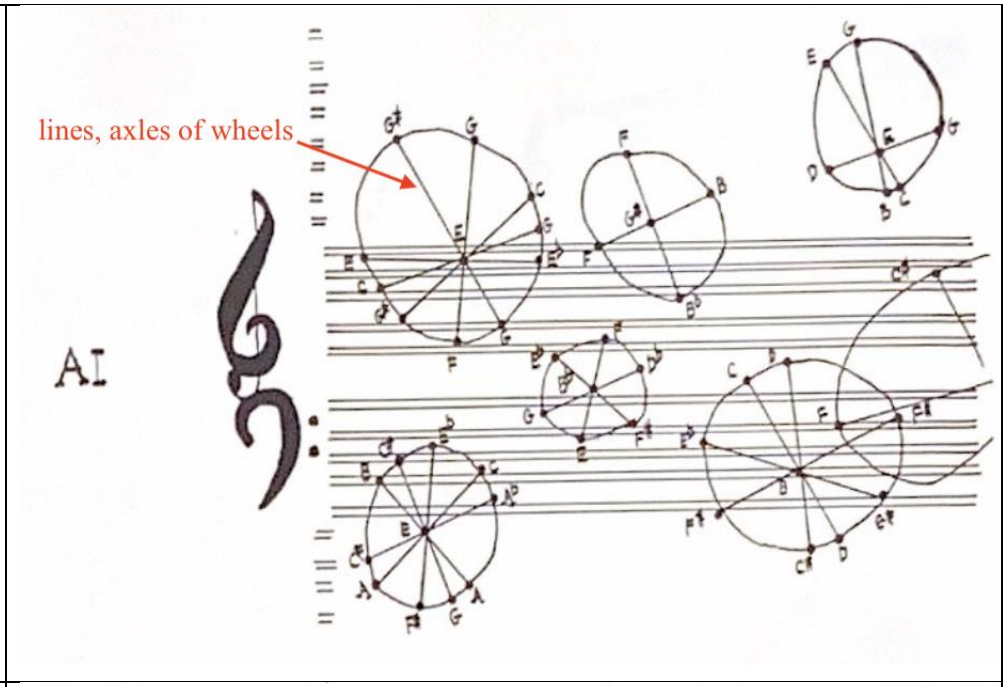
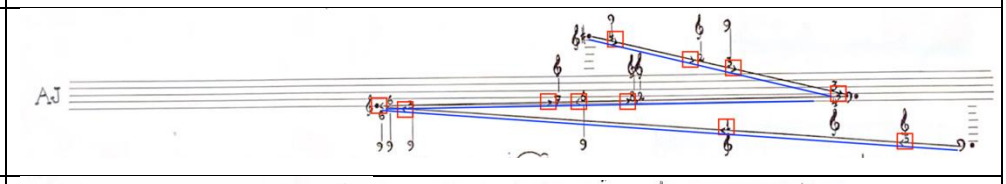
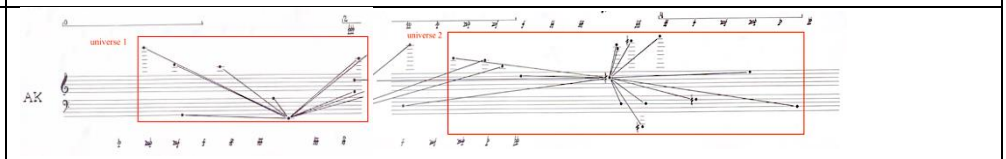
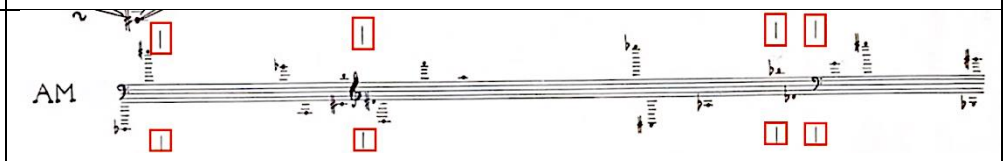
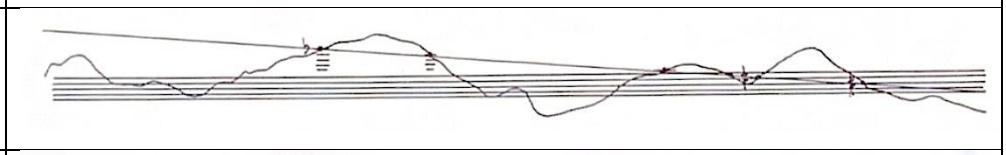
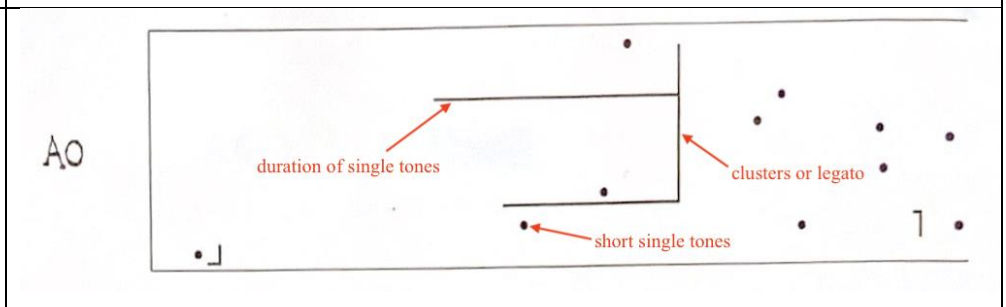
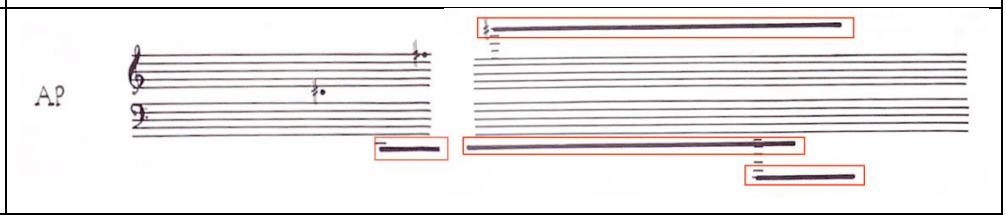
The lines in Notation AA Cage called ‘sticks’, which he used for both annotating the beginning and end of pitch area and indicating the stick’s duration. This means that lines have two functions: delimiting pitch limit and demonstrating the performing time. As the example in Table 14 below shows, two red lines connect four notes, indicated by blue squares, together, with one note as a grace located at the intersection of two red lines. For this realisation, performers may choose notes from B5 to D7#, take F6 as a grace, or choose notes from G4 to D7, and use F6 as a grace (Table 14). In Notation AB, lines connect notes together, as in Notation Z. Notation AE takes performers straight to the delimit pitch area for the playing range of notes; Table 14 also shows five red lines constructing a pitch area. In Notation AF, Cage used lines to connect two notes as one event. In Notation AH, lines not only connect

notes, but also show performing directions. In Notation AI, lines are shown as the axles of wheels where they not only connect notes together, but also show the performing directions. Notation AJ uses lines with arrow to show the direction of notes and offer the pitch limit; as the example shows, several red squares on the blue lines. With the arrows and lines, performers follow assigned directions and given pitch limits. Notation AK uses lines to demonstrate universes; the example in Table 14 shows two universes consisting of multiple lines. Cage stated that performers should realise this notation by selecting one note from one universe, the lines functioning as pitch limits.

Notation AM includes another function for lines, where they separate the duration of notes and intervals from the various sections. As the example in Table 14 shows, short lines order notes into different sections; in the same section, the notes should be performed to the same duration of time. Notation AN uses lines in two ways: showing both pitch limits and notes direction. As with Notation AJ, Notation AN shows two functions of lines, performers follow the lines to choose performing notes, regardless of their numbers. Notation AO uses vertical and horizontal lines to show the performing approach of notes, which can be either cluster or legato, and which demonstrate the performing time. Lines in Notation AP, which have the same function as in Notations AA and AO, are used to demonstrate performing time. Notation AQ, applies dotted lines to divide it into two parts, thereby allowing performers to decide which part to perform; a similar function operates in Notation U. Notation AT uses lines to delimit the pitch area, as in Notations AA, AE, AJ, and AK. In Notation AU Cage used lines to show performers directions, where to apply intersections and that they are free to change direction if they wish. In Notation AZ, lines connect notes; notes with lines should be performed as a cluster; this function can be found in Notations AB, AF, AH, and AI.

In this section, Cage applied lines in the following sixteen notations – AA, AB, AE, AF, AH, AI, AJ, AK, AM, AN, AO, AP, AQ, AT, AU, and AZ – it also describes seven different functions, such as delimiting pitch limits, demonstrating performing times, connecting notes, showing performing directions, differentiating the duration of notes and intervals from the different sections, acting as the performing approach of notes, and creating intersections. These seventeen notations construct the first type of relationship, while notations with the same functions construct the second type of relationship.

Notation	Presentation of lines
Notation AA	
Notation AB	
Notation AE	
Notation AF	
Notation AH	

Notation AI	
Notation AJ	
Notation AK	
Notation AM	
Notation AN	
Notation AO	
Notation AP	

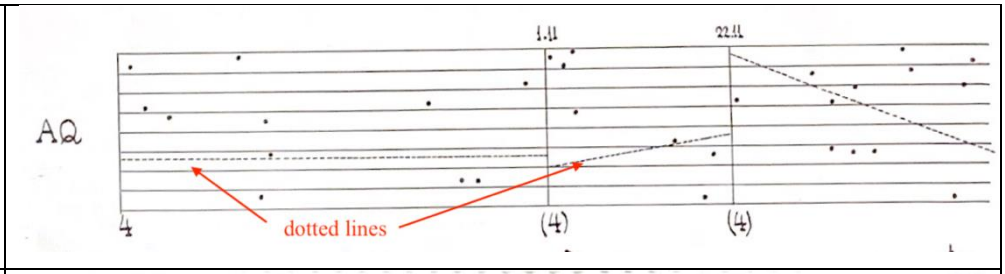
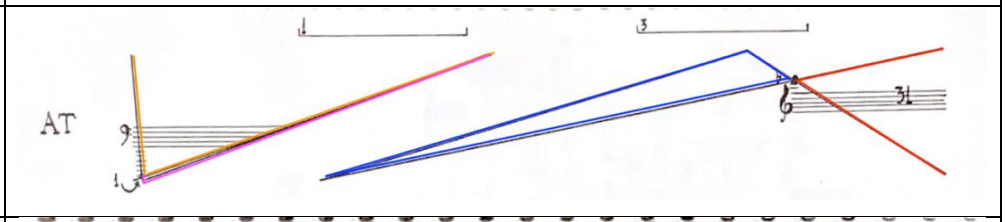
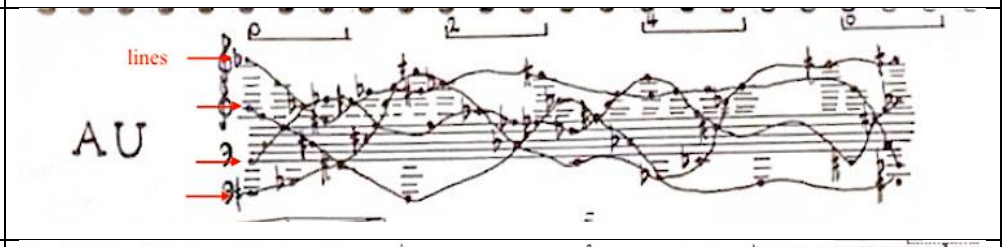
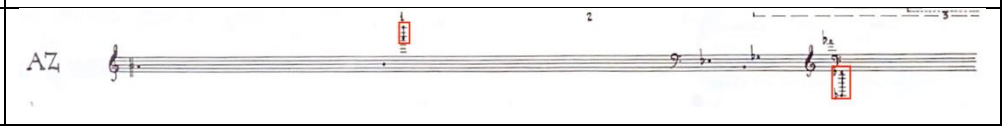
Notation AQ	
Notation AT	
Notation AU	
Notation AZ	

Table 14 Different types of lines in notations.

The next compositional material is shapes (Table 15). Along with Notations A, E, G, K, L, O, R, T, W, and Z, shapes are as important as they are in Notations AA to AZ. For instance, AA uses circles to differentiate the degrees of emphasising notes; triangles in Notation AB show performing approach of notes, such as which notes should be played as a cluster and which as a single note. In Notation AI, the shapes are showed as wheels with two to five axles, and used for grouping notes; however, since Cage required performers to realise AI from left to right, the wheel does not show the order of notes. Notation AR uses different sized squares, which are relatively ambiguous, suggesting potential performing directions, performing approaches, pitch limits, and duration. Notation AV uses shapes to demonstrate the same information as in AR, however, performers are limited in assigned amplitudes. Notations AA, AB, AI, AR, and AV all use shapes, which include circles, triangles, wheels, and squares, to show functions. The functions of this compositional material include (i) differentiating the degrees of emphasising notes, (ii) showing performing approaches, (iii) grouping notes, (iv) suggesting performing directions, (v) showing pitch limits, and (vi) suggesting durations.

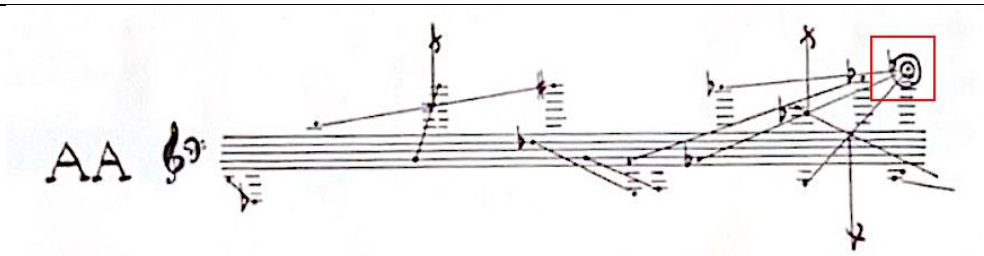
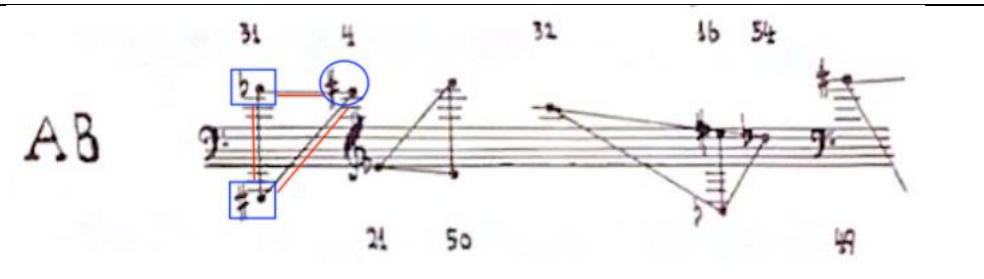
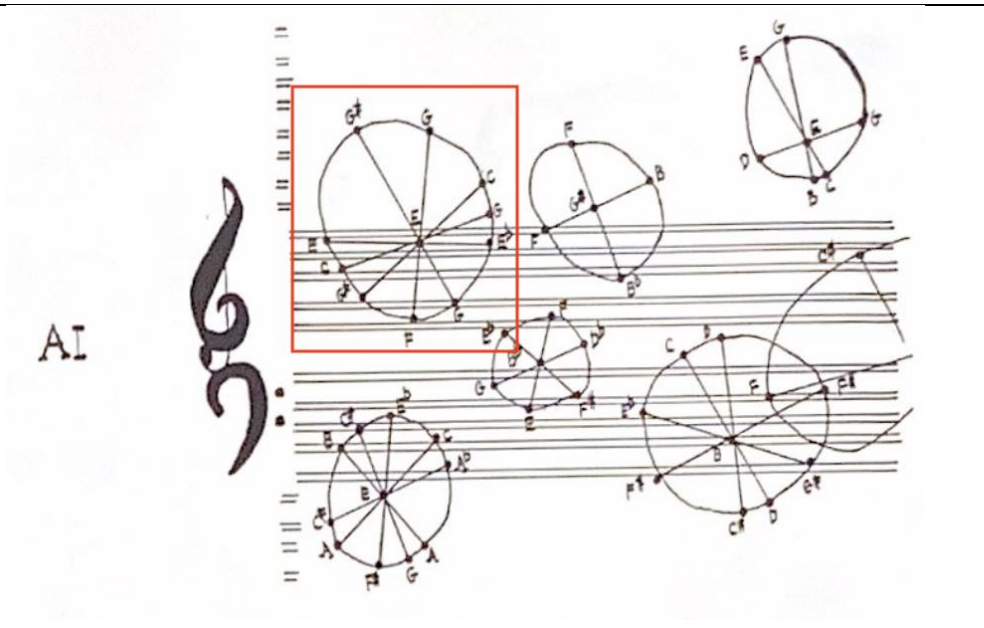
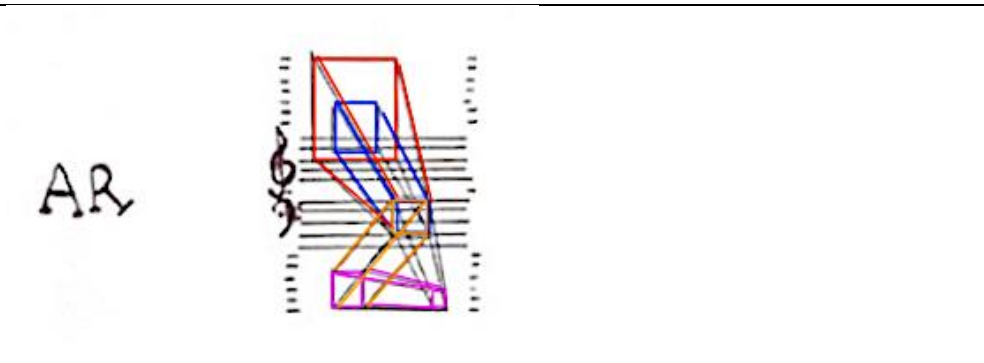
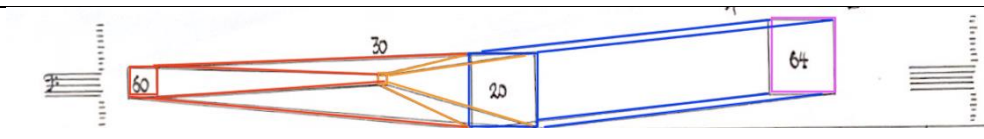
Notation	Presentation of shapes
Notation AA	
Notation AB	
Notation AI	
Notation AR	
Notation AV	

Table 15 Different types of shapes in notations.

The next compositional material consists of arrows, which Cage used to show performing directions in both Notations AH and AJ. Meanwhile, he used arrow as indicators in Notation AT (Table 16). The following compositional material is areas, which can be found in Notations AC, AE, AO, AT, and AX; the functions of this compositional materials include: (i) showing different way to produce noises, (ii) delimiting pitch areas, and (iii) showing the frequency and duration of single notes (Table 17). In AC, the three areas show different ways of producing noises, including through the interior and exterior piano construction and using auxiliary tools. In AE, Cage applied lines and notes to delimit the pitch areas. Notation AO uses area to show the frequency and duration of single notes. As in AE, Cage used lines in AT to delimit pitch areas; the example below shows areas marked in four colours, the performer needs to choose the given number of notes in certain pitch areas. Thus, performers are free to choose thirty-one notes from the area with red lines. Cage used areas in Notation AX as he did for Notation AC, demonstrating different ways of producing noises; the areas in Notation AX are wood, plastic, metal, and miscellaneous.

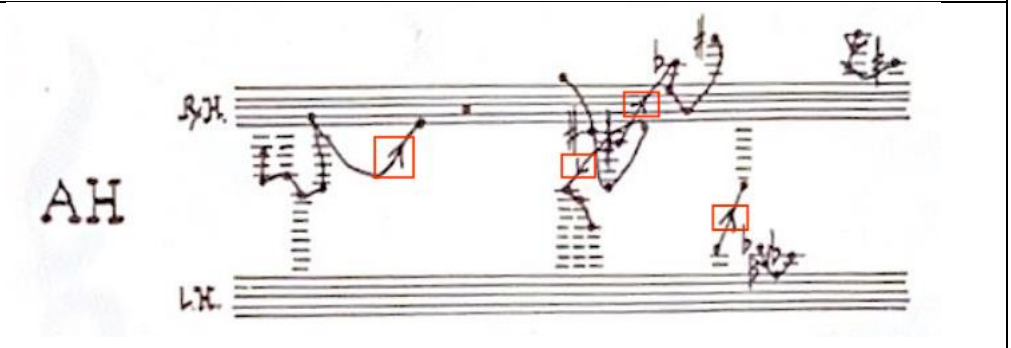
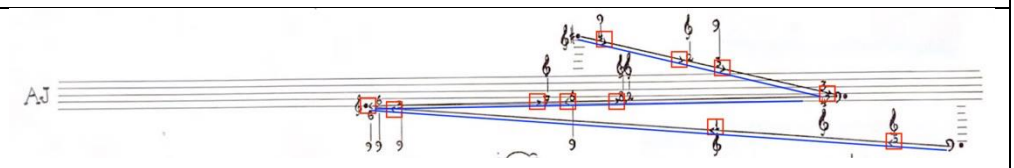
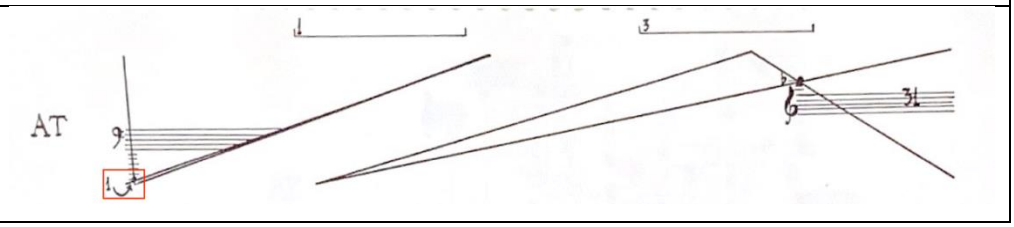
Notation	Presentation of arrows
Notation AH	
Notation AJ	
Notation AT	

Table 16 Different types of arrows in notations.

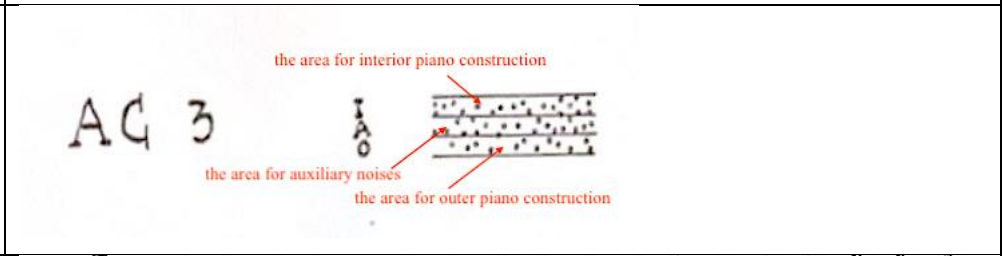
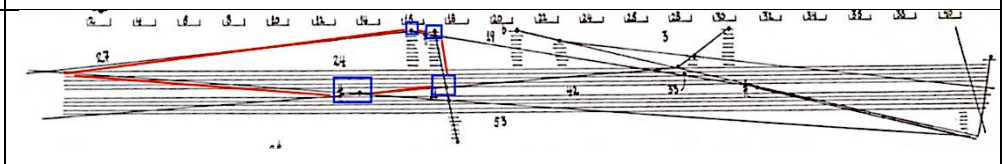
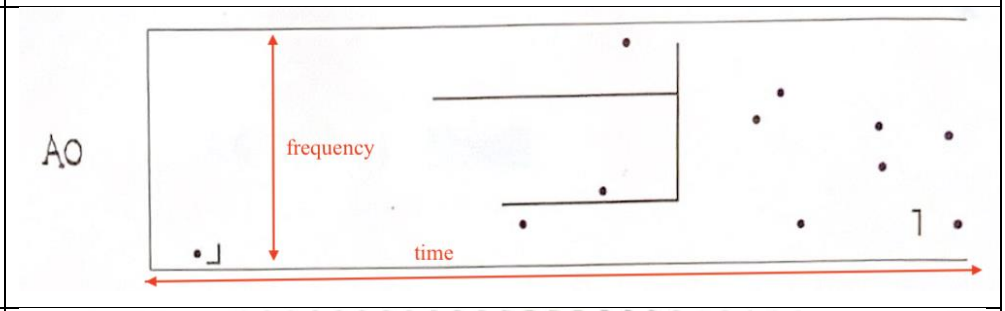
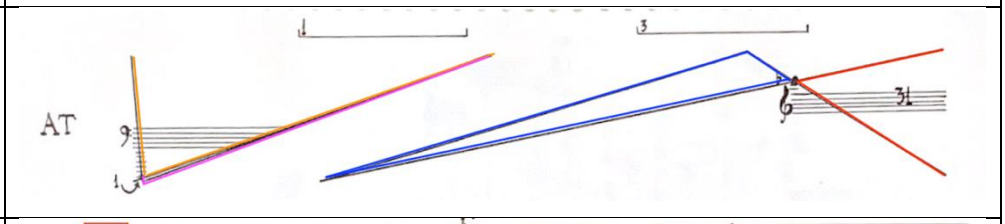
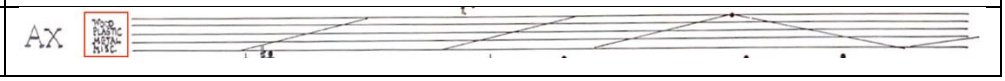
Notation	Presentation of area(s)
Notation AC	
Notation AE	
Notation AO	
Notation AT	
Notation AX	

Table 17 Different ways of using areas in notations.

The final common compositional material Cage used is dots (Table 18). As mentioned previously, this needs to be understood alongside other materials. In Notation AC dots indicate the positions and timing of noises, however, without the areas pointing out I, A, and O, initial letters meaning interior piano construction, auxiliary noises, and outer piano construction, the dots cannot be understood. A similar use of dots can be found in Notation AL, where they show both the attack timing of notes and the duration of notes of the chosen notes. Notation AO uses dots as short single tones, meaning that one dot represents a single chosen note; performers need to comprehend the dots through vertical space (frequency) and horizontal space (time), to realise the notes. Notation AX uses dots as in Notations AC and AL, to indicate the positions and timings of noises.

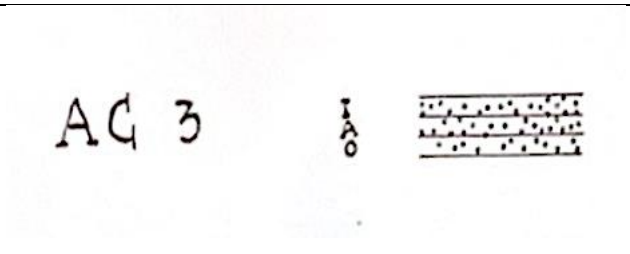
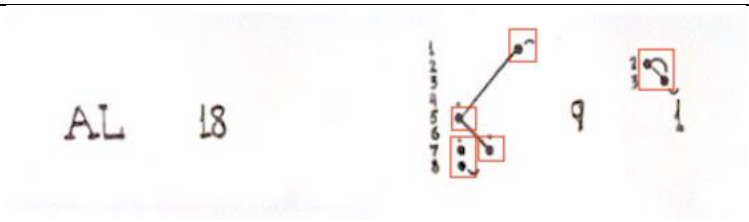
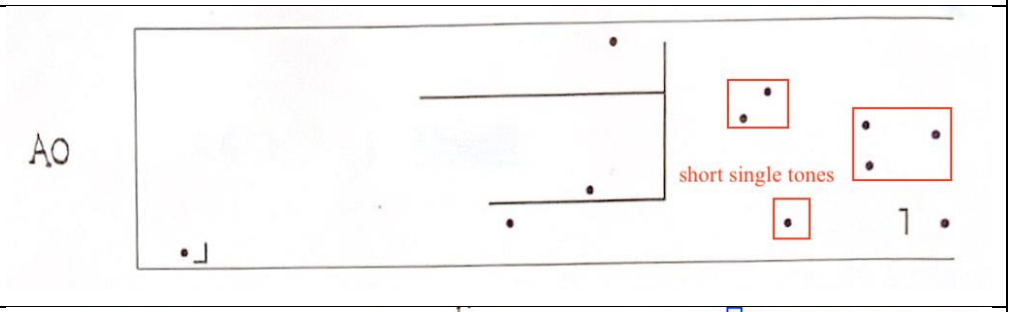
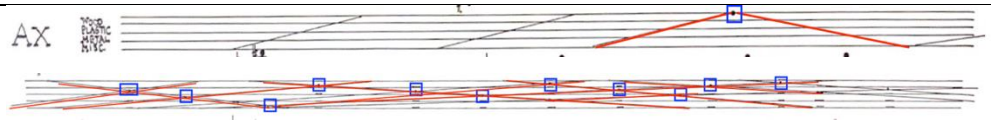
Notation	Presentation of dots
Notation AC	
Notation AL	
Notation AO	
Notation AX	

Table 18 Different ways of using dots in notations.

Alongside the compositional materials in Section One, Cage also used other compositional materials, such as intersections, in Section Two. Notation AU is his first notation use of intersections between Notations AA to AZ. Similar to Notations M and Q, notes here are placed at intersections, therefore performers need to decide whether to maintain, or change, the performing direction. For instance, intersections in AU indicate decision-making regarding performing directions. However, in AU, the intersection notes are ambiguous, not only because of the indeterminate performing directions, but also because of ambiguous clef signs. Although this compositional material does not commonly apply in Notations A to Z (he only used it in Notations M and Q), it can be found in AU serving the same function. This demonstrates the first and the second type of relationships in Notations M, Q, and AU (Table 19).

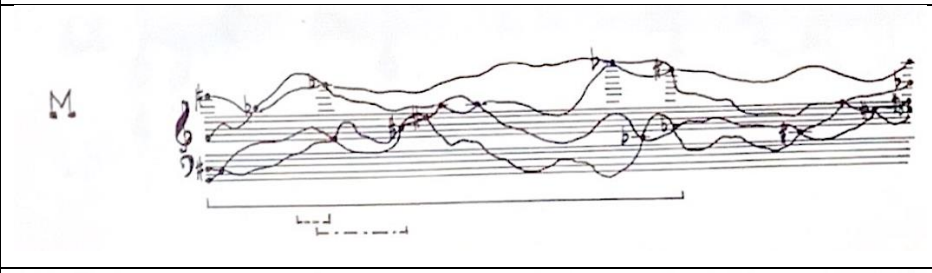
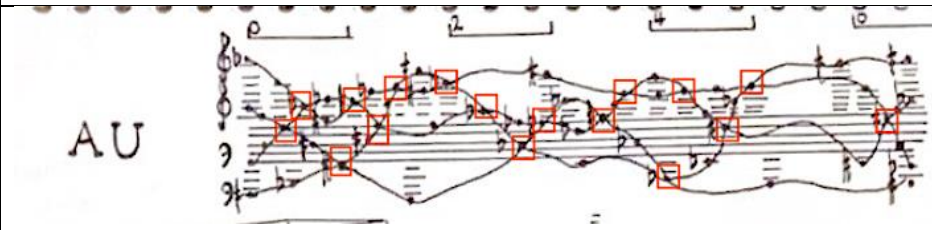
Notation	Presentation of intersections
Notation M	
Notation Q	
Notation AU	

Table 19 The uses of intersections in notations.

The first type of relationships building on compositional materials – numbers, shapes, arrows, areas, and dots – are observable across Notations A to AZ, which demonstrates an inner notation connecting structure. Numbers have been used in twenty-three notations – Notations A, B, D, F, G, J, L, Q, R, T, U, Y, Z, AB, AC, AE, AJ, AK, AL, AT, AU, AV, and AZ. The functions of this compositional material indicate: (i) different degrees of dynamics, (ii) showing the numbers of playing noises, (iii) time, and (iv) chosen notes.

Cage used lines to (i) show pitch limits/areas, (ii) group notes together, (iii) divide areas, and (iv) indicate noises in Section One, while in Section Two he added functions of (v) demonstrating the performing time, (vi) show the performer performing directions, (vii) differentiating the duration of notes and intervals from the different sections, (viii) the performing approach of notes, and (ix) delimit the notation. The speciality of lines is that one notation may use lines in two different ways, this can be seen in Section Two. For instance, Notations AH and AI both use lines to connect notes and to show performing directions; Notations AJ and AN both show the performing directions and pitch limits. Thus, one compositional material carries two functions and is applied in two notations to demonstrate potentially a second type of relationship. Along with the second type of relationships, lines construct the first type of relationships among Notations J, O, R, S, U, V, W, Z, AA, AB, AE, AF, AH, AI, AJ, AK, AM, AN, AO, AP, AQ, AT, AU, and AZ.

Shapes are used in Notations A, E, G, L, K, T, AA, AB, AI, AR, and AV the functions of which include showing, (i) notes (ii) pitch areas/limits, (iii) performing approach, (iv) time, (v) duration, (vi) grouping notes, (vii) emphasis on notes, and (viii) suggestions for potential performing directions. This compositional material shows eight functions among eleven notations. Arrows, which he applied to Notations D, G, I, J, N, S, AH, AJ, and AT, whose functions he used for (i) directing pitch, (ii) directing time, (iii) showing performing directions, and (iv) acting as indicators. Among these, functions of showing performing directions and acting as indicators have been applied in both notation sections. In this way, Cage built the first type of relationships across nine notations and enhanced the relationships between Notations I, J, N, S, AH, AJ, and AT through the second type of relationships.

Cage applied areas in Notations O, P, T, U, V, Y, AC, AE, AO, AT, and AX. The functions include (i) showing the performing approach – Notations O, AC, and AX, (ii) showing the degree of loudness – Notations P, V, and Y, (iii) delimiting pitch areas – Notations T, U, AE, and AT, (iv) guiding the performing speed – Notation V, and (v) showing the frequency and duration of single notes – Notation AO. When comparing the functions in both notation sections, I notice that Cage applied the function of showing the performing approach and the function of delimiting pitch areas in both notation sections, and that he only used the functions of 'showing the degree of loudness' and 'guiding the performing speed' in Notation Section One, and that the function of 'showing the frequency and duration of single notes' is only applied in Section Two. This pattern seems to show us that Cage's function arrangements may be not throughout the whole musical piece, but that they change within it.

Dots are used in Notations P, S, V, Y, AC, AL, AO, and AX. The major difference between dots and other compositional materials is that dots appear to be meaningless without supporting compositional materials, where they are used to point out, (i) the timing of sounds, (ii) the levels of loudness, (iii) the level of speed, (iv) to indicate ways of producing noises, and (v) to indicate short single tones.

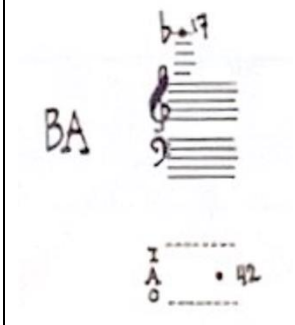
3.1.3 Compositional Materials in Notation Section Three

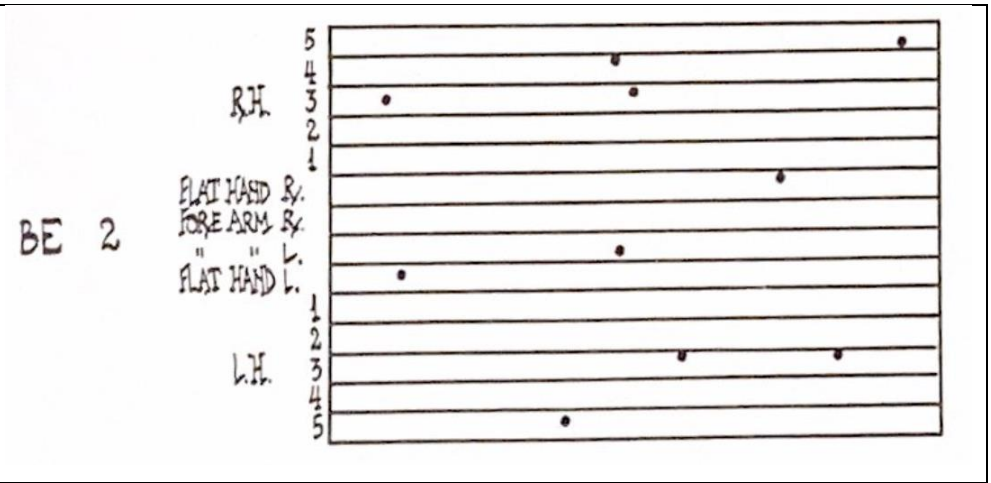
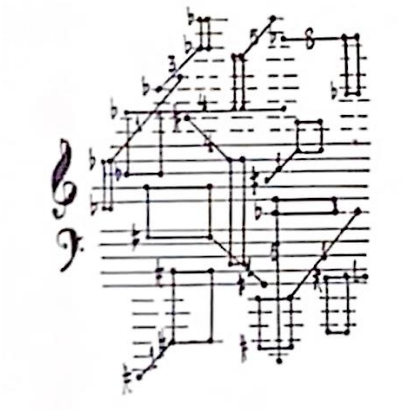
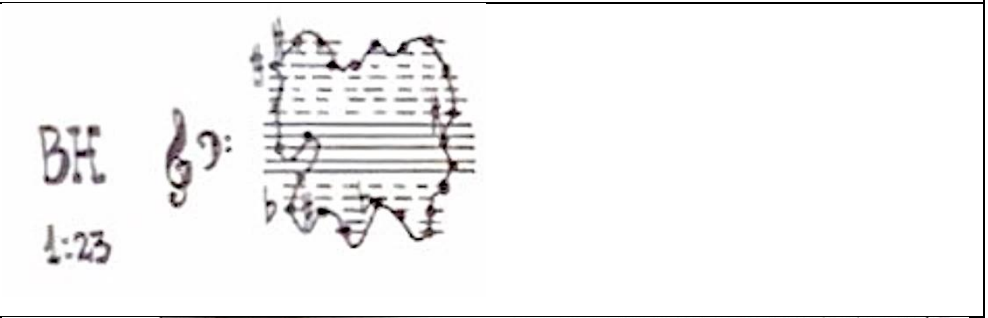
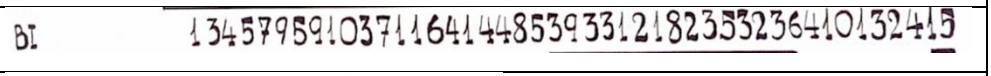
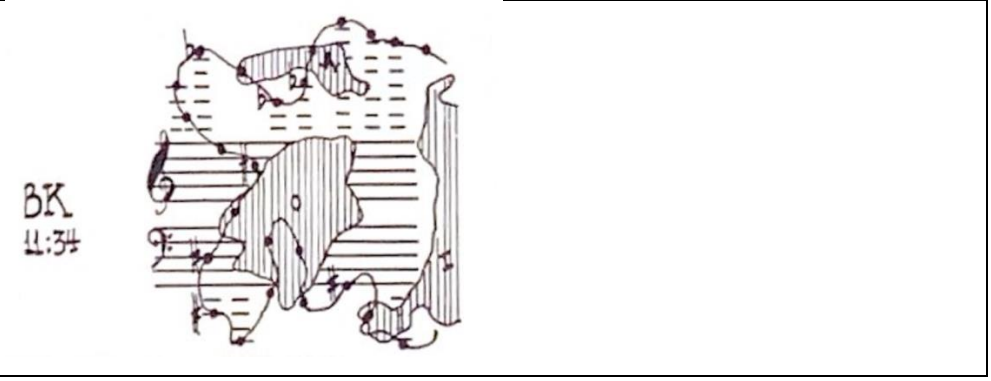
This section examines the use of numbers, lines, shapes, areas, and dots from Notations BA to BZ. The first compositional materials, numbers, Notation BA uses numbers to indicate the degrees of loudness for notes and noises. Notation BC uses numbers to show performing

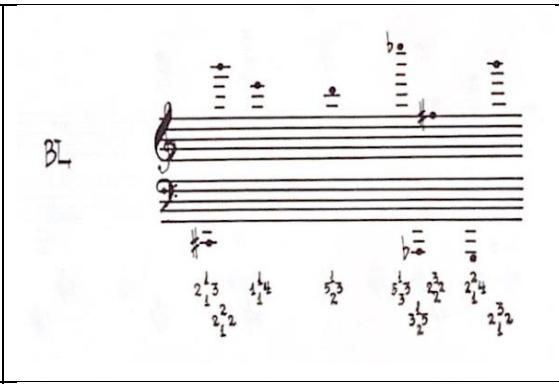
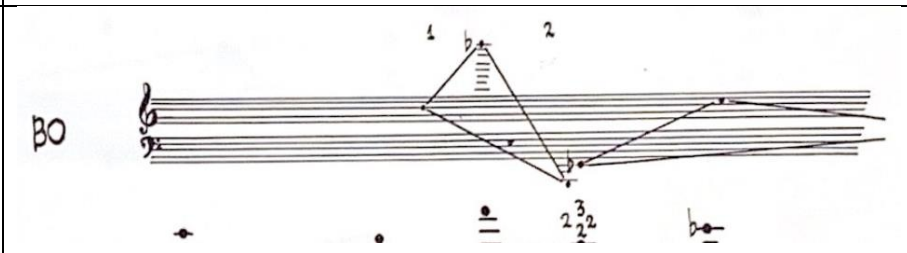
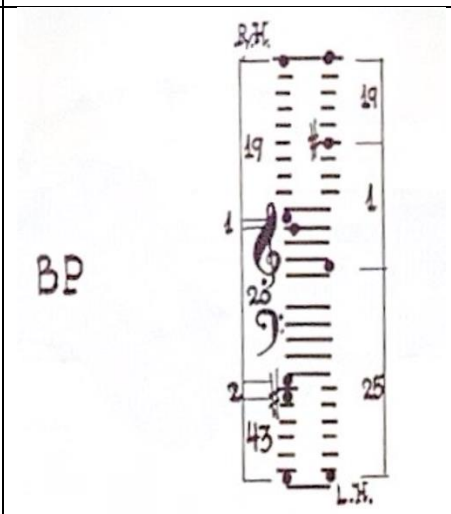
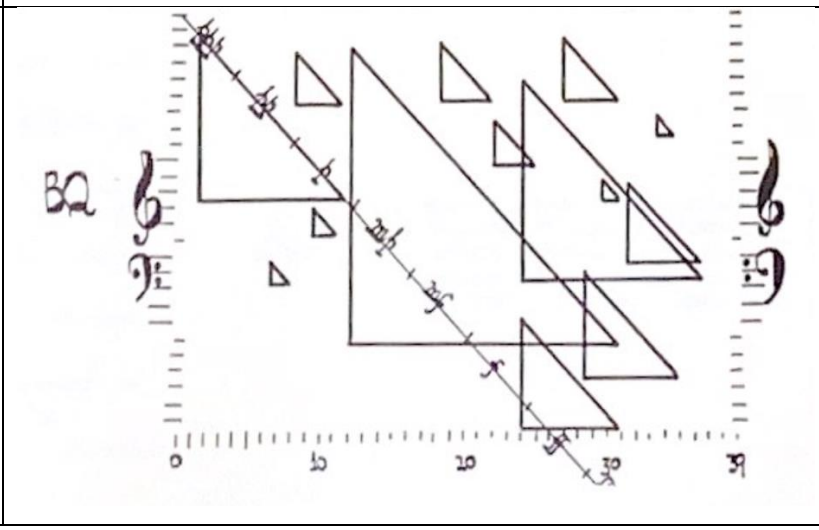
numbers of notes and represent the fingers; the function of showing performing numbers of notes can be found in Notations BE, BG, BH, BK, BL, BP, BR, and BU (Table 20).

The number's secondary function is to indicate time; these can be seen in Notations BO, BQ, and, BI. In BO, the numbers are listed above the staff; the distances between numbers are different, and performers should consider the distance to decide how they should perform the notes between each number. Dissimilar to Notation BO, BQ shows the following numbers – 0, 10, 20, 30, and 39 – with scales; performers need to use the scales and numbers in order to determine the performing time and other quality of notes. The last notation with this function is BI, which also brings out the final function of numbers: showing frequencies. Cage did not apply notes or points to formulate Notation BI, rather, he chose to apply serial numbers to indicate performing timing and frequencies. Consequently, performers need to decide how to use the serial numbers (the possible realising process was discussed in the analysis of Notation BI). Based on the position of numbers and the performers' arrangements, the duration and the frequencies of notes can be varied.

In summary, numbers are applied in Notations BA, BC, BE, BG, BH, BI, BK, BL, BO, BP, BQ, BR, and BU. These notations form the first type of relationships, while Notations BC, BE, BG, BH, BK, BL, BP, BR, and BU form the second type of relationships by using the function of showing performing numbers of notes; Notations BO, BQ, and, BI also form the second type of relationships among them through the function of indicating performing time.

Notation	Presentation of numbers
Notation BA	

Notation BE	
Notation BG	
Notation BH	
Notation BI	
Notation BK	

Notation BL	
Notation BO	
Notation BP	
Notation BQ	

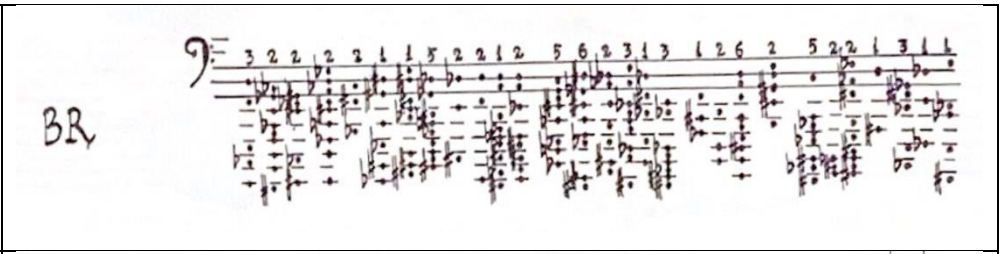
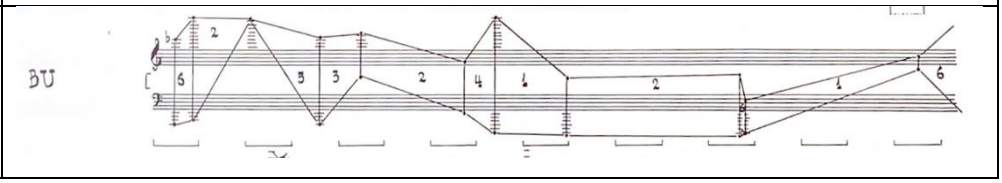
Notation BR	
Notation BU	

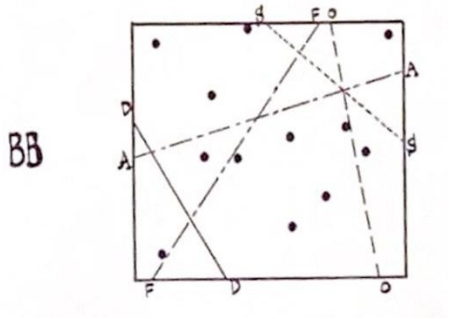
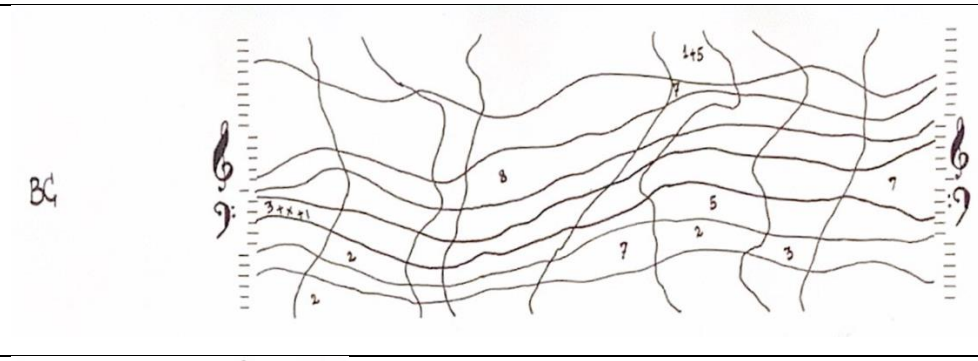
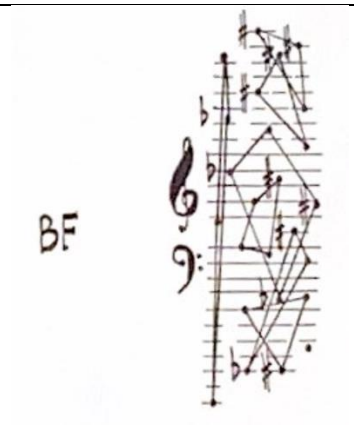
Table 20 Different ways of applying numbers in notations.

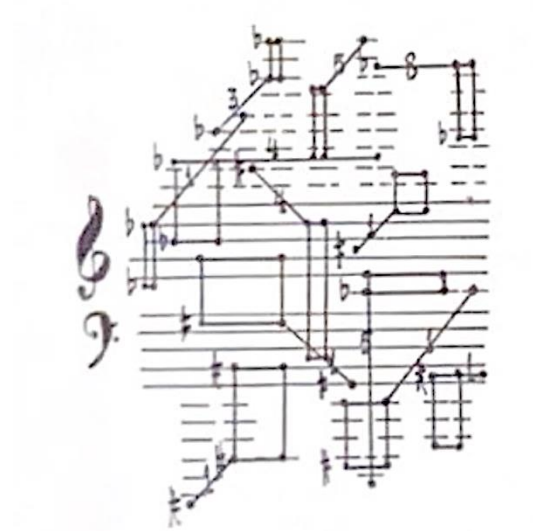
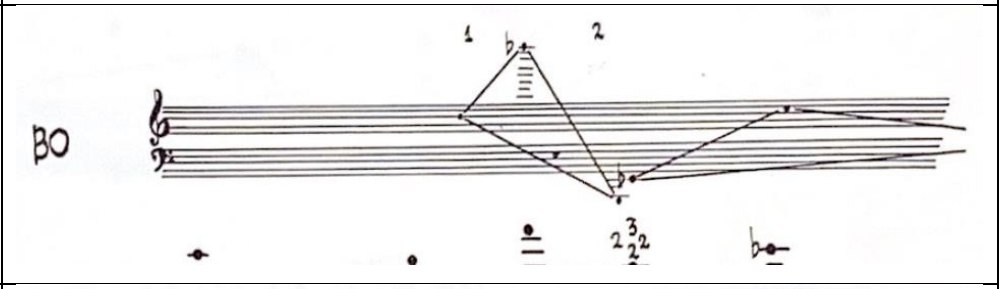
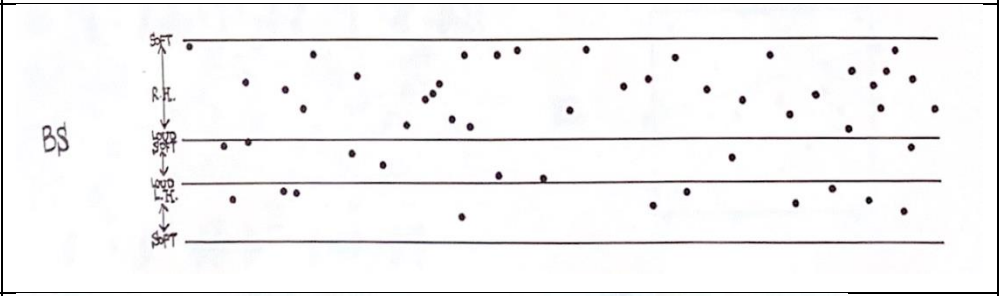
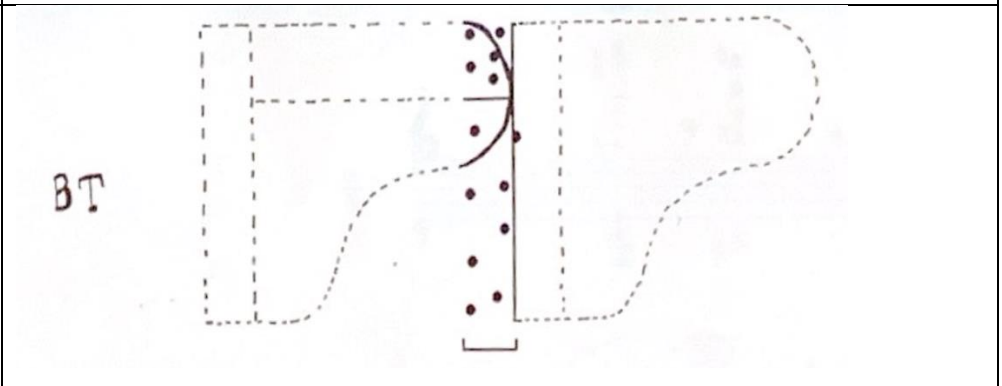
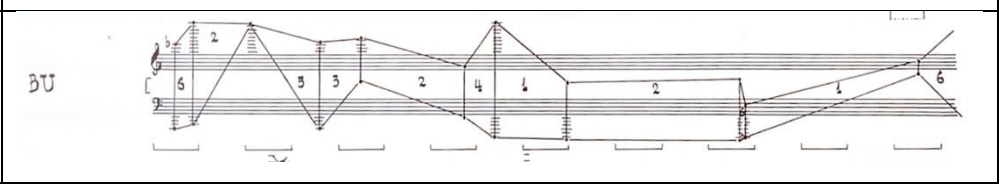
The next compositional material is lines that are applied in Notations BB, BC, BF, BG, BO, BS, BT, BU, BV, BX, and BZ (Table 21). The first notation in this section is BB, which applies lines that show the quality of sounds, such as, duration, frequency, overtone structure, amplitude, and occurrence (succession); here the performers need to apply them to obtain conditions of the sound qualities;[356] therefore, the function of the lines in this notation is to show the quality of sounds. Notation BV applied the lines in the same way as BB. Lines in Notation BC show another use of them: showing pitch areas. The same use can be found in Notations BG and BU. Notation BS used lines to divide the area in order to indicate the quality of sounds for both hands; thus, by referring to the positions of dots in the dynamic areas, performers will obtain the relative relationships between notes as regards dynamics. Notations BF and BO show similar uses to this compositional material as Notations R, W, Z, and AB, which is grouping notes together.

The lines in Notation BF are used for differentiating legato and staccato and indicating superimpositions of notes. When two notes are connected by a line, the notes should be performed as legato; on the other hand, when a note is isolated, it should be performer as staccato. The lines in Notation BT uses the shapes of pianos with dots and solid lines, which point out which part of piano construction the performers can use in their realisations. The lines in Notation BX are unique. Cage used them to illustrate the image of a plant. For this notation he only stated that the performer should express this notation like a moment of a plant, meaning they have the freedom to decide the quality of sound, but they have to present the acoustic

[356] For the approach of obtaining the conditions, refer to Notation BB in Section 2 that Analyses the Compositional Materials in Chapter 5.

effects ‘like a plant’. The last notation in this section is BZ, where the lines indicate where the performers should follow the ups and downs of the pedals.

Notation	Presentation of lines
Notation BB	
Notation BC	
Notation BF	

Notation BG	
Notation BO	
Notation BS	
Notation BT	
Notation BU	

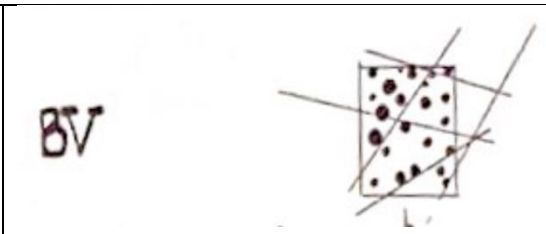
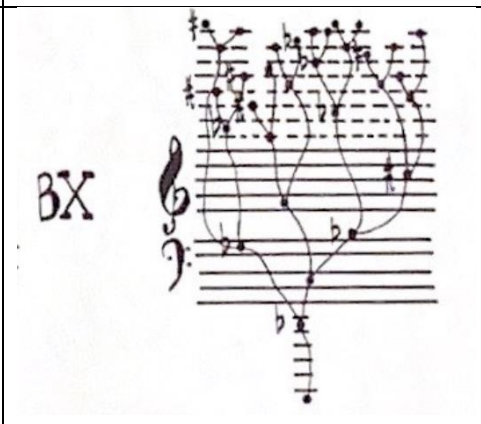
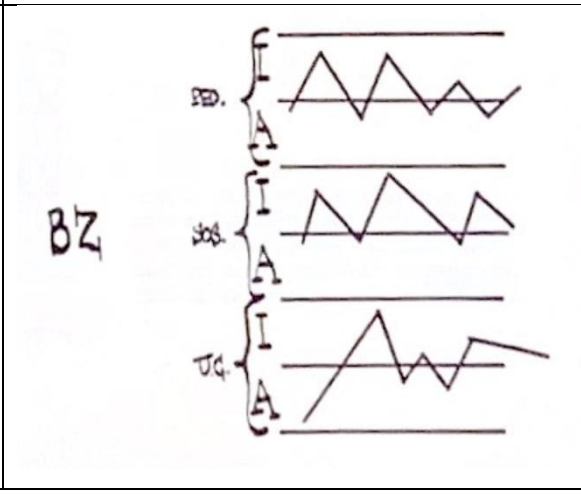
Notation BV	
Notation BX	
Notation BZ	

Table 21 Different ways of using lines in notations.

The notations that use shapes include BH, BN, BK, and BT. In BH, BN, and BK, the shape shows directions and sequences, as in Notation A. The shape in BT shows an image of a piano; as explained in the section of lines' functions, the shape in BT is used to indicate the applicable part of the piano construction (Table 22).

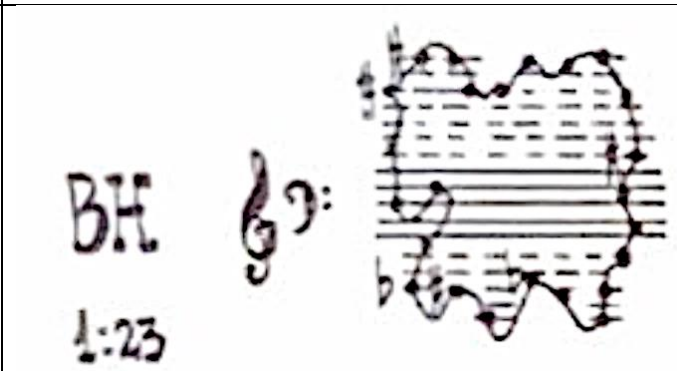
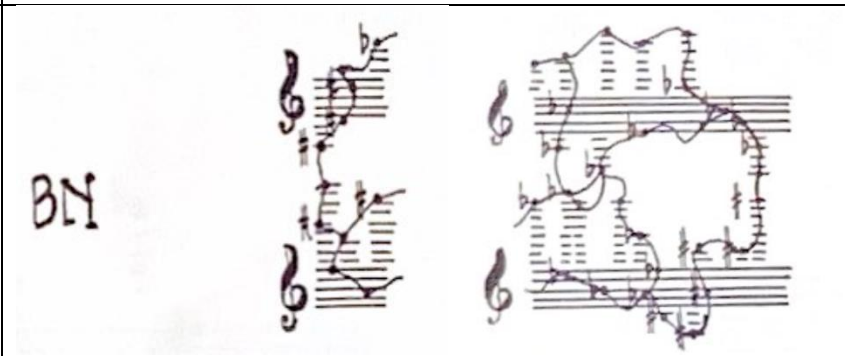
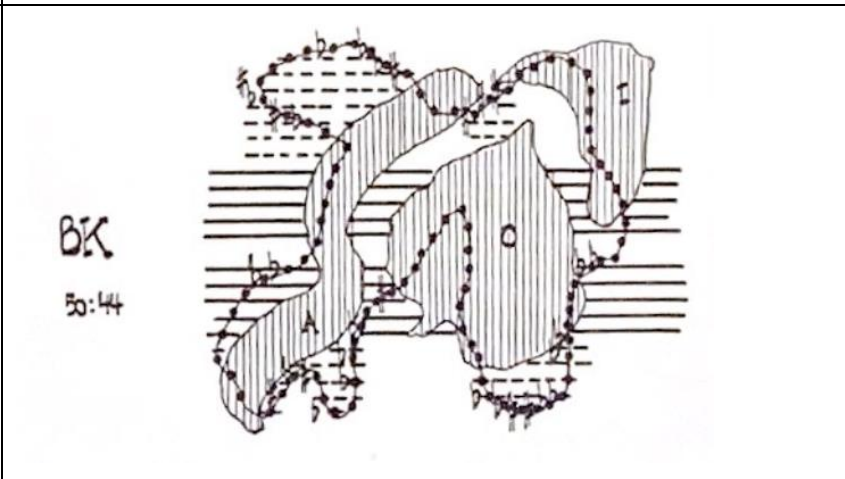
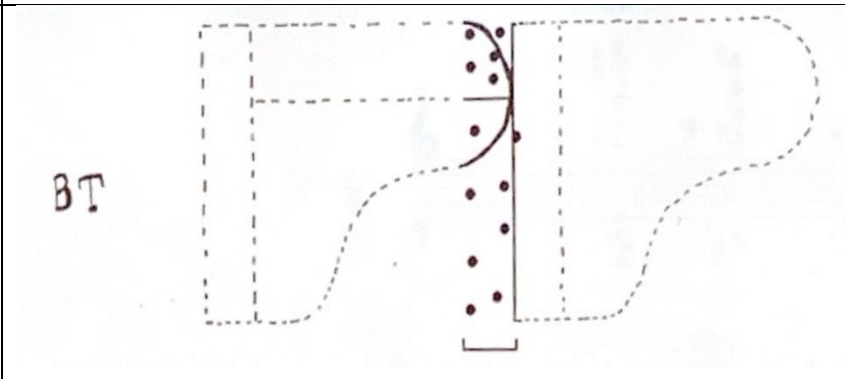
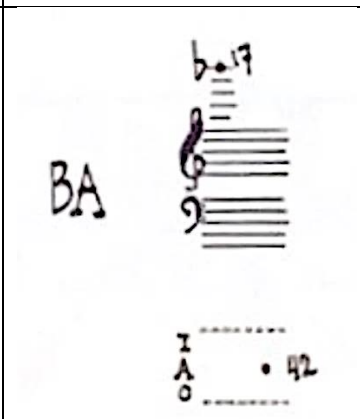
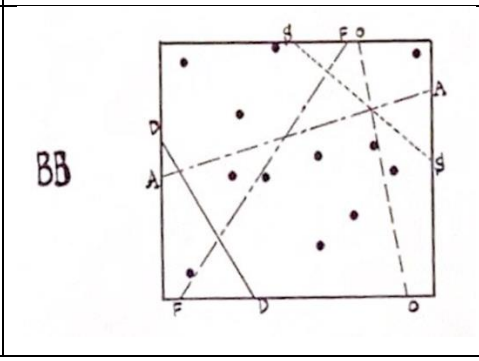
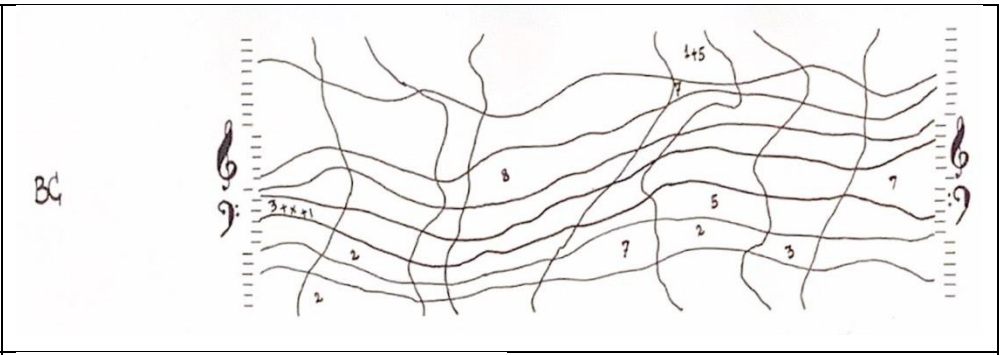
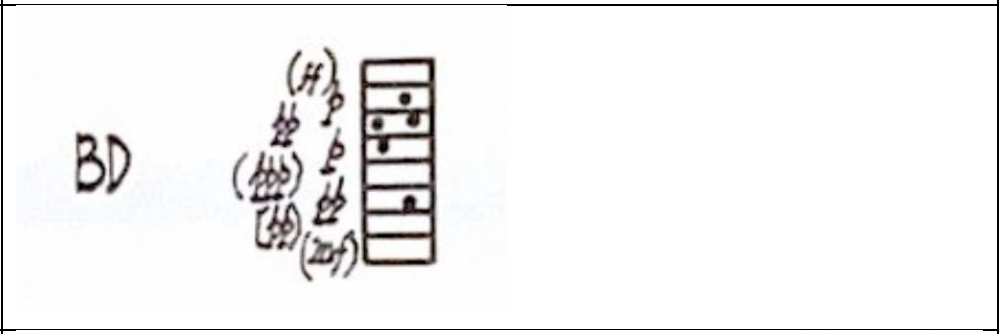
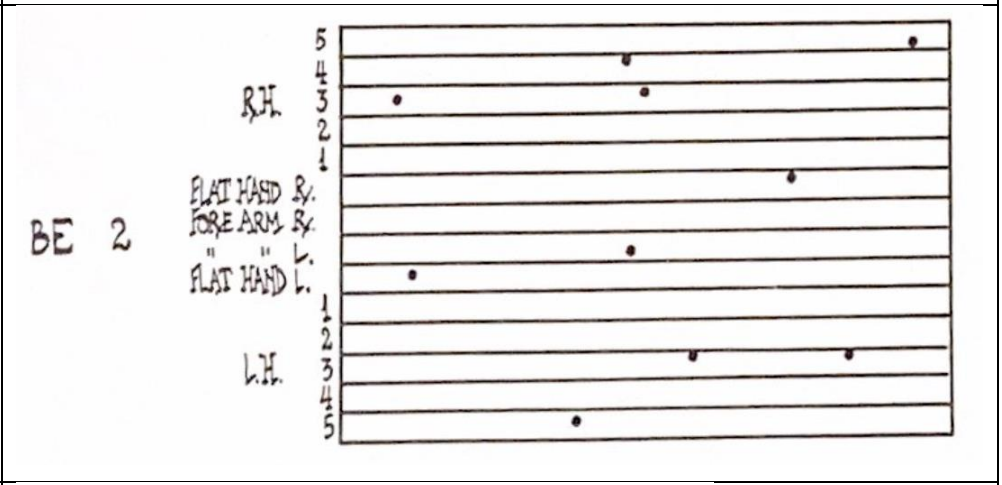
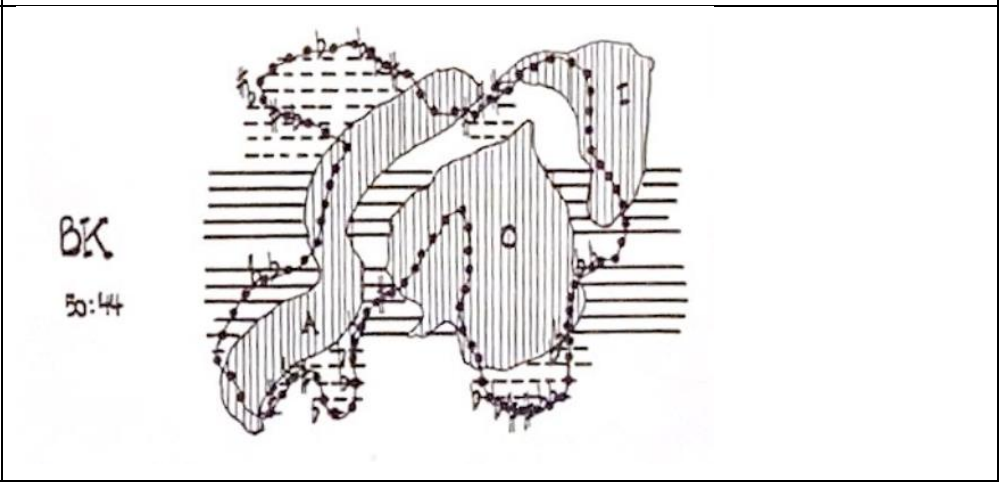
Notation	Presentation of shape(s)
Notation BH	
Notation BN	
Notation BK	
Notation BT	

Table 22 Different types of shapes in notations.

Areas have been applied in Notations BA, BB, BC, BD, BE, BK, BS, BJ, BU, BV, and BY (Table 23). Areas in Notation BA demonstrate the same meaning, as in Notation AC, which uses areas to differentiate different ways of producing noises. A similar function can be found in Notation BK, but with different representations. Two types of areas are found in Notation BK, one that is white and the other is coloured shapes; in this notation, the performers need to follow the abbreviations, A, I, and O, to produce sounds among the indicated notes. The next use of this compositional materials shows the pitch area, which can be found in Notations BC and BU, in both of which Cage applied areas to offer choices of notes. Notations BD and BS present another use of areas in which he clearly stated different levels of dynamics; Notation BD it is *ppp* to *ff*, while BS is soft to loud, and loud to soft. Notation BE uses a similar presentation style to BD to show the chosen notes in grids, which means the performers should read the grid for their chosen notes to proceed with their realisations. The final function of areas is in Notations BB, BJ, BV, and BY, where Cage delimited an area by applying the boundaries of the area in order to show the quality of sounds or applying different dots in a certain area to process the measurement.

Notation	Presentation of areas
Notation BA	
Notation BB	

Notation BC	
Notation BD	
Notation BE	
Notation BK	

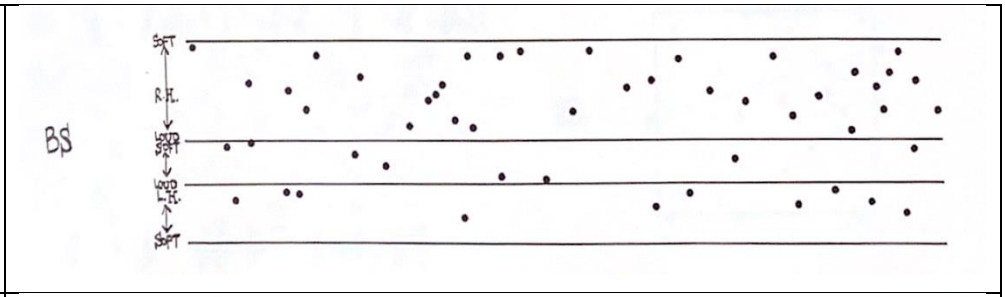
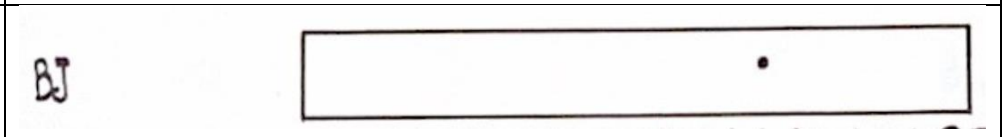
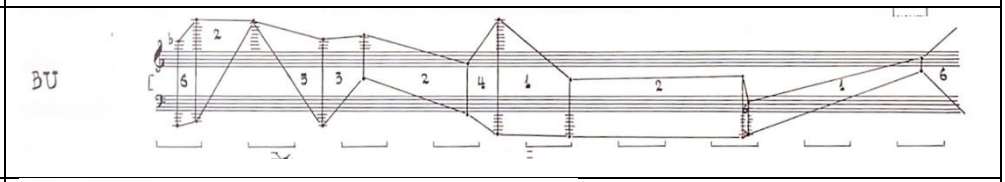
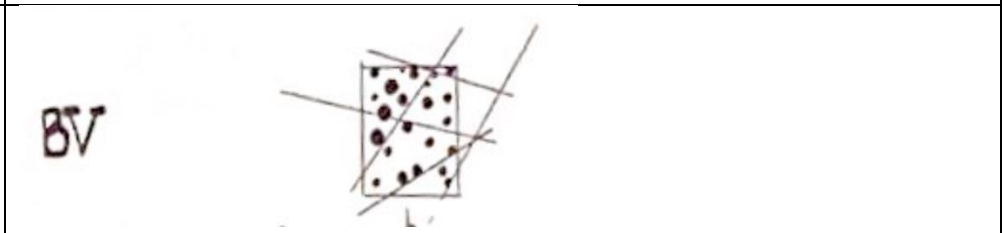
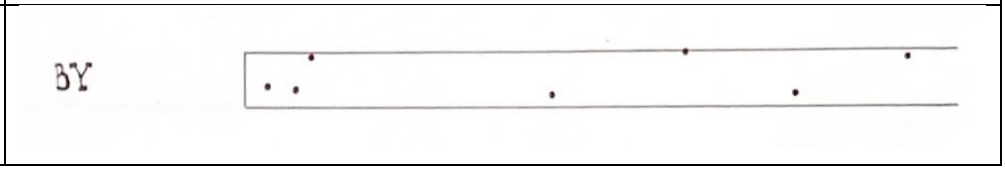
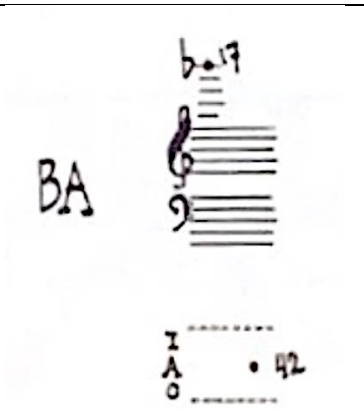
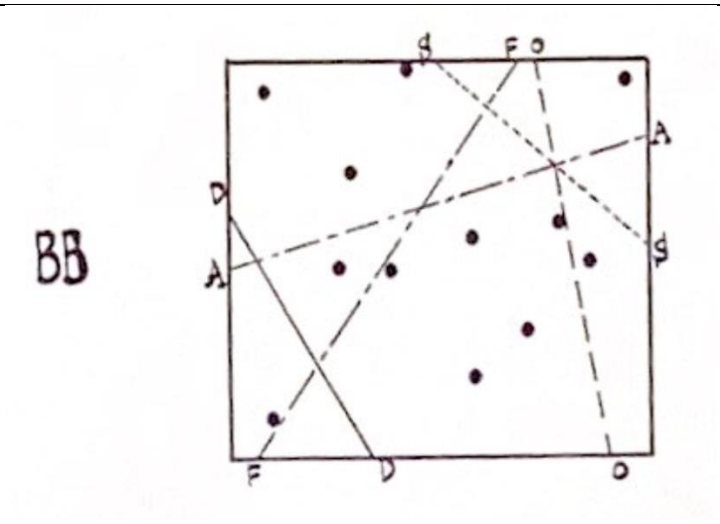
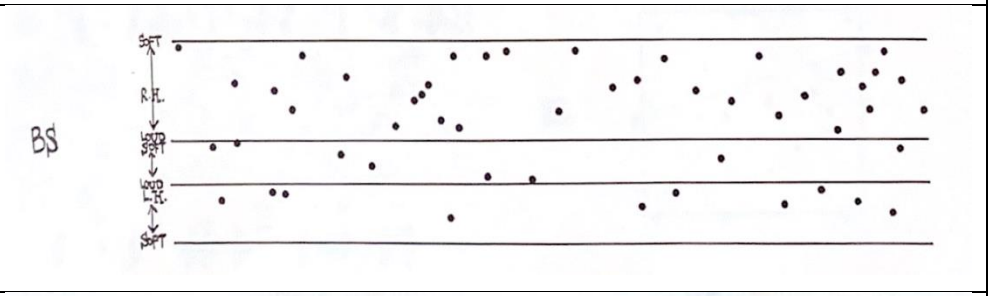
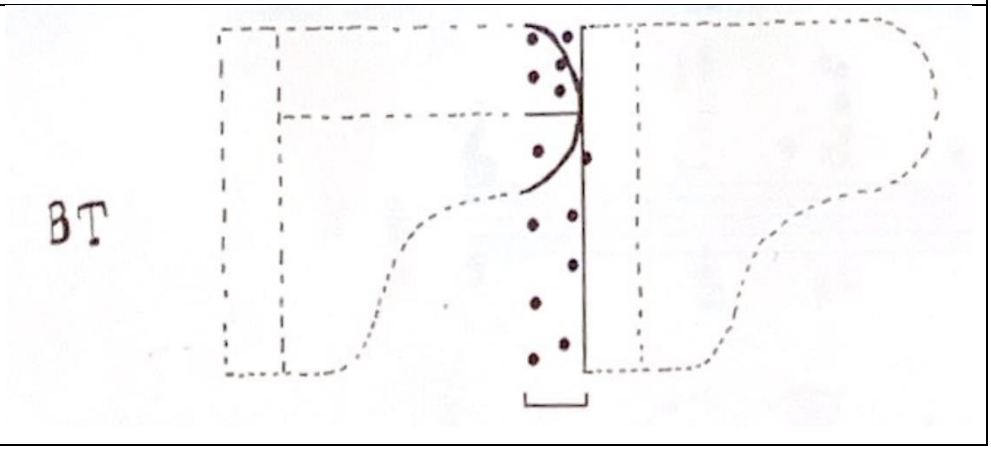
Notation BS	
Notation BJ	
Notation BU	
Notation BV	
Notation BY	

Table 23 Different uses of areas in notations.

The last compositional material of this notation section is dots. Cage used it in Notations BA, BB, BS, BT, BV, and BY. Their functions include: showing the performing approaches, showing the timing of noises, the demonstration of single sounds, and the presentation of different amounts of sound events. Notations BA and BT apply the first function to indicate the sounds should be performed through which part of piano construction or auxiliary tools. The second function can be discovered in Notations BA, BS, and BY. Notation BB applies this compositional material to demonstrate single sounds, while the same function can be found in Notation BY; however, Notation BY also shows the timing of noises and pitch. Notation BV shows the final function, in which large dots, lesser dots, smaller dots, and very small dots are used to present different numbers of sound events (Table 24).

Notation	Presentation of dots
Notation BA	
Notation BB	
Notation BS	
Notation BT	

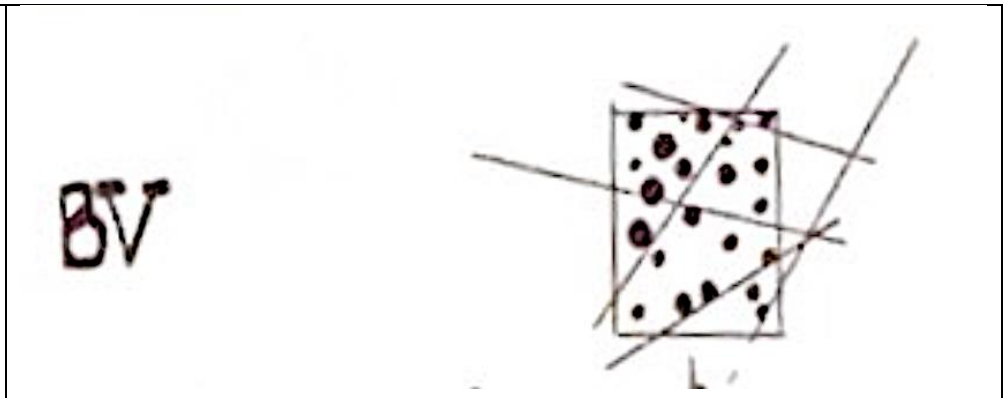
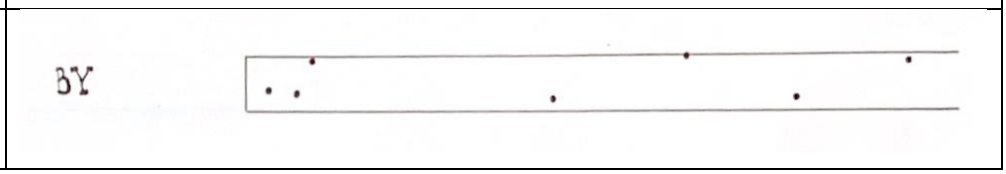
Notation BV	
Notation BY	

Table 24 Different uses of dots in notations.

3.1.4 Compositional Materials in Notation Section Four

This section examines the uses of numbers, lines, shapes, areas, and dots in Notation Section Four, from Notations CA to CF (Table 25). Numbers are used in Notations CB and CC, which use numbers in two different ways; CB uses numbers to present the numbers of performing notes, while CC uses numbers to show time.

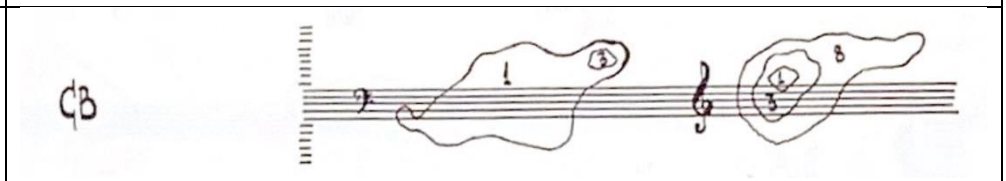
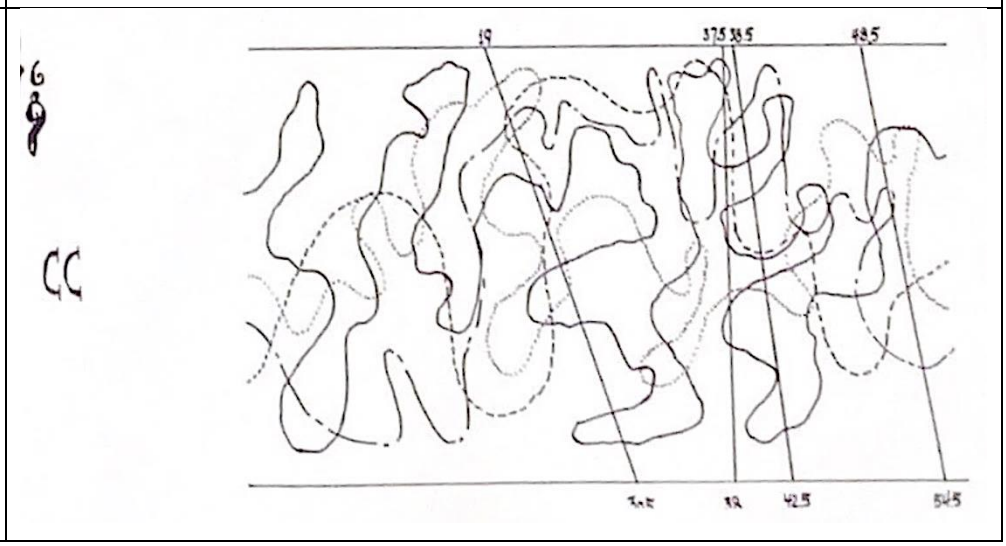
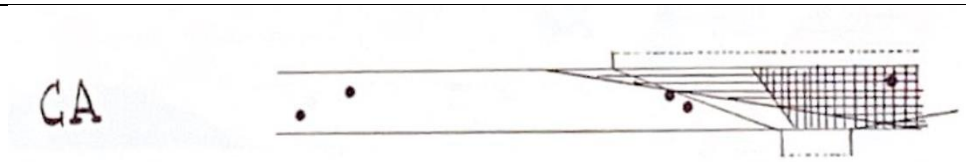
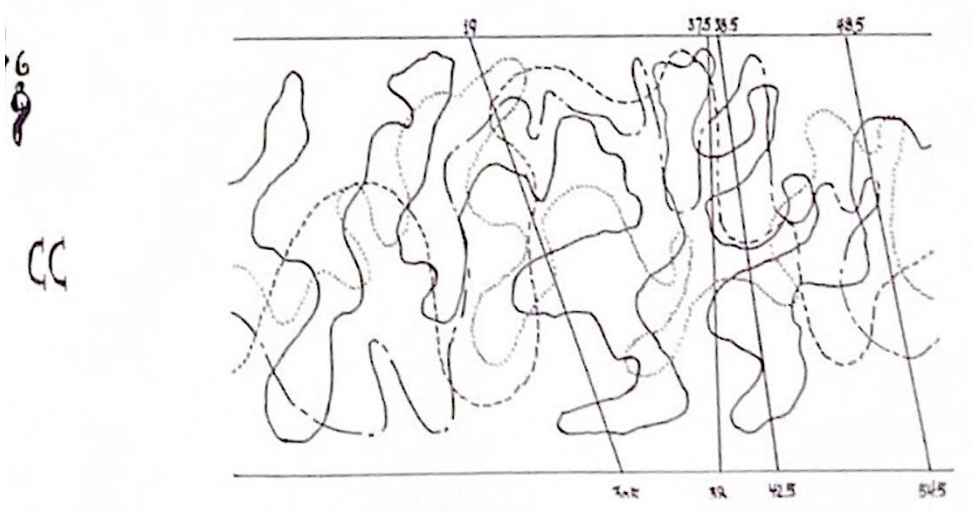
Notation	Presentation of numbers
Notation CB	
Notation CC	

Table 25 Different uses of numbers in notations.

Lines are applied in Notations CA, CC, and CF. In Notation CA, the lines are used to demonstrate different ways of performing notes. For instance, in the area of vertical lines, the notes should be mute; in the area bracketed by dotted lines, the notes should be pizzicato; and in the area with horizontal lines, the notes should be performed as friction. With this type of use, the boundary between lines and areas is blurred; the two compositional materials in this notation seem to assist each other to guide the performers. In Notation CC, four types of lines represent frequency, duration, amplitude, and overtone structure, and the performers are free to decide the meaning of each line. Lines in Notation CF serve the same function as those in Notation BZ, indicating the movement of the pedals. Thus, lines in Notations CA, CC, and CF represent three functions: showing performing approach, representing the quality of sounds, and indicating movements of pedals (Table 26).

Notation	Presentation of lines
Notation CA	
Notation CC	

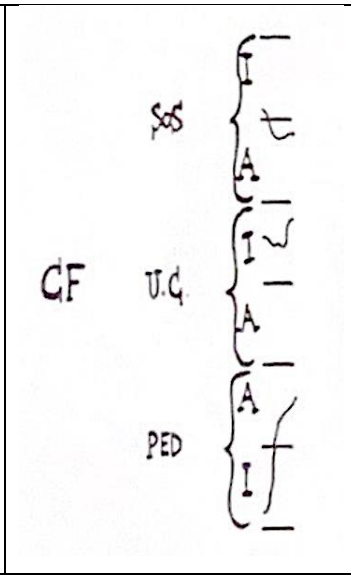
Notation CF	

Table 26 Different types of lines in notations.

Shapes is only applied in Notation CB, which uses shapes to indicate the pitch areas, as the same as the areas in this notation. Areas can be found in Notations CA and CB. As mentioned above, CA applies lines to constitute different areas in order to show different performing approaches. CB uses areas as pitch areas, meaning that performers should choose the given numbers of notes within the assigned areas (Table 27).

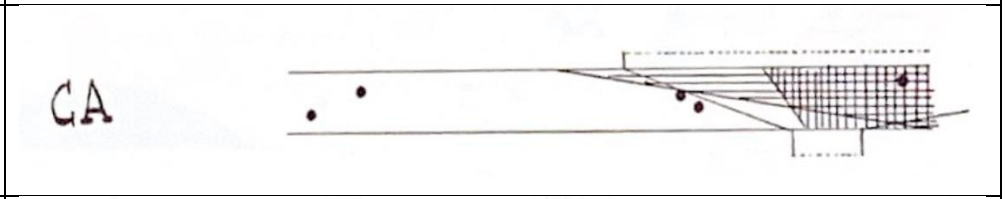
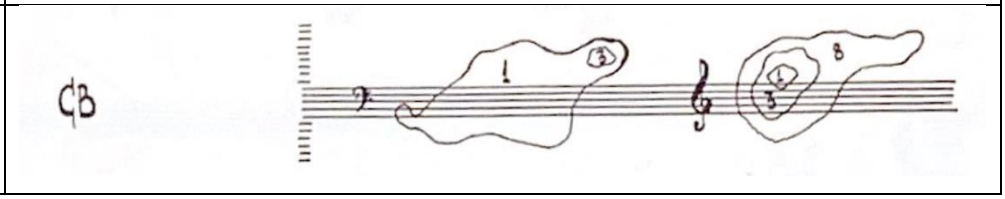
Notation	Presentation of areas
Notation CA	
Notation CB	

Table 27 Different types of shapes and areas in notations.

Dots are applied in Notation CA (Table 28), where they indicate notes that should be realised in the allocated areas. For instance, four dots are located in the white area, meaning that performers should play four notes with the keyboard. One dot is indicated in the area by vertical and horizontal lines, the area is also bracketed by dotted lines, meaning that the performers may realise the dot as mute, pizzicato, or friction.

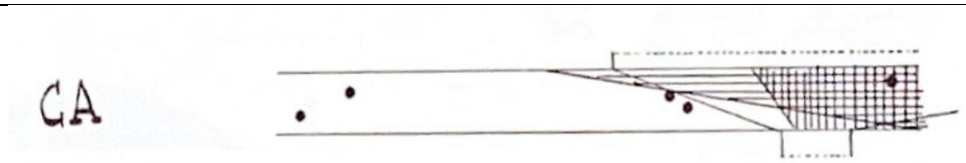
Notation	Presentation of dots
Notation CA	

Table 28 The use of dots in notation.

It is noticeable, from the above discussions, that Cage's uses of compositional materials are varied; for instance, one may carry two or more functions and include different ways of presenting. With the diverse uses of compositional materials, it seems that the *Solo for Piano* is far away from rational features, since rational features of European art music, which are: (i) methodical structure based on harmonic and melodic progression, (ii) affectual functions in intervals and chords, and (iii) the free interpretation of musical works. Delving into the three rational features, I suggest, these rational features are from consistent uses of relevant musical elements in a musical work. As Thomas suggests, different levels of indeterminacy show degrees of conformity within notations, as well as lead the performers to consider various aesthetic needs in their realisations.[357] Hence, I propose that the six compositional materials are the fundamental materials for constructing the rational features within this piece. Among eighty-four types of notations, I conclude that Cage used six compositional materials – numbers, lines, shapes, arrows, areas, and dots – frequently in the *Solo for Piano*, thereby creating the sense of consistency and potential rational features.

Numbers, Cage used this compositional material for showing the numbers of playing notes in Notations A, B, D, G, J, L, U, AC, AE, AJ, AT, AU, BC, BE, BG, BH, BK, BL, BP, BR, and BU; in Notations F, Q, R, Y, AE, AK, AL, AT, AZ, BO, BQ, and, BI, numbers indicated time, while in Notations T, Z, AB, AV, and BA the numbers is to show the degree of loudness of sounds. Two additional function appears in Notation BI and Notation BE; for Notation BI,

[357] Thomas, 'Understanding Indeterminate Music through Performance', 110–11.

Cage applied numbers to show frequency, while in Notation BE he also applied numbers to represent which fingers should be used in realisations.

Lines, he used this compositional material to: (i) show pitch limits/areas, (ii) group notes together, (iii) divide areas, (iv) indicate noises, (v) show performing directions, (vi) demonstrate performing time, (vii) present performing approaches, (viii) act as reference objects for measurements, (ix) show imagine and performing suggestions, and (x) indicate the movements of pedals. He applied lines to show pitch limits and pitch areas in Notations J, O, U, AA, AE, AJ, AK, AN, AT, BC, BG, and BU; he also used them for grouping notes together in Notations O, R, W, Z, AB, AF, AH, AI, AZ, BF, and BO. To divide areas, he used them in Notations S, AM, and AQ. To indicate noises, he used them in Notations V and AX. For directions he used them in Notations M, Q, AH, AI, AJ, AN, and AU. To demonstrate performing time, he used them in Notations AA, AO, and AP. To assign performing approaches he applied them in Notations AO, AZ, and CA. To act as reference objects for measurements he used them in Notations BB, BV and CC; and, to show imagine for performing suggestions he included them in Notations BT and BX, and to indicate the movement of pedals, he applied them in Notations BZ, and CF.

Shapes, the third compositional material, which Cage applied to Notations A, E, G, K, L, T, AA, AB, AI, AR, AV, BH, BN, BT, BK, and CB. The functions of shapes are diverse, in that they involve eight elements: (i) directions, (ii) sequences, (iii) groups of notes, (iv) pitch ranges, (v) time, (vi) differentiating the degrees of emphasising notes, (vii) showing performing approaches. For showing performing directions he applied them in Notations A, L, AR, AV, BH, BN, and BK; for showing performing sequences they can be found in Notations A, L, AI, BH, BN, and BK; for grouping notes together can be found in Notations A, E, G, L, W, and Z; for presenting pitch limits he used them in Notations T, AR, AV, and CB; for showing duration of notes he applied them in Notations T, AR, and AV; for indicating performing notes he used them in Notation K; for differentiating degrees of emphasising notes they appear in Notation AA, and to show the performing approach of notes he applied them in Notations AB, AR, AV, and BT.

Arrows, the next compositional materials, the functions of which involve: (i) the relationship between notes in terms of pitch, (ii) the relationship between notes in terms of time, (iii) performing directions, (iv) directions of dynamics, and (v) indications. They are found in

Notations D, G, I, J, N, S, T, AH, AJ, and AT; they are not used in Notations BA to CF. The first two functions be found in Notation D, which is also the only notation that uses arrows in this way. The third function, showing a performing direction, is applied in Notations G, J, AH, and AJ; the fourth function, showing the directions of dynamics, is applied in Notation G; the final function as indications can be found in Notations I, N, S, T, and AT.

Areas, the next compositional materials, which he applied to Notations O, P, T, U, V, Y, AC, AE, AO, AT, AX, BA, BC, BD, BE, BJ, BK, BS, BU, BY, CA, and CB. After deconstructing the notations, I have concluded that their functions are, (i) for assigning performing approach to particular areas, (ii) to show levels of loudness, (iii) to show pitch areas, (iv) to show different ways of producing noises, (v) to show the quality of sounds, and (vi) to present the chosen notes. They perform the function of: assigning performing approaches to certain areas, as in Notations O and CA; showing the level of loudness as in Notations P, V, Y, BD, and BS; showing pitch areas as they are applied in Notations T, U, AE, AT, BC, BU, and CB; producing noises in different ways, as used in Notation AC, AX, BA, and BK; showing quality of sounds, as in Notations AO, BJ, and BY, and presenting the chosen notes, as is applied in Notation BE.

Dots, the final compositional material, can be found in Notations P, S, V, Y, AC, AL, AO, AX, BA, BB, BD, BE, BJ, BS, BT, BV, BY, and CA. As I described in the analyses in Section 2 Analyses of Compositional Materials in this chapter, dots do not carry meaning in themselves; they need to be understood together with other compositional materials, however, this feature does not distance them from other compositional materials, rather, it shows that dots are closed to other materials. Dots demonstrate seven types of decoding that involve, (i) the timing of noises and notes, (ii) levels of loudness, (iii) performance speed, (iv) chosen notes, (v) showing performing approach of noises and notes, (vi) numbers of sounding events, and (vii) pitch. Showing and timing of noises and notes is used in Notations P, S, V, Y, AC, AL, AO, AX, BA, BB, BS, BV, BY, and CA; presenting the levels of loudness can be seen in Notations P and V; in V, Cage also applied the function of showing performing speed. Representing the chosen notes, can be found in Notations AL and AO; showing performing approach of noises and notes is applied in Notations AC, BA, BT, and CA; showing numbers of sounding events is used in Notations BB, BY, and CA; and the final function, indicating pitch, can be found in Notation BY.

The previous sections conclude the descriptions of the uses and functions of six compositional materials: numbers, lines, shapes, arrows, areas, and dots. From the conclusion, I notice that one compositional material can present two and more functions, while the same function can be presented by different compositional materials. For example, numbers are used in Notations A, B, D, G, J, L, U, AC, AE, AJ, AT, AU, BC, BE, BG, BH, BK, BL, BP, BR, and BU to show the numbers of playing notes; it is also used for indicating time in Notations F, Q, R, Y, AE, AK, AL, AT, AZ, BO, BQ, and, BI; and showing the degree of loudness of sounds in Notations T, Z, AB, AV, and BA. As these thirty-five notations all apply numbers, the first type of relationship – the uses of compositional materials – is built into them.

On the other hand, the function of showing the degree of loudness of sounds can also be presented as numbers – as in Notations T, Z, AB, AV, and BA; as shapes in Notation AA, as areas in Notations P, V, Y, BD, and BS, and as dots in Notations P and V. Hence, Notations P, T, V, Y, Z, AA, AB, BA, BD, and BS build the second type of relationships – the compositional materials' functions – by sharing the same function of different compositional materials. This finding indicates the overlapping of relations building among compositional materials, functions, and notations, and thus, no matter how diversely the compositional materials are used in the *Solo for Piano*, they do not distance notations from one another, instead, they reveal a sort of invisible relationship across them. Thus, I suggest, through the uses of compositional materials and their functions, two rational features can be revealed – (i) method/methodical and (ii) function/functional.

3.2 The Third Relationships and the Fourth Relationship: Types of Freedom and Performing Approach

In the previous two sections, I deconstructed eighty-four notations and concluded six main compositional materials that Cage applied in the *Solo for Piano*. In this process I noticed that he not just used new compositional materials, but also used them repetitively, which means that one may show multiple times. Moreover, by delving into the function of numbers, lines, shapes, arrows, areas, and dots, I found that a single one may serve multiple functions, thereby revealing a sense of conformity and continuity throughout this piece.

Once again, Cage's intention appears to have been to compose a work with a shifting centre in order to reflect his interpretation of Zen Buddhism, which makes the performers' role essential. In terms of the revised tripartite model, the performers are responsible for determining all

decisions for the realisations at the new poietic level, which means that compositional materials are not the only crucial elements in the analyses of the *Solo for Piano*, types of freedom and the performing approaches are equally important. From the analyses of eighty-four notations in Section 2 Analyses of Compositional Materials in this chapter, I discussed Cage's performing instructions and how performers can decode these notations, and then explained the first and the second type of relationships in Section 3.1 The First and the Second Types of Relationship. Following these discussions, the aim of this section is to gather freedom and performing approaches in order to present the third and the fourth types of relationships. The following sections will focus on the following five types of freedom and performing approaches: (i) notes, (ii) quality of sounds, (iii) formation of notes and sounds, (iv) body movement, and (v) noise.

3.2.1 Freedom and Performing Approaches of Notes

In Notations A to Z, I noticed that Cage gave eleven types of freedoms and twenty-two performing approaches, include: (i) in Notations A, B, D, F, G, J, L, R, T, U, and Y, to free to choose notes; (ii) in Notation A, the performing direction; (iii), in Notation D, the order of notes; (iv), in Notation H, to choose the staff; (v) in Notations H and R, to choose the clef sign; (vi) in Notation K, to perform either odd, or even, numbers of notes; (vii) in Notation K, how to use the given figures; (viii) in Notations M and Q, to decide directions; (ix) in Notations M and Q, to decide on one or more voices; (x) in Notations M, O, and X, to use pedals; and (xi) in Notation X, the timing of punctuations and different amplitudes.

In terms of performing approaches, there are twenty-two types: (i) in Notation A to follow the perimeter; (ii) in Notations A, B, G, J, U, and Y to perform the given proportion of notes; (iii) in Notation A, to play in an opposite direction; (iv) in Notations D, to follow the rule of the series of numbers; (v) in Notations E and L, to play with the assigned hands; (vi) in Notations E, L, M, Q, and U, to perform notations in the assigned directions; (vii) in Notations F, Q, R, and Y, to perform based on the time arrangement; (viii) in Notation G, to perform by keys or harp; (ix) in Notation G, to follow the arrow to perform the beginning and ending; (x) in Notation H, to perform selected staffs in sequence; (xi) in Notation H, to perform based on the first chosen clef; (xii) in Notation J, to follow the arrows to decide performing directions; (xiii), in Notation N, to perform with pedal; (xiv), in Notation R, to perform the notes with assigned clef signs; (xv) in Notation S, to perform notes with the assigned piano construction; (xvi) Notations T and Y, to follow positions of materials to perform; (xvii) in Notations U and Y, to

choose notes from the pitch areas; (xviii) in Notation V, to perform notes based on indications; (xix), in Notation V, to read the indications from up to down; (xx) in Notation W, to follow the notation to decide the order of performing notes; (xxi), in Notation X, to follow signs to perform the assigned sounding requirements, and (xxii), in Notation Z, to perform single notes at the ends.

From the Notations AA to AZ, the types of freedom that Cage applied in Notations A to Z, are also apparent, along with some new types of freedom. The freedom to choose notes, was also applied in Notations AN and AQ; the freedom to choose clef sign was also used in Notations AA, AD, AF, AH, AU, AW, and AY; the freedom both to decide directions and to decide one or more voices occur in Notation AU. Along with the freedoms that had been applied in Notations A to Z, Cage also applied other types; for instance, the freedoms to perform any number of notes in Notation AN; to choose the performing areas in Notation AQ, and to choose graphs in order to perform in Notation AY.

With regard to performing approaches, in his performing instructions, Cage offered diverse approaches for guidance – for instance (i) in Notations AQ, AE, AT, and AY, the given proportion of notes; (ii) in Notations AF, AH, and AN, playing with assigned hands; (iii) in Notations AH, AI, AN, and AU, performing notations in the assigned direction; (iv) in Notations AE, AK, AL, AM, AQ, AT, AU, AX, and AZ, performing based on the time arrangement; (v) in Notation AJ, following the arrows to decide performing directions; (vi) in Notation AD, AJ, and AW, performing the notes with assigned clef signs; (vii) in Notation AC, performing notes with the assigned piano construction; (viii) in Notations AC, AO, and AQ, following positions of materials to perform; (ix) in Notations AA, AE, AJ, AK, AN, AO, AQ, and AT, choosing notes from the pitch areas, and (x) in Notation AB, performing single notes at the ends.

The performing approach that applied in the section of Notations AA to AZ, but not applied in Notations A to Z, follow a circle to emphasise the notes in Notations AA; performing single notes at the beginning, if applicable, in Notation AB; omitting notes in Notation AG; choosing one performing area in Notation AQ; and performing a given note in Notation AS. From Notations BA to CF, performers are: (i) in Notations BB, BC, BD, BE, BH, BK, BN, BP, BQ, BR, BU, BV, CA, and CB, given the freedom to choose notes; (ii) in Notations BH, BN, and

BK, free to decide the performing direction, and (iii) in Notation BH, free to choose the clef sign.

Meanwhile, Cage applied new types of freedom, such as, (i) in Notations BB, the freedom to decide how to measure BJ, BV, and CC; (ii) in Notation BC, the freedom to decide numbers; in Notation BE, the freedom to choose events; (iii) in Notation BG, the freedom to decide how to perform intervals; (iv) in Notation BI, the freedom to decide the uses of numbers, and (v) in Notation BN, the freedom to decide the path for both hands.

The performing approaches of the next notation section include, (i) following the perimeter in Notations BH, BN, and BK; (ii) performing the given proportion of notes in Notations BC, BG, BH, BK, BL, BP, BR, and CB; (iii) in Notations BH and BK playing in opposite directions; (iv) in Notation BS, playing with the assigned hands (v) in Notations BO, BQ, BU, and CC, performing based on the time arrangement; (vi) in Notations BZ and CF, performing with pedal; (vii) in Notations BA, BK, and BT, performing notes with the assigned piano construction; (viii) in Notations BA, BB, BD, BE, BJ, BS, BT, BV, BW, BX, BY, and CA, following positions of materials; (ix) in Notations BC, BG, BP, BU, and CB, choosing notes from the pitch areas; (x) in Notations BF, BO, BU, and BX, following the notation to decide the order of performing notes; as well as the newly applied notes related to the performing approaches from Notations BA to CF, such as (xi) in Notations BB and BV, measuring the distance between perpendiculars and the given lines; (xii) in Notations BB, BJ, BV, and CC, following the measurement results to determine the quality of notes; (xiii) in Notation BI, choosing either one or two number(s); (xiv) in Notation BN, ending the realisation at the centre of this notation; (xv) in Notation BO, following the distances among numbers; (xvi) in Notation BQ, choosing notes for each triangle; and (xvii) in Notation BV, following the size of notes to perform.

3.2.2 Freedom and Performing Approaches of Quality of Sounds

Among Notations A to Z, six types of freedom are found: (i) in Notations A, E, F, I, J, and L, the freedom to decide, in a general sense, the quality of notes; (ii) in Notations B, C, D, H, N, O, S, U, W, X, and Z, the freedom to decide the duration of notes/events; (iii) in Notations C, S, U, and W, the freedom to decide the dynamics of notes; (iv) in Notations F and Q, the freedom to decide time units; (v) in Notation Q, the freedom to decide the range of increased speed; (vi) in Notations T and Z, the freedom to decide the relationship between numbers and dynamics.

The performing approaches include five types, (i) in Notation D follows the horizontal arrows to determine the beat of sounds; (ii) in Notations G, P, T, and Z, it is to follow dynamic indications; (iii) in Notations I and N is to perform the reappearance of notes as the original; (iv) in Notations K and L, is to ignore time; and (v) in Notation Q, is to increase speed when changing the performing direction.

In addition to Notations A to Z, (i) in Notations AD, AE, AF, AG, AH, AI, AJ, AM, AN, AS, AT, AW, AZ, BC, BF, BG, BH, BK, BL, BN, BP, BU, BX, BZ, CA, CB, and CF, Cage broadly applied, in a general sense, the freedom to decide the quality of notes; (ii) in Notations AB, AF, BD, BE, and BO, he applied the freedom to decide the duration of notes/events; (iii) in Notations AP, AR, and BO, he granted the freedom to decide dynamics of notes; (iv) in Notations AL, AU, AZ, and BI he allowed the freedom to decide time units; in Notation AU, he gave the freedom to decide the range of increased speed; (v) in Notations AB, AV, and BA he allowed the freedom to decide the relationship between numbers and dynamics.

Along with the above types of freedoms, he developed four new types of freedom: (i) how to emphasise notes, (ii) the duration of realisations, (iii) the use of boundaries, and (iv) the use of lines; all of which he applied in the *Solo for Piano*. The first one was applied in Notation AA, the second was used in Notations BI and BS, the third was used in Notations BJ and BV, and the fourth was applied in Notation CC.

Returning to performing approaches: in addition to the five types mentioned above, another two types, regarding lines to decide duration and pitch indications, were also applied in the *Solo for Piano*. The dynamic indications were not only used in Notations G, P, T, and Z, but he also applied them in Notations AB, AK, AV, BA, BD, and BS. The lines in order to decide duration can be found in Notation AP, and the pitch indications appears in Notation BY.

3.2.3 Freedom and Performing Approaches of Formation of Notes and Sounds

The types of freedom relate to the formation of notes and sounds, include: (i) to produce certain types of sounds, (ii) to realise overlapping and interpenetrations, (iii) to decide performing manner, (iv) to decide the performing approach of the remaining notes. The performing approaches include: (i) playing the unplayable notes as harmonics, (ii) performing as ictus, (iii) performing notes as pizzicato, (iv) performing notes as mute, (v) performing as single notes, (vi) performing arpeggiation the notes that have a single stem, (vii) performing ascending or

descending gamut, (viii) applying the given figures as graces or punctuations, (ix) performing certain format of sounds, (x) not to perform as harmonic(s), (xi) realising the space as frequency and time, and (xii) realising the notation as the drawing suggests. These types of freedom and performing approaches can be found in Notations B, C, D, E, G, H, I, J, K, N, O, S, W, AA, AD, AI, AL, AM, AO, AP, AR, AV, AW, AY, AZ, BF, BG, BO, BR, BT, BX, BY, BZ, CA, and CF.

Cage applied the first type of freedom, to produce certain types of sounds, in Notations B, D, O, AA, AO, BR, and CA; he used the second type to realise overlapping and interpenetrations, in Notations B, D, O, and BR; he used the third type to decide performing manner, in Notations G, BT, BY, BZ, and CF, and he applied the fourth type of freedom, free to decide the performing approach of the remaining notes, in Notation O.

In terms of performing approaches, (i) performing the unplayable notes as harmonics was applied in Notations B, D, O, and BR; (ii) performing as ictus was applied in Notations B, D, and O; (iii) and (iv) performing notes as pizzicato and mute were applied in Notations C, H, I, N, and S; (v) performing as single notes was applied in Notations C, H, N, and S; (vi) performing arpeggiation the notes having a single stem was only applied in Notation E; (vii) performing ascending or descending gamut was applied in Notation J; (viii) applying the given figures as graces or punctuations was applied in Notation K; (ix) performing certain format of sounds was applied in Notations W, AD, AL, AM, AO, AP, AW, AZ, BF, BG, and BO; (x) not to perform as harmonic(s) was only applied in Notation AI; (xi) realising the space as frequency and time was applied in Notation AO, and (xii) realising the notation as the drawing suggests was applied in Notations AR, AV, AY, and BX.

3.2.4 Freedom and Performing Approaches of Body Movements

Unlike notes and quality of sounds related freedom and performing approaches, body movement related performing approaches can only be found as two types: (i) in Notation BE, realising the notes with the given body parts, and (ii) in Notations BZ and CF, following movements of lines. At the same time, I did not find this type of freedom in the performing instructions.

3.2.5 Freedom and Performing Approaches of Noises

The noise-related freedoms include five types: (i) how to produce noises, (ii) the duration of noises, (iii) choosing noises, (iv) the quality of noises, in a general sense, and (v) the amplitude of noises. There are three types of performing approaches: (i) performing notes around lines as noises, (ii) performing the given proportion of noises, and (iii) performing with the given materials. In Notations P, V, and BS, Cage allowed the freedom to decide how to produce noises, and in Notation P, their duration. A third type of freedom can be found in Notation AC, where Cage applied numbers to indicate the proportion of noises; a fourth in Notations AC, BA, BS, BT, and BU; and a fifth in Notation AX. In Notations S and V, Cage applied the first type of performing approaches, assigning the performing notes around lines as noises; in Notations AC and BU, he also allowed the performers to perform the given proportion of noises from the given ranges; in Notation AX, he directed the performers to perform with the given materials.

3.3 The Final Relationship: Similarity of Appearance

The similarity of appearance does not come out of nowhere, rather, it is produced by the similarity of compositional materials in notations; this means that notations with similar looks are based on a similar use of compositional materials. On the other hand, as I clarified in the introduction of this chapter, the inclusion of notations' appearances is due to the speciality of graphic scores, and hence, notations sharing similar uses of compositional materials and performing instructions may suggest another layer of relationship simply in terms of their appearances. In Appendix B: under 'Appearance of Each Notation', I list eighty-four notations in a table to clearly present their looks.

Notations with similar uses of compositional materials did not always appear at the same place, whether they are the variations, or notations, with similar compositional materials. On the contrary, these notations are scattered over the *Solo for Piano* the musical work. For instance, there are six types of Notation A on page 1, at pages 5–6, 45–47, 49, 49–51, and 53. On the other hand, Notation A is not the only notation on page 1, Notations B and C are also allocated in this page. This arrangement – various types of notations appear on the same page – can be hard to recognise similarities without using comparisons. For example, page 9 of the *Solo for Piano* consists of Notations B, G, K, M, N, P, and R (alphabetical order) which comprise of various compositional materials, types of freedoms, and performing approaches, including, but

not limited to, different types of shapes, lines, numbers, and dynamic indicators (Figure 190). Containing a great variety of composing elements, these notations demonstrate a sense of fragmentariness. Based on these specifics, I suggest that those notations with similar looks can produce a further level of closeness that enhances the inner structure of the *Solo for Piano*.

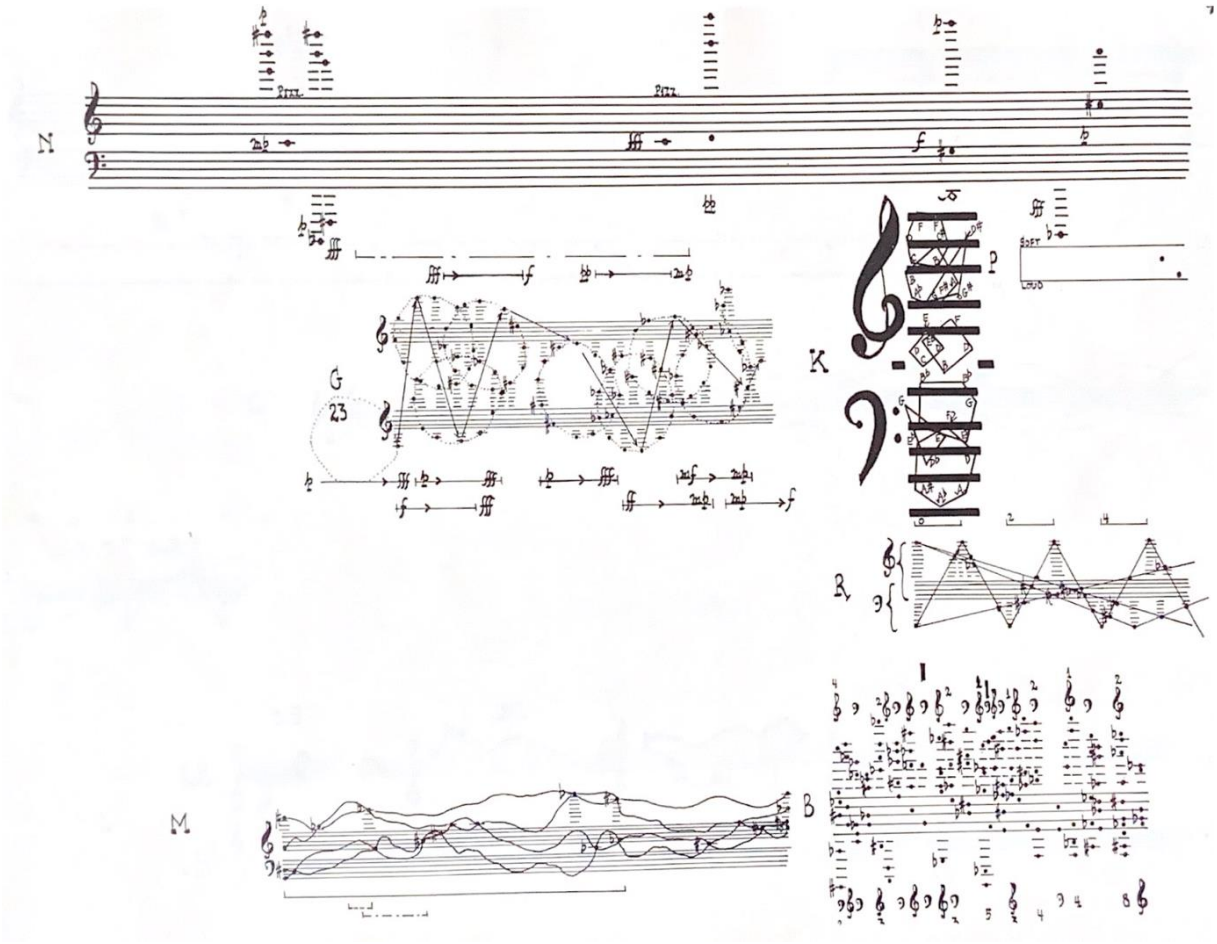
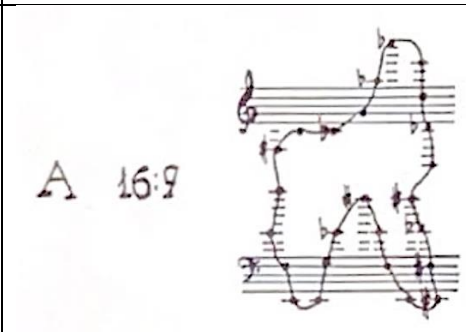
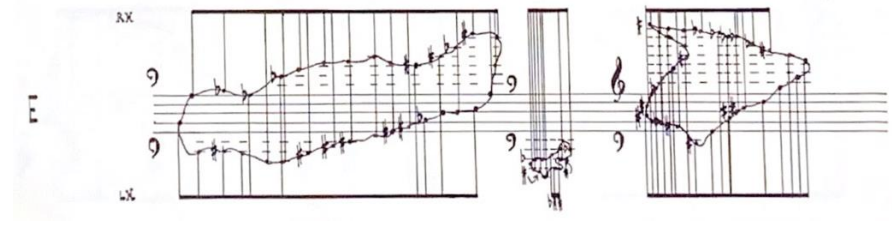


Figure 190 Page 9 of *Solo for Piano*.

The appearance can be categorized into the following types: (i) perimeter, (ii) ambiguous clef signs with numbers, (iii) symbols to indicate mute and pizzicato, (iv) lines and intersections, (v) lines to indicate noises, (vi) the notes should be performed as the same way as the first time in the notation, (vii) triangles to indicate clusters, (viii) show information for noises, (ix) ambiguous clef signs with staff, (x) numbers with bracket to indicate time, (xi) pitch areas, (xii) eight pitch areas with numbers indicating time, (xiii) lines showing pitch areas and numbers showing proportions of notes, (xiv) similar type of drawing, (xv) the grid, (xvi) shapes to differentiate legato and staccato, (xvii) areas with dots, (xviii) area with boundaries, lines, and dots, and (xix) lines to show pedals' movements.

In Notations A, E, L, BH, BK, and BN, the perimeter is the common feature that shows the similarity (Table 29). In Notations B, D, and L, Cage applied ambiguous clef signs with numbers (Table 30); in Notations C, H, and S, the symbols of mute and pizzicato can be found (Table 31); in Notations M, Q, and AU, the performers need to follow the lines with intersections to produce the realisations (Table 32); in Notations S and V, the lines to indicate the noises can be found (Table 33); in Notations I and X, both use symbols to show the notes should be performed in the same way as they were presented the first time in the notation (Table 34); in Notations Z and AB triangles are used for indicating clusters (Table 35); in Notations V, AC, and BA, the area for indicating the quality of noises can be found (Table 36); in Notations R, AD, and AW, an ambiguous clef sign with staff has been applied (Table 37); in Notations R, AE, and AU, numbers in bracket indicate time (Table 38); in Notations J and AE, Cage used lines to show pitch areas (Table 39); in Notations Y and AQ both show eight pitch areas with time (Table 40); in Notations AE, and AT contain lines showing pitch areas and numbers for the proportions of notes (Table 41); in Notations AR and AV, similar drawings can be found (Table 42); in Notations BD and BE both used grids but convey different information (Table 43); in Notations W, BF, and BO shapes have been applied to differentiate legato and staccato (Table 44); in Notations P, BJ, and BY applied areas are indicated by dots (Table 45); in Notations BB and BV, the area with boundaries, lines, and dots indicate the main compositional materials (Table 46); in Notations BZ and CF Cage used lines with different curves to show the movements of the pedals (Table 47).

Notation	Appearance of the notation
Notation A	
Notation E	

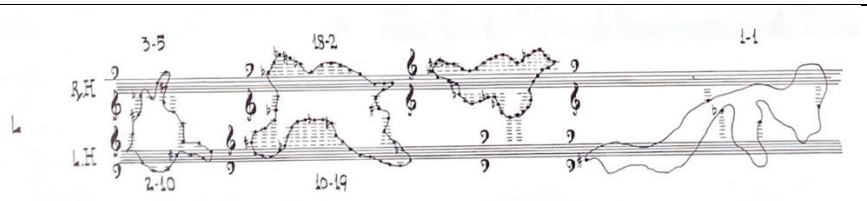
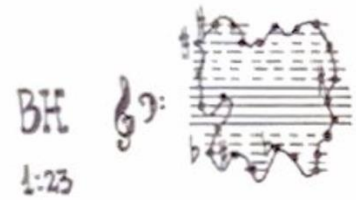
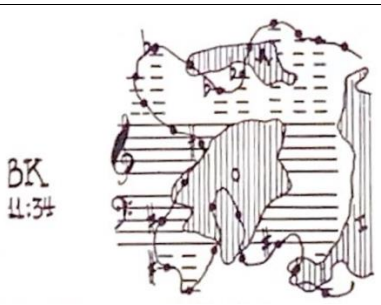
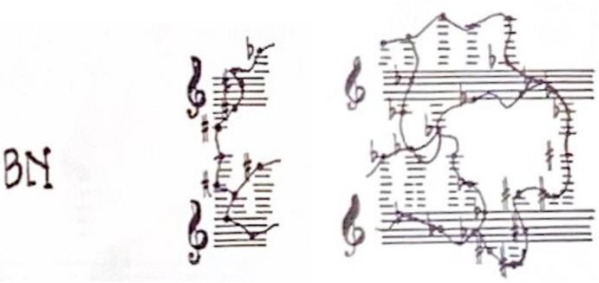
Notation L	
Notation BH	
Notation BK	
Notation BN	

Table 29 Notations with the perimeter.

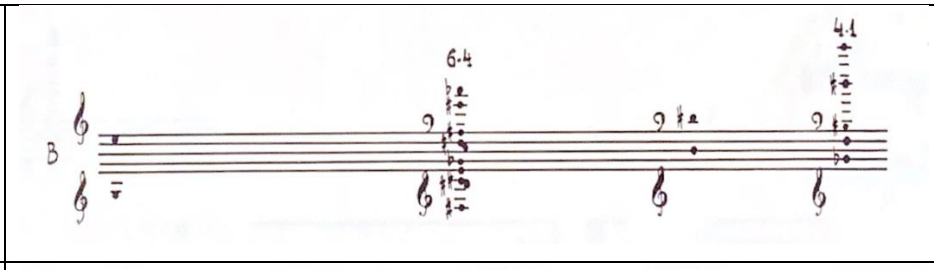
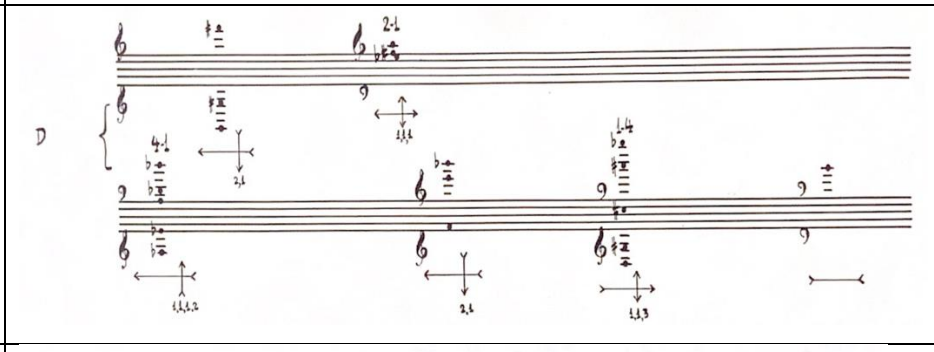
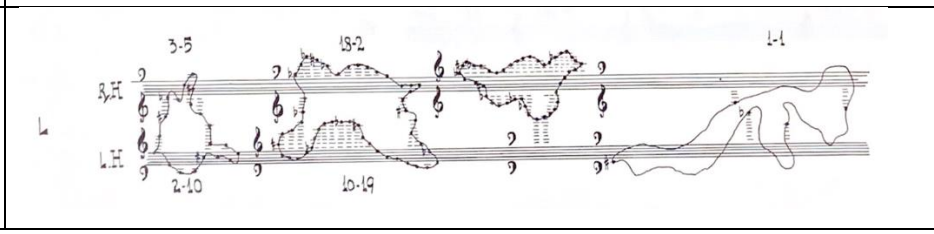
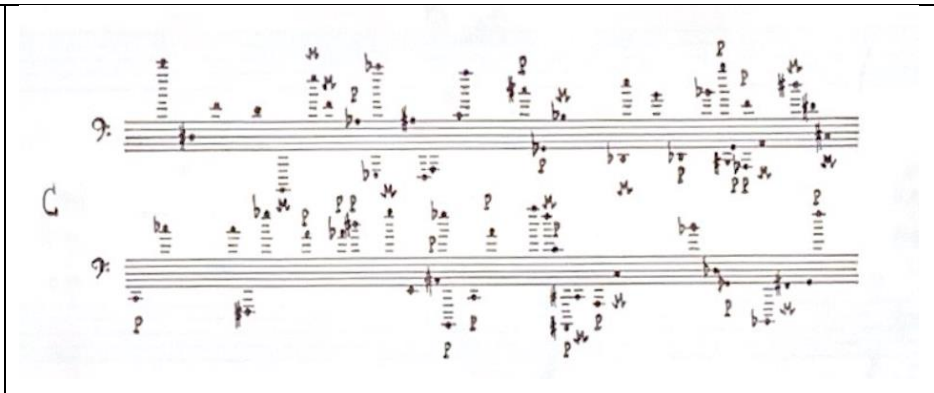
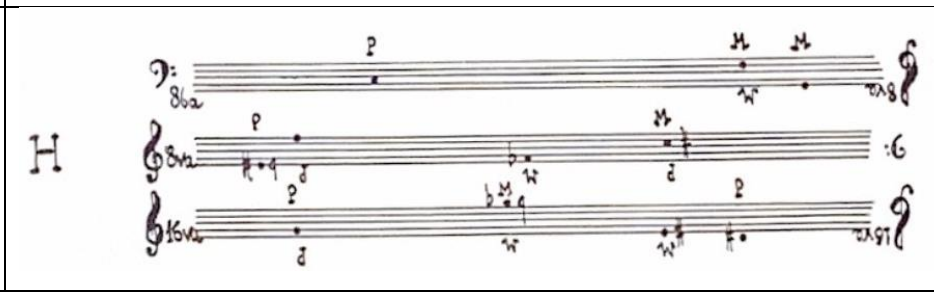
Notation B	
Notation D	
Notation L	

Table 30 Notations use ambiguous clef signs with numbers.

Notation C	
Notation H	

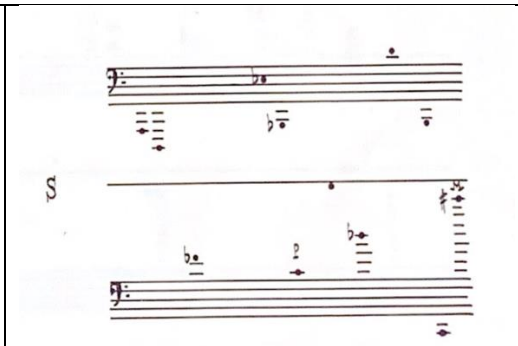
Notation S	

Table 31 Notations use symbols to indicate mute and pizzicato.

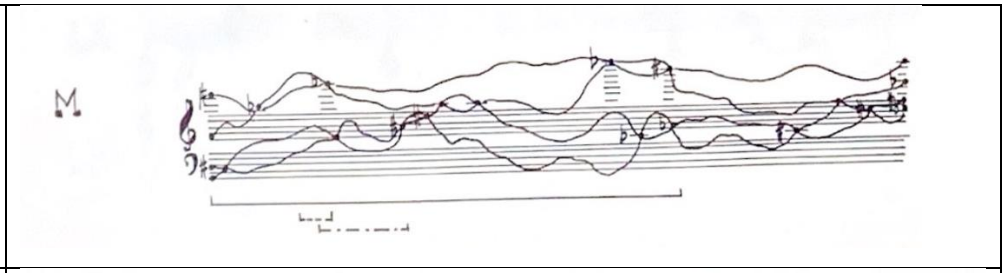
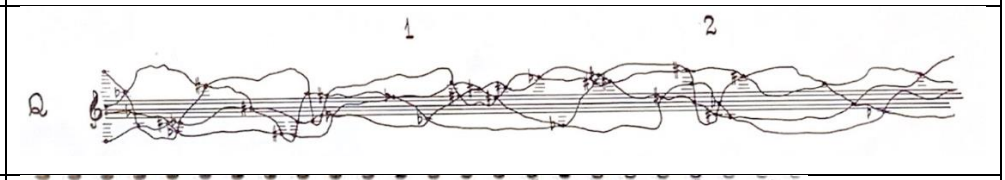
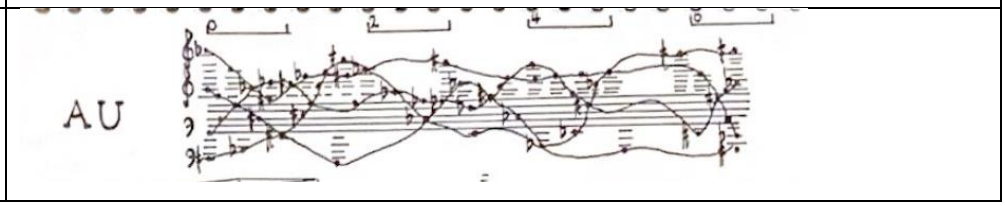
Notation M	
Notation Q	
Notation AU	

Table 32 Notations use lines and intersections.

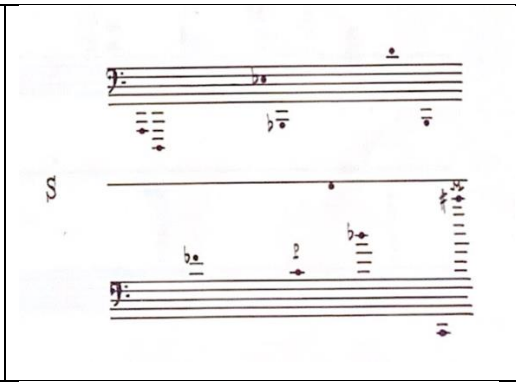
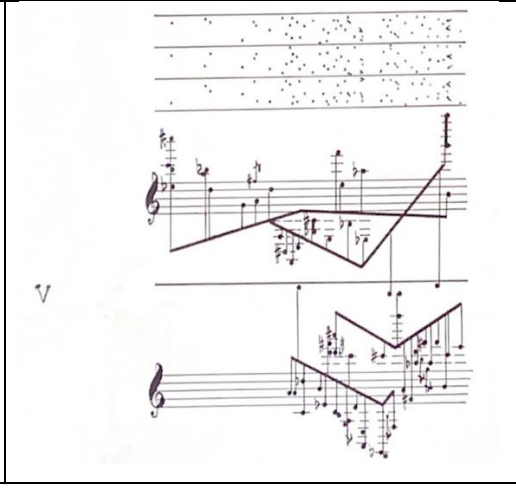
Notation S	
Notation V	

Table 33 Notations use lines for indicating noises.

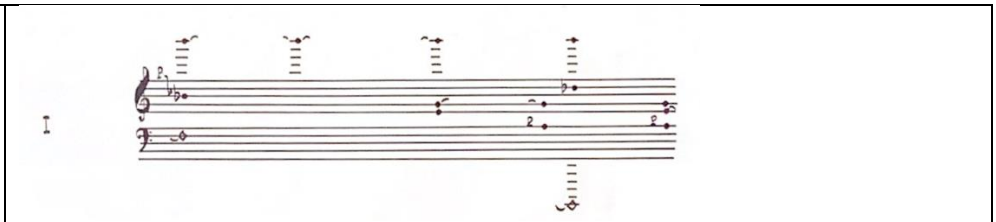
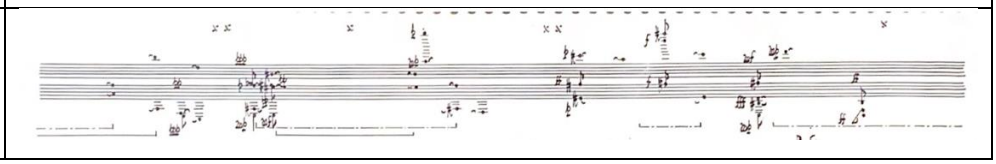
Notation I	
Notation X	

Table 34 Notations use symbols to show the notes should be performed as the same way as the first time in the notation.

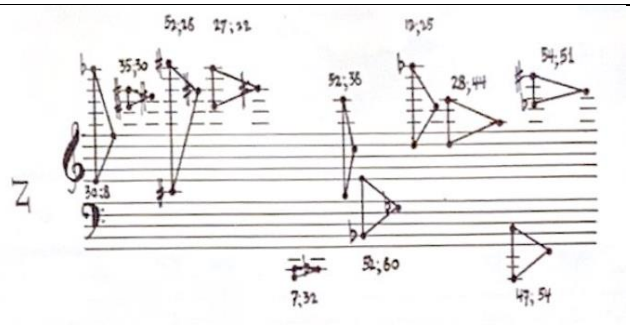
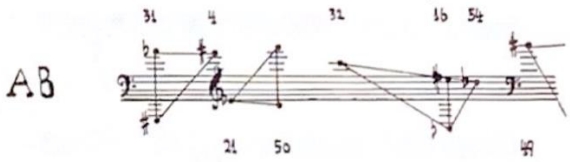
Notation Z	
Notation AB	

Table 35 Notations use triangles to indicate clusters.

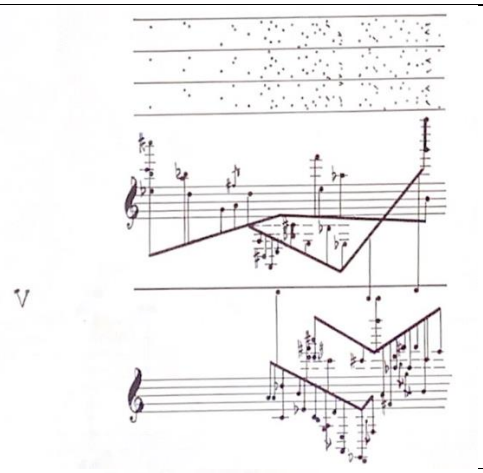
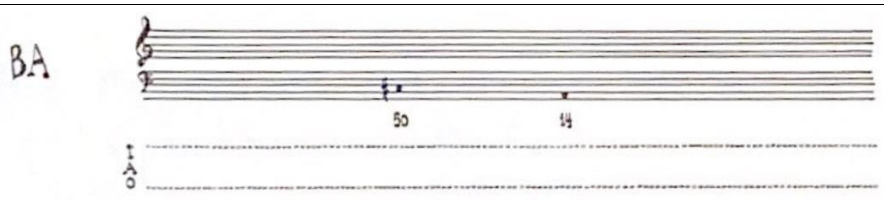
Notation V	
Notation AC	
Notation BA	

Table 36 Notations use areas to indicate information for noises.

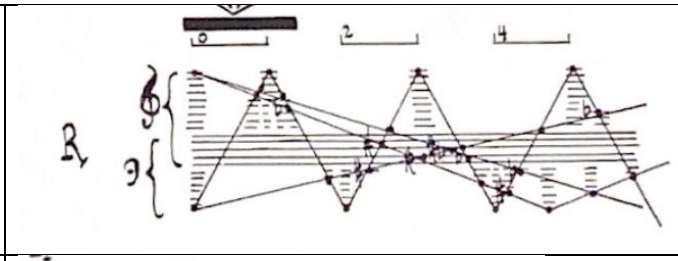
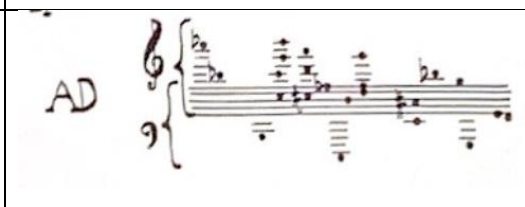
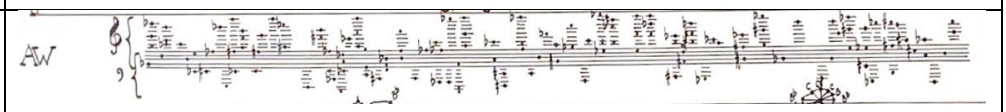
Notation R	
Notation AD	
Notation AW	

Table 37 Notations use ambiguous clef signs with staff.

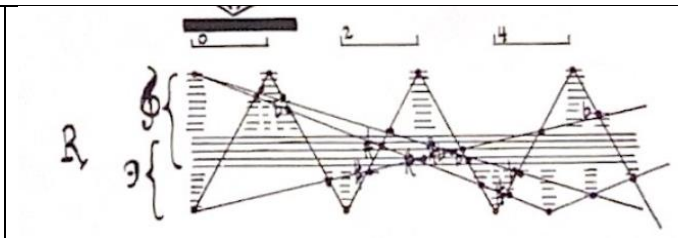
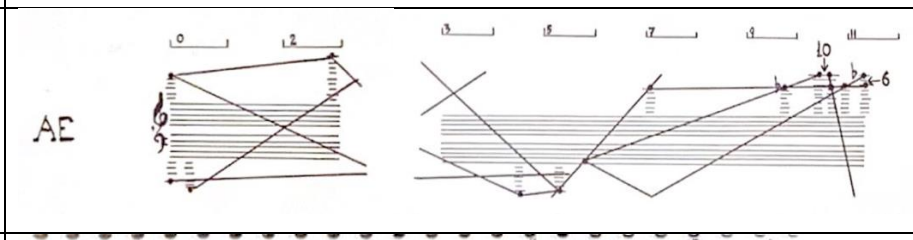
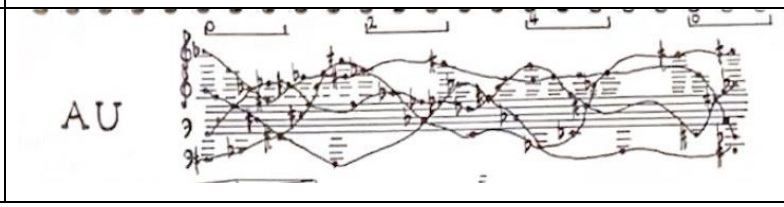
Notation R	
Notation AE	
Notation AU	

Table 38 Notations use numbers with bracket to indicate time.

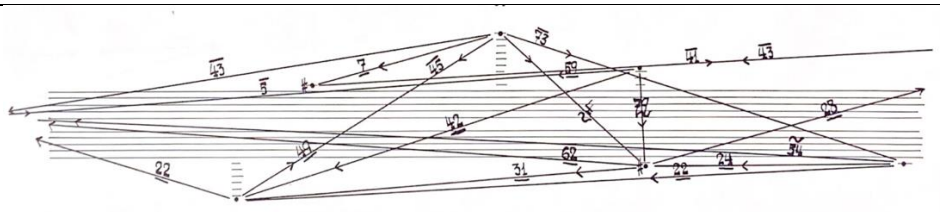
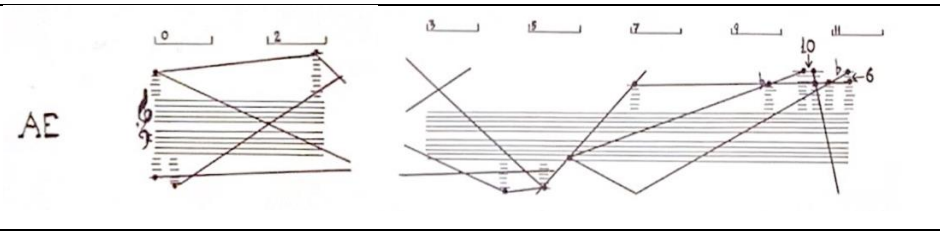
Notation J	
Notation AE	

Table 39 Notations use lines shows pitch areas.

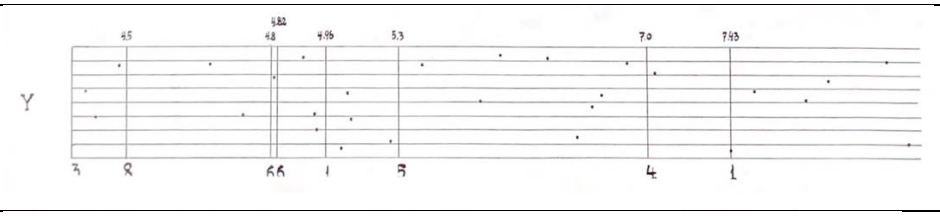
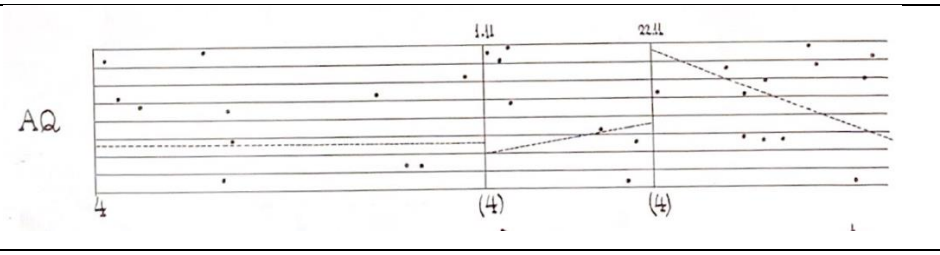
Notation Y	
Notation AQ	

Table 40 Notations use eight pitch areas with number indicating time.

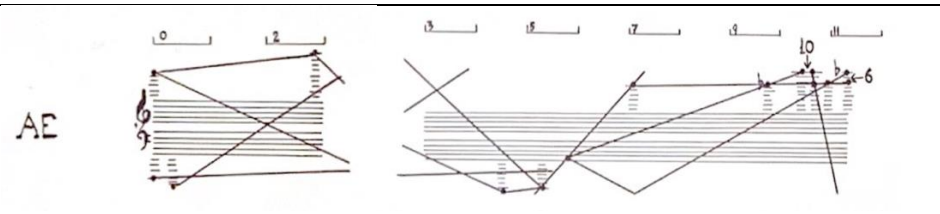
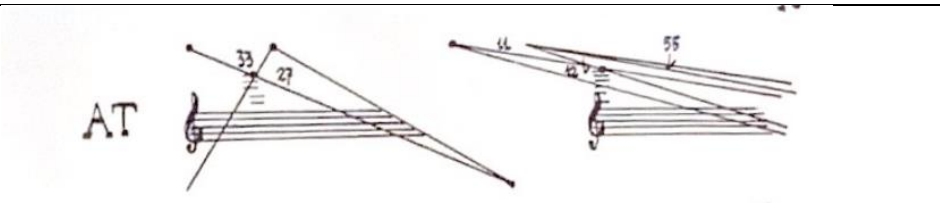
Notation AE	
Notation AT	

Table 41 Notations use lines showing pitch areas and numbers showing proportions of notes.

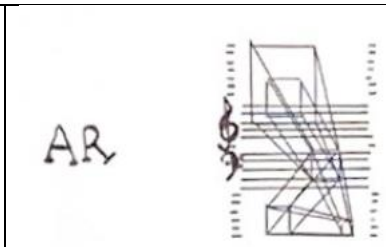
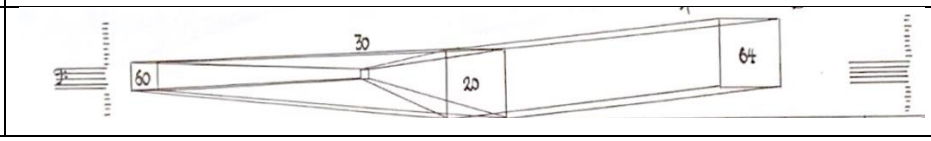
Notation AR	
Notation AV	

Table 42 Notations use the similar type of drawing.

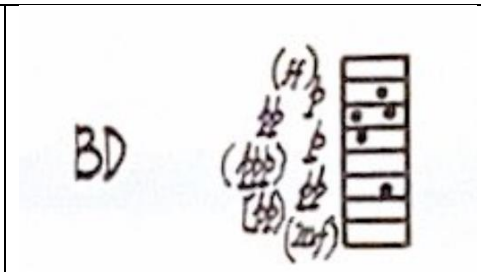
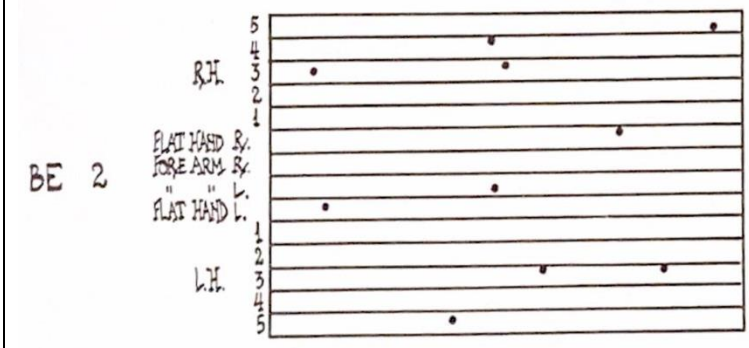
Notation BD	
Notation BE	

Table 43 Notations use grids as the form of areas.

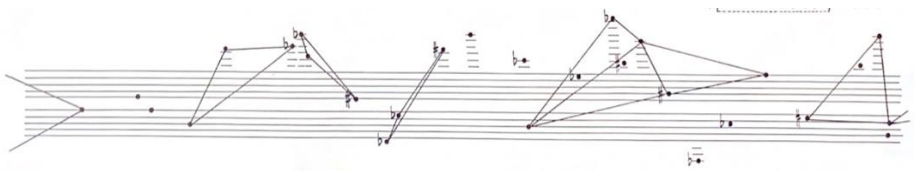
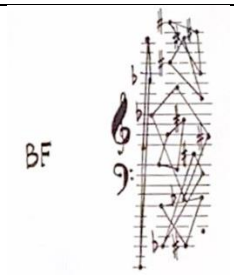
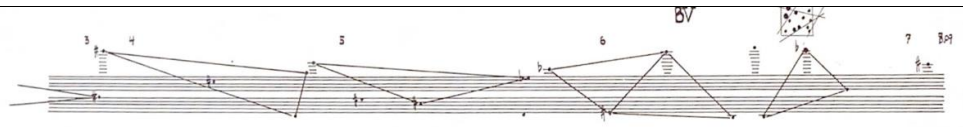
Notation W	
Notation BF	
Notation BO	

Table 44 Notations use shapes to differentiate legato and staccato.

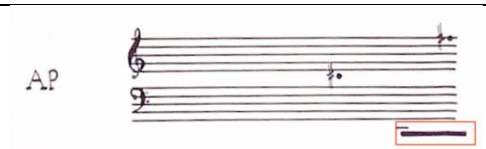
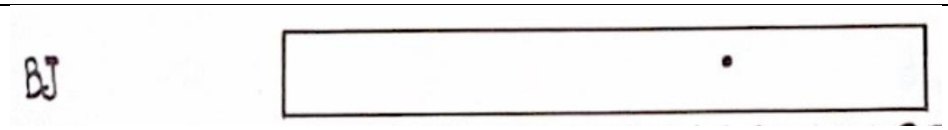
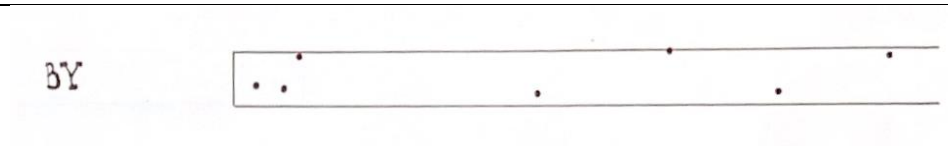
Notation P	
Notation BJ	
Notation BY	

Table 45 Notations use areas with dots.

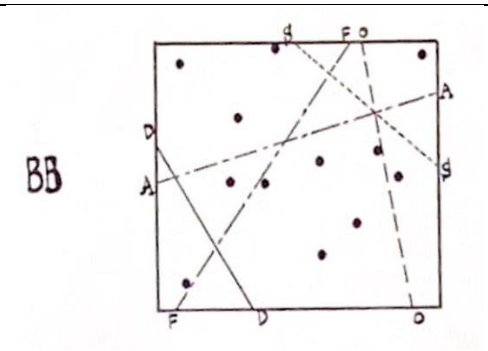
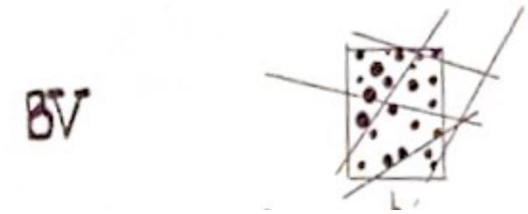
Notation BB	
Notation BV	

Table 46 Notations use area with boundaries, lines, and dots.

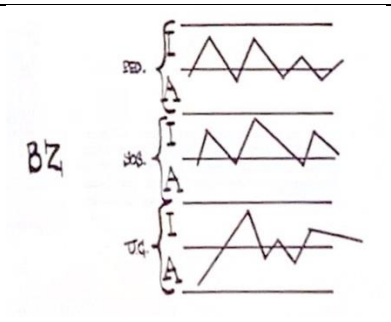
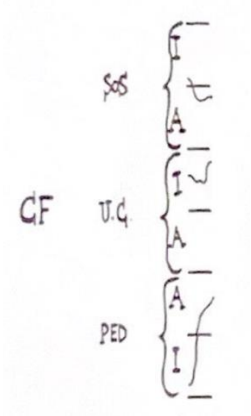
Notation BZ	
Notation CF	

Table 47 Notations use lines to show pedals' movements.

Chapter 6. Results of Visualisation: Individual Systems and the Inner Structure

1 Introduction

John Cage's arrangements of compositional materials seem to be contradictory to the idea of Zen Buddhism since Zen emphasises the pure facts of being, rather than offering the means of attaining it through any other means. By following this, in Chapter 4, I questioned Cage's intentions and purposive actions may not practise the idea of Zen, as emptiness and nothingness concern the awareness of reality, instead of intentional persistence on nothing.[358] This means that his interpretations – unimpededness and interpenetration – may twist the original meanings of Zen.[359] Nevertheless, Cage still effectively brought original perspectives in forming new relationships between the composer and the performers, as well as in forming the musical work. In the *Solo for Piano*, Cage interpreted Zen, in his own fashion by marking uneven surfaces of paper sheets with dots and creating drawings in order to communicate his compositional material to the performers; specifically, he applied lines, numbers, shapes, arrows, and other conventional musical symbols to his notation scheme. In Chapter 5, I deconstructed notations into the compositional materials, functions of compositional materials, types of freedom, performing approaches, and appearance of notations, then suggested the five types of relationships represent the methodical feature of this musical work. By following this logical flow, in Chapter 6, I propose to discover the individual systems and the inner structure within the *Solo for Piano*, and I shall visualise the relationships between notations and these materials in order to discover the piece's systems and the inner structure.

Visualisation, like an infographic, provides information that shows what variables exist that might influence the datum;[360] therefore, visualisation can be an effective way of showing the rational features in Cage's graphic compositional system, in which, Cage solidified his interpretation of Zen by transforming chance-oriented existences, such as paper imperfections, into semi-determinate compositional materials. As I explained in 4.2 Value Formation in Graphic Compositional System in Chapter 4, he marked on the paper imperfections and offered

[358] Low, 'Religion and the Invention(s) of John Cage', 170.
[359] Pong, 'True or False as It Is: Zen Enlightenment in John Cage's Music 真謬之間如如觀：凱吉音樂中的禪意解讀', 231.
[360] Shapiro, 'Once Upon a Stacked Time Series', 16.

them meaning through his drawings; this process is the first step of rationalisation, as Cage arranged the materials based on his value concepts at the poietic level, meaning that his arrangements build relationships between compositional materials and notations. However, the relationships are not distinct enough to be recognised. Thus, I suggest using Gephi to visualise the relationships with focuses of clusters and individual systems.

2 Application of the *Solo for Piano*

As I illustrated in Chapter 1 Section 1.1 The Scope of the Study, the final step of my thesis is to visualise the results through Gephi, a visualisation software that gives a visual analysis,[361] by using its own terms of (i) source, (ii) target, and (iii) weight. The details of what is Gephi and how will Gephi work has been explained in Chapter 1 Section 3.2.2 Analysis Method: Gephi. In summary, I consider three situations: (i) notations are sources nodes, while compositional materials, functions of compositional materials, types of freedom, and performing approaches are targets nodes, (ii) compositional materials are sources nodes, while the functions are target nodes, and (iii) in discussions of individual systems, notations are both source nodes and target nodes. With this setting, the weights link the source nodes and the target nodes together to illustrate the relationships among them. Thus, by visualising these relationships, I can reveal the rational features within the *Solo for Piano.*

Considering the nature of visualisation, there are three elements involved in rational features, (i) clusters, (ii) individual systems, and (iii) the inner structure. Clusters are how it usually refers to a group of analogous objectives. Therefore, in regard to the visualisation of the *Solo for Piano*, they refer to nodes representing notations, compositional materials, functions, types of freedom, and performing approaches clustering around each other.[362] Individual systems, which refer to a more concrete relationship among notations; Cage built his performing instructions in such a way that he clearly desired that one notation should follow another

[361] Venturini, Jacomy, and Jensen, 'What Do We See When We Look at Networks', 1.

[362] Despite clusters and ForceAtlas2 emphasise the gathering of compositional materials, functions, performing approaches, and types of freedom, results of clusters still show a sense of randomness. The cause of randomness is because repetitive uses of elements, meaning that the repetitively used elements may show up in any positions where the elements are also applied in any other notations. Nevertheless, it is possible to recognise similar distribution of nodes based on Cage's assigned relationships – I term these relationships as 'individual system' in this thesis. Further discussions about other versions of visualisations will be provided in Appendix C, where I shall provide three visualising results of the *Solo for Piano*. Regardless, I would like to emphasise the clusters in Chapter 6 are one of possible interpretations.

notation's performing approach by function or types of freedom. I have classified these notations into one individual system; by following these cluster and systems, I achieved the point where the hidden structure of the *Solo for Piano* becomes visible.

2.1 Visualisation of the *Solo for Piano*: Clusters and Modularity Class

I expect to visualise the relationships to demonstrate the rational features within the *Solo for Piano*. Before explaining each relationship, I shall provide a visualising result that shows the relationships between five compositional elements: (i) notations, (ii) compositional materials, (iii) functions, (iv) types of freedom, and (v) performing approaches. I applied ForceAtlas2 to produce the layout, and I applied closeness centrality for the nodes' sizes, with a setting of 50 (minimum) to 150 (maximum).

ForceAtlas2 is based on attraction force, repulsion force, and gravity force to keep the connected nodes closer, prevent nodes from clumping together, and attract nodes to the centre of the spatialisation space. According to Jacomy et al., the attraction force keeps the relevant nodes together, it applies the formula: $F_a(n_1,n_2) = d(n_1,n_2)$, F_a indicates the attraction force between nodes n_1 and n_2; the attraction force are decided by the distance between two nodes; the distance is presented as d.[363] The repulsion force prevents nodes from overcrowding, it applies the formula: $F_r(n_1,n_2) = k_r \frac{(deg(n_1)+1)(deg(n_2)+1)}{d(n_1,n_2)}$. F_r stands for repulsive force between nodes n_1 and n_2; K_r stands for the scaling factor, which increases the strength of the repulsive force between nodes to keep nodes from overlapping; $(deg(n_1) + 1)$ means the degree of node n_1, while the $(deg(n_2) + 1)$ stands for the degree of node n_2; the +1 is an adjustment to provide a baseline value and so prevent the potential error from zero degree.[364] The final force is gravity force, which applies $F_g(n) = k_g(deg(n) + 1)$. $F_g(n)$ refers to the gravity force applies to the node n, and it ensures the nodes stay in centre of the spatialisation space; $k_g(n)$ stands for the scaling factor that applies to the node n, meaning that the value of k_g will influence the value of F_g, higher k_g will pull the nodes to the centre, and vice versa.[365] With these forces, ForceAtlas2 visualises the relationships between compositional elements; when the nodes are close to one another,

[363] Jacomy et al., 'ForceAtlas2, a Continuous Graph Layout Algorithm for Handy Network Visualization Designed for the Gephi Software', 3.
[364] Ibid.
[365] Ibid., 4.

they share more communal elements; on the contrary, when the nodes are distant from one another, they share less communal elements.

The result is shown as Figure 191, in which I used the colour red for notations, purple for compositional materials, orange for functions, green for performing approaches, and blue for types of freedom. The visualising result revealed eight clusters, (i) Notations B, D, and BR, (ii) Notations C, H, I, and N, (iii) Notations G, BZ, and CF, (iv) Notations W, K, AP, BO, and BF, (v) Notations F, Q, Y, AL, AQ, BI, BQ, and BU, (vi) Notations P, V, AX, BB, BD, BJ, BS, BV, CA, and CC, (vii) Notation BE, and (viii) Notations T, Z, AA, AB, AC, AR, AV, BA, and BT.

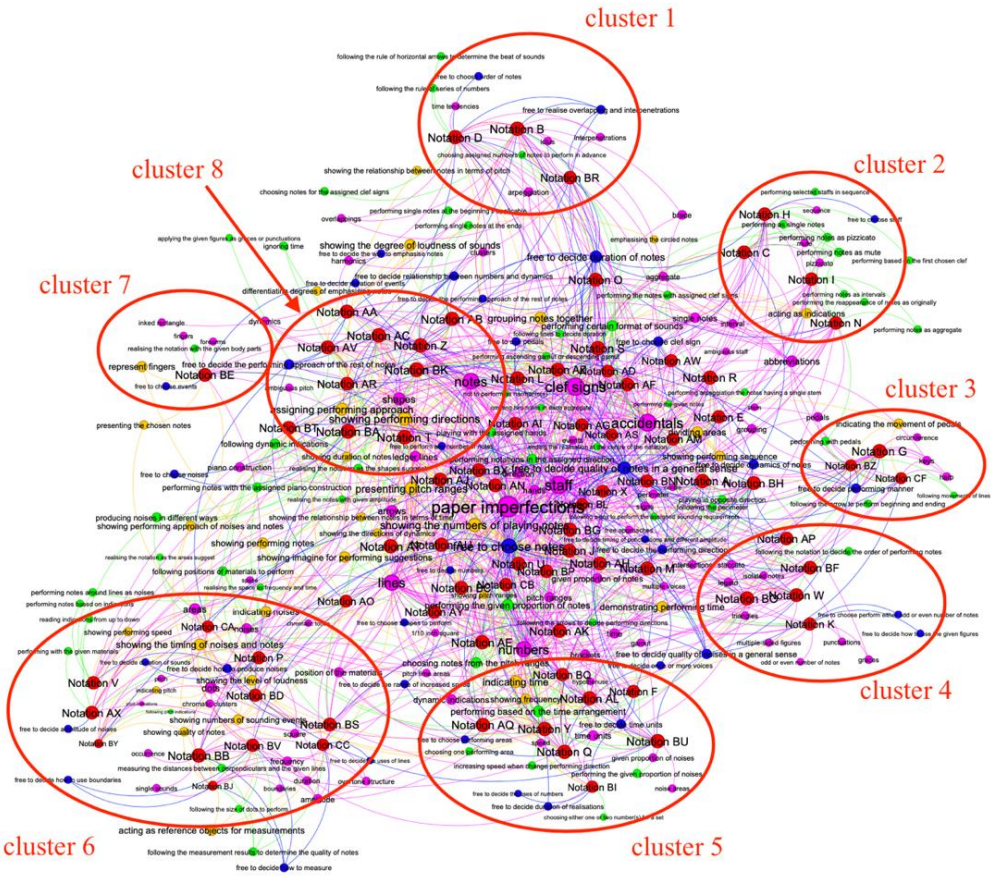


Figure 191 The visualising result of eighty-four notations, which revealed eight clusters.

The first cluster – Notations B, D, and BR (Figure 192) refers to the deconstruction of three notations, which share these compositional materials: overlapping, harmonics, notes, accidentals, staff, numbers, ictus, interpenetrations, and paper imperfections, all of which perform assigned numbers of notes in advance and produce an assigned format of sounds. Three of these offer the freedom of realising overlapping and interpenetrations, deciding the

duration of notes, and the choice of notes in terms of types of freedom. Finally, the compositional material and numbers, direct performers to choose the given proportion of notes for the clef signs or performing in advance. Given these communal elements, Notations B, D, and BR create a relatively more closed relationship than the other notations.

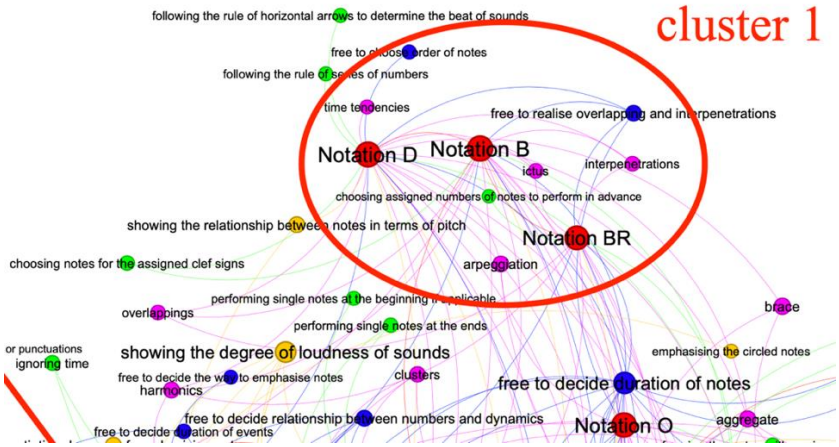


Figure 192 Cluster 1 includes: Notations B, D, BR, and their elements.

The difference between the Cluster 2, which includes Notations C, H, I, and N (Figure 193) and the first cluster, is that these notations seem to share fewer communal elements; the compositional materials they share are: paper imperfections, clef signs, accidentals, notes, single notes, and pizzicato, while the only performing approach they share is pizzicato; also, they do not share any types of freedom. At this point, it seems to be unreasonabe to locate these notations together. Nevertheless, the compositional material – pizzicato – plays an important role by connecting Notations C, H, I, and N. This compositional material was only used in these notations, therefore, by comparing them with other notations, four of them connect with each other through pizzicato.

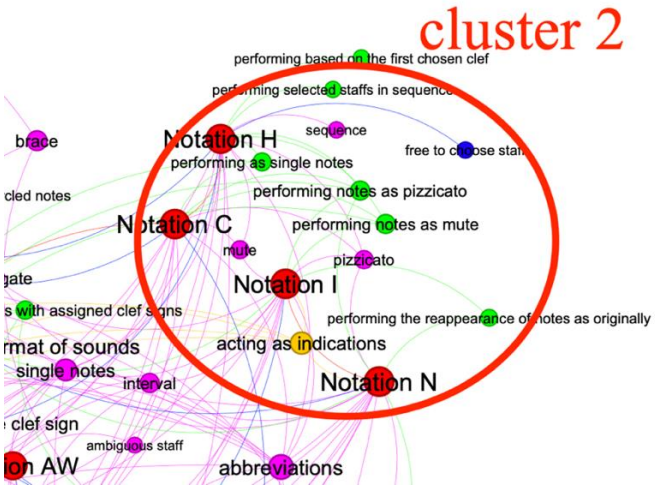


Figure 193 Cluster 2 includes: Notations C, H, I, and N, and their elements.

A similar situation can be found in Cluster 3, which consists of Notations G, BZ, and CF (Figure 194). The compositional materials between the three notations include: paper imperfections, keys, and harp, the one freedom they share is to decide the performing manner. With these communal nodes, three notations are closed to one another than other notations. Another finding here is that these elements appear not to demonstrate a strong relationship in terms of quantity. Based on Clusters 2 and 3, I noticed that, although the frequency of elements used is the basis from which five types of relationships are formed (i.e. compositional materials, functions, types of freedom, performing approach, and appearances), the uses of elements should not consider the numbers of the elements in order to avoid the bias regarding quantity. For this reason, I have input one in every weight of nodes in the Gephi setting.

For instance, Notation A on page 1 uses twenty-five paper imperfections, twenty-five notes, two staffs, two clef signs, two numbers, eleven accidentals, and one perimeter; Notation G on page 4 uses twenty-five paper imperfections, twenty-five notes, two staffs, two clef signs, one number, three dynamic indications, twelve accidentals, one line, two shapes, and two arrows; Notation I on page 29 uses fifty-seven paper imperfections, fifty-seven notes, two staffs, two clef signs, one arrow, and one mute symbol. Regardless the numbers of each element of three notations, the input of numbers to Gephi remain one, e.g. one paper imperfection, one note, one staff, one clef sign, one number, one accidental, and one perimeter, etc. With this setting, the bias showing that Notation I has a stronger relationship with notes and paper imperfections than Notations A and G can be avoided, and therefore, the relationships between notations, compositional materials, functions, types of freedom, and performing approaches can be revealed.

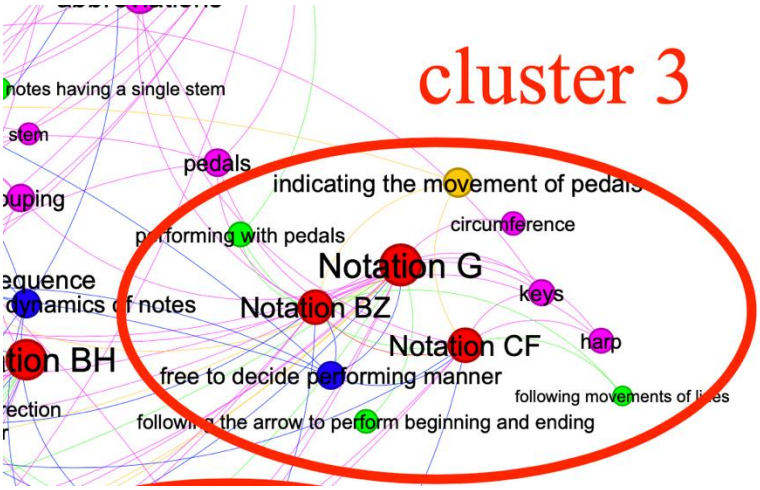


Figure 194 Cluster 3 includes: Notations G, BZ, and CF, and their elements.

Cluster 4 contains Notations W, K, AP, BF, and BO (Figure 195). Here I found multiple links. For instance, W, AP, BF, and BO, used staccato and the need to produce a certain sounds format; Notations W, BF, and BO applied the performing approach of following the notation to decide the performing order. In Notations W, BF AP and BO there are two common types of freedom – to decide both the dynamics and duration of notes; Notations W and BO both use staccato, legato, isolated notes, and triangles; and finally, multiple-sided figures were applied in Notations W, K, BF, and BO. Along with these links, all five notations in this cluster used paper imperfections, staff, accidentals, and clef signs.

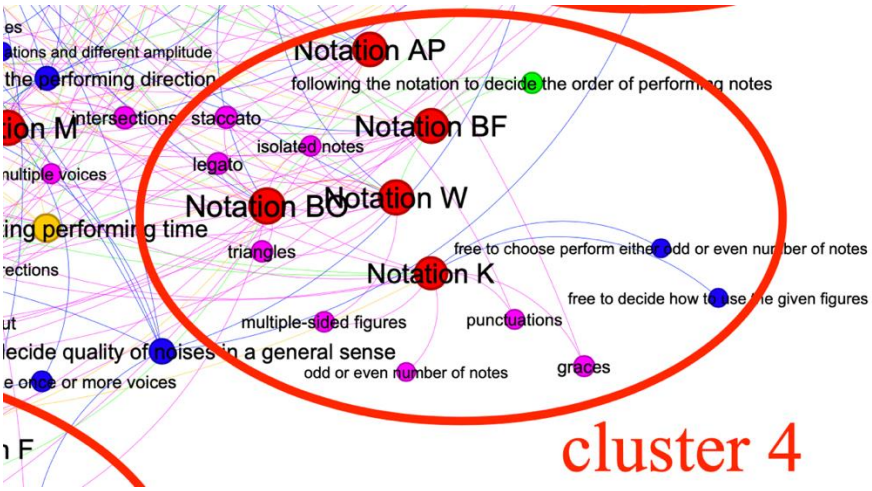


Figure 195 Cluster 4 includes: Notations W, K, AP, BF, BO, and their elements.

The next cluster contains Notations F, Q, Y, AL, AQ, BI, BQ, and BU (Figure 196), all of which apply numbers for three functions: indicating time, showing frequency, and showing the playing numbers of sounds. Although numbers connect these together, the decisive element of this link is the function of number – indicating time – which is applied in F, Q, Y, AL, BI, and BQ. Following this use, I noticed the compositional material – time units – is broadly used in Notations F, Q, Y, BI, and BU. However, even though time units are also applied in Notations AA (Cluster 8), AY (no cluster), and BO (Cluster 4), these notations do not belong to the same cluster. The application across clusters reveals another relationship level constructing the individual system of the *Solo for Piano*.

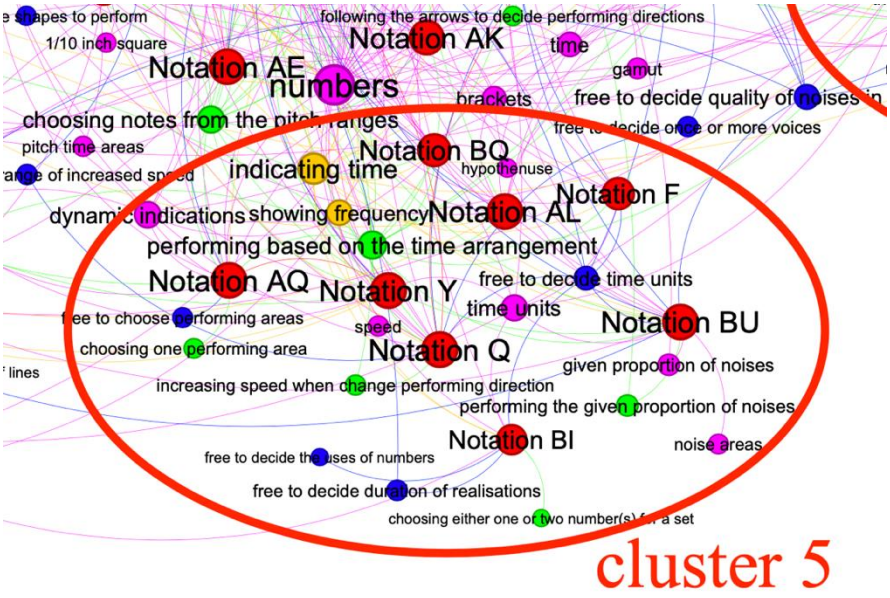


Figure 196 Cluster 5 includes: Notations F, Q, Y, AL, AQ, BI, BQ, BU, and their elements.

The Cluster 6 contains P, V, AX, BB, BD, BJ, BS, BV, BX, CA, and CC (Figure 197). Here, I noticed that it is based on the function of areas. This compositional material is used in Notations P, V, AX, BD, and BS to offer the level of loudness, and in Notations BB, BJ, BV, and CC they are used to represent the quality of sounds, in BY for showing pitch only, while in CA, for showing performing approaches, such as with keyboard, and performing as mute or pizzicato.

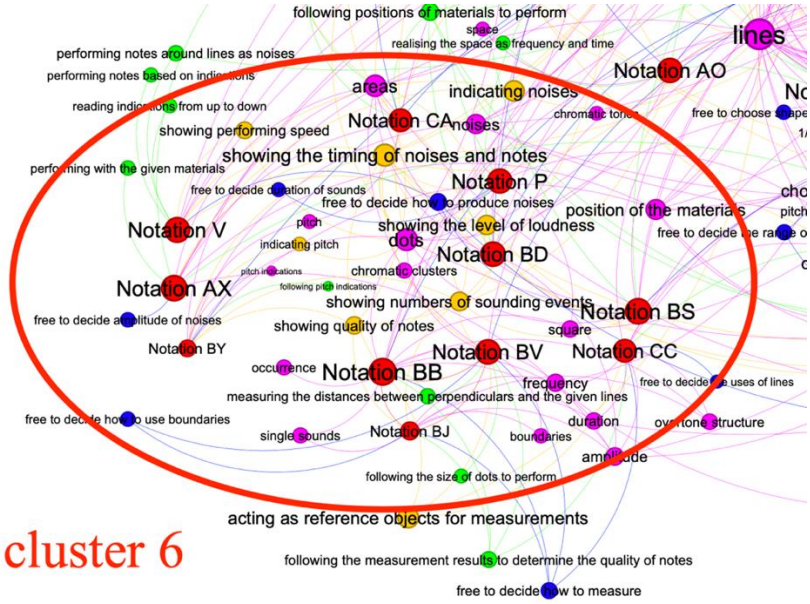


Figure 197 Cluster 6 includes: Notations P, V, AX, BB, BD, BJ, BS, BV, BY, CA, CC, and their elements.

The Cluster 7 only includes Notation BE (Figure 198). This is because this notation consists of the elements that are only used in this notation, such as, fingers, forearms, representing fingers, free to choose events, and realizing the notation with the given body parts. The Cluster 8 contains Notations T, Z, AA, AB, AC, AR, AV, BA, BK, and BT (Figure 198). Between T, Z, AB, AV, and BA, they share the freedom to decide relationships between number and dynamics. Applying the piano construction is the communal compositional material in AC, BA, and BK, while the function of presenting pitch ranges is used in T, AA, AV, and AR by using shapes or lines.

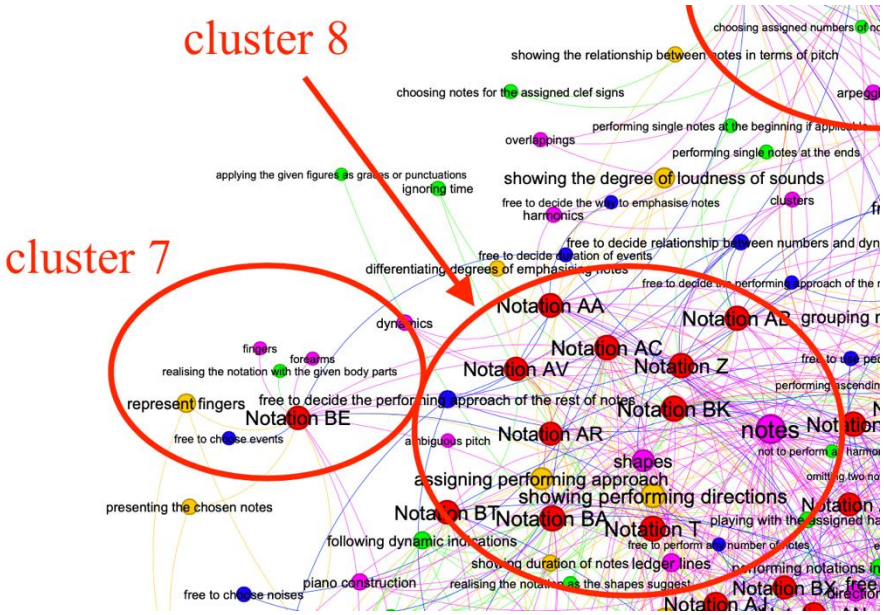


Figure 198 Clusters 7 and 8 include: Notations BE, T, Z, AA, AB, AC, AR, AV, BA, BK, BT, and their elements.

Although the positions of nodes show the relationships between elements, this visualising result is still not clear enough to show where those relationships are. Modularity class, a measure of the quality of a particular division of networks, interactions, or structural constructions can clearly present the relationships. The modularity class is based on the formula: $Q=\frac{1}{2m}\sum_{i,j}\left[A_{ij}-\frac{k_i k_j}{2m}\right]\delta\left(c_i,\ c_j\right)$. N. E. J Newman and Blondel et al. explain, c_i represent the community where the node *i* belongs, the δ-function $\delta(u,v)$ is 1 if the nodes are from the same community, or it is 0, if the nodes from different communities. *m* stands for total number of edges in the graph, it shows as $m=\frac{1}{2}\sum_{ij}A_{ij}$, thus, $\frac{k_i kj}{2m}$ is the expected number of edges between the nodes i and j, while k_i shows the number of edges of the node i; A_{ij} represents the connections between the nodes i and j, and therefore, $\left[A_{ij}-\frac{k_i k_j}{2m}\right]$ indicates whether there are

more or fewer edges than expected between nodes in the same community.[366] In essence, modularity class detects the communities based on the numbers of nodes and edges, and hence it can present the methodical feature of the *Solo for Piano*. To produce the visualisation result, the settings I apply are: Forceatlas2 for layout, modularity class for node's colours, value 50 to 150 for node size, the default value 1 for resolution, and the starting point as 0 for modularity class. The modularity result is 0.432,[367] which is a good indication showing a clear community structure,[368] with ten detected communities (Figure 199).

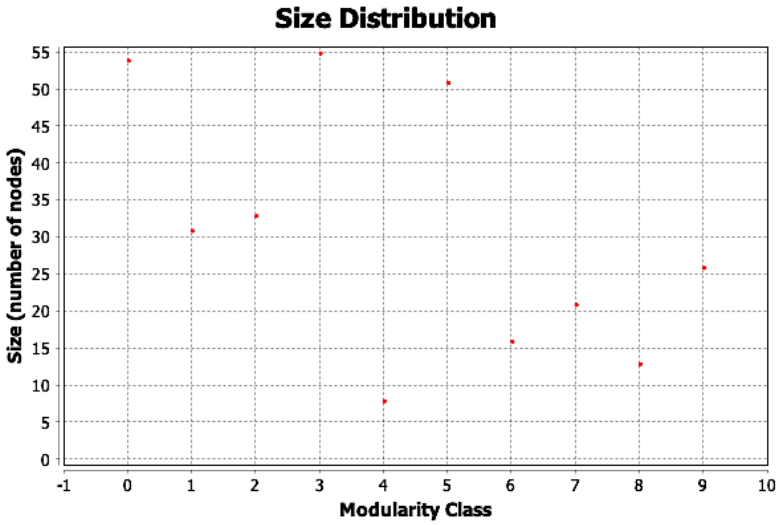


Figure 199 The modularity result shows the numbers of communities and the nodes.[369]

The visualisation result appears as Figure 200. From this result, I noticed the division of clusters was more detailed than it was in Figure 191. This result includes ten clusters: Cluster 1, Notations B, D, and BR (grass green); Cluster 2, Notations H, I, and N (purple); Cluster 3, Notations G, BZ, and CF (grey and brown); Cluster 4, Notations W, BO, AO, AP, K, and BF (red); Cluster 5, Notations F, AL, U, BC, BG, BP, BK, CB, AE, AT, BQ, BU, Y, AZ, and BI (brown and yellow); Cluster 6, Notations P, V, AX, BB, BD, BS, BV, BJ, BY, CA, and CC (violet); Cluster 7, Notation BE (green); Cluster 8, Notations, S, T, Z, AC, BA, BT, AB, AR,

[366] Newman, 'Analysis of Weighted Networks', 7; Blondel et al., 'Fast Unfolding of Communities in Large Networks', 2. For detailed formula explanation, refers to Newman and Blondel et al.'s articles.
[367] The results can be downloaded through: Modularity Report (Accessed: 4 December 2024).
[368] Newman, 'Analysis of Weighted Networks', 7; Fortunato and Barthélemy, 'Resolution Limit in Community Detection', 37.
[369] Figure 199 shows the numbers of communities and the numbers of nodes in each community. For instance, the number 4 of Modularity Class corresponds to number 9 of Size (numbers of nodes), meaning that the Community 4 includes nine nodes. Comparing it with the visualisation result (Figure 200), Community 4 refers to Cluster 7, which consists of Notation BE and its constituent elements (Figure 212).

AA, and AV (grey, purple, and blue); Cluster 9, Notations A, E, J, L, AN, AI, AJ, AH, BN, BH, and BK (brown); Cluster 10, Notations M, O, Q, R, X, AD, AU, and AW (orange and grass green).

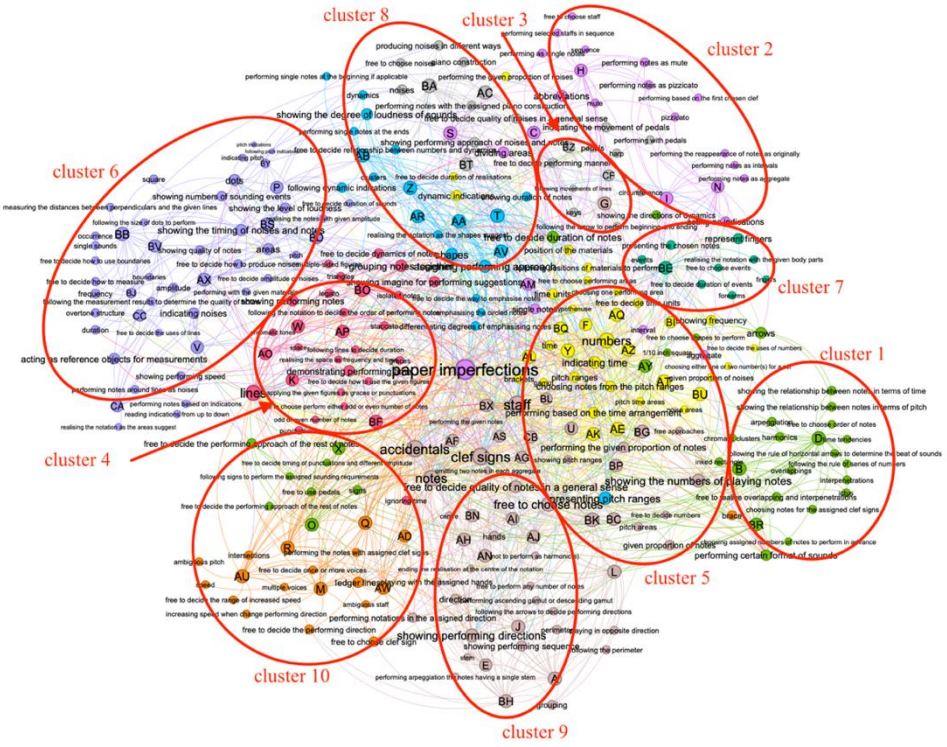


Figure 200 This visualising result shows compositional materials, functions, types of freedom, and performing approaches of eighty-four notations, with the of setting as modularity class.

The Cluster 1 (below) shows Notations B, D, and BR, and the elements they share (overlapping, harmonics, notes, accidentals, staff, numbers, ictus, interpenetrations, and paper imperfections) (Figure 201). As I explained previously, because of the uses of communal elements, three notations create a relatively close relationship. However, such a cluster does not mean that it shares no communal elements with other clusters. When deconstructing the compositional materials, I noticed that arrows were used repeatedly in Notations D, G, N, AE, AH, AJ, AT, and AY with the functions of showing performing directions, showing relationships between notes in terms of time and pitch, acting as indications, and showing the direction of dynamics. This connection occurs across Cluster 1 (Notation D), Cluster 2 (Notation H), Cluster 3 (Notation G), Cluster 5 (Notations AE, AT, and AY), and Cluster 9 (Notations AH and AJ); thus, the arrows act as a connection between them all (Figure 202).

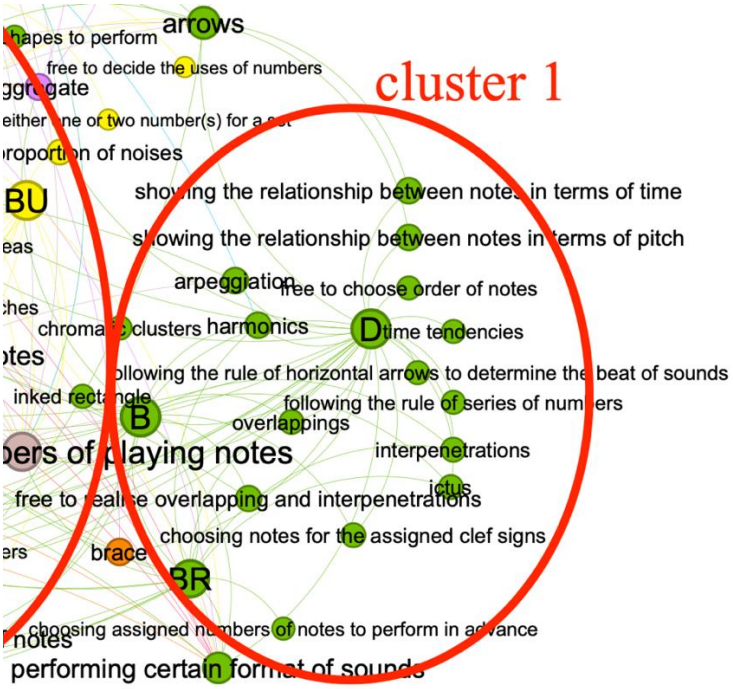


Figure 201 Notations B, D, BR, and their corresponding elements.

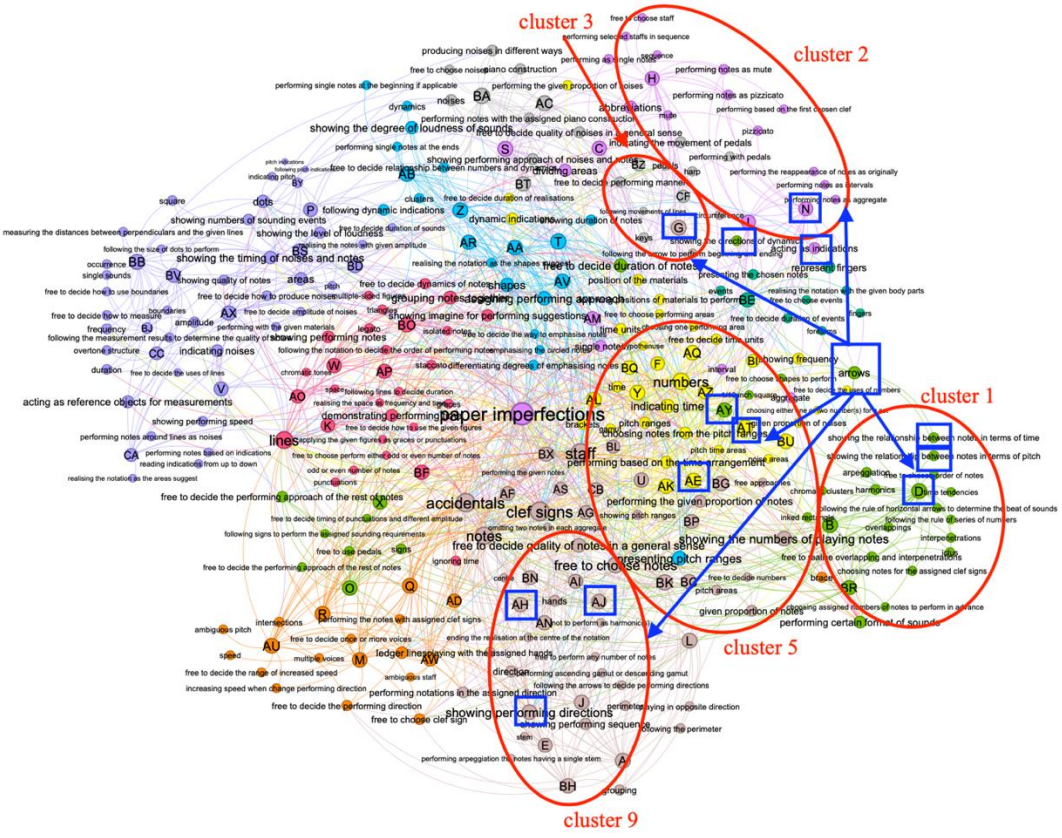


Figure 202 The connection based on uses of arrows in Notations D, G, N, AE, AH, AJ, AT, AY, and their corresponding functions.

Looking into Cluster 2 (below) (Figure 203), which includes Notations H, I, and N. The difference between this Cluster 2 below (Figure 203) and the Cluster 2 above (Figure 193) is the inclusion of Notation C. In the Cluster 2 above (Figure 193), Notation C is included, but it is excluded in this Cluster 2 below (Figure 203). This difference does not exclude Notation C from the cluster consisting of Notations H, I, and N, rather, it suggests Notation C is acting as a connection among the Clusters 2, 8, and other compositional materials that are commonly used in the compositions, such as, paper imperfections, staff, clef signs, and accidentals (Figure 204). The reason for this change is due to Cage assigning Notations H and S 'as in C' in his performing instructions; consequently, Notations C, H, and S are built upon a fundamental system, based on which, three notations construct other types of relationships, as other notations do.

For instance, Notation S shares compositional materials (noises and piano constructions) and a performing approach (performing noises with the assigned piano constructions) with Notations AC and BA; hence, three notations construct Cluster 8. From this system and relationships, I found that the composer's arrangements build relationships step by step – thus, each notation has its own structure, and Cage added the other layer of structure based on the original structure through the notations, shared compositional materials, functions, performing approaches, or types of freedom.

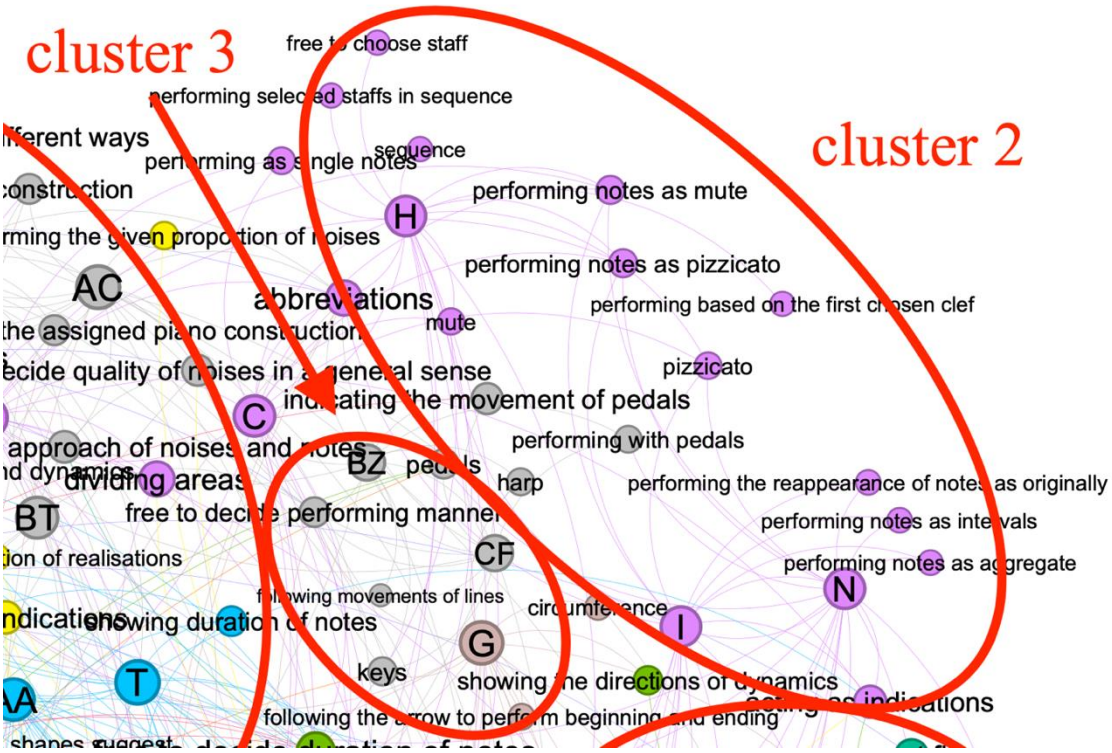


Figure 203 Notations G, I, H, N, BZ, CF, and their corresponding elements in Clusters 2 and 3.

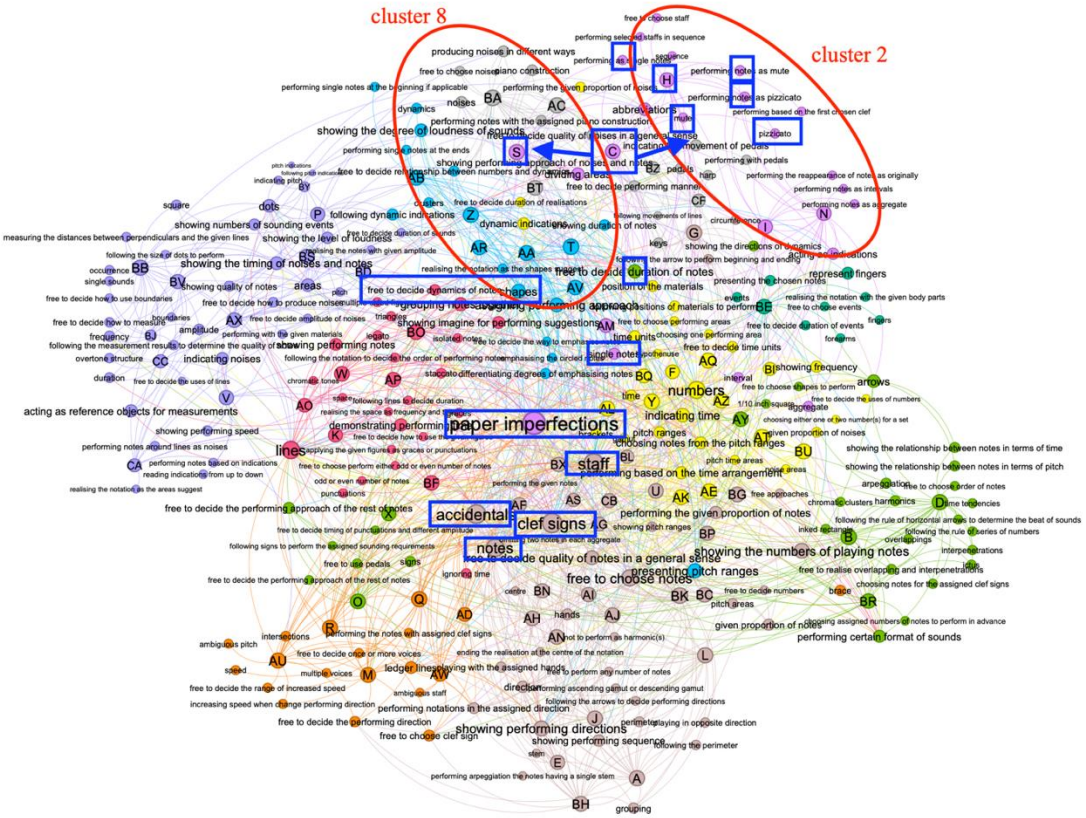


Figure 204 Notation C acts as a conjunction between Clusters 2, 8, and other compositional elements.

Cluster 3 includes: Notations G, BZ, CF, and their corresponding elements (Figure 203). As previously explained, the decisive compositional materials and types of freedom (keys, harp, and freedom to decide performing manner) bind three notations together. Looking into other compositional materials, I noticed that they share elements with notations in Clusters 2, 8, and 10 (Figure 205). For instance, the compositional material, pedals, also apply in Notation N of Cluster 2, and Notations M, O, and X of Cluster 10; the freedom to decide performing manner is the key to connect with Notation BT of Cluster 8.

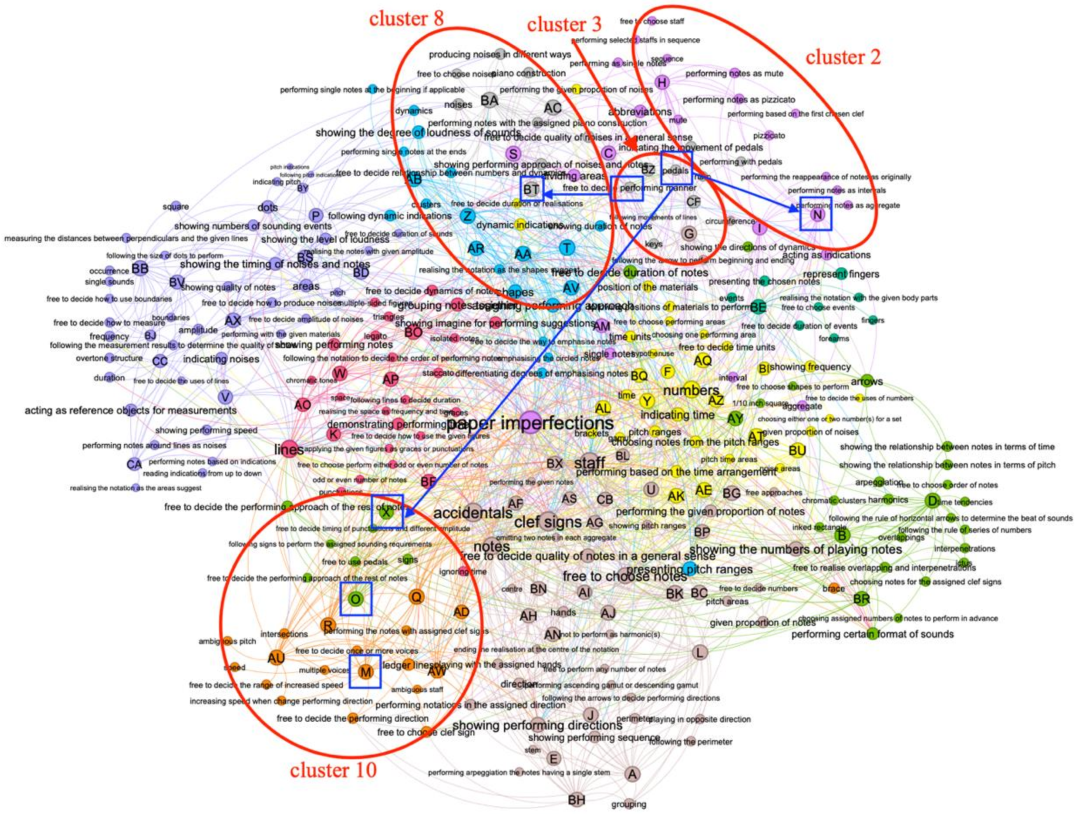


Figure 205 Pedals connect Clusters 2, 3, and 10, and the freedom to decide the performing manner connect Clusters 3 and 8.

Cluster 4 includes: Notations W, BO, AO, AP, K, and BF (Figure 206). Along with the relationships that have been mentioned previously, this cluster includes Notation AO and its elements, which share the function of showing performing notes with Notation K, which, in turn, shares the function of demonstrating performing time with Notation AP, while also sharing pitch ranges with Cluster 5 (Figure 207).

Figure 206 Notations K, W, AP, BF, and BO, and their corresponding elements in Cluster 4.

Cluster 5 includes Notations F, U, Y, AE, AK, AL, AQ, AT, AY, AZ, BC, BG, BI, BK, BL, BP, BQ, BU, and CB. Comparing with the Cluster 5 above (Figure 196), which contains Notations F, Q, Y, AL, AQ, BI, BQ, and BU, with a clear focus of three functions (indicating time, showing frequency, and showing the playing numbers of sounds), and the compositional material, time units. Delving into Notations U, Y, AE, AK, AT, AZ, BC, BG, BK, BP, and CB, which are not included in the Cluster 5 above (Figure 196), I noticed the connections of these notations are based on pitch, time, and noises. For example, the pitch ranges are applied in Notations U, Y, AK, AQ, BC, BG, BP, BQ, BU, and CB; the time function is applied in Notations F, Y, AE, AT, AK, AL, AZ, BI, and BQ; the given proportion of noises is applied in Notations AL, AQ, AK, and BU. The compositional materials and performing approach – given proportion of notes and performing the given proportion of notes – connect Notation BK with most of the notations in this cluster and also in notations in Clusters 1 and 9 (Figure 208, Figure 209).

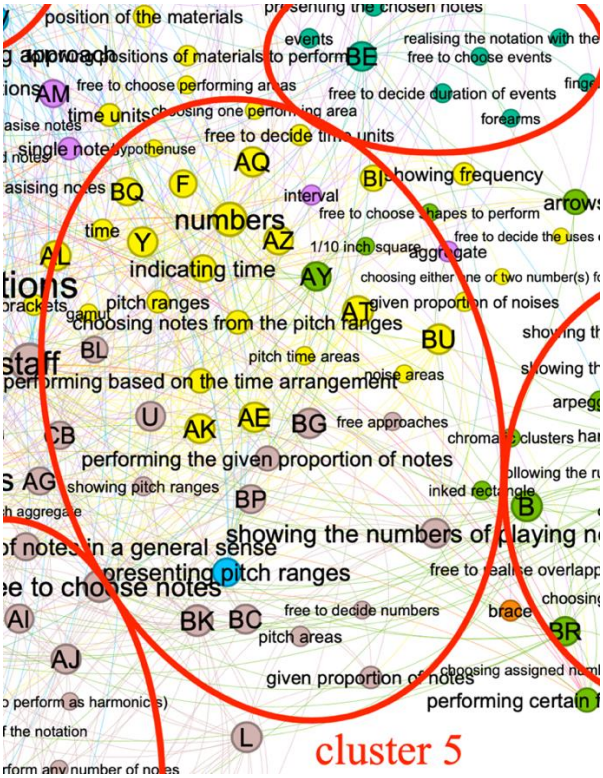


Figure 207 Notations F, U, Y, AE, AK, AL, AQ, AT, AY, AZ, BC, BG, BI, BK, BL, BP, BQ, BU, and CB, and their corresponding elements in Cluster 5.

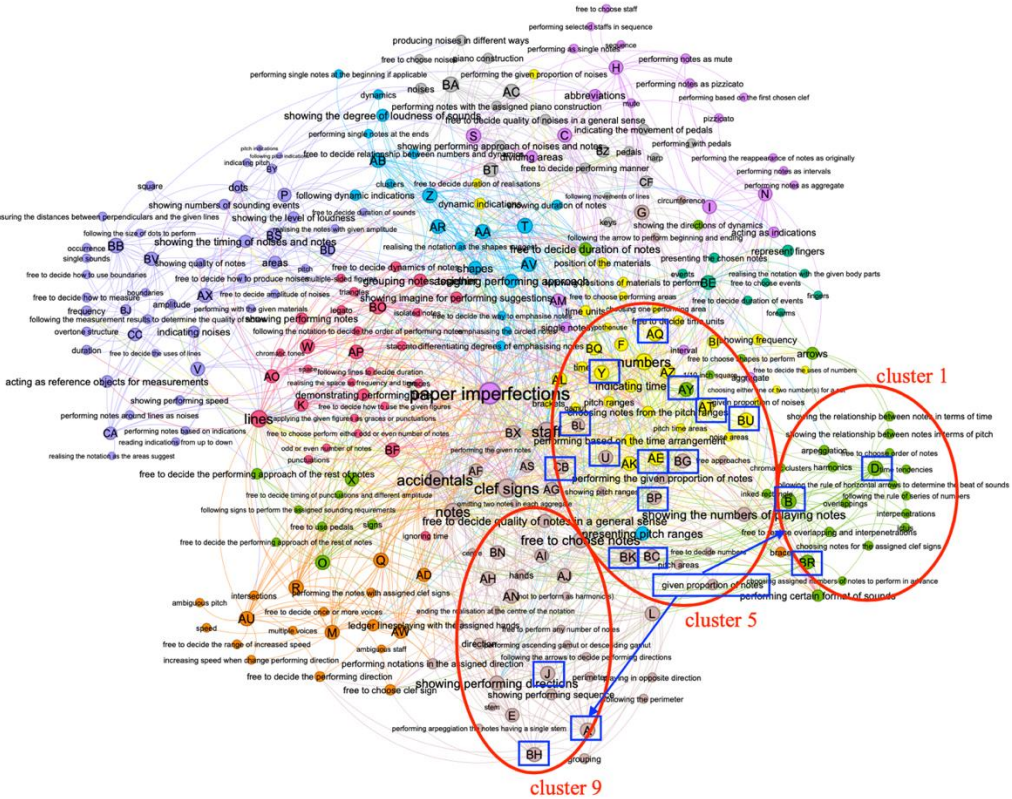


Figure 208 The compositional material, the given proportion of notes connects notations in Clusters 1, 5, and 9.

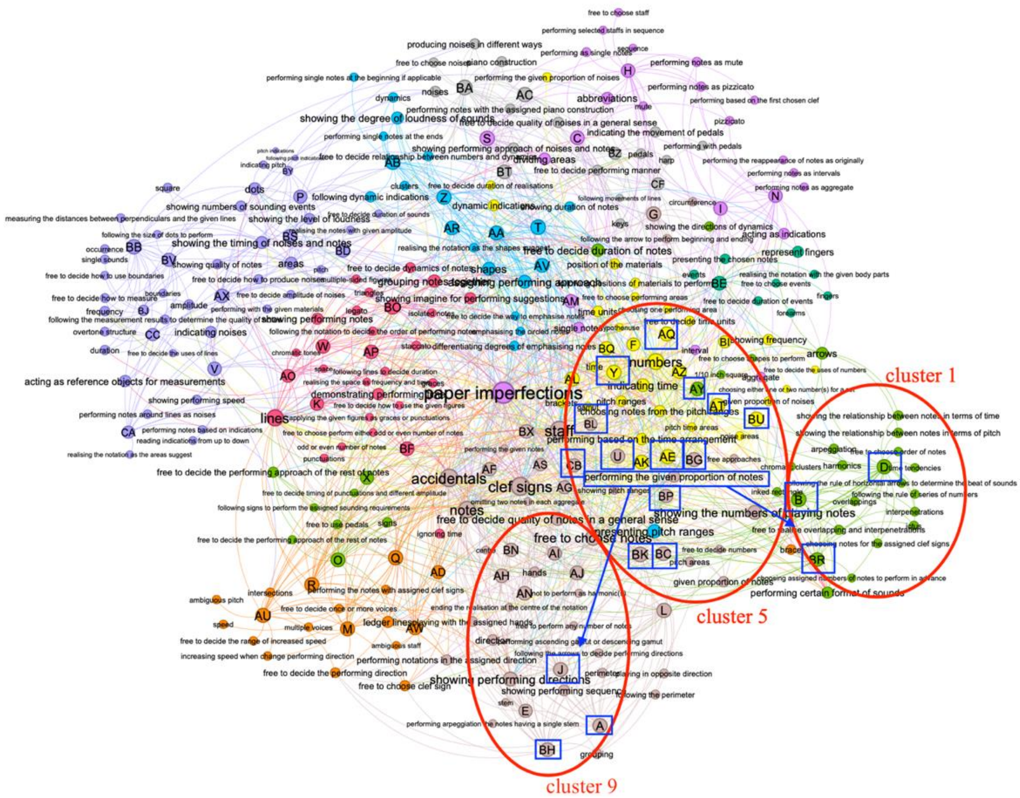


Figure 209 The performing approach, performing the given proportion of notes, connects notations in Clusters 1, 5, and 9.

Cluster 6 includes Notations P, V, AX, BB, BD, BJ, BS, BV, BY, CA, and CC (Figure 210). It shows a relatively focused use of compositional materials. From the deconstruction of notations, I noticed that these notations are based on the uses of areas, which are used in multiple functions, including offering the level of loudness in Notations P, V, AX, BD, and BS, and representing quality of sounds in Notations BB, BJ, BV, and CC; showing pitch in Notation BY, and showing performing approaches (i.e. keyboard, mute, or pizzicato) in Notation CA. To repeat, a closed cluster does not imply that it has no connections with other clusters, rather, it shows that one or more nodes can be acting as key conjunctions to connect other clusters. The node representing the performing approach, following dynamic indication, is used in Cluster 6 (Notations P, BD, and BS), Cluster 2 (Notation N), Cluster 3 (Notation G), Cluster 5 (Notation Y, AK, AQ, and BQ), Cluster 8 (Notations T, Z, AB, AV, and BA), and Cluster 10 (Notation X) (Figure 211).

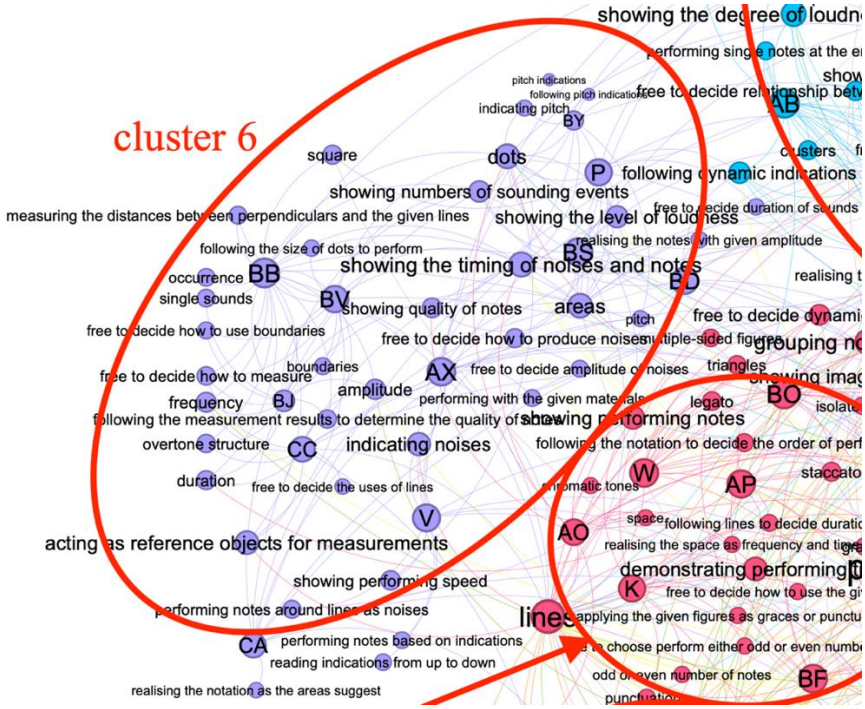


Figure 210 Notations P, V, AX, BB, BD, BJ, BS, BV, BY, CA, CC, and their corresponding elements in Cluster 6.

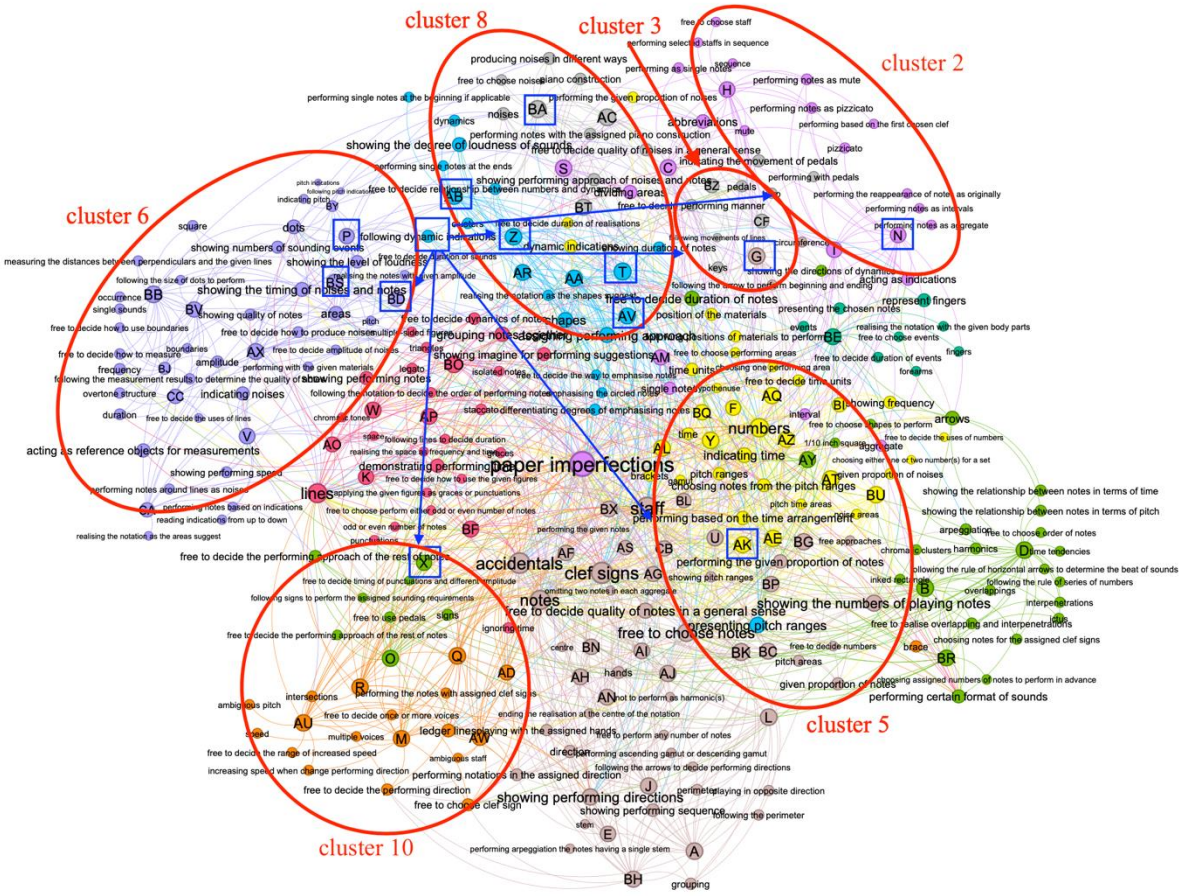


Figure 211 The performing approach, following dynamic indications, acts as a conjunction connecting Clusters 2, 3, 5, 6, 8, and 10.

The seventh cluster consists of Notation BE and its correlated elements (Figure 212). Looking into its elements, I noticed this cluster is Notation BE exclusive. However, other compositional materials (areas, dots, and numbers) and functions (showing numbers of playing notes, and showing performing approach of noises and notes) build relationships with other clusters. Its compositional material – dots, connect Notation BE with Notations P, BB, BS, BV, BY (from Cluster 6), Notations BA, BT (from Cluster 8), Notations AO and BO (from Cluster 4) (Figure 213). The compositional material – areas connect Notation BE with Notations P, AX, BB, BD, BS, BJ, BV, BY, and CA (from Cluster 6), Notation BA (from Cluster 8), and Notations BU and BK (from Cluster 5) (Figure 214). The function of showing numbers of playing notes builds relationships between Notation BE and other notations, such as B, D, BR (from Cluster 1), Notation G (from Cluster 3), Notations U, AE, AT, BC, BG, BK, BP, BL, BU (from Cluster 5), Notation AC (from Cluster 8), and Notations A, J, L, AJ, and BH (from Cluster 9) (Figure 215). Another function – showing performing approach of noises and notes – relates Notation BE with AC, BA, BT, and CA (Figure 216).

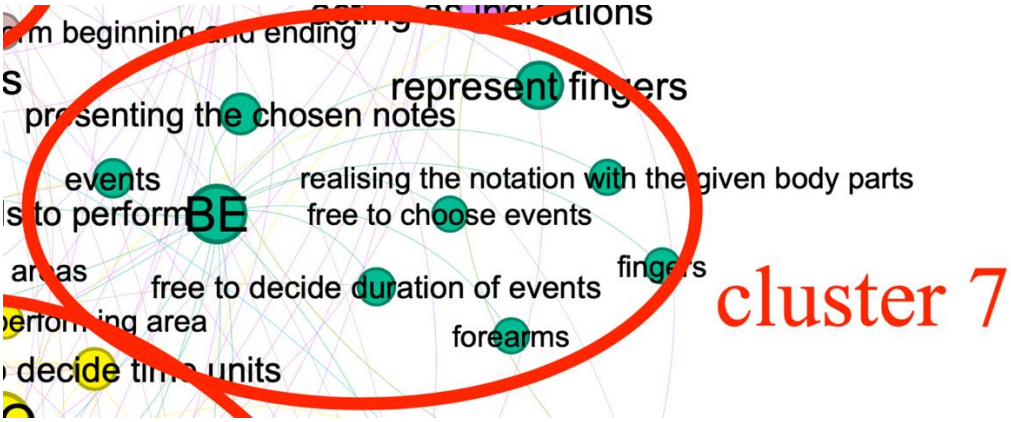


Figure 212 Notation BE and its corresponding elements in Cluster 7.

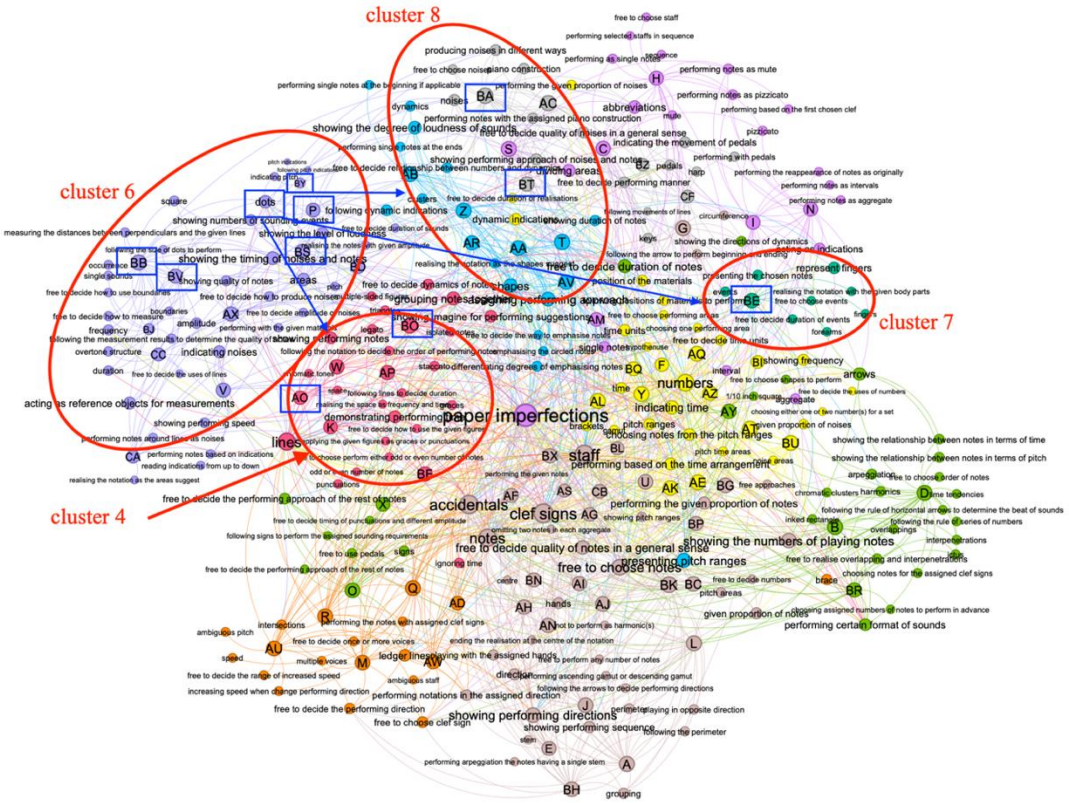


Figure 213 The dots connect notations from Clusters 4, 6, and 8.

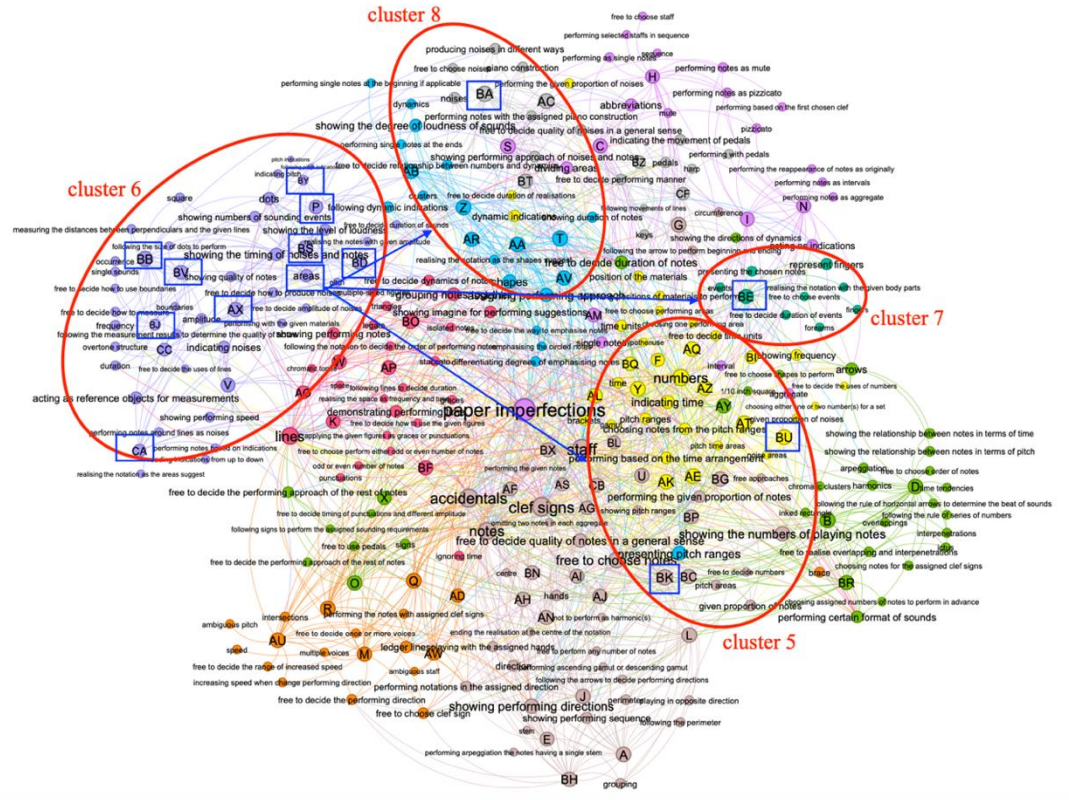


Figure 214 The areas connect Notation BE with other notations.

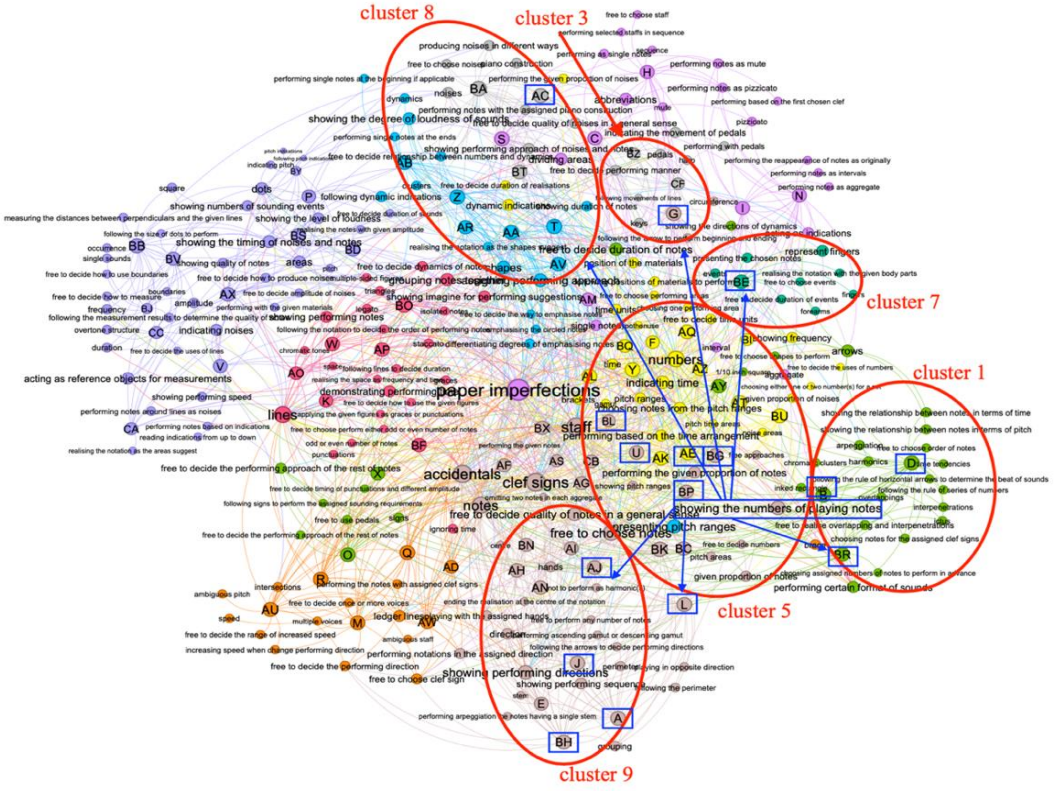


Figure 215 The function of giving the numbers of playing notes connects Clusters 1, 3, 5, 7, 8, and 9.

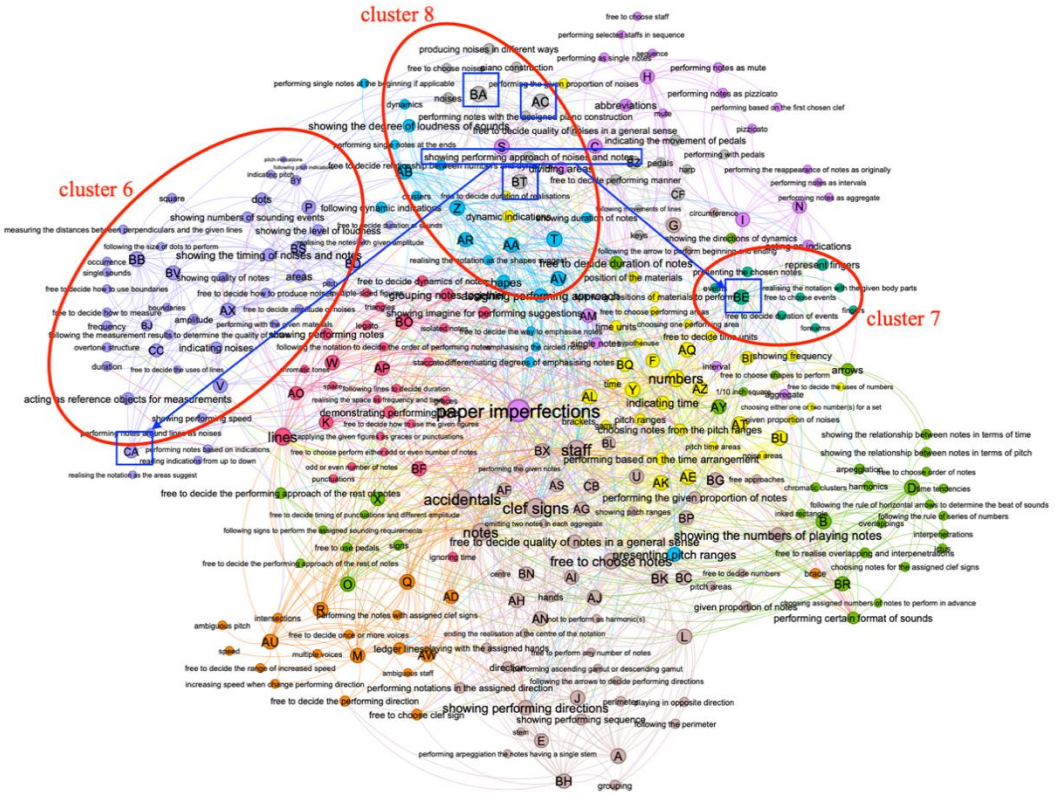


Figure 216 The function of showing the performing approach of noises and notes relates Notation BE with Notations AC, BA, BT, and CA.

The cluster 8 includes Notations S, T, Z, AA, AB, AC, AR, AV, BA, and BT (Figure 217). This cluster mainly focuses on compositional materials about noises, notes, dynamics, and piano construction. The arrangement of compositional materials and performing approaches bring Notations AC, BA, and BT into Cluster 8. Specifically, Notations S, AC, and BA all use noises as one of the compositional materials; meanwhile, their noises are based on the other compositional material – piano constructions – which is also applied in Notation BT. This means that two compositional materials – noises and piano constructions – tie Notation S with Notations AC, BA, and BT (Figure 218). Also, in his performing instruction for Notation S, Cage stated, '*like C, but with noises*', thus he directly assigned a relationship between them; therefore, Notation S acts as an important conjunction to connect with Notation C. Looking at the type of freedom in Notation S – to decide dynamics of notes – I noticed it also applied in Notations W, AP, and BO (from Cluster 4). Hence, this freedom connects four notations together, as well as Clusters 8 and 4 (Figure 218).

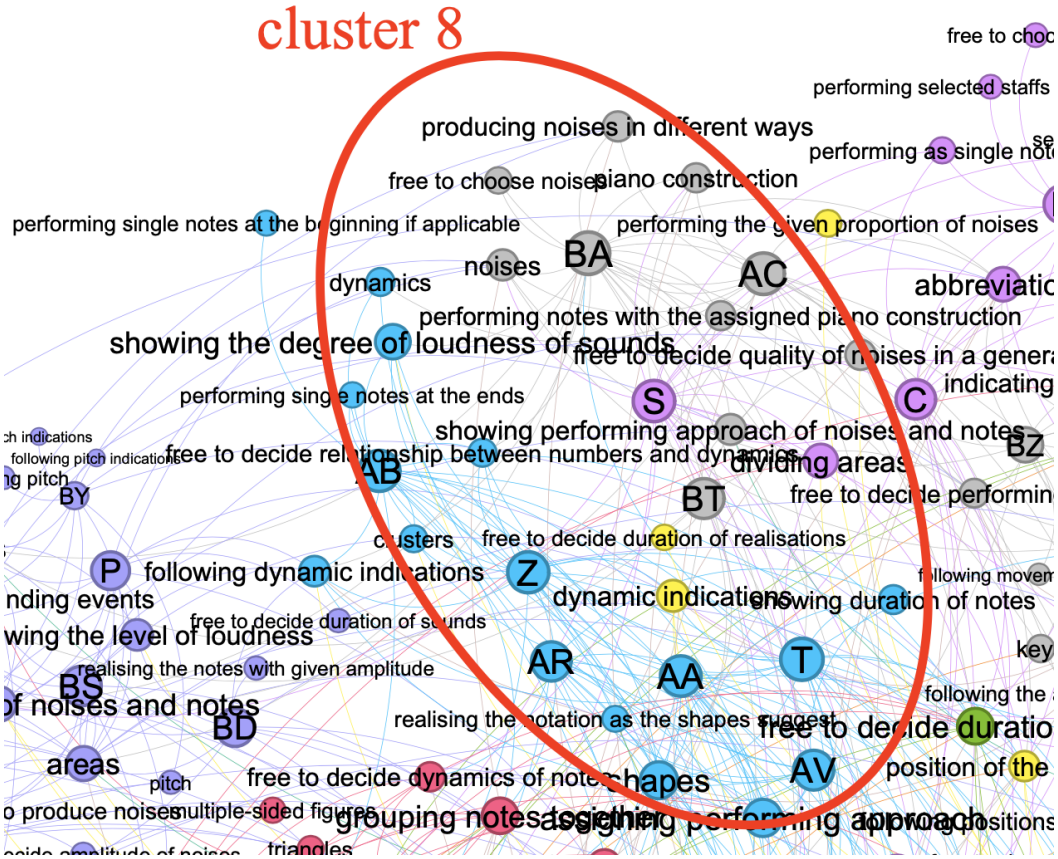


Figure 217 Notations S, T, Z, AA, AB, AC, AR, AV, BA, BT, and the corresponding elements in Cluster 8.

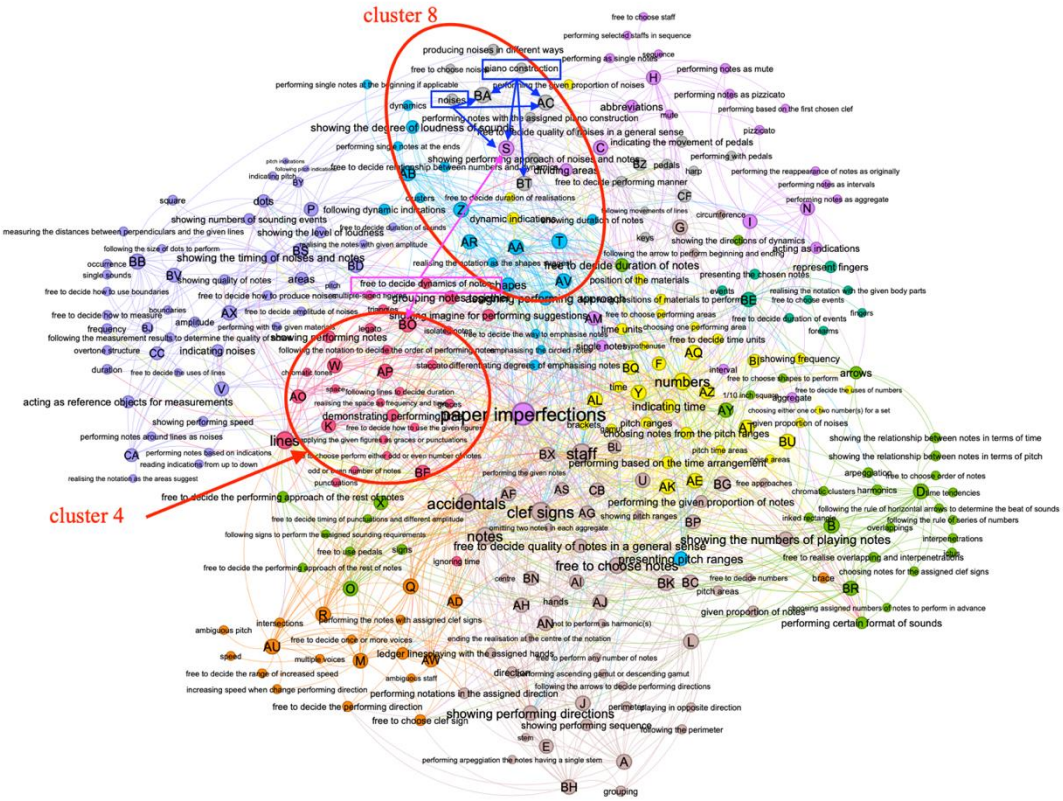


Figure 218 The noises and piano constructions connect Notations S, AC, BA, and BT; the freedom to decide the dynamics of notes that connect Clusters 4 and 8.

Cluster 9 contains notations that are based on direction and sequence, such as Notations A, E, J, L, AN, AI, AJ, AH, BN, BH, and BK; Cluster 10, consists of notations that are constructed by intersections, ambiguous staff, and pedals, such as, Notations M, O, Q, R, X, AD, AU, and AW (Figure 219).

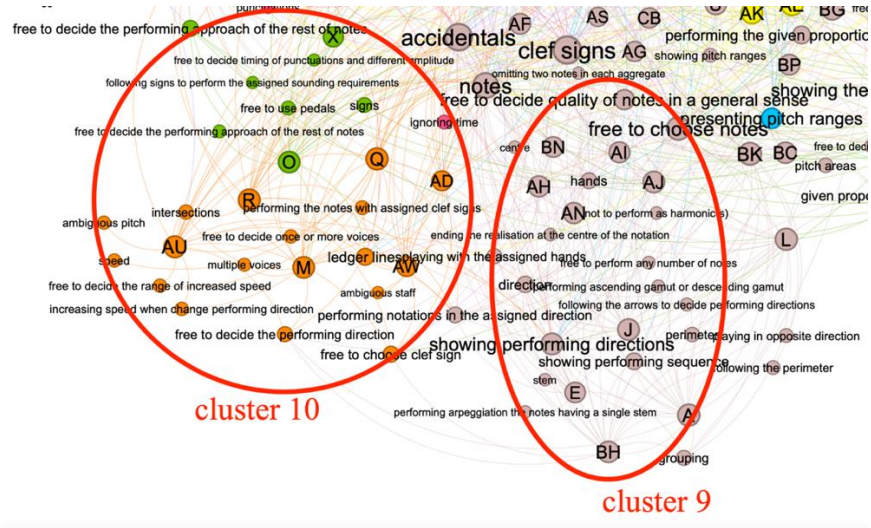


Figure 219 Notations A, E, J, L, AH, AI, AJ, AN, BK, BN, and their corresponding elements in Cluster 9; Notations M, O, Q, R, X, AD, AW, AU, and their corresponding elements in Cluster 10.

In the above discussions, I have explained the relationships between notations through the applications of compositional materials and its functions, types of freedom, and performing approaches. Based on these relationships, I have visualised ten clusters, from which I have observed their intertwined relationships between all the compositional elements. Relating these relationships to the first of the rational features: method/methodical, I propose that this rational feature can be found in notations. As I explained in Chapter 4 Section 4.2 Value Formation in Graphic Compositional System, John Cage first applied chance-derived materials, the paper imperfections, in the graphic compositional system, and then he applied indeterminacy, by offering diverse choices for the performers; through this two-stage process, he rationalised the chance-derived materials into semi-determinate materials, showing the two-stage process to be a methodical way to create valuable compositional materials. For this reason, the feature of methodical can also be recognised among eighty-four notations.

In the visualisation process, nodes represent notations, compositional materials, functions, types of freedom, and performing approaches. A node can act as an essential conjunction to relate two or more nodes, such as notations, compositional materials, functions, types of freedom, and performing approaches, together. A node can also be responsible for connecting two or more clusters. Thus, the relationships between notations, compositional materials, functions, types of freedom, and performing approaches are methodically constructed – from the nodes of the same notation to the nodes of different notations – and then moving on into different clusters.

Regarding the second rational feature – function/functional – I suggest that this feature can be discovered through the functions of compositional materials – therefore I argue that this rational feature does not serve emotional purposes, rather it is concerned with the formation of performers' realisations. In Chapter 3 Section 2.1 Rationality or Irrationality: Affectual Function and Affectual Result, I argued that the affectual function of musical elements is a part of rationality, whilst the affectual result is a part of irrationality, since musical elements serve expressive purposes in the composers' composition, but these elements then become unpredictable by producing affectual results in performers' interpretations.

Similar circumstance can be discovered in the compositional materials' functions. Cage clearly assigned one compositional material – shape – for a certain purpose by assigning seven functions, including performing approach, showing performing directions, presenting pitch

ranges, showing duration of notes, differentiating degree of emphasising notes, showing performing sequence, and showing performing notes. These performing instructions clearly show the function of shapes; however, listeners are unable to recognise which notations or functions performers choose in their realisations or are able to predict the acoustic results before they actually hear the realisations. Hence, I notice the similarity to the rational feature of European art music and the similarity of irrationality between two types of music. The next section will focus on Cage's assigned relationships among notations, in order to construct the individual systems.

2.2 Visualisation of the *Solo for Piano*: Individual Systems[370]

As I illustrated above, Cage arranged some notations' performing approaches to be based on other notations, either in identical ways or with some variations. For instance, he referred Notation B in Notation D's performing instruction, also referred Notation C's performing approaches in Notation H's performing instruction. These arrangements can be discovered throughout the *Solo for Piano*. By the way of referring notations in the other notation's performing instruction, these arrangements build relationships among them and their relevant elements. Thus, I suggest, these assigned relationships act as essential spines of the inner structure; with such, the methodical feature of the *Solo for Piano* can be distinctly recognised.

Based on his overall instructions, I have found twenty-seven relationships; hence, here are seventeen systems in the instructions: System A (Notations A–L, A–BK A–BH), System B (Notations B–D, B–BR, B–L), System C (Notations C–H, C–S), System D (Notations D–O), System M (Notations M–O, M–Q), System I (Notations I–N), System Q (Notation Q–AU), System T (Notations T–Z, T–BA), System W (Notation W–BO), System Y (Notations Y–AQ), System Z (Notations Z–AB), System AC (Notations AC–BK, AC–BA), System AD (Notations AD–AW), System AE (Notations AE–AT), System AR (Notations AR–AV), System BB

[370] The systems here refer to Cage's assigned relationships between notations across the *Solo for Piano.* Considering their relationships are sometime more than two notations and some notations bridge notations with one another, I named these relationships as systems. The title of each system will follow the first notation of the system, for instance, one system includes Notations A, L, BK, and BH, this system will be named as system A.

(Notations BB–BJ, BB–BV, BJ–BV), and System BZ (Notations BZ–CF).[371] Inputting these individual systems into Gephi, with the setting: directed weight as two; the result shows in Figure 220.

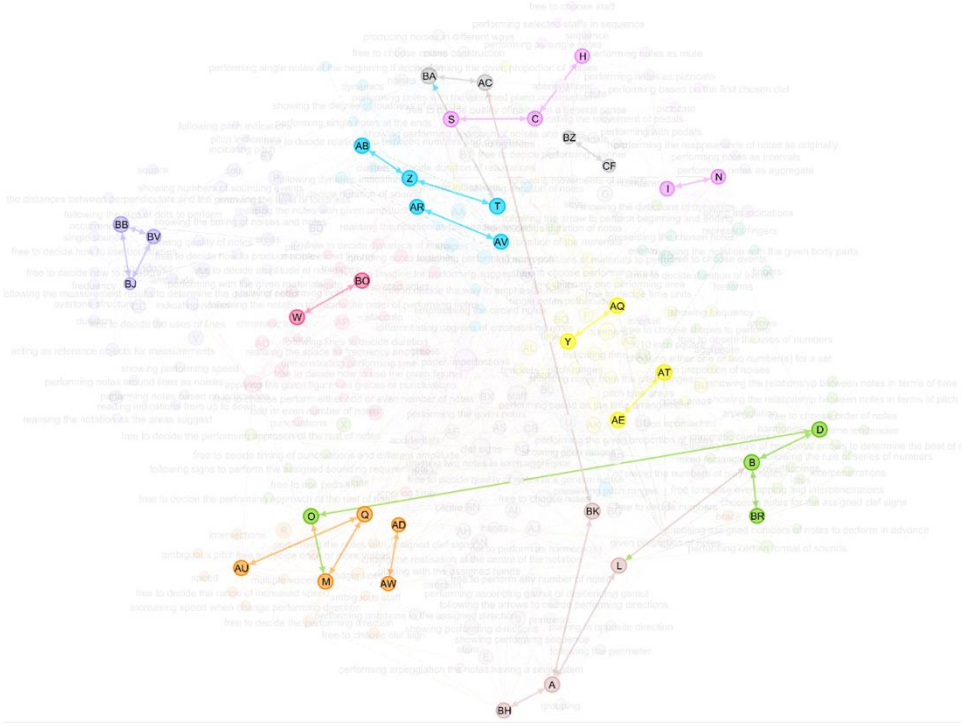


Figure 220 The individual systems that were formed by Cage's assigned relationships in his performing instructions.

[371] Notation D, Cage mentioned, is like Notation B, which is also mentioned in Notation BR. In Notation H, he explained, should be performed like Notation C, and he referred to Notation C in Notation S, but with noises. For Notation L, he referred to Notation A for the perimeter and to Notation B for using clef signs; Notation N, he used Notation I to indicate notes to be performed as mute and pizzicato, but with assigned dynamics. In Notation O, he referred to Notation D for the vertical arpeggiations. In Notation M he mentioned both the uses of pedals, referred to Notation Q for the uses of intersections; also, in Notation Q, he mentioned Notation AU. In Notation Z, he used Notation T to indicate the dynamics, Notation Z is then mentioned in Notation AB for the performing approaches of the clusters. In Notation AQ, he explained the performing approaches by referring to Notation Y. Notation AT refers to Notation AE. Notation AD is mentioned in Notation AW. In Notation BA, he used Notations T and AC to explain how to use numbers and the source of noises. In Notation BH he mentioned Notation A to explain the use of the perimeter; Notation A is also mentioned, along with Notation AC in Notation BK. In Notation BJ he explained its boundaries are similar to Notation BB; two notations are mentioned in Notation BV. Notation BO refers to Notation W but with given time. Notation CF refers to Notation BZ for the movements of the pedals. For the full performing instructions for eighty-four notations, refer to Chapter 5 Section 2 Analyses of Compositional Materials: Notation A to Notation CF, in which I deconstructed notations into compositional materials, functions of compositional materials, types of freedom, and performing approaches, and explain how the performers may conduct the realisations.

From the visualisation of these individual systems, I noticed that, while some notations apply to only one notation, others apply to more than one. For instance, Notation W is applied in Notation BO, Notation Y is applied in Notation AQ, Notation AD is applied in Notation AW, Notation AE is applied in Notation AT, and Notation BZ is applied in Notation CF, meaning that these notations are connected only with each other; however, those connected with two show the features from both. Looking into these types of relationships, I realise it includes two situations: (i) one notation is applied to two notations, and (ii) two notations are applied to one notation. In Situation 1, Notation A applies to Notations L and BK, Notation B applies to both Notations D and BR, Notation C applies to both Notations H and S, Notation T applies to both Notations Z and BA, Notation M applies to both Notations Q and O.

In Situation 2, Notation L uses both Notations A and B, Notation O uses both Notations D and M, Notation BA uses Notations T and AC, Notation BK uses both Notations A and AC, and Notation BV uses both Notations BB and BJ. Despite the differences that I found these notations act as similar roles of conjunction.

Looking into the individual system that based on Notation A. Based on the relationships in the performing instructions, Notation A is the foundation of Notations L, BH, and BK; therefore, the visualisation shows the elements in Notation A along with these notations (Figure 221). Referring to four notations, they all use numbers with a ratio format to indicate the numbers of playing notes and use perimeters to indicate the performing sequences and direction. The performing approach requires the performers to perform in opposite directions; they need to follow the perimeters and perform the required proportion of notes. They are free to decide the quality of notes, to choose notes and to decide the initial performing directions for hands. However, the numbers of Notation L do not use the same numbers as the same performing approach in Notations A, BH, and BK.

As I previously stated, some notations apply two notations, Notation L is one of these, as it uses both Notations A and B; its numbers show the same performing approach as in Notation B, choosing notes for the assigned clef signs. Due to this shared performing approach, choosing notes for the assigned clef signs, Notations B and L are connected to each other (Figure 222), hence Notation L acts as a conjunction between systems A and B (Figure 223).

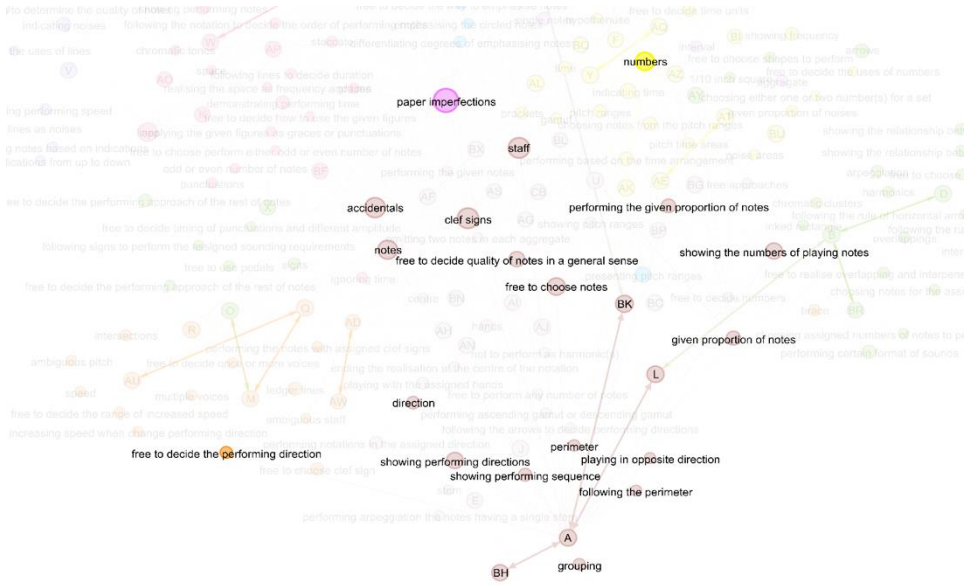


Figure 221 The individual system of Notation A, its corresponding elements, and the notations connecting with Notation A.

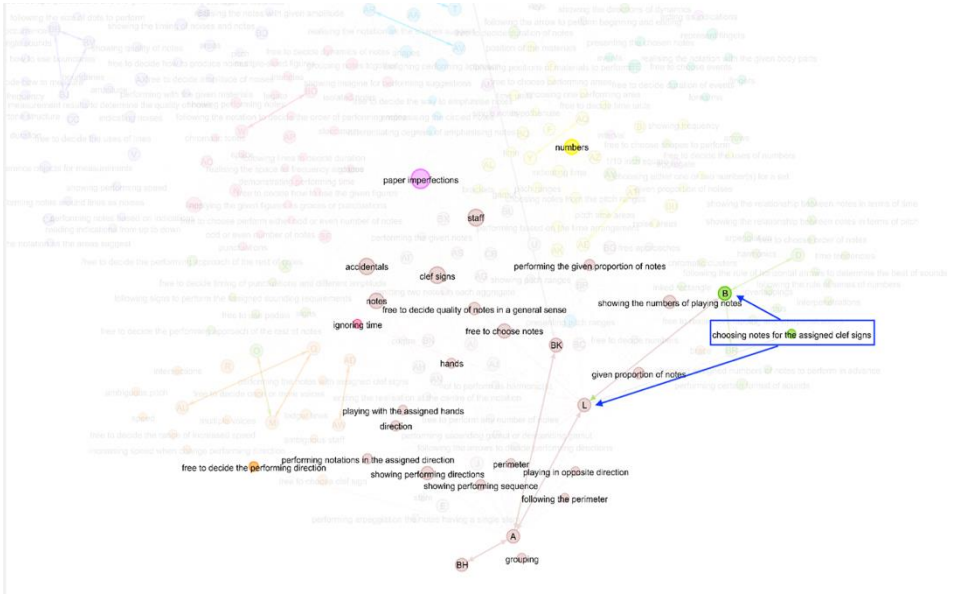


Figure 222 The choosing note for the assigned clef signs connects Notations B and L.

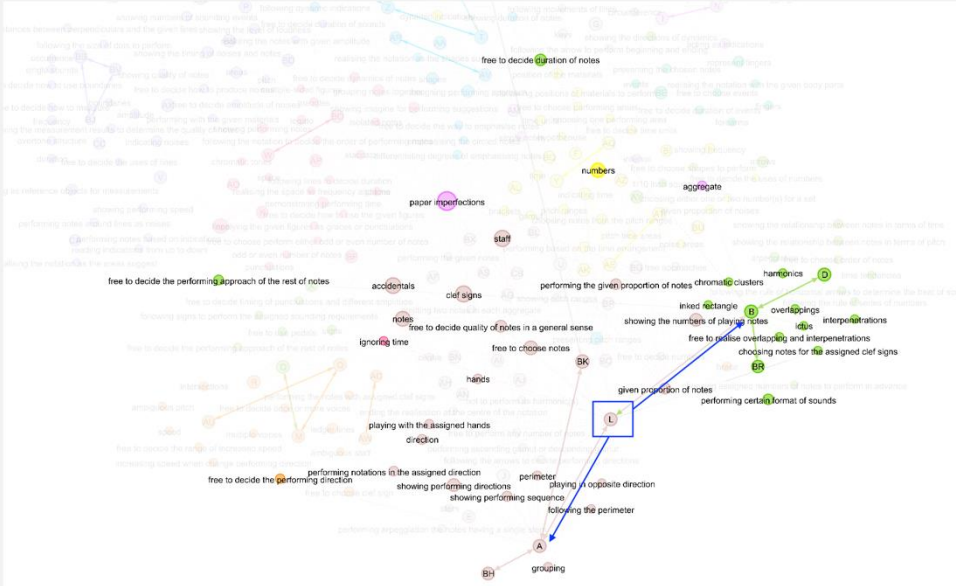


Figure 223 Notation L connects systems A and B.

System B, as Figure 224 shows, includes Notations D, L, BR, and the elements that are used in Notation B. I examined the performing instructions of these; Cage explained that Notation D is 'Like B, but with vertical arpeggiations and time tendencies making use of harmonics unnecessary', he stated that Notation L is '[an]Ambiguity as in B' and indicated Notation BR should 'Play as in B'.[372] Thus, it is clear that these notations have specific relevance to Notation B. For instance, Notation D shares with Notation B the compositional materials of: (i) overlapping, (ii) harmonics, (iii) ictus, (iv) interpenetration, (v) aggregates, (vi) given proportion of notes, (vii) numbers, (viii) paper imperfections, together with other commonly used musical signs, such as, clef signs, staff, accidentals, notes. Two notations also share the function of showing the numbers of playing notes, four types of freedom: (i) realising overlapping and interpenetration, (ii) deciding duration of notes, (iii) choosing notes, (iv) producing certain format of sounds, and two performing approaches: (i) performing certain format of sounds, and (ii) choosing the notes for the assigned clef signs.

The shared compositional materials, function, types of freedom, and one of the performing approaches (performing certain format of sounds) can also be found in Notation BR. Together with these shared elements, this visualisation shows the links between three notations (Figure 225). Meanwhile, Notation L shares both elements from Notations A and B; it shares with

[372] Cage, *Concert for Piano and Orchestra: Solo for Piano*. For the full performing instructions, refer to the section 2 Analyses of Compositional Materials: Notation A to Notation CF, in Chapter 5.

Notation B the given proportion of notes, choosing notes for assigned clef signs, showing numbers of playing notes. Hence, as I clarified above, Notation L acts as a conjunction between Systems A and B (Figure 223).

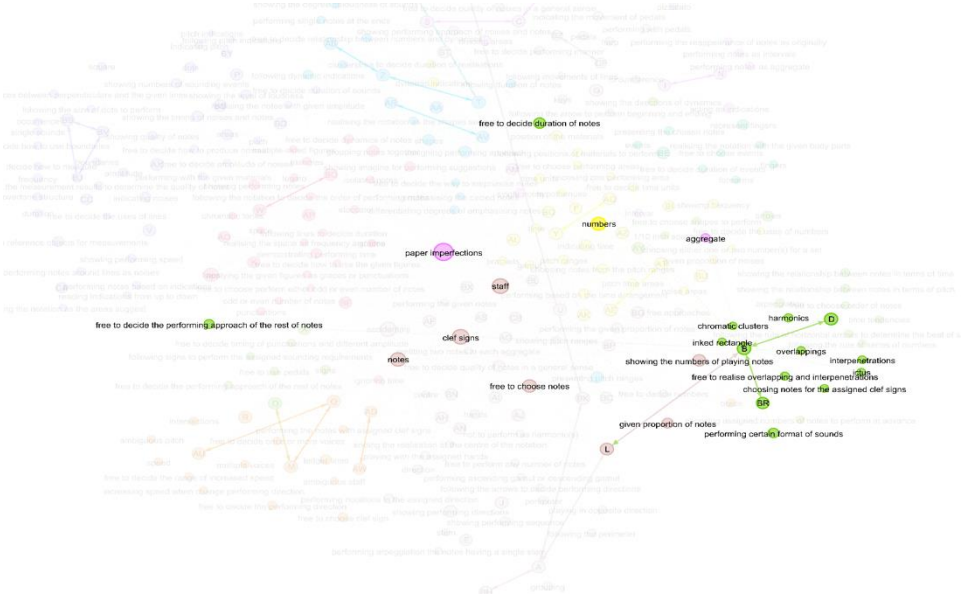


Figure 224 The individual system of Notation B, its corresponding elements, and the notations that connect with it.

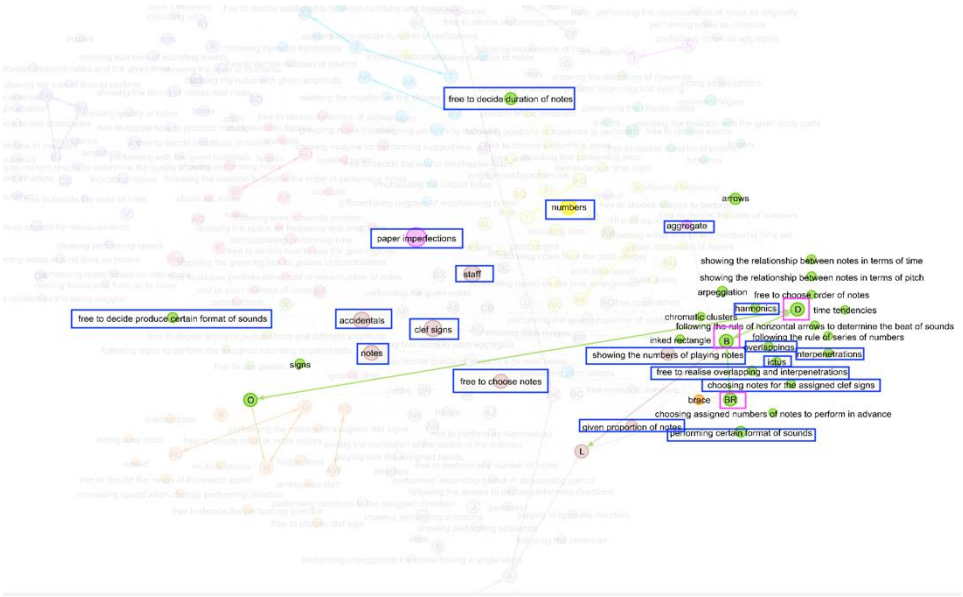


Figure 225 The shared elements between Notations B, D, and BR.

System C, which includes Notations C, H, and S. In Notation C, I noticed that the compositional materials focus on mute, pizzicato, and single notes, that the performing approaches are performed as single notes, mute, and pizzicato, and that the type of freedom here is the freedom to decide the duration of notes (Figure 226). Cage applied these elements to Notations H and S, and therefore, three notations connect with each other. From Figure 227, the connections between this system and other notations can be noticed. For instance, the function of dots, showing the timing of noises and notes, is also applied in Notations P, V, Y, AC, AL, AO, AX, BA, BB, BS, BV, BY, CA, and S; the freedom to decide dynamics of notes is applied in Notations C and S of System C and Notations W, U, AP, AR, and BO (Figure 228); and the freedom of deciding the duration of notes is largely applied in notations that are in System B (Notations B, D, and BR), System Z (Notations Z and AB), System W (Notations W and BO), and Notations O, U, X, AD, AP, and BD (Figure 229). In short, the proposing elements show that Notations C, H, and S are linked together, while other elements circle them together with other notations. These links show the three types of relationships, (i) the uses of compositional materials, (ii) functions of compositional materials, and (iii) types of freedom.

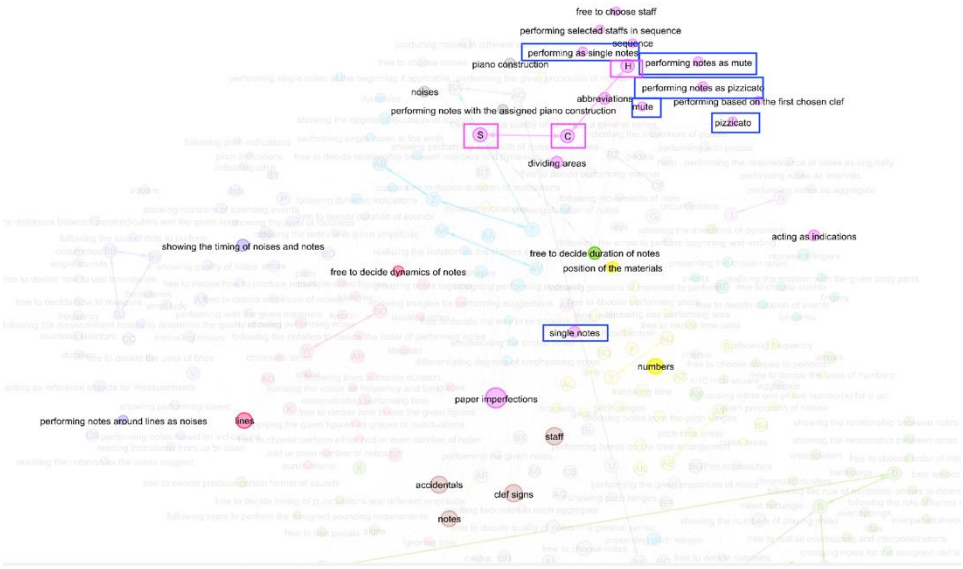


Figure 226 The elements that are shared between Notations C, H, and S.

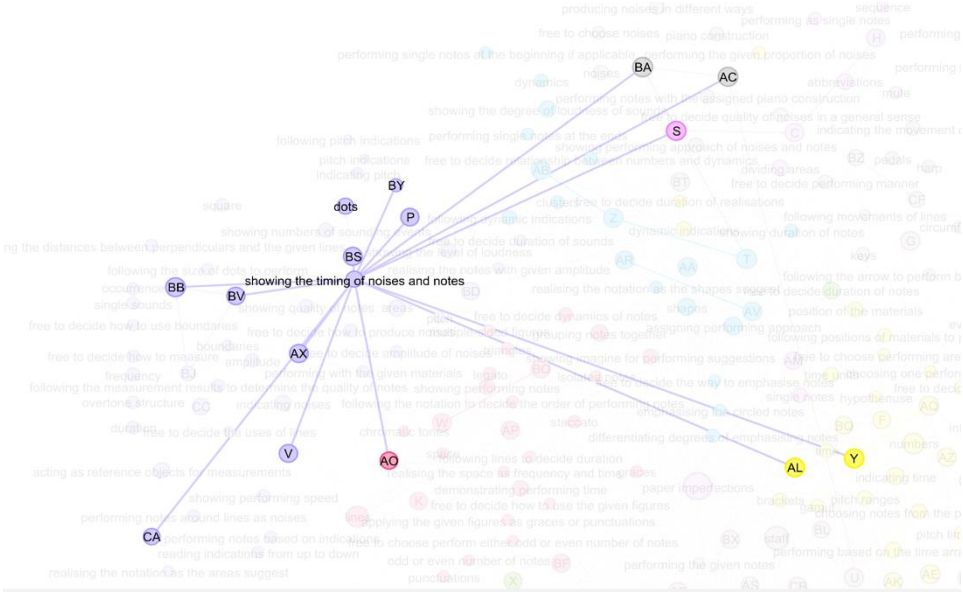


Figure 227 Notations that apply the function of dots, showing the timing of noises and notes.

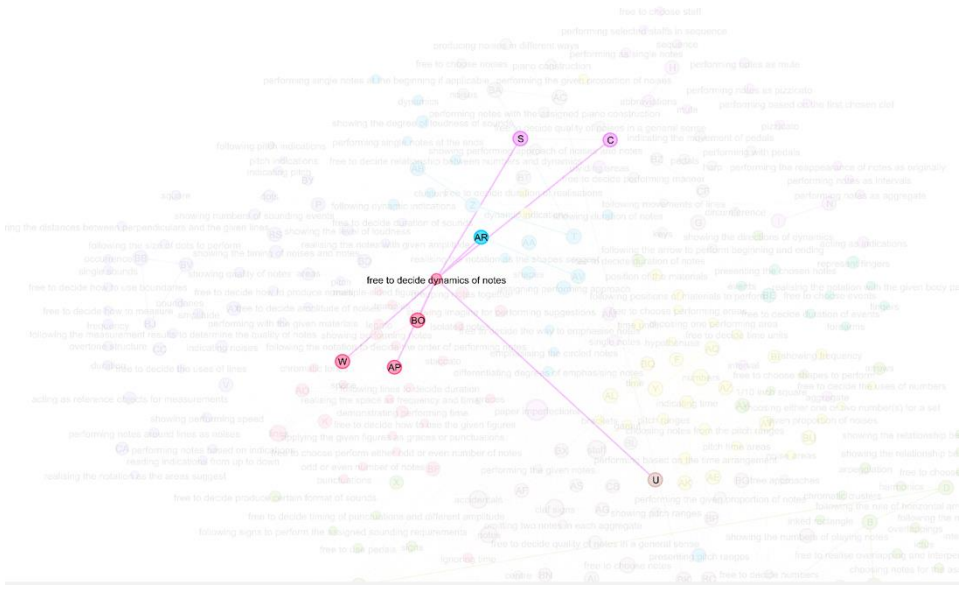


Figure 228 The notations that apply the type of freedom of deciding the dynamics of notes.

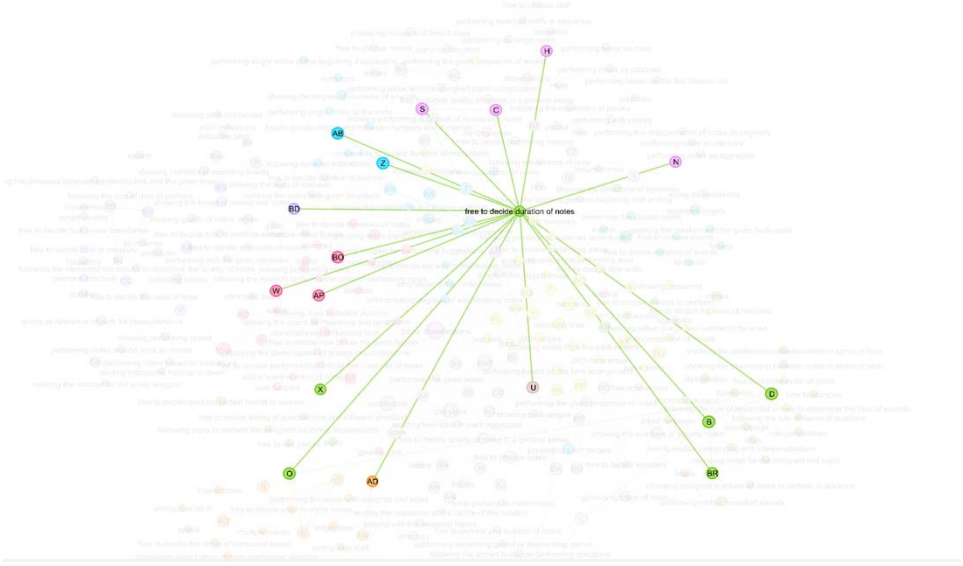


Figure 229 The notations that apply the type of freedom of deciding the duration of notes.

System D includes Notations D and O (Figure 230). The performing instruction of Notation O, showing the assigned relationship between two notations, is based on: (i) two communal compositional materials: arpeggiation and harmonics; (ii) two types of freedom: free to decide certain format of sounds, and free to decide duration of notes; and (iii) the performing approach, performing certain formats of sounds. (Figure 231). The freedom of deciding certain formats of sounds is not only used in Notations D and O, but also in other notations, such as those in System B (Notations B, D, and BR), and in Notations AO, AA, and CA (Figure 232). With the relationship based on this type of freedom, these notations form the third type of relationship. Meanwhile, the performing approach, performing certain formats of sounds, also connects this system with Systems B (Notations B, D, and BR), I (Notations I and N), W (Notations W and BO), AD (Notations AD and AW), and also Notations AL, AM, AO, AP, AZ, BF, and BG (Figure 233).

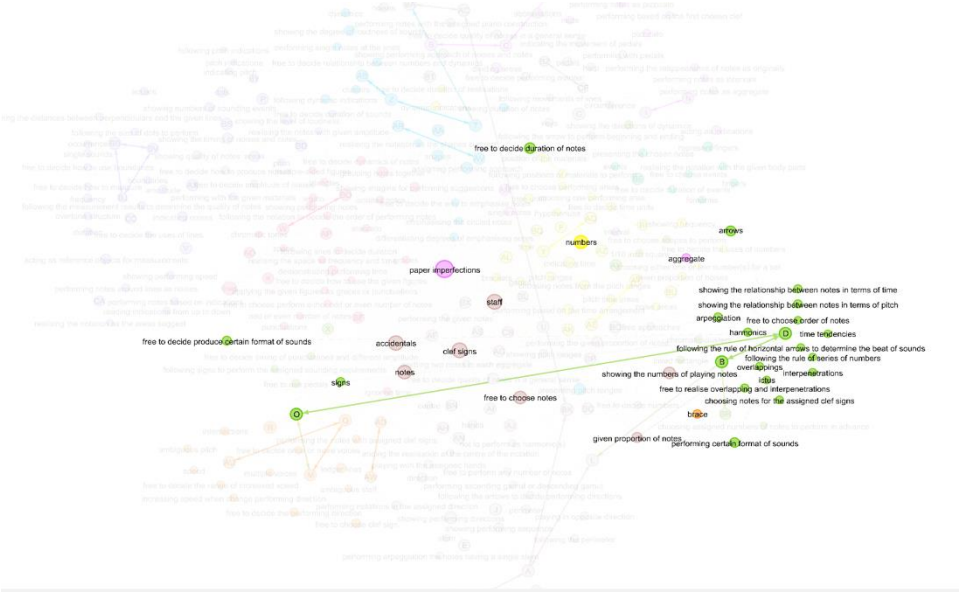


Figure 230 The System D includes Notations D and O.

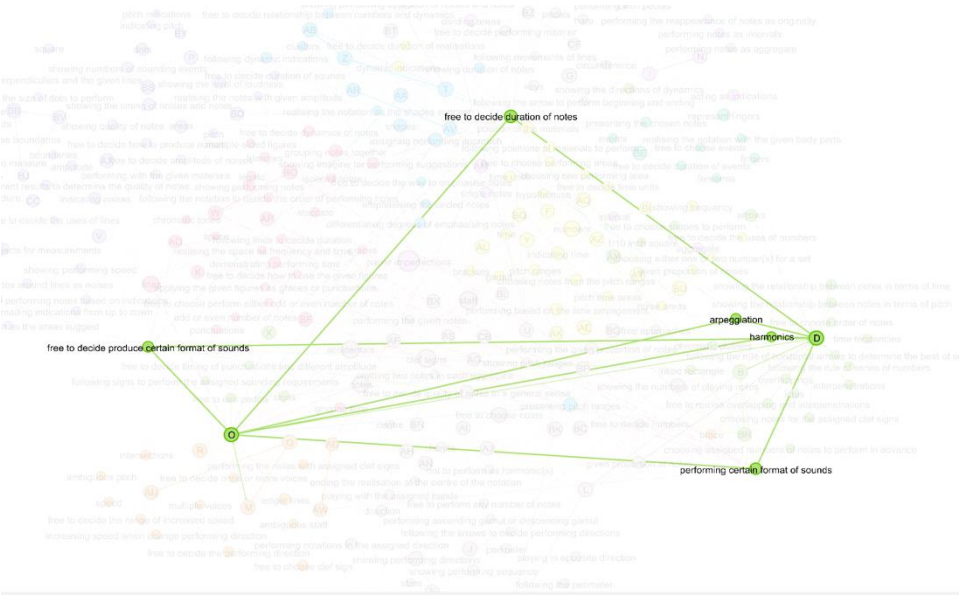


Figure 231 The communal elements between Notations D and O.

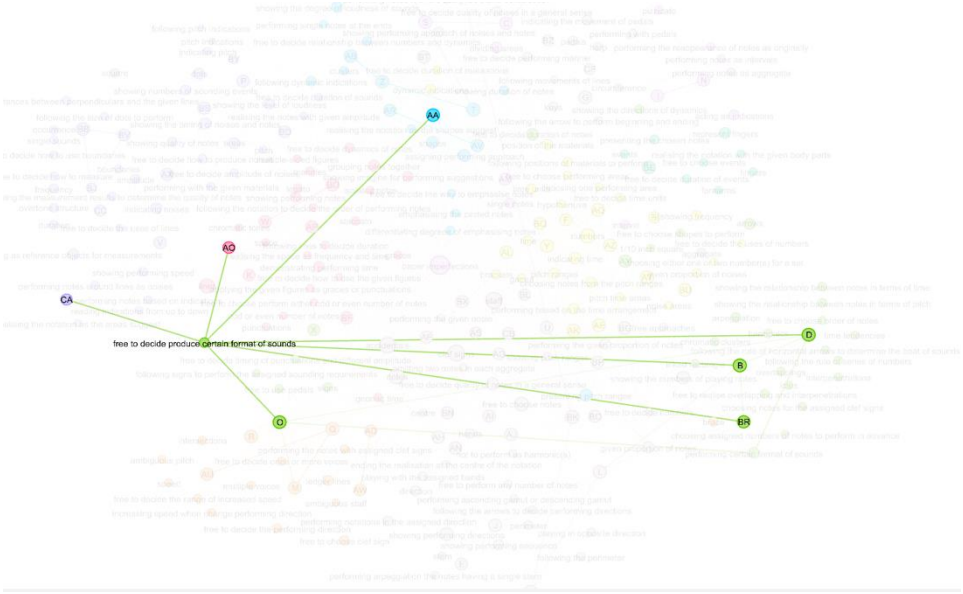


Figure 232 The freedom to decide the format of sounds that connect Notation O with System B, Notations AA, AO, and CA.

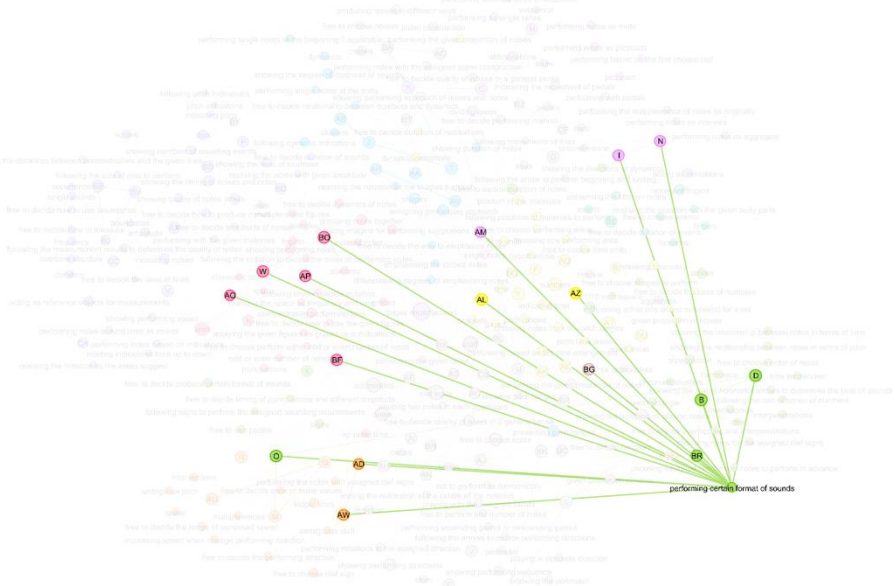


Figure 233 The performing approach, performing certain formats of sounds, connects system D with systems B, W, I, AD, and Notations AL, AM, AO, AP, AZ, BF, and BG.

The performing instruction of Notation O also indicates its relationship with Notation M, as it describes how its pedals are applied as in Notation M; meanwhile, Notation M is also the referential notation for Notation Q, therefore, System M includes Notations M, O, and Q (Figure 234). System M focuses on two key elements, intersections and pedals, meaning that its compositional materials functions, types of freedom and performing approach are based on those elements. The intersections are applied in Notations M, Q, R, AU, and CC, the use of this

material concerns the performing direction in Notations M, Q, and AU, performing notes in Notation R, and the quality of sounds in Notation CC. Looking into intersections, I notice that this compositional material is constructed by lines, and it can represent direction in some notations. In terms of lines, which is applied in Systems M (excluding Notation O), Q (Notations Q and AU), and AR (Notations AR and AV), and Notations J, AH, AN, and AJ, while the directions is applied in Systems A (A, L, and BK) and M (excluding Notation O), and Notations E, J, R, U, AH, AN, AI, and AJ. Based on a combination of intersections, directions, and lines, the function of showing performing direction is applied in Systems A, M (excluding Notation O), Q, and AR, and Notations J, AH, AN, and AJ; the type of freedom of deciding performing direction is applied in Systems A and M (excludes Notation O), and Notation BN; the performing approach, performing notations in the assigned direction, is applied in Systems M and Q, and Notations E, U, AH, AN, AI, and AJ (Figure 235). From Figure 235, the relationships based on compositional materials, functions, type of freedom, and performing approach can be observed.

These relationships not only connect notations within System M, but they also build relationships with systems and notations that are outside it. Another main compositional material is pedals, and the relevant freedom is the use of pedals. Pedals is applied in Notations M, O, N, X, and System BZ (Notations BZ and CF), while the freedom is applied in System M (excluding Notation Q) and Notation X. With the connection based on pedals and their relevant freedom of use; System M, once again, builds the first and the third type of relationships with other notations; meanwhile, the performing approach, performing with pedals, connect System BZ with Notation N (Figure 236).

System M includes Notations M, O, and Q, however, when looking into the compositional material, functions, types of freedom, and performing approach. These relationships are not limited to System M but expand to other systems that apply the same elements. For instance, System Q, including Notations Q and AU, which largely applies to elements from System M, such as, intersections, lines, directions, showing performing directions, freedom to decide performing directions, and performing notations in the assigned directions. When referring to the performing instruction of Notation Q, it is clear that Cage assigned Notation Q to be performed like Notation M with time arrangements; meanwhile, the performing instruction for Notation AU also shows Cage's assigned relationship between Notations Q and AU. When

form their relationships, I noticed a gradational formation among them, which extended the relationship from a notation to a system, and then extending it to another system.

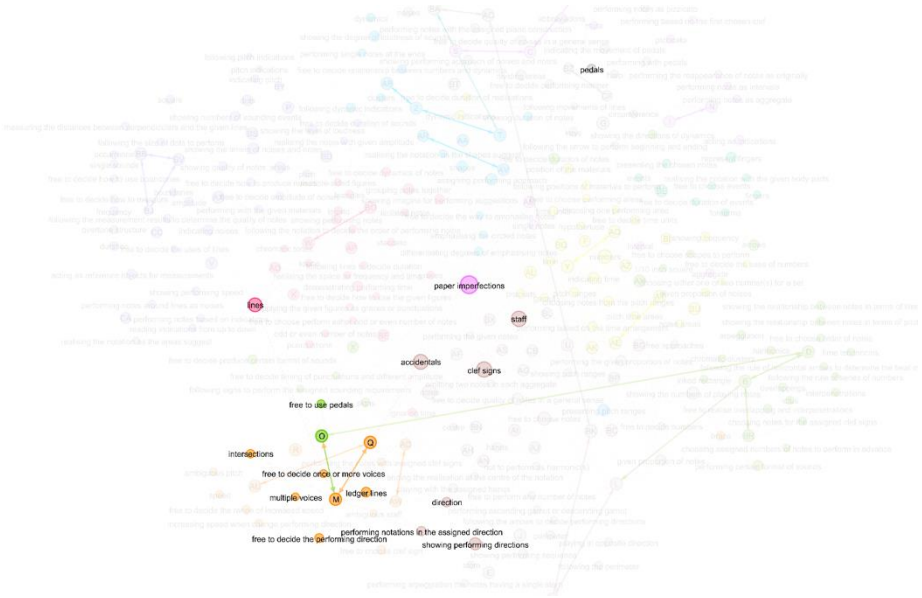


Figure 234 System M, including Notations M, O, Q and the corresponding compositional elements.

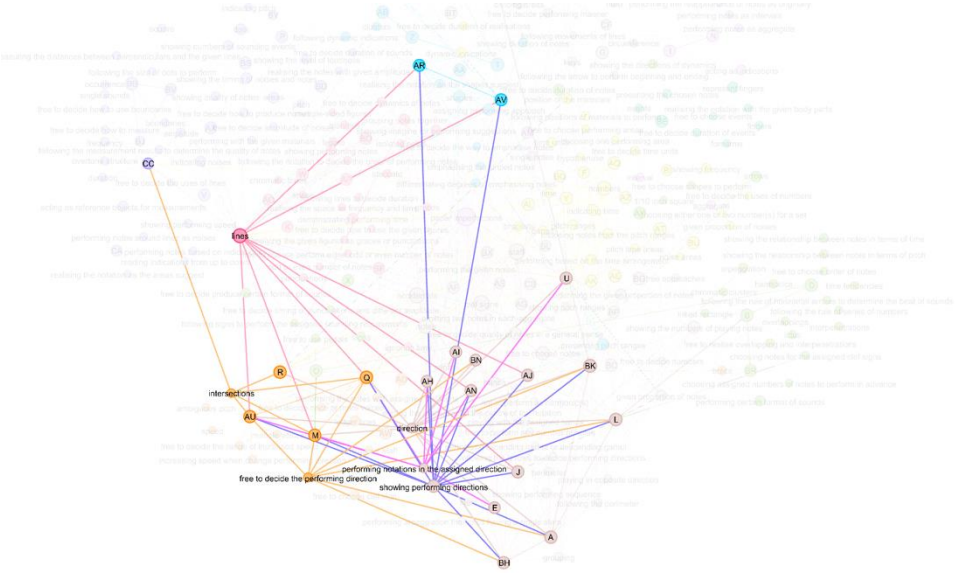


Figure 235 The compositional materials, intersections, and its relevant compositional materials, function, types of freedom, and performing approaches connect notations.

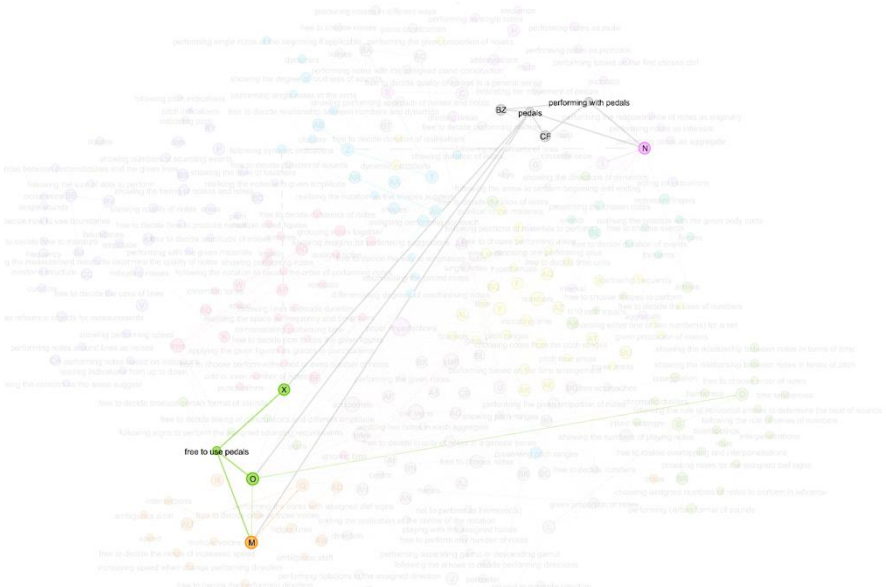


Figure 236 The relationships based on pedals and their relevant freedom and performing approach across Systems M and BZ, and Notations N and X.

System I includes Notations I and N (Figure 237). From the compositional materials of Notation I, I noticed two composition materials – pizzicato and mute – and two performing approaches – performing notes as mute and performing notes as pizzicato – from System C, which are also applied in this system. Referring to two notations, it is clear that Notations I and N both apply mute and pizzicato, while the performing instructions show the performing approaches are playing the notes as either mute or pizzicato where indicated. For this reason, Systems C and I are connected through these elements (Figure 238). One additional connection here is Notation N's compositional material (dynamic indications) and the performing approach (following the dynamic indications) connect itself with Notations T, Z, and BA (System T), Notation Y and AQ (System Y), and Notations Z and AB (System Z) (Figure 239).

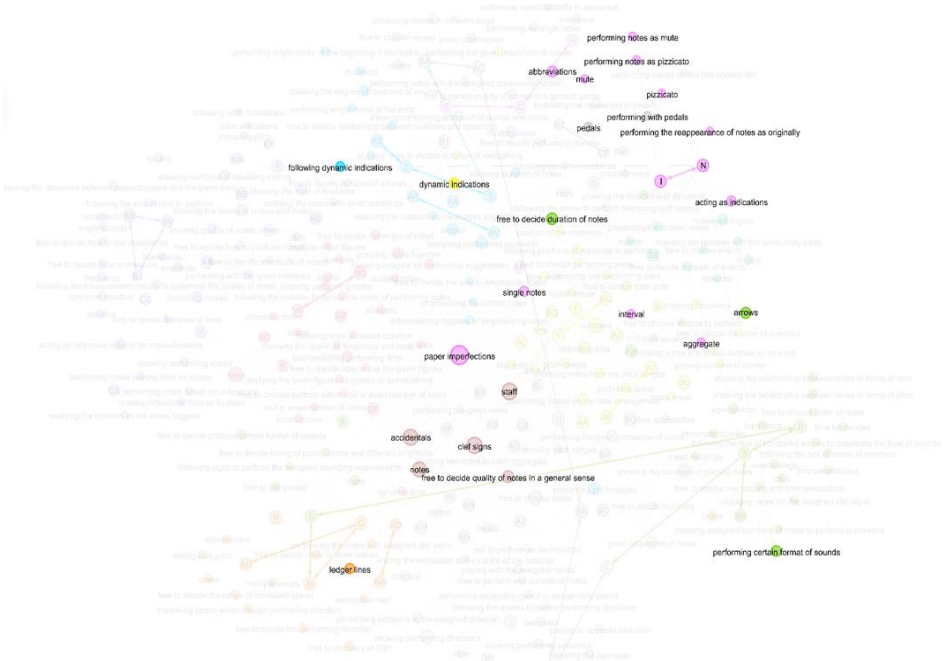


Figure 237 System I and its constituent notations and elements.

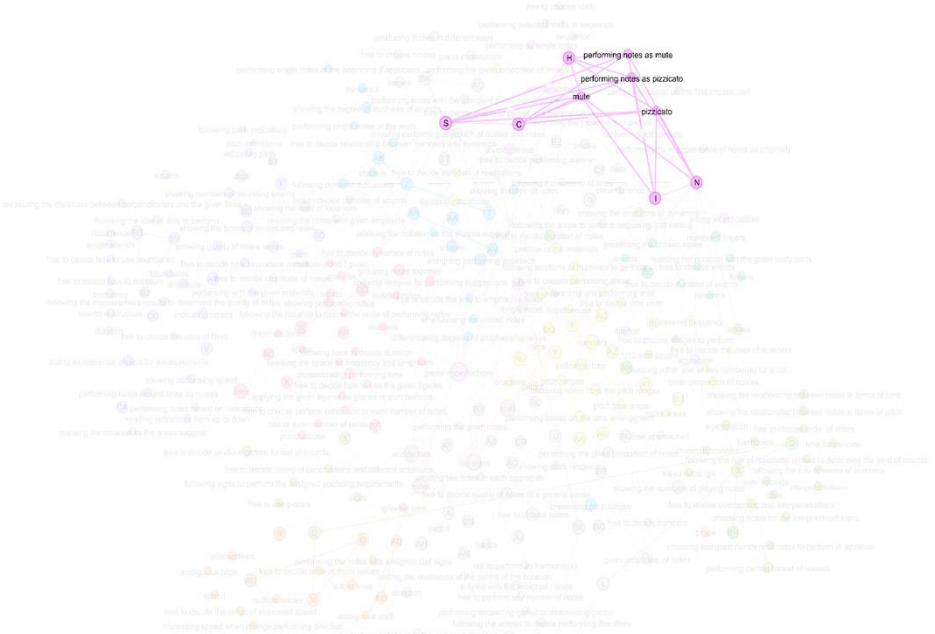


Figure 238 The relationships between Systems C and I based on mute, pizzicato, and the relevant performing approaches.

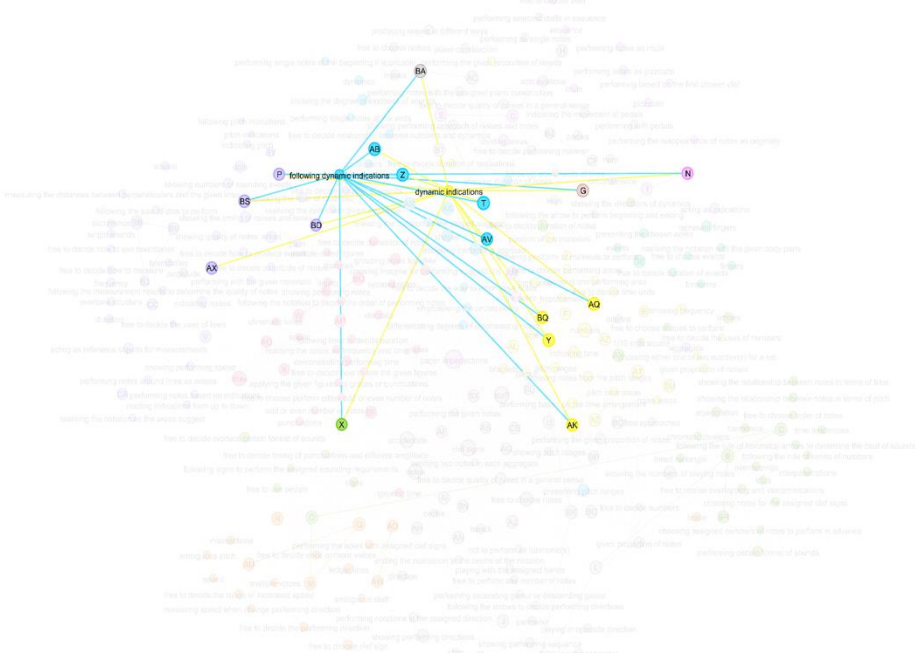


Figure 239 The relationships between Notation N and Systems T, Y, and Z based on the compositional material and the performing approach.

System T consists of Notations T, Z, and BA (Figure 240). Along with, and following, the dynamic indications, the communal element connecting notations within this system is the function of numbers, which shows the degree of loudness. Referring to the performing instructions of Notation T, Z, and BA, Cage stated ‘numbers refer to loudness’ in Notation T, then ‘dynamic as in T’ in Notation Z, and finally he referred to the use of Notation T in Notation BA. Thus, three notations are connected to form the second type of relationship. This function is not only applied in this system, it also occurs in System Z, which consists of Notations Z and AB, along with Notations P, AV, BD, and BS. The application of this function in Notations P, BD, and BS reveals the second type of relationship between these notations and the compositional materials, dots and areas. In Systems T and Z, the function of showing the degree of loudness is based on the numbers, however, in Notations P, BD, and BS, this function is presented by dots and areas; hence, this function not only bridges notations, it also connects to other compositional materials (Figure 241).

Returning to System Z, despite Cage not directly mentioning Notation T in Notation AB, as he did in other notations, he assigned the performing approach of this notation to be the same as Notation Z, and stated, ‘numbers are dynamics’ in the performing instruction of Notation AB, while Notation Z’s performing instruction shows ‘dynamics as in T’. Thus, Notation Z acts as

a conjunction between Notation AB and Notation T to connect two systems. Moreover, the description, ‘numbers are dynamics’, means that the performers can decide their uses in terms of dynamics. Hence, a type of freedom – to decide relationship between numbers and dynamics – connects Systems T and Z, and Notation AV (Figure 241).

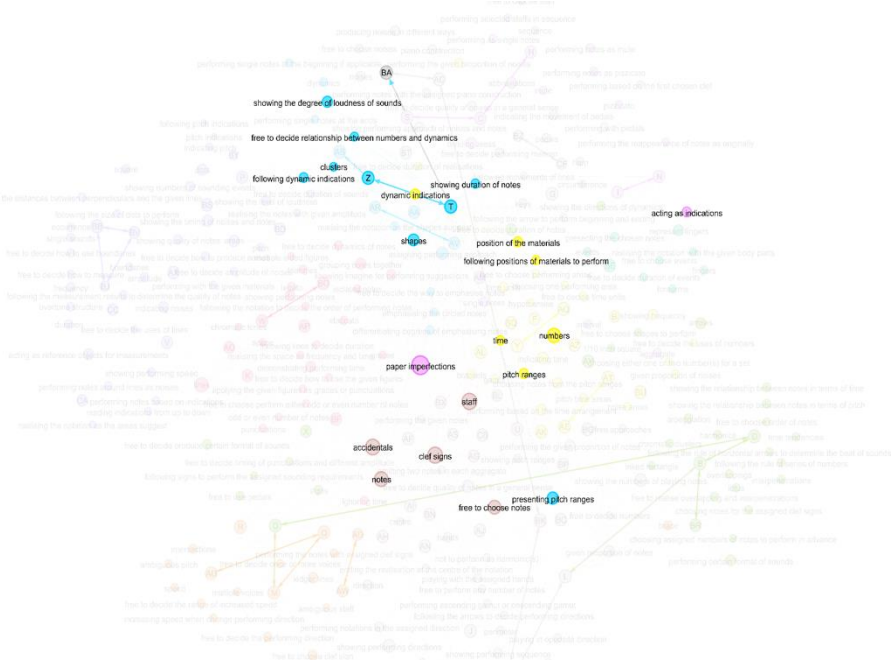


Figure 240 The constituent elements of System T.

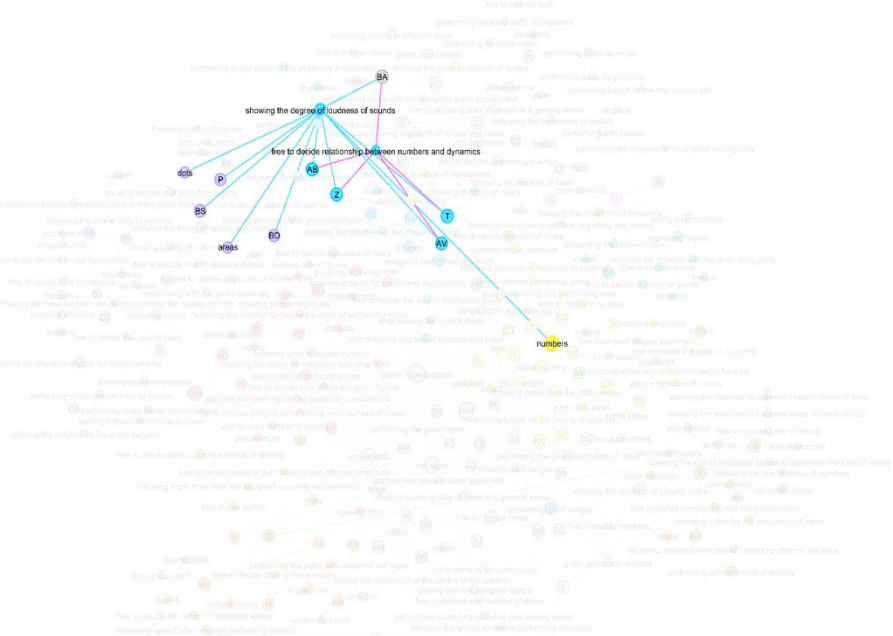


Figure 241 The second and third type of relationships based on the degree of loudness and the freedom to decide relationship between numbers and dynamics.

System W includes Notations W and BO. From the performing instructions of two notations, the main compositional materials here are legato, staccato, shapes, and isolated notes (Figure 242). From these elements, the known connections are the first type of relationships – the application of compositional materials. By following these, Cage assigned the performing approaches to this system, such as performing as legato, performing as staccato, and performing certain formats of sounds (as single notes).[373] With these elements – legato, staccato, performing as legato, performing as staccato, and performing certain formats of sounds – connect System W and Notations BF, AL, and AP (Figure 233; Figure 243); while the function of lines and shapes (grouping notes together) largely connect the system with System Z and other notations, such as, O, R, AF, AH, AI, and BF (Figure 243).

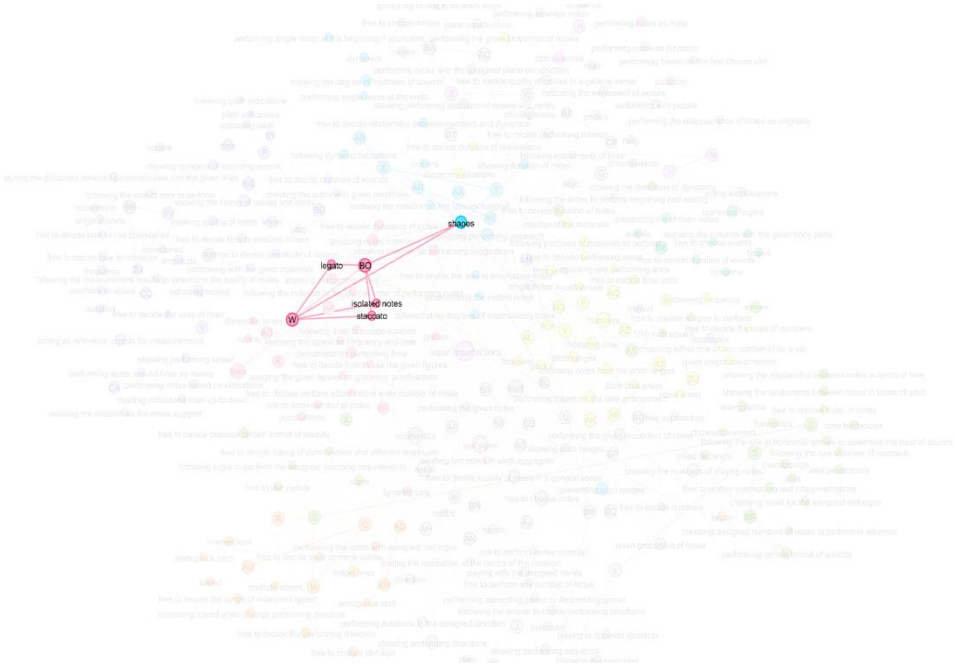


Figure 242 The first type of relationships within System W, based on their constituent compositional materials.

[373] This relationship has been explained in Figure 233.

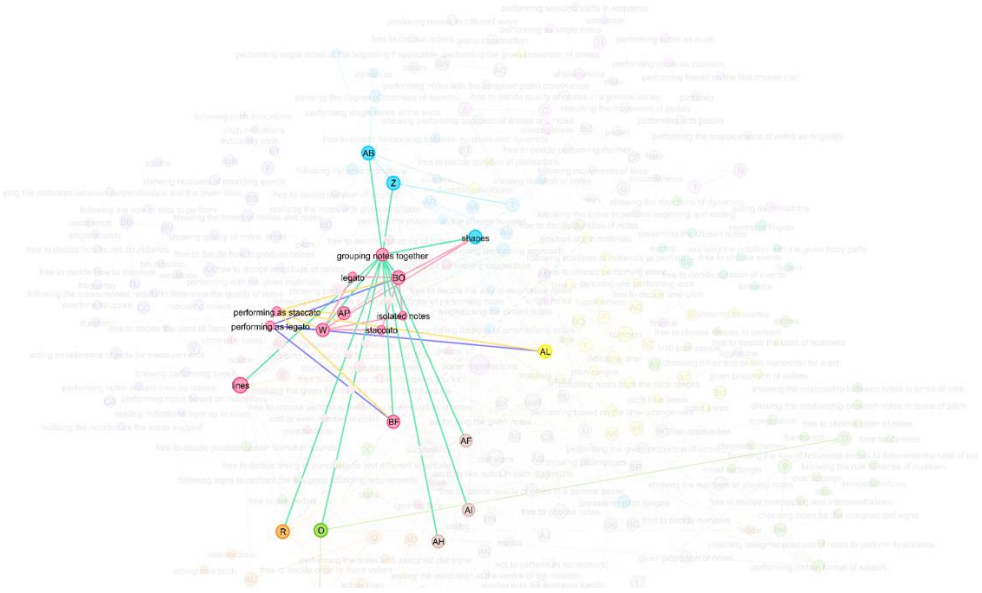


Figure 243 The first type of relationships (red), the second type of relationships (green), and the fourth type of relationships (yellow and blue) between System W and other notations.

System AC includes Notations AC, BA, and BK (Figure 244). The compositional material shared are noises and piano construction, which form the first type of relationships and provide the third type, based on the type of freedom to decide the quality of noises, and the fourth type of relationships based on performing notes/noises with assigned piano construction. With these relationships, Notations AC, BA, and BK not only build relationships within their system, but also connect to System BZ and other notations (Figure 245). The speciality of this system is that Notations BA and BK both refer to other notations, not just Notation AC; the performing instructions of the two notations show Notation BA refers to Notation T for the functions of numbers, while Notation BK refers to Notation A for its performing approaches, types of freedom, and the functions of its compositional materials. Therefore, I suggest two notations – Notations BA and BK – act as conjunctions building relationships with their referential notations, as well as with Systems A and T.[374]

[374] For the two systems, refer to the previous discussions in terms of Systems A and T.

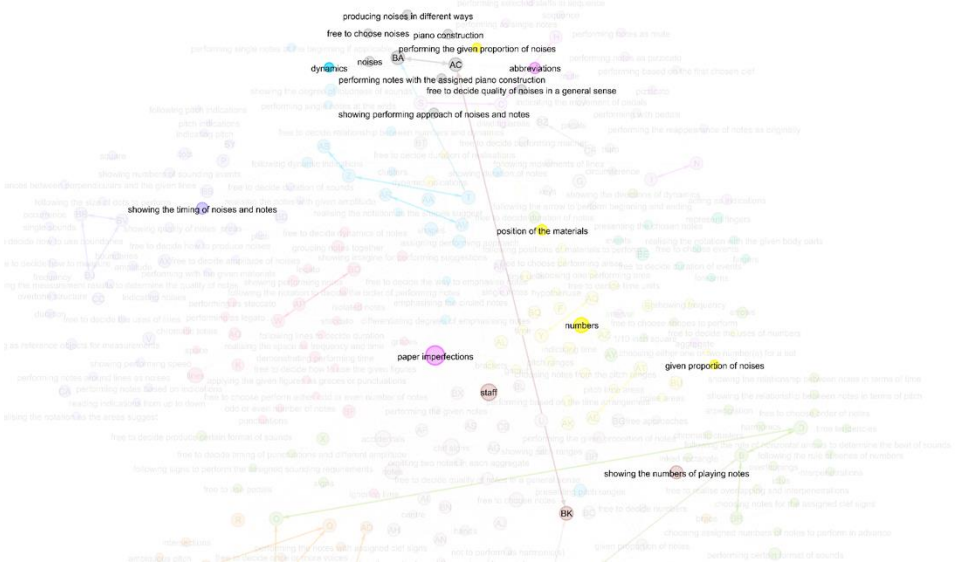


Figure 244 The constituent elements of System AC.

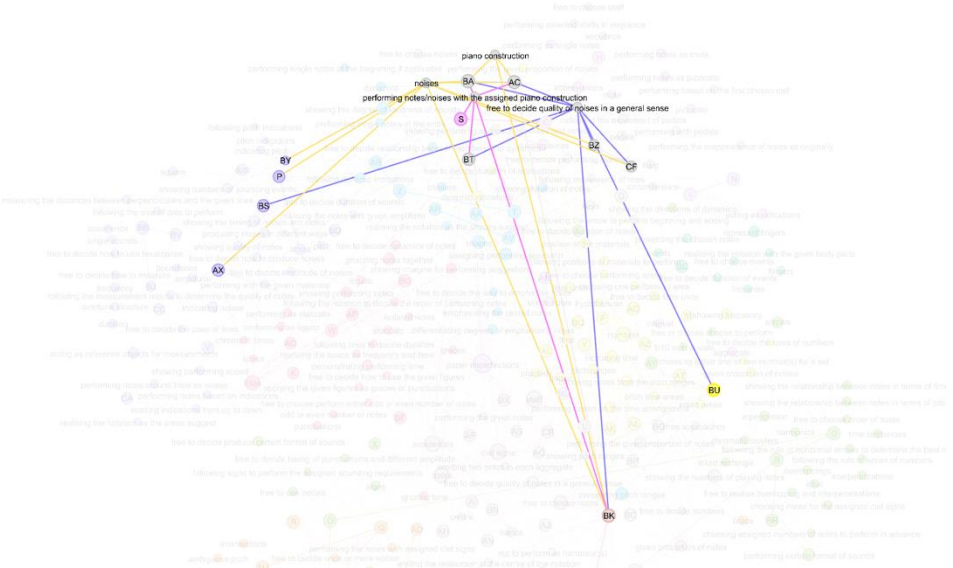


Figure 245 The first type of relationships (yellow), the third type of relationships (blue), and the fourth type of relationships (purple) between Systems AC, BZ, and other notations.

From the visualisation of Systems M and AC, I notice that System BZ shares their main compositional materials: noises and pedals (Figure 236; Figure 245). Another unique compositional material of this system is keys and harp, which is only applied in System BZ and Notation G. Referring to the performing instructions of Notations G, BZ, and CF, Cage states that the shared use of keys and harp can be recognised; ‘given notes play in any manner (keys, harp)’ in Notation G, ‘any or no keyboard, harp, or noise sounds’ in Notation BZ, and

‘as in BZ’ in Notation CF.[375] Thus, three notations construct the first type of relationships upon the application of keys and harp. Cage offered the keys and harp as performing approaches in three notations, he also offered a type of freedom to decide performing manners, hence the third type of relationships appear in three notations. Looking into other performing instructions, I notice that this freedom is applied in Notations BG, BT, and BY, where Cage stated, ‘intervals with free approaches’, ‘notes give place of performance with respect to piano’, and ‘any noises’, respectively. Consequently, the third type of relationship builds across System BZ and Notations G, BG, BT, and BY (Figure 246).

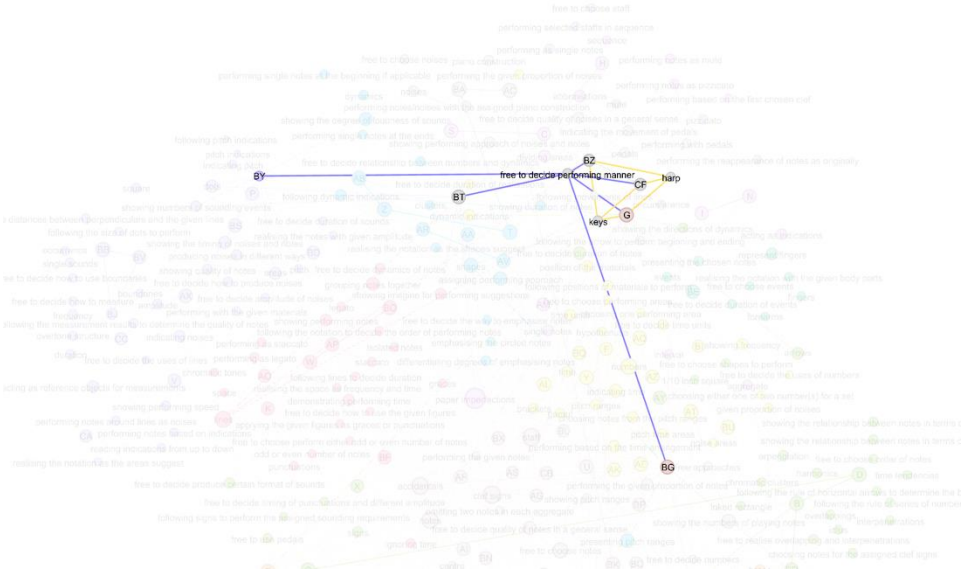


Figure 246 The first type of relationship (yellow) and the third type of relationship (blue) between System BZ and other notations.

System AD consists of Notations AD and AW, which share ambiguous staff, thus developing the type of freedom to choose clef signs. However, this freedom is only applicable to notes on the staff; Cage assigned the notes above and below it to G-clef and F-clef, respectively. Thus, a performing approach, performing notes with assigned clef signs, emerged. Based on these elements, System AD constructs the third type of relationship with Notations R, AF, AH, AU, AY, and BH, while it builds the fourth type of relationship with Notations R and AJ (Figure 247).

[375] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

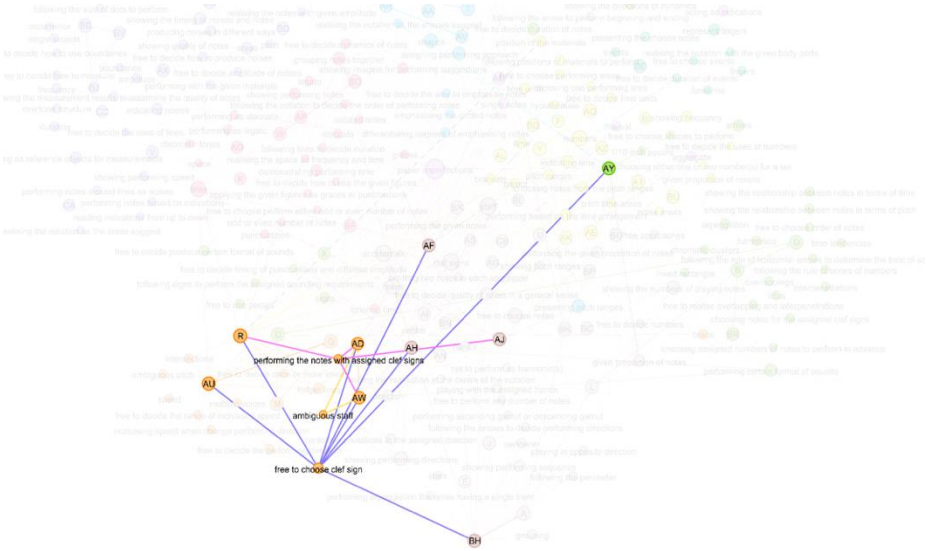


Figure 247 The first type of relationship (yellow), the third type of relationship (blue), and the fourth type of relationship (purple) between System AD and other notations.

System AE includes Notations AE and AT, both of which use pitch-time areas as the main compositional material. These include instructions of pitch and time, meaning that two performing approaches can be found: (i) performing based on time arrangement and (ii) choosing notes from pitch ranges. Through the former performing approach, this system connects with Systems Q and Y, Notations F, R, AK, AL, AM, AX, AZ, BO, BQ, BU, and CC, while the latter performing approach builds another third type of relationship between System AE with System Y and Notations U, AA, AN, AO, AK, AY, BC, BG, BP, BQ, BU, and CB (Figure 248).

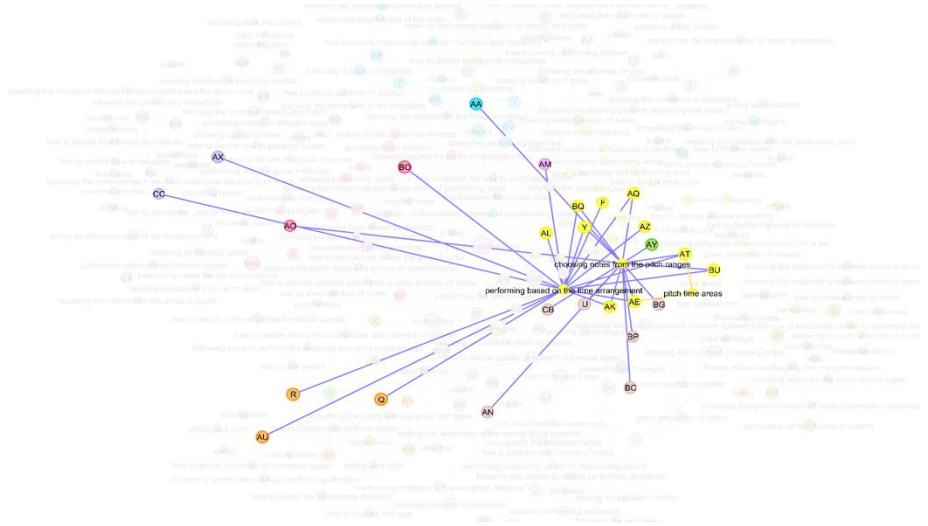


Figure 248 The first type of relationship (yellow) and the third type of relationship (blue) between System AE and other notations.

System AR consists of Notations AR and AV. The uniqueness of this system is its appearance, or, rather, Cage's arrangement of shapes. In the performing instruction of Notation AR, he stated 'played in any way that is suggested by the drawing', while in Notation AV he explained 'AR with the addition of numbers giving amplitude'.[376] These instructions clearly show the performing approach is based on the shapes; therefore, in this system, it is realising the notations, as the shape suggests, which is also applied in Notations AY and BX. Looking into the shapes in System AR, I notice they are used to show the duration of notes; this function is also applied in Notation T. In short, the function of showing duration of notes connects this system with Notation T, while the performing approach builds the fourth type of relationships between this system with Notations AY and BX (Figure 249).

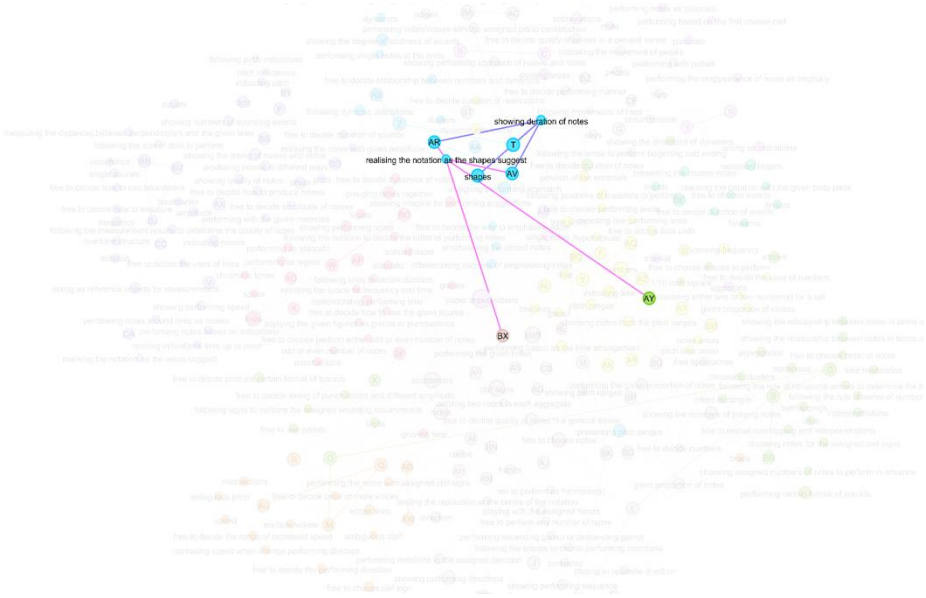


Figure 249 The second type of relationship (blue) and the fourth type of relationship (purple) between System AR and other notations.

The last assigned relationship from Cage is System BB,[377] which contains Notations BB, BJ, and BV. The communal element of three notations is their performing approach, which is based on their measurement result. Performers need to decide how to use referential lines (in Notation BB) or boundaries (in Notations BJ and BB), and then measure the distance between the dots and the lines or boundaries. Thus, the type of freedom of this system is to decide how to measure, while the relevant performing approach is to follow the measurement results to

[376] Ibid.
[377] System BZ (Notations BZ and CF) has been explained in the discussion of System M, refer to Figure 236.

determine the quality of notes. Both the type of freedom and performing approach apply in Notation CC (Figure 250).

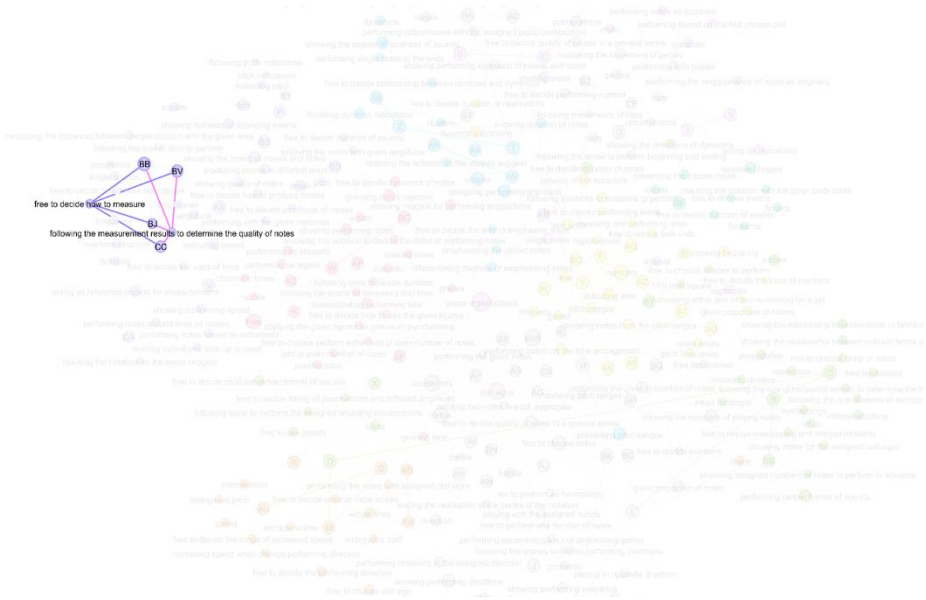


Figure 250 The third type of relationship (blue) and the fourth type of relationship (purple) between System BB and Notation CC.

The visualisations of clusters and individual systems illustrate the relationships between notations and their constituent elements; the former is based on the applications of elements across the *Solo for Piano*, while the latter is based on Cage's assigned relationships. Regardless the difference between two types of visualisations, they effectively demonstrate systematic and structural features, which reveal how clusters/individual systems connect with one another. In the visualisations of clusters, I explain the relationships between them based on compositional materials and their functions, types of freedom, and performing approaches; in the visualisation of individual systems, I build the four types of relationships based on Cage's individual systems, notations, and their elements. Based on these discussions, I suggest that the repetitive uses of elements demonstrate Cage's arrangements, and from which, it provides an opportunity to discover rational features.

3 Findings of Visualisation

The above discussions focus on clusters, individual systems, and the four types of relationships among notations and I recognise that their importance for the *Solo for Piano* is not only about their novelty, but also their repetition. When deconstructing the eighty-four notations in

Chapter 5, I recognised a potential pattern regarding the structure of Cage's musical work, therefore, in this chapter, I have visualised my finding.

In the above visualisations, nodes represent compositional materials, their functions, types of freedom, and performing approaches, while the edges between these nodes represent the relationships between one another. When Cage repeatedly applied one element, he intended it to connect the different nodes; the more repetitions that occur, the more relationships show up; thus, they form an invisible structure throughout the *Solo for Piano*. However, these repetitions do not mean that Cage did not use new elements, or only used the same elements over again; instead, they reveal how he built new notations based on previous ones. By this, I do not mean in alphabetical order, such as Notation A, followed by Notation B, etc, but in the sense of a 'former and latter' relationship across notations. Therefore, once he used a new element Cage intended to reuse it in future notations.

In other words, Cage often referred to an earlier performing instruction, which he built into a later one; for instance, he mentioned Notation A in Notations L, BH, and BK, and Notation B he mentioned in Notations D, L, and BR, and many other notations that I have discussed above. With such a distinct relationship among notations, a sense of systems can be seen. Hence, the way Cage assigned relationships between notations and his selections of compositional materials, their functions and types of freedom, together with his performing approaches, reveals his compositions to possibly possess rational features.

3.1 Rational Features

As mentioned above, Cage's compositional arrangement offers a possibility of discovering rational features in the *Solo for Piano*. Here, they include two features, (i) method/methodical and (ii) function/functional. As I explained in Chapter 2 Section 3, titled '*The Rational and Social Foundations of Music* (1921)', method/methodical shows the system and structure of musical works, the examples that Max Weber proposed was about the arithmetical structures of the musical elements, the notational system, and the harmonic and melodic structures of musical works. In Chapter 2, I have followed Weber's ideas and have demonstrated the method/methodical features through the harmonic progression of J. S. Bach's BWV. 846 Prelude in C Major. In terms of Cage's *Solo for Piano*, I started with rationalisation in his composing technique. As I suggested in Chapter 4 Section 4.2 Value Formation in Graphic Compositional System, the graphic compositional system transformed chance-derived

materials (paper imperfections) into semi-determinate compositional materials, which were then arranged according to Cage's intentions. This means that rationalisation represent value formation. Speaking of Cage's arrangements regarding the compositional materials, they involve the following elements: functions of compositional materials, types of freedom, and performing approaches, and these are the essential conditions to demonstrate the method/methodical feature of the *Solo for Piano*.

The feature of method/methodical can be discusses through two further features: systematic and structural, which are revealed through analysing musical elements; for instance, my analyses of BWV. 846 includes harmonic progressions and Schenkerian analysis that show two layers of musical structures. However, in Cage's *Solo for Piano*, harmonic progressions and Schenkerian analysis are certainly impractical analytical methods; nevertheless, it is still feasible to analyse this work. Analysis involves the interpretations of analysts, where they discover the constituent elements and structures in music, and the structure can be a part of, or as a whole of, a musical work.[378] Therefore, I choose to deconstruct the constituent materials in the *Solo for Piano*, then discover Cage's application of these elements, and finally, put them together to demonstrate the systems and the inner structure of this musical work.

At this point, the system is not an inflexible delimitation; instead, it means demonstrations of behaviour or meaning, which can be formed by way of multiple objects.[379] The objects can be varied; in this discussion, they are the compositional elements carrying Cage's arrangements. When I have pieced these together, I suggest they will construct relationships and individual systems, which will comprise the inner structure (Figure 251).

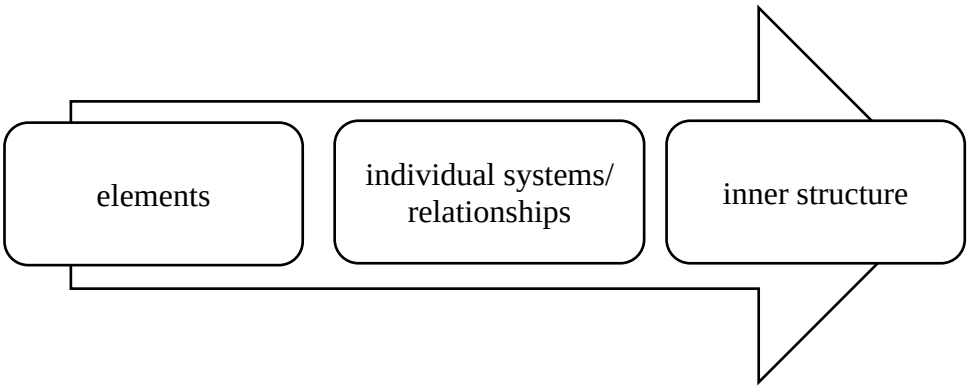


Figure 251 Piecing the elements together to discover systems and inner structure.

[378] Bent, 'Analysis'.
[379] Dori et al., 'System Definition, System Worldviews, and Systemness Characteristics', 1540.

For instance, the constituent elements of Notation M include direction, intersections, multiple voices, pedals, staff, clef signs, accidentals, ledger lines, lines, paper imperfections, showing performing directions, performing notations in assigned directions, freedoms to decide: performing directions, one or more voices, and to use pedals. These elements construct Notation M, which is referred to in Notations O and Q. Together, three notations construct the System M. As I explained in the visualisation of System M, the compositional material, direction, is applied in Systems A and M, and Notations E, J, R, U, AH, AN, AI, and AJ; lines are applied in Systems M, Q, and AR, and Notations J, AH, AN, and AJ; the function of showing performing direction is applied in Systems A, M, Q, and AR, and Notations J, AH, AN, and AJ; the freedom of deciding performing directions is applied in Systems A and M, and Notation BN. The performing notations in the assigned directions are applied in Systems M and Q, as well as in Notations E, U, AH, AN, AI, and AJ. In short, constituent elements construct notations; the notations then construct individual systems, if applicable, or the notations sharing the same constituent elements construct the four types of relationships among them. Together, the individual systems and four types of relationships construct the inner structure. The feature of method/methodical is reflected in the construction of these notations, in terms of how these elements form the notation; it also reflects in individual systems constructed by notations, and the four types of relationships based on the use of the elements.

The second rational feature, function/functional, is reflected in the functions of compositional materials. Unlike the feature of function/functional in European art music, the feature here does not carry the function of emotional expressions or serve the purpose of emotional expressions; rather it refers to the compositional materials' functions. For instance, the functions of this musical work can be categorised in (i) numbers, (ii) lines, (iii) shapes, (iv) arrows, (v) areas, and (vi) dots. Numbers are used for showing the numbers of playing notes, indicating time, showing the degree of loudness, showing frequency, and representing fingers. Lines are applied to present pitch ranges, group notes together, divide areas, indicate noises, show the performing directions, demonstrate performing time, assign performing approaches, act as reference objects for measurement, show imaginings for performing suggestions, and indicate the movements of pedals.

Shapes are used for grouping notes together, showing duration of notes, showing performing directions, showing performing sequences, presenting pitch ranges, showing performing notes, differentiating the emphasising notes, and assigning performing approaches. Arrows, are used

for showing the relationships between notes in terms of pitch and time, showing directions of dynamics, showing performing directions, and acting as indications. Areas are applied for assigning performing approaches, showing degree of loudness of sounds, showing different ways of producing noises, showing the quality of notes, showing the performing approaches of notes and noises, presenting the chosen notes, and representing fingers. Finally, dots are used for showing the timing of noises and notes, showing performing speed, showing performing notes, showing performing approaches of notes and noises, showing numbers of sounding events, indicating pitch, and showing degree of loudness of sounds.[380] With these functions, the third rational feature, interactivity/being interactive, can be revealed.

The special nature of the third rational feature is that interactivity between Cage and the performers no longer focuses on interpretations but concentrates on realisations. Thus, the performing instructions are crucial for performers to enable them to process their realisations. In graphic notations, the composer may design sounds through various elements, therefore, for performers to decode the notations, they must read the performing instructions in advance. Moreover, the performing instructions and the arrangements of notations carry intentions, purposes, and values, since Cage deliberately offered various types of freedom, and he also inserted ambiguity in his compositional materials and performing approaches. These actions represent Cage's value and purposive rationality regarding his interpretations of Zen; hence, intentional interactivity is part of the rational feature in the *Solo for Piano*. However, these rational actions also bring out the irrational features, such as unpredictability and randomness.

3.1.1 Micro-, Meso-, and Macro-Structures of the Solo for Piano

After analysing the notations based on the compositional materials, materials' functions, performing approaches, and types of freedom, I noticed that the constituent elements of notations construct the individual systems and relationships, and then they form the inner structure. Interpreting this methodical feature through visualisation, the above discussions demonstrate the connections between elements are not limited in one notation or one cluster but expand to other notations and elements in other clusters. This means that the relationships between elements and clusters are across the musical work, and hence, the inner structure of

[380] The detailed analyses refer to Chapter 5 Section 3.1 The first and second types of relationships: compositional materials and their functions.

the musical work can be observed. This finding also reveals the *Solo for Piano*'s structures at micro, meso, and macro-levels.

In European art music, the small dimensions include motives, subphrases, and phrases; the elements of the middle dimensions include sentences, paragraphs, sections, and parts; the large dimensions focus on movements and works.[381] Despite the fact that the actual elements of each dimension are subject to the particular musical work, the three dimensions offer a basic yardstick of what micro-, meso-, and macro-structures are in music. Applying three structures to the *Solo for Piano*, the judgement of which can be varied and based on the analysts' analytical methods. In this thesis, I have built my analyses based on compositional materials, materials' functions, types of freedom, and performing approaches, and therefore, the structures of three levels are associated with them. In the *Solo for Piano*, the performers are free to decide which notations they wish to include in their realisations, meaning that each notation is independent from one another. Nevertheless, independence does not imply no conjunctions among notations. When Cage applied compositional materials, functions, freedom, and performing approaches in different notations, he also built the relationships between them, and hence, despite notations being complete objects, the relationship between them is still recognisable.

When taking the *Solo for Piano* as a complete work to discover its micro-, meso-, and macro-structures, the discussion shall widen to all compositional elements and graphic notations and focus on how these elements connect with one another. Thus, the micro-structure refers to compositional materials, materials' functions, types of freedom, and performing approaches of one notation; the meso-structure refers to the notations and elements in the same cluster; the macro-structure refers to the connections between ten clusters that form the *Solo for Piano*.

Taking Notations T and Z to explain the micro-structure. Notation T applies pitch, time, shapes with centre notes, clusters, numbers, dynamic indications, clef signs, accidentals, paper imperfections, staff, and notes as its compositional materials; the materials' functions of this notation include showing degree of loudness by numbers, showing time and pitch by shapes, and connecting the numbers with notes by arrows; its freedoms include free to decide relationships between numbers and dynamics and free to choose notes for the clusters; its

[381] LaRue, *Guidelines for Style Analysis*, 6.

performing approaches are following positions of shapes and notes to perform and following numbers to decide loudness. The mentioned compositional materials, functions, freedoms, and performing approaches construct Notation T. Another example, Notation Z uses clusters, single notes, dynamic indications, clef signs, accidentals, lines, numbers, paper imperfections, staff, and notes as its compositional materials; the functions of Notation Z include showing degree of loudness by numbers and grouping notes together by lines and shapes; its freedoms include deciding relationship between numbers and dynamics and deciding duration of notes; the performing approaches are following numbers to decide loudness, performing clusters, and performing single notes at the end. These elements construct Notation Z. By analysing two notations, the micro-structure of the *Solo for Piano* can be effectively revealed and explored.

The next level is meso-structure, which refers to the connection between notations from the same cluster. For example, Cluster 8 consists of Notations S, T, Z, AA, AB, AC, AR, AV, BA, and BT, and the compositional materials, freedom, and performing approaches that are related to noises, dynamics, and the piano construction. Within this cluster, Notations S, AC, BA, and BT apply the compositional material piano construction and performing the notes with the piano construction; Notations S, AC, and BA use noises as one of the compositional materials; Notations AC, BA, and BT apply the function of showing performing approaches of noises and notes; Notations T, Z, AA, AB, and AC use dynamics as the compositional material, while Notations T, Z, AB, and BA use the same function of showing the degree of loudness in numbers and shapes; Notations AC, BA, and BY apply the function of showing performing approaches of noises and notes; Notations AR and AV include the performing approach of realising the notation based on the shapes; and Notations AC and BA include the freedom of choosing noises. From the applications of the compositional elements, it is noticeable that the connections at the meso-level are across notations and within the cluster.

The macro-structure reveals the relationships between elements in different clusters. For instance, freedom of deciding performing manner is applied in Notation G, BZ, and CF from Cluster 3 and Notation BT from Cluster 8 (Figure 205); the performing instruction of following dynamic indication is used in Notation N from Cluster 2, Notation G from Cluster 3, Notation Y, AK, AQ, and BQ from Cluster 5, Notations P, BD, and BS from Cluster 6, Notations T, Z, AB, AV, and BA from Cluster 8, and Notation X from Cluster 10 (Figure 211); the compositional material, dot, is applied in Notations AO and BO from Cluster 4, Notations P, BB, BS, BV, BY from Cluster 6, Notation BE from Cluster 7, and Notations BA and BT from

Cluster 8 (Figure 213); the compositional material, area, is applied in , Notations BU and BK from Cluster 5, Notations P, AX, BB, BD, BS, BJ, BV, BY, and CA from Cluster 6, Notation BE from Cluster 7, and Notation BA from Cluster 8 (Figure 214); the function of showing numbers of playing notes is used in Notations B, D, BR from Cluster 1, Notation G from Cluster 3, Notations U, AE, AT, BC, BG, BK, BP, BL, and BU from Cluster 5, Notation BE from Cluster 7, Notation AC from Cluster 8, Notations A, J, L, AJ, and BH from Cluster 9 (Figure 215); the function of showing performing approach of noises and notes is applied in Notation CA from Cluster 6, Notation BE from Cluster 7, and Notation AC, BA, and BT from Cluster 8 (Figure 216); the freedom of deciding dynamics of notes is used in Notations W, AP, and BO from Cluster 4 and Notation S from Cluster 8 (Figure 218). Based on the connections across clusters, the macro-structure can be observed. In summary, the micro-structure indicates the construction of graphic notations; meso-structure is based on the connections within a cluster; the macro-structure refers to the connections between clusters.

My observation of the micro-, meso-, and macro-structures of the *Solo for Piano* particularly focuses on the musical work itself, it takes all graphic notations, compositional elements, and performing instructions into consideration. Thus, this observation may not be applicable if the analyst particularly focuses on one performer's realisations or performances of this musical work. When the performer has processed the second stage of rationalisation, the realisations/performances should be considered as the products from the performer's esthesic and new poietic levels, and hence, the analyses of the micro-, meso-, and macro-structures are subject to the performer's rationalisation.

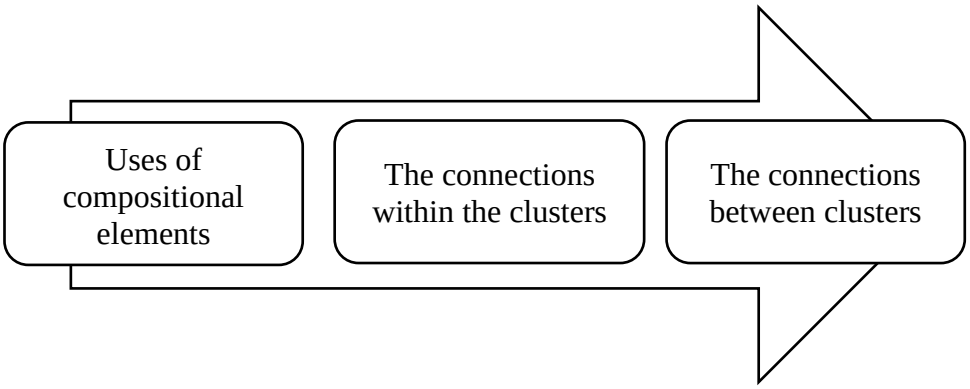


Figure 252 The micro-, meso-, and macro-structures of the *Solo for Piano*.

3.2 Irrational Features

In Chapter 4 Section 4.1 Jean-Jacques Nattiez's Tripartite Model, I applied the revised tripartite to explain that the intentional ambiguity in the *Solo for Piano,* emphasises participation of its performers. They play a crucial role of turning semi-determinate compositional materials/notations into determinate realisation. In this circumstance, the rational feature of interactivity/being interactive as well as the irrational features of unpredictability and randomness can be recognised.

For instance, the performing instruction for Notation A, 'Following the perimeter, from any note on it, play in opposite directions in the proportion given. Here and elsewhere, the absence of indications of any kind means freedom for the performer in that regard'.[382] This provides freedom of choosing notes, deciding the performing directions, and deciding quality of notes, while the performers ought to follow the performing approaches of performing in the opposite directions for both hands, performing the given proportion of notes for both hands, and following the perimeter. This means that performers are free to start with any notes, as long as they perform sixteen notes for the right hand, nice notes for the left hand, or vice versa. This freedom substantially prevents potential duplications and foreseeable performing results. In other words, unpredictability and randomness are produced by the semi-determinate compositional materials/notations.

On the other hand, the choices are based on performers' preferences, or, rather, their intentional, purposive, and conscious decisions. Therefore, despite unpredictability and randomness are from this rational feature (interactivity/being interactive), this feature (interactivity/being interactive) also produces the first rational feature (method/methodical) in performers' choices and realisations. Reflecting this to the revised tripartite model, the performers' participation shows at the esthesic level, where they determine the details for the realisations, while at the poietic level, the performers not only produce the realisation according to their perception at the esthesic level (rational), but also when they produce the unpredictable acoustic results (irrational) (Figure 253).

[382] Cage, *Concert for Piano and Orchestra: Solo for Piano*.

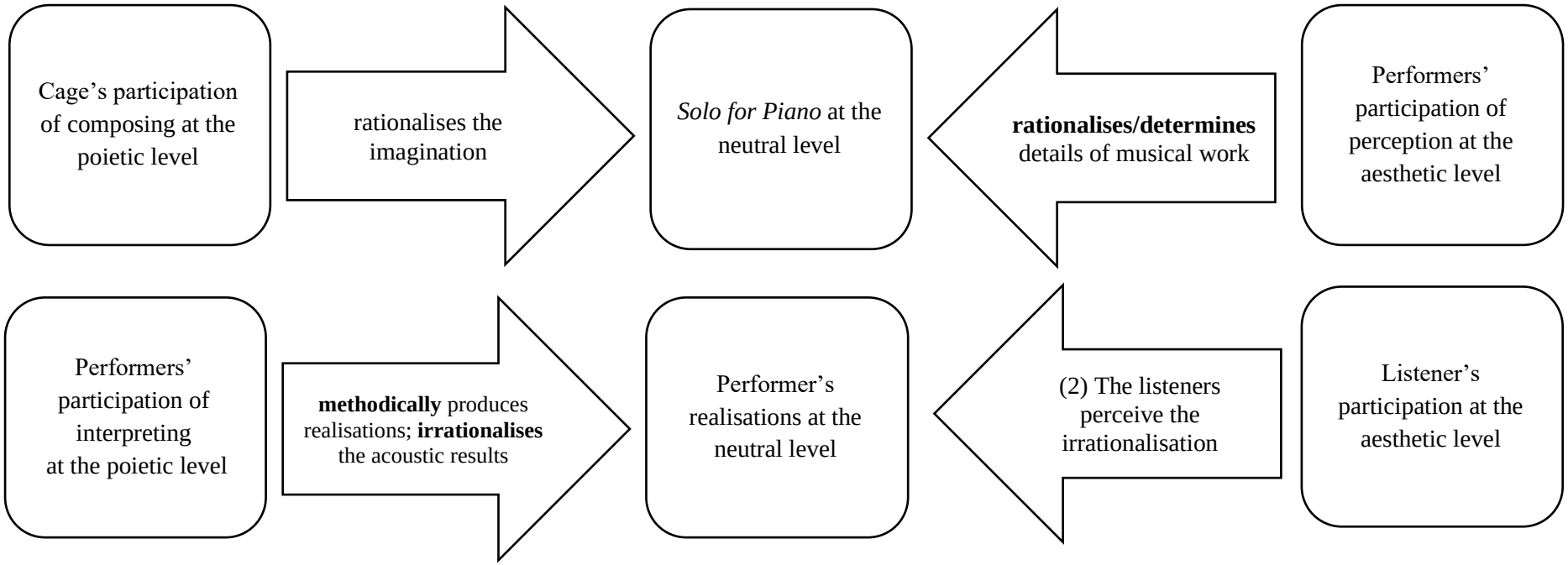


Figure 253 A bilateral participation across the composer, the performer, and the listeners in indeterminate music.

Chapter 7. Conclusion

Weber was the first who identified rationalisation in European art music through its developments along with the historical, cultural, and societal factors, he recorded his finding in the publication, *The Rational and Social Foundations of Music*.[383] When he applied the ideas of the theory of rationality to European art music, he noticed that the rationality showed in mathematic basis of musical elements, developments of musical instruments, and theoretical rules of composition, while irrationality appeared in its emotional expressions.[384] Drawing conclusions from his findings, I propose that the rational features in European art music are: method/methodical, function/functional, and interactivity/being interactive, and the irrational features include spontaneity, impulsiveness, and emotional expressiveness.

Valerie Ann Malhotra and James Wierzbicki propose a relevant question about Weber's rationalisation in music – what would Weber think if he was able to witness developments of music after World War II, when music featured as the ultra-rationalised structure through pitches, rhythms, sound colours, and volume levels.[385] Moreover, when Weber's rationalisation undoubtedly showed up in features of the neatness and accuracy of modern musical notations,[386] does it suggest that other forms of notations possessed no elements of rationalisation? Such questions inspired me to rethink the applicability of the theory of rationality in music. Thus, this thesis aims to discover rational and irrational features in John Cage's indeterminate music work, the *Solo for Piano*, and present the transformation of the theory of rationality. Firstly, I examine the ideas of Weberian rationality from *The Protestant Ethic and the Spirit of Capitalism* and *The Rational and Social Foundations of Music*; secondly, I investigate Cage's interpretation of the ideas of Zen to research his rationalisation; thirdly, I apply the revised tripartite model and deconstruct the *Solo for Piano* to examine the three rational features of this musical work; finally, I input the qualitative data that I obtained from the analyses into Gephi to visualise the relationships between notations and their constituent elements.

The research findings suggest that the theory of rationality is an evolving idea that develops along with the development of music; the transition of the theory of rationality suggests that Cagean indeterminacy results from a deeper level of bilateral participation of Cage and the

[383] Adorno, *Sound Figures*; Konoval, 'Max Weber and the Sociology of Music', 468.
[384] Weber, *The Rational and Social Foundations of Music*, li.
[385] Malhotra, 'Weber's Concept of Rationalization and the Electronic Revolution in Western Classical Music'; as cited in Wierzbicki, 'Max Weber and Musicology', 282.
[386] Attinello, 'The Possible Sign', 13.

performers, while the application of the theory of rationality reveals the micro-, meso-, and macro-structures through the relationships between notations, compositional materials, materials' functions, types of freedom, and performing approaches.

The bilateral participation starts with Cage's graphic compositional system, which he applied the chance-derived materials in the drawing process, and then offered them meaning by various types of functions, freedoms, and performing approaches. The randomness of paper imperfections has been rationalised by Cage and so they turn into semi-determinate notations. This process is the first stage of rationalisation that happened in Cage's poietic level, which produces the *Solo for Piano*, the musical trace at a neutral level. Despite the graphic notations remain semi-determinate, they manifest the methodical feature in their relationships. To present the relationships within the musical work, I take compositional materials, materials' functions, types of freedom, and performing approaches as the metalanguage to analyse the *Solo for Piano*.

The analyses show that Cage presented different functions and performing approaches by the same compositional materials, and he also assigned the same functions and performing approaches in different notations, and therefore, the notations constructed by the same materials, functions, or performing approaches demonstrate a sense of consistency. The analyses answer the research question: How can we understand the deeper relationship between rationality and irrationality resulting from the composer's actions? In Chapter 4 Zen Buddhism and Rationality in Cage's *Solo for Piano*, I suggest discovering value and purposive rationalities in Cage's intentions and actions of applying the ideas of Zen in his composition, as Cage interpreted the intrinsic Zen values – looking for pure fact and experience, rather than constructing concepts[387] – as unimpededness and interpenetration and applied the chance-derived materials in his composition, he successfully utilised chance as a tool in his composition.[388] As a consequence, his actions fulfilled the conditions of value and purposive rationality.

Most importantly, as this musical work is not based on the conventional notational system or subjective senses, such as tastes, emotions, and the memories of performers and listeners,[389] this radical change points out that irrationalisation and irrationality no longer concern emotional expressions but focus on unpredictability and unforeseen acoustic results. This irrationality

[387] Suzuki, *An Introduction to Zen Buddhism*, 42,44,51.
[388] Pong, 'True or False as It Is: Zen Enlightenment in John Cage's Music 真謬之間如如觀：凱吉音樂中的禪意解讀', 225.
[389] Saladin, *Tacet: Experimental Music Review, N° 1. Qui Est John Cage?. Who Is John Cage?*, 45.

relates to the second stage of rationalisation, in which the performers determine the performing details and so turning the semi-determinate notations into determinate realisations.

The bilateral participation invites us to reassess the meaning of realisations of indeterminate music and interpretations of European art music. Is realisations of the *Solo for Piano* truly liberating, or is it another form of control? Is the interpretations less free than realisations? Cage's purposive ambiguity let the performers adopt a deeper level of autonomy and emphasises every decision from the performers. Based on this circumstance, the free realisations do not go along with the binary idea of either under control or free from control, but to jump out of the box and stress on the performers' experiences. The performers' experiences indicate corresponding creative processes, which refers to their unique rationalisation. Building upon this idea, the unpredictable acoustic results of indeterminate music and emotional expressions of European art music are analogous with each other. Emily Payne suggests that when the performer produces the performance based on a 'fixed' notation, the performance is not merely execution, but involves imagination and full consideration, and therefore, each performance brings out nuanced differences from one another.[390] Specifically, realisations of graphic notations and interpretations of conventional notations both require the performers to experience the process of rationalisation, where they shall determine how to conduct their realisations/interpretations; every action they take are not simply iterative but actively involve in the completion of the works.[391] Putting it differently, both the realisations and interpretations lay stress on the performers' experiences; the difference between the experiences is that the unpredictability of realisations shows in acoustic dissimilarity, and the unpredictability of interpretations manifests in performing styles.

This reassessment leads me to reconceptualise the structure of musical works. As Attinello proposes, the development of notations offers the possibility of delving into the philosophical roots of notations to rethink and reinvestigate the idea of notations.[392] In the case of Cage's *Solo for Piano*, Zen inspired Cage to take chance and indeterminacy as tools, and therefore, performers' realisations no longer focused on correctness or authenticity but turned the focus to the participation of individuals. Following this idea, Cage's composition shed new light on analysing the systems and structures of the musical work. Such an analysis is dependent on the

[390] Payne, 'Creativity beyond Innovation', 333, 341.
[391] Ingold, *Being Alive*, 216; Payne, 'Creativity beyond Innovation', 330.
[392] Attinello, 'The Possible Sign', 15.

interpretations of analysts, meaning that the analysts are led to rethink what the constituent elements are in the *Solo for Piano*.

In Chapter 5, as I deconstructed the graphic notations to obtain the constituent elements, I realised that the structure was not limited in the notations themselves; instead, the structure can be micro-, meso-, and macro-levels. Interestingly, the multilayered structure only became visible after visualising and clustering. As Chapter 6 explained, ForceAtlas2 and modularity class are the two computational methods that I apply in the Gephi; ForceAtlas2 allocates the nodes' positions based on the relationships between the nodes, and modularity class colours the nodes based on the same condition. With this setting, the visualising result reveals the hidden relationships among notations and their constituent elements. These relationships are the fundamental and the small-scale structures of the *Solo for Piano*. The middle-scale structure can be observed after clustering. As Figure 200 illustrates, ten clusters can be discovered, while each cluster consists of particular elements. For instance, Cluster 2 (Figure 203) focuses on mute and pizzicato, Cluster 6 surronds with the uses of areas (Figure 210), and Cluster 8 is about noises and piano construction (Figure 217). Thus, the meso-structure points to the notations and constituent elements in the same cluster. When Cage composed this work, he repeatedly applied materials, functions, freedom, and performing approaches, and therefore, the connections are across ten clusters. The connections bridge two or more clusters together and form the macro-structure of the *Solo for Piano*. In summary, the micro-structure indicates the construction of graphic notations; meso-structure is based on the connections within a cluster; the macro-structure refers to the connections between clusters.

Nevertheless, the multilayered structure is not fixed. As I emphasised in Chapter 6, my observation of the structures may not be applicable if the analyses focus on the performers' realisations/performances, since those are the products of performers' rationalisation. Meanwhile, different understanding of performing instructions will influence the analyses of the structure, as my analytical process involves my act of decoding, meaning that I have rationalised the notations while analysing. This means that when the purposive ambiguity in Cage's composition offers the performers a deeper level of autonomy, this autonomy also produces 'free' analysis that depends on analysts' understanding towards the musical work. My analysis only concentrates on the *Solo for Piano* at the neutral level, and it is based on the metalanguage consisting of compositional materials, materials' functions, types of freedom, performing approaches; if other analysts have their own aspects to examine this work, their

analyses will be different from mine. Regardless, my argument remains in the fact that Cage's composition has widened our comprehension of what the structure of musical works can be.

Following this, one might suggest that the analysis should encompass performer's realisation – a guaranteed realising result – otherwise the validity seems to be questionable. By acknowledging this idea, I encourage that the future research may investigate the performers' realisations at the esthesic level and performers' executions at the new poietic level to refine the preliminary idea of free analysis. To address the prospective directions, the rationalisation in performers' realisations and the irrationalisation during performances may worth our attention. As the theory of rationality considers individuals' purposes, intentions, and actions, delving into the performers' points of views may enrich the data of how they determine their realisations and what are the actual situation when they execute their realisations. With these directions in mind, the application of the theory of rationality in indeterminate music may be beneficial for scholars who wish to investigate analyses and performance research of indeterminate music.

At the end of this thesis, I wonder if this might be the end of the theory of rationality. Referring to the discussions across this thesis, I have argued that the theory of rationality is not a fixed idea but Weber's observations under certain social conditions. Thus, the ideas of Weberian rationality are changeable. As the societies develop, the ideas will be pushed a little bit further, just as it changed across Protestantism and European art music to reach indeterminate music.

Appendix

Glossary

Terms	Definition
European art music	European art music refers to the tradition of composed music in Europe that is rooted in formal structures and written notation. In this thesis, I take J.S. Bach's Prelude in C major BWV. 846 as the example to discuss rationality and irrationality.
Common Practice	Common practice refers to the standardised rules of composition that were commonly used in European art music in Baroque, Classical, and Romantic periods, such as tonal harmony, functional harmony, voice leading, modulation, binary form, sonata form, and fugue, etc.
Rationality in European art music	Rationality in European art music shows as harmonic structure, notational system, harmonic functions, and developments of musical instruments.
Irrationality in European art music	Irrationality in European art music shows as asymmetrical structure, melodiousness, and intuitiveness, which lead to emotional expressions.
Rationality in indeterminate music	Rationality in indeterminate music shows as multilayered structure, composer's intention, bilateral participation of the composer and performers.
Irrationality in indeterminate music	Irrationality in indeterminate music shows as chance-derived materials, unpredictability, and randomness, which lead to unpredictable acoustic results.

Appendix A: Tables listing compositional materials, types of freedom, and performing approaches of eighty-four notations

The compositional materials from the performing instruction	The perimeter, notes, direction, the given proportion.
The compositional materials from the notation	Numbers, clef signs, accidentals, grouping, paper imperfections, and staff.
Freedom	Free to choose notes as the start; free to determine the performing direction; free to decide quality of sounds.
The performing approach	Following the perimeter; playing in opposite direction; performing the given proportion.

Table 48 The compositional elements in Notation A.

The compositional materials from the performing instruction	An aggregate, ictus, harmonics, overlappings, interpenetrations, clef signs, the given proportion, numbers, staff, inked rectangles, and a chromatic cluster.
The compositional materials from the notation	Paper imperfections, and notes.
Freedom	Free to decide to produce certain format of sounds; free to decide duration of notes; free to realise overlapping and interpenetrations; free to choose notes.
The performing approach	Performing certain format of sounds; choosing notes for the assigned clef signs.

Table 49 The compositional elements in Notation B.

The compositional materials from the performing instruction	Abbreviations and single notes.
The compositional materials from the notation	Clef signs, accidentals, staff, paper imperfections, and notes.
Freedom	Free to decide duration of notes; free to decide dynamics of notes.
The performing approach	Performing mute as annotations suggest; performing pizzicato as annotations suggest; realising all notes as single tones.

Table 50 The compositional elements in Notation C.

The compositional materials from the performing instruction	Time tendencies, harmonics, the arpeggiation, arrows, numbers, sign, and Notation B.
The compositional materials from the notation	An aggregate, ictus, overlappings, clef signs, staff, accidentals, brace, paper imperfections, and notes.
Freedom	From Notation B: free to decide to produce certain format of sounds; free to decide duration of notes; free to realise overlapping and interpenetrations; free to choose notes. From Notation D: free to choose the order of notes; free to choose the beat of sounds.
The performing approach	From Notation B: performing certain format of sounds; choosing notes for the assigned clef signs. From Notation D: following the rule of series of numbers to choose notes; following the rule of horizontal arrows to determine the beat of sounds.

Table 51 The compositional elements in Notation D.

The compositional materials from the performing instruction	Hands, clef signs, notes, stem, arpeggiation, direction.
The compositional materials from the notation	Accidentals, grouping, staff, abbreviation, and paper imperfections.
Freedom	Free to decide quality of sounds.
The performing approach	Playing with the assigned hands; arpeggiating the notes having a single stem (up or down); performing notations in the assigned direction.

Table 52 The compositional elements in Notation E.

The compositional materials from the performing instruction	Numbers and time units.
The compositional materials from the notation	Clef signs, accidentals, notes, and paper imperfections.
Freedom	Free to decide time units; free to decide quality of sounds.
The performing approach	Performing based on the time arrangement.

Table 53 The compositional elements in Notation F.

The compositional materials from the performing instruction	Numbers, keys, harp, arrows, dynamic indications, circle, circumference, and notes.
The compositional materials from the notation	Clef signs, accidentals, grouping, and paper imperfections.
Freedom	Free to decide performing manner; free to choose notes.
The performing approach	Performing by keys or harp; following dynamic indications; following the arrow to perform beginning and ending; performing the given proportion of notes.

Table 54 The compositional elements in Notation G.

The compositional materials from the performing instruction	Clef signs, staff, numbers, sequence, and Notation C.
The compositional materials from the notation	Accidentals, staff, abbreviations, paper imperfections, mute, pizzicato, and single notes.
Freedom	From Notation C: free to decide duration of notes. From Notation H: free to choose staffs; free to choose the clef sign.
The performing approach	From Notation C: performing mute as annotations suggest; performing pizzicato as annotations suggest; realising all notes as single notes. From Notation H: performing selected staffs in sequence; performing based on the first chosen clef.

Table 55 The compositional elements in Notation H.

The compositional materials from the performing instruction	Pizzicato, single note, interval, aggregate.
The compositional materials from the notation	Clef signs, accidentals, staff, abbreviations, notes, mute (on page 29), and paper imperfections.
Freedom	Free to decide quality of sounds.
The performing approach	Performing mute, pizzicato, intervals, and aggregate as annotations suggest; performing the reappearance of tones as originally.

Table 56 The compositional elements in Notation I.

The compositional materials from the performing instruction	Numbers, notes, lines, direction, backwards, forwards, straight lines, ascending gamut, descending gamut, sign.
The compositional materials from the notation	Clef signs, accidentals, staff, and paper imperfections.
Freedom	Free to choose notes; free to decide quality of sounds.
The performing approach	Performing the given proportion of notes, following the arrows to decide performing directions; performing ascending gamut or descending gamut.

Table 57 The compositional elements in Notation J.

The compositional materials from the performing instruction	Odd or even number of notes, multiple-sided figures, graces, and punctuations.
The compositional materials from the notation	Clef signs, accidentals, ledger lines, staff, and paper imperfections.
Freedom	Free to choose perform either odd or even number of notes; free to decide how to use the given figures; free to decide duration of notes.
The performing approach	Ignoring time; performing only either odd or even numbers of notes; applying the given figures as graces or punctuations.

Table 58 The compositional elements in Notation K.

The compositional materials from the performing instruction	Perimeter, direction, clef signs, abbreviations, Notation A, and Notation B.
The compositional materials from the notation	Numbers, grouping, notes, hands, and paper imperfections.
Freedom	From Notation A: free to start at any notes; free to decide quality of sounds. From Notation B: free to choose notes.
The performing approach	Ignoring time; played with the assigned hands; performing notations in the assigned direction.

Table 59 The compositional elements in Notation L.

The compositional materials from the performing instruction	Direction, intersections, one or more voices, pedals.
The compositional materials from the notation	Clef signs, accidentals, ledger lines, staff, curved lines, and paper imperfections.
Freedom	Free to decide performing direction (decide whether change direction at the intersection or not); free to decide once or more voices; free to decide use pedals or not.
The performing approach	Performing notations in the assigned direction.

Table 60 The compositional elements in Notation M.

The compositional materials from the performing instruction	Dynamic indications, pedals.
The compositional materials from the notation	Clef signs, accidentals, arrows, pizzicato, ledger lines, staff, Notation I, and paper imperfections.
Freedom	From Notation I: free to decide duration of notes.
The performing approach	From Notation I: performing mute, pizzicato, intervals, and aggregate as annotations suggest; reappearances of tones to be played as originally. From Notation N: play with sostenuto pedal.

Table 61 The compositional elements in Notation N.

The compositional materials from the performing instruction	Arpeggiations, chords, lines, pedals, Notation D, and Notation M.
The compositional materials from the notation	Clef signs, accidentals, pedals, ledger lines, staff, notes, and paper imperfections.
Freedom	From Notation B and Notation D: free to decide produce certain format of sounds; free to decide duration of notes; free to realise overlapping and interpenetrations. From Notation M: free to decide use of pedals. From Notation O: free to decide the performing approach of the rest of notes.
The performing approach	From Notation B and Notation D: performing certain format of sounds. From Notation O: None.

Table 62 The compositional elements in Notation O.

The compositional materials from the performing instruction	Noises, and dynamic indications.
The compositional materials from the notation	Square, dots, paper imperfections.
Freedom	Free to decide how to produce noises; free to decide duration of sounds.
The performing approach	Following dynamic indications.

Table 63 The compositional elements in Notation P.

The compositional materials from the performing instruction	Time units, direction, speed, Notation M.
The compositional materials from the notation	Numbers, lines, ledger lines, clef signs, accidentals, staff, notes, paper imperfections.
Freedom	From Notation M: free to decide performing direction; free to decide once or more voices. From Notation Q: free to decide which time units; free to decide the range of increased speed.
The performing approach	From Notation M: performing notations in the assigned direction. From Notation Q: performing based on the time arrangement; when change performing direction at the intersections, need to increase speed.

Table 64 The compositional elements in Notation Q.

The compositional materials from the performing instruction	Time, notes, staff, clef signs, location of notes, lines, intersections, keys.
The compositional materials from the notation	Numbers, brackets, brace, ledger lines, accidentals, staff, paper imperfections.
Freedom	Free to choose clef signs; free to decide quality of sounds.
The performing approach	Performing based on the time arrangement; performing the notes with assigned clef signs.

Table 65 The compositional elements in Notation R.

The compositional materials from the performing instruction	Noises, lines, location of notes, piano construction, brace, and Notation C.
The compositional materials from the notation	Abbreviations, clef signs, accidentals, paper imperfections, staff, and notes.
Freedom	From Notation C: free to decide duration of notes; free to decide dynamic of notes.
The performing approach	From Notation C: performing mute as annotations suggest; performing pizzicato as annotations suggest; realising all notes as single tones. From Notation S: performing notes with the assigned piano construction; performing notes around lines as noises.

Table 66 The compositional elements in Notation S.

The compositional materials from the performing instruction	Pitch, time, shapes with centre notes, clusters, numbers, dynamic indications.
The compositional materials from the notation	Clef signs, accidentals, paper imperfections, staff, and notes.
Freedom	Free to decide relationship between numbers and dynamics (loud to soft or soft to loud); free to choose notes for the clusters.
The performing approach	Following positions of materials (shapes and notes) to perform; following the dynamic indications (numbers).

Table 67 The compositional elements in Notation T.

The compositional materials from the performing instruction	Areas, lines, direction, numbers.
The compositional materials from the notation	Clef signs, square, paper imperfections, staff.
Freedom	Free to choose performing notes but with given proportions; free to decide dynamic and duration of notes.
The performing approach	Performing notations in the assigned direction (left to right); performing the given proportion of notes; choose notes from the pitch areas.

Table 68 The compositional elements in Notation U.

The compositional materials from the performing instruction	Performance indications, degree of force, vertical distance, speed of attack, noises, line, staff.
The compositional materials from the notation	Clef signs, accidentals, paper imperfections, and notes.
Freedom	Free to decide how to produce noises.
The performing approach	Performing notes around lines as noises; reading indications from up to down; performing notes based on indications.

Table 69 The compositional elements in Notation V.

The compositional materials from the performing instruction	Legato, shapes, staccato, and isolated notes.
The compositional materials from the notation	Clef signs, accidentals, lines, staff, paper imperfections.
Freedom	Free to decide duration of notes; free to decide dynamic of notes.
The performing approach	Performing certain format of sounds (performing the shapes as legato; perform isolated notes as staccato); following the notation to decide the order of performing notes.

Table 70 The compositional elements in Notation W.

The compositional materials from the performing instruction	Signs (♪ and X) , punctuations, amplitude, and pedals.
The compositional materials from the notation	Clef signs, accidentals, dynamic indications, staff, and paper imperfections.
Freedom	Free to decide duration of notes; free to use pedals; free to decide timing of punctuations and different amplitude.
The performing approach	Following signs to perform the assigned sounding requirements – (♪) to perform punctuations, and (X) to decide repeated notes with different amplitude.

Table 71 The compositional elements in Notation X.

The compositional materials from the performing instruction	Pitch areas, numbers, staff, lines, time units, dynamic indications, position of notes.
The compositional materials from the notation	Paper imperfections.
Freedom	Free to choose notes.
The performing approach	Performing the given proportion of notes; performing based on the time arrangement; following positions of materials to perform (relationships between pitch areas, and loudness of notes).

Table 72 The compositional elements in Notation Y.

The compositional materials from the performing instruction	Clusters, single notes, dynamic indications, and Notation T.
The compositional materials from the notation	Clef signs, accidentals, lines, numbers, paper imperfections, staff, and notes.
Freedom	From Notation T: free to decide relationship between numbers and dynamics (loud to soft or soft to loud). From Notation Z: free to choose duration of notes.
The performing approach	From Notation T: following the dynamic indications (numbers). From Notation Z: performing clusters; performing single notes at the ends.

Table 73 The compositional elements in Notation Z.

The compositional materials from the performing instruction	Clef signs, lines (sticks), pitch areas, time units, notes, graces, dynamics, harmonics, circle.
The compositional materials from the notation	Accidentals, paper imperfections, staff.
Freedom	Free to choose clef signs; free to decide produce certain format of sounds (based on circles); free to decide the way to emphasise notes.
The performing approach	Choosing notes from the pitch areas (based on sticks); emphasising the circled notes, whether by dynamics, harmonics, repetition or other ways.

Table 74 The compositional elements in Notation AA.

The compositional materials from the performing instruction	Clusters, single notes, numbers, dynamic indications, and Notation Z.
The compositional materials from the notation	Clef signs, accidentals, lines, numbers, paper imperfections, staff, and notes.
Freedom	From Notation T: free to decide relationship between numbers and dynamics (loud to soft or soft to loud). From Notation Z: free to decide duration of notes. From Notation AB: none.
The performing approach	From Notation T: following dynamic indications. From Notation Z: performing single notes at the ends. From Notation AB: performing single notes at the beginning if applicable.

Table 75 The compositional elements in Notation AB.

The compositional materials from the performing instruction	Noises, numbers, given proportion of noises, abbreviations (I, A, and O), piano construction (interior and outer), position of materials, dynamics.
The compositional materials from the notation	Paper imperfections, and staff.
Freedom	Free to choose noises; free to decide quality of noises in a general sense.
The performing approach	Performing the given proportion of noises; performing notes with the assigned piano construction.

Table 76 The compositional elements in Notation AC.

The compositional materials from the performing instruction	Single notes, interval, aggregate, staff, ledger lines, clef signs.
The compositional materials from the notation	Accidentals, paper imperfections.
Freedom	Free to choose clef signs; free to decide quality of notes.
The performing approach	Performing the notes with assigned clef signs (notes are on ledger lines above the staff with treble, notes are on ledger lines below the staff with bass); performing certain format of sounds (single tones, intervals, and three-note aggregate).

Table 77 The compositional elements in Notation AD.

The compositional materials from the performing instruction	Pitch time areas, and numbers.
The compositional materials from the notation	Clef signs, accidentals, lines, ledger lines, arrows, brackets, paper imperfections, notes, and staff.
Freedom	Free to decide quality of notes in a general sense; free to choose notes.
The performing approach	Performing based on the time arrangement; performing the given proportion of notes; choosing notes from the pitch areas.

Table 78 The compositional elements in Notation AE.

The compositional materials from the performing instruction	Events (line-connected notes), lines, clef signs, hands.
The compositional materials from the notation	Staff, accidentals, and paper imperfections.
Freedom	Free to decide quality of notes; free to decide the duration of events; free to choose clef sign (for the events).
The performing approach	Playing with the assigned hands (the line-connected notes should be played by one hand, clef signs above the staff are for the right hand, while below are for the left hand).

Table 79 The compositional elements in Notation AF.

The compositional materials from the performing instruction	Notes, aggregate.
The compositional materials from the notation	Staff, clef sign, accidentals, and paper imperfections.
Freedom	Free to choose notes (to omit them); free to decide quality of notes.
The performing approach	Omit two notes in each aggregate.

Table 80 The compositional elements in Notation AG.

The compositional materials from the performing instruction	Hands, lines, direction, and arrows.
The compositional materials from the notation	Staff, accidentals, notes, and paper imperfections.
Freedom	Free to decide quality of notes; free to choose clef sign.
The performing approach	Performing notations in the assigned direction; playing with the assigned hands.

Table 81 The compositional elements in Notation AH.

The compositional materials from the performing instruction	Wheels, axles, and direction.
The compositional materials from the notation	Staff, accidentals, clef signs, notes, and paper imperfections.
Freedom	Free to choose notes (form wheel and axle); free to decide quality of notes.
The performing approach	Performing notations in the assigned direction; not to perform as harmonic(s).

Table 82 The compositional elements in Notation AI.

The compositional materials from the performing instruction	Number, notes, direction, arrow, clef signs.
The compositional materials from the notation	Staff, accidentals, lines, and paper imperfections.
Freedom	Free to choose notes; free to decide quality of notes.
The performing approach	Performing the notes with assigned clef signs; performing notations in the assigned direction.

Table 83 The compositional elements in Notation AJ.

The compositional materials from the performing instruction	Notes, pitch areas (universe), time, and amplitude.
The compositional materials from the notation	Staff, accidentals, lines, numbers, bracket, and paper imperfections.
Freedom	Free to choose notes.
The performing approach	Performing based on the time arrangement; choosing notes from pitch areas; following dynamic indications.

Table 84 The compositional elements in Notation AK.

The compositional materials from the performing instruction	Gamut, numbers, time, events, legato, and staccato.
The compositional materials from the notation	Lines, brackets, and paper imperfections.
Freedom	Free to choose notes (for eight tone gamut); free to decide time units.
The performing approach	Performing based on the time arrangement; performing certain format of sounds.

Table 85 The compositional elements in Notation AL.

The compositional materials from the performing instruction	Single notes, interval, time, signs, and staff.
The compositional materials from the notation	Accidentals and paper imperfections.
Freedom	Free to decide quality of notes.
The performing approach	Performing certain format of sounds (single notes and intervals); performing based on the time arrangement.

Table 86 The compositional elements in Notation AM.

The compositional materials from the performing instruction	Clef signs, lines, direction (left and right), notes, and hands.
The compositional materials from the notation	Accidentals and paper imperfections.
Freedom	Free to choose notes; free to perform any number of notes; free to decide quality of notes.
The performing approach	Choosing notes from the pitch areas; performing notations in the assigned direction; playing with the assigned hands.

Table 87 The compositional elements in Notation AN.

The compositional materials from the performing instruction	Pitch areas, chromatic tones, space, lines (horizontal lines and vertical lines), and dots.
The compositional materials from the notation	paper imperfections.
Freedom	Free to choose notes (for at least twenty chromatic tones); free to decide produce certain format of sounds (vertical lines as clusters or legato).
The performing approach	Realising the space as frequency and time; performing certain format of sounds (performing single notes at where the dots located); choosing notes from the pitch areas.

Table 88 The compositional elements in Notation AO.

The compositional materials from the performing instruction	Lines, duration, notes, and staccato.
The compositional materials from the notation	Staff, clef signs, accidentals, and paper imperfections.
Freedom	Free to decide dynamic of notes.
The performing approach	Following lines to decide duration; performing certain format of sounds (single notes performed as staccato).

Table 89 The compositional elements in Notation AP.

The compositional materials from the performing instruction	From Notation Y: Pitch areas, numbers, staff, line, time (second as the unit), dynamic indications, position of materials.
The compositional materials from the notation	From Notation Y: Paper imperfections.
Freedom	From Notation Y: Free to choose notes (but follow the relationship between notes). From Notation AQ: Free to choose the performing areas (either below or above the dotted lines).
The performing approach	From Notation Y: performing the given proportion of notes; performing based on the time arrangement; following positions of materials to perform; choosing notes from the pitch areas. From Notation AQ: Choosing one performing area (either below or above the dotted lines).

Table 90 The compositional elements in Notation AQ.

The compositional materials from the performing instruction	Drawing.
The compositional materials from the notation	Lines, square, rectangles, staff, clef signs, ledger lines, and paper imperfections.
Freedom	Free to decide dynamics of notes.
The performing approach	Realising the notation as the drawing suggests.

Table 91 The compositional elements in Notation AR.

The compositional materials from the performing instruction	A single note.
The compositional materials from the notation	Staff, clef sign, accidentals, and paper imperfections.
Freedom	Free to decide quality of the notes.
The performing approach	Performing the given notes.

Table 92 The compositional elements in Notation AS.

The compositional materials from the performing instruction	From Notation AE: Pitch-time areas, and numbers.
The compositional materials from the notation	From Notation AE: clef signs, accidentals, lines, ledger lines, arrows, brackets, paper imperfections, notes, and staff.
Freedom	From Notation AE: free to decide quality of notes.
The performing approach	From Notation AE: performing based on the time arrangement; performing the given proportion of notes; choosing notes from the pitch areas.

Table 93 The compositional elements in Notation AT.

The compositional materials from the performing instruction	Notation Q, lines, clef signs, ambiguous pitch, direction, intersection, notes.
The compositional materials from the notation	Numbers, ledger lines, accidentals, staff, and paper imperfections.
Freedom	From Notation M: free to decide performing directions. From Notation Q: free to decide time units; free to decide the range of increased speed. From Notation AU: free to choose clef sign.
The performing approach	From Notation M: performing notations in the assigned direction. From Notation Q: performing based on the time arrangement; increasing speed when change performing direction.

Table 94 The compositional elements in Notation AU.

The compositional materials from the performing instruction	Drawing, numbers, and amplitude.
The compositional materials from the notation	Lines, squares, rectangles, staff, clef signs, ledger lines, Notation AR, and paper imperfections.
Freedom	Free to decide relationship between numbers and dynamics.
The performing approach	From Notation AR: realising the notation as the drawing suggests. From Notation AV: following dynamic indications (numbers).

Table 95 The compositional elements in Notation AV.

The compositional materials from the performing instruction	Notation AD.
The compositional materials from the notation	Single tones, interval, aggregates, ambiguous staff, ledger lines, clef signs, accidentals, paper imperfections, and staff.
Freedom	From Notation AD: free to choose clef sign; free to decide quality of notes.
The performing approach	From Notation AD: performing the notes with assigned clef signs; performing certain format of sounds.

Table 96 The compositional elements in Notation AW.

The compositional materials from the performing instruction	Noises, amplitude, position of the materials (beaters), lines, and areas.
The compositional materials from the notation	Staff, and paper imperfections.
Freedom	Free to decide amplitude of noises; free to choose materials for producing noises.
The performing approach	Performing based on the time arrangement; performing with the given materials.

Table 97 The compositional elements in Notation AX.

The compositional materials from the performing instruction	1/10 inch square, time unit, numbers, notes, graphs (horizontal and vertical), frequency, clef signs.
The compositional materials from the notation	Staff, arrows, paper imperfections.
Freedom	Free to choose graphs to perform; free to choose clef sign.
The performing approach	Realising the notation as the drawing suggests (the horizontal graph for time unit, and the vertical graph for frequency); performing the given proportion of notes.

Table 98 The compositional elements in Notation AY.

The compositional materials from the performing instruction	Numbers, time (any units), notes, lines, and clusters.
The compositional materials from the notation	Staff, and accidentals.
Freedom	Free to decide time units; free to decide quality of notes.
The performing approach	Performing based on the time arrangement; performing certain format of sounds (perform the vertical lines as clusters).

Table 99 The compositional elements in Notation AZ.

The compositional materials from the performing instruction	Numbers, Notation T, noise, Notation AC.
The compositional materials from the notation	Dot, F-clef, G-clef, accidentals, abbreviations (I, A, and O), area, interior piano construction, auxiliary noises, outer piano construction, paper imperfections, staff, and notes.
Freedom	From Notation T: free to decide relationship between numbers and dynamics (loud to soft or soft to loud). From Notation AC: free to decide quality of noises in a general sense. From Notation BA: none.
The performing approach	From Notation T: following dynamic indications; following positions of materials to perform. From Notation AC: performing notes with the assigned piano construction. From Notation BA: none.

Table 100 The compositional elements in Notation BA.

The compositional materials from the performing instruction	Dots, single sounds, lines, duration, frequency, overtone structure, amplitude, and occurrence (succession), and abbreviations.
The compositional materials from the notation	Square, paper imperfections.
Freedom	Free to choose notes (based on the results of measurements); free to decide how to measure when there are more than one contact between perpendiculars and lines.
The performing approach	Measuring the distances between perpendiculars and the given lines; following the measurement results to determine the quality of notes.

Table 101 The compositional elements in Notation BB.

The compositional materials from the performing instruction	Given proportion of notes, pitch areas, and numbers.
The compositional materials from the notation	Staff, clef signs, lines, paper imperfections.
Freedom	Free to decide numbers; free to decide quality of notes; free to choose notes.
The performing approach	Performing the given proportion of notes; choosing notes from the pitch areas.

Table 102 The compositional elements in Notation BC.

The compositional materials from the performing instruction	Notes, amplitude, and areas.
The compositional materials from the notation	Brackets.
Freedom	free to choose notes; free to decide duration of notes.
The performing approach	Realising the notes with given amplitude.

Table 103 The compositional elements in Notation BD.

The compositional materials from the performing instruction	Numbers, events, dots, fingers, hands, and forearms.
The compositional materials from the notation	Areas.
Freedom	Free to choose events; free to choose notes; free to decide duration of events.
The performing approach	Realising the notation with the given body parts.

Table 104 The compositional elements in Notation BE.

The compositional materials from the performing instruction	Notes, lines, legato, single notes, staccato.
The compositional materials from the notation	Shapes, clef signs, staff, accidentals, and paper imperfections.
Freedom	Free to decide quality of notes.
The performing approach	Performing certain format of sounds (single notes as staccato, connected notes as legato); following the notation to decide the order of performing notes.

Table 105 The compositional elements in Notation BF.

The compositional materials from the performing instruction	Intervals, free approaches, numbers, given proportion of notes, pitch limits.
The compositional materials from the notation	Shapes, clef signs, staff, accidentals, and paper imperfections.
Freedom	Free to decide performing manner; free to decide quality of notes; free to choose notes.
The performing approach	Performing certain format of sounds; performing the given proportion of notes; choosing notes from the pitch limits.

Table 106 The compositional elements in Notation BG.

The compositional materials from the performing instruction	Notation A, and clef signs.
The compositional materials from the notation	Numbers, accidentals, grouping, perimeter, notes, the given proportion of notes, paper imperfections, and staff.
Freedom	From Notation A: From any note on it; free to choose notes; free to decide the performing direction; free to decide quality of notes. From Notation BH: free to choose clef sign.
The performing approach	From Notation A: following the perimeter; playing in opposite directions; performing the given proportion of notes.

Table 107 The compositional elements in Notation BH.

The compositional materials from the performing instruction	Numbers, frequency, and time units.
The compositional materials from the notation	Numbers (has shown in the performing instruction).
Freedom	free to decide the uses of numbers; free to decide time units; free to decide the duration of realisations.
The performing approach	choosing either one or two number(s) for a set.

Table 108 The compositional elements in Notation BI.

The compositional materials from the performing instruction	Single sound, boundaries, frequency, duration, amplitude, overtone structure, and Notation BB.
The compositional materials from the notation	The performing instruction shows all compositional materials.
Freedom	From Notation BB: free to decide how to measure. From Notation BJ: free to decide how to use boundaries.
The performing approach	From Notation BB: following the measurement results to determine the quality of notes.

Table 109 The compositional elements in Notation BJ.

The compositional materials from the performing instruction	Noises, abbreviations (A, I, and O), piano construction, Notation A, and Notation AC.
The compositional materials from the notation	The perimeter, direction, the given proportion of notes, numbers, areas, clef signs, accidentals, grouping, paper imperfections, staff, and notes.
Freedom	From Notation A: free to choose notes; free to decide the performing direction; free to decide quality of notes.
The performing approach	From Notation A: following the perimeter; playing in opposite direction; performing the given proportion of notes. From Notation AC: performing notes with the assigned piano construction. From Notation BK: none.

Table 110 The compositional elements in Notation BK.

The compositional materials from the performing instruction	Single notes, numbers (above, below, before and after).
The compositional materials from the notation	Clef signs, staff, and accidentals.
Freedom	Free to decide quality of sounds.
The performing approach	Performing the given proportion of notes (based on the central note).

Table 111 The compositional elements in Notation BL.

The compositional materials from the performing instruction	Pitch, amplitude, and time.
The compositional materials from the notation	Lines, areas, clef sign, staff, accidental, and paper imperfections.
Freedom	Free to decide the frequency of performing two given notes
The performing approach	Following dynamic indications; performing based on the time arrangement; performing the given notes.

Table 112 The compositional elements in Notation BM.

The compositional materials from the performing instruction	Hands, the perimeter, the centre.
The compositional materials from the notation	Clef signs, accidentals, staff, notes, shapes, and paper imperfections.
Freedom	Free to choose notes (two notes as starting point); free to decide quality of notes; free to decide the performing direction.
The performing approach	Ending the realisation at the centre of this notation.

Table 113 The compositional elements in Notation BN.

The compositional materials from the performing instruction	Notation W, and time units.
The compositional materials from the notation	Legato, triangles, staccato, isolated notes, clef signs, accidentals, lines, staff, and paper imperfections.
Freedom	From Notation W: free to decide dynamic of notes.
The performing approach	From Notation W: following the notation to decide the order of performing notes; performing certain format of sounds. From Notation BO: performing based on the time arrangement.

Table 114 The compositional elements in Notation BO.

The compositional materials from the performing instruction	Numbers, pitch limits, given proportion of notes, and hands.
The compositional materials from the notation	Clef signs, accidentals, staff, and paper imperfections.
Freedom	free to choose notes; free to decide quality of sounds.
The performing approach	performing the given proportion of notes; choosing notes from the pitch ranges.

Table 115 The compositional elements in Notation BP.

The compositional materials from the performing instruction	Single notes, triangles, hypothenuse, and dynamics.
The compositional materials from the notation	Clef signs, staff, numbers, pitch ranges, and paper imperfections.
Freedom	Free to choose notes.
The performing approach	Choosing notes from the pitch ranges; performing based on the time arrangement.

Table 116 The compositional elements in Notation BQ.

The compositional materials from the performing instruction	From Notation B: ictus, harmonics, overlappings, aggregate, and interpenetrations.
The compositional materials from the notation	Numbers, accidentals, staff, paper imperfections, notes, single ictus, overlappings, and interpenetrations.
The performing approach	From Notation B: performing certain format of sounds. From Notation BR: choosing assigned numbers of notes to perform in advance.
Freedom	From Notation B: free to decide to produce certain format of sounds (harmonics); free to realise overlapping and interpenetrations; free to choose notes (the assign proportion, and for playing in advance); free to decide duration of notes.

Table 117 The compositional elements in Notation BR.

The compositional materials from the performing instruction	Dynamics, and hands.
The compositional materials from the notation	Lines, areas, dots, and paper imperfections.
Freedom	free to decide how to produce noises; free to decide quality of noises.
The performing approach	following dynamic indications; playing with the assigned hands.

Table 118 The compositional elements in Notation BS.

The compositional materials from the performing instruction	Dots, piano construction.
The compositional materials from the notation	Lines, shapes, and paper imperfections.
Freedom	Free to decide performing manner; free to decide the duration of realisations; free to decide quality of noises.
The performing approach	Performing notes with the assigned piano construction.

Table 119 The compositional elements in Notation BT.

The compositional materials from the performing instruction	Notes, given proportion of notes, given proportion of noises, areas, brackets, staff, noise area, and time units.
The compositional materials from the notation	Paper imperfections.
Freedom	Free to choose notes; free to decide quality of notes; free to decide quality of noises; free to decide how to produce noises.
The performing approach	Performing based on the time arrangement; performing the given proportion of noises; performing the given note; choosing notes from the pitch ranges.

Table 120 The compositional elements in Notation BU.

The compositional materials from the performing instruction	Dots (large, less large, small, and very small dots), lines, boundaries, Notation BB, and Notation BJ.
The compositional materials from the notation	Area.
Freedom	From Notation BB: free to choose notes; free to decide how to measure. From Notation BJ: free to decide how to use boundaries.
The performing approach	From Notation BB: measuring the distance between perpendiculars and the given lines; following the measurement results to determine the quality of notes. From Notation BJ: following the measurement results to determine the quality of notes From Notation BV: following the size of dots to perform.

Table 121 The compositional elements in Notation BV.

The compositional materials from the performing instruction	Four-sided figures/shapes, frequency, amplitude, duration, overtone structure, occurrence, and illusion.
The compositional materials from the notation	Dots, paper imperfections.
Freedom	Free to decide the representatives of the sides (i.e., frequency, amplitude, duration, and overtone structure); free to decide which dot corresponds with which shape.
The performing approach	Observing the order of three shapes; following the observing result to decide the performing order.

Table 122 The compositional elements in Notation BW.

The compositional materials from the performing instruction	Shape (a plant).
The compositional materials from the notation	Lines, clef signs, staff, accidentals, notes, and paper imperfections.
Freedom	free to decide quality of notes.
The performing approach	Realising the notation as the shapes suggest; performing the given proportion of notes.

Table 123 The compositional elements in Notation BX.

The compositional materials from the performing instruction	Noises, position of materials, and pitch.
The compositional materials from the notation	Dots, and area.
Freedom	free to decide performing manner.
The performing approach	following pitch indications.

Table 124 The compositional elements in Notation BY.

The compositional materials from the performing instruction	Pedals, abbreviations (for inactivity and activity), keys, harp, and noise.
The compositional materials from the notation	Lines.
Freedom	free to decide performing manner (keyboard, harp, or noises); free to decide quality of notes; free to decide quality of noises.
The performing approach	performing with pedals; following movements of lines.

Table 125 The compositional elements in Notation BZ.

The compositional materials from the performing instruction	Areas (keyboard area: white, mute area: vertical lines, pizzicato area: bracketed by dotted lines, friction area: horizontal lines), notes, and pitch.
The compositional materials from the notation	The performing instruction shows all compositional materials.
Freedom	free to choose notes; free to decide to produce certain format of sounds.
The performing approach	Realising the notation as the drawing suggests; following positions of materials to perform.

Table 126 The compositional elements in Notation CA.

The compositional materials from the performing instruction	Numbers, notes, and pitch ranges.
The compositional materials from the notation	Clef signs, staff, and ledger lines.
Freedom	free to choose notes; free to decide quality of notes.
The performing approach	performing the given proportion of notes; choosing notes from the pitch ranges; following the clef signs.

Table 127 The compositional elements in Notation CB.

The compositional materials from the performing instruction	Lines, frequency, duration, amplitude, overtone structure, intersections, numbers, and time.
The compositional materials from the notation	The performing instruction shows all compositional materials.
Freedom	Free to decide the uses of lines; free to decide how to measure.
The performing approach	following the measurement results to determine the quality of notes; performing based on the time arrangement.

Table 128 The compositional elements in Notation CC.

The compositional materials from the performing instruction	Ambiguous clef signs, signs (· , ◦ , and X), keys, and harp.
The compositional materials from the notation	Accidentals, staff, notes, and paper imperfections.
Freedom	Free to choose clef signs; free to decide quality of notes.
The performing approach	Performing by keyboard or harp; choosing given numbers of clef signs.

Table 129 The compositional elements in Notation CD.

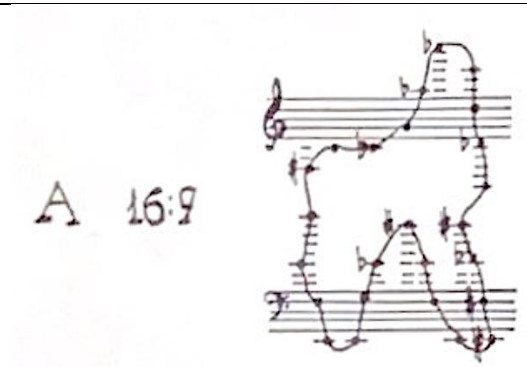
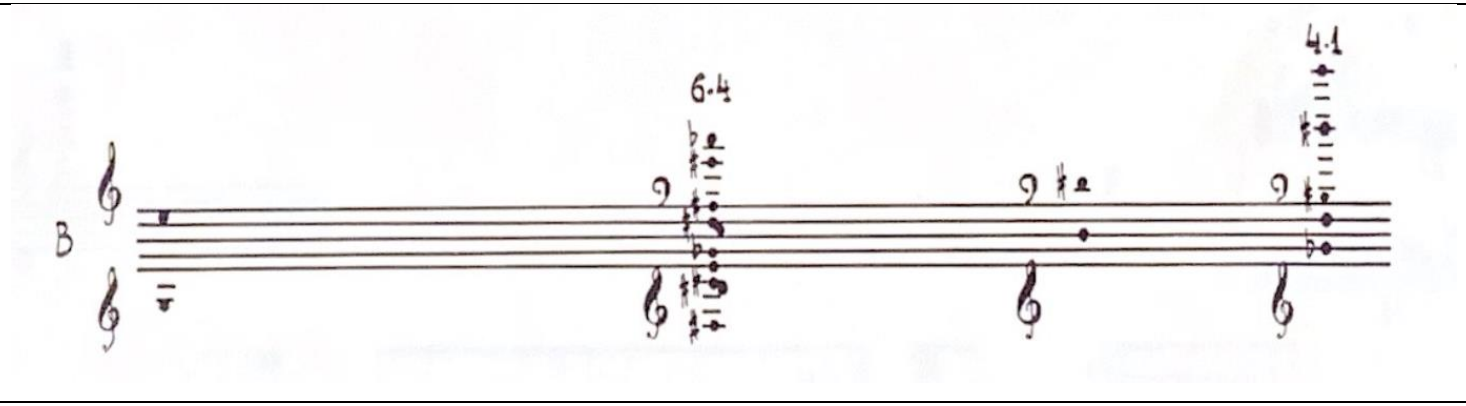
The compositional materials from the performing instruction	Ambiguous clef signs, ledger lines, numbers (given proportion of notes), intervals, aggregates, notes.
The compositional materials from the notation	Accidentals, staff, and paper imperfections
Freedom	Free to decide the quality of notes; free to choose clef sign; free to choose notes.
The performing approach	Performing certain format of sounds.

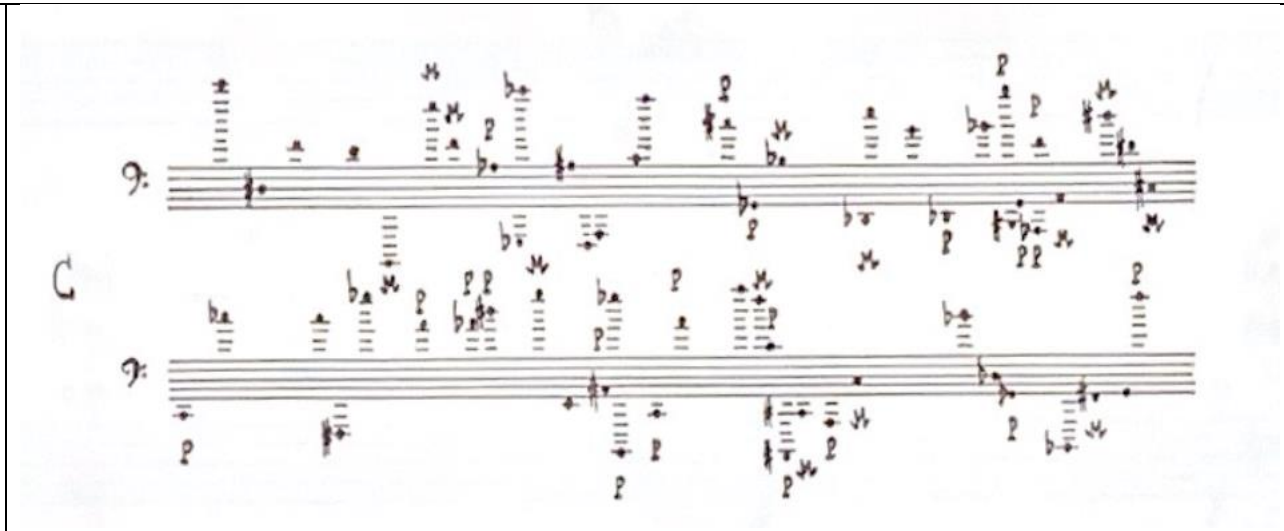
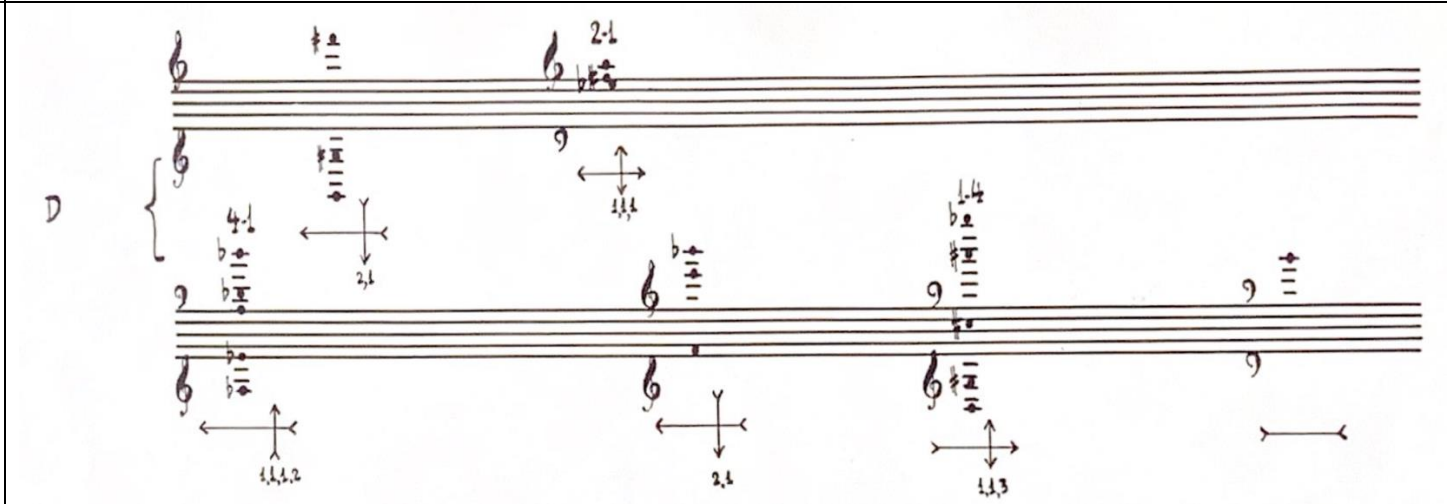
Table 130 The compositional elements in Notation CE.

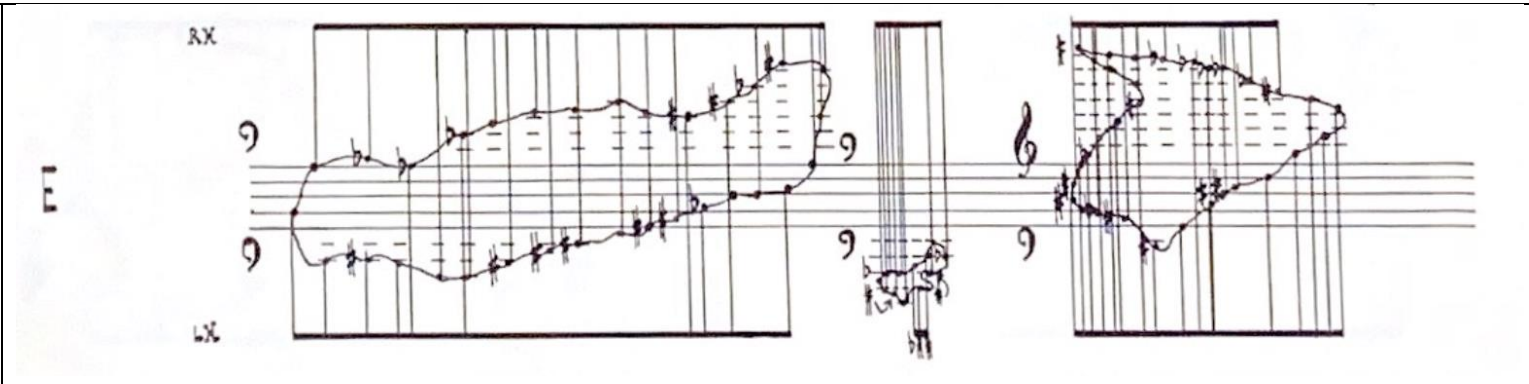
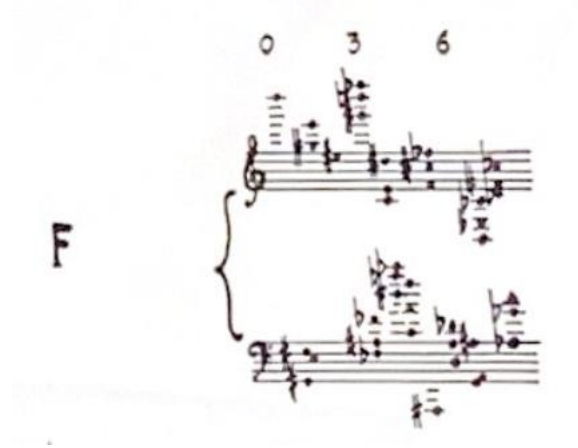
The compositional materials from the performing instruction	Notation BZ.
The compositional materials from the notation	Pedals, abbreviations (for inactivity and activity), lines, keys, harp, and noise.
Freedom	From Notation BZ: free to decide performing manner (keyboard, harp, or noises); free to decide quality of notes; free to decide quality of noises.
The performing approach	From Notation BZ: performing with pedals; following movements of lines

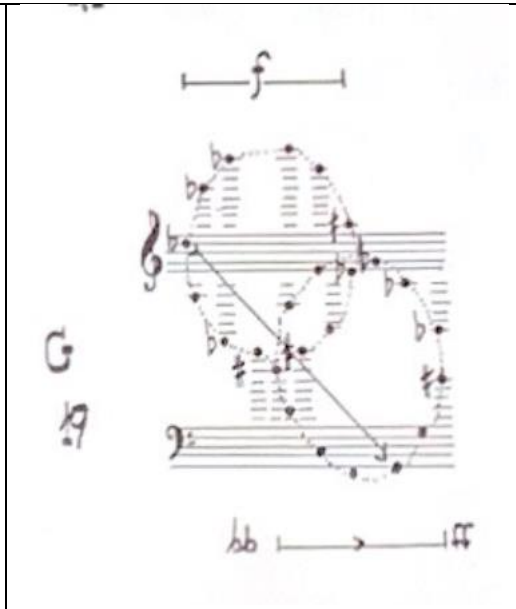
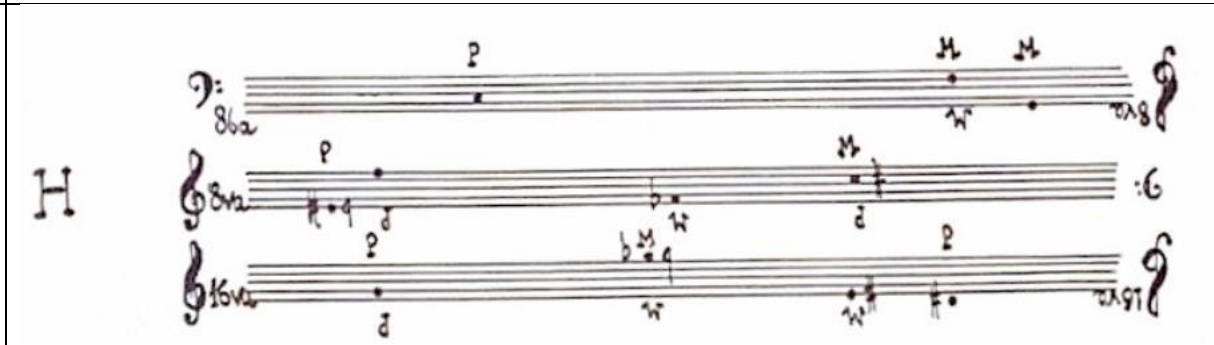
Table 131 The compositional elements in Notation CF.

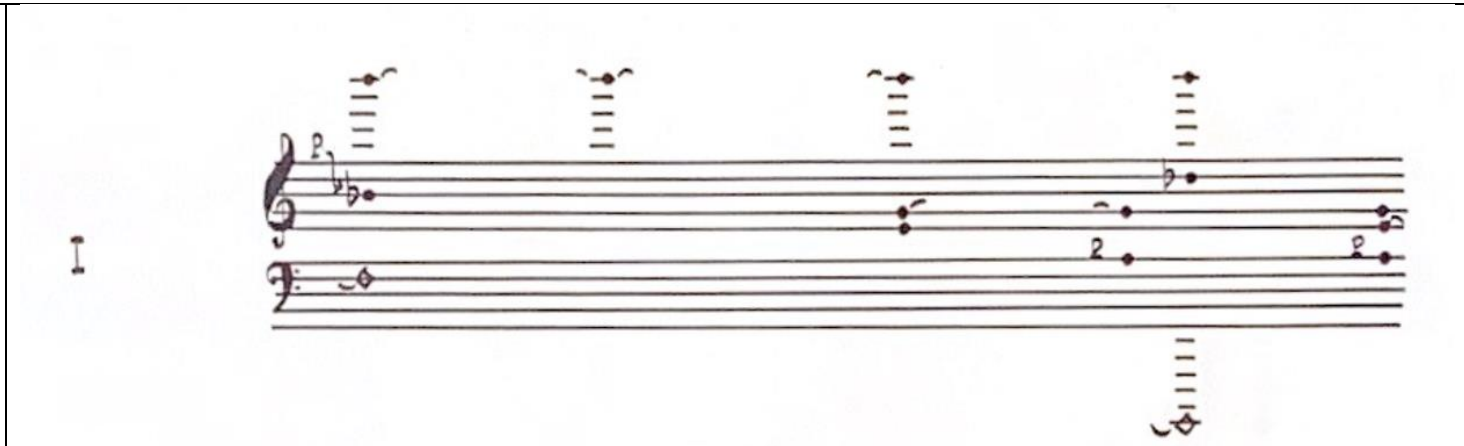
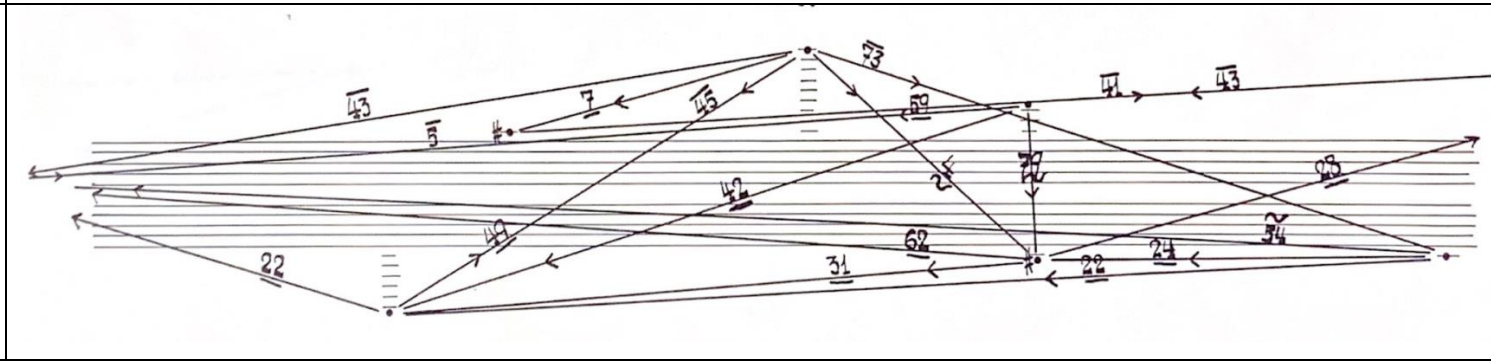
Appendix B: Appearance of Each Notation

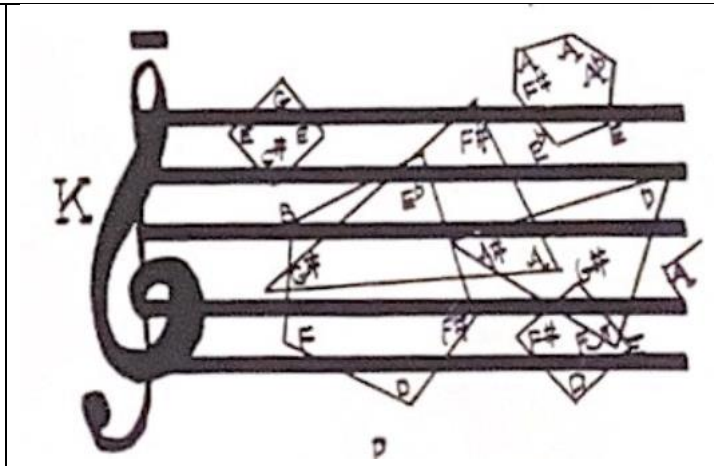
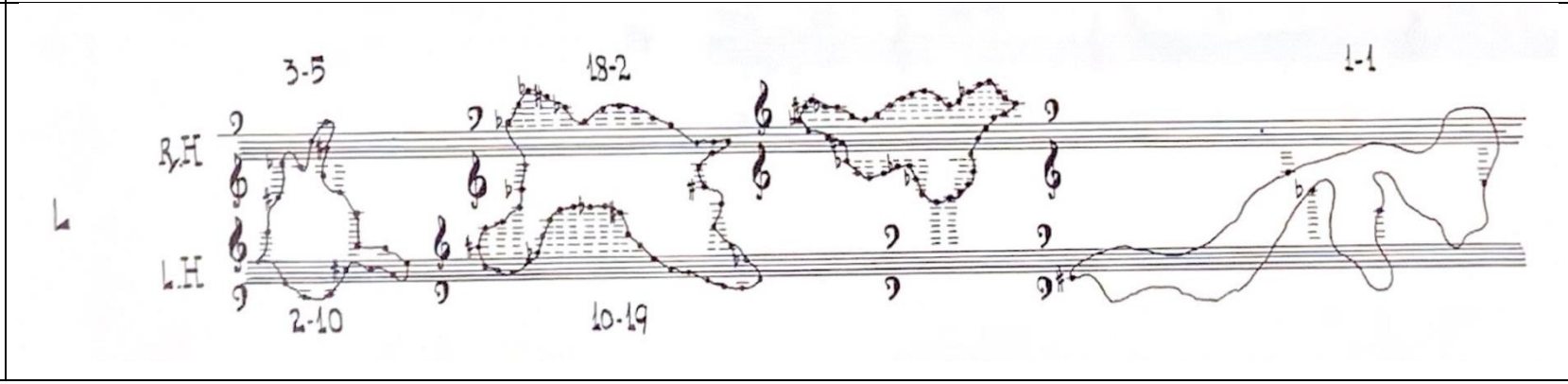
Notation	Appearance of the notation
Notation A	
Notation B	

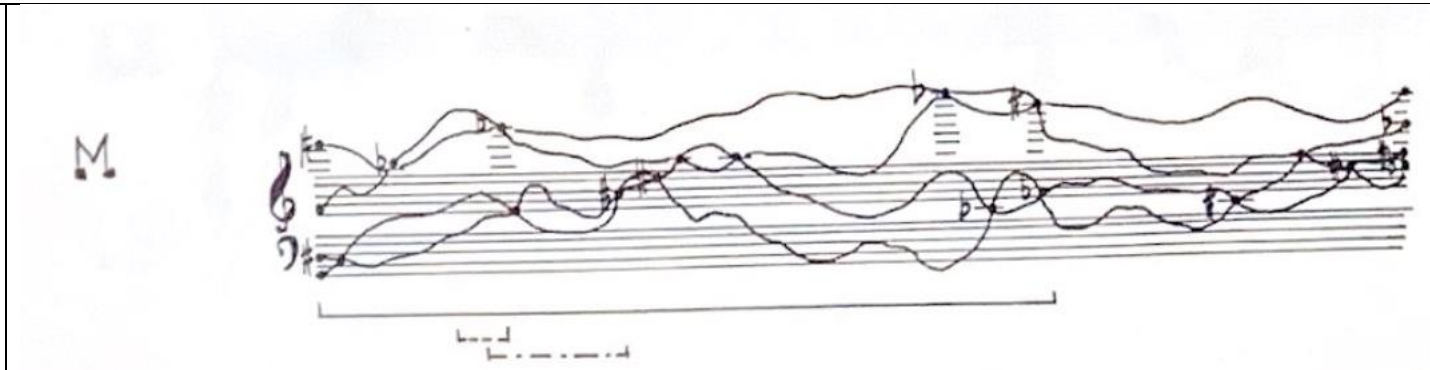
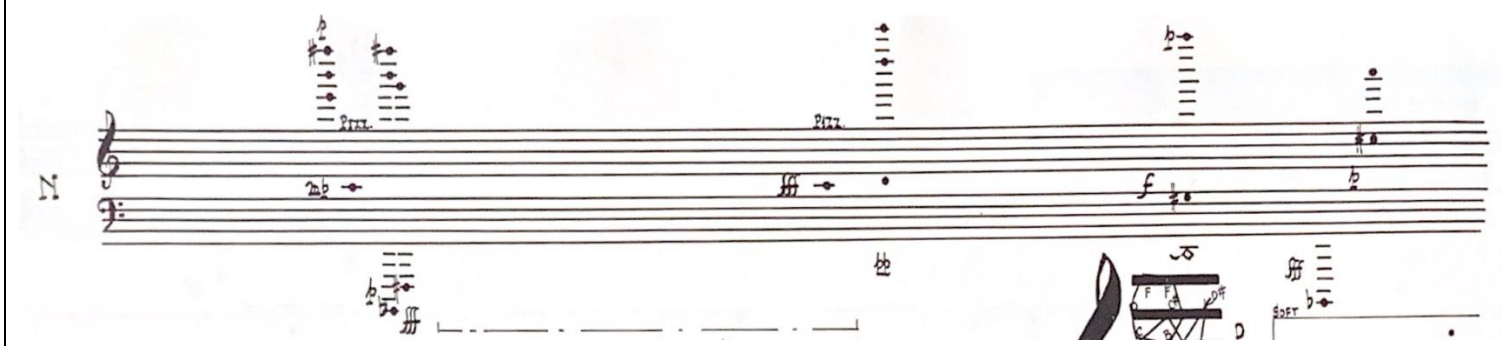
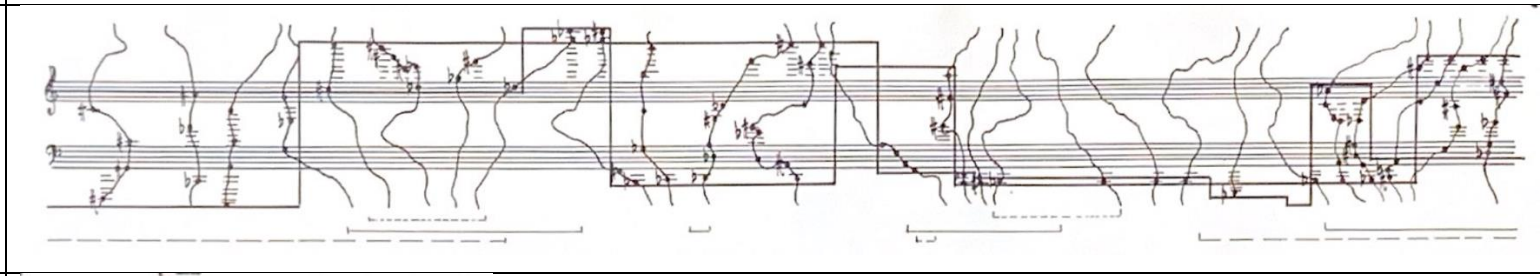
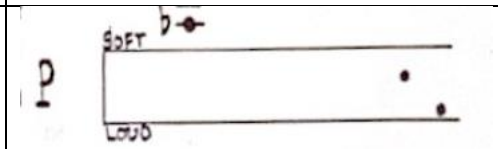
Notation C	
Notation D	

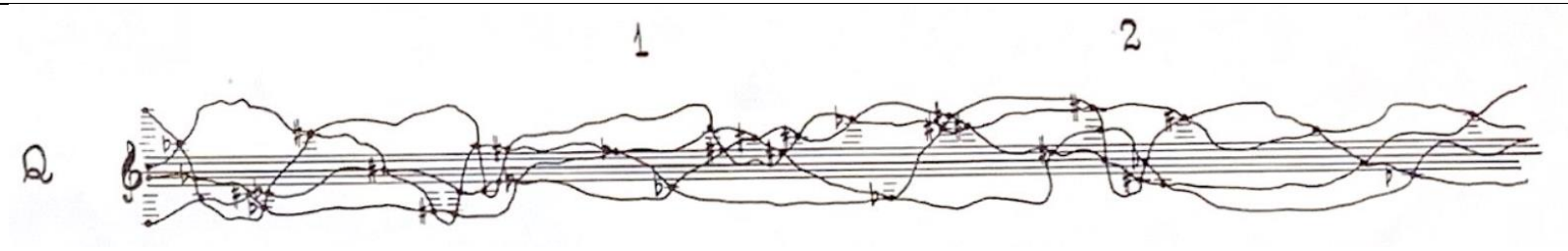
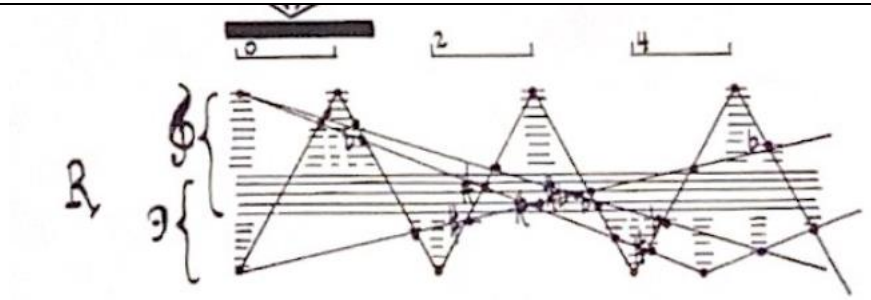
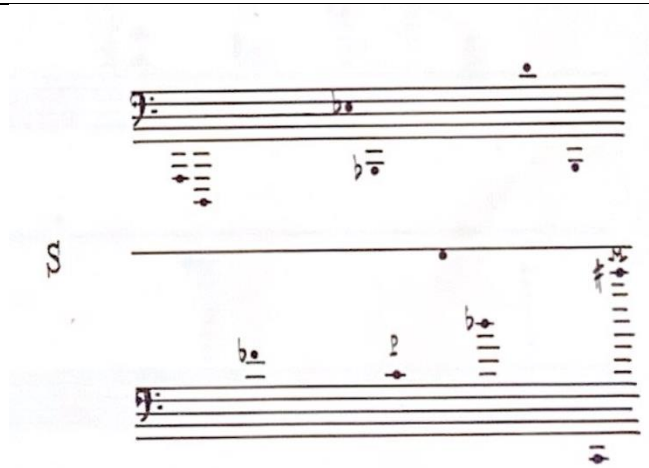
Notation E	
Notation F	

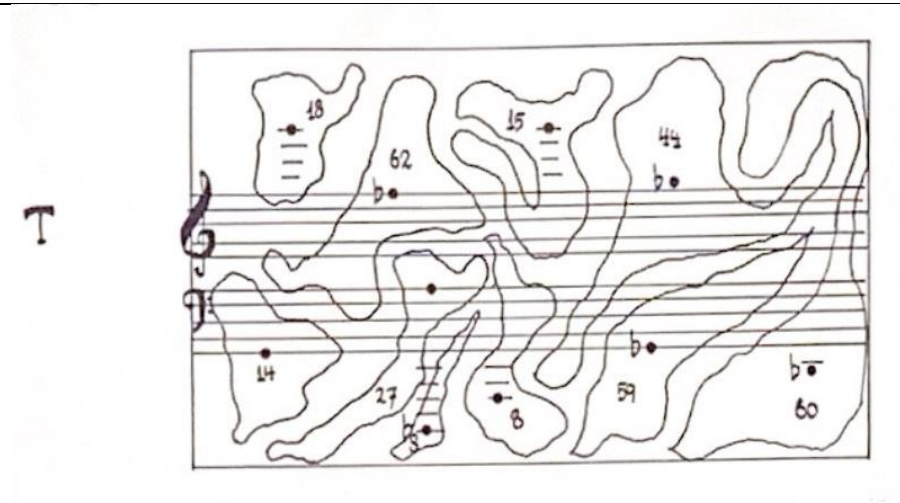
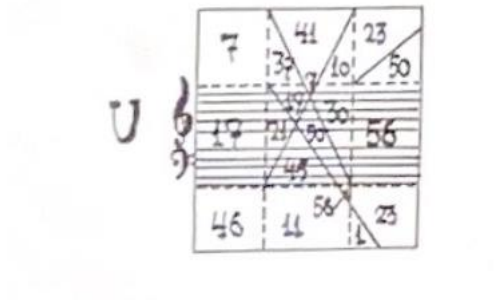
Notation G	
Notation H	

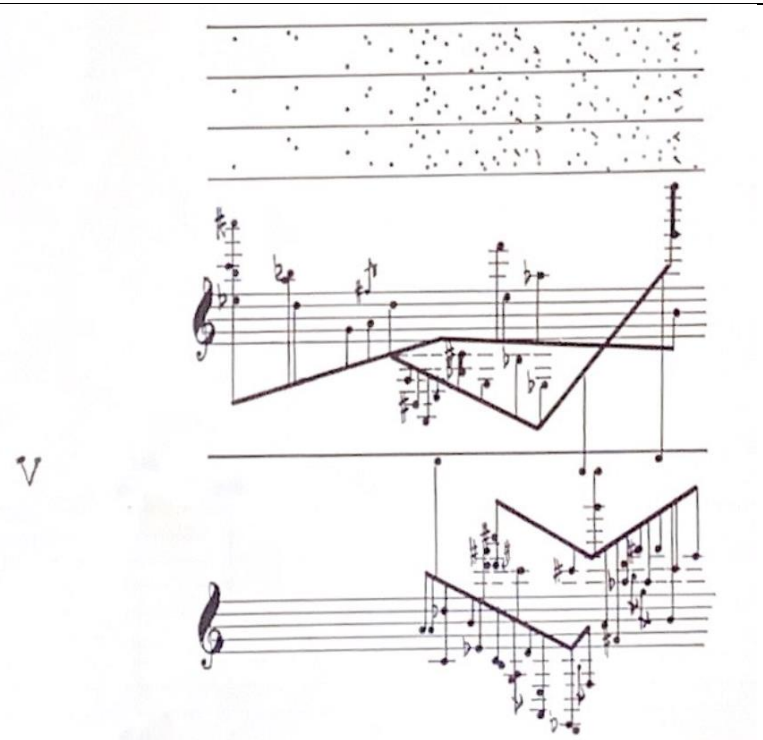
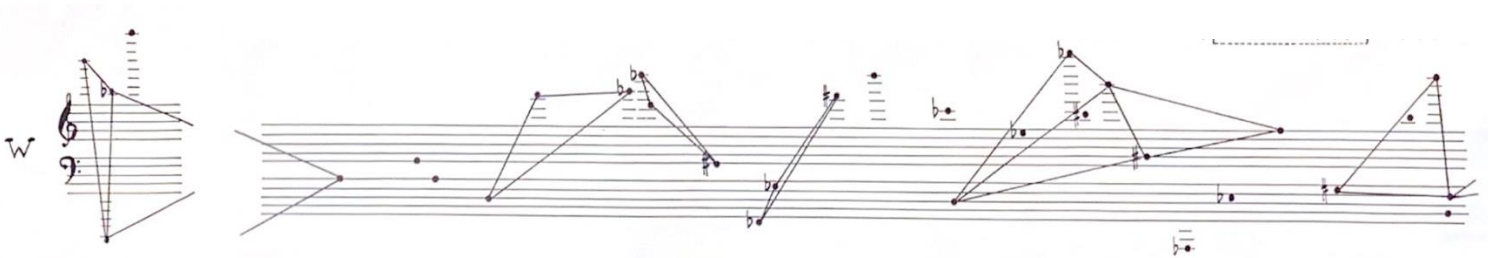
Notation I	
Notation J	

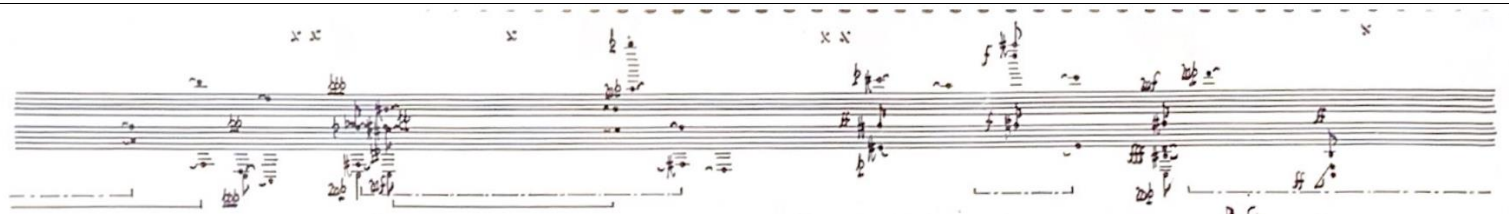
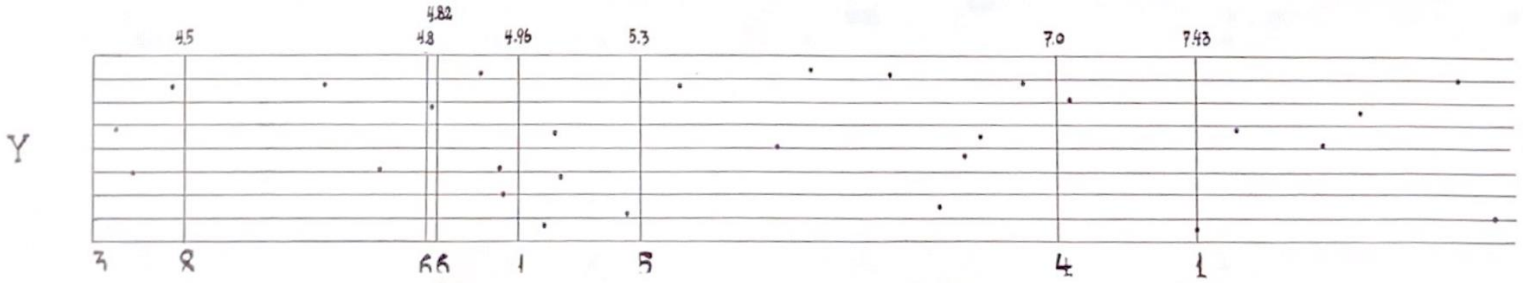
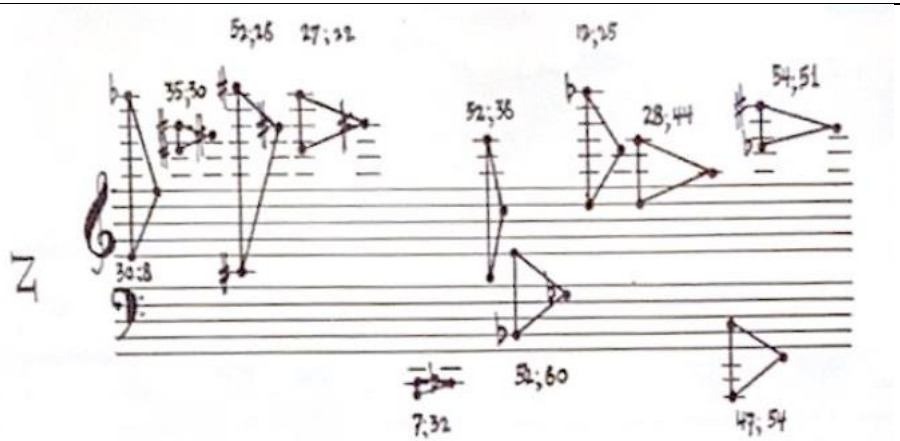
Notation K	
Notation L	

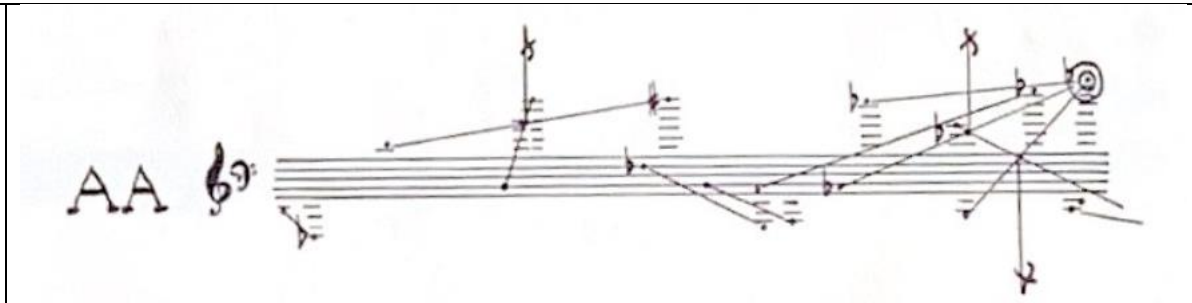
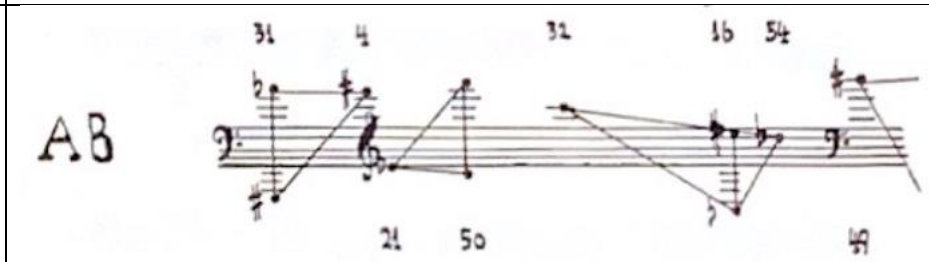
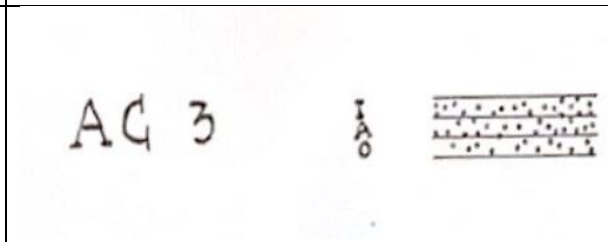
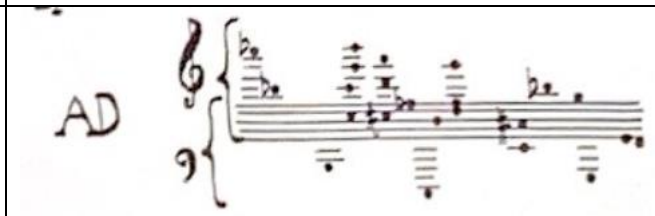
Notation M	
Notation N	
Notation O	
Notation P	

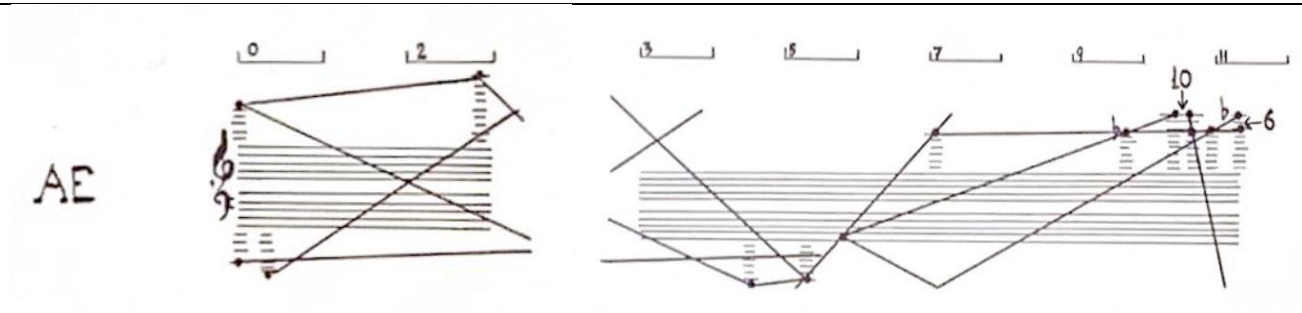
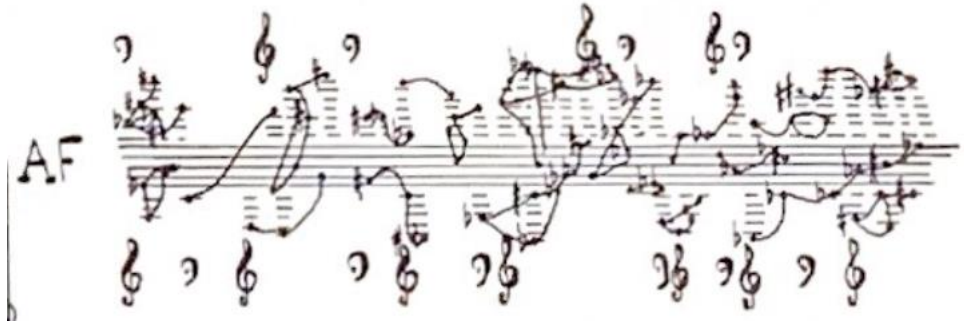
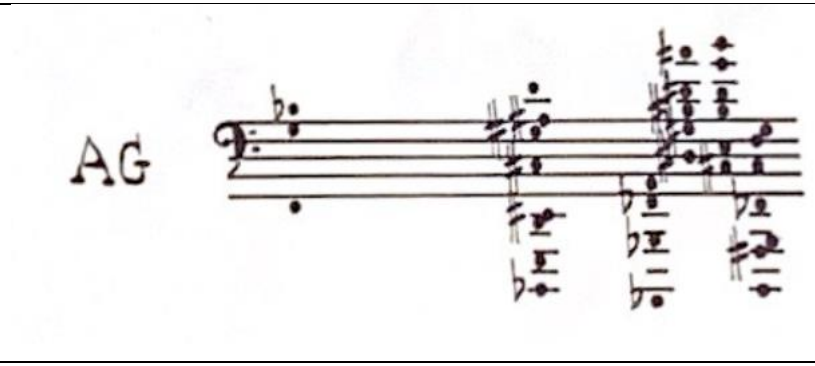
Notation Q	
Notation R	
Notation S	

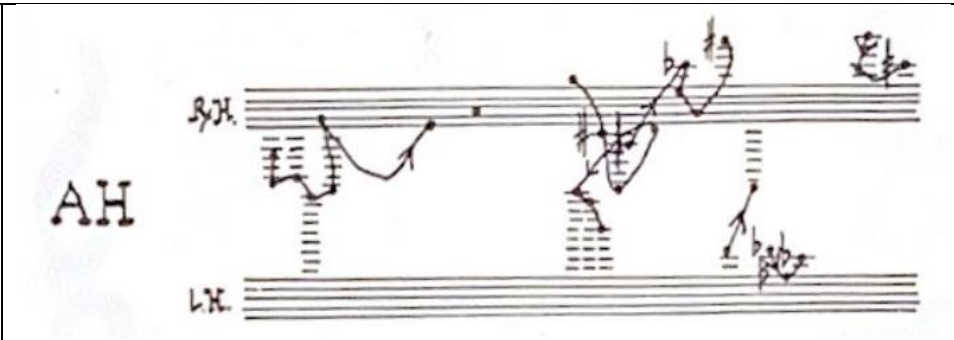
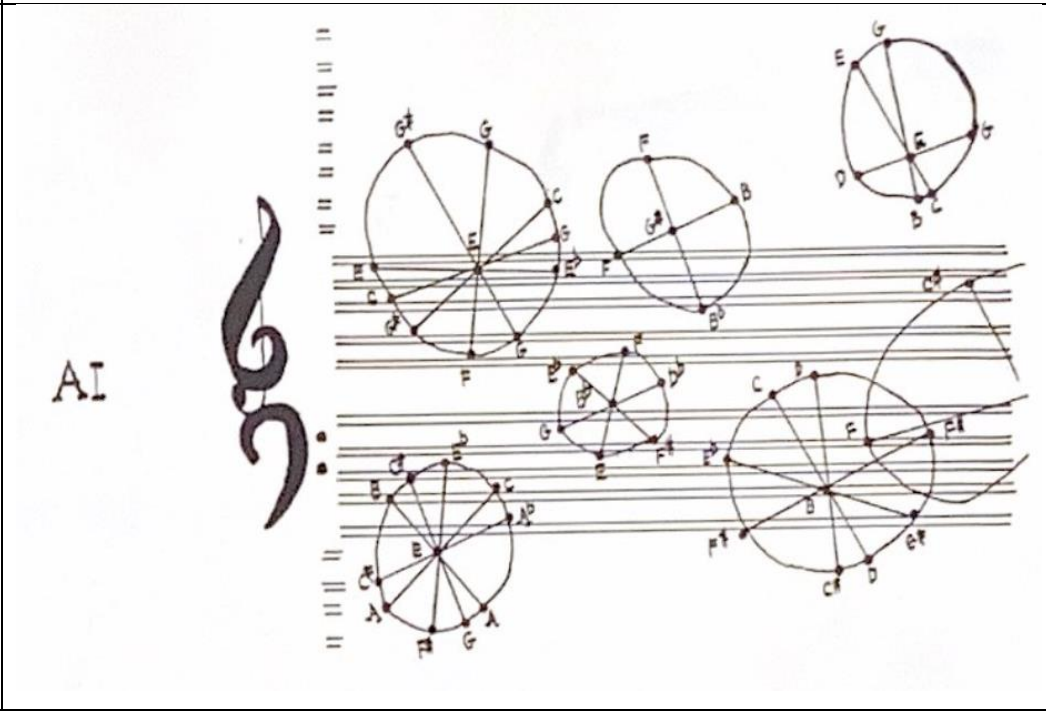
Notation T	
Notation U	

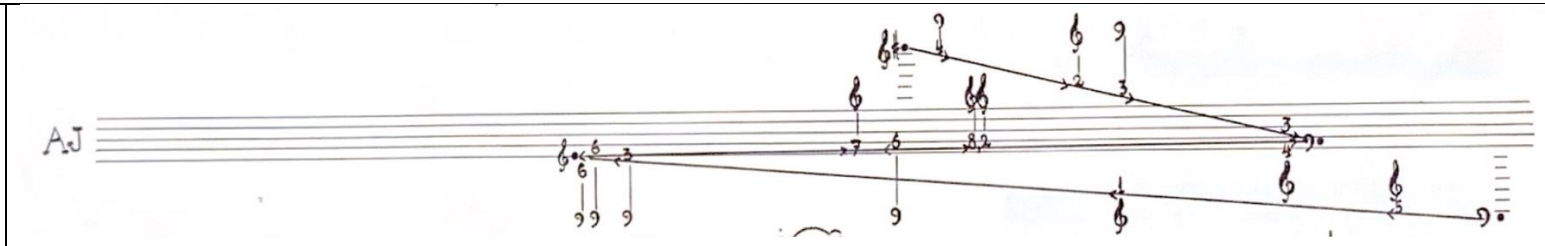
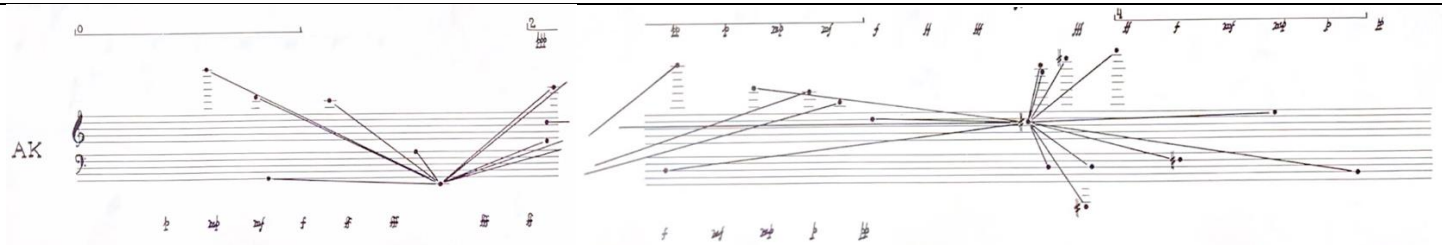
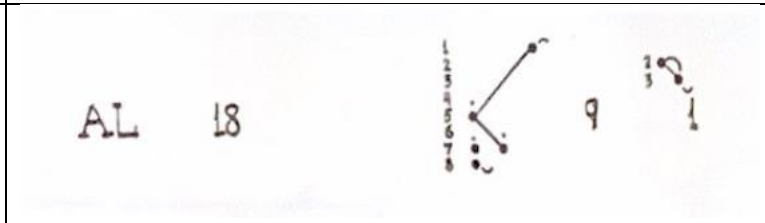
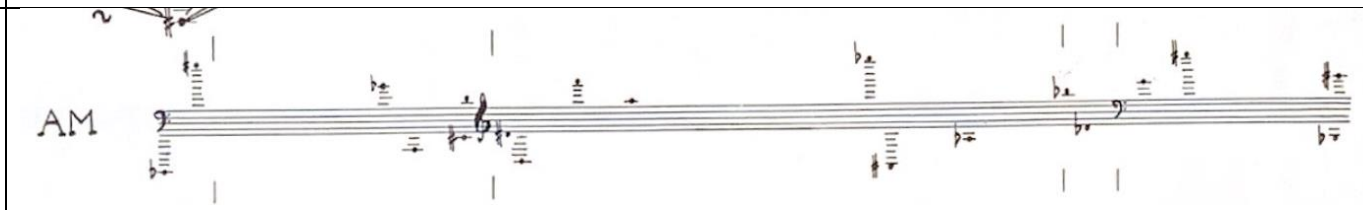
Notation V	
Notation W	

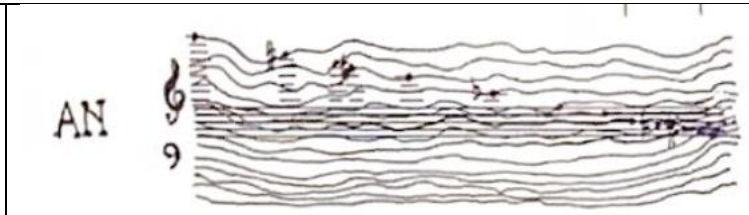
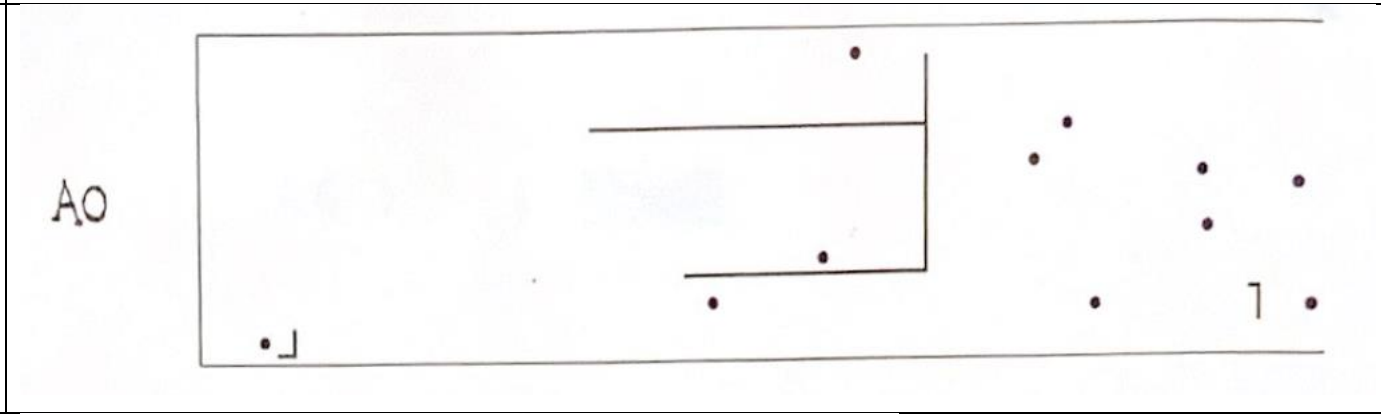
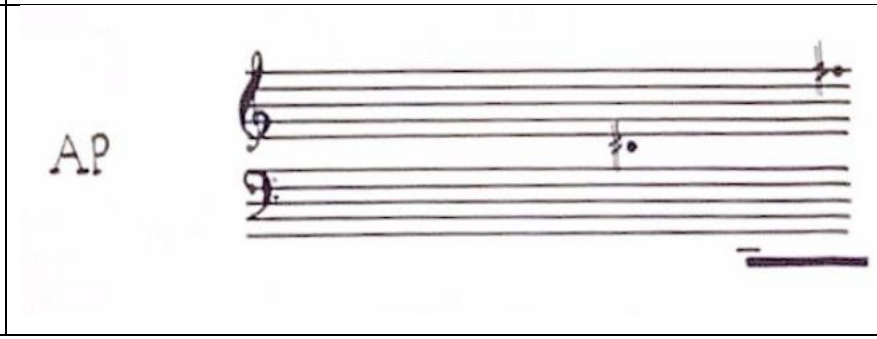
Notation X	
Notation Y	
Notation Z	

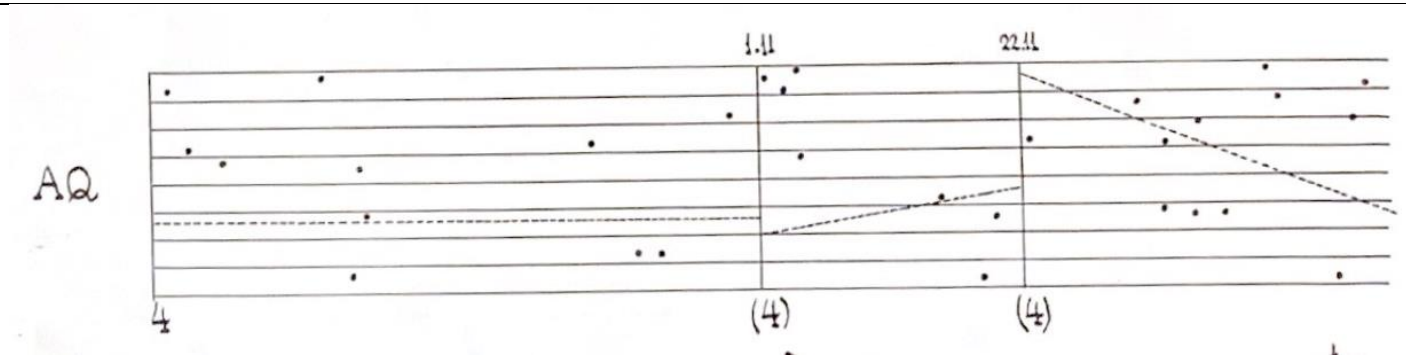
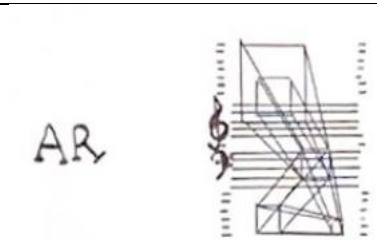
Notation AA	
Notation AB	
Notation AC	
Notation AD	

Notation AE	
Notation AF	
Notation AG	

Notation AH	
Notation AI	

Notation AJ	
Notation AK	
Notation AL	
Notation AM	

Notation AN	
Notation AO	
Notation AP	

Notation AQ	
Notation AR	
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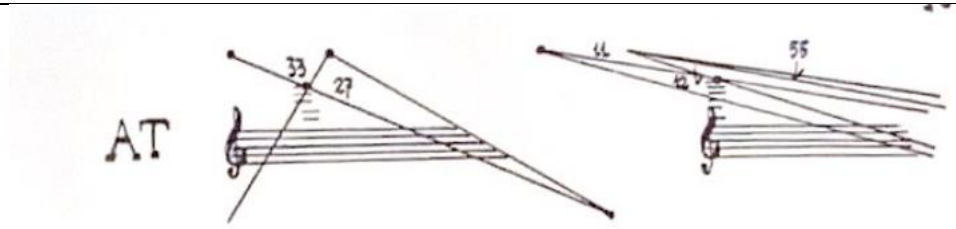
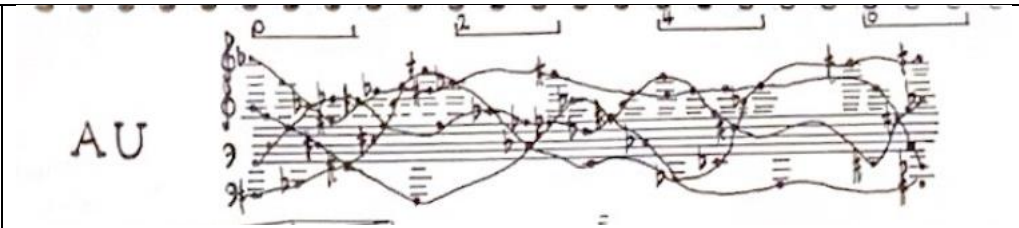
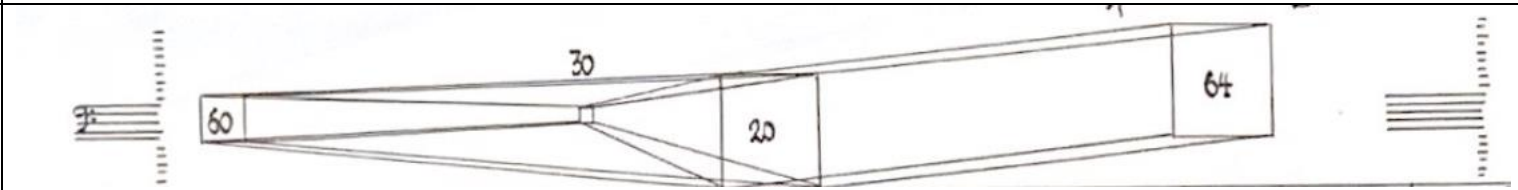
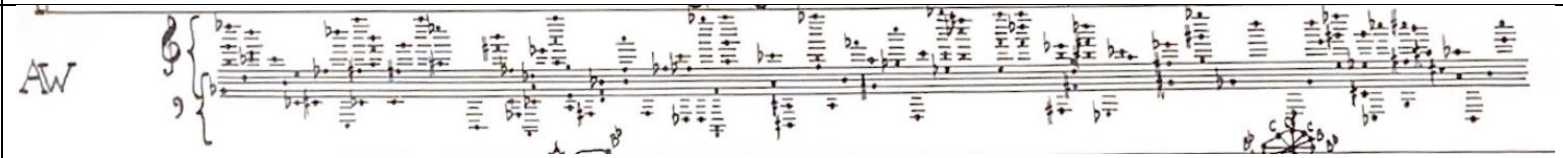
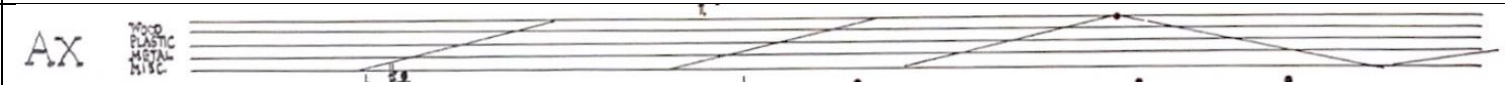
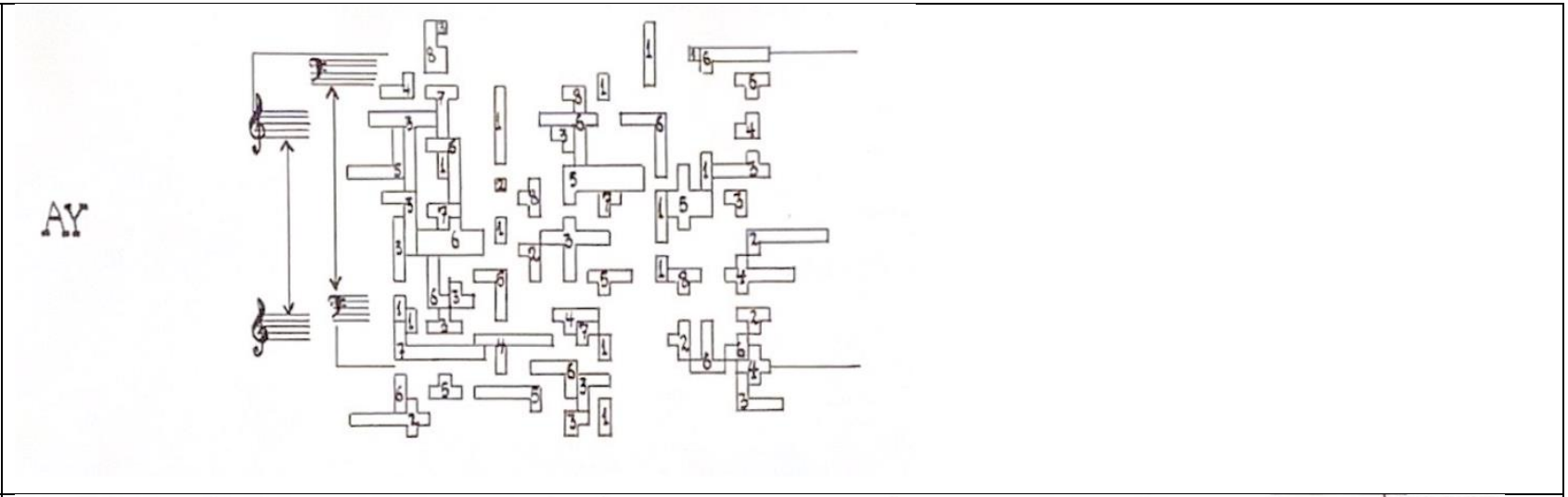
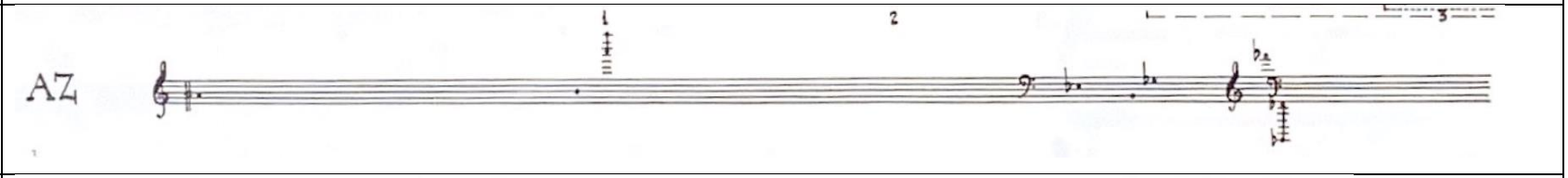
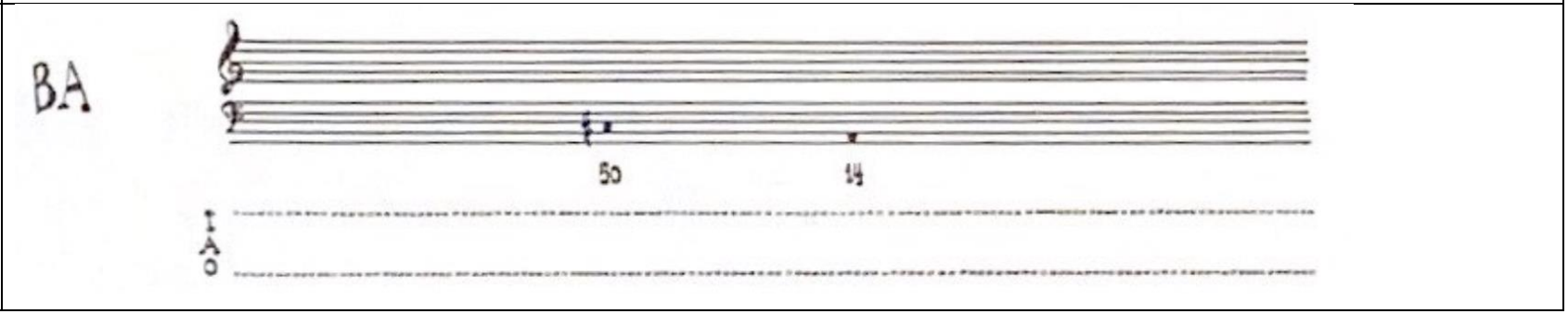
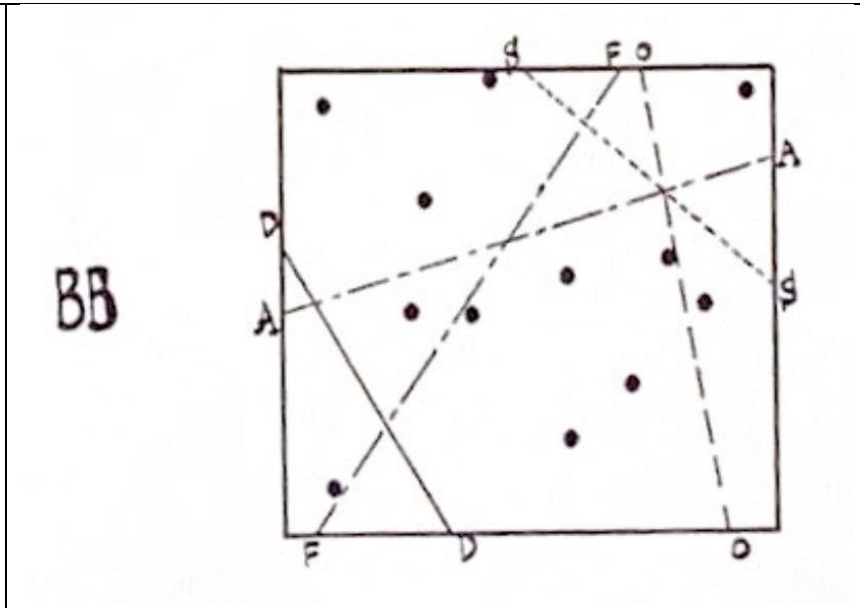
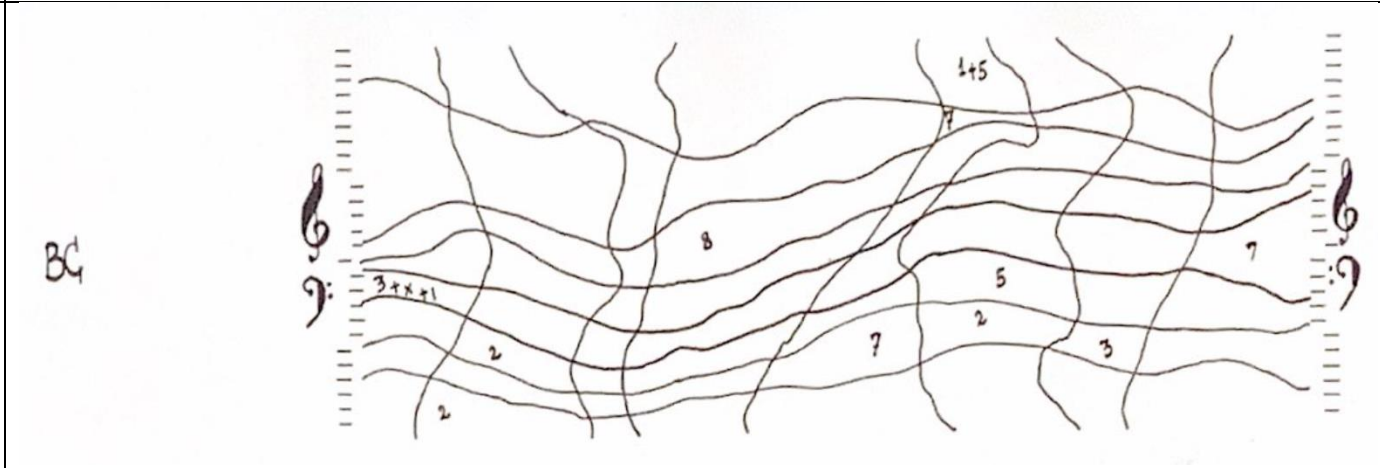
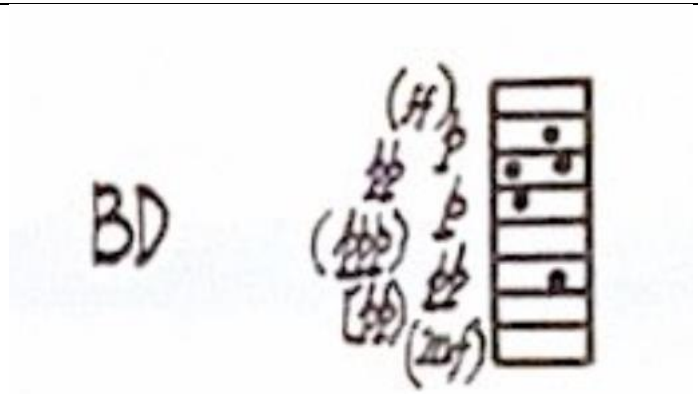
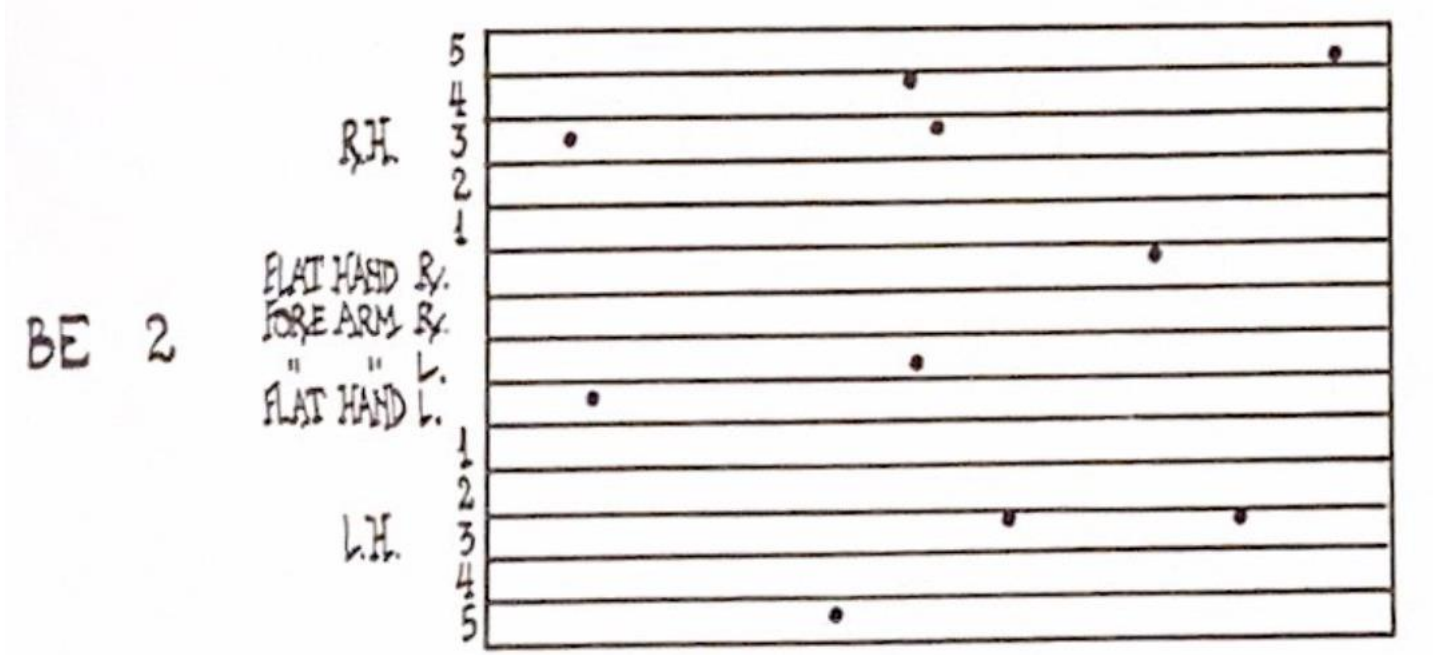
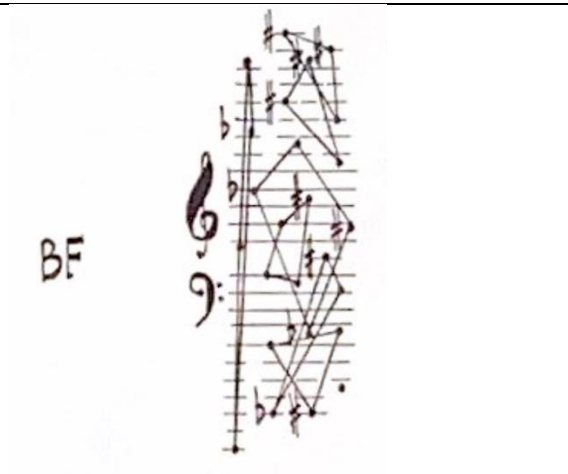
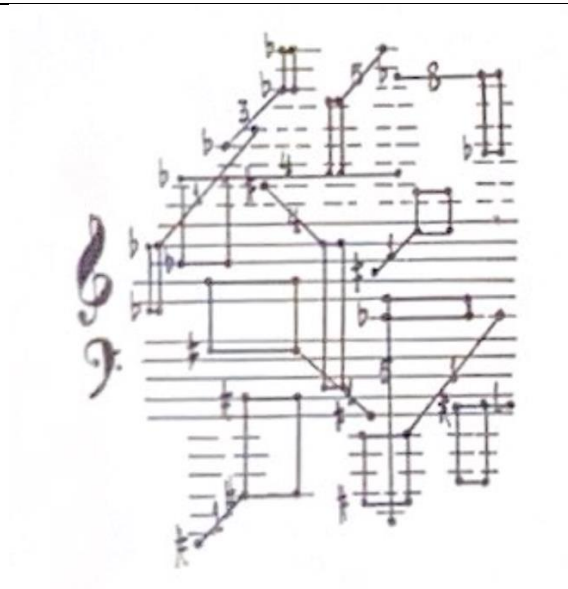
Notation AT	

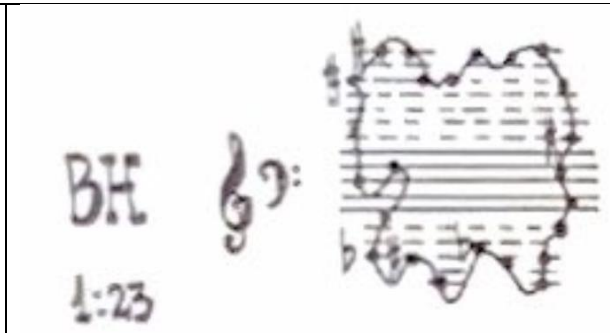
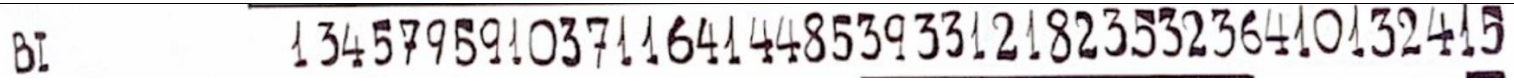
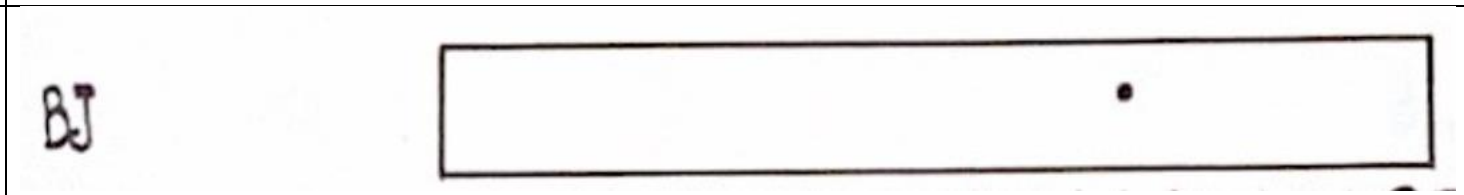
Notation AU	
Notation AV	
Notation AW	
Notation AX	

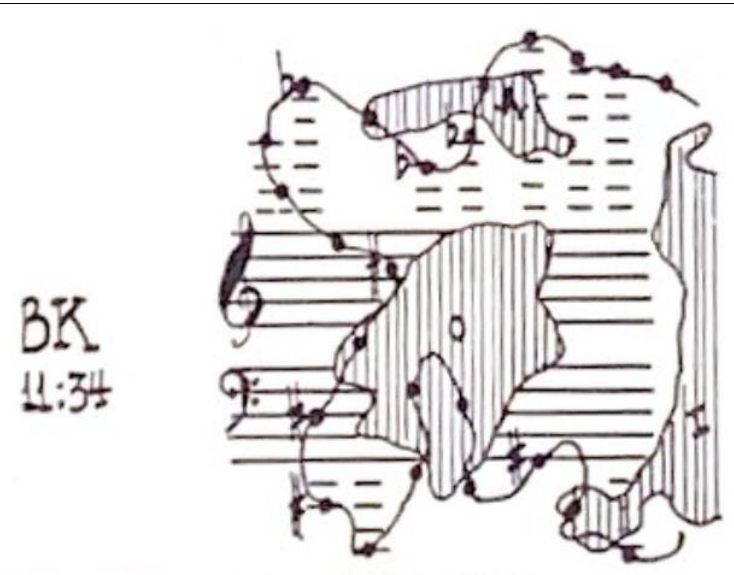
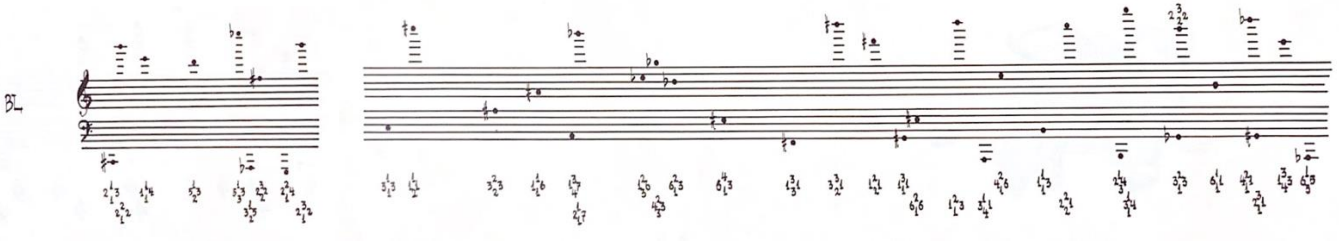
Notation AY	
Notation AZ	
Notation BA	

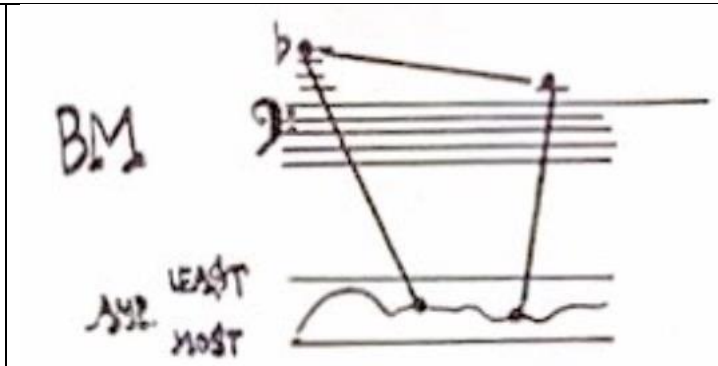
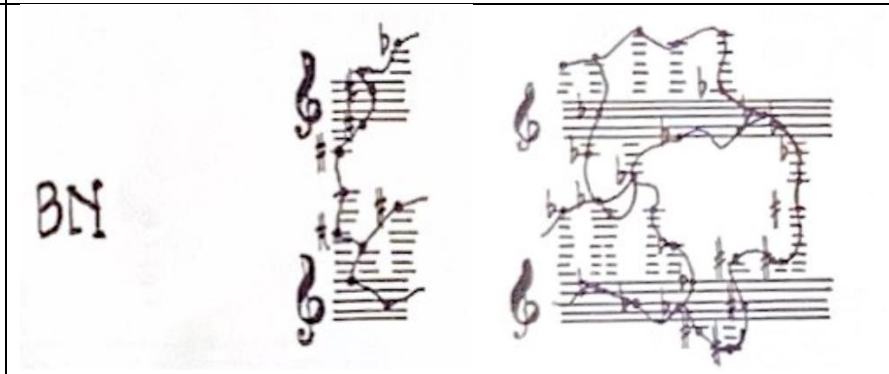
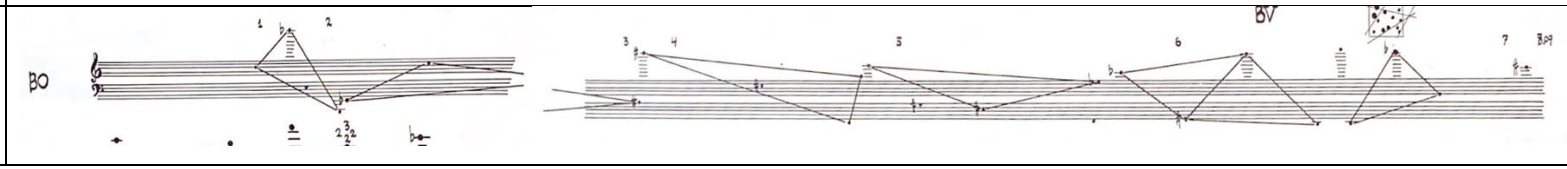
Notation BB	
Notation BC	

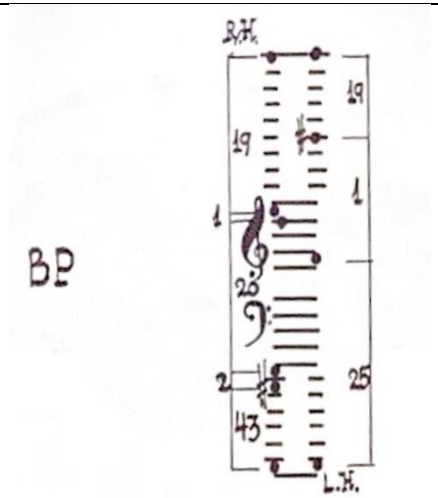
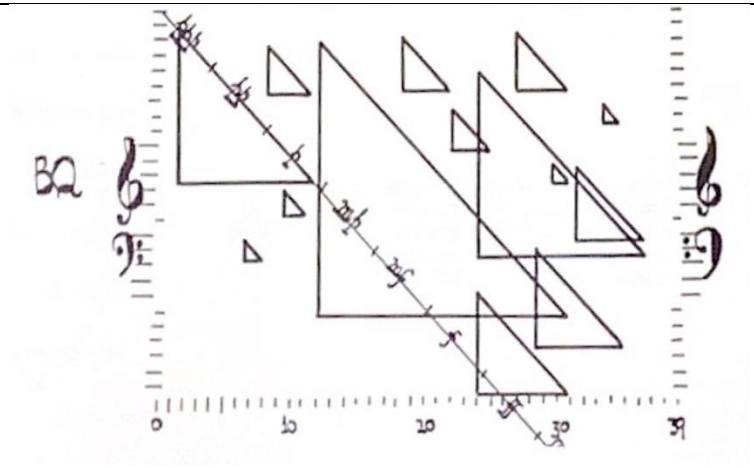
Notation BD	
Notation BE	

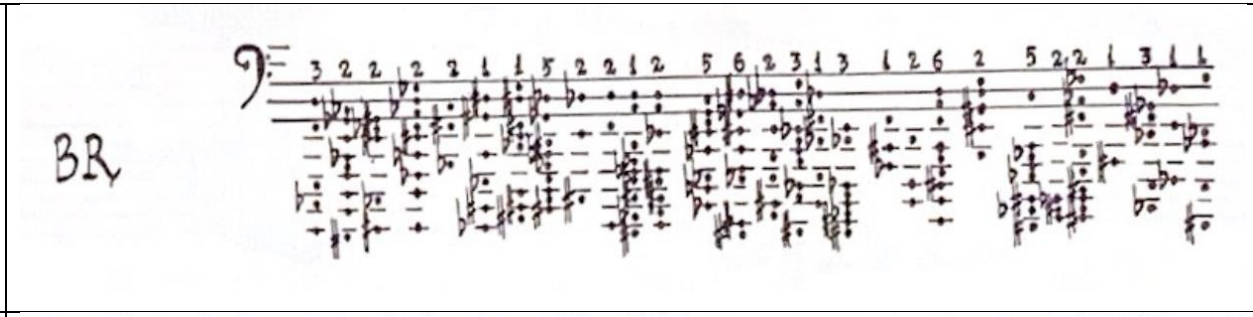
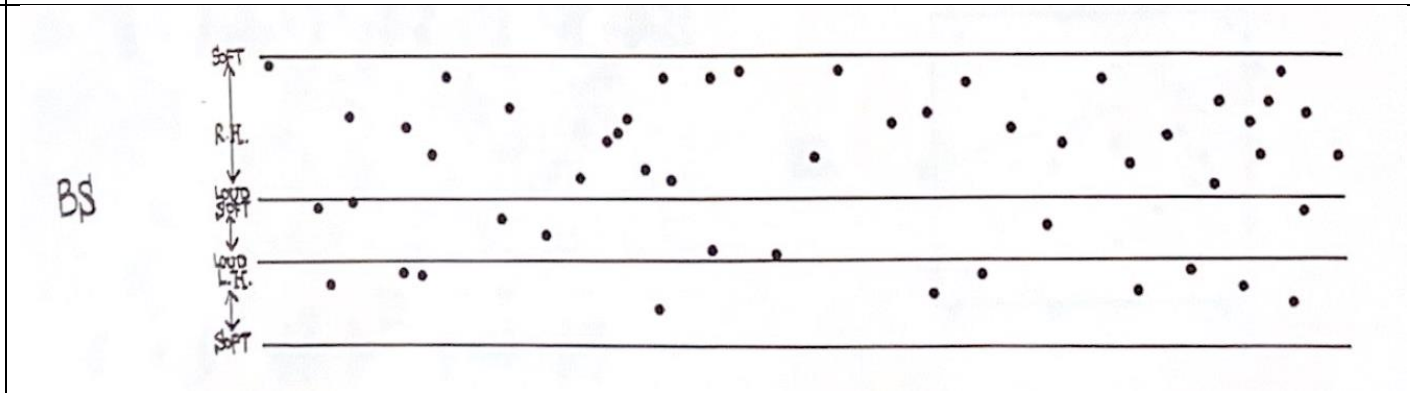
Notation BF	
Notation BG	

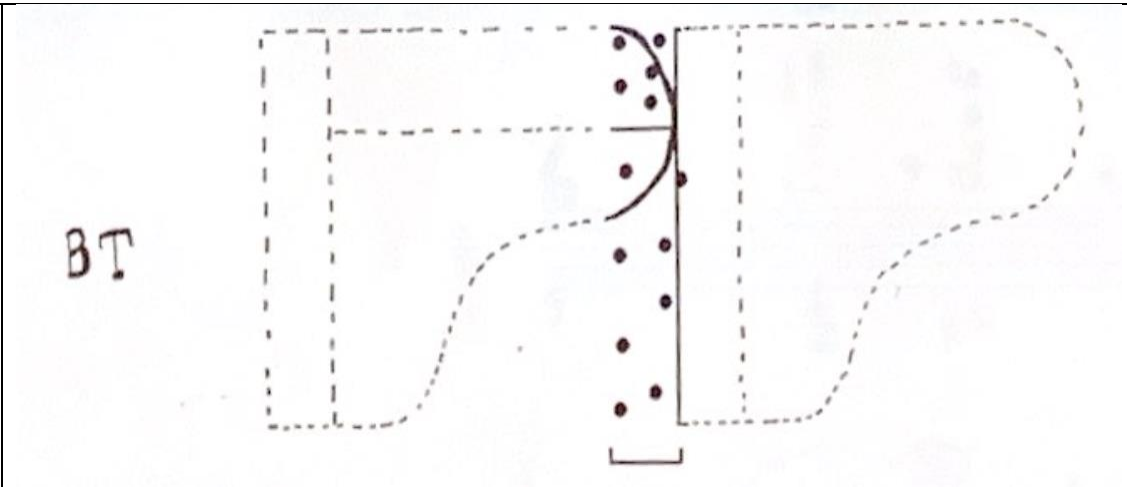
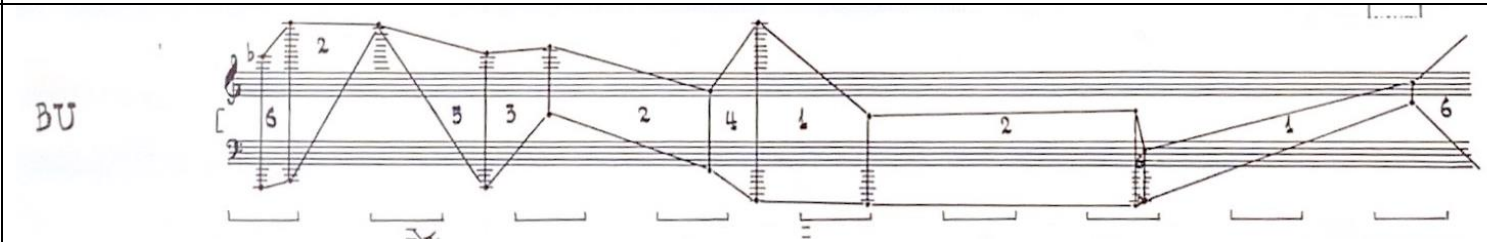
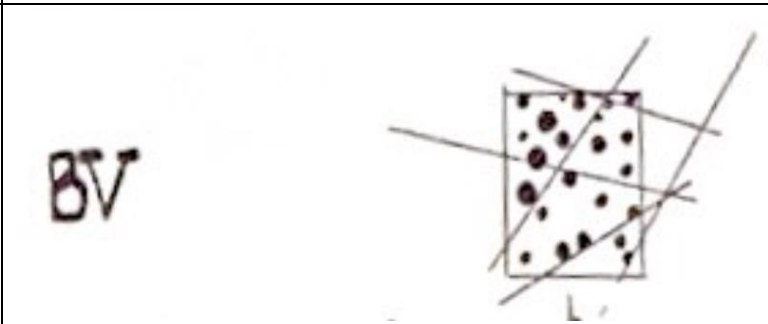
Notation BH	
Notation BI	
Notation BJ	

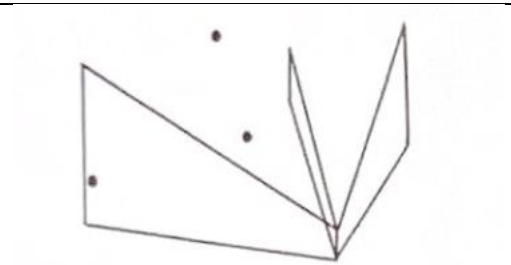
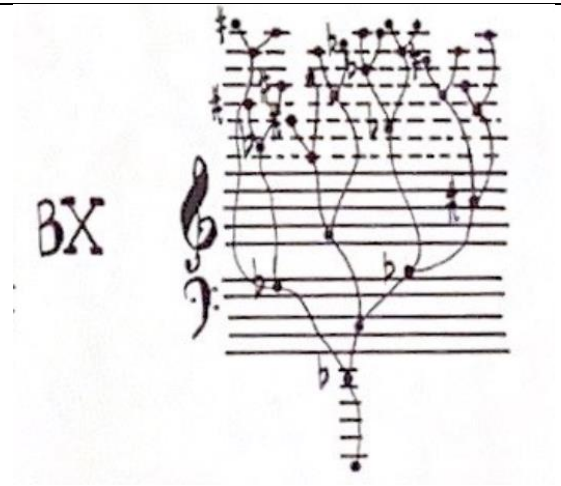
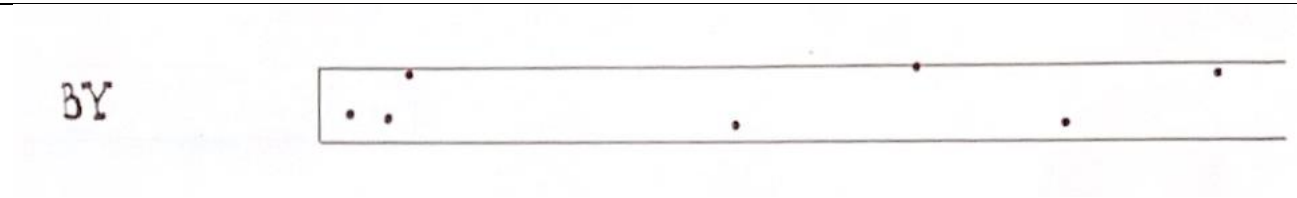
Notation BK	
Notation BL	

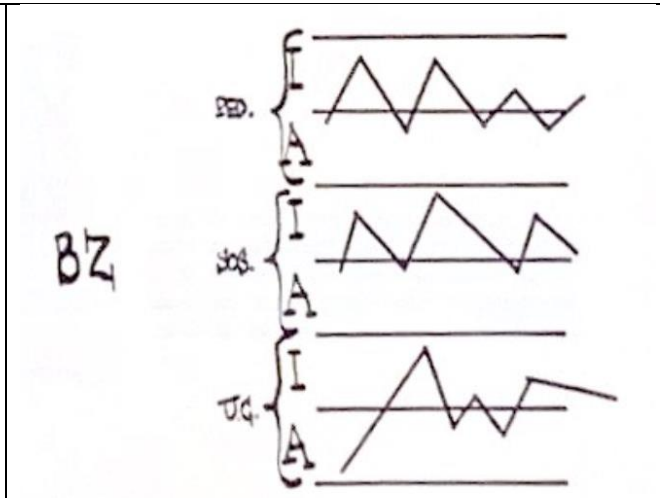
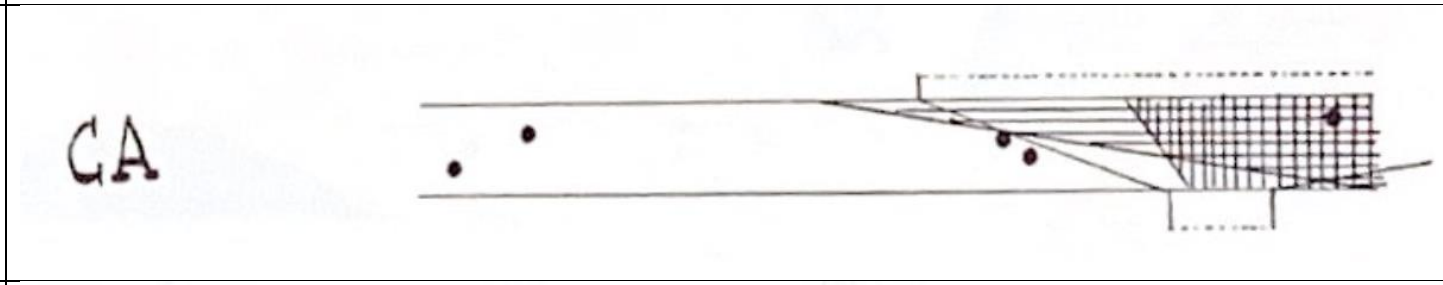
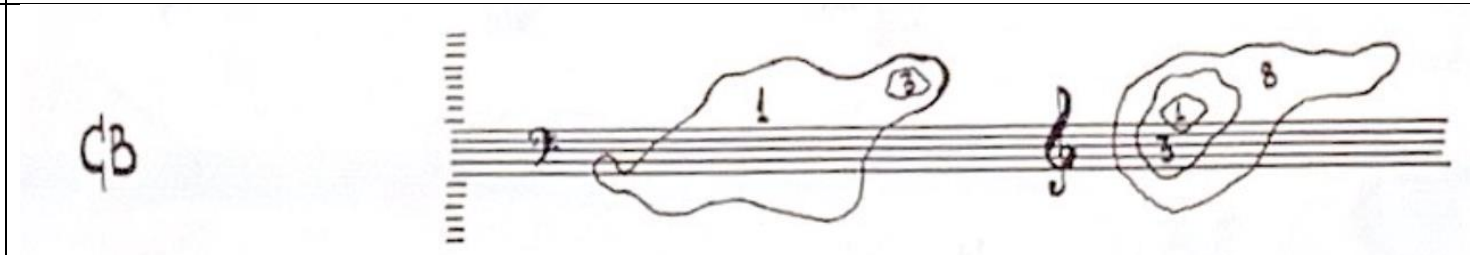
Notation BM	
Notation BN	
Notation BO	

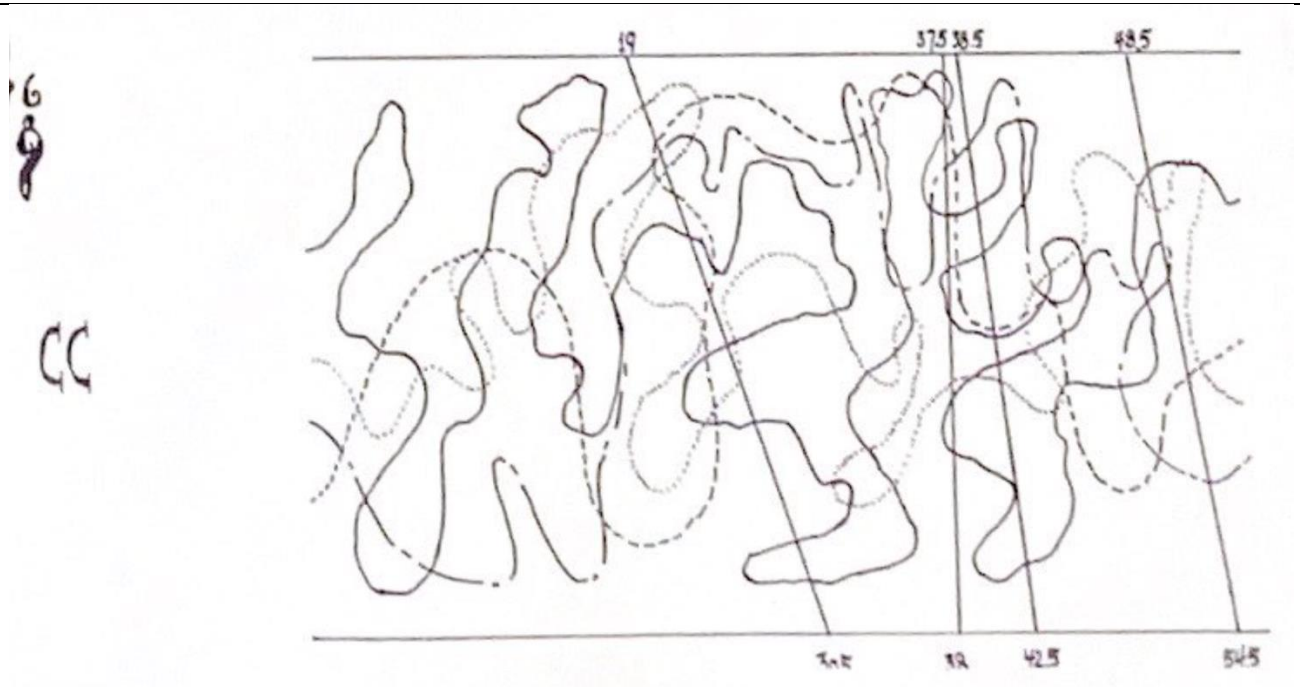
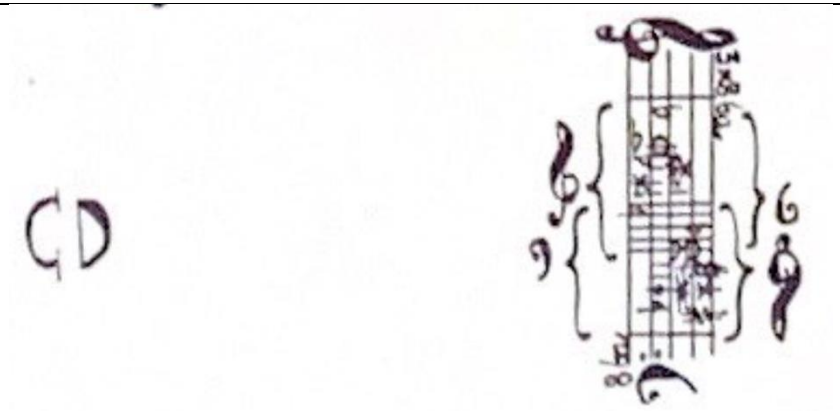
Notation BP	
Notation BQ	

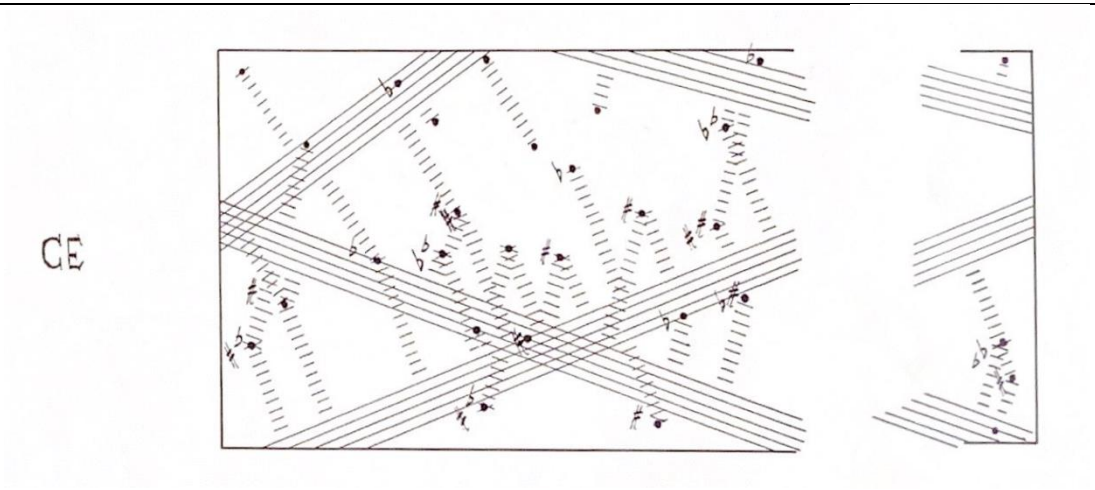
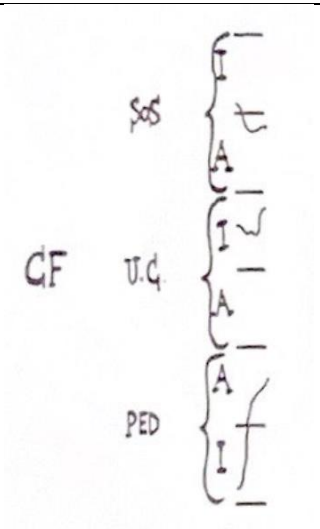
Notation BR	
Notation BS	

Notation BT	
Notation BU	
Notation BV	

Notation BW	
Notation BX	
Notation BY	

Notation BZ	
Notation CA	
Notation CB	

Notation CC	
Notation CD	

Notation CE	
Notation CF	

Appendix C: Other Visualisations

As I explained in Chapter 6 Section 2 Application of *Solo for Piano*, the clusters in discussions are one of many possibilities, because the repetitive uses of compositional materials, functions, types of freedom, and performing approaches influence the position of nodes; with this influence, the individual systems take an important role to firm the relations between nodes. The Appendix C provides additional three versions of visualisations[393] to discuss clusters and individual systems. As Versions 2–4 visualisations (Figure 254; Figure 255; Figure 256) show, the numbers of clusters are different – Version 1 includes ten clusters, Version 2 includes eleven clusters, Version 3 includes nine cluster, and Version 4 includes nine clusters – while the looks of four visualisations are also different from one another. Despite the differences, the individual systems firm the correlations between notations, meaning that the inclusions of notations are influenced by individual systems. For instance, System A gathers Notations A, E, J, L, AN, AI, AJ, AH, BN, BH, and BK in Cluster 9 of Version 1; it also gathers Notations A, E, J, L, Q, AN, AI, AJ, AH, AU, BN, BH, and BK in Cluster 2 of Version 2; also, Notations A, E, J, L, Q, AE, AN, AJ, AH, AU, BC, BN, BH, BP, and BK in Cluster 3 of Version 3; and Notations A, E, J, L, AN, AI, AJ, AH, BH, and BN in Cluster 3 of Version 4.

Comparing the listed notations of four versions, I noticed that Version 2 includes Notations Q and AU in the cluster, Version 3 includes Notations Q, AE, BC, and BP in the cluster, and Version 4 does not include Notation BK. Delving into these differences, I found the differences are because the applied materials. Version 2 includes the two notations belong to System Q, as System A includes three nodes: (i) free to decide performing directions, (ii) showing performing directions, and (iii) directions in its system, and these nodes are also included in System Q (Figure 257). Version 3 includes Systems Q and AE, and Notations BC and BP. Along with the inclusion of the three nodes mentioned above, the three decisive elements of this visualisation results are: (i) given proportion of notes, (ii) performing given proportion of notes and (iii) showing numbers of playing notes. The three elements are the communal nodes between Systems A and AE and Notations BC and BP (Figure 258). Version 4 does not include Notation BK. This is because Notation BK acts as conjunction between Systems A and AC. It shares elements with both systems, such as perimeter, direction, given proportions of notes, numbers, showing numbers of playing notes, and showing performing directions, etc. from Notation A; piano construction, abbreviations, and performing with assigned piano

[393] For Version 1 visualisation refers to Figure 200 in Chapter 6.

construction, etc from Notation AC. Table 132 lists all clusters from four versions visualisations and the corresponding systems.

From the four clusters of four versions visualisations, I found that despite the positions of nodes are effected by repetitive uses of elements, the individual systems locate the nodes' positions in a much methodical way, meaning that the structural correlations can be recognised based on individual systems. Consequently, I discuss the visualisation in an order 'clusters, individual systems, and inner structure' in Chapter 6 Section 2 Application of the *Solo for Piano*.

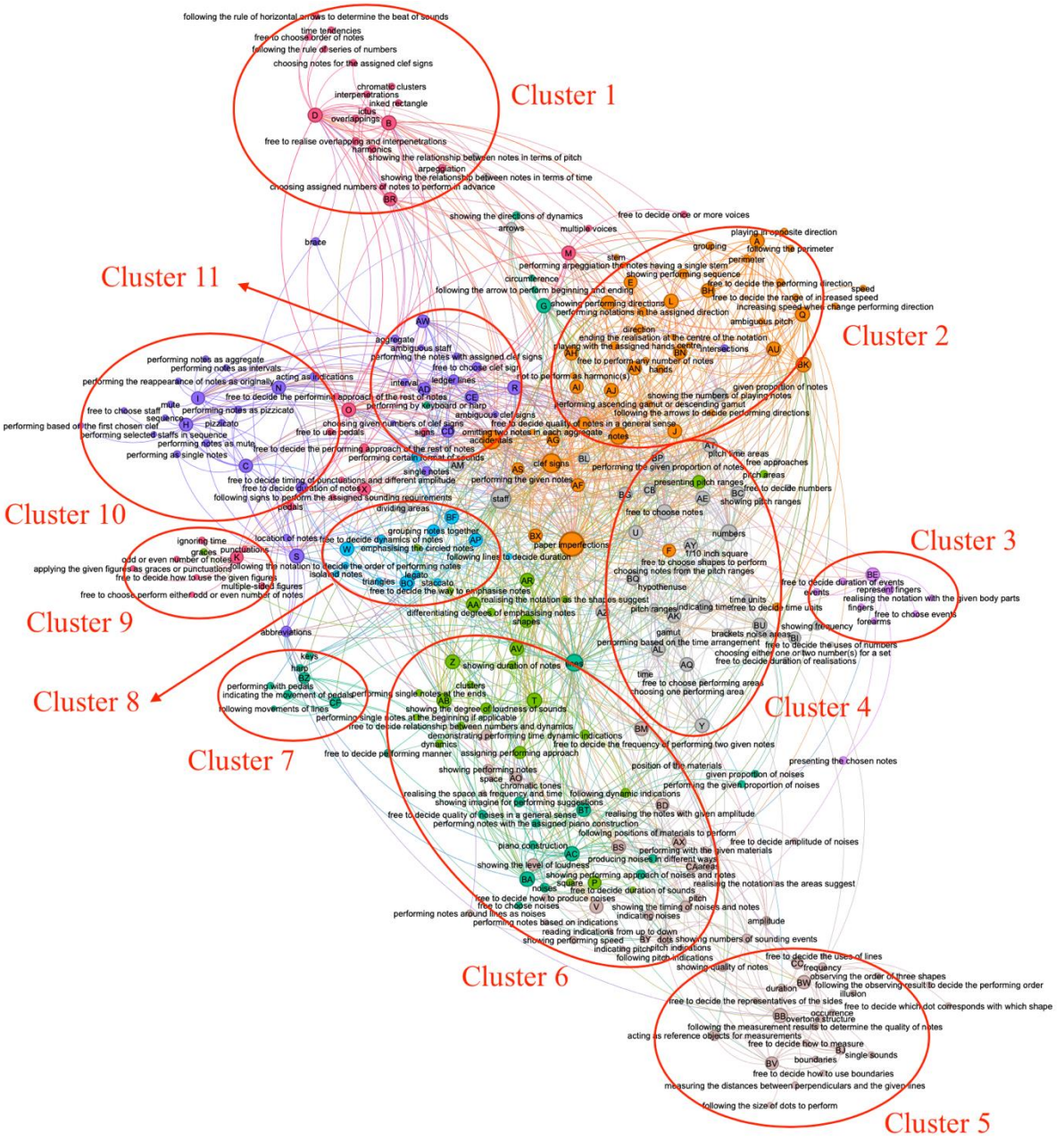


Figure 254 Version 2 visualisation shows eleven clusters.

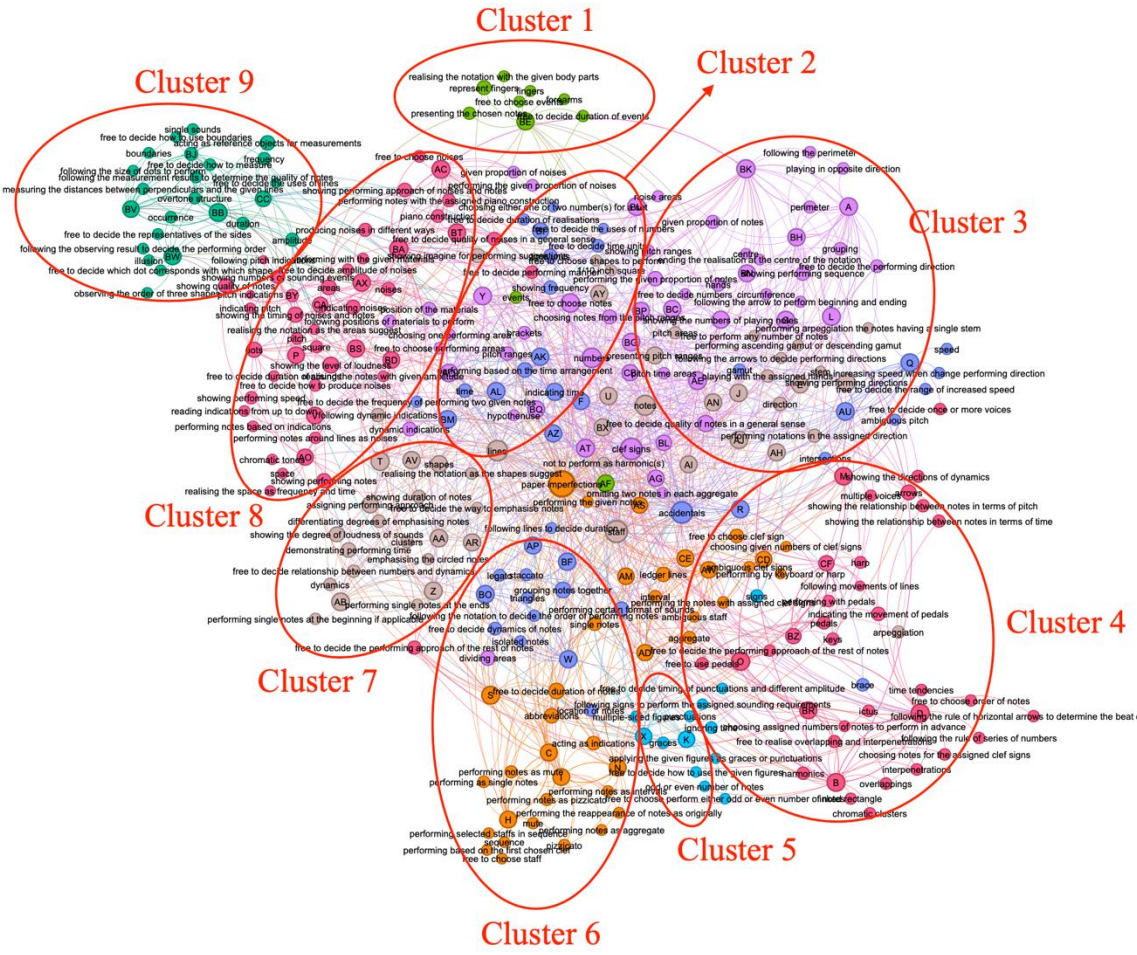


Figure 255 Version 3 visualisation shows nine cluster.

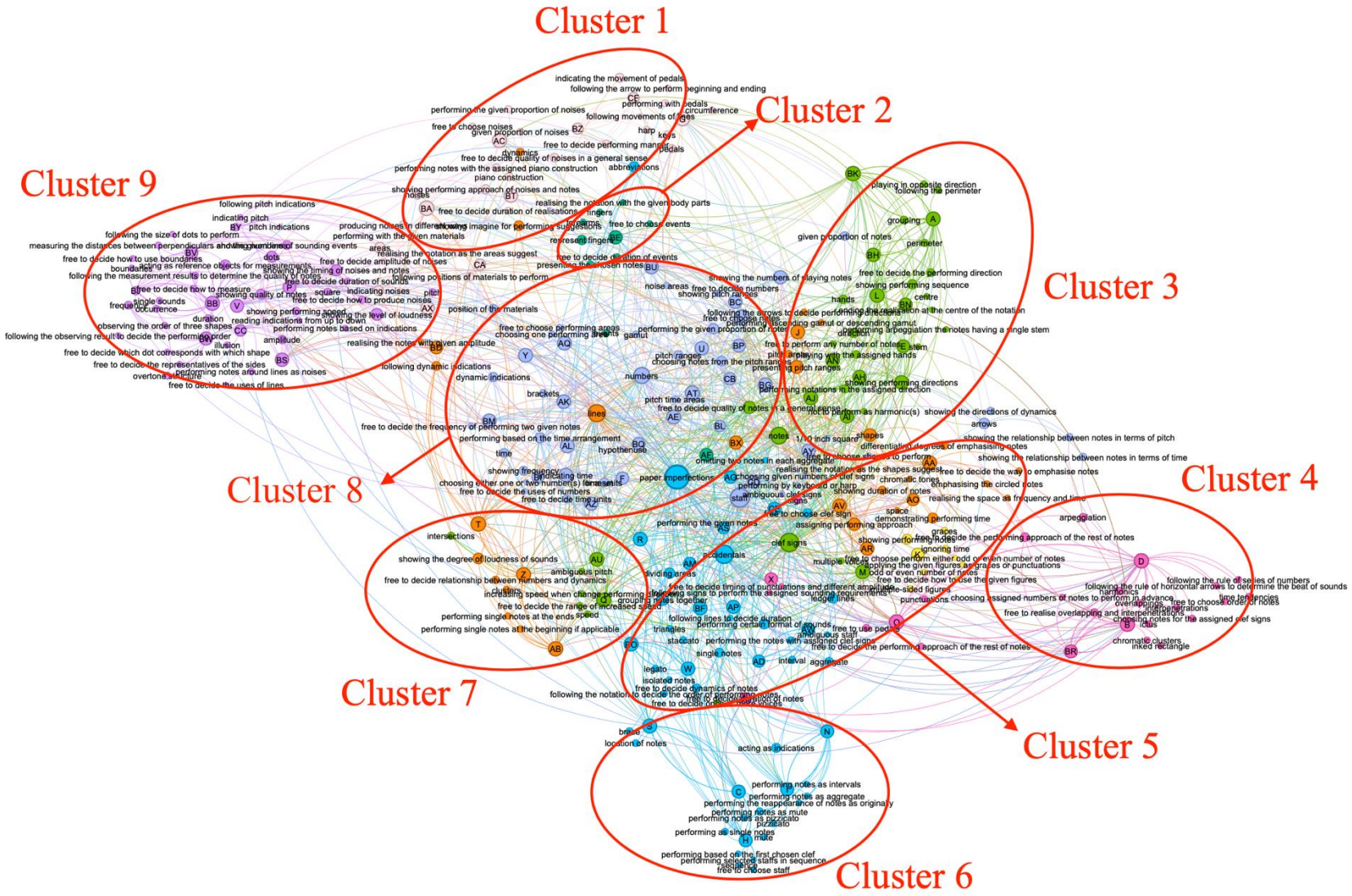


Figure 256 Version 4 visualisation shows nine clusters.

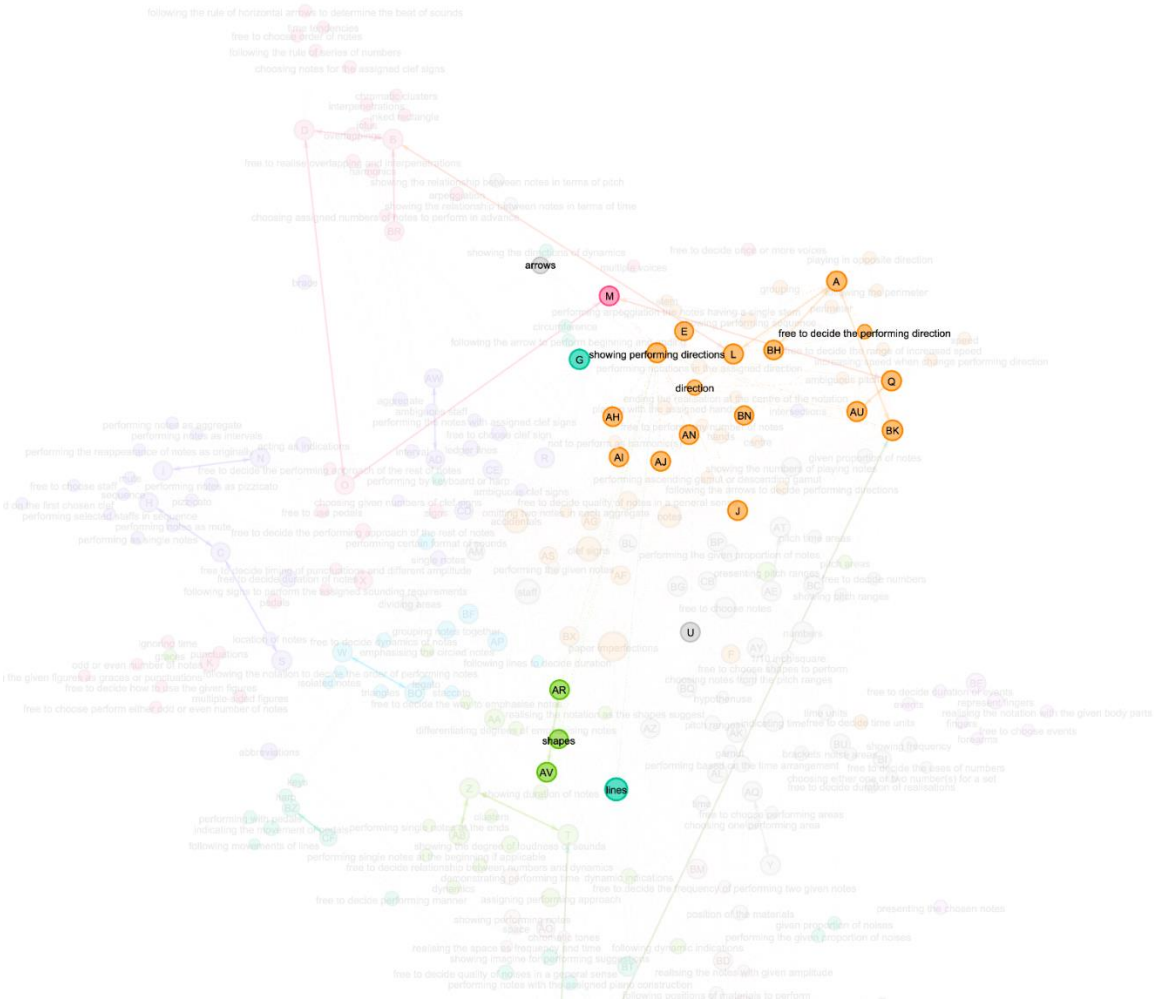


Figure 257 Three elements are applied in Systems A and Q.

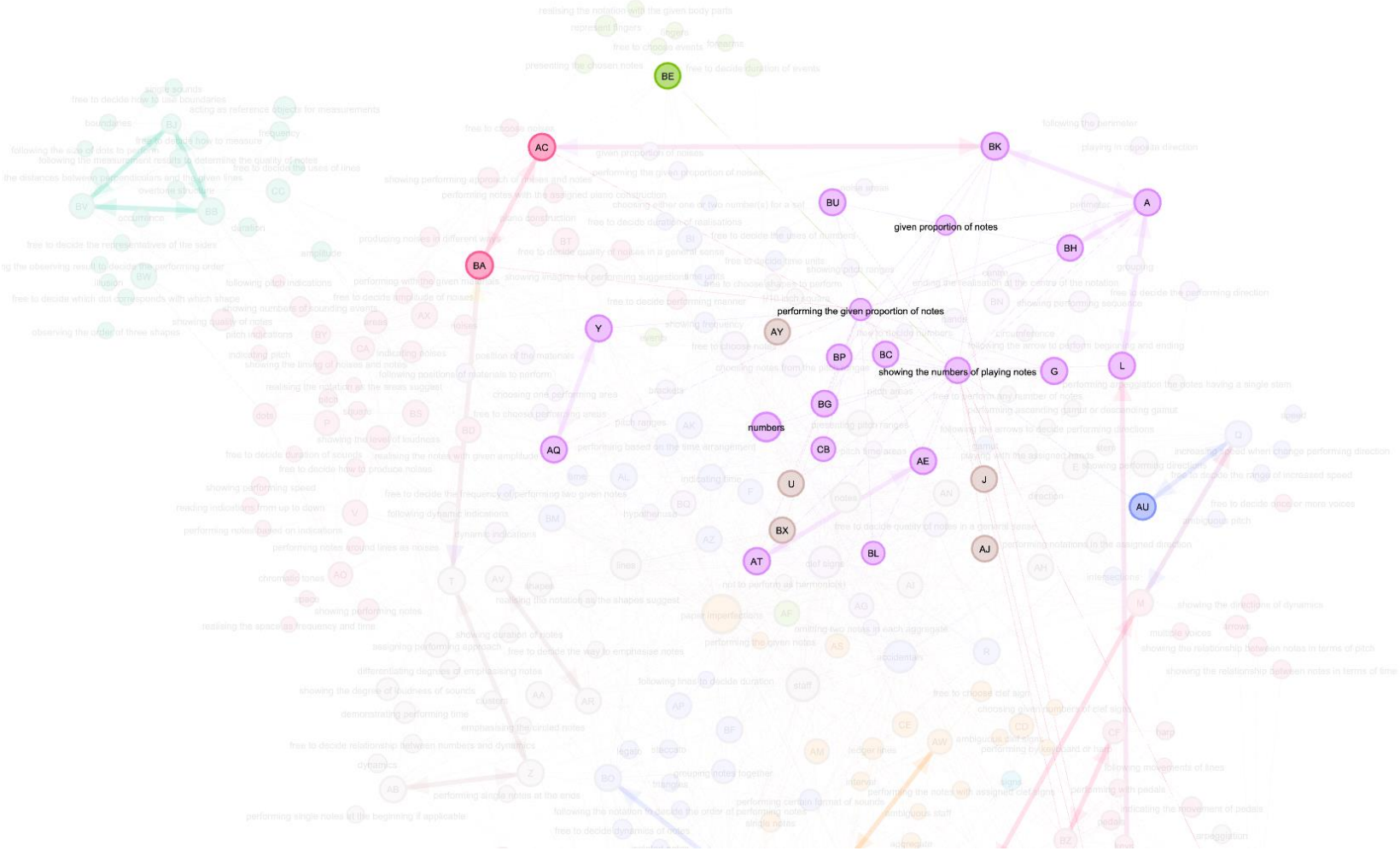


Figure 258 Three elements relate Systems A and AE and Notations BC and BP.

Version	Cluster	Notations	Individual systems
1 (Figure 200)	Cluster 1	Notations B, D, and BR	Systems B and D
	Cluster 2	Notations H, I, and N	Systems C and I
	Cluster 3	Notations G, BZ, and CF	System BZ
	Cluster 4	Notations W, BO, AO, AP, K, and BF	System W
	Cluster 5	Notations F, AL, U, BC, BG, BP, BK, CB, AE, AT, BQ, BU, Y, AZ, and BI	Systems Y and AE
	Cluster 6	Notations P, V, AX, BB, BD, BS, BV, BJ, BY, CA, and CC	System BB
	Cluster 7	Notation BE	n/a
	Cluster 8	Notations, S, T, Z, AC, BA, BT, AB, AR, AA, and AV	Systems T, Z, AC, and AR
	Cluster 9	Notations A, E, J, L, AN, AI, AJ, AH, BN, and BK	System A
	Cluster 10	Notations M, O, Q, R, X, AD, AU, and AW	Systems M, Q, and AD
2 (Figure 254)	Cluster 1	Notations B, D, and BR	Systems B and D
	Cluster 2	Notations A, E, J, L, Q, AN, AI, AJ, AH, AU, BN, BH, and BK	Systems A, M, and Q
	Cluster 3	Notation BE	n/a
	Cluster 4	Notations F, U, Y, AT, AE, AL, AK, AQ, AY, AZ, BC, BG, BP, BQ, BU, and CB	Systems Y and AE
	Cluster 5	Notations BB, BV, BJ, BW, and CC	System BB
	Cluster 6	Notations T, P, V, Z, AB, AC, AD, AV, AX, BA, BD, BT, BM, BS, BY, and C	Systems T, Z, AC, and AR
	Cluster 7	Notations BZ, and CF	System BZ
	Cluster 8	Notations W, AP, BO, and BF	System W
	Cluster 9	Notation K	n/a
	Cluster 10	Notations C, H, I, and N	Systems C and I
	Cluster 11	Notations R, AD, AW, CD, and CE	System AD
3 (Figure 255)	Cluster 1	Notation BE	n/a
	Cluster 2	Notations Y, AL, AK, AQ, AY, BI, BM, BQ, and BU	System Y
	Cluster 3	Notations A, E, J, L, Q, AE, AN, AJ, AH, AU, BC, BN, BH, BP, and BK	Systems A, Q, and AE
	Cluster 4	Notations B, D, M, O, AW, BR, BZ, CD, and CF	Systems B, D, M, AD, and BZ

	Cluster 5	Notation K	n/a
	Cluster 6	Notations C, I, H, N, S, W, AP, BF, and BO	Systems C, I, and W
	Cluster 7	Notations T, Z, AA, AB, AR, and AV	Systems T, Z, and AR
	Cluster 8	Notations P, V, AC, AO, AX, BA, BD, BS, BT, BY, and CA	System AC
	Cluster 9	Notations BB, BV, BJ, BW, and CC	System BB
4 (Figure 256)	Cluster 1	Notations G, AC, BA, BT, BZ, and CF	Systems AC and BZ
	Cluster 2	Notation BE	n/a
	Cluster 3	Notations A, E, L, J, AN, AH, AI, AJ, BH, and BN	System A
	Cluster 4	Notations B, D, and BR	Systems B and D
	Cluster 5	Notations K, M, O, X, W, AA, AO, AV, AR, AW, AD, AP, BF, BO, and CE	Systems M, W, AD, and AR
	Cluster 6	Notations C, I , H, N, and S	Systems C and I
	Cluster 7	Notations T, Q, Z, AB, and AU	Systems T, Q, and Z
	Cluster 8	Notations F, U, Y, AE, AF, AT, AI, AL, AK, AQ, BC, BG, BL, BI, BM, BP, BU, BQ, BX, and CB	Systems Y and AE
	Cluster 9	Notations P, V, AX, BB, BS, BJ, BV, BW, BY, and CC	System BB

Table 132 The notations of each cluster of four versions of visualising results.

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