SOIL REACTION FORCES ON AGRICULTURAL DISC IMPLEMENTS

Volume I

by

Md. Monjurul Alam
B. Sc. Agric. Engng. (BAU)

A thesis submitted in fulfilment of the requirements
for the degree of Doctor of Philosophy
in
Agricultural Engineering

The University of Newcastle upon Tyne
October 1989

Abstract

Theoretical models for the prediction of the performance of wide cutting blades have been in existence for some time. These models characterise the soil as a rigid-plastic Mohr-Coulomb material and depend on advanced mathematical techniques developed for the solution of the complex equations of equilibrium of soil elements in two-dimensional plane-strain failure. Less rigorous techniques have been employed for developing mathematical models for the behaviour of deep narrow tines. In both cases the soil-implement contact boundary is assumed to be a plane surface of simple geometrical shape. The extension of these methods to deal with three-dimensional failure generated by curved loading boundaries is of comparatively recent origin. The soil failure patterns associated with disc soil cutting implements fall into this category. The thesis describes the development of a mathematical model for predicting the performance of such implements.

The method employed follows the technique used by Godwin et al for reducing three-dimensional failure into two-dimensional components. The present analysis caters for discs implements having both inclination and disc angles. Soil contact in such implements takes place on complex curved surfaces and the geometry of these were analysed. These surfaces were approximated by plane elements which were then assumed to generate two-dimensional failure in planes parallel to the direction of translation of the disc. The rupture geometry and the forces acting on these elements are then computed using the Newcastle adaptation of Sokolovski's rigorous solution to soil failure. The force acting on the soil contact surface is then obtained by a version of the method of slices used for analysing slip surfaces.

The model developed can predict the quasi-static soil reactions on disc implements from a knowledge of the disc geometry, soil properites and depth of

cut. The computer programme was used to investigate the sensitivity of the input parameters to the final predicted results and this information was used to develop a set of non-dimensional force coefficients which can be used in a simple additive algebraic equation to predict the three orthogonal force components acting on the disc. Empirical correction factors have been devised to cater for deviations between these and computed valus and prediction to within 5 percent of the computer prediction are possible by this relatively simple method.

In order to check the performance of this theoretical model a special dynamometer rig was designed and built to assess all three orthogonal force components acting on a disc tool. A single plough disc was tested in a loam soil and the computer prediction was in good agrement with experimental values. The model was also used to check other published experimental results and once again the prediction was good.

Acknowledgements

This work has been carried out under the supervision of Dr. D. R. P. Hettiaratchi. The author acknowledges with gratitude his invaluable help, advice, guidance and constructive criticism during all stages of this investigation.

The author wishes to express his appreciation for the support and interest shown by professor J. R. O'Callaghan.

Acknowledgement is also due to the Commonwealth Scholarship Commission for its financial support and the Bangladesh Agricultural University for granting study leave.

The author would like to extend his appreciation to his wife, beloved daughter and parents for their sincere inspiration.

Contents

Chapter I

INTRODUCTION

1.1 Mechanics of Tillage Tool

Tillage tools are mechanical devices used to apply forces to the soil in order to cause some desired effect, such as pulverization, cutting, inversion or movement of the soil. The ultimate aim of tillage is to manipulate a soil from a given known condition into a different desired condition by mechanical means. Soil manipulation depends on three abstract factors:

1. Initial soil condition.

2. Tool shape or geometry.

3. Manner of tool movement.

The result of these three input factors are evidenced by two output factors, namely:

1. The final soil condition.

2. The forces required to manipulate the soil.

Of the three input factors, the tillage tool designer has a complete control only over the tool shape. The user may vary the depth or speed of operation and may use the tool over a wide range of initial soil conditions. However, tool shape or geometry cannot be considered independently of the manner of movement or initial soil conditions. The orientation of a tool shape with respect to the direction of travel must be defined. Different initial soil conditions may sometimes require different tool shapes.

The shape that is of concern in design is the surface over which the soil moves as the tillage is operated. Gill and Vanden Burg (1968) classify three shape characteristics:

1. Macroshape

2. Edgeshape

3. Microshape

The term macroshape designates the shape of the gross surface, whereas edge-shape refers to the peripherial and cross-sectional shapes of the boundaries of the soil working surface. Notched and smooth disc blades have different edgeshape but the macroshape may be the same. Microshape refers to surface roughness.

The geometry of the cutting edge can materially affect draught as well as vertical and lateral components of soil forces. For example disc blades sharpened from the concave side penetrate more readily than blade sharpened from the convex side.

The roughness of a surface (i.e. its micro-geometry) over which soil slides influences friction forces. Surface roughness is related to the initial polish and the effect of abration and wear may result locally from rust, scratches or small depressions.

The forces on tools remain constant only as the geometric conditions of the tool are maintained. It may not be possible to maintain constant conditions in a tillage tool operating under field conditions. Rock, roots or layering in non-homogeneous soil may cause point loading. In addition, alteration of tool geometry may take place due to various reasons namely:

- The wear of the tool.
- The formation and adherence of soil bodies.

If abrasion changes the geometry of the tool, the forces on the tool may not remain fixed. Analytical studies of the alteration in forces due to the changes of the tool geometry by wear, shows that wear usually occurs along the underside of the tool and changes the degree of sharpness. As wear increased on the underside of the cutting edge, the clearance angle decreased and finally disappeared under the tip of the tool and tended to lift the tool out of the soil.

Soil adhering to the tool can form a body of soil that acts as a part of the tool. The effect of the presence of this soil body on the forces and soil reactions is of great importance. Any description of the shape of a tool that does not include the geometry and action of the soil boundary will not present the true system. The shape and size of the soil mass that sticks to the tool is determined by the direction and movement of the tool and the state of tool operation. The absence of scouring creates the situation in which soil bodies are formed. If the coefficient of soil-soil friction is greater than the coefficient of soil-metal friction along the tool, the draught of the tool will increase. In addition, the draught of the tool may increase because the adhering soil increases the size of the tool. Payne (1958), Tanner (1960) and Nichols et al. (1958) observed and reported different types of soil bodies. Sokolovski (1954) reported some concept of general wedge formation. Later Hettiaratchi (1968) developed the mechanics of wedge formation and quantified the effect of wedge on the forces and soil reaction especially on tine shaped tillage tool.

1.2 Physical Properties of a Disc Tool

One of the most difficult processes in cultivation of crops is the working of soil. To perform this difficult task disc implements can be used to advantage. The working parts of a disc execute not only forward motion along with the machine but also rotate on its own axis. Hence it easily rolls across obstacles without becoming jammed by plant remnants, does not stick and wears out at a slow rate. These advantages are apparent during operations on heavily weeded or thick shrub areas where the upper soil layer is abundantly populated with roots of grassy plant growths.

The common disc plough is geometrically a section of a sphere cut off by a plane. The radius of curvature of the disc is the radius of the sphere. The edge of the disc is therefore a circle and the diameter of this circle is called the diameter of the disc. The diameter and the depth of concavity of the disc depend on the distance of the disc face plane from the centre of the sphere and the radius of the sphere. In common practice a disc plough operates at an angle with the direction of travel in horizontal plane, which is called disc angle. In addition to disc angle, disc plough operates with an inclination. As the disc working surface is a part of a sphere, the lower part of the disc face cuts the soil layer, deforms it and moves along the working surface. When the disc operates at very low disc angle or with large inclination angle the rear face of the disc start scrubbing with the furrow wall or at the bottom of the furrow. Under these circumstances the soil compaction takes place rather than soil loosening, which causes a great increase in draught force as well as vertical and lateral forces. The effect of the scrubbing reaction reduces to zero as the disc angle increases to the critical disc angle, which is equivalent to the disc clearence angle and may be defined as the horizontal angle between the plane of the circumference of the disc and the chamfer at the soil surface. Similarly, scrubbing at the bottom of the disc due to the large inclination can be avoided by choosing the disc inclination to be less than the clearence angle at the bottom of the disc.

The working surface of the disc performs a complex motion. Besides horizontal forward movement with the machine, the disc rotate on its own axis under the action of active forces of the soil. Technical reviews contain many conflicting views on conditions affecting disc roll in the soil with slipping and skidding. However, the absolute velocity of each point on the disc working surface varies continuously in magnitude and direction with the change in relation of the disc, and depends on disc parameters and the location of points on its working surface. This has an influence on the nature of displacement of the soil by the spherical disc and on the energy expended in the process.

1.3 Importance of Force Prediction Model

The soil conditions to be produced by tillage must be expressed in criteria that characterize the soil in terms associated with its intended use. This underlines the need for soil dynamic aspects to be taken into consideration. The dynamic properties of the soil get attention from the fact that the soil is placed in motion during manipulation and its dynamic behaviour becomes important. Evaluation of tillage performance requires:

- Measurement of soil conditions to evaluate the changes performed by the manipulation.

- Measurement of the forces required to cause the manipulation.

Generally, soil conditions cannot be controlled without tillage, so that all changes must come from the use of the tool.

Quantitative evaluation of the soil conditions becomes difficult as there is no adequate method for describing desired soil conditions. Moreover, the soil properties vary over a wide range of weather and geographical conditions. Thus, any changes in the soil's physical, chemical and dynamic properties has an obvious effect on the requirement of force system for proper soil manipulation. So any means of determination of the soil reaction force on the basis of soil and tool properties is of great importance.

This investigation is specifically concerned with the disc tool, which has a complex geometry with more complex manner of movement. Due to the complexity even an efficient field operation need some basic information for proper setting of the disc with the variation of operating conditions. This underlines the obvious need of the force prediction model to provide useful information for scientific investigation, sound design and efficient field operation. Although a considerable amount of research has been carried out upon the effect of disc geometry, only one effective mathematical model has been developed by Godwin et al. (1985) for vertical disc. Unfortunately there is no soil mechanics model for discs operating at an inclination. Thus, evaluation of disc performance through mathematical model with soil of known Mohr-Coulomb properties will provide a useful operation and design aid in estimating the effect of different disc setting and geometric configurations.

1.4 Scope of Present Investigation

This investigation is designed to evaluate the cutting forces on a disc tool. There is only one known prediction model of disc forces as a function of the Mohr-Coulomb parameters. This significant contribution was made by Godwin et al. (1985) and is based on the passive pressure theory developed by Hettiaratchi and Reece (1974). However, this model only caters for discs which have their central axis lying on a horizontal plane (i.e., without inclination angle).

Literature survey shows that the disc geometry and attitude angles have a great influence on the soil reaction force components. Any changes in disc geometry or attitude angles affect the shape of the cutting surface in contact with the soil as does the geometry of the rupture soil block. So by incorporating any changes in disc geometry and attitude angles into consideration an analysis will be carried out to determine the shape of the cutting surface as well as the rupture geometry of the soil block.

Godwin et al. (1985) simplified this disc problem by assuming that the total horizontal force component always acts normal to the disc face. This is only an approximation of the real situation, and the forces generated by an imaginary cutting surface represented by a chord of the disc has to be transferred to the soil-to-metal interface of the disc. There are several ways of achieving this, and a quasi-static analysis of the soil trapped between the imaginary plane cutting surface and the disc can be carried out using Bishop's (1954) simplified "method of slices".

In order that the complex calculation can be reduced to a system which is of practical use, a system of Disc Force Charts have been developed. This reduces the force calculations to a simple algebraic procedure.

For reasons of time only a limited amount of experimental verification of the theory was attempted. To this end a 3-Axis dynamometer suspension was designed and built to test a single disc in one soil type. Additionally, other published data was used to check the performance of the theory developed.

Chapter II

REVIEW OF LITERATURE

2.1 Geometry and Kinematics of a Disc

To understand the relationship between disc and soil reaction, it is necessary to consider certain inherent features of the disc. McCreery et al. (1956) and Kepner et al. (1975) in their discussion on disc geometry, indicated that the common disc is geometrically a section of a sphere cut off by a plane. The edge of the disc is therefore a circle and the radius of curvature of the disc is the radius of the sphere. The diameter and depth of concavity of the disc depend on the distance of the disc face plane from the centre of the sphere and the radius of the sphere. Gill et al. (1968) mentioned that, though in general disc blades are spherical but it may be of reversed curvature or a cone shaped.

McCreery et al. (1956) studied the disc geometry and showed that if the disc is set so that the disc angle ϕ, between the line of travel and the disc face plane is grater than or equal to θ, which is the angle in horizontal plane at the soil surface between the tangent to the cutting edge and the disc face plane, there will be no bearing area on the back of the disc. If the disc angle is less than θ, the back or convex side of the disc will exert pressure on the soil. This pressure is usually a compressive force and resist penetration. As the setting of the disc 1s changed so that disc angle ϕ become greater than θ (clearence angle), the curvature of the disc causes the soil to move upward along its path across the disc surface producing the 'suction' effect of an inclined plane. They also pointed out that in forward motion the bearing on the back or convex side of the disc not only depends on the angle of travel, the radius of curvature and depth of cut has an influence on the bearing. When the line of forward travel is parallel with the plane cutting off the disc (i.e., at zero disc angle) half of the convex side below the soil surface has bearing on the soil. As the disc is angled for cutting, the bearing on the convex side is the zone between the cutting edge and the locus of the point where the line of travel is tangent to the disc.

In studying the effect of micro-geometry on bearing of the disc McCreery et al. (1956) reported that the bearing on the edge of the disc is materially affected by the method of sharpening the disc. If the disc is sharpened on the back or convex side, the bearing may be increased by an amount depending on the thickness of the disc. If the sharpening is on the concave side, the bearing depends on the setting of the disc. Sharpening on the face side gives the minimum bearing for a disc at any given angle.

Abo El Ees et al. (1986) studied the geometry of a vertical curved disc. They expressed soil-disc interaction in terms of projected area and volume both on the concave and convex sides of the disc in relation with disc parameters. They classified radius of curvature of the sphere and the diameter of the disc face circle as direct disc parameters, whereas the disc angle and depth of penetration were defined as attitude parameters. As their analysis was limited in vertical disc the effect of disc inclination which has a considerable influence on spherical disc performance remained unresolved.

McCreery et al. (1956) examined the performance of six discs of 8 inches in diameter, one flat and the other five having radius of curvature 5, 7, 9, 11, and 13 inches respectively. They measured the travel distance per revolution and established the ratio of forward movement to rotation as a function of the disc angle (ϕ), so that:

$$T = \frac{c}{\cos\phi} \qquad 2.1$$

Where T is the forward travel and c is the circumference of the disc. They recommended the above equation as an index of disc rotation. The above relationship is valid when the disc angle ϕ, is greater than θ (clearence angle) and when ϕ is less than θ, there will be appreciable bearing on the back of the disc and the disc behaves rather like a wheel. They also noted that when the back of the disc does not exert pressure on the soil, the velocity of rotation decrease with an increase in disc angle. Similarly, an increase in soil density reduce the velocity of rotation.

Gill et al. (1979) studied the capability of a rotating disc, with a view that it imparts motion to the soil particles and may have an influence in soil and residue handling. They used three shapes of disc namely, conventional spherical disc (spherical I), conical disc and a compound spherical disc having two distinct radii of curvature (spherical II) and three disc sizes of 610mm, 710mm and 810mm with depth 100mm, 134mm and 202mm respectively. They defined kinematic parameter λ, as the ratio of rotational to forward velocity $\lambda = \frac{V_c}{V_f}$. Where V_c ($V_c = \omega r$, ω is the angular velocity and r is the radius of the disc) is the disc rotational velocity, and V_f the forward velocity. Comparison between different size and shapes of disc revealed that disc size has no important effect on λ. Whereas, disc shape has a little effect on the magnitude of the kinematic parameter λ. One evident trend shows that both the cone and the spherical (II) disc rotate faster than the spherical (I) and disc at low disc angles, which refute the findings of McCreery et al. (1956).

The comparison between sandy loam and clay loam indicate that the soil properties has an effect on the kinematic parameter. At a forward velocity of 1.3 m/s and a normal operating angle of 0.38 rad, the average values of kinematic parameter λ for sandy loam and clay loam were 0.85 and 1.0 respectively. Gill et al. (1979) explained the higher value of λ in the clay loam as the cohesive soil slice rides higher up on a disc, closer to the centre, causing the disc to rotate faster. Their experimental result shows that an increase in forward velocity to 2 m/s, with the same operating angle 0.38 rad, most values of λ are greater than 1.0. Generally, λ is about 10 percent greater for the clay loam than for the sandy loam, which indicate that an increase in forward velocity affects the kinematic parameter differently for the cohesive and non-cohesive soils. The increase in rotational velocity is greater in cohesive soil than in non-cohesive soil, because the force applied by the more rigid cohesive soil slice acts closer to the centre of rotation and increase the rotational velocity of the disc.

Nartov (1985) studied the kinematics of the disc working parts in relation to the basic disc parameters and was the first who mathematically analyse the kinematic performance of the disc. In action the working surface of the spherical disc performs a complex motion. Besides horizontal forward movement with the machine the disc rotates on its axis under the action of reactive forces of the soil. During movement, the disc not only moves forward but rotates on its axis as a result of which it executes a rolling motion. Nartov (1985) considered the disc rolling accompanied by slipping or skidding. He divided the disc displacement into two parts. One is in the plane of the cutting edge of the disc along the line I-II (Fig. 2.1), and the second one is perpendicular to the first, along the line II-III. In pure rolling, he suggested that the linear circumferential speed of the disc V_1 is equal to the forward speed and this relation represented by the equation:

$$V_1 = V \cos \alpha \qquad 2.2$$

Where V is the forward speed of the machine. The angular velocity of rotation of the disc in this case equals:

$$\omega = \frac{2V \cos \alpha}{D} \qquad 2.3$$

Where D is the diameter of the disc face circle and α is the disc angle. On the other hand, when disc rolling accompanied by slipping or skidding, he mentioned that the circumferential linear velocity of the disc cutting edge is less or more than V_1. Hence, he worked out that ω becomes:

$$\omega = \frac{2V \cos \alpha}{D(1 \pm \eta)} \qquad 2.4$$

where η is the slipping or skidding co-efficient:

$$\eta = \frac{S - S_0}{S_0} \qquad 2.5$$

S is the actual distance covered by the disc in the plane of the cutting edge per revolution and S_0 ($S_0 = \pi D$) is the distance covered by the disc in the same plane per revolution with pure rolling. If the disc rotates in soil with slipping, then $S > S_0$ and η is introduced with a plus sign. In case of rolling with skidding $S < S_0$ and η has a negative sign.

Nartov's (1985) experimental results agreed well with the theoretical configuration of pure rolling with slipping or skidding of the disc. He observed that the magnitude of the disc angle α has a major effect on the nature of the disc rolling. At small disc angles, the disc rolled in the soil with slipping. Whereas, at higher disc angles the disc begin to roll with skidding. His observation revealed that as the disc angle increases the pressure on the working surface of the disc also increases and initially reduces the disc slipping, afterwards the disc starts rolling with slipping and eventually rolls with skidding.

Nartov (1985) also studied the effect of other disc parameters on disc rotation such as disc inclination, radius of curvature and disc diameter. His experimental results revealed that disc inclination leads disc rotation with higher slipping since the normal pressure on the disc is reduced. Reduction in the radius of curvature leads the disc to rotate with higher skidding, and a reduction in the value of disc angle α corresponding to pure rolling of the disc. This occurs because the rate of deformation of the soil and the value of the component of the soil normal pressure on the plane of rotation of the disc increases with curvarure. An increase in the disc diameter for a given depth of cut causes an increase in the area of contact with the soil and in friction force, hence shows an increase in angular speed of rotation of the disc.

Nartov (1985) used a system of moving co-ordinates to determine the absolute velocity of any point on the spherical working surface of the disc. Each point of the working surface of the disc describes a circle during the rotary motion on its axis. As it moves forward with the machine the path of each point on the working surface describe a cycloid. He presented the projections of any point on spherical surface of the disc on the co-ordinate axes as:

$$V_x = V - \rho\theta(\sin\beta\sin\alpha\sin\theta + \cos\alpha\cos\theta) \tag{2.6}$$

$$V_y = \rho\theta(\sin\alpha\cos\theta - \sin\beta\cos\alpha\sin\theta) \tag{2.7}$$

$$V_z = \rho\theta\cos\beta\sin\theta \tag{2.8}$$

Where V_x, V_y, and V_z are the velocity components at X, Y, and Z axes, α is disc angle, β is angle of inclination, θ is the angle of turn of the disc and ρ is the distance from the axis of rotation of the disc to the given point of its working surface. The magnitude of the absolute velocity of an individual point on the working surface of the disc represented as:

$$V_{abs} = \sqrt{V_x^2 + V_y^2 + V_z^2} \tag{2.9}$$

Nartov (1985) also worked out the direction of the absolute velocity and characterised by the angle described by the velocity vectors V_{abs} with co-ordinate axes as:

$$\cos \lambda_{abs} = \frac{V_x}{V_{abs}} \qquad 2.10$$

$$\cos \gamma_{abs} = \frac{V_y}{V_{abs}} \qquad 2.11$$

$$\cos \delta_{abs} = \frac{V_z}{V_{abs}} \qquad 2.12$$

The analysis of the above absolute velocity equation indicates that the value of the absolute velocity of points varies within wide limits. At the moment of initiation of contact with the soil of a given point on the working surface of the disc, the absolute velocity is close to the velocity of the implement. At the lowest position of the point, the absolute velocity is minimum. When the point leaves the soil, the absolute velocity again is close to the value of the velocity of the assembly and at the upper region of the disc, the absolute velocity of any point is much higher than the speed of the implement. Nartov (1985) also noted that the maximum amplitude of the deviation of absolute velocity equals 1.95V. With the increase in the disc angle and with the decrease in the distance ρ, this amplitude decreases. Inclination of the disc also leads to a certain reduction in amplitude. The rolling of the disc with slip reduces the deviation of absolute velocity from the mean value, whereas skidding is accompanied by an increase in this deviation. As it appeared in the above discussion, the absolute velocity of each point on the working surface of spherical discs varies continuously in magnitude and direction with the change in rotation of the disc parameters and the location of points on its working surface. This has an influence on the nature of displacement of the soil by the spherical discs and on the energy expended in the process.

2.2 Effect of Disc Parameters on Soil Reaction Forces

The features of disc geometry and the operational parameters which govern the soil handling capabilities are disc diameter, shape or radius of curvature, disc angle, angle of inclination, edge shape, velocity, depth of cut and width of cut (Gill et al. (1978)). Abo El Ees (1978) classified the radius of curvature of the sphere, the diameter of the disc face circle and the disc cutting edge shape as direct disc parameters whereas, the disc angle, angle of inclination, velocity, depth of cut and width of cut as disc attitude parameters.

Gill et al. (1980) studied three disc shapes and sizes such as conventional spherical disc (spherical I), compound spherical disc (spherical II), and conical disc. Disc sizes were 610mm, 710mm and 810mm in diameter. They observed that the draught of spherical II was always lower than that of the standard spherical I disc, and generally, was lower than that of the conical disc. They concluded that the lower force was probably caused by the spherical II discs larger radius of curvature, which had two effects. Firstly, the bearing on the back side of the disc was less, so longitudinal compression of the soil by the disc was lower. Secondly, the reduced curvature of the front face of the disc kept the disc from lifting and throwing as much as the conical and spherical I discs threw.

Gill et al. (1981) tested a series of seven 610mm diameter discs including one flat disc (infinite radius of curvature) and six spherical discs with different radius of curvature and investigated the influence of the radius of curvature of the disc as a function of disc angle and velocity. Their experimental result revealed that with the exception of the curve for flat disc, each draught curve has a minimum force. The disc angle at which this minimum draught force occurs is always greater than the clearance angle of the disc at the specified depth of cut. The draught decreases progressively at small disc angle (0.2 rad) as the shape of the disc is flattened, but at large disc angles (0.7 rad) the effect is just the opposite.

With the exception of the flat disc, an increase in disc angle from 0.2 rad to 0.3 rad caused a large decrease in upward vertical force exerted on the disc. They noticed that the disc angle at which the force curve reached the constant value was a function of radius of curvature. The small concavity discs attained constant vertical force at smaller disc angles than the discs with larger concavity. From the viewpoint of reduction in upward vertical force on the disc they recommended the large discs with ratios of radius of curvature to diameter greater than 1.0, which could permit the construction of light weight discs and could be operated at the desired depth without additional weight.

Gill et al. (1981) found that the discs with large concavity receive a large back force (negative force) in the normal range of disc angles between 0.2 rad and 0.3 rad. Whereas, the discs with smaller concavity receive low back force, and a flat disc receives no back force. As the disc angle increases the force curves become constant. This finding confirmed Gordon's (1941) experimental results. His results show that the draught force tends to increase with the increase in concavity. In the heavier soil (Decatur clay) the disc having the greatest concavity is reacted upon by a considerable upward thrust when compared with discs of moderate concavity.

McCreery (1959) reported that an increase in the radius of curvature of a disc with diameter 610mm from 478mm to 660mm resulted in a 17 percent decrease in the force applied by the soil when the disc was operated at a disc angle of 0.21 rad and 0.40 rad, but the increase had no influence on the forces when the disc was operated at a disc angle of 0.61 rad.

Nartov (1985) reported that a reduction in radius of curvature in most cases leads to an increase in working resistance. At large disc angles this is due to more intense deformation of the soil by discs of larger curvature. At small disc angles it is the result of an increase in area of contact of the rear side of the disc with the furrow wall. However, this dependence exists only within certain limits. His experiments with flat disc shows that their resistance is much higher than spherical discs especially when steeply oriented to the direction of the motion of the implement.

From the standpoint of soil reactions, Gordon's (1941) experimental results revealed that the smaller discs requires slightly more draught in comparison to the disc of higher diameter. Furthermore, the smaller discs has a greater side thrust as well as higher vertical upward force, which requires more weight to hold the discs to the same depth of cut than the larger discs.

Nartov (1985) made a series of experimental observations with discs of diameter 360mm to 900mm. His observation confirmed Gordon's (1941) findings and showed that the disc diameter has little effect on longitudinal specific resistance. With the increase in diameter there is a certain decrease in longitudinal and vertical specific resistance. When the working disc is mounted steeply with respect to the direction of motion there is an increase in specific resistance. A reduction in disc diameter marginally increases the transverse specific resistance.

2.3 Effect of Disc Attitude Parameters on Soil Reaction Forces

The effect of the disc angle on the soil reaction of disc tool may be considered as the most important attitude factor in a given soil condition. It affects some other variables such as the width of cut and the area of contact between the furrow wall and the convex side of the disc. Moreover, the disc angle controls the relation between the forward and rotational movement of a freely rotating disc (McCreery (1956)). Gordon (1941) measured the draught, vertical and side force components of the soil reactions on a disc tool at different disc and inclination angles with constant width and depth of cut 7 inch and 6 inch respectively. The disc angles ranged from 36° to 51° for two specific settings of angle of inclination 0° and 15°. The diameter and radius of curvature of the disc were 26 inch and 22.4 inch respectively. He observed that the draught force on the disc attains a minimum for a disc angle of about 45°, and that above 45° the draught force increases rather sharply. On the other hand for smaller disc angles there is an increased area of contact between the furrow wall and the convex side of the disc, which consequently increases the draught force. He also noted that when the angle of inclination increased to above 15° a marked increase occured in the draught force.

From experimental observation with 610mm, 710mm and 810mm disc sizes Gill et al. (1978) suggested that for an optimum range of operation in terms of draught force, the disc angle would be between 25° and 32°, which is much lower than that recommended by Gordon (1941). Later, Gill et al. (1980) studied the spherical and conical disc with large radius of curvature and confirmed that due to the pressure on the back of the disc, the optimum disc angle ranges between 25° to 32°. They also noted that if the disc is operated below 25° it creates compaction and is nonproductive.

Kepner et al. (1978) in their discussion on the performance of standard agricultural disc plough suggested the range of disc angle for vertical disc from 35° to 55°, commonly 40° to 45°; and for the inclined disc from 42° to 45°, with a range of inclination angle of 15° to 25°. They also noted that an increase in inclination with in 15° to 25° increases the draught and vertical upward force but decreases the side force. Thus, penetration is better at the smaller tilt angles. Similar to Kepner et al. (1978), Cuplin (1986) suggested the optimum disc angle to be about 45° as a general rule and indicated that upto this angle the penetration tends to increase. He also pointed out that an increase in angle of inclination from 15° to 25° increases the draught slightly and tends to reduce penetration, but assists in turning the furrow slice efficiently.

Nartov (1985) studied a series of discs with diameter 360mm to 900mm and radius of curvature 560mm to 1200mm, and represented the value of forces acting on the disc in terms of specific resistance per unit area of cross-section of the soil being worked. His observation established that the specific working resistance varies within wide limits as a function of the disc angle. The minimum value of longitudinal specific resistance occurs at disc angle between 25° and 35°. In general, the disc angle above, or below this range causes an increase in specific resistance. He noted that the increase in specific resistance at large disc angle is due to the steeper mounting of the disc working surface with respect to the direction of motion of the disc unit, causing a more intensive deformation of the slice and a longer longitudinal displacement of the soil mass. At smaller disc angles resistance increases as a result of scrubbing at the rear of the disc, as well as an increase in the number of longitudinal cuts per unit width of cut of the implement.

Data obtained from his experiment suggested that the maximum value of the specific transverse force occurs at disc angles between 30° and 35°. A sharp reduction in this value occurs at small disc angles due to the counter pressure exerted by the furrow wall on the rear side of the disc. At large disc angles, it is reduced as a result of decrease in the transverse component of the normal pressure of the slice on the disc. The observation for specific values of vertical reactive forces revealed that at small disc angles this specific resistance has a large value. With the increase in disc angle it decreases rapidly and at a certain disc angle the specific vertical force has changed its direction. Thus, he concluded that the widespread opinion in literature of the poor penetrability of the disc implement is incorrect. On the other hand he emphasises the correct mounting of the disc working tool for its effectiveness.

Observation made by Nartov (1985) agreed that the inclination of disc usually results in great reduction of working resistance. Since there is a smoother creeping of the slice onto the disc due to inclination, the soil mass is less deformed and moves a shorter distance. He noted that at very small disc angles an inclined disc has a higher value of specific longitudinal resistance than a vertical disc, due to the friction of the rear side of the disc cutting edge with the furrow wall. His observations show that an increase in angle of inclination from 0° to 30° results in a more than twofold decrease in transverse specific resistance, due to a reduction in the transverse distance to which the soil slice is thrown and an increase in contact width of the rear side of the disc with the furrow wall. At large disc angles the vertical specific resistance decreases as a rule with the increase in the angle of inclination. However, when the inclination reaches such a limit that there is contact at the rear side of the disc with the furrow bottom, soil vertical resistance increases with the increase in the angle of inclination. At small disc angles this resistance has a larger magnitude for an inclined disc than for a vertical disc.

Taylor (1967) studied the effect of disc angle, tilt angle, furrow width and speed of operation on the three components of soil reactions. He measured the three forces by using a trailed rig composed of two discs, the rear disc was set independently of the attitude by moving the front disc laterally. The instrumented disc was carried on an L-shaped subframe, which was connected to the main frame by six links with each link terminating in a strain gauge load cell. Three of the links were vertical and the other three were horizontal, such that the force components were oriented representing relative to the ground plane and the direction of travel. He used two different types of soil namely, Gray silty clay and Sandy clay loam with discs of 24 inch diameter and approximately 20 inch radius of curvature. He statistically analysed the results obtained from experimental measurements. From the experimental analysis, he obtained polynomial regression equations between each soil force components and the above mentioned variables taking their interaction effect into account. His observation shows that in general, the draught force (L), increases with the disc angle (α). In contrast, the width of cut decreases with the increase in disc angle. There is a simple geometric relationship between the disc angle and the furrow width, and its choice has a great influence on the resistance of the disc plough. The combined effects of these variables is thus most important. Accordingly, draught force would be minimized at maximum width of cut by using the lowest possible disc angle, subject to innerfurrow ridging considerations and possible compaction effects. The draught per unit width shows a marked decrease from minimum to maximum width of cut.

Taylor (1967) also observed that the draught force always increased with the speed, but the power required to operate the plough was not raised proportionately to speed and a tilt angle of about 15° was generally suitable for draught. Hence, to avoid this effect and to take advantage of a decreasing draught per unit width, he suggested to operate the disc plough at greater width for the same rate of work.

Taylor (1967) found that the minimum side force occurs at high tilt angles, possibly combined with low disc angles. He also noted that the side force increased more than proportionately with furrow width, and that it increased with speed by a mean amount of 18 lbf/mph on lighter soil and 35 lbf/mph on heavier soil. He also observed that the penetration became increasingly difficult as the plough was opened up because the upward soil force on the disc rises sharply at low disc angles and more weight is required to keep the plough in work. Tilt angle interacts with this effect in such a way that low tilt angles mitigate it and high ones accentuate it. He also observed that speed had no effect on vertical force except for a small interaction with tilt angle to give easier penetration at higher speeds with 25° tilt.

Godwin et al. (1985) studied with deep spherical and shallow spherical discs of diameter 610mm, concavity 140mm and 70mm respectively with a depth of cut 100mm and 40mm. They observed that the minimum draught force occurred at disc angles in the range of 20° to 30° both in 40mm and 100mm depth of cut. They noted that the point of minimum draught was ranked according to the clearance angle of the disc. The vertical resistance to penetration decreased rapidly with disc angle up to the clearance angle of the disc, above which the vertical resistance to penetration was relatively insensitive to the disc angle. The comparison between predicted and experimental specific resistance (i.e., draught force per unit area of soil disturbance), it appears that specific resistance is the realistic parameter with which to judge tillage efficiency. On the basis of the observation it is interesting to note that the minimum specific resistance occurs at similar but marginally larger disc angles than those of the minimum draught force, which allows the disc to cover larger width of cut with the same rate of soil resistance.

At the National Tillage Machinery Laboratory (NTML), USA, Reaves et al. (1981) studied the effect of width and depth of cut on soil reaction force acting on three shapes and sizes of discs. One shape was a conventional spherical disc (spherical I). The second shape was a compound spherical disc with distinct radii of curvature (spherical II). The third shape was a conical disc. The discs diameters were 610mm,710mm and 810mm for each shape. Since the depth of cut was constant in each run, the change of forces in magnitude and direction was attributed directly to the change in width of cut. The increase in width of cut was accompanied by a simultaneous increase in the amount of soil cut by the disc. Their experimental results revealed that the relative order of the force curves of the three shapes is constant for all disc sizes. The draught of the spherical (I) disc has a constantly higher value than that of the spherical (II) disc and the conical disc always lag in between two. The vertical force components of different disc sizes and shapes has showed the same relative order as draught. As the slope of the curves are small, they recommended to use fewer number of discs with large width of cut, therefore the total disc weight could be reduced.

The sperical (I) and conical discs generally has a negative side force, caused by more pressure on the back surface of the disc than on the front surface. In contrast, the spherical (II) disc has a neutral to relatively higher positive side force, which could create unstable operating condition. The depth of cut has the same effect as width of cut on the three force system acting on the discs except that the slope of the force curves are steep. They concluded that the steepness of the curves may have been due to one or both of the following factors: Firstly, an increase in depth is frequently accompanied by appreciable increase in the soil strength; secondly, an increase in depth is accompanied by simultaneous increase in the width of cut and an increased amount of soil being cut by the disc. The variation in depth of cut occurs because the disc projects an elliptical shape in the direction of the movement.

Harrison et al. (1976) conducted field studies in which they used 508mm diameter (635mm radius of curvature) vertically mounted spherical disc, and measured forces while operating at two depths 51mm and 76mm. The width of cut was maintained at 178mm. They observed that with the increase in depth of cut in a clay loam, the increases the draught, vertical and side force were 42 percent, 31 percent and 29 percent respectively.

Harrison (1977) conducted field experiments with spherical discs having diameter of 50mm and radius of curvature 635mm. In those experiments two different soil/site were used namely, silty loam and clay loam. He used a multicomponent sensor to measure the soil reaction forces and determined the location of the wrench or screw axis of the disc. He observed that the differences in the draughts and lateral reactions for the soil were large and noted that it was largely attributed to the difference in the soil types. His experimental results showed that the increase in the draught and lateral reaction for an increase in depth was substantial, and even more for clay loam site. He noted that the above differential response may be due to a difference in the angle of the failure surface causing larger primary and secondary failure planes in the clay loam site and consecquently a larger draught and lateral reaction. He also noted that the increase in the lateral reaction with speed may be caused by a change in the soil strength associated with the loading rate, the acceleration of the soil or both.

He also observed that the maximum draught per disc occurred for a disc angle of 35°, whereas the maximum draught per unit width of cut occurred for a disc angle of 45° or greater. He noted that the decrease in the draught per unit width causes an increase in the disc angle from 35° to 45° which may be due to the decrease in the width of cut of the disc or the volume of tilled soil. He also observed that the draught of the implement can be reduced by increasing the disc angle from 35° to 45°, but this results in a greater draught per unit width of cut; that is, the work done to till a given acreage will be greater at a disc angle of 45° than 35°, even though the draught of the implement would be less. Increasing the disc angle from 30° to 40° decreases the vertical and lateral reactions, improving the penetration and lessening the skewing if the implement is a one way disc harrow. As the operating and capital cost are the function of the draught, he suggested that the disc angle should be kept near the minimum

(30°) for the implement unless it is necessary to increase penetration, reduce skewing or both. The wrench or screw axes of different disc angles shows that the resultant of the soil reaction forces intercepts the disc near its centre and not in the vicinity of the disc which engages the untilled soil. Apparently the untilled soil ahead of the disc is loaded or stressed by the disc surface directly, that is, the geometry of the disc was affected by a soil body (Harrison (1977)).

Thompson and Kemp (1958) made a graphical analysis to illustrate the effect of disc diameter, angle of operation and the distance between the discs on their common bar on a clean cut of vertical discs. They pointed out that the discs cut elliptical shaped furrows which leave ridges between the bottoms of the furrows. As they ignored the behaviour of the soil failure, the ridges between the furrow bottom may not exist with the exact shape developed by drawing the projections of the discs cuts. In other words, the soil failure behaviour should be considered as well in order to get a reliable complete picture of the furrow cross section. However, Thompson and Kemp (1958) emphasised that the projections of the discs cuts would be fairly representative on smooth surfaces. On rough surfaces they suggested to increase the actual depth of cut to compensate for any unevenness so that the machine would make a clean cut across its entire width. They also discussed the depth of operation of the disc implements required for a complete cut of the soil surface and suggested that the disc implement should be operated at least $\frac{1}{2} inch$ deeper than the required depth. They also suggested an increase in the depth of cut with an increase in spacing and with a decrease in the disc diameter.

The effect of disc angle, tilt angle, speed of operation, disc diameter and concavity were studied by Johnston and Birstwistle (1963). They measured the draught, side and vertical components of the soil reaction forces and presented as a force per unit area of furrow, based on a cross-sectional area equals wd, where, w is the furrow width and d is the depth of cut. They reported that the minimum rearward draught force L, may occur over a wide range of disc angles from the usually accepted 45°, depending on furrow width, the manner of edge sharpening and soil type, whereas the inclination angle appeared to have little effect on draught in the 10° to 15° range commonly used for wider furrow widths. The effect of the disc diameter and concavity, furrow width and depth was reported to be insignificant on the draught per unit area. On the other hand, they found a highly significant effect of speed on draught per unit area. Speed also increased side force but not vertical force. Johnston and Birstwistle (1963) also observed that the side force S per unit area of furrow cross-section increased with the disc angle and the disc diameter. They noted that the effect of concavity was consistently negative, since the rear of the disc furnished a considerable side support.

In general, weight pushing the disc into the soil is a major element in penetration and its effectiveness is governed by the angle of the line of pull, inclination, diameter, concavity, thickness, sharpness and the method of sharpening the disc cutting edge (McCreery et al. (1956)). In their discussion on disc penetration they pointed out that weight is the most important factor in disc penetration, on the other hand the bearing on the convex side of the disc has been found to be an important factor in compaction and in resisting penetration. If the setting of the disc is such that there is no scrubbing at the rear of the disc, the curvature of the disc causes the soil to move upward along the path across the disc surface producing the suction effect of an inclined plane. They suggested that this penetration action can be increased by setting the disc at an inclination. They also suggested that the disc penetration can also be increased by cutting notches at the edge of the disc, which may reduce the area of the bearing at the circumference rather than setting the disc at an inclination, as the inclination increase the bearing at the back of the disc at smaller values of the disc angle, which results compaction.

Gordon (1941) carried out experiment to compare the reactions of soil on discs having a notched edge and plain edge. His observations are similar to that of McCreery et al. (1956). Gordon's (1941) observation shows that the notched edge disc has required slightly less draught than that of plain edged disc in the vertical position. However, in Decatur clay soil with a disc inclination of 20°, his experimental data shows that the draught of the plain edged disc is lower and soil penetration can accomplished with less weight on the discs. With the discs held vertically, the upward thrust is slightly less on the notched disc.

In contrast, Nartov (1985) shows that the notches on the cutting edge of the disc led to an increase in longitudinal specific resistances. This is due to a somewhat increased power expenditure for cutting the slice by notched-discs. In a given case each tooth of the disc moves its own screw shaped groove on the furrow wall. This observation contradicts the previously discussed performance study of the notched discs, by Gordon (1941) and McCreery et al. (1956). Nartov (1985) explained this contradiction as a systematic error involved in the experiments conducted by the above authors. He noted that during the operation of notched-discs, there are uncut segments at the bottom of the furrow, leading to reduction in the total volume of cultivated mass. The uncut volume of the soil should be considered in determining soil resistance forces, otherwise systematic errors will mislead the performance of the notched-discs. He also observed that a disc with internal cutout have almost the same specific working resistance at a disc angle of 35° to 45° as a disc with a continuous cutting edge. However, with the increase in disc angle upto 65°, the specific longitudinal resistance does not increase appreciably, since a large portion of the soil mass freely passes through the notches without pressing into them and moves a shorter distance along the disc movement.

Nartov's (1985) experimental data also shows that the notches on the disc cutting edge results in some increase both in transverse and vertical specific resistance. This is specially noticeable in the steeply mounted disc. Notches on the disc working surface reduce the transverse force. Cutouts on the disc in the form of holes increase the vertical resistance force, since the slice exerts pressure on the disc and the vertical component of the cutting force directed upward remain as in the discs with continuous working surface.

Gill et al. (1981) conducted experiment to determine the influence of the radius of curvature on the depth of penetration of a vertically mounted disc as a function of disc angle, velocity and mass on the discs. They observed that in general, increase in the mass on the discs increases the penetration depth at all disc angles and velocities. At smaller disc angles the discs with smaller radius of curvature penetrated less with the increased mass than the discs with large radius of curvature. But above certain disc angle, discs with smaller radius of curvature penetrated deeper than those with larger radius of curvature. They explained the above behavior as an increase in disc angle permitting the curvature of the front of the disc to develop an increased penetration, the magnitude of which was proportional to disc concavity.

In general, the data from the velocity phase of their studies indicated that an increase in velocity caused only small difference in the nature of the disc penetration. The study with the Norfalk sandy loam and Decatur clay loam revealed that the change in penetration depth was influenced by the soil type, not by the velocity. In contrast, Gordon's (1941) experimental data revealed that the draught force increases with the speed not at a uniform rate but at a slightly accelerated rate. The vertical reaction on the disc indicates that the disc tends to penetrate better at higher speed. Similarly, experimental data of Singh et al. (1978) shows that the penetration of a vertically mounted discs increases with the velocity and with the increase in the disc angle from 0 rad to 0.5 rad. Increase in mass also increases the depth of penetration.

Singh et al. (1978) developed an equation for depth factor and determined the effect of velocity, disc angle and mass on the disc penetration in field condition. They represented the depth factor in nondimensional form as:

$$\frac{d}{D} = f(\theta, \frac{V}{\sqrt{gD}}, \frac{W}{CD^2}) \qquad 2.13$$

Where $\frac{d}{D}$ is the depth variable, d is the depth, D is the diameter of the disc, θ is gang angle, V is the speed of operation, C is the cone index and g is the acceleration due to gravity. All factors were found to be additive and the theoretical relationship agreed with the experimentally determined depth of penetration.

Realistically, the tractor and disc have both lower and upper limits in speed for satisfactory performance. Travel speed lower than 2 m/s are not recommended for most wheel tractors. Travel speed higher than 3 m/s cause excessive soil throwing by the disc and can create an unlevel field (Sommer et al. (1983)). Their experience in field tests shows that changing the speed from 2.5 m/s to 3 m/s would result in no change in depth of cut but a five to ten percent increase in draught. Soil bin test results confirm their field observations that depth does not change with speed (no change in vertical force) and that a five to ten percent increase in draught occurs when speed is increased from 2.5 m/s to 3 m/s.

Sommer et al. (1978) also studied the effect of disc curvature in the soil bin. The result showed that the depth increases 110 percent when curvature has been increased from 648mm to 1238mm. Their analysis with constant vertical force shows a 24 percent increase in depth by changing from spherical disc to conical disc. But the field test shows no significant difference in penetration with same variables.

Gill et al. (1981) studied the soil handling capabilities of discs with different radius of curvature. They observed that flatter disc (large radius of curvarure) displace the soil slightly and that it gives a little or no inversion of the soil slice. Forward velocity has very little effect on these characteristics; whereas, the disc with large concavity (small radius of curvature), displace and invert the soil considerably and excessively at higher forward speed. They recommended discs with moderate concavity since these they provide adequate soil handling, coupled with the force advantages. Their experimental results also indicated that the disc shape (various radius of curvature) has little influence on the change in draught, at different velocities and at a specific disc angle. With the increase in disc angle the draught force increases, so that the soil confinement increases on the front surface of the disc. Soil confinement increases because of two geometric factors: the increase of disc angle increases soil confinement in the horizontal plane, and an increase in the steepness of the disc itself increases soil confinement in the vertical plane. The increase in the radius of curvature (i.e., the increase in disc flatness) creates a steeper disc surface at large disc angles, thereby increasing the soil confinement in the vertical plane.

2.4 Semi-empirical Methods for Soil Forces on a Disc

McCreery et al. (1956) studied the reaction of the soil to the disc and assumed that the direction of the pressure of the soil on the disc is in all cases parallel with the line of travel, and that the reaction is the resultant of the pressure force and is normal to the disc surface at each point where force is applied. This is an oversimplification however, since the soil moves across the disc and the actual pressure at any point is the result of the force in the line of travel plus the pressure of the soil being forced across the disc surface, which includes the effects due to friction and adhesion. They proposed a system of determining the relationship between the forces acting on the disc in three co-ordinate axes. In their proposed system they determined the vertical force from the vertical component of a line that is the intersection of a plane tangent to the disc at that point and the vertical plane through the point parallel to the line of travel. The longitudinal force (L) and side force (S) at that point in horizontal plane through the point. They computed side force 'S' as a component of 'L' using the general formula:

$$X^2 + Y^2 + Z^2 = R^2 \qquad 2.14$$

The equation describes the surface of the disc sphere with its centre at the origin of the sphere and the disc angle which determines the disc setting. Since the radius of the sphere R and Z (depth of cut) are known the equation has two unknowns and the changes in side force (S) with respect to X (longitudinal force) in a plane determined by a fixed value of Z is:

$$\frac{\partial y}{\partial x} = \frac{X}{\sqrt{(R^2 - Z^2 - X^2)}} \qquad 2.15$$

Gill et al. (1980) calculated the values of force ratios, n=side force/draught and m=vertical force/draught for spherical discs of 610mm, 710mm and 810mm diameters and plotted these ratios as a function of velocity for each disc size, shape and angle of operation. They obtained generalized curves of n and m for discs operating in the normal range of disc angles by averaging the nine individual curves for the disc angles of 16°, 19°, 22° and 25° in the Decatur clay loam. Observation shows that the values of n are curvilinear, increasing initially as velocity increases and levelling off at about 1.5 m/s to 2.5 m/s. They noted that at a given velocity the curve of some individual discs vary from the average curve by as much as 70 percent. At disc angle above 28° the curves overlapped to form a horizontal dense grouping of curves with an average n value of 0.8 for all velocities. The values of m as a function of velocity are linear; m values for each disc angle decreases 0.07 per 1 m/s increase in velocity. The m values of some individual curves at any specific velocity vary from the average curve by as much as 25 percent. Above 28°, all curves overlap with each other and form a horizontal dense group of curves averaging m=0.3 for all velocities. Finally they concluded that the two force ratios n and m are regular enough that they may have practical value in estimating side and vertical force for specific operating condition if the draught force is known either experimentally or theoretically. Nartov (1985) confirmed the general nature of n and m mentioned in the results above, but he also mentioned that the ratio varies with disc parameters and many other factors. So, the ratio of n and m can not be used as a general function unless they are determined for a specific operating condition.

Godwin et al. (1985) proposed a mathematical model to predict soil reaction forces on a vertical disc. They separated the soil reactions into two components:

1. The cutting reactions on the concave section or face of the disc, in which they used two-dimensional cutting theory to estimate passive soil resistance as introduced by Hettiaratchi et al. (1974).

2. The scrubbing reaction on the convex section or rear of the disc, in which they used bearing capacity theory as introduced by Meyerhof (1961).

Their experimental observations with 305mm radius, shallow spherical, deep spherical and conical discs for known soil and disc properties agreed well with the predicted forces in the normal working range. But some of their assumptions were not entirely logical. They assumed that the total soil reaction force on the concave side of the disc calculated on the basis of two-dimensional soil cutting theory were always in the plane perpendicular to the disc face plane and at an inclination δ with a line which is normal to the disc concave working surface at the point of total soil reaction. This approximation does not reflect the relationship between the direction of travel of the disc and the direction of total soil reaction acting on the concave side of the disc.

They also assumed that the constrained adhesion is zero ($C_a = 0$), which is only possible if disc surface (i.e., the soil-metal interface) is perfectly smooth. As they used the soil-metal friction angle δ to partition the total soil reaction force into vertical and horizontal soil reaction components, so the approximation of constrained adhesion $C_a = 0$ leads to an error in calculation. Moreover, their model can only handle vertical discs (i.e., the model does not allow any disc inclination), but normal disc ploughs always operate with a disc angle as well as tilt angle. Despite the limitations, this is the only semi-rigorous mathematical model which can fairly represent the disc performance for known soil and disc properties.

2.5 Soil Reaction Force Representation on a Disc Tool

The geometry of the tillage tool and the soil-tool interface are of great importance in investigating the soil reactions. In operation at constant speed a tillage tool is subjected to three different force systems, which must be in equilibrium (Kepner et al. (1978)). These are:

- Weight of the implement, acting at the centre of gravity of the tool.
- Forces acting between the tool and prime mover.
- Soil reaction forces acting on the implement.

The weight of the implement is an independent force system acting on the implement. The second and third force system are interdependent on each other and each of them is dependent of the weight of the implement. The well defined implement weight can be located and controlled. The prime mover forces can be adjusted by the adjustment of the pulling linkage. The resulting soil reaction forces mobilized to active equilibrium represent the third system of forces acting on the implement.

Clyde (1936) subdivided the total soil reaction into useful and parasitic forces. He defined useful soil forces as those, which a given tool must overcome in cutting, breaking and moving the soil or the resistance force that the working face and edge of the tool must overcome. Parasitic forces are those including friction or rolling resistance along the sliding surface of a tool, such as scrubbing at the rear face of a disc. The magnitude of the parasitic forces can be largely controlled by the design of the tool or by controlling the operating condition. The distinction between useful and parasitic forces for disc tool is therefore difficult because the friction drag and rolling resistance take place on the same tool simultaneously with the useful soil forces during tillage operation.

The total soil reaction on a tillage tool, is generally expressed in one of the following ways, each of which may be more desirable than the others in a particular situation depending upon the intended use (Kepner et al. (1978), Vanden Berg (1966)).

- A wrench (i.e., one force plus a couple in the plane perpendicular to the force).
- Three forces on mutually perpendicular axes and three couples in the plane of intersection of the axes.
- Three forces in the major planes.

Furthermore, Kepner et al. (1978) reported another two ways of expressing the soil reaction forces on a tillage tool. These are:

- One force plus a couple in a plane perpendicular to the line of motion.
- Two non-intersecting forces; one is in vertical and the other lies in a horizontal plane but inclined with the direction of travel due to the side thrust resulting from the non-symmetry of the tillage tool about the vertical directional plane.

The net effect of all soil forces acting on a disc blade as a result of the operations of cutting, pulverizing, elevating and inverting the furrow slice, plus any parasitic forces acting on the disc can be expressed in any one of the above mentioned ways. Kepner et al. (1978) discussed the methods of representation of the soil and parasitic forces on a disc blade introduced by Clyde (1939). The resultant effect can be expressed by two non-intersecting forces, one being a thrust force T, parallel to the disc axis and the other being a radial force U (Fig. 2.2(a))This method is particularly advantageous in calculating loads on disc support bearings. The thrust force is always well below the disc centerline because the soil acts against the lower part of the disc face. The radial force which includes the vertical support force on the disc blade must pass slightly to the rear of the disc centerline to provide the torque necessary to overcome bearing friction and cause rotation of the disc.

The resultant effect can also be expressed by the method illustrated in Fig. 2.2(b), which is based on the longitudinal (L), lateral (S) and vertical (V) components and resultants of these forces. This type of force representation, illustrated in Fig. 2.2(b), is more useful than the other, when considering the effects of soil forces upon an implement as a complete unit. In Fig. 2.2(b), the L and

S components are combined into the horizontal resultant R_h so that the entire effect is represented by the two non-intersecting forces V and R_h. Because these two forces do not intersect, they introduce a couple V_a that tends to rotate the implement about the axis of forward travel. This couple is always clockwise for a right-hand disc plough as viewed from the rear. The forces indicated in Fig. 2.2(a) can be obtained directly from those in Fig. 2.2(b) by proper application of the method of statics.

Clyde (1936) suggested two methods for measuring and locating the useful soil forces and their line of action:

- The pulling method, in which the tool is pulled by means of a cable, chain or link. When the tool is in working position, then by changing the angle and position of the pulling force, the pull is measured and located. The useful soil forces are then determined according to the principles of statics with known pull and weight of the implement.

- The tillage meter method, in which soil forces are balanced by several forces located elsewhere.

Clyde (1936) used a triangular frame to measure six unknown forces to determine their resultant. He tested an 18 inch disc using both method and indicated that the pulling method was more satisfactory. He adjusted the line of pull, so that the vertical soil reaction was equal to the weight of the implement. The intersection point of the line of pull with the plane of the disc edge in the horizontal plane was considered as the point of concurrence of the longitudinal force L and the side force S. The location of the vertical soil reaction V and the horizontal plane containing L and S were determined, so that the resultant of V and L passed through the point of intersection of the weight W and the horizontal plane containing the line of pull. As concluded by Clyde (1936) the moment of the couple W and V must be balanced by the component of the pull in the vertical plane normal to the direction of travel. He also located and determined the soil force system by determining d and e (Fig. 2.3) in connection with the values L, S and V, and by assuming that L and S are in the same horizontal plane.

Taylor (1967) represented the soil reaction as three forces and three moments in three major planes and took the origin of the reference co-ordinate system to be at the centre of the disc for calculating forces. He considered the positive direction of the soil reactions on the disc, so that the draught component F_x was positive opposite to the direction of pull, the side force component F_y was positive towards the unploughed land and the vertical component F_z was positive in the upward direction. He used a moving co-ordinate system and determined the force components F'_x, F'_y and F'_z in the disc axes X', Y' and Z' and similarly to the moment components.

$$F'_x = F_x \cos\alpha - F_y \sin\alpha \qquad 2.16$$

$$F'_y = F_y \cos\alpha \cos\beta - F_z \sin\beta + F_x \sin\alpha \cos\beta \qquad 2.17$$

$$F'_z = F_z \cos\beta + F_y \cos\alpha \sin\beta + F_x \sin\alpha \sin\beta \qquad 2.18$$

He denoted by α and β the disc angle and tilt angle representing the shifted co-ordinate axes Y' along the disc shaft, X' along the horizontal diameter of the disc and Z' along the diameter of the disc in the vertical plane.

The presentation of soil reactions in two non-intersecting forces as described by Taylor (1967) and Kepner et al. (1978) is seriously disadvantageous, because the line of action of the resultant force in each plane cannot be considered as a projection of the total resultant force in space. As a result it does not give any information about where or how the soil force acts on the disc working surface. To avoid the above mentioned disadvantages as viewed by Taylor (1967), he suggested the wrench method as an alternative. He indicated that the line of action of the wrench in space is unique and its projections on the three planes are compatible and that a point on the disc body can be located through which the resultant force and moment may considered to act.

Vanden Berg (1968) has used vector analysis to show that reducing forces to a wrench indicates an unique line of action. The true line of action of a resultant force represents the minimum couple for a system of distributed forces, and hence the wrench is independent of any arbitrary reference point or reference plane. Any of the above mentioned methods of reporting can be reduced to a wrench to locate the unique line of action.

Most of the above mentioned methods of soil force representation are used mainly for locating the soil reactions on the disc body for different purposes. The net forces on the disc are of great importance along with locating the position of the force systems and their couples on the disc. So, simply the measurement of the longitudinal L, side S and vertical V force components may be adequate for general purposes.

Chapter III

SOIL-DISC INTERFACE GEOMETRY

3.1 Variables and Assumptions for Soil-Disc Interface

In soil cutting operations, three basic types of variable are involved, namely, the soil, the interface and the soil boundary. Each has a distinct influence on the soil reaction forces acting on the cutting tool.

3.1.1 The Soil

The soil is assumed to be an isotropic, homogeneous, rigid-plastic Mohr-Coulomb material. In soil cutting action, as the tool advances, the soil in its path is subjected to compressive stresses and this results in a shearing action. The maximum shear stress at which the soil fails can be expressed in terms of cohesion and internal friction of the soil by Coulomb's equation:

$$\tau = c + \sigma \tan \phi \qquad 3.1$$

Where τ is the shearing stress at the soil failure, c is the cohesion, σ is normal stress to the plane of shear failure and ϕ is the angle of internal friction.

Kepner et al. (1978) referred to cohesion and internal friction as real physical properties of the soil. The soil moisture content is also an important factor; an increase in moisture content reduces the cohesive strength while leaving the angle of internal friction relatively constant (Moller (1975)), although some exception has been observed in a dry crumby clay soil with good structure by Payne (1958). To take account of these various influences, the soil bulk density γ has also to be taken as a variable in calculation of soil reaction on the tool.

3.1.2 The Interface

The 'Interface' may be defined as the surface which forms the boundary between the loading structure and the soil. This is the boundary, where it is desirable to determine the stresses in order to calculate the forces acting on the tool. As the conventional disc is a part of a sphere, the working surface of a disc is a circle. The geometry of the disc working surface is fixed by two sets of parameters (the disc parameter and the disc attitude parameter) and its depth 'd' from the sphere centre to the horizontal soil surface. The radius of the sphere R (this is also radius of the curvature of the disc) and 'a' (the distance between the disc and sphere centres) are considered as the disc parameters. The disc operating angle β (disc angle), and the inclination angle α (tilt angle) are considered as the attitude parameters (Fig. 3.1).

Unlike plane tines, a disc is a three-dimensional problem having a complex geometry. For simplicity, it is assumed that the soil-disc interface is composed of a finite number of narrow tines parallel to the direction of motion of the implement, which has variable rake angle (α_r), rake length (L) and depth of cut (Z), defined by the geometry of the disc working surface (Fig. 3.2). Furthermore, the interface is assumed to be rigid, and the movement of the interface into the soil takes place at uniform velocity V, and as an approximation disc rotation is neglected. The tangential shear stress τ on the interface can be related to the normal stress σ_n by making a variation similar to Coulomb's strength expression:

$$\tau = C_a + \sigma_n \tan\delta \qquad 3.2$$

Where C_a is the soil-interface adhesion independent of normal pressure and δ is the angle of friction between the soil and interface. A perfectly rough interface would have $\delta = \phi$ and $C_a = c$ (Hettiaratchi (1968)), and in this case equation 3.2 reduces to 3.1, because such a surface simulates soil to soil stress conditions.

3.1.3 The Soil Boundary

In this investigation the soil is considered to be semi-infinite with a horizontal soil surface. The soil surface may be free of any applied stress or it may have a uniformly distributed surcharge pressure representing any possible loading on the soil surface, whether from a machine or the presence of loose material. This surcharge is assumed to have no discontinuities and extend over at least the entire rupture zone of the soil. The surcharge material must be much weaker than the soil below the surface and required a negligible force to cut, that is, it is assumed that the soil shear stresses do not pass from the free surface into the mechanism producing surcharge.

3.2 Geometric Parameters of a Disc

The common disc is a section of a sphere cut off by a plane as shown in Fig. 3.1 and that plane is called the disc face plane. The radius of the sphere R is the radius of curvature of the disc. In this investigation R has been taken as unity for non-dimensional analysis. As the disc is a part of a sphere, the edge of the disc is a circle. The diameter of the disc edge circle is called the diameter of the disc and the radius of the disc edge circle is represented by r_3. The disc radius r_3 and the depth of concavity depend on the distance 'a' between the centre O of the sphere and the centre of the disc edge circle as shown in Fig. 3.1. Thus, from Fig. 3.3. the relation between sphere radius R, disc radius r_3 and the distance between the disc and sphere centre can be expressed as:

$$r_3 = \sqrt{1 - a^2} \qquad 3.3$$

As the soil-disc interface is assumed to be composed of a finite number of narrow tines, essentially each elemental segment of the interface has different depth and rake angle due to the disc geometry. Thus, to calculate the depth of cut in different segments a constant depth 'd' for all the segements in a given condition is considered as the distance between the horizontal soil surface and the centre of the sphere.

In this investigation the centre of the sphere O, has been taken as the origin of the reference co-ordinate system and directions are chosen in accordance with the right-hand rule as shown in Fig. 3.4. Furthermore, the disc angle (angle of attack) β, is defined as the included angle in the horizontal plane between the lines passing through the centre of the disc and the x-axis at the sphere centre. Similarly, the angle of inclination (tilt angle) α, is defined as the included angle in the vertical plane between the lines passing through the disc centre and the X-axis at the sphere centre.

The working part of a disc has a spherical surface. At any instant the lower part of the disc cuts the soil layer, deforms it and moves it in a particular way, depending on the forward and rotational movement of the disc. The geometry of soil-disc interface has been illustrated in the following sections. A comprehensive geometrical analysis in three dimensions has been attempted to incorporate the effect of various disc geometry and attitude parameter on the soil-disc interface. The boundary of the soil-disc interface at any particular instant is fixed by the horizontal soil surface, disc attitude angles and the section of the disc working surface in contact with the soil. For desirable manipulation of this disc interface section, three interscting planes are to be considered. They are (Fig. 3.5):

1. Horizontal soil surface plane.

2. Any vertical plane along the disc interface parallel to the direction of motion.

3. Disc face plane.

The evaluation of the co-ordinates of intersecting points A, B, D, and E of the above mentioned planes and the disc centre C allows the intended manipulation of the disc interface.

From Fig. 3.4. the projected disc angle β' and inclination angle α' in relation to β and α can be written as discussed in Appendix A.1.

$$\beta' = \tan^{-1} \frac{\tan\beta}{\sqrt{1+\tan^2\alpha}} \qquad 3.4$$

$$\alpha' = \tan^{-1} \frac{\tan\alpha}{\sqrt{1+\tan^2\beta}} \qquad 3.5$$

3.3 Co-ordinates of Disc Centre C

In addition to the disc angle and inclination angle, the disc centre is fixed by the distance 'a' from the origin of the reference co-ordinate system as shown in Fig. 3.4. The co-ordinates of the disc centre X_c, Y_c and Z_c can be analytically determined (Appendix A.2) with reference to Fig. 3.4 by the expression:

$$X_c = a\cos\alpha' \cos\beta \qquad 3.6$$

$$Y_c = a\cos\alpha'\sin\beta \qquad 3.7$$

$$Z_c = a\cos\beta'\sin\alpha \qquad 3.8$$

3.4 Co-ordinates of A

Two intersecting planes have been considered to specify the co-ordinates point of A, shown in Fig. 3.6. They are:

1. Horizontal soil surface at a distance 'd' from the sphere centre.
2. Any vertical plane at a distance 'e' from the sphere centre (the reference co-ordinate system).

As the boundary of both intersecting planes is fixed by the sphere, they are essentially circles of different diameter. Thus, from Fig. 3.5 r_1 and r_2 can be written as:

$$r_1 = \sqrt{1-d^2} \qquad 3.9$$

$$r_2 = \sqrt{1-e^2} \qquad 3.10$$

Where r_1 is the radius of the horizontal circle on the soil surface at a distance 'd' from the centre of the sphere defined by the sphere geometry, r_2 is the radius of any vertical circle at a distance 'e' from the sphere centre, sphere radius R is unity. As explained in Appendix A.3, with reference to Fig. 3.6 and Fig. 3.7 the co-ordinates point of A can be expressed as:

$$X_a = \sqrt{1-(d^2+e^2)} \qquad 3.11$$

$$Y_a = e \qquad 3.12$$

$$Z_a = d \qquad 3.13$$

3.5 Co-ordinates of D and E

To specify the intersecting points D and E of disc face plane with the horizontal soil plane, three intersecting planes have to be considered, which are defined by the sphere. The three intersecting planes are:

1. Horizontal soil plane, at a distance 'd' from the sphere centre.

2. Any vertical plane at a distance 'e' from the sphere centre.

3. Disc face plane at a distannce 'a' from the sphere centre.

A detail analysis of the intersecting planes has been carried out in Appendix A.4. From Fig. 3.5 the distance between the disc centre to any of the intersecting point D or E can be expressed in terms of ϵ:

$$\epsilon = (X - X_c)^2 + (Y - Y_c)^2 + (d - Z_c)^2 - {r_3}^2 \quad 3.14$$

Where X_c, Y_c and Z_c are the co-ordinates of disc centre C; X, Y and Z (d) are the co-ordinates of intersecting point D or E depending on the value of θ.

From the intersecting planes (1) and (3) in Fig. 3.5 and from the Fig. 3.8 the co-ordinates of D and E can be expressed as:

$$X = r_1 \cos\theta \quad 3.15$$

$$Y = r_1 \sin\theta \quad 3.16$$

$$Z = d \quad 3.17$$

Where r_1 is the radius of the horizontal circle on the soil surface defined by the sphere. To find the appropriate value of θ, basic equations 3.14, 3.15, 3.16 and 3.17 have been considered to develop a computer programme (Programme P1) on the basis of bisection and root finding technique.

3.6 Co-ordinates of B and B'

To specify the intersection points B and B' of the disc edge circle, two intersecting planes have been considered, which are defined by the sphere geometry. The intersecting planes are plane (2) any vertical plane at a distance 'e' from the sphere centre, and plane (3) disc face plane at a distance 'a' from the sphere centre. As illustrated in the Appendix A.5 with reference to Fig. 3.5, the distance between disc centre and any one of the intersecting point B and B' can be expressed in terms of ϵ:

$$\epsilon = (X - X_c)^2 + (e - Y_c)^2 + (Z - Z_c)^2 - {r_3}^2 \quad 3.18$$

Where X_c, Y_c and Z_c are the co-ordinates of the disc centre C; X, Y (e) and Z are the co-ordinates of intersecting points B or B' depending on the value of θ.

From the intersecting planes (2) and (3) in Fig. 3.5 and Fig. 3.8 the co-ordinates of B and B' can be expressed as:

$$X = r_2 \cos\theta \tag{3.19}$$

$$Y = e \tag{3.20}$$

$$Z = r_2 \sin\theta \tag{3.21}$$

Where r_2 is the radius of the circle defined by the sphere and lies on the vertical plane at a distance 'e' from the sphere centre.

To determine the appropriate value of θ, basic equations 3.18, 3.19, 3.20 and 3.21 have been considered to develop a computer programme (Programme P1) on the basis of bisection and root finding technique.

3.7 Determination of d

In earlier discussion 'd' has been defined as the distance between the sphere centre and horizontal soil surface. In general, a particular setting of disc is fixed by the disc angle (β), inclination angle (α) and the depth of cut (Z). Among them the depth of cut and inclination angle has an influence on 'd'. The relation between above mentioned variables and 'd' has been explained in Appendix A.6 with reference to Fig. 3.9 and can be presented as:

$$d = R\sin(\alpha + \cos^{-1}\frac{a}{R}) - z \tag{3.22}$$

Where Z is the depth of cut of the disc and R is the radius of the sphere. In present analysis d accounted as a dimensionless number, thus it can be expressed as:

$$\frac{d}{R} = \sin(\alpha + \cos^{-1}\frac{a}{R}) - \frac{z}{R} \tag{3.23}$$

3.8 Disc Rear Face Scrubbing Analysis

From observation the soil reaction on disc can be divided into two parts:

1. Soil reaction on the concave section of the disc.

2. Soil reaction (scrubbing) on the convex or rear section of the disc.

The nature of the soil reaction mainly depends on the disc setting, which is fixed by the disc angle, inclination angle and depth of cut. The disc geometry has also an effect on the nature of soil reaction. The transition between the concave to convex soil reaction can be explained by the criterion of scrubbing. This can be explained from Fig. 3.10. In the first instance Fig. 3.10(a), the disc setting is such that the total soil reaction is on concave side; in the second instance Fig. 3.10(b), the disc is at the point of start scrubbing and in the third instance Fig. 3.10(c), the disc has both concave and convex (scrubbing) soil reactions. So it is evident that if any point on the disc edge circle crosses the Y-Z axis plane the rear face of the disc starts scrubbing.

A moving co-ordinate system CXYZ is used to study the scrubbing criterion with the changes in disc angle, disc inclination and depth of cut. The origin coincides with the centre of the circular cutting edge of the disc; CX is the direction of motion of the assembly, CY and CZ are the transverse and vertical directions respectively (Fig. 3.11). An auxiliary system of co-ordinates CX_1Y_1Z is formed by rotating the axes CX and CY through an angle of $90° - \beta$ with respect to the axis Z and the system $CX_1Y_2Z_1$ is formed by rotating the axes CY_1 and CZ through an angle of α with respect to CX_1. Axes CX_1 and CZ_1 are in the plane of the disc cutting edge.

Each point on the edge of the disc describes a circle. The equation of the point have the following form in the system of co-ordinates $CX_1Y_2Z_1$.

$$X_1 = r_3 \cos\theta \tag{3.24}$$

$$Y_2 = 0 \tag{3.25}$$

$$Z_1 = -r_3 \sin\theta \tag{3.26}$$

Where r_3 is the radius of the disc face circle, θ is an angle measured clockwise between the radius of the disc parallel to the horizontal and the radius passing through the given point on the working surface.

Rotating co-ordinate axes CY_2 and CZ_1 with respect to CX_1 axis through an angle and then axes CX_1 and CY_1 about the CZ axis through an angle $90^\circ - \beta$ (Fig. 3.11), the equation of co-ordinate points on the working surface of the disc in the system of co-ordinates CXYZ can be determined as:

$$X = r_3 \cos\theta \sin\beta + r_3 \sin\theta \sin\alpha \cos\beta \qquad 3.27$$

$$Y = r_3 \sin\theta \sin\alpha \sin\beta - r_3 \cos\theta \cos\beta \qquad 3.28$$

$$Z = -r_3 \sin\theta \cos\alpha \qquad 3.29$$

The co-ordinate points of the disc centre (X_c,Y_c,Z_c) with respect to the co-ordinate axes at the centre of the sphere have been determined earlier. Now, the position of a given point on the disc edge with respect to the reference co-ordinate system at the sphere centre can be easily determined as follows:

$$X_0 = X_c - X \qquad 3.30$$

$$Y_0 = Y_c - Y \qquad 3.31$$

$$Z_0 = Z_c - Z \qquad 3.32$$

It has been observed from the cutting action of a disc, that scrubbing on the rear face of the disc occurs when the disc edge crosses the Y-Z axes plane i.e., when the value of X_0 is negative for any point at the edge of the disc cutting surface in contact with soil. Therefore, scrubbing will occur when $X_0 < 0$ for any point on the disc cutting edge in contact with soil. A computer programme P1 (Subroutine Scrub.) has been developed to predict the scrubbing with the changes in disc and attitude parameters.

3.9 Rake Angle, Length and Depth of Cut at the Concave Side

McKyes (1985) proposed an approximate method of analyzing curved shape cutting implements by using the angle of the cutting point at the lower blade tip as the effective rake angle of the tool, provided that the tool is symmetrical about the X-Z plane of the travel direction. In addition to the previous assumption of narrow tines parallel to the direction of motion of the implement, a straight plane has been considered through the tip of the cutting edge and the intersection point of the horizontal soil plane with disc working surface (Fig. 3.5) along each tine, which will act as an imaginary straight tine. Therefore rake angle, rake length and depth of cut can be easily determined from previously determined co-ordinates of intersecting points A and B (Fig. 3.5) as follows:

$$L = \sqrt{(X_a - X_b)^2 + (Y_a - Y_b)^2 + (Z_a - Z_b)^2} \qquad 3.33$$

$$Z = Z_b - d \qquad 3.34$$

$$\alpha_r = \sin^{-1} \frac{Z}{L} \qquad 3.35$$

Where L, Z and α_r are the length of rake, depth of cut and rake angle. X_a, Y_a and Z_a are the co-ordinates of intersecting point A; X_b, Y_b and Z_b are the co-ordinates of intersecting point B and 'd' is the distance between the sphere centre and the horizontal soil surface.

3.10 Determination of Width of Cut

In operation a particular setting of disc implement is fixed by the disc angle (β), inclination angle (α) and the depth of cut (Z). These variables have a significant influence on the width of cut performed by a disc at a particular setting. The relation between the above mentioned variables and the width of cut 'W' can be expressed with reference to Appendix A.7 and Fig. 3.12 as:

$$W = \frac{2\cos\beta}{\cos\alpha}\sqrt{Z(D\cos\alpha - Z)} \qquad 3.36$$

Where D is the diameter of the disc.

3.11 Rake Angle, Length and Depth of Cut at the Convex Side

To determine the area of scrubbing at the rear face of the disc implement three intersecting planes need to be considered (Fig. 3.13). They are:

1. Horizontal soil plane at a distance 'd' from the sphere centre.

2. Any vertical plane at Y-Z axes plane.

3. Disc face plane at a distance 'a' from the sphere centre.

The spherical triangle described by the intersecting points E, F and G in Fig. 3.13 represent the area of scrubbing at any instant for a particular setting of disc. The area of scrubbing is a function of both disc and attitude parameters. From the intersecting planes (1) and (2) in Fig. 3.13 the co-ordinates of F can be expressed as:

$$X_f = 0 \quad 3.37$$

$$Y_f = \sqrt{1 - d^2} \quad 3.38$$

$$Z_f = d \quad 3.39$$

The co-ordinates of intersecting point E of the planes (1) and (3) has been determined in the section 3.5. In reference to the Appendix A.8, the distance between disc centre and the intersection point G of the planes (2) and (3) can be expressed in terms of ϵ:

$$\epsilon = (X - X_c)^2 + (Y - Y_c)^2 + (Z - Z_c)^2 - {r_3}^2 \quad 3.40$$

Where X_c, Y_c and Z_c are the co-ordinates of the disc centre C; X, Y and Z are the co-ordinates of intersection point G (X co-ordinate of G is zero) and r_3 is the radius of the disc. In reference to the Appendix A.8, the co-ordinates of the intersection point B of the planes (2) and (3) can be expressed as:

$$X = 0 \quad 3.41$$

$$Y = \cos\theta \quad 3.42$$

$$Z = \sin\theta \tag{3.43}$$

To determine the appropriate value of θ, basic equations 3.40, 3.41, 3.42 and 3.43 have been considered to develop a computer programme (Programme P1) on the basis of bisection and root finding technique.

As in the case of the concave section soil-disc interface it is assumed that the disc rear scrubbing surface is also composed of a finite number of narrow back-ward raked tines defined by the geometry of the scrubbing surface parallel to direction of motion of the implement. So the width of each back-ward raked tine can be expressed as:

$$b = \frac{(Y_f - Y_g)}{N} \tag{3.44}$$

Where b is the width of each tine, Y_f and Y_g are the Y co-ordinate of intersection point of F and G, and N is any arbitrary number of back-ward rake tine.

The distance y_m between the vertical plane along the interface of the back-ward rake tine and the sphere centre can be determined from the expression:

$$y_m = Y_f - bm \tag{3.45}$$

Where m is any positive integer (m=1...N).

The boundary of the vertical plane along the back-ward rake tine is fixed by the disc sphere and the distance between this imaginary plane and the sphere centre is y_m. Thus, the radius of this imaginary circle r_m can be expressed as:

$$r_m = \sqrt{1 - {y_m}^2} \tag{3.46}$$

Where r_m is radius of the vertical circle representing the interface of back-ward rake tine m (m=1....N).

In reference to Appendix A.8 and Fig. 3.14. disc scrubbing rake angle, depth of cut and rake length can be expressed as:

$$\alpha_r = 180^\circ - \alpha_r{}' \tag{3.47}$$

$$Z = r_m - d \tag{3.48}$$

$$L = \frac{Z}{\sin \alpha_r} \tag{3.49}$$

Where α_r is the rake angle, Z is the depth of cut, L is the rake length and $\alpha_r{}'$ is an angle explained in the Appendix A.8.

As an alternative to the above values of rake angle, length and depth of cut. it may be more realistic to consider each scrubbing rake angle along the broken line shown in Fig. 3.14, because at scrubbing, the soil failure takes place along the disc outer working surface.

Chapter IV

RUPTURE GEOMETRY OF SOIL BLOCK

4.1 Introduction

The working surface of the common disc is a section of a sphere. For simplicity it is assumed that at any instant the soil-disc interface is composed of a finite number of narrow tines along the direction of motion of the implement. Unlike the straight tine, each imaginary tine has a curved interface defined by the geometry of the disc, attitude angles (disc and inclination angle) and the direction of motion. As discussed in determining rake angle and rake length of individual imaginary tine in the previous chapter, a straight plane has been considered through the tip of the cutting edge and the intersection point of the horizontal soil plane with the disc working surface, which act as an imaginary straight tine. With the above assumption, the three dimensional disc problem has been resolved into two dimensional type problem. So that, the established tine theory can be applied to estimate the rupture geometry of the soil block as well as the performance of each individual tine. Therefore, disc performance can be determined by integrating all individual tine characteristic performances.

The analytical methods of solving two dimensional soil mechanics problems have been evolved from the earlier works of Coulomb (1776), Rankine (1857) and Terzaghi (1943), and a significant contribution has been made by Sokolovski (1960). Later Hettiaratchi and Reece (1974) developed an improved numerical solution on a more realistic approach to soil-machine mechanics problems of complicated nature. In case of solving problem with very narrow tine, a three dimensional approach has been made successfully by Payne (1956), Hettiaratchi and Reece (1967), Godwin and Spoor (1977) and McKyes and Ali (1977). These methods are based on the important assumption that the limiting equilibrium of soil mass under loading can be quantified mathematically. This approach leads to the solution of the stress within the soil mass under load from certain known boundary conditions. The central assumption in this type of analysis is that the soil is a rigid-plastic (Mohr-Coulomb) material which fails under zero strain.

4.2 Sokolovski Analysis

The most rigorous approach to the methods of classical soil mechanics for the solution of two dimensional soil failure is Sokolovski's analysis of the limiting equilibrium of a soil mass. Essentially, this analysis is based on the failure properties of a rigid Mohr-Coulomb material and combines the known relationship between the stresses at failure with the partial differential equations of equilibrium for a cohesive frictional material with weight are:

$$\frac{\partial \sigma_x}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} = \gamma \qquad 4.1$$

$$\frac{\partial \tau_{xy}}{\partial x} = \frac{\partial \sigma_y}{\partial y} = 0 \qquad 4.2$$

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy} = \sin^2\phi(\sigma_x + \sigma_y + c\cot\phi)^2 \qquad 4.3$$

The total distribution of stress throughout a mass or failing soil can be obtained by integrating the above equations from known boundary stresses using the method of characteristics. Three basic types of boundary value problems are encountered, which depends on the failure geometry.

1. The Cauchy Problem: This involves within the boundary OA round to the failure line or characteristics direction OB (Fig. 4.1(b)). The distribution of stresses can be obtained by integrating the equations within the above mentioned boundary.

2. The Goursat Problem: This boundary proceeds from the characteristics direction OB (Fig. 4.1(b)), to the neighbouring characteristic OC. The stress distribution can be obtained by proceeding from the characteristics direction OB to OC. This is feasible because the 'singular point' O is a convenient mathematical focus where information required for the solution of the equations can be transferred from the failure line OB, through OC to that pertaining to the interface OD itself.

3. The Mixed Boundary Value Problem: This involves the region ODC (Fig. 4.1(b)), where OC is a characteristic direction and OD is a non-characteristic line along which charecteristic direction are known, because its roughness is quantifiable.

4.3 Newcastle Analysis

A comprehensive set of the passive slip-line fields predicted by Sokolovski's method has been developed by Hettiaratchi and Reece (1974). The main slip-line fields are built up from a small number of basic zones, each of which has simple mathematical properties. A detail soil rupture configuration on the basis of slip-line fields has been represented by Hettiaratchi (1986). Those basic zones are basically divided into four distinct types of slip-line fields which depends on the failure geometry controlled by the interface rake angle α_r, the direction of translation β_t, the soil internal friction angle ϕ and the soil-interface friction angle δ. These shapes are illustrated in Fig. 4.2(a)-(d) and each of these modes of failure has to be considered separately, although (a) and (c) use almost identical calculation procedures.

The main assumptions of his analysis are as follows:-

- Soil failure takes place in two-dimensional field
- Soil is an isotropic rigid-plastic material which fails according to the Mohr-Coulomb criterion such that $\tau_f = \sigma \tan\phi + c$
- The shear stresses mobilised on the soil-structure interface can be described by the relationship $\tau_f = \sigma \tan\delta + a$. The mobilised values of the angle of friction δ_m and the tangential adhesion a_m may lie in the range $-\delta < \delta_m < \delta$ and $-a < a_m < a$ respectively. These values depend on kinematic considerations associated with the slip-line field and the direction of translation of the interface.
- The rigid plane soil-structure interface extends at least up to the free soil surface.

- Any normal surcharge pressure q applied to the soil surface is uniformly distributed over an area at least as great as the rupture zone.

- In the development of the shape of the slip-line field it is assumed that $a = c_a = c \tan \delta \cot \phi$.

- It is assumed that along any straight slip-line the frictional stresses due to soil self weight increase linearly with depth and those due to cohesion and surcharge remain independent of depth. This is predicted by Sokolovski's rigorous solutions for $\frac{c}{\gamma z} = \infty$.

4.3.1 Basic Rupture Surface

The basic slip-line fields appropriate to passive failure in front of an interface with a large rake angle is shown in Fig. 4.3(a) and consists of three distinct zones. The zone OCD adjacent to the soil surface is a passive Rankine zone [R] in which the direction of the slip lines is controlled by the gravitational field, the slope of the soil surface and the nature and direction of action of the surcharge pressure. In the basic problem the last two variables are restricted to a horizontal soil surface and a vertically applied surcharge pressure only. For these restrictions the slip lines are straight and make angles of $45° - \frac{1}{2}\phi$ with the horizontal surface.

Adjacent to the interface is zone OAB (referred to as the interface zone) and this is also composed of straight slip lines whose inclination with the interface is governed by the values of δ and ϕ. Its geometry is controlled by the angle $OAB = \theta^+$ and the magnitude of this can be readily determined from the Mohr's diagram given in Fig. 4.3(b). The point F on the Mohr's diagram corresponds to the shear plane AB and the point H represents the stress condition along the inerface OA and the angle $FGH = 2\theta^+$. Solving triangle GHJ leads to the following:-

$$\Delta = \sin^{-1} \frac{\sin \delta}{\sin \phi} \qquad 4.4$$

$$\theta^+ = 45° + \frac{1}{2}\phi + \frac{1}{2}(\delta + \Delta) \qquad 4.5$$

It is evident from equation (4.4) that Δ is indeterminate when $\delta = 0$ and $\phi = 0$. This condition will be dealt with as a special case in a later section and as far as the present discussion is concerned it will be assumed that $\delta > 0$ and $\phi > 0$.

The transition zone OBC (labelled [T]) simply fills up the slip-line field, if any, between the interface and Rankine zones. This zone is composed of curved slip lines, which are logarithmic spirals, and straight radii and the slip-line field merges smoothly into the adjacent two zones.

This slip-line field satisfies the boundary conditions everywhere, with the exception of the interface where $a = C_a = c \tan\delta \cot\phi$ and all the slip lines intersect at $(90^\circ + \phi)$ and $(90^\circ - \phi)$. It may be thought therefore that it is the only possible slip-line field. This is not generally so, there are other possibilities in which the slip lines in the interface zone are slightly curved as are the radial slip lines in the transition zone. These fields without any straight slip lines give lower values of the passive force the actual values depending on the magnitude of the dimensionless numbers $\frac{c}{\gamma z}$ and $\frac{q}{\gamma z}$. This effect can only be taken into account by a rigorous numerical solution of Sokolovski's equations or by using the correction factors presented by Hettiaratchi et al. (1974). However, it should be emphasised that the slip-line fields discussed in this section are correct for values of soil numbers $\frac{c}{\gamma z}$ and $\frac{q}{\gamma z}$ in excess of a threshold value of 10.

4.3.2 Rupture Surface with a Stress Discontinuity

It will be observed from equation (4.5) that the angle θ^+ and hence the shape of the interface zone is independent of the rake angle α_r. As the rake angle is made smaller from the situation shown in Fig. 4.3 the transition zone decreases in size until it finally disappears at a limiting value α_d given by:

$$\alpha_d = 90^\circ - \frac{1}{2}(\delta + \Delta) \qquad 4.6$$

It is interesting to note that in practice α_d has quite a large value.

At values of α_r less than α_d there is no transition zone to provide a smooth transition between the slip-lines and the stress conditions in the Interface and Rankine zones. Instead there has to be a sudden jump in one normal stress across a boundary of stress discontinuity. This boundary is a straight line when $\frac{c}{\gamma z}$ is large giving the slip-line field shown in Fig. 4.4(a).

The stresses on a pair of elementary cubes R and I, lying in contact with each other on either side of the discontinuity are shown in Fig. 4.4(b). The normal stress σ_a must be the same for both elements R and I as also are the shear stresses τ_i and τ_r. The two normal stresses σ_b and σ_c are, however, different since they are generated by contact with separate soil elements lying on either side of R and I. It will be seen that the straight slip lines in the two zones are 'refracted' across the plane of discontinuity.

The slip-line field with a discontinuity is fully defined once the inclination ω made by the plane of discontinuity with the horizontal is known, the angle θ^+ being obtained from equation (4.5). The value of ω can be obtained from the Mohr's circle diagram shown in Fig. 4.5. The common point A shared by the two Mohr's circles for the Interface and Rankine zones represents the stresses (σ_a, τ_r) whilst the points B and C characterize the stresses (σ_b, τ_r) and (σ_c, τ_i) respectively. From the geometry of the Mohr's circle labelled 'R':

$$\sigma_a = \sigma_r[1 - \sin\phi\cos 2(90^\circ - \Omega)] - H$$

$$\tau_r = \tau = \sigma_r \sin\phi \sin 2(90^\circ - \Omega)$$

Similarly from the Mohr's circle labelled 'I':

$$\sigma_a = \sigma_i[1 - \sin\phi\cos 2(\zeta_i - \Omega)] - H$$

$$\tau_i = \tau = \sigma_i \sin\phi \sin 2(\zeta_i - \Omega)$$

These four equations can be combined to give:

$$\frac{1 - \sin\phi\cos 2(90^\circ - \Omega)}{\sin 2(90^\circ - \Omega)} = \frac{1 - \sin\ \phi\cos 2(\zeta_i - \Omega)}{\sin 2(\zeta_i - \Omega)} \qquad 4.7$$

The inclination ζ_i made with the vertical by the direction of the major principal stress in the interface zone is given by:

$$\zeta_i = 180^\circ - [\alpha_r + \frac{1}{2}(\delta + \Delta)]$$

Equation (4.7) can thus be solved for Ω and the required inclination $\omega = (90^\circ - \Omega)$ of the plane of discontinuity can be obtained as:

$$2\omega = \xi - \sin^{-1}(\sin\phi\sin\xi) \qquad 4.8$$

$$\xi = \alpha_r + \frac{1}{2}(\delta + \Delta)$$

Once ω is known the entire slip-line field can be constructed and the frictional soil resistance force can be determined .

4.3.3 Rupture Surface with a Boundary Wedge

It has been shown by Hettiaratchi and Reece (1975) that under certain circumstances fixed bodies of soil, referred to as 'boundary wedges', build up in front of the interface. For appreciable interface movements the boundary wedges form purely as a result of the kinematic incompatibility between the basic slip-line field developed in the previous sections and the direction of translation of the interface zone. These wedges are formed at the larger values of rake angle when the lower boundary of the Interface zone in the basic field points down below the direction of translation. In practice such a situation would cause cavities to open up in the soil and under these conditions the interface zone develops into a wedge of soil which does not slide relative to the interface and thus alters the boundary conditions at the interface.

Boundary wedge formation is illustrated in Fig. 4.6 where the basic slip-line field predicted by the methods previously described is shown in Fig. 4.6(a). This field is appropriate only for interface motion in the range shown. The limits of this range are fixed by the directions parallel to the interface and the bottom boundary of the interface zone.

As shown in Fig. 4.6(b), translation in directions nearer to the horizontal results in the formation of a boundary wedge with its base parallel to the direction of translation and having an apical angle of $90° - \phi$. The broken lines on this diagram outline the basic slip-line field shown in Fig. 4.6(a) and emphasises the changes introduced by the boundary wedge. It will also be noticed that the usual Transition and passive Rankine zones lie ahead of the wedge and upward sliding of these zones relative to the wedge can take place along the face of the wedge which is itself fixed relative to the interface. The direction of translation is defined by the angle β_t measured positive downwards from the horizontal reference datum. The rupture blocks shown in Fig. 4.6(b) to 4.6(d) demonstrate how the geometry of the boundary wedge is altered as the direction of translation is changed. The limit of this analysis is reached when $\beta_t = -(45^\circ - \frac{1}{2}\phi)$.

It is evident from Fig. 4.6 that , like the interface zone, the shape of the boundary wedge depends on the included angle θ^+ between the bottom of the wedge and the interface. It is also evident that this angle is now kinematically determined by the rake of the interface and its direction of translation and is given by:

$$\theta^+ = 180^\circ - (\alpha_r - \beta_t) \qquad 4.9$$

The way this angle is constrained to change as the rake angle of the interface is altered whilst it is translating horizontally ($\beta = 0$) is illustrated in Fig. 4.7. It will be recalled that the value of θ^+ given by equation (4.5) is expressed as a function of known values of the peak interface roughness angle δ. In the case where boundary wedge θ^+ is known beforehand from kinematic considerations alone, it follows that δ must take on a value comensurate with the known value of θ^+. This implies that the actual *mobilised* angle of friction δ_m is other than the maximum value of δ and must lie in the range $-\delta < \delta_m < +\delta$.

The limit at which the interface zone just converts into a boundary wedge with the interface friction still fully mobilised ($\delta_m = \delta$) can be obtained by combining equations (4.5) and (4.9) to give:

$$\alpha_w{}^+ = (135^\circ - \frac{1}{2}\phi) + \beta_t - \frac{1}{2}(\delta + \Delta) \qquad 4.10$$

As the rake angle α_r is increased or the direction of translation β_t decreased the degree of mobilization of the friction and adhesion between the boundary wedge and the interface falls off and a point is reached when these become zero. The wedge is then in neutral equilibrium with no tendency to move either towards or away from the soil surface. Further change in α_r and β_t will *reverse* the mobilization of friction and adhesion and the boundary wedge will be loaded for incipient downwards slip away from the soil surface. This process finally ends when the friction and adhesion are fully mobilised in the reverse or negative sense. At this point the rake angle $\alpha_w{}^-$ for any specified β_t is given by:

$$\alpha_w{}^- = (135^\circ - \frac{1}{2}\phi) + \beta_t + \frac{1}{2}(\delta + \Delta) \qquad 4.11$$

At values greater than $\alpha_w{}^-$ the friction and adhesion are fully mobilised and negative and become independent of both α_r and β_t.

4.3.4 Rupture Surface with a Rankine Wedge

In the case of small rake angle and perfectly rough interface with $\delta = \phi$, $\phi > 0$ the value of the angle $OAB = \theta^+$ in Fig. 4.3(a) is given by the equation (4.5) as $\theta^+ = 90° + \phi$. For this case OB coincides with OA and the Interface zone vanishes from the slip-line field. Furthermore, equation (4.8) gives $\omega > \alpha_r$ and equation (4.6) shows that the interface will lie entirely within the passive Rankine zone because $\alpha_d = 45° - \frac{1}{2}\phi$. Under these circumstances the rupture diagram given in Fig. 4.4(a) is inappropriate. The situation is then represented by the slip-line field given in Fig. 4.2(d).

Chapter V

PREDICTION OF SOIL REACTION FORCES ON A DISC

5.1 Introduction

On the basis of the assumptions and analysis set out in the previous chapters, the soil reaction force calculation on a disc is handled through the following steps:

- Soil reaction force calculation on each imaginary tine, in which two dimensional passive force calculation theory (Newcastle theory) has to be used.
- Effective reaction force calculation on each interface slice, in which soil slice conception (Bishop's simplified method) has to be used.
- Effective reaction force calculation on the disc.

5.2 Soil Reaction Force Calculation on Imaginary Tine

In previous chapters an imaginary straight plane has been assumed to replace the actual curved interface, which is passes through the tip of the cutting edge and the intersection points of the horizontal soil surface with the working surface of the disc. The soil body bounded by this assumed interface and the real curved interface is assumed to be a fixed soil block at any instant. Therefore, in calculating soil reaction forces at the imaginary interface, soil frictional resistance is considered as soil-soil friction.

5.2.1 Force Calculation for a Basic Rupture Surface

The passive soil reaction on the imaginary interface is made up of two components, the frictional soil resistance P acting at an angle ϕ to a line which is perpendicular to the tangent of the interface and a tangential cohesive force A_c along the imaginary interface. The line of action of P can be determined by separating it into its gravitational-frictional component P_γ, cohesive component P_c and surcharge component P_q, assuming that P_γ acts two thirds the way along the interface from O and the corresponding position of P_c and P_q are half way from O (Fig. 5.1). The main task is therefore the determination of the forces P_c, P_q, P_γ and A_c from the static equilibrium of all forces acting on the soil body OABCE in Fig. 5.1(a) and Fig. 5.1(b).

The assumptions made are sufficient to determine the magnitude, location and direction of action of the forces F_1, F_1', F_2 W, W_1, Q and A_1. The magnitude of the moment M_c about O of the cohesive forces acting along the spiral sector BC can also be established but the location and magnitude of the corresponding frictional component F_3 is indeterminate. From the properties of the logarithmic spiral surface BC the line of action of F_3 is known to pass through the pole O of the logarithmic spiral surface BC.

A solution for P_c, P_q and P_γ can be obtained by first considering the static equilibrium of the transition zone and half of the passive Rankine zone forming the body OBCE in Fig. 5.1(b), followed by the static equilibrium of the interface zone OAB and the zone bounded by the imaginary interface OA and real curved interface of the disc in Fig. 5.1(a) This requries the introduction of three new internal forces F_4 F_5 and A_1; the latter being easily determined as the product of the length OB with the soil cohesion c.

The static equilibrium of the body OBCE is analysed in three stages as follows:

1. under the action of gravitation and friction forces only, so that c=0, q=0 and $\frac{c}{\gamma z} = \frac{q}{\gamma z} = 0$.
2. under the action of cohesion and friction with $c > 0$ and $\frac{c}{\gamma z} > 0$.
3. under the action of surcharge and friction with $q > 0$ and $\frac{q}{\gamma z} = 0$.

This assumes that the principle of superposition holds good and is a well known device introduced by Terzaghi (1959). The technique is not applicable in a rigorous analysis because it assumes that the slip-line field remains unchanged in the two cases. However, it is perfectly applicable to the present method and introduces no additional error. The analysis then proceeds as follows:

Stage I: Equilibrium of OBCE

1. Gravitation and Friction. Moment equilibrium about O:

$$F_4 d_4 = F_2 d_2 + W_1 d_w + (W_d) \tag{5.1}$$

2. Cohesion and Friction. Moment equilibrium about O:

$$F_{5c} d_5 = F_1 d_1 + M_c \tag{5.2}$$

3. Surcharge and Friction. Moment equilibrium about O:

$$F_{5_q} d_5 = F_1' d_1 + Q d_q \tag{5.3}$$

The moments $M_w = W_d$ and M_c can be evaluated from the following expressions in which the boundary radii of the logarithmic spiral zone are $OB = r_1$ and $OC = r_2$ and the included angle $BOC = \eta$.

$$M_w = \frac{\gamma {r_2}^3}{3(m^2+1)}[e^{m\eta}[m\cos(\varepsilon+\eta) + \sin(\varepsilon+\eta)] - m\cos\varepsilon - \sin\varepsilon] \tag{5.4}$$

$$M_c = \frac{c({r_2}^2 - {r_1}^2)}{2\tan\phi} \tag{5.5}$$

Where $m = -3\tan\phi$ and $\varepsilon = (45^\circ - \frac{1}{2}\phi)$

Stage II: Equilibrium of OAB

Considering the equilibrium of the force components perpendicular to F_6 it is helpful to note that A_1 is parallel to F_6 and hence does not enter into the calculations and F_5 and F_4 are parallel to A_2.

$$P_\gamma = \frac{[W_2 \sin(\theta^+ - \phi - \alpha_r) + F_4\cos\phi]}{\sin(\theta^+ - 2\phi)} \tag{5.6}$$

$$P_c = \frac{[A_c\cos(\theta^+ - \phi) + (F_{5c} + A_2)\cos\phi]}{\sin(\theta^+ - 2\phi)} \tag{5.7}$$

$$P_q = \frac{F_{5q} \cos\phi}{\sin(\theta^+ - 2\phi)} \quad 5.8$$

Stage III (A). Equilibrium of the Section Bounded by OA and Interface (Soil reaction occur on the disc concave face):

The weight of the soil body bounded by OA and the interface is negligible. Although the direction of the adhesive force is changing continuously along the curved interface, for convenient in computation it may be assumed that the resultant adhesive force A acting half-way from O and tangent to the interface. Thus, in analysing equilibrium of the force components acting on the soil body the weight of the soil is neglected and forces has been resolved normal to the adhesive and cohesive force components as explained in the Appendix B.1 with reference to Fig. 5.2 and Fig. 5.3.

1. Gravitation and Friction:

$$P_{\gamma r} = \frac{P_\gamma \cos(\phi + \theta_3)}{\cos\delta} \quad 5.9$$

2. Cohesion-Adhesion and Friction:

$$P_{ca} = \frac{P_c \cos\phi}{\cos\delta} \quad 5.10$$

3. Surcharge and Friction:

$$P_{qr} = \frac{P_q \cos\phi}{\cos\delta} \quad 5.11$$

Stage III (B). Equilibrium of the Section Bounded by OA and Interface (Soil reaction occur on the disc convex face):

As discussed in section 3.11, the imaginary tine interface has been considered along the broken line in Fig. 3.14. The weight of the soil bounded between the imaginary interface and convex working surface of the disc is negligible and this weight force component has been neglected in analyzing equilibrium on the soil body bounded by the interface and the tine as explained in the Appendix B.2 with reference to Fig. 5.4.

1. Gravitational component:

$$P_{\gamma r} = \frac{P_\gamma \cos(\phi - \theta_4)}{\cos\delta} \qquad 5.12$$

2. Cohesion and Adhesion component:

$$P_{ca} = \frac{P_c \cos\phi}{\cos\delta} \qquad 5.13$$

3. Surcharge component:

$$P_{qr} = \frac{P_q \cos\phi}{\cos\delta} \qquad 5.14$$

Stage IV. Passive Soil Reaction Force on the Interface.

The frictional soil resistance force P acting on the interface can be simply determined by adding the gravitation, cohesive-adhesive and surcharge force components acting to the interface. Which will act at an angle δ to a line perpendicular to the tangent of the interface.

$$P = P_\gamma + P_{ca} + P_q \qquad 5.15$$

The magnitude of the tangential adhesive force A can be determined as:

$$A = ar_4\theta \qquad 5.16$$

Where 'a' is the adhesion, θ is the arc angle in raddians (fixed by the interface) at the centre of the imaginary circle passes along the interface and r_4 is radius of that circle derermined in the Appendix B.1 and B.2.

Due to the curved interface the direction of adhesive force is changing continuously along the interface. However, this force can be combined with the cohesive-adhesive force component acting half-way from O (Fig. 5.5), by a simple assumption that the resultant adhesive force A is acting at the same point as a tangential force to the interface. From Fig. 5.5 the combined cohesive-adhesive force and its direction with the horizontal can be evaluated as:

$$P_{cx_1} = P_{ca}\sin(\delta + \alpha_r) + A\cos\alpha_r \qquad 5.17$$

$$P_{cz_1} = P_{ca}\cos(\delta + \alpha_r) - A\sin\alpha_r \qquad 5.18$$

$$P_{cr} = \sqrt{{P_{cx_1}}^2 + {P_{cz_1}}^2} \qquad 5.19$$

$$\theta_c = \tan^{-1}\frac{P_{cz_1}}{P_{cx_1}} \qquad 5.20$$

Similarly, the gravitational and surcharge force components can be divided as:

$$P_{\gamma x_1} = P_{\gamma r}\cos\theta_\gamma \qquad 5.21$$

$$P_{\gamma z_1} = P_{\gamma r}\sin\theta_\gamma \qquad 5.22$$

$$P_{qx_1} = P_{qr}\cos\theta_q \qquad 5.23$$

$$P_{qz_1} = P_{qr}\sin\theta_q \qquad 5.24$$

Where $P_{\gamma x_1}$ and $P_{\gamma z_1}$ are the horizontal and vertical conponents of gravitational force and θ_γ ($\theta_\gamma = 90° - \delta - \alpha_r$) is the angle between $P_{\gamma r}$ and the horizontal (Fig. 5.5), P_{qx_1} and P_{qz_1} are the horizontal and the vertical components of surcharge force $P_{\gamma r}$ and θ_q is equal to θ_γ.

5.2.2 Force Calculation for a Stress Discontinuity

The limiting conditions and the construction of slip-line field has been discussed throughly in section 4.3.2. It is evident from previous discussion that once ω is known the entire slip-line field can be constructed and the frictional soil resistance are readily determined by considering the static equilibrium of the soil body OAFE in Fig. 5.6. All the forces marked on this sketch can be determined with the exception of P_γ, P_c, P_q and F_3. The required force P_γ, P_c and P_q can be calculated by evaluating the equilibrium of all forces normal to the force F_3:

1. Gravitational and Frictional Component:

$$P_\gamma = \frac{[W_3 \sin(\theta^+ - \phi + \alpha_r) - F_2 \cos(\theta^+ - \phi + \alpha_r)]}{\sin(\theta^+ - 2\phi)} \qquad 5.25$$

2. Cohesive Component:

$$P_c = \frac{[A_2 \cos\phi + A_c \cos(\theta^+ - \phi) - F_1 \cos(\theta^+ - \phi + \alpha_r)]}{\sin(\theta^+ - 2\phi)} \qquad 5.26$$

3. Surcharge Component:

$$P_q = \frac{[Q \sin(\theta^+ - \phi - \alpha_r) - F_1' \cos(\theta^+ - \phi - \alpha_r)]}{\sin(\theta^+ - 2\phi)} \qquad 5.27$$

Then stage III.A and IV of section 5.2.1 has to be followed to calculate the soil reaction forces acting on the interface.

5.2.3 Force Calculation for a Boundary Wedge

The limiting conditions and construction of the rupture soil block has been discussed throughly in section 4.3.3. It is evident from previous discussion that in the case of boundary wedge θ^+ being known beforehand from kinematic considerations alone, it follows that δ must take on a value commensurate with the known value of θ^+. When the value of θ^+ is fixed by the equation (4.9) the corresponding value of the mobilised angle of friction δ_m can be obtained from the Mohr's diagram given in Fig. 4.3(b) as:

$$\delta_m = \frac{\sin\phi \cos[\phi + 2(\alpha_r - \beta_t)]}{\sin\phi \sin[\phi + 2(\alpha_r - \beta_t)] - 1} \qquad 5.28$$

The actual mobilised angle of friction δ_m is other than the maximum value of δ and must lie in the range $-\delta < \delta_m < +\delta$.

The analysis of the forces acting on an interface with a boundary wedge is identical to that of the basic slip-line field, except that δ is replaced by the mobilised value δ_m obtained from the equation (5.8).

5.2.4 Force Calculation for a Rankine Wedge

The limiting conditions and construction of slip-line configuration have been discussed in section 4.3.4. The analysis to determine the forces acting on the interface in this case follows the steps already outlined. However, in this instance, the soil block OAF (Fig. 5.6) is missing and the equilibrium equations are obtained by resolving all the force in the direction of forces P_γ, P_c and P_q.

1. Gravitation and Friction Component:

$$P_\gamma = (W_4 \cos\psi + F_2 \cos\varepsilon)\cos(\psi - \alpha_r - \phi) + W_5 \cos(\alpha_r + \phi) \qquad 5.29$$

2. Cohesive and Friction Component:

$$P_c = (F_1 \cos\varepsilon - A_3 \sin\phi)\cos(\psi - \alpha_r - \phi) + A_3 \sin(\varepsilon - \alpha_r - \phi) - A_c \sin\phi) \qquad 5.30$$

3. Surcharge and Friction Component:

$$P_q = (F_1' \cos\varepsilon + Q_1 \cos\psi)\cos(\psi - \alpha_r - \phi) + Q\cos(\alpha_r + \phi) \qquad 5.31$$

Then stage III.A and IV of section 5.2.1 are to be followed to calculate the soil reaction force acting to the interface.

5.2.5 Special Considerations when $\phi = 0$

The angle of friction δ depends both on soil properties and interface surface conditions such as its hardness, degree of roughness and lubrication. In practice δ varies between about $\frac{1}{2}\phi$ for hard polished surfaces up to a maximum of ϕ for interfaces which are very rough or very soft so that $\frac{1}{2}\phi < \delta < \phi$.

The above restriction raises a problem when ϕ tends to zero because δ also tend to zero. It may then appear that there can be no difference between the interface acting as a principal plane ($\delta = 0$) and a failure plane ($\delta = \phi$). This ignores the important role played by the tangential adhesion acting between the soil and the interface. In the analysis discussed so far the magnitude of the interface adhesion is fixed by δ because of the basic assumption that the linear laws for soil failure and the interface sliding set out in assumptions (section 4.3) pass through a common point in the $\sigma - \tau - plane$. This stipulation requires $a = C_a = c \tan\delta \cot\phi$.

As δ tends to zero the influence of adhesion plays a dominant role in controlling the orientation of the slip-planes at the interface in contrast with the part played by the diminished value of δ. When $\phi = 0$ the product $\tan\delta \cot\phi$ and the ratio $\frac{\sin\delta}{\sin\phi}$ in equation (4.4) become indeterminate and it is then necessary to use the ratio a/c to estimate the angle Δ in Fig. 4.3(b). The revised value of this angle can be evaluated from the Mohr's diagrams in Fig. 5.7(a) as:

$$\Delta = \sin^{-1}(a/c) \tag{5.32}$$

This value must be used in all the equations in which it appears when $\phi = 0$ introduces an indeterminacy.

When a boundary wedge is formed the degree of mobilization of the adhesion has to be taken into account as was the case for δ when $\phi > 0$. The mobilised value of adhesion δ_m can lie in the range $-a < a_m < +a$. The minimum value of θ^+ is reached for the condition shown by the Mohr's diagram in Fig. 5.7(b) and corresponds to the point where the wedge has mobilised the full negative adhesion with incipient downwards motion of the wedge. In order to estimate the magnitude of the adhesive force A it is necessary to determine δ_m from equations (4.9) and (5.32) as:

$$a_m = c \cos 2(\beta_t - \alpha_r) \tag{5.33}$$

5.3 Effective Horizontal Force on each Interface Slice

A generalized procedure of slices for composite slip-surface has been widely used in stability analysis. In most of the cases this has been carried out in terms of effective stress along the slip-surface. Due to the variations in the stresses along the trial slip-surface, in common practice the slip mass is considered as a series of slices. This procedure of determining effective stress on slices was introduced by Bishop (1954) and Janbu (1954).

In the present alalysis the conception of Bishop's (1954) simplified method has been considered to determine the effective force acting on each interface slice. It has been seen from the previous discussion that the resultant forces determined from two-dimensional tine theory are acting half way (P_{ca} and P_q) and two-thirds of the way (P_γ) from the soil surface along the interface with an angle δ to a line, which is perpendicular to the tangent of the interface. Two slip-circles have been selected with centres O_1 and O_2 on the Z-axis in the plane of the forces, so that each set of forces lie in one slip-circle plane. As the width of each imaginary tine is very small, it is assumed that the interslice forces are equal and opposite and cancel each other out , that is, $E_1 = E_2$ and $X_1 = X_2$ in Fig. 5.8.

Therefore, the forces acting on the interface are, P determined from previous calculation, the transverse adhesive force T and a reaction force N normal to the interface transverse section as shown in Fig. 5.8. As explained in Appendix B.3 the effective force on the interface slice can be determined as:

$$N = P \cos \eta \qquad 5.34$$

Where N is the effective force on the interface slice, P is the horizontal component of P_γ, P_{ca} or P_q determined from tine theory, η is the angle between the line of action of the forces P and N as explained in Appendix B.4 with reference to Fig. 5.9.

5.4 Force Calculation Above the Limiting Value $\alpha_w{}^-$

The values of the force components acting on a narrow tine above $\alpha_w{}^-$ limit can be calculated satisfactorily using Newcastle theory with appropriate considerations. Due to the simplification of the complex disc geometry, the calculation of the scrubbing soil reactions above the $\alpha_w{}^-$ limit, especially when the back-ward rake angle is near to 180° produces very low in magnitude. Terzaghi's (1948) bearing capacity conception has been considered to overcome the problem. The modified rupture geometry is shown in Fig. 5.10. The force components P and A_c in Fig. 5.10 can be determined by considering $\angle AOC$ as modified rake angle

α_m and fully rough interface. So the vertical resistance force acting on the middle of back-ward rake (Fig. 5.10) can be expressed in reference to Appendix B.5 as:

$$P_v = 2\cos\epsilon(P + A) - W \qquad 5.35$$

The resultant soil reaction P_r on each slice has been considered acting at $-\delta$ and normal to the tangent of the disc underside. So the horizontal component P_h of the resultant reaction force can be determined as:

$$P_r = \frac{P_v}{\cos(\alpha_r{}' + \delta)} \qquad 5.36$$

$$P_h = P_r \cos(90^\circ - \delta - \alpha_r{}') \qquad 5.37$$

The resultant force obviously passes through the vertical axis of the reference co-ordinate axes. Then the draught and lateral component can be expressed in terms of the horizontal component as:

$$BP_{rx} = P_h \cos\eta \qquad 5.38$$

$$BP_{ry} = P_h \sin\eta \qquad 5.39$$

Where BP_{rx} and BP_{ry} are the draught and side force components and η is the angle between P_h and the x-axis of the reference co-ordinate axes as explained in Appendix B.4.

In accordance to the reference co-ordinate system BP_{rx} is positive and BP_{ry} is negative as the slices under scrubbing lie in (-X)-Y quadrant. The polarity of BP_{rz} is negative as it acts vertically upward.

5.5 Calculation of the Total Force on a Disc

A. Disc Concave Section: In calculating effective horizontal forces on each interface slice the centre of the slip-circle lies on the vertical axis of the reference co-ordinate axes. Essentially all the effective forces on the slices passes through the vertical axis. From the knowledge of the point of action and direction the effective horizontal force for cohesive-adhesive, gravitational and surcharge components can be divided into draught and side force components from Fig. 5.9 :

$$P_{ix} = N \cos \eta \qquad 5.40$$

$$P_{iy} = \pm N \sin \eta \qquad 5.41$$

Where P_{ix} and P_{iy} are the draught and side force components, [i stands for ca (cohesive-adhesive), γ (gravitational) and q (surchagre) components], η is the angle between effective horizontal force and the X-axis.

The direction of the side force P_{iy} is positive towards the face of the disc and negative when the direction is away from the disc face. In other words, with reference to the reference co-ordinate system when the slice is in X-Y quadrant P_{iy} is positive, and when in X-(-Y) quadrant P_{iy} is negative. The direction of P_{iy} can be determined by the polarity of e (calculated in chapter 3). The polarity of P_{iy} and e is the same in all the cases.

The vertical force components for cohesive-adhesive, gravitational and surcharge force are same as P_{cz_1}, $P_{\gamma z_1}$ and P_{qz_1} as determined in section 5.2.1, stage IV.

Then each force component is multiplied by the tine width to get the real magnitude of the force component. To determine the total force on the concave side of the disc, force components of all the tines has been integrated as draught, side and vertical forces as:

$$FP_x = P_{cx} + P_{\gamma x} + P_{qx} \qquad 5.42$$

$$FP_y = P_{cy} + P_{\gamma y} + P_{qy} \qquad 5.43$$

$$FP_z = P_{cz} + P_{\gamma z} + P_{qz} \qquad 5.44$$

B. Disc Convex Section: The scrubbing force calculation procedure is the same as the concave side force calculation except when the back-ward rake angle is greater than the limiting value of rake angle α_w^-. The calculation procedure above this limiting rake angle has been discussed in section 5.3. As all the slices under scrubbing are in (-X)-Y quadrant the polarity of the side force component is always negative.

As with the concave side, the force components on all the back-ward raked tines have been integrated as draught, side and vertical forces:

$$BP_x = BP_{cx} + BP_{\gamma x} + BP_{rx} \qquad 5.45$$

$$BP_y = BP_{cy} + BP_{\gamma y} + BP_{ry} \qquad 5.46$$

$$BP_z = BP_{cz} + BP_{\gamma z} + BP_{rz} \qquad 5.47$$

C. Total Force: The total draught P_x, side P_y and vertical P_z reaction forces on the disc can be determined from the following equations:

$$P_x = (FP_x + BP_x)R_s \qquad 5.48$$

$$P_y = (FP_y + BP_y)R_s \qquad 5.49$$

$$P_z = (FP_z + BP_z)R_s \qquad 5.50$$

Where R_s is radius of the sphere in appropriate unit.

The resultant force on the disc can also be determined as:

$$P = \sqrt{{P_x}^2 + {P_y}^2 + {P_z}^2} \qquad 5.51$$

Chapter VI

EXPERIMENTAL METHODOLOGY AND INSTRUMENTATION

6.1 Experimental Apparatus and Instrumentation

All experiments were carried out in the University of Newcastle upon Tyne soil mechanics laboratory with the objective of measuring the forces acting on a working disc in three orthogonal directions, along with disc rotation. The experiments were conducted in the soil bin with a special power carriage, which hold the dynamometer frame and furnished power for holding and pulling the disc under study. The disc was mounted on a dynamometer frame and in this manner the force system which the soil exerted on the disc was measured. Various dynamometer arrangements allowed continuous measurement of forces on the disc by an Oscillograph recorder connected through an amplifier (Fig. 6.1). The disc rotation was also monitored simultaneously by an event marker with a special arrangement of a micro-switch through a voltage regulator.

More sophisticated techniques aiding the measurement of the soil reaction force include Godwin's (1975) extended Octagonal ring transducer, used for measuring two mutually perpendicular force components (a horizontal force F_x and a vertical force F_y) and a moment (M_y) in the plane of these two forces, and O'Dogherty's (1986) Tri-axial dynamometer, used for measuring three orthogonal forces acting on a soil working tool.

Godwin (1975) pointed out the following advantages of his newly developed technique over the conventional one, especially when a multi-dynamometer suspension system is needed :

- problems associated with the dynamometer suspension bushes.
- single component construction, compactness and simple mounting aid an advantage, where precision allignment is necessary for accuracy.

- ability to monitor two force components and the moment in the plane of these two forces simultaneously.

Taking these facts into consideration it would have been desirable to use an Octagonal ring transducer as developed by Godwin (1975) for measuring the disc forces. However, these advanced transducers were not available for use in the proposed test rig. As a result, the rather cumbersome and possibly less accurate, compound dynamometer system was designed and developed for the measurement of disc forces. Within the limitations of such an arrangement the system performed with an acceptable degree of accuracy and consistency. Regular calibration checks were incorporated into the test procedures to ensure the set standards.

6.1.1 Existing Apparatus and Experimental Facility

The Universal Soil-tool Interaction Testing Machine consists of four main functional components namely: the soil bin, the dynamometer and tool suspension carriage, the soil compaction and levelling carriage and the power unit as shown in Fig. 6.2.

The size of the soil bin was 6.1mX0.76m with a depth of about 40cm. The soil bin was placed parallel to the movement of the tool suspension carriage.

The dynamometer and tool suspension carriage was suspended from and ran alongside the soil bin on two parallel rails located in the same vertical plane. The distance between the rails was 1.22m. This carriage had a sub-carriage which was able to move in the vertical plane on two parallel vertical rails operated by an electric motor. This vertical motion carriage had a vertical displacement of about 50cm, which carried the testing equipment and the dynamometer assembly frame. The carriage was strong enough to support any reasonable amount of longitudinal, vertical and side force. The main carriage was also capable of any speed in the range 0.0061-1.83m/sec. The carriage was driven by a looped chain from variable speed 7.5 H.P. D.C. motor.

The soil compaction and levelling carriage was also suspended and ran on the same parallel rails in vertical plane. The compaction of soil was carried out by a vibrator, which could move vertically on two vertical parallel rails and was connected to a flat horizontal plate fixed by two legs. The leveller could also move similarly to the vibrator on two separate parallel vertical rails, and levelling was carried out with a metal plate connected to the vertically movable frame by two legs. Both the vibrator and the leveller were powered by two electric motors.

Before present modification on the dynamometer rig, two dynamometers were connected between the two right vertically mounted parallel plates (see Fig. 6.3). The first plate from the right was rigidly bolted to the vertical moving carriage and the second plate was suspended from the first plate by three links, which allowed the second plate to move vertically and horizontally but prevented any side-ways movement. The dynamometer arrangement between the plates was such that one dynamometer could monitor any horizontal movement, and the other could monitor any vertical movement of the second plate relative to the first plate. Any tool which had to be tested had be suspended rigidly from the second plate. So the testing facility provided by this arrangement could handle only two dimensional soil failure problem by the tillage tool (i.e., only draught and vertical force component can be measured). However, our purpose was to measure the three force components on the disc implement along three orthogonal directions (i.e., draught, vertical and side force components), which clearly needed some means to measure the sideway force component.

6.1.2 Design and Modification of the Existing Dynamometer Rig

In addition to the previous dynamometer rig a third plate had been introduced with the attitude to arrange additional three dynamometer between the second and third plate to measure the side force component (Fig. 6.4). The thickness of the plate was selected as 25mm, so that it could withstand without bending under large range of force system. The third plate was suspended from the second plate through three linkage, so that the third plate could move only sideway. Both ends of each linkage were connected with a ball joint, which allowed the linkage to transmit horizontal and vertical force without significant friction. The linkage was also designed so that its length could be varied if necessary for proper alignment. One dynamometer was placed at the top and other two were placed along the side between the two plates parallel to the horizontal

(Fig. 6.3), so that any side force F_y could be measured in relation to the forces measured by the three transducers:

$$F_y = R_1 - R_2 \qquad 6.1$$

Where R_1, ($R_1 = R_l + R_r$) was the combined reaction force of the two side transducer and R_2 was the reaction force of the top transducer. Explained in Appendix C.1 with reference to Fig. 6.5. The direction of the side force F_y could be determined from the polarity of the force R_1 and R_2 displayed on the Oscillograph recorder. The dimension of all brackets, pivot pins and bolts had been chosen so that they could withstand 8 KN of side force. The disc under test was to be mounted on the third plate with a disc clamp.

6.1.3 Design of the Disc Leg Assembly

In field operation disc ploughs are set at an angle to the direction of the travel as well as at an inclination with the vertical. In order to fulfil the field operating condition the disc leg assembly had been designed so that the disc could be set at any disc angle (β) from $0°$ to $90°$ and inclination algle (α) from $0°$ to $25°$ with an increment of $5°$ (Fig. 6.6 and Fig. 6.7).

A bar 120mmX30mm in cross-section and 463mm in length had been chosen for disc leg. A 30mm diameter machined rod was welded at the upper end of the leg and the bottom end was machined so that it could swing without friction with the base of the disc holder when pivoted with the disc collar (Fig. 6.8). Two strips of metal plate were welded on both side of the leg so that it could be aligned properly between the disc collars. A single hole was drilled at the bottom strip to pivot with the disc collar by a bolt and several holes were drilled on the other strip to allow the disc setting at an inclination with the vertical in the range $0°$ to $25°$ at an increment of $5°$ when fixed to the disc collar at specified holes.

A 16mm slot was cut on the upper plate (Fig. 6.6) with a radius of 150mm and centre at the disc leg vertical axis and rigidly bolted to the dynamometer rig outer plate. The lower plate containing two holes of 16mm was welded to the disc leg so that those holes could be aligned properly to the slot of the upper plate.

This arrangement allowed the disc to be set at any disc angle between 0° to 90°. The disc leg assembly was mounted on the outer plate of the dynamometer rig with a clamp.

6.1.4 Design of the Event Marker

A micro-switch was mounted on the disc collar so that the four equally distant L-shaped fingers fixed on the disc carrier could trigger the micro-switch by actuating the micro-switch trigging lever, when the disc in operation performed rotation. This signal was carried out through a voltage regulator to the Oscillograph event marker channel. The circuit diagram of the designed voltage regulator is shown in Fig. 6.9.

6.2 Experimental Procedures

The main objective of the experimental work was to assess the performance of the calculation procedures developed. Longitudinal (draught), vertical and side force components of soil reaction were measured together with the disc rotation and forward travel. These measurements were made at different combination of the two independent variables, namely: the disc angle(β) and inclination angle (α). In all cases the depths were kept at 100mm and cutting speed of the disc was kept very low so that inertia force could be neglected.

To exclude the effects of any other variables, the soil conditions were kept unchanged in all the test runs. The soil moisture content was checked in regular interval. To maintain moisture content at the same level the soil bin was kept covered with a polythene cover between the test runs and the soil was slightly sprayed with water when necessary to prevent a fall in the moisture content. Same number of vibrator runs were maintained for the preparation of the soil bed to keep the soil bulk density at the same level. A preliminary experiment was carried out to determine the minimum number of vibrator passes with same depth beyond which the soil bulk density did not significantly increase. This minimum number of passes plus one more pass of the vibrator for confirmation was made in each soil bed preparation. Nevertheless, the soil bulk density was checked before each test run to ensure the same level of bulk density in all test runs.

6.2.1 Calibration of Dynamometers

Dynamometers used in this experiment were calibrated individually and collectively. Each dynamometer was hung freely from a rigid support through flexible wire and loaded incrementally. This dynamometer was connected to the specified Oscillograph channel through an amplifier and the sensitivity was chosen so that each division of the galvanometer deflection on the Oscillograph record represent 100N. Then the dynamometer was unloaded in the same fashion to check its performance with loading. The calibration for all the dynamometers were found satisfactorily linear in normal working load.

To check the cross-sensitivity of the dynamometer arrangement in the dynamometer rig, they were calibrated collectively. After mounting the disc on the dynamometer rig a horizontal pull was provided by loading the disc horizontally forward through flexible wire and pulley arrangement. The response of each dynamometer was recorded by the Oscillograph keeping the sensitivity unchanged. The dynamometer (4) responsible for monitoring horizontal force deflected properly as it was intended. There was a little deflection in each of the three dynamometers (1) placed at the top, (2) at the left and (3) at the right (Fig. 6.3); responsible for monitoring side force, when added or subtracted according to the nature of the force (tension or compression) cancelled each other out. Similarly, application of the vertical load deflected the dynamometer (5) responsible for monitoring vartical force as intended and a little deflection in dynamometers (1), (2) and (3) cancelled each other out. In side force calibration, a side-way pull provided by loading the disc horizontally side-way through flexible wire and pulley arrangement. There was no deflection in dynamometers (4) and (5). The deflection in dynamometers (1), (2) and (3) responsible for monitoring side force, when added or subtracted according to the nature of the force displayed the loading characteristics as it was intended.

6.2.2 Soil Processing

The soil chosen for this experiment was a Ryton sand, the physical and mechanical properties of this soil will be discussed later in this chapter. Three days before the test run, the soil was sprayed with water and allowed to reach the soil moisture content at an optimum level. Soil preparation was carried out by thoroughly pulverizing the soil by moving the vibrator blade in opposite direction and then with a spade. Then the soil was raked to a uniform depth over the entire area of the soil tank. The levelling blade was then pulled along the soil tank to level the soil surface. The vibrator was then set to the lower level and passed along the soil tank. The top layers of soil were subsequently compacted in a second pass with the vibrating plate close to the surface. A level compacted surface was then produced using two passes to compact the soil to its optimum bulk density. Finally levelling blade was pulled along the soil tank to level the soil surface by skimming off a very thin layer of soil. The uniformity of the soil surface was then checked by passing the disc along the soil tank so that the disc edge was just about to touch the soil surface.

6.2.3 Measurement of the Soil Reaction Force Components

A 660mm 'Parmiter' DPM disc having 600mm radius of curvature and 96mm height of concavity (Parmiter Product Manual (1987)), was used to conduct this experiment. The disc was allowed to rotate freely to its axis and was mounted on the dynamometer rig through disc leg. Before test run the disc angle (β) and inclination angle (α) were set as intended and disc clamp bolts were tightened properly. Then all galvanometers positions were set to zero by adjusting the galvanometer position adjusting screws. Having satisfactory soil preparation the disc was set at 100mm depth and the carriage was allowed to move slowly. When the trigging finger attached to rear of the disc about to trig the micro-switch the carriage was stopped and a mark was put down on the soil tank parallel to a particular point on the carriage. Then the Oscillograph was allowed to run, subsequently the carriage was pulled at a slow speed (0.5 m/min). When the disc travelled a resonable distance and the trigging finger was about to trig the micro-switch the carriage was stopped. Before lifting the disc out of the soil the surcharge was accounted by calculating the height of the tilled soil accumulated in front of the disc. In doing so, the cross-section of the accumulated soil along the face of the disc was approximated by a triangle and three arms of that triangle were measured. Then the disc was lifted out of the soil and after a while the Oscillograph recorder was stopped. Another mark was put down on the soil tank parallel to the corresponding point on the carriage and the distance between the two marked points on the tank was measured. All the forces being measured by the dynamometers were recorded along with the disc rotation as a event mark on the Oscillograph record. All steps discussed above were repeated for each combination of disc angle (β) in the range $0°$ to $80°$ at an increment of $10°$ and disc inclination angle (α) in the range of $0°$ to $15°$ at an increment of $5°$. Each combination was duplicated.

6.2.4 Measurement of the Surcharge

The surcharge stress (q) changes with the disc angle (β). Soil bin experiment shows that at a disc angle of $\beta = 0°$ the soil builds up in front of the disc and overflow from the disc top, and that soil building up gradually decreases with the increase of β (Fig. 6.10) until at high disc angle, no surcharge was observed. It was also observed from the computation of the experimental measurements of the accumulated soil mass that the changes in surcharge with the disc angle is a cosine function. So the surcharge stress can be determined as:

$$q = \frac{(D - Z)}{2} \gamma \cos \beta \qquad 6.2$$

Where D is the diameter of the disc and Z is the depth of cut.

6.2.5 Measurement of the Soil Parameters

The soil moisture content was checked twice a day. Each time five samples were taken from the soil tank. The samples were weighed before putting in the drying oven at $110°C$ for a period of about 24 hours. Then the soil samples were weighed again after drying. Weights of the soil samples before and after the drying gave the necessary parameters to determine the soil moisture content. The moisture content was calculated by the equation:

$$m = \frac{W_w - W_d}{W_d - W_c} X100 \qquad 6.3$$

Where m is the moisture content in percent, W_c is the weight of container (gm), W_w is the weight of container plus wet soil (gm) and W_d is the weight of container plus soil after drying (gm). The average moisture content determined from all soil samples was 6.34 percent.

The soil bulk density was measured for each test run. After the preparation of soil for a test run, a cylindrical cutter of 38mm diameter and 76.2mm in length was inserted in the soil, dug out and emptied, and its soil content was weighed. Three samples were taken from different position along the soil tank before each run and the average bulk density was measured. The bulk density was measured as the weight of the soil sample divided by the volume of the cylinder and represented in KN/m^3. The average bulk density of all the samples was found as 16 KN/m^3.

The shear box apparatus was used to determining soil cohesion, soil internal friction angle, adhesion and soil-metal friction angle. A 60mmx60mmx38mm brass box, open top and bottom was used in the shear apparatus. The box was horizontally divided into two halves, which could be made to slide relative to each other. This box was placed in a rectangular brass container so that the bottom half of the shear box was rigidly held in position relative to the container. The container rode upon a frictionless bearing and was restricted to move in the longitudinal direction. Then the sample (soil with above mentioned moisture content and bulk density) was sheared at a constant rate of strain by a geared jack, driven by an electric motor. The jack was pushed against the container and hence the bottom of the shear box. This movement was transmitted through the sample to the upper half of the box and hence to a force transducer. The transducer was connected to the Y axis of a X-Y plotter through appropriate amplifier. The displacement of the geared jack was measured by another displacement transducer connected to the X axis of the X-Y ploter through amplifier. This test was repeated with a different normal load. The failure shear load was detected when the shear load either started to fall off or remained constant for a significant jack displacement. This failure shear load was divided by the cross-sectional area of the shear box to determine the shear stress. Similarly, normal stress was determined by dividing the total normal load with the same cross-sectional area. The shear stress was then plotted against the normal stress (Fig. 6.11). The resulting straight line made an angle ϕ (soil-soil friction angle) with the X-axis and soil cohesion was measured from the origin to its interception with the vertical axis.

The soil interface properties was measured by replacing the bottom of the shear box with a sample strip of disc blade material. The above mentioned procedures were followed to determine the angle of soil-metal friction (δ) and adhesion 'a' (Fig. 6.12). From shear box test the value of a and δ were found as 2.5 KP_a and 24.8° respectively.

Undrained triaxial compression test was carried out to compare the values of cohesion (c) and soil internal friction angle (ϕ) with that obtained from the shear box test. Standard procedures were followed to make soil samples of 38mm diameter and 76.2mm length with proper density and moisture content mentioned above.

The loading frame, proving ring and the triaxial cell provided a system for applying both the hydrostatic pressure and the axial load to the sample. The axial load was monitored automatically by a transducer, which was mounted on the proving ring. The lateral strain was monitored by another transducer, which recorded the upward movement of the pedestal.

The changes in the sample volume were monitored by measuring the amount of water entering or leaving the triaxial cell. This movement of water was monitored automatically by another transducer. These three transducers were connected to a X-YY plotter through an amplifier, which then produce a continuous recording of the three variables.

The compression stresses were calculated using the proving ring constant and the new area of the sample at a particular interval of strain. The failure compressive stress was detected when the stress either started to fall off or remain constant for certain interval. Four samples were tested at increasing cell pressure and were plotted on a Mohr's stress diagram (Fig. 6.13). In this diagram normal stresses were plotted as abscissa and shear stresses as ordinate with the centres located on the abscissa axis at a distance from the origin=(cell pressure+$\frac{1}{2}$ maximum compressive stress). The diameter of the circle=(maximum compressive stress).

The condition of failure of the sample was approximated by a straight line drawn as a tangent to the circles, the equation of which is the Coulomb equation $\tau_f = c + \sigma \tan\phi$. The values of cohesion (c) was read off the shear stress axis, where it was cut by the tangent to the Mohr's circles and the angle of shearing resistance (ϕ) was the tangent between the tangent and a line parallel to the shear stress axis. Both the shear box test and the triaxial test come out with the same value of cohesion c=3 KP_a and soil internal friction angle ϕ=32.5 deg.

6.2.6 Analysis of the Oscillograph Records

During calibration of each dynamometer, the sensitivity of the amplifier channels was chosen so that the deflection of each division on the Oscillograph record represent 100N. The direction of each galvanometer deflection depended upon the nature of the force acting on each dynamometer. From calibration it was seen that the dynamometers (1), (2) and (4) deflected rightward in response to tensile forces whereas, the dynamometers (3) and (5) deflected leftward. In response to compressive force all dynamometers deflected in opposite direction. The dynamometers (1), (2) and (3) monitored side force ((1) placed at the top, (2) at the right and (3) at the left). The dynamometers (4) and (5) monitored horizontal and vertical forces respectively. The side force was calculated from the equation 6.1 with appropriate polarity. The linear distance covered by the disc rotation was calculated as:

$$S = \frac{\pi D}{4}(E-1) \qquad 6.4$$

Where S is the actual distance covered by disc rotation, D is the disc diameter and E is the number of event marks. The co-efficient of slipping or skidding was determined from the equation 2.5.

6.3 Characteristics of Experimental and Predicted Force Curves

Fig. 6.14 to Fig. 6.16 shows the comparison between the predicted and experimental reaction force curves for 0.10m depth in the Sandy loam (Ryton sand) soil. The force curves are presented disc angle (β) as abscissa from 0° to 80° at an intervel of 10° and force in KN as ordinate with an inclination angle of 0°, 5° and 10°. The soil and disc properties are presented in Appendix C.2. In general the magnitude of the draught force for both predicted and experimental decreases with an increase in the disc angle (β). As the disc start scrubbing around disc angle of 60° – 70° depending upon the depth of cut and inclination angle (α), the magnitude of the draught force increases rapidly.

The magnitude of the side force P_y in both predicted and experimental results is zero at 0° disc angle. This can be explained by the fact that at disc angle of 0° the disc cutting edge is lies in symmetry with the direction of pull, so the magnitude of the side force on both symmetrical side cancel each other out. As the disc angle increases the side force increases and reached at its peak level at a disc angle of about 30°. Then with the increase of disc angle the magnitude of the side force decreases and after scrubbing the direction of the side force become reverse and the magnitude increases rapidly with the further increase of disc angle.

In general the predicted values of vertical force component P_z show a small downward vertical reaction at disc angles before scrubbing, whereas the experimental results indicate an upward reaction. After scrubbing both predicted and experimental results show an upward reaction and the magnitude increases rapidly with the further increase of disc angle.

6.4 Comparison Between Predicted and Experimental Forces

A series of tests were conducted on a disc of 650mm diameter and 600mm in radius of curvature at a working depth of 0.10m and inclination angle of 0°, 5°, and 10°, in Sandy loam (Ryton sand) soil, whose properties are given in Appendix C.2. Figures 6.14 to 6.16 show the comparison between predicted and experimental results. In general the predicted model adequately reflects the changes in the magnitude of the draught, side and vertical forces due to the interaction between passive cutting and scrubbing forces with changes in disc and inclination angles.

In general the minimum value of the draught force occurs in the range of disc angle (β) $60^\circ - 70^\circ$. This minimum draught force range is dependent on the disc angle at which the disc starts scrubbing. At the point the disc starts scrubbing, draught is lowest. The curves shows that the draught force is 10 percent over-estimated at $\beta = 10^\circ$ whereas, at $\beta = 70^\circ$ the draught force is under-estimated as much as 50 percent. The practical field operation is confined in the range of $\beta = 35^\circ - 55^\circ$. In that range the variation between predicted and experimental draught force is below 8 percent.

The reversal in the direction of the side force occurs in the range of $\beta = 65^\circ - 75^\circ$. This happens at an angle just after the disc starts scrubbing as the scrubbing component of the side force is always in the negative direction. In general the predicted side force is over-estimated in the range of $\beta = 0^\circ - 45^\circ$. The worse case occurs at $\alpha = 0^\circ$ and $\beta = 30^\circ$, the predicted side force is as much as 25 percent over-estimated. As the inclination angle increases the difference between the predicted and experimental results become minimized in the normal working range of the disc angle. After scrubbing, at a higher inclination angle side force component is slightly under-estimated.

The predicted values of vertical forces generally shows a small downward vertical reaction before scrubbing, whereas the experimental results indicate an upward reaction. This can be explained by the fact that the disc has a compound rake angle and a small amount of scrubbing would take place on the underside of the disc edge. In Fig. 6.14 with $\alpha = 0^\circ$, the vertical force is slightly negative at $\beta = 0^\circ$, then it increases in magnitude in the positive direction with an increase in the disc angle. With the increase in inclination angle α, the downward force slightly increases. This can be explained by the fact that with the increase in α, the rake angle of the assumed tines decreases, which increase the penetration of the disc slightly. It is evident from Figures 6.14 to 6.16. that in practical working range (i.e., $\beta = 35^\circ - 55^\circ$) predicted results are adequate enough to reflect the changes in vertical force component.

The general trend shows that the experimental results agreed well with the predicted results are as follows:

- A lower draught force, at a disc angle just before scrubbing starts.
- Equlibrium of the side force occurs at disc angle $\beta = 0^\circ$ and at a disc angle when scrubbing starts.
- Vertical force is more or less constant upto a disc angle when scrubbing starts.

These findings are in agreement with the experimental results of Godwin et al (1985), shown in Fig. 6.17 for a shallow spherical disc in a sandy loam soil. However, there is a variation observed in the draught force where the predicted force shows an over-estimation in the range of disc angle $\beta = 0° - 40°$. Furthermore, $\beta = 80°$, the vertical and side force components are over-estimated.

Although the minimum value of draught force were observed in the disc angle between 60° to 70°, the specific resistance curve (Fig. 6.18) in combinaton with the width of coverage curve (Fig. 6.19) shows that the marginally smaller disc angles than those of the minimum draught force can manipulate substantially higher volume of soil with a slight penalty in specific resistance. These finding agreed with the experimental results of Godwin et al. (1985) and Nartov (1985). It is worthwhile to mention here that their suggestion to judge tillage efficiency in terms of specific resistance is quite realistic.

Chapter VII

NON-DIMENSIONAL DISC PARAMETERS

7.1 Introduction

The main difficulties in computing earth pressure on agricultural implements are due to their complex shape and the nature of application of force to the soil. Even in simple tines the existing less complex analytical methods for calculating soil reaction forces needs a fair amount of arithemtical work. The complex geometry and the nature of application of force to the soil by an agricultural disc implement makes the analytical computation more complicated. As discussed in section 2.3 only a few attempts were made to compute the soil forces analytically on a disc implement, yet these analysis do not cover the whole range of disc application. The analytical computation of the soil reaction forces on the disc implement discussed in chapters III, IV and V needs a vast computer facility which is not readily available to all engineers. To an engineer a rapid 'calculator solution' would be of the utmost value, and the ensuing analysis is carried out to meet this need.

7.2 Dimensional Analysis

The variables involved in estimating soil reaction forces in soil cutting operation were discussed in section 3.1. It can therefore be stated that the force P per unit projected width of the interface involving in plane soil failure is the function of all other variables involved and can be related as:

$$P = f(\gamma, c, C_a, \phi, \delta, \beta, \alpha, Z, q, \frac{a}{R}, g, V)$$

A comprehensive dimensional anslysis has been carried out in Appendix D.1. There are 13 variables involving three fundamental dimensions (M), (L) and (T) which results in 10 (i.e. 13-3) dimensionless groups. If the values of P, c, C_a, and q are specified in appropriate engineering units, then the equation D.5 becomes:

$$\frac{P}{\gamma Z^2} = f(\frac{c}{\gamma Z}, \frac{C_a}{\gamma Z}, \frac{q}{\gamma Z}, \phi, \delta, \beta, \alpha, \frac{a}{R}, \frac{V^2}{gZ}) \qquad 7.1$$

This equation does not specify the actual influence of each dimensionless group. In computing general earth pressure on simple tines, the total soil reaction force on the interface can be calculated as the vector sum of the respective gravitational, cohesive, adhesive and surcharge components. This has been generally done on the basis of the following assumptions:

- The resultant stresses due to the cohesive, adhesive and surcharge are uniformly distributed over the interface.
- The corresponding stress distribution due to gravitational force varies linearly with depth.

By analogy with above assumptions Hettiaratchi (1968) suggested to express equation 7.1 in the form:

$$\frac{P}{\gamma Z^2} = K_\gamma + \frac{c}{\gamma Z}K_c + \frac{C_a}{\gamma Z}K_a + \frac{q}{\gamma Z}K_q + \frac{V^2}{gZ}K_V \qquad 7.2$$

Where the K-factors are dimensionless numbers which are functions of ϕ, δ, β, α and $\frac{a}{R}$ the five dimensionless groups not accounted for in equation 7.2. Equation 7.2 can be rearranged as suggested by Reece (1965) which yields:

$$P = \gamma Z^2 K_\gamma + cZK_c + C_a Z K_a + qZK_q + \frac{\gamma V^2}{g} Z K_V \qquad 7.3$$

As a further simplification it can be assumed that $V \approx 0$, so that the inertia effects on the soil can be neglected as we consider very slow movement of the interface. Thus, the last term in equation 7.3 can be neglected.

Hettiaratchi (1968) estimated the remaining four groups of dimensionless factors by two different methods. In the first case there was a complete freedom of choice of the values of adhesion. The comparison between the computed K-factors showed an almost identical set of results, the only deviation being of the order of 5 percent at very large rake angle. He noted that this variation was due to the second method of calcultion, which did not accomodate the variation in adhesive component. He suggested to substituting K_{ca} for K_c. This composite grouping is necessary because the limitation set by the relationship $C_a = c \tan\delta \cot\phi$ constrains C_a to take on specific values dependent on δ and ϕ. This is in effect K-factor for a 'restrained' adhesion and the general equation 7.3 for this case gives:

$$P_{ca} = cZK_c + c\tan\delta\cot\phi K_a = cZK_{ca}$$

Where $K_{ca} = (K_c + \tan\delta\cot\phi K_a)$ and K_c and K_a are the K-factors specified in section 7.2. By combining these two K-factors K_c and K_a as K_{ca} the equation 7.3 can be written as:

$$P_j = \gamma Z^2 K_{\gamma j} + cZK_{C_a j} + qZK_{qj} \qquad 7.4$$

Where (j=x, y, z); K-factors of draught, side and vertical force components are represented by x, y and z respectively.

The equation 7.4 yields force component per unit projected width of the disc interface. Therefore, in estimating actual magnitude of that force component right hand side of the equation 7.4 must be multiplied by the projected width of the disc interface.

7.3 Estimation of K-factors

Rapid estimation of forces on the interface would be possible if we are able to evaluate the K-factors in equation 7.4. Hettiaratchi (1968) suggested that if the values of the K-factors can be separated as individual force components P_γ, P_{ca} and P_q, then the appropriate K-factors can be worked out from the product of each of those components with $\frac{1}{\gamma Z^2}$, $\frac{1}{cZ}$ and $\frac{1}{qZ}$ respectively. His analysis also showed that the stress component at the singular point was independent of gravitational force. Thus the absolute stress ordinate is a function of cohesion and surcharge only, and these earth pressure are uniformly distributed over the interface. It is therefore evident that the cohesive and surcharge components of earth pressure on the interface can be readily obtained purely from the magnitude of the stress ordinate at the singular point (Fig. 7.1).

On the basis of the above justification the soil reaction force on the disc was calculated in chapter IV and V, separated into the individual force components P_γ, P_{ca} and P_q. The computer programme (Programme P2) has been used to compute the values of K_γ, K_{ca} and K_q for unit projected width of the disc interface as $\frac{P_\gamma}{\gamma Z^2}$, $\frac{P_{ca}}{cZ}$ and $\frac{P_q}{qZ}$ for various combination of ϕ, β and α for any specified values of Z, γ and $\frac{a}{R}$. Considering the number of variables in all cases δ, has been taken as $\frac{3}{4}\phi$. This simplification is justified because the normal range of δ lies between $\frac{1}{2}\phi$ and ϕ.

The difficulty however is to present the values of K-factors in an intelligible form considering that there is a large number of combinations of the variables. This may be intervened by representing the variation of K-factors with the disc angle (β) by a family of curves for finite steps in the value of ϕ, at a specific interface condition $\delta = \frac{3}{4}\phi$ and at a specific disc inclination angle (α) and $\frac{a}{R}$.

On the basis of the previous works discussed in section 2.3 and the experimental and predicted reaction force curves discussed in section 6.3 and section 6.4, the range of β and α has been chosen as $35° - 60°$ and $10° - 20°$ respectively at an increment of 5° and also for the vertical disc. The range of soil internal friction angle (ϕ) also been chosen in the range of $0° - 40°$ at an increment of 5°. From Fig. 7.2 to Fig. 7.7 it is evident that a group of discs having the same radius of curvature shows no significant variation in the magnitude of K-factors for draught and side force components, whereas, K-factors of the vertical force component shows slightly higher variation in magnitude. So in representing K-factor charts for most of the British Standard Discs recommended by the British Standard Institution (1979), K-factors for draught and side force components have been represented as a group and K-factors for vertical force component expressed in tabular form.

7.4 Characteristics of the K-factors Curves

The K-factors computed by the method described in the preceding sections are displayed in Charts 1 to 72 and in Tables 1 to 27. In general at smaller values of β the magnitude of the K-factors are at the highest level in all cases of draught and side force components. As the disc angle β increases the magnitude of the K-factors decreases. The K-factors for the vertical force component (which is tabutated in Tables **1** to **27**) shows the same trend at smaller values of ϕ. At higher values of ϕ (e.g., $\phi = 40°$) the trend of the K-factors with β become opposite and this occurs especially in the cohesive-adhesive component of the vertical force. In general the soil internal frictional angle ϕ has an exponential effect on the magnitude of the K-factor. Keeping all other condition unchanged, the magnitude of the K-factors increase as ϕ increases, except in the K-factors for the vertical force component. The increase in the magnitude of the K-factors for vertical force component is approximately linear rather then exponential, but at higher value of ϕ, (e.g., $\phi = 40°$) the trend is just opposite.

Similar to the force curves discussed in the section 6.3, the magnitude of the K-factors for draught and side force components decreases as the disc inclination angle increases. The K-factors for the vertical force component shows opposite trend as α increases.

In section 2.2, the discussion on the effect of radius of curvature on disc force from previous works revealed that with the increase in radius of curvature the magnitude of the force components decreases. Similar effect of the radius of curvature has been observed in the magnitude of the K-factors for draught and side force components. The magnitude of the K-factors for vertical force component increases as the radius of curvature increases. This behaviour can be explained by analyzing the effect of radius of curvature on the rake angle of each narrow tines considered in calculating vertical force on the disc. As the radius of curvature increases, for same depth of cut the rake angle of the assumed tine decreases, which helps in incre^a^sing suction as well as the downward vertical force.

7.5 Semi-Empirical correction in K-factors

Strictly speaking the K-factors are not independent of the depth of cut. This is an additional complication which results from the complex feature of the soil-disc geometry leading to small changes with depth of cut in the so-called rake angle of the soil elements. Fig. 7.8 to Fig. 7.13 show the effect of different depths of cut on the K-factors. An involute and comprehensive investigation of these discrepancies was carried out using the computer programes developed. These data were then analysed and empirical rules were developed to provide a correction technique for these deviations.

An empirical correction technique has been introduced with a view to get the approximate value of K-factors within 5 percent error level. the approximate correction equation are tabulated below:

Components	Correction equation
K_{gx} K_{cx} K_{qx}	$K = K_0 + \xi n$
K_{gy} K_{cy} K_{qy}	$K = K_0 + \xi n \tan\beta$
K_{gz}	$K = K_0 + a(e^{bn} - 1)$
K_{cz} K_{qz}	$K = K_0 - \xi \frac{n}{m} \cot\beta$

Where K is the appropriate K-factor, K_0 is the K-factor obtained from the chart, $m = \frac{Z_0}{Z}$ and $n = \frac{Z - Z_0}{Z_0}$, Z_0 is the depth of cut at which the K-factor charts have been prepared, $a = -\tan\phi\tan(\beta - \frac{\pi}{9}) + 1.2\tan(\beta - \frac{\pi}{5.625})$ and $b = 1.57\tan\beta\tan\phi + \tan(\frac{\pi}{4} - \beta)$. The K-factor charts presented in Chart 1 to Chart 72 for the Z value of 0.10m. Z is the intended depth of cut at which K-factor values are being calculated, ξ is the dimensionless number presented in Chart 73 to Chart 104. In all the cases the error is within 5 percent level, except in K_{gz}, in which case some of the values show around 10 percent error. A computer programme (Programme P3) was developed to determine coefficients 'a' and 'b' in the correction equation for K_{gz}. These values were used to develop empirical equation for 'a' and 'b' in terms of variables involved. As the K-values varies unproportionately with ϕ and β for K_{gz}, it is very difficult to get those K-values closer than 10 percent error level. On the other hand, the influence of

K-values for K_{gz} in the total force is very small. So the 10 percent error in some of the K-values may be considered as an acceptable range.

7.6 Interpolation

As shown in section 7.2, the K-factors are the function of disc angle β, inclination angle α, the soil internal friction angle ϕ, the soil interface friction angle δ and the disc characteristics ratio $\frac{a}{R}$. To reduce the very large number of permutations of these variables, K-factors were computed at limiting value $\delta = \frac{3}{4}\phi$. The computed interval for ϕ and α in the chart is 5°. Evidently a system of interpolation has to be devised in order that intermediate values of ϕ and α may be obtained from the charts presented.

7.6.1 Interpolation for ϕ

An analysis of the computed K-factors shows that the variation of the K-factors for different ϕ follows the following exponential variation:

$$K_{ij} = ae^{b \tan \phi}$$

Where $(i = \gamma, C_a, q)$, (j=x, y, z), a and b are constants dependent on the disc angle β, inclination angle α and $\frac{a}{R}$.

In the charts K-factors are specified at 5° intervals of ϕ and K-factors scales are linear. Although both ϕ and α show an exponential variation the magnitude of the variation is not too large. Thus, linear interpolation off the K-factors ordinates on the charts are sufficient for ϕ at 5° intervals. Such an interpolation assumes that the variations of the K-factors in the 5° intervals are proportional to ϕ and not $\tan \phi$. However, the interpolation can be carried out for any ϕ in the range $\phi_1 < \phi < (\phi_1 + 5°)$ as:

$$K = K_1 (\frac{K_2}{K_1})^{n_1} \qquad 7.5$$

Where $n_1 = \frac{\phi - \phi_1}{\phi_2 - \phi_1}$, $\phi_2 = \phi_1 + 5°$, K is the intended K-factor value, K_1 and K_2 are the K-factor values corresponding to ϕ_1 and ϕ_2 and the disc angle β. The detail analysis has been carried out in Appendix D.2 with reference to Fig. 7.14.

7.6.2 Interpolation for α

The variation in K-factor for different inclination angle α also shows exponential variation and follows the expression:

$$K_{ij} = ae^{-b\tan\alpha}$$

Where $(i = \gamma, C_a, q)$, $(j = x, y, z)$, a and b are the constants dependent on disc angle β, soil internal friction angle ϕ and disc characteristics ratio $\frac{a}{R}$. The K-factor charts have been presented for 10°, 15° and 20° of α and for vertical disc. So interpolation similar to ϕ can be carried out for computing K-values at any specified α in the range $\alpha_1 < \alpha < \alpha_2$, $(\alpha_2 - \alpha_1 = 5^{\circ})$ as:

$$K = K_5(\frac{K_6}{K_5})^{n_2} \qquad 7.6$$

Where K is the intended K-value, K_5 and K_6 are the corresponding K-values of α_1 and α_2, and soil internal friction angle ϕ and the disc angle β are specified, and $n_2 = \frac{\alpha-\alpha_1}{\alpha_2-\alpha_1}$.

Chapter VIII

WORKED EXAMPLE

8.1 Use of Disc Performance Charts

Example : This is to calculate the passive soil reaction force on a disc. The disc is assumed to translate horizontally with a disc angle as well as an inclination angle.

DISC PROPERTIES:	
Disc sphere radius (R) :	600mm
Disc characteristic ratio $(\frac{a}{R})$:	0.866
Depth of cut (Z):	100mm
Disc angle (β) :	$45°$
Disc inclination angle :	$14°$
SOIL PROPERTIES:	
Bulk unit weight :	$\gamma = 12.0 \frac{KN}{m^3}$
Cohesion :	$c = 5.0 \frac{KN}{m^2}$
Surcharge :	$q = 4.5 \frac{KN}{m^2}$
Angle of internal friction :	$\phi = 32°$
Angle of soil-metal friction :	$\delta = 24°$

The solution of this problem needs interpolation in both ϕ and α, as well as correction in K-values for the depth of cut.

Determination of K_{gx}:

From chart 7 with $\beta = 45°$ and $\alpha = 14°$:

$$K_1 = 0.650$$

$$K_2 = 0.813$$

and

$$n_1 = 0.4$$

Therefore, K-value for this case can be obtained from the equation 7.5.

$$K_{\alpha_1} = 0.711$$

Similarly, the value of $K_{\alpha_2} = 0.496$ can be calculated from chart 13.

A further interpolation is needed for the disc inclination angle ($\alpha = 14°$), which can be obtained from equation 7.6.

$$n_2 = 0.8$$

$$K = 0.533$$

Correction in K-value for the depth of cut:

To obtain the actual value of K_{gx}, correction equation in section 7.5 ($K = K_0 + \xi n$) is to be considered. The value of correction coefficient with respect to charts 81 and 89, and linear interpolation for $\alpha = 14°$ results, $\xi = 0.498$. The value of the depth factor in correction equation n=0.5.

Thus, the actual value of the K-factor $K_{gx} = 0.782$.

Similarly, the required K-factor values obtained from charts 7 to 18, tables 16 to 18, and charts 81 to 96 are shown in the table below:

K_{gx}	K_{cx}	K_{qx}	K_{gy}	K_{cy}	K_{qy}	K_{gz}	K_{cz}	K_{qz}
0.782	1.778	1.561	0.660	1.443	1.268	0.281	-0.153	0.573

The next step is to calculate the force components in three orthogonal directions by using equation 7.4. The projected width of cut (W) of the disc along the direction of pull can be calculated from equation 3.36.

$$W = \frac{2\cos 45°}{\cos 14°}\sqrt{0.15(0.6\cos 14° - 0.15} = 0.371\ m$$

$$P_x = [12.0 \times 0.15^2 \times 0.782 + 5.0 \times 0.15 \times 1.778 + 4.5 \times 0.15 \times 1.561]0.371 = 0.964 \;\; KN$$

$$P_y = [12.0 \times 0.15^2 \times 0.66 + 5.0 \times 0.15 \times 1.443 + 4.5 \times 0.15 \times 1.268]0.371 = 0.785 \;\; KN$$

$$P_z = [12.0 \times 0.15^2 \times 0.281 + 5.0 \times 0.15 \times -0.153 + 4.5 \times 0.15 \times 0.573]0.371 = 0.129 \;\; KN$$

The resultant force can also be calculated from equation 5.51.

$$P = \sqrt{0.964^2 + 0.785^2 + 0.631^2} = 1.25 \;\; KN$$

Chapter IX

SUMMARY AND CONCLUSIONS

The analysis of the interactions between machine elements and soil seeks to (a) provide calculation procedures for estimating soil reactions, (b) identify the boundaries of soil disturbance and (c) evaluate the deformations imposed on the soil. In the necessary process of simplifying what would otherwise be an impossibly complex set of conditions, the soil is considered as a rigid-plastic material and consequently little information on (c) is forthcoming. However, the other major simplification introduced by assuming that the soil fails according to the Mohr-Coulomb criterion opens up a whole range of analytical techniques for the solution of many practical soil-machine mechanics problems involving (a) and (b).

Useful solutions in these two categories have been developed using both Sokolovski's (1954) analysis and its less accurate derivatives for purely two-dimensional failure problems such as wide cutting blades [Hettiaratchi et al. (1974)]. A less rigorous but practical approach has been carried out for three dimensional problems induced by plane surfaces such as cultivating tines [e.g. Payne (1956), Godwin and Spoor (1977), McKyes et al. (1977)]. However, three-dimensional problems developed by machine elements having curvature in two orthogonal planes has eluded the theoreticians for some considerable period. The first viable theory for such problems was proposed by Godwin et al. (1985) and the present investigation is an extension of the basic concepts of this approach.

The main analysis presented in the thesis deals with disc soil cutting tools which is section isolated by a small circle on a thin hollow sphere. The disc is assumed to engage soil having a flat horizontal free surface at a predetermined depth of cut. The disc element is allowed two degrees of freedom in its setting; the inclination angle in a vertical plane and a disc angle in the horizontal plane. This represents the most general case and is applicable both to the special case of harrow discs (no inclination angle) and to plough discs (with both disc and inclination angles).

The initial part of the investigation was devoted to the complex solid geometry of the disc surfaces in contact with the soil. The main cutting surface consists of the concave contact surface whilst the convex contact areas are involved only when parts of the disc operates in the scrubbing mode. Involved iterative solutions were required in establishing the mathematical relationships describing these working surfaces and the tedious nature of such solutions require the use of a computer.

Once the soil-implement interface has been defined the next step was to develop a cutting theory by approximating the curved surfaces by elemental plane strips. As these strips are in vertical planes parallel to the direction of translation of the disc, the approximation replaces the curved disc by a number of raked cutting blades which are chords of the disc segments. Although in theory the width of the segments could be reduced to infinitely thin ones the analysis is not a strict infinitesimal technique.

Each of these elements is then treated as a plane strain two-dimensional problem and involved computer programmes were developed to predict the force on these 'chordal cutting elements'. The total force acting on the disc surface was then estimated using a simplified version of the method of slices commonly employed for solving slip surface problems. The total force on the disc is then the summation of these chordal elemental forces. The same technique can be used to build up the volume of soil disturbed by the curved cutting surface.

The self-contained computer software developed has the capability of predicting the three orthogonal soil force components from a knowledge of soil failure parameters (c and ϕ), disc geometry and depth of cut. Although most modern design offices and research laboratories are equipped with computers which can handle such calcutations, it was felt that a more widely accessible solution technique, using dimensional force coefficients, would be a practical alternative. To this end the computer programmes were used to develop a set of force coefficients to be used in a simple additive equation such as that proposed by Reece

(1965). This method holds good in the case of two-dimensional failure involving plane interfaces because the rupture boundaries preserve geometric similarity with changes in cutting depths. However in the present context the imaginary rake angle of each cutting element and the associated rupture block shape are not independent of disc orientation and cutting depth.

The presentation of computational charts to cover all these variables would lead to an unacceptably large number of charts thus defeating the whole object of this type of solution. The computer programme was used in a comprehensive investigation to assess the relative sensitivity of the force coefficients to changes in disc orientation, geometry and depth of cut. The results of this investigation were then used to select the variables for the 'base set' of dimensionless force coefficients and these were supplemented with a full set of empirical correction factors to allow for departures from the base values. These provide comprehensive cover over a wide range of input parameters giving chart values to within 5 percent of the computer values.

The specific draught predicted by the model developed shows that the minimum values are attained with disc angles of around 60° to 70°. However these minima are very near the scrubbing limit and in practice much lower values would have to be employed. It would appear that the inclination angle would have a similar effect but once again considerations of penetration and scrubbing would have to be taken into account in determining a practical value.

This investigation includes a check to the performance of the theoretical model for one single plough disc working on a loam soil. A rather crude dynamometer suspension was designed and built specially for this part of the work. Results of other published experimental data could have been checked but for the fact that crucial input parameters on soil strength were generally not specified. The only recent tests carried out by Godwin et al. (1985), which specified all the necessary parameters, was checked against this model. The performance of the theoretical model was found to be good in both cases.

The work presented in this thesis is by no means comprehensive. There are many areas which have been developed by the introduction of drastic simplifications and future models may be capable of improving on these. Some of the topics which need attention are listed below:

1. The effect of disc rotation has been neglected. If this is allowed for the direction of action of soil-to-metal friction forces will no longer lie in vertical planes and a correction may be required.

2. The influence of cutting edge dimensions and sharpening angle has not been dealt with, although this aspect has been checked in previous investigations [McCreery et al. (1956)].

3. The current analysis is a quasi-static equilibrium analysis. The whole question of inertia may prove important as disc tools are quite often used in high-speed operations.

4. The cutting edge configuration used in the present analysis assumes that the disc makes a line contact with the soil, except when scrubbbing takes place. In reality the bottom of the disc is probably in contact over a patch of finite area. That this is the case, even in the laboratory tests, was suspected from inconsistent readings of the vertical force components.

5. The current analysis is applicable to discs which are parts of a hollow sphere. Theoretical models to look into other shapes may be worthwhile to check whether there are any posible improvements in cutting efficiency.

6. The performance of the model needs to be checked in several soil types. A perticularly important evaluation would be in clay soils.

Appendix A

A.1 Determination of Projected Angle β' and α'

The centre of the sphere O, has been taken as the origin of the reference co-ordinate system and right hand rule also be chosen to represent the direction of the co-ordinate axes, as shown in Fig. 3.4. The disc is fixed by a given β and α.

Hence, from $\triangle ABC$:

$$Y_c = X_c \tan\beta$$

From $\triangle OAB$:

$$Z_c = X_c \tan\alpha$$

From rectangles OBCE and OACD in Fig. 3.4.

$$\tan\beta' = \frac{Y_c}{\sqrt{{X_c}^2 + {Z_c}^2}} \qquad A.1$$

$$\tan\alpha' = \frac{Z_c}{\sqrt{{X_c}^2 + {Y_c}^2}} \qquad A.2$$

Substituting the values of Y_c and Z_c in equation A.1 and A.2, the equation for β' and α' becomes:

$$\beta' = \tan^{-1}\frac{\tan\beta}{\sqrt{1+\tan^2\alpha}} \qquad A.3$$

$$\alpha' = \tan^{-1}\frac{\tan\alpha}{\sqrt{1+\tan^2\beta}} \qquad A.4$$

A.2 Determination of the Co-ordinates of Disc Centre C

The disc centre is fixed by the given β, α and the distance 'a' from the origin of the reference co-ordinate system as shown in Fig. 3.4.

From $\triangle OAB$: $\frac{OA}{OB} = \sin\alpha$ and $OA = Z_c$

Hence,

$$Z_c = OB \sin\alpha$$

Similarly, from the $\triangle OBC$: $\frac{OB}{a} = \cos\beta'$ and $OB = a\cos\beta'$.

Therefore,

$$Z_c = a\cos\beta' \sin\alpha \qquad A.5$$

Similarly, from $\triangle ABC$ and $\triangle OAC$: $Y_c = AC\sin\beta$ and $AC = a\cos\alpha'$

Hence,

$$Y_c = a\cos\alpha' \sin\beta \qquad A.6$$

From $\triangle OAB$: $X_c = OB\cos\alpha = a\cos\alpha\cos\beta'$

Again from $\triangle ACB$: $X_c = CA\cos\beta = a\cos\alpha'\cos\beta$.

Therefore,

$$X_c = a\cos\alpha\cos\beta' = a\cos\alpha'\cos\beta \qquad A.7$$

A.3 Determination of the Co-ordinates of A

From Fig. 3.6 $R^2 = {r_1}^2 + d^2$ and hence,

$$r_1 = \sqrt{1 - d^2} \qquad A.8$$

Where, sphere radius R is unity.

From Fig. 3.7.

$$X_a = \sqrt{{r_1}^2 - e^2} \qquad A.9$$

Substituting the value of r_1 in equation A.9.

$$X_a = \sqrt{1 - (d^2 + e^2)} \qquad A.10$$

From Fig. 3.6 and Fig. 3.7

$$Y_a = e \qquad A.11$$

$$Z_a = d \qquad A.12$$

A.4 Determination of the Co-ordinates of D and E

From the intersecting planes (1) and (3) in Fig. 3.5 $CD = CE = \sqrt{R^2 - a^2} = \sqrt{1 - a^2} = r_3$. Sphere radius R is unity. From Fig. 3.5 CD and CE may be written as:

$$CD^2 = CE^2 = (X - X_c)^2 + (Y - Y_c)^2 + (Z - Z_c)^2 = 1 - a^2 = {r_3}^2$$

Therefore,

$$\epsilon = (X - X_c)^2 + (Y - Y_c)^2 + (d - Z_c)^2 - {r_3}^2 \qquad A.13$$

Where X_c, Y_c and Z_c are the co-ordinates of disc centre C, X, Y and Z (d) are the co-ordinates of intersecting point D or E.

From Fig. 3.5 and Fig. 3.8.

$$X = r_1 \cos\theta \qquad A.14$$

$$Y = r_1 \sin\theta \qquad A.15$$

$$Z = d \qquad A.16$$

To find out the appropriate value of θ a computer programme (Programme P1) has been developed on the basis of bisection and root finding technique with reference to equations A.13, A.14, A.15 and A.16.

A.5 Determination of the Co-ordinates of B and B'

From the intersecting plane (2) and (3) in Fig. 3.5.

$$CB = CB' = \sqrt{R^2 - a^2} = \sqrt{1 - a^2} = r_3$$

Sphere radius R is unity. From the Fig. 3.5 CB and CB' can be written as:

$$CB^2 = CB'^2 = (X - X_c)^2 + (Y - Y_c)^2 + Z - Z_c)^2 = 1 - a^2 = {r_3}^2$$

Therefore, the above equation can be presented in terms of ϵ:

$$\epsilon = (X - X_c)^2 + (e - Y_c)^2 + (Z - Z_c)^2 - {r_3}^2 \qquad A.17$$

Where X_c, Y_c and Z_c are the co-ordinates of the disc certre C; X, Y (e) and Z are the co-ordinates of intersecting points B and B'.

From Fig. 3.5 and Fig. 3.8.

$$X = r_2 \cos\theta \qquad A.18$$

$$Y = e \qquad A.19$$

$$Z = r_2 \sin\theta \qquad A.20$$

Similar to the Appendix A.5 the value of θ has been determined form the computer programme (Programme P1) based on bisection and root finding technique in reference to equations A.17, A.18 A.19 and A.20.

A.6 Determination of d

From Fig. 3.9 the angle θ' can be expressed in relation with R and 'a' as:

$$\cos\theta' = \frac{a}{R}$$

$$\theta' = \cos^{-1}\frac{a}{R} \qquad A.21$$

So the angle θ can be determined as:

$$\theta = 90^\circ - (\alpha + \cos^{-1}\frac{a}{R}) \qquad A.22$$

Therefore,

$$\cos\theta = \frac{(d + Z)}{R} = \cos[90^\circ - (\alpha + \cos^{-1}\frac{a}{R})]$$

$$d + Z = R\sin(\alpha + \cos^{-1}\frac{a}{R})$$

$$d = R\sin(\alpha + \cos^{-1}\frac{a}{R}) - Z \qquad A.23$$

In this analysis 'd' has been presented in terms of non-dimensional number. To do so, both side of the equation A.23 has been divided by sphere radius R. Therefore 'd' in terms of non-dimensional number becomes:

$$\frac{d}{R} = \sin(\alpha + \cos^{-1}\frac{a}{R}) - \frac{Z}{R} \qquad A.24$$

A.7 Determination of Width of Cut 'W'

From Fig. 3.12, Z' can be expressed in terms of depth of cut (Z) and inclination angle (α) as:

$$Z' = \frac{Z}{\cos\alpha}$$

Therefore,

$$r_3' = r_3 - Z'$$

From the projection in Fig. 3.12.

$$W' = 2\sqrt{r_3{}^2 - r_3'^2} \qquad A.25$$

Substituting the value of r_3' and Z' in the equation A.25, the expression becomes:

$$W' = 2\sqrt{\frac{Z}{\cos^2\alpha}(D\cos\alpha - Z)} \qquad A.26$$

From the projection in Fig. 3.12 the actual width of cut can be expressed as:

$$W = W'\cos\beta \qquad A.27$$

Substituting the value of W' in the equation A.27, the expression becomes:

$$W = \frac{2\cos\beta}{\cos\alpha}\sqrt{Z(D\cos\alpha - Z)} \qquad A.28$$

A.8 Rake Angle, Rake Length and Depth of Cut on the Disc Convex Side

Two intersecting planes are to be considered to determine the co-ordinates of the intersecting point F (Fig. 3.13). The planes are (1) horizontal soil surface plane at a distance 'd' from the centre of the sphere, and (2) any vertical plane lies in Y-Z axes plane. The geometry of the intersecting planes are fixed by the sphere. Therefore each plane essentially describe a circle. From Fig. 3.13 the co-ordinates of intersection point F can be expressed as:

$$X_f = 0 \qquad A.29$$

$$Y_f = \sqrt{R^2 - d^2} = \sqrt{1 - d^2} \qquad A.30$$

$$Z_f = d \qquad A.31$$

Where X_f, Y_f and Z_f are the co-ordinates of intersection point F, and sphere radius R is unity.

From the intersecting planes (2) and (3) in Fig. 3.13.

$$CG = \sqrt{R^2 - a^2} = \sqrt{1 - d^2} = r_3$$

$$CG^2 = (X - X_c)^2 + (Y - Y_c)^2 + (Z - Z_c)^2 = 1 - a^2$$

$$CG^2 = (X - X_c)^2 + (Y - Y_c)^2 + (Z - Z_c)^2 = {r_3}^2 \qquad A.32$$

As the plane (2) lies on Y-Z axes plane the X co-ordinate of G is zero. Therefore, the equation A.32 can written as:

$$\epsilon = {X_c}^2 + (Y - Y_c)^2 + (Z - Z_c)^2 - {r_3}^2 \qquad A.33$$

From intersecting planes (2) and (3) in Fig. 3.13 and Fig. 3.8 the co-ordinates of G can be expressed as:

$$X = 0 \qquad A.34$$

$$Y = R\cos\theta = \cos\theta \qquad A.35$$

$$Z = R\sin\theta = \sin\theta \qquad A.36$$

Where X, Y and Z are the co-ordinates of intersecting point G, sphere radius R is unity, θ is an angle shown in Fig. 3.8. To determine the appropriate value of θ, basic equations A.33, A.34, A.35 and A.36 have been considered to develop a computer programme (Programme P1) on the basis of bisection and root finding technique. The distance Y_m of the vertical plane from the sphere centre can be determined as:

$$y_m = Y_f - bm \qquad A.37$$

Where $b = \frac{(Y_f - Y_g)}{N}$, N is any arbitrary number of back ward rake tine and m is any number (m=1....N).

The vertical plane along the back-ward rake tine describe a circle and its boundary is fixed by the disc sphere and the distance y_m between the sphere centre and the plane. Thus, the radius of this circle $r_m = \sqrt{1 - y_m{}^2}$. From Fig. 3.14, $BC = \sqrt{r_m{}^2 - d^2}$.

Therefore,

$$\tan\theta = \frac{BC}{d}$$

$$\theta = \tan^{-1}\frac{BC}{d}$$

From $\triangle O'AB$: $\angle O'AB = \angle O'BA = \frac{(180^\circ - \theta)}{2}$ and $\alpha_r{}' = 90^\circ - \frac{(180^\circ - \theta)}{2}$

From Fig. 3.14:

$$\alpha_r = 180^\circ - \alpha_r{}' \qquad A.38$$

$$Z = r_m - d \qquad A.39$$

$$L = \frac{Z}{\sin\alpha_r} \qquad A.40$$

Where α_r is the rake angle, Z is the depth of cut and L is the rake length.

Appendix B

B.1 Equilibrium of the Section Bounded by OA and the Interface

When soil reaction occur on disc concave section:

The distance 'y' of any imaginary tine from the centre of the sphere can be determined on the basis of the assumption in chapter III and IV as:

$$y = e + \frac{b}{2}$$

Where e is the inner edge distance of the tine from the sphere centre and b is the width of the tine.

The radius of the imaginary circle passes along the interface can be determined as:

$$r_4 = \sqrt{1 - y^2}$$

From the known geometry of the sphere and the imaginary circle passes through the tine at a distance y, the relevant angles can be evaluated in reference to Fig. 5.2:

$$r_d = \sqrt{{r_4}^2 - (\frac{L}{2})^2}$$

$$\theta = 2\tan^{-1}\frac{\frac{L}{2}}{r_d}$$

Where L is the rake length discussed in section 3.9.

The direction of the adhesion force is changing continuously along the curved interface for convenience in computation it may be assumed that the resultant adhesive force A acting half-way from O and a tangent to the interface. The weight of the soil body bounded by OA and the interface is negligible. In computation the effect of this soil body weight is neglected.

Considering the equilibrium of the force components perpendicular to the adhesive and cohesive force components acting on the soil body (Fig. 5.2), we get:

$$P_{ca} \cos \delta = P_c \cos \phi$$

$$P_{ca} = \frac{P_c \cos \phi}{\cos \delta} \qquad B.1$$

Similarly, from the equilibrium of the surcharge components acting on the soil body bounded by OA and the interface can be expressed as:

$$P_{qr} = \frac{P_q \cos \phi}{\cos \delta} \qquad B.2$$

From the known geometry of the sphere and the imaginary circle passes through the tine at a distance y, the relevant angles can be evaluated in reference to the Fig. 5.3:

$$r'_d = \sqrt{{r_d}^2 + (\frac{L}{6})^2}$$

$$\theta' = \sin^{-1} \frac{r_d}{r'_d}$$

$$\theta_3 = (90^\circ - \theta')$$

The equilibrium of the gravitational force component on the soil body can be expressed in reference to Fig. 5.3:

$$P_{\gamma r} = \frac{P_\gamma \cos(\phi + \theta_3)}{\cos \delta} \qquad B.3$$

B.2 Equilibrium of the Section Bounded by OA and the Interface

When soil reaction occur on the disc convex or rear section: The radius of the imaginary circle passes along tine interface can be determined as:

$$r_m = \sqrt{1 - {y_m}^2}$$

Where y_m is the distance from the centre of the sphere to the tine, discussed in Appendix B.1.

Similar as Appendix B.1 equilibrium of the cohesive-adhesive force components acting on the rear section of the disc can be expressed with reference to Fig. 5.4:

$$P_{ca} = \frac{P_c \cos\phi}{\cos\delta} \qquad B.4$$

Similarly, the equilibrium of the surcharge components acting on the soil body can be expressed as:

$$P_{qr} = \frac{P_q \cos\phi}{\cos\delta} \qquad B.5$$

From Fig. 5.4:

$$r'_m = \sqrt{{r_m}^2 + (\frac{L}{6})^2}$$

$$\theta_4 = \cos^{-1}\frac{r_m}{r'_m}$$

The equilibrium of the gravitational force components on the soil body can be expressed in reference to Fig. 5.4:

$$P_{\gamma r} = \frac{P_\gamma \cos(\phi - \theta_4)}{\cos\delta} \qquad B.6$$

B.3 Effective Force Calculation on Each Tine Interface Slice

In reference to Fig. 5.8 the effective stress at the base of the slice is:

$$\tau_f = a + \sigma_n \tan\delta \qquad B.7$$

Where τ_f is the effective shear stress, σ_n is the normal stress,'a' is the adhesion and δ is the angle of soil-metal friction.

The shear force can be determined by multiplying both side of the equation D.7 with the width of silce b as:

$$\tau_f = T = ab + N\tan\delta$$

Therefore, the static equilibrium along the base of the slice at any instant is:

$$T = P\sin\eta \qquad B.8$$

Interslice forces are assumed to be equal and opposite and cancel each other out, that is, $E_1 = E_2$ and $X_1 = X_2$ (Fig. 5.8).

So, the static equilibrium along the vertical direction

$$P - N\cos\eta - T\sin\eta = 0 \qquad B.9$$

Substituting equation B.8 in equation B.9

$$P - P\sin^2\eta = N\cos\eta$$

$$P(1 - \sin^2\eta) = N\cos\eta$$

$$N = P\cos\eta \qquad B.10$$

B.4 Dtermination of η

In section 2.2.1 the line of action of P_γ was assumed to be acted two third the way along the interface from the soil surface and the corresponding position of P_{ca} and P_q are half-way from the soil surface. Moreover, in present analysis at any instant the distance between the soil surface and the centre of the sphere is presented as d. Thus, in reference to Fig. 5.9 $u_c = u_q = \frac{Z}{2}$ and $u_g = \frac{2}{3}Z$ and corresponding $Y = e + \frac{b}{2}$. Where Z is the depth of cut, e is the distance of the tine from the centre of the sphere and b is the width of each tine.

From $\triangle OAL$ in Fig. 5.9.

$$OA^2 = AL^2 + OL^2$$

$$R^2 = AL^2 + (u_i + d)^2$$

$$AL = \sqrt{R^2 - (u_i + d)^2} = \sqrt{1 - (u_i + d)^2} = R_i$$

Where i= c, q, γ and in present analysis sphere radius R is unity.

Therefore, from $\triangle ALM$:

$$\frac{ML}{AL} = \frac{Y}{R_i} = \sin\eta$$

$$\eta = \sin^{-1}\frac{Y}{R_i} \qquad B.11$$

In the case of rear face scrubbing the calculation of η is similar to that in concave section.

B.5 Force Calculation Above the Limiting Value $\alpha_w{}'$

Terzaghi's(1948) theoretical study revealed that if the base of a continuous footing rests on the surface of a weightless soil possessing cohesion and friction the loaded soil fails but because of the friction and adhesion between the soil and the base of footing, the soil under the footing remain in an elastic state and penetrate the soil like a wedge. He also found that the wedge boundaries rise at an angle of $45^\circ + \frac{\phi}{2}$ with the horizontal. Thus, the rupture geometry could be illustrated as in Fig. 5.10.

The passive soil reaction on the pseudo-interface is made up of two components, the frictional soil resistance P acting at an angle ϕ to a line which is perpendicular to the tangent of the pseudo-interface and a tangential cohesive force A along the pseudo-interface. If the forces P and A are known then from the static equilibrium of all forces acting on the soil body OAB, the force P_v can be determined.

The force P and A can be determined readily as the procedures discussed in the section 2.1 to 2.2.5 by simple recalculation of the values of modified rake angle, rake length and depth of cut as:

$$\alpha_m = 180^\circ - (45^\circ + \frac{\phi}{2})$$

$$Z_m = \frac{L}{2}\tan\epsilon$$

$$L_m = \frac{L}{2} \sin \epsilon$$

Where L is the back-ward rake length calculated in Appendix A.8 and $\epsilon = 45^\circ - \frac{1}{2}\phi$.

From Fig. 5.10 static equilibrium of the forces acting on the soil body bounded by OAB:

$$P_v + W = 2P \cos \epsilon + 2A \cos \epsilon$$

$$P_v = 2 \cos \epsilon (P + A) - W \qquad B.12$$

The resultant soil reaction force P_r on each tine slice has been considered to be acting at $-\delta$ to a line which is normal to the tangent of the disc underside. Thus the horizontal force component of the resultant force can be determined from Fig. 5.10 as:

$$P_r = \frac{P_v}{\cos(\delta + \alpha_r{}')} \qquad B.13$$

$$P_h = P_r \cos(90^\circ - \delta - \alpha_r{}') \qquad B.14$$

Where $\alpha_r{}'$ is the inclination angle of the actual back-ward rake from the horizontal soil surface discussed in Appendix A.8.

Appendix C

C.1 Determination of Side Force from Side-way Mounted Transducers

In reference to Fig. 6.5, taking moment at the point of action of the force R_2 measured from the top transducer:

$$Fb = R_1 a$$

$$R_1 = \frac{Fb}{a}$$

The resultant force R_1 measured from the two side transducers always acting at the middle between the two transducers. Taking moment at the point of action of force R_1 we get,

$$F(b-a) = R_2 a$$

$$R_2 = \frac{Fb}{a} - F = R_1 - F$$

$$F = R_1 - R_2 \qquad C.1$$

C.2 Properties of Disc and Soil in Experimental Analysis

Disc Properties:	
Disc sphere radius (R) :	600mm
Disc characteristic ratio $(\frac{a}{R})$:	0.866
Depth of cut :	100mm
Soil Properties:	
Soil type :	Sandy loam (Ryton sand)
Bulk unit weight :	$\gamma = 16.0 \frac{KN}{m^3}$
Cohesion :	c=3.0 $\frac{KN}{m^2}$
Adhesion :	a=2.5 $\frac{KN}{m^2}$
Surcharge :	q=7.4 $\frac{KN}{m^2}$
Angle of internal friction :	$\phi = 32.5^\circ$
Angle of soil-metal friction :	$\delta = 24.8^\circ$

Appendix D

D.1 Dimensional Analysis

The variables involved in the evaluation of the soil resistance on a disc implement can be expressed in general as:

$$f(P,\gamma,c,C_a,q,\phi,\delta,\beta,\alpha,Z,\frac{a}{R},g,V)=0 \qquad D.1$$

The meaning of these abbreviations are given in Appendix E.

The dimensions of the above variables are as follows:

P	$[\frac{M}{T^2}]$
γ	$[\frac{M}{L^3}]$
$c,\ C_a,\ q$	$[\frac{M}{LT^2}]$
$\phi,\ \delta,\ \beta,\ \alpha,\ \frac{a}{R}$	*Dimensionless*
Z	[L]
g	$[\frac{L}{T^2}]$
V	$[\frac{L}{T}]$

The total number of variables (n) involved is 13 and all three fundamental dimensions appear in the variables (r=3). So there are (n-r)=10 dimensionless products as $N_1, N_2, ...N_{10}$. By choosing γ, Z and g as the first variables, the dimensionless numbers can be determined by combining these three variables with the remaining (n-r) variables as:

$$N_j=\gamma^{uj}Z^{vj}g^{wj}(v)_j=1,2,3,....n \qquad D.2$$

Where $(v)_j$ represents the remaining (n-r)=10 variables P, c,C_a, q,ϕ,δ,β,α,$\frac{a}{R}$ and V. In dimensional form we can write the equation D.2 as:

$$(\frac{M}{L^3})^{uj}\ (L)^{vj}\ (\frac{L}{T^2})^{wj}\ [M^m\ L^l\ T^t]=[\quad] \qquad D.3$$

Where m, l and t are appropriate indices for the (n-r) variables. The dimensional equation D.3 in general case can be written as the following equations.

$$
\begin{aligned}
M: &\qquad u + m = 0 \\
L: &\qquad -3u + v + w + l = 0 \qquad\qquad D.4 \\
T: &\qquad -2w + t = 0
\end{aligned}
$$

More specifically 10 equations corresponding to equation D.4 can be obtained for the appropriate values of m, l and t and their solution for the u, v and w are given in tabular form below:

Variables	m	l	t	Equation D.4	u	v	w	Group
P	1	0	-2	u+1=0 -3u+v+w=0 -2w-2=0	-1	-2	-1	$\frac{P}{\gamma Z^2 g}$
c C_a q	1	-1	-2	u+1=0 -3u+v+w-1=0 -2w-2=0	-1	-1	-1	$\frac{c}{\gamma Z g}$ $\frac{C_a}{\gamma Z g}$ $\frac{q}{\gamma Z g}$
	0	0	0	u+0=0 -3u+v+w=0 -2w+0=0	0	0	0	
v	0	1	-1	u+0=0 -3u+v+w+1=0 -2w-1=0	0	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{v}{(gZ)^{\frac{1}{2}}}$ or $\frac{v^2}{gZ}$

Equation D.1 can therefore be written as:

$$
f_1[(\frac{P}{\gamma Z^2 g}),(\frac{c}{\gamma Z g}),(\frac{C_a}{\gamma Z g}),(\frac{q}{\gamma Z g}),\phi,\delta,\beta,\alpha,\frac{a}{R},(\frac{V^2}{gZ})] = 0
$$

Re-arranging terms this may also be written as follows:

$$[\frac{P}{\gamma Z^2 g}] = f_2[(\frac{c}{\gamma Z g}),(\frac{C_a}{\gamma Z g}),(\frac{q}{\gamma Z g}),\phi,\delta,\beta,\alpha,\frac{a}{R},(\frac{V^2}{gZ})] \qquad D.5$$

D.2 Interpolation for ϕ

The variation in K-factors for ϕ follow the approximate expression as:

$$\log(K_i) = a\tan\phi + b \qquad D.6$$

Where $i = \gamma, c, C_a, q$. The constant a and b are the function of disc angle β, inclination angle α and disc characteristic ratio $\frac{a}{R}$. All the K-factor curves represented in the earth pressure charts are at 5° intervals for ϕ. Hence, within any 5° interval there will be no great loss of linearity if the abscissae are plotted as ϕ rather than $\tan\phi$. From Fig. 7.14 $c = \ln K_2 - \ln K_1 = \ln(\frac{k_2}{K_1})$ and $\frac{a}{b} = \frac{\phi_2-\phi_1}{\phi-\phi_1}$. Therefore $b = a\frac{(\phi-\phi_1)}{(\phi_2-\phi_1)}$. If $n_1 = \frac{\phi-\phi_1}{\phi_2-\phi_1}$ then $b = an_1$. Therefore,

$$\ln K = \ln K_1 + b = \ln K_1 + an_1$$

Substituing the value of 'a' we get,

$$\ln K = a\ln(\frac{K_2}{K_1}) + \ln K_1 = \ln K_1(\frac{K_2}{K_1})^{n_1}$$

We may write this equation as:

$$K = K_1(\frac{K_2}{K_1})^{n_1} \qquad D.7$$

Appendix E

E.1 Notation

Only important generally used symbols are listed here. A special list of main subscripts and their appropriate meaning has been compiled in section E.2.

A	Tangential adhesive force per unit width of interface.
a	Distance between the disc and the sphere centres. Also as the interface adhesion in $\tau_f = a + \sigma \tan\delta$.
b	Width of each imaginary tine.
C	Centre of the disc face circle.
c	Cohesion in Coulomb's equation $\tau_f = c + \sigma \tan\phi$.
C_a	Constrained adhesion $C_a = c \tan\delta \cot\phi$.
D	Diameter of the disc.
d	Distance between the horizontal soil surface and the sphere centre.
e	Distance between the vertical circular plane along a tine and the sphere centre.
F	Soil reactions in rupture zones usually used with the subscripts in E.2.
g	Acceleration due to gravity.
K	K-factor used with appropriate subscripts in E.2.
L	Rake length of an imaginary tine.
[L]	Dimension of length.
[M]	Dimension of mass.
M	Moment, used with subscripts.
m	Ratio of the depth of cut presented in K-factor charts to required depth of cut ($\frac{Z_0}{Z}$).
N	Effective horizontal soil reaction force on interface slice.
n	Ratio of depth cut ($\frac{Z-Z_0}{Z_0}$).
n_1	Interpolation coefficient of soil internal friction angle ϕ, ($n_1 = \frac{\phi-\phi_1}{\phi_2-\phi_1}$).

n_2	Interpolation coefficient of inclination angle α, $(n_2 = \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1})$.
O	Origin of the reference co-ordinate system (sphere centre).
P	Frictional soil resistance component per unit width of interface.
q	Uniform normal surcharge pressure on horizontal soil surface.
R	Radius of the disc sphere.
r_1	Radius of the horizontal circle on the soil surface defined by the sphere.
r_2	Radius of any vertical circle along the tine interface defined by the sphere.
r_3	Radius of the disc face circle.
r_m	Radius of any vertical circle along the interface of the scrubbing back-ward rake tine.
[T]	Dimension of time.
T	Tangential or shear stress component on interface slice.
V	Velocity of interface.
W	Weight of a sector of rupture zone.
X	X-axis in physical plane.
Y	Y-axis in physical plane.
Z	Z-axis in physical plane.
α	Disc Inclination angle.
α_r	Rake angle of the imaginary tine interface measured from the horizontal.
α_d	Limiting angle of transition zone.
α_w	Rake angle of the imaginary tine at leading limit of plane wedge formation.
α_w^-	Limiting rake angle in plane wedge, when $\delta_m = -\delta$.
α'	Projected inclination angle.
β	Disc angle.
β_t	Direction of translation of interface measured from the horizontal (downward positive).
γ	Soil bulk weight.

δ	Angle of soil-internal friction in $\tau_f = a + \sigma \tan \delta$.
δ_m	Mobilised angle of friction between interface and boundary wedge $(-\delta < \delta_m < +\delta)$.
ϵ	$Angle = 45^\circ - \frac{1}{2}\phi$.
η	Angle between horizontal soil reaction force calculated from tine theory and effective horizontal force of each soil slice.
ξ	Constant used in empirical correction equations.
θ	Angle of interscetion point from reference axis. Used to find out the co-orditates of intersection point between two planes.
θ^+	Included angle between lower slip-line and interface.
σ	Normal stress.
τ	Shear stress.
ϕ	Angle of friction in Coulomb's equation $\tau_f = c + \sigma \tan \phi$.
ψ	$Angle = 45^\circ + \frac{1}{2}\phi$.
ω	Inclination angle from the horizontal to the plane of stress discontinuity.

E.2 Subscripts

The following subscripts appear frequently in the text and have the specified meaning in each instance. These subscripts appear with some of the symbols listed in E.1 and indicate that these symbols are related to the following properties:

a	Adhesion, adhesive.
c	Cohesion, cosive.
C_a	Combined cohesion and adhesion.
γ	Gravitational.
q	Surcharge.
r	Resultant.
v	Velocity.
w	Plane soil wedge.
x	Longitudinal direction in physical plane.

y	Lateral direction in physical plane.
z	Vertical direction in physical plane.

REFERENCES

1. Abo El Ees, N. A. E. H. (1978). "The use of curved discs as furrow openers on direct drills", *Ph. D. thesis.* University of Newcastle upon Tyne.

2. Abo El Ees, N. A. E. H. and Wills, B. M. D. (1986). "An analysis of the geometric parameters of a curved vertical disc", *J. Agric. Engng. Res. Vol.* **35**, 277-286.

3. Bishop, A. W. (1954). "The use of the slip-circle in the stability analysis of slopes", *Euro. Conf. on Stability of Earth Slopes, Vol.* **1**, Stockholm.

4. British Standard Institution. (1979). "Specification for Agricultural discs", *BS.* **5822:1979.**

5. Clyde, A. W. (1936). "Measurement of forces on soil tillage tool", *Agric. Engng. Vol.* **17**.

6. Clyde, A. W. (1939). "Improvement of disc tools", *Agric. Engng. Vol.* **20**, 215-221.

7. Cuplin, C. (1986). "Farm machinery", *Elevelth Edition, Collins,* 8 Grafton Street, London W1.

8. Gill, W. R. and Vanden Berg, G. E. (1968). "Soil dynamics in tillage and traction", *USDA. Agric. Handbook No.* **316**, 122-123.

9. Gill, W. R., Reaves, C. A. and Bailey, A. C. (1978) "The effect of geometric parameters on disc forces", *ASAE paper No.* **78-1535**.

10. Gill, W. R., Reaves, C. A. and Bailey, A. C. (1979). "The influence of velocity and disc angle on the kinematic parameter λ of discs", *ASAE paper No.* **79-1522**.

11. Gill, W. R., Bailey, A. C. and Reaves, C. A. (1981). "Harrow disc curvature influence on soil penetration", *ASAE paper N0.* **81-1532**.

12. Gill, W. R., Reaves, C. A. and Bailey, A. C. (1981). "The influence of harrow disc curvature on forces", *Transaction of the ASAE. Vol.* **24**, No. 4, 579-583.

13. Godwin, R. J. (1975). "An extended Octagonal ring transducer for use in tillage studies", *J. Agric. Engng. Res. Vol.* **20**,347-352.

14. Godwin, R. J. and Spoor, G. (1977). "Soil failure with narrow tines", *J. Agric. Engng. Res. Vol.* **22**, 213-228.

15. Godwin, R. J., Seig, D. A. T. and Allott, M. (1985). "The development and evaluation of a force prediction model for agricultural discs", *International Conf. on soil dynamics*, Auburn, Alabama, USA.

16. Godwin, R. J., Seig, D. A. and Allott, M. (1987). "Soil failure and force prediction for soil engaging discs", *Soil use and management, Vol.* **3**, No. 3, 106-114.

17. Gordon, E. D. (1941). "Physical reaction of soil on plough discs", *Agric. Engng. Vol.* **22**, No. 6, 205-208.

18. Harrison, H. P. and Thevavarnongs, K. (1976). "Soil reacting force from discs", *Canadian J. Agric. Engng. Vol.* **18**, No. 1, 49-53.

19. Harrison, H. P. (1977). "Soil reacting forces for discs from field measurements", *Transaction of the ASAE. Vol.* **20**, 836-838.

20. Hettiaratchi, D. R. P. and Reece, A. R. (1967). "Symmetrical three dimensional soil failure", *J. Terramechanics. Vol.* **4**, No. 3, 45-67.

21. Hettiaratchi, D. R. P. (1968). "The calculation of passive earth pressure", *Ph. D. thesis*, University of Newcastle upon Tyne.

22. Hettiaratchi, D. R. P. and Reece, A. R. (1974). "The calculation of passive soil resistance", *Geotech. Vol.* **24**, No. 3, 289-310.

23. Hettiaratchi, D. R. P. and Reece, A. R. (1975). "Boundary wedges in two-dimensional passive soil failure", *Geotech. Vol.* **25**, No. 2, 197-220.

24. Hettiaratchi, D. R. P. (1986). "A simple slip-line method for estimating passive earth pressure", *Dept. of Agric. Engng.*, University of Newcastle upon Tyne.

25. Janbu, N. (1954). "Application of composite slip surface for stability analysis", *Euro. Conf. on Stability of Earth Slopes, Discussion, Vol.* **3**, Stockholm.

26. Johnston, R. C. R. and Birtwistle, R. (1963). "Wheatland disc plough investigations. II disc forces", *J. Agric. Engng. Res. Vol.* **8** No. 4, 312-326.

27. Kepner, R. A., Bainer, R. and Barger, E. L. (1978). "Principles of Farm Machinery", *3rd Edition, AVI publishing company, INC.*, Westport, Cannectient, USA, 116-117.

28. McCreery, W. F. and Nichols, M. L. (1956). "The geometry of discs and soil relationships", *Agric. Engng. Vol.* **37**, No. 12, 808-812, 820.

29. McCreery, W. F. (1959). "Effect of design factors on soil reactions", *ASAE paper No.* **59-622**.

30. McKyes, E. and Ali, O. S. (1977). "The cutting of soil by narrow blades", *J. Terramechs. Vol.* **14**, No. 2, 43-58.

31. McKyes, E. (1985). "Soil cutting and Tillage", *Elsevier Science Publishers B. V.*, Science and Technology Division, P. O. Box. 330, 1000 AH Amsterdam, Netherland.

32. Meyerhof, G. G. (1961). "The ultimate bearing capacity of wedge-shaped foundations", *Proc. 5th Int. Conf. on Soil Mechanics & Foundation Engng. Vol.* **2**, Divisions 3B, DUNOD, Paris.

33. Moller, N. (1975). "Conventional coulters for small grain drilling", *Dept. of Agric. Engng. & Rationalization*, Agricultural College of Sweden, S-750 07 Uppsala 7,19-20.

34. Nartov, P. S. (1985) "Disc soil-working implements", *Russian Trans. Series, No.* **27**, Printsman Press, Faridabad, India.

35. Nichols, M. L., Reed, I. F. and Reaves, C. A. (1958). "Soil reaction: To plough shear design", *Agric. Engng. Vol.* **39**, 336-339.

36. O'Dogherty, M. J. (1986). "A Tri-axial Dynamometer for soil trough measurement of forces on soil working components", *J. Agric. Engng. Res. Vol.* **34**, 141-147.

37. "Parmiter Product Manual, Mounted disc plough Model DPM. Issue No. 3", (1987). *P. J. Parmiter & Sons Ltd.*, Station Works, Tisbury, Wiltshire, England.

38. Payne, P. C. J. (1956). "The relationship between the mechanical properties of soil and the performance of simple cultivation implements", *J. Agric. Engng. Res. Vol.* 1, 23-50.

39. Payne, P. C. J. (1958). "The mechanical properties of soil in relation to the design of cultivation implements", *Inst. of British Agric. Engrs. Vol.* **15**, N0. 4, 86-92.

40. Reaves, C. A., Gill, W. R. and Bailey, A. C. (1981). "Influence of width and depth of cut on disc forces", *Transaction of the ASAE. Vol.* **24**, No. 3, 572-578.

41. Reece, A. R. (1965). "The fundamental equation of earth-moving mechanics", *Proc. Symp. Earthmoving Machinery, Instn. Mech. Engrs. Pt. 3E.*

42. Singh, J., Ganguly, P. and Singh, K. N. (1978). "Effect of soil implement parameters on depth of penetration of disc harrow", *Transaction of the ASAE. Vol.* **21**, NO. 4, 620-622, 627.

43. Sokolovski, V. V. (1954). "Statics of solid Media", *Butterworths Scientific Publication*, 88 Kings Way, London, W. C. 2, 143-151.

44. Sommer, M. S., Chen, S. H. and Bierl, J. F. (1983). "Disc blade performance", *ASAE paper No.* **83-1537**.

45. Tanner, D. W. (1960). "Further work on the relationship between rake angle and the performance of simple cultivation elements", *J. Agric. Engng. Res. Vol.* **5**, 307-315.

46. Taylor, P. A. (1967). "Field measurement of forces and moments on wheatland plough discs", *Transaction of the ASAE.*

47. Terzaghi, K. (1943). "Theoretical soil Mechanics", *John wiley & Sons, Inc.*, New York, USA.

48. Thompson, J. L. and Kemp, J. G. (1958). "Analyzing disc cuts graphically", *Agric. Engng. Vol.* **39**, 285-287.

49. Vanden Berg, G. E. (1966). "Analysis of forces on tillage tools", *J. Agric. Engng. Res. Vol.* **11**, No. 3.