Shares Valuation Throughout the IPO Procedure: A Perspective from Game Theory

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Abstract

Ever since the appearance of economic activities, the idea of game theory has been closely connected to the development of economic framework and applications. For years, the game theory, as an economic topic, has been rarely applied in the research regarding the financial market. As one of the most intriguing topics in the financial market, asset valuation has consistently developed with theoretical knowledge. The initial public offering (IPO) which has been regarded as the most lucrative equity investment, the share valuation is the pivotal issue of the financial service industry. As observed in my industrial interviews, pricing the shares for the IPO is not the skill of the valuation, but the art of the valuation.

The first essay is mainly focus on the IPO contract designing issue of the company which initiates the IPO. Different from the conventional one-by-one contract design, the first essay proposes the cross application of the auction theory and bargaining theory to facilitate the channel coordination. Compared with the conventional bargaining theory which commonly uses the ‘price’ as the strategic variable, here uses the ‘quantity’ as the strategic variable. This strategic variable setting will make the bargaining more suitable to the practical situation in the IPO. In the model, there will be two different types of contracts, the first is the customised type and the standardised type. The customised contract enables the seller and the underwriter to reach their own contract while the standardised contract refers to identical contract for every underwriter. The research finds that the different strategic variable setting would shape the initial demand function in great degree, while the seller and the underwriters will have different preferences toward the types of the contract based on their relative bargaining power. As for the auction setting, if all the underwriters are risk-neutral, then the equilibrium result would be the same regardless of the auction types. But for the future planning of the seller, the seller tends to adapt the second price auction which could better help the seller to get the insight regarding the true valuations.

The second essay is concentrating on the area of the facilitation of channels. For different negotiation mechanisms, the bargaining parties usually consider themselves. Under the assumption that each participant is rational, the bargaining results tend to be beneficial for individual participant. This would result in the low efficiency throughout the whole channel. In articles that discuss the possible solution of improve the channel efficiency, some possible methods have been proposed including side payment, buyback contract etc. However, the fundamental idea of the channel efficiency improvement remains untouched. In the essay, it

has been found that once the model has been set, the total profit that can be earned by all participants is set. What side payment act throughout the channel coordination is first create a bigger pie and split the pie based on the relative negotiation power of participants. In another word, the side payment helps both participants to be better off compared to the initial independent bargaining situation, but the individual participant cannot fully enjoy the incremental profit, and the partial profit sacrificed under the side payment condition is dependent upon relative bargaining power.

The third essay is concerning the bargaining situation regarding the seller and the underwriter when the market demand is a stochastic information and remains private to the underwriter. The bargaining mechanisms concern price matching and simultaneous negotiation. The model comprises belief construction procedure regarding the underwriter and offer proposition process under an alternating negotiation mechanism. The model illustrates the appropriate disclosure behaviour of the underwriter and preferences of the seller regarding the negotiation mechanism has certain deviations compared with the circumstance of the certain market demand base. The existence of the outside option could help the seller to avoid the exploitation from the underwriter who possess the private information. This essay provides managerial insights to both the seller and underwriters regarding the appropriate behaviours throughout the bargaining procedure that comprises stochastic elements.

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Chapter 1. Introduction

Ever since the emergence of business activities, interactions consistently occurred between participants. The most encountered interactions between business participants are bargaining over the price. The very first bargaining between the seller and the buyer is time immemorial, but the research regarding the bargaining has evolved with the development of modern business activities. In this research, the intention is cross applying the game theory from the conventional bargaining situation of the supply chain to the field that previous bargaining research rarely concerned, the initial public offering (IPO) process in the financial market.

Initial public offering (IPO) refers to the procedure that a company transfers from a private owned company to a public company. One of the most significant features of the public company is that the shares can be openly traded through the stock market. Researchers believe that IPOs have an importance that outweighs other dollar amount of money raised including bond offering, bank loans and private equity investments etc (Huang, et al., 2023). The peak number of IPO emerged before the 2000 and this figure has been lower in the preceding two decades. The US stock market is widely regarded as the standard to judge the global economy and becomes one of the ideal markets for companies to initiate the IPO. As per data from the Centre for Research in Security Prices (CRSP), the number of public companies that domestic operating within US dropped from almost 8,000 in 1997 to below half of that number from 2012 to 2020. The reasons that attributable to this phenomenon is concluded as both the paucity of IPOs and increasing level of merger activities (Doidge, et al., 2017). On the other hand, the development of the venture capital provides the possibility for the company to stay private for a longer time due to an increasing availability of liquidity.

It can be saying that the completion of the IPO signifies the initiation of trading shares in the secondary market. However, the research regarding the share price typically uses the stochastic analysis techniques with a lot of corresponding assumptions. It seems that finding the so called ‘fair’ value of the share price has become one of the most popular topics related to share price descriptions.

Ever since the emergence of share in the financial market, the fair value of the share has been consistently a key topic for investors. Since every investor wants to know the ‘true’ value of the share, so he can benefit from the transaction of the share determined by the state of undervalue or overvalue. Hence, the valuation of the share has become a topic that draws the attention of the researchers.

Till now, there are several theories regarding the valuation of the share. The predominant thought of valuing the share is discounting the future value of the company back to the present. One commonly encountered example is calculating the fair value of the shares by using the dividend. If it is assumed the company's value is a constant number and all the value of the company is distributed to the shareholders through dividends, then discount all the dividend payment to the present time to determine the price of the shares. If, on the other hand, the future of the company is changing, the key is analysing the effect of information exert over the company value. In other word, the share price in certain degree, reflects investors' expectation regarding company's future development based on the interpretation of information regarding the company.

With this idea, the share price can be interpreted as monetary expression of certain company's future information. But the share price is not always fully reflecting the information, based on this circumstance, there is a famous theory to describe this situation, the efficient market hypothesis (EMH). Based on the type of information that share price reflected, the EMH proposes three types of market.

The first type of market is the weak form market. Under this form of market, all the past information will be reflected in the share price. Here the past information refers to information that comprises historical trading volumes and prices that can be found through trading history. Under this condition, the technical analysis which refers to the analysing technique with the idea that occasions in the past will repeat themselves in the future will not provide consistent abnormal return to the investors. To achieve a satisfying result, investors could adapt the fundamental analysis which refers to the method of using the public available source such as company's annual report, analysis from professional institutions, and related financial news to forecast company's future performance.

The second type of market is the semi-strong market. Under this market, the share price reflects all the public information. Under this situation, the fundamental analysis is no longer effective for the investors. Since all the information that is publicly available is quickly reflected, the fundamental analysis could no longer provide any edge for the investors. Under this situation, the investor who possess the private information could benefit from the transaction since the private information remains unknown to the public which gives the advantages to the information owner.

The third type of market is the strong form market. Under this type of market, even the private information will be quickly reflected in the share price. This refers to the condition that even the owner of the private information cannot consistently achieve abnormal benefit. Hence, the strong form market can be called as a perfect financial market as well. But unfortunately, there does not exist such a market.

For the scholars, the composition of share price is not as attractive as the determination of market type under the EMH theory. For the determination of the market type, researchers tend to investigate the effectiveness of technical analysis in the market. Under the weak form market, it is generally assumed that the share price follows the random walk. This indicates that the price of the share at day 1 is independent of the share price at day 0. Under this assumption, the daily payoff would be independent. Hence, the general research method would be finding the historical data and investigating whether or not there exist a relationship between those daily payoffs.

Following this rhythm, most of the research regarding the share price description has been concentrated on the effectiveness of various investment strategies given the share price follows certain pattern.

Among current academic research regarding a successful investment strategy, a majority of them described the term ‘success’ are standardized for every player which generally do not consider the specific circumstance of individual investor, for example, an investor with risk aversion of his Neumann-Morgenstern utility function (Brennan, et al., 2005). But modern scientific investment rules are formulated based upon the precise prediction of investor utility functions even though it could be extremely difficult in real life (Campbell & Viceira, 2002). Hence, the common academic prescription should not be optimal for any given utility function.

The most fundamental decision that has to be made by the investor is the proportion of wealth allocation to risk assets. Early modern theories regarded this issue as a complicated function of the expected returns and investors’ risk preferences (Markowitz, 1959) (Sharpe, 1963). A famous folk investment guidance that echoed this thought was the simple 60-40 rule where prescribes 60% of wealth to equities and the remaining 40% to cash. In practice, this simple method achieved a large proportion of the potential gains for a risk aversion (Brennan & Torous, 1999). In a more general way of speaking, when there are N classes of financial asset, an equal distribution of $1 / N$ asset allocation rule usually has better performance compared to those sophisticated rules of invest in a single equity based on financial theories echoed in an investment common practice ‘do not put all the eggs in one basket’.

On the basis of the simple 60-40 rule, following evolutionary variant prescribes that percentage of an investor’s should allocate his wealth to equities equal to 100 minus his age in years (Malkiel, 1996). Such an age-dependent allocation strategy conflicts with the result of early analytical models based on dynamic investment planning that developed by Samuelson in 1969 that stated for standard iso-elastic utility functions, the optimal equity allocation should be age independent (Samuelson, 1969). But after taking consideration of the depreciating endowment of human capital, later more sophisticated models validate the principle of the heuristic age dependent rule (Bodie, et al., 1992) (Cocco, et al., 2005).

Regarding the advises from investment advisors, scholars conducted research regarding the popular asset allocation rules. The result noted that in contrast to implication of standard Tobin Separation Theorem which implies that an investor can separate the problem into first finding that optimal combination of risky securities and deciding whether to hold a long or short position depending on the attitude toward risk (Canner, et al., 1997). Investment advisors recommended that investors with risk averse attitudes have proportionately larger distribution to bonds. Compare to the fact that Tobin’s theory was deducted under the setting of static interest rate, investment advisors’ recommendations are under dynamic setting when interest rate is stochastic which implies that those advices have been based on a more realistic and sophisticated model than academic critics were employing (Brennan & Xia, 2000).

The predominating investment advice to investors is that they should cut their losses and keep their profit running. However, this advice finds no support from the classical scientific theory unless this recommendation is based on tax considerations (Dammon, et al., 2001). Actually, the random walk theory of the efficient market hypothesis and mean variance analysis would rather suggest the opposite rebalancing strategy. But after entering 1990, price momentum theory could provide some evidence to support this maxim. The emergence of price momentum theory raised the probability that ‘cut losses and keep profit run’ has certain advantages in the absence of utilisation of tax policy (Jegadeesh & Titman, 1993). Later study found that individual investors tend to do the opposite compare the price momentum theory which implies that following the maxim is profitable (Odean, 1998).

The description of share price has been divided into various ramifications. The currently popular theories can be divided into two categories. The first category refers to the believe that the share price follows the random walk pattern. Under this category, researchers tend to investigate the effectiveness of various investment strategies. While the second category refers to the believe that the share price is determined by several elements including demand and supply relationship, company's future development prospectives, management decisions etc.

However, the negotiation process before the share trades for the first time in the financial market has been rarely discussed by researchers. The bargaining situation between the IPO company and the underwriters has been rarely considered in the price determination but to mention there does not have an adequate mathematical model which could demonstrate how the negotiation between the IPO company and the underwriters affect the upcoming price determination of shares before public trading.

From the previous sections, it can be said that the research concerning the bargaining theory is primarily related to the supply chain structure from the commercial world. It focuses on solving practical problems including the optimal price setting, the appropriate contract dealing, and sequential bargaining situations between participants. On the other hand, the research regarding the share price determination mainly concentrates on applying conventional statistics knowledges and stochastic analysis regarding the price of shares that have already traded in the financial markets. However, regarding the share price determination for the newly issued shares, there are rare amount of papers that have covered this topic. This might because the initial public offering procedure contains large amount of bargaining between the company initiates the IPO and underwriters and there does not exist adequate amount of historical data used for stochastic and statistical analysis.

The existence of this research gap motivates the initiation of this thesis to cross apply the knowledges from the game theory including the auction theory and the bargaining theory which suits the situation in the IPO circumstances to facilitate the price determination of shares.

From the conventional research papers, the mathematical models that cover the share price descriptions follows the main idea that the traditional mathematical analysis including the conventional calculus is no longer applicable since the share price itself even though is continuous but nowhere differentiable. And in lot research papers, the researchers tend to assume the investors are rational which leads to the condition that the buy and sell decisions follow a rigid standard which concerns the cost of purchasing and rate of return. For the IPO activities in the financial markets, the procedure of determining share price before the official trading still lacks mathematical model to describe the overall process.

By employing the knowledges from the bargaining theory, they fully consider different situations between bargaining participants including their relative negotiation power, the supply chain structure or to be precise, the IPO company and underwriters' concerns. Utilising the bargaining theory knowledges, it could help to resolve the issues that lacks mathematical models to describe the pre-IPO situation, the price determination of the newly issued shares, and contract design between the IPO company and underwriters.

The research from this thesis has the following objectives. The first objective is filling the current research gap in the area of IPO share price determination by employing the knowledges from the game theory to provide a new mathematical model. Different from the current predominant mathematical models in the research field of the share price description, this mathematical model assumes each participant of the bargaining has its own valuation and this valuation will change over time to help both participants to be better off. The mathematical model deducted from this thesis hopes to fill the theoretical gap currently exists in the field of pre-IPO share price determination.

The second research objective is modelling different contexts for different types of IPO companies. It is commonly known that with more assumptions regarding the model, the model would encounter more difficulties to apply in practical situations. The model deducted from this thesis tries to model elements throughout the IPO procedure as much as possible including the timing pressure faced by the IPO company, the asymmetrical bargaining power against the underwriters, the existence of the outside options, and the existence of asymmetrical information which requires belief construction regarding the counterparty based on bargaining behaviour.

The third research objective is providing managerial insights for participants in the IPO procedure. Not only considering the IPO company, but the mathematical model also comprises behaviour deduction of the counterparty, the underwriters. Every participant could find certain guidance regarding their theoretical best response when the counterparty behaves in certain ways. With the guidance from this thesis, it would provide managerial insights from participants with the objective of maximising profit, designing appropriate contract terms, and optimal bargaining offering proposals.

In this thesis, there proposes three research questions:

1. For the pre-IPO bargaining procedure between the IPO company and the underwriters, does there exist an appropriate way to help the IPO avoid the circumstance of being taking advantage by the underwriters when they on purposely propose a low price for the newly issued shares?

2. Are there methods for the IPO company to change the relationship with the underwriter from competing with each other to cooperate with each other?

3. In case from the practical situations, the information possessed by both parties tend to be asymmetrical, what will happen when one party has private information regarding the market and how to construct the belief regarding the unknown information based on counterparty's reaction?

In order to answer the questions above, a theoretical model is developed to illustrate the bargaining procedure between the IPO company and the underwriters with different settings. The number of assumptions for the model is gradually reduced in order to make the model more fit to practical situations.

For the first question, we initially propose the mathematical model which comprises the auction setting by the IPO company and underwriters could propose their offers to win the contract. By comparing different types of auction settings, the payoff of the IPO company is indifferent as long as the auction is set properly. In the first essay, two different contract types are proposed to be auctioned, different from the conventional contract setting which sets the price as the strategic variable, two new contract types use the quantity as the strategic variable. This is because the vendor problem is commonly encountered in the commercial world which the retailer and the seller has a long-term relationship which make them familiar with the quantity used for the production. However, in the pre-IPO phase, the IPO company and the underwriter are not familiar with each other which requires setting quantity of shares purchased prior to the determination of price.

Subsequently, in the second essay, the mathematical model considers the possibility of switching the relationship of IPO company and the underwriter from competing with each other to cooperate with each other. This brings the possibility of creating more profit which could make both parties better off compared to original situations proposed by the first essay. After comparing the profit received by each party after introducing the side payment which generally regarded as the cooperation facilitation tool, it has been found that once the preliminary settings of the model have been set, the payoff for each participant will be received after the side payment is regardless of the direction of the side payment. The essence of the side payment is redistributing the incremental profit, and the proportion of redistribution is related to certain factors.

To answer the third question of this thesis, we initially designed the model where the underwriter has more information regarding the market demand compared to the IPO company. By comparison with the perfect information situation, it is evident that the underwriter will mimic the low demand situation regardless of the true market demand and has no incentive to mimic a high demand when the actual situation is low demand. However, with the adaptation of alternating bargaining mechanism, it would enable the IPO to construct his own belief regarding the market situation based on the behaviour of the underwriter. To make this mathematical model even more close to real life situation, the existence of outside option has been considered as well.

The investigation of this thesis fills the theoretical gap in the share price determination in the pre-IPO phase by cross applying the knowledges from the game theory. The mathematical model provided avoids the situation that makes the conventional mathematical analysis become pointless in the field of share price description. This is because the mathematical model from this thesis treats the shares as a commodity throughout the valuation procedure, unlike treating the share price as a stochastic random variable. This fills the gap of lacking adequate mathematical model in studying the pre-IPO situation.

The second contribution is providing a mathematical model that does not require a lot of assumptions regarding the share price itself, and the financial market. The conventional mathematical model regarding the share price description usually assumes the share price is a random variable with certain mathematical attributions can be described by certain statistical methods. However, this assumption itself is controversial in certain way as certain scholars think the share price reflect more than the past information, people's expectation regarding the company's future performance will always reflect on the current share price. The model in this thesis, however, assumes the share price finally agreed by both parties from the negotiation contains individual valuation, adjustments when new information is known by each party, and effects from other elements.

The third contribution is providing the guidance to managers in both IPO companies and underwriters regarding what they should do to maximise their profit when the negotiation in practice fits certain characteristics from the mathematical model proposed by this thesis. Different from the empirical research that tells the relationship between certain element and result regarding how the evolution of certain element causes influence over the result from the past history, the results conducted from this thesis are future-orientated. This thesis focuses more on what should be done by managers in the future if the negotiations in pre-IPO phases satisfy the characteristics stated in the mathematical model from this thesis.

Chapter 2. Literature Review

2.1 What is initial public offering (IPO)

Initial public offering (IPO) refers to the procedure that a company goes from a private owned company to a public company. Through trading in the opening financial market, the company could raise fund through the open market. Since raising capital has been one of management's fundamental responsibilities (Blankespoor, et al., 2023), to accomplish this target, managers often present summaries of the company performance and investment opportunities to potential investors. Even though there are various sources for raising the capital, conducting the IPO has been considered as one of the best ways for entrepreneurs (Ritter & Welch, 2002). It is regarded as one of the most important steps in the life cycle of an enterprise as it comes under the scrutiny of the regulatory institutions and the public for the first time (Helbing, 2019).

Current literature regarding the IPOs generally classified the process into three phases, name them as the input, process, and the output of the enterprise's journey of going public (Carbone, et al., 2022). The first phase, or widely known as the pre-IPO, involves the options of raising capital considering whether the firm should go public or adopt other methods of raising capital. Throughout the consideration regarding whether or not to conduct the IPO, commonly considered issues including the initial cost of capital, enterprise's liquidity situation (Roell, 1996), information considerations (Subrahmanyam & Titman, 1999), and the willingness of the board of management to diversify the current ownership structure (Pagano & Röell, 1998). The IPO decision should be agreed to be proceeded if and only if the benefits of going public (higher company visibility and brand recognition, improvement in liquidity situation, better access to other sources of financing) outweigh the cost involved (Carbone, et al., 2022).

Once the decision is made, the company could submit the application of IPO to the SEC and begin the second phase. From the SEC's policy, the IPO firm first files the S-1 statement (the SEC registration statement). In this statement, the company is required to provide extensive written information about the firm's business plan, past performance, current capital structure, management team and governance policies. This S-1 filing is created collaboratively by a wide range of participants with distinctive functions: multiple management teams, firm's internal and external legal counsel, company accountants, external auditors, and investment bankers. The S-1 is reviewed and ultimately accepted by the SEC. The firm then presents a roadshow pitch which the top management summarizes the views on the most important aspects of the company and offering. Once the offering price is determined, the company's share is officially traded in the financial markets. The third phase usually refers to one month after the IPO is issued. It measures the long-run performance of the IPO.

Once the firm goes public, the second phase will be initiated. And this second phase could be further divided into three steps. The first step is hunting for external advisors. These so called 'external advisors' include investment banks, underwriters and auditors. The past experiences showed that hiring reputable external advisors has a positive influence over the underpricing and long-term performance of IPOs (Carter, et al., 1998). The price determination is the second step in the second phase is one of the topics that researchers have dedicated their efforts to for a long time. Currently, there are two principal methods of deciding the offer price: the fixed price method and the book-building method. In the fixed price method, the company fixes the rate at which shares will be offered. While in the book-building method, the underwriter sets the offer price based on the proposals received by investors (Khurshed, et al., 2014).

In this thesis, the research question is primarily focused on the second phase of the IPO activity. Different from the current dominated method of considering the company's situation and determining the offer price to the general investors. This thesis concentrates on the bargaining procedure between the company that initiates the IPO and the underwriters prior to the determination of the offer price. Based on mathematical models, this thesis provides managerial insights to both IPO companies and underwriters regarding their best responses when the counterparty behaves in a certain way.

The third step is measuring the short-run performance of the IPO from the first day of trading until the share price's stabilization (Jhawar & Seal, 2023). From past literature, researchers have observed that the first-day initial returns have been dissatisfactory (underpricing) and given several explanations. One of the commonly stated explanations is the existence of information asymmetry. This information asymmetry could exist either between the issuing company and the underwriter (Baron, 1982) or between the various types of investors (Rock, 1986). Other researchers considered the existence of underprice as a signify of the issuing company's quality (Allen & Faulhaber, 1989). Besides those seminal research papers in the past, researchers have proposed other explanations for the share price underpricing in modern era. For example, researchers have observed that different ownership structures could cause the underpricing of shares, the principal-agent conflicts (Arthurs, et al., 2008), and different features of the corporate governance (Teti & Montefusco, 2022). Besides aforementioned microeconomic factors, researchers found that certain macroeconomic elements could cause the underpricing as well. For instance, researchers have demonstrated that throughout the time when the economic policy uncertainty is high, the share price tends to be underpriced because of the greater information disparity (Boulton, 2022).. Furthermore, this positive relationship will significantly improve during elections (Colak, et al., 2021). Recent papers have proved that the political risk arising due to excessive political intervention at a local level has a positive impact on IPO underpricing (Song & Kutsuna, 2023).

The third phase usually called post-IPO phase common begins one month after the IPO is issued. In this phase, researchers generally measure the long-run performance of those IPO companies. For the long-run underperformance of IPO companies, researchers have proposed several reasons including the divergence of investors hypothesis (Miller, 1977), the overreaction or fads hypothesis (Teoh, et al., 1998). The IPOs of companies that have politically connected CEOs have demonstrated relatively poor long-term performance compared with companies with no political connection due to the presence of bureaucracy (Fan, et al., 2007).

2.2 Applications of game theory in IPO and financial market

The starting cornerstone of the game theory is generally recognised as Nash's seminal works. In his paper 'Non-cooperative Games', he proposed the very first prototype of the bargaining model and in his later work, 'Two-person Cooperative Games' he further demonstrated the distribution of profit situation encountered by two players. The mathematical model conducted from those work initiated the later research regarding game theory. In a word, the game theory is primarily asking one basic question: What is my best move if my counterparty acts in certain ways?

In the financial market, from debt to shares, from futures to options, trading happens all the time. For each trading to happen, there must be at least two players. The concept of game theory is applied in the financial market all the time. From the past to the current era, there existed several ramifications of game theory application in the financial market.

The first type of application of the game theory is the repeated games. Originating from Nash's 'Two-person Cooperative Games', repeated games refer to the situation in which players interact multiple times with each other in numerous rounds or sequences which the profit in one stage could be influenced by the result in previous rounds. In a stock market comprises the strategic use of private information, a repeated auction model is used to analyse the evolution of the price system on a market with asymmetric information (De Meyer & Saley, 2003). The model turns out to be a zero-sum repeated game with one party possessing the information. The researchers have demonstrated that the stochastic evolution of the price system tends to be a continuous time martingale related to a Brownian Motion.

In the financial market, information is usually one of the most important determinants of share price. As mentioned in previous content, the information asymmetry leads to the share price underpricing in the market. To send the message from the IPO company to the market leads to the second application of game theory in the financial markets, the cheap talk. This concept was originally introduced by Maskin and Tirole, and in their paper, cheap talk refers to the condition when the cost of sending a message is low and the credibility associated with that message is limited as well. In this condition the communication does not directly affect the payoff but could exert influence over decision made by players (Maskin & Tirole, 1988). Using this model in the modern financial markets, researchers found that unexaggerated claims could communicate favourable unverifiable information if buyers are not too likely to verify claims, and sellers with better information care more about future prices compared with sellers with worse information (Bloomfield & Kadiyali, 2005).

Extending the traditional game theory stated by Nash, researchers further developed the game theory named hyper game theory considering modern financial market situations such as misperceptions in strategic interactions between players in a game. Employing this theory, researchers illustrated that a market crash constitutes an equilibrium state if players misperceive the true game. When the ambiguity is resolved, prices readjust to the appropriate level, this will cause flash crashes which are perceived as sharp drops in market prices that rebound shortly after. Through analyzing the interaction with herd behaviour, it has been found that flash crashes may be an unavoidable systematic problem of modern financial markets (Brandt & Neumann, 2015).

In order to deal with uncertainty and vagueness in data, researchers have developed a soft set model for better simulation of real-world situation (Molodtsov, 1999). Applying the game theoretic solution concepts in the language of soft set theory. By defining the Nash equilibrium in pure strategies, mixed strategies and cooperative bargaining games with the framework of soft set theory, researchers have illustrated practical applications to an over-the-counter financial market (Kollias, et al., 2024).

One of the commonly well-known game theory models is the prisoner's dilemma. This refers to the type of non-zero-sum game there two players can either cooperate or defect the other player with the sole purpose of maximizing individual payoffs. Through different settings of corresponding payoffs, the equilibrium strategies of players could either cooperate or compete with each other. Researchers have documented that the behaviours of the players are more likely to cooperative when the game is called the Community Game than when it is called the Stock Market Game. These findings are inconsistent with the hypothesis that the Community label triggers a desire to cooperate, but consistent with the hypothesis that social frames are coordination devices (Ellingsen, et al., 2012).

The concept of zero-sum game is introduced when the term game theory is introduced. It is a condition where the winnings of one player are directly equal to the losses of another player. The total wealth in the game is fixed, and for each condition of the result the net change of the total wealth is zero. For the modern insurance companies, they commonly face optimal investment-reinsurance problems. For those insurance companies that invest in a capital market index, their risk comes from the dynamics of share index which follows a geometric Brownian motion and is governed by either a compound Poisson process or its diffusion approximation. Researchers formulated the optimal investment-reinsurance problems with model uncertainties as two-player, zero-sum, stochastic differential games between the insurance company and the market. They provided verification theorems for the Hamilton-Jacobi-Bellman-Isaacs (HJBI) solutions to the optimal investment-reinsurance problems and derive closed-form solutions to the problems (Zhang & Siu, 2009).

Another application of game theory in the financial market comprises certain concepts that are proposed way earlier compared with Nash's work. The Pareto Efficiency which states the condition of the allocative efficiency where no one person can be made better off without making someone else worse off. Under the context in which an industry with knowledge spillover and debt financing, researchers found the equilibrium investments in research and development (R&D) projects are subject to three economic factors: free dissemination of new technologies, free riding, and incentive for risk shifting. Through research regarding how interactions among these forces affect a company's investment decisions and equilibrium industry outcomes, researchers characterized subgame perfect, Pareto-dominant equilibria and demonstrated that for some parameter choices, free riding and risk shifting cancel each other in the decisions of individual firms, resulting in the first-best investments (Ning & Babich, 2018).

For the determination of asset prices, the information update plays a crucial role for a continuous time manner. The Bayesian theory which is a mathematical framework that allows probabilities to be updated based on the release of new information. To exam the post-forecasting issue where predictions influence behaviour and render original forecasts obsolete, researchers used game-theory models and analysed two player types: econometricians with predictions and normal individuals. Their research shows that late-moving individuals should not participate, while early mover probabilities in Bayesian games are beneficial. The improvements in the accuracy of prediction will benefit the econometrician but has little impact on normal individuals (Song & Wu, 2023). With the research regarding the information release, researchers also devoted effort to investigating the condition of perfect and imperfect information games. This application is developed based on Bayesian theory. The concept of perfect and imperfect information games refers to conditions where all players have complete and accurate knowledge about the factor affecting returns in the game and outcome and are a result of decisions or players do not have complete information and are unaware of the fact that certain players have more information regarding actions or decision that lead to preferred outcomes (Harsanyi, 1968). In the financial market, the share price and option price are two financial assets that are highly sensitive to information release and imperfect information is commonly found in those transactions. By constructing a game-theoretical model characterized by perfect market, researchers put an additional element, the frictionless framework. In the theoretical level, the research result leads both the Binomial option pricing model and Black-Scholes-Merton Model to misprice the vanilla options written on these assets (Toraubally, 2022).

Based on the conventional game theory, researchers further developed the content to fit their observations regarding the financial market that investors tend to act differently compared with the assumption of rational agent. Behavioral game theory is an extension of traditional game theory that socioeconomic practices such as psychology and experimental economics to better model behavior in a strategic interaction (Ellsberg, 1961). The price bubbles, one of the commonly encountered topics in the area of applying behavioral game theory in practice, researchers developed countless papers trying to explain the reasons behind. In a bubble game that involves sequential trading of an asset which is commonly known to be valueless. Researchers found that because no trader is ever sure to be last in the market sequence, the game allows for a bubble at the Nash equilibrium if there is no price cap. Through the experiments, structural estimation of behavioral game theory models suggests that quantal responses and analogy-based expectations are important drivers of speculation (Moinas & Pouget, 2013).

2.3 Early research regarding the game theory

In general speaking, the game theory is used to model situations where players take actions to maximize their benefit while taking into consideration that other players are doing the same and their decision making could jointly affect each other’s utility (Nagarajan & Sosic, 2008). Current research streams can be classified into two divisions: the cooperative and non-cooperative approaches. Even though these two approaches adapt distinctive theoretical contents and methodologies, in essence, they are just two ways of dealing with the same problem.

Starting from the all-time well-known example, the prisoners’ dilemma, the prisoners’ attitude towards cooperative or non-cooperative is all dependent upon the setting of the corresponding punishment. For example, when the reward of both prisoner's confess is overwhelming the punishment of only one prisoner confess, then both prisoners will choose to cooperate and confess. Hence it is always said that the game itself is a coin and the cooperative and non-cooperative approach are two sides of this coin.

The non-cooperative theory of games is strategy oriented which comprises considering what the other player will do to maximise personal utility when taking actions. On the other hand, however, the cooperative approach examines the set of possible outcomes, the research is primarily focused on what participants can achieve, will they form any coalitions and how the coalition will shape the final result.

In general, the modelling of the game can be classified into two categories. The first category is commonly referred to as 'zero sum' game where participants compete over fixed total benefit, the gain of one party is accompanied by other parties' lose. While the second category refers to situations where participants could take actions to gain a better total payoff compared to a purely competitive situation.

The nature of the game theory brings new method to the analysis regarding situations within the supply chain structure which includes multiple participants have to interact with each other with consideration that each participant's strategy sticks with maximise personal benefit given other participants with identical motive. The application of game theory within the supply chain analysis is closely related to two themes. The first theme is the feasible outcomes which indicates the total set of all possible results that participants could achieve, even if they have no incentive to achieve certain outcomes. This is the first step of analysing situations within the supply chain which prepares the foundation of deducting participants' action towards the final result (Nagarajan & Sosic, 2008). The second theme is the stability of the result. After the participants decide on allocation of benefit from the feasible set, regardless of in-between procedure, some other potential participants could pursue options to follow the same result or initiate a new independent negotiation (Shang & Cai, 2022).

The model used to analysis the interactions within the supply chain structure is referred to 'bargaining model' which addresses the problem which indicates two or more participants face a feasible set of results, any single outcome could be the final result if it is universally agreed by all participants.

The idea of bargaining model is inspired by Nash's work which first introduced the concept of disagreement outcome and Nash equation to model the bargaining circumstance (Nash, 1951). In this paper, Nash proposed the idea that if the feasible outcomes can make every participant better off compared to the disagreement result then there is an incentive to reach an agreement. And he mathematically modelled the overall bargaining situation with the equation to explore the overall equilibrium result which indicates the resulting payoff allocation that each party unanimously agrees upon.

Under the conventional supply chain structure, the seller and buyer often bargain over price, quantity and corresponding delivery schedules. The negotiation mechanism is widely applied in the construction industry, it is so popular that nearly half of the projects are awarded using negotiation mechanisms (Bajari, et al., 2009). They found that certain aspects such as supplier reputation, complexity of the project, and absence of large supplier base will shape the preference over the bargaining mechanism. Similar studies have been conducted by other researchers as well, the results demonstrated similar trend which suggests the more uncertainties faced by the overall supply chain, the bargaining mechanism is more likely to be chosen including agricultural, medical procurement, and other commodity industry (Iskow & Sexton, 1992; Elyakime, et al., 1997; Worley, et al., 2000).

The conventional articles focus on the application of game theory in the supply chain structure has been developed a variety of papers that describe how the risk neutral agents take the roles of suppliers and retailers facing both vertical and horizontal competition (Nagarajan & Sosic, 2008). The supply chain aims to coordinate the profit distribution across the overall structure using various contract settings. The most commonly used method including transfer price, revenue sharing, side payment design, and quantity discounts and etc. Followed with the innovative contract setting ways, the research regarding the adaptation of game theory in the supply chain structure has been flourishing and brings countless variations to the theoretical development of this subject.

2.3.1 The development of game theory papers

Game theory can be regarded as a philosophy which involving adapting mathematical representation to model the interactions concerning conflicts and cooperations among two or more parties within a certain situation (Von Stengel, 2022). The game theory could aid in strategic thinking, predicting other parties' actions and outcomes, and provides decision-makers and policy-makers recommendations after considering the possible situations (Baniak & Dubina, 2012). The game theory was firstly developed by the field of microeconomics and later researchers have expanded the application of game theory to other fields including sociology, psychology, computer and political science (Piraveenan, 2019).

The originality of game theory is difficult to judge, but it was formally proposed by John von Neumann and Oskar Morgenstern in 1944 and John Nash's work in 1950s formed the foundation for later research of the field (Carmichael, 2005). Nash proposed the mathematical model to describe the cooperative bargaining procedure (Nash, 1951). In his paper, Nash engaged in axiomatic derivation of a bargaining model which comprises two negotiating parties. The solution refers to the allocation of payoff between the negotiating parties that each party unanimously agreed upon (Nagarajan & Sosic, 2008). The axiomatic approach of the Nash solution requires the final solution to have certain characteristics and one of the most essential requirements is that under the rational agent assumption both parties will agree over the result that one party's incremental of payoff will cost the sacrifice of the counterparty's payoff. This is the oft-quoted phrase Nash Equilibrium. The following content will illustrate the detailed constituents of the Nash Bargaining (NB) model.

In his paper, Nash defined a two-person game as a fundamental model which can be expanded to more than two participants. For each participant, his payoff can be written as (F, d). The first factor F represents participant's payoff if both participants reach an agreement and receive the best outcome. While the letter d represents the payoff if the negotiation between the participants failed, this commonly referred as disagreement point. F is a set that is non-empty, with clear upper and lower boundaries that indicates the possibility to calculate a clear maximum value. The disagreement point, on the other hand, can be regarded as the opportunity cost to reach an agreement. Throughout the paper, Nash explores the method to find the unique bargaining solution in the feasible set that satisfy the axioms. In other words, what Nash explores is finding the Pareto equilibrium among feasible outcomes. The whole paper proposed that there exists a unique solution that satisfies all the axioms. For any bargaining game, the solution is obtained by resolving the following first order condition (FOC):

$$argmax\,(x_1 - d_1)(x_2 - d_2)$$

For the letter x, $x = (x_1, x_2) \in F, x \geq d$. This approach even though looks rather simple through the structure as it is the multiplication of two differences, it can be used as the building block for a more complex situation. Since complicated formation of profit still follows this track and Nash just uses an abstract letter to represent it. Based on Nash's work, researchers developed more complicated negotiation models, such as descriptive non-cooperative games

(Kagel & Roth, 1995) and alternating offer game (Rubinstein, 1982) reached similar conclusion as Nash did. These later findings prove the universal applicability of Nash's thought regarding the axiomatic bargaining solution.

2.3.2 The Nash bargaining in a newsvendor structure

The newsvendor structure is the setting that brings the Nash Bargaining model to the application of the conventional supply chain problem. In the Nash Bargaining model, two players are negotiating over certain issue. This can be regarded as a bargaining between the seller and the retailer. However, in practical life, this situation is far away from the real situation. In reality, the retailer has to consider the demand situation in the market before he enters the negotiation with the seller. Hence, to make the Nash Bargaining model more suitable for the application to real life, the newsvendor setting becomes rather necessary.

The newsvendor setting begins with one seller with utility function U_S sells the raw material to a retailer with utility function U_R. The negotiation between the seller and the retailer is in essence the capable allocations of utilities between these participants. The seller realises profit by selling certain quantities of raw material to the retailer. On the other hand, the retailer has to face the market demand which is a random variable with corresponding probability density function $f(x)$. The retailer has to determine the optimal level of raw material required based on the information regarding the market demand. The overall procedure includes purchasing raw material from the seller, holding those materials, storing finished products etc. The retailer realises profit through selling finished products to the customer and the revenue figure is proportional to the quantity sold. In the following content, the wholesale price paid by the retailer is denoted as w, holding cost and shortage cost are h & v respectively. The unit cost for the seller is c, and any redundant products manufactured by the retailer are assumed to have zero residual value. The retailer and the seller are negotiating over the quantity q and wholesale price w.

For the seller, the profit π_S subject to the demand level of market x can be written as the function $\pi_S(w, q, x)$ and profit for the retailer is $\pi_R(w, q, x)$. Since the market demand is random variable, taking the expectation of both players are $E[\pi_S(w, q, x)] = \pi_S$ and $E[\pi_R(w, q, x)] = \pi_R$. To focus on the negotiation result between the players, it is temporary assumed that the stochastic demand situation is ignored for this part.

Since the Nash Bargaining model requires the feasible set of payoff and corresponding disagreement points, this requires certain analysis prior to the formulation of bargaining model.

For the disagreement point, it is assumed to be zero if the negotiation between the seller and the retailer failed. This is because the failure to reach an agreement refers to the situation that retailer has no material to be manufactured as the finished product. Hence, the seller and the retailer are negotiating over the contract (w, q). With this, first define:

$$\Delta = \{E[U_S(\pi_S(w, q, x))], E[U_R(\pi_R(w, q, x))]\}$$

Now let the Ω indicate the convex set of all Δ and thereafter refer Ω as the feasible set of the bargaining. The Ω is all randomised feasible pairs of utility allocation options of the total profit.

When both players are risk neutral and negotiate over the wholesale price and corresponding quantity of raw materials to be purchased, certain observations can be gathered. The first is that the Pareto optimal solution for both players is not randomised even though the construction of feasible set includes certain stochastic elements. The second observation is that the unique bargaining solution requires certain degree of coordination from both players, in a way, the Pareto optimal contract can be expressed as (w^c, q^c), the upper-scription c refers to the condition of players' coordination rather than the cost.

This analysis simplifies the overall analysis procedure by removing the double marginalisation effect (Nagarajan & Sosic, 2008). The negotiated result w allocates the overall surplus between the seller and the retailer. The observations gathered from this fundamental two-player model still holds as the model evolves to be much more complicated as long as the assumption of all parameters of the contract are simultaneous negotiated still holds.

2.3.3 The introduction of bargaining power

The previous content has introduced the bargaining model and newsvendor setting to suit the condition of real supply chain structure. However, in real life, the two players usually have distinctive bargaining power which refers to the fact that negotiations are usually dictated by one party rather than the other. With common knowledge, it is reasonable to conduct that the party with relative stronger negotiation power will take advantage over the bargaining procedure and receives a larger pie over the total surplus compared to the relative weaker counterparty. With this fact, researchers developed further sophisticated elements to reflect this situation.

It is clear for the situation where two identical players receive equal payoff through the negotiation. This is because the Nash Bargaining theory requires symmetrical negotiation power as a preliminary setting. In the paper, Nash sets both players with similar risk preferences

(Nash, 1951). The formulation of the Nash Bargaining model is the geometric mean of allocation options minus the disagreement point of each participant. The result concluded by Nash implies when both players are risk neutral and solely chase the profit maximization, they will eventually receive identical payoff. Hence, thinking backwards, the circumstance that both players receive identical payoffs indicates the risk neutral attitude of each player and implies that they are equally powerful.

With this underlying conclusion, the result regarding various player's attitudes begins evolving. The unique bargaining solution will change as players have different attitudes or in other words, different utility functions. The researchers have illustrated that within a two-person negotiation model, one player's utility will be increasing as the opponent demonstrates a risk averse attitude (Roth, 1989). Hence, it can be said that the more risk averse of the opponent, the more powerful the player will become. As for the measurement of player's comparative risk aversion attitude, researchers have given certain measurement method, it is recommended that these papers could be rather helpful (Pratt, 1964) (Yarri, 1969).

So far, the effect of risk attitude of the player has a general direction of affecting the negotiation result. However, the detailed influence over the mathematical configuration still remains vague as the risk preference usually implies a nonlinear change over the cost figure. In the conventional newsvendor setting, the risk aversion attitude makes the utility allocation function difficult to be clearly written down. Furthermore, since the negotiation between the seller and the retailer requires the contract to be negotiated implies specific contract, it would make the traditional newsvendor setting becomes less capable of capturing the influence of risk preference and bargaining power. The generalized Nash bargaining (GNB) formulation provides a recipe to solve this issue. This concept was proposed by Roth and becomes the dominant formulation for the Nash Bargaining model to capture the effect of bargaining power over the negotiation procedure (Roth, 1985). The formulation is:

$$argmax\ (x_1 - d_1)^{\alpha}(x_2 - d_2)^{\beta}$$

The restriction $x \geq d, x\epsilon\Omega$, and the exponential power $\alpha + \beta = 1$ refers to the bargaining power of two players. For example, when two players are negotiating over the total pie of 1, assuming the disagreement points for both players are 0, the GNB implies that the first player will receive the payoff $x_1 = \alpha$. And the counterparty will receive $1 - \alpha = \beta$. The assumption of the disagreement point of each player is zero may be unusual when both players have distinctive bargaining power. The explanation regarding this assumption is given by Muthoo as commitment tactics (Muthoo, 1996). The meaning of the commitment is that players are not willing to accept the share of utility that is smaller than the commitment. But it is important to note that the commitment is not directly connected to the determination of disagreement points.

The genius point of this GNB model is providing a method to explore the effect of asymmetrical negotiation power while preserving the simplicity and linearity of the newsvendor setting. With the commitment explanation, it is clear to conduct the negotiation knowing that each player will not accept the contract that is inferior to the disagreement point. Hence, the GNB model can be regarded as the mechanism to explore how much extra utility compared to the disagreement point the player could earn by taking advantage of the negotiation power. However, it is worth noting that if the total commitment of both players is more than available surplus, then one of the players has to revoke the previous stated commitment as refusing to do so will result in a failure of the negotiation and both players receive nothing. If this occurs, a subgame emerges as the player has to decide how much to be revoked in order to ensure the success of the negotiation while minimize the sacrifice of the profit. When all these have been accomplished, both players enter the Nash Bargaining procedure to negotiate the deal and determine the utility allocation option. Such a situation is not rare case, and in most cases, the party with relative lower bargaining power is the one who makes the sacrifice to ensure the success of the bargaining. Researchers have given certain amount of examples in practical life regarding how the asymmetrical negotiation power tilts the bargaining results (Bacharach & Lawler, 1981) (Schelling, 1960) (Hercus, 1997). The later research still applies this model and the GNB becomes dominant among studies regarding the circumstances of the supply chain structure.

2.3.4 Bargaining within a supply chain structure

With contents from previous sections, the Nash Bargaining model becomes capable of comprising multiple participants, commitment tactics, and negotiation power which illustrates its suitability for the application within the supply chain structure. In this section, the content will concentrate on the application of bargaining theory within the supply chain structure as the supply chain evolves from a simple one seller one retailer situation to multiple retailers and more complicated negotiation mechanism framework.

The first category of applying Nash Bargaining theory in the supply chain structure is cooperative bargaining theory. Certain papers adapt the idea of cooperative negotiation which comprises coordination of total profit among participants but without using the GNB model.

The bargaining activity, to a certain degree, is regarded as the sub-step to facilitate the distribution of ex-post gains. A representative paper that adapts this approach is van Mieghem's work which values the option of subcontracting to improve the overall performance of the supply chain. Through analysing three types of contracts (price-only contract, incomplete contract, and state-dependent contract), Mieghem proves that subcontracting with these three types of contract could coordinate production decisions within the system. While only the state-dependent contract is capable of eliminating all decentralization costs. The game theoretic model proposed in this paper allows the further analysis regarding the role of transfer prices and of the bargaining power of the buyer and supplier (Van Mieghem, 1999).

The possible earliest known instance of the application of cooperative bargaining theory within a supply chain context is Kohli and Park (1989) (Nagarajan & Sošić, 2008). In the paper, authors explore the situation in which quantity discounts are offered by a monopolist under the bargaining problem which the buyer and the seller negotiates over the quantity and corresponding unit price. The effects of risk sensitivity and bargaining power over the quantity discounts are detailly discussed in this paper (Kohli & Park, 1989). The underlying negotiation model is a variant of the model proposed by earlier article named 'Other Solutions to Nash's Bargaining Problem'. In this article, the classic two-person bargaining problem is considered. When the axioms are different compared to Nash's proposal, the unique solution will change (Kalai & Smorodinsky, 1975). Another paper that builds the fundamental block for Kohli and Park's work is Jehoshua Eliashberg's paper regarding the arbitrating disputes, this paper studies the choice over the optimal subset of the Pareto optimal frontier. This paper proposes the decision-analytic approach which is inspired by allocation-function approach (Eliashberg, 1986). These studies initiate the thought that allocations among participants of the negotiation can be regarded as a function of risk aversion and bargaining power. Later on, an increasing amount of research begin to investigate the effect of various elements throughout the supply chain structure. Researchers model the effect of contract parameters over the finished product quality through both noncooperative and cooperative setting. The research of Reyniers and Tapiero highlights the importance of price and after sale warranty over the quality management issues (Reyniers & Tapiero, 1995).

With those content regarding the cooperative bargaining within the supply chain structure, the model regarding the whole situation begins to evolve including more contract types and negotiation mechanisms. Now consider a simple supply chain structure which comprises one seller and one retailer who purchase materials from the seller in anticipation of market demand.

When both players are risk neutral and the contractual agreements are conducted through a Stackelberg game which refers to the situation of the oligopoly market which one player moves first and then the other firm decide how to proceed afterwards, the existence of the double marginalization will cause the channel inefficiency by using the simple wholesale price contract (Nagarajan & Sošić, 2008). To eliminate the channel inefficiency, the models later developed illustrated the possibility of adapting the buy-back contract (Pasternack, 2008) or a revenue sharing contract (Cachon & Lariviere, 2005).

In previous content, the feasible set and utility allocation of the bargaining model have been discussed. It has been illustrated that when both players are risk neutral, there does not exist any double marginalization problem. However, when the player type is not risk neutral, especially when they are risk aversion, the situation will be changed. And the specific situation will be discussed in the following paragraphs.

Now suppose a simple supply chain structure that has one seller S and one retailer R. It is assumed that the market demand is stochastic but the distribution and the density function $f(x)$ are known to both seller and retailer. The negotiation between the seller and retailer is the contract regarding the wholesale price and quantity of the material (w, q). But in addition to that, the seller will pay the retailer a buy back price b which is the price that retailer could sell the unsold material back to the seller. To simplify the model, it is assumed that there is no shortage cost.

With this preliminary setting of the model, the analysis regarding the players could be initiated. As illustrated before, the player's feasible set that represents the utility as the function of the price $P = (w, b, q)$. When both players are risk neutral, any negotiation process where the contract terms are negotiated simultaneously can be regarded as the procedure to share the fixed pie. This is because, for any quantity q, contract parameters have a one-one mapping to the profit distribution option when q units are transacted.

But for the case where the seller is risk neutral while the retailer is risk averse with the utility function $u_R(\cdot)$, and the bargaining between the seller and the retailer is concentrating over wholesale price w, buy back price b, and quantity q. The following result is conducted based on the circumstance that buy back price and order quantity are known.

Hence, suppose using the expression $\hat{q}$ to represent the specific quantity that maximizes the channel profit. Let the letter r represent the unit revenue of the retailer, when the buy-back price equals to the summation of holding cost and the unit revenue $b = r + h$, the retailer's profit will be $\pi_R = (r - w)q$ and the seller's expected profit would be:

$$E_x(\hat{\pi}_s) = (w - c)q - (r + h)\int_0^q (q - x)f(x)dx$$

Hence, the Nash Bargaining formulation becomes:

$$argmax\left\{(w - c)q - (r + h)\int_0^q (q - x)f(x)dx\right\}\{u_R[(r - w)q]\}$$

The result of $\hat{q}$ indicates that for any price $P = (w, b, q)$, the total payoff of the players dominates all other options:

$$E_x\big(\pi_s(P, x)\big) + E_x\big(\pi_R(P, x)\big) \leq E_x(\hat{\pi}_s) + \hat{\pi}_R$$

Hence, for a give price P, the players can choose the optimal wholesale price $\widehat{w}$ such that each player's profit is the highest $E_x\big(\pi_s(P, x)\big) \leq E_x(\hat{\pi}_s)$ and $E_x\big(\pi_R(P, x)\big) \leq \hat{\pi}_R$. With this finding, it can be said that all the possible allocation options can be found that until the total payoff reach its maximum (Pareto Frontier). By setting the buy-back price as the summation of unit revenue and holding cost, the retailer becomes risk sensitive. Because the utility function of the retailer is increasing and concave, it indicates that there exists only one choice which makes the utility of the retailer reaches the maximum. Any other options can not become the Nash Bargaining solution. With this idea, it is rather straightforward to compute the optimal wholesale price $\widehat{w}$.

Through the above content, certain observations can be gathered. First, the existence of the buy-back contract, in essence, provides certain degree of insurance for the retailer. Second, when there are more than two players that are risk aversion in the bargaining situation, the overall negotiation procedure is not equivalent to the negotiation over certain total payoff. By comparing cooperative and noncooperative negotiations with various types of contracts, it can be concluded that different setting of the contract terms will bring different Nash Bargaining solutions. In the mathematical perspective, let $\hat{q}$ be the optimal quantity that coordinates the channel efficiency of one risk neutral seller and one risk aversion retailer, if both players become risk aversion, then $\hat{q}$ will no longer be the Nash Bargaining solution for the new circumstance.

Hence, the risk attitude of the player can be expressed as one of the constituents of the bargaining power. The more risk aversion the retailer, the higher utility can be received by the seller. Now consider a three-tier system that has a risk-averse seller and a risk averse retailer. The distributor placed in the middle plays the role of the insurance provider as same as the existence of the buy-back price. More interestingly, the application of buy-back contract is used by the risk neutral seller to encourage the risk averse retailer to think risk neutrally. That is saying, the risk neutral distributor could bring more utility to both the upper stream seller and downstream retailer if they are both risk-averse.

2.3.5 The sequential bargaining

Previous sections have illustrated the situations of the bargaining between one seller and one or two retailers. For this section, it is time to introduce the model which includes one seller negotiates with multiple retailers (more than two retailers). Now consider a situation where there is one seller sells products to n retailers. To single out the effect of second stage competition between retailers, assuming each retailer will compete in distinctive markets. Same as previous negotiation setting, the retailer and seller bargain over the wholesale price and corresponding quantity. At the same time, the seller promises to buy back any redundant products once the retailer cannot sell to the customer. Hence, the contract negotiated is expressed as (w, b). If these n retailers are identical, all the negotiations can be separated into nth single negotiation with same result.

If, however, suppose those n retailers are different, the negotiation procedure will become rather different compared to the simple situation. Let $A = \{R_1,\ R_2, R_3, \dots, R_n\}$ denote the group of retailers that are differentiated by selling products in different market with unique selling price. In this section, it is assumed that a sequential negotiation will be conducted between the seller and those retailers as Nash Bargaining game. In the first stage, the seller will negotiate with the first retailer R_1 and reach an agreement with wholesale price, buy back price, and corresponding quantity. The result from the first negotiation forms the contract parameters that are negotiable for the following bargaining. To make the following analysis content to be simple and straightforward, those contract parameters are simply denoted as 'price'. In the second round of the negotiation, the seller will bargain the 'price' with the second retailer as well. In other words, the price reached from the first round becomes a vector that could shape the result of the following round. In conclusion, for n retailers, the seller will get n prices.

For these n round of negotiations, the re-negotiation between the seller and certain retailer is not permitted which means once the contract is agreed there is no room for regret. Furthermore, it is assumed that each retailer that has been participating in previous negotiations is no worse than the retailer in the forthcoming round. With these settings, it can be calculated that the profit of the seller is dependent upon the sequence of negotiations. Hence, denoting the profit of the seller as Π_s^n which is the set of all the permutations of n prices negotiated with all the retailers, and the seller as a rational participant will naturally choose the sequence that generates the highest profit.

Let $\Theta(n)$ represents the set of all subsets of A such that each subset will have exactly n retailers. For $\lambda_i^n \in \Theta(n)$ $i = 1,2,\dots,n,$ let $\Pi(\lambda_i^n)$ denotes all the permutation of λ_i^n. When the seller decides to negotiate with the set of retailers λ_i^n, the seller could choose the best outcome from $\Pi(\lambda_i^n)$. Let the best sequence denoted as $\hat{\lambda}_i^n$ with the corresponding price $\hat{P}(\hat{\lambda}_i^n)$. Now with the negotiation mechanism that enable the seller and the retailer reach the agreement over the wholesale price and buy back price, the vector 'price' has the following expression:

$$\hat{P}(\hat{\lambda}_i^n) = [\hat{w}(\hat{\lambda}_i^n), \hat{b}(\hat{\lambda}_i^n)]$$

Now let P_i represents the price at the end of the ith round of negotiations. P_n becomes the final price at the end of all the negotiations. The corresponding profit expression of the seller and jth retailer are denoted as $\Pi_s^n(P_i)$ and $\Pi_{R_j}^n(P_i)$. The resulting price satisfy the following constraints:

$$\sum_{i=1}^{n} \Pi_s^n(P_i) \geq \sum_{j \in \lambda} \Pi_{R_j}^n(P_i)$$

The overall sequential negotiation conclusion illustrates that for the seller, it would be more profitable to negotiate with all available retailers rather than ignore certain retailers. With the final price P_n, each retailer will receive at least what he promised with the retailer during the round that only comprises the seller and himself.

Throughout the above analysis, there is one factor that seems to be neglected, the disagreement point. In the above analysis, it is assumed to be zero. However, one of the settings in the model states that the retailer will not accept an offer that is less than what he promised before. Hence, the disagreement point can be regarded as a gradually increasing factor at each round of the negotiation.

Hence, the complete n retailers' sequential negotiation can be described as the followings. When the 'price' satisfies the above criteria, it would be profitable for the seller to negotiate will all retailers and reach agreements with them. Moreover, if the profit set of the seller is convex, the result can be calculated using the Nash Bargaining solution when the seller finishes the negotiation with the last retailer. On the other hand, if the set is not convex, the maximum profit can be found by using the largest convex subset. In extreme case, if the subset is an empty, this indicates that the seller and retailers failed to reach agreement over any payoff options and all participants receive zero payoff.

2.4 The research concerning the auction theory

Ever since the emergence of game theory, the interactions between individuals have consistently drawn the attention of the researchers. Stated from previous section, researchers have found that bargaining is a choice when both participants have symmetrical information while increasing amount of uncertainties would tilt the preference over the auction mechanism. For example, 43% of projects within the construction industry are awarded by negotiation mechanisms while the remaining adapt certain degree of auction (Bajari, et al., 2009). It can be said that just as the cooperative and non-cooperative games forms the coin of the game theory, auction theory and bargaining theory share similar relationships.

Auction theory is an important cornerstone of the game theory due to practical and theoretical reasons. In practical terms, auction theory has widely applied for vast amount of economic transactions including selling treasury bills, foreign exchange rate determination, and mineral rights for certain degree of assets. From the perspective of theoretical development, the simple and well-defined environment of auction setting makes it become an ideal testing ground for economic theories (Klemperer, 1999).

The first application of auction in the economic activities is time immemorial, but it is widely acknowledged that William Vickery formally illustrated the essence of auction and founded the modern research regarding auction theory (Vickery, 1961). In his work, he proposed that there are four elementary types of auctions are widely used: the ascending-bid auction (also known as English auction), the descending-bid auction (commonly used in the sale of flowers in Netherland, also called the Dutch auction), the first-price sealed-bid auction, and the second-price sealed-bid auction. To better introduce the rules of these four types of auctions, the overall procedure is simplified as the sale of one object.

In the ascending auction, the price of the object will be increased consistently until there is only one buyer left who is willing to buy this object with the currently proposed highest price. This type of auction can be held by the seller calls out the price repetitively or let the buyers propose their offers. The overall procedure can be described as the price continuously rises and bidders step by step quit the auction. The bidder who quits the auction is not allowed to come back and the seller tend to set a fixed increase price interval for the bidder to propose the new bid which could prevent one bidder from offering a new bid with too much increase in the price.

Opposite to the ascending auction, the bidding behaviour of the descending auction starts from the auctioneer propose a high price and gradually reduce this offer until the first bidder agrees the offer. Different from the ascending auction which the last standing bidder wins the object, the winner of the descending auction is the first emerged bidder.

In the first-price sealed-bid auction, each bidder independently submits his bid without knowing other bidders' offer. The object is selling to the bidder with the highest bid and the winner pays for what he offers. While in the second-price sealed-bid auction, the offering procedure is the same, but the winner only needs to pay for the price offered by the second-highest bidder. This kind of auction is proposed by William Vickery in his work back in 1961 and hence some economists refer the second-price sealed-bid auction as Vickery auction.

A key characteristic that shapes participants' preference over the auction mechanism is the existence of information asymmetry (Ashenfelter, 1989). This information asymmetry is commonly found in the model of auctions named as private-value model which each bidder knows how much he or she values the object, but this information is private to himself or herself. The other type of information asymmetry refers to the situation where actual value is same for every bidder, but each bidder has distinctive private information regarding what that value actually is. This kind of model is named as the pure common-value model. A practical example of the pure common-value model is the auction regarding the oil extraction license. Bidders would value this license based on how much oil is under the ground. The exact value of oil is impossible to be known, bidders could depend on various signals to form their own estimation. Throughout the auction process, bidder could change his estimation based on other bidders' offer. If the model is set as the private-value model, then bidder's offer will not be changed even though he could learn other bidders' information. Current mainstream of research usually set the model as the private-value model but allow the bidder to change the estimation based on available signals (Maskin & Riley, 1984).

To theoretically study bidders' behaviours throughout different types of auctions, researchers introduced the concept of behaviour function which primarily comprises mathematical set to the field of research. Consider the first type of auction introduced, the first descending auction. Even though the overall procedure is dynamic, the problem for individual bidder is rather static. That is buying the object with the price that is as low as possible but make sure call out the price before other bidders. In essence, the bidding strategy is the same compared to the first-price sealed-bid auction which is bidding truthfully.

With the private information, in ascending auction, it is sensible for the bidder to bid truthfully until the price reach the private valuation. This is because when the price is below the private valuation, bidder's utility of getting the object is positive with a negative rate of increments when price increases. When the price reaches the private valuation, the bidder is just indifferent between winning and not winning. When there is only one bidder left, the price will be equal to the valuation of the second highest price bidder. In other words, in a second-price sealed-bid private-values auction, the appropriate bidding strategy for the bidder is to bid the true value regardless of other bidders' offer (Klemperer, 1999). Since the winner of the ascending auction usually wins at the second highest price, sometimes the ascending auction is occasionally referred as an open second-price auction.

A well-known circumstance from the common-value auction model is called the winner's curse. This refers to the situation where the winner pays more than the prize really worth because of failure of taking account that other bidders' information could be false. This happens a lot in real life and hence, in equilibrium, to prevent this from happening, bidder usually adjust the valuation lower based on other bidder's signals.

Because of the bidding strategy and corresponding bidding function of the descending and first-price sealed-bid auction, the ascending and second-price sealed-price auction are equivalent, researchers in the literature usually refers two categories of auctions as first-price and second-price respectively.

Another type of auction that worth mentioning is named as double auction. Under this auction mechanism, both sellers and buyers submit their ideal price regarding certain object. Then those offers will be ranked from the highest to the lowest to match information from both sides and generate demand and supply profiles. With available profiles, the highest amount exchanged can be determined by matching the offers with the lowest price and moving up (Feldman & Mehra, 1993). While on the other side, the demand bids, starting with the highest price and moving down. The equilibrium price is determined by this sort of matching mechanism. This kind of auction is commonly found in the stock exchange house as the market-clearing mechanism. In the stock exchange process, the bid offer from the investor who wishes to buy stock with corresponding quantity and the ask offer from those who wishes to sell are recorded in a 'specialist's book' and the difference between the bidding price and asking price is well-known as 'bid-ask spread'.

To make these contents to be straightforward and easy to be understood, the following table is made to aid understanding:

Table 1. Comparison of different types of auctions

Type	Rules	Strategy	Expected profit
Ascending-price, open-bid auction (English auction)	Seller announces the initial low bid price, buyers offer their prices gradually to increase the final deal price until no other buyer offers a higher price.	Bidder's strategy is a function of personal valuation regarding the object, information regarding competitors' valuations and information gained throughout the price offering procedure.	Bidder's highest bid minus the price he paid for the object.
Descending-price, open-bid auction (Dutch auction)	Seller announces the initial high bid price, and buyers offer their prices to gradually lower the initial bid price until the first buyer agreed to buy the object.	The strategy of the bidder is the function of personal valuation, competitors' valuation before the auction, and no new information gained	Bidders' valuation of the auctioned object minus the price he paid for the object.

		throughout the price offering procedure.	
First-price, sealed-bid auction	Bidder submit a written bid to the seller in ignorance of other bidders. The bidder with the highest price wins the auctioned object.	Same as Dutch auction	Same as Dutch auction
Second-price, sealed-bid auction	Bidder submit a written bid to the seller in ignorance of other bidders. The bidder offers the highest price wins the object but only needs to pay the price of the second highest price.	Same as Dutch auction.	Bidder's valuation of the auctioned object minus the second highest price bid.

2.5 The application of auction theory in practice

Even though the auction theory and bargaining theory can be described as the two sides of the coin over the game theory, the application of auction theory is rather late compared to the bargaining theory.

The era of applying the auction theory to practice began in 1993-1994 to sell the license of using radio spectrum in the United States (Milgrom, 2000). This application has received vast amount of attention due to the fact that it symbolizes the reducing regulation from the federal government to enable the valuation conducted by the market to determine who could use this resource. Another intriguing point of this auction is because of the large amount money involved, this very first application of auction rules resulted in 617 million dollars of sale of 10 paging licenses at that time. A few months later, the auction of the broadband personal communications services in December sold for a total price of approximately 7 billion dollars. Since then, the tremendous value involved from the auction activity in the practical activity makes it impossible to be ignored and the successful experience encourages other activities to conduct similar auction activities.

Different from the monetary consideration from the commercial world, the academic economists are more interested towards applying auction theory in practice because the auction design adapts the ideas and recommendations rooted from economic theories. The fact is that nearly all the vital rules adapted by the U.S. communications regulator come from valuable suggestions from research academy.

After years of development, the auction is widely applied in the financial industry. The most well-known case of adapting auctioning mechanisms to set the 'price' of the financial asset is the government securities. From the theoretical analysis, the researchers focus on the situation in which the bidder only demand one indivisible unit of the auctioned object. However, in practical cases, especially the auction of government securities, it is permitted to submit multiple bids. In real life, the bids submitted enable the bidder to demand differing quantities and prices of securities at the same auction. When the practical setting is deviating from the academic framework, the theoretical guidance could only provide limited insights, and extra care must be taken to consider those differences.

Consider the U.S. government securities market, the weekly auction of the treasury securities is constructed differently compared to the theoretical format from the academic papers. This provides an example to illustrate the gap between those stylized models and real-world mechanisms. The U.S. Treasury's offering around trillions of dollars in new debt on an annual basis that is auctioned in a multiple-priced, sealed-bid auction with active trading on both prior and after each event (Feldman & Mehra, 1993). This market is, in essence, a forward market for those securities since the actual issuance date is the delivery date for the forward contract. And this forward market for government securities serves two important functions: allocative and evaluative.

Currently, there are many primary dealers who are capable of participating in the U.S. treasury auction. The submit sealed bids indicating price and corresponding number of securities they are willing to buy. Those bids are referred to as competitive bids. With this setting, the tremendous amount of government securities can be split between several capable dealers to prevent the situation in which certain dealer does not have equivalent amount of fund to purchase. Another advantage is setting the price of the securities at the fair value calculated from the available dealers in avoidance of failing to raise sufficient amount of money.

In addition to the forward market and auction, there is a repurchase and reserve market which enables the short-term borrowing and lending with collateral of those securities. An individual could borrow money overnight by selling securities with corresponding agreement to buy back the security next day with a predetermined price.

The profit throughout the auction of securities coms from the interactions of the three mentioned platforms – the auction itself, the forward market of the government securities, and the repurchase and reserve market. Even though researchers criticized the current auction setting of the U.S. securities could causing the 'winner's curse' and propose to adapt the second-price auction to eliminate the existence of 'winner's curse', the economic efficiency seems to be triumph compared to simple profit maximization.

Besides the applications in the financial market, auctions have been widely applied in various industries. It helps the seller to identify the potential seller and with appropriate setting of the auction itself, the seller could get the maximum value of the assessment from the bidders. In addition to its function in value exploration, the allocative function of the auction makes it become predominant since it could improve the efficiency of allocating the resources to match demand from various agents. Till now, the auction has been applied from the antique market to financial markets involving trillions of dollars of transactions.

Looking at the auction and bargaining theory, in essence, they are two important applications of the game theory. Sometimes they are two mechanisms that require comparisons to determine which is optimal to suit the circumstances. However, in some cases they can work together to resolve certain tricky issues. These two topics are intertwined, sometimes, the bargaining can be regarded as a special case of auction where there exist only one seller and one buyer. Hence, in the following chapters, instead of illustrating auction settings, the negotiation mechanisms will be demonstrated to explore the strategy that each participant should adapt, and managerial insights gathered from the conduction process. In general, the discussion regarding the negotiation mechanism is under the background of financial market comprises a company initiate the IPO, the underwriters, and the individual investors.

2.6 The introduction of negotiation mechanisms

Similar to the different auction settings, negotiation mechanisms could shape participants' appropriate strategy and corresponding payoffs. The choice of the negotiation mechanisms usually depends upon the preference over the party with stronger relative negotiation power over the bargaining procedure, while in some countries, there exist certain legislative regulations to prevent the occurrence of price discrimination. In the following content, several negotiation mechanisms will be introduced.

The most straightforward negotiation mechanism is the seller and the buyer participant in the negotiation. Even though the number of buyers increases, the seller still insists to negotiate with one buyer for each negotiation. In another word, when there is one seller and n buyers, the seller will conduct the negotiation n times. If those n buyers are identical, it can be said that the seller conducts the same negotiation n times. If those buyers are different, then the seller could receive n results after finishing all the negotiations. Considering the relative negotiation power, the situation could refer to the content from previous section 'The introduction of bargaining power'. Depending on the circumstance of conducting the negotiation, if the seller conducts those negotiations at the same time, then this situation is commonly referred as 'Simultaneous Negotiation'. If, however, those negotiations are conducting in a certain rank, then this negotiation mechanism is often referred as 'Sequential Negotiation'. For the content of this negotiation, the adequate content can be found in the section 'Sequential Bargaining'.

Now, suppose each buyer is different with distinctive negotiation power, this indicates that for each negotiation the seller could reach a contract that is different compared to other negotiation's result. Assuming the seller to be rational, then for the negotiation which the seller has stronger negotiation power the seller will exploit this advantage to the maximum. Put this situation in a supply chain structure, the retailer with less negotiation power will have a high cost which could causing retailers transfer this cost to the customers. To prevent retailer being exploited by the seller and customers bearing cost, some countries introduce legal requirement to specify that each retailer should receive the same price contract (e.g. Robinson-Patman act) (Nagarajan & Sošić, 2008). Hence, to follow this act, some seller could introduce a negotiation mechanism named 'Price Matching'. In a common price matching negotiation, the seller will negotiate with one buyer first, if they successfully reach an agreement, then this contract will apply for all afterwards buyers. For those buyers, instead of bargaining with the seller, they only need to choose whether to accept this contract or refuse and exit the market. For the seller, it needs to be considered which buyer to be negotiated to determine the contract for other buyers to consider. While for the buyer to be chosen, the primary consideration is whether the contract reached is beneficial for other buyers who could be competitors under the supply chain structure and lose advantage when selling products to customers or the personal strong negotiation power being taken advantage by other buyers without receiving any perks.

These two negotiation mechanisms are rather one-time setting and lack consideration regarding the interaction between the seller and the buyer. To better describe the circumstances invovling both buyer and the seller, it is appropriate to introduce the Rubinstein Model.

The Rubinstein Model was proposed in his work describing the situation where the buyer and the seller take turn to propose the contract to the counterparty, and the counterparty should only choose to accept or not (Rubinstein, 1982). The Rubinstein model provides two insights regarding the research of bargaining situation. The first insight is that frictionless bargaining procedures are indeterminate. The meaning of 'frictionless' under the context of bargaining procedure refers to the circumstance that the participant does not incur any cost by haggling. Another insight the bargaining power of the participant is determined by the relative magnitude of the counterparty's respective costs of haggling since the absolute magnitude of the haggling cost has been proven to be irrelevant to the bargaining outcome (Muthoo, 2002).

A vital reason that the Rubinstein Model plays an important role among the research regarding the bargaining situation has had, and continues to have, is that it provides the elementary framework which provides infinite possibilities to be adapted, extended and modified to suit the research contexts with different settings. This will become evident in later content of this thesis.

The basic alternating-offers model proposed by Rubinstein includes two players, the seller S and the buyer B. These two players bargain over the distribution of the total payoff π. Of course this total payoff must be positive. The bargaining procedure starts from one party proposes the offer to the counterparty, the first party to propose can be either the seller or the buyer. For example, if it starts by the seller proposes the contract to the buyer at time 0, if the buyer accept this contract, then the agreement is struck and both players take the corresponding partition of the payoff as per the contract. On the other hand, if the buyer rejects the offer at time 0, then makes the counteroffer at time Δ $(\Delta > 0)$. If this counteroffer is accepted by the seller, then the agreement is struck. Otherwise, the seller makes a counter-offer at time 2Δ. This procedure of making offers and counteroffers will not stop until one of the participants accept the offer.

A more precise mathematical description over the bargaining procedure is the following. Contracts are proposed at discrete time points: $0, \Delta, 2\Delta, 3\Delta, \dots, t\Delta, \dots (\Delta > 0)$. An offer is a partition or can be saying a number that is greater or equal to zero but less than or equal to π. The distribution proposal, the contract itself, is the share of the total payoff proposed to be distributed to the proposer, therefore, the total payoff π minus the contract is the share for the responder. At the time point $t\Delta$ where t is an even number, it is the turn for the seller to propose the offer to the buyer. If the buyer accepts the contract, then the negotiation ends with an agreement. If the buyer, on the other hand, rejects the offer proposed by the seller, then at time point $(t + 1)\Delta$, the buyer proposes the counter-offer to the seller. If the seller accepts this counter-offer, then the negotiation ends with an agreement. On contrary, if the seller refuses the contract at time point $(t + 1)\Delta$, then the seller will propose the new offer at time point $(t + 2)\Delta$. The whole negotiation ends if and only if either participant accepts an offer.

The payoffs of the participants are as following. If the participants reach an agreement at time $t\Delta$ $(t = 0, 1, 2, \dots)$ over the partition that yields the player i $(i = S, B)$ a share x_i $(0 \le x_i \le \pi)$ of the total payoff. Then player i's payoff is $x_i \exp(-r_i t\Delta)$, where $r_i > 0$ represents the discount rate of the player i. In extreme case, if the players perpetually can not reach an agreement over the partition of the payoff, then each player's payoff is zero.

2.7 Applications of game theory in modern business ecosystem

In the research of business ecosystems, the game theory is widely applied to the field of business innovations. Baniak and Dubina (2012) proposed that the applications of game theory in innovation can be classified as three levels, namely: meta-organisational games, inter-organisational games and intra-organisational games. The participants of meta-organisational games commonly comprise policy makers and enterprises agents. Those games have been found rather helpful for policy makers to develop the effective policy configuration that could improve enterprises' innovation capabilities and create value (Zhao & Bai, 2021).

Descending from the macro-level of the business environment, the game at the level of intra-organisational concentrates on the innovation strategy within the scale of the company. The players are commonly individuals or departments from the same company. The overall objective is optimizing the structure to form the appropriate structural configuration to enhance the over capability of innovation (Pandher, et al., 2017).

And the last type of game, the inter-organisational games comprise various companies with potentials to be cooperative or competitive relationships. The purpose of this category is to analyse the best strategies for a particular company regarding the optimal strategy given certain predictions or actions have already been conducted by other competitors or alliances. The representative examples of this kind of game can be found relating to innovation decisions such as finding suitable time to publish new product, exploring appropriate financial investment opportunities with potential innovations collaborations (Qiu, 2023). From those games, the managerial insights can be concluded regarding policymaking, cooperation or competition.

In current research field, a majority of research are concentrating on the intra-organisational games. This is because this type of game is the most commonly encountered among three kinds of games and the model could be modified to suit various environmental requirements. However, this does not imply other types of games lack corresponding research.

Innovation literature regarding the meta-organisational games provide various cases regarding appropriate policy-makings. For example, researchers developed a quality ladder model to research the impact of incentives behind research and development over the intellectual property rights with 'research exemption' and 'experimental use' provision. The overall innovation procedure is setting as a sequential and cumulative while production is set under an infinite-horizon. The researchers found that firms, ex ante, always prefer the full protection over the patent under the condition of the relevant Markov perfect equilibria, and profit and welfare effects of the research exemption (Moschini & Yerokhin, 2008). Besides the motivation for R&D activities, researchers successfully modelled the innovation contest with endogenous innovation height by the aid of Tullock contest success function. Researchers have proved that the stability for the prize contest come from the influence of the winning firm exert over the innovation height and the life span of the temporary monopoly achieved by the success from the R&D activities (Schmidt, 2008). The motivation for the R&D activities is one of the predominant topics for the meta-organisational games. This commonly including one policymaker considers tax policy regarding enterprise activities and a firm considers decisions regarding R&D activities. This all time classic model, same as the prisoners' dilemma, illustrate the well-known result that without any incentive, this game will end with the worst possible result because both parties achieve the non-cooperative situation which common refers to the circumstance that government charges a high tax over the enterprise profit while the company has to no intention to conduct the R&D activities (Carayannis & Dubina, 2014).

The methodology adapted for the meta-organisational game has been followed by the inter-organisational game regarding the competition and cooperation between companies. Game theory provides valuable insights for the analysis regarding situations where specific results cannot be conducted. The research regarding R&D decisions which with such significant repercussions over other players has been favoured by game theoretic analysis techniques. These analyses are valuable for the policy-makers to understand company's rational behind the innovation strategies, and provide guidance regarding policy making considering both market situations and enterprise internal issues.

For the majority of research articles, the application of game theory stays at a enterprise level which indicates the condition that single company is regarded as a player within the strategic game. Following the rhythm of Baniak and Dubina (2002), the applications of game theory for the enterprise level are divided into three categories (1) entrepreneurial policy applications, (2) inter-firm applications, and (3) entrepreneurship theory applications (AIOmari, 2024).

The entrepreneurial policy applications comprise models that involving government and entrepreneurs as main players that share similar thought as the meta-organisational games. The second category of applications including games played among private entities. And the third category of applications comprise theoretical research regarding entrepreneur behaviour by utilising relevant concepts and models from the game theory.

The purpose of articles of entrepreneurship policy application is to understand entrepreneurs' reactions against legislation or governmental policies. For example, Gonzalo Castaneda conducted research regarding the Mexican political economy from 1940 to 1988. The model has three players (government, entrepreneurs and works). The application of game theoretic concepts provides rational explanations of several stylised circumstances at that time. A noticing result is that the model suggests that the policy swings in various administrations moving from one side to the other side of the political spectrum are not necessarily a result of the ideology confliction, but a compromised result of the political-environment at that moment (Castaneda, 1995). Different from the economic environment in the west hemisphere, the unique economic environment and policy in China promote the flourish of another type of business, the rural Township-Village Enterprises (TVEs) industry. Applying the game theory framework, researchers formalised the reason behind the popularity of joint ownership among Chinese rural industry from 1990s to 2010s (Lu, 2012). Besides the research regarding the partnership business, one special type of business draws the attention of the researchers as well, the evasive enterprise. One of the most well-known evasive business is the pirate Bay. The emergence of the evasive business is circumventing and disrupting the existing institutional frameworks which eventually lead to responses from regulators. By adapting the conceptual model of the game theory, researchers illustrate and successfully map the interdependency between the evasive entrepreneurship and the regulatory response it provokes. The value of the research is bringing several managerial insights. The first is that under a fast-changing marketing, the actions of the regulators are always later compared to the evasive actions. Hence, waiting for the governmental response seems to be rather not feasible for entrepreneurs. The second insight is that the slow reaction from the regulatory institution could lead to certain degree of demotivation as the entrepreneur might fear the innovations fall into the late new legislation. And the third insight is that the fast-changing environment requires the relevant regulatory institutions need to improve their pace of reaction regarding innovations and demonstrate the necessities of continuous adjustments to current legislation (Elert, et al., 2016). After researching the innovative behaviours, researchers turned their attention towards the bigger picture, the business ecosystem. Researchers adapted the idea of sustainable development from the standpoint of a nonlinear dynamic stability of open system through information exchange, addressing the research question regarding how a regional or national innovation and entrepreneurial ecosystems could function in a sustainable model under uncertain external environment as a multi-criteria decision problem, integrating the concept of 'Innovative helix' with corresponding modifications which including the interaction of science, government and business, as well as game theory methodology, they conclude that the successful implementation of the system is only possible through certain degree of compromise given the complexity of the whole ecosystem and the divergence of its constituents' objectives (Dubina, et al., 2017). The rather recent research regarding the model of applying game theory framework for the enterprise level application can be found regarding the application of peer-to-peer financing choice for small and medium sized enterprises. Formulating a three level

Stackelberg game model including the local supply chain, peer-to-peer financing platform and local government, researchers present a scenario-based decision-making framework to jointly evaluate various supply chain financing problems (Reza-Gharehbagh, et al., 2020). As for the factors that could affect the foreign direct investment, researchers adapt the evolutionary game theory and data from China to study the main drives to boost the business entrepreneurship and innovation. The research considers both internal and external perspectives regarding the entrepreneurship-institutional quality and foreign direct investment. Different from the common thoughts, researchers concluded that the institutional quality has a positive effect over the promotion of business entrepreneurship, but the foreign direct investment does not necessarily stimulate the foster of business entrepreneurship. Situations in different countries could deviate from each other, in some instances, these two factors could inversely affect the condition of business entrepreneurship. In general, it is found that the relationship between the foreign direct investment and the business entrepreneurship has an inverted-U shape, which is quite the opposite circumstance compared to the situation of the institutional quality and the business entrepreneurship (Feng, 2021). Compared to the conventional business activities, the development of green entrepreneurship, as a rather special type of entrepreneurship that is capable of achieving sustainable development, has drawn attention of lot of researchers. Large-scale green entrepreneurship activities start to emerge all around the world. Studying how to promote the diffusion of eco-innovation among green entrepreneurship activities, researchers used the evolutionary game models to research the rationale behind ventures' eco-innovation and greenwash behaviour that is influenced by market mechanisms and government regulations. The researchers concluded that it is rather difficult for the promotion of new ventures' eco-innovation behaviours in the early stage of the green industry, hence the government regulatory guidance is deemed to be rather vital. The flexible application of both government subsidy and penalty mechanism could effectively boost the development of the new venture's eco-innovation behaviours under certain circumstances (Yang, et al., 2021).

For the inter-firm applications, it refers to the application range of the game theory model to describe the involvement between entrepreneurs and other private organisations. Based on the market situation for technology, researchers provide a new explanation for the phenomenon that new entrants tend to be superior to incumbents in originating radical innovations. With the aid of the game theory, researchers explained the reason of the success regarding the entrepreneurial firms whose technologies are acquired by incumbents and then commercialise the innovation. In the two staged game model comprises one incumbent and a large number of entrants, the firms make choice over their R&D approach with corresponding success probabilities and payoffs, while for the second stage, the successful entrants bid to be acquired by the incumbent. Since the preliminary assumption is that the entrants can not survive on their own, hence the equilibrium is the incumbent preforming the least radical projects. With an increasing number of entrants, there will be more the most radical projects. In general, entrants tend to choose more radical R&D approaches that as a result will generate the highest value innovation when the R&D succeeded. And the theoretical finding is supported by the qualitative empirical study and derive managerial insights for the further research and management (Henkel, et al., 2015). Similar thought can be found regarding the collaboration between the innovative and entrepreneurial firms as well, researchers characterise the strategic game between a rather smaller innovative company and a larger entrepreneurial enterprise when they have the opportunity to form a strategic alliance to commercialise the technological invention. The researchers have concluded that the situation between these two companies could be similar to the stag hunt game with two equilibria as long as the small innovative company is not too overconfident, and the entrepreneurial company is not too complacent. The successful formation of the strategic alliance not only require sufficient fund but also complementary efforts from both companies when they choose to cooperate (Chou, et al., 2016). Regarding the distribution of multifaced benefits created by crowdfunding, Bade studies the distribution of benefits created with the help of the game theory model comprises crowdfunders, entrepreneurs, and the venture capitalists. The model illustrates that the higher entrepreneurial bargaining power with regard to the crowd may not always be beneficial for the venture capitalists. This is caused by the reducing success probability of crowdfunding due to higher bargaining power of the entrepreneur. The predominant issues that dictate the distribution of the multifaced benefits are bargaining power and the value of the outside options determine the equilibrium condition, expected venture value and expected wealth. This research provides managerial insights for the entrepreneurs when facing the trade off between the venture quality gains and worse outcomes from crowdfunding campaigns. When the purpose is enhancing the overall social welfare, then the success of the crowdfunding and the venture quality gains are the ultimate objective of the policy maker (Bade, 2018). Regarding the bank's choice over the private equity partner, researchers develop a formal game-theoretic model to analyse the economic and behavioural factors that influence the choice of the private equity partner when investing into the entrepreneurship. Since three parties are constantly involved in the procedure of value creation, the triple-sided moral hazard issues involving the bank, PE-manager, and the entrepreneurs are occurring throughout the process. The crucial factors that will affect the bank's investment choices come from two sides. The first factor is the relative abilities and the potential level of empathy, excitement and passion that generated between the PE-manager and the entrepreneur which is concluded as the behavioural factors. The second factor refers to the personal emotional attachment that the bank has against the private equity company. Researchers concluded that the more severe of the triple sided moral hazard, the less efficiency the decision-making progress becomes (Fairchild, et al., 2019). Besides the research regarding the financing procedure of the entrepreneurship, researchers' paid attention to the optimal contract configuration, since the interactions between to enterprises could be explained by the game theory model as well. Employing the game theory and Markov decision process approach, researchers analyse the contract between the entrepreneur and an investor. The model they set is a non-zero-sum game which the entrepreneur is interested in company survival while the investor is more interested in maximisation of the expected net present value of the project. The theoretical results illustrate that both parties could benefit more by reaching a contract that including repayments and a share of the start-up company. For the practical managerial insights, the researchers observed that when the company meets difficulties in survival, they tend to take risker actions as the repayments are insufficient to support the daily operation (Archibald & Possani, 2021). Considering the situation where there is one entrepreneur designs a crowdfunding campaign for an innovative product with fixed funding rewards, researchers studied how the entrepreneur can signal the information that is only private to himself to other participants. After employing the game theoretic model of signalling information between the entrepreneur and other campaign backers, researchers found that the entrepreneur should signal the high-quality information by proposing a high fund-raising target which is above the full information optimal level. They have demonstrated that the high target affects the quality choice of entrepreneurs. More precisely, they conducted the result that it is rather difficult and costly for the entrepreneur with low quality innovations to mimic the behaviour of a high-quality condition. For the managerial insights, this work proposed regarding the optimal behaviour for the entrepreneur to convey the private information of the high-quality innovative product by design an effective crowdfunding campaign. They also illustrate that the existence of information asymmetry and signalling will affect product quality decisions which ultimately becomes the interest of the platform designer who seeks to solicit high quality products for platform (Chakraborty & Swinney, 2020).

The other mainstream of the inter-firm application of the game theory relates to the research regarding to the venture capital companies. Different from the other research mentioned in previous paragraph, Fairchild conducted a game-theoretic model which puts the entrepreneur at the focal point of the game. This model primarily studies the entrepreneur's choice over the available financiers (venture capitalist or angel investors). For each choice is made, each dyad faces a double-sided moral hazard in the form of ex ante effort-shirking and ex post project expropriation. Throughout the financier choice, the entrepreneur has to face the trade off condition between the venture capitalist with a higher value creating capability and the angel investor with a closer more empathetic and trusting relationship. This research contributes to the existing literature by providing insights regarding the choice of financier under the condition of double-sided moral hazard problems and effects of behavioural factors exerted over the creation of relational rents (Fairchild, 2011). For the venture capital syndication, Agarwal presents a modified version of the game theory model to conduct the crucial situations that capable of leading to sub-optimal venture performance in a syndicate. Quite contrary to previous conventional studies, Agarwal's research proposes that syndication with large amount of partners, entering the maturing life of the projects, the already big portfolio of existing investment do not add significant value to the ventures and could make the situation becomes even worse (Agarwal, 2012). For the venture capital terms, Roger Bowden believes that they are the result of cooperative bargaining between project owners and the financiers. His research examines the conditions under which bargains can be consummated, the very nature of the bargains, and how bargains are influenced by venture fund size. By adapting the game theory, Bowden uses the Edgeworth box construction to establish the Pareto optimal efficient frontier in order to demonstrate the Nash bargaining solution. Since the contracting is mutually beneficial, the specific term depends upon the relative bargaining power of the funds versus the owners. This research reveals how venture capitalists negotiate over the contract terms considering the fund size (Bowden, 1994). Regarding the venture capitalist's decision regarding the optimal investment opportunities, researchers develop a continuous time model from the game theory framework to investigate a complete two-stage decision procedure for the venture capitals, comprising the first stage of investment in the private market and the second stage of exiting through IPO in the public market. The specific decision factors including optimal timings, investment terms, and exit decisions are investigated by real options model facing trade-offs between the same required returns in both the public market and private market and higher required return in the private market than in the public market. The results demonstrated that the identical required rate of returns in the public and private market generate the optimal investment decision at the first stage while then the required return in the private market is superior compared to the public market, the exit decision will affect the investment decision. The reasons that tilts the equilibrium results come from the influence exerted from the size of the initial capital, ownership structure, growth rate and risk industry, required returns in both markets, extent of lock-up period price pressure, and transaction cost of financing (Chen, et al., 2021). Another research that pays attention between the venture capitalist and the entrepreneur from the inception to the exit stage is conducted by Elitzur and Gavious. They use a multi-period game theoretic model comprises moral hazard to conduct their investigation. The research provides insights vis-à-vis optimal contract setting and characterisation of an endogenous exit point. To be precise, it illustrates that the appropriate incentive scheme should backload the incentive payments to the entrepreneur rather than kept them. Thus, the straight debt contract would be more optimal in venture financing (Elitzur & Gavious, 2003). Rather recent research concerning the exiting strategy of the venture capital is based on the paradigm of quantum game theory and classic game theory. Researcher explores the strategic choices of venture capitalists for external investment and constructs the optimal venture capital exit strategy that capable of achieving the unification of Nash equilibrium and Pareto equilibrium. The research provides empirical support for the choice of venture capital exit strategies and expands the theoretical support from a rather innovative perspective (Yuan, 2021).

The last group of articles concentrate on the application of game theory to the field of entrepreneurship theory by utilising the game theory to the theory of entrepreneurship in order to demonstrate problems regarding entrepreneurial behaviour and mechanisms in the existing market. Researchers studied how do private entrepreneurs transform the local social capital into economic capital. Combined the classic Prisoner's Dilemma game theory with in-depth studies regarding entrepreneurs in the rural area of Denmark. Researchers has identified a correlation between the strategies adapted by those entrepreneurs and game theory's prisoner's dilemma. By conducting four case studies, researchers successfully mapping the four studies results into four dimensions of the prisoner's dilemma matrix. With this result, they explained the reason why only of the four case studies succeeds in the capitalising social capital as implied by the theoretical conclusion (Haase Svendsen, et al., 2010). Regarding the effect of stability level exert over the level of entrepreneurial activities, researchers adapt an evolutionary game theoretic approach. Researchers illustrated that under the evolutionary stable equilibria, certain conditions will be played by a population including agents regardless of whether they engage in entrepreneurship or not. The result demonstrates that entrepreneurship could persist even though the assuming strategic complementarities or group selection are missing. Last but not least, they explained how information regarding equilibrium payoffs to both self and paid employment could aid to address the question regarding whether entrepreneurs are different from other economic agents (Kuechle, 2011). Based on this foundation, Kuechle develops the study regarding how economically similar regions could end up having distinctive conditions regarding the level of entrepreneurial activities. This research contributes to the field of entrepreneurial agglomerations by introducing the market entry game model in an evolutionary setup. After examining the long run dynamics of this model and evaluate the impact of the economic, social and demographic exchange over the regional entrepreneurial activities, Kuechle concluded two agglomeration equilibria that including in the case of early events determining region specialisation and one non-agglomeration equilibrium (Kuechle, 2014). After reviewing the existing theoretical propositions over the equilibrating and disequilibrating effects of the entrepreneurship, researchers introduce a game theoretical model of the market process and employ computer simulation to investigate the creation as entrepreneurial mechanisms to advance the theory over disequilibrium and entrepreneurial rents. The analysis suggests that entrepreneurship as the creation of new opportunities may not always be disequilibrating while the entrepreneurship as the discovery and exploitation of current opportunities may not always be equilibrating. Counterexamples to the previous entrepreneurial research are created by certain specific conditions. For the managerial insights, the research largely supports previous ideology regarding how entrepreneurs help the markets by discovering opportunities or disrupting the market through creative destruction (Keyhani & Lévesque, 2016). By considering the entrepreneurs as a iconoclast, researcher analysed the theoretical relationship between rule-breaking by entrepreneurs and realised advantage. Introducing the experimental strategic business game which including entrepreneurs, researcher found that entrepreneurs often break the rules since breaking existing rules by a smart way will help them realise greater benefits. This research intends to explain the complicated relationship between entrepreneurs, rule-breaking actions, and inter-enterprise competitiveness (Arend, 2016). Motivating from the concern regarding the entrepreneurial sustainability, researcher investigates issues regarding the three dimensions of reputational management of the entrepreneurs by using game theory model. The author demonstrated the reputational management behaviour through three attitudes of the entrepreneurs towards the reputation as risk threats, competitive advantage and strategic asset. The research provides a multidisciplinary analysis regarding the reputational management by linking the famous game theory results with organisational realities (Pineiro-Chousa, et al., 2016). With a game-theoretic model, researchers studied the tactical level time allocation decision regarding how technology entrepreneurs should allocate their time to potential customers. For the current two important dynamics that affect the customers' purchase decision regarding the new technology products, consumer peer learning and incumbent reaction, researchers provide the economic rationale for the optimal time allocation for various levels of these two dynamics (Huang, et al., 2018). Regarding the cognitive and behavioural approach, researchers conducted studies vis-à-vis the trust behaviours of entrepreneurs and non-entrepreneurs under a dynamic environment. Researchers argued that due to differences in the business contexts, educational background and corresponding thinking frameworks, entrepreneurs have a distinctive trust behaviour compared to non-entrepreneurs under the volatile business environment throughout out their decision-making procedure. Adopting the established paradigm introduced from the behavioural game theory, researchers examined the evolution of trust behaviours over two groups regarding trust building, trust violation, and trust recovery. Based on the observation from Singapore, they found out that entrepreneurs demonstrated relative quicker behaviour in those three aspects compared to non-entrepreneurs. The researchers speculate that this kind of behaviour from the entrepreneurs are shaped by the more volatile business environment they operate. The results contribute to a better understanding regarding the research of the entrepreneurs' behaviours (Bi, et al., 2021).

2.7.1 Mathematical models from supply chain management context

The two commonly encountered models of the supply chain strategies are vertical integrating (VI) and Manufacturer Stackelberg (MS) (Baron, et al., 2016). In a VI supply chain structure, the variables would be the production quantity and retail price when the wholesale price is identical to the production cost. However, if the supply chain uses the MS strategy, then it indicates that the manufacturer and the retailer operate under a non-cooperative environment. A usual condition of the MS supply chain is that the manufacturer proposes a take-it-or-leave-it contract with regard to the wholesale price, on the other side, the retailer who act as the follower who aims to maximises the personal profit by selecting the optimal retail price given the accepted wholesale price. But a vast amount of research papers focus on the situation where the retailer and the manufacturer bargain over the determination of wholesale price. This process is attractive for researchers since many factors that directly affect the bargaining results can be introduced such as bargaining power, patience level, information advantages etc. For example, Iyer and Villas-Boas develop the framework of exploring the effect of bargaining power over the negotiation between the manufacturers and retailers. They found out that the bargaining procedure will affect the degree of coordination, and the two-part tariffs will not become the part of the market contract regardless of the complexity of the supply chain structure. The result shows that the greater bargaining power the retailer has, the more likely the contract will be beneficial to all channel members (Iyer & Villas-Boas, 2003). To be more precise, Ertek and Griffin explore the impact of power structure on price, sensitivity of market price, and profits under a two-stage supply chain structure which comprises one supplier and one buyer of a single product. They examined conditions when either the supplier or the buyer has the dominant bargaining power and consider a pricing scheme which enables both a multiplier and a constant mark up for the buyer. Their result illustrate that it would be optimal for the buyer to set the mark-up to zero and adapt only the multiplier and the market price itself and its corresponding sensitivity are higher when the distribution and inventory costs exist. Beyond the optimal behaviour of the buyer, they also found that the sensitivity of the market price increases in a nonlinear manner as the wholesale price increases and derive a lower bound for it. Their model tells that marginal impact of increasing shipment cost and carrying charge over prices and profits are decreasing regardless of whom dominates the overall supply chain (Ertek & Griffin, 2002). Instead of researching the situation of a simple supply chain structure, researchers' attention has been drawn to the circumstance of asymmetrical retailers as the flourish of consumer products market. The manufacturer of consumer products complains the diminishing profit due to the emergence of the dominant retailers such as Wal-Mart, Home Depot, and etc. The researchers develop an analytical supply chain with competing environment which refers to members include competing manufacturers and retailers, they illustrate that manufacturers might experience an increased profit when the retailer gains an exogenous cost advantage compared to its rival retailer which make their complains become invalid. The improve in the channel efficiency is captured by transferring the market share to the more efficient retailer through the bilateral bargaining, and ultimately increasing channel profits (Dukes, et al., 2006). With the purpose of providing insights into the determinations of channel profitability and the relative bargaining power within the supply chain channel by incorporating the customer demand, researchers developed the equilibrium framework regarding the bargaining between the manufacturers and retailers. By adapting the Nash bargaining model, the split of margins between channel members is determined by the wholesale price determination. The resulting equilibrium margin is a function of demand primitives and the negotiation power of both the manufacturer and the retailer. While the bargaining power itself, is a function of exogenous retailer and manufacturer features. This function of the bargaining power tells that the more they have to lose in the negotiation relative to the outside option, the weaker the bargaining position. In the model that is developed to investigate the role of the three main factors (firm size, branding effect, and service level differentiation) of shifting bargaining power between the manufacturer and the retailer, the empirical analysis shows that the bargaining power is not an inherent feature of the firm but rather depend upon the counterparty's situation (Draganska, et al., 2010). Hence, the word 'relative bargaining power' gradually replaced 'bargaining power' over the field of application of bargaining theory in the supply chain structure.

The definition of a coordinated supply chain sometimes is similar to the notion for monopolistic chains that a centralised VI chain is coordinated. In the coordinated VI chain, each participant is striving to maximise the profit of the entire supply chain. While under the competitive setting, those choices made do not necessarily guarantee the optimality of the whole supply chain. After investigating the effect of product substitutability under the Nash equilibrium distribution structure of the duopoly structure which indicate each manufacturer distributes the goods through one exclusive retailer. Given the assumption of static linear demand and cost function, it is found that product substitutability does influence the equilibrium distribution structure. Throughout the analysis, both VI and MS chains are considered, it is found that when the competition is intense, the equilibrium profit will be higher for the channel when members use the MS rather than VI contracts (McGuire & Staelin, 2008).

With these research articles, most of the research regarding application of game theory in the context of supply chain structure follows the previous stated framework. The primary models are the conventional bargaining model between two players based on Nash (1950) and preliminary settings (Binmore, et al., 1986). While for the competition between two supply chains based on the framework proposed by McGuire and Staelin (2008).

Ever since the proposal of the idea of bargaining power, it has been a famous factor that affect the bargaining outcome. Previous mentioned Ertek and Griffin (2002) investigated the effect exerted from dominant bargaining power of either the manufacturer or the retailer. This research, in certain degree, inspires later research regarding this field in depth. To be more practically describe the real situation, Dukes et al. (2006) considers the situation of asymmetrical retailers in the supply chain structure. But they model the source for the asymmetry between retailers as different outside options which greatly restricted their attention to the symmetrical negotiation power between the manufacturer and the retailer. The 'relative bargaining power' is gradually adapted by researchers in later articles. Researchers propose an analytical approach that combines with behavioural experiment for a joint examination of both competitive and cooperative relationship between a buyer and a seller. They use the concept of relative bargaining power by considering the scenario which the buyer and the seller invest in strategic capabilities to increase personal relative bargaining power. The overall research is a dynamic procedure which examines how decisions are affected by the locus of bargaining power and by consideration between synergistic or adversarial of the interfirm relationship. The model employed, dynamically examines the evolution of investment strategies in critical resources, yields the equilibrium expressions for the investment strategies of each member within the supply chain structure (Nair, et al., 2011). The result of this research demonstrated that the behavioural experiment can be designed to investigate how relative bargaining power evolves over a dynamic evolution.

After considering the case of the first-time interaction between a simple supply chain structure which comprises one buyer and one supplier, researchers use a Nash bargaining model to address the incentive compatibility where both the buyer and the supplier have asymmetric information. For different cases of certain party possess the private information, corresponding contract would be appropriate to maximise the channel profit (Gurnani & Shi, 2006). In a decentralised supply chain model, which one assembler purchase complementary components from multiple suppliers in anticipation of the demand, researchers consider a sequential bargaining procedure to explore the effect of bargaining power over the structural coalitions (Nagarajan & Bassok, 2008).

In a monopolistic market with a single chain, the VI supply chain is rather coordinated. Under the demand uncertainty, for a decentralised supply chain with competing retailers, researchers address a two-echelon supply chains with a single manufacturer servicing several competing retailers. They conclude that when retailers face the random demands, the distribution is depend only on its own retail price or other competing retailers' prices as per general stochastic demand functions (Bernstein & Federgruen, 2005). Hence, the mechanisms that could coordinate the VI supply chain draws the attention of the researchers.

As per Robinson-Patman Act, the manufacturer is required to treat retailers equally. Ingene and Parry developed the model comprises single manufacturer with competing retailers. They show that there does not exist a single two-part tariff with the same unit cost that could duplicate the behavioural results that are obtained by a vertically integrated system. However, instead of providing same contract to retailers, researchers found that an appropriately specified quantity discounted schedule could make the channel to earn identical profit (Ingene & Parry, 1995). Considering the pricing decision that has to be faced by a producer of the commodity with short demand life, Pasternack develops a hierarchical model to study the effect of channel coordination. It is found that currently employed policy by the manufacturer such as providing full credit for unsold products or no returns for unsold goods could achieve the channel cooperation but is highly dependent upon the retailer demand. This would be efficient when there is only one retailer but will become incompetent when there are multiple retailers. To achieve the channel efficiency under the context with multiple retailers, the policy employed by the manufacturer should be trimmed to provide a partial credit for all unsold products (Pasternack, 2008). Same as the model setting from Pasternack (2008), Weng proposes a different possible method of achieving the channel coordination which is quantity discount policy. In a system comprises only one supplier and multiple homogeneous buyers, the joint decision of this channel is characterised by the unit selling price and the corresponding quantity ordered. From the perspective of the buyer, the annual demand rate and operating cost are depended upon the joint decision policy. Through the analysis, Weng shows that the optimal all-unit quantity discount policy has the same effect as the optimal incremental quantity discount policy regarding achieving the channel coordination. Beyond this finding, it also demonstrates that employing the quantity discounts alone is not sufficient to guarantee the joint profit maximisation (Weng, 1995). By incorporating the uncertain market demand to the supply chain, Tsay develops the model which requires the manufacturer and the retailer to commit resources to production based on forecasted demand rather than realistic situation. The model developed studies the incentives of the two parties, successfully identifies the causes of the inefficiency and suggests corresponding remedies, the quantity flexibility contract, to fix the situation. The contribution of the quantity flexibility contract provides an effective allocation of the cost of forecasting the market demand and motivate both the manufacturer and the retailer to act optimally to achieve channel efficiency (Tsay, 1999). Instead of considering those sophisticated designed mechanisms, Cachon and Lariviere pay their attention to a more straightforward way of promoting channel coordination, the revenue sharing contract. This type of contract refers to the condition that the retailer not only pays the supplier the wholesale price for each unit of material purchased, but also a certain percentage of revenue earned. Compared with the conventional commodity industry, researcher found that the revenue sharing contract is rather prevalent in the videocassette rental industry. After employing the model with revenues determined by retailer's purchase price and corresponding quantity with demand that could be either deterministic or stochastic, researchers found that the revenue sharing coordinates the whole supply chain with a single retailer and arbitrarily allocates the profit across the whole supply chain structure. However, this rather straightforward contract is limited due to its relative insignificant improvement when the wholesale price contract is administratively cheaper. Also, revenue sharing is incompetent of coordinating the supply chain when the demand is depending on the costly retail effort (Cachon & Lariviere, 2005). Encountering the newsvendor problem, researchers consider a simple supply-chain contract which the lone contract parameter is the wholesale price. Researchers found the coefficient of variation is the key issue throughout the channel coordination. The analysis illustrates that when the relative variability decreases. The retailer's price sensitivity decreases and wholesale price increases, this would make the decentralised system becomes more efficient and resulting in an increase of the manufacturer's share of realised profit. This research sets the worst-case analysis of the supply chain performance, and the corresponding thought has been commonly adapted in the research employing the Nash bargaining theory to conduct the disagreement joint (Lariviere & Porteus, 2001).

For these articles, researchers primarily put their attention over the supply chain structure that does not exist any competing relationships. For the field of research regarding two competing supply chains, the seminal work of McGuire and Staelin (2008) establishes the fundamental framework regarding the research when products are highly substitutable. Their research demonstrates that the Manufacturer Stackelberg (MS) supply chain Nash equilibrium is preferred by both manufacturers. By considering the problem of choosing a vertical marketing channel in a product differentiated duopolistic situation, Coughlan extends the research of the MS supply chain to the electronic industry. It is shown that the integration of the marketing function could lead to a high level of price competition and lowers the price compared with the adaptation of independent marketing middlemen (Coughlan, 1985). By taking the perspective of the manufacturer, Moorthy develops the research regarding the effect of strategic interaction over the manufacturer's channel-structure decision. Researcher found that the existence of the strategic interaction, the manufacturers' preference regarding the decentralised channels will be influenced. The answer to the question of how strategic interaction makes the decentralization a Nash equilibrium strategy is that the existence of strategic interaction makes it possible for the manufacturer's retail demand curve to rise. The raising of the demand curve can happen when either the manufacturer's products are demand substitutes at the retail level or strategic complements at the manufacturer or retailer levels or those products are demand complements at the retail level and strategic substitutes at the manufacturer or retailer level. These two conditions are mutually exclusive (Moorthy, 1988).

When there is competition, there always exist the room for the cooperation. For the cooperation literature, researchers aim to study the structure choice of supply chains under the competitive environment to face the uncertain demand. By including two competing supply chains, researchers assume each of them could choose either the vertically integrate or decentralise with the coordinating contracts. After comparing different types of supply chain structures with corresponding equilibrium structural choice, researchers found that the product characteristics exert influence over the equilibrium structure. To be precise, the two decentralised chains are suitable for the substitutable products whereas for complementary products, two integrated chains are the appropriate structure. Furthermore, the higher level of demand uncertainty the more stable those equilibrium choices become (Wu & Chen, 2016).

When companies choose to not manufacture products by themselves but to procure products from the upstream suppliers, this is commonly referred as the upward channel decentralization. Rather than putting attention over the predominantly idea of cost benefits from the marketing scholars and practitioners, researchers switch their interest over the effects of upward channel decentralization where competitors are capable of outsource their production to the upstream suppliers. Researchers demonstrated that the downstream firms could benefit from the upward channel decentralization given their products are endogenous even though those upward suppliers do not possess any advantages on production cost (Liu & Tyagi, 2011). Thinking of the outsourcing literature overlook the cost differential and contract negotiations between manufacturers and suppliers, researchers raise the question regarding whether the upstream supplier's cost efficiency is always beneficial to the downstream participants when there exists competition and negotiation. To address this problem, researchers develop a multiunit bilateral bargaining framework to investigate the competing manufacturer's sourcing decisions. The result shows that low-cost outsourcing could lead to a win-lose outcome which the supplier wins while the manufacturer loses. This is caused by the potential backfire of the supplier's cost advantage which is the result as the relative weaker negotiation power (Feng & Lu, 2012). For most research papers, researchers believe the coordination and harmony between the manufacturing and marketing firms are achieved by eliminating the suboptimal practices within the firms. However, researchers propose a contrasting view regarding the manufacturing-marketing interface. By modelling a duopoly situation where firms compete over price and quality dimensions, the managers of the manufacturing and marketing firms are presented with conflicting incentives as cost minimization and revenue maximization respectively. The managers bargain over the price-quality contract. The analysis illustrates that firm's resulting profits under the conflict-of-interest setting could be higher than those obtained when decisions of the managers are coordinated which is a rather intriguing result compared to the conventional cognition (Balasubramanian & Bhardwaj, 2004).

The current research articles over the application of game theory in the supply chain structure can be concluded as two categories. The first category is the application in the vertical integration which refers to one line of relationship comprises both the manufacturer and the retailer that have either competing or cooperative status to achieve certain outcome. The other category refers to the manufacture Stackelberg models. In this field, there usually exist more than one line of supply chain relationships. Among those supply chains, there is a competing relationship. Some research papers provide results that is quiet contradictory to the conventional cognition which infers to the situation that the competition among participants could result in a higher total profit compared with the coordination circumstance.

In the model analysis, the Nash bargaining is commonly adapted with corresponding adjustments to suit the assumption of the situation. The importance of relative bargaining power is consistently stated while the negotiation situation among the seller and the retailer is assuming to be rather straightforward. These research articles are primarily focus on the rationale behind participant's behaviour while the effect caused by various negotiation mechanisms do not receive adequate attention.

2.7.2 Research regarding channel coordination and competition

The previous research articles concentrate over the condition where the supply chain usually works as a whole, or participants interact with each other to persuade for the maximisation of personal profit. The behaviour of the retailer, who acts as the intermediate player between the seller and the final customer, is rather neglected as the research regarding the relative bargaining power shade the light of this topic. However, with the economic development, the economic activity becomes increasingly specialised, and the effect of the retail platform has gradually drawn the attention of researchers in recent years.

As a newly raised economic body, China has experienced rapid economic growth over the past decades. With the development of in the level of economic activities, some traditional business routines have been challenged by new patterns. Conventionally, the consumer appliances were distributed through various small regional retailers. However, with the rapid development of economic, a new pattern of retailing emerges. Currently, there are two famous self-owned chain stores, Gome and Suning, who still are dominant appliance retailers across the country (Shen, et al., 2019). Different from the traditional retailers of buying the product from the seller and then resale to the final customer, Gome and Suning will provide a space for the manufacturer within their store to enable them directly to sell the products to the final customer. They are more likely a provider of the platform to let the manufacturer to sell products to the customer rather than simply purchase and resell. As a retailer, Gome or Sunning benefits from sharing a part of the sales revenue from the manufacturers and the slotting fee paid by those manufacturers. The functions of the retailers gradually evolve from independent business agent of making profit through price differences between purchase raw materials and sell to customers to an intermediate agent who capable of providing a platform in convenient of facilitating business between the manufacturers and the consumers.

The foundation of researching the function of retailer within the supply chain structure originated from the seminal work of McGuire and Staelin (1983) which provides explanations for the sellers' behaviour of choosing an intermediate retailer. The earlier achievements of research including famous models and corresponding applications are concluded by Ingene and Parry in their excellent book named 'Mathematical Models of Distribution Channels' (Ingene & Parry, 2004). For the new business pattern in the recent two decades, researchers take a different perspective compared to previous studies. Regarding the existence of the upfront payment, despite it exists for a long time between manufacturers and retailers, the effect exerted over the competitive edges in procuring the item remains unexplored. To understand this issue, Marx and Shaffer develop the model to address the issue of exclusive dealing in retailing which comprises two competing retailers make take-it-or-leave-it offers to a common manufacturer. They found that the small manufacturers are unable to obtain the widespread distribution of the products due to the existence of upfront payment because the less powerful retailer could be excluded from the market (Marx & Shaffer, 2007). With the development of internet technology, the flourishment of the e-business brings challenges to the conventional reseller structure as the online platform forms the direct channel between the manufacturer and the customer. To study the influence of the direct channel, Cai studies the effects of channel structures and channel coordination over the participants and the entire supply chain in the context of two single-channel and two dual-channel supply chains. It is found that, in the channel-adding Pareto zone, the supplier and the retailer will benefit from adding the new channel to the conventional supply chain model while in the channel-implementing Pareto zone, it would be mutually beneficial for the supplier and the retailer to utilise the contract coordination policy. The results suggest that the preference of the supplier and the retailer regarding the channel structures are different with and without the channel coordination. The preferences are dependent upon various parameters including demand base, channel operational costs and the substitutability between channels (Cai, 2010). To research the appropriate design of the dual channel supply chain under the background of the advent of the e-commerce, researchers develop a model that conceptualises the influence of customer acceptance of a direct channel over the design of the supply chain. The direct marketing is used as a strategic control even though itself is inefficient. Constructing a price setting game between the manufacturer and the independent retailer, the direct marketing would increase the flow of the profits and help the manufacturer to improve the overall profitability by reducing the degree of inefficient price marginalisation. The result demonstrates that the threat of introducing the direct channel could increase manufacturer's negotiating power over the share of cooperative profits even though the price efficiency is acquired by adapting other business practices (Chiang, et al., 2003). In recent years, it is found that an increasing number of retailers has transformed from a conventional 'brick-and-mortar' retailer to a new era of the 'brick-and-click'. Under the environment of e-commerce, the decision made by the retailer is commonly regarding two aspects when the manufacturer presents a new product to the retailer: the first aspect is whether or not to carry the new product and the second is the channel outlet that the product will be carried in. Based on this trend, researchers investigate the manufacturer's product design strategy when the conventional retailer expands the online business. It is found that the addition of the online store could incentivize the retailer to adjust the participation criterion to be less than the determination of the outside option (Luo & Sun, 2016).

With the emergence of increasing amount of dominant retailer, researchers begin to study how the presence of those dominant retailer shape the new pattern of supply chain coordination. To study this circumstance, researchers develop the model with the presence of the dominant retailer in the middle of the supply chain structure to study how a manufacturer could do to optimise the overall channel. It is demonstrated that such a channel could be coordinated to benefit the manufacturer by either the quantity discount contract or two-part tariffs. This is because those two contracts could make the manufacturer to charge distinctive prices and receive different surpluses from different types of retailers. But a noticing point is that even though these two types of contract could help the manufacturer to coordinate the supply chain, they do not share equal efficient throughout the coordination procedure which requires the manufacturer to take judicious decisions based on practical situations (Raju & Zhang, 2005). Similar study has been conducted by other researchers as well. Through present a theoretical model comprises a dominant and a weak retailer compete for the sale of a single product supplied by a single dominant manufacturer. In the model setting, the dominant retailer has the power to set the wholesale price while this power belongs to the manufacturer when he faces the weak retailer. With the partial ability to transfer demand between these two retailers, the manufacture could achieve the profit maximisation target by strategically raising wholesale price for the weak retailer and then transfer demand to weak retailer by adapting joint promotions and advertising (Geylani, et al., 2007).

Regarding to the slotting fee, it has become one of the topics that draw the attention of the researchers vis-à-vis the emergence of the dominant retailer. The concept of slotting fees or slotting allowances is not a recent coined term, the introduction of them can be traced back to the mid of 1980s. In current research field, there exist two schools regarding this topic. One school considers the slotting allowances as the tool for improving distribution efficiency while the other school proposes that the slotting allowance operates as a mechanism for enhancing marketing power and diminish the level of competition (Bloom, et al., 2000). Researchers agree with the first school indicate the slotting fee can be regarded as the tool to signal the quality of the new product. In an asymmetric information model where one manufacturer has the private information regarding the demand for the new product competing with the manufacturer that is less confident regarding the demand by conducting high levels of pre-launch advertising. The retailer could stipulate the take-it-or-leave-it slotting allowance to screen the potential high demand from the low demand products. It is shown that manufacturers prefer to signal the demand condition through advance advertising activities and wholesale price while the retailer could screen the demand condition by slotting allowances (Chu, 1992). With the slotting allowance has gradually become a vital part of promotional agreements between the manufacturer and the retailer, it forms a significant cost of launching a new entry in wide range of product categories. Considering the situation where the manufacturer introduces the new product to a retail channel, the retailer is independent of the manufacturer and would only accept the product if the expected revenue could recover the positive fixed cost. Researchers illustrated that under the condition of equally informed regarding the product, the terms of the trade will never include the slotting fee. While under information asymmetric, if the manufacturer is better informed, he would convey this information through wholesale price rather than slotting allowance. However, when signalling with the wholesale price alone fails, and retailer has high fixed costs, then the terms of trade must include the positive slotting allowance to convey the information assure retailer participation. The slotting allowance serves two objectives throughout the launch of the product: conveying information down to the retailer and shifting costs up to the manufacturer (Lariviere & Padmanabhan, 1997). With regard to the variations of the slotting allowances, researcher conduct a study under the consumer search cost model and shows that the slotting allowances are consistent with the competitive behaviour and could be caused by the increase in the products. The model predicts that when the increase in the supply of products is not accompanied by the corresponding increase in sales per store, then the equilibrium slotting fee will increase as well (Sullivan, 1997).

For the second school, the researchers believe that the slotting allowances are payment transferred from the manufacturer to the retailers to acquire the retail shelf space. A well-known point of view states that the reason for the rising slotting allowances is due to the scarce shelf space that retailer can profitably carry given an increasing amount of product. Researchers demonstrated that the scarcity of shelf space could cause by the feasibility of slotting allowances which can be anticompetitive even if they have no effect over retail prices (Marx & Shaffer, 2010). A rather distinctive point of view is proposed in recent years regarding this topic, researcher proposes that slotting fee can be part of an equilibrium solution when the unit margins are different among downstream and upstream, especially when downstream margin is smaller than upstream margin. Researcher illustrates that slotting fees will be larger if the products sold if the product sold by the retailer are complementary rather than substitute products. Under the channel bargaining model, it is found that for contracts with slotting fees under the full vertical coordination, the upstream marginal cost functions are required to be increasing (Dhar, 2013). Regarding the prevalent adaption of slotting fee among the grocery retailing, researchers investigate the slotting contract between the manufacturer and the retailer. It is found that retailers are compensated for providing promotional shelf space that at minimum level compared with other inter-retailer price competition over product makes compensation with a lower wholesale price a more costly way of generating equilibrium retailer space rents. The result implies that slotting fee is positively related to the manufacturer's incremental profit margins. This result explains both the growth and the incidence across slotting contracts among grocery retailing (Klein & Wright, 2007). While under the optimal two-part tariff contract between one manufacturer and one retailer, it is shown that the retail competition could lead to slotting allowances in an optimal contract, even though the manufacturer has the monopolistic position and no information asymmetry. On the other hand, the slotting allowances do not arise when the retailer has the monopolistic position without information asymmetry regardless of the monopolistic manufacturer or not. The result also shows that the increasing level of competition among retailers, the larger retailer fixed costs with corresponding lower marginal cost of retailing, and larger relative retailer size will together cause a positive impact over the incidence and the magnitude of slotting allowances (Kuksov & Pazgal, 2007). With the flourish development of Chinese electronic appliance retail market, researchers consider a model which the newsvendor product's demand is dependent upon the retailer price and sales effort. Researchers demonstrate that the slotting fee could empower the dominant retailer to specify contract terms that could benefit all stakeholder groups, but the slotting fee itself could cause unfair competition (Wang, et al., 2012).

The research regarding the emergence of dominant retailer has primarily concentrate over the optimal contract designing and corresponding effect over each participant. Overall, the powerful retailer does not necessary implies the damage to others and the manufacturer could have several strategic actions to achieve certain objectives.

2.8 Research regarding auction settings

The theory of auction is primarily concerning the asymmetric information between the bidder and the seller. It would be very easy to predict the final result if both sides are fully aware of the information. The analysis regarding the symmetrical information of equilibrium is generally known as the Bayesian Nash Equilibrium (Chatterjee, 2013).

In a more general condition of auctioning, it is assuming that each bidder has the private valuation regarding the object being auctioned. Each bidder's valuation is the private information for himself while other bidders have no source of knowing that. The valuation will remain as the private information as long as the other bidders' valuation is not revealed. But another type of valuation structure is rather prevalent in the field of auction theory as well, that is known as interdependent valuations. This type of valuation structure refers to the procedure that each bidder constructs his own valuation based on not only his knowledge but also the function of valuations from all the other bidders.

Auctions can be classified in several distinctive ways: in terms of number of units up for sale, the number of sessions involved in the auction, the openly conducted auction or sealed bids auctions. In terms of number of units, the auction could be single unit auction which indicates only one unit is for sale or multiple units auction where more than one unit of the identical or different units are auctioned. The second type of auction is only concerning with multiple unit auctions. The multiple units of objects can be sold in one round or they can be sold separately through parallel stages. In general, the equilibrium bidding behaviour under auctions with several parallel sessions will deviate from single unit auction due to the level of information availability in multiple stages of auctions. And finally, the auction could be either openly bid which bidders call out offers publicly or sealed bid which each bidder submits his own offer to the auctioneer. In different auction settings, the participants are commonly assumed to have different attributions. In the following content, the settings that are constantly stated in the research regarding the auction theory will be illustrated.

2.8.1 Risk aversion participant

One of the common assumptions among the auction theory is that the bidders are risk neutral. Although this assumption would greatly simplify the analysis of the auction situation, the bidders in real life tend not to behave as assumed. If, however, bidders are assumed to be risk aversion, then the revenue equivalence theory will not hold in general cases. To explain this situation, an example is provided.

Considering the second price sealed bid auction. In this case, the risk aversion bidders will not exert any influence over the equilibrium condition which is every bidder bids the true valuation. But in the case of first price sealed bid auction, the marginal increase in the bid from the equilibrium bid for the risk neutral bidder will increase the probability of winning the object and marginally reduce the expected payoff at the same time. This would be preferrable for a risk averse bidder which indicates the outcome that the risk averse bidder will bid more aggressively compared to the bidding behaviour of the risk neutral bidder in the first price sealed bid auction. With this outcome, it can be concluded that if the bidders are risk averse, then the first price sealed auction could generate more profit compared to the second price sealed bid auction which could one of the justifications that create the popularity of the first price sealed bid auction in practical circumstances. For a rational seller who aims of maximizing personal revenue, the first price sealed auction would be more preferrable give the bidders are risk averse (Krishna, 2002).

The above content is relating to the condition where the bidders are risk averse but the seller is risk neutral. Now suppose the seller is risk averse while the bidders are risk neutral. The previous stated revenue equivalence theory suggests the first price sealed bid auction and second price sealed bid auction will yield the same expected revenue to the seller. But in the case of second price sealed bid, the winner only needs to pay the second highest price to the seller. This second highest price is a certain outcome for the seller compared to the first price sealed auction. For a risk averse seller, it is preferrable to receive a certain outcome under the same expected payoff. Hence, for a risk averse seller, he would prefer the second price sealed bid auction over the first price sealed bid auction.

In the paper named 'Optimal Auctions with Risk Averse Buyers', Maskin and Riley studied the situation which the bidder is risk averse and proposed the optimal auction setting framework in response to this situation (Maskin & Riley, 1984). Through the analysis in the above paragraphs, it can be concluded that for the revenue equivalence theory to be effective, the seller and the bidder must be risk neutral. Under this condition, the payoff functions of the bidders are functions that are quasilinear to the payment and the valuation. But when any party is risk averse, the revenue equivalence theory will not hold, the payoff function under this situation is the utility of the valuation minus the payment. If the utility function is plotted on a diagram, it would be concave, but the point which maximized the utility is not the point where the difference between the expected valuation and the actual payment.

2.8.2 The existence of interdependent valuations and common value

In the previous content, the valuations from the bidders are assumed to be the private information to themselves and each bidder's valuation will not be affected by other bidders' valuation. However, in practical auctions, it is commonly observed that the valuation of one bidder is affected by the valuations from other bidders. In other words, one bidder's valuation function is constituted by other bidders' valuations. An example of this situation is the auction regarding the coalmine. In practical, no bidder could know the exact amount of coal in the coal mine. But the amount of coal will remain the same regardless of who wins the auction. Hence, even though each bidder could conduct the research regarding the coalmine, this private valuation could be adjusted based on other bidders' offers throughout the auction. Hence, the valuation function of each bidder has the parameter containing other bidders' valuations. Since no bidder could know the actual valuation of the coalmine while it is the same for every bidder, then it can be saying that the true value of the coalmine is the common value for every bidder (Bulow, et al., 1999).

The interdependent value, on the other hand, has several intriguing observations regarding the formulation of each bidder. As told in the previous paragraph, the interdependent valuation refers to the condition that bidder could adjust his own valuation based on information observed from other bidders. The bidder fails to adjust the valuation, he might be suffered from the winner's curse. The concept of winner's curse will be explained in the following paragraph.

Supposing a bidder is bidding based his own valuation in a first price sealed bid auction, the equilibrium bidding strategy is $E[V|X = x]$. The letter x refers to the private valuation of the bidder and the letter V is the actual valuation that is a function of all bidders' private valuations. If the bidder ignores that his true valuation is the function of all other bidders' valuations, then assumes the auctioneer announces this particular bidder as the winner of the auction. Then the bidder will know that he offers the highest price among all the bidders. Now if the letter Y is denoting the highest price among the remaining bidders, then the price offer by the winner is higher than that, $Y < x$. The updated estimation of the winner now becomes $E[V|X = x, Y < x]$. Then, the comparation has the following relationship:

$$E[V|X = x, Y < x] < E[V|X = x]$$

This indicates that the winner actually pays more for the bidding object given he neglects the valuations from other bidders.

The well-known example in the financial market could be the subscription for the new shares in the IPOs. When the company goes to the public for the first time, there is no market price for the precise stock valuation, for the purpose of buying the stock, the investors might pay more than the fair value of the share. However, the rather informed investors tend not to subscribe for the share. This is one of the explanations of IPO short-term underperformance (Chen & Zheng, 2021).

The other vital aspect of the interdependent valuation is that it breaks down the appropriate bidding strategy between the second price sealed bid auction and the English auction. In the second price sealed bid auction, the appropriate bidding strategy is still bidding the own true valuation since winning the bid still has to pay the second highest price which is still lower than winner's valuation. But in English auction, if one bidder exits the market, then he will left the information regarding his valuation to other remaining bidders. It can be saying that with every drop out of one bidder, the remaining bidders will update their valuations once. But there are two situations that makes the drop out of bidder becomes irrelevant to the others' valuations. The first situation is there does not exist the interdependence relationship while the other situation is there only existing two bidders. The drop out of one bidder will automatically give the win of the other bidder which leaves no room for recalculation.

For the procedure of adjusting the valuation based on the available information, Myerson (1981) has provided a process of adjustment. In the later research regarding the design of the optimal resource allocation mechanism in the presence of the asymmetric information, researchers found that result concluded by Myerson was too general and lack certain applicability in real situations (Crémer & McLean, 1985). In another seminal paper, Milgrom and Weber developed the general theory of auction with affiliation. Their result demonstrated that in the model comprises competitive bidding in which the winning bidder's payoff may depend upon the personal preferences, the preferences of others, and the intrinsic qualities of the object being auctioned, the English auction generates higher average prices than does the second price sealed bid auction. And under the condition where the bidders are risk-neutral, the second price auction tends to generate higher average prices than the Dutch and first price auctions. In above auctions, the seller could raise the expected price by introducing the policy of providing appraisals from experts regarding the quality of the objects being auctioned (Milgrom & Weber, 1982).

2.8.3 The setting of reserve price, entry fees, and number of bidders

In previous discussions, it is assumed that the seller has a zero valuation regarding the object being auctioned. As a result, any positive offer in the price from the bidder will resulting in a positive profit for the seller. This assumption will simplify the analysis in the behaviours of the participants, but this assumption is commonly untrue in real life. Hence, in this part this seller's zero valuation assumption is not holding anymore. This would be resulting the occurrence of a circumstance that the seller will not sell the auctioned object if the equilibrium price proposed by the winner is less than the valuation from the seller.

In a second price sealed bid auction, if the seller could impose a reserve price for the auction, then all the bidders with a lower valuation below that reserve price will automatically exit the auction. But for the remaining bidders, bidding the true valuation is still the strategic appropriate bidding behaviour. The situation will be the same for the first price sealed bid auction. Hence, resulting the identical expected revenue from two types of auctions for the seller. This confirms the revenue equivalence theory under the existence of reserve price regardless of the formats of the auction. Then the question evolves to be: What would be the appropriate reserve price that could effectively help the seller to achieve the revenue maximization?

To resolve this issue, the first assumption is that no seller will set the reserve price which is lower than his own valuation. To be more precise, a seller with the objective of maximizing the expected revenue will never set a reserve price that is lower than the valuation due to the reason that the expected profit of the seller by introducing the reserve price above the valuation that is strictly higher will guarantee expected profit higher than potential loss. This is commonly known as ‘exclusion principle’, as the seller could excludes some potential bidders by imposing the reserve price.

Besides setting the reserve price could help the seller to exclude potential bidders with lower valuations, the introduction of entry fee could bring the similar effect to the seller. The entry fee is a one-time payment which is non-refundable from the bidder to the seller before the auction started. Since the entry fee will not be refunded to the bidder even if the bidder can not win the bid, this indicates that the bidder with relative low valuation will tend to exit the auction by avoiding paying the entry fee as the sunk cost. For the perspective of generating profit for the seller, setting the entry fee or introducing the reserve price has the similar effect of excluding bidders with low valuation (Krishna, 2002).

Although setting the reserve price that is higher than the seller’s valuation will guarantee the profit, it will cause certain negative effect over the efficiency of the auction. Under a simple structure of the auction, the efficiency of the auction is determined by whether the auction could allocate the object being auctioned to the bidder who proposes the highest price. However, imposing the reserve price could lead to the situation that the auction failed to allocate the object to a winner even there is a bidder who has a higher valuation regarding the object. This could happen in the second price sealed bid auction for the condition that some bidders have the valuation lower than the reserve price while other bidders have opposite valuations. Researchers conducted research regarding the aforementioned circumstances. Harstad studied various auction formats with reserve prices and entry fee and conducted their social and private implications that are consistent with previous outcomes (Harstad, et al., 1990). The situation of including entry fee in the auctions are researched by Levin and Smith, they modelled entry incentives with risk neutral bidders and characterise the symmetric equilibrium when the number of bidders is stochastic. Their work show that the seller and the society could benefit from policies that reduce market thickness, such a result extends the famous revenue equivalence and ranking theorems but more importantly, they illustrated that the different auction environment could exert influence over the optimal policies (Levin & Smith, 1994).

In most auctions, the number of bidders is usually remaining unknown until the auction starts. This condition is very common in sealed bid public procurement auctions. When one bidder submits his offer, he cannot get information regarding how many rivals submit the offer at the same time. However, studies have shown that even though the number of bidders is stochastic, the revenue equivalence theory still holds. Under the second price sealed bid auction with unknown number of bidders, the optimal bidding strategy for the bidder is still bidding the true valuation regarding the object. Under the first price sealed bid auction, the equilibrium bid is a weighted average of the others' equilibrium bids if every bidder knows the amount bidders. But one bidder could affect other bidders through the strategic behaviour. For example, the research conducted by Fishman shows that bidder could on purposely propose a high premium initial bid to discourage other bidders from entering the auction which enable a bidder to acquire costly information after the bidding has begun (Fishman, 1988).

To answer the question of comparing a more profitable method of selling a company between an auction with no reserve price or an optimally-structured negotiation with one less bidder, researchers found that under reasonable assumptions that the auction is always preferable when bidders' signals are independent. This means that the seller's profit is increasing as the number of bidders is increasing. This finding is holding for certain common value auctions as well (Bulow & Klemperer, 1996).

2.8.4 Asymmetries among bidders

Previous paragraphs assumes that bidders are symmetrical among each other. Hence the equilibrium bidding strategy for one bidder is generally assumed to be optimal for other bidders as well. But in real life, bidders tend to be asymmetrical which will bring more complexities in auctions. Here the meaning of asymmetries among bidders refers to the condition that valuations of different bidders are conducted from distinctive distributions. For a second price sealed bid auction, bidding the true valuation is still the optimal strategy even though there exist an asymmetry among bidders. But for the first price sealed bid auction, things could become complicated if bidders are asymmetrical. Since every bidder has a different distribution of valuation, it would be intractable to derive a closed form strategy. Researchers have demonstrated that in the case of asymmetrical bidders, the revenue equivalence theorem will no longer holds, and the first price sealed bid auction could lose its efficiency while the second price sealed bid auction might remain effective (Lorentziadis, 2020).

Another important result regarding the asymmetric conditions among bidders is the relative weak bidder tend to bid more aggressively than the strong bidder under a first price auction. The terms 'weak' and 'strong' are relative terms, if the value distribution of bidder i dominates the value distribution of bidder j in terms of the reverse hazard rate, then bidder i is the strong bidder. This result is rather beneficial for the seller who aims to maximize the expected revenue, and commonly government will adapt the price-preference policy to exploit this advantage throughout a government procurement auction (McAfee & McMillan, 1989).

2.8.5 Multi-unit auctions

One preliminary setting in previous sections is that there is only object being auctioned. In this section, the discussion will concentrate over literatures regarding multi-unit auctions. In general, the multi-unit auction could be divided into two categories based on whether or not the objects being auctioned are homogenous or heterogeneous. For the circumstance of heterogeneous objects, they can be either close substitutes of each other or they could be complements.

Another vital perspective of the multi-unit auction is that the likelihood of a single object and multiple objects can be sold in many different ways is very low. In the case of sequential and simultaneous auctions, the number of objects that will be allocated in each of the single round is a decision taken by the auctioneer. It is worth noticing that under the sequential auction, the result in current round could exert influence for the forthcoming round while this would not happen in the simultaneous auctions. The sequential auction could be efficient if the auction is designed under the guidance of the efficient rules, but the possibility of simultaneous auction achieving the efficient status is rather remote. This is due to the fact that the existence of the asymmetrical information makes the simultaneous auction becomes very difficult to update each bidder's valuation. Since whichever group of bidders is constructed, it is very difficult to ensure that each group's winner is the bidder with the highest valuation. To be accurate, there still exist the possibility that those winners are bidders with the highest valuation, but this phenomenon occurs as merely a coincidence.

A situation is that for sequential auction, the auctioneer could adapt different types of auctions for each round. For example, in a conventional two-stage sequential auction, the auctioneer could adapt a first price sealed bid auction for the first round and a second price sealed bid auction for the forthcoming round.

Suppose the seller wants to sell an object as quick as possible, which in general refers to conduct a single round auction. Three types of auctions, the discriminatory auction, the uniform price auction, and the Vickrey auction will be discussed in the following contents. The discriminatory auction or commonly known as pay-your-bid auction and uniform price auction are rather general in practical life, while the Vickrey auction is vital in the field of theoretical research. In the following content, it is assumed the marginal value of the object being auctioned has a decreasing function regarding the number of objects. This refers to the condition that the valuation of the first object is higher than the second one and so on for all the bidders.

The Vickrey auction, in theory, allocates the objects efficiently and bidding the true valuation is the equilibrium bidding strategy for a bidder (Vickrey, 1961). The uniform price auctions are, in general, inefficient since the bidders have the tendency to shade the true valuation after the first unit which such a circumstance is called 'demand reduction' (Chatterjee, 2013). For the discriminatory auction, its generally equilibrium is rather inefficient since it could help to select the bidder with the highest valuation, but it is impossible to reach the highest valuation when there is only one bidder left. Hence, throughout the three auction formats, only the Vickrey auction provides the efficiency. The researchers have demonstrated that the Vickrey auction and the Groves-Clarke extension are efficient for the auction of one good and multiple goods respectively. If each buyer's information could be represented as a one-dimensional signal, then those two auctions can be generalised to attain efficiency given there are common values (Dasgupta & Maskin, 2000). However, situations tend to be rather different when the signals are multi-dimensional. After researching the efficient, Bayes-Nash incentive compatible mechanisms in a social choice setting which allows for informational and allocative externalities, researchers demonstrated such a mechanism exist if and only if a congruence condition relating private and social rates of information substitution is satisfied. When signals are multi-dimensional, the congruence condition is determined by an integrability constraint which can hold on in non-generic cases where the values are private, or certain symmetry assumption holds. If the signals are one-dimensional, then the congruence condition would reduce to a monotonicity constraint which satisfy previous result. However, under the case of the multi-dimensional signals, the efficiency of the auction cannot be guaranteed (Jehiel & Moldovanu, 2001). Regarding the condition that bidders exhibit multi-unit demand, the standard auction methods tend to yield inefficient outcomes. To resolve this issue, researcher proposes an ascending-bid auction for that case. In this auction, the auctioneer announces a price and bidders respond with quantities. The items are awarded at the current whenever they are agreed and the price is incremented until the market clears. When the values are private, this dynamic auction will have the same outcome compared with the Vickrey auction, but it has the advantage of simplicity and privacy preservation. But when valuations are interdependent, then the auction might retain efficiency while the Vickrey auction could suffer from a generalized winner's curse (Ausubel, 2004).

There are other open formats for the multiple unit auction. Those formats including Dutch auction, English auction, and aforementioned Ausubel auction. In the case of Dutch auction, the auctioneer starts by announcing a high price, if no bidder is willing to buy the object, then this price will gradually decrease until the first bidder agreed upon. When a bidder expresses the willingness to purchase the object at certain price, then he is awarded a unit of the concerned object subject to the payment of the announced price. After the first deal with the bidder, the price level is lowered further until another unit is sold in the same way and it continues like this until all the objects are sold off. In this way, the multiple unit Dutch auction will yield the outcome that is equivalent to the discriminatory auction (Krishna, 2002). For the English auction, the auctioneer starts to announce the price level at low which is contrary with the Dutch auction. The price will increase step by step, the bidders will express their willingness of purchasing the object with corresponding price levels. For higher price levels, some bidders will drop out in various stages. This upward version of price announcement will continue until the bidders left match the number of objects. The result of the English auction tends to yield the result equivalent to the uniform price auction (Krishna, 2002).

For the Ausubel auction, it is in theory, equivalent to the Vickrey auction. It also refers to the condition that the auctioneer starts to announce the price at the low level, as well as the English auction. The allocation of the objects occurs through the computation of the residual supply at various price levels.

2.8.6 Contests & Tournaments

Another well-known variation of auction theory is the contest. It is a widely used mechanism to allocate the prize. Under any contest, there are two agents: the contest designer and the contestants. The sequence of action for those two agents is working as the following: the contest designer sets the rules for the contest, and those rules are obeyed by every contestant. Then all the contestants put efforts for winning the contest. The contest designer will announce the winner of the contest based on the rules designed. Under the contest, any cost that each contestant spends in order to win the contest is regarded as the sunk cost which is irrecoverable after the contest initiates. Hence, the contestants are similar to the bidders who participated in an auction with entry fee. They all paid certain fees to the auctioneer before the winner is announced and have no right to claim those fees back if they are not the winner. But on the other hand, those contestants and all-pay auction bidders have certain dissimilarities. In an all-pay auction, the auctioneer sets the payment rule of the auction under the condition of not knowing the valuation of any bidder. Then the payment of any bidder in an all-pay auction solely depends on the bid rather than the type of the bidder. But for a contest, the capabilities of all contestants are constrained by nature which resulting the cost functions are given to the contestants. Therefore, the cost efforts of a contestant could depend on the capability as well as the valuation.

The theory of contest begins in the 1980s when Tullock proposed his seminal paper regarding the rent seeking the lobbying in practical life (Tullock, 1982). The research regarding this topic has been flourished. Regarding the circumstance that economists belatedly recognize that the law of one price is no law at all in real life, Varian proposed an appropriate model of sales (Varian, 1980). With the rapid technological competition in the 1980s, researchers found that the explanations of features regarding the technological innovations is not self-evident while it is less evident that the development and inventive activities are related to the structure of economic organisations. Researcher managed to provide an explanation regarding those subjects from the perspective of the contest circumstance under the auction theory (Dasgupta, 1986). With the development of game theory, Moulin outlined the fundamental concepts of game theory and demonstrated its applications in the field of economic and political discourse. In the book, the behavioural scenarios underlying various equilibrium concepts are discussed and a self-contained exposition of elementary equilibrium concepts for strategic games are provided (Moulin, 1986). For the contests that shares similarities with the all-pay auction with complete information (e.g. rent seeking, R&D races, political contests, and job promotion tournaments), researchers fully characterised equilibrium and illustrated there is no revenue equivalence across the equilibria, the asymmetric equilibria imply higher expected revenues than symmetric equilibria (Baye, et al., 1996).

In the 1990s, more theoreticians put their attention over the contest situation. Under the competitive credit market where banks use imperfect and independent tests to evaluation the capability of a potential creditor to repay the loan, banks compete with each other by announcing interest rates which they willing to provide credit to those applicants. Researcher shows that in a situation where all banks charge the same interest rate to the creditor, one bank will always have the incentive to undercut that interest rate to improve the average credit-worthiness of its own clientele. This feature is the major difference compared with the results of Bertrand and Bertrand-Edgeworth models (Broecker, 1990). Robert Wilson compared the sale prices resulting from two different types of auctions. The first type of auctions refers to the ordinary 'unit' auction which an item is sold to the bidder who submitting the highest bid at the price equal to the highest bid. While the other type of auction refers to a 'share' auction that bidders receive fractional shares of the item at a sale price that equates the demand and supply of shares. The main conclusion is that a share auction can yield a significantly lower sale price and in some extreme cases the share-auction sale price is only half of the unit-auction sale price (Wilson, 1979). Regarding the split award procurement auctions which a buyer divides full production between two suppliers or awards all production to a single supplier, and suppliers have private cost information, researchers found an interesting feature that split awards is that the equilibrium bids are implicitly coordinated. This is due to the situation that the split award price is the sum of offered split prices and each supplier will ultimately veto a split award by bidding very high for the split. The need for coordination is reflected in a split price that regardless with the existence of private information (Anton & Yao, 1992). By taking the example of providing an incentive to make workers exert extra effort without breaking uniform wage restrictions, researchers suggested that the complete information version of the all-pay auction is extended to allow for multiple prizes. Such an extension can be applied at the theoretical level to investigate whether the established properties of the single-prize all-pay auction could be carried over to the more general cases (Clark & Riis, 1998).

Assuming the case that the seller could only offer a single prize in the contest, the equilibrium behaviour in an incomplete information all-pay auction is studied by various researchers. Regarding the case of politically rents and transfers, researchers studied the behaviour of the contestants when contenders place different valuations on the politically allocated prize. Their model explains the phenomena that small numbers of active participants in contests to exert political influence and low lobbying and other influence-seeking outlays relative to the value of politically allocated prizes (Hillman & Riley, 1989). Through the study regarding the war of attrition and the all-pay auction given players' signals are affiliated and symmetrically distributed, researchers found that sufficient conditions for the existence of symmetric monotonic equilibrium bidding strategies and the war of attrition raises greater expected revenue than all other known sealed-bid auction (Krishna & Morgan, 1997).

In this literature, the amount of prize available in the contest is determined by the contest designer. The information is complete for the contestant. Different from those contests that prizes are exogenously offered by the contest designer, Moldovanu and Sela conducted research regarding the contest with multiple, nonidentical prizes. In their model, the contestants are privately informed about a parameter (ability) affecting their costs of effort. The rule of the contest is working as the following: the contestant with the highest effort wins the first prize, the contestant with the second-highest effort wins the second prize, and so on until all the prizes are allocated. The cost function of the contest is separable in effort and ability and researchers studied the equilibrium behaviour under linear and concave cost functions. The result illustrates that under the assumption of the cost function, it is optimal to allocate the entire prize sum to a single first prize. But when the cost functions are convex, several positive prizes may be optimal (Moldovanu & Sela, 2001).

Based on the preliminary framework of the contest, tournaments are actually contests with multiple rounds. In a tournament, whether players are allowed to proceed or win something in later rounds of the game is decided by the result in the previous rounds. An essential feature of the tournament is that the participant needs to put substantial effort in order to participate in the future stages, hence it would be reasonable for the participant to devote as little effort as possible when he knows that he could proceed to next round. On the other hand, a participant would put zero effort or just give up the current round if he performs very badly in the previous rounds, even though there might exist a chance of success in the later rounds if he is willing to put in high effort. This circumstance is rather common in several perspectives in life. For example, Warneryd conducted research regarding the endogenous formation of jurisdictions, assuming the political procedure is a costly fight to acquire shares of the economic development. Warneryd found that in such a setting, the stability of a unified jurisdiction is not necessarily determined by the preferences of the richest member region. Besides this, the result also shows that a system of federalism is especially significant in ameliorating distributional competition and conflict (Warneryd, 1998). Based on this research, Muller and Warneryd cooperated research regarding the comparation of inside and outside ownership of the firm. They show that outside ownership may alleviate the deadweight losses associated with such costly distributional conflict, even if all it does is add another level of conflict. In the case when managers are required to provide with incentives to make firm-specific investments, there is a trade-off between minimizing conflict costs and maximizing output. This provides the managerial insights which explains the reason why some firms are organized as partnerships and others as stock corporations (Muller & Warneryd, 2001). Considering the candidates for U.S. presidential elections are determined through sequential elections in single state, the primaries, researchers consider the situation in the procedure of presidential elections as a tournament. After developing a model in which candidates can influence their winning probabilities in electoral districts by spending money on campaigning, the equilibrium replicates several stylized facts: Campaigning is very intensive in the first district; the outcome of the first election then creates an asymmetry in the candidates' incentives to campaign in the next district, which endogenously increase the equilibrium probability that the first winner wins the further districts. The model provides a possible explanation for the sequential organization: it leads to a lower level of advertising expenditures than simultaneous elections. And if one of the candidates is the more effective campaigner, the sequential elections perform better with regarding to the selection of the best candidate (Klumpp & Polborn, 2006). This research illustrates the effect of discouragement over the tournaments.

The aforementioned literature studied the tournaments under incomplete information. However, under complete information with no noise the discouragement effect is very strong. In these cases, the contestant putting in the highest effort to win for sure. Such studies can be found in Hilman and Riley (1989) and Baye et al. (1996). If on the other hand, the noise is allowed, which makes the winning becomes a random function of the effort levels of the contestant, than the discouragement effect is prevalent as well (Tullock, 1982). For the research regarding the tournaments with incomplete information and multiple rounds, it can be found in Moldovanu and Sela (2001).

2.9 Research gaps and opportunities

Through the review from above sections, this chapter creates a comprehensive description regarding existing research covering the theoretical foundations and practical implementation focusing on applying the game theory in the financial markets especially the IPO activities. These research papers concentrate on aligning conflicting objectives, channel coordination, contract discussion and financial asset pricing combined with distinctive practical situations in the financial market could effectively provide solutions to certain research questions.

First and foremost, the research regarding the applications in the financial markets, which plays an important role in bridging theoretical contents to real life problems. However, lacking adequate mathematical models make it difficult to analyse the situation in the pre-IPO phase. Although there are many models proposed to describe the share price behaviour in the public market, the share value determination before the company goes to public still remain rather vague. Our further investigation regarding the situation of the pre-IPO phases fills the blank that there lacks the model that could describe the situation between the IPO company, underwriters, and individual investors.

Besides numerous research regarding the share price description, this research employs the model from the conventional economic game theory papers. This kind of model is different compared with traditional models used which mainly employ complicated stochastic analysis and statistical techniques to describe share price movement, the model used in this research still adapt the traditional mathematical framework which enables certain elements such as participant's emotions and asymmetrical information that have never considered in share price description being added into the consideration.

In this research, the core model employed come from the Shang and Cai (2022). In this paper, they proposed the model regarding different types of negotiation mechanisms and introduced the backwards induction framework to analyse participants' behaviour in certain stage. Based on their research, this thesis further considers the possibility of setting up an auction between the IPO company and the underwriters, the existence of asymmetrical information which makes the model employed in this thesis further suitable for practical situations. Different from the one-time negotiation stated by Shang and Cai, this thesis proposes the alternating negotiation mechanism to help the participants to construct their beliefs regarding the counterparty.

As we look through the above publications, none of them covers the situation between the IPO company, underwriters, and investors. The negotiations between the supply chain structure have been researched by different scholars, but none of them have applied the game theory in the pre-IPO phase. The research conducted by Shang and Cai (2022) provides an inspiration for analysing the pre-IPO situation. The capability of adding artificial elements into the model solve the critics that current predominant mathematical model in share price description always neglect those issues. This cross application of game theory and share price determination provides an opportunity to study how share price is determined before the public trading. Furthermore, traditional supply chain modelling provides a chance to further consider the existence of asymmetrical information in the financial market which is usually considered to be non-exist in conventional financial modelling assumptions.

Chapter 3. Methodology

3.1 The mathematical framework of research

3.1.1 The introduction of basic mathematical frameworks

The conventional mathematical method of finding the maximum or minimum value of certain object requires certain attributions. For example, under the supply chain which comprises seller, retailer and customer. Assuming the seller and the retailer are rational, this indicates both participants will try to find the maximum profit they can obtain. Adapting the mathematical thought, it first requires the profit figure can be expressed by certain elements. One simple example would be the following: for a retailer, his material comes from the purchase from the seller and then resell those materials to the final customer. If the seller charges the wholesale price w, and the retailer can charge P for customer. Hence the profit π of the retailer given the sales volume is q is:

$$\pi = (P - w)q$$

If the wholesale price is a fixed number and the sale volume q is a function dependent upon the price P, then the overall profit can be written as a function that only has one independent variable P. Then the maximum value and the minimum value of the profit π can be found by conducting the first order derivative and the second order derivative. If the overall function is concave or convex, then the point which makes the first order derivative to be 0 is the maximum or minimum value of the general function. The preliminary assumptions are the function has to be continuous and overall smooth.

However, what if the function itself is not continuous with few jumps, more abstractly, what if the component itself is not a continuous function but a random variable which can not be expressed as a function. To resolve this issue, it requires the stochastic integration regarding the random variable.

The concept of stochastic integration is originated from the research regarding the asset pricing. The first step is to set up the fundamental concept for the model. An investment starts from any time $t = t_0$ ends at time T has a correspondence mathematical notation of its cashflow account. The standard notation is as follows: recording the time $0 = t_0 < t_1 < \cdots < t_n = T$when there is a cashflow occurrence with corresponding cashflow amount $x_1, \ldots, x_T$. If $x_i > 0$ then it denotes an cash-inflow and in opposite when $w_i < 0$, it denotes a total cash-outflow.

From economic perspective, the same amount of money has different values to compare between present and future. In a simple sentence: 100 dollars today is more expensive compare to 100 dollar one year later. In order to compare the present value and future value of cashflow, the effect of interest must be considered. All the interest payment can be cumulated to present t_0 or to any future time $t^* = T$. In reality, the 'time' must be divided into intervals in order to compare economic differences between time t_1 and t_2 denoted as $\tau(t_1, t_2)$.

Suppose a fixed annual interest rate r and the payment of the interest does not take the compounding style, at time t_1, …, t_n, with corresponding payment X_1,…, X_n has the following expression of value at $t = T$:

$$V_T = \sum_{i=0}^{n} x_i(1 + \tau(t_i, 1)r)$$

When $t = 0$ the present value can be calculated as the formula

$$V_0 = \sum_{i=0}^{n} x_i D(0, t_i)$$

The expression of $D(0, t_i)$ is $D(0, t_i) = \frac{1+\tau(t_i,T)r}{1+rT}$. The term $D(0, t_i)$ represents the discount factor of the correspondence payment $x_i$at time t_i.

Usually the interest will be paid at some fixed points throughout the year, such as quarterly or monthly manner. If the time in a year can be divided into *m* intervals with interest rate *r/m*, then a single unit of investment passed *k* intervals will have an additional value:

$$1 + \frac{r}{m}k$$

If the compounded interest circumstance is considered, then this value will change into:

$(1+r/m)^k$

When k=m→∞, this discrete interest sequence will tend to be a continuous compounding circumstance, which indicates:

$$\lim_{m\to\infty} (1 + {}^{r}/_{m})^m = e^r$$

Hence, if an investment has a duration of infinite years through *tm* time intervals, the cumulative discount factor is:

$$\lim_{m\to\infty} (1 + r / m)^{mt} = e^{rt}$$

Now consider the continuous growing of account balance $S_0(t)$ given that interest $r=r(t)$ is a function of time t and satisfy $r(t)>0, t>0$. There are two methods to describe the relationship between these elements, the first one is use the model with $S_0(t)$ and the other is $r(t)$.

First suppose the $S_0(t)$ in a known factor, then the rate of increase throughout time interval $[t, t+h]$ is:

$$\frac{1}{h}\frac{s_0(t+h)-s_0(t)}{s_0(t)}$$

Suppose the account balance $S_0(t)$ is a differentiable function, then the formula below has a definition. And $r(t)$ is spot exchange rate.

$$r(t)=\lim_{h\to 0}\frac{1}{h}\frac{s_0(t+h)-s_0(t)}{s_0(t)}$$

From the above equation: $r(t)=\frac{s_0'(t)}{s_0(t)} \leftrightarrow S_0'(t)=r(t)S_0(t)$

Corresponding differential form is: $dB(t)=r(t)B_{(t)}\,dt$

The general solution for this constant differential equation is $S_0(t)=C\,exp\left(\int_0^t r(S)\,dS\right)$, C belongs to the real number set **R**. For this example, the particular solution for the initial value $S_0(t)=1$ is $S_0(t)=exp\left(\int_0^t r(S)\,dS\right)$, $t\geq 0$. For any random t, there is always $r(t)=r$, then the equation $S_0(t)=e^{rt}$ which conforms previous context.

In the financial model, the interest rate usually takes spot exchange rate, hence the expression of $S_0(t)$ is commonly used as the expression of account balance.

An account with initial endowment 1, take spot exchange rate $r(t)$ and compounding interest has a balance at time t:

$$S_0(t)=exp\left(\int_0^t r(S)\,dS\right), t\geq 0$$

If the initial endowment is x, at time t the balance is $xS_0(t)$. In opposite, in order to have 1 unit of payoff, at $t=0$, the initial requirement for the fund is $x=1/S_0(T)$, at any time [0, T] the cumulative amount is:

$$xs_0(t)=\frac{s_0(t)}{s_0(T)}$$

Throughout time t to T, the discount factor D that makes D unit fund at time t has a payoff equals to 1 unit fund at time T under risk-free condition. The expression of D is:

$$D(t,T) = \frac{s_0(t)}{s_0(T)} = exp\left(-\int_0^t r(S)\,dS\right)$$

3.1.2 Asset pricing model

For the bond with predetermined payoff, the value can be calculated before its expiration due to known interest rate. That is one of the reasons that bond usually regarded as risk-free asset. But for other risky financial assets, for example stocks, their rate of return is determined by the prices.

Suppose S_t denotes the share price at time *t*, since the share price is usually quoted at times with identical intervals, common practice is assigning time to a discrete set of natural numbers **N,** if an individual investor has one share from time *t*-1 to *t*, the changes of share prices can be expressed as:

$$S_t = S_{t-1}(1 + R_t)$$

$$R_t = \frac{s_t - s_{t-1}}{s_{t-1}} = \frac{s_t}{s_{t-1}} - 1$$

R_t is denoted as net payoff and total payoff is: $1 + R_t = \frac{s_t}{s_{t-1}}$

Suppose investor hold one share from time s to *t*= *s*+*k* where *k* is the number of identical time interval, *s, t, k* belong to set **N**, in more general form $s, t, k \in [0, \infty)$. Define the payoff throughout time interval [*s, t*] or *k* intervals is:

$$R_t(k) = \frac{s_t - s_s}{s_s} = \frac{s_t}{s_s} - 1$$

Hence the relationship between total payoff and individual payoff for each identical time interval:

$$1 + R_t(k) = \frac{s_t}{s_s} = \prod_{i=s+1}^{t} \frac{s_i}{s_{i-1}} = \prod_{i=s+1}^{t} (1 + R_i)$$

If the risky asset can be held for k years, then the annual average payoff is the following geometric average formula:

$$R_{t,k} = \left[\prod_{i=0}^{t}(1 + R_{t+i})\right]^{\frac{1}{k}} - 1$$

An investment with fixed annual rate of return $R_{t,k}$ will generate the same cumulative payoff, hence the previous equation is equivalent to the logarithm form of total payoff:

$$R_{t,k} = exp\left[\frac{1}{k}\sum_{i=0}^{k-1} log\ (1 + R_{t+i})\right] - 1$$

The logarithmic return is:

$$r_t = log(1 + R_t) = log\frac{S_t}{S_{t-1}}$$

From time *s* to *t*=*s*+*k*, the logarithmic return throughout *k* intervals is

$$r_t(k) = log\big(1 + R_t(k)\big) = \sum_{i=s+1}^{t\cdot} log\ (1 + R_i) = \sum_{i=s+1}^{t} r_i$$

Comparing to payoff R_t, logarithmic return has the preferrable time additivity. With above definitions, the asset pricing multiplication formular can be written as:

$$S_t = \prod_{i=1}^{t}(1 + R_i)S_0 = \prod_{i=1}^{t} exp\ (r_i)S_0$$

When the stocks are trading in the market, denote S_0, S_1, … as the quoting prices (assume there is no quoting prices before going public), and those quoting prices are usually their closing prices of corresponding trading days. The $S_0>0$ represents the initial quoting price and usually assume it as a constant. Sometime to prevent the influence brought by the initial price, S_0 can be assumed as 0.

The first model to describe the stochastic share pricing is assume the change of stock prices satisfy:

$$\Delta + \mu_n, n = 1,2, \dots$$

In the above expression, $\Delta \in R$ is a non-stochastic constant factor, u_n $(n \in N)$ is an independent random variable with a distribution function F and satisfies:

$E(u_n) = 0$, Var $(u_n) = \sigma^2 \in (0, \infty)$, $\forall n \in N$

The series of u_n usually denoted as $\{u_n, n \in N_0\}$, when the parameter n is clear, the series can be simplified as $\{u_n\}$. Based on the above share price modelling, the process to describe the share price is:

$S_t = S_0 + \sum_{i=1}^{t}(\Delta + u_i) = S_0 + t\Delta + \sum_{i=1}^{t} u_i,\ t = 0,1,2 \ldots$

In the above formula, $\mu_0 = 0$ and suppose that for any series $\{a_n\}$there is a condition that $\sum_{i=1}^{0} a_i = 0$. S_t is named as a random walk. If $\Delta \neq 0$, then S_t is named as a random walk with drift. The expectation and variance of S_t is:

$E(S_t) = S_0 + \Delta t$

Var $(S_t) = t\sigma^2$

This asset pricing formula come from the work of Bachelier back in 1990.

Another share price model is developed on logarithmic return, denoted as:

$R_i = \log(S_i/S_{i-1}),\ i \geqslant 1$

Then S_t can be expressed as:

$$S_t = \prod_{i=1}^{t} \frac{S_i}{S_{i-1}} S_0 = \prod_{i=1}^{t} exp\,(R_i) S_0$$

The corresponding logarithmic pricing process is:

$log(S_t) = log(S_0) + \sum_{i=1}^{t} R_i,\ t = 0,1,\ldots$

Regarding the logarithmic return R_i, the classic assumptions think that it follows the normal distribution:

$$R_i \sim N(\mu, \sigma^2)$$

In the above expression, $\mu \in \mathbf{R}$ and $\sigma^2 > 0$ and similarly, the logarithmic price also follows the normal distribution:

$log(S_t) = log(S_0) + \sum_{i=1}^{t} R_i \sim N(log(S_0) + t\mu, t\sigma^2)$

For the logarithmic normal distribution, if a random variable X satisfy Y=log(X) ~N(μ, σ^2), then X follows the logarithmic normal distribution with parameters μ∈ $\boldsymbol{R}$, σ>0 with a domain (0,∞) and

P(logX⩽y)=$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{y} e^{-(t-\mu)^2/2\sigma^2}$dt, y ∈ $(0, \infty)$

Substitute $\mu = e^t$,

P(X⩽e^y) =P(logX⩽y)=$\int_{-\infty}^{e^y} \frac{1}{\sigma\mu\sqrt{2\pi}} e^{-(\log\mu-\mu)^2/2\sigma^2}\, d\mu$

Take y=$log(x)$, then the probability density function's (p.d.f) $f(x)$expression is:

$f(x)$=$\frac{1}{\sigma x\sqrt{2\pi}} e^{-(\log x-\mu)^2/2\sigma^2}$(x>0), x∈ **R**

The expectation and variance from the above expression of p.d.f can be written

E(X) = $e^{u+\sigma^2/2}$ Var(X)=$e^{2u+\sigma^2}\left(e^{\sigma^2}-1\right)$

3.1.3 The exponential Levy process

Scholars have dedicated their effort to develop an appropriate model to describe the price movement of stocks. In the field of mathematical finance, the stock price are commonly regarded as a stochastic process and Brownian Motion has been widely used for the stock price movement description.

The very first concept of Brownian Motion was stated by botanist Robert Brown for the description regarding pollen movement in the water. In 1905, Albert Einstein developed the first mathematical theory about Brownian motion. In 1923, Wiener putted Brownian motion into the measure-theoretic framework which enables the utilization of probability theory to analyse the stock price movement.

The stochastic process is regarded as the result of the state of nature in which each state has the capability to affect the value of the stochastic process. The asset price process is an essential example of stochastic process.

Normally, the probability space is usually denoted as (Ω,$\mathcal{F}$,P) and explanations regarding these three notations will be included in the following content.

The set of all states is denoted as Ω with the name state space. A single element within Ω is denoted as ω and is called sample point. The set Ω which contains all possible outcomes is called as a sample space.

The system of $\mathcal{F}$ of subsets of Ω is called a σ-algebra if it fulfils the following properties:

- $\Omega \in \mathcal{F}$
- $A^c \in \mathcal{F}$ whenever $A \in \mathcal{F}$
- $A_n \in \mathcal{F}$ for all n = 1,2,3,… implies that $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$

Let Ω be a non-empty set and let $\mathcal{F}$ be a σ-algebra on Ω, then (Ω, $\mathcal{F}$) is called a measurable space. A probability measure P is a real-valued function $P: \mathcal{F} \rightarrow R$ satisfying

- $P(E) \geq 0$ for all $E \in \mathcal{F}$
- (countable additivity) Let (E_n) be a sequence of countable collection of disjoint sets in F then
- $P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$
- P(Ω) = 1 (without this condition, P refers to the measure in normal cases)

A random variable is a variable whose value is subject to the randomness. It represents a numeric value to the result s of an experiment. Hence, it can be regarded as a function.

$X: \Omega \rightarrow R^n : \omega \rightarrow X(\omega)$

In the case for asset price, X(ω) can be regarded as discrete share prices and the general price movement can be regarded as a function to time in a continuous manner. Under this circumstance, the probability that a random variable will assume a specific value will always be zero for any ω ∈ Ω. Hence, probabilities have to be given to parts of Ω or in a mathematical perspective subset of Ω. Subsets of Ω which probabilities can be assigned are σ-algebras.

For this part, some new concepts from Real Analysis and probability theory should be introduced which will be helpful to understand the mathematical application in financial aspect.

The first concept covered is measure which is the third part P defined in previous probability space. In general, the measure can be understood as a function which generates a mapping from the original Euclid space R^n to a new defined space. In a general explanation, in a one, two and three dimensional space, the measure generates corresponding 'length' 'area' and 'volume'.

But in a probability space, all the measure must be assigned to 1 as the probability of certain event's occurrence cannot be larger than 1.

As for its application in financial asset pricing theory, the measure indicates how each discrete random variable which can be share price form a bigger picture indexed by time. With the help of measure theory, corresponding theories of stochastic process can be applied to the description of share price behaviour. Moreover, the measure theory developed based on French mathematician Henri Lebesgue. He developed a new form of integral that is different from traditional Riemann integral which provide a method to deal with functions that can not be calculated through a Riemann integral. The main thought of Lebesgue integral is integrating the function based on non-intersective intervals rather than a strict continuous interval. This idea is particular helpful to understand probability space in the area of asset pricing.

A random variable is adequate when the uncertainty at a single point in time requires to be described. Taking the consideration of asset price, they dynamics are important as well. By putting a number of random variables altogether, a stochastic process can be constructed. For the description of share price, a stochastic process is a family of random variables X = $\{x_t\}_{t\in T}$ defined on a given probability space indexed by T which represents time.

Under the circumstance of stock price description, the information available in the market is considered to be connected with the σ-algebra named filtration. The filtration is defined to reflect the accumulation and destruction of information available on the market. As time passes, more information will be revealed to the market.

Given a probability space (Ω, $\mathcal{F}$, P), a filtration is regarded as a non-decreasing family $\{\mathcal{F}_t\}_{0\leq t\leq T}$ of sub-σ-algebras of $\mathcal{F}$ satisfying

$\mathcal{F}_s \subset \mathcal{F}_t \subset \mathcal{F}_T \subset \mathcal{F}$, $0\leq s<t\leq T$

Where $\mathcal{F}_t$ represents the information available at time t, and $\{\mathcal{F}_t\}_{0\leq t\leq T}$ represents the process of information evolution in a timely manner.

A sequence formed by random variables (r.v) can be regarded as a stochastic process. The r.v itself can be discrete or continuous. In order to distinguish them in the following content, a discrete r.v. will be used as X and a continuous r.v. will be used as Y. For a discrete r.v, it has a probability mass function (p.m.f) $f_x: \Omega \rightarrow R_{\geq 0}$ and all the possible outcomes will assign to 1, writing in a mathematical expression $\sum_{\Omega} f(x) = 1$.

For a continuous r.v, it has a probability distribution function which mapping the r.v to the Euclid space $f_Y \Omega \rightarrow R_{\geq 0}$. But as regard all the possible outcomes the mathematical expression has some deviation compared to discrete r.v. Instead of adding all the outcomes, the non-dividable attribution of continuous r.v requires integration to express as $\int_{\Omega} f_Y(y)\, dy = 1$.

To calculate probability and expectation of event for each circumstance the expression is following:

$$P(A) = \sum_{x \in A} f_x(x) = \int_A f_y(y)\, dy$$

$$E = \sum_{x \in \Omega} x\, f(x) = \int_{\Omega} y f(y)\, dy$$

Through this expression, the contribution of Lebesgue integral can be concluded as providing an innovative method of integration regarding the whole relative abstract probability space as an expansion regarding integration on intervals.

For two random variables $x_1\ x_2$, if they satisfy the following expression then their relationship can be justified as independent.

$P(x_1 \in A\ \& x_2 \in B) = P(x_1 \in A) * P(x_2 \in B)$ for all events A & B

3.1.4 Stochastic process and Markov chain

For a stochastic process, it can be regarded as a collection of r.v indexed by time or in a mathematical way a probability distribution over a space of paths. A fundamental category of stochastic processes is simple random walk. Let Y_i be an identical independent distribution (iid) r.v with equal probability to be 1 or -1 at any time. Define another variable X and let t represents the time. Then let $X_t = \sum_{i=1}^{t} Y_i$ and suppose $X_0 = 0$. Then the stochastic process formed by X is a one-dimensional simple random walk. This simple random walk has following characteristics:

I. $Ex_k = 0$
II. Independent increments
III. Stationary: for all $h \geq 0$, $t \geq 0$ the distribution of $x_{t+h} - x_t$ is the same as the distribution of x_h

A common example of the simple random walk is tossing a fair coin. Suppose two players A and B are playing a coin tossing game. With equal probability of head and tail. If it is a head, then player A wins \$1 from B and otherwise, player A loses \$1 to B. Then the balance of each player starting from \$0 can be regarded as an example of simple random walk. So for each player, the probability of player quit playing this game when his balance first reaches \$100 or -\$100 is $\frac{1}{2}$ by symmetry.

If we examine the situation of account balance of each player, the new account balance of the player will only depend on current balance and the situation of next tossing rather than previous situation. And this kind of simple random walk is a Markov chain which has a distinctive attribution that a stochastic process whose effect of the past on the future is summarised only by current state. A discrete time stochastic process $x_{0,}x_{1,}x_{2,} \dots x_n$ is a Markov chain if

$$P\left(x_{t+1} = s | x_0, x_1, x_{2,\dots} x_t\right) = P(x_{t+1} = s | x_t)\ \forall t \geq 0\ \forall s$$

The next definition is stopping time which has a very important application in financial investment field. This can be understood as an investment strategy as when to sell the shares and exit the market in order to achieve the predetermined target return. For a stochastic process formed by discrete r.v $\{x_1, x_2, x_3 \cdots x_n\}$, a non-negative integer r.v 'τ' is called a stopping time if ∀ integer k≥0 τ≤k depends only on $x_1, x_2, \cdots x_k$.

The example could refer to the coin toss game as well, let τ be the first time when the balance is \$100 or -\$100 then τ is a stopping time. However, if τ is set as the first peak of the balance then it is not a stopping time as the peak of the balance can only be known when a player loss for the first time and when the lose occurred the peak was just a past.

3.1.5 The Brownian Motion

For the real condition, especially applying stochastic knowledge to describe stock price behaviour, the discrete time stochastic process is not adequate since the stock price are commonly regarded as a function over time which plays as a continuous factor that make the description at singular point become meaningless. Under this condition, the requirement to introduce a continuous time stochastic process is essential. Mathematically, such a category of stochastic process are written in the form: $\{x_t\}_{t\geq 0}$.

To develop a model that could describe the randomness of share price behaviour, a model which was developed in the first place to describe the pollen particle movement in water was utilised as one of the most well-known models in this field. Brownian Motion was developed by the botanist Robert Brown back in 1827. The Brownian motion was first introduced to the area of financial analysis by Louis Bachelier in 1900. In his paper 'Théorie de la spéculation', he analysed the price of option and corresponding underlying stock by modelling their trends as a Brownian motion. Five years after his paper, Albert Einstein explained the Brownian motion through a physical perspective that indicate the random movement of pollen was caused by the collision with the surrounding water molecule. Compared to the development of Brownian motion theory in physics, Brownian motion's mathematical development was relative slower. The rigorous definition and description of Brownian motion was stated by Norbert Wiener in 1918. Hence, Brownian motion is also known as Wiener process.

A stochastic process $W = (W_t)_{t\geq 0}$is called a standard Brownian Motion if

- $W_0 = 0$
- (W_t) has independent increments for example, for $0 \leq t_1 \leq t_2 \leq \cdots \leq t_m$ W_{t1}, W_{t2}-W_{t1}, W_{t3}-W_{t2}, …, W_{tm}-$W_{tm-1}$are independent
- For $0 \leq s < t$, $W_t - W_s \sim \mathcal{N}(0, t-s)$

Brownian motion has many intriguing properties that have important meaning for describing stock price behaviour.

1. An individual path of a Brownian motion will cross the t-axis frequently,
2. At any time t, the position $w(t)$ will not deviate too much from $\pm\sqrt{t}$ (positive & negative standard deviation)
3. Suppose $M(t)$ is the maximum value of a Brownian motion from interval 0 to t, $M_{(t)} = \underset{0\leq s\leq t}{max}\, w_{(t)}$, then the probability of $M(t)$is larger or equal to a set value α equals to two times of the probability of $w(t)$ is larger or equal to α, in a more direct mathematical expression:$P(M(t) \geq \alpha) = 2P\left(w_{(t)} \geq \alpha\right)$.
4. Even though the Brownian motion is continuous, but it is nowhere differentiable (this attribution is very essential)

The property 1&2 can be easily understood by applying them into a case of daily stock price description. Suppose s_0 denotes the opening price of a stock in a trading day. Property 1&2 indicates that the following stock price on this day will move up or down based on a horizontal axis equals to s_0 rather than staying on the top or the bottom of the opening price. At a specific time t, the stock price will not deviate too much from $s_0 \pm \sqrt{t} \cdot \sigma$ (σ represents the standard deviation of stock price movement). This implication is so vital for the high-frequent speculative traders since these properties indicates that there is a very high probability that stock price will change constantly over a set price rather than changing only on one side of it.

The property 3 provides the quantified model of Brownian motion's extreme value. Since $w_{(t)}$ satisfies a normal distribution with mean 0 and standard deviation t, the property 3 can be proven through the attributions of Markov chain.

To proof the property of Brownian motion with the help of Markov chain attributions, the relationship between them must be addressed. Suppose in a close interval [0,1] equally divided this interval into n non-intersective intervals. Let $Y_0, Y_1, Y_2, \ldots Y_n$ be a simple random walk in corresponding interval. Let Z denotes a specific path of this random walk, then Z can be expressed as $z\left(\frac{t}{n}\right) = Y_t$, then linearly extend in intermediate value of this discrete stochastic process. Taking $n \to \infty$, the resulting distribution of this Markov chain is the Brownian motion.

With this relationship, the proof of property 3 would be reasonable.

Proof:

Let τ_α (stopping time) represent certain investment strategy indicates a value of the Brownian motion $w_{(t)}$. So the first time when the path hit α can be written as:

$$\tau_\alpha = \min_t \{w_{(t)}\}$$

Through the symmetric attribution of the Markov chain, the probability of the price that will go up or down is the same at τ_α.

$$P(w_{(t)} - w_{(\tau_\alpha)} | \tau_\alpha < t) = P(w_{(t)} - w_{(\tau_\alpha)} < 0 | \tau_\alpha < t)$$

Hence for the maximum value $M(t)$:

$P(M(t) > \alpha) = P(w_{(t)} - w_{(\tau_\alpha)} > 0 \cap \tau_\alpha < t)$+ $P(w_{(t)} - w_{(\tau_\alpha)} < 0 \cap \tau_\alpha < t)$

=2 $P(w_{(t)} - w_{(\tau_\alpha)} > 0 \cap \tau_\alpha < t)$

Since the value of the Brownian motion at time τ_α is α, then $w_{(\tau_\alpha)}$= α, substrate this in above equation:

$P(M(t) > \alpha)$= 2 $P(w_{(t)} - \alpha > 0 \cap \tau_\alpha < t)$

Based on the continuity of the Brownian motion, if the final value is higher than α, then the path must hit α before, hence the condition that $\tau_\alpha < t$ must occur and have no necessity to stay in the equation. Given the final equation:

$P(M(t) > \alpha)$=2 $P(w_{(t)} > \alpha)$

If the Brownian motion is used as the description for stock price behaviour, then property 3 can be used to quantify the probability distribution of the extreme value of stock price. This is particular helpful for risk management.

The last property of Brownian motion is a non-trivial property which indicates that even though it is continuous but it is nowhere differentiable. It is completely different from any smooth and continuous functions in classical calculus. Hence, the property 4 make the most commonly used analytical tool, classical calculus, failed to work. Researchers dedicated countless effort to find a simple understanding and practical stochastic process but without adequate method to study it. This situation was significantly changed after Japanese mathematician Ito Kiyoshi developed his Ito calculus. In a word, Ito calculus formed the foundation of modern financial mathematics.

3.1.6 The Quadratic variation

Consider a closed time interval $[0,T]$ and divide it into the following form: $\{0=t_0 < t_1 < t_2 < \cdots < t_N = T\}$, then for any continuous function $f(t)$, its quadratic variation is defined as

$$\sum_{i=0}^{N-1} [f(t_{i+1}) - f(t_i)]^2$$

For any function that is continuous and differentiable in $[0, T]$, the mean value theorem from classic calculus gives following inequality:

$$\sum_i [f(t_{i+1}) - f(t_i)]^2 \leq \sum_i (t_{i+1} - t_i)^2 f'(S_i)^2$$

$$\leq \max_{s\in[0,T]} f'(s)^2 \cdot \sum (t_{i+1} - t_i)^2$$

$$\leq \max_{s\in[0,T]} f'(s)^2 \cdot \max_i \{t_{i+1} - t_i\} \cdot T$$

This indicates that when the time interval $[0, T]$ being chopped into more intervals, $\max_i \{t_{i+1} - t_i\}$ tend to be 0, hence the quadratic variation of this continuous and differentiable function is 0.

Then what if substituting $f(t)$ by $w_{(t)}$? Will the quadratic variation of the Brownian motion shares the same story? The fourth property of Brownian motion indicates that it is continuous but not differentiable. So regarding the quadratic variation, Brownian motion has the following circumstance:

With more intervals on $[0, T]$ has been divided, each interval become smaller and smaller causing $\max_i\{t_{i+1} - t_i\}$ tend to be 0, and the quadratic variation of $w_{(t)}$equals to T, that is:

$$\lim_{|\Pi|\to 0} \sum_i [w(t_i + 1) - w(t_i)]^2 = T$$

$$|\Pi| = \max_i\{t_{i+1} - t_i\}$$

This can be proven by the law of large number regarding random variables with identical independent distribution. In a word, as a stochastic process, the quadratic variation of Brownian motion is T rather than 0.

For a Brownian motion, $[w(t_i + 1) - w(t_i)]^2$ denontes the squared position difference between two specific time and the quadratic variation is the cumulative summation of those squared position differences. For a ordinary continuous function, when the intervals are chopped to be finer and finer, its quadratic variation tend to be 0. However, for the Brownian motion, its non-zero quadratic variation indicating its position change is so frequent that no matter how small those intervals being created, the cumulation of the squared position difference will never disappear, it will not become 0 but the length of the interval T.

Hence the formula of the quadratic variation of a Brownian motion can be written in the form of an infinitesimal difference $(dw)^2 = dt$ or $(dB)^2 = dt$. Since Brownian motion is also called Wiener process, using the letter Bor w has the same meaning. The quadratic variation has an important meaning in Ito calculus which will be covered later.

3.1.7 The description of share price by using the geometric Brownian motion

After introducing the standard Brownian motion which is a normal distribution with mean 0 and standard deviation t for any length of time t. Now consider adding a drift μt which is only related to time t and a scale parameter σ resulting in a Brownian motion with drift, denoted in following expression:

$$x(t) = \mu t + \sigma B(t)$$

At any time t, it satisfies a normal distribution with mean μt and variance $(\sigma^2)t$. Considering the infinitesimal form, it can be expressed as:

$$dx(t) = \mu\, dt + \sigma dB(t)$$

This is a stochastic differential equation is an expansion of regular differential equation but contains at least one stochastic process. A notable point is that even though the Brownian motion is nowhere differentiable, $dB(t)$ still has a meaning. It represents the change of the Brownian motion within an infinitesimal time interval.

The Brownian motion $x(t)$ with a drift and a scale parameter is still not the best option to describe the stock price behaviour since the value of $x(t)$ or $B(t)$ can be negative regarding the changes in t. But in reality, the stock price can never be a negative figure. Hence to better fit the realistic circumstance, $x(t)$ are chosen to describe the rate of return.

Suppose $s(t)$ represent the price of stock, then $ds(t)$ indicate the change of stock price in an infinitesimal time interval. Then the rate of return in this time interval can be written as

$$ds(t) \,/\, s(t)$$

Combining both parts:

$$\frac{ds(t)}{s(t)} = \mu\, dt + \sigma dB(t)$$

The stochastic differential equation form is:

$$ds(t) = \mu s(t)\, dt + \sigma s(t) dB(t)$$

Such a form of stochastic differential equation $s(t)$ is a geometric Brownian motion (GBM). GBM is the most favourable model to describe the stock price because:

1. Normal distribution: empirically, scholars found that the continuous compound interest rate is approximately normal distributed
2. Markov process: Based on the attribution of Brownian motion, the stock price that follows the model is a Markov process which indicate that current price contain all the information required to predict future performance which is conform with Weak Form of Efficient Market Hypothesis
3. The nowhere differentiable and non-zero quadratic variation of Brownian motion conforms with the circumstance that rate of return exist sharp turning point regarding time

In order to analyse the behaviour of stock price $s(t)$, the stochastic differential equation must be solved which required Ito calculus.

3.1.8 The Ito Lemma

Brownian motion provides a foundation to study the stock price but regarding the research on financial derivatives, their prices are regarded as a function of stock price. Let $f(B_t)$ is a continuous and smooth function of Brownian motion B_t. In the area of mathematical finance, one of the most important subjects is studying how f changes over a infinitesimal time interval which is the property of df. Based on previous content, the Brownian motion itself is nowhere differentiable which caused makes classic calculus lost its applicability to solve df. But Japanese mathematician Ito Kiyoshi developed the innovative Ito calculus which becomes a game changer and provides an effective tool for stochastic analysis.

First is understanding why classic calculus failed. In order to solve df, applying the chain rule:

$$df = \left(\frac{dB_t}{dt} f'(B_t) \right) dt$$

Due to the property that Brownian motion is nowhere differentiable, derivative $dB_t \,/\, dt$ does not exist which makes the above expression has no meaning which is a failure.

So is there any possibility to bypass $dB_t \,/\, dt$ and only use dB_t? Since previous stated that dB_t has a clear meaning of changes of Brownian motion in a infinitesimal interval. Hence, there is an expression:

$$df = f'(B_t)\, dB_t$$

For this expression, $f'(B_t)$ can be calculated since f is a continuous and smooth function and dB_t is solvable as well. It seems that this expression bypasses the issue that B_t is nowhere differentiable but the whole expression is incorrect.

The whole expression actually come from Taylor expansion, considering the Taylor expansion for a normal function $f(x)$:

$$f(x + \Delta x) - f(x) = f'(x)(\Delta x) + \frac{f^{(2)}(x)}{2}(\Delta x)^2 + \frac{f^{(3)}(x)}{3!}(\Delta x)^3 + \cdots$$

In fact, for an ordinary function, the Taylor expansion does show $df = f'(x)\, dx$. This is because when $\Delta x \to 0$, on the right-hand side, except the first term, all the remaining term all become the high-order minim of the first term. Under this circumstance they can be omitted. Hence the infinitesimal form of the above expression is

$$df = f'(x)\, dx$$

But when $x = B_t$, this property does not hold. Substituting x as B_t for the expansion:

$$\Delta f = f(B_t + \Delta B_t) - f(B_t)$$

$$= f'(B_t)(\Delta B_t) + \frac{f^{(2)}(B_t)}{2}(\Delta B_t)^2 + \frac{f^{(3)}(B_t)}{3!}(\Delta B_t)^3 + \cdots$$

In the above expression, the first term $f'(B_t)(\Delta B_t)$ is important but other terms can not be omitted since the quadratic variation $(dB)^2 = dt$. Because of the non-zero quadratic variation of Brownian motion, the second term is rather not the high-order minim of the first term but the same order which cannot be omitted. So in the infinitesimal form to neglect high-order minim starting from the third term, the basic form of the Ito lemma is:

$$df(B_t) = f'(B_t)\, dB_t + \frac{1}{2} f^{(2)}(B_t)\, dt$$

More generally, if f is a smooth function of time t and some variable x, then the classical partial differentiation is:

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dx$$

If x is the Brownian motion B_t, then the Ito calculus has the form:

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dB_t + \frac{1}{2}\frac{\partial^2 f}{\partial^2 x}(dB_t)^2$$

$$= \left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial^2 x}\right) dt + \frac{\partial f}{\partial x} dB_t$$

Hence, the quadratic variation of Brownian motion requires to add an extra term based on classic calculus to solve df. The extra term is the second derivative of f or the second order partial derivative. This conclusion helps scholars to apply calculus in the analysis regarding stochastic process.

3.1.9 The general form of Ito Lemma

After introducing the Brownian motion with drift and diffusion, the stochastic differential equation of Brownian motion can be written as:

$$dx(t) = \mu\, dt + \sigma dB(t)$$

In the above equation, μ and σ are usually assumed as constants. More generally, the parameter of drift and diffusion can be the function of stochastic process $x(t)$ over time t. Suppose

$\alpha(x(t),t)$ and $b(x(t),t)$ represent the parameter of drift and diffusion [$\mu = \alpha(x(t),t)$, $\sigma = b(x(t),t)$]. Then the stochastic process that satisfy this stochastic differential equation (SDE) is called as Ito drift-diffusion process:

$$dx(t) = \alpha(x(t),t)\,dt + b(x(t),t)dB(t)$$

Let $f\left(x_{(t),t}\right)$ is the second order differentiable function of $x(t)$, hence based on Ito lemma:

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx + \frac{1}{2}\frac{\partial^2 f}{\sigma^2 x}(dx)^2$$

Substituting $dx = \alpha(x(t),t)\,dt + b(x(t),t)dB(t)$ in the expression, neglect all the higher-level minim of dt and eventually the general form of Ito lemma is:

$$df = \left(\frac{\partial f}{\partial t} + \frac{\sigma f}{\partial x}\alpha + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}b^2\right)dt + \frac{\partial f}{\partial x}b\,dB$$

Known from the SDE, as the function of x and t, f itself is a Ito process as well. More importantly, the Ito lemma indicates that the Brownian motion at the right-hand side of the expression is exactly the Brownian motion in the expression of dx. In a simple word, the randomness of f and dx are determined by the same Brownian motion rather than two independent ones. This is essentially vital throughout the deduction of Black-Scholes differential equation.

The idea of stochastic integration comes from the research regarding the changes in share price and corresponding pricing theories. In the following contents, various models regarding different settings under the game theory framework will be introduced.

Previous introduction part has been mentioned the common bargaining models. It evolves from a simple structure which two players bargaining over a total payoff to the complicated supply chain structure. In this chapter, the mathematical calculation regarding those models will be illustrated. Besides the descriptive contents, the analysis regarding the bargaining models would include a majority amount of abstract mathematical expressions over each bargaining situation.

3.2 Bargaining over the partition of the cake

Starting from the most basic bargaining model, two players bargaining over the fixed total payoff regarding the distribution options. Suppose the total payoff (the cake) is π which is larger than 0. The possible set of distribution plan among the two players i & j is $X = \{(x_i, x_j),$

$0 \leq x_i \leq \pi\}$ and $x_j = \pi - x_i$. x_i indicates the share of the cake distributed to the player i. For each distribution plan $x_i \in (0, \pi)$. The corresponding utility function of the player i over its distribution plan is $U_i(x_i)$. The utility of the player is increasing as the share distributed from the total payoff increasing. That is, the utility function $U_i: (0, \pi) \rightarrow R$ is increasing and concave. If both players fail to reach an agreement, then each player will receive the disagreement point d_i & d_j respectively. And the disagreement point is at least equal to the utility of zero payoff, $d_i \geq U_i(0)$. To ensure that there still exist adequate motivation for both players to reach an agreement through the bargaining, it is assumed that there will a mutual beneficial distribution plan for both players compared to their disagreement points $U_i(x_i) > d_i$ and $U_j(x_j) > d_j$.

To define the Nash bargaining solution, the first step is to define the set of possible utility pairs $\Omega = \{(u_i, u_j)\}$ the mapping relationship is that there exist $x \in X$ such that $U_i(x_i) = u_i$ and $U_j(x_j) = u_j$. The range of utility $u_i \in [U_i(0), U_i(\pi)]$. After defining the monotonicity of the utility function, the inverse function of the distribution plan is $x_i = U_i^{-1}(u_i)$. Hence, the utility of player j when the player i has the distribution plan of x_i is:

$$f(u_i) = U_j[\pi - U_i^{-1}(u_i)]$$

Hence, the unique Nash bargaining solution (NBS) is described as the maximization problem of the multiplication of the players' utility functions:

$$\max(u_i - d_i)\left(u_j - d_j\right)$$

There exist a unique solution as the Nash product above is continuous and strictly quasi-concave. And the function f is decreasing and concave. Hence, for both players, they have the motive to reach an agreement since $u_i^N > d_i$ and $u_j^N > d_j$, and the agreement is reach at the point that

$$\left(x_i^N, x_j^N\right) = [U_i^{-1}(u_i), U_j^{-1}(u_j)]$$

The following diagram is using to aid the explanation of exploring the NBS:

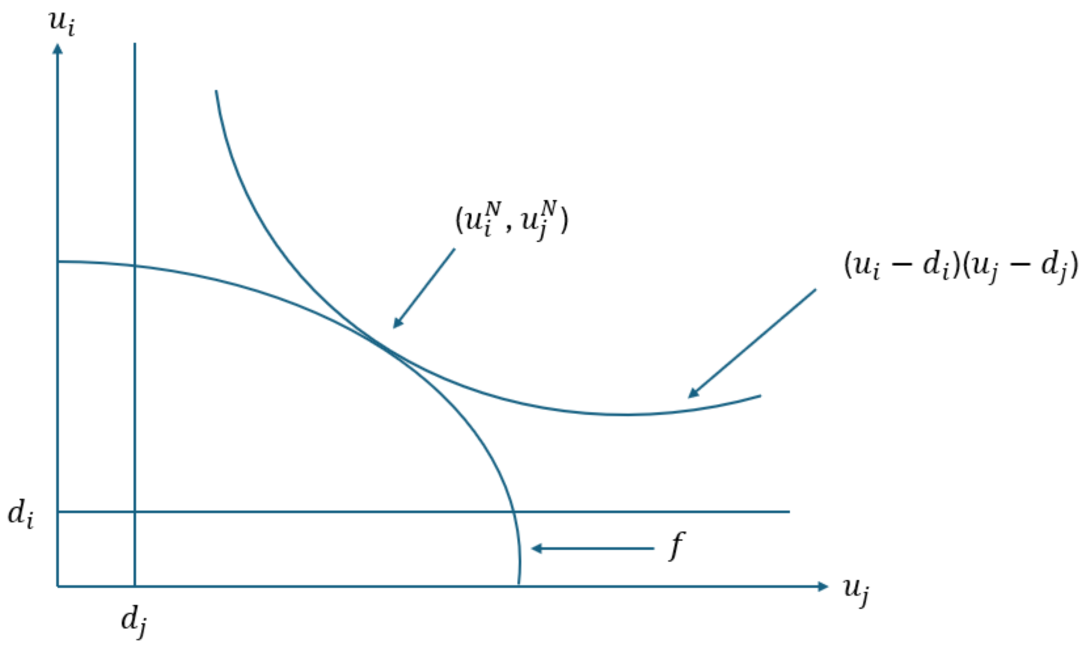

Diagram 1. Nash Bargaining Solution

From this diagram, it can be seen that the Nash bargaining product is higher than the disagreement points of two players which indicates the motive to reach an agreement over the bargaining. The function f is decreasing and concave, while all the Nash bargaining solutions (u_i^N, u_j^N) can be found along the curve.

Hence, it can be saying that the unique solution to the Nash Bargaining can be obtained as long as the function f is differentiable. This solution refers the maximum utility that any party could obtain without hurting the counterparty's profit. Hence, writing in the mathematical way, the unique Nash Bargaining Solution (NBS) is:

$$-f'(u_i) = \frac{u_j - d_j}{u_i - d_i} \ and \ u_j = f(u_i)$$

Since the NBS is one of the solutions that $u_i^N > d_i$ and $u_j^N > d_j$, it can be saying as finding the value of u_i such that maximizes $(u_i - d_i)(f(u_i) - d_j)$. This makes the process of obtaining the unique NBS becomes finding the first order condition (FOC) of the equation.

In the diagram above, the differentiable point is actually the tangent point over the curve, while the unique NBS indicates the unique point u^N is on the curve and the function f has the property such that the line starting from the disagreement point and passes through the point u^N. And at the point of u^N, the unique tangent line intersects with the curve.

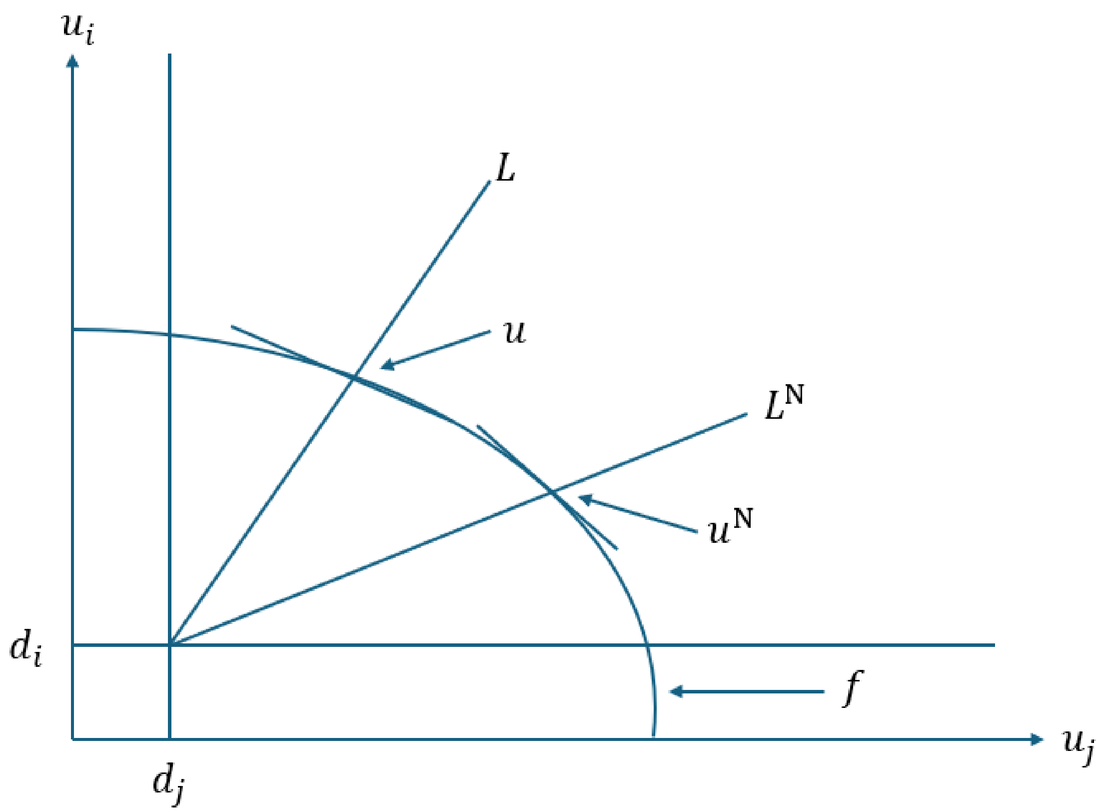

Diagram 2. The illustration of unique NBS

Through the above diagram, it can be observed that the point u is also at the curve and the line L also passes through the disagreement point. But the slope of the line L is much more steeper compared to the line L^N. This means the slope of the line L is increased from the line L^N while the absolute value of the slope of the tangent at the point u is smaller than the absolute value of the slope of the tangent at the point u^N.

With that result, it can be saying that under the Nash Bargaining situation, if the function f is differentiable, then the partition of the cake obtained by the player i, x_i^N is the unique solution to the equation:

$$\frac{U_i(x_i) - d_i}{U_i^{'}(x_i)} = \frac{U_j(\pi - x_i) - d_j}{U_i^{'}(\pi - x_i)}$$

Hence, these are the content of the characterization of the NBS when the function f is differentiable and concave.

Now it is time to expand the characterization of the NBS to a more general definition.

A bargaining problem, is actually, describing a pair (Ω, d), where both Ω & d belongs to the set of real numbers $\boldsymbol{R}$. The meaning of the letter Ω is commonly refers to the possible set of utility pairs that are obtainable through reach an agreement between the participants. And the letter d represents the utility that each participant will receive if the agreement failed to be reached. And the calculation of the unique NBS requires certain assumptions prior to the mathematical analysis.

The first assumption is that the Pareto frontier is the set of feasible utilities Ω that is concave function whose domain is a closed interval and exist possible utilities of both parties reach an agreement and receive higher utility compared to the disagreement points.

The second assumption is that the weakly Pareto efficient utility pairs are closed. If we use the letter Σ to denote all the utility pairs that satisfy these two assumptions. Then the bargaining problem can be described as:

$$\Sigma = \{(\Omega, d) \colon \Omega \in \boldsymbol{R}, d \in \boldsymbol{R} \ and\ any\ pair\ of\ (\Omega, d)\ satisfies\ above\ assumptions\}$$

The Nash bargaining solution (NBS) is the function $f^N \colon \Sigma \to \boldsymbol{R}$, defined as the following. For each utility pairs (Ω, d) that satisfies above assumptions, the NBS $f^N(\Omega, d) = (f_i^N(\Omega, d), f_j^N(\Omega, d))$ is the unique solution to the maximization problem:

$$argmax\ (u_i - d_i)(u_j - d_j)$$

3.3 Inventory games with fixed unit cost

The inventory games comprise competition between either horizontal levels regarding various retailers or vertical levels regarding different participants. The example of competition in horizontal channels was developed based on a single-period context game theoretic model between two players (Parlar, 1988). In this model, the products are sold by two retailers. Those products are substitutable while those retailers choose their quantities sold at the same time to maximise their expected profits. If the profit is denoted as π, and corresponding quantities supplied are denoted as u & v, then those two retailers' profit are: $\pi_1(u, v)$ and $\pi_2(u, v)$. Then the objective of the first retailer is:

$$\pi_1(u,v) = (s_1 + p_1)\left[\int_0^u xf(x)dx + u\int_u^{\infty} f(x)dx\right] - p_1E(X) + q_1\int_0^u (u-x)f(x)dx$$
$$+ (s_1 - p_1)\int_0^u \left[\int_v^B b(y-v)g(y)dy + \int_B^{\infty}(u-x)g(y)dy\right] f(x)dx - c_1u$$

The functions $f(x)$ and $g(y)$ are the demand density functions of demand faced by each retailer, a & b where $(a \geq 0, b \leq 1)$ are the substitution rates between two retailers' products. The expressions s_1, c_1, q_1, p_1 represent the unit selling price, purchase cost, residual value of unsold product, and shortage penalty cost for the first retailer's product. And $B = \left[\frac{u-x}{b}\right] + v$ while $A = \left[\frac{v-y}{a}\right] + u$. For this model proposed, Parlar proved the existence and the unique Nash equilibrium. The result illustrates that the cooperation between those retailers could lead to higher profit for both of them which creates the motivation for cooperation.

Another famous model regarding the single-period inventory procurement model comprising multiple competing retailers is proposed by Nti. The model considers the stockpilers of commodities in short supply under the game-theoretic ramifications (Nti, 1987). Under the single-period demand condition, it is assumed that a retailer j faces the demand:

$$D_j = g_j(P) + \xi_j$$

The component $g_j(P)$ is the deterministic component of the demand when the price of the product is setting at P. While the other component ξ_j has the range $[0, \bar{x}_j]$ which refers as a nonnegative random variable. It has the cumulative distribution and probability density functions as $F_j(\cdot)$ and $f_j(\cdot)$ respectively.

At the beginning of the period, each retailer has to order a nonnegative quantity I_j from the seller in anticipation of the demand based on the price setting P_j. Hence, for each retailer, the strategic option becomes demand management and stockpiling in case of a crisis. Assuming the purchasing or procurement cost of each unit of the commodity is depending upon the aggregated quantity ordered by all the competing retailers, then the unit purchasing cost is denoted as $c(I)$ where $I = \sum_{j=1}^{n} I_j$. The unit purchasing cost will never be negative, twice differentiable function of the I will be increasing and convex based on observation regarding practical situations. Not only the total demand will affect the unit purchasing price, the crisis itself will exert influences over the cost. Hence, for each fixed I, $c(I, s)$ tends to be increasing as the seriousness of the crisis level s increases.

Under the rational expectation, it is assumed that the retailers of the stockpile act to maximise the expected profits. Suppose the retailer j sets the price level of P_j and orders the quantity as I_j, the remaining retailers order I_i, $i = 1, \dots, j-1, j+1, \dots, n$. Let $I = \sum_{j=1}^{n} I_j$, then the social profit Π_j of the retailer j given the realized demand is D_j is given as:

$$\Pi_j(D_j, P_j, I_j, I) = P_j D_j - C(I)I_j - h_j(I_j - D_j) \; if D_j < I_j$$

$$= P_j I_j - C(I)I_j - r_j(D_j - I_j) \; if \; I_j < D_j$$

To get this expression, it is assumed that the retailer holds zero inventory at the beginning of the period. Hence, it will incur the holding cost h_j if the quantity ordered exceeds the realized demand. On the other hand, if the quantity order can not satisfy the realized demand, then it will incur the shortage cost r_j.

The expected profit for the retailer j when he chooses the strategy (P_j, I_j) and the total inventory requirement is I is denoted as $\Pi_j(D_j, I_j, I) = E\Pi_j(D_j, P_j, I_j, I)$. The expectation is taken vis-à-vis to the demand distribution D_j, then:

$$\pi_j(P_j, I_j, I) = P_j g_j(P_j) + P_j \mu_j - c(I)I_j - (P_j + r_j)\int_{z_j}^{x_j}(x - z_j)f_j(x)dx - h_j\int_0^{z_j}(z_j - x)f_j(x)dx, \qquad j = 1,2, \dots, n$$

$\mu_j = E(\xi_j)$ and $z_j = I_j - g_j(P_j)$.

In this paper, Nti showed that there exists a unique Nash equilibrium. Following the similar thought, scholars consider a competitive version of the classical newsboy problem in which the company needs to come up with the optimal solution regarding the inventory and production level for a perishable product facing the random demand. The splitting rule clarifies how initial demand is allocated among various retailers and how any excess demand is allocated among the retailers with remaining inventory. The result illustrates that if all excess demand is reallocated, then competition never leads to a decrease in the inventory level (Lippman & McCardle, 1997). Similar research has been conducted by other scholars as well, under a single-period inventory problem comprising three retailers who aims to determine the personal optimal order quantity to maximise personal profit. It has been shown that the Nash equilibrium exist when each retailer is making their decision independently. The optimal order quantity would be larger than the result under the newsboy model. The reason is because that substitution could be a source of sales which gradually becomes the justification for inventory orders (Wang & Parlar, 1994). Expanding the model to a more general situation, researchers consider the competition among multiple firms providing competing, substitutable products. The customers choose dynamically based on the current available products in the market which leads to the outcome that inventory levels at one firm will affect the demand of all competing firms. Based on previous stated framework, researchers modelled the demand in a more realistic style by incorporating the thought of stochastic sequence of heterogeneous consumers who dynamically make decisions regarding available products as per utility maximization criterion. The result demonstrates that there is a bias towards overstocking because of the existence of competition. To be more precise, reducing the quantity stocked at any equilibrium of the game will increase total systematic profits, and at each individual joint-stocking levels, each company has the incentive to increase its own stock (Mahajan & van Ryzin, 2001).

This category of modelling is commonly referred as the decentralized distribution system which each retailer make independent decision regarding the stock level and the decision make might act intertwined with other participants to exert influence over the final outcome. For the research regarding this category, researchers propose a general framework to conduct efficient research regarding multiple retailers who face stochastic demands and are required to take strategic decisions regarding stock level. The general framework is divided into two stages. In the first stage, or in another name, the non-cooperative stage, each retailer makes the decision over the order quantity to satisfy the individual demand. While in the second stage (cooperative stage), those retailers reallocate products for the residual demands and allocate the corresponding incremental profits. For the first stage, researchers develop various conditions to guarantee the existence of the Nash equilibrium, and for the second stage, the idea of core for allocation of profit would ensure the occurrence of profit reallocation (Anupindi, et al., 2001). Based on this framework, researchers further extend the research regarding a three-stage model. The first and the third stages remain the same compared to Anupindi's work, but for the second stage, each retailer decides the amount of residual demand that he wants to share with other competing retailers (Granot & SoSic, 2003).

A few other papers also propose a cooperative inventory system. For the allocation of joint inventory control costs among multiple consumers of a single supplier, researchers prove that the centralization is always beneficial (Gerchak & Gupta, 1991). The centralized inventory model is working as the following:

$$C(\hat{Q}, \hat{r}) \leq \sum_{i=1}^{N} C_i(Q_i^*, r_i^*)$$

The $C(Q, r)$ represents the inventory relevant costs including ordering, holding and shortage costs while Q_i^* indicates customer i's quantity and r_i^* refers to customer i's optimal recorder point. Hence, $\hat{Q} = \sum_{i=1}^{N} Q_i^*$ and $\hat{r} = \sum_{i=1}^{N} r_i^*$. Researchers demonstrated that the control costs for the model has the feature of super additive. However, some researchers propose that customers not necessarily benefit from the centralized distribution system which indicates the best of allocation approaches might be unstable (Robinson, 1993). In his work, Robinson proposes the idea of Shapely value which is characterized by a collection of desirable properties as the scheme of allocating to satisfy the stability such that the cost allocated to customer i as

$$X_i = t^{-1}[C_{\{i\}} + \sum_{S \subseteq T \setminus \{i\}} \frac{C_{S \cup \{i\}} - C_S}{\binom{t-1}{|s|}}]$$

In this expression, the set of t customers is denoted by $T = \{1,2, \dots, t\}$, $S \subset T$ represents a non-empty subset of T, C_S is the joint control cost for the subset S, while $|s|$ is the cardinality (the number of elements) of the subset S. From the above expression, the second term can be regarded as the Shapely value and the term C_S could be interpreted as the characteristic value of the coalition S.

Regarding the evaluation of cost allocation methods, researchers propose three necessary criteria – stability (core of a related cooperative game), justifiability (consistency of benefits with costs), and polynomial computability. The predominant cost allocations methods have been investigated and scholars present a method that is possible to meets all three criteria that could facilitate manager to evaluate the trade-offs in selecting an allocation scheme for the cost of inventory centralization (Hartman & Dror, 1996).

As indicated in above content, the mathematical model regarding the strategic inventory selection when retailers facing the stochastic demand can be divided into two ramifications. The first category refers to the competing situation among retailers, each of them independently and individually make decisions regarding order quantity under the anticipation of demand.

The research concerning this category primarily focus on the exploration of unique Nash equilibrium and potential cooperation between retailers in the case of profit reallocation. While for the other category which primary concentrate over the centralized inventory system. For this category, the primary research question is about finding the appropriate cost allocation methods that could satisfy multi-dimensional requirements.

3.4 Inventory games with quantity discounts

The quantity discounts refer to the condition that the unit cost of the product will decrease as the quantity ordered increases. Such a policy usually introduced by the seller to motivate retailers to order more products. The discussion regarding this topic becomes rather popular in recent years and several famous papers have been introduced to discuss this topic.

The conventional quantity discount problem is studied through the perspective of game theory, comprising both the points of view regarding noncooperative and cooperative models. For the noncooperative perspective, the Stackelberg equilibrium is conducted while for the cooperative perspective, the Pareto optimality criteria are adapted to find the set of optimal strategies (Chiang, et al., 1994). The price quantity discount policy itself has been shown to provide opportunities for the buyers rather than explicit discount schedule itself. Based on this attribution, researchers propose a taxonomy of price quantity discount schedules and examine the implications of various price quantity discounts for the ordering behaviour of the buyers and the corresponding formation of the alternative channels of distribution (Wilcox, et al., 1987).

The fundamental framework of the inventory games with quantity discounts can be traced back to Monahan's work which develops and analyses a discount model of determining the optimal quantity discount schedule for a vendor (Monahan, 1984). In this paper, Monohan proposes a model comprises a vendor and a buyer under a sequential-move Stackelberg model. It assumes the vendor requires the buyer to increase the order size by a factor K ' and performs the analysis to determine the response of the buyer. The total annual demand faced by the buyer is defined as D_1. The S_1 & S_2 are the fixed order processing cost for the buyer and vendor respectively. Q_1 is the buyer's current order size, and H_1 is the buyer's annual inventory holding cost which is a percentage of the value of the item and P_1 is the current price paid by the buyer. With these expressions, the vendor's annual net profit becomes:

$$D_1(M_2P_1 - d_k) - (\frac{D_1}{Q_1k})S_2$$

The M_2 refers to the vendor's gross profit based on sales revenues. The d_k is the break-even price discount with detailed expression as $\left(\sqrt{\frac{2S_1H_1P_1}{D_1}}\right)(K-1)^2/2K$. Then vendor's optimal value for the factor K is expressed as:

$$K^* = \sqrt{[(2D_1S_2)/(Q_1\sqrt{2D_1S_1H_1P_1})]} + 1$$

As the early research regarding the quantity-discount policy, Monahan's work has important contribution to the later research. However, there still exist certain disadvantages that make the results conducted become unpractical (Joglekar, 1988). However, the shortcomings in the assumptions do not shade the light of Monohan's work which provides an introductory model for this area. Another note regarding Monohan's work proposes extensions regarding the model by incorporating vendor's inventory carrying costs (Banerjee, 1986). These papers build a solid foundation for later research regarding the topic of quantity discount.

To generalize the model proposed by Monohan to a practical situation which comprises inducing the buyer to alter the order schedule and size so the supplier could benefit from the lower set up, ordering, and inventory holding costs, researchers extend Monohan's work by addressing two vital issues. The first issue is imposing certain restrictions on the amount of price discount, so the discount is less than the selling price of the product. The second issue is relaxing the implicit assumption of a lot-for-lot (or order-for-order) policy adopted by the seller. With these assumptions, researchers successfully develop the optimal discount schedule for the seller under the general context (Lee & Rosenblatt, 1986). A much simpler method of understanding Lee and Rosenblatt's work is proposed by Goyal to simplify the calculation of the overall procedure (Goyal, 1987). Based on the same problem as Monohan proposed, Lal and Staelin discuss the rationale and appropriate actions that should be taken by the seller to develop the discount pricing structure. The model is extended to resolve conflicts between variable ordering and shipping costs when the seller faces various groups of buyers, each of them with distinctive ordering policies (Lal & Staelin, 1984).

Following the framework of Lal and Staelin's model, researchers further examined the cooperative game theory model of the quantity discount to analyse the determination of discounts offered in the bargaining context based on the consideration of transaction-efficiency. Under the model, the buyer and the seller negotiate over the quantity and the average unit price. Researchers applied the Pareto optimality approach to study the behaviours of participants throughout the negotiation process (Kohli & Park, 1989). Like the conventional inventory game, the quantity discount game has also two perspectives: cooperative and noncooperative games. Researchers studied the effects of quantity discount over supplier's profit and buyer's cost both under competitive and cooperative contexts. The research studies supplier's behaviour of setting the discount schedule under the assumption that the buyer always act rationally to determine the economic order quantity (EOQ) which refers to the optimal order quantity that meets the demand while minimizing the total cost related to the ordering, receiving, and holding inventory (Kim & Hwang, 1989). The research regarding the quantity discount model agrees that it acts as a mechanism of coordinating channel members if the effect of the quantity discount policy remains rather independent.

The studies that propose the idea that the quantity discount could further influence the demand of the customer comes from Parlar and Wang. They investigated the discounting scheme of the seller and a linear ordering decision from a group of homogeneous customers under the framework of the game theory. They assumed that the seller's quantity discount policy could exert influence over the buyer's demand and they applied the Stackelberg model to conclude two results. The first result is the gains from the discount schedule will provide incentives for the seller to propose the discount policy which encourages the buyers to order more compared with the EOQ. The second result illustrates that the benefit from the discount policy comes from the decrease in the inventory-related costs and the resulting incremental market demand (Parlar & Wang, 1994). This research can be regarded as the study of the joint decision policies of the quantity discount topic.

In later research, a new model is presented by Weng to study the impact of joint decision policies over the channel coordination of the supply chain that includes one supplier and a group of homogeneous buyers (Weng, 1995). In this model, the annual profit of the supplier and the buyer are:

$$G_s(p{:}\,x, Q) = (p - c)D_{(x)} - \frac{S_s D_{(x)}}{Q} - \frac{1}{2}h_s Q$$

$$G_b(x, Q; p) = (x - p)D_{(x)} - \frac{S_b D_{(x)}}{Q} - \frac{1}{2}h_b Q$$

In the above equation, p denotes the unit purchase price paid by the buyer to the supplier, x & Q are the buyer's selling price and order quantity respectively, $D_{(x)}$ is the annual demand conditional to the selling price charged by the buyer, h_s & h_b are the holding cost per year of the seller and buyer, S_s & S_b are the supplier's and buyer's fixed cost per order.

Utilizing this model, Weng proved that the quantity discount itself is not sufficient to guarantee the joint profit maximization and all-unit and incremental discount policies have the identical influence over the coordination given the complete information.

Different from the previous stated inventory game, the appearance of the quantity discount incorporate the thought from the seller of promoting selling by giving a favourable price to the buyer if the quantity ordered exceeds the EOQ. The inventory games with the quantity discount incorporates more considerations from both the supplier and the buyer compared to conventional inventory vendor framework. With these models, the research regarding the game theory further developed to include more considerations regarding the channel coordination rather than simple individual conditions.

3.5 Production and pricing competition

Ever since the emergence of the game theory, it has been widely applied in the production and pricing competition situations. With the development of the supply chain concepts, the production and pricing decisions play a vital role in the profitable operation.

The early publications regarding the production and pricing competition starting from Cournot and Bertrand. Their names become famous across the field of research regarding this topic due to the models proposed. Their works were finished in the 19th century, however, their models has been continuously applied and evolved in the field of research. Cournot develops the production equilibrium model in a market with two producers supply similar products to the same market while Bertrand concentrates on the pricing equilibrium.

In the Cournot model, the quantities chosen by firms 1 & 2 is denoted by q_1 & q_2 respectively. The aggregate demand Q is the summation of all products provided $Q = q_1 + q_2$. The underlying assumption is that all the products manufactured by the firms can be sold to the customers. For an individual company i, its corresponding total cost of producing certain quantity of products is $C_i(q_i) = cq_i, i = 1,2$. The lower capital c represents the marginal cost and the price charged when there is Q units of products available is $p(Q) = a - Q$ $(Q < a, c < a)$. Hence, the profit function of each firm is:

$$\pi_i(q_i, q_j) = q_i[p(q_i + q_j) - c] = q_i[a - (q_i + q_j) - c]$$

For this model, the two firms' best response functions are:

$$q_1 = \frac{1}{2}(a - c - q_2), and\ q_2 = \frac{1}{2}(a - c - q_1)$$

By resolving these two equations, the Nash equilibrium (q_1^N, q_2^N)under the Cournot model is

$$q_1^N = q_2^N = \frac{1}{3}(a-c)$$

On the other hand, the Bertrand's model, two companies choose their prices p_1 & p_2 independently at the same time. And the market demand q is allocated to the company who charges the lower price. It is assumed that the demand is a linear function of the prices charged by those two companies: $q = a - p_1 - p_2$ where $a \leq p_1 + p_2$, $c \leq p_1$ and $c \leq p_2$. Hence, the profit function for each firm in the Bertrand model in terms of prices p_1 and p_2 are:

$$\pi_i(p_i, p_j) = (p_i - c)(a - p_1 - p_2), if\ p_i < p_j$$

When $p_i = p_j$, then the profit figure is:

$$\pi_i(p_i, p_j) = \frac{1}{2}(p_i - c)(a - p_1 - p_2)$$

And the profit will be zero, if $p_i > p_j$.

For the Bertrand model, the Nash equilibrium (p_1^N, p_2^N) is conducted as $p_1^N = p_2^N = c$. Under this condition, both firms earn zero profit. But in real life, firms compete in prices and could make some positive profits. Sometimes, this deviation between the academic findings and practical situation is commonly known as the 'Bertrand paradox'.

Based on these foundations, the Austrian researcher von Stackelberg extended the Cournot model by assuming the firm 1 is the leader and the firm 2 is the follower. In this leader-follower game, the firm 1 could determine the optimal production quantity q_1 by solving the equation:

$$max\,\pi_1\,[q_1, q_2^R(q_1)] = \max\frac{1}{2}[q_1(a - q_1 - c)]$$

Resolving the above Stackelberg equations, the equilibrium result yields as $q_1^s = \frac{1}{2}(a-c)$ and $q_2^s = \frac{1}{4}(a-c)$.

Here is a table that summarizes the constraints of the production and pricing competition among firms.

Year	Author	Summary
1972	(Levitan & Shubik, 1972)	The equilibria conducted from the Bertrand's and Cournot's models under the assumption of capacity restraints for firms face a linear demand are examined.
1991	(Hviid, 1991)	The paper analyses the effect of uncertainty under the duopoly model which the firms are competing with each other while their capacities are constrained. Researchers demonstrated that there is no pure strategy Nash equilibrium exist in this game.
1991	(Gal-Or, 1991)	In the industry with the competition exists at the level of the wholesale price, the vertical restraints are not necessarily desirable from the perspective of the manufacturer. When the vertical restraints are employed, the form tends to be Franchise Fee Pricing rather than Retail Price Maintenance.
1997	(Butz, 1997)	A manufacturer distributes output before knowing demand and risks making more than its rivalrous

		independent retailers can sell at the monopoly price. The manufacturer controls the vertical relationship with retailers by using many levers (e.g. vertical integration, buyback contract)

Table 2. Representative articles of various strategic variables

Ever since the emergence of the Cournot and Bertrand's models, researchers consistently evolve those models based on practical situations. When the demand is assumed to be a random component, in a duopolistic or in more general speaking, the oligopolistic market which companies provide differentiated products has the competitive effect of increasing stability in the sense that the market without random component could exist no noncooperative equilibrium point while the market with the random component might have a noncooperative equilibrium point (Levitan & Shubik, 1971). With the emergence of the online information servers that provide access to different databases that users could search for, browse through, and download the information, researchers develop a pricing model for this innovative product. It has been found that the pricing strategy adapted by the server are dependent upon the variation in consumer expertise and valuation of information. The result is conducted based on the examination regarding the conditions regarding pricing schemes that including connect-time-based pricing, search-based pricing, and subscription-free pricing are optimal (Jain & Kannan, 2002). To study the effect of business reputation exert over the bargaining procedure, researchers develop the model regarding the repeated-transactions bargaining with two-sided uncertainty. The key question of the research is to determine the appropriate types of reputation that is the best for a buyer or a seller to bring to the bargaining table and their corresponding effects over the equilibrium strategies and payoffs. The results illustrate that the best reputation for the seller to take to the bargaining table is one that makes the buyer nearly certain in the belief that the cost of the seller is high. While counterintuitively, researchers found that an increase in the buyer's reputed willingness to pay could cause the seller to offer a lower price. Such a result shows that the best reputation for the buyer to take to the bargaining table is the one that makes the seller believe that there is a significant chance that he is willing to pay a high price (Banks, et al., 2002).

The aforementioned articles concentrate over the individual decision procedure of certain participant within the supply chain structure. The first publication of the joint decision regarding the channel coordination is the work of Zusman and Etgar. In their paper, they combine the theory of economic contract theory and Nash bargaining theory to investigate the situation of a three-level channel and corresponding equilibrium set of contracts that are attainable (Zusman & Etgar, 1981). For channel situations, researchers tend to compare the effectiveness among various channels by considering various types of channel situations and appropriate strategies. The products provided by different channels have substitutability in the market.

To study the effect of existence of channel intermediary over the intensity of the horizontal competition between two manufacturers. Choi considers three non-cooperative structures including two Stackelberg games and one Nash game between two manufacturers and one retailer (Choi, 1991). Under these three conditions, the manufacturer i's and the common retailer's profit functions are:

$$\Pi_{M_i} = (w_i - c_i)q_i, i = 1,2, and\ \Pi_R = \sum_{i=1}^{2} m_i\, q_i$$

In the above expression, w_i denotes the manufacturer i's wholesale price charged to the common retailer; m_i refers to the retailer's margin over the product which is calculated as the difference between the retail price charged by the retailer against the customer and the wholesale price paid to the manufacturer; c_i is manufacturer i's variable cost of producing the product; and q_i is the demand from the market for the product i given the price of p_i while the other brand j has the price of p_j. The linear duopoly demand function is consistent with the model proposed by McGuire and Staelin (2008) which including the product substitutability denoted by γ (McGuire & Staelin, 2008):

$$q_i = a - bp_i + \gamma p_j$$

The parameters $a, b, \& \gamma$ satisfy that $a > 0$ and $b > \gamma > 0$. In the model, Choi assumes that the cost of manufacturing the product is the same $c_1 = c_2 = c$ and the corresponding Nash equilibrium as

$$w_1 = w_2 = \frac{a + 2bc}{3b - \gamma}$$

$$p_1 = p_2 = \frac{a(2b-\gamma)}{(3b-\gamma)(b-\gamma)} + \frac{bc}{3b-\gamma}$$

With these results, Choi concluded that a manufacturer is better off through maintaining exclusive retailers while in contrary, the retailer would prefer to have multiple manufacturers. Another counter-intuitive result indicates that the more profits will be generated when products are less differentiated. When the demand function is assumed to be non-linear, an exclusive retailer channel provides higher profits for all members.

The research regarding the production and pricing competition is based upon the foundation of Bertrand and Cournot models that were all-time classic in the field of research. Those models have been consistently improved and adjusted to suit different conditions in the supply chain situations. The level of sophistication of model is consistent with the evolution of the supply chain circumstances. The research questions go beyond from any single member's personal profit maximization decision to a channel coordination cooperation.

3.6 Games with other factors

In previous sections, the inventory game models with fixed unit purchase cost, quantity discounts and games with production and price competition have been introduced. However, the game models concern other factors as well. In the following content, different models with distinctive factors will be introduced.

3.6.1 The Capacity Decisions

Given the condition that a supplier's total demand from the retailers that he supplies frequently exceeds his capacity, to resolve this issue, the supplier has to allocate the capacity in certain manners. Researchers consider three allocation schemes: proportional, linear and uniform. Through the exploration of the Nash equilibria regarding this condition, researchers found that the behaviour in this game with either of those allocation schemes can be very unpredictable due to the reason that there might not exist a Nash equilibrium under either proportional or linear allocation. As a result, the retailer with a high need may be allocated less than a retailer with a low need which is an ex post inefficient allocation. But through the uniform allocation, there always exist a unique Nash equilibrium (Cachon & Lariviere, 1999). Rather than an information symmetrical condition, Cachon and Lariviere consider a condition where the optimal stocking levels are private information possess by the retailers. They studied the behaviour of the participants through both the manipulable and truth-inducing capacity allocation schemes. It has been found that a manipulable mechanism may lead to supplier to choose a higher level of capacity than he would under the truth-inducing mechanism (Cachon & Lariviere, 1999). To make the research more close to practical life, Cachon and Lariviere study the behaviours of the suppliers and the retailers under the condition that market demand is a stochastic element. The retailer provides the initial forecast to the supplier along with a contract, then the supplier constructs the capacity, the retailer receives an updated forecast and submit the final order. The model illustrates that the optimal supply chain performance requires the retailer to share the initial forecast truthfully, but the retailer has the motivation to inflate the forecast to induce the supplier to build more capacity. While the supplier is aware of this condition, hence he may not believe the retailer's forecast. As a result this would damage the performance of the supply chain (Cachon & Lariviere, 2001). Motivated by the experiences of a major US-based semiconductor manufacturer, researcher present an integrated model of incentive problems arising in forecasting and capacity allocation. The model involves multiple product managers and manufacturing managers who forecast the means of their respective demand and capacity distributions. There is a central coordinator who is responsible for allocating capacities to product lines. Researchers propose a game theoretic model and design a mechanism that motivates truthful reporting by all managers. The result illustrates that the structure of the truth-eliciting bonus schemes is rather simple with easily calculable parameters. The result also shows that large classes of allocation rules, including the current allocation practice of the firm, are manipulable. The bonus is usually required for elicitation of truthful information (Mallik & Harker, 2004).

The current predominant mathematical model regarding the capacity decisions comes from the article of Hall and Porteus. In their paper, they consider a game where firms compete on the capacity investment for market share. In the model, there is a fixed total market of customers whose demands for the good or service are random and who divide their patronage between the companies in each period. They assume that the expected level of customer service can be expressed as a function of the capacity of the firms' service delivery systems, and that service declines as the capacity decreases (Hall & Porteus, 2000). Based on the assumption with two firms i and j the expected market share of firm i in month $t + 1$ is:

$$E\left(\lambda_{i,t+1} | \lambda_{it}, \lambda_{jt}\right) = \lambda_{it} - \lambda_{it}\gamma_i h_i(y_{it}) + \lambda_{jt}\gamma_j h_i\left(y_{jt}\right)$$

In the above expression, λ_{it} is the fractional market share for firm i in month t; y_{it} is the normalized capacity of firm i in month t; γ_i is the switching rate of customers experiencing service failure from firm i to firm j $(0 \leq \gamma_i \leq 1)$. In their paper, Hall and Porteus denoted the letter μ_{it} as the capacity selected by firm i in month t which is expressed as $\mu_{it} = y_{it}\lambda_{it}$. The letter h is defining as the customer service, $\lambda_{it}h_i(y_{it})$ is the expected number of firm i's customers that experience service failure in month t when firm i has a normalized capacity of y_{it}. The term $\lambda_{jt}\gamma_j h_i(y_{jt})$ refers to the expected number of firm i's customers that switch to firm j in month $t+1$. Then the result derived an appropriate capacity choice and the conditions under which the Nash equilibrium capacity levels scale directly and linearly in the number of customers being served. The model developed by Hall and Porteus also applied for two contexts: competition between Internet service providers and inventory availability competition.

3.6.2 Service Quality

The objective of the supply chain is to deliver products to a consuming market with satisfaction of ultimate consumers. Hence, the consumers not only pay attention to the sale price of the product but also to the service quality. Compared with service quality, product quality is the concept that is easily to be understood. The service quality may involve issues including firm's response time to customer demand, waiting time of customers, after sale service. In order to construct the loyalty of the existing customers and attract more attention from new customers, channel members tend to strengthen their market power through improving the quality of their products and service. Hence, the trade-off between expenditures spending on improving quality of products and service and benefits received are constantly considered by competing firms. In the following content, the game theoretic approaches for service quality are considered.

A company's service speed or commonly referred as response time to customer demand is an important issue implicitly affecting the profitability of the company. The idea of game theory has been applied to service speed decisions of firms. Regarding the study of economic behaviour of vendors of service in competition, researchers propose a simple model with two competing exponential servers and Poisson arrivals is considered. For each server, it is free to choose his own service rate at a cost that is strictly convex and increasing. For each customer served, there is a fixed reward. The model is designed to study the competition in speed of service as means for capturing a larger market share in order to maximize long-run expected profit per time unit (Kalai, et al., 1992). For the condition that customer choice in response to random variation in quality, researcher develops a model to study the behaviour. The choice model yields closed-form expressions which reflect the effect of competing suppliers' service quality on the long-run fraction of purchases a customer makes at the various competitors. Those expressions are used as the foundation of simple normative models for suppliers seeking to maximize the long-term average profits. The results demonstrate that the consumer's switching behaviour forces suppliers to maintain the industry norm that increases with the number of competitive suppliers and a competitor with cost advantage could compete for higher market share by increasing investment for quality improvement (Gans, 2002).

Besides the models regarding the strategic thinking, researchers also consider the effect of service quality in other types of models. In a product life-cycle model which studies a set of strategic choices facing manufacturers as they design the joint product/service bundle for a product which might require maintenance and repair after the sale, researchers conduct their research under the guidance of the Stackelberg theory. The choice parameters of interest including the product price, the quality of after-sales service and the price to be charged for the after-sales service. In this sequential game, the product price and the service quality/price are characterized by an equilibrium result. The resulting outcome is applied to support the valuation of alternative product designs in explicit consideration of the trade-off between profit from product sale and from the provision of after-sale service (Cohen & Whang, 1997). Regarding the two categories of motivations by which a manufacturer can incentivize its retailers to provide high customer satisfaction, the first is the assistance from the manufacturer that reduces the retailer's cost of providing customer satisfaction (CS assistance) and the second is customer satisfaction index (CSI) bonus, researchers found that if a retailer has a long-term orientation, the CS assistance would be a more effective coordination mechanism. But CSI could effectively facilitate the short-term orientated operation. Through the comparison between two types of retailers, researchers illustrate that a long-term orientated retailer is more valuable to a manufacturer than a short-term oriented one. In general, the result shows that the use of customer satisfaction could resulting in both manufacturer and retailer receive greater profits (Chu & Desai, 1995). Taking the perspective of the customer, researcher develops a game theoretic model to study the optimal queuing system for different types of customers competing for service. Different types of customers are divided based on actions when he arrived when the server is busy. The first type of customer will immediately departs while the other type of customer will conduct a retail after an exponential period of time and persists this way until gets served. For both types of customers, the costs for waiting and conducting retrials and wish to find optimal retrial rates are linear. This problem is studied as a two-person nonzero sum game and noncooperative strategies are studied (Kulkarni, 1983).

Even though the idea of incorporating the service quality into the model of strategic thinking is rather distinctive compare to previous sections' contents, the model used to study this issue has no special diversion compared with predominant models employed. The service quality is constantly employed as a particular element rather than a special form mathematical model.

3.6.3 Product Quality

In the above section regarding the effect of service quality exerted over the participants of the supply chain, the product quality and service quality is constantly combined together. If isolate the product quality as a single factor, then the amount of literature is rather limited. As one of the few papers that discuss the product quality under the supply chain structure, Reyniers and Tapiero modelled the effect of contract parameters over the quality of the end product in a vertical channel that contains a supplier and a producer. In their model, the supplier and producer bargains over the price rebates and after-sale warranty for a material to be delivered and parts from the supplier. The bargaining condition in this paper concerning a bi-matrix (A, B) which has single element (a_{ij}, b_{ij}), the subscript i refers to the quality. The number 1 represents the low quality while the number 2 denotes the high quality. The letter j is the producer's decision over whether or not to test the incoming parts (1 for test and 2 for no test). In the bi-matrix, a_{ij} and b_{ij} denote a risk-neutral producer's and supplier's expected payoffs respectively. The bi-matrix is written as:

$$\left(a_{ij}, b_{ij}\right) = (\theta - m - [\pi - p_i \Delta\pi], \pi - p_i(\Delta\pi + C) - T_i), j = 1$$

$$\left(a_{ij}, b_{ij}\right) = (\theta - [\pi + p_i(1 - \alpha)R], \pi - p_i \alpha R - T_i), j = 2$$

In the above expression, the letter θ denotes the producer's profit after subtracting the manufacturing costs, p_i is the probabilities of a defective part with conditions when $i = 1,2$ respectively. m is the cost of testing an incoming part, π is the unit sale price of the producer, $\Delta\pi$ is the reduction in sale price once there is a defective unit, C is the producer's repair cost, R is the post-sales failure cost, α is a parameter in sharing R between producer and the supplier, and T_i is the unit cost of production taken by the supplier such that $T_1 < T_2$. For different values of those parameters, researchers conducted various Nash solutions including one mixed strategy (Reyniers & Tapiero, 1995). Extending Reyniers and Tapiero's work into the condition with incomplete information regarding the quality of the products, Lim developed the first game-theoretic model of quality control which captures this informational asymmetry. The model concentrates on two compensation schemes embedded in the contract, namely the price rebate and warranty. The result shows that when a full-price rebate is not possible and the producer and the supplier has to share the damage cost, an optimal contract is such that the supplier compensates the producer by the same amount, regardless of the product quality. For the behaviour of the supplier, when the quality is low then he is more likely to be offered a contract with an inspection scheme, while under the high-quality condition, a warranty scheme is preferred. Also, when the producer does not need to share the cost in exactly one of the compensation schemes, he might still provide other types of compensation schemes to the supplier depend upon the relative cost involved, the maximum compensation cost acceptable by all supplier types and the ex-ante beliefs about the quality level of the supplier (Lim, 2001).

A research paper that concentrating on the product quality signalling mechanism is written by Chu and Chu, in their paper, they proposed a game theoretical model of the manufacturer selling products through a reputable retailer to signal its product quality. They show that in a 'maximally' separating equilibrium, manufacturers of high-quality products distribute through retailers with strong reputation, while the manufacturers of low quality products distribute through retailers who has no reputation. Such a method, even if high quality manufacturers have no reputation of their own to post as bond, they could signal quality by posting the reputation of the chosen retailers (Chu & Chu, 1994).

3.7 Games with joint decisions

In many practical situations, the members of the supply chain structure would encounter the issue that involves decisions that require two or more decisions that must be made simultaneously. For instance, the price of the product concerns the price decisions from multiple retailers. In the following section, the models concerning the joint decisions will be introduced.

3.7.1 Joint inventory and pricing decisions

In the paper written by Eliashberg and Steinberg, they present a model that considers the interactions between marketing and production decisions in a channel of distribution of industrial goods comprised of a manufacturer and a distributor. The focus point in this article is the dynamic nature of the coordination aspects of different joint policies. The model proposed is providing explicit answers to various questions regarding joint decisions such as the pricing strategies, the nature of the contractual price in the channel, and appropriate conditions for various inventory decisions (Eliashberg & Steinberg, 1987). Regarding the optimal pricing and inventory policies in a supply chain structure concerning joint decisions, researchers adapt both centralized and decentralized decision-making procedures to investigate the effectiveness of each mechanism. The optimal trajectories for inventories, replenishment rates, and retail price are derived through the application of phase diagrams and formal synthesizing procedures under both cooperative and non-cooperative games (Jørgensen & Kort, 2002). For the vendor-buyer system, researcher considers a model for decentralized dynamic production distribution control. Under the vendor-buyer system, the vendor produces a product in a batch production environment and supplies it to a buyer under deterministic conditions. The construction of Nash equilibrium strategies is given in the proof of the existence theorem (Bylka, 2003).

Under a two-echelon distribution system in which a supplier distributes a product to N competing retailers, the demand rate of each retailer depends upon all of the retailers' prices, or the price each retailer can charge for its product depends upon the sales volumes targeted by all of the retailers (Bernstein & Federgruen, 2003). In this paper, researchers first characterized the solution to a centralized supply chain structure. Assuming the linear wholesale pricing schemes by the supplier, this paper investigates the decentralized systems under Cournot and Bertrand competition. In either game, retailer i's profit function $\pi_i(p_i|\boldsymbol{p}_{-i}, w_i)$ with the optimal economic order of quantity replenishment policy is given as:

$$\pi_i(p_i|\boldsymbol{p}_{-i}, w_i) = (p_i - c_i - w_i)d_i(\boldsymbol{p}) - \sqrt{2d_i(\boldsymbol{p})h_i K_i^r}$$

In the above expression, p_i denotes the retailer i's price, while $\boldsymbol{p}_{-i} = (p_1, p_2, \dots, p_{i-1}, p_{i+1}, \dots p_N)$, w_i is the constant wholesale price charged by the supplier to retailer i at per-unit base, c_i is the unit transportation cost from the supplier to the retailer i, h_i is the annual holding cost per unit inventory at retailer i, K_i^r is the fixed cost incurred by delivery process by retailer i and $d_i(\boldsymbol{p}) = a_i - b_i p_i + \Sigma_{j \neq i} \beta_{ij} p_j$ is the demand function for retailer i where the elements a_i and b_i are both positive and $\beta_{ij} \geq 0$. Under the Bertrand price competition, it was shown that if $[d_i(\boldsymbol{p})]^{\frac{3}{2}} \geq \frac{1}{8} b_i \sqrt{2h_i K_i^r}$, then the retailer game has a Nash equilibrium p^*. The authors also found that a similar result would also exist if the game is a Cournot quantity competition. Based on this result, researchers further extend the result to infinite-horizon models under demand uncertainty. To be specific, researchers consider a periodic review which two echelon system with a single supplier servicing a network of competing retailers (Bernstein & Federgruen, 2004).

A rather recent paper concentrating on the allocation of inventory risk in a supply chain is proposed by Gernard Cachon, in his paper, he analysed the issue of allocating inventory risk between the supplier and the retailer through three different types of wholesale price contracts namely push, pull, and advance purchase discount. Under a push contract, there is a single wholesale price and the retailer orders the entire supply before the selling which indicates all the inventory risk is taken by the retailer. The pull contract has a single wholesale price as well, but the supplier will take the inventory risk since the supplier holds inventory while the retailer replenishes as needed during the season. An advance purchase discount is distinctive by having two wholesale prices: a discounted price for inventory purchased before the selling season and a regular price for replenishments during the selling. This would enable the intermediate allocations for inventory risk: The retailer takes the risk on inventory ordered before the selling season while the supplier takes the risk on any production in excess of that amount. Cachon found that the efficiency of a single wholesale price contract is considerably high as long as firms consider both push and pull contracts. Also, if firms could consider advance purchase discounts, then the channel coordination of the supply chain and the arbitrary allocation of the profit is possible (Cachon, 2004).

Regarding the quantity discounts and buyback pricing decisions, Su and Shi developed a game theoretic model to study the optimal behaviour. They incorporated the buyback contracts into the conventional quantity discount problems in a two-stage game which comprises one manufacturer and one retailer. In the first stage two supply chain members determine the inventory level in a cooperative way as

$$Q^* = F^{-1}\{(p + s - m)/(p + s)\}$$

In the above expression, p, s & m denote unit retail price, unit goodwill loss and unit production cost respectively. $F(\cdot)$ refers to the distribution function of the market demand D. In the second stage, the manufacturer bargains with the retailer regarding the quantity discount and return schemes to maintain the channel efficiency. The quantity discount Δw is given as:

$$\Delta w = (w_0 - w^l) - u[\frac{1}{Q^*} E(Q^* - D)^+]$$

The w_0 denotes the baseline wholesale price determined in first stage, and u is the unit buyback price, and

$$w^l = \frac{1}{Q^*}\{pE\{min(Q^*, D)] - sE(Q^* - D)^+ - \pi_r(w_0, \hat{Q})\}$$

$$\pi_r(w_0, \hat{Q}) = pmin(\hat{Q}, D) - w_0\hat{Q} - (Q^* - D)^+$$

$$\hat{Q} = F^{-1}\{(p + s - w_0)/(p + s)\}$$

It has been shown that all feasible set $(\Delta w, u)$ combinations satisfying the Pareto efficiency. The result shows that return policy can be considered as mirror images of quantity discount strategy. This indicates that options with more generous return privileges are coupled with higher wholesale prices, while the lowest wholesale price comes with rather strict limits on returns and a restocking fee for any returned products (Su & Shi, 2002).

3.7.2 Joint inventory and Capacity decisions

Different from the joint decisions regarding the pricing policy, now the attention is turned to the joint decisions regarding the capacity issues. The early development of this topic is originated from the work from Cachon and Lariviere, in their paper they consider a situation where a supplier selling to multiple retailers when demand varies across periods with fixed supplier's capacity and wholesale price. If demand is high, then retailers' needs will exceed capacity, and the supplier must implement an allocation mechanism to dole out production. After analysing the turn-and-earn allocation, researchers showed that it will induce the retailers to increase their sales when demand is low and the supplier's capacity is otherwise underutilized which will increase the profit (Cachon & Lariviere, 1999). Under a non-cooperative environment. Two make-to-order firms provide distinctive values of service and have firm-dependent unit costs of waiting. Researchers concluded the sufficient conditions for the existence of a Nash equilibrium and characterized the equilibrium analytically for certain cases and numerically for some other cases. The results confirmed that the firm with the higher speed of service could usually charge a premium price and does take a larger market share. Also, the firm with the higher value of service and lower cost of waiting can usually charge a premium price and take a larger market share (Chen & Wan, 2003).

Caldentey and Wein developed a supply chain game-theoretic model where a supplier selects the production capacity v and a risk neutral retailer adopts a $(s-1, s)$ base-stock replenishment policy. In the paper, they assumed that retailer faces the Poisson demand process, the author derived the retailer's and the supplier's expected cost functions respectively as

$$C_R(s, v) = s - \frac{1-e^{-vs}}{v} + \frac{\alpha b e^{-vs}}{v}$$

$$C_S(s, v) = (1-\alpha)\frac{be^{-vs}}{v} + cv$$

In the above expressions, the letter b denotes the per unit backorder cost share, the letter c represents the supplier's capacity cost per unit of product. Caldentey and Wein characterized the unique Nash equilibrium solution to the system-wide cost $C_R(s, v) + C_S(s, v)$ (Caldentey & Wein, 1999).

3.7.3 Joint Production/Pricing and Capacity decisions

Different from previous stated joint decisions regarding the pricing policy or capacity arrangement, the content in this section will concern joint decisions regarding production, pricing and capacity decisions. Such a type of joint decisions is commonly encountered under a two stages supply chain structure. Kreps and Scheinkman developed a two-stage game model, in the first stage, two companies simultaneously and independently determine their production capacities, and in the second stage they will engage in a Bertrand style price competition. Their result showed that the unique equilibrium production capacities for both companies are actually the Cournot solutions (Kreps & Scheinkman, 1983). Under a competitive stochastic investment game with resource, researcher values the option of subcontracting to improve financial performance and system coordination. The manufacturer and subcontractor decide separately on their capacity investment levels. It has been showed that sometimes firms may be better off leaving some contract parameters unspecified ex-ante and agreeing to negotiate ex-post (Van Mieghem, 1999). Building on that foundation, Van Mieghem and Dada developed a two-stage decision model of postponing strategies. In their model, there are various strategies such as no postponement, production postponement and others. For the no postponement strategy, the value functions of each company involve joint decisions on capacity investment and price is

$$V(K, p) = -(c_K + c_q + c_h)K + pEmin(K, (\varepsilon - p)^+)$$

And the value function for the production postponement is

$$V(K, p) = -c_K K + (p - c_q)Emin(K, (\varepsilon - p)^+)$$

In the above expressions, K and p are production capacity and price set by the firm, c_K, c_q & c_h are denoting the unit capacity investment cost, constant marginal production cost and constant marginal inventory holding cost rate of ex-ante production, and the random variable ε represents the uncertainty in the market demand D. The authors showed how competition, uncertainty and the timing of operational decisions affect the strategic investment decision of the firm and its value (Van Mieghem & Dada, 1999).

Chapter 4. Share valuation throughout IPO procedure – A cross application of bargaining approach and auction approach

4.1 Introduction

Auctions are broadly regarded as an economic mechanism which transfers the right of control of an asset while determine a price for the transaction at the same time (Dasgupta & Hansen, 2007). This economic mechanism is ubiquitous around the world and has been widely applied in the financial market. One of the most well-known auctions in the financial market is the determination of the interest rate regarding US Treasury bond. The total annual volume of treasury securities auctioned has kept a steady rate of increase since 1981 from a level of $670 billion to a scale of trillions in current stage (Sundaram & Das, 1997). The most recent example of auction that has been applied in the IPO (Initial public offering) procedure is the internet search company Google offers its shares and auctioned its shares via a Dutch auction.

Initial public offering (IPO) is one of the methods that private owned enterprises move to public owned ones. The whole process represents a significant shifts of ownership structure, a channel to raise additional capital, and a lucrative liquidation for private owners (Poulsen & Stegemoller, 2008). It is said by entrepreneurs that an IPO is the most in favoured form of harvest (Kensinger, et al., 2000). From 1980 to 2021, there has been 9088 IPOs in the US financial market and in 2021 there were 311 IPOs (Ritter, 2022). Entering the 21st century, the flourishment of US financial market made the aggregate proceeds of IPOs constitute more than half of the total value ($711.38 billion out of $1121.3 billion) accounted from 1980 to 2021. In 2021, the annual aggregate proceed ($119.36 billion) is worth more than the summation of 2019 and 2020 results that were $39.18 billion and $61.87 billion respectively (Ritter, 2022).

For years, researchers are dedicated to developing the theoretical framework regarding auction mechanism designing while the issue regarding value determination of the object seems to be neglected. Throughout the procedure of the IPO, the price determination, in the field of academic research, researchers generally choose to subjectively give a number or vaguely represented by expressions like ‘valuation high’ and ‘valuation low’. However, in the real world, valuation of the shares is regarded as the most important issue throughout the IPO as it determines whether the company initiates the IPO could raise sufficient fund it required, and the large percentage of payoff underwriter could earn.

Different from the theoretical framework, practical features of IPO and financial markets make the whole process become rather intertwined with various topics. For the auction in IPO, shares are rather divisible which indicates the possibility of more than one bidder could get them with different offers. This comprises the application of another topic, the Bargaining theory. The whole process can be regarded as a combination of both auction and bargaining. On the one hand, the seller would expect to receive a as high as possible price that satisfying its fund-raising target, on the other hand, however, buyers would try to offer a price that as low as possible to maximise their profit after the shares are traded publicly. The auction is set when a pricing mechanism is determined by the seller while the bargaining procedure is occurring with the price offering from the time to time. This interesting circumstance brings a unique opportunity to utilise the cross-topic knowledges to explore a rather practical question and discuss the inherent relationship between zero sum and non-zero-sum game.

Based on the well-established foundation of the auction theory, this research intends to describe a rather complete story compared to papers regarding conventional auction theoretical framework. Not only analysing the theoretical designing of the auction, this paper provides more specific method to determine the value regarding the bidding shares. This happens to be a practical question in the real world while being neglected by the related academia. Starting from Vickrey's work, the research regarding the designing of the auction has been the core issue of auction papers (Vickrey, 1961). Developed on the foundation of Vickrey's paper, later research has developed more sophisticated and refined models to explore the equilibrium circumstances in auctions (Griesmer, et al., 1967). Later research primarily focused on participants' behaviours within the auction when there exist various factors including private information (Milgrom, 1979). However, a rather practical question regarding the valuation of the object being auctioned has not been drawn the attention of researchers. For years, it gradually forms a tacit practice that researchers vaguely use the expression of valuation high or low to abstractly value the object throughout the analysis. However, the precise valuation of share is essential throughout the IPO and this research intends to provide managerial insights for the company who initiates the IPO regarding optimal behaviour in response to certain conditions. To facilitate the price determination process, this paper configures the constitutions of the price of shares auctioned in IPO procedure aided by the bargaining theory and explore the behaviours of participants throughout the procedure. Different from the existing papers in the field of the bargaining theory, the model construction would primarily focus on practical circumstance and the valuation of the object is lies downs within a range rather than several specific points (Shang & Cai, 2021).

4.2 Literature Review

4.2.1 The modern auction theory

This research is connected to several research streams. The first area is related to the auction theory. The very first application of auction has been used from time immemorial, but the formal theory has entered the economics literature rather recently. The first recognition of the game-theoretic perspective of the problem traced back to Vickrey's work back in 1961. In his work, he made substantial contribution in analysing the nature of the auction and developed the all-time famous Revenue Equivalence Theorem (Vickrey, 1961). Later related research articles focus on analysing the equilibrium circumstance of the first-price auction that assumes the bidders' valuations are drawn from a uniform distribution (Griesmer, et al., 1967). Milgrom initiated the later development of the auction theory, in his paper he first proposed the model to investigate the behaviour of the winning bid where within the auction each bidder has private information (Milgrom, 1979). This very paper provides the fundamental framework for the later analysis regarding the behaviour of participants. Current predominate auction types are 'English Auction' and 'Dutch Auction'. Different from the conventional thought, Dutch auction takes a descending order to state price by the seller until there is a buyer that is willing to takes the object at the quoted price (Azevedo, et al., 2020). The researchers have demonstrated that Dutch auctions have a vital advantage of providing bidders with a useful price guarantee as the auction proceeds (Kleinberg, et al., 2016). However, the researchers use an abstract representation of valuation high or low to obscurely conduct the result without giving a specific indication of how the valuation should be constructed. This seems plausible in the theoretical study as the majority of papers focus on the design of the auction mechanism and corresponding behaviour upon participants caused by inherent characteristics of the auction setting. However, in the IPO procedure, one of the most important topics is price determination which requires dedicated technical analysis and specific range rather than a subjectively assumed amount. This paper intends to bridge this gap between the theoretical research and practical application.

4.2.2 Financial management of the supply chain structure

To help facilitate the determination of the high and low valuation of the object, this article takes the inspiration from the operation research area. By incorporating supply chain structure into the conventional credit rating models, researchers have developed framework that could provide guidance for potential value construction for strategic suppliers (Moretto, et al., 2019). To address the supply chain finance challenge of commodity price volatility, the model named 'Real Option Valuation (ROV)' was developed and tested on real cases (Pellegriono, et al., 2019). Most articles tend to set the ‘price’ as the strategic variable throughout the analysis due to actual condition in the commodity industry. For instance, regarding the influence of channel structures and channel coordination on the supply chain, researchers found that some Pareto zones can be achieved by setting appropriate price (Cai, 2010). In a negotiation circumstance where there exists a leading buyer, researchers have found that even though alliance could obtain a lower wholesale price, the follower could be worse off if the competition intensity between those buyers falls within a certain region (Hsu, et al., 2017). Driven by the common practices of cost-drive outsourcing, researchers further explored the strategic impact of outsourcing decisions through examining contractual forms and industry structure. The results illustrate that wholesale price contracts could mitigate competition among retailers (Feng & Lu, 2013). Considering a highly competitive industry, researchers have determined that wholesale price negotiations and the cost reduction brought by scale of economies could shape manufacturers’ decision of outsourcing (Heese, et al., 2021). For a supply chain comprises manufacturer and retailer, researchers have found that their preferences deviate over the form of the wholesale price contract due to the potential conflict of interest (Lu & Wu, 2015). For vertical bargaining condition, it is found that the condition in one channel could bring dramatic effect over other channels and researchers determined that effect could comprises input prices and welfare (Lozzi & Valletti, 2014). However, the research regarding the ‘price’ of the shares adapted a complete distinctive mathematical method. The dominant mathematical thought regarding share price in financial market is simulating price behaviour by stochastic process theories. Ever since the well-known Geometric Brownian Motion (GBM) model was stated (Black & Scholes, 1973), researchers have dedicated to formulating the appropriate expression regarding stock price behaviour. Some of the most famous models comprises Merton’s jump diffusion model (Merton, 1976) and Kou’s jump diffusion model (Kou, 2002). However, there is no research available that applied the knowledge of supply chain management to the field of financial market since it has become the common knowledge that conventional mathematical analysis techniques cannot be applied. However, regarding IPO process which requires the company to determine the price of the stock prior to the first day of trading and its unique underwriting mechanism enables the application of supply chain theories regarding negotiation progress prior to IPO.

4.2.3 Comparison of various negotiation mechanisms

The third related research stream is comparing different negotiation mechanisms. The negotiation mechanism itself has been a popular topic among articles. Scholars have established various negotiation mechanisms to simulate real life circumstance and explore their effects. In a vertical supply relationship, researchers illustrated that simultaneous bargaining is optimal for the manufacturer when retail prices are similar while sequential bargaining is preferred if dispersion in retail prices are sufficiently large (Guo & Iyer, 2013). Similar thoughts can be traced back to the paper developed by Horn and Wolinsky that combines a bargaining model with a duopoly model to examine how input prices and profits are affected by the structures of the supply chain (Horn & Wolinsky, 1988). Regarding the different price setting for participants in the negotiation, researchers have concluded that forbidding discriminatory discounts makes the retailer's bargaining power useless in mitigating manufacturers' market power (O'Brien & Shaffer, 1994). These papers tend to concentrate on the negotiation mechanism which enable each retailer and manufacturer to form their own contract whether in a timely manner or simultaneously. While the other well-known negotiation mechanism named price matching does not receive as much attention compared to the simultaneous negotiation mechanism. A recent paper that concerns the comparison of two negotiation mechanism is developed by Shang and Cai(2021). In their paper, wholesale prices negotiation under two mechanisms named price matching and simultaneous negotiation are calculated and compared (Shang & Cai, 2021). These already matured negotiation mechanisms were developed based on industrial practices, however, the situation that the bargaining theories have not been applied to the financial market caused a circumstance that there neither exist a term to describe the negotiation mechanism in financial market where the independent variable is quantity nor specific research to compare the efficacy regarding those mechanisms.

4.3 The model description

To conduct the behaviour of participants within the auction, the auction is first to be assumed to proceed before the shares are published to general investors. We consider a situation where the company that initiates the IPO will be the seller and provides shares to be auctioned. While underwriters will be buyers who propose their offers based on their valuation regarding shares. The IPO is conducted by a fully underwriting which indicates that all shares would be purchased by underwriters before shares are issued to general investors. Seller has the only option to auction all the shares to underwriters in order to satisfy its fund-raising target.

Consider an IPO is conducted between a seller and two underwriters, the early-stage preparation has been accomplished and proceeded to the value determination stage. The value of the share will be determined through a fully underwriting via the auction. Both underwriters propose their offer b_i & b_j regarding quantity of shares they would like to purchase and corresponding price. To better deduct the procedure of the auction, the offer from each underwriter is completely available to all participants with no time delay. The auction type will include both first and second price auctions. For each underwriter, their own valuation regarding the shares would be v_i & v_j respectively. The seller has the capability to choose either proposal offered by individual underwriter.

For the financial market, the nature of the IPO process caused the circumstance that the market price for an asset is not as widely known compared to commodity industry. Usually, some relevant information is held by various parties and the goal of auction design is to facilitate the acquisition, revelation, and integration of those information (Azevedo, et al., 2020).

4.3.1 Bidding behaviours of participants

The current predominant types of the auction can be divided into two general types namely first and second price auction. For the first-price auction, it works rather clear. The bidders win the object with the highest bidding price and pay this price. While for the second-price auction, the winner is still the one who offers the highest price, but the winner only needs to pay the second highest price. For years, academia has developed rather mature deduction regarding the 'game' of auctions and in this article several corollaries are deducted.

Lemma 1: In the second-price auction, the participants will always offer their true valuation regarding the object since winning the bid will be profitable for sure.

For a first-price auction, the circumstance would be more complicated compared to the second-price auction due to reason that each bidder would need to speculate others' strategy to secure the object. If the IPO underwriting auction adapted this style, bidding the valuation is clearly not a clever option since whether win the auction or lost the auction, there is no surplus for the buyer. Hence no bidder will bid for a price higher than their valuation which leads to an optimal bidding strategy slightly lower than their own valuation. For each buyer, as a strategic game, their bidding strategy is actually the best response to that they speculate the other bidder does.

Lemma 2: In the first-price auction, if there are only two bidders, then the best response function would be half of the valuation.

If the condition expands to N buyers, then some corresponding corollaries can be deducted.

Lemma 2.1: The best response bidding function would be:

$$\hat{b}_i = \frac{N-1}{N} v_i$$

Lemma 2.2: Bidders will never bid for their valuation regarding the shares, the shading coefficient is $\frac{N-1}{N}$. The coefficient has range from 0 to 1.

Lemma 2.3: The optimal bidding strategy is increasing in valuation, that is when a buyer values the shares more than any other buyers, he would be willing to offer a price higher than others and he will take the object, since the first order derivative of the bidding function is

$$\frac{\partial \hat{b}_i}{\partial v_i} = \frac{N-1}{N} > 0$$

Lemma 2.4: The more bidders participate in this auction, the higher bid will be received by the seller

$$\frac{\partial \hat{b}_i}{\partial N} = \frac{v_i}{N^2} > 0$$

$$\lim_{N \to \infty} \hat{b}_i = v_i$$

When the number of buyers increase, the seller tends to receive the true valuation of the shares.

4.3.2 Expected revenue of the seller

The above section is regarding the revenue of the buyer, and for the seller of the auction, the perceived revenue will be discussed in this section.

In the previous discussion, when the auction is set as the second-price auction, the bidders offer the price based on their own valuation. For the seller, its expected revenue would be the second-highest valuation of the bidders. From the perspective of the seller, this revenue is a continuous random variable and in this part, it is denoted by $v_{(2nd\ highest)}$.

Theory 1: (sellers' revenue equivalence theory) The seller's expected revenue is the same for the seller regardless of the auction type.

In the first-price auction, the seller expects to receive the winning bid equal to $\frac{N-1}{N}$ of the highest valuation. Under the model with two bidders, this coefficient would be 0.5. The perceived highest valuation among the bidders is a random variable as well which is denoted by $v_{(1st\ highest)}$.

The two random variables has some attributions, the first attribution is that for both random variables will be fall within the [0,1] interval of possible valuations as no bidders will bid for a price higher than their valuation. The second attribution is that by definition $v_{(1st\ highest)} > v_{(2nd\ highest)}$. To better compute the properties of these random variables, these variables are assumed to be uniformly distributed.

Considering the probability density function of these random variables, $v_{(2nd\ highest)}$ is the half-way point of the [0, $v_{(1st\ highest)}$] and $v_{(1st\ highest)}$ is the half-way point of $[v_{(2nd\ highest)}, 1]$.

From the previous deduction process, it can be concluded that the seller should expect on average to receive the second highest valuation. In the second-price auctions this is rather easy to understand. In a first-price auction, bidders would shade their offer but each of them would try to outbid the average second-highest valuation in order to win the object.

The induction in previous part illustrated that the expected revenue from the auctioneer with N bidders whose valuations are independent lies between the interval from $[v_{(low)}, v_{(high)}]$, the expected revenue is:

$$v_{(low)} + \frac{N-1}{N+1}(v_{(high)} - v_{(low)})$$

From the articles regarding the auction theories, these results are presented as the conclusion. However, the key issue to apply these results to the actual financial market, the determination of $[v_{(low)}, v_{(high)}]$ is the crucial issue that company who initiated the IPO eager to know in order to figure whether the IPO could raise enough fund for the company.

To avoid the subjective assignment of valuation, this article intends to conduct the specific formulation for those values. For the determination of the valuation, the essence of the action is to formulate the expression regarding the buyers' willingness to pay for the object which is the shares under the circumstance of the IPO procedure. Under the circumstance of the auction theory, all the potential bidders are participating in a zero-sum game which indicates the buyers' valuation could vary in a large scale. To specifically determine the formulation, it would require understanding the rationale behind every bidder's behaviour which implies to conduct the mathematical deduction procedure under an assumption that each bidder will act for their best interest. Hence, the bargaining theory would be helpful to solve this condition since its underlying framework is determining participants' behaviour when their actions' corresponding results are known. The IPO procedure can be regarded as a combination of both auction and bargaining. On the one hand, the seller would expect to receive a as high as possible price that satisfying its fund-raising target, on the other hand, however, buyers would try to offer a price that as low as possible to maximise their profit after the shares are traded publicly. The auction is set when a pricing mechanism is determined by the seller while the bargaining procedure is occurring with the price offering from the time to time.

However, throughout the deduction process of auctions in IPO, it is noticeable that the specific price for the object is rather vague. To aid the whole procedure, it is usually represented by a term or a special character. To answer the practical question regarding the determination of the value regarding the object of bidding, this requires the facilitation from the bargaining theory which primarily focus on the determination of objects within a supply chain structure.

4.4 Bargaining in the IPO procedure

To simulate the IPO procedure in the financial market, the whole bargaining follows a conventional supply chain structure where the company who initiate the IPO is the seller of the product, and the underwriters are buyers in the market. When they buy the shares from the seller, they could construct the share into a portfolio and sell it to the investor.

In this research, the whole supply chain follows the conventional IPO underwriting procedure and a common one seller, two buyers and final individual investor will construct the complete structure. The whole story proceeds as the following: the seller which is the qualified company that decides to conduct the IPO offers the identical products (the shares) to both buyers (professional financial institutions act as underwriter), who then construct those shares into their own portfolios and sell those portfolios to individual investors. Followed by the US Securities and Exchange Commission's (SEC) regulation, before the first day's trading, the issuing company is required to finalising the issued price and number of issued shares. Before the first day of trading, financial institutions could purchase shares in advance and sell those shares to investors.

The negotiation between the 'seller' and the 'buyer' follows the conventional process of the supply chain theories. But the distinctive feature of the financial market makes the 'quantity' of shares that buyer intends to buy as the key factor as intention of the IPO is raising additional capital rather than maximising revenue. To avoid the circumstance of failing to raise required capital, the capitalisation of the issued shares usually higher than actual requirement. Underwriters' actions of purchasing shares in advance could in some degree ease company's concern of failing to raise enough fund. Hence there is a motive for both participants to reach agreement before the first trading day. Taking the thought from the current derivative trading, there are two categories of contract available. One is called the standard form of contract which implied identical contract forms for any potentially interested participants. The other can be called as the customised contract which is widely applied in Over the counter (OTC) market where participants of the contract could construct their own terms of the unique contract. Due to the concern of the parsimony, it is assumed that for one unit of the final portfolio to be constituted, one unit of the issues share is required.

Since two buyers compete over the quantity rather than price, the model follows the theory of a Cournot competition model. In this research, the variable P_i is used to represent the corresponding price that Buyer i will receive for the quantity he chose. The following demand function is followed by the theory of the Cournot competition model from the paper developed by Ingene and Parry back in 1995. (Ingene & Parry, 1995)

$$x_i = a - P_i + \gamma P_j$$

$$j = 3 - i, i = 1,2$$

In the above equation, a denotes the initial demand base, x_i represents the quantity of shares that Buyer i decides to purchase from the seller, and $\gamma \in [0,1)$ represents the substitutability of the portfolio. From the equation it can be inducted that the supply quantity of the shares has a negative relationship with its own price and a positive relationship with the price of its substitution. A most recent event to demonstrate this relationship is regarding Elon Musk's decision over acquiring Twitter. When he sells his shareholding in Tesla to raise fund for this acquisition, the share price of Tesla drops. Unlike the common commodity products, the share price determination in large degree closely related to its intrinsic value which mainly comes from investors' expectation over its future capability of earning money. When there is a large supply which indicates investors sell a large volume of shares would create a herding effect over the general confidence of share's future earning capability. This would motivate more investors to clear their shareholding of the shares and create an oversupply of the share and result in a decrease in the share price (Jin, et al., 2016). On the other hand, however, the supply of the share has the positive relationship with its substitution's share price. Consider an industry that is having a great potential, investors tend to purchase shares of the company with the greatest potential to offer satisfying return. Assume the total fund hold by an investor is constant, he or she has to sell the shares to retrieve cash in order to invest in the target company. The over-demanding condition would make the share price of the target company increase while supply of shares of the substitute company increase as well. This phenomenon can be explained by the substitution effect (Allen, 1950).

The underwriter's negotiation power is denoted by θ and the seller's relative negotiation power against the underwriter is $1-\theta$. The negotiation power has the range from 0 to 1.

For the negotiation mechanism, there are two forms of negotiation. One is called a standardised purchase which is commonly applied in the futures contract which implies each contract has identical terms regarding amount, time to maturity and current price etc (Jongadsayakul & McMillan, 2020), and the other is named as customised purchase. The standardised purchase concerns the seller and one buyer negotiate for a form of purchase agreement, once that negotiation is successful, the other buyer should choose only to accept that contract or refuse to get no share. The other kind of negotiation is called customised purchase, it refers to the condition where seller has to negotiate separately with each buyer regarding the amount of shares purchased. This negotiation mechanism is inspired from the forward contract which enable both parties of the contract to tailor the terms to satisfy the unique requirements and has been widely applied in the negotiation of electricity market (Anderson, et al., 2007).

The game sequence is working as the follows. In the first stage, the seller negotiates with the buyers over the wholesale price corresponding with the chosen number of shares determined by the underwriter. In the second stage, the underwriters construct their own portfolio and sell it to the investors. The first stage is realised as a Cournot competition and the second stage demand is realised in the differentiated Bertrand competition between both underwriters. The whole model will be solved through a backward induction method.

4.4.1 Analysis with symmetric negotiation power

To highlight the influence of the two kinds of contract (standard form vs customised form), the analysis first concentrates on the circumstance of the symmetric negotiation power ($\theta_i = \theta_j = \theta$) and then explore the analysis to the area where the negotiation power between underwriters is different. By analysing the game backwards, the second stage of the game will be analysed and then the analysis regarding the first stage subgame will be conducted.

Analysis of the second-stage game

For this part of the analysis, it is assumed that the negotiations in the first stage are successful which indicates that both underwriters have purchased shares from the seller and successfully constructed their own portfolios.

The second-stage Bertrand competition

For any given wholesale price pairs (w_i, w_j), the result of the second-stage game is independent regarding which form of contract that have been chosen by the underwriters in the first stage. The revenue functions for the buyers and the sellers are:

$$\pi_i = (P_i - w_i)x_i$$

$$\pi_s = w_i x_i + w_j x_j$$

The subscript of i & s represent the underwriter i and the seller respectively. Taking the rational perspective of conducting business, the buyer seeks to maximise its own profits by determining the appropriate price to the investor. Hence the procedure to determine the optimal price is following:

$$x_i = a - P_i + \gamma P_j$$

$$\pi_i = (P_i - w_i)x_i$$

By solving the equation, the detailed expressions are: (1)

$$x_i = \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_i + 2\gamma(1-\gamma^2)w_j}{(4-2\gamma^2)}$$

$$x_j = \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_j + 2\gamma(1-\gamma^2)w_i}{(4-2\gamma^2)}$$

$$P_i = \frac{a(1+\gamma) - \gamma x_j - x_i}{(1-\gamma^2)}$$

Substitute both components into the expression of P_i: (2)

$$P_i = \frac{a(1+\gamma)(2+\gamma) + 2(1+\gamma)(1-\gamma)\big(w_i + w_j\big)}{(1+\gamma)(4-2\gamma^2)}$$

Given the value of the quantity and price are the determination when the payoff of the participant maximises, hence the unique value of the equilibrium retail quantity and prices are:

$$\hat{x}_i\big(w_i, w_j\big) = \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_i + 2\gamma(1-\gamma^2)w_j}{(4-2\gamma^2)}$$

The resulting price are: (3)

$$\hat{P}_i(w_i, w_j) = \frac{a(2+\gamma) + 2(1-\gamma)(w_i + w_j)}{(4-2\gamma^2)}$$

The equilibrium profits are: (4)

$$\pi_i(w_i, w_j) = \frac{a(2+\gamma) - 2(1+\gamma+\gamma^2)w_i + 2(1-\gamma)w_j}{(4-2\gamma^2)}\hat{x}_i$$

$$\pi_s(w_i, w_j) = w_i x_i + w_j x_j$$

Single-channel Monopoly market

In general, the disagreement point in a dyad of negotiation is the worst payoff of the participant. Hence, to determine that point it is assumed that the negotiation between the seller and the Buyer i failed in either a customised contract or the first-round bargain of standardised contract, which results the seller sells to Buyer j only. This condition would empower the bargaining position of the Buyer j as it is the last hope for the seller to earn revenue. The resulting demand would be

$$P_j(x_j) = a - x_j$$

Given w_j, the buyer aims to maximise

$$(a - x_j - w_j)x_j$$

Resulting the optimal quantity

$$x_j^M = \frac{a - w_j}{2}$$

And the firms profits

$$\pi_j^M(w_j) = \frac{(a - w_j)^2}{4}$$

$$\pi_s^M(w_j) = \frac{w_j(a - w_j)}{2}$$

The superscript M represent the monopoly market.

4.4.2 Equilibrium analysis

Customised purchase

In a customised purchase contract, the seller negotiates with both buyers simultaneously regarding the specific terms in the contract. The firms' profits have deducted by the equations in the previous part. The bargaining solution (w_i^{CP}, w_j^{CP}) is an equilibrium when the buyer i and the seller agreed on the wholesale price w_i^{CP} given that buyer j and the seller have reached an agreement on w_j^{CP}. With a known w_j^{CP}, it could be deducted that the set of revenue pairs $\{[\pi_i(w_i, w_j^{CP}), \pi_s(w_i, w_j^{CP})] | w_i \geq 0\}$ has a potential maximum value. And the unique bargaining solution (w_i^{CP}, w_j^{CP}) that maximises the total payoffs of the negotiation participants satisfies: (5)

$$w_i^{CP} = argmax\ [\pi_i(w_i, w_j^{CP}) - 0]^{\theta_i}[\pi_s(w_i, w_j^{CP}) - \pi_s^M(w_j^{CP})]^{1-\theta_i}$$

In the above equation, the disagreement point is $(d_i, d_j) = (0, \pi_s^M(w_j^{CP}))$. The reason for using w_j^{CP} in the expression of π_s^M rather than w_j^M is that negotiations between the seller and the buyers are separate and simultaneous. The information of buyer i failed to reach an agreement with the seller can not be known by the buyer i until they have accomplished the negotiation.

For a symmetric case where $\theta_i = \theta_j = \theta$), which implies $(w_i = w_j = w)$ it can be shown that the unique solution of the wholesale price is:

Lemma 1: (6)

$$w_i^{CP} = \frac{a(1+\gamma)(2+\gamma) - a\sqrt{\theta^2(1+\gamma)^2(2+\gamma)^2 + 4\theta\gamma(\gamma-2)^2(\gamma-1)(1-\gamma^2)(2+\gamma)(1+\gamma)}}{4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)\theta}$$

Standardised purchase negotiation

Under a standardised purchase contract, to keep the generality of the negotiation, the seller initially negotiates with Buyer i. If they successfully reach a deal over the wholesale price W_i^{SP}, the remaining buyer (Buyer j) does not negotiate with the seller but entitled for the option of either accept the existing W_i^{SP} or refuse this offer and exit the market. It is easy to be conducted that in an equilibrium circumstance, the Buyer j would accept the wholesale price if the wholesale price lies within a reasonable domain which is the entire feasible domain $W_i^{SP} \in (0, a)$.

However, if the first negotiation between the seller and Buyer i failed, the bargain will be conducted between the seller and the Buyer j to determine the wholesale price $\widehat{w}_j$ that is uniquely applied for Buyer j given that Buyer i exits the market. The participants' revenue functions are still follow the equilibrium condition in previous section. Backward induction is still used to analyse this bargaining. Given the negotiation between the seller and Buyer i failed, the seller will reach a deal with Buyer j or otherwise failed both negotiation and earn nothing.

$$\widehat{w}_j = argmax\ [\pi_j^M(w_j) - 0]^{\theta_j}[\pi_s^M(w_j)]^{1-\theta_j}$$

For the seller, it earns $\pi_s^M(\widehat{w}_j) = \frac{(1-\theta_j{}^2)a^2}{8}$. From the results in the previous deduction procedure, the determination factors in the equilibrium wholesale price mainly depends on the buyers' relative negotiation power (customised purchase & standardised purchase) to the seller and final product substitutability (only in customised purchase). Given the assumption of reasonable participants in the negotiation, the seller could conduct the identical process which indicates that the importance of reaching a contract in the first-round negotiation with the Buyer i. Otherwise, in the second-round negotiation with the remaining buyer, the seller would be exploited by the only available buyer because failing to reach a contract in both rounds would indicate a 0 payoff. Hence consider the negotiation between the seller and Buyer i both understand the subsequent outcome if their negotiation failed. The bargaining problem is described by the payoff condition $\{[\pi_i(w_i, w_i), \pi_s(w_i, w_i)] | w_i \geq 0\}$, which forms the boundary of the following expression. The unique equilibrium wholesale price w_i^{SP} satisfies:

$$w_i^{SP} = argmax[\pi_i^{SP}(w_i, w_i)]^{\theta_i}[\pi_s^{SP}(w_i, w_i) - \pi_s^M(\widehat{w}_j)]^{1-\theta_i}$$

Lemma 2: (7)

$$w_i^{SP} = \frac{-2\theta_i a(1+\gamma)(2-\gamma) + a\sqrt{4\theta_i(1+\gamma)^2(2-\gamma)^2 + 2\theta_i(1+\gamma)(2-\gamma)(4-2\gamma^2)(1-\theta_j{}^2)}}{8(\gamma-1)(1-\gamma^2)}$$

The wholesale price in the standard purchase contract is intuitively depending on the buyers' relative power and product substitutability. According to the expression, the higher the buyer's bargaining power the lower the wholesale price.

Comparison of firms' preferences

From the previous part's calculation, it can be concluded that the customised purchase and standardised purchase could result in the identical wholesale price when both buyers have the identical negotiation power and conduct the negotiation without knowing the result of the other negotiation (the extreme condition). Besides this special case, the participants' best-response functions are distinctive because of the different attributions of the negotiation mechanisms.

$$w_i^{CP} = \frac{a(1+\gamma)(2+\gamma) - a\sqrt{\theta^2(1+\gamma)^2(2+\gamma)^2 + 4\theta\gamma(\gamma-2)^2(\gamma-1)(1-\gamma^2)(2+\gamma)(1+\gamma)}}{4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)\theta}$$

$$w_i^{SP} = \frac{-2\theta_i a(1+\gamma)(2-\gamma) + a\sqrt{4\theta_i(1+\gamma)^2(2-\gamma)^2 + 2\theta_i(1+\gamma)(2-\gamma)(4-2\gamma^2)(1-{\theta_j}^2)}}{8(\gamma-1)(1-\gamma^2)}$$

The above results demonstrated that the relationship of the wholesale price in either negotiation mechanism has a negative correlation to the relative negotiation power. For a symmetric case where $\theta_i = \theta_j = \theta$, the result shows that $w_i^{SP} \geq w_i^{CP}$ and the two values will equal to each other if the negotiation takes the extreme condition stated above. Thus the wholesale prices are higher in the standardised purchase than in customised purchase and the difference is closely related to the value of the relative negotiation power θ.

Theorem 2 (Participants' preferences between CP & SP) With symmetric negotiation power, the seller (IPO company issues the new shares) always prefers SP to CP while both buyers (underwriters) would always prefer CP to SP.

In the above calculation, the participants' preferences regarding CP and SP are differ, even though the symmetric negotiation power implies an identical wholesale price. Hence, the preference of the company is purely dependent upon the differences between two negotiation mechanisms rather than the relative negotiation power asymmetry. As previous demonstration part stated that under the extreme condition, the disagreement points are the same for both participants and under this unique condition the price matching characteristics could be found in the customised purchase contract as well.

From the deduction in the previous part, the higher level of wholesale price in standardised purchase contract would demotivate the buyer to bargain for a more favourable wholesale price since the result of taking advantage of the buyer's own negotiation power could benefit the other buyer without consuming any available resources. Taking the perspective of the seller, offering the lower price to both buyers is significantly unfavourable due to a potential loss of profit compared to offering a lower wholesale price to the only buyer that has the relative strong negotiation power. Because of these factors, the buyer-side and the seller-side motivation contributes to the condition where the negotiated wholesale price in SP is higher than CP.

To understand this feature from the perspective of the Nash bargaining model, the very nature of the Nash bargaining framework should be discussed. From the previous Nash bargaining expression, for both participants of the negotiation, each party negotiate for its marginal gain against the counterparty with the same objective. Given one party's marginal gain is the same while the other party's marginal gain is diminishing and lower than that number. The negotiation result will tend to be more favourable to the party with constant marginal return. As the party with higher marginal return will have higher opportunity cost which motivates the party to ask for more regarding the additional profit otherwise the negotiation will fail. The standardised purchase contract lowers the marginal gain of the buyer when it negotiates for a lower wholesale price because the lower wholesale price can equally be received by the rival buyer. In contrast, the standardised purchase contract will enable the seller to have a higher marginal gain because by offering both buyers a relative higher wholesale price. Hence, the bargain result will favour the seller.

Corollary 3: When the bargaining power of buyers are asymmetric, the seller still prefers SP to CP. The buyer tends to discard the right to exploit the relative stronger bargaining power in CP.

The corollary can be explained by both financial and economic perspectives.

The nature of the corollary can be explained through an option exercise scenario. The buyer with relative stronger bargaining power can be regarded as a holder of the call option which empowers the holder to buy the underlying asset in a predetermined price. However, after the buyer exercise this option, other buyers could buy the same underlying asset at the same price without requiring having the option. To exploit the relative stronger bargaining power will make the price of the underlying asset to be lower which leading to a narrower profit margin of exercising the call option.

To explain this corollary through an economic perspective, the role of the buyer should be considered. Vertically, the buyer positions at the middle of the whole supply chain structure. After determining the wholesale price, the buyer will manufacture the raw material into the finished product and sell it to the final customer. Under the assumption of producing substitute products from both buyers, a unitary lower wholesale price of the material will cause both buyers enjoy this favourable term at the cost of exploiting a single seller's negotiating power. As for selling the manufactured products, the same low cost could make the buyer lost its pricing advantage against other competitors. Taking the perspective of the buyer, this circumstance would be reluctant to be occurred.

Hence the above reasons make the buyer with relative stronger negotiation power chooses not to exploit its advantage in first stage of the negotiation. This corollary provides an insight regarding the channel efficiency of the negotiation situation. The standard purchase contract will give more power to the seller which could damage the revenue of the buyers regardless the relative bargain power of both buyers is symmetric or not.

In a conventional financial market structure, the company that applies for the IPO is much more concerned regarding the wholesale price of the shares because these wholesale prices are directly paid from the underwriters to the issuing company. As for the retail price which indicates the underwriter construct the purchased shares into the portfolio and sell the portfolio to the general investor, the seller does not share the same level of sensitivity. For the underwriters, their revenue comes primarily from the later share price increase rather than price disparity from the IPO process. Hence, they can never charge the investor a price that is high enough to compensate the payment they made to the seller. As the result, the underwriters tend to prefer the customised purchase contract rather than the standardised purchase.

Hence for the seller, to compute the result of the equilibrium wholesale price for both standard purchase and customised purchase contracts, substitute those results into the expected revenue function of the auction, the seller's revenue would be

$$w^{CP} + \frac{N-1}{N+1}(w^{SP} - w^{CP})$$

In the model there is only two buyers, hence the specific expression of the revenue would be

$$w^{CP} + \frac{1}{3}(w^{SP} - w^{CP})$$

From the previous part, it can be known that the wholesale price under either customised purchase or standard purchase are both positive and the equilibrium wholesale price equals to the lower valuation (the wholesale price of the customise purchase) plus the difference between the higher valuation (the wholesale price of the standardise purchase) and the lower valuation multiplied by the coefficient closes to 1 when the number of buyers tend to be infinity.

The auction avoids the buyer paying too high to the seller while seller could take a relative favourable result due to the fear from the buyer of losing the contract. The equilibrium result would be higher than the result in customised purchase but lower than the standard purchase. By setting the auction properly, the result could be satisfying for all participants.

4.5 Discussion and Conclusion

This article studies the procedure of shares underwriting in the IPO process and proposes a game-theoretic framework to determine the high and low valuation of the bidders in the auction setting process. Different from existing literature, this research concerns cross application from two research fields. The research regarding the auction theory have been profoundly influenced by Vickery's work (Vickrey, 1961), this filed never lacks outstanding papers regarding auction designing and corresponding economic theories. Researchers can always find them through the rather recent publication (Azevedo, et al., 2020). However, when apply this matured theory to the practical world, the issue that is frequently regarded as trivial becomes the vital question and hard to found a solution via auction theory alone. The intention to introduce the bargaining theory to facilitate the value determination is exploring buyers' true valuation regarding the shares under a conventional supply chain structure. This circumstance is commonly encountered in the financial market comprising company that initiates the IPO, underwriters, and investors. The cross application of both topics enables participants to understand what their acceptable price range are rather than simple two results from the conventional bargaining theory computation (Shang & Cai, 2021). Instead of comparing channel efficiency (Cai, 2010), this paper provides several options to companies that initiate the IPO regarding potential options over share issuing methods and their corresponding payoff which would be helpful to determine whether their fund-raising target can be achieved.

The mathematical framework is constructed based on various assumptions. The first assumption is that the underlying object for the auction is divisible and identical for any single unit of the object. The second assumption is the number of shares required to construct the portfolio. In the framework of the bargaining circumstance, it is assumed one unit of shares purchased from the seller can be constructed into one unit of the portfolio sold to the investor for the sake of mathematical calculation. However, in the real life, conditions may be different as different portfolios have different weighting philosophy of each constituent of shares. The third assumption is unlimited number of shares will be available to purchase for the buyer. To compute the high and low valuation from the buyer, the willingness to pay for the share under a quantity-oriented bargaining condition require the buyers to freely choose their desired amount of shares and corresponding price. In real life, the number of shares available for buyers to purchase is usually limited.

The differences between the financial market and commodity market makes the current mechanism which used to facilitate the channel efficiency lost its effectiveness in financial market. The bargaining theory related research most adapted a price-oriented negotiation framework since it is commonly discovered situation in commodity market. However, in the financial market, the cost of shares is difficult to be quantified compared to the tangible material in conventional market. In another word, the cost of shares is nearly zero and the value of the share most dependent upon investors' confidence or expectation of company's future development. Each investor has his own valuation regarding the share and in an efficient financial market, the share price is the reflection of the available information. And usually, the time lag between underwriter purchases the shares from the company initiates the IPO and sell the shares to the investor are much longer compared to the conventional supply chain structure. The time lag is long enough for the market to know the contract between the seller and underwriters and the price of the share would reflect this information.

The findings in this article conforms with the previous literature. For the existing literature regarding the channel efficiency under the supply chain structure, this article has found that the price mechanism (price matching in conventional supply chain or standard purchase in this article) that imposes equalised term as the first contract has agreed on every buyer could demotivate buyer's incentive to exploit its relative stronger negotiation power regardless of the strategic variable as price or quantity. This mechanism would promote the buyer's fear of enabling other competitors to enjoy the advantageous term without any cost. This would result in a higher wholesale price and lower channel efficiency. For other negotiation mechanism which require seller and individual buyer to achieve customised contract, the wholesale price would depend upon the relative negotiation and the seller would bear some intangible cost including the time for negotiation. Hence the mechanism that provides the equalised treatment for all buyers would be preferred by the seller while the customised contract would be preferred by the buyers.

In this article, the whole mathematical model is developed based on the idea that IPO procedure is regarded as a combination of both auction and bargaining. The facilitation of bargaining theory is primarily used to determine a rather subjective question in the auction theory which is the price determination. For years, researchers are dedicated to study to mechanism design throughout the auction. It is very common in the papers to subjectively give a number to the valuation range of the buyer. However, to apply the auction theory to the IPO procedure, the seller should always expect to receive the same offer as long as the auction is set properly. However, what is the specific number of this offer becomes a crucial issue as each participant is more concerning about maximisation of personal payoff. Hence the constituency of the price and the factors that affect the price. With the facilitation of the bargaining theory, both buyers and seller could understand the price determination process and understand whether the current price offered could be satisfying. This could provide some managerial insights for all the participants regarding their best reaction throughout the IPO procedure.

Judging from the wholesale price of the stock, the standard purchase contract could bring the highest result while the customised purchase contract yields the lowest wholesale price. Hence for the seller, the application of the standard purchase contract is always preferred compared to other options. The unique equal treatment characteristic could help the seller to determine the term for all the interested underwriter and save time regarding repetitive negotiations. On the other hand, the fear from the buyer regarding the 'free rider' issue would encourage the buyer to discard exploiting the relative stronger bargaining position and result in a favourable contract term for the seller. Due to the reason that the wholesale price is depending on the relative negotiation position between the seller and the buyer, hence for the seller, it would be always preferrable to initiate the standardised contract with a buyer with relative weaker bargaining position.

As illustrated in the previous paragraph, the standardised contract is always preferable for the seller in the IPO procedure. However, if this form of contract is rejected by potential buyers, holding an auction for the shares appears to be optimal. As long as the auction is setting appropriately, the seller should be expected to receive the same revenue which implies the price would be the upper limitation of the lower valuation plus the difference between the high and low valuation with an increasing number of buyers.

This paper proposes a combination of auction theory and bargaining theory to facilitate the efficiency in the financial market. Usually, the zero-sum game and the non-zero-sum game are divided into distinctive circumstance for discussion. But in some cases, the result computed from one condition could provide some insights to clarify the range of equilibrium result for the other circumstance. The auction itself can be regarded as a bargaining between the seller and various buyers occurring at the same time while there exists a competition between buyers.

This provides a possibility of intertwining the results from both auction theory and bargaining theory.

However, there still exist some areas that still could require further refine and discussions. In this article, the underwriting procedure assumes a fully underwriting circumstance which indicates underwriters will purchase all the shares from the company that initiates IPO and resell it to the investor who wish to purchase the shares when the company goes public. Under this condition, the revenue of the seller is determined completely and exclusively based on the contract with underwriters. However, there still exist other forms of underwriting contract namely: best effort. Under this contract, underwriter will help the company to find any interested investors but make no promise regarding either how much the shares will be sold at nor the quantity to be sold. Hence under this circumstance, there provides an opportunity to consider the effect brought by the existence of outside option. The seller could choose to sell the shares directly to the final customer without the underwriter. This price could be either higher or lower than the contract terms signed with the underwriter.

Another potential development of current model could be the discussion regarding the two stages competition. Since current literature primarily focus on the channel efficiency which indicates a large volume of articles select utility as the measurement for the condition of the second stage. For this area, there still exist some blank that worth filling.

Chapter 5. The effect of outside option in IPO process and construction of the side payment

5.1 Introduction

Inspired from the flourishment of ecommerce in the last decade, the effect of outside option throughout the auction has drawn the attention of researchers (Maslov, 2020). Outside option, an option provided to quit the haggle in search for a delayed agreement and complete the deal (Chang, 2021). Commonly found in the eBay auction, seller could choose the type of auction including a buy-it-now price or best offer until certain time. In essence, this is a first-price auction. Deducted from existing papers, when the number of buyers is sufficient enough, the offer price will approach the true valuation of buyers' recognition (Vickrey, 1961). However, different from the framework proposed by Vickrey, the existence of the outside option could shape buyers' behaviour significantly especially when the outside option is lower than buyer's true valuation. Rather than gaining benefit (when the outside option is lower than buyer's true valuation), exercising the outside option prevent buyer spending time on negotiation. Different from the common commodity, the value of time is always essential for a participant in the financial market.

It has been commonly found that the participation of underwriter throughout the IPO process could be helpful for the company's preparation for the investigation from authority. Usually, underwriter's job concerns three aspects including advisory, selling and protective. Advisory function contains providing counselling suggestions to the company regarding legislative compliance, financial reports preparation, and share issuing timing and price determination. While selling function primarily refers to aid the company sell shares to the investors and protective function indicates the permission under related laws to stabilise financial market (Chinese Securities Regulatory Commision, 2006). The service provided by the underwriter is essential for the company and could be expensive especially for the company which urgently require fund raising.

Throughout the IPO process, inevitably, the company wishes to go public would face a bargaining circumstance with the underwriter. Under this conventional sales negotiation, it has been commonly found that parties often face a choice between haggling in search for a delayed agreement or promptly quitting the current negotiation (Chang, 2021). Even though most IPO comprises the participation of underwriter, there are some occasions that company chooses to initiate the public offering procedure without the underwriter namely conduct the outside option. One of the most famous examples is Facebook which initiate the IPO in 2012 without reaching a contract with the underwriter.

Even though the outside option shares some similarities compared with the conventional stock options in the financial markets, the research regarding this subject in the field of supply chain coordination commonly regarded it as a potential option which the participant could adapt throughout the negotiation procedure. The value of the existence of the outside option and other potential equivalent tools that could exert similar effects tends to be neglected by existing literature.

Different from existing literature where the financial papers tend to propose the specific valuation for options and papers from the supply chain management tend to investigate the effect of channel efficiency when adapt the outside option, this research intends to propose the theoretical framework regarding company's choice over an underwriter and outside option which more appropriately suits the financial market rather than common commercial circumstance stated by Maslov (Maslov, 2020). By incorporating the outside option, the negotiation would be distinctive compared to current structure (Shang & Cai, 2021) since the company is capable of holding a direct selling of product or being forced to do so. In this article, a more comprehensive situation will be considered to analyse the participants throughout the IPO progress. On the other hand, the side payment will be considered as well but under a different setting up condition to discuss the rationale behind the side payment contract.

5.2 Literature Review

5.2.1 The behaviour of the participants throughout the auction process

The introduction of the formal decision theoretic approach to analysis bidders' behaviour within an auction was proposed by Friedman's work (Friedman, 1956). Then Vickrey's seminal paper was the first to examine the auction situation under a game theoretic perspective (Vickrey, 1961) and formed the foundation for the modern development of this topic. Based on this common knowledge, researchers follow the thought that all players maximise their expected profit which being generally speaking, their utilities. The later analysis regarding private value auction is postulating that each bidder knows how much he or she values the object (Lorentziadis, 2016). The early research primarily focuses on effect of auction design over participants' behaviour under the independent private value paradigm. Researchers have developed theories regarding asymmetry in the probability distribution regarding the value of the auctioned object (Bulow, et al., 1999) and risk aversion of players (Maskin & Riley, 1984). However, the existence of the outside option and its corresponding effect over participants' behaviour are not thoroughly discussed by researchers. In some degree Vickrey's work formed the foundation but in some degree, it limits researchers' thought of what if participants could get the auctioned object without bidding in the current auction. Not to mention discussing the condition of the IPO since this topic is not deeply related to economy.

5.2.2 The appropriate mechanism design comprising the outside options

The research regarding the mechanism design concerning the outside option and quitting option has been started from the point of view regarding the Agency Theory. Researchers have found that the creation of countervailing incentives due to private information could enhance aggregate welfare (Lewis & Sappington, 1989). The characteristics of the optimal contract provided by an uninformed principal to an informed agent when the latter's reservation utility is dependent upon the type of the outside option (Jullien, 2000). Regarding the design of the mechanism, researchers found that some of the received wisdom from the designing and nonlinear pricing is not robust and the richer model is capable of affording a more general empirical specification (Rochet & Stole, 2002). The idea of mechanism design comprising an outside option is commonly found in the topic of wage determination, similar work can be found regarding modern agriculture (Dippel, et al., 2020). However, these papers focus on the mechanism design without a quitting right where the buyer cannot exit for an outside option when he or she has participated in a mechanism. The essence of these papers is designing a mechanism to be chosen rather than researching participant's behaviour when the negotiation proceeds. They focus on a specific point of time rather than within the flow of time.

Later one, researchers started to consider combining the quitting rights (also known as veto constraints) in the designing of the mechanism. Researchers have found that when the quitting rights and contract design are no longer separated, some of the previous academic conclusions does not hold. Compte and Jehiel have developed the framework to include quitting rights in the mechanism and the term veto constraint is formally introduced in their work. Their work has illustrated that inefficiencies are inevitable regardless of the form of correlation as long as private information is dispersed. Also, they have demonstrated how veto constraints differ from ex post participation constraints (Compte & Jehiel, 2007) (Compte & Jehiel, 2009). More importantly, their work proved previous acknowledged full-extraction theory developed by Cremer and McLean regarding the auction comprising private information (Cremer & McLean, 1988) does not hold if agents are entitled to quitting rights. Different from existing literature, this research intends to simulate participants' decision when company intends to initiate its IPO in the financial market, the company does have an outside option however the information is asymmetry and whether the outside option is a premium or inferior for the company will be discussed and factors that may affect the result will be analysed.

5.2.3 The research regarding the channel efficiency

The last research stream concerns in this research is channel efficiency which concerning channel competition and coordination. Research regarding this topic is commonly to be found for bargaining topic in a conventional supply chain structure. Applied this structure in the IPO procedure, the company that initiates the IPO would be the seller and underwriters will act as retailers. The investor would become the buyer. Traditionally, papers focus on the analysis of rationale regarding various behaviours. The research conducted by Ingene and Parry formed the foundation for later research, in their paper, an elegant formula was stated to explain the relationship between the price, quantity and total demand within a supply chain structure (Ingene & Parry, 1995). Based on their formula, researchers explained seller's motivation to adapt an intermediary retailer within a bilateral channel with exclusive channels without revenue sharing (McGuire & Staelin, 2008). Step by step, researchers dedicated to refining the fundamental model by adding some practical factors. Considering the effect of relative negotiation power, researchers have found that a manufacture prefer to choose the contract with one beneficiary term over the other given there is a presence of a dominant retailer (Raju & Zhang, 2005). With this same dominant retailer model, researcher illustrated that the utilisation of trade promotion will benefit the seller and the channel because of retailer's behaviour regarding different level of inventory (Cui, et al., 2008). With an increasing amount of research have been conducted regarding the supply chain with channels, researchers' attention have drawn regarding effect upon sellers' product line decision given the channel is centralised or decentralised (Liu & Cui, 2010). In recent research, it gradually becomes the dominant trend to discover the Pareto zone which could facilitate channel efficiency through revenue sharing among channel members (Cai, 2010). Based on this work, researchers provide sophisticated bargaining solutions regarding revenue sharing rates after investigating firms' channel selection preference among four channel structures (Cai, et al., 2012).

Retail price, it seems to become one of the key issues among the vast amount of economics and marketing literature. However, these papers are inherently influenced by economic and marketing natures, the relationship between retail price and other factors are discussed. For instance, retail price and channel competition (Chen, et al., 2001); retail price and price discrimination (Coughlan & Shaffer, 2009); retail price and price signalling (Moorthy & Winter, 2006) and so on. However, none of these papers include the factor that values a lot in financial perspective, time. How much does time play a role in the determination of retail price? This is a topic that neither economic and marketing intend to explore as the effect of time in the model of these areas is tend to be neglected. However, in the financial market, the value of time is an essential factor that could influence asset value. Lots of financial terms concerns the value of time (e.g. yield to maturity, time related discounted factor). For a company initiates the IPO that intends to bypass the underwriter before the first issue of shares, the time it takes to prepare for the IPO should be considered. As the longer it needs to prepare, the later that company could raise fund for the entity. Hence, this research intends to tailor the model from bargaining research to make it more appropriate for the application in the financial market by adding the time factor in the model.

5.3 The model

In this research, the situation of a company initiates the IPO in the financial market is simulated by a common-seller two buyer channel model with outside option. The seller (the company initiates the IPO) offers the identical material to the buyers (the underwriter) who then sell them to general investors after constructing those shares as a portfolio or the seller could sell shares directly to the general investors as an outside option. For parsimony concern, it is assumed that constructing one unit of final portfolio requires one unit of newly issued share. To simplify the circumstance in convenience of discussing how does the existence of outside option affect participants' behaviour, the strategic variable is set as the most direct factor: price. Inspired from Ingene and Parry (1995), the reverse demand function is written as:

$$P_i = a - x_i - \gamma x_j$$

In the above equation, P_i indicates the price sold from the underwriter to the general investor, x_i represents total supply from one underwriter to the general investor, a represents the total demand, Greek letter γ indicates the substitutability between the portfolio construct by underwriters and has a range $\gamma \in [0,1]$, i & j represents the underwriter within this channel model. The relationship between i & j is:

$$j = 3 - i$$

$$i = 1,2$$

In the above equation, the price sold to the general investor has an inverse relationship with volume of shares available for sale in the market. Ipso facto, when there are more similar products on the market, the price for the product would be high especially when those products are not that differentiated. For the amount of shares issued, this research followed Horn and Wolinsky's thought of sufficient capacity to satisfy the total demand (Horn & Wolinsky, 1988).

The underwriters negotiate wholesale prices of the intermediate supply through a Nash bargaining. To better illustrate the effect of the relative negotiation power throughout the bargaining procedure, it is assumed that underwriter i's bargaining power relative to the seller is denoted as $\theta_i \in (0,1)$, hence the seller's bargaining power relative to the underwriter i is $1-\theta_i$.

5.3.1 Analysis with symmetric negotiation power

Inspired from existing literature regarding analysis of channel negotiation, this research takes a conventional backwards conducting framework (Shang & Cai, 2021). To single out the effect of the outside option, this research first concentrates on the case with symmetrical negotiation power ($\theta_i = \theta_j = \theta$). To start with a common condition, both underwriters successfully reach a contract with the seller which indicates no existence of the outside option. Both underwriters committed a fully underwriting contract which means underwriters will buy all the shares from the seller and resell them to general investors.

Conducting the whole situation backwards, given the wholesale prices (w_i, w_j) the revenue function for participants would be:

$$\pi_i = (P_i - w_i)x_i$$

$$\pi_s = w_i x_i + w_j x_j$$

The letter π represents the revenue of participants and the subscripts i & s indicates the revenue of underwriter i and seller respectively. Hence, for the underwriter, he or she chooses to maximise individual profits by selecting the appropriate price P sold to the general investor. By solving the first order conditions (FOCs) the equilibrium price sold to the general investor would be: (1)

$$\hat{P}_i(w_i, w_j) = \frac{(2-\gamma-\gamma^2)a + 2w_i + \gamma w_j}{(4-\gamma^2)}$$

The resulting sales quantities are (2)

$$\hat{x}_i(w_i, w_j) = \frac{(2-\gamma-\gamma^2)a - (2-\gamma^2)w_i + \gamma w_j}{(1-\gamma^2)(4-\gamma^2)}$$

Substituting both expressions into the revenue function, the equilibrium profits are: (3)

$$\pi_i(w_i, w_j) = (1-\gamma^2)[\hat{x}_i(w_i, w_j)]^2$$

$$\pi_s(w_i, w_j) = w_i \hat{x}_i(w_i, w_j) + w_j \hat{x}_j(w_i, w_j)$$

5.3.2 Bargaining without outside option

In a conventional supply chain structure, one of the most important issues within the analysis procedure is determining the disagreement point. Under the thought of conventional supply chain structure without outside option, when the first negotiation failed between the seller and underwriter, the remaining underwriter is empowered in the negotiation as both participants known that without an agreement this time indicates zero revenue for everyone. This could be disastrous for the seller since initiate the IPO is the key to raise fund for the entity or in some cases, going public is the requirement in the Bet-on agreement that company signed with existing investors. In another word, when the remaining underwriter known he or she is the last resort for the company to initiate the IPO, he or she will gain the sufficient power to dictate the whole negotiation.

Given when there is only one underwriter j remaining on the market, this situation is often referred as a single channel monopoly market. In this time, the demand function would be:

$$x_j(P_j) = a - P_j$$

The underwriter intends to maximise the revenue:

$$\pi_j^M = (P_j^M - w_j^M)(a - P_j^M)$$

Solving this FOC, the resulting optimal price will be:

$$P_j^M = \frac{a + w_j^M}{2}$$

And the profit for the underwriter would be:

$$\pi_j^M(w_j) = \frac{(a - w_j^M)^2}{4}$$

$$\pi_s^M(w_j) = \frac{w_j^M(a - w_j^M)}{2}$$

The superscript M represents the situation of the single channel monopoly market.

Hence, in the negotiation without outside option, the underwriters' profits are stated above, the bargaining solution result (w_i^T, w_j^T) is an equilibrium when both the seller and underwriters choose under the traditional bargaining structure. The superscript T represents the condition of the traditional condition. The unique bargaining solution (w_i^T, w_j^T) satisfies: (4)

$$w_i^T = argmax\left[\pi_i(w_i^T, w_j^T)\right]^{\theta_i}\left[\pi_s(w_i^T, w_j^T) - \pi_s^M(w_j^M)\right]^{1-\theta_i}$$

For the symmetric negotiation power case, it can be conducted that: (5)

$$w_i^T = \frac{(1-\theta)(1-\gamma)(2+\gamma)a}{2(1-\gamma)(2+\gamma)+\gamma^2(1+\gamma)(2-\gamma)\theta}$$

5.3.3 Bargaining with outside option

For the seller, in a traditional negotiation structure, it is assumed that participants can earn nothing if negotiation fails. In reality, seller could bypass the underwriter and sell shares directly to general investors. Facebook has done so back in 2012 which is rather rare but not impossible in financial market. Since there has a precedent case, it indicates that bypass the underwriter is not always an inferior choice. Based on Shang and Cai (2021)'s work, this research proposes a model that combines conventional supply chain characteristics and factors from financial market.

In the model, it is assumed that sellers could choose to sell shares directly to general investors, but comparing with underwriter, one investor cannot afford to buy sufficient amount of shares. Hence it may take longer time for the seller to reach the final deal, after all, the seller needs to negotiate with various individual investors. For the underwriter, he or she can still purchase shares from the open market. Assuming the overall average price is w_0. Then the bargain circumstance would be:

$$w_i^T = argmax\left[\pi_i\left(w_i^T, w_j^T\right) - \pi_i(w_0, w_j^T)\right]^{\theta_i}\left[\pi_s(w_i^T, w_j^T) - \pi_s(w_0e^{-r\Delta t}, w_j^T)\right]^{1-\theta_i}$$

The term $e^{-r\Delta t}$ is the discount factor for the seller if he or she encounter the outside option circumstance. r represents the interest rate which usually used by one year US treasury bills interest rate. Δt indicates time taken by the seller to reach the final deal with various general investors.

In the above expression, the underwriter and the seller aim to maximise

$$\left[\pi_i\left(w_i^T, w_j^T\right) - \pi_i(w_0, w_j^T)\right]^{\theta_i}\left[\pi_s(w_i^T, w_j^T) - \pi_s(w_0e^{-r\Delta t}, w_j^T)\right]^{1-\theta_i}$$

For the underwriter i, it is assumed that $\pi_i(w_i^T, w_j^T) > \pi_i(w_0, w_j^T)$ if $w_i^T \leq w_0 e^{-r\Delta t}$. For the seller, the difference of the profit $\pi_s(w_i^T, w_j^T) - \pi_s(w_0 e^{-r\Delta t}, w_j^T)$ would be: (6)

$$(w_0 e^{-r\Delta t} - w_i^T)\frac{(2-\gamma^2)w_i^T - [(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 e^{-r\Delta t} + 2\gamma w_j^T]}{(1-\gamma^2)(4-\gamma^2)}$$

This difference is positive if

$$w_i^T \geq \frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 e^{-r\Delta t} + 2\gamma w_j^T}{(2-\gamma^2)}$$

Given the assumption that $w_i^T \leq w_0 e^{-r\Delta t}$. Then if $w_0 e^{-r\Delta t} \leq \frac{(1-\gamma)(2+\gamma)a-(2-\gamma^2)w_0 e^{-r\Delta t}+2\gamma w_j^T}{(2-\gamma^2)}$, then under this circumstance, w_i^T reaches its highest value $w_i^T = w_0 e^{-r\Delta t}$. Otherwise, w_i^T becomes the interior point $w_i^T < w_0 e^{-r\Delta t}$ if $w_0 e^{-r\Delta t} > \frac{(1-\gamma)(2+\gamma)a-(2-\gamma^2)w_0 e^{-r\Delta t}+2\gamma w_j^T}{(2-\gamma^2)}$.

In another word, the wholesale price of the outside option has a relationship with the remaining underwriter:

$$w_0 e^{-r\Delta t} > \frac{(1-\gamma)(2+\gamma)a + 2\gamma w_j^T}{2(2-\gamma^2)}$$

In the market of share issuing, this condition illustrate that the seller has the option to choose either selling the shares to underwriters and take all the money raised from underwriters and immune to any effect when underwriter sell their shares to the general investor. Or otherwise, seller could directly sell shares to general investors and bypass the underwriters. To determine the threshold wholesale price for the seller that help them make the decision, it may take to consider some extreme case to compute for those thresholds.

Considering an extreme circumstance when the seller chooses not to negotiate with any underwriter and directly sell the newly issued shares to the general investor while enable underwriters to buy the shares as well. This means that both underwriters will buy shares at the price of $(w_0 e^{-r\Delta t}, w_0 e^{-r\Delta t})$.

Hence the equilibrium equation would be: (7)

$$\frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 e^{-r\Delta t} + 2\gamma w_0 e^{-r\Delta t}}{(2-\gamma^2)} \geq w_0 e^{-r\Delta t}$$

The final result reveals that

$$w_0 e^{-r\Delta t} \leq \frac{a}{2}$$

Recalling from the previous part, the expression for the whole sale price under a traditional negotiation circumstance is:

$$w_i^T = \frac{(1-\theta)(1-\gamma)(2+\gamma)a}{2(1-\gamma)(2+\gamma)+\gamma^2(1+\gamma)(2-\gamma)\theta}$$

It contains two elements: relative negotiation power and product substitutability. But under the condition which seller bypasses both underwriters, indicating an irrelevant condition of relative negotiation power. And since both underwriters purchase the exact product as the final investor, the product substitutability becomes irrelevant as well. The whole expression will become $\frac{a}{2}$. This indicates that $\frac{a}{2}$ is not only one of the equilibrium result for the outside option wholesale price but also the boundary point of equilibrium.

To better understand the effect brought by the outside option to the negotiation, the following discussion is developed based on the valuation of outside option.

Circumstance 1: $\frac{a}{2} < w_0 e^{-r\Delta t} < a$:

The above deduction illustrates a threshold exist such that when the price from the outside option is sufficiently higher compared to a conventional negotiation result. This condition would motivate all the participants to negotiate for an optimal contract and avoid entering the outside option. In another word, the outside option would become the last resort for the seller. It is rather reasonable for underwriters since negotiate for a lower wholesale price would directly benefit for them. It would be rather difficult to understand the rationale behind seller's action to discard a higher wholesale price.

Seller's consideration regarding the wholesale price concerns the trade-off between a higher profit margin with smaller demand base and lower profit margin with bigger demand. Remember the very first demand function:

$$P_i = a - x_i - \gamma x_j$$

Without any underwriter, the seller directly faces the general investors as well as underwriters. The relationship between the quantity and price of the product has an inverse relationship. Given the equilibrium price is higher than half of the total demand. Under the basic demand and supply condition, the higher equilibrium price above the half point of the total demand brings unproportional decrease of the consumer surplus. This indicates that the revenue of the seller is increasing in a relative smaller scale compared to the price. In a simple word, seller would always prefer a lower profit margin with larger demand base give the above trade off.

While for the underwriters the logic is rather simple, the lower wholesale price represents a lower cost and potentially larger profit margin when they sell those shares to general investors. Hence when the wholesale price of the outside option is sufficiently higher, it would be mutually beneficial for both parties to negotiate for an optimal contract.

Circumstance 2: $w_0 e^{-r\Delta t} < \frac{a}{2}$:

Under this circumstance, the wholesale price from outside option would be sufficiently low. The seller would prefer to directly sell their shares to general investors rather than negotiate with underwriters. Taking the point of view of the seller, negotiating with underwriters not only takes time but could not bring a much more favourable result for the seller. The relative lower profit margin will be fully compensated by the expansion of demand base. While for the underwriter, if they think the shares from the seller is worth investing they can buy shares from a lower price compared to conducting a negotiation. If the wholesale price is sufficiently high, then underwriters would prefer to conduct a negotiation in order to pursued for a relative lower wholesale price. In another word, the wholesale price of the outside option determines underwriters' attitudes towards whether to conduct the negotiation with the seller.

5.3.4 Comparison of wholesale price in conventional negotiation and outside option

From previous part, the condition in both conventional negotiation situation and outside option are discussed. Hence, it would be appropriate to compare conditions from both circumstances to explore effects brought by the existence of the outside option and conduct effective managerial insights.

Under a conventional negotiation structure, when the information of the first negotiation failed between underwriter i and seller being received by the remaining buyer j, the advantageous negotiation power immediately tilts to the remaining underwriter since both the underwriter and the seller know that if the negotiation failed for the second time, the seller cannot issue shares to the market and will not accomplish the fund raising target. Hence, this would give the remaining underwriter j the ultimate power to select the most favourable wholesale price for itself. From the previous part it can be seen that the response function of underwriter j is solely focus on maximising personal revenue and the sell has to take the very low wholesale price as the last resort of raising fund.

However, when the seller has an outside option in negotiation, this will bring tremendous difference during the negotiation between the underwriter j and the seller. This time during the negotiation, they both know the existence of outside option. The underwriter afraid that proposing the wholesale price too low will push the seller abandon the current negotiation and switch to the outside option which will make the time and effort contributed to the negotiation in vain. Even though these factors did not have specific expressions in the current model, they still have economic value in real life practice. By taking those consideration, when the remaining underwriter j negotiates with the seller knowing that seller has an outside option, the boundary of the best response proposal should be as low as the wholesale price from the outside option. Since any price lower than that will motivate the seller to issue shares through an outside option where underwriter j will lose the negotiation power gained by knowing that the other underwriter has failed in the negotiation.

Under the scenario that wholesale price in outside option is higher than half of the total demand ($\frac{a}{2} < w_0 e^{-r\Delta t} < a$), the negotiated wholesale price proposed by underwriter j should be positioned with the following boundary:

$$\frac{a}{2} < w_j^T \leq w_0 e^{-r\Delta t} - \frac{(1-\gamma)(2+\gamma)a}{2(2-\gamma^2)}$$

In another word, the existence of the outside option could help the seller to avoid the condition where the participant has no negotiation power in a bargaining condition. The existence of the outside option could prevent the remaining underwriter cutting down the wholesale price and damage the revenue of the seller. It can be saying that outside option is one of the protections that can be utilised by the seller to defend its revenue in a negotiation circumstance.

The above discussion is primarily focus on the condition that the existence of outside option is an inferior option for both sides of the participation. It would be pretty straightforward to be understood as the existence of the outside option empowers the seller to refuse the unacceptable offer from the only underwriter remaining in the market. But what if the existence of outside option provides a premium option for the seller but stays as an inferior option for the underwriter? The new argmax function would be

$$w_i^T = argmax\left[\pi_i\left(w_i^T, w_j^T\right) - \pi_i\left(w_0, w_j^T\right)\right]^{\theta_i}\left[\pi_s\left(w_0 e^{-r\Delta t}, w_j^T\right) - \pi_s\left(w_i^T, w_j^T\right)\right]^{1-\theta_i}$$

From previous induction content, the criteria of judging whether the outside option is the premium result is solely rely on the comparison between wholesale prices from both conditions. Hence, if the new condition remaining the same criteria would be contradictory. Since for the underwriter, the wholesale price from the outside option must be higher compared to the traditional negotiation. For the underwriter i, it is assumed that $\pi_i\left(w_i^T, w_j^T\right) > \pi_i(w_0, w_j^T)$ if $w_i^T \leq w_0 e^{-r\Delta t}$. But for the seller, if the outside option becomes the premium condition, the discounted wholesale price should be lower compared to traditional negotiation price but could bring higher quantity sold. So for each condition to be real, the wholesale price condition in the outside option would be contradictory. In another word, the above argmax formula does not exist since the negotiation cannot be conducted if both participates have various premium options and disagreement points.

Lemma 1: To proceed a negotiation, both parties must in consent regarding the premium option and disagreement point. Otherwise, the negotiation will never be initiated without any common ground.

5.4 Side payment

From the previous deduction, it is known that the existence of outside option provides a last resort for the seller when the negotiated price is far too low to be accepted. In the reality, the whole process of the IPO has two distinctive stages. The first stage primarily contains the negotiation between the seller and underwriters regarding the quantity of shares and price of the shares. However, for the second stage, it mainly comprises the underwriter and the general investor under the circumstances of a fully underwriting. Noticing that the seller would only encounter in one of the stages throughout the procedure. This very nature is dependent upon the objectives of the participants. The objective of the seller to conduct the IPO is raising money for the entity to ensure sound cash reserve for future development. Hence, the seller will accept the offer from the underwriter as long as the terms proposed by the underwriter meet the 'bottom line' set by the seller if it is rational.

Considering the scenario with outside option throughout the IPO procedure, the seller knows that he has the last resort of encountering the outside option if the negotiation failed. However, encountering in the outside option would bring the uncertainty regarding the price of shares as well. In another word, the seller has the right to turn down the unfavourable offer from the underwriter with the cost of bearing price uncertainty and risk of failing to raise sufficient fund for the entity. Hence, it is possible that seller would still prefer to conduct a negotiation with the underwriter though there exists an outside option because of an early agreement with the underwriter indicate an assurance regarding fund raising. To ascertain this assurance, it is acceptable for the seller to transfer or in other word sacrifice some of the benefit in order to ensure an accomplishment of the target.

In the field of the theoretical research, this kind of transfer of benefits between the participants is often referred to as 'side payment'. It is used to facilitate the negotiation in order to make every participant better off compared to the conventional situation. Commonly practices can be found in the real world with different names but with identical essence. In the financial world, this type of side payment is commonly found as price discount, premium etc. All of them have the same priority, facilitate the transaction as quickly as possible since in the financial world, the value of time sometimes can be a mystery to be interpreted.

Considering the situation where both the seller and the underwriters know the existence of the outside option but without knowing the exact value of shares once seller decides to encounter the outside option. However, the seller knows that once decided to encounter in outside option, the time cost and uncertainty of failing to raise sufficient fund would be solely undertaken by himself. When both parties encountering the negotiation, the fear of fund raising failure promote the seller has more motivation to conduct a contract with the seller. Hence, the seller would be willing to sacrifice part of the potential benefit by paying the side payment to both underwriters in hope of reaching a reliable contract before the IPO officially initiates.

In a traditional negotiation with side payment (TS) conducted between the seller and underwriters, the equilibrium wholesale price and the side payment (w_i^{TS}, T_i^{TS}) solve the following question:

$$max\left[\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}\right]^{\theta_i}\left[\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - \pi_s^M(w_j^{TS})\right]^{1-\theta_i}$$

In the above expression, the underwriters' profit would be $\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}$ and sellers' profit would be $\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - T_j^{TS}$ with the disagreement point $(d_i, d_s) = \left(0, \pi_s^M\left(w_j^{TS}\right) - T_j^{TS}\right)$. Hence the appropriate side payment would be:

Taking the side payment as the strategic variable, let the FOC equals to zero:

$$\theta_i\left[\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}\right]^{\theta_i - 1}\left[\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - \pi_s^M(w_j^{TS})\right]^{1-\theta_i} - \left[\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}\right]^{\theta_i}(1-\theta_i)\left[\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - \pi_s^M\left(w_j^{TS}\right)\right]^{-\theta_i} = 0$$

$$\theta_i\left[\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}\right]^{\theta_i - 1}\left[\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - \pi_s^M\left(w_j^{TS}\right)\right]^{1-\theta_i}$$
$$= \left[\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}\right]^{\theta_i}(1-\theta_i)\left[\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - \pi_s^M\left(w_j^{TS}\right)\right]^{-\theta_i}$$

$$\theta_i\left[\pi_s\left(w_i^T, w_j^{TS}\right) - T_i^{TS} - \pi_s^M\left(w_j^{TS}\right)\right] = \left[\pi_i\left(w_i^T, w_j^{TS}\right) + T_i^{TS}\right](1-\theta_i)$$

$$T_i^{TS} = \theta_i\left[\pi_s\left(w_i^T, w_j^{TS}\right) - \pi_s^M\left(w_j^{TS}\right)\right] - (1-\theta_i)\pi_i\left(w_i^T, w_j^{TS}\right)$$

In the negotiations, the firm chooses the w_i^{TS} to maximise $\pi_i\left(w_i^T, w_j^{TS}\right) + \pi_s\left(w_i^T, w_j^{TS}\right)$, the first order condition with respect to w_i^{TS} is

$$\frac{\gamma^2(2+\gamma)(1-\gamma)a - 4(2-\gamma^2)w_i + 4\gamma w_j}{(1-\gamma^2)(4-\gamma^2)^2} = 0$$

Solving the FOCs jointly, the following equilibrium results can be obtained:

$$w_i^{TS} = \frac{\gamma^2 a}{4}, p_i^{TS} = \frac{(2-\gamma)a}{4}, x_i^{TS} = \frac{(2+\gamma)a}{4(1+\gamma)}$$

Hence the underwriter's profit after considering the side payment would be:

$$\pi_i\left(w_i^{TS}, w_j^{TS}\right) - T_i^{TS}$$

The full expression would be:

$$\pi_i\left(w_i^{TS}, w_j^{TS}\right) - \theta_i\left[\pi_s\left(w_i^T, w_j^{TS}\right) - \pi_s^M\left(w_j^{TS}\right)\right] + (1-\theta_i)\pi_i\left(w_i^T, w_j^{TS}\right)$$

Substituting all the known elements into the expression, the profit of the underwriter after considering the side payment is:

$$\frac{\theta_i(2+\gamma)(4-2\gamma-\gamma^3+\gamma^4)a^2}{32(1+\gamma)}$$

The condition would be the same for the other underwriter. Hence the profit of the issuing company would be: (8)

$$\frac{(2+\gamma)[8-4\gamma-(4-2\gamma-\gamma^3+\gamma^4)(\theta_i+\theta_j)]a^2}{32(1+\gamma)}$$

This result is the consistent with deduction conducted by previous researcher but the side payment was designed to be paid from the buyer to the seller (Shang & Cai, 2021). It has been proved that the application of side payment could help to create a bigger cake for all the participants while the distribution of the total profit would be dependent upon the relative negotiation position. However, the intriguing point is the mathematical result from both circumstances. They both lead to the same conditions even though the side payment is set as a completely opposite direction.

To better understand this phenomenon, the essence of the side payment shall be discussed in the following paragraphs.

The side payment usually acts as the coordinator which facilitates participants' profits within the negotiation. The function of the side payment is sacrificing part of the benefit from the initial inferior equilibrium in order to promote the achievement of a premium equilibrium. In the paper developed by Shang & Cai (2021), their side payment is setting as a payment from the buyer to the seller within a channel negotiation situation in order to negotiate for a beneficiary contract term. However, in this research's framework setting, the side payment is paid from the seller to the underwriter in order to achieve an early agreement regarding share underwriting terms to avoid an inferior outside option. It could be argued that the intention from both conditions is the same which is negotiating for a better contract term, but different payment structures simultaneously lead to same equilibrium result. Judging from the condition from Shang & Cai (2021), the result conducted from the side payment helps the seller to receive a higher wholesale price and total revenue. And from the model set in this article, the side payment is paid from the retailer to the seller and yield the same result. It is saying that no matter how the side payment is set, once the fundamental model is set, the result is the same regardless of the side payment direction. This is because the total revenue that could be generated from the bargaining is set at the beginning, and the side payment is the method used to facilitate the distribution of revenue between the seller and retailers.

To better understanding how the side payment work under both circumstances does, the following exploration content will contain mathematical results from previous deductions to aid explanation.

The existence of the outside option only set a lower boundary for participants within a traditional negotiation that if the negotiation failed, there exist a option where seller would necessarily has to face the condition of failure to raise sufficient fund for the entity. And for the underwriter, it gives a turning point for the price offering indicates any price lower than that would trigger the exit of the seller. However, entering the outside option means that the seller has to take the risk that it would take more time to find enough investors who would like to purchase the initially issued shares. And the time is one of the most important factors within the world of finance. Since the wholesale price from the outside option would be $w_0 e^{-r\Delta t}$ and when the time becomes sufficiently large, the wholesale price becomes smaller as the time is a positioned on the exponential power with a negative sign. When the wholesale price is low enough, both underwriters will exist the current negotiation and purchase shares from the outside option market. Hence to prevent this condition from happening, the seller would like to sacrifice some of its benefit in exchange for an early assurance regarding share insurance.

The side payment itself has an expression of

$$T_i^{TS} = \theta_i\left[\pi_s\left(w_i^T, w_j^{TS}\right) - \pi_s^M\left(w_j^{TS}\right)\right] - (1-\theta_i)\pi_i\left(w_i^T, w_j^{TS}\right)$$

Mathematically speaking, the side payment is constructed by relative negotiation power and excess payoff between the equilibrium and the disagreement point. The coefficient multiplied by each excess payoff is the relative negotiation power of the counterparty within the negotiation. And the result is the payer's relative negotiation power multiplies the counterparty's excess return minus the counterparty's relative negotiation power multiplies the payer's excess return.

In the language of the bargaining theory, the payer of the side payment is willing to sacrifice part of the personal benefit in order to persuade the counterparty to discard the current equilibrium payoff and reach a new equilibrium where both parties are better off. After considering the side payment, the new equilibrium profit of the payer would be:

$$\theta_i\pi_i\left(w_i^T, w_j^{TS}\right) - \theta_i\left[\pi_s\left(w_i^T, w_j^{TS}\right) - \pi_s^M\left(w_j^{TS}\right)\right]$$

This means that payer's new profit would be the relative negotiation power multiplies personal excess return from a successful negotiation minus the counterparty's excess return under the framework of conventional negotiation. This result is consistent with previous work which indicates a potential lemma.

Lemma 2: Once the conventional negotiation framework is set, the profit of the participants after the considering the side payment would be always the same regardless of the side payment structure. The amount of the side payment is solely dependent upon the relative negotiation power of the participants.

5.4.1 Side payment with outside options

The previous analysis is based on the conventional negotiation framework, the side payment is constructed with the idea that failure in the negotiation will lead to zero payoff. However, with the existence of the outside option, the construction of the side payment may demonstrate some variations.

The existence of the outside option provides an alternative choice for the seller when the negotiation terms are too harsh to be accepted. In another word, if the situation from the outside option is superior compared to the underwriter's monopoly circumstance, the seller is motivated to discard the negotiation with the only remaining underwriter and switch to the outside option. In the mathematical word, the seller will engage in the outside option when the first negotiation failed and outside option provides a result that

$$\pi_s^O(w_0 e^{-r\Delta t}) > \pi_s^M(w_j^{TS})$$

The upper script O indicates the condition in the outside option.

In a more direct relationship regarding the wholesale price, the outside option price has the condition that:

$$w_0 e^{-r\Delta t} > \frac{\gamma^2 a}{4}$$

And the new construction of the side payment would be less compared to the conventional negotiation. The new side payment would be:

$$T_i^O = \theta_i[\pi_s(w_i^T, w_j^{TS}) - \pi_s^O(w_0 e^{-r\Delta t})] - (1-\theta_i)\pi_i(w_i^T, w_j^{TS})$$

The new profit for the underwriter would be:

$$\theta_i \pi_i(w_i^T, w_j^{TS}) - \theta_i[\pi_s(w_i^T, w_j^{TS}) - \pi_s^O(w_0 e^{-r\Delta t})]$$

And the seller's profit after considering the side payment would be:

$$\pi_s(w_i^T, w_j^{TS}) - 2\theta_i[\pi_s(w_i^T, w_j^{TS}) - \pi_s^O(w_0 e^{-r\Delta t})] + 2(1-\theta_i)\pi_i(w_i^T, w_j^{TS})$$

While compared to the conventional negotiation with side payment, the payoff was:

$$\pi_s(w_i^T, w_j^{TS}) - 2\theta_i[\pi_s(w_i^T, w_j^{TS}) - \pi_s^M(w_j^{TS})] + 2(1-\theta_i)\pi_i(w_i^T, w_j^{TS})$$

For both the seller and the underwriter, the existence of the outside option will improve their profit compared to traditional circumstance since the outside option provides a superior condition to the monopoly condition which will make the second term becomes smaller. And the final result will become larger.

It is consistent with the argument from Shang and Cai (2021) regarding the side payment has the functions of create a bigger cake as well as split the cake but the existence of the outside option provides a low-boundary cap for the transfer of benefit between participants. The side payment would be smaller as long as the outside option provides a premium return compared to the traditional monopoly situation. And the key of the determination, more precise than other factors within the conventional negotiation framework, is the discounted factor $e^{-r\Delta t}$ which implies the interest rate and the time taken to reach an appropriate contract through outside option.

Hence there leads to a new lemma:

Lemma 3: when the outside option provides a superior return compared to conventional monopoly situation, the existence of the outside option will decrease the scale of side payment by providing a low boundary to the profit transformation between participants. Otherwise, when outside option is an inferior option, the situation embedded side payment will converge to a universal result regardless the side payment direction.

5.5 Conclusion & Discussion

In this research, the outside option is introduced to the conventional negotiation structure regarding an IPO negotiation between the seller and underwriters. Compared to conventional negotiation, the existence of the outside option gives the seller an available choice when the seller thinks terms proposed by the underwriter is remotely acceptable. At the same time, when the information is transparent within the negotiation, the underwriter would be frightened to offer a price that solely maximise personal benefit as the counterparty could switch to outside option and lead to a zero-payoff condition from the negotiation. Entering the outside option also indicates that the underwriter could bearing more cost as the wholesale price in the outside option is higher than what underwriter proposed and got refused by the seller. In another word, the existence of the outside option empowers the seller by providing an alternative to refuse the harsh terms. However, when the outside option price is too low, the underwriters will also lose interest in negotiating with the seller in the first place as they know they could purchase the identical shares from the outside option but with a sufficiently low price. Later this research tries to deduct situation where both parties have opposite disagreement points with each other and deduction process demonstrated that one of the most upfront factors is both parties have an agreement regarding the premium condition and the disagreement point.

Since this research introduces the time factor and interest rate into the construction of share price, the framework of the negotiation model considers more practical issues such as time consumed to reach a suitable contract with various investors. Hence, there is a time pressure on the seller to reach a contract as soon as possible. The better facilitate a successful negotiation with underwriters to ascertain an early ascertain regarding fund raising target, it is assumed that seller would be willing to pay side payments to both underwriters. The payment direction is completely opposite to existing literature while the final result demonstrates a consistent expression regarding the profit after considering side payment. This leads to the discussion regarding the side payment mechanism and the construction of side payment.

The side payment in essence is the transfer of personal excess benefit from a previous agreed equilibrium condition to reach a circumstance that everyone is better off compared to otherwise without the side payment. Regarding the coefficient of the benefit transferred, it is the relative negotiation power. For the negotiation with side payment, once the framework is set, the result is the same regardless the payment direction. This is because of the essence of the mechanism. Side payment is the sacrifice of personal interest which the full range is the excess return of the successful negotiation minus counterparty’s situation. The percentage of the transfer can also be regarded as a negotiation which determines the exact profit transferred which solely determined by the bargaining power.

After considering the outside option, the construction of the side payment will be different. Since the constitution of the side payment includes the excess return from a successful negotiation, in another word, it is the difference between the result of the successful negotiation and the disagreement point. The existence of the outside option provides an alternative when the negotiation failed. This illustrates that when the outside option is better off than the disagreement point, the outside option will replace the previous monopoly situation and provides aa lower boundary for the transfer of benefit within the side payment. Once this happens, this will provide a better off situation for both seller and underwriter. Under this situation, the existence of outside option is not only empowering the seller but also provides a better result for the underwriter.

Managerial insights

The outside option does not always exist for a conventional commodity market, but for the financial market, it will be frequently encountered by both the company who initiates the IPO and underwriters. The unique feature of the financial market makes the situation constantly require consideration regarding the interest rate and time.

The existence of the outside option is beneficiary for the seller. It provides an alternative option for the seller to turn down the offer proposed by the underwriter. And it would be benefit for the seller to let the counterparty knows the existence of the outside option. This will provide an extra caution for the underwriters when they lower their offers. The seller will always have the choice to engage in outside option throughout the bargaining which is a valuable tool to empower the relative negotiation power of the seller.

Judging from the model from previous model, the outside option price requires discounted time issue which concerns interest rate and time taken to accomplish the deal. When both issues become higher, their exponential position will make the wholesale price from the outside option becomes smaller which makes the initial negotiation with underwriters lost the attractiveness.

The side payment still acts as a facilitator throughout the negotiation since the seller bears the timing pressure and uncertainty regarding wholesale price from the outside option. The relative negotiation power compared with the underwriter should be consistently considered by the manager of the company initiates the IPO. This issue determines the percentage of profit transferred. When the seller is powerful relative to the underwriter, the value of the side payment could be rather lower compared to a company with weaker negotiation power. In the real world, the side payment is rarely showed as the monetary transaction but as alternative forms such as share price discount. It would be beneficiary for the company to exploit its negotiation power to gain advantage over the IPO negotiation.

The side payment is a powerful tool to facilitate the agreement of the contract. Its formation is profoundly affected by the circumstance of the outside option. It reshapes the contents of the side payment compared to traditional negotiation framework. Under the circumstance of outside option is higher than the monopoly underwriter situation, the higher the interest rate and the longer the time taken, the diminishing wholesale price from the outside option would make the side payment becomes larger.

In the real world, the managers of the company that initiates the IPO should carefully choose the time of the initiation. Since both internal and external factors will significantly affect the result of the IPO negotiation. The existence of the outside option acts as a double edges sword, when the interest rate is high and rising, it will make the traditional negotiation lost the attractiveness for the underwriter. However, when the interest rate is at a rather moderate level, the outside option could be a tool to enhance company's negotiation power against the underwriter and limit the amount of side payment if the seller intends to adapt this facilitation.

Chapter 6. The value determination of shares with stochastic demands and private information throughout the bargaining process

6.1 Introduction

Many negotiations take place between participants over a period which indicates a repetitive procedure of bargaining. Unlike a one-time offering, a repetitive and sequential negotiation would be rather representative in the real life. Studies regarding those sequential negotiation has been developed since the late of the 20th century and a lot of researchers published their elegant articles to build a firm foundation for the later research. This type of research initially starting from an analysis under an oligopoly market structure since it gives the most direct relationship that is sufficient to construct a bargain: a simple two layers supply chain which including one supplier with two retailers. Dobson has demonstrated that the union gains by bargaining firstly with a firm with relative weak bargaining position or a firm with relatively large profit under a conventional bargaining framework which consists a sequentially negotiation between the supplier and the retailer (Dobson, 1994). Inspired from the idea of Dobson, later researchers have generalized the founding to a commonly found sequential negotiation between a monopolist retailer with two suppliers over substitute products. The research conducted by Marx and Shaffer illustrates that the application of below cost pricing by one supplier would allow the retailer to extract rents from the second supplier which explains how one supplier and the retailer are capable of increasing their joint profit at the expense of the remaining participant (Marx & Shaffer, 1999). With more research investigating how the negotiation is set, researchers have dedicated an increasing amount of effort to study how the unique characteristics of the participant could shape the result of the negotiation. Two of the commonly recognized features are bargaining power and competitive intensity and Alexander Raskovich's work demonstrated how these two features jointly affect the negotiation result when buyers are capable of choosing the order when they negotiate with suppliers of known characteristics (Raskovich, 2007). Besides exploring how participants' characteristics could twist the negotiation, another research stream regarding effects caused by negotiation mechanisms is fascinated by researchers as well. The effects of price matching and simultaneous negotiation over the channel efficiency are discussed by Cai and Shang (Shang & Cai, 2021).

The research regarding the circumstances of different negotiation mechanisms has been a popular topic. These research comprises exploration of appropriate behaviours when there exist a retailer which can either has relative strong or weak negotiation power compared to the seller. The research regarding the so called 'optional strategies' including the determination of quantity purchased, price settlement and contract type determination. For the negotiation mechanism, researchers have dedicated efforts to explore the preference of participants based on the results conducted from the mathematical models (Shang & Cai, 2021). In this article, it is assumed that information is symmetrical for all participants. The results conducted is based on the assumption that every participant is rational. If, on the other hand, there exist some information asymmetry between the participants, researchers have conducted the process regarding how the less informed participant could construct the belief regarding the stochastic information based on the behaviour of the counterparty during the sequential negotiation (Feng, et al., 2015). But the situation which concerns stochastic information under different negotiation mechanisms remains unintended. Commonly, the bargaining between two participants with asymmetric information is referred as the dynamic bargaining. In this chapter, the framework of dynamic bargaining will be applied to research the channel preferences of the participants under different negotiation mechanisms.

6.2 Literature Review

6.2.1 The sequential negotiation model

This research relates to several research streams. The first stream is regarding the exploration of bargaining within a sequential negotiation model. Starting from the idea of reaching the equilibrium among participants (Rubinstein, 1982), the study regarding the sequential negotiation has formed the theoretical foundation. Based on this thought, researchers built a simple two-person, two-period bargaining model and resolves its by utilizing the concept of the perfect Bayesian equilibrium (Fudenberg & Tirole, 1983) which the backward analysing approach has brought profound effect to the later research. By introducing the mathematical expression for a participant's behaviour, researchers have developed the concept so called 'the perfect equilibrium' supported by the beliefs which prevent a player from deviating to an unreached node (Grossman & Perry, 1986). A major direction of the research in this field is regarding the availability of information. A strategic game with incomplete information concerning the time preference which introduced by Rubinstein to demonstrate the clear connection between the equilibrium circumstance and the first mover's belief regarding his opponent's type (Rubinstein, 1985). Different from Rubinstein's model of two players propose sequentially regarding cutting a pie, a common trade model is created to discuss buyer's and seller's proposal behaviour when the buyer knows the gains from the trade while the seller does not (Grossman & Perry, 1986). Inspired from the unequal bargaining positions of participants, researchers developed an infinite horizon bargaining model incorporating two-sided incomplete information-revealing, illumination of interesting qualitative bargaining issues and plausible equilibria, the model illustrates the relationship between player's relative bargaining strength and likelihood of capturing the gains from bargaining (Chatterjee & Samuelson, 1987). The research regarding the availability of information promoted the emergence of signalling game which comprises the procedure of convey private information and its effect over the general equilibrium over the whole model, researchers have founded that by restricting the out-of-equilibrium beliefs, participant can sometimes eliminate several unintuitive equilibria. By investigating participants' behaviour in specific examples, researchers have successfully related those restrictions to Kohlberg and Merten's notion of stability (Cho & Kreps, 1987). Incorporating the signalling effect to the bargaining with asymmetrical information, researchers found that a repetitive negotiation may perform worse than the outcome from one-period bargaining game. The more frequent offers proposed, the outcome is more likely to be characterized by recurring bursts of high profitability of agreement and the profitability becomes negligible due to long periods of delay (Deneckere & Liang, 2006). However, the research regarding the sequential negotiations seems to be halted after the 21st century, the theoretical framework remains consistent over the streams of the articles. But the models are limited by themselves due to three types of commonly encountered economic forces contributed to the shape of the outcomes. These forces contain buyers' option of deferring the purchase, seller's anticipation over buyers' strategic behaviour, and the possibility of making multiple price offers would allow the seller to engage in price discrimination to extract more surplus from buyers (Rosato, 2017). Beyond these conventional economic models, this research proposes a new model by adding the availability of outside option to empower the buyer under an asymmetric information circumstance.

6.2.2 Different effects over the supply chain structures caused by various negotiation mechanisms

The second research stream concerns the structure of the supply chain and results associated with various negotiation mechanisms. Previous research regarding this research stream mostly concerning various negotiation designs regarding a conventional bilateral bargaining and Stackelberg style bargaining. By analysing behaviours in a two-tier supply chain system contains competing manufacturers and retailers, researchers found that the bargaining structure will critically affect firms' preferences over contract types and thus their equilibrium contract types (Feng & Xiaoyuan, 2013). Further research considers the option of outsourcing within the bargaining framework, researchers discovered the behaviours of the manufacturers regarding the outsourcing option regarding various types of suppliers (Feng & Lu, 2013). With the introduction of dynamic bargaining, researchers proposed a new model with asymmetric demand information. Under rational assumptions, researchers characterised the perfect Bayesian equilibrium of the bargaining game (Feng, et al., 2015). Associated with various negotiation situations, researchers start to consider the negotiation power over the complete supply chain. Utilising the dynamic Nash equilibrium, researchers successfully demonstrate participants' predictive behaviours with empirical support (Baron, et al., 2016). The research regarding the bargaining structure usually contains only manufacturer and retailer. While the analysis regarding the customer which constitutes the final piece of a complete supply chain does not to be included in the studies regarding the supply chain structure until Shang and Cai's work which includes discussions over negotiation mechanisms and corresponding computation of customers' utilities (Shang & Cai, 2021). This article builds the bridge over the existing research regarding effects caused by negotiation mechanisms and computation regarding channel efficiency. The research regarding the channel efficiency of the negotiation has been concentrated over the concept of 'platform' due to the flourishing development of e-commerce. Researchers studied manufacturers' behaviour regarding the engagement with a platform retailer and a traditional reseller (Shen, et al., 2019). The later emerged dominant retail platforms operating using a store-within-a-store strategy motivates researchers to study the interactions of such a retailer's decision over the selling format and a manufacturer's decision on the channel selection (Shen, et al., 2019). By adding more factors to the model, researchers have illustrated factors that could shape platforms' behaviour over revenue sharing (Zhang, et al., 2019). With an increasing number of platforms emerges in the business world, scholars investigated the circumstance of a monopoly manufacturer facing various downstream platforms under a game model. The results demonstrated the criticality of platforms' competition intensity and order-fulfilment costs (Liu, et al., 2021). However, current research uses the economic concept 'utility' to measure the payoff of the participants, and the existence of outside option has been neglected by researchers. Current mainstream of models primarily concerning bargaining between manufacturers and retailers and rarely included customers within the model. For those models do contain customers, three participants operate strictly over a chain. The existence and the value of the outside option which commonly refers to direct sale from the manufacturer to the customer are not computed by scholars.

6.2.3 The mathematical model of share valuations

The third research stream is relating to the mathematical framework of the model. The mathematical computation for the financial asset price is started from the all-time famous paper written by Black and Scholes, in this paper, Black and Scholes derived a theoretical valuation formula for options under the principle of zero profit making circumstances by creating portfolios consist of long and short positions if options are correctly priced in the market (Black & Scholes, 1973). The concept of binomial tree was proposed for the first time following Black and Scholes's work in order to simulate the option price under a simple two-outcome multi-stage scenario (Cox, et al., 1979). One of the most iconic features of the binomial tree model is the application of risk neutral probabilities, it helps the whole model evaded the shortage of lacking real probability associated with each outcome (Rubinstein, 1994). The research regarding the differences between the European and American option under the binomial tree model is proposed with a Monte Carlo simulation regarding optimal early exercise condition (Grant, et al., 1997). With later development regarding the simulation of share price behaviour, it has gradually reached a common agreement among scholars to model the share price under the assumption of continuous variable and the valuation of option is commonly computed based on the stochastic analysis and full space probability calculus. Even though the binomial tree method is not the main approach in the field of option valuation, the thought of backwards calculation enlightened the thought of computation for other fields. Current articles of operation research commonly adapt the backwards calculation approach to work out the equilibrium situation for the bargaining. A rather recent article uses this approach is the article written by Shang and Cai in which they adapt the backwards analysis regarding retailer and seller's behaviour under different negotiation mechanisms (Shang & Cai, 2021). Similar approach is commonly found in articles regarding negotiations with different channels and exploration regarding effectiveness of negotiation mechanisms (e.g. (Cai, et al., 2012) (Cai, 2010)). However, the current research articles regarding the bargaining theory is profoundly influenced by the idea of Nash equilibrium which commonly refers to a certain static circumstance. This referred static circumstance is concluded based on the underlying assumption of maximising personal profit. With this idea, the discussion regarding the negotiation model is commonly developed based on the conventional economic framework. However, the existence of outside option may create the opportunity of cross application of financial option valuation and negotiation computation. The concept of outside option is concerned in Shang and Cai (2021), however, it was not the main discussion of their work and they choose to give a static value of the outside as a separate outcome within the bargaining result. The value of outside option when it acts as an option for the investor is neglected by current research steam and this will be discussed in the following parts.

6.2.4 The application of sequential negotiation in practical activities

The fourth research stream concerns is the application of sequential negotiation theory in practical activities. Conventional research articles in the early stage primarily focus on theoretical analysis regarding economic situation. Starting from Steve Nash's article, finding the equilibrium conditions among participants has been become the core of research. The preliminary mathematical expression regarding the analysis of the bargaining situation was proposed which enable further mathematical analysis of participant's behaviour (Ingene & Parry, 1995). Later articles started to set the sequential negotiation model concerning single issue (Marx & Shaffer, 1999) and multi-issue (Flamini, 2007). These previous papers are concerning products when they are substitutes which commonly refers to an inverse relationship between competitors' performance and product price. The comparison regarding the complementary product and substitute product is discussed under a sequential negotiation and demonstrated suppliers' preference regarding each type of the product (Reme & Sørgard, 2016). A rather practical application of the sequential negotiation is the discussion of effectiveness regarding issues within an auditor-client relationship. Some accounting related research has illustrated that bargaining is a pervasive feature of audit engagement and the outcome of the bargaining could have rather material effect over the financial statements (Bame-Aldred & Kida, 2007). As a practical sequential negotiation, scholars has illustrated the effect of pre-negotiation over negotiated outcomes (Trotman, et al., 2005). Vast amount of research articles reveals that characteristics of the negotiating parties and the manner in which those negotiated items could affect the amount of concessions offered throughout the bargaining process (Perreault, et al., 2017). However, the application of the bargaining theory in the financial filed seems to be an unexplored area. There never lacks negotiation in the financial market and yet it has not drawn any attention of the scholars. This research intends to explore the effect of the existence of outside option over the sequential negotiation over the pre-IPO procedure consisting of the company that initiated the IPO (Seller), the underwriters (Retailer) and investors (Customer). Different from existing negotiating articles that proposed a rather separated staged negotiation framework, these three parties will be consistently encountered with each other in every stage of the negotiation. And the computation procedure would give a specific valuation method for the existence of outside option rather than simply focus on each participants' payoff conditions.

6.2.5 Participants' behaviours within the oligopoly situation

The fifth research stream relates to participants' behaviours within an oligopoly situation. For years, the research has been developed within this theoretical circumstance which typically comprises one seller, two buyers or retailers and final customer. Previous researches regarding this supply chain are usually separated, the research regarding the behaviours of the seller and buyers are generally based on the bargaining theory starting from Ingene and Parry's elegant formula (Ingene & Parry, 1995) while the discussion regarding the retailer and final customer is primarily based on the utility theory from the economic field (Shang & Cai, 2021). For years, it has been reached a common recognition that the behaviours of the retailers in the middle part of this conventional supply chain could make a large impact over the whole structure (Cai, 2010). The research that concentrate on the condition over the oligopoly situation primarily discuss the strategy between the two participants. It has been found that the communication between firms could improve profits through comparing pricing behaviour with and without the possibility of communicating (Fonseca & Normann, 2012). The research regarding the oligopoly has rather practical applications in real life including: determination of damage multipliers (Baumann & Friehe, 2015), influence over the quality standards in food supply chains (Schlippenbach & Teichmann, 2012), guidance of reformation regarding merger policy (Duso, et al., 2013) and etc. But current research still demonstrated a rather independent condition when they encounter the circumstance of the IPO. Since the IPO procedure contains not only separated negotiation between the seller and retailers, retailers and final investor. Throughout the procedure, it is rather flexible since final investor could directly purchase shares from the seller instead of following the rigid supply chain structure. On the other hand, adapting the utility theory to calculate the payoff of the participants under the IPO condition seems inappropriate as the payoff most come from the later increase in the price of the share which is rather a different logic compared to economic utility formulation. Hence, this research adapts a combination of both financial and economic methods to conduct participants' condition throughout the negotiation and compute for the value and effect of the outside option from the beginning till the end of the bargaining.

6.3 The model description

6.3.1 The fundamental model

Considering the situation where there is one seller and two retailers supply chain model. The seller, sells identical raw materials to both retailers and those retailers will manufacture those raw materials into the final product and sell to the final customer. Due to the consideration regarding the parsimony issues, it is assumed that one unit of the raw material will be manufactured into one unit of the final product. In the following analysis procedure, the quantity of product will be expressed as q. The preliminary setting of the model is as the following: the seller denoted as s, sells the raw material to the retailer i & j. After the retailers manufactured those raw material to the final product, they will sell the final product to the market, the reverse demand function is: $P_i = a - q_i - \gamma q_j$. In this reverse demand function, it is indicating that the relationship of the price charged by any retailer has a negative relationship regarding the quantity sold from both himself and the other retailer. The letter a represents the total market demand and γ implies the product substitutability between the final product of i & j. The letter γ has a range from $[-1, +1]$, when it lies below 0, it indicates products provided by retailers are complementary goods while above 0 indicates the condition of substitutes. Consistent with the assumption proposed by Horn and Wolinsky, the seller will always have sufficient capacity to satisfy the total market demand base a (Horn & Wolinsky, 1988).

To make the model more suitable for the practical life, the market demand is assumed to be a dynamic state rather than a static stage. Hence, it is assumed that the market demand can be either a high or a low demand condition. Through the model, these two conditions are denoted as a^H or a^L and $a^H > a^L$. The retailers will negotiate the wholesale price w with the seller. Throughout this negotiation, the negotiation power will be denoted as θ. For instance, the relative negotiation power of the retailer i against the seller is denoted as θ_i. And the relative negotiation power of the seller, the counterparty, is denoted as $1 - \theta_i$.

Within this supply chain structure, the negotiation mechanisms are setting as simultaneous negotiation and price matching. The simultaneous negotiation (SN) refers to the common situation where the situation between the seller and one of the retailers is independent regarding the result of another negotiation. In another word, SN can be regarded as two independent negotiations that happen at the same time. While the other negotiation mechanism, price matching (PM) is distinctive compared to SN. This refers to the condition where the first negotiation has significant influence over the upcoming bargaining. If the seller and the retailer reach an agreement over the first negotiation, then the remaining retailer should only choose to accept the same agreement or refuse and exit the market. If the first negotiation failed, then the seller and the remaining retailer will negotiate over the specific contract terms. Failure in both negotiations will result in zero finished product available for the final customer.

To research the effect of negotiation mechanisms imposed over participants' behaviour, the analysis first assumes that the relative negotiation power is symmetric $(\theta_i = \theta_j = \theta)$. Adapting the Bertrand competition framework, the second stage game is analysed before the first stage subgames in the first stage.

The analysis regarding the second-stage condition is based upon the assumption that both first-stage games are successful. While the condition of the first negotiation failed, the monopoly condition will be clearly illustrated.

If the negotiation between the seller and the retailer succeeded under the SN, then the contract will be (w_i, q_i) and (w_j, q_j) respectively. The profit functions given the market demand is static for the seller and the retailers are:

$$\pi_s = w_i q_i + w_j q_j$$

$$\pi_i = (P_i - w_i) q_i$$

And when the market situation is a stochastic state, then the strategy for each participant will be examined in detail. Hence, the situation under the static demand base is setting the baseline for the later analysis.

First, under the assumption of the rational agent, the retailer who lies in the middle stage of the supply chain structure, will seek to maximise the personal profit by setting the optimal selling price P given the wholesale price w. Therefore, solving the first order condition (FOC) gives the equilibrium retail price:

$$P_i = \frac{a(2 - \gamma - \gamma^2) + 2w_i + \gamma w_j}{4 - \gamma^2}$$

And the resulting quantity is:

$$q_i = \frac{a(2 - \gamma - \gamma^2) - (2 - \gamma^2)w_i + \gamma w_j}{(1 - \gamma^2)(4 - \gamma^2)}$$

Now to form the Nash Bargaining situation, the disagreement points are required. The disagreement points are the worst situation when the negotiation failed. Under this condition, this refers to the situation when the first negotiation failed. Considering the negotiation failed, there will be only one retailer for the seller. Under this circumstance, the new reverse demand function given the negotiation between the seller and the retailer i failed becomes $P_j = a - q_j$.

With the agreement reached between the seller and retailer j succeeded as w_j, the retailer j maximizes the personal profit function $(P_j - w_j)(a - P_j)$. Solving the FOC, the optimal price becomes $\frac{a+w_j}{2}$ and retailer's profit is $\frac{(a-w_j)^2}{4}$ and the seller's profit will be $\frac{w_j(a-w_j)}{2}$.

With the corresponding profit function and disagreement points, then the Nash Bargaining solution can be expressed as the following:

$$argmax\left[(1-\gamma^2)[\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_i+\gamma w_j}{(1-\gamma^2)(4-\gamma^2)}]^2\right]^{\theta}\left[w_i\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_i+\gamma w_j}{(1-\gamma^2)(4-\gamma^2)} + w_j\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_j+\gamma w_i}{(1-\gamma^2)(4-\gamma^2)} - \frac{w_j(a-w_j)}{2}\right]^{1-\theta}$$

After solving the FOC, the resulting wholesale price under the simultaneous negotiation would be:

$$w_i^{SN} = \frac{[a(1-\theta)(1-\gamma)(2+\gamma)]}{[\,\theta(1-\gamma)(2+\gamma)\gamma^2 + 2(1-\gamma)(2+\gamma)]}$$

For the condition of the price matching mechanism, the situation will be slightly different compared to SN. Since the successful agreement from the first negotiation will automatically become the terms for the second negotiation which indicates that if the second retailer agrees the contract term, then the wholesale price for both retailers will be the same. But different from the SN condition, the calculation regarding the equilibrium wholesale price would require more calculations regarding the Nash Bargaining Solution (NBS).

Given the negotiation between the seller and the retailer i failed, the Nash Bargaining formulation between the seller and the remaining retailer j becomes:

$$w_j = argmax\left[\frac{(a-w_j)^2}{4}\right]^{\theta_j}\left[\frac{w_j(a-w_j)}{2}\right]^{1-\theta_j} = \frac{(1-\theta_j)a}{2}$$

And the corresponding profit for the seller with this wholesale price contract would be $\frac{(1-\theta_j^2)a^2}{8}$. Now given this is the result that the seller knows that has to be faced if the first negotiation with retailer i failed, then the negotiation between the seller and the retailer i under the price matching mechanism becomes:

$$w_i^{PM} = argmax\left[(1-\gamma^2)[\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_i+\gamma w_i}{(1-\gamma^2)(4-\gamma^2)}]^2\right]^{\theta_i}\left[2w_i\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_i+\gamma w_i}{(1-\gamma^2)(4-\gamma^2)} - \frac{(1-\theta_j^2)a^2}{8}\right]^{1-\theta_i}$$

After solving the above FOC, the result yields as: (1)

$$w_i^{PM} = \frac{\left(3 - \theta_i - \sqrt{{\theta_i}^2 + 1 + \theta_i\theta_j^2(1+\gamma)(2-\gamma) + \gamma^2\theta_i - \gamma\theta_i}\right)a}{4}$$

These results are built upon the assumption that the market demand is a static state. Now the analysis should proceed to the stage where the market demand is a stochastic situation.

6.3.2 The condition of stochastic information

To make the model more suitable for the practical conditions, the stochastic condition is introduced. In the market, it is supposed that the market demand can be either in high state or low state. While this information is known to the retailers but not to the seller. The seller has to construct his own belief based on the behaviours of the retailers.

When the market demand is at a high demand condition, then the expression will be a^H with corresponding reverse price function as: $P_i(H) = a^H - q_i(H) - \gamma q_j(H)$. The logic follows the same rhythm, with the corresponding selling quantity as: (2)

$$q_i(H) = \frac{a^H(2-\gamma-\gamma^2) - (2-\gamma^2)w_i(H) + \gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)}$$

The following analysis does not have any deviations, and hence, the expressions regarding the wholesale price are (3)

$$w_i^{SN}(H) = \frac{[a^H(1-\theta)(1-\gamma)(2+\gamma)]}{[\,\theta(1-\gamma)(2+\gamma)\gamma^2 + 2(1-\gamma)(2+\gamma)]}$$

$$w_i^{PM}(H) = \frac{\left(3 - \theta_i - \sqrt{{\theta_i}^2 + 1 + \theta_i\theta_j^2(1+\gamma)(2-\gamma) + \gamma^2\theta_i - \gamma\theta_i}\right)a^H}{4}$$

While when the market demand is low, the expressions will be: (4)

$$q_i(L) = \frac{a^L(2-\gamma-\gamma^2) - (2-\gamma^2)w_i(L) + \gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)}$$

$$w_i^{SN}(L) = \frac{[a^L(1-\theta)(1-\gamma)(2+\gamma)]}{[\,\theta(1-\gamma)(2+\gamma)\gamma^2 + 2(1-\gamma)(2+\gamma)]}$$

$$w_i^{PM}(L) = \frac{\left(3 - \theta_i - \sqrt{\theta_i{}^2 + 1 + \theta_i \theta_j^2 (1+\gamma)(2-\gamma) + \gamma^2 \theta_i - \gamma \theta_i}\right) a^L}{4}$$

Through above expressions, it can be concluded that the wholesale price is an increasing function from zero to half of the market demand. And since the profit of the seller is closely connected to the variation of both wholesale prices and corresponding quantities, it can be concluded that the seller would prefer price matching mechanism compared to the simultaneous negotiation.

The above results are constructed under the condition that both the seller and retailers have the perfect information. The above conduction procedure is based on the circumstances that the seller has successfully construct the correct belief regarding the market demand and the retailer tells the truth throughout the bargaining process. Under this condition, the appropriate bidding strategy for both seller and retailer is bidding truthfully. But it is worth noting that the retailer who possesses the private information has the incentive to lie to the seller if lying could bring more profit to the retailer.

Instead of assuming the perfect information symmetry among the participants of the negotiation, the market demand situation remains a private information that is only available for the retailer. That is indicating that when the seller initiates the bargaining with the retailer, he needs to form his own belief regarding the market demand based on information gathered. After forming the belief, the seller should choose to accept or not regarding the contract of wholesale price and corresponding quantity under either price matching or simultaneous negotiation. It is worth mentioning that the following analysis is still conducted under the condition of identical relative negotiation power among two retailers i & j.

Now assume the market demand is high, whether or not there exist a motivation for the retailer to mimic a low demand condition is conducting as the follows.

Under the simultaneous negotiation, still adapts the backward analysis technique, the selling price of the finished product would be:

$$P_i(H) = a^H - q_i(L) - \gamma q_j(L)$$

Then the profit for the retailer i if he successfully persuades the seller that the market demand is low becomes:

$$\pi_i(H) = [a^H - q_i(L) - \gamma q_j(L) - w_i(L)] q_i(L)$$

$$\pi_s(H) = w_i(L)q_i(L) + w_j(L)q_j(L)$$

Solving the FOC, the optimal selling price for the retailer to maximise the personal profit would be:

$$P_i(H)\left[w_i(L)w_j(L)\right] = \frac{a^H(2-\gamma-\gamma^2)+2w_i(L)+\gamma w_j(L)}{4-\gamma^2}$$

And the resulting selling quantity would be:

$$q_i(H)\left[w_i(L)w_j(L)\right] = \frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(L)+\gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)}$$

Hence, the formulation for the unique Nash Bargaining solution is:

$$argmax\left[(1-\gamma^2)\left[\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(L)+\gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)}\right]^2\right]^{\theta}\left[w_i(L)\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(L)+\gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)} + w_j(L)\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_j(L)+\gamma w_i(L)}{(1-\gamma^2)(4-\gamma^2)} - \frac{w_j(L)\left(a-w_j(L)\right)}{2}\right]^{1-\theta}$$

Solving the first order condition, the equilibrium result of the wholesale price is:

$$w_i^{SN}(L) = \frac{[a^H(1-\theta)*(1-\gamma)(2+\gamma)]}{[\,\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}$$

And the profit for the retailer i becomes:

$$\left\{\frac{a^H(2-\gamma-\gamma^2)+2w_i(L)+\gamma w_j(L)}{4-\gamma^2} - \frac{[a^H(1-\theta)*(1-\gamma)(2+\gamma)]}{[\,\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}\right\}\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(L)+\gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)}$$

Since the quantity has to be positive, the existence of the motivation for the retailer to mimic the low-demand behaviour under the high-demand situation depends on the positivity of the result regarding the selling price minus the wholesale price compared to the truth telling condition.

Comparing these two situations, the profit difference between the two options is:

$$\frac{a^H(2-\gamma-\gamma^2)+2w_i(L)+\gamma w_j(L)}{4-\gamma^2}\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(L)+\gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)}$$
$$-\frac{[a^H(1-\theta)*(1-\gamma)(2+\gamma)]}{[\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(L)+\gamma w_j(L)}{(1-\gamma^2)(4-\gamma^2)}$$
$$-\frac{a^H(2-\gamma-\gamma^2)+2w_i(H)+\gamma w_j(H)}{4-\gamma^2}\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)}$$
$$+\frac{[a^H(1-\theta)*(1-\gamma)(2+\gamma)]}{[\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)}$$

The result has the same sign as:

$$\begin{aligned}&a^H(2-\gamma-\gamma^2)(2-\gamma^2)[w_i(H)-w_i(L)]+a^H(2-\gamma-\gamma^2)\gamma[w_j(L)-w_j(H)]\\&+2a^H(2-\gamma-\gamma^2)[w_i(L)-w_i(H)]+2(2-\gamma^2)[w_i^2(L)-w_i^2(H)]\\&+4\gamma^2w_i(L)w_j(L)w_i(H)w_j(H)+a^H(2-\gamma-\gamma^2)\gamma[w_i(L)-w_j(H)]\\&-\gamma(2-\gamma^2)[w_i(L)-w_i(H)][w_j(L)-w_j(H)]+\gamma^2[w_j^2(L)-w_j^2(H)]\end{aligned}$$

Since the wholesale price under the low-demand condition is always higher than the high-demand condition, that is saying that the above expression is always positive.

This is saying that there exist an incentive for the retailer to mimic the low-demand condition when the actual market demand is a high demand condition.

In contrast, is there any motivation for the retailer to fake the high-demand state given a low demand situation?

Following the same rhythm, the low demand situation yields the corresponding price function when retailer mimics the high demand condition:

$$P_i(L)[w_i(H)w_j(H)]=\frac{a^L(2-\gamma-\gamma^2)+2w_i(H)+\gamma w_j(H)}{4-\gamma^2}$$

And the resulting quantity would be:

$$q_i(L)[w_i(H)w_j(H)]=\frac{a^L(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)}$$

Hence, the formulation of fining the unique NBS is:

$$argmax\left[(1 - \gamma^2)[\frac{a^L(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)}]^2\right]^\theta \left[w_i(H)\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)} + w_j(H)\frac{a(2-\gamma-\gamma^2)-(2-\gamma^2)w_j(H)+\gamma w_i(H)}{(1-\gamma^2)(4-\gamma^2)} - \frac{w_j(H)\big(a-w_j(H)\big)}{2}\right]^{1-\theta}$$

Solving the above FOC, the resulting equilibrium wholesale price under this condition is:

$$w_i^{SN}(H) = \frac{[a^L(1-\theta)*(1-\gamma)(2+\gamma)]}{[\,\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}$$

Hence, the profit difference between the truth telling situation and retailer fake the high demand given the low demand condition becomes:

$$\frac{a^L(2-\gamma-\gamma^2)+2w_i(H)+\gamma w_j(H)}{4-\gamma^2}\frac{a^H(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)} - \frac{[a^L(1-\theta)*(1-\gamma)(2+\gamma)]}{[\,\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}\frac{a^L(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)} - \frac{a^L(2-\gamma-\gamma^2)+2w_i(L)+\gamma w_j(L)}{4-\gamma^2}\frac{a^L(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)} + \frac{[a^L(1-\theta)*(1-\gamma)(2+\gamma)]}{[\,\theta\gamma^2(1-\gamma)(2+\gamma)+2(1-\gamma)(2+\gamma)]}\frac{a^L(2-\gamma-\gamma^2)-(2-\gamma^2)w_i(H)+\gamma w_j(H)}{(1-\gamma^2)(4-\gamma^2)}$$

The result has the same sign as:

$$a^L(2-\gamma-\gamma^2)(2-\gamma^2)[w_i(L)-w_i(H)] + a^L(2-\gamma-\gamma^2)\gamma\big[w_j(H)-w_j(L)\big] + 2a^L(2-\gamma-\gamma^2)[w_i(H)-w_i(L)] + 2(2-\gamma^2)[w_i^2(H)-w_i^2(L)] + 4\gamma^2 w_i(L)w_j(L)w_i(H)w_j(H) + a^L(2-\gamma-\gamma^2)\gamma\big[w_i(H)-w_j(L)\big] - \gamma(2-\gamma^2)[w_i(H)-w_i(L)]\big[w_j(H)-w_j(L)\big] + \gamma^2[w_j^2(H)-w_j^2(L)]$$

Since the wholesale price under the high demand condition is lower than the wholesale price under the low demand condition, hence, this result would be negative. That is saying that, there does not exist any motivation for the retailer to mimic a high demand behaviour when he knows the market is actually low demand.

Through the above content, it can be saying that for the price matching negotiation mechanism, it can be regarded as a special form of simultaneous negotiation which both retailers pay equal amount of wholesale price to the seller for identical amount of material. The logic of whether or not the retailer has the motivations to mimic the opposite condition behaviour under certain market demand situation remains unchanged compared to the simultaneous negotiation.

Hence, the above conduction forms a corollary regarding the behaviour of the retailer given he possess the private information relative to the seller.

Lemma 1: Under the condition of having private information regarding the market demand, the retailer will always have the motivation to mimic the low demand condition if the actual market demand is high. But the retailer will never have the incentive to mimic the high demand condition if the market demand is low. And the dominant strategy for the retailer is always bidding as a low demand condition.

6.3.3 The preference of the seller

From the condition of perfect information, it can be concluded that the seller would prefer the price matching mechanism over the simultaneous negotiation because the price matching mechanism would yield a higher wholesale price compared with the result from the simultaneous negotiation. As for the negotiation order in the price matching mechanism, the seller would prefer to negotiate first with the retailer with relative lower negotiation power to reach a favourable contract that is rather beneficial to the seller.

But when the market demand is stochastic and remains as the private information to the retailer, then the whole bargaining is totally dominated by the retailer. Under the rational assumption of the retailer, he will only propose the contract that is appropriate under the low-demand condition. If the seller only has the opportunity to bargain once with the retailer, then he has to suffer from the information disadvantage and may never construct the belief regarding the actual market demand. To enable the construction of belief regarding the market demand, the one-time bargaining framework is no longer appropriate for the seller. A rather optional bargaining mechanism would be the alternating bargaining model which was originally proposed by Rubinstein. Throughout the repetitive price offerings, the seller could construct his own belief regarding market demand based on the reactions from the retailer.

6.3.4 The alternating bargaining model

To update the belief regarding the market, the seller could adapt the alternating bargaining mechanism introduced by Rubinstein which gives the seller opportunity to find out the true market demand based on the behaviour of the retailer (Rubinstein, 1982). This bargaining mechanism enables the seller and the retailer to offer their own contract to the counterparty, but for each round, only one party could propose the contract. For example, in the first round, the seller proposes the contract to the retailer. If the retailer refuses the contract, then the negotiation proceeds to the second round which enables the retailer to propose the contract to the seller, and the seller should only choose to accept or not. The overall negotiation will proceed until there is a contract to be agreed or one of the parties exit the negotiation.

Applied the same setting as previous paragraphs, the market demand base has two states, either high demand (H) or low demand (L). Under each state, the retailer negotiates with the seller regarding the quantity q to be delivered by the seller, and the corresponding payment wq from the retailer to the seller. Since the negotiation model is the alternating negotiation mechanism, the time series variable $\tau(\tau = 1,2, \dots)$ is introduced to signal the round of the negotiation. If an agreement is failed to be reached prior to the round τ, then one party offers a contract to the other party, and the later responds with acceptance or rejection. If the contract is accepted, then the contract will be executed, in contrast, if the contract is rejected, then the negotiation will evolve to the next round. In the beginning of the overall alternative negotiation, it is assumed that the negotiation starts with the seller proposes the offer to the retailer.

Without any restriction, the alternating negotiation will last forever. Hence, to make it more suitable for practical applications, the patience level is introduced to this article. In the following content, the letter $\delta \in (0,1)$ to denote the patience level. The expressions of δ_S & δ_R are denoting the patience level of the seller and the retailer correspondingly. This patience level is acting as the discount factor and a potential risk of the overall failure of the negotiation (Binmore, et al., 1986). With all these preliminary settings, the following contents contain analysis regarding the alternating negotiation procedure.

The commonly deducted equilibrium result is the bargaining equilibrium under the complete information. Let $\tilde{F}_k(a) = 1 - F_k(a)$, $k \in (H, L)$ denotes the market demand base regarding the quantity of material demand. Given a quantity q, the revenue of the k-type retailer is $R_k(q) = pExp[\min(q, a_k)] = p\int_0^q \tilde{F}_k(a)da$. The cost for the seller would be cq. Hence the total trade surplus would be $\pi_k = R_k(q) - cq$. The overall trade surplus would be maximised at the quantity $\hat{q}_k = \tilde{F}_k^{-1}\left(\frac{c}{p}\right)$. Hence the expression of the maximum trade surplus when the underwriter's type is i is: $\hat{\pi}_k = R_k(\hat{q}_k) - c\hat{q}_k$, and the bargaining problem is all about deciding the appropriate wholesale price $\widetilde{w}_k$.

For the stochastic market demand condition, if the probability of high demand is β and the corresponding probability of low demand would be $1 - \beta$. Under the condition of the complete information, the distribution of the total profit is determined solely on the patience level of participants. This result has been deducted by Rubinstein stated that there exist a unique subgame perfect equilibrium in which the seller would offer $(\hat{q}_k^S, \widetilde{w}_k^S) = (\hat{q}_k, \frac{1-\delta_U}{1-\delta_U\delta_S}\hat{\pi}_k + c\hat{q}_k)$. In equilibrium, the seller's expected profit is

$$\hat{\pi}^S = \frac{1-\delta_R}{1-\delta_R\delta_S}[\beta\hat{\pi}_H + (1-\beta)\hat{\pi}_L]$$

And the corresponding i type retailer's profit is

$$\hat{\pi}^R = \frac{\delta_R(1-\delta_S)}{1-\delta_R\delta_S}\hat{\pi}_k$$

From the above result, it could be told that under the complete information, the split of the trade surplus between the seller and the retailer is dependent on the patience level. The information transparency would eliminate retailer's advantages regarding the understanding of the market conditions. Hence, there is a motive for the retailer to on purposely provide information that could mislead the seller in order to maximise personal benefit. With acknowledgement regarding this information, it is essential that seller could deduct the real information regarding market demand of shares through the sequential negotiation with the retailer.

To understand the information gathered from the alternating negotiation, the logic of the retailer's action must be understood. From the previous model configuration, it is easy to conclude that there exists no incentive for the retailer to pretend a high demand condition given the market is low demand regarding the shares. Because the excess shares bought from the seller would remain redundant for the retailer and reserve no value. With the private information, it is reasonable for the H-type retailer to mimic a L-type since the shortage in the supply of the amount of shares could result in a drastic increase of the price and improve the total trade surplus. Within the model, it is assumed that the L-type retailer will always tell the truth while the H-type retailer could choose to mimic a L-type one dependent on its patience level relative to the seller's. This conforms with previous corollary and illustrate that the appropriate behaviour of the retailer remains the same if the relative negotiation power of the retailer and the seller keep the same.

Since in previous settings have introduced the new parameter, the patience level. Then the following content is discussing how the patience level could affect the participant's behaviour.

Comparison of L-type's profit under $(\hat{q}_L, \widehat{w}_L^R)$ with under $(\hat{q}_H, \widehat{w}_H^R)$:

$$R_L(\hat{q}_L) - \widehat{w}_L^R - [R_L(\hat{q}_H) - \widehat{w}_H^R]$$

Since the payment from the retailer to the seller is:

$$\widehat{w}_i^R = R_i(\hat{q}_i) - \frac{1-\delta_S}{1-\delta_R\delta_S}\hat{\pi}_i$$

The comparison expression becomes;

$$R_L(\hat{q}_L) - R_L(\hat{q}_L) + \frac{1-\delta_S}{1-\delta_R\delta_S}\hat{\pi}_L - \left[R_L(\hat{q}_H) - R_H(\hat{q}_H) - \frac{1-\delta_S}{1-\delta_R\delta_S}\hat{\pi}_H\right]$$

Eliminating the same item, the result is:

$$\frac{1-\delta_S}{1-\delta_R\delta_S}\hat{\pi}_L - \left[R_L(\hat{q}_H) - R_H(\hat{q}_H) - \frac{1-\delta_S}{1-\delta_R\delta_S}\hat{\pi}_H\right]$$

$$R_H(\hat{q}_H) - R_L(\hat{q}_H) - \frac{1-\delta_S}{1-\delta_R\delta_S}(\hat{\pi}_H - \hat{\pi}_L)$$

Since the $\hat{q}_L$ maximises the trade surplus $R_L(q) - cq$, hence the relationship can be concluded:

$$R_H(\hat{q}_H) - c\hat{q}_H - [R_L(\hat{q}_H) - c\hat{q}_H) = R_H(\hat{q}_H) - R_L(\hat{q}_H) \geq \hat{\pi}_H - \hat{\pi}_L \geq 0$$

Hence the above comparison indicates that the profit difference of the L-type retailer under the high demand condition and low demand condition with high demand purchase is non-negative. This shows that there never exist any motivation for the retailer to mimic a high demand condition when the actual situation is low demand.

Now the comparison of H-type retailer's profit under $(\hat{q}_H, \widehat{w}_H^U)$ with under $(\hat{q}_L, \widehat{w}_L^U)$:

$$R_H(\hat{q}_H) - \widehat{w}_H^R - [R_H(\hat{q}_L) - \widehat{w}_L^R] = \frac{1-\delta_S}{1-\delta_R\delta_S}(\hat{\pi}_H - \hat{\pi}_L) - [R_H(\hat{q}_L) - R_L(\hat{q}_L)]$$

To single out the effect of the patience level of retailer within the comparison, the indifferent expression would be: (5)

$$\delta_R = \frac{R_H(\hat{q}_L) - R_L(\hat{q}_L) - (1-\delta_S)(\hat{\pi}_H - \hat{\pi}_L)}{\delta_S[R_H(\hat{q}_L) - R_L(\hat{q}_L)]}$$

Any increase over this basis would result in a negative result of the comparison which indicate a worse condition. In another word, this is the lower threshold of retailer's patience level to mimic the low demand condition under a high demand circumstance.

By repeating these procedures, but replace $\widehat{w}_L^R$ by $\widehat{w}_L^S$, it can be concluded that the L-type retailer could earn a larger profit with $(\hat{q}_L, \widehat{w}_L^S)$ than $(\hat{q}_H, \widehat{w}_L^S)$ and the H-type retailer could earn a larger profit with $(\hat{q}_L, \widehat{w}_L^S)$ than $(\hat{q}_H, \widehat{w}_H^S)$ when the patience level lower than the expression: (6)

$$\frac{R_H(\hat{q}_L) - R_L(\hat{q}_L)}{(1-\delta_S)(\hat{\pi}_H - \hat{\pi}_L) + \delta_S[R_H(\hat{q}_L) - R_L(\hat{q}_L)]}$$

Hence, both the upper and lower threshold of the patience level is conducted. With these results, it can be concluded that in any round τ of the sequential negotiation, there always exists two thresholds of the retailer's patience level such that:

Lemma 2: The upper and lower thresholds for the patience

i) If is the seller's turn to propose the offer, the low demand offer under the low demand condition is always better than the high demand offer $(\hat{q}_L^S, \widehat{w}_L^S)\tau > L(\hat{q}_H^S, \widehat{w}_H^S)\tau$, and when the patience level of the retailer is lower than the upper threshold, the proposed low demand offer is better than the high demand offer under the high demand condition $(\hat{q}_L^S, \widehat{w}_L^S)\tau > H(\hat{q}_H^S, \widehat{w}_H^S)\tau$

ii) If it is the retailer's turn to propose the offer, the low demand offer under the low demand condition is always better than the high demand offer $(\hat{q}_L^R, \widehat{w}_L^R)\tau > L(\hat{q}_H^R, \widehat{w}_H^R)\tau$, and when the patience level of the retailer is lower than the lower threshold, the proposed low demand offer is better than the high demand offer under the high demand condition $(\hat{q}_L^R, \widehat{w}_L^R)\tau > H(\hat{q}_H^R, \widehat{w}_H^R)\tau$

The above conclusions suggest that when the retailer knows the market demand is at a low demand level, there never has any incentive for the retailer to mimic a H-type retailer. While a H-type retailer is willing to reveal the truth to the seller when the patience level is relatively high. Throughout the whole procedure, the profits of the participants can be regarded as a distribution over the total trade surplus and the coefficient of individual profit is solely dependent upon the patience level. Hence, it is the retailer's burden to balance the relationship between the benefit of lying, e.g. reduced wholesale price paid to the seller, against the potential loss due to the potential gain caused by shortage of shares. A noticing point is that when the retailer proposes the offer, he can always take more profit from the total trade surplus compared to choice regarding whether to accept or reject offer from the seller.

With these understanding regarding the retailer's logic behind the actions, the seller could build his own belief based on the behaviour of the retailer. Throughout the procedure of the sequential negotiation, it is crucial for the seller to construct retailer's type regarding market demand based on the behaviour of sequential offering. Under the model of the sequential negotiation, rejecting an offer is closely followed by the proposition of the new offer from the counter party. Hence, the seller could construct its own belief regarding the retailer's type based on the action of the retailer. Based on previous conduction, it is worth noticing that there is an incentive for the H-type retailer to mimic a L-type action, while the L-type retailer finds it beneficial to signal its type. When constructing the belief regarding the retailer, the previous conducted result should be considered.

For the seller's construction of belief regarding the market demand, there are following lemmas:

Theory 1

Suppose the seller proposes an offer (q^S, w^S) in round τ, but there is no agreement, and then the retailer offers (q^R, w^R) in the following round $\tau + 1$. If $(\tau + 1)(q^R, w^R) \geq S(\tau + 1)(\hat{q}_L, \widehat{w}_L)$, $(q^U, w^U) \geq L\tau(q^S, w^S)$, but $(\tau + 1)(q^U, w^U) \leq H\tau(q^S, w^S)$, then the seller could ascertain the L-type of the retailer.

This lemma is used rather helpful for the seller to figure out the type of the retailer through two round's negotiation. Stay in mind that the retailer tends to accept the offer when it is beneficial for him. This lemma is saying that when the retailer refuses previous contract and offers a new contract which is more attractive to the retailer himself for one type but this contract is less attractive to the retailer of the other type than the offer provided in the previous round, then the seller could ascertain that the retailer is the type that would be beneficial if the second-round proposed contract is accepted. For an example, if the retailer proposes an offer that is more beneficial compared to the contract offered by the seller in the previous round when the seller assumes the retailer is a L-type. Then the seller could confirm that the retailer is a H-type.

Theory 2

Suppose the seller has a belief $\beta \epsilon (0,1)$, offers a contract in the round τ, but the retailer refuses and proposes a new counteroffer (q^R, w^R) in the following round $\tau + 1$. If the new counteroffer $\tau(q^R, w^R) \geq S\tau(\hat{q}_L, \widehat{w}_L^R)$, and there also exist an alternative contract such that

$\tau(q,w) \geq S\tau((\hat{q}_H, \widehat{w}_H^U)$ and $\tau(q,w) \geq H\tau(q^R, w^R)$, but $\tau(q,w) \leq L\tau(q^U, w^U)$. Then the seller updates the retailer's type to L with probability of 1.

For this lemma, the seller is convinced that the retailer is a type of low demand circumstance if the counteroffer proposed guarantees seller's profit under the complete information condition but the high demand condition would be better compared to current contract with distinguish offer. Any offer other than the range stated in previous lemmas cannot credibly reveal the type of the retailer.

These conduction contents can be applied to the condition of the simultaneous negotiation as this negotiation mechanism can be regarded as two independent negotiations. What the seller could do is applying these lemmas separately. However, the situation would be rather different when the negotiation mechanism is chosen to be price matching. Since the agreement reached with the first retailer will be the identical for the other retailer, the contract agreed is crucial for the seller. To conduct the appropriate strategy for the seller, the thought of the retailer when he enters the price matching negotiation should be conducted before the consideration regarding seller's behaviour.

6.3.5 Retailer's thought under the price matching negotiation

To single out the effect exerted from the quantity and corresponding wholesale price over the decision philosophy, it is assumed that the product sold by both retailers are completely identical for the final customer.

For the retailer, when he enters the negotiation with the seller, the contract regarding the wholesale price and the quantity of material purchases is not the only factor to be considered by the retailer. Moreover, the retailer needs to consider the afterwards selling condition of the finished product since it directly relates to the revenue figure. Under the assumption of identical finished product, this means under the condition of both retailers provide more products than the market demand base, both retailers will have same residual unsold products. This will cause both retailers the equal loss. So based on the idea of maximizing the personal profit, what retailer persuade is the condition that the contract reached between him and the seller becomes unacceptable to the other retailer which will become the competitor when they sell finished products to the customer while at the same time the quantity of finished products can not satisfy the overall market demand which will create a shortage of the supply and resulting in an abnormal profit for the retailer.

Now, assuming the seller conducts the negotiation with the retailer i that is relative stronger compared to the remaining retailer j under the price matching mechanism. Remember the contract proposed by the retailer comprises both the wholesale price and quantity (w_i, q_i). Hence, working the overall procedure backwards, what retailer i wants to achieve is to put the retailer j into the situation between the choice of refusing the contract and receiving zero profit and accept the existing term and receive a relative low level of profit. Hence the negotiation between the seller and the retailer j becomes:

$$argmax\left[\pi_j(w_i, q_j) - 0\right]^{\theta_j}\left[\pi_S(w_i, q_i + q_j) - \pi_S(w_i, q_i)\right]^{1-\theta_j}$$

At this point, the total quantity should be the market demand: $q_i + q_j = a$. Solving the above FOC, the result is:

$$\theta_j\left[\pi_j(w_i, q_j) - 0\right]^{\theta_j - 1}\left[\pi_S(w_i, q_i + q_j) - \pi_S(w_i, q_i)\right]^{1-\theta_j} + \left[\pi_j(w_i, q_j) - 0\right]^{\theta_j}(1 - \theta_j)\left[\pi_S(w_i, q_i + q_j) - \pi_S(w_i, q_i)\right]^{-\theta_j} = 0$$

$$\theta_j\left[\pi_S(w_i, q_i + q_j) - \pi_S(w_i, q_i)\right] + (1 - \theta_j)\pi_j(w_i, q_j) = 0$$

$$\theta_j\left[w_i q_j\right] + (1 - \theta_j)(1 - \gamma^2)q_j^2 = 0$$

To make this equation stands, the variable q_j has to be zero, which means the first contract agreed with the retailer i is (w_i, a).

In another word, under the price matching mechanism, the bargaining not only exist between the seller and the retailer, but also exist between the retailers. If the seller chooses to negotiate with the retailer with the relative high negotiation power (in other word, the lead retailer)compared to the other retailers. Then to prevent the other retailers entering the market, the lead retailer will propose the quantity to satisfy all the market demand base with the wholesale price. This would be rather helpful if the seller wants to understand the true demand base from the market. The opportunity to negotiate with the seller before other retailers yields the first move advantage to the lead retailer.

But in opposite, if the seller chooses to negotiate with the retailer with relative lower negotiation power, the quantity proposed by the retailer might be lower than the market demand due to lack of manufacturing capability, as for the lead retailer, he might reach an agreement with the seller as well with the thought of initiating the price war with the other retailer when they face the final customer. However, the seller has to bear the risk of the second retailer refuse the contract resulting in a potential loss for the seller. In essence, the profit realisation of the seller when he chooses to negotiate first with the retailer with weaker negotiation power, the profit is depending on the willingness of the lead retailer regarding the initiation of the price war.

This result is quite contrary compared to what Shang & Cai (2010) found regarding the preference of the seller over the price matching mechanism. Under the private information regarding the market demand base, the preference of the seller depend upon the purpose of the seller and his risk attitude. If the seller wants to find out the true market demand and seeks a safe profit, then he should negotiate first with the lead retailer. But if the seller decides to take some risk and persuade for a potential higher profit figure than the true demand base, then the preference would be the same compared what Shang and Cai (2010) found in their research. In a general speaking, the preference of the seller maintains the same given the seller is rather risk neutral and the market demand is high, but when the demand is low and the seller is risk adverse, then the preference of the seller would be distinctive compared to Shang and Cai's result.

6.3.6 The effect of the outside option

In previous sections, to explore the effects of negotiation mechanisms exert to the bargaining outcome, it is assumed that the retailer will earn nothing if the negotiation between the seller and the retailer failed. However, in practical life, there might exist the outside option. For example, under the situation of the IPO, the company could choose the directly offering which refers to the condition that company sells newly issued shares directly to the investors. This does not indicate the retailers, the underwriters, earns nothing. They can purchase shares from the company as well. This refers to the condition that the seller and the retailer are still capable of selling and purchasing the products from the spot market respectively.

Now assuming the seller could sell the products to the spot market when the negotiations with the retailers failed. The corresponding wholesale price would be w_0. Following previous model setting, the wholesale price would be lower than market demand base because otherwise there will not exist any trading. The existence of the outside option will provide new disagreement points for participants given the negotiation between the seller and the retailer failed. To explore the effect of outside options over the bargaining outcomes of price matching and simultaneous negotiation mechanisms, the Nash Bargaining equation becomes:

$$w_i^{PM} = argmax\left[\pi_i(w_i, w_i) - \pi_i\left(w_0, w_j\right)\right]^{\theta_i}\left[\pi_s(w_i, w_i) - \pi_s\left(w_0, w_j\right)\right]^{1-\theta_i}$$

The factor w_j is the maximiser of the following equation:

$$w_j = argmax[\pi_j(w_j, w_0) - \pi_j(w_0, w_0)]^{\theta_j}[\pi_s(w_j, w_0) - \pi_s(w_0, w_0)]^{1-\theta_i}$$

For the simultaneous negotiation, if the negotiation succeeds, then the bargaining solution (w_i^{SN}, w_j^{SN}) satisfies:

$$w_i^{SN} = argmax[\pi_i(w_i, w_j^{SN}) - \pi_i(w_0, w_j^{SN})]^{\theta_i}[\pi_s(w_i, w_j^{SN}) - \pi_s(w_0, w_j^{SN})]^{1-\theta_i}$$

The general cases of the relative bargaining power are intractable, and in the following analysis, it is assumed the symmetrical negotiation power ($\theta_i = \theta_j = \theta$).

Now suppose both retailers can sell and buy the identical products from a spot market at the wholesale price w_0 if the negotiations between the seller and retailers failed. Under a symmetrical negotiation power assumption, there exists certain theorem.

1. If the wholesale price in the spot market is higher than half of the market demand base, that is $\frac{a}{2} \leq w_0 \leq a$, then both retailers would prefer simultaneous negotiation over the price matching mechanism because previous conclusion gives $w_i^{SN} \leq w_i^{PM} \leq \frac{a}{2}$. In contrast to the retailer, the seller would prefer price matching over the simultaneous negotiation.
2. If $w_0 < \frac{a}{2}$, retailers and the seller would trade through the spot market at the price w_0.

This theory illustrates that there exists a threshold of $\frac{a}{2}$, when the price in the spot market is sufficiently high, this will push the retailers to negotiate with the seller to get an ‘appropriate’ wholesale price which could be lower than the spot market and create the profit margin. One noticing point is that the seller could be motivated to reach an agreement with the retailer as well as long as the total revenue is higher compared to the revenue received from the spot market. Rember the lower wholesale price could lead to an increasing demand size from the consumers. Choosing between the high unitary profit margin with low demand base and relative lower unitary profit margin with high demand base, a rational seller will always choose the larger one to maximise personal profit. Hence, when the spot market has a sufficiently high wholesale price, there exist motivations for the seller and retailers to conduct the negotiation to persuade for a lower wholesale price with high demand from consumers. Hence, when the outside spot market has a price lies between half of the demand and total demand, conducting negotiation is a mutually beneficial solution for all participants.

For the second part of the theory, when the spot market has the price under the half of the market demand, then both retailers and the seller will trade through the spot market regardless of either PM or SN. Remember from previous content, the wholesale price reached through either PM or SN has the upper boundary of $\frac{a}{2}$, if the spot market has the price higher than wholesale price reach by either mechanism, the retailer will sell all products they purchased from the seller to exploit the riskless profit from this price differences. Given both retailers are more sensitive to changes in the price over the demand from consumers, the existence of the outside option does not shape the preference of those retailers regarding the negotiation mechanism.

But things would make a lot of differences to the seller which would significantly improve the seller's negotiation power when the first negotiation with one of the retailers failed. Recalling from previous deduction content, under the price matching mechanism, when the first negotiation between the seller and the retailer i failed, the second negotiation is totally dominated by the retailer j. The retailer j dominates the wholesale price setting procedure as both the seller and the retailer j knows that if the second negotiation failed, the seller will earn nothing. With the idea of anything will be better than nothing, the seller has to accept the wholesale price proposed by the retailer j as long as the agreement guarantees profit for the seller. However, the existence of the outside option gives seller the encourage to turn down the contract proposed by the retailer j if the proposal generates lower profit compared with the situation from the spot market. The mathematical conduction procedure is the following:

Under the SN, participants aim to maximise:

$$\left[\pi_i(w_i, w_j) - \pi_i(w_0, w_j)\right]^{\theta_i}\left[\pi_s(w_i, w_j) - \pi_s(w_0, w_j)\right]^{1-\theta_i}$$

For the retailer i, the profit figure $\pi_i(w_i, w_j)$ will be equal or larger than $\pi_i(w_0, w_j)$ if the wholesale price w_i is equal or smaller than wholesale price in spot market w_0.

For the seller's condition:

$$\pi_s(w_i, w_j) - \pi_s(w_0, w_j) = (w_0 - w_i)\frac{(2-\gamma^2)w_i - [(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j]}{(1-\gamma^2)(4-\gamma^2)}$$

If $w_i \leq w_0$, the profit difference would be positive if

$$w_i \geq \frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j}{(2-\gamma^2)}$$

That can be saying that w_i is a boundary point ($w_i = w_0$) if

$$\frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j}{(2-\gamma^2)} \geq w_0$$

And w_i would be lower than w_0 if $\frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j}{(2-\gamma^2)} < w_0$.

The boundary equilibrium is (w_0, w_0) if $\frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j}{(2-\gamma^2)} \geq w_0$ which is equivalent to $w_0 \leq \frac{a}{2}$.

Hence, for any non-(w_0, w_0) equilibrium, $\frac{a}{2} \leq w_0 \leq a$, and corresponding requirement:

$$\frac{(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j}{(2-\gamma^2)} \leq w_i \leq w_0$$

It can be saying that w_i is an interior solution given the upper boundary w_0, it requires:

$$(1-\gamma)(2+\gamma)a - (2-\gamma^2)w_0 + 2\gamma w_j \leq 0$$

The FOC of the objective function has the same sign as:

$$\begin{aligned} &2(2-\gamma^2)^2 w_i^2 - (2-\gamma^2)\big[(5-\theta_i)(1-\gamma)(2+\gamma)a - 2(2-\gamma^2)^2 w_0 + 6\gamma w_j\big] w_i \\ &\quad + 2(1-\gamma)^2(2+\gamma)^2 a^2 \\ &\quad + \big[6\gamma(1-\gamma)(2+\gamma)w_j - (1-\gamma)(2+\gamma)(2-\gamma^2)(1+\theta_i)w_0\big]a + 2\gamma(-2w_0 \\ &\quad + \gamma^2 w_0 + 2\gamma w_j)w_j \end{aligned}$$

Resulting the function of $w_i(w_j)$ is:

$$(5-\theta_i)(1-\gamma)(2+\gamma)a - 2(2-\gamma^2)^2 w_0 + 6\gamma w_j - \frac{\sqrt{N}}{4(2-\gamma^2)}$$

Where N is:

$$\begin{aligned} &(1-\theta_i)(9-\theta_i)(1-\gamma)^2(2+\gamma)^2 a^2 - 12(1-\theta_i)(1-\gamma)(2+\gamma)\big(2w_0 - \gamma^2 w_0 - \gamma w_j\big)a \\ &\quad + 4(2w_0 - \gamma^2 w_0 - \gamma w_j)^2 \end{aligned}$$

The lower boundary equilibrium $(0, 0)$: when the $w_j = 0$, then $w_i = 0$ is the best response if the above FOC is negative at $w_i = 0$. This is equivalent to:

$$\theta_i \geq \frac{(2+\gamma)(1-\gamma)a-(2-\gamma^2)w_0}{(2-\gamma^2)w_0}$$

Under the price matching mechanism, suppose the seller negotiates with the retailer i first, if the negotiation failed, then the seller will conduct the negotiation with the retailer j. With the similar thought, working backwards from the second negotiation. The negotiation between the seller and the retailer j, the Nash Bargaining product becomes:

$$\left[\pi_j(w_j,w_0)-\pi_j(w_0,w_0)\right]^{\theta_j}\left[\pi_s(w_j,w_0)-\pi_s(w_0,w_0)\right]^{1-\theta_j}$$

The result $\pi_j(w_j,w_0)$ is deceasing as w_j decreases, and $\pi_j(w_j,w_0)\geq\pi_j(w_0,w_0)$ if $w_j\leq w_0$. On the other hand, $\pi_s(w_j,w_0)$ is increasing in w_j when $w_j\leq\frac{(2-\gamma-\gamma^2)a+2\gamma w_0}{2(2-\gamma^2)}$, and $\pi_s(w_j,w_0)\geq\pi_s(w_0,w_0)$ if the contract price $w_0\leq w_j\leq\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)}$. If $w_0\leq\frac{a}{2}$, then $w_0\leq\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)}$. Then when the spot market has the price below half of the demand, $w_0\leq\frac{a}{2}$, then $w_0\leq\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)}$, then the seller would only accept the wholesale price $w_j\in[w_0,\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)}]$. And the retailer will only accept the wholesale price $w_j\leq w_0$. The result will end at w_0. On the other hand, if $w_0\geq\frac{a}{2}$, then the situation becomes $w_0\geq\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)}$. The wholesale price w_j falls into the range of the following: $[\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)},\frac{a}{2}]$.

Now consider the negotiation between the seller and the retailer i, if they successfully reached a wholesale price w_i, the retailer j faces the trade off between accepting the contract or reject and enters the spot market. If retailer j accepts it, the profit will become $\pi_j(w_i,w_i)$. If, on contrast, retailer j rejects the contract, then the profit figure will be $\pi_j(w_0,w_i)$. It has been known that retailer i will accept the contract regarding w_i if $w_i\leq w_0$.

Considering the Nash bargaining product:

$$\left[\pi_i(w_i,w_i)-\pi_i(w_0,w_j)\right]^{\theta_i}\left[\pi_s(w_i,w_i)-\pi_s(w_0,w_j)\right]^{1-\theta_i}$$

The retailer i has the incentive to reach for an agreement if:

$$(2-\gamma-\gamma^2)a-(2-\gamma^2)w_i+\gamma w_i\geq(2-\gamma-\gamma^2)a-(2-\gamma^2)w_0+\gamma w_j$$

$$w_i \leq \frac{(2-\gamma^2)w_0 - \gamma w_j}{(2-\gamma-\gamma^2)}$$

The retailer has the incentive to reach for the contract if (7)

$$\frac{2w_i(a-w_i)}{(2-\gamma-\gamma^2)} \geq w_j\left(\frac{(2-\gamma-\gamma^2)a-(2-\gamma^2)w_j+\gamma w_0}{(1-\gamma^2)(4-\gamma^2)}\right) + w_0\left(\frac{(2-\gamma-\gamma^2)a-(2-\gamma^2)w_0+\gamma w_j}{(1-\gamma^2)(4-\gamma^2)}\right)$$

If $w_0 \leq \frac{a}{2}$, the participants will end up with $w_i^{PM} = w_0$. If $w_0 \geq \frac{a}{2}$, then the w_i^{PM} will fall between $\left[w_i^{SN}, \frac{a}{2}\right]$ based on previously conducted property $w_j \epsilon [\frac{(2-\gamma-\gamma^2)a-(2-2\gamma-\gamma^2)w_0}{(2-\gamma^2)}, \frac{a}{2}]$.With the wholesale price below $\frac{a}{2}$, the seller would prefer a high wholesale price while the retailer still prefer low wholesale price, then the preferences of the retailers remain the same compared to the circumstance without the existence of outside option.

From the above content, it can be concluded that the existence of outside option will not shape the preference of the retailer regarding the negotiation mechanisms due to the sensitivity of retailer is higher regarding to the changes in price rather than quantity. However, for the seller, the existence of outside option gives the last resort when the contract proposed by the retailer is too low. That is saying that the outside option could provide a lower boundary which could guarantee the profit of the seller.

When the market demand is a private information, the existence of the outside option could help the seller to guarantee the profit when the market demand is low and in certain degree prevent the retailer deliberately propose a contract that too low.

This is because when the market demand is actually high, there exist the motivation for the retailer to mimic the low demand condition to take advantage of shortage in supply. However, the existence of the spot market will provide a low boundary of the contract as both the seller and the retailer know that if the contract proposed is lower than the spot market, the seller will switch to the spot market resulting in the retailer loses the advantage of possessing private information. On the other hand, when the market demand is low, the retailer has no incentive to mimic the high demand situation but the existence of the spot market will guarantee the wholesale price no lower than the spot market. Hence, the existence of the outside option helps the seller to prevent from exploiting by the retailer through providing low boundary of the revenue.

6.4 Discussion and conclusion

In a supply chain which has the seller, retailers and customers. Or to be more specific, in the financial market where there are company that initiates the IPO, underwriters and general investors, the information is the key element throughout the negotiation between the company and the underwriters.

If the information is perfectly symmetrical between the company and the underwriters, the company could raise more fund through the application of price matching negotiation mechanism. And the first-round negotiation has to be conducted between the company and the underwriter who has relative weak negotiation power. While those underwriters would prefer the application of simultaneous negotiation.

However, if there is information asymmetry between the company and underwriters, especially when underwriters have private information. Then any one-time negotiations will be dictated by the underwriters. They will on purposely propose the contract that is suitable for the low demand market regardless of market demand type. For the company who wish to the true type of market demand, he has to adapt the alternating bargaining mechanism. Throughout the alternating bargaining, the patience of the participants play an important role regarding the behaviour of the underwriter. Usually, the less patience the underwriter has, the more likely that the underwriter tends to tell the truth.

Moreover, if there exist an outside option, which refers to the possibility of conducting the direct offering to the general investors, then this would be the resort that could help the company who initiates the IPO to prevent from underwriters dictating the price of the shares being issued. However, direct public offering can only guarantee the share price that will not be too low to be accepted by the seller. But it does not generate the share price to be very high. In another word, the existence of outside option remains the underwriters that they can not propose the price that is too low since the seller has the option to seller shares by himself. With that thought in mind, the existence of outside option could, in certain degree, encourage the underwriter and the company to reach an agreement.

In practical life, the situations might have certain deviations compared to the model prosed in this article. The first issue is the market demand, in this article, the market demand is simply expressed in a letter and simply divided into a high demand and low demand type. But in real life, the demand could be deterministic based on various elements and the state of demand would be more common that this element follows a stochastic procedure which the specific constituents are difficult to be clearly illustrated.

The second issue is the relationship between retailers. In the financial market, it is commonly encountered that one big investment bank plays the leading underwriter and other investment banks forms a syndicate to help the company to initiate the IPO. The relationships between those underwriters have both cooperation and competition. However, in the model, this complicated relationship is very difficult to be illustrated.

The third issue is related to the outside option. In practical life, company bypasses the underwriters to directly offering shares to the general investors is very rare. On the one hand, conducting the direct public offering requires a rather high level of confidence regarding the company's future prospect. It would be suitable for company that has been widely known by general investors before the IPO. The well-known example of conducting the direct public offering is the Facebook (now named Meta). Besides this example, there lacks aftercoming examples. Hence, for most public companies, they still choose to conduct the IPO with the help from underwriters. That is saying the functions of the outside option remain theoretical rather than practical.

In general, this research proposes new bargaining mechanism compared to existing commonly encountered mechanisms, price matching and simultaneous negotiation. The alternating bargaining mechanism is particularly useful to the seller when there is information asymmetry which requires the seller to construct the belief regarding the private information. Part of the results conducted from the model conforms with existing literature while certain results show opposite outcomes given certain settings of model changed.

Chapter 7. Conclusion and Contribution

Over the past decades, the development of game theory has originated from solving situations between two players and grown to be a subject that comprises various practical subjects. In recent years, the mathematical techniques used in share price determination are primarily concerned with the stochastic analysis regarding the random variables. The conventional mathematical analysis techniques are commonly used in the research regarding the supply chain management issues. Though the idea of game theory is developed based on certain situations in the financial markets and some well-known game theory concepts are originated from those papers regarding the application of game theory in the financial markets. For instances, the repeated games (Meyer and Saley, 2003), the cheap talk issues (Bloomfield and Kadyali, 2005), the soft set theory (Kollias et al, 2024), the Pareto efficiency (Ning and Babich, 2018), the Bayesian theory (Song and Wu, 2023) etc. However, those papers rarely concern the bargaining situation between the IPO company, underwriters and individual investors. Regarding this situation, this thesis proposes to use bargaining model to describe the situation of the pre-IPO phase regarding the share price determination.

In the first technical chapter (chapter 4), we investigate the possibility of setting an auction which is host by the IPO company and let the underwriters propose their biddings for the afterwards underwriting activities. After comparing different auction types and expected profit that will be received by the IPO company, the result shows that as long as the auction is set fairly, the expected payoff would be the same regardless of auction types. This is consistent with the result conducted by researchers in previous times (Nagarajan and Sosic, 2008). In the following contract determination progress, this thesis proposes two different types of contracts. Different from other papers describing the bargaining situation among the supply chain structure (Shang and Cai, 2022) that usually set price as the strategic variable, the mathematical model in thesis sets the quantity as the strategic variable. This is because the supply chain management papers usually study the 'vendor problem'. This refers to the condition that the retailer and the seller have constructed a long-term relationship which the quantity is a rather stable element. In this thesis, however, the IPO company negotiates with the underwriter regarding the number of shares and corresponding share price for the first time. Hence, it would be more appropriate to set the quantity of shares as the strategic variable rather than price. The result from chapter 4 indicates that under different negotiation powers, the participants would behave differently regarding the contract selection in order to fully take advantage of their negotiation power.

In the second technical chapter (Chapter 5), the focus of the mathematical model is mainly focus on the channel efficiency. Following the thought from the first technical chapter, the mathematical model in chapter 5 considers the possibility of switching the relationship between the IPO company and the underwriter from competing with each other to co-operate with each other. By introducing the outside option, it has successfully helped both parties becomes better off compared to situation without the outside option. With the outside option, both parties could receive more profits than before, and the distribution of the incremental profit would be proportionate to the relative negotiation power. These findings are consistent with previous article (Shang and Cai, 2022) regarding side payment's function including creating a bigger pie and splitting the pie. However, different from Shang and Cai's setting, the side payment setting in the second technical chapter is taking an opposite direction. The result shows that even though the setting is different, the result is the same. The model finds that once the preliminary model is set, the equilibrium result after the introduction of side payment will always be the same regardless of how the side payment is transferred. This result brings the discussion regarding the composition of side payment. The function of the side payment, in essence, is a transfer of partial incremental profit generated from the condition that both parties cooperate rather than compete. Regarding the expression of the equilibrium side payment that should be transferred, the model illustrates that specific amount is directly related to the relative negotiation power. In general, the higher relative negotiation power a party possesses, the less (more) side payment he shall sacrifice (receive). In the practical situation, for the participate of the negotiation, if the current reached equilibrium situation seems to be dissatisfactory for himself, he could consider the application of side payment, while the proposed amount of the side payment should be considered based on the negotiation power he possesses.

In the third technical chapter (Chapter 6) the model is conducted under the stochastic element. This setting is inspired from the paper written by Qi Feng, Guoming Lai, and Lauren Lu. In this paper, they proposed a setting where the information between the IPO company and the underwriter is asymmetrical and the information regarding the market demand is possessed by the underwriter (Feng, et al., 2015). In this chapter, it has been concluded that no matter what the negotiation mechanism is, the underwriter would always have the motivation to fake a low demand condition under an actual high demand circumstance. With the one-time negotiation, the seller would be always exploited by the underwriter who possess the private information. To construct the belief regarding the true market demand, the alternating negotiation is introduced to facilitate the construction of the seller's belief. After construct the belief regarding the market demand, the seller conducts the simultaneous negotiation and price matching bargaining with underwriters. Different from the conventional situation where the seller prefers to conduct the price matching negotiation first with the underwriter with weaker bargaining power relative to the seller, the seller tends to conduct the price matching negotiation with the underwriter that has stronger bargaining power. For the element, the bargaining power that has been consistently mentioned in these three chapters, we proposed a factor, namely the patience level, which is a diminishing element regarding the time, to represent a partial explanation of the relative bargaining power. And the specific conduction of alternating bargaining process can be found in the appendix. In addition to the previous mentioned model which the seller only has to sell his shares to the underwriters, the third chapter considers the existence of the outside option which enables the seller to directly sell shares to the general investors. The mathematic illustration shows that the existence of the outside option is incapable of improving the profitability of the seller but could provide a guarantee of the minimum profit received. The outside option is beneficial for the seller, and before conducting the negotiation, it would be preferrable to let the underwriters know the existence of the outside option.

This thesis contributes to the literature in the following stages. First aspect is to provide a different share valuation philosophy for the newly issued shares compared with current mathematical methods (Braselton, et al., 1999). It considers the expectation from the underwriters and the seller. Different from the commonly encountered situations that underwriters tend to purchase shares from the seller and resell those shares to the common investors, this thesis provides the auction theory to conduct the IPO procedure. The second aspect is to propose different types of contracts that use different strategic variables. The current literature tends to consider the ‘price’ as the independent strategic variable (Shang and Cai, 2022), while in this thesis, the strategic variable is setting as ‘quantity’ which makes this thesis more suitable for the application in practical situations. The third aspect is decomposing the idea behind the side payment which commonly acts as the channel facilitator. This thesis provides a specific range which the side payment lies and proposes the rule of splitting the incremental profit caused by the application of the side payment which is different compared to conventional side payment settings in the commercial practices (Goranko, 2022). The fourth aspect is providing managerial insights for the participants when the practical situation contains stochastic elements (Feng, et al, 2015). How they should construct their own belief regarding the stochastic demand and how to appropriately negotiate with the counterparty. This provides insights for both sides regarding what they should do in order to maximise their personal profit by taking advantage of their advantages.

However, this thesis still has certain limitations regarding the assumptions for the mathematical models' deduction. The first issue is the discount factor in chapter 4. In the section 5.3.3 in chapter 4, the term $e^{-r\Delta t}$ is used as the discount factor to illustrate that if the IPO company decides to take the outside option, the longer it takes to settle the outside option, the lower profit will be received by the IPO company. However, for different companies that could have different attributions, this discount factor could add more company-specific factor to better demonstrate different situations of the company to better reflect their risk profile.

The second limitation is regarding the assumptions of models used in the thesis. For the models employed in the thesis, the game theory, the bargaining models and the auction dynamics, we assumed that participants tend to be rational throughout the process. Even though bidders have different valuations regarding the object being auctioned, they could update their valuation based on other bidders' offers. However, in real world, there exist possibilities that bidders could insist their thought regardless of other information newly obtained. Assuming each participant to be rational is rather theoretical, which may make the model loses certain applicability in practical situations.

The third limitation is the applicability test of the models. Throughout the thesis, the models proposed are highly theoretical. The applicability of these models in real IPO situations lacks certain test. These are the future work that requires certain empirical test in the real-world situations to better understand their theoretical disadvantages and practical improvements.

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Appendices

Appendix 1. Classification of supply chain models:

Non-cooperative game models	Collaborating to Compete (Amaldoss, Wilfred ; Meyer, Robert J ; Raju, Jagmohan S ; Rapoport, Amnon, 2000); Centralization of Stocks: Retailers vs. manufacturer (Anupindi, Ravi ; Bassok, Yehuda, 1999); Supply Contracts with Quantity Commitments and Stochastic Demand (Ravi Anupindi & Yehuda Bassok); Inventory control under substitutable demand: A stochastic game application (Avsar, Zeynep Müge ; Baykal-Gürsoy, Melike, 2002); A framework for decentralized multi-echelon inventory control (Axsater, S, 2001); A Quantity Discount Pricing Model to Increase Vendor Profits (Monahan, James P, 1984); Advertising Competition under Consumer Inertia (Bibek Banerjee and Subir Bandyopadhyay, 2003); Reputation in Marketing Channels: Repeated-Transactions Bargaining with Two-Sided Uncertainty (Banks, Darryl T ; Hutchinson, J. Wesley ; Meyer, Robert J, 2002); Pricing and Replenishment Strategies in a Distribution System with Competing Retailers (Bernstein, Fernando ; Federgruen, Awi, 2003); Dynamic inventory and pricing models for competing retailers (Bernstein, Fernando ; Federgruen, Awi, 2003); Théorie mathématique de la richesse sociale (Bertrand, J, 1883) ; Vertical price controls with uncertain demand (Butz, David A, 1997); Competitive and cooperative policies for the vendor–buyer system (Bylka, Stanistaw, 2003); Competitive and cooperative inventory management in a two-echelon supply chain with lost sales (Gerard. P. Cachon, 1999); Stock Wars: Inventory Competition in a Two-Echelon Supply Chain with Multiple Retailers (Gerard. P. Cachon, 2001); The Allocation of Inventory Risk in a Supply Chain: Push, Pull, and Advance-Purchase Discount Contracts (Gerard. P. Cachon, 2004); Competition and Outsourcing with Scale Economies (Cachon, Gerard P ; Harker, Patrick T, 2002); Capacity Allocation Using Past Sales: When to Turn-and-Earn (Cachon, Gerard P ; Lariviere, Martin A, 1999); Capacity Choice and Allocation: Strategic Behavior and Supply Chain Performance (Cachon, Gerard P ; Lariviere, Martin A, 1999); An equilibrium analysis of linear, proportional and uniform allocation of scarce capacity (Cachon, Gerard P ; Lariviere, Martin A, 1999);

	Contracting to Assure Supply: How to Share Demand Forecasts in a Supply Chain (Cachon, Gerard P ; Lariviere, Martin A, 2001); Competitive and Cooperative Inventory Policies in a Two-Stage Supply Chain (Cachon, Gerard P ; Zipkin, Paul H, 1999); Analysis of a Decentralized Production-Inventory System (René Caldentey, Lawrence M. Wein, 2003); Price Competition of Make-to-Order Firms (Chen, Hong ; Wan, Yat-Wah,2003); Direct Marketing, Indirect Profits: A Strategic Analysis of Dual-Channel Supply-Chain Design (Chiang, Wei-yu Kevin ; Chhajed, Dilip ; Hess, James D, 2003); A Game-Theoretic Approach to Quantity Discount Problems (Wen-Chyuan Chiang, James Fitzsimmons, Zhimin Huang, Susan X. Li, 1994); Price Competition in a Channel Structure with a Common Retailer (Choi, S. Chan, 1991); Demand Signalling and Screening in Channels of Distribution (Chu, Wujin, 1992); Signaling Quality by Selling Through a Reputable Retailer: An Example of Renting the Reputation of Another Agent (Chu, Wujin ; Chu, Woosik, 1994); Channel Coordination Mechanisms for Customer Satisfaction (Wujin Chu, Preyas S. Desai, 1995); Collusion and the Incentives for Information Sharing (Clarke, Richard N, 1983); Competing in Product and Service: A Product Life-Cycle Model (Cohen, Morris A ; Whang, Seungjin, 1997); Stochastic Inventory Systems in a Supply Chain with Asymmetric Information: Cycle Stocks, Safety Stocks, and Consignment Stock (Corbett, Charles J, 2001); Shared-Savings Contracts for Indirect Materials in Supply Chains: Channel Profits and Environmental Impacts (Corbett, Charles J ; DeCroix, Gregory A, 2001); A Supplier's Optimal Quantity Discount Policy Under Asymmetric Information (Corbett, Charles J ; de Groote, Xavier, 2000); Competition and Structure in Serial Supply Chains with Deterministic Demand (Corbett, Charles J ; Karmarkar, Uday S, 2001); Researches into the mathematical principles of the theory of wealth (A.A. Cournot, 1897); Optimizing Advertising Expenditures in a Dynamic Duopoly (Deal, Kenneth R, 1979); Strategic commitment versus postponement in a two-tier supply chain (Cvsa, Viswanath ; Gilbert, Stephen M, 2002); Multiple Messages to Retain Retailers: Signaling New Product Demand (Desai, Preyas S, 2000);

	Marketing-Production Decisions in an Industrial Channel of Distribution (Eliashberg, Jehoshua ; Steinberg, Richard, 1987); Information Sharing in Oligopoly (Gal-Or, Esther, 1985); Information Transmission—Cournot and Bertrand Equilibria (Gal-Or, Esther, 1986); Duopolistic vertical restraints (Gal-Or, Esther, 1991); Customer Loyalty and Supplier Quality Competition (Gans, Noah, 2002); Fair transfer price and inventory holding policies in two-enterprise supply chains (Gjerdrum, Jonatan ; Shah, Nilay ; Papageorgiou, Lazaros G, 2002); A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits (Lee, Hau L ; Rosenblatt, Meir J, 1986); A Three-Stage Model for a Decentralized Distribution System of Retailers (Granot, Daniel ; SoSic, Greys, 2003); Price and Delivery Logistics Competition in a Supply Chain (Ha, Albert Y ; Li, Lode ; Ng, Shu-Ming, 2003); Customer Service Competition in Capacitated Systems (Joseph Hall, Evan Porteus, 2000); The Competitive Implications of Relevant-Set/Response Analysis (Hauser, John R. ; Wernerfelt, Birger, 1989); Co-op advertising models in manufacturer–retailer supply chains: A game theory approach (Zhimin Huang, Susan X. Li, 2001); Capacity constrained duopolies, uncertain demand and non-existence of pure strategy equilibria (Hviid, Morten, 1991); Pricing of Information Products on Online Servers: Issues, Models, and Analysis (Jain, Sanjay ; Kannan, P. K, 2002); Note--Comments on "A Quantity Discount Pricing Model to Increase Vendor Profits" (Joglekar, Prafulla N, 1988); Optimal production, purchasing and pricing: A differential game approach (Jorgensen, Steffen, 1986); Optimal pricing and inventory policies: Centralized and decentralized decision making (Jørgensen, Steffen ; Kort, Peter M, 2002); Manufacturer-Retailer Channel Interactions and Implications for Channel Power: An Empirical Investigation of Pricing in a Local Market (Kadiyali, Vrinda ; Chintagunta, Pradeep ; Vilcassim, Naufel, 2000); Optimal Service Speeds in a Competitive Environment (Kalai, Ehud ; Kamien, Morton I ; Rubinovitch, Michael, 1992);

	The Value of Market Share and the Product Life Cycle--A Game-Theoretic Model (Karnani, Aneel, 1984); Simultaneous Improvement of Supplier's Profit and Buyer's Cost by Utilizing Quantity Discount (Kim, Kap Hwan ; Hwang, Hark, 1989); Price Competition vs. Quantity Competition: The Role of Uncertainty (Klemperer, Paul ; Meyer, Margaret, 1986); Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes (Kreps, David M. ; Scheinkman, Jose A, 1983); A game theoretic model for two types of customers competing for service (Kulkarni, V.G, 1983); An Approach for Developing an Optimal Discount Pricing Policy (Lal, Rajiv ; Staelin, Richard, 1984); Pricing, Production, Scheduling, and Delivery-Time Competition (Lederer, Phillip J ; Li, Lode, 1997); A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits (Lee, Hau L ; Rosenblatt, Meir J, 1986); Price variation duopoly with differentiated products and random demand (Levitan, Richard ; Shubik, Martin, 1971); Price Duopoly and Capacity Constraints (Levitan, Richard ; Shubik, Martin, 1972); Cournot Oligopoly with Information Sharing (Li, Lode, 1985); The Role of Inventory in Delivery-Time Competition (Li, Lode, 1992); Information Sharing in a Supply Chain with Horizontal Competition (Li, Lode, 2002); Pricing and Delivery-Time Performance in a Competitive Environment (Li, Lode ; Lee, Yew Sing, 1994); Producer-Supplier Contracts with Incomplete Information (Lim, Wei Shi, 2001); Channel Strategies and Stocking Policies in Uncapacitated and Capacitated Supply Chains (Mahajan, Jayashree ; Radas, Sonja ; Vakharia, Asoo J, 2002); Inventory Competition Under Dynamic Consumer Choice (Mahajan, Siddharth ; van Ryzin, Garrett, 2001); Competition to Retain Customers (McGahan, A. M ; Ghemawat, Pankaj, 1994); An Industry Equilibrium Analysis of Downstream Vertical Integration (McGuire, Timothy W ; Staelin, Richard, 2008); A Quantity Discount Pricing Model to Increase Vendor Profits (Monahan, James P, 1984); Product and Price Competition in a Duopoly (Moorthy, K. Sridhar, 1988); Strategic Decentralization in Channels (Moorthy, K. Sridhar, 1988);

	Centralized and Competitive Inventory Models with Demand Substitution (Netessine, Serguei ; Rudi, Nils, 2003); Competitive Procurement Under Demand Uncertainty (Nti, Kofi O, 1987); Game theoretic analysis of the substitutable product inventory problem with random demands (Mahmut Parlar, 1988); Discounting decisions in a supplier-buyer relationship with a linear buyer's demand (Mahmut Parlar & Qinan Wang, 2007); A game theoretical analysis of the quantity discount problem with perfect and incomplete information about the buyer's cost structure (Mahmut Parlar & Qinan Wang, 1995); Endogenous Quality Differentiation in Congested Markets (Reitman, David, 1991); The Delivery and Control of Quality in Supplier-Producer Contracts (Reyniers, Diane J ; Tapiero, Charles S, 1995); Improving Profitability with Quantity Discounts under Fixed Demand (Meir J. Rosenblatt & Hau L. Lee, 2007); Stability and Chaos in Input Pricing for a Service Facility with Adaptive Customer Response to Congestion (Rump, Christopher M ; Stidham, Shaler, 1998); Price strategy oligopoly with product variation (Shapley, Lloyd ; Shubik, Martin, 1967); Implicit Understandings in Channels of Distribution (Shugan, Steven M, 1985); A manufacturer's optimal quantity discount strategy and return policy through game-theoretic approach (Su, C-T ; Shi, C-S, 2002); Distribution Channels: An Extension of Exclusive Retailership (Minakshi Trivedi, 1998); Price Versus Production Postponement: Capacity and Competition (Van Mieghem, Jan A ; Dada, Maqbool, 1999); Duopoly information equilibrium: Cournot and Bertrand (Vives, Xavier, 1984); Determination of suppliers' optimal quantity discount schedules with heterogeneous buyers (Wang, Qinan, 2002); Improving a supplier's quantity discount gain from many different buyers (Wang, Qinan ; Wu, Zhang, 2000); A duopolistic model of dynamic competitive advertising (Wang, Qinan ; Wu, Zhang, 2001); Information Transparency of Business-to-Business Electronic Markets: A Game-Theoretic Analysis (Zhu, Kevin, 2004)

Cooperative game models	Centralization of Stocks: Retailers vs. Manufacturer (Anupindi, Ravi ; Bassok, Yehuda, 1999); A General Framework for the Study of Decentralized Distribution Systems (Ravi Anupindi, Yehuda Bassok, Eitan Zemel, 2001); Dynamic inventory and pricing models for competing retailers (Bernstein, Fernando ; Federgruen, Awi, 2004); Competitive and cooperative policies for the vendor–buyer system (Bylka, Stanisław, 2003); The allocation of inventory risk in a supply chain: Push, pull, and advance-purchase discount contracts (Cachon, Gerard P, 2004); Contracting to Assure Supply: How to Share Demand Forecasts in a Supply Chain (Cachon, Gerard P ; Lariviere, Martin A, 2001); Competitive and Cooperative Inventory Policies in a Two-Stage Supply Chain (Cachon, Gerard P ; Zipkin, Paul H, 1999); Coordination Mechanisms for a Distribution System with One Supplier and Multiple Retailers (Chen, Fangruo ; Federgruen, Awi ; Zheng, Yu-Sheng, 2001); A Game-Theoretic Approach to Quantity Discount Problems (Wen-Chyuan Chiang, James Fitzsimmons, Zhimin Huang, Susan X. Li, 1994); A Supplier's Optimal Quantity Discount Policy Under Asymmetric Information (Corbett, Charles J ; de Groote, Xavier, 2000); On apportioning costs to customers in centralized continuous review inventory systems (Gerchak, Yigal ; Gupta, Diwakar, 1991); Cost allocation in continuous-review inventory models (Bruce C. Hartman, Moshe Dror, 1996); Optimizing centralized inventory operations in a cooperative game theory setting (Hartman, Bruce C. ; Dror, Moshe, 2003); Co-op advertising models in manufacturer–retailer supply chains: A game theory approach (Huang, Zhimin ; Li, Susan X, 2001); An Analysis of Manufacturer-Retailer Supply Chain Coordination in Cooperative Advertising (Huang, Zhimin ; Li, Susan X. ; Mahajan, Vijay, 2002); Managing Channel Profits (Jeuland, Abel P ; Shugan, Steven M, 2008); Optimal pricing and inventory policies: Centralized and decentralized decision making (Jørgensen, Steffen ; Kort, Peter M, 2002); Retail promotions with negative brand image effects: Is cooperation possible? (Jørgensen, Steffen ; Taboubi, Sihem ; Zaccour, Georges, 2003);

	Simultaneous Improvement of Supplier's Profit and Buyer's Cost by Utilizing Quantity Discount (Kim, Kap Hwan ; Hwang, Hark, 1989); A Cooperative Game Theory Model of Quantity Discounts (Kohli, Rajeev ; Park, Heungsoo, 1989); An Approach for Developing an Optimal Discount Pricing Policy (Lal, Rajiv ; Staelin, Richard, 1984); Supply Chain Contracting and Coordination with Stochastic Demand (Martin A. Lariviere, 1999); Cournot Oligopoly with Information Sharing (Li, Lode, 1985); Managing buyer-seller system cooperation with quantity discount considerations (Li, Susan X. ; Huang, Zhimin, 1995); Cooperative advertising, game theory and manufacturer–retailer supply chains (Li, Susan X. ; Huang, Zhimin ; Zhu, Joe ; Chau, Patrick Y.K, 2002); The Competitive Newsboy (Lippman, Steven A ; McCardle, Kevin F, 1997); Coordinating supply chains with competition: Capacity allocation in semiconductor manufacturing (Mallik, Suman ; Harker, Patrick T, 2004); Impact of demand correlation on the value of and incentives for information sharing in a supply chain (Raghunathan, Srinivasan, 2003); The Delivery and Control of Quality in Supplier-Producer Contracts (Reyniers, Diane J ; Tapiero, Charles S, 1995); A Two-Location Inventory Model with Transshipment and Local Decision Making (Nils Rudi, Sandeep Kapur and David F. Pyke, 2001); A Game Theoretic-Mathematical Programming Analysis of Cooperative Phenomena in Oligopolistic Markets (Hanif D. Sherali and Roby Rajan, 1986); Coordinating Investment, Production, and Subcontracting (Jan A. Van Mieghem, 1999); Coordinating Independent Buyers in a Distribution System to Increase a Vendor's Profits (Qinan Wang, 2001); Coordinating independent buyers with integer-ratio time coordination and quantity discounts (Wang, Qinan, 2004); A game-theoretical cooperative mechanism design for a two-echelon decentralized supply chain (Wang, Hongwei ; Guo, Min ; Efstathiou, Janet, 2004); A three-person game theory model arising in stochastic inventory control theory (Wang, Qinan ; Parlar, Mahmut, 1994); Channel Coordination and Quantity Discounts (Weng, Z. Kevin, 1995);

	Coordination of joint pricing-production decisions in a supply chain (Zhao, Wen ; Wang, Yunzeng, 2002); The Marketing Channel as an Equilibrium Set of Contracts (Zusman, Pinhas ; Etgar, Michael, 1981)

Appendix 2. Bargaining models with strategic variables:

Article Names	**Model Construction**	**Strategic variable in First Stage Competition**	**Strategic variable in Second Stage Competition**	**Adaptation of Bargaining Theory**	
Bilateral monopoly, identical distributors, and game-theoretic analyses of distribution channels (Charles A. Ingene, Mark E. Parry; 2007)	Manufacture - Distributor	Quantity		No	
Channel selection and coordination in dual-channel supply chains (Gangshu Cai,;2010)	Supplier - Retailer - Customer	Price	Price	No	
Negotiations in competing supply chains: The Kalai-Smorodinsky bargaining solution (Qi Feng, Yuanchen Li, J. George Shanthikumar; 2022)	Supplier - Retailer	Price		Yes	
Leader-Based Collective Bargaining: Cooperation Mechanism and Incentive Analysis (Vernon N. Hsu, Guoming Lai, Baozhuang Niu, Wenqiang Xiao; 2016)	Supplier - Retailer	Price		Yes	
Channel selection and contracting in the presence of a retail platform (Yuelin Shen, Sean P.	Supplier - Retailer - Customer	Quantity	Price	No	Concentrate on platform profit sharing rather than product substitutability

Willems, Yue Dai; 2019)					
Bargaining for an Assortment (Goker Aydin, H. Sebastian Heese; 2015)	Supplier - Retailer	Assortment of product		No	
The strategic perils of low cost outsourcing (Feng Qi, Lauren Xiaoyuan Lu; 2012)	Supplier - Retailer	Price		Yes	
Supply chain contracting under competition: Bilateral bargaining vs Stackelberg (Feng Qi, Lauren Xiaoyuan Lu:2013)	Supplier - Retailer	Price		Yes	
The role of contract negotiation and industry structure in production outsourcing (Feng Qi, Lauren Xiaoyuan Lu; 2013)	Supplier - Retailer	Price		No	
Exclusive channels and revenue sharing in a complementary goods market (Gangshu Cai, Yue Dai, Sean X. Zhou; 2012)	Supplier - Retailer - Customer	Price	Price	No	
Outsourcing under Competition and Scale Economies: When to Choose a Competitor as a Supplier （Hans Sebastin Heese, Eda Kemahlioglu-Ziya, Olga Perdikaki; 2020)	Supplier - Retailer	Price		No	

Advertising in Asymmetric Competing Supply Chains (Liu Bin, Gangshu Cai, Andy A Ssay; 2014)	Supplier - Retailer	Quantity		No	
Online manufacturer referral to heterogeneous retailers (Hao Wu, Gangshu Cai, Jian Chen, Chwen Sheu; 2015)	Supplier - Retailer	Quantity		Yes	
Preferences for contractual forms in supply chains (Lijian Lu, Yaozhong Wu; 2015)	Supplier - Retailer	Price		No	
A Bargaining framework in supply chians: The assembly problem (Mahesh Nagarajan, Yehuda Bassok; 2008)	Supplier - Retailer	Price/Quantity		Yes	Only provide framework, no calculation
Agency selling or reselling? Channel structures in electronic retailing (Vibhanshu Abhishek, Kinshuk Jerath, Z. John Zhang; 2016)	Supplier - Retailer	Quantity		No	
RFQ, sequencing, and the most favorable bargaining outcome (Leo Yang Chu, Ying Rong; 2017)	Supplier - Retailer	Quantity		Yes	Set as two suppliers one buyer
Vertical bargaining and countervailing power (Alberto Lozzi, Tommaso Valletti; 2014)	Supplier - Retailer - Customer	Price	Price	Yes	Focus on negotiation power rather than negotiation mechanisms
Bargaining chains (William S. Lovejoy 2010)	Supplier - Retailer - Customer			Yes	Focus on the theories and ways to conduct calculation

					without any modelling
Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes (Gangshu Cai, Zhe George Zhang, Michael Zhang; 2009)	Supplier - Retailer - Customer	Price	Price	No	
Impact of downstream competition on innovation in a supply chain (Jingqi Wang, Hyoduk Shin; 2012)	Supplier - Retailer	Quantity		No	
Coordinating project outsourcing through bilateral contract negotiations (Chengfan Hou, Mengshi Lu, Tianhu Deng, Max Shen; 2019)	Supplier - Retailer	Price/Quantity			
Nash bargaining with asymmetric bargaining power: Bargaining over profit in a simple supply chain (Opher Baron, Oded Berman; 2014)	Supplier - Retailer	Price/Quantity		Yes	Only theories without models
Game-theoretic analyses of strategic pricing decision problems in supply chains (Feimin Zhong, Zhongbao Zhou, Mingming Leng; 2020)	Supplier - Retailer	Price		Yes	

Game-theoretic coordination mechanisms in distribution channels: Intergration and extensions for models without competition (Charles A Ingene, Sihem Taboubi, Georges Zaccour; 2012)	Supplier - Retailer - Customer	Quantity	Price	Yes	The model has no factor of competition nor discussion regarding the negotiation mechanism
Channel conflict and coordination in the E-commerce age (Andy A Tasy, Narendra Agrawal; 2004)	Supplier - Retailer - Customer	Price	Price	No	Focus on the comparisons between various conditions of intermediate channels
Bargaining in competing supply chains with uncertainty (Desheng Wu, Opher Baron, Oded Berman; 2009)	Supplier - Retailer	Price		Yes	
The strategic role of third-party marketplaces in retailing (Benny Mantin, Harish Krishnan, Tirtha Dhar; 2014)	Supplier - Retailer	Price		Yes	No competition in the modell
Harmony in competition: On preferences for contractual forms in supply chains (Lijian Lu, Yaozhong Wu; 2012)	Supplier - Retailer	Price		No	Calculation in matrix
The strategic benefit of request for proposal/quotataion (Leon Yang Chu, Ying Rong, Huan Zheng; 2022)	Supplier - Retailer	Quantity		Yes	One buyer but two sellers
Contract design in a cross-sales supply chain with demand information asymmetry (Xiaojing	Supplier - Retailer - Customer	Price	Price	No	

Li, Jing Chen, Xingzhen Ai; 2019)					
Modeling multichannel supply chain management with marketing mexes: A survey (Gangshu Cai, Yue Dai, Wenzhu Zhang; 2015)	Supplier - Retailer - Customer	Price	Price	No	
Pricing decisions and strategies selection of dominant manufacturer in dual-channel supply chain (Lidan Ma, Rong Zhang, Sandang Guo, Bin Liu; 2012)	Supplier - Retailer	Price			
A larger pie or a larger slice? Contract negotiation in a closed-loop supply chain with remanufacturing (He Huang, Yu Xiong, Yu Zhou; 2020)	Supplier - Retailer	Quantity			Focus on manufacturer

Appendix 3. Proofs of main results in chapter 4

Bidding behaviour in a second-price auction

Proof of corollary 1:

Suppose the IPO underwriting process takes a conventional Vickrey auction which is a second price sealed-bid auction. It works as the following, bidders with the highest bidding price win the object but only required to pay the second-highest price (Vickrey, 1961).

Starting with the two buyers i & j and assume the valuation from both buyers are denoted as v_i & v_j respectively. The valuation is a continuous function, and it is uniformly distributed.

Each buyer knowns their own valuation regarding the newly issued shares, and each of them will try to maximise their expected surplus from claim the underwriting:

$$Max(v_i - b_j)\Pr\,(i\ wins)$$

Where b_j denotes the second highest bid price offered by buyer j. The buyer i would win the bid if he puts the highest bid but he is only required to pay the second-highest bid offered by buyer j. Hence throughout the process of offering bidding price, what he can affect is only the probability of winning Pr $(i\ wins)$ instead of the surplus from wining.

It can be concluded that the buyer i will offer a bid that is equal to his valuation $b_i = v_i$ to win the bid. Since underbidding $(b_i < v_i)$ would lead to him lost the bid but winning the bid is still profitable for the buyer as long as the second highest bid is lower than the true valuation from buyer i (if $b_i < b_j < v_i$). On the other hand, overbidding $(b_i > b_j > v_i)$ will result in a winning of the bid but making a loss throughout the underwriting which seems a not reasonable choice.

Given the above consideration, both under-bidding and over-bidding are weakly dominated strategies resulting the truthfully bidding strategy is a dominant strategy and the optimal bidding strategies for both buyers would be $b^* = v$ for any valuation. In an alternative way of speaking, neither bidder would need to speculate what the opponent's bidding strategy but offer their true valuations regarding the shares since winning the bid is profitable for sure.

Throughout the auction process, the bidding price keeps going higher until one of the buyers is not willing to offer a higher price and exit. The whole process is essentially a process when buyers leave the bid when the bidding price reaches their valuation. Since the winner also bid truthfully, given he is the last person standing, his valuation must be higher than the winning price he is required to pay.

To expand this circumstance to more than two buyers, the above argument still holds for any N number of bidders. The dominate strategy is to bid truthfully or not quit the bid until the price reaches the valuation, hence for any valuation there is $b_i^* = b_j^* = \cdots = b_N^* = v$.

In this way, the final bidding price would lead up to the customised purchase contract in previous part, but the final price paid by the buyer is the lower one between the results.

Bidding in a first-price auction

Proof of corollary 2:

According to the work from Vickery, the condition would be more complicated due to the reason that each bidder would need to speculate others' strategy to secure the object. If the IPO underwriting auction adapted this style, bidding the valuation is clearly not a clever option since whether win the auction or lost the auction, there is no surplus for the buyer. Hence no bidder will bid for a price higher than their valuation which leads to an optimal bidding strategy slightly lower than their own valuation.

For each buyer, as a strategic game, their bidding strategy is actually the best response to that they speculate the other bidder does. This is equivalent to the previous part of either standardised purchase contract or customised purchase contract and b_i^* & b_j^* need to form a Nash equilibrium.

The strategy for the bidder with its own valuation v has the following bidding strategy $b(v)$ which is a function increasing in v: the higher one buyer values the new issued shares, the higher bid he would willing to offer to the seller. This condition still applies for the other buyer as a conform of symmetry axiom.

From the induction from the previous condition, the buyer will not truthfully bid under the first-price auction: $b(v) < v$. To be more specific, it would be convenient to define a new variable called the shading multiplier z with a range from 0 to 1. Hence, the expression of the bidding function has a basic form:

$$b(v) = zv$$

Starting the bidding with two buyers i & j. Under the reasonable assumption, both of them believe their opponent will bidding in a similar style.

Buyer i with his own valuation v_i will choose to maximise the revenue function if he wins the bid:

$$Max(v_i - b_i)\Pr\,(i\ wins)$$

The above function has two parts: the first part is the payoff if buyer i takes the bid and the second part is the probability that buyer i wins the bid. The two parts has a contrast feature as the increase in the bidding price will increase the probability of winning the bid but lowers the payoff of taking the bid.

For the second part, the probability of buyer i wins the bid, this part has the following expression:

$$\Pr(i\ wins) = \Pr\big(b_i > b_j\big) = \Pr\big(b_i > zv_j\big) = \Pr\,(v_j < \frac{b_i}{z})$$

If the bidding function has uniform distribution function from 0 to 1 then the revenue function of the buyer i could be rewrite into

$$Max(v_i - b_i)\frac{b_i}{z}$$

Hence the optimal bid would be:

$$\frac{v_i}{z} - \frac{2b_i}{z} = 0$$

$$\hat{b}_i = \frac{v_i}{2}$$

By a symmetric assumption, the situation would be the same for the buyer j which is:

$$\hat{b}_j = \frac{v_j}{2}$$

Overall, the general expression for the bidding function would be:

$$\hat{b} = \frac{v}{2}$$

Expected revenue for the seller

Proof of corollary 3:

$$v_{(2nd\ highest)} = \frac{0 + v_{(1st\ highest)}}{2}$$

$$v_{(1st\ highest)} = \frac{1 + v_{(2nd\ highest)}}{2}$$

By solving these equation the result is

$$v_{(2nd\ highest)} = \frac{1}{3}$$

$$v_{(1st\ highest)} = \frac{2}{3}$$

The expected revenue from the second-price auction is $\frac{1}{3}$ while for the two-bidder first-price auction the expected revenue is $\frac{1}{2}v_{(1st\ highest)} = \frac{1}{3}$ which essentially the same under both circumstances. This is consistent with Revenue Equivalence Theorem which is one of the most important economic theories of auctions. This theory states that all the 'standard' auctions yield the same expected revenue for the auctioneer.

Expressions of quantity and price

$$\pi_i = (P_i - w_i)x_i$$

$$\pi_s = w_i x_i + w_j x_j$$

$$x_i = a - P_i + \gamma P_j$$

$$\pi_i = (P_i - w_i)x_i$$

If both underwriters continue to face the condition where they have to decide final selling quantity rather than price:

$$x_i = a - P_i + \gamma P_j$$

$$x_j = a - P_j + \gamma P_i$$

Then

$$P_i = a + \gamma P_j - x_i$$

Substitute this expression into x_j:

$$x_j = a - P_j + \gamma(a + \gamma P_j - x_i)$$

$$P_j = \frac{a(1+\gamma) - \gamma x_i - x_j}{(1-\gamma^2)}$$

Hence

$$P_i = \frac{a(1+\gamma) - \gamma x_j - x_i}{(1-\gamma^2)}$$

Because

$$\pi_i = (P_i - w_i)x_i$$

Substitute P_i into the revenue function

$$\pi_i = [\frac{a(1+\gamma) - \gamma x_j - x_i}{(1-\gamma^2)} - w_i]x_i$$

Setting x_i as the strategic variable, and let the first order partial derivative of revenue function to 0:

$$\frac{d\pi_i}{dx_i} == \left[\frac{a(1+\gamma) - \gamma x_j - x_i}{(1-\gamma^2)} - w_i\right] - \frac{x_i}{(1-\gamma^2)} = 0$$

$$a(1+\gamma) - \gamma x_j - x_i - (1-\gamma^2)w_i - x_i = 0$$

$$x_i = \frac{1}{2}[a(1+\gamma) - \gamma x_j - (1-\gamma^2)w_i]$$

Hence

$$x_j = \frac{1}{2}[a(1+\gamma) - \gamma x_i - (1-\gamma^2)w_j]$$

Substitute the expression of x_j back to the expression of

$$a(1+\gamma) - \gamma x_j - x_i - (1-\gamma^2)w_i - x_i = 0$$

It will be:

$$a(1+\gamma) - \gamma\frac{1}{2}[a(1+\gamma) - \gamma x_i - (1-\gamma^2)w_j] - x_i - (1-\gamma^2)w_i - x_i = 0$$

$$x_i = \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_i + 2\gamma(1-\gamma^2)w_j}{(4-2\gamma^2)}$$

$$x_j = \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_j + 2\gamma(1-\gamma^2)w_i}{(4-2\gamma^2)}$$

$$P_i = \frac{a(1+\gamma) - \gamma x_j - x_i}{(1-\gamma^2)}$$

Substitute both components into the expression of P_i:

$$P_i = \frac{1}{(1-\gamma^2)}[a(1+\gamma) - \gamma\frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_j + 2\gamma(1-\gamma^2)w_i}{(4-2\gamma^2)} - \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_i + 2\gamma(1-\gamma^2)w_j}{(4-2\gamma^2)}]$$

$$P_i = \frac{a(1+\gamma)(2-\gamma-\gamma^2) + 2(1-\gamma^2)(1-\gamma)(w_i+w_j)}{(1-\gamma^2)(4-2\gamma^2)}$$

To factorise the term $(2-\gamma-\gamma^2)$ and $(1-\gamma^2)$:

$$2-\gamma-\gamma^2 = (1-\gamma)(2+\gamma)$$

$$(1-\gamma^2) = (1-\gamma)(1+\gamma)$$

$$P_i = \frac{a(1+\gamma)(2+\gamma) + 2(1+\gamma)(1-\gamma)(w_i+w_j)}{(1+\gamma)(4-2\gamma^2)}$$

Lemma 1: wholesale price in customised purchase contract

$$[\pi_i(w_i, w_j^{CP})]^{\theta}[\pi_s(w_i, w_j^{CP}) - \pi_s^M(w_j^{CP})]^{1-\theta}$$

Take the first order derivative of w_i regarding the expression and let it equals to 0:

$$\theta[\pi_i(w_i, w_j^{CP})]^{\theta-1}\frac{d\pi_i}{dw}[\pi_s(w_i, w_j^{CP}) - \pi_s^M(w_j^{CP})]^{1-\theta} + \left[\pi_i(w_i, w_j^{CP})\right]^{\theta}(1-\theta)\left[\pi_s(w_i, w_j^{CP}) - \pi_s^M(w_j^{CP})\right]^{-\theta}\left[\frac{d\pi_s}{dw} - \frac{d\pi_s^M}{dw}\right] = 0$$

After eliminating the same term with lower power

$$\theta\frac{d\pi_i}{dw}\left[\pi_s(w_i, w_j^{CP}) - \pi_s^M(w_j^{CP})\right] + \left[\pi_i(w_i, w_j^{CP})\right](1-\theta)\left[\frac{d\pi_s}{dw} - \frac{d\pi_s^M}{dw}\right] = 0$$

From the previous deduction, the equilibrium quantity has the expression

$$\hat{x}_i(w_i, w_j) = \frac{a(1+\gamma)(2-\gamma) - 2(1-\gamma^2)w_i + 2\gamma(1-\gamma^2)w_j}{(4-2\gamma^2)}$$

$$\hat{P}_i(w_i, w_j) = \frac{a(2+\gamma) + 2(1-\gamma)(w_i+w_j)}{(4-2\gamma^2)}$$

When $(w_i = w_j = w)$, the above equilibrium expression would become:

$$\hat{x}_i(w, w) = \frac{a(1+\gamma)(2-\gamma) + 2(\gamma-1)(1-\gamma^2)w}{(4-2\gamma^2)}$$

$$\hat{P}_i(w,w) = \frac{a(2+\gamma)+4(1-\gamma)w}{(4-2\gamma^2)}$$

$$\pi_i = (P-w)\hat{x}_i = \left[\frac{a(2+\gamma)+4(1-\gamma)w}{(4-2\gamma^2)} - w\right]\frac{a(1+\gamma)(2-\gamma)+2(\gamma-1)(1-\gamma^2)w}{(4-2\gamma^2)}$$
$$= \frac{a(2+\gamma)+2\gamma(\gamma-2)w}{(4-2\gamma^2)} * \frac{a(1+\gamma)(2-\gamma)+2(\gamma-1)(1-\gamma^2)w}{(4-2\gamma^2)}$$
$$= \frac{a^2(2+\gamma)(1+\gamma)(2-\gamma)-2aw(1+\gamma)(2+\gamma)+w^2[4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)]}{(4-2\gamma^2)^2}$$

$$\frac{d\pi_i}{dw} = \frac{-2a(1+\gamma)(2+\gamma)+2[4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)]w}{(4-2\gamma^2)^2}$$

$$\pi_s(w,w) = 2w\hat{x}_i = 2w\frac{a(1+\gamma)(2-\gamma)+2(\gamma-1)(1-\gamma^2)w}{(4-2\gamma^2)}$$
$$= \frac{2a(1+\gamma)(2-\gamma)w+4(\gamma-1)(1-\gamma^2)w^2}{(4-2\gamma^2)}$$

$$\pi_s^M = \frac{w(a-w)}{2}$$

$$\pi_s - \pi_s^M = \frac{2a(1+\gamma)(2-\gamma)w+4(\gamma-1)(1-\gamma^2)w^2}{(4-2\gamma^2)} - \frac{w(a-w)(2-\gamma^2)}{2(2-\gamma^2)}$$
$$= \frac{a(2+2\gamma-\gamma^2)w+[4(\gamma-1)(1-\gamma^2)+(2-\gamma^2)]w^2}{(4-2\gamma^2)}$$

$$\frac{d\pi_s}{dw} = \frac{2a(1+\gamma)(2-\gamma)+8(\gamma-1)(1-\gamma^2)w}{(4-2\gamma^2)}$$

$$\frac{d\pi_s^M}{dw} = \frac{a-2w}{2}$$

$$\frac{d\pi_s}{dw} - \frac{d\pi_s^M}{dw} = \frac{2a(1+\gamma)(2-\gamma)+8(\gamma-1)(1-\gamma^2)w}{(4-2\gamma^2)} - \frac{(a-2w)(2-\gamma^2)}{2(2-\gamma^2)}$$
$$= \frac{a(2+2\gamma-\gamma^2)+[8(\gamma-1)(1-\gamma^2)+2(2-\gamma^2)]w}{(4-2\gamma^2)}$$

Hence, substitute each part into the previous equation:

$$\theta\frac{d\pi_i}{dw}\left[\pi_s\left(w_i,w_j^{CP}\right)-\pi_s^M\left(w_j^{CP}\right)\right]+\left[\pi_i\left(w_i,w_j^{CP}\right)\right](1-\theta)\left[\frac{d\pi_s}{dw}-\frac{d\pi_s^M}{dw}\right]=0$$

Substitute each part into the above expression to solve the equation. Because the common denominator is $(4-2\gamma^2)^3$ if the final result equals to 0, this implies that

$$\theta\{-2a(1+\gamma)(2+\gamma)+[8\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)]w\}\{a(2+2\gamma-\gamma^2)w + [4(\gamma-1)(1-\gamma^2)+(2-\gamma^2)]w^2\} + (1-\theta)\{a^2(2+\gamma)(1+\gamma)(2-\gamma)-2aw(1+\gamma)(2+\gamma) + w^2[4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)]\}\{a(2+2\gamma-\gamma^2) + [8(\gamma-1)(1-\gamma^2)+2(2-\gamma^2)]w\} = 0$$

Utilising the general solution of the quadratic function, the wholesale price at the equilibrium condition is

$$w_i^{CP} = \frac{a(1+\gamma)(2+\gamma) - a\sqrt{\theta^2(1+\gamma)^2(2+\gamma)^2 + 4\theta\gamma(\gamma-2)^2(\gamma-1)(1-\gamma^2)(2+\gamma)(1+\gamma)}}{4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)\theta}$$

Standardised purchase contract negotiation:

$$\widehat{w}_j = argmax\ [\pi_j^M(w_j) - 0]^{\theta_j}[\pi_s^M(w_j)]^{1-\theta_j}$$

$$\theta_j[\pi_j^M(w_j)]^{\theta_j-1}\frac{d\pi_j^M}{dw_j}[\pi_s^M(w_j)]^{1-\theta_j} + [\pi_j^M(w_j)]^{\theta_j}(1-\theta_j)[\pi_s^M(w_j)]^{-\theta_j}\frac{d\pi_s^M(w_j)}{dw_j} = 0$$

$$\theta_j\frac{d\pi_j^M}{dw_j}\pi_s^M(w_j) + \pi_j^M(w_j)(1-\theta_j)\frac{d\pi_s^M(w_j)}{dw_j} = 0$$

$$\pi_j^M(w_j) = \frac{(a-w_j)^2}{4}$$

$$\pi_s^M(w_j) = \frac{w_j(a-w_j)}{2}$$

$$-\theta_j\frac{(a-w_j)}{2}\frac{w_j(a-w_j)}{2} + (1-\theta_j)\frac{(a-w_j)^2}{4}\frac{(a-2w_j)}{2} = 0$$

$$-\theta_j\frac{w_j}{4} + (1-\theta_j)\frac{(a-2w_j)}{8} = 0$$

$$2\theta_j w_j = (1-\theta_j)(a-2w_j)$$

$$\widehat{w}_j = \frac{a(1-\theta_j)}{2}$$

Lemma 2: wholesale price in standardised purchase contract

$$w_i^{SP} = argmax[\pi_i^{SP}(w_i,w_i)]^{\theta_i}[\pi_s^{SP}(w_i,w_i) - \pi_s^M(\widehat{w}_j)]^{1-\theta_i}$$

$$\pi_i^{SP}(w_i, w_i) = \frac{a^2(2+\gamma)(1+\gamma)(2-\gamma) - 2aw_i(1+\gamma)(2+\gamma) + w_i{}^2[4\gamma(\gamma-2)(\gamma-1)(1-\gamma^2)]}{(4-2\gamma^2)^2}$$

$$\pi_s^{SP}(w_i, w_i) = \frac{2a(1+\gamma)(2-\gamma)w_i + 4(\gamma-1)(1-\gamma^2)w_i{}^2}{(4-2\gamma^2)}$$

$$\theta_i[\pi_i^{SP}(w_i, w_i)]^{\theta_i-1}\frac{d\pi_i^{SP}}{dw_i}[\pi_s^{SP}(w_i, w_i) - \pi_s^M(\widehat{w}_j)]^{1-\theta_i} + [\pi_i^{SP}(w_i, w_i)]^{\theta_i}(1-\theta_i)[\pi_s^{SP}(w_i, w_i) - \pi_s^M(\widehat{w}_j)]^{-\theta_i}\frac{d(\pi_s^{SP} - \pi_s^M)}{dw_i} = 0$$

$$w_i^{SP} = \frac{-2\theta_i a(1+\gamma)(2-\gamma) + a\sqrt{4\theta_i(1+\gamma)^2(2-\gamma)^2 + 2\theta_i(1+\gamma)(2-\gamma)(4-2\gamma^2)(1-\theta_j{}^2)}}{8(\gamma-1)(1-\gamma^2)}$$

Appendix 4. Proof of main results in chapter 6

The comparison of L-type's profit under $(\hat{q}_L, \hat{T}_L^U)$ under the condition of $(\hat{q}_H, \hat{T}_H^U)$. The expression here T refers to the payment from the retailer to the seller, the subscription L & H indicate the low demand and high demand market condition and the letter U refers to the underwriter.

The expression of the payment to the seller is $T_i^U = R_i(\hat{q}_i) - \frac{1-\delta_s}{1-\delta_s\delta_U}\hat{\pi}_i$. The letter i here does not refer to the underwriter but the state of the market demand of high or low. Hence, the difference of revenue applying optimal low type contract in low market demand and high market demand is:

$$R_L(\hat{q}_L) - \hat{T}_L^U - [R_L(\hat{q}_H) - \hat{T}_L^H] = \frac{1-\delta_s}{1-\delta_s\delta_U}\hat{\pi}_L - \left[R_L(\hat{q}_H) - R_H(\hat{q}_H) + \frac{1-\delta_s}{1-\delta_s\delta_U}\hat{\pi}_H\right]$$
$$= R_H(\hat{q}_H) - R_L(\hat{q}_H) - \frac{1-\delta_s}{1-\delta_s\delta_U}(\hat{\pi}_H - \hat{\pi}_L)$$

Since the optimal quantity $\hat{q}_L$ is the quantity that maximizes the revenue of the underwriter, which is $R_L(q) - cq$. The letter c refers to the unit cost. Hence there is the expression of:

$$0 \le \hat{\pi}_H - \hat{\pi}_L \le R_H(\hat{q}_H) - c\hat{q}_H - [R_L(\hat{q}_H) - c\hat{q}_H] = R_H(\hat{q}_H) - R_L(\hat{q}_H)$$

Hence, it can be concluded that the above expression will be always larger than 0 this indicates that if the market demand is actually low demand the underwriter will never has the motivation to mimic the high demand condition including the patience level in the model.

Now comparing the H-type profit under $(\hat{q}_H, \hat{T}_H^U)$ with under $(\hat{q}_L, \hat{T}_L^U)$:

$$R_H(\hat{q}_H) - \hat{T}_H^U - \left[R_H(\hat{q}_L) - \hat{T}_L^U\right] = \frac{1-\delta_s}{1-\delta_s\delta_U}[\hat{\pi}_H - \hat{\pi}_L] - [R_H(\hat{q}_L) - R_L(\hat{q}_L)]$$

Hence, when $\delta_U < (>)\delta_U(\delta_S) = \frac{R_H(\hat{q}_L) - R_L(\hat{q}_L) - (1-\delta_S)(\hat{\pi}_H - \hat{\pi}_L)}{\delta_S[R_H(\hat{q}_L) - R_L(\hat{q}_L)]}$, then the H-type is better (worse) off by providing $(\hat{q}_L, \hat{T}_L^U)$.

Now replacing $\hat{T}_L^U$ by $\hat{T}_L^S$ in the above content, it can be found that the L-type earns a larger profit with $(\hat{q}_L, \hat{T}_L^S)$ than with $(\hat{q}_H, \hat{T}_H^S)$ when $\delta_U < (>)\delta_U(\delta_S) = \frac{R_H(\hat{q}_L) - R_L(\hat{q}_L)}{(1-\delta_S)(\hat{\pi}_H - \hat{\pi}_L) + \delta_S[R_H(\hat{q}_L) - R_L(\hat{q}_L)]}$

The bargaining result for any one dyad:

If the bargaining process with alternating offers has an infinite time horizon game, the this bargaining may never come to an end. But with the patience level, this seems infinite bargaining procedure may come to the end in a rather quick style.

In an equilibrium, an agreement could be reached after at most two rounds of negotiation, and

i) the underwriter will reject any offer (q, T) made by the seller with $T - cq > \frac{1-\delta_U}{1-\delta_s\delta_U}\hat{\pi}_H$;

ii) the seller will accept any offer (q, T) made by the underwriter with $T - cq \geq \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_H$ and turn down offer (q, T) with $T - cq < \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L$

To prove the part (i), suppose the $\widetilde{\omega}$ be the highest profit that the seller could earn on any equilibrium path and the associate contract is $(q_{\widetilde{\omega}}, T_{\widetilde{\omega}})$. Hence $\widetilde{\omega} = T_{\widetilde{\omega}} - cq_{\widetilde{\omega}}$. Based on the above content, $\widetilde{\omega} = T_{\widetilde{\omega}} - cq_{\widetilde{\omega}} \leq \frac{1-\delta_U}{1-\delta_s\delta_U}\hat{\pi}_H$. Assume this is not true. Then there exists some variable $x > 0$ such that any type-i underwriter must accept a contract that leads to the condition that the seller's profit being close to:

$$\widetilde{\omega} = \frac{1-\delta_U}{1-\delta_s\delta_U}\hat{\pi}_H + x$$

Let $\tilde{q}$ be the quantity that is specified in the contract and $\tilde{T} = \widetilde{\omega} + c\tilde{q}$.

Suppose this buyer underwriter deviates and counteroffers the contract $(\tilde{q}, \dot{T})$ with $\dot{T} - c\tilde{q} = \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_H + x$. Then it has:

$$\begin{aligned} R_i(\tilde{q}) - \tilde{T} - \delta_U\left[R_i(\tilde{q}) - \dot{T}\right] &= (1-\delta_U)[R_i(\tilde{q}) - c\tilde{q}] - \left[\tilde{T} - c\tilde{q} - \delta_U\left(\dot{T} - c\tilde{q}\right)\right] \\ &\leq (1-\delta_U)\hat{\pi}_H - \frac{1-\delta_U}{1-\delta_s\delta_U}\hat{\pi}_H - x + \frac{\delta_U\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_H + \delta_U x \\ &= -(1-\delta_U)x < 0 \end{aligned}$$

In the above calculation, it is clear that the buyer is strictly better off by rejecting the contract $(\tilde{q}, \tilde{T})$ and counteroffering $(\tilde{q}, \dot{T})$.

$$\dot{T} - c\tilde{q} - \delta_s(\tilde{T} - c\tilde{q}) = \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_H + x - \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_H - \delta_s x = (1-\delta_s)x > 0$$

This means the seller should accept the counteroffer from the underwriter. This would be contradicted to the assumption that some type-i underwriter accepts the contract $(\tilde{q}, \tilde{T})$.

To testify the second part of (ii), it is worth noting that it is true if the underwriter reveals his type of market demand. In this case, the seller will reject the underwriter's offer if the contract proposed yields a lower profit compared to the first equilibrium situation. Hence, what should be considered here is the case that when the underwriter's offer can not reveal any information of the market demand.

In previous content, it has been shown that any equilibrium contract (q, T), if the seller deems it to be acceptable, then it has to satisfy the $\hat{q}_L \leq q \leq \hat{q}_H$. Now given that contract (q, T) is acceptable, then the seller should accept any other offer $(q^{'}, T^{'})$ with $T^{'} - cq^{'} \leq T - cq$ such that $(q^{'}, T^{'})$ does not reveal the type of the underwriter. For any $q < \hat{q}_L$, $R_i(q) - cq \leq R_i(\hat{q}_L) - c\hat{q}_L$. Under this condition, both underwriter types are at least better off offering $(\hat{q}_L, T - c(q - \hat{q}_L))$ instead of (q, T) while keep their type unrevealed. Hence $q \geq \hat{q}_L$ and likewise, this method can be used to prove that $q \leq \hat{q}_H$.

Since the contract (q, T) does not reveal the buyer's type, the conditions for revealing the type of information (q^S, T^S) implies that first there is no contract will yields a higher profit for the L-type and a lower profit for the H-type and the second there is no contract that yield a higher profit for the L-type, a higher profit for the seller, and a lower profit for the H-type, than (q, T) does.

These conditions imply that $q \leq \hat{q}_L$. Suppose $q > \hat{q}_L$. Then it can be constructed a contract (q^0, T^0) such that it equals to $(\hat{q}_L, T - c(q - \hat{q}_L))$ hence that the seller is indifferent between the new contract and (q, T). For the L-type underwriter, $R_L(q^0) - T^0 = R_L(\hat{q}_L) - c\hat{q}_L - T + cq > R_L(q) - T$ because $\hat{q}_L$ maximizes the total supply chain profit under the L signal. For the H type underwriter, $R_H(q^0) - T^0 = R_H(\hat{q}_L) - c\hat{q}_L - T + cq < R_H(q) - T$ because the total supply chain profit decreases as the q decreases from $\hat{q}_H$ to $\hat{q}_L$. Hence, the contract (q^0, T^0) satisfies the condition in the second condition mentioned in previous paragraph. Together with previous paragraphs, it can be concluded that if the underwriter's offer does not reveal the type, the contract quantity must be $q = \hat{q}_L$.

Now the proof of the seller will reject any contract that offers the profit strictly less than $\frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L$ in equilibrium. Following the same rhythm as above, first suppose it is not true and the seller accepts a offer $(\hat{q}_L, T)$ with $\overline{\omega} = T - c\hat{q}_L < \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L$. Consider the situation that the seller deviates and counteroffers the contract $(\hat{q}_L, \overline{\omega} + c\hat{q}_L)$ with $\tilde{\overline{\omega}} = \frac{(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L - x$ and x satisfies:

$$x < \frac{(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L - \frac{\overline{\omega}}{\delta_s}$$

Then $\overline{\omega} < \delta_s\tilde{\overline{\omega}}$ and thus the seller would make more profit if this counteroffer is accepted by the underwriter.

Then the i type underwriter is willing to accept this counteroffer if it yields a larger underwriter profit:

$$0 < R_i(\hat{q}_L) - (\tilde{\overline{\omega}} + c\hat{q}_L) - \delta_U[R_i(\hat{q}_L) - (\overline{\omega} + c\hat{q}_L)] = (1-\delta_U)[R_i(\hat{q}_L) - c\hat{q}_L] - \tilde{\overline{\omega}} + \delta_U\overline{\omega}$$

Since $R_L(\hat{q}_L) < R_H(\hat{q}_L)$, the H-type would accept the counteroffer whenever it is acceptable to the L-type underwriter. While for the L-type underwriter to accept the offer, it must have

$$x > \frac{\delta_s\delta_U(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L - \delta_U\overline{\omega}$$

Hence, as long as there exist a x that satisfying the above conditions, then the seller will deviate form accepting the offer $(\hat{q}_L, \overline{\omega} + c\hat{q}_L)$. The conditions imply that:

$$\frac{\delta_s\delta_U(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L - \delta_U\overline{\omega} < x < \frac{(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L - \frac{\overline{\omega}}{\delta_s}$$

Or $\overline{\omega} < \frac{\delta_s(1-\delta_U)}{1-\delta_s\delta_U}\hat{\pi}_L$, which will always hold under the hypothesis. Hence, the second part of the lemma has been proved.