THE CHARACTERIZATION OF UNSATURATED SOIL BEHAVIOUR FROM PENETROMETER PERFORMANCE AND THE CRITICAL STATE CONCEPT

Volume 1 of 2

BY

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ABSTRACT

The proportions of the Critical State surfaces of a given soil depend on its stress and moisture history, its current moisture status and soil type. The influence of these factors on the geometry of the state surfaces, and their bearing on the mechanical behaviour of the soil itself, cannot be readily perceived because of the very large number of interacting effects present in the model. The use of sophisticated computer graphics for the three-dimensional visualization of critical state surfaces would therefore provide a ready means for obtaining an insight into the complex changes taking place in critical state space. The initial part of the thesis deals with the formulation of mathematical models and computer software for integrating numerical computations, data reduction and visualization techniques for analysing critical state surfaces. The programmes developed were used for interpreting the influence of moisture status on the key behavioural patterns of three different British Soils. Systematic changes to state space due to the combined influence of moisture content variations and soil type were readily traced.

It is well known that collating data for the above analysis is difficult and requires advanced measuring techniques. An attempt was therefore made to establish a connection between the data obtained from a simple field measuring device, such as a cone penetrometer, with the volume-change behaviour of soil, as modelled by its critical state surfaces. This is attempted in two stages. The first stage presented in the thesis assumes the soil to be a rigid-plastic Mohr-Coulomb material and deals with the formulation of a mathematical model to predict cone index as a function of cone geometry, penetration depth, c, φ and soil-to-metal parameters c_a and δ. This model is based on the extension of the basic two-dimensional Sokolovski solution to the three-dimensional slip-line field developed during the deep penetration phase of a cone. Shallow penetration depths, at which the standard Sokolovski rupture surface interacts with the soil surface, cannot be dealt with by this approach.

The second stage of the investigation attempts to connect cone index with the stress (p, q) and pore space (v) parameters of the soil on the cone surface. The model developed in the thesis is based on identifying a *state parameter* ψ (defined by Been and Jefferies for dry sands). This establishes the position, within critical state space, of the cone surface stress and pore space parameter (p, q and v) relative to the critical state wall. The state parameter ψ is then associated with soil type and moisture

content by a two-parameter linear function. Once these two parameters are found experimentally, cone index can be readily translated into pore-space estimates. The thesis presents the mathematical analysis which provides the basis for this correlation.

The thesis describes the experimental investigations carried out to verify the performance of the theoretical models developed. The validation of the state parameter concept required the design and development of a special calibration chamber which could apply controlled boundary stresses to a cylindrical soil sample into which the penetrometer is advanced. Ideally very large sample diameters are required to minimise boundary interference, but a compromise had to be made by using miniature penetrometers and a realistic sample diameter of 100 mm. The cone penetrometer performance model was tested under laboratory conditions in an indoor soil tank. Both these investigations required tedious back-up laboratory experimentation to establish the basic Mohr-Coulomb and critical state parameters of the test soil over a wide range of moisture contents. All the soils were dealt with in a remoulded state as consistently reproducible stress and moisture histories for this case can be easily maintained in each of the very large number of samples required in the experimental programmes.

The experimental work shows very clearly that the state parameter concept is applicable to partly saturated c-φ soils over a wide range of moisture contents and that it is possible to quantify the systematic changes in the state parameter ψ with soil moisture content. The predictive performance of the cone penetrometer model, within the specified penetration range, was also good. Data reduction charts for interlinking these two models are presented and the use of these charts for the derivation of pore space particulars from cone index data predicted satisfactory trends. However, this procedure appears to over-predict dry bulk density by a considerable margin.

The validation presented in this study is for a single sandy loam soil. Even though the overall predictive performance of the mathematical models in this particular soil is most encouraging, it should be borne in mind that the models developed are bound to be influenced by the drastic simplifications required to interlink two disparate models, one which ignores volume change with one which does not. Further work is required to remove any detrimental consequences of these compromises and to introduce confidence in extending the findings to other soil types.

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CHAPTER - ONE

INTRODUCTION

1.1 Mechanical Behaviour of Soils

There are several recent developments which have helped to advance our understanding of the complex processes involved in soil machine interactions. Of these, two stand out clearly as having made the most significant impact on the discipline of Soil-Machine Mechanics. These are:

(a) Sokolovski's solution (1960) of the basic partial differential equations of static equilibrium of soil mass, and

(b) The development of the Cambridge Critical State concept (Roscoe *et al.,* 1958).

The former is a purely theoretical application of a branch of higher Mathematics to a Soil Mechanics problems. The latter, though based on advanced Plasticity Theory, is essentially a semi-empirical technique which relies on intricate experimental investigations. Both these developments are of such complexity as to make their application to any but the simplest field conditions, clearly impracticable. Evidently significant improvements to the functional capabilities of these powerful techniques can be made with the help of modern computers. The present investigation is a preliminary attempt to interlink the best features of these two mathematical models in a computer simulation of the performance of cone penetrometers. This should provide the basis for the ready analysis of cone index data as a potential method for characterizing the volume change behaviour of agricultural soils.

1.2 Reconciling Conflicting Requirements

It should be borne in mind that the alliance between the Critical State concept and the Sokolovski solution proposed in the previous paragraph cannot be achieved without introducing certain inconsistencies. These difficulties have to be countenanced mainly in the interest of simplicity. The acceptance of these conflicts can be fully justified if the predictive performance of the resulting models are within acceptable limits.

Sokolovski's analysis, which can be classified as a Limit Equilibrium method, is one of three theoretical techniques currently available for evaluating soil reactions on machine elements. The other two are the Limit Analysis and the Finite Element Methods. Each of these methods has their own relative advantages and disadvantages. On balance, the Sokolovski solution, particularly the simplified version (referred to as the "Newcastle method"), presents a solution which has the following advantages.

(a) the method provides the user with a clear insight into the mechanics of rupture surface development.
(b) it leads to statically admissible slip-line fields which are readily quantifiable, and
(c) it provides acceptable stability solutions to quasi-static problems and is based on a simple two-parameter Mohr-Coulomb failure criterion ($\tau = c + \sigma \tan \varphi$).

A drawback of the Sokolovski solution is that it deals exclusively with two-dimensional stress fields and this requirement is somewhat restrictive as it cannot be applied directly to the soil failure mode induced by cone penetrometers. The solution applies to rigid-plastic materials which fail according to the Mohr-Coulomb failure criterion. Consequently it takes no account of the intermediate principal stress, nor does it does allow for any volume-change phenomena. These latter factors are obviously not compatible with the Critical State model which does specify a special condition for the intermediate principal stress ($\sigma_2 = \sigma_3$) and allows for all changes in pore space during loading.

Turning next to the Critical State model, this was originally developed for describing the mechanical behaviour of **saturated** soils. There is a wealth of recent evidence that the model can be readily adapted to deal with partly saturated soils (Hettiaratchi, 1987; Kirby, 1989; Petersen, 1993; O'Sullivan *et al.,* 1994). It is the only satisfactory model currently available for predicting the volume-change behaviour of such soils. It is essentially a 7-parameter model when dealing with unsaturated soils and the experimental evaluation of these parameters presents considerable practical difficulties. Additionally, these parameters are also functions both of the moisture and stress history of the soil. In the present experimental investigations the soil is dealt with as a re-moulded material mainly because reproducible initial conditions can be readily established in such laboratory samples. The conclusions drawn from both the theoretical and experimental work presented here are, therefore, applicable only to soils approaching this state in the field. It is not inconceivable that the techniques developed could be extended to other field conditions such as cemented states (Hatibu, 1987).

1.3 Scope of Current Study

The basic objective of the present study is to link the best available theoretical and semi-empirical mathematical models, discussed earlier, into a composite model of a simple field measuring device, such as a cone penetrometer. There is a specific need to quantify the performance of this simple and effective instrument so that it can be used as a useful adjunct for the characterization of the volume-change behaviour of soils.

The basic objectives of the present study break down into following categories:

(a) ***Visual Graphics of Critical State Surfaces***

Research workers experience little difficulty in associating the physical shape of the critical state space of a given soil with its mechanical behaviour. However, a recurring practical problem is the difficulty in visualizing the precise influence of the Critical State parameters on the actual three-dimensional geometry of Critical State space itself. These changes in geometry are of particular significance when assessing the influence of moisture regimes in partly-saturated soils.

The initial part of this study is directed at using computer graphic techniques for plotting the Critical State space for any given soil from a known set of parameters. It is then a simple matter to inspect the resulting three-dimensional image (either on a VDU screen or on a paper plot) from any convenient viewpoint by suitable rotation of axes. A logical extension of this part of the work is to trace state paths on and within this surface. The original intention was to plot, within Critical State space, the state paths of soil elements lying on the surface of a cone penetrometer. However, this extension could not be made in the time available.

(b) ***Prediction of Cone Penetrometer Performance***

The Sokolovski analysis has been used in previous investigations to evaluate the failure load on cone and wedge indenters (Houlsby, 1982; Liang, 1986; Hoque, 1991). These studies are essentially of "drop-cone" tests where the top rim of the cone does not penetrate below the soil surface. In this event the geometry of the contact surface is independent of the penetration depth.

In a cone penetrometer test the cone rim is forced below the soil surface and consequently the boundary geometry changes as penetration proceeds. Practical two-dimensional problems with such boundary conditions have been investigated using a "non-singular point" Sokolovski solution (Witney, 1966; Albuquerque, 1975; Sarker, 1984). The current investigation also takes these changes into account with the proviso that the soil surface does not interfere with the standard Sokolovski rupture boundaries. In effect this places a minimum penetration limit below which the present analysis is not valid. The penetration depth range for cone penetrometers having a projected area of 320 mm^2 lies well beyond this limit and the analysis presented is therefore of practical significance.

(c) *Linking Penetrometer Performance with Critical State*

In order to establish a link between objectives (a) and (b) it is necessary to introduce a recent development which identifies a "*State parameter*" in Critical State space (Been and Jefferies, 1985). This parameter defines the stress and specific volume states of soil elements on the cone surface relative to the Critical State Wall. So far this concept has been established for dry sands only. Once this state parameter is defined, then the volume change behaviour of the soil in the neighbourhood of the cone surface can be characterized. The present study investigates this concept with a view to associating cone index with the pore-space regime of the soil. The basic objective is to remove the current empiricism associated with the interpretation of cone index data.

(d) *Design and Development of a Calibration Chamber*

A suitable calibration chamber is required to validate the state parameter concept outlined in objective (c) above for unsaturated c-φ soils. Ideally, soil samples of very large physical size (1.0 m dia.) are required to eliminate boundary effects. In the present context this is impracticable and as a compromise a miniaturized cone penetrometer calibration rig was designed and developed by adapting an existing triaxial compression machine.

(e) *Experimental Validation of Models*

The performance of the mathematical models developed are evaluated under laboratory conditions. This validation exercise necessitates the following extensive laboratory investigations.

(i) a programme of triaxial compression tests for the evaluation of the variation of the Critical State parameters with moisture content of remoulded soil samples;
(ii) evaluation of soil-metal properties required for defining cone surface parameters;
(iii) estimation of standard moisture-tension curves of the soils used in the experiments together with their mechanical composition;
(iv) calibration chamber tests to validate the state parameter concept for a partly saturated loam soil;
(v) a penetrometer test in an indoor soil tank to simulate the field performance of the penetrometer.

Actual field trials were avoided because of the difficulty in estimating the relevant parameters.

CHAPTER - TWO

REVIEW OF THE WORK RELATED TO THE PRESENT INVESTIGATION

2.1 Critical state model

The critical-state framework originally developed from plasticity theory for saturated soils (Roscoe *et al.*, 1958) offers a theoretical basis for predicting not only the volume change behaviour but also the shear deformation taking place during triaxial compression. Although this powerful conceptual model of soil behaviour has been found acceptable with many civil engineers it was not widely used before the last two decades by those concerned with soil deformation by agricultural vehicles and implements. One main reason is that civil engineers are concerned with saturated soils, whereas agricultural engineers are interested in unsaturated soils, representative of the condition encountered in agricultural practice.

The relevance of the concept to agricultural soils has been reviewed by Kurtay and Reece (1970), Reece (1977) and Hettiaratchi & O'Callaghan (1980) and they suggested that the theory can be applied to the study of compaction and tillage problems in agricultural practice. The preliminary investigations by Potamias (1976) and Hettiaratchi & O'Callaghan (1980) have indicated that the theory can be extended to include unsaturated soils. There is now available a substantial body of experimental evidence to show that the basic concepts of the model are quite applicable to partly saturated soils (Leeson and Campbell, 1983; Hettiaratchi and O'Callaghan, 1985; Hettiaratchi, 1987; Hatibu and Hettiaratchi, 1986; Kirby, 1989; Toll, 1990; Wheeler and Sivakumar, 1992, 1995).

McKyes (1986, p.33) stated that the concept is not applicable to agricultural soils because it was developed specially for saturated soils. Kirby (1989) pointed out that there are difficulties associated with the use of effective stresses in unsaturated soils. Towner (1983) raised a question that the concept might be difficult, if not impossible, to apply quantitatively because of the difficulties with quantifying the effective stresses and he suggested to develop critical state parameters on the basis of effective stress. It is indeed essential in case of saturated soils, but for unsaturated soils Hettiaratchi and O'Callaghan (1985) have justified that it is perfectly valid to work in terms of total stress and the experimental work was reported on this basis (Hettiaratchi, 1987).

Quantitative agreement between laboratory shear and compression tests and the critical state description of soil deformation was demonstrated by the major contribution by the Newcastle group described by Hettiaratchi (1987) and the recent work by Kirby (1989, 1991) and Bakker & Harris (1992) and more recent painstaking observation by Petersen (1993, 1994).

This incremental constitutive model provides the agricultural engineers and soil scientists with a powerful tool for analysing and understanding many aspects of field soil behaviour (Hettiaratchi, 1990). The main difficulties to the application of this model to practical situations is the need for elaborate experimental investigation to determine the critical state parameters over the normal range of field moisture contents.

Kirby (1989, 1991) avoided some of the measurement difficulties by using a direct shear testing device. The lateral stresses imposed on the soil by these devices were ignored and the model was accordingly restated in terms of normal stress and shear stress. This simplification make the critical state model more usable because the tests are easily carried out with standard equipment. However, ignoring the lateral stresses leads to a lack of generality in the results. A further problem is that the results are determined to some extent by the geometry of equipment. The model has general applicability when expressed in terms of spherical stress (p) and deviatoric stress (q). As a prelude to applying the critical state more widely, Hettiaratchi *et al.* (1992) simplified triaxial testing by developing a constant cell volume (CCV) triaxial apparatus. A meticulous experimental investigation was carried out by O'Sullivan *et al.* (1994) for deriving the critical state parameter from this CCV triaxial apparatus.

2.2 State parameter

The basic idea developed in recent under sea penetrometer studies is concerned with the definition of a 'state parameter' designated by the symbol ψ. This concept of state parameter was developed by Been and Jefferies (1985) in relation to critical state of sands. It is regarded as important because it addresses the question of how to characterise sand behaviour. This parameter basically embodies a combination of void ratio, ambient stress level and orientation relative to the critical state line.

Been and Jefferies (1985) postulated from extensive testing on sand that its bulk characteristics are not sufficient to predict the mechanical behaviour of granular materials. Similar evidence can be found in other published studies of sand properties (e.g. Lee, 1965; Lade, 1972). In particular, confining pressure modifies material behaviour of sands to the point that even dense sand, if tested at sufficiently high confining pressure, will behave similar to loose sand. Therefore, properties of sands cannot be expressed in terms of relative density alone; a description of stress level should also be included.

One of the exciting developments from the sand testing in the recent years was the state parameter concept. This fundamental physical concept has wide applicability both as an empirical normalising parameter and for constitutive modelling of soil behaviour. The commonly used sand behaviour models normalise rather well to the state parameter which is the utility of the concept to the practising engineers. The state parameter incorporates information which orients the ambient stress and current specific volume relative to the critical state wall. Hence, this parameter is a useful single index of the conditions required to bring the soil element to failure (i.e. its strength).

Sladen *et al.* (1985) have shown that the behaviour of very loose, potentially liquefiable sands, including undrained brittle index, can be rationalised by considering initial state in relation to the critical state line. Because of the importance of the state parameter to sand behaviour, the possibility of correlating it directly to CPT (Cone Penetration Test) tip resistance is of considerable interest and has been explored by Been and his co-workers in a series of papers (Been *et al.*, 1986; Been *et al.*, 1987a; Been *et al.*, 1987b). They have proposed that for a given sand, normalised tip resistance is a unique function of state parameter.

2.3 Cone penetrometer

Penetrometers consist of any device that can be forced into the soil and its resistance to penetration can be measured. A wide variety of such instruments has been developed to measure either static penetration (when penetrometer is pushed into the soil at a constant rate) or dynamic penetration resistance (when it is driven by a series of blows). The penetration resistance so measured has been used as an index of a wide range of soil physical and mechanical properties in empirical soil studies in civil engineering, vehicle mobility, and agricultural soil mechanics.

Perhaps the most frequent use of penetrometers is to assess soil strength. Recognising the need to standardise both the apparatus and test procedure where empirical tests are concerned, the ASAE (1969) have specified penetrometer with 30^0 conical tip either 20.27 mm diameter or 12.83 mm diameter which has become widely used to give an index of soil strength from static penetration tests. A 3.0% decrease in diameter is allowed for cone wear. The diameter of the shafts are 15.88 mm for 20.27 mm diameter cone and 9.53 mm for 12.83 mm diameter cone. The cone index is measured as the force per unit base area required to penetrate this cone into the soil at the rate of 30 mm/s.

It is well known that the adhesion of soil to the penetrometer shaft and friction between shaft and soil both act to increase cone resistance. If the shaft diameter is small, compared with that of the cone, movement of soil into the cavity behind the cone may decrease cone resistance by relieving the pressure on the cone face (Freitag, 1968). Interaction between soil and shaft can have a significant effect on a standard ASAE penetrometer, specially in wet clay (Mulqueen *et al.,* 1977; Freitag, 1968). Freitag (1968) found that the cone resistance of a clay was approximately constant with depth when a 20.27 mm diameter cone was used with a small (9.53 mm) diameter shaft, but increased with depth below 130 mm to three times its original value at 300 mm depth when standard (15.88 mm) diameter shaft was used. An 8.0 mm diameter shaft was used with a 20.27 mm diameter cone by Reece and Peca (1981) to eliminate the clay-shaft interaction. Reece and Peca (1981) suggested that the 8.0 mm diameter shaft be adopted as standard. The difference in radius between the cone (6.4 mm) and the shaft (4.8 mm) on the smaller ASAE penetrometer is only 1.6 mm, decreasing to 1.5 mm at maximum acceptable cone wear. Thus, a reduction in standard shaft diameter may be desirable to avoid problems of soil-shaft interaction. However, a very small diameter shaft may not withstand the stresses imposed on it without bending.

The cone penetrometer has been employed for various applications, including prediction of the tractive capability of an off-road vehicle (Freitag and Richardson, 1968; Wismer and Luth, 1973); characterisation of soils in terms of crop growing ability (Raghavan and McKyes, 1977); determination of resistance to root penetration and seedling emergence (Bowen, 1976; Taylor and Gardner, 1963; Morton and Buchele, 1960); prediction of draft force (Johnson *et al.*, 1978; Gill and Vanden

Berg, 1967) and assessment of compaction caused by vehicle traffic (Soane, 1973; Raghavan and MeKyes, 1977; Chesness *et al.*, 1972). Cone indices were also used to determine the effect of wheel size and vehicle weight on soil compaction, as well as earth embankments and foundations.

The technique of indentation, basically developed for testing metals, has been employed to penetrate soil to measure different soil properties. A variety of devices are used in soil engineering practices. The drop cone penetrometer has been evaluated recently for field use (Bradford and Grossman, 1982) which measures deformation in response to applied loads (Scholefield and Hall, 1986). Hansbo (1957) undertook the drop-cone tests on remoulded clay in the laboratory for determining undrained shear strength of soil. Towner (1973) compared the measurements of unconfined shear strength and drop-cone penetration over a range of moisture contents on seven remoulded agricultural soils and he found that it is specific to soil type. He suggested that drop-cone apparatus could be used to derive such limit by assuming that the liquid and plastic limits of a soil correspond to two fixed strengths. Sherwood and Ryley (1970) proposed to use drop-cone penetrometer method as an alternative to the Casagrande device for determining the liquid limit to correspond to a standard penetration of 20 mm after 5 seconds of release, which has been adopted as a definitive method of liquid limit determination (*BS 1377*:1975).

Campbell (1975) stated that drop cone penetrometer method for liquid limit determination is more reproducible and easy to conduct than the Casagrande method. Campbell (1976) showed that the drop cone apparatus could also be used to determine the plastic limit of soil giving more reproducible results than the tests done by the Casagrande apparatus. In predicting soil behaviour and its classification, plasticity index is more acceptable than the plastic limit and the drop cone has the capacity to measure both plastic limit and the liquid limit simultaneously which saves about 50% of the time period required to complete the same by Casagrande apparatus (Campbell, 1976). Wood in his subsequent two papers (1982, 1983) elucidated the way the fall cone test can be used to deduce information concerning the compressibility and water content relationships for soils.

The theoretical analysis of Houlsby (1982) examined the various factors, such as effects of cone angle, bluntness of cone, roughness of cone, heave formed around the cone, size of the container and motion of the cone during indentation and it was found that the single most important factors affecting liquid limit is cone roughness.

Queiroz de Carvelho (1986) tested soils for liquid limit using the cone penetrometer and Casagrande method, in accordance with British Standard (*BS 1377*:1975) and found slightly better results with the cone penetrometer method. The frequent use of cone penetrometer is to characterise the soil strength. It measures shear strength easily, rapidly and is widely used for assessment of the compacting and loosening effect of agricultural machines (Soane *et al.*, 1972). Cone Index , penetration resistance force per unit base area of the cone, provides the relative indications of soil strength conditions (Smith and Dumas, 1978). The cone penetrometer is widely used in tillage and off-road mobility research as an indicator of soil strength and density characteristics (Wells *et al.*, 1981).

Knight and Freitag (1962) first suggested that the cone penetrometer can be used as a means of evaluating surface strength trafficability related to the mobility of certain military vehicles and described a corresponding soil cone index. Threadgill (1982) used cone indices to measure the soil compaction. Turnage (1972) showed that in frictional soil the rate of increase of cone index is a measure of its density. Mulqueen *et al.* (1977) extended his experiments in sandy and clayey soils with blunt and sharp cone penetrometer probe to relate penetration resistance to bulk density and cohesion over a range of moisture contents. The effect of bulk density, moisture content and soil type on cone index was investigated and discussed by Ayers and Perumpral (1982). They found some relationships among bulk density, moisture content and soil types. Ayers (1980) also found that the cone index and dry density relationship was independent of moisture content for pure sand.

The quantitative and detailed interpretation of the results of cone penetrometer tests has not yet been achieved. Mulqueen *et al.* (1977) suggested the use of penetrometers for comparing the relative strength of soils under conditions of similar moisture content and structural state only.

O'Sullivan and Ball (1982) compared the performance of five instruments, torsional shear box, shear vane, cone penetrometer, drop-cone penetrometer and pocket penetrometer for measuring soil strength at several depths less than 150 mm both in cultivated and uncultivated general seedbeds and found that each of these instruments had its own merits over particular soil properties. The development of recording penetrometers have been proved to be useful in evaluating in-situ soil strength (Carter, 1967). The recent electronic developments greatly enhanced the accuracy of the results. Prather *et al.* (1970) developed a hand operated recording penetrometer which can be operated reliably and provides an accurate measurement of force versus depth and can be easily operated by one man. Soane *et al.* (1972) developed a hand held penetrometer which is mostly used in Britain for field studies.

Spencer *et al.* (1977), Smith and Dumas (1978) and Anderson *et al.* (1980) contributed by developing recording penetrometer separately. The main distinctive features of these penetrometers are that the data (penetration depth and its corresponding force) are recorded on magnetic tapes, analysed by computer for statistical processor and / or transcribed by printer in the form of diagrams showing the key variation of the penetration resistance recorded on a vertical plane. Penetrometer advancements have resulted in greater convenience and time efficiency relative to the collection and analysis of soil penetrometer data (Wells *et al.*, 1981).

Besides the studies on top soil, cone penetration tests are carried out for geotechnical and terrain investigations into various soils and rocks. The principle of operation is pushing of an instrumented cone tip into soil and simultaneous measurement of cone tip resistance. It can be used in making a detailed profile of soil properties either in sands or clays (Houlsby and Withers, 1988) and is becoming a more popular test for site investigation and geotechnical design (Ismayel and Jeragh, 1986).

The existing technique for site investigation for interpreting cone penetration tests is based on the stress measurement in large diameter calibration chamber test correlating it with the behavioural properties obtain from laboratory test (Been *et al.*, 1986). Lunne and Eide (1978) suggested a method to correlate between cone resistance and vane shear strength for the determination of the shear strength from a cone penetration test. Sladen (1989) stated that unique relationships exist between sand void ratio, vertical effective stress, cone tip resistance for normally consolidated sand. Dr. Meigh stated that undrained shear strength of a cohesive soil can be obtained from cone penetration tests (Holden and Pang, 1987).

The standard penetration test (SPT) is another in situ test used for site investigation. The parameter actually measured by this test is the number of standard blows (SPT N) necessary to advance a predetermined sampler 300 mm into a soil at the bottom of a bore hole. This analysis of this test is entirely empirical (Johnson, 1983). de Mello (1971) postulated that the SPT N value is predominantly a function of shear strength. The general review of standard penetration tests are given by Nixon (1982), Johnston (1983) and Fletcher (1965), and that of the cone penetration test by Ruiter (1982) and Johnston (1983) and are discussed by Holden and Pang (1987).

The two modern in situ tests are self boring pressuremeter and the piezo-cone tests. The pressuremeter test was first introduced by Menard (1955) which can measure the strength and stiffness of soils and rocks. Houlsby and Hitchman (1988) presented an analytical solution to the undrained shear strength and shear modulus from the full displacement pressuremeter test. This test is expensive and sophisticated one and there is an uncertainty of obtaining overestimated values (Wroth, 1984). The piezocone measures both the mechanical resistance and induced pore pressure near the tip during penetration into the soil. Konrad (1987) developed an interpretation technique for piezo-cone penetrometer results in order to obtain undrained shear strength in cohesive soils.

The interpretation of data obtained from in situ tests is difficult and for most test it is both incomplete and imprecise (Wroth, 1984). Wroth (1984) emphasised the difficulties of the interpretation of in situ testing observations. The mode of these difficulties are due to the complex behaviour of soils together with the lack of control and of choice of boundary conditions in any field test. Instrumental error may disturb the in situ test which is worse in the field than in the laboratory. On the other hand, the development of the electronic computer has enabled to analyse complex solution to resolve the soil boundary problems by numerical methods resulting in obtaining more in situ test.

2.4 Theoretical prediction of cone performance

In spite of the wide use of cone penetrometers in studies, remarkably little work has been done on the theoretical basis of the operation. Civil engineers have drawn their attention to the development of bearing capacity theories for the limiting case of a foundation resting on the surface of the soil and only two major attempts have been made to consider sinkage of the foundation (Meyerhof, 1951; Balla, 1962). Both these theories suffers from serious deficiencies which preclude their application in mobility theories (Witney *et al.*, 1966).

There are two approaches for obtaining engineering properties from the results of cone penetration testing; either through empirical correlation or theoretically using an appropriate constitutive model in a numerical (or analytical) solution of the particular boundary value problem (Been *et al.*, 1986). Konrad and

Law (1987) reviewed the theories relating unit resistance (q) to the undrained shear strength (S_u) for incompressible fully saturated soils. Konrad and Law (1987) categorized most of the studies into the interpretation of cone penetration tests as follows.

(a) the classical plasticity approach to bearing capacity of deep foundation;
(b) cavity expansion theory combined with classical plasticity;
(c) principle of conservation of energy with cavity expansion;
(d) analytical and numerical approaches using linear and non-linear stress-strain relationships;
(e) empirical studies attempting to relate cone resistance to undrained shear strength derived from different tests and some cases, to additional soil index parameters.

Hansbo (1957) studied the region of failure created around the cone when dropped into a clay both theoretically and experimentally. He related the depth of cone penetration, h, and the undrained shear strength τ_f of clay in the following way:

$$\tau_f = \frac{\mathrm{K}\,Q}{\mathrm{h}^2}, \tag{2.1}$$

where Q is the weight of the cone and K is a constant whose magnitude depends upon the cone angle.

Similar expression for the same was given by Wood and Wroth (1978). On the basis of dimensional analysis the penetration depth d and the undrained shear strength C_u of soil, in penetration tests using a cone of weight W and different moisture contents, are connected through the equation (Hansbo, 1957; Wood and Wroth, 1978):

$$\frac{C_u\, d^2}{W} = \mathrm{K}, \tag{2.2}$$

where K is a constant. Hansbo (1957) suggested the value of K = 1.2 for 30° cones. Houlsby's (1982) analyses suggest that a value of K of 0.96 might be appropriate for a perfectly smooth cone, or 0.51 for a perfectly rough cone. Gardiner (1982) used a miniature remoulding vane to estimate shear strengths and suggests that a value of about 1.0 may be reasonable.

Houlsby (1982) presented a theoretical analysis of the fall cone test and a direct evaluation of the undrained strength at the liquid limit. The solution is made using the lower bound theorem of plasticity theory. The lower bound analysis of the static case involves calculation of the load at a particular penetration of the load and the effects of change of geometry need not to be considered. The geometry of the cone is described simply by the two variables; the cone angle and the depth of penetration h. The vertical load exerted by the cone on the soil is P and properties of soil are given by the undrained shear strength C_u and the bulk unit weight γ, and the surface properties of the cone are specified by adhesive value a_u which specifies maximum allowable shear stress on the surface. A simple dimensionless analysis showed that the load on the cone must be given by an expression of the form:

$$\frac{P}{C_u h^2} = f\left(\frac{\gamma h}{C_u}, \frac{a_u}{C_u}, \alpha\right). \tag{2.3}$$

The analysis is carried out in terms of total stresses as the indentation is assumed to take place under undrained conditions, the yield criterion is assumed to take the Tresca form (σ_{max} - σ_{min}) = $2C_u$ and the Haar Karman hypothesis is adopted for axial symmetric deformation i.e. hoop stress is equal to one of the principal stresses.

Durgunoglu and Mitchell (1973) presented a solution with the penetration or ultimate tip resistance, Q, as follows:

$$Q = c\,S_c\,N_c + B\,\gamma\,N_{\gamma q}\,S_{\gamma q}, \tag{2.4}$$

where B is the penetrometer base width or diameter, γ is the soil unit weight, N_c and $N_{\gamma q}$ are bearing capacity factors and S_c and $S_{\gamma q}$ are shape factors. Durgunoglu (1972) presented the solutions for N_c and $N_{\gamma q}$ with shape factors S_c and $S_{\gamma q}$. The penetration resistance is defined in the model to be a function of cone geometry, penetration depth, soil unit weight, soil-metal friction ratio and the soil strength parameter, cohesion c and angle of internal friction φ. Ayers and Bowen (1987) adapted this analysis to model the soil failure mechanism during cone penetration to predict the soil bulk density and soil moisture profiles. Firstly, they determined the c and φ by torsional shear tests at different dry densities and moisture contents. The relationships of these values c and φ dry densities and moisture contents are considered as input parameters into a computer programme to determine the bulk density of soils at different depths of penetrations. Durgunoglu and Mitchell (1975), Janbu and Seneset (1974), Vesic (1972) and Baligh & Scott (1976) have also used various theories to develop relations between the friction angle and cone tip resistance.

Cavity expansion theory have been used to solve problems related to foundation and bearing capacity of soils (Gibson, 1950; Ladanyi, 1963) and adapted to simulate the deformation during cone penetration (Ladanyi, 1967 and Vesic, 1972).

Farrell and Greacen (1966), dissatisfied with the use of soil mechanics theory of the bearing capacity of piles to explain cone penetration, developed a new theory for predicting soil moisture to penetration by roots and cones. Their treatment allowed not only for soil cohesion and internal friction but also for soil compressibility, which is clearly important in agricultural soils but may be irrelevant during pile driving in soft, saturated, cohesive soils. Their theory is based on a model in which a spherical cavity is created in the soil at the penetrometer tip as penetration takes place. A homogeneous, isotropic soil is assumed, with no allowance being made for soil structure.

The spherical cavity is created by two types of soil deformation, each of which occurs in a soil distinct zone. In the inner zone, the radius of which can be up to 6-10 times the radius of the probe, plastic failure occurs. Beyond this zone is a second zone in which elastic compression occurs. The total resistance to penetration is made up to a component associated with formation of the cavity and a component due to soil-probe friction. The mean stress acting on the surface of the cone is assumed to equal that required to form a cavity in the soil large enough to accommodate the probe. Neglecting the frictional resistance between the shaft of the penetrometer and the cone they proposed the normal point resistance (p_n) as:

$$P_n = \frac{P_r}{1+\tan\delta\ \cot(\alpha/2)}, \tag{2.5}$$

where P_r is the total point resistance or cone resistance, δ is the angle of soil-metal friction and α is the included angle of the cone. Gill (1968) stated that the magnitude of this stress (i.e. specific resistance), P_s is given by

$$P_s = \frac{P_n}{\cos\delta} \tag{2.6}$$

Voorhees *et al.* (1975) found that the root extension rate is more closely correlated with normal point resistance. Specific resistance was found to be better than cone resistance when predicting cultivation tine performance in a fine-grained soil, although cone resistance was the better predictor in a fine-grained soil (Gill, 1968).

When a blunt probe is pushed in to soil, a cone of soil builds up on the probe tip and subsequently acts as part of the probe. Koolen and Kuiper (1983) stated that this occurs once the cone angle exceeds (90-φ). In studies of the effect of cone angle on cone index, Gill (1968) found that cone index decreased with cone angle until a minimum was reached, after which cone index increased with decreasing cone angle. The effect was attributed to the large frictional force on the surface of sharp cones. Although the relative proportions of shear, compressive, and tensile strength that contribute to cone resistance vary with soil water content (Mulqueen *et al.*, 1977), in saturated, fine-grained soils where the angle of internal friction φ is zero, cone resistance increases with cohesion c (Freitag, 1968) as follows:

$$P_r = a.c \tag{2.7}$$

The value of a may not be constant, but is likely to vary with soil type between 10 and 20 (Reece and Peca, 1981). Freitag (1968) found a = 12.5, whereas Reece and Peca (1981) found a = 11. In air-dry sand, cone resistance is empirically related to bulk density and φ (Freitag,1968), but the relationship is specific to sand types.

Steinhardt (1974) deduced from soil mechanics theory that the increase in cone resistance with decreasing matric potential should be approximated linear in wet soils. Such a relationship has been found experimentally for a range of soils at matric potentials above about -15 kPa (Steinhardt and Trafford, 1974; Paul and De Vries, 1979; Van Wijk, 1980). The change in cone resistance with change in matric potential, when both are measured in the same units, varies between 20 and 50, depending on soil type and measurement depth (Steinhardt and Trafford, 1974; Paul and De Vries, 1979). It was greater in a cultivated silt clay loam (Snedecor and Cochran, 1967) than in the same soil under grass (Cassel and Nelson, 1979) and, in a clay loam, was greater in unploughed land (Anderson *et al.*, 1980) than in ploughed land (Davies, 1985; Steinhardt and Trafford, 1974).

Plasticity theory has been used to investigate into the penetration test analytically. The differential equations of plastic equilibrium were solved by numerical analysis and the predictions were found well with experimental results. The finite element analysis to assess the static penetration test shows better results for shallow foundation problems (Borst and Vermeer, 1982).

Been *et al.* (1986) stated that there is no satisfactory general solution for interpreting cone penetration tests to develop a relationship between friction angle, cohesion and cone tip resistance. In an attempt to find out cohesion and frictional angle from indentation tests, Liang (1986) used two indenters simultaneously. He chose two dissimilar indenters and used two entirely different theoretical approaches for these two indenters for finding rigorous mathematical models for characterising soil failure during indentation tests. These were

(a) cavity expansion models in finite soil medium;

(b) slip line geometry analysis defined by the rigorous Sokolovski solution for two dimensional soil failure.

Liang *et al.* (1985) performed some experimental investigations which showed that the predicted values could not produce conclusive results. The slip line analytical method (Hettiaratchi and Reece, 1974) has also been used by Riva (1982) to characterise drop-cone penetration.

Ayers and Perumpral (1982) developed a mathematical relationship connecting cone index model with moisture content and density for different soil types. The equation developed is of the form:

$$CI = \frac{C1 * DD^{C4}}{[C2 + (MC - C3)^2]}, \tag{2.8}$$

where CI is cone index (kPa), DD is dry density (g/cm^3), MC is moisture content, C1, C2, C3 and C4 are constants to be estimated depending on soil type. They (1982) used the penetration rate, the maximum possible in their experimentation, as 21 mm/s. Turnage (1970, 1974) found no effect on the penetration resistance with the change of rate of penetration in coarse-grained soil and minimal effect for fine-grained soils.

Camp and Gill (1969) proposed that cohesion and the angle of internal shearing resistance were directly proportional to moisture content and that bulk density was a quadratic function of moisture content for silt and clay soils. On sandy soils, Harrison and Chang (1966) found that soil strength changed little with moisture content in contrast with the heavier soils where cohesion and angle of shearing resistance decreased dramatically above the liquid limit. Kuipers and Kroesbergen (1966) obtained high correlation coefficients between observed and predicted values of the cohesion over a number of soils ranging from sands to clays by means of linear regression equation involving moisture content and pore space such that:

$$c = i + j M' + k P' + l M' P', \qquad (2.9)$$

where *i, j, k* and *l* are coefficients and *M'* and *P'* are the deviation moisture content and pore pressure from standard values, respectively. Collins (1971) suggested a logarithmic form of the equation for cone cone index, such that:

$$\ln CI = a_{CI} + b_{CI} \ln MC \qquad (2.10)$$

and attempted with limited success to define the coefficients a_{CI} and b_{CI} in terms of commonly measured soil physical parameters. Wells and Treesuwan (1977) indicated the influence of bulk density on Collins's (1971) equation and fitted it to experimental data at two different bulk density values.

Eradat Oskoui and Witney (1982) proposed that the cone penetration resistance of the soil is a function of soil moisture content and soil specific weight which jointly represent the cohesive and frictional components, such that:

$$\mathrm{CI} = f(\mathrm{c}) + f(\gamma), \qquad (2.11)$$

where CI is cone index (MPa), c is soil cohesion (kPa), γ is soil specific weight (kN/m^3). It was urgued that the cohesive strength of the soil is substantially influenced by soil moisture content whilst both the specific weight and the angle of internal shearing resistance are affected to a lesser extent. Based on the results of three soils ranging from sandy loam through to clay loam Witney *et al.* (1982) deduced an empirical equation of cone index of the form:

$$\mathrm{CI} = 450.5\ (\theta)^{-2} + 0.019\ (\gamma), \qquad (2.12)$$

where θ is soil moisture content (%). Witney *et al.* (1984) proposed a general form of cone index equation by taking into account the clay fraction as the clay has the cohesive properties by virtue of its chemical bonds. The proposed equation is of the form

$$\mathrm{CI} = \mathrm{K_c C_r}\ \exp(-\mathrm{n}\theta) + \mathrm{K}_\phi \gamma\ \exp\{\pi/(1 + \mathrm{C_r})\}, \qquad (2.13)$$

where C_r is clay ratio, K_c and K_ϕ are coefficients and n is exponent. Based on this equation he found a very good compromise between the theoretical prediction and experimental results of four soils described by Eradat Oskoui and Witney (1982). Witney *et al.* (1984) concluded that this form of equation is more appropriate for inclusion in a compaction penalty index.

CHAPTER - THREE

COMPUTER SIMULATION OF CRITICAL STATE SPACE

3.1 The Critical State Models

The advancement in the critical state soil mechanics has established the connection between stress and volume-change behaviour of saturated soils. The analytical pathways relevant to interactions between most machine elements and soils can be set out in Fig.3.1. With the progressive development, a number of different theories for the prediction of plastic strains in soils have been published, mostly by research workers at Cambridge, but the essential characteristics of these theories are the same. These are the original Cam-clay model (Roscoe *et al.*,1958; Schofield and Wroth, 1968) and the modified Cam-clay model (Roscoe and Burland, 1968). Reviews of the critical-state and Cam-clay theories have been given by Atkinson and Bransby (1978) and Britto & Gunn (1987). A short account of the critical-state theory including a discussion of the applicability to unsaturated soils has been given by Hettiaratchi (1987).

(a) *The Cam-clay Model*

The simple Cam-clay theory is the basis for several more advanced theories which, although more complicated, give a better fit to experimental data. This theory has been developed for normally consolidated and lightly overconsolidated soils. One of the key assumptions of this non-mathematical Cam-clay theory is that the flow rule follows the normality condition. Thus, if the plastic strain increment vector in Fig.3.2(a) is everywhere normal to a yield locus, it is only necessary to specify either the shape of the yield curve or the relationship between $\delta\varepsilon_s^p/\delta\varepsilon_v^p$ and the stress state (the flow rule) in order for both the flow rule and the yield curve to be fully specified.

A second key assumption, which arises from a consideration of the work dissipated during shear, is that the flow rule is given by

$$\frac{d\varepsilon_v^p}{d\varepsilon_s^p} = M - \frac{q}{p}. \qquad (3.1)$$

This equation has the consequence that the associated yield curve is given by

$$\frac{q}{Mp}+\ln\left(\frac{p}{p_x}\right)=1, \tag{3.2}$$

where p_x is the value of p at the intersection of the yield curve with the projection of the critical state line at point x as shown in Fig.3.2(b). The whole array of yield curves together form a three-dimensional surface in p-q-v space which limit possible states of samples. The equation of the Cam-clay state boundary surface can be obtained using the results that the yield curve, and in particular the highest point on it, point x at $v = v_x$, $p = p_x$, lies on a single swelling line, or

$$v_k = v + k\, ln(p) = v_x + k\, ln(p_x), \tag{3.3}$$

and the highest point x also lies on the critical state line

$$v_x = \Gamma - \lambda\, ln(p_x), \tag{3.4}$$

$$q_x = Mp_x. \tag{3.5}$$

Combination of equations (3.2) - (3.5) can be used to eliminate v_x and p_x to give

$$q=\frac{Mp}{\lambda-\kappa}[\Gamma + \lambda - \kappa - v - \lambda\, ln\,(p)], \tag{3.6}$$

which is the equation for the Cam-clay boundary surface.

(b) *The Modified Cam-clay Model*

The modified Cam-clay model falls within the theory of hardening plasticity for materials which exhibits temperature- and time-independent properties. The volume change as a function of the logarithm of the mean stress is characterised by an elastic (i.e. reversible) rebound/ recompression parameter, κ, and a plastic (i.e. non-reversible) compression parameter, λ as shown in Fig.3.3. The change in slope from reversible to irreversible compression occurs at the maximum preconsolidation stress, P_{pc}. In the deviator-stress (q) - mean-stress (p) plane (shown in Fig.3.4a) soil deforms elastically in stress state within elliptical yield surface (yield being taken as the onset of permanent deformations). Elastic behaviour is according to elasticity theory (for example, Jaeger, 1962). Once soil reaches the yield surface it deforms plastically, experiencing irreversible volume change. The volume may increase or decrease during yielding, according to the stress conditions. At higher mean stresses, yielding is accompanied by volume decrease and thus the soil becomes stronger and exhibits strain hardening. As the soil becomes stronger, the yield surface itself expands (shown in Fig.3.4a) and exhibits strain hardening. At lower mean stresses, yielding is accompanied by volume increase and thus the soil becomes weaker and exhibits strain softening. In this case, the yield surface contracts, reflecting the weaker state of the soil. The critical state is the intermediate mean stress at which yielding proceeds with shear distortion but without volume change. The locus of critical state states in this plane is given by the critical state line, which has a slope of M and passing through the origin of the co-ordinate system.

The type of distortion experienced during yielding is labelled 1-5 (shown in Fig. 3.4a) are normal to the yield surface, in fulfilment of the normality requirement (e.g. Britto and Gunn, 1987). At 1, the stresses are purely compressive and yielding is purely compression without shear distortion; conversely, at 5, the stresses are purely tensile and yielding is purely expansion without shear distortion. Thus, an arrow that lies parallel to the mean stress axis indicates volume change without shear; if it points to the right, the volume decreases whereas to the left the volume increases. At 3 (i.e. at the critical state line), the arrow is parallel to the q-axis and the yielding is pure shear with no volume change. The intermediate cases 2 and 4 show yielding which is a mixture of shear and volume change, case 2 being mostly shear with some compression and case 4 being some shear with a lot of expansion. The arrows are normal to the yield surface and thus the normal to the yield surface indicates the relationship between stresses and strains on the yield surface.

The loading function of that part of modified Cam-clay model which deals with strain-hardening behaviour is expressed mathematically as follows:

Critical state line: $$q = Mp; \tag{3.7}$$
Yield curve: $$q = M[p(p_i - p)]^{1/2}, \tag{3.8}$$

such that $0.5\, p_i \leq p \leq p_i$ (shown in Fig. 3.4b).

3.2 Considerations for the Development of the Model

As the model deals with partly saturated soil it is necessary to consider some special conditions required for developing the model.

(a) *Total and Effective Stress*

The symbolic phase diagram for both saturated and partly saturated soil is shown in Fig.3.5. In partly saturated soil the pore volume is occupied by air and water at their respective pressure u_a and u_w. The resulting curvature of the water interfaces ensures that $u_w < u_a$ (soil water suction). The pore water in partly saturated soils does not carry any of the external loads and any change in pore volume has a negligible influence on u_w. The main role of soil water suction is to increase intergranular contact stresses in some complex manner (Hettiaratchi and O'Callaghan, 1985; Hettiaratchi, 1987).

The stress state variables relevant to unsaturated soils are $(\sigma - u_a)$, $(u_a - u_w)$ and τ (Fredlund & Morgenstern, 1977). In general, pore space in top soils has a free path to the atmosphere and hence $u_a = 0$. The stress variables thus reduce to σ, τ and $-u_w$. The magnitude of u_w for a soil at any specific volume v, is a function of its moisture content w. Critical state space discussed here in this investigation is thus a function of w and the **total stress** component σ and τ. The variation of the relevant critical state parameters with w (or S_r) reflect the manner in which u_w controls the deformation and displacements taking place at the intergranular contact sites in the soil microstructure.

(b) *Moisture and Stress History*

The scatter in the measured values of critical state parameters of undisturbed samples is quite appreciable (Kirby, 1991). Triaxial compression tests on reconstituted soil samples appear to present the best compromise for obtaining representative values of the critical state parameters. A ***remoulded*** sample at specific volume v_1 and moisture content w_1 can be made by mixing the required mass of distilled water with appropriate dried mineral components of the soil. Alternatively a ***cemented*** sample at specific volume v_1 and w_1 can be prepared by allowing a wet specimen to dry out without disturbing the inter-particle contact sites. Both these specimens have nearly identical values of v_1 and w_1, but because their stress and moisture histories are different their microstructural states, and their critical state parameters are not necessarily identical. In assessing published critical state parameters it is therefore essential to take into account the stress and moisture history involved in preparing the test specimens (Hettiaratchi, 1987).

3.3 Development of the Model and the Critical State Parameters

The main features of the critical state model are well documented. In general the ***state*** of a soil is defined in terms of its pore space status (v) and its ambient total stress level p and q. For triaxial loading conditions ($\sigma_2 = \sigma_3$) the stresses are defined as:

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3) \qquad (3.9)$$

and $$q = (\sigma_1 - \sigma_3) \qquad (3.10)$$

(a) *Intersection Boundaries between State Surfaces*

The Roscoe, Hvorslev and Tension surfaces of critical state space (shown in Fig.3.6) intersect in three distinct curves in p-q-v-space. The projections of these curves on the p-v-plane in Fig.3.7(a) are the isotropic compression line (ICL), the critical state line (CSL) and the tension cut-off line (TCL). Note that in the literature the ICL has been variously designated as the virgin or normal consolidation line. The revised designation identifies this boundary from anisotropically compressed samples with $q = \eta p$, $\eta \neq 0$. The ICL is a special case of this when $\eta = 0$ (i.e. $q = 0$).

The linearised form of these curves on a v-ln(p)-plot [Fig.3.7(b)] and their projections on the q-p-plane [Fig.3.7(c)] provides the basis for quantifying these boundaries:

ICL: $v = N - \lambda_N \ln(p)$, $q = 0$; (3.11)

CSL: $v = \Gamma - \lambda \ln(p)$, $q = Mp$; (3.12)

TCL: $v = T - \lambda \ln(p)$, $q = 3p$. (3.13)

State paths of recoverable elastic deformations traverse on curved ***elastic walls*** (EW) within state space [Fig.3.7(a)]. Neglecting hysteresis effects, the projection of these walls on the v-$ln(p)$-plane also plot as a straight lines [Fig.3.7(b)]:

EW: $v = S - \kappa \ln(p)$. (3.14)

(b) *Surface Geometry of State Space*

Points on the Tension (TS) and Hvorslev (HS) state surfaces for any specified v are quantified as follows:

TS: $q = \eta p$; $\eta = 3$; (3.15)

HS: $q = (M - h) \exp[\Gamma - v)/\lambda] + hp.$ (3.16)

The parameter T in Equation (3.13) can be evaluated by putting $q = 3p$ in Equation (3.16):

$$T = \Gamma - \lambda \ln [(3 - h)/(M - h)]. \quad (3.17)$$

It follows from Equations (3.12) and (3.17) that $T \equiv v$ on the CSL at $p = (3 - \mathrm{h})/(M - h)$ [shown in Fig.3.7(b)].

According to modified Cam-clay theory (Roscoe and Burland, 1968) for saturated soils ($\lambda_N = \lambda$) typical Roscoe surfaces, such as C_1I_1 and C_2I_2 in Fig.3.8(a), project as ellipses on the *q-p*-plane with abscissa values ae = 2 (ac) and ad = 2(ab). All such ellipses pass through the origin with bC_1 and cC_2 as their minor axes and their common major axes lie on the abscissa. However, for unsaturated soils ($\lambda_N \neq \lambda$) typical abscissa values such as *ae* are of the form *ae* = R_s (*ac*) where R_s is a function of v and not necessarily of magnitude 2. Not all the elliptical Roscoe surfaces are therefore required to pass through the origin [typical point *f* in Fig.3.8(b)].

3.4 Computer Model of Critical State Space

The seven parameters required for computing values of p, q and v for all points on the state surface are: N, λ_N; Γ, λ, M; κ; h. These are defined in the critical state space shown in Fig.3.9. The value of T can then be obtained from Equation (3.17) and for any chosen value of v_i on the ICL the value of S for that particular EW is given by Equation (3.14). The pairs of co-ordinates (p_c, v_c) and (p_t, v_t) where the EW intersects the CSL and the TCL respectively can be evaluated off Equations (3.11) - (3.16). The semi-major and semi-minor axes of the elliptical Roscoe surface (shown in Fig.3.10) are respectively (p_i - p_c) and (Mp_c). The co-ordinates of state surfaces are built up for values of p in the range (0 to p_t) for the Tension surface, (p_t to p_c) for the Hvorslev surface and (p_c to p_i) for the Roscoe surface (shown in Fig.3.11).

A 'FORTRAN-77' coded computer software **'Programme-1'** has been developed to compute q with supplied values of p and selected values of v at every point on the state surface. This process simplifies somewhat if κ is set to zero so that $v_t = v_c = v_i = v$. Once values of p, q and v are known for a reasonably fine mesh a computer graphics package can be invoked to plot out all the state surfaces, complete with iso-stress contours of equal values of p and q.

3.5 The Package for Graphical Modelling of Critical State Space

The graphics package Unimap-2000 has been employed to plot out the state surfaces. This Unimap package, a part of Uniras software family, is a technical mapping system that can capture, model, and analyse data in 2D and 3D. Its built-in analysis functions allow the computation of surface models from sparse data points, and it can perform relatively complex mathematical calculations on multiple surfaces simultaneously. The input data consists of points with three X-, Y-, Z-values, in which Z is a function of the parameters X and Y. The X-, Y-co-ordinates represent the 2D location of a point, and the Z-co-ordinate represents the factor to be visualised.

The Unimap operates by using the processes of either triangulation or rectangular gridding. In triangulation, the Z-value of each point is connected to the Z-values of its two nearest X-Y-neighbours to create a flat triangular plane that approximates the surface of the area. Each data point is a node of more than one triangle, and each side of the triangle is shared by two different planes. This creates a continuous Z-based surface across the X-Y-field. On the other hand, the rectangular gridding produce remarkably accurate 3D maps although it is significantly slower than triangulation gridding. This rectangular gridding system has the better performance over triangular gridding in producing 3D maps from the data points relatively sparse. In rectangular gridding, a grid is laid over the field of data points, and a Z-value is calculated for each grid node. This calculation evaluates and averages the data in a specific area surrounding each node. This results in a 3D grid or surface reconstruction that can be plotted as a smooth continuous surface. Since this grid is a two-dimensional matrix of values, most operations can be performed on the entire surface at once. This package has the capacity to offer a practical solution by making it very easy to choose alternative algorithms and display the results in a variety of different styles and orientations. The colour gradations, using different colours to represent Z-values, can highlight subtle physical variations.

3.6 Application of the Model Developed to Three British Soils

The model is applied to generate state space for two Scottish soils (Darvel sandy loam and Winton clay loam) and an English soil (Evesham clay) over a range of moisture contents. The soils has been described by O'Sullivan *et al.* (1994). The relevant properties of the soil are given in Table-3.1. The soil parameters are obtained from CCV triaxial compression tests (Hettiaratchi *et al.*, 1992) on 76mm diameter remoulded soil samples. There are no published record of data for values of h. Order of magnitude values of h were interpolated from separate experiments on similar soils (Hatibu, 1987). The soil parameters at different moisture content is given in Table-3.2.

Typical output of the model in 3D- and 2D-contour maps, using Unimap package, are shown respectively in Fig.3.12(a) and Fig.3.12(b) for Evesham clay at 24.5% moisture content (w). The colour gradations show the variation of q-values as a function of p and v at every points on the state surfaces. The 3D-line and the 2D-line for the same soil at identical w are shown in Fig.3.12(c) and Fig.3.12(d) respectively. The composite diagram [Fig.3.12(c)] shows the constant p-, q- and v-contours. The extremity of the Roscoe-surface lying on q=0 plane constitutes the ICL along ABC [Fig.3.12(c)]. But it can be seen clearly in the 2D-line diagram [Fig.3.12(d)] that the line ABC is not a smooth curve.

The main difficulty in plotting a smooth curve is due to the selection of the number of grid-cells in X-Y-direction. The higher the grid numbers, the smoother the curve. The production of such maps with high grid numbers is very time-consuming. It demands vast numbers of points to be specified in X-Y-field and the corresponding Z-values to draw a representative surface. In addition, the number of grid-cells plays an important role in evaluating and averaging the data surrounding each node. This in turn controls the calculation of the Z-values in each node required to construct a smooth continuous surface. As a practical compromise, for the present investigation, the grid numbers were chosen, by trial and error, to be 80x20 in X-Y-direction for a polynomial fit.

The typical 3D-line Unimap plot shown in Fig.3.13 and Fig.3.14 are respectively for Winton clay loam and Darvel sandy loam soils. These composite drawings, consisting of crowded lines, are simplified by a hand drawn smooth ICL. A typical plot of such a drawing for Winton clay loam at 15.6% w is shown in Fig.3.15 with all the identified features in it. The constant p, q and v contours with contour intervals Δp = 100 kPa, $\Delta q = 50$ kPa and $\Delta v = 0.1$ are shown accordingly.

The complete set of diagrams of critical state space for these three soil at 5 different levels of moisture contents from dry to wet are shown in Fig.3.16 (for

Evesham clay), Fig.3.17 (for Winton clay loam) and Fig.3.18 (for Darvel sandy loam). Contours of equal *p, q* and *v* are shown on the state space diagrams for each individual moisture content.

The typical outputs of the model, using the Unigraph graphic package, in *v-p*-plane for the three soils are shown in Fig.3.19 through Fig.3.21. These diagrams identify the Roscoe-, Hvorslev- and Tension-surfaces. These surfaces are characterised by the stress ratios η (= q/p) starting at η= 0 for the ICL, to 3 for the TCL and $\eta=M$ for the CSL. The $\Delta\eta$ interval used was 0.2. The '**Programme-2**' has been developed for this purpose. It calculates the values of *p* with supplied values of *v* at different η in 2D-plane. The typical outputs of the same in *v-ln(p)*-plane are shown in Fig.3.22 through Fig.3.24.

The typical output of the model for constant *v*-lines in *q-p*-plane are shown in Fig.3.25 through Fig.3.27. The normalised plots on q/p_i - p/p_i-plane are also shown in Fig.3.28 through Fig.3.30. The values of q/p_i and p/p_i has been calculated by modifying the model. The '**Programme-3**' has been developed to serve this purpose.

3.7 Interpretation of the Graphically generated Critical State Space

The variation of state space with soil moisture content shown in Fig.3.16, Fig.3.17 and Fig.3.18 is brought about rather dramatically. Based on this diagrams following broad conclusions can be drawn.

(a) The state space proportions are least sensitive to moisture content in the sandy loam soil. Evidently clay content plays a significant part in this behaviour.
(b) The clay loam soil exhibits the largest variation in state space with moisture content.
(c) Comparatively speaking, the clay soil encloses the largest volume within state space. Elastic deformations are therefore significant, particularly in the dry state.
(d) Increase in moisture content swings the ICL, CSL and the TCL of all the soils towards the origin of co-ordinates in a systematic fashion.
(e) The tension surface is comparatively small in the loamy soils, but appreciable in the clay soil.
(f) In all three soils the slopes of the CSL increase as the soils dry out. This is indicative of a corresponding increase in the Mohr-Coulomb friction angle φ.

(g) The TS occupies a significant proportion of state space in the clay soil. This is particularly so for this soil in its dryer states and indicates a tendency to develop cracks on drying.

(h) The state space for the Winton clay loam soil at high moisture content is small indicating likely damage if worked very wet.

These observations are specific to the soils described here. However, a preliminary analysis of all the variable data in this manner would suggest that these trends may be of general applicability. The logical extension of this modelling technique is the introduction of state path tracing for actual field operations.

CHAPTER - FOUR

THEORETICAL BACKGROUND AND THE CONE PENETROMETER MODEL

4.1 Indentation

A number of theoretical approaches for developing mathematical models characterising soil failure have been published. The evaluation goes back to Coulomb (1776), who introduced the basic concept of shear strength, which is still used to assess earth pressure problems. Rankine (1857) developed an equilibrium equation for a small soil element at failure to find the solution for earth pressure problems. Kotter (1903) considered a curved surface for which he derived the differential equations governing the stresses. Prandtl (1921) applied these solutions to study the penetration of the hard bodies and deduced a rupture geometry for a weightless frictional soil. Rendulic (1937) developed a single logarithmic spiral solution for the rupture surface to estimate the earth pressure on vertical walls. Ohde (1938) proposed a boundary of a rupture zone within which shear and normal stresses are at incipient failure and the stresses at the boundaries are in equilibrium with the external forces acting on them. This is known as the logarithmic spiral solution.

Terzaghi (1943) and Tailor (1948) modified Prandtl's (1921) original bearing capacity solution and introduced a new dimension to it. Terzaghi (1943), Hansen (1961) have contributed by incorporating some factors into this equation. The most rigorous approach to the method of classical soil mechanics for the solution of two dimensional soil failure is Sokolovski's (1960) analysis of 'limiting equilibrium' of a soil mass. A simplified approach, based on Sokolovski's analysis, has been developed here in Newcastle as reported by Hettiaratchi *et al.* (1966), Hettiaratchi and Reece (1967, 1974, 1975), Hettiaratchi (1988) and Reece & Hettiaratchi (1988, 1989). This solution takes the Sokolovski's solution a step further in analysing the soil failure applicable to the agricultural engineering situation. Hettiaratchi and Reece (1974) made charts for this analysis to provide easy calculation of earth pressure problems. They (1975) allowed for the kinematics of the motion of the plain interface to allow for the boundary wedges formed during soil failure.

4.2 Sokolovski's Analysis

Sokolovski (1960) considered the 2-dimensional static equilibrium of a Mohr-Coulomb soil by relating the two basic partial differential equations of equilibrium together with the relationship of the stresses at failure which provide the necessary three equations for the complete solution of the three unknown stresses $(\sigma_x, \sigma_y, \tau)$ everywhere in a loaded soil.

The three equations for a cohesive frictional material with weight are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \gamma, \tag{4.1}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} = 0 \tag{4.2}$$

and $$(\sigma_x - \sigma_y)^2 + 4\tau^2 = \sin^2 \varphi (\sigma_x + \sigma_y + 2c \cot \varphi)^2, \tag{4.3}$$

where c is the cohesion, φ is the internal friction angle of soil, δ is the angle of friction between the soil and the interface and γ is the specific weight of the soil.

The total distribution of stresses throughout the failing material is obtained by integrating the above equations from known boundary stresses using the method of characteristics. The essential features of the boundary of the soil in front of an interface in shown in Fig.4.1a which consists of three distinct zones; and the nodes in the numerical solution corresponding to this figure is shown in Fig.4.1b. Three form of boundary problems occurring in these zones can be described as follows.

The first zone OCD adjacent to the soil surface is a passive Rankine zone and its boundary OD can carry an optional normal surface load. The numerical solution commences from the boundary OD, a non-characteristic direction, to the characteristic direction OC and stresses everywhere within the zone OCD can be calculated by considering Cauchy boundary problem.

The second zone OBC is the transition zone bounded by two characteristic direction OC and OB. Once the stresses on the characteristic boundary OC are known, the solution method proposed by Goursat can be applied to solve the equations between this boundary and the other characteristic boundary OB.

The third zone OAB adjacent to the interface is the interface zone where OB is a characteristic direction and OA is a non-characteristic line. The stresses on OA are not known but the direction of the characteristics along it is predetermined by the soil interface properties. The equations are integrated as a mixed boundary-value problem in this zone.

4.3 Newcastle Analysis

Hettiaratchi and Reece (1974) used Sokolovski's solution to study the normal soil failure of a rigid plastic soil as a two-dimensional problem and where no soil boundary wedges are formed. The condition where wedges may be formed was dealt by them in a separate analysis (1975). The following assumptions necessary for the basic failure are:

(a) Soil failure takes place in two dimensional field.

(b) The soil is considered to be an isotropic, rigid plastic material which fails at zero strain as described by Mohr-Coulomb criterion, where:

$$\tau = c + \sigma \tan \varphi \qquad (4.4)$$

(c) The motion of the interface into the soil is that no boundary wedges are formed.

(d) The shear stresses mobilised on the soil-interface are described by:

$$\tau = a + \sigma \tan \delta \qquad (4.5)$$

(e) The rigid plane soil-interface extends at least up to the free soil surface.

(f) The normal surface, if any, should be uniformly distributed over an area as great as the rupture zone on the soil surface.

(g) The constrained adhesion is the measure of the adhesion between the soil and the interface such that:

$$c_a = a = c \tan \delta \cot \varphi \qquad (4.6)$$

(h) The frictional stresses component increases linearly with depth along the slip lines whereas the cohesive and surcharge components remain independent of depth as proposed by Sokolovski for $c/\gamma z = \infty$.

The detail soil rupture configuration on the basis of slip line fields has been presented by Reece and Hettiaratchi (1989). There are four distinct types of slip line fields which depends on the failure geometry controlled by the interface rake angle α, direction of translation β, the soil internal friction angle φ and the soil-interface friction angle δ. Depending on these factors the basic soil failure can have the interface (I) zone, the transition (T) zone and the Rankine (R) zone as shown in Fig.4.2(a). Under certain conditions, such as small rake angles, a stress discontinuity will be present between (**I**) and (**R**) with the disappearance of the transition as shown in Fig.4.2(b). Depending on the kinematics of the system, soil wedges may form. With large rake angles a plane boundary wedge (**W**) may be present as shown in Fig.4.2(c). With small rake angles a Rankine wedge may form as shown in Fig.4.2(d). The detailed rupture surface and force analysis by Newcastle method is given in **Appendix A.1**.

4.4 Shallow and Deep Penetration

The rupture surface geometry assumed by Terzaghi (1966) for surface foundation and buried footings are shown in Fig.4.5. For the buried footings, the soil above the level of the footings was assumed to be free of shear deformations and was replaced as surcharge pressure. Meyerhof (1951) assumed modified rupture surfaces overcoming the depth limitations of Terzaghi's method as shown in Fig.4.6 by including a mixed radial and plane shear zone on both sides of the footing. The general rupture surface composed of three zones:

(a) A central wedge under the footing.
(b) Two shear zones of the form BCD.
(c) A mixed shear zone BDEF, where the shear stresses vary between that at the radial boundary and a plane shear zone, depending on the roughness and depth of the foundation. The inclination of BD varies with foundation depth.

Thus Meyerhof's rupture geometry not only depends on footing geometry but also on the depth of embedment. Meyerhof (1953) studied the effect of eccentricity and inclination of load on the total bearing capacity value. He concluded that the bearing capacity factors decrease with the eccentricity as well as load inclination from vertical for a horizontal base footing. He observed that the bearing capacity factors increases with the inclination of the base of foundation. The load was applied normal to the base of inclined foundations.

Both the above investigators considered one boundary as an equivalent-free surface in their rupture geometry, which is not entirely satisfactory. This problem was overcome by Witney (1966), who developed a general theory based on the revised rupture geometry shown in Fig.4.7 for shallow and deep sinkages. The rupture geometry consists of three zones:

(a) A central wedge under the footing which is independent of footing roughness and is function of φ only.

(b) Two radial shear zones, consists of logarithmic spirals whose poles lie at the corner of the footing. This allows complete compatibility of the slip line field from the central wedge into the radial zone.
(c) Two plane shear zones whose shape and extent depend only on considerations of static equilibrium of forces acting along its boundaries.

This failure is therefore more consistent with the boundary conditions for such footings. The central wedge moves as a continuous part of the footing itself and the two faces AC and BC of it acts a pseudo-interfaces translating parallel to the direction of loading. Because of the facts that Witney's (1966) analysis considered both static equilibrium of forces and the compatibility of slip line field throughout surface, it would be appropriate to extend his analysis to the more general case of sub-surface interfaces. Albuquerque (1975), in his theoretical solution for horizontally translating sub-surface cutting blades, applied Witney's (1966) analysis successfully with slight modification.

Albuquerque and Hettiaratchi (1980) developed a method of analysis for the interfaces which do not extend up to the soil surface, and they restricted the interface translation only to the horizontal direction. This statically admissible analysis was initially for soils with weight but no cohesion, and subsequently extended to cater for soil cohesion.

4.5 Theoretical Concepts in Formulating Model

It is possible to use the Newcastle Method, described earlier, to extend Sokolovski's solution to the development of a predictive model for cone penetrometer performance. The two main requirements to be satisfied are:

(a) The slip-line field generated by the cone-soil interface must be compatible with the rules set out in the Newcastle Method. This can, therefore, involve continuous or discontinuous stress fields.
(b) The slip-line field must be compatible with the frictional properties at the boundaries. The boundary at the cone-soil interface is fairly straightforward. However, the contact zone between the penetrometer shaft and the soil requires special consideration.

There are two further complications that influence the development of the model and these are:

(c) The Sokolovski solution (and its derivative, the Newcastle Method) applies to **two-dimensional** stress fields. Clearly, the cone penetrometer generates three-dimensional, rotationally symmetric, stress fields.

(d) The variation of the geometry of the contact surfaces of the penetrometer with depth of penetration.

The method of overcoming difficulty (c) above is discussed in section 4.3. The problems associated with (d) requires some clarification.

There are essentially three geometrical phases in the indentation process of a cone penetrometer. The depth of penetration has no effect on the interface geometry in the initial phase shown in Fig.4.8(a) so long as the top of the cone lies above the soil surface. This case applies to the well known drop-cone test and has been extensively studied [Hansbo, 1957; Houlsby, 1982; Wroth, 1984; Wood, 1985; Hoque, 1991 and others]. When this limit is exceeded the rupture surface develops an additional boundary - the shaft contact surface. This boundary alters with depth of penetration [see Fig.4.8(b)]. The analysis of this configuration was first dealt with by Witney (1966) and subsequently extended by Albuquerque and Hettiaratchi (1980) and Sarker *et al.* (1985).

A third phase is reached in deep penetration when the cone tip depth is in excess of the limit f discussed in Appendix A.2 [see Fig.4.8(c)]. In this event the rupture surface configuration, once again, reverts to one which is independent of depth of penetration. In the present analysis this third phase only is considered. In most practical cases this is the range of particular significance in Agricultural Engineering field practice.

4.6 Development of Slip-Line Field

Consider first the two extreme limits where the shaft-soil interface is (a) perfectly smooth ($\delta = 0$) and (b) when it is perfectly rough ($\delta = \varphi$). As shown in Figs.4.9(a) and 4.9(e) the rules of the Newcastle Method can be applied directly to these two limiting cases to give acceptable **continuous** slip-line fields provided that the cone surface roughness **matches exactly** the shaft roughness.

Secondly consider the case when intermediate roughness states obtain for both the shaft and the cone. Referring to Fig.4.9(b), which is for $\delta \neq 0$, $\delta < \varphi$, it will be seen that the slip line field ODE is the segment which agrees with the boundary condition at the shaft. The corresponding transitional zone is OB'E. This leaves a segment DEC, which is common to the transitional zone and contiguous with it at EC. However the boundary DE is not continuous with the Rankine zone OED along the surface DE. This boundary can be accommodated as a field with a stress discontinuity where there are finite jumps in the stress vector in the direction perpendicular to DE. This expedient allows for a stress field with a stress discontinuity along DE to be developed. Note that the change from $\delta = 0$ in Fig.4.9(a) to $\delta = \varphi$ in Fig.4.9(e) is traced in the diagrams and shows how this line of discontinuity DE elongates and demonstrates how the $\delta = \varphi$ zone gradually spreads to completely take over the slip-line field at the end of the limit.

The above analysis shows that a compatible slip-line field can indeed be developed for the deep-penetration case and hence the outer boundary of the rupture zone constitutes a statically acceptable boundary for any state of roughness between soil and metal. Once this boundary has been established the methods outlined in section 4.7 (details given in Appendix A.2) can be applied to calculate the cone load in terms of the Mohr-Coulomb parameter c, and φ and soil-metal roughness parameter c_a and δ for any specified cone geometry and penetration depth ($> f$). The proposed skeleton diagram with the necessary parameters for the cone penetrometer model is shown in Fig.4.10.

4.7 Equilibrium of Cone and Shaft

The equilibrium of cone and shaft is shown in Fig.4.25. The total gravitational, cohesive-adhesive and surcharge force components can be summarised as follows:

$$TF_\gamma = F_{V\gamma} + T_\gamma - (W_c + W_s) \tag{4.7}$$

$$TF_{ca} = F_{Vc} + T_c + A_s + Vc_a \tag{4.8}$$

$$TF_q = F_{Vq} + T_q \tag{4.9}$$

The total load exerted onto the cone penetrometer is therefore

$$Q_{cone} = TF_\gamma + TF_{ca} + TF_q \tag{4.10}$$

which can be written in the form

$$Q_{cone} = (\gamma\, z^2\, N_\gamma + c\, z\, N_{ca} + q_s\, z\, N_q)\, d_c \tag{4.11}$$

and hence the non-dimensional soil resistance coefficients are respectively:

$$N_\gamma = \frac{TF_\gamma}{\gamma z^2\, d_c}, \tag{4.12}$$

$$N_c = \frac{TF_{ca}}{c\, z\, d_c} \tag{4.13}$$

and $$N_q = \frac{TF_q}{q_s\, z\, d_c}. \tag{4.14}$$

A 'FORTRAN -77' coded computer software has been developed to compute the total load exerted onto the cone penetrometer and will be discussed in chapter seven. The programme contains two subroutines. The subroutines are of the form:

SUBROUTINE ZLIM (φ, c, c_a, θ_c, d_c, δ_c, f) and
SUBROUTINE CONE (φ, c, γ, θ_c, d_c, W_c, d_s, W_s, δ_c, δ_s, c_a, c_{as}, q_s, z, TF_γ, TF_{ca}, TF_q, Q_{cone}).

In the first subroutine, the first 6 variables are to be supplied those which fix the value of critical depth limit f and enters to the second subroutine as input with all other soil parameters feed from a main programme and come up with the gravitational, cohesive-adhesive and surcharge force components with the total cone load. From these force components the cone index and accordingly the non-dimensional soil resistance coefficients can be calculated.

CHAPTER - FIVE

STATE PARAMETER

5.1 State Parameter Concept

The term 'State' is a description of the physical conditions under which a material exists; the material behaviour is controlled by these conditions. Specific volume (the volume of soil containing unit volume of soil solids) and stress level are the most important physical conditions which define the current state of the soil and therefore control its behaviour. It is a significant parameter for describing material behaviour because many material properties vary as a direct function of state. However, it is also known that sand matrix structure is an important controlling factor in soil behaviour. Mitchell (1976) used the word structure, in a wider sense, to include fabric, void ratio and composition. Soil 'fabric', on the other hand, can be described as the geometric arrangement of particle contacts. Recent studies have shown that a given cohesionless soil may have different fabric at the same specific volume or relative density. It is postulated that the behaviour of a soil may be characterised in terms of two variables. Firstly, a state parameter which combines the influence of specific volume and stress and secondly, a fabric parameter which characterises the arrangement of the soil grains.

The 'state' of soil should be defined in terms of specific volume (v) and stress (p), but it must also be measured against a reference condition. The physical considerations for an appropriate state parameter for soil are therefore what is the appropriate combination of specific volume and stress and what is the appropriate reference condition?

The selection of a reference condition on which to define state requires that the reference condition should have a unique structure which is not influenced by the original test conditions. It is further postulated that the soil has a unique structure at the critical state. This postulate is quite common in the published literature as described in chapter three. Some authors (Rowe, 1962; Schofield and Wroth, 1968) have postulated that granular material has no structure while others (Poulos, 1981; Casagrande, 1975) have postulated that a 'flow' structure exists. Been and Jefferies (1985) elucidated that this uniqueness does not depend on the nature of the sand structure at the critical state. Rather it depends on there being a unique, repeatable particle arrangement at the critical state condition. It is proposed that the spherical pressure p, [= $(\sigma_1 + 2\sigma_3)/3$ for triaxial loading condition] described in chapter three, is a suitable stress measure for incorporation into the state parameter. This choice is based on the assumption that the deviatoric component of stress will be reflected directly in the soil fabric parameter. These ideas lead to the kernel concept that the critical state defines a reference state and the distance of a soil from this reference state in *v-p* space characterizes the soil's ambient state. This measure of state is called the state parameter. The symbol ψ has been used to represent the state parameter.

However, the state parameter ψ embodies a combination of specific volume, ambient stress level and orientation relative to critical state line (CSL). The definition of state parameter is shown in *v-ln(p)*-plane [Fig.5.1(a)] and its association with the critical state wall (CSW) and Elastic walls (EW) is shown in *p-q-v* space [Fig.5.1(b)]. Essentially this parameter is the difference between the in-situ specific volume v of a soil element at an ambient pressure p and the specific volume v_c corresponding to this pressure on the CSL. This is essentially the distance, in the direction of the v-axis, of the CSW from a point on the EW on which a soil element lies in its ambient state. Let us consider the soil element at state D in Fig.5.1(a) which is at a specific volume v and ambient isotropic stress p. The soil has been compressed elastically along a swelling line from some initial specific volume v_0 at A to point B on the isotropic compression line (ICL), then normally consolidated to C (with $p = p_i$) and subsequently allowed to swell to D. The soil at D represents a typical soil element in an over-consolidated state (over-consolidation ratio $R = p_i/p$). The specific volume on the critical state line (point E) corresponding to the ambient mean pressure p is v_c. By definition the state parameter of point D is then given by:

$$\psi = v - v_c \qquad (5.1)$$

For the particular over-consolidation ratio R shown in Fig.5.1(a) the value of ψ is negative ($v_c > v$). As the over-consolidation ratio is decreased the point D moves towards C and at F ($p = p_0$) the state parameter $\psi = 0$. In the interval $p_0 < p < p_i$ the state parameter ψ is positive. Referring to the pictorial view of critical state space [Fig.5.1(b)] the point D lies on the intersection between the "constant pressure wall" labelled W_1 and the "elastic wall" W_2. The wall W_1 meets the critical state wall W_3 at E. Thus the state parameter ψ is a measure of the position of these "walls" relative to the CSW. Intrinsically the state parameter incorporates information of the ambient stress level (p), the current specific volume (v) and the how far away the critical state line is, which is a measure of the stress required to bring this element to failure (i.e. its strength). Therefore the state parameter, a single index, can be used to describe much of the behaviour of soils over a wide range of stresses and densities. A practical application of the relationships between state parameter and behavioural properties is dependent on the ability to measure ψ in-situ.

5.2 Cone Performance and State parameter

In two subsequent papers (Been *et al.*, 1986 and Been *et al.*,1987) the authors try to establish a connection between the state parameter ψ, described in section 5.1, with cone index q_c for sands. Referring to Fig.5.2(a) the well known cone index is obtained from the cone resistance force F_c as:

$$q_c = F_c / A_p, \tag{5.2}$$

where A_p is the projected area of the cone normal to its axis. On a proposal by Wroth (1986), the absolute value of q_c is normalised relative to the ambient stress level. There are two convenient reference stress level that can be used to normalise q_c. The first possibility is to use the vertical geostatic stress σ_v on horizontal planes [Fig.5.2(b)] at the cone level and the other is the critical state mean normal stress p. The latter is simply given by:

$$p = \sigma_v + 2\sigma_H. \tag{5.3}$$

It will be seen that Equation (5.3) makes the common assumption regarding the intermediate principal stress where the conditions are equivalent to those in a triaxial test ($\sigma_2 = \sigma_3 = \sigma_H$). The normalized cone index relative to the two possible ambient stress levels are:

$$Q = (q_c - p)/p \tag{5.4}$$

and $$Q = (q_c - \sigma_{v0})/\sigma_{v0}. \tag{5.5}$$

In the analysis that follows, either of these normalised forms of the cone index can be used and, in the present investigation, the definition of Equation (5.4) will be employed.

Been and others found that for a given sand there is a close correlation between the normalised cone index Q and the state parameter ψ. In their analysis of a very large number of cone penetrometer tests on five different **sands** they found that the fundamental relationship between Q and ψ takes the following form:

$$Q = (q_c - p)/p = k \exp(-m\psi), \qquad (5.6)$$

where the two coefficients k and m depend mainly on the type of sand. Sladen (1989), in a rather negative (his own description!) critique of the work of Been and his colleagues, points out that the coefficients m is also sensitive to the ambient pressure p. Ignoring this complication, Equation (5.6) presents a significant relationship between cone index, ambient mean stress and current specific volume and opens up a possible route for using cone index data to arrive at a measure of the pore volume and hence bulk density of a soil. This has exciting implications in the interpretation of the thousands of cone penetrometer data frequently encountered in published literature. Of course, this analysis would be possible only if the position of the critical state line for the soil in which the penetrometer was used is known beforehand. This requirement then points to the practical relevance of the recent emphasis on the development of rapid methods for the measurement of critical state parameters.

The two coefficients m and k appearing in Equation (5.6) determine how the normalized cone index Q varies with the state parameter ψ. The main behaviour of these two coefficients for sands is shown in Fig.5.3. Equation (5.6) can be restated thus:

$$\ln(Q) = \ln(k) - m\,\psi \qquad (5.7)$$

The typical linear relationship represented by Equation (5.7) is shown in Fig.5.3(a) and experimental evidence (Been *et al.*, 1986 and Been *et al.*,1987) shows that this relationship is indeed an acceptable simplification. Been and his colleagues (and several others) have found that the coefficient m for sands also bears a linear relationship to the slope λ of the critical state line [Fig.5.3(a)]. The corresponding variation of the coefficient k is shown in Fig.5.3(c). Thus for all practical purposes we can assume that the coefficients m and k are unique to a particular soil. Thus, for any specified λ_1 there are two unambiguous values of m_1 and k_1. As the experiments to establish the variations shown in Fig.5.3(b) and Fig.5.3(c) have been conducted exclusively on saturated sands this uniqueness must, at this stage, be attributed only to such soils.

5.3 Practical use of State parameter

A basic requirement to make practical use of the state parameter concept in the interpretation of cone penetrometer data is the need to extend the validation of the currently available findings in saturated marine sands to partly saturated land-based soils, ranging from clays to sands. In essence this exercise would be to validate Equations (5.6) and (5.7) for agricultural soils. This is clearly a major research undertaking in itself. Anticipating a favourable outcome of such investigations the steps required for developing the interpretation procedure are as follows:

(a) **Critical State Parameters:** The measurement of the critical state parameters of the soil at the moisture content of interest. In general a knowledge of the critical state line parameter λ and Γ are sufficient.

(b) **Calibration:** The calibration of the cone penetrometer in the particular soil. This is required to establish the state parameter coefficients m and k.

(c) **Normalized Cone Index:** Any cone penetrometer reading q_c at depth z has to be normalised with reference to p. This requires an estimate of the geostatic stress on horizontal planes at depth z which has to be approximated as $\sigma_v = \gamma z$. A further approximation has to be introduced to arrive at σ_H and this requires a knowledge of the coefficient K_0 of earth pressure at rest.

The steps required for (a) has been elucidated by Hettiaratchi *et al.* (1992). The steps involved in (b) and (c) is explained in the next chapter.

CHAPTER - SIX

CALIBRATION CHAMBER

6.1 Existing Calibration Chambers

Various workers have drawn their attention in quantifying the relationships between sand density (or specific volume), effective stress level and CPT tip resistance by using large-scale chamber tests (Schmertmann, 1977; Villet and Mitchell, 1981; Baldi *et al.*,1982; Parkin *et al.*, 1980).

In principle, this chamber contains a large triaxial sample of soil, enclosed in a rubber membrane and loaded laterally by a water jacket. In order to control the lateral deformation of the sample a very rigid enclosing pressure vessel is necessary which, of course, is not physically possible. However, by using a cavity wall, and by maintaining a cavity pressure equal to the chamber pressure, full rigidity of the inner-wall can be effectively established. These test chambers have ranged in size from 0.76 to 1.2m diameter and generally allow the lateral and vertical effective stresses to be varied independently.

The original concept for test rig is due to Holden, at the Road Construction Authority of Victoria, Australia, and resulted, in 1969, in the construction of a chamber housing a sample 0.75m diameter by 0.90m height (Parkin, 1988). The base piston was inflated by water from an air/water cylinder, with deformations being derived from water level observations. Sample formation was by travelling sand spreader (after the principle of Kolbuszewski and Jones, 1961), and the results of this investigation were reported by Veismanis (1974).

A great effort has been paid for the subsequent developments that occurred in various countries with progressive development in technology. In 1970, a chamber for samples 1.20m by 1.20m was built at the University of Florida, essentially similar to its predecessor but with a view to accommodating a large cone (Holden, 1971; Laier *et al.*, 1975). Further developments from the Florida design, and the need for increased travel, led to the construction of 1.20m diameter by 1.80m height chamber at Monash University in Australia, with travelling spreader (Chapman, 1974), followed soon after by 1.2m by 1.5m chamber at the Norwegian Geotechnical Institute (NGI), Oslo. Holden (1977), in association with Jacobsen (1976), developed a calibration chamber at the University of Aalborg in Denmark which has been considered to be the satisfactory method of preparing soil samples.

The two other spectacular developments occurred in Italy, first one was at the Italian Electricity board (ENEL), Milan (in 1977) and the second one was at the ISMES research Institute, Bergamo (in 1983) as reported by Parkin (1988). Both the chambers are of same size as the NGI chamber, and use similar soil spreaders. In the case of the ENEL chamber (Bellotti *et al.*, 1982), significant developments were made in respect of precision servo-controlled mechanical drive for penetrometer (replacing the hydraulic ram), a highly sensitive device for volume change measurement, and advanced methods for saturating samples. In the case of the ISMES chamber, the principal development is in modifications to the soil spreader to enable samples to be "rained" in high vacuum. This has particular value in the preparation of samples at lower densities, and for use with finer materials. However, with the progressive development of technology, procedures have been developed that allow samples of soil to be prepared at various densities with reasonable uniformity.

Detailed accounts of the effect of boundary conditions in large calibration chamber test results have been reported by various researcher at a seminar held at Southampton University and is compiled by Last (1984). The importance of chamber size and boundary conditions has been recognised (Bellotti *et al.*, 1979; Parkin and Lunne, 1982) and correction factors have been developed to allow data from different sized chambers and varying boundary conditions to be compared with field conditions (Been *et al.*, 1986). The larger the calibration chamber the smaller these correction factors are, and intuitively, the more reliable the chamber test results are likely to be. Although different workers have used different soils, most of the published chamber tests have been performed on pure medium sands in a dry state.

The calibration chambers used to develop cone resistance correlation vary significantly in terms of chamber size and imposed boundary conditions. Descriptions of the chambers and cones used to develop cone calibrations are provided by Bellotti *et al.* (1982), Villet and Mitchell (1981), Chapman (1974) and Parkin *et al.* (1980). Parkin and Lunne (1982) have clearly illustrated that both chamber size and imposed boundary conditions affect the cone resistance depending on the soil density.

Construction and operational details of a typical large calibration chamber have been described by Sweeney and Clough (1990).

Al-Mukhtar (1988) adopted a small calibrated chamber by modifying a classical triaxial cell. The prediction of the tests allowed him to check the validity of constitutive soil models of complex non homogeneous paths.

6.2 Chamber Size and Boundary Effect

It is essential to consider how representative the calibration chamber test results are for in-situ conditions. There are a number of factors that need to be considered for the interpretation of the calibration chamber test results in practical situation. These factors are: chamber boundary conditions, size of the chamber in relation to cone diameter, cone size, fabric of the soil grain and ageing effects, stress and strain history of soil sample, and degree of saturation of the sample. The most obvious constraint with calibration chamber testing is that the chamber is not infinitely large, and therefore the penetration resistance may be influenced by the boundary conditions in the chamber. Parkin and Lunne (1982) found that for loose soil (relative density $D_r < 30\%$), chamber size effects are not significant, provided that the chamber-to-cone diameter ratio is greater than 20. For dense sands ($D_r = 80$ to 90%), the chamber-to-cone diameter ratio must be greater than about 50 to reduce the influence of chamber size on the test results. Bellotti *et al.* (1985) concluded that it is impractical to conceive apparatus that would give a value of D_r sufficiently favourable to completely eliminate such boundary effects.

The lateral boundary condition of constant volume, rather than constant stress, minimizes chamber size effects. Using a vertical stress-controlled boundary minimizes end effects. Provided that the chamber-to-cone diameter ratio is larger than 50, lateral boundary conditions are not significant. Vertical boundary (end effect) are readily observed in the individual test data and are effectively eliminated by using test data from the central portion of the chamber.

6.3 Considerations for the Calibration Chamber for Present Investigation

The arrangement in Fig.6.1(a) shows schematically a suitable calibration chamber where the boundary stresses σ_H and σ_V can be controlled. Essentially this is similar to a triaxial cell with a very large diameter specimen into which the cone penetrometer can be advanced. The requirement for a large diameter specimen is essential to minimize the boundary effects as discussed in section 6.2 and the specimen diameter should be about 50 times the cone diameter. In practice the larger the chamber the more difficult it is to prepare samples of uniform density. An acceptable compromise has to be made between end effects and experimental convenience. As a practical compromise the size of such a calibration chamber can be around 15 to 20 times the cone diameter. For the present investigation a modified triaxial cell is used as a calibration chamber which can accommodate a maximum of 100 mm dia. soil sample with 10 mm space free for the water jacket around the sample. This, rather smaller sample, allows us to prepare a sample of uniform density. The maximum height of the sample is 185 mm with slenderness ratio (L_s/D_s) equal to 1.85 which is slightly lower than the standard triaxial sample slenderness ratio of 2. As an experimental convenience, the dia. of cone is selected to be 10mm with a sample-to-cone diameter ratio of 10, the maximum possible, for the present investigation. This is not entirely satisfactory, but practical constraints determine this ratio.

6.4 Data Calibration

The calibration chamber values of σ_H, σ_V together with measured cone index q_c provide the information necessary to estimate p and Q. Referring to Fig.6.1(b), a knowledge of the in-situ specific volume v_1 (preferably arrived by isotropic compression along path ABCD from initial sample preparation specific volume v_0) and p leads to an estimate ψ from known values of λ and Γ of the critical state line. As shown in Fig.6.1(c) the values of Q and ψ for a range of values of p and v can be plotted on log-linear scales to give the straight line EE$'$ from which the values of the coefficients m and k can be derived for any degree of saturation of the soil.

6.5 Data Interpretation

The stages in the interpretation of a field measurement of q_c are outlined in Figs.6.2(a) to 6.2(c). At this point it is necessary to make a further approximation by assuming a value for K_0 in order to calculate p from site values of $\sigma_{v0} = \gamma z$. The value of coefficient of earth pressure at rest, K_0 is reasonably well approximated by (1-sinφ) , the value of φ being obtained from triaxial tests. With this assumption the mean stress p is obtained as follows:

$$\begin{aligned} p &= \sigma_v + 2\sigma_H \\ &= \gamma z\,(1 + 2K_0) \qquad (6.1) \end{aligned}$$

The normalized cone index Q can then be calculated from measured values of q_c as:

$$Q = (q_c - p)p. \qquad (6.2)$$

Then ψ, associated with this estimate of Q, is extrapolated from Fig.6.2(b). Transferring this value of ψ together with p from Fig.6.2(b) to critical state space in Fig.6.2(c) leads to the required specific volume v of the soil. This rather tortuous process allows us to convert the field cone index to a measure of field bulk density and pore space.

6.6 Calibration Chamber Design for Present Investigation

The schematic diagram of the calibration chamber for present investigation is shown in Fig.6.3. Essentially it is a modified triaxial chamber. The objectives to design such calibration chamber are:

(a) to test a miniature cone penetrometer under controlled conditions.
(b) to measure the cone index at different boundary conditions (varying hydrostatic stress at chamber boundary and varying soil sample moisture content).

The calibration chamber design for the present investigation is the modification of an existing ELE 100mm x 50 kN triaxial testing machine. The revised design components are shown in Fig.6.4. These are:

(a) Modified base plate
(b) Top seal (connected to axial hollow shaft)
(c) Axial hollow shaft (modification of existing loading ram)
(d) Provision for ease of assembly without pre-loading or damaging the prepared sample (a 400mm long hollow-shaft assembler and a wooden frame)
(e) Penetrometer aligner (push shaft).

The design details of all this components are presented in Fig.6.5 through Fig.6.7.

(a) *Modified Base Plate*

This is a 100mm dia. MS plate with a 5mm dia. hole at the centre to ensure the water to be drained out during testing. The soil sample is seated on this plate. The detailed design is shown in Fig.6.5(a).

(b) *Top Seal*

This is a 100mm dia. MS plate with threaded hole of ϕ16mm at the centre. The detailed design of top seal is shown in Fig.6.5(b). The axial hollow shaft is screwed hermetically about half-way with it from the top. The remaining bottom unscrewed portion of the hole is left for accommodating the cone during assemblage. The circular grooved portion of the top seal metal is removed to lighten its weight.

(c) *Axial Hollow Shaft*

This is a 25.4mm (with 0.01mm clearance) dia. hollow shaft. The internal dia. of the shaft is 8mm (0.02 clearance) to allow the easy movement of cone penetrometer shaft. The detailed design of the axial hollow shaft is shown in Fig.6.6(a). The externally-threaded end of the shaft can be screwed about half-way with a dowty-washer hermetically with the top seal to stop water passage at high confining pressure within the triaxial cell during testing. The tapered end is for easy and safe insertion of the shaft through the upper hole of the triaxial cell. The internal-thread at the tapered end is for coupling the 400mm hollow-shaft assembler.

(d) *Provision for Ease of Assembly*

The components necessary for the assembly includes a 400mm long (25mm dia.) hollow-shaft assembler with 9mm internal dia. and threaded at one end [detailed design is shown in Fig.6.6(b)] which can be screwed with the axial hollow shaft and hold it. And a wooden frame [detailed design is shown in Fig.6.7(a)] which is held fixed with triaxial testing machine frame and hold the triaxial cell and it also hold the axial hollow shaft (hermetically connected with the top seal) through 400mm long hollow-shaft assembler.

(e) *Penetrometer Aligner (Push Shaft)*

This is either of 300mm in length (for longer distance between the top end of penetrometer shaft and the loading ring) or 50mm (for shorter distance for the same) long and 25mm dia. shaft with ϕ10mm x 10mm deep bore at one end to keep the penetrometer shaft in position and to push the penetrometer concentrically and the other end is screwed with the loading ring. The detailed design is shown in Fig.6.7(b).

The detailed design of miniature cone is shown in Fig.6.8(a). The length of penetrometer shaft is 300mm with ϕ5mm dia. for 100mm length and ϕ8mm (0.02mm clearance) for 200mm length for increasing the shaft stiffness. The detailed design of the cone penetrometer shaft is shown in Fig.6.8(b). The cone is screwed at the 100mm end. The cone surface was machine finished to a smooth surface.

The design components for the calibration chamber and sample preparation accessories are shown in Fig.6.9(a).

6.7 Steps in the Working Principle of the Designed Calibration Chamber

The sequential steps in placing the sample in the calibration chamber ready for testing are shown in Figs.6.9(b) through 6.9(d). The operating principles were as follows:

(a) Referring to Fig.6.9(b), the base plate was screwed and fixed with the triaxial chamber base. The wooden frame was then held fixed with the triaxial machine frame tightly by the butterfly-nut from both ends. The 400mm assembling hollow-shaft was extended through the central hole of the wooden frame and was clipped from upper side. The wooden frame comprises four steel 'U'-shaped hooks at the end of flexible rubber cord connected to it (two nos. at the left side and the other two nos. at the right side, 100mm away, from the central hole). Triaxial cell was then anchored with the wooden frame with the 400mm assembling hollow-shaft passing through the upper hole of the triaxial cell. The axial hollow shaft attached with the top seal was then screwed with the 400mm hollow-shaft assembler. Having done that, the cone penetrometer shaft was inserted into the axial hollow shaft from the bottom and was held it in position with a 'C'-shaped cone penetrometer holder accessory placed horizontally and anchored with the top seal edges when unscrewed half-way the top seal hole can accommodate the cone in it.

(b) Referring to Fig.6.9(c), the prepared sample was placed on the base plate and was encapsuled in a rubber membrane and sealed with two nos. sealing-ring with the base plate. Two other sealing rings were held with split sealing-ring former and was kept with the soil sample. Now the 400mm hollow-shaft assembler was lowered and the 'C'-shaped cone penetrometer holder was removed and allow the top seal to be seated, with the cone penetrometer inside, gently on the top end of the soil sample. The top end of the sample was then sealed with the top seal by two nos. sealing rings from split sealing-ring former placed with it beforehand.

(c) Referring to Fig.6.9(d), triaxial cell was unhooked and lowered carefully by maintaining proper vertical alignment so that the portion of the axial hollow shaft extended through the upper hole of the triaxial cell. To do this smoothly and to stop the leakage of water, high vacuum grease was used on the axial hollow shaft surface. Next, the triaxial cell was fixed tightly with the chamber base. The assembling arrangement was then dismantled and the penetrometer shaft was connected to the penetrometer aligner (i.e. the push shaft). The encapsuled sample was then submerged with water and the water inside the chamber was pressurised up to desired confining pressure. The sample within the calibration chamber was then ready for miniature cone penetration testing.

CHAPTER - SEVEN

EXPERIMENTAL INVESTIGATION AND ANALYSIS

7.1 Triaxial Tests

The objective of triaxial tests were to find the value of c and φ of the experimental soil as well as to measure its critical state parameters. For the present analysis, the value of Γ and λ are sufficient as described in step (a) of section 5.3 (chapter - 5).

(a) *Soil Type*

The soil in the Soil-Machine Mechanics Laboratory of soil testing facility was used in this investigation. The soil was a Sandy clay loam (sand = 65.2%, silt = 14.5% and clay = 20.3%). The Cone penetrometer Plastic limit (CPPL) and Liquid limit (LL) for the soil were respectively 18.0% and 33.0% and is shown in Fig.7.1. The particle size distribution curve for this soil is shown in Fig.7.2. The moisture characteristics of the soil using both Haine's apparatus and Pressure plate extractor (experimental set-up presented in Fig.7.3) are shown in Fig.7.4. All these soil properties were determined according to *BS 1377*: 1990.

(b) *Soil Preparation*

The soil clods were broken down to ensure the particles pass through *BS* 2mm sieve. The sieved soil were used for preparing the specimen for triaxial testing. Five different moisture levels were selected to provide a reasonable number of observations. The different selected moisture levels were 7.0%, 11.0%, 15.0%, 18.0% and 22.0%, more or less maintaining the equal interval between the two successive moisture contents. Among the moisture contents, one was chosen at comparatively dry state (7%), the other at 18.0% (i.e. at CPPL) and the highest one at 22.0% which is far below the LL so as to maintain the unsaturated specimen with the degree of saturation S_r not exceeding 80%. As the experiment was done on remoulded samples, the lowest workable moisture content was selected to be 7.0%, below which the preparation of the sample was not convenient with loose sample (dry density 1.2 Mg/m^3 in this investigation).

Initially soil was allowed to dry in the oven for 24 hours where the temperature was maintained at 55°C. Soil was cooled and distilled water was then added (by weight) to obtain the required level of moisture content. The thorough mixing of water with soils was done carefully by hand as well as with a spatula. The mixture was then sealed in a polythene bag and then thoroughly remoulded by hand for about 10 minutes and left overnight to equilibrate the moisture throughout the soil mass. All the tests were accomplished by maintaining the same dry density of 1.2 Mg/m^3. After completing one set of experiment, the average moisture content of the soil mass was determined by oven dry method by taking a representative sample. The moisture content was increased by adding the excess water to reach the next higher moisture content level for another set of test.

(c) *Triaxial Test Apparatus*

The standard triaxial test apparatus for unsaturated soil, as described by Bishop and Henkel (1962), was used to determine the soil parameter. The triaxial testing machine was essentially the ELE 100mm x 50 kN model. The basic component of the system was a 120mm dia. perspex chamber wherein an unsaturated sample of 38mm diameter and 81mm length sample, encapsuled in a rubber membrane capped at the top and at the bottom with a porous plate, was seated on a pedestal and submerged in a liquid. Load to the sample was applied triaxially by pressurising the chamber fluid and uniaxially through a loading ram whose shaft was extended through the chamber via a hermetically tight coupling.

The triaxial apparatus was fully instrumented. The axial load was monitored automatically by a transducer which was mounted on the proving ring. The axial displacement was monitored by another transducer which recorded the upward movement of the pedestal. The volumetric change of the sample within the triaxial cell (as inflow or outflow of water) was monitored by another transducer mounted on a constant volume cell. The signals were recorded on an X-Y-Y-plotter.

(d) *Calibration of the Apparatus*

An accurate calibration of the apparatus was necessary in order to convert the signal on the X-Y-Y-plotter for actual measurements of load, volume change and axial displacement. All the three transducers were calibrated by comparing the distance traversed by the pens in X and Y direction on the plotter to actual known changes on the triaxial equipment. It was possible to amplify the output signals to any desired magnitude, by the signal conditioning unit and the gain control available on the plotter itself. This allowed the maximum pen deflection to be set to the expected maximum output from each transducer, thus optimizing the accuracy of the record for all ranges of transducer output. Calibration was checked time to time during the long run of experiments. The calibration (compression) of the proving ring is shown in Fig.7.5. The expansion in volume of the triaxial cell at different cell pressure is shown in Fig.7.6. In practice, this was a measure of expansion of the triaxial cell and all hydraulic connections down-stream of the volume measuring transducer.

There were two sources of volume change which occurred simultaneously during either triaxial compression testing or isotropic compression. The volume change sensed by the transducer (VC_t) during isotropic compression was a result of change in the specimen volume and change in the volume of the cell due to expansion (VC_{cx}). The actual volume change of the specimen due to isotropic compression (VC_{ai}) was, therefore, given by:

$$VC_{ai} = VC_t - VC_{cx}. \tag{7.1}$$

The volume change VC_t, recorded during triaxial compression tests, was due to change in specimen volume and water expulsion due to the loading piston entering the cell. Since the loading piston has a diameter of 25.4 mm the volume change (VC_{pm}, in cm^3) due to the piston movement was given by:

$$VC_{pm} \cong 0.507 \text{ dH}, \tag{7.2}$$

where dH is the axial movement (in mm) of piston. Because the piston expelled water from the cell, this was recorded as specimen expansion. Hence the actual volume change in the specimen (contraction +ve) due to triaxial compression (VC_{at}) was given by:

$$VC_{at} = VC_t + 0.507 \text{ dH} \tag{7.3}$$

(e) *Soil Sample Preparation*

To determine the soil parameters 6 tests were chosen at different cell pressure and at a particular moisture content. The accuracy of this measurement depended on maintaining equal and uniform density in the samples. For this purpose, the split sampler was weighed. The volume of sample was known (= 91.9 cm^3) and the dry density to be maintained was 1.2 Mg/m^3. Therefore, the weight of soil required for different moisture contents were known [= 91.9x1.2(1.0+w) gms]. The known weight (with a little excess) soil from the sealed plastic-bag was put into the sampler by a spoon in several steps (5 nos.) each of equal amount. Spoon-fed soil was compacted evenly after each addition. After compacting each layer, the top surface of it was scratched with nail gently and then the soil for the next layer was added.

The idea of the scratching of soil-layer top was to ensure the proper bonding between the two successive layers. When the sampler was full a trimmer was used to trim the excess soil from the top to make the sample height 81mm. The weight of the sampler with soil was taken again. The difference between these two weights was the weight of soil which was adopted to determine its dry density and that was maintained as far as possible equal to 1.2 Mg/m^3. This low dry density (i.e. loose sample) was adopted with a view to obtain an appreciable volumetric change of the specimen in triaxial testing. The test was performed at a particular cell pressure. For test at another cell pressure an identical amount of soil was taken, filled up into the sampler and the soil specimen was prepared in the same way as explained above. The tests were completed in a similar way for different cell pressure and at different moisture contents.

(f) *Test Procedure and Data Processing*

The testing technique for performing the triaxial tests on unsaturated soil described by Head (1986) was followed in this investigation. The sample was first compressed isotropically and was allowed it to swell by reducing the cell pressure to a certain level and then sheared at that particular cell pressure. The axial force was applied on the specimen by raising the top cross head at a continuous rate of 0.7mm/min. The vertical displacement, resultant load and the volumetric change of the sample were continuously recorded on the X-Y-Y-plotter. The failure of the sample can be detected either by a sudden drop in reading (for compacted specimen) or when it remained constant for a certain interval (for loose specimen). In the present contest, situation for loose specimen was applied. In samples where a clear failure point was not observed, the failure value was obtained by calculating the stress after allowing the sample for barrelling. A total of six samples were tested by consolidating at 250kPa pressure and reducing the cell pressures to 25-, 50-, 100-, 150-, 200-, 250-kPa for the same moisture content and dry density. These tests were repeated on each individual moisture content. A typical output on the X-Y-Y-plotter from triaxial test is shown in Fig.7.7. The plot shows the load [Fig.7.7(a)] and volume change [Fig.7.7(b)] with respect to axial displacement of the sample. The recording for the volume change of the specimen during isotropic compression and subsequent swelling is shown in Fig.7.7(c). The line OA [Fig.7.7(b)] is the zero volume-change line during triaxial compression. This is due to the intrusion of loading ram into the triaxial cell as explained in section 7.1(d).

For the calculation of the deviatoric stresses (q), the effect of the increase in the cross-sectional area due to barrelling of the specimen was considered. A computer programme (**programme-4**) was employed to find the soil parameters. The detailed calculation of the stress on the sample due to change in diameter is given in **Appendix-B**. A typical output from the programme with available data is given in Table-7.1. For isotropic compression, the specific volume (v) at different cell pressure ($\sigma_3 = p$) was calculated from sample data as well as the information from X-Y-Y-plotter. In case of triaxial test, the programme calculated the major principal stress (σ_1), spherical pressure [$p = (\sigma_1 + 2\sigma_3)/3$], the specific volume($v$), deviatoric stress ($q = \sigma_1 - \sigma_3$) and the central distance of Mohr's circle from origin ($= \sigma_3 + q/2$) for different cell pressure (σ_3). From these available data, the centres were located on the abscissa as normal stress (σ) on σ-τ-plane. The Mohr's circles (top part) for the six different cell pressures were drawn.

The condition of failure of the sample was approximated by a straight line drawn as a tangent to the circles, the equation of which is the Coulomb's equation $\tau = c + \sigma \tan\varphi$. The value of cohesion (c) was read off the shear stress axis, where it cut by the mean tangent to the Mohr's circles and the angle of internal friction (φ) was the angle between the tangent and a line parallel to the shear stress axis. The best fit tangent line to these circles was drawn by hand (typical example shown in Fig.7.9) and was used to estimate the value of c and φ. The variation of c and φ with respect to water content of the specimen is shown in Fig.7.10 and Fig7.11 respectively.

The spherical pressure (p) and the specific volume (v) from triaxial test data were plotted in v-ln(p)-plane with ln(p) as abscissa. This formed a straight line known as CSL from which the value of Γ (at 1 kPa spherical pressure) and its the slope (λ) were measured. The values of Γ and λ for different moisture content will be discussed in section 7.3 in conjunction with the calibration chamber test results. However, the value of M (slope of CSL on q-p-plane) was calculated from q-p-plot by fitting the regression line passing through the origin and measuring its slope. The

CSL and its slope M at different moisture content is shown in Fig.7.12. The variation of M [values from q-p-plot as well as from equation, $M = 6 \sin\varphi/(3-\sin\varphi)$] with moisture content is shown in Fig.7.13.

7.2 Modified Shear Box Tests

The objective of the shear box test was to find the value of soil-metal frictional angle (δ) and the constrained adhesion (c_a). In this test soil was allowed to slide over the metal (both rough and smooth surface) by the action of a steadily increasing horizontal shearing force, while a constant load was applied normal to the plane of relative movement. These conditions were achieved by placing the soil in a rigid metal box (60- x 60- x 50-mm), square in plan, consisting of two halves (each with 25mm deep) with the bottom half completely occupied by metal block. The lower half of the box could slide relative to the upper half when pushed or pulled by a motorised drive unit while a yoke supporting a load hanger provided the normal load.

(a) *Soil Sample Preparation*

The soil was prepared in the same way described in section 7.1(b). For this test five moisture levels (by weight) were selected with equal interval. These were 5.0%, 10.0%, 15.0%, 20.0% and 25.0%. The dry density of the sample was kept constant (= 1.2 Mg/m^3) as triaxial samples. The soil compactor used in this case was a square steel plate of size just to insert into the box. Metal block was placed into the shear box which covered the bottom half. Soils were placed into the upper portion in three different layers, approximately one-third of total volume contained in each layer. After compacting evenly the first layer, its top was scratched with a knife gently and then the soil for the second layer was added. The scratching of soil made to ensure the proper bonding between the two layers. Similar steps were followed for the subsequent layers. Tests were completed for different levels of moisture content.

(b) *Test Procedure and Data Processing*

The calibration (compression) of the proving ring for shear box test is shown in Fig.7.14. Because the cone penetrometers used in the soil tank experiment (will be discussed in section 7.4) were with both rough and smooth surface, a block of cone penetrometer material (MS) was chosen for this test. This block was available in the departmental soil mechanics laboratory with its one side roughen by bonded sand and the opposite side was smooth. The size of the block was just to cover the bottom part of the shear box. To perform the experiment the bottom part of the shear box was replaced by this block and the rest of the box was filled up with soil. The experiments were carried out at different moisture content both with rough and smooth soil-metal interfaces. A normal load was applied and shearing was continued at a rate of 0.7mm/min. The maximum gauge reading was recorded at which the soil-metal shear-failure occurred. The experiments were performed on five different moisture contents as explained above and on seven different normal loads (incremental) for each individual soil moisture content.

The normal stress was obtained by dividing the normal load with the cross-sectional area (60- x 60-mm) of the shear box. The maximum recorded gauge reading was converted to shear load by using calibration chart (Fig.7.14). The maximum shear load was divided by the cross-sectional area (i.e. the sliding area) of the shear box to obtain the shear stress at soil-metal failure.

The normal stress (σ) and shear stress (τ), both for rough and smooth soil-metal interfaces were plotted on the σ-τ-plane with σ as abscissa. The regression line was drawn to fit the points. The soil-metal frictional angle (δ) was calculated from the slope (μ) of the line following equation $\tan(\delta) = \mu$. The intercepts of these lines along τ-axis were the constrained adhesion (c_a). A typical example is shown in Fig.7.15. The variation of δ and c_a with water content is shown in Fig.7.16 and Fig.7.17 respectively.

7.3 Calibration Chamber Tests

The objective of mini-calibration-chamber test, representative of laboratory conditions, was to obtain the cone index (q_c) for a miniature cone under controlled condition. This will establish the state parameter coefficient m and k as explained in chapter five.

(a) *Soil Sample Preparation*

The sample preparation for the triaxial calibration chamber test was similar to that of triaxial tests but with a large diameter sample (100mm). The height of the sample was 185mm. The volume of sample was, therefore, 1453 cm^3. and the dry density was maintained to be 1.2 Mg/m^3. Hence, the weight of soil required for different moisture content is known [= 1453x1.2(1.0+w) gms]. The preparation of soil was the same as explained in section 7.1(b). For this test a split-mould (100mm dia. and 200mm height), a wooden rammer (96mm dia. with a 30cm long handle) and a trimmer were made (discussed in chapter six) to prepare the sample. The weight of the split-mould was taken. The known weight (with a little excess) soil from the sealed plastic-bag was put into the mould in several steps (10 nos.) each of equal amount and was compacted evenly after each addition. After compacting each layer, the top surface of it was scratched with nail gently and then the soil for the next layer was added. When the sampler was full the trimmer was used to trim the excess soil from the top to make the sample height 185mm. The weight of the sampler with soil was taken again. The difference between these two weights was the weight of soil which was adopted to determine its dry density and that was maintained as far as possible equal to 1.2 Mg/m^3. A number of trials were needed initially to prepare a uniform sample.

(b) *Test Procedure and Data Processing*

The sequential steps for placing the sample into the calibration chamber is explained in section 6.7 (chapter - 6). The sample was compressed isotropically and was allowed it to swell by reducing the cell pressure to a certain level and then the penetration by miniature cone was started. The penetrometer was advanced to the specimen by raising the top cross head at a continuous rate of 0.7mm/min. The penetration depth, load on the cone penetrometer and the volumetric change of the sample due to advancement of cone-penetrometer through the soil sample in the controlled chamber were continuously recorded on the X-Y-Y-plotter. The cone penetrometer was advanced up to 55mm. This was the maximum limit of penetration depth for the calibration chamber (modified triaxial cell) test. A total of six penetration tests were performed by consolidating the samples at 250 kPa pressure and reducing the cell pressures to 25-, 50-, 100-, 150-, 200-, 250-kPa for the same moisture content and dry density. The tests were repeated and performed on three replications at each individual moisture content. A typical output on the X-Y-Y-plotter from triaxial calibration chamber test is shown in Fig.7.18. The plot shows the load [Fig.7.18(a)] and volume change [Fig.7.18(b)] with respect to penetration depth. The recording for the volume change of the specimen during isotropic compression and subsequent swelling is shown in Fig.7.18(c). The line OA [Fig.7.18(b)]' is the zero volume-change line during penetration. This is due to the intrusion of the cone penetrometer into sample within the triaxial cell. As the diameter of the penetrating portion of the shaft was only 5mm the volume of water displaced by the penetrometer shaft was neglegible.

A computer programme (**programme-5**) was employed to find the soil parameters. A typical output from the programme with available data is given in Table-7.2. For isotropic compression, the specific volume (v) at different cell pressure ($\sigma_3 = p$) was calculated from sample data as well as the information from X-Y-Y-plotter [Fig.7.18(c)]. These values of v and p were plotted on v-ln(p)-plane. The values of v and p, for triaxial tests as explained in section 7.1, were also plotted on this v-ln(p)-plane [Fig.7.19 through Fig.7.23] from which the values of N, λ_N, Γ, λ, and k were calculated. The degree of saturation S_r is also shown in these plots. A computer programme (**Programme-5A**) was employed to calculate S_r at different ambient state. The state parameters (ψ) were measured from these v-ln(p)-plot and are shown accordingly at different stress levels and moisture contents.

The cone tip resistance (q_c) for miniature cone was calculated from cone load divided by cone-base area. The cone load was calculated (from X-Y-Y-plotter) by averaging the three replicates for each individual moisture content. This q_c was normalized by cell pressure ($\sigma_3 = p$) as discussed in chapter five. The normalized cone index ,Q [= $(q_c - p)/p$] and ψ were plotted on ln(Q)-ψ-plane with ψ as abscissa. A computer programme (**Programme-5B**) was employed to calculate ln(Q) at different cell pressure (= p). A typical example of ln(Q)-ψ-plot for 7% moisture content is shown in Fig.7.24. The regression line was fitted through the points. The slope of the line is m and the intercept at ψ=0 is ln(k). The values of m and k at different moisture content are shown in summarised Fig.7.25. The variation of m and k with moisture content is shown in Fig. 7.26 and Fig.7.27 respectively. The variation of m and k with the slope of critical state line are also shown in Fig. 7.28 and Fig.7.29 respectively.

The cone penetrometer model (**programme-6**) was employed to predict the miniature cone load for the calibration chamber test. The typical output of the model is given in Table-7.3. A typical experimental and model predicted values of miniature cone load at different cell pressure with moisture content is shown in Fig.7.30. The critical depth limits (cd = f) at different moisture content are also shown in it.

7.4 Soil Tank Experiment

The objective of the soil tank experiment, representative of approximate field situation, was to measure the cone index (q_c) and to validate the prediction from cone penetrometer model using cones with different dimensions.

(a) *Existing Apparatus and Experimental Facility*

The universal Soil-Machine Testing Facility consists of four main functional components namely: the soil bin, the dynamometer and tool suspension carriage, the soil compaction and levelling carriage and the well arranged automatic power systems.

The size of the soil bin is 610-x 76cm with a depth of about 40cm and is made of steel plates. The soil bin is placed parallel to the movement of the tool suspension carriage.

The dynamometer and tool suspension carriage is suspended from and run alongside the soil bin on two parallel rails located in the same vertical plane. The distance between the rails is 122cm. This carriage has a sub-carriage which is able to move in the vertical plane on two vertical rails operated by an electronic motor. This vertical motion carriage has a vertical displacement of about 50cm, which carry the tool and the dynamometer assembly frame. The main carriage is also capable of moving with any speed in the range 0.0061 - 1.83m/sec. The carriage can be driven by a looped chain from variable speed by a 7.5 H.P. motor (D.C.).

The soil compaction and levelling carriage is also suspended and run on the same parallel rails in vertical plane. The compaction of soil can be carried out by a vibrator, which move vertically on two vertical parallel rails and is connected to a flat horizontal plate fixed by two legs. The leveller can also move similarly to the vibrator on two separate parallel vertical rails, and levelling can be carried out with a metal plate connected to the vertically movable frame by two legs. Both the vibrator and the leveller can be powered by two electronic motors.

(b) *Calibration of the apparatus*

Both the transducers were connected to X-Y-plotter through appropriate amplifier. The displacement transducer was fixed with the upper frame of the vertical motion carriage. The thin plastic cord of the transducer was anchored to the main carriage frame from the top. The transducer was then calibrated by comparing the distance traversed by the pens in X direction on the plotter to actual known movement of the vertical motion carriage. The dynamometer (2.5 kN capacity) was calibrated by putting known weight (tension) through flexible wire with its one end fixed with the tool holder assembly and load was suspended incrementally on the other end passed over a frictionless pulley. The dynamometer was connected to the X-Y-plotter through an amplifier and the sensitivity was chosen for appropriate amplification. The dynamometer was then unloaded in the same fashion to check its performance with loading.

(c) *Soil Processing in the Tank*

The preparation of soil in the tank was carried out by thoroughly pulverizing the soil by moving both the leveller and the vibrator plate in opposite direction and then with a spade. The soil was thus raked to a uniform depth over the entire area of the soil bin. The levelling blade was then pulled along the soil tank to level the soil surface. The vibrator plate was set to the soil surface level and passed along the soil tank. The top layer of soil were subsequently compacted in a second pass with the vibrating plate. The vibrating plate level and its compacting speed was marked. Six representative samples were taken to measure its bulk density and moisture content. Representative samples were analysed and checked to maintain the dry density to be 1.2 Mg/m^3. If different, the soil compaction arrangement was re-done. An workable situation was produced, by trial and error, to process the soil with reasonable accuracy for dry density ($\cong$ 1.2 Mg/m^3). In order to conserve the moisture, the bin was kept covered with a polythene between the test runs.

(d) *Test Procedure and Data Processing*

The schematic arrangement of the experimental set-up for the cone penetrometer test in the soil tank is shown in Fig.7.31. The penetrometer shaft was 110cm long and 16mm dia. The cones with different dimensions and surface roughness used in this tests are shown in Fig.7.32(a). These cone penetrometers were screwed and fixed with the shaft. The penetrometer shaft was clamped with tool holder and dynamometer assembly.

The penetrometer was advanced to the soil by lowering the vertical motion carriage electrically at a continuous rate of 7.5 mm/s. The depth of penetration and load due to the advancement of cone-penetrometer within the soil mass were continuously recorded on the X-Y-plotter through amplifier. The complete experimental set-up is shown in Fig.7.32(b). The cone penetrometer was advanced up to 300mm. This was the safe limit to avoid the touching of cone-tip at the bottom of soil bin. The penetrometer was moved with the horizontal carriage along the soil bin to another point with a reasonable distance between two test points (about 50cm c/c) in order to reduce the interference of the failure boundaries. Three to five penetrations were carried out to obtain an average load at different depth at the same moisture content and dry density.

The penetration speed was increased to 42.5 mm/s (maximum possible) to see the effect of penetration rate but no significant difference in the value of penetration load was noticed. This might be due to loose soil. The test was accomplished in the existing moisture content of the soil in the bin and that was 17.5%. A typical output on the X-Y-plotter from this test is shown in Fig.7.33. Note that the soil in the bin beyond 250 mm depth could not be processed completely and it remained as compacted base layer due to which the cone load was inconsistent and unexpectedly larger beyond this depth (Fig.7.33). The soil was reprocessed following the steps as above. The cone was replaced by another cone with different dimensions and surface roughness and the reading for another set of tests were taken.

The cone penetrometer model (**programme-7**) was employed to find the predicted values of cone load. **Programme-7** is similar to **programme-6** but without any boundary conditions. The typical output of the programme for different cone dimensions and surface roughness is shown in Table-7.4. A typical experimental and model predicted values of cone load for 25 mm diameter smooth cone with different cone angles is shown in Fig.7.34.

CHAPTER - EIGHT

COMMENTARY ON EXPERIMENTAL RESULTS

8.1 Triaxial Tests

The effect of moisture content on both c and φ were observed in this test. Referring to Fig.7.10 the cohesion of the experimental soil shows a peak value at a moisture content of 15.0% and falls off as moisture content varies from this value. The peak value occurs at a moisture content very close to the CPPL. Liang (1986) and Hoque (1991) also found similar trend in results but with Birtley clay.

The characteristic variation of cohesion with moisture content can be related to the soil behavioural changes with moisture content. As discussed by Harris (1971) the clay particles in soil interact mainly through the layers of adsorbed water, the diffused ion-layers and mineral contact. The forces holding the water to the clay surface are due to both the clay and the water. In fact, water is a bi-polar molecule with a separation of centres of positive and negative charges. Therefore, water is attracted by the charges on the clay surface. In addition hydrogen ions of water will lead to hydrogen bonding of water molecules to the exposed atoms of the clay mineral surface. The first layer of water molecules are held by hydrogen bonding to the clay surface. The second layer is held to the first layer by hydrogen bonding but the force becomes weaker with distance as the orient influence of the surface on the water molecule decreases. Each successive layer is held less strongly and the bonding quickly decreases to that of free water.

The reduction of cohesion at lower moisture can thus be explained by the lack of water to surround all the particles, which results in less force of attraction. With the increase in moisture more particles have access to water, so the bonding force increased, and hence the cohesion was increased. When the first layer of all the particles were filled up with water, the cohesion reaches its peak value. When more water was applied a second layer was formed, and reduced the bonding force and accordingly the cohesion. As the moisture content was increased the cohesion decreased.

The effect of moisture content on the angle of internal friction in the experimental soil is shown in Fig7.11. It is seen that the φ-value decreased with the increase in moisture content. The trend of these results agree with the findings of Liang (1986) and Hoque (1991). Their experiments were on both Birtley clay and Ryton sand.

As this experiment was conducted to find c and φ at different moisture content at a particular dry density for the developed cone penetrometer model, Fig7.10 and Fig7.11 will be attributed only to such experimental soil condition.

8.2 Shear Box Tests

The effect of moisture content on both δ and c_a, as model parameters, were observed in this test. Referring to Fig.7.16 the δ-value in both smooth and rough soil-metal interface decrease with the increase in moisture content. The value of δ was assumed to be equal to 0.75φ for smooth soil-cone penetrometer interface and equal to φ for a perfectly rough interface. These assumptions were found reasonably consistent with the experimental values.

8.3 Calibration Chamber Tests

It is observed from ln(Q)-ψ-plot (Fig.7.25) that the slope, m and intercept at $\psi = 0$, ln(k) of best-fit lines at different moisture content changes in a systematic fashion. These values of m (Fig.7.26) and k (Fig.7.27) with moisture content can be characterized precisely. Referring to Fig.7.28, it produces a straight line in m-λ-plot which is consistent with the findings of Been *et al.* (1986) and Been *et al.* (1987) but their studies was on pure saturated sand. The trend of k-λ-plot (Fig.7.29) also agree the findings of the above researchers. Referring to Fig.7.30 it is observed that the experimental and model predicted values of cone load hold good within the critical depth limit (f = cd) but this trend is not found satisfactory beyond the critical depth limit. The discrepancy beyond the critical depth limit can be explained by considering an arbitrary example (Fig.8.1) where the sample boundary (ABCD), failure boundary (OEFGIO) and the critical depth (OG) are clearly indicated. The cone should be advanced within the soil sample up to the level at which the failure boundary touch the penetrometer shaft (i.e. point G) to get the actual cone load exerted onto the cone penetrometer. As will be seen in this case, the gravitational moment (under area OEFGIO) and cohesive moment (along failure line OEFG) was considered in calculating the theoretical cone load. But in the laboratory experiment, the cone penetrometer could not be advanced beyond 55 mm (maximum possible) and the diameter of the sample was limited to 100 mm as a practical compromise. Because the gravitational moments due to the area OEBHIO and the cohesive moment along failure line OE only are taken into account it under estimated the cone load. Intuitively, it would be reasonable if the cone could be advanced at least up to critical depth for the actual cone load to be sensed by the load transducer.

8.4 Soil Tank Experiment

In this test, an approximate field situation was created by reprocessing the soil in the tank. Referring to Fig.7.34, a typical result abstracted from numerous penetration tests in the soil bin, it is observed that an excellent correlation exists between the model predicted values and the experimental results of cone load. The cone penetration depth (cp), in all cases of cone angle, is higher then the critical depth. Similar trend in results were also observed using other dia. cone with varying cone angles. The effect of penetration rate on penetration load can be observed from Fig.8.2 both for 7.5 mm/s (curve-A) and 42.5 mm/s (curve-B) penetration speed. The difference is not noticeable and these results can be compared to the results of Turnage (1970, 1974) who found no effect on the penetration resistance with the change of rate of penetration in coarse-grained soil and minimal effect for fine-grained soil. In the present context, the experiment was performed on loose soil and this could be due to this soil condition.

CHAPTER - NINE

PREDICTION OF PENETROMETER PERFORMANCE

9.1 Polynomial Curve Fittings to Experimental Data

To establish precisely the state parameter coefficient m and k as a function of w (Figs.7.28 and 7.29) interpolation within the experimental w range and extrapolation up to near saturation (about 27.0% w) was required. This was achieved by fitting a curve through the experimental data points. The nature of the curve is fitted to a polynomial minimax, the equation to which is of the form:

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{r+1}x^r \qquad (9.1)$$

such that $2|P(x_i) - y_i|$ is minimum for a given set of data points (x_i, y_i) where $i = 1, 2, 3, \ldots, n$ and x-values are in ascending order. The NAG FORTRAN Library Routine - **EO2ACF** has been used to find the polynomial coefficients. This routine uses the exchange algorithm to compute rth order polynomial. A computer programme **(Programme-8)** was employed which connected the Routine-**EO2ACF** as calling programme and solved the supplied data to find up to 4th order polynomial coefficients A_0, A_1, A_2, A_3, A_4 (for k) and B_0, B_1, B_2, B_3, B_4 (for m). The coefficients with analytical data are given in Table-9.1.

The measured and the programme-predicted data are plotted both in m-w-plane (Fig.9.1) and k-w-plane (Fig.9.2). From this observation, m and k can be characterized as a function of w and take the following forms.

$$m = 6.658403 + 0.378318\,x - 0.067790\,x^2 + 0.002117\,x^3 - 0.000015\,x^4 \qquad (9.2)$$

and

$$k = -17.226213 + 15.062773\,x - 1.433905\,x^2 + 0.048894\,x^3 - 0.000570\,x^4 \qquad (9.3)$$

9.2 Prediction

Once the equations for the state parameter coefficients m and k are established in terms of w then it is possible to interpret the field measurement of cone index (q_c) as follows:

$$q_c = Qp + p, \tag{9.4}$$

where $Q = k\exp(-m\psi)$ explained in details in section 5.2 (chapter-5) and $p = \gamma z\,(1 + 2K_0)$ such that $K_0 = 1 - \sin\varphi$ described in section 6.5 (chapter-6). The stages in the interpretation of a field measurement of q_c are outlined in Figs.6.2(a) to 6.2(c) and will not be repeated here. A computer programme (**Programme-9**) was developed to predict q_c against dry density at different moisture content with m and k obtained from equations 9.2 and 9.3. A typical output of the programme is shown graphically in Fig.9.3. The Fig.9.3 shows the predicted values of q_c at a depth of 200 mm for dry densities ranging from 1.1 to 1.8 Mg/m^3 and moisture contents from 7.0 to 22.0%.

Another programme (**Programme-10**) was developed to predict q_c against moisture content at different dry density with m and k obtained as above. A typical output of the programme is shown graphically in Fig.9.4. The Fig.9.4 shows the predicted values of q_c at a depth of 200 mm for moisture contents ranging from 5.0 to 25.0% and dry densities from 1.33 to 1.78 Mg/m^3.

The **programmes-9** and **10** can be used to predict q_c against dry density as well as to predict q_c against moisture content similar to Figs.9.3 and 9.4 for any specified depth of penetration z ($>f$).

Typical outputs from the selected cone penetrometer runs in the soil tank are converted to cone-index vs depth plot and are shown in Fig.9.5 and Fig.9.7 respectively for a 20.0 mm diameter 30° smooth cone and a 25.0 mm diameter 60° smooth cone. Once charts equivalent to Fig. 9.5 (or Fig.9.7) can be prepared for a given field soil, at any moisture content of interest, then it is possible to predict the dry density using Fig.9.3 at that particular depth of penetration for which the q_c vs dry density plot in Fig.9.3 is produced.

The procedural stages to predict the dry density are as follows:

(a) Find the value of cone index corresponding to cone penetration depth from Fig.9.5 (or Fig.9.7) .

(b) With this value of cone index, find the dry density at the moisture content of interest from Fig. 9.3.

Typical examples of the predicted values of dry density at 17.5% moisture content (interpolating data between 15% and 18.0% m.c.) are shown in Fig. 9.6 and Fig.9.8 for the above mentioned two different cones.

In both instances the predicted dry density curve lies well above the in-situ estimated dry density of the soil in the tank. The latter value was obtained from core-sampler data carried out on the surface layer of the soil in the tank. Although the procedure appears to over-predict the dry density the deviation reflects the higher in-situ densities in the deeper layers of the tank. On this basis the performance of the model would appear to be acceptable.

CHAPTER - TEN

SUMMARY AND CONCLUSIONS

10.1 Visual Graphics of Critical State Surfaces

The driving programme for the graphical presentation of Critical State space was developed by incorporating a major change to the mathematical description of the Roscoe surface. In saturated soils the spacing ratio in the Cam-clay and modified Cam-clay models was tacitly assumed to be 2. This implies that the ratio R_s of the equivalent spherical pressure on the ICL (p_i) and the corresponding value (p_c) on the CSL have a fixed value of $R_s = p_i / p_c = 2$. As a consequence of this the elliptical Roscoe surface always passes through the origin ($p = 0$ line) and requires that $\lambda_N = \lambda$. In partly saturated soils the latter equality does not hold and the model was developed for the most general case when $\lambda \neq \lambda_N$.

It was found that the UNIMAP Graphics package available on the University Computing network had insufficient resolution to plot smooth curves for the chosen contours of the Critical State surfaces. Decreasing the interpolating interval did improve matters but did not eliminate the problem entirely, especially near the ICL. The diagrams given in Figs.3.15-3.18 had to be hand drawn off unsmoothed plots, which defeated the main objective of the exercise. However, the two-dimensional plots, using UNIGRAPH Graphics package, did not suffer from this difficulty and anisotropic compression lines have been plotted, probably for the first time, for the general case on both the Roscoe and Hvorslev surfaces (Figs. 3.19 - 3.24).

The best available data on three unsaturated soils (O'Sullivan *et al.,* 1994) was used to prepare graphic images of their Critical State Surfaces. These show, most dramatically, the influence of moisture content changes on Critical State Space. This aid provides a ready means for qualitative interpretation of the changes in the behaviour of these soils with moisture content (see Section 3.7). The graphic package is therefore another useful tool for the analysis and interpretation of soil data. However, the major difficulty of actually measuring the parameters necessary to prepare the graphic images still remains unresolved.

10.2 Cone Penetrometer Model

The basic Sokolovski solution applies to boundary conditions where the singular point (see for example point O in Fig. 4.1) is accessible. A cone penetrometer passes through three boundary condition phases. In the initial phase (drop-cone case) the solution is clearly with a singular point. As the cone rim penetrates below the soil surface the boundary conditions at the singular point are not clearly defined. The solution to this phase has not been attempted here. The case when the outer rupture boundary clears the soil surface once again returns the analysis to a singular-point solution. In this case the penetration depth exceeds the critical depth f . A detailed analysis of the slip-line field for this case has been presented [see Figs. 4.9 (a) to (e)].

This analysis allows for the degree of roughness at the shaft and the cone surface-boundaries which have important consequences in determining the proportions of the rupture surface. An incompatibility occurs between the slip-lines associated with the shaft boundary and those generated by the cone surface. This was overcome by introducing a plane of discontinuity, which is a legitimate part of such boundaries. The diagrams show that as the roughness ratio $\rho = \delta/\varphi$ is increased from the unrealistic perfectly smooth case ($\rho = 0$) to the more practical values approaching perfect roughness ($\rho = 1$) the slip-line fields from the two boundaries transits in a systematic manner.

Once an acceptable slip-line field was formulated the problem of converting the two-dimensional field to the three-dimensional one was tackled. The empirical method employed is outlined in Fig. 4.23. The final computer model is capable of predicting the cone load (and hence cone index) for known values of c and φ. The solution is a comprehensive one which caters for a variety of soil-to-metal parameters of both the shaft and the cone.

The model, however, suffers for the fact that it becomes operational only when penetration depth $z > f$. The early phase, when the penetrometer is passing the surface layers, cannot be modelled. The theoretical basis for this phase exists but, considerations of time, precludes an attempt to resolve it in this study. However, this is not a serious drawback because the top 50 mm of soil in a field is, in any event, in a very variable state and initial penetrometer readings in the soil crust are generally unreliable.

10.3 State parameter ψ

The main concept of a state parameter to describe the "state" of a soil element is a powerful one and it is surprising that it has not been utilised to tackle problems in soil-machine interactions before. Published data on its use are limited and confined to dry sands. It was therefore felt that its use in partly-saturated c-φ soils might be suspect. The calibration chamber investigations helped to allay some of this uncertainty and its performance was most encouraging.

Essentially the state parameter ψ of a soil element in Critical State space is the distance (in non-dimensional *v-units*) of a soil element from the Critical State wall (see Fig. 5.1). Thus ψ is negative when the element is under the Hvorslev surface, zero at Critical State and positive when the element is under the Roscoe surface. The sign of ψ thus gives a direct indication as to whether the soil will dilate or compact when sheared. Actual values of ψ obtained from the calibration chamber are shown in Figs. 7.19 - 7.23. The degree of saturation of the samples are also shown on these diagrams.

It is remarkable that the plot of $\ln(Q)$ vs ψ (where Q is the non-dimensional cone index) is consistently **linear** for the range of moisture contents used in the tests (see Figs. 7.24 and 7.25). This gives a high degree of confidence in extending the state parameter concept to partly saturated soils, at least in the sandy loam soil used in the experiments. There is no reason to doubt that the performance would deviate from this even in a clay soil, but obviously further clarification would be desirable.

The linear coefficients m and k of the $\ln(Q)$ vs ψ plots also show a systematic variation with moisture content (Figs. 7.26 and 7.27) and an unexpected linear correlation of m with the Critical State Line slope λ. The investigation thus provides the basis for developing a mathematical relationship for the variation of these parameters with soil moisture contents.

10.4 Computer Simulation

The basic strategy for the prediction of pore-space changes from cone penetrometer tests is set out in Figs. 6.1 and 6.2. The calibration process shown in Fig. 6.1 using a miniature penetrometer and a soil sample from the soil tank, in which simulated field tests were to be conducted, yielded values of m and k for a range of moisture contents. The Sokolovski model was then used to predict the miniature cone load and the results are given in Fig. 7.30. The agreement is, once again, surprisingly good when the penetration depth exceeds the limiting value f. The theory over-predicts the cone index when $z < f$ and this is not entirely unexpected.

The simulated "field test" of the computer models was carried out in a laboratory soil tank, which had the same soil as that used in all the other tests, including the calibration chamber runs. It was felt that a test carried out under real field conditions would be self-defeating as there would be no possibility of obtaining reliable information of all the parameters required by the predictive models. In contrast all these parameters were known to a very high degree of confidence for the soil in the test facility tank.

The tank tests utilized a range of cone geometries with cone angles ranging from the standard value of 30° to blunt cones with apical angles of 135°. The prediction of the Sokolovski model was exceptionally good over the full range of cone angles (see Fig. 7.34).

The translation of cone index into a more useful evaluation of in-situ bulk density is embodied in the charts given in Figs. 9.3 and 9.4. These have been prepared from the calibration chamber data and can be used to "convert" cone index to actual bulk density values for a range of moisture contents. The application of this technique to a selected cone penetrometer run in the soil tank is shown in Figs. 9.5 and 9.6. The prediction performance is encouraging. The technique appears to over-predict the bulk density. Hence, if charts equivalent to Fig. 9.5 can be prepared for a given field soil, then penetrometer readings can be readily "converted" to bulk density or pore space estimates. This appears to be a massive undertaking for even a single soil and may prove impracticable on this account. Nevertheless it is theoretically feasible and it may be possible to develop some short-cuts, which may make this technique more manageable.

10.5 General Comments

The investigation has demonstrated that it is indeed feasible to formulate theoretical mathematical models of both Critical State space and the performance of cone penetrometers. The analyses presented are, in a sense, as rigorous as can be expected when dealing with such complex conditions and does not violate basic mathematical requirements. Empirical simplifications and compromises have been introduced in the interest of simplicity. The experimental validation bears this out as order of magnitude predictions can be made with confidence. However, this view must be tempered by the fact that the validation presented in this study has been carried out for a single soil and that too under strictly controlled laboratory conditions.

The basic idea of using state parameter to define the condition of a partly saturated soil shows considerable promise. This drift is reinforced by the experimental work in the calibration chamber. Encouraging though this may be, the trends found are only for a single soil.

The software development and the formulation of the mathematical background associated with it are both time consuming activities, but when completed require little modification and is generally a once and for all process. However, the experimental work is repetitive, tedious and time consuming in the extreme and each new soil will require this form of attention. There is no respite from this aspect of this investigations.

10.6 Future Work

The present investigation has only been able to touch on the periphery of the problems it set out to study. Much further work needs to be done to reinforce the present findings. The main areas that merits fresh attention are summarized below:

(a) A theory is required to fill in the missing penetrometer range $0 < z < f$. There is evidence that the Sokolovski analysis can be readily adapted to do this. A computer model can then be presented for the full range of indentation devices.

(b) The Critical State space simulation requires a Graphics package which will eliminate the smoothing problems. This may prove to be more difficult than appears at first because the problem is most acute at the junction of the Roscoe surface with the ICL, where the changes in surface gradients are steepest. [During the preparation of this thesis the University Computing Service has just introduced the latest Advanced Visual System. This package may be able to overcome the shortcomings of UNIMAP package].

(c) The visual impact of the Graphics presentation scheme would be enhanced if the programme could be extended to automatically trace specified state paths. This extensions could be used to obtain quantitative data in soil-machine interactions; the scheme developed can only supply qualitative information.

(d) The experimental investigations have been confined to a single soil. The work needs to be repeated for other soil types, preferably a clay soil.

(e) Comprehensive charts of bulk density variations with cone index and moisture content (similar to Figs. 9.3 and 9.4) need to be prepared for known soils for a more comprehensive evaluation of the state parameter concept. If such charts are available for field soils, then it would be a simple matter to convert cone index to soil bulk density.

Notations:

φ	=	Angle of internal friction, deg.
γ	=	Bulk unit weight, kN/m^3
Γ	=	CSL intercept on *v-ln(p)* plane at $p = 1$ kPa
σ	=	Normal stress, kN/m^2
α	=	Opened-up cone angle, deg.
τ	=	Shear stress, kN/m^2
λ	=	Slope of CSL on *v-ln(p)* plane
κ	=	Slope of swelling line on *v-ln(p)* plane
ψ	=	State parameter
η	=	Stress ratio (= q/p)
δ	=	Soil interface friction angle, deg.
ρ	=	Roughness ratio = δ/φ
$\sigma_\gamma, \sigma_c, \sigma_q$	=	Normal stresses on the cone element, kN/m^2
$\tau_\gamma, \tau_c, \tau_q$	=	Shear stresses on the cone element, kN/m^2
θ^+, θ^-	=	Inclination of S^+ and S^- slip direction with interface, deg.
$\sigma_1, \sigma_2, \sigma_3$	=	Principal stresses, kN/m^2
σ_V	=	Vertical geostatic stress, kN/m^2
σ_{V0}	=	Stress at cone level, kN/m^2
σ_H	=	Horizontal geostatic stress, kN/m^2
δ_c	=	Cone-soil friction angle, deg.
θ_c	=	Half of cone angle, deg.
λ_N	=	Slope of ICL on *v-ln(p)* plane
δ_s	=	Shaft-soil friction angle, deg.
A_p	=	Projected area of cone base, m^2
A_s	=	Adhesion force on the shaft surface, kN/m^2
c	=	Cohesion, kN/m^2
c_a	=	Soil-cone constrained adhesion, kN/m^2
c_{as}	=	Soil-shaft constrained adhesion, kN/m^2
CSL	=	Critical state line
D_s	=	Sample diameter, m
d_1, d_2, d_3, d_4	=	Moment arms
d_c	=	Cone diameter, m
d_s	=	Shaft diameter, m
f	=	Critical depth, m
F_c	=	Cone resistance force, kN
$F_{H\gamma}, F_{Hc}, F_{Hq}$	=	Normal force components per unit width acting on the shaft surface
h	=	Slope of Hvorslev surface on *q-p* plane
H_c	=	Cone height, m
ICL	=	Isotropic compression line

k, m	= State parameter coefficients
N_γ, N_c, N_q	= Dimensionless factors
K_0	= Coefficient of Earth pressure at rest
L	= Cone face length, m
L_s	= Sample length, m
M	= Slope of CSL on q-p plane
M_c	= Cohesive moment
M_w	= Gravitational moment
N	= ICL intercept on v-$ln(p)$ plane at $p = 1$ kPa
p	= Mean normal stress, kN/m^2
p_c	= Stress level at CSL, kN/m^2
p_i	= Stress level at ICL, kN/m^2
P_{pc}	= Preconsolidation pressure, kN/m^2
P_γ, P_c, P_q	= Soil reactions on interface
q	= Deviatoric stress, kN/m^2
Q	= Normalized cone index, $(q_c - p)/p$
q_c	= Cone index, kN/m^2
Q_{CONE}	= Total cone load, kN
q_s	= Surcharge, kN/m^2
r	= Interface length, m
R_s	= Stress (mean) ratio $= p_i / p_c$
R	= Over-consolidation ratio $= p_i / p$
R_γ, R_c, R_q	= Resultant force acting on the cone surface
S^+, S^-	= Slip direction in physical plane
T	= TCL intercept on v-$ln(p)$ plane at $p = 1$ kPa
T_γ, T_c, T_q	= Tangential force components per unit width acting on the shaft surface
TCL	= Tension cut-off line
TF_γ	= Total gravitational component
TF_{ca}	= Total cohesive-adhesive component
TF_q	= Total surcharge component
u_a	= Pore air pressure (gauge)
u_w	= Pore fluid pressure (-ve = suction)
v	= Specific volume
v_c	= Specific volume at CSL for p_c
v_i	= Specific volume at ICL for p_i
w	= Gravimetric moisture content
W_c	= Weight of cone
W_s	= Weight of shaft
z	= Penetration depth, m

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Appendix - A

A.1 Detailed Analysis of Forces by Newcastle Method

(a) *Basic Rupture Surface*

The basic slip-line field appropriate to passive failure in front of an interface with a large rake angle is shown in Fig.4.3(a). The direction of the slip lines on the Rankine zone OCD is governed by the gravitational field, the slope of the soil surface and the nature and direction of action of the surcharge pressure. In the basic problem, if the last two variables are restricted to a horizontal soil surface, the slip-lines are straight and make angle of

$$\varepsilon = 45^0 - \frac{\varphi}{2} \tag{A. 1}$$

with the horizontal surface. The zone OAB which is adjacent to the interface is known as the interface zone. The slip lines in this zone are straight lines and controlled by the values of δ and φ. The geometry of this zone governed by angle OAB = θ^+ which can be seen from the Mohr diagram [Fig.4.3(b)] and its magnitude can be readily determined as follows:

$$\theta^+ = 45^0 + \frac{\varphi}{2} + \frac{(\Delta + \delta)}{2} \tag{A. 2}$$

with Δ written as:

$$\Delta = \sin^{-1} \frac{\sin \delta}{\sin \varphi}. \tag{A. 3}$$

The value of θ^- can be calculated as

$$\theta^- = 45^0 + \frac{\varphi}{2} - \frac{(\Delta + \delta)}{2}. \tag{A. 4}$$

In analysing the magnitude of the interface, adhesion is fixed by δ as it is assumed that $a = c_a = c \tan\delta \cot\varphi$. As δ tends to zero the interface of adhesion plays a dominant role in controlling the orientation of the slip planes at the interface in contrast with the part played by the diminished value of δ. When $\varphi = 0$ the product $\tan\delta \cot\varphi$ and the ratio ($\sin\delta/\sin\varphi$) becomes indeterminate and it is then essential to use the ratio a/c to estimate the angle Δ. The revised value of this angle can be evaluated from the Mohr's diagram in Fig.4.3(c). as:

$$\Delta = \sin^{-1}\frac{a}{c} \tag{A. 5}$$

The transition zone OBC simply fills up the slip line field, if any, between the interface and Rankine zones. This zone is composed of curved slip lines, which are logarithmic spirals, and straight radii and the slip line field merges smoothly into the adjacent two zones and has the form:

$$f = r\, e^{\eta \tan\varphi}, \tag{A. 6}$$

where η is the angle between the radii of length f and r as shown in fig 4.4. Hence the three variables φ, δ and the rake angle α would suffice to make the rupture shape and size of soil at failure.

(b) *Calculation of Forces for Basic Rupture Surface*

The forces on the interface are made up of two components, the frictional soil resistance P acting at an angle δ to the interface and the tangential adhesion force A as shown in Fig.4.3(a). The latter can be easily calculated from $A = a\, z\, \mathrm{cosec}\alpha$. The main task is therefore the determination of P from the equilibrium of all the forces acting on the soil body OABCD shown in Fig.4.3(a). The line of action of P (Fig.4.4) can be determined by separating it into its gravitational-frictional component P_γ, cohesive-adhesive component P_c, and surcharge component P_q in accordance with the assumption (h) mentioned in section 4.3 (chapter - 4).

The other assumptions in section 4.3 (chapter - 4) are sufficient to determine the magnitude, location and direction of action of the forces F_1, F_2, W, W_1, Q and A_1. The forces F_1 and F_2 are two Rankine passive stresses acting on the vertical side CE (Fig.4.4) can be given as:

$$F_1 = \left[2\, c \tan(45^0 + \frac{\varphi}{2}) + q \tan^2(45^0 + \frac{\varphi}{2})\right] CE \tag{A. 7}$$

and $$F_2 = \frac{\gamma}{2}\tan^2(45^0 + \frac{\varphi}{2}) CE^2. \tag{A. 8}$$

The magnitude of cohesive moment M_c about O acting along the spiral sector BC can also be estimated. Notice here that F_3, due to properties of logarithmic spiral surface BC, will pass through the pole O. Therefore F_3 will disappear from force equilibrium calculation when taking moment about O. A solution of P can be obtained by considering the static equilibrium of the transition zone and half the passive Rankine zone forming the body OBCE in Fig.4.4(b), followed by the static equilibrium of the interface zone OAB in Fig.4.4(a). This requires the introduction of three new internal forces F_4, F_5, and A_1, the latter being easily determined as the product of length OB with soil cohesion c.

The static equilibrium of the body OBCE is analysed in two stages as follows:

(a) under the action of gravitational and frictional forces only, so that $c = 0$, $q = 0$ and $c/\gamma z = q/\gamma z = 0$.

(b) under the action of cohesion, surcharge and friction with $c > 0$, $q > 0$ and $c/\gamma z > 0$, $q/\gamma z > 0$.

This assumes that the principle of superposition holds good and is a well known tool introduced by Terzaghi (1959). The technique is not applicable in a rigorous analysis because it assumes that the slip-line field remains unchanged in the two cases. However, it is perfectly applicable to the present methods introducing no additional error. The analysis then proceeds as follows:

(i) *Stage - 1*: Equilibrium of OBCE

(a) Gravitational and Friction; Moment equilibrium about O:

$$F_4 d_4 = F_2 d_2 + W_1 d_w + (Wd) \tag{A. 9}$$

(b) Cohesion and Friction; Moment equilibrium about O:

$$F_{5c} d_5 = F_1 d_1 + M_c \tag{A.10}$$

(c) Surcharge and Friction; Moment equilibrium about O:

$$F_{5q} d_5 = F'_1 d_1 + Qd_q \tag{A. 11}$$

The moments M_w (= Wd) and M_c can be evaluated from the following expressions in which the boundary radii of the logarithmic spiral zone are OB = r and OC = f and the included angle BOC = η.

$$M_w = \frac{\gamma f^3}{3(m^2+1)}\{e^{m\eta}[m\cos(\varepsilon+\eta)+\sin(\varepsilon+\eta)]-m\ \cos\varepsilon-\sin\varepsilon\} \qquad (A.\ 12)$$

$$M_c = \frac{c(f^2-r^2)}{2\tan\varphi}, \qquad (A.\ 13)$$

where m = -3 tanφ and $\varepsilon = (45^0 - \frac{\varphi}{2})$.

(ii) *Stage - 2*: Equilibrium of OAB

Considering the equilibrium of the force components perpendicular to F_6 it is helpful to note that A_1 is parallel to F_6 and hence does not enter into the calculations and F_5 and F_4 are parallel to A_2. The equilibrium can, therefore, be written as:

$$P = \operatorname{cosec}(\theta^+ - \varphi - \delta)\{W_2\sin(\theta^+ - \varphi + \alpha) + A\cos(\theta^+ - \varphi) + (F_4 + F_5 + A_2)\cos\varphi\} \qquad (A.14)$$

The values of frictional-gravitational component P_γ, cohesive-adhesive component P_c and surcharge component P_q can be separated and their lines of action can be located assuming P_γ acts two-thirds the way along the interface from O and the corresponding positions of P_c and P_q are half way from O. Then

$$P_\gamma = \frac{W_2\sin(\theta^+ - \varphi + \alpha) + F_4\cos\varphi}{\sin(\theta^+ - \varphi - \delta)} \qquad (A.\ 15)$$

$$P_c = \frac{A\cos(\theta^+ - \varphi) + (F_{5c} + A_2)\cos\varphi}{\sin(\theta^+ - \varphi - \delta)} \qquad (A.\ 16)$$

and $$P_q = \frac{F_{5q}\ \cos\varphi}{\sin(\theta^+ - \varphi - \delta)}. \qquad (A.\ 17)$$

Thus the resultant passive soil force P is the vector sum of the above forces:

$$P = P_\gamma + P_c + P_q \qquad (A.\ 18)$$

or $$P = \gamma z^2 K_\gamma + (cz\,K_c + az\,K_a) + qz\,K_q, \qquad (A.\ 19)$$

where K_γ, K_c, K_a, and K_q are called 'K-factors' and they are non-dimensional soil resistance coefficients, each of them is a function of the following variables:

$$K = f(S_c, S_a, S_q, \varphi, \delta, \alpha), \tag{A. 20}$$

where $S_c = c/\gamma z$, $S_a = a/\gamma z$ and $S_q = q/\gamma z$. These three groups of numbers are called 'Soil Numbers'. The equation (A.19) was first suggested by Reece (1965) and is called the general soil resistance equation. Hettiaratchi and Reece (1974) prepared charts to calculate the soil resistance coefficients for a wide range of soil and interface loading conditions.

A.2 Detailed Analysis of the Developed Cone Penetrometer Model

In analysing the stresses in the shaft boundary [Fig.4.11(a)] it has been assumed a plane shear zone which necessarily means that the principal stresses are not vertical or horizontal (i.e. modified Rankine zone). Mohr's diagram for this case is shown in Fig.4.11(b). From the triangle OAC [Fig.4.11(b)]

$$\Delta_s = \sin^{-1}\frac{\sin\delta_s}{\sin\varphi} \tag{A. 21}$$

and from ΔACD ($\equiv\Delta$AFE)

$$\mathrm{AD} = \mathrm{AE} = \mathrm{R}\cos(\delta_s + \Delta_s). \tag{A. 22}$$

As $\mathrm{R} = \mathrm{OA}\sin\varphi$ therefore,

$$\mathrm{R} = (c\cot\varphi + \mathrm{S})\sin\varphi. \tag{A. 23}$$

From equations (A.22) and (A.23) with the assumption $(\delta_s + \Delta_s) = \mu_s$

$$\mathrm{AE} = c\cos\varphi\cos\mu_s + \mathrm{S}\sin\varphi\cos\mu_s. \tag{A. 24}$$

Now, $\mathrm{S} = \gamma z + q + \mathrm{AE}$. Putting the value of AE and rearranging, it can be written as

$$\mathrm{S} = \frac{\gamma z + q + c\cos\varphi\cos\mu_s}{1 - \sin\varphi\cos\mu_s}. \tag{A. 25}$$

The magnitude of σ_x can be written as:

$$
\begin{aligned}
\sigma_x &= S + AE \\
&= S\,(1 + \sin\varphi \cos\mu_s) + c \cos\varphi \cos\mu_s \\
&= \left[\frac{\gamma z + q + c\ \cos\varphi\ \cos\mu_s}{1 - \sin\varphi\ \cos\mu_s}\right](1 + \sin\varphi \cos\mu_s) +\ c \cos\varphi \cos\mu_s \\
&= (\gamma z + q + c \cos\varphi \cos\mu_s)\, K_1 + c \cos\varphi \cos\mu_s.
\end{aligned}
$$

i.e. $\sigma_x = \gamma z K_1 + c\ \cos\varphi\ \cos\mu_s (K_1 + 1) + qK_1$ (A. 26)

such that $\dfrac{1 + \sin\varphi\ \cos\mu_s}{1 - \sin\varphi\ \cos\mu_s} = K_1$.

or $\sigma_x = \gamma z K_1 + cK_2 + qK_1,$ (A. 27)

where $\cos\varphi\ \cos\mu_s (K_1 + 1) = K_2$. The coefficients K_1 and K_2 are functions of δ_s and φ. The three components in the right hand side of equation (A.27) are respectively the gravitational, cohesive-adhesive and surcharge components and these are shown in Fig.4.12.

The normal force per unit width dF, acting on the shaft surface, can be written as

$dF = \sigma_x\, dz.$ (A. 28)

Therefore total force

$$
F = \int_{z-f}^{z} \sigma_x\, dz
$$

$$
= \gamma K_1 \int_{z-f}^{z} z\, dz + cK_2 \int_{z-f}^{z} dz + qK_1 \int_{z-f}^{z} dz
$$

or

$F = \dfrac{1}{2} \gamma K_1 f (2z - f) + cfK_2 + qfK_1.$ (A. 29)

Separating the force components for gravitational ($F_{H\gamma}$), cohesive (F_{Hc}) and surcharge (F_{Hq}) we get:

$F_{H\gamma} = \dfrac{1}{2} \gamma K_1 f (2z - f)$ (A. 30)

$F_{Hc} = cfK_2$ (A. 31)

$F_{Hq} = qfK_1$ (A. 32)

and the corresponding tangential components per unit width can be written respectively as:

$$T_\gamma = F_{H\gamma} \tan \delta_s \tag{A. 33}$$

$$T_c = F_{Hc} \tan \delta_s \tag{A. 34}$$

$$T_q = F_{Hq} \tan \delta_s. \tag{A. 35}$$

The forces acting on the shaft surface are shown in Fig.4.13. The tangential force components T_γ, T_c and T_q will be required when the equilibrium of cone and shaft is calculated. Adhesion along the shaft surface A_s is simply the product of c_{as} and area of the shaft.

i.e. $$A_s = c_{as}\, \pi\, d_s\, z \tag{A. 36}$$

The cohesive moment (M_c) and gravitational moment (M_w) for the rupture surface shown respectively in Fig.4.14 and Fig.4.15 can be calculated as follows:

$$M_c = \frac{c\,(f^2 - r^2)}{2\ \tan\varphi} \tag{A. 37}$$

for $\varphi > 0$. For the condition $\varphi = 0$, $M_c = cr^2\eta$.

$$M_w = \frac{\gamma f^3}{3(m^2+1)}\{e^{m\eta}(m\ \sin\eta - \cos\eta) + 1\}, \tag{A. 38}$$

where $m = -3\tan\varphi$. The value of η and rupture distance f (designated as critical depth) can be calculated from Fig.4.16 as follows:

$$\begin{aligned} \eta &= 270° - (90° - \theta_c) - \theta^- \\ &= 180° + \theta_c - \theta^- \end{aligned} \tag{A. 39}$$

$$f = r\, e^{\eta \tan\varphi}, \tag{A. 40}$$

where $\theta^- = 45^0 + \frac{\varphi}{2} - \frac{(\Delta_c + \delta_c)}{2}$ with $\Delta_c = \sin^{-1}\frac{\sin\delta_c}{\sin\varphi}$ as discussed in section A.1(a).

From Fig.4.17 the value of S_c, r and t_c can be calculated as follows:

$$S_c = \frac{d_c}{2\ \sin\theta_c}, \tag{A. 41}$$

$$r = S_c \frac{\sin\theta^+}{\cos\varphi} \tag{A. 42}$$

and $$t_c = S_c \frac{\sin\theta^-}{\cos\varphi}, \tag{A. 43}$$

where $\theta^+ = 45^0 + \frac{\varphi}{2} + \frac{(\Delta_c + \delta_c)}{2}$ with $\Delta_c = \sin^{-1}\frac{\sin\delta_c}{\sin\varphi}$ as discussed in section A.1(a).

Referring to Fig.4.18 the moment arms are:

$$d_1 = \frac{f}{2} \tag{A. 44}$$

$$d_2 = f\left[\frac{3z-2f}{3(2z-f)}\right] \tag{A. 45}$$

$$d_3 = \ = \frac{1}{2} r\cos\varphi \tag{A. 46}$$

$$d_4 = \ = \frac{2}{3} r\cos\varphi, \tag{A. 47}$$

and the forces can be calculated (taking moment at point O) as follows:

$$P_\gamma = \frac{F_{H\gamma}\, d_2 + M_w}{d_4} \tag{A. 48}$$

$$P_c = \frac{F_{Hc}\, d_1 + M_c}{d_3} \tag{A. 49}$$

$$P_q = \frac{F_{Hq}\, d_1}{d_3}. \tag{A. 50}$$

The forces acting on the soil wedge is shown in Fig.4.19(a). Adhesion forces A_1, A_2 and A_c per unit width can be calculated as follows:

$$A_1 = r.\ c \tag{A. 51}$$

$$A_2 = t_c.\ c \tag{A. 52}$$

$$A_c = c_a.\ S_c. \tag{A. 53}$$

The weight of soil wedge W per unit width can be calculated [Fig.4.19(b)] as:

$$W = \frac{1}{2}\gamma . S_c . h \qquad (A. 54)$$

such that $h = t_c \sin\theta^+$. The equilibrium of all these forces acting on the soil wedge is shown in Fig.4.20. At equilibrium condition,

$$(P_\gamma + P_c + P_q)\cos\varphi + W\sin\varepsilon_2 + A_c\sin\theta^- + A_2\cos\varphi - (R_\gamma + R_c + R_q)\sin\varepsilon_1 = 0 \qquad (A. 55)$$

such that $\varepsilon_1 = (90 - \delta - \theta^-)$ and $\varepsilon_2 = (\theta^- - \theta_c)$. These follows that the resultant forces acting on the cone surface are:

$$R_\gamma = \frac{P_\gamma \cos\varphi + W \sin\varepsilon_2}{\sin\varepsilon_1} \qquad (A. 56)$$

$$R_c = \frac{P_c \cos\varphi + A_c \sin\theta^- + A_2 \cos\varphi}{\sin\varepsilon_1} \qquad (A. 57)$$

$$R_q = \frac{P_q \cos\varphi}{\sin\varepsilon_1}. \qquad (A. 58)$$

Now, if the cone is opened-up (Fig.4.21) then it follows the angle $\alpha = 2\pi\sin\theta_c$ in radians and the slant surface area:

$$A = \frac{\pi d_c^2}{4\sin\theta_c}. \qquad (A. 59)$$

As shown in Fig.4.22 the force components normal to the cone surface for gravitational, cohesive and surcharge components are respectively

$$R_{N\gamma} = R_\gamma \cos\delta \qquad (A. 60)$$

$$R_{Nc} = R_c \cos\delta \qquad (A. 61)$$

and $$R_{Nq} = R_q \cos\delta, \qquad (A. 62)$$

and the corresponding tangential components are

$$R_{T\gamma} = R_\gamma \sin\delta \qquad (A. 63)$$

$$R_{Tc} = R_c \sin\delta \qquad (A. 64)$$

and $$R_{Tq} = R_q \sin\delta. \qquad (A. 65)$$

The stresses on the element of cone; for gravitation (Fig.4.23)

$$\sigma_\gamma = K_s . l \qquad \text{(A. 66)}$$

such that $K_s = \dfrac{2 R_{N\gamma}}{L^2}$; for cohesion and surcharge

$$\sigma_c = \frac{R_{Nc}}{L.1} \qquad \text{(A. 67)}$$

and $$\sigma_q = \frac{R_{Nq}}{L.1}. \qquad \text{(A. 68)}$$

Therefore, the total normal component of the forces to the cone surface

$$F_N = \int_0^L \int_0^S (\sigma_\gamma + \sigma_c + \sigma_q) ds\, dl, \qquad \text{(A. 69)}$$

where $ds = (L - l)\, d\alpha$ or $S = \int_0^S ds = \int_0^\alpha (L - l)\, d\alpha = (L - l)\alpha$. Hence,

$$F_N = \int_0^L (\sigma_\gamma + \sigma_c + \sigma_q)(L - l)\alpha\, dl$$

$$= \int_0^L \sigma_\gamma (L - l)\alpha\, dl + \int_0^L \sigma_c (L - l)\alpha\, dl + \int_0^L \sigma_q (L - l)\alpha\, dl$$

$$= \frac{1}{3}\alpha L\, R_{N\gamma} + \frac{1}{2}\alpha L\, R_{Nc} + \frac{1}{2}\alpha L\, R_{Nq}. \qquad \text{(A. 70)}$$

Similarly, the total tangential component of the forces to cone surface

$$F_T = \frac{1}{3}\alpha L\, R_{T\gamma} + \frac{1}{2}\alpha L\, R_{Tc} + \frac{1}{2}\alpha L\, R_{Tq}. \qquad \text{(A. 71)}$$

The vertical component of forces (normal plus tangential) for gravitational component [Fig.4.24(a)] is

$$F_{V\gamma} = \frac{1}{3}\alpha L\, (R_{N\gamma} \sin\theta_c + R_{T\gamma} \cos\theta_c). \qquad \text{(A. 72)}$$

Putting the values of $R_{N\gamma}$ and $R_{T\gamma}$ in the above equation it can be written as:

$$F_{V\gamma} = \frac{1}{3}\alpha L\,(R_\gamma \cos\delta \sin\theta_c + R_\gamma \sin\delta \cos\theta_c)$$

$$= \frac{1}{3}\alpha L\, R_\gamma \sin(\theta_c + \delta). \qquad \text{(A. 73)}$$

Similarly, the same for cohesive and surcharge components are

$$F_{Vc} = \frac{1}{2}\alpha L\, R_c \sin(\theta_c + \delta) \qquad \text{(A. 74)}$$

and $$F_{Vq} = \frac{1}{2}\alpha L\, R_q \sin(\theta_c + \delta), \qquad \text{(A. 75)}$$

where $L = d_c/2\,\sin\theta_c$ and $\alpha = 2\pi\,\sin\theta_c$ in radians.

The total vertical component of adhesion [Fig.4.24(b)] can be calculated as:

$$\begin{aligned} Vc_a &= c_a \cos\theta_c \,.\, A \\ &= c_a \cos\theta_c \left[\frac{\pi\, d_c^2}{4 \sin\theta_c}\right] \\ &= c_a \pi H_c^2 \tan\theta_c, \qquad \text{(A. 76)} \end{aligned}$$

where $H_c = d_c/2\,\tan\theta_c$.

Appendix - B

Calculation of Deviatoric Stress for the Triaxial Sample

The trace on the X-Y-Y-plotter attached to the triaxial compression testing machine will give values of the axial load N for an axial compressive displacement x. In calculating the deviatotic stress (q), an allowance has to be made for the fact that during compression loading the cylindrical specimen [Fig.7.8(a)] usually barrels outwards [Fig.7.8(b)]. In heavily over-consolidated specimens failure can take place on a slip plane as shown in Fig.7.8(c). Now, let the area of cross-section at mid plane of the sample be denoted by A, and the initial area by A_0, where $A_0 = \pi D^2/4$. The area A can be roughly estimated by assuming the volume of sample remains unchanged during barrelling. Referring to Fig.7.8 it can be written as:

$$A_0 L_0 = AL = A(L_0 - x) \tag{B.1}$$

such that $$A = A_0 \frac{L_0}{L_0 - x}. \tag{B.2}$$

Now the deviatoric stress, $q = (\sigma_1 - \sigma_3) = \dfrac{N}{A}$, where N = applied load to the sample. Putting the value of A from equation (B.2), the corrected equation of q will be as follows:

$$q = \frac{N(L_0 - x)}{A_0 L_0}$$

or $$q = \frac{N(1-\varepsilon)}{A_0}, \tag{B.3}$$

where $\varepsilon = \dfrac{x}{L_0}$, the axial strain of the cylindrical specimen.